Linking retirement age to life expectancy does not lessen the demographic implications of unequal lifespans

Jesús-Adrián Álvarez, Malene Kallestrup-Lamb and Søren Kjærgaard

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Department of Economics and Business Economics
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 9352 1936

Linking retirement age to life expectancy does not lessen the demographic implications of unequal lifespans

Jesús-Adrián Álvarez*[1], Malene Kallestrup-Lamb[2], and Søren Kjærgaard[1]

[1]Interdisciplinary Centre on Population Dynamics, University of Southern Denmark

[2]Department of Economics and Business Economics, Aarhus University

Abstract

The fact that individuals are living longer and thus spending more time in retirement challenges the sustainability of pension systems. This has forced policy makers to rethink the design of pension plans to mitigate the burden of increased longevity. Countries such as the Netherlands, Estonia, Denmark and Finland have implemented reforms that link retirement age to changes in life expectancy. However, the demographic and financial implications of such linkages are not well understood. This study analyses the Danish case, using high-quality data from population registers during the period 1985-2016. We identify trends in demographic and actuarial measures after retirement by sex and socio-economic group. We also introduce a new decomposition method to disentangle the demographic sources of socio-economic disparities in pension costs per year of expected benefits. We reach two main results. First, linking retirement age to life expectancy increases uncertainty about length of life after retirement, with the financial cost becoming more sensitive to changes in mortality. Second, socio-economic disparities in lifespans persist regardless of the age at which individuals retire. Males from lower socio-economic groups are at a greater disadvantage, because they spend fewer years in retirement, pay higher pension costs per year of expected benefits and are exposed to higher longevity risk than the rest of the population. This disadvantageous setting is magnified when retirement age is linked to life expectancy.

Keywords: Danish pension system, longevity, socio-economic disparities, lifespan inequality, pensions, mortality heterogeneity

JEL codes: J26; J11; H55

*alvarez@sdu.dk

1 Introduction

Recently observed survival trajectories for individuals posit unprecedented challenges to existing pension systems. First of all, life expectancy at birth has increased over time (Medford, 2017; Oeppen and Vaupel, 2002; Pascariu et al., 2018). This measure has gone up chiefly because of a reduction in the risk of dying at young ages (Burger et al., 2012). As a consequence, a high proportion of individuals from recent cohorts survive to retirement ages. Secondly, the risk of dying during post-retirement ages has also trended downwards (Rau et al., 2008; Zuo et al., 2018), such that individuals from more recent cohorts spend a greater number of years in retirement in comparison to those from previous ones (Sanderson and Scherbov, 2010, 2017). As more people survive to retirement and live for longer once retired, there is an increased need to reform pension systems in many countries (Chomik and Whitehouse, 2010).

To ensure sustainability in their pension systems, many countries including Denmark[1], Estonia, Finland, Italy, the Netherlands, Portugal and the Slovak Republic have passed reforms that modify the way pension systems operate (OECD, 2017). A common feature among these countries is that their pension systems will gradually take into account increases in life expectancy, either by modifying retirement age or by adjusting payouts (OECD, 2018; Whitehouse, 2007). In Denmark, the statutory retirement age will gradually increase, targeting the age at which remaining life expectancy is 14.5 years[2] (hereafter referred to as *target retirement age*, see Danish Ministry of Economic Affairs and Interior (2017, 2018) and OECD (2015) for more details.). Rather than focusing on the number of years already *lived* (fixed chronological age), the target retirement age is based on the number of years a person is *expected* to spend in retirement. This scheme implies that: *(i)* regardless of the year in which they retire, Danish retirees can expect to receive pension payments for an average period of 14.5 years and *(ii)* the number of years gained in life expectancy will be spent in work rather than in retirement.

In increasing retirement ages (in this case, by linking them to life expectancy), the demographic circumstances in which individuals can retire changes. Pension payments are now conditioned upon surviving to an older age, at which the risk of dying is higher than at younger ages (Rau et al., 2008). Additionally, not all individuals experience the same age-specific survival probabilities at retirement; it has been shown that the probabilities differ considerably with respect to different socio-economic groups (Cairns et al., 2019; Sasson, 2016; Villegas and Haberman, 2014). Increasing retirement age might have unanticipated consequences for lower socio-economic groups, as they exhibit lower life expectancies and higher lifespan inequality[3](van Raalte et al., 2018). Individuals from lower socio-economic groups might be more affected by changes in retirement age than those with higher survival chances (Chetty et al., 2016). An analysis of the demographic circumstances in which individuals from different socio-economic groups retire is necessary to understand how increasing retirement age affects them, but also to determine the extent of socio-economic disparities.

From the perspective of the pension provider, large socio-economic differences in longevity also repre-

[1] The outcome of other reforms at the beginning of the 1990s was the strengthening of actuarial fairness in the Danish pension system by shifiting people into mandatory defined contribution schemes (Andersen, 2015).

[2] The way statutory retirement age will be determined in the coming years will be more complicated than merely matching it to the target retirement age (see Section 2 for further details). While it is possible that the statutory retirement age will not exactly match the target retirement age right away, the latter represents a long-term goal. The target retirement age serves as a reference of the future development of the statutory retirement age. Hence, its study is the starting point in understanding the implications of linking retirement ages to life expectancy.

[3] Lifespan inequality is an indicator of uncertainty about the number of years a person will spend in retirement. Haberman et al. (2011) showed that, in a context of low interest rates, high lifespan inequality indicates that the financial cost of a pension (actuarially represented by life annuity factors) is highly sensitive to fluctuations in mortality rates. Thus, linking retirement age to life expectancy could increase the exposure to longevity risk in particular for those socio-economic groups that exhibit a higher degree of lifespan inequality.

sent important implications in the financial sustainability of pension schemes (Villegas and Haberman, 2014). Given that pension providers and insurance companies price their annuities, calculate reserves and evaluate their exposure to longevity risks using mortality rates for national populations, neglecting socio-economic disparities in longevity could result in inadequate funding for pension liabilities (Coughlan et al., 2011). Further, socio-economic differentials in longevity could also distort redistribution properties of both defined benefit and defined contribution pension schemes (Brown, 2002, 2003; Liebman, 2002). Depending on the type of pension scheme and on the extent of such socio-economic disparities, undesirable transfers of wealth from lower socio-economic groups (with shorter lifespans) to higher socio-economic groups (with longer lifespans) could arise (Ayuso et al., 2017; Holzmann et al., 2019). Socio-economic differentials in longevity and their effect on pension schemes could be exacerbated when increasing retirement ages.

The aim of this study is to determine the implications of linking retirement age to life expectancy in terms of demographic inequalities and their influence on the financial cost of pensions across socio-economic groups. We use the Danish pension system to illustrate these issues. Specifically, we compare the previous demographic setting at retirement (in which individuals retire at a fixed chronological age) to the demographic setting, based on the Danish pension reform (Danish Ministry of Economic Affairs and Interior, 2018). A strategy to compare these two scenarios could be to forecast age-specific mortality rates into the future and determine the demographic differences between the target and current pension schemes (assuming that the current pension scheme will also be maintained into the future). However, this set-up implies additional assumptions about the future path of age-specific mortality rates, and our results would hinge to a great extent on the model used to forecast. Instead, our comparisons are made retrospectively; we analyse the current and target pension schemes with historical data (during the period 1985-2016). The advantage of this strategy is that we are able to describe the current demographic scenario at retirement and compare it with the target setting by using observed data without making any additional assumptions about the future path of age-specific mortality rates. We inspect trends over time in demographic (life expectancy and lifespan inequality) and actuarial measures (the cost of life annuity at retirement and its entropy). Furthermore, we introduce a new decomposition method to determine the sources of the gap in the entropy of a life annuity between socio-economic groups.

In summary, we find that socio-economic disparities in longevity persist, regardless of the age at which individuals retire. Males from lower socio-economic groups are at a greater disadvantage because they have lower probability of surviving to retirement, spend fewer years in retirement, pay higher pension costs per year of expected benefits and are exposed to higher longevity risk. While linking retirement age to life expectancy might contribute to ensuring the sustainability of national pension systems, this policy introduces higher inequality and positions males from lower socio-economic groups in a more disadvantageous retirement setting. A key contribution of this paper is in bringing together a demographic and an actuarial perspective on socio-economic differences in pension outcomes, by using high-quality micro data. Our findings serve as a reference for the demographic and actuarial implications that might arise in other countries undergoing similar pension reforms to Denmark, in which retirement age will be linked to developments in life expectancy (OECD, 2015, 2018).

We have organized the remainder of the paper as follows. First, we describe the components of the Danish pension system in Section 2. Then, in Section 3, we describe the data and summarize the methodology used in the paper. In Section 4, we show the results. Lastly, we conclude with a discussion of the implications of our results for the Danish pension system in Section 5.

2 The Danish pension system

The main objective of a pension system is to provide income security at old ages (OECD, 2017) while ensuring financial sustainability, actuarial fairness, limited inter/intra-generational redistribution, and adequate replacement rates (OECD, 2019b). To achieve these objectives, the Danish pension system follows a multi-pillar system that combines public and labour market pensions (World Bank, 1994). The first pillar consists of a basic public old age pension (OAP) financed on a Pay-As-You-Go (PAYG) basis combined with a public mandatory and contributory pension linked to earnings (the Labour Market Supplementary Pension Scheme, ATP, see Andersen (2015) for further details). Old age pension benefits consist of a universal basic amount (independent of the individual's labour market history) and a supplement, which is means-tested against all other pension- and labour income. Finally, pillar one contains a number of early retirement options; early retirement pay (in Danish: efterløn), disability benefits, and senior pension (OECD, 2018).

The second pillar consists of defined contribution (DC) labour market pension schemes, which are fully funded. They cover more than 90% of wage earners between the ages 25 and 59 (Balter et al., 2020; OECD, 2015). Moreover, the second pillar contains tax-financed earnings-related civil servant pensions (constituted as defined benefit (DB) schemes) which are currently being phased out (Danish Ministry of Economic Affairs and Interior, 2018). The third pillar consists of individual, private pension savings schemes independent of employment conditions. Unlike pillar two, individuals make their own choices about the size of contributions and composition of benefits in pillar three.[4]

The eligibility age for OAP has significantly changed over time. Until 2004, the OAP age was 67 and thereafter it was reduced to 65. The main rationale for doing so was to substitute old age retirement for early retirement among 65 and 66 year olds and thereby save on public finances, because the early retirement benefit entitlement exceeded the OAP benefit, (Bingley et al., 2020). Starting from 2019, the OAP age was increased again with 0.5 year steps and from 2025, the OAP (and early retirement) age will be raised each 5th year depending on increases in life expectancy.[5] Moreover, since 1999, older workers have had the option to postpone OAP retirement by up to 10 years resulting in an actuarial adjustment (OECD, 2015).

The Danish pension system effectively prevents old age poverty and ensures reasonable replacement rates for most pensioners (Andersen, 2015). In pillar 1, redistribution and actuarial fairness objectives are achieved through floors and ceilings applied to contributions, benefits and accrual rates (Barr and Diamond, 2006). In pillars two and three, redistribution and inter/intra-generational fairness are unlikely to constitute an issue, as each individual is responsible for their contributions and benefits (OECD, 2018).[6] However, inequality in longevity between socio-economic groups potentially affects the pension objectives of the pension system. For instance, to the extent that the system is designed to be progressive, and intended to re-distribute income from rich to poor, longevity differences might reduce or even reverse the direction of redistribution (Sánchez-Romero et al., 2020). These issues are further discussed in Section 5.

[4] Individual pension schemes can be set up as DC plans with banks, insurance companies or pension funds (OECD, 2015).

[5] The indexation will be done every 5th year with 15 years notice, conditional on the life-expectancy of 60 year-olds increasing by at least 0.6 years over the notice period (Danish Ministry of Economic Affairs and Interior, 2018).

[6] These objectives might be distorted in DC schemes with pension guarantees, see Balter et al. (2020)

3 Data and methods

3.1 Data

We use the newly developed affluence measure by Cairns et al. (2019), based on individuals' income and wealth, to divide the population into five socio-economic groups (quintiles) at each age and each point in time. This measure is an attractive choice, given that income and wealth are highly correlated with mortality rates and health outcomes (Duncan et al., 2002). The affluence index $A(j,t,x)$ for individual j aged x in year t is defined as

$$A(j,t,x) = W(j,t-1,x-1) + K \cdot I(j,t-1,x-1), \tag{1}$$

where $W(j,t-1,x-1)$ is the wealth for individual j at age $x-1$ in the year $t-1$, $I(j,t-1,x-1)$ is the income of individual j at age $x-1$ in year $t-1$, and K is the capitalization factor (approximately a value of an annuity factor calculated at retirement age). Cairns et al. (2019) show that the allocation procedure is robust to different choices of K in the interval from 10 to 20 and argues that $K = 15$ is a reasonable choice. Individuals' incomes are measured as the gross annual income which includes wage income, unemployment benefits, social assistance and pension income. Net wealth is defined as total assets minus total liabilities. The assets include real property, bank deposits, stocks, bonds, and cash holdings. Liabilities include all types of debt to private companies and the government. For married couples, wealth is assigned by 50% to each spouse.

Individuals are ranked according to their affluence and thereafter the total population is divided into five equally sized socio-economic groups (quintiles). The five different groups are denoted group 1 (G1) to group 5 (G5), with G5 being the most affluent group with the highest combination of wealth and income. We refer to G1, G2,..,G5 as the low, low-middle, middle, middle-high, and high affluence group, respectively. A benefit of the affluence measure is that it is possible for individuals to have either high wealth or high income independently, or both. If, for example, only income is used, individuals with a low income but high wealth would wrongly be assigned to a low affluence group. Thus, by using both variables, it is possible to more effectively distinguish survival trajectories between different socio-economic groups. This is seen as the capacity of the affluence index to capture heterogeneity in longevity (Cairns et al., 2019).

As in Cairns et al. (2019), individuals can move between socio-economic groups but are locked into their assigned socio-economic group at age 67. This procedure avoids compositional effects between socio-economic groups and allows for the calculation of demographic and actuarial measures, such as life expectancies and annuity prices. Kallestrup-Lamb et al. (2020) performed a sensitivity analysis of the affluence index and the movement between socio-economic groups. They show that, every year since 1985, only 6.1% of females have moved up by one group and 5.7% have moved down by one socio-economic group. The percentage of individuals moving up or down by more than one group is negligible (less than 2%).

3.2 Methods

Our analysis is based on two different retirement ages: the current (c) and the target (t) retirement age. The current retirement age was operational during the whole study period. The target retirement age is in line with the Danish pension reform of 2007 (Danish Ministry of Economic Affairs and Interior, 2018) and it implies that remaining life expectancy after retirement should be constant at 14.5 years. Thus, t is the age that solves the equation $e(t) = 14.5$, where $e(t)$ denotes remaining life expectancy at age t. Both ages are calculated based on the demographic regime of the total Danish population. We calculated them under a period life table perspective: every year from 1985 to 2016.

To determine the exact age t, we smooth annual death rates over the age axis by sampling 100,000 individual lifetimes from an exponential distribution with piece-wise constant rate (Willekens, 2009). We performed this procedure for every single year from 1985 to 2016. The results obtained are equivalent to period life tables calculated directly from the observed death rates (Preston et al., 2000). However, with this procedure, we are able to obtain continuous estimates of the force of mortality. Another common approach to smoothing death rates is to use splines (Camarda et al., 2012), which serves as an interpolation procedure of lifetable outcomes. We decided to simulate individual lifespans (Willekens, 2009) instead of using splines given that the calculation of demographic and actuarial measures arises naturally. For example, the calculation of remaining life expectancy at a specific age (e.g., 71.57) could be easily done with our approach by simply taking the mean of all the lifespans above that age. Conversely, the calculation of the same remaining life expectancy with splines will result in an interpolation between life expectancy values at ages 71 and 72, retrieved from the lifetable.

3.2.1 Demographic measures

The demographic indicators used in this analysis are life expectancy and lifetable entropy. Life expectancy measures the average length of life after retirement (Preston et al., 2000) and it is defined as

$$e(x) \equiv \frac{\int_x^\infty l(y)dy}{l(x)}, \tag{2}$$

where $l(x)$ is the survivorship function at age x.

Lifetable entropy measures lifespan inequality after retirement (Aburto et al., 2019; Keyfitz and Caswell, 2005). It is defined as

$$H(x) \equiv \frac{\int_x^\infty e(y)d(y)dy}{e(x)} = \frac{e^\dagger(x)}{e(x)}, \tag{3}$$

where $e^\dagger(x) = \int_x^\infty e(y)d(y)dy$ denotes the number of years lost due to death (Vaupel and Canudas-Romo, 2003) and $d(x)$ is the distribution of lifespans after age x. Keyfitz and Caswell (2005) defined $H(x)$ as the elasticity of $e(x)$ due to changes in the force of mortality. For example, if $H(x) = 0.3$, then a uniform reduction of one percent in the force of mortality at all ages above x will yield an increase of 0.3 percent in $e(x)$. Goldman et al. (1986) and Vaupel (1986) reformulated $H(x)$ as a measure of relative lifespan inequality: if everyone dies at the same age (minimum inequality) then $H(x) = 0$ and in the case of high lifespan inequality, $H(x) = 1$. An important characteristic of $H(x)$ is that it does not depend on the level of mortality (Wrycza et al., 2015). This property is particularly important in our study because we compare the shape of the distribution of lifespans after retirement ages c and t, which changes over time. The use of measures of absolute inequality (e.g., standard deviation) could lead to different results depending on the onset age of the calculation, because these measures do not control for differences in the average remaining lifespan; in our case the average remaining lifespan also varies according to age c and t.

3.2.2 Actuarial measures

To evaluate the financial implications of linking retirement age to developments in life expectancy, we calculate the financial transaction of a pension in terms of life annuities. For this purpose, we calculate the present value of a series of monetary payments of $1 made until the pensioner dies, also called the cost of a life annuity (Bowers et al., 1997; Dowd et al., 2011). We assume that there are no periods of guarantees to the life annuity and that pension payments of one monetary unit are paid continuously per year starting from age x. Further, pension payments carry interest, which is determined by the force of interest, δ. Thus, the cost of a life annuity is defined by:

$$\bar{a}(x) \equiv \frac{\int_x^\infty l(y)e^{-\delta y}dy}{l(x)e^{-\delta x}}, \tag{4}$$

If the force of interest is equal to zero, i.e. money carries no interest, the cost of a life annuity is only determined by the survival of the pensioners and equal to the expected number of years they have to live. This means that if $\delta = 0$ then $\bar{a}(x) = e(x)$.

We also calculate the entropy of the cost of a life annuity (Haberman et al., 2011). At a given value of δ, the entropy measures the sensitivity of $\bar{a}(x)$ to changes in death rates and it is defined as:

$$\bar{H}(x,\delta) \equiv \frac{\int_0^\infty \bar{a}(y)d(y)e^{-\delta y}dy}{\bar{a}(y)} \equiv \frac{\bar{a}^\dagger(y)}{\bar{a}(y)}, \tag{5}$$

where $\bar{a}^\dagger(x) = \int_x^\infty \bar{a}(y)d(y)e^{-\delta y}dy$ can be interpreted as the average number of life annuity payments lost due to death (see Appendix, Section B for more details on the derivation of Equation 5). It is also straightforward to show that, in the case of $\delta = 0$, then $\bar{H}(x,\delta = 0) = H(x)$ and $\bar{a}^\dagger(x) = e^\dagger(x)$.

As the objective of this paper is to analyse the socio-economical differences in a pension system where the retirement age is linked to life expectancy, we evaluate relative differences in $\bar{a}(x)$ and $\bar{H}(x,\delta)$ to the total:

$$\Delta\bar{a}_s(x) = \frac{\bar{a}_s(x) - \bar{a}_p(x)}{\bar{a}_p(x)}, \tag{6}$$

$$\Delta\bar{H}_s(x,\delta) = \frac{\bar{H}_s(x,\delta) - \bar{H}_p(x,\delta)}{\bar{H}_p(x,\delta)}, \tag{7}$$

where s and p denote values of $\bar{a}(x)$ and $\bar{H}(x,\delta)$ for a specific socio-economic group and for the total population, respectively.

For simplicity, we assume that all individuals in the population have accumulated the same amount of savings at retirement, that they all retire at the same retirement age (either c or t) and that they face the same interest rate δ. Thus, differences in $\bar{a}(x)$ and $\bar{H}(x,\delta)$ between socio-economic groups arise solely through differences in the distribution of lifespans for each group. Socio-economic groups with positive values of $\Delta\bar{a}(x)$ have a higher annuity cost and hence are expected to receive higher expected pension benefits compared to the population average. Socio-economic groups with positive values of $\Delta\bar{H}(x,\delta)$, exhibit more variable costs of the annuity and hence experience more uncertainty about their expected pension payments. The uncertainty is a consequence of more unequal lifespans and the expected pension payments thus vary because the times of death for these groups are more uncertain than for the population average. Hence, groups with a positive value are exposed to higher micro longevity risk (Hari et al., 2008) than the population average. The underlying demographic mechanisms $\bar{H}(x,\delta)$ are disentangled with a new decomposition method, introduced in Section E of the Appendix.

4 Results

Figure 1 summarizes the two demographic scenarios at retirement that are analysed in this study. The first scenario portrays the pension scheme that was in force during the period 1985-2016. We denote this scenario as the *current* pension scheme (measures indicated in black). The second scenario assumes that the target retirement age was operational from 1985, such that remaining life expectancy is always 14.5 years at the retirement age ($e(t) = 14.5$). We denote this scenario as the *target* pension scheme, and it is indicated in Figure 1 by the measures in green. All the calculations shown in Figure 1 consider demographic information of the total population with no distinction between the sexes, because the reform does not make such distinction (Danish Ministry of Economic Affairs and Interior, 2018).

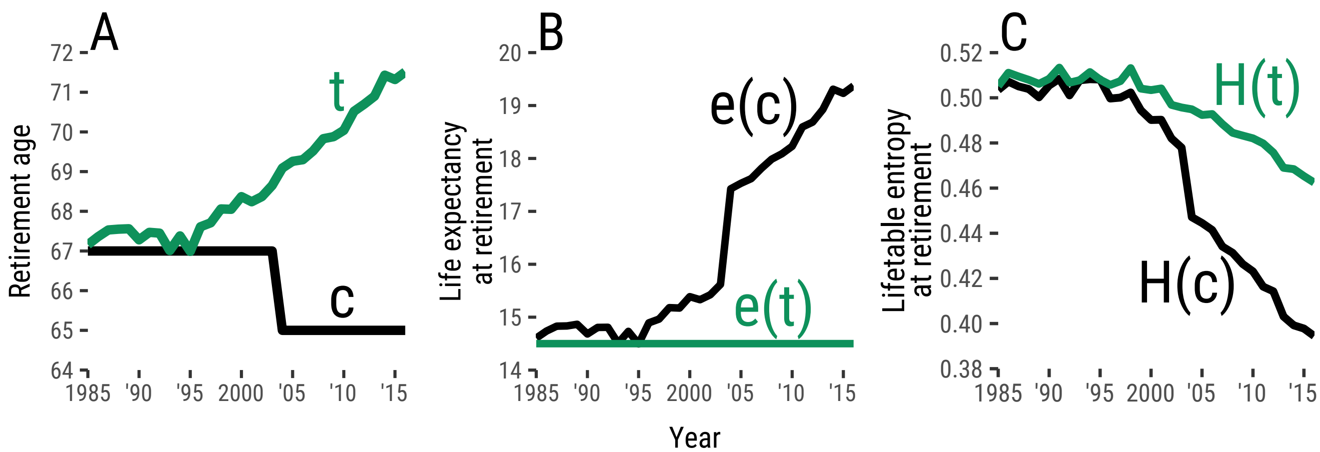

Figure 1: *Trends over time in statutory retirement ages (Panel A), life expectancy (Panel B) and lifespan inequality (Panel C) for the total population under the current and target pension schemes. 1985-2016.*

The current retirement age, c, was set at age 67 during the period 1985-2004. From 2004 onwards, c was fixed at age 65. In Panel B of Figure 1, we observe that life expectancy after age c trended upwards. This indicates that, under the current pension scheme, individuals from more recent generations spend more time in retirement than those from previous generations. The increasing trajectory of $e(c)$ was the major reason for the Danish pension reform in 2007 (Danish Ministry of Economic Affairs and Interior, 2017, 2018). Panel A of Figure 1 also shows the trajectory over time of the target retirement age, t. During the period 1985-1995, t remained constant around age 67. From 1995 onwards, age t went up, reaching values of 71 years in 2016. Given that t indicates the age at which $e(t) = 14.5$, the increasing trajectory of t implies that a person aged 71 years in 2016 enjoyed the same remaining life expectancy after retirement as a person aged 67 years in 1995 (14.5 years in this case). In Panel B of Figure 1, we observe that the increasing pattern in $e(c)$ is offset by setting retirement age to t, since $e(t)$ remains constant over time. This implies that, under the target pension scheme, individuals are expected to spend the same average number of years in retirement regardless of the year in which they retire. This mechanism controls for unexpected changes in life expectancy, reducing exposure to macro-longevity risk (Blake et al., 2018; Hari et al., 2008).

Panel C of Figure 1 shows trends in lifespan inequality measured by the lifetable entropy (Keyfitz and Caswell, 2005), calculated at the current and target retirement ages $H(c)$ and $H(t)$, respectively. We observe that $H(c)$ and $H(t)$ remained roughly constant during the period 1985-2000. Thereafter, both measures declined toward values of 0.39 for $H(c)$ and 0.46 for $H(t)$, respectively. The downward trend of $H(c)$ is more pronounced than the trend of $H(t)$ (the annual average decline of $H(c)$ was about 1.06% and 0.50% for $H(t)$). Furthermore, $H(c)$ is lower than $H(t)$, implying that lifespan inequality is lower after retirement age c than it is after age t. Lifespan inequality is also an indicator of uncertainty about the length of life at the individual level (Alvarez et al., 2019; van Raalte et al., 2018). Therefore, the results shown in Figure 1 indicate that, at population level, pensioners are more uncertain about the number of years they spend in retirement when they retire at age t than when they retire at age c.

The patterns described in Figure 1 are calculated for the total population. However, not all socio-economic groups have the same probability of surviving to retirement ages and not all of them exhibit the same $e(x)$ and $H(x)$. The levels and trends over time of these measures largely depend on the distribution of lifespans of each socio-economic group (see Section A in the Appendix). In the following sections we present trends over time in the probability of surviving to retirement ages, together with trends in life expectancy and lifespan inequality according to specific demographic profile of each socio-economic group. We analyse differences between them and their impact on the cost of pensions.

4.1 Differences in survival at retirement between socio-economic groups

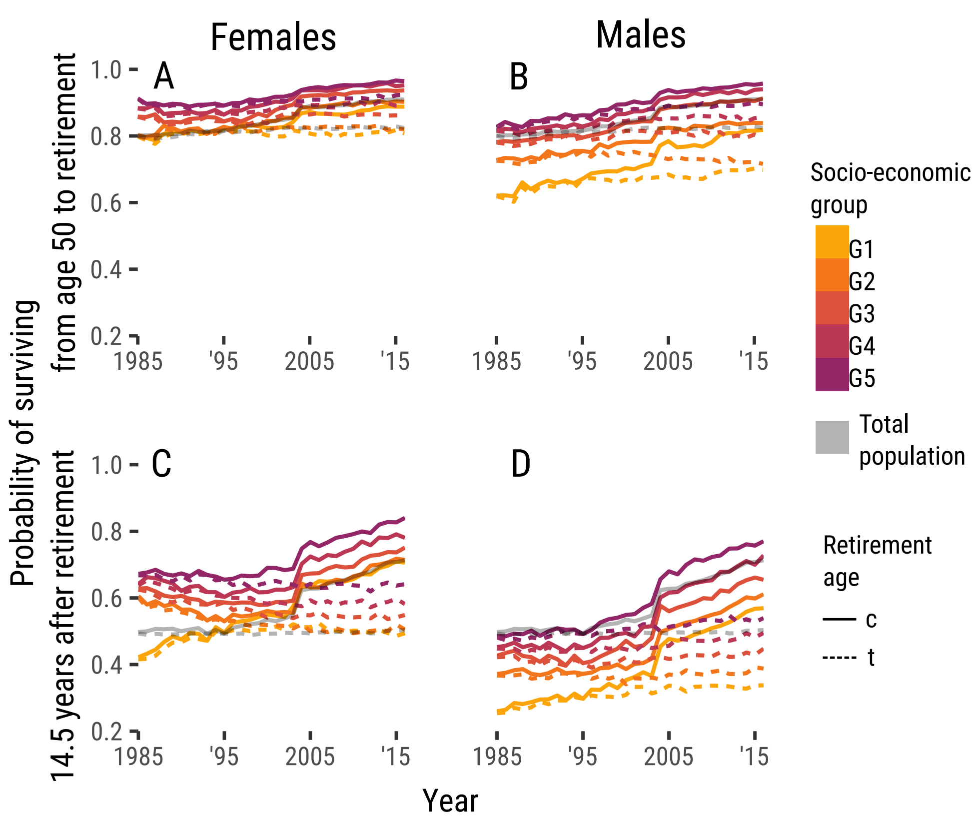

Figure 2: *Survival probabilities by socio-economic groups. Panels A and B indicate the probability of surviving from age 50 to retirement age (c and t). Panels C and D indicate the probability of surviving 14.5 years after retirement (c and t). Solid and dashed lines indicate the survival probability of retirement ages c and t, respectively. Grey lines indicate probabilities calculated for the total population. Both sexes, 1985-2016.*

Figure 2 shows that *i)* a clear socio-economic gradient in survival exists at retirement, and *ii)* such gradient varies depending on whether retirement age is set at age c or at age t. Panels A and B of Figure 2 depict sex-specific probabilities of surviving from age 50 to retirement ages (to age c in solid lines and to age t in dashed lines) for the total population and by socio-economic groups. Females in the highest socio-economic group (G5) have similar probabilities of surviving to both retirement ages c and t (0.96 and 0.93 under retirement ages c and t in 2016). This is not the case for males in the lowest socio-economic group (G1), where probabilities of surviving to retirement ages c and t are lower than those for females and differ greatly (0.82 to retirement age c and 0.69 to retirement age t in 2016).

Panels C and D of Figure 2 depict probabilities of surviving for 14.5 years after retirement ages c and t for females and males, respectively. Given that the pension reform (Danish Ministry of Economic Affairs and Interior, 2017) implies that, on average, pensioners will receive pension payments during an expected period of 14.5 years, these probabilities are indicators of who is more likely to survive to the end of the expected time of pension payments. We also observe a socio-economic gradient in these probabilities; however, in this case, the magnitude of differences between socio-economic groups is much bigger than in Panels A and B. In 2016, the probabilities of surviving 14.5 years after retirement were 0.84 at age c and 0.64 at age t for females in the highest socio-economic group (G5). For males in the lowest socio-economic group (G1), these probabilities were 0.57 at age c and only 0.33 for males at age t. Figure 2 entails that males in lower socio-economic groups have lower probability of surviving after retirement in comparison to females and that this disadvantageous setting is magnified when retirement age is linked to life expectancy (age t).

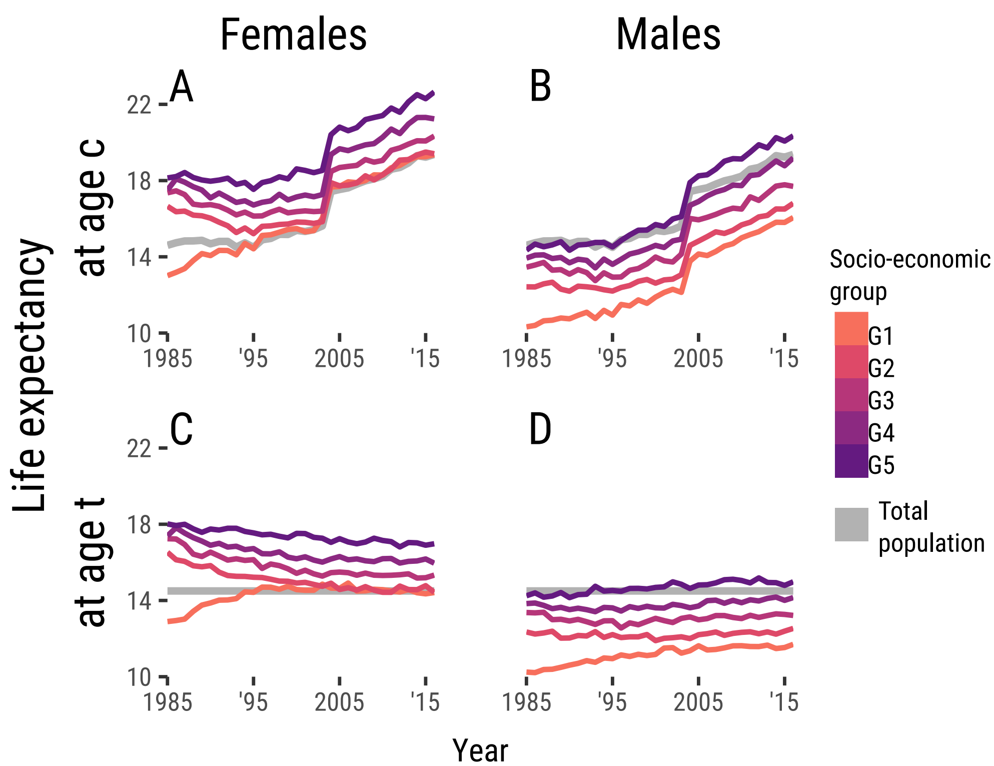

Figure 3: *Life expectancy at the current and target retirement ages by socio-economic groups. Both sexes, 1985-2016.*

The socio-economic gradient in survival probabilities at retirement documented in Figure 1 is reflected in summary measures such as life expectancy (Figure 3) and lifespan inequality (Figure 4). Figures 3 and 4 show that higher socio-economic groups enjoy longer lives (higher life expectancy, $e(x)$) and experience less lifespan inequality (lower entropy, $H(x)$) than lower socio-economic groups. At the retirement age c (Panels A and B of Figures 3 and 4), females in G5 exhibit $e(c)$ of 22.71 and $H(c)$ of 0.31 in 2016. Males in G1 exhibit a $e(c)$ of 16.04 and $H(c)$ of 0.47 during the same year. These disparities between higher and lower socio-economic groups prevail throughout all the period. In particular, males from

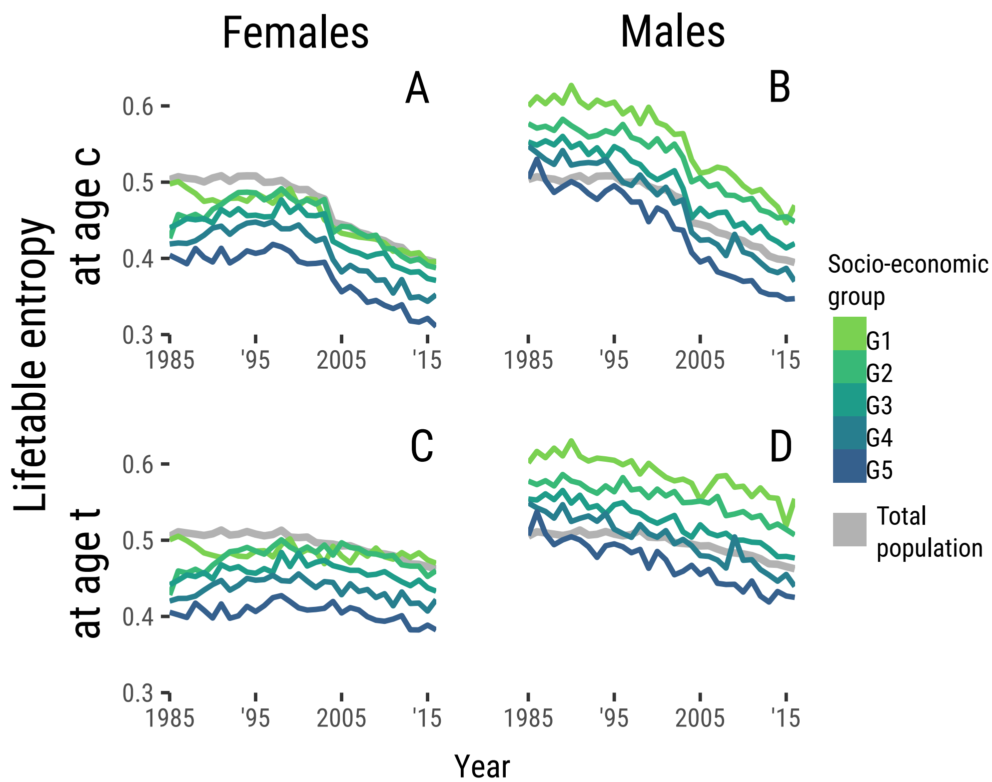

Figure 4: *Lifespan inequality at the current and target retirement ages by socio-economic group. Both sexes, 1985-2016.*

lower socio-economic groups under-perform in life expectancy and lifespan inequality.

The absolute disadvantages in $e(x)$ and $H(x)$ for lower socio-economic groups are magnified when these measures are calculated at the target retirement age t (Panels C and D of Figures 3 and 4). For example, in 2016, males in G1 exhibit $e(t)$ of 11.69 and $H(t)$ of 0.55. Thus, by retiring at age t rather than at age c, males in G1 experience a reduction of 4.35 years in the average prospect of life after retirement (life expectancy) and an increase of 0.08 in its uncertainty (lifespan inequality) for the year 2016. This is a demographic implication of setting t at a higher age than c (t at age 71.51 and c at age 65 in 2016). From the analysis of Figures 3 and 4, it is clear that linking retirement age to life expectancy sets individuals from lower socio-economic groups in a more disadvantaged demographic setting, compared to the current pension scheme, in which the statutory retirement age is constant.

Trends in $e(x)$ and $H(x)$ across socio-economic groups follow similar patterns over time to the ones observed for the total population (Panels B and C of Figure 1 and grey lines in Figures 3 and 4). The absolute level of these trends (in the total population and in socio-economic groups) depends on how retirement ages are defined. For example, in 2004, c was reduced from age 67 to age 65 (Danish Ministry of Economic Affairs and Interior, 2017). This change in retirement age c implied an increase in life expectancies at retirement for the total population (Figure 1) and for all socio-economic groups. However, this increase in life expectancy at retirement is an artefact of reducing age c from 67 to 65. Therefore, it is necessary to control for the absolute level in $e(x)$ and $H(x)$ of the total population to quantify the extent of the socio-economic gap.

4.2 Actuarial perspective on socio-economic inequalities

In this section we analyse relative differences in the financial cost of pensions between each socio-economic group and the total population. In Figure 5 panels A and B, we show the relative difference in the cost of an annuity and its associated entropy for the socio-economic groups and for the total population, calculated using Equations (6) and (7). The cost of an annuity is higher for all female socio-economic groups after 1990, compared to the total population meaning that the present values of their pensions payments are higher. Hence, for the same annuity, females are expected to receive higher payments than the average of the population. On the contrary, only the top affluence group for males after 1995 has payments higher than the average for the population. The force of interest is assumed to be zero in Figure 5, for simplicity but the results are robust to higher interest levels, see Figure 8 in the Appendix. A low force of interest reflects the current financial situation in Denmark (OECD, 2019a).

Panels C and D show the associated entropy to the cost of an annuity. For all the female socio-economic groups, the entropy is lower than the total population meaning that the uncertainty related to pension payments for the females group is lower than the average. This is a result of a lower uncertainty about their average life span as shown in Section 4.1. The reverse is again true for the male group where only the highest group has an entropy value below the average. Hence, most of the male groups have lower expected pension payments for the same annuity with higher uncertainty related to the payments.

For both females and males there is a clear rank in Figure 5 between the socio-economic groups meaning that most affluence groups for each sex have the highest expected payments with the lowest uncertainty from the same annuity. This shows that the socio-economic differences in life expectancies and lifespan inequality directly affect life annuities. This is especially true for a low interest regime, because the annuity payments with zero interest only are determined by an individual's length of life, which is the financial setting prevailing in Denmark and many other countries (OECD, 2019a). However the results are robust to higher to higher interest levels, as shown in Figure 8 in the Appendix.

To understand the underlying mechanisms behind changes in $\bar{H}(x,\delta)$, in Appendix B we have decomposed the differences in relative changes over time in $\bar{H}(x,\delta)$ between the highest and the lowest socio-economic groups. The main conclusion is that both, the level of life expectancy and absolute lifespan inequality play an important role in the uncertainty related to the cost of an annuity.

Figure 5: *Relative differences by socio-economic group in life annuities (Panles A and B) and in life annuity entropies (Panels C and D) by sex. Both sexes, 1985-2016.*

4.3 Differentiation in retirement ages by socio-economic groups

In this article, we quantify the socio-economic disparities at retirement that arise under retirement ages c and t, which are computed using demographic information of the total population. In this regard, previous studies (Ayuso et al., 2017; Sánchez-Romero et al., 2020) argue that socio-economic disparities at retirement can be reduced by differentiating retirement ages according to the demographic profile of each socio-economic group. In the Danish case, the differentiation could occur in terms of the target retirement age, where retirement ages are calculated such that each socio-economic group spends 14.5 years (in average) according to its own demographic profile. In Figure 6, we show that this policy implies a difference of almost eight years between the highest socio-economic group for females and the lowest socio-economic group for males. We anticipate that a policy of this type could imply major legal issues and implementation challenges. It could also influence individuals to move between socio-economic groups (people can move to lower socio-economic groups to retire at an earlier age) and this could have a major impact in the sustainability of the pension system. Further, a differentiation of retirement ages by socio-economic group could be perceived as discriminatory. because, in practice, it might be difficult to allocate individuals to a specific socio-economic group (Arcanjo, 2019). Hence, while differentiating retirement ages by socio-economic group could reduce socio-economic disparities at retirement due to heterogeneity in longevity, other important issues might arise from this policy.

Then, the question arises as to how to set retirement ages. In this study we show that, while linking retirement age to life expectancy (of the total population) might contribute to ensuring the sustainability of national pension systems (Danish Ministry of Economic Affairs and Interior, 2018), this policy introduces higher inequality. Therefore, retirement ages should be seen as a trade-off between constant life expectancies and lower lifespan inequality. In addition, one possible policy to reduce socio-economic

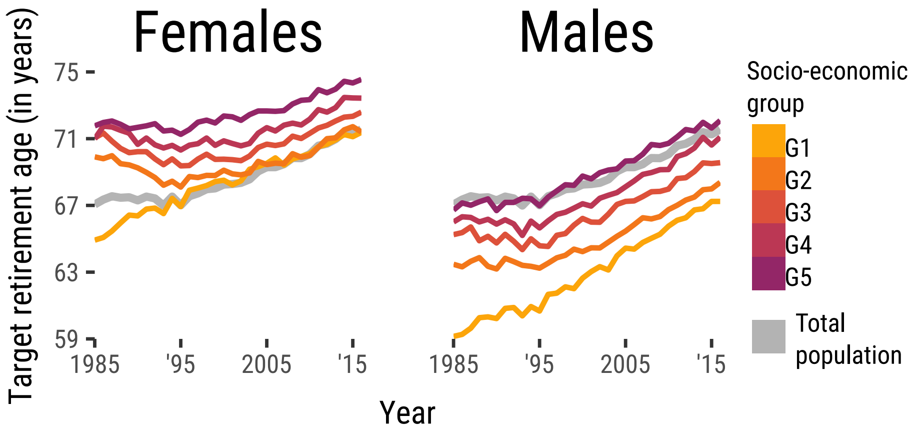

Figure 6: *Target retirement ages calculated according to the demographic profile of each socio-economic group. 1985-2016*

disparities is to link early retirement to the timing of individuals' first full-time jobs[7]. Early retirement could depend on the number of years since the time the individual enters to the labour market, by taking into account the years of unemployment. This type of policy tends to benefit individuals in lower socio-economic groups, as they tend to enter the labour market earlier than those in higher socio-economic groups (Bravo and Herce, 2019).

In this study, we explore the implications of unequal lifespans by looking at Danish pension reform. However, these issues are not restricted to Denmark. In a similar fashion, other countries, such as Estonia, Finland, Italy, the Netherlands, Portugal and the Slovak Republic will also modify retirement ages, by linking them to life expectancy (OECD, 2018, 2019b) in a context of low interest rates (OECD, 2019b). Therefore, it is likely that demographic imbalances after retirement will also arise in those countries.

5 Discussion

In this study, we analyse the implications of linking retirement age to life expectancy, reflected in socio-economic disparities in longevity and their influence on the financial cost of pensions. We advance the following findings in terms of lifespan inequality, longevity risk and socio-economic differences in pension outcomes.

Lifespan inequality at retirement and longevity risk

Under the target pension scheme, the upward trend of life expectancy (calculated for the total population) is offset and the exposure of pensions to macro-longevity risk (Blake et al., 2018) is reduced, because it is expected that retirees spend similar average times in retirement. However, the linkage rule increases lifespan inequality after retirement and, in consequence, the sensitivity of life annuities to changes in mortality rates is also increased (micro-longevity risk, see Hari et al. (2008)). Allowing for life annuities, backed up by riskier financial products that yield higher interest rates, can reduce the exposition to micro-longevity risk that is embedded in the target pension scheme. Even in this case, individuals will deal with uncertain pension payments under the target pension scheme, either through increased micro-longevity risk or through higher financial risk.

[7] A policy of this type, which allows people to retire earlier was implemented in Denmark in 2020, see https://bm.dk/media/15057/aftaletekst-tidlig-pension.pdf in Danish

Another implication of high lifespan inequality after retirement relates to ineffective financial planning. At an individual level, the high levels of lifespan inequality imposed by the target pension scheme can blur the perception about the future, such that individuals might take important financial decisions lightly (Aronsson and Blomquist, 2018). In particular, high lifespan inequality at retirement can discourage individuals from participating in schemes that require voluntary contributions to pension funds (pension schemes from pillar 2 and pillar 3). Consequently, individuals might incur shortages of pension wealth to face post-retirement years.

Socio-economic disparities in longevity and impact on pension schemes

In this study we also show that socio-economic disparities in longevity prevail, regardless of the age at which individuals retire. Males from lower socio-economic groups are at a clear disadvantage, since they spend less time in retirement, pay higher pensions costs per expected benefits and are exposed to higher longevity risk than upper socio-economic groups. These disparities are magnified when linking retirement age to developments in total life expectancy.

The question thus arises: how do these socio-economic disparities in longevity affect pension objectives, such as redistribution[8] of income in the Danish pension system? To understand this issue, it is necessary to distinguish between the different pension schemes operating in pillars 1 and 2. In the case of old age pensions of pillar 1 (PAYG scheme), individuals receive pension payments regardless of the time spent actively participating in the labour market (Andersen, 2015). In the case of non-participation and not having any income other than pension payments (i.e. lowest socio-economic groups), pensioners would receive the highest amount of old age pension. Furthermore, the amount of old age pension is reduced based on pension income from pillar 2. This means that old age pensions payments from pillar 1 are regulated by the other pillars. For example, individuals receiving higher pension income from labour market pensions (pillar 2), receive a small amount of old age pension (pillar 1). These progressive factors built into pillar 1 aim at reducing the financial inequality in pensions payments experienced by the lower socio-economic groups. In consequence, potential income redistribution might occur from higher to lower socio-economic groups via taxation, because people in high socio-economic groups pay higher taxes from higher earned income during their working life and receive less old age pension at retirement.

Pillar 2, on the other hand, is mostly constituted by mandatory defined contribution schemes (labour market pensions). It has been shown that pension objectives in defined contribution schemes are little affected by policies aiming at increasing retirement age (Alonso-García et al., 2018; Sánchez-Romero et al., 2020). However, redistribution issues could arise within highly heterogeneous defined contribution pension schemes that cover a large number of individuals from very different socio-economic groups (Brown, 2003; Holzmann et al., 2019; Milevsky, 2020).

Policies that allow for early and late retirement options also play an important role in either reversing or magnifying socio-economic inequalities in pension schemes. For old age pensions (pillar 1), individuals have the option to postpone retirement up to 10 years which results in a higher amount of pension (OECD, 2015). Thus, additional contributions to the old age pension translate into additional benefits, while annuity payments will run for a shorter period. For the case of labour market pensions (defined benefit schemes of pillar 2), individuals can postpone retirement, which results in higher contributions to their pension accounts. Early retirement on these schemes, however, is heavily penalized because pension

[8] in the Appendix we discuss how socio-economic disparities in longevity affect other pension objectives, such as actuarial fairness, sustainability and adequacy.

payments are subject to high taxation rates[9] (Arnberg and Barslund, 2014). This setting aims to promote retirement at older ages, but it also has detrimental implications for lower socio-economic groups. If they choose to retire early, they get penalized through high taxation rates, but if they decide to retire at the target retirement age, they would spend fewer expected years receiving pension (as shown in our results). Further research that would consider data on accumulation of pension wealth in Denmark is required, to properly quantify the interaction between early retirement, taxation and benefits (perhaps in a similar fashion to the analysis performed by (Sánchez-Romero et al., 2020)).

Robustness towards cohort measures

In this study we use period data to calculate retirement ages and to quantify socio-economic differences in demographic and actuarial outcomes. Alternatively, the same analysis could have been performed using a cohort life table approach (Ayuso et al., 2020). We replicate the same analysis by using cohort measures to quantify the differences between both approaches. The complete analysis can be found in Section D of the Appendix. Our results show that the target retirement age is higher under the cohort perspective than under the period perspective (Figure 10 in the Appendix). However, the magnitude of the gap between both measures is not overwhelmingly large, as also observed in other populations (e.g., the USA, see Goldstein and Wachter (2006)). In terms of lifespan inequality, entropies are remarkably similar from both the period and cohort perspectives. Further, we show that the socio-economic gap in life annuity factors is remarkably similar from perspectives (Figure 11 in the Appendix). This shows that our calculations (using period data) correctly depict the implications of linking life expectancy to retirement age.

[9] If individuals started saving in their defined contribution schemes before May 2007, they can retire some years before the official retirement age without being taxed (Arnberg and Barslund, 2014)

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Appendix

A Distributions of lifespans by socio-economic groups

Figure 7 shows the distributions of lifespans after age 65 for the lowest and highest socio-economic groups (G1 and G5) by sex, for selected years during the period 1985-2016. The survival trajectories of Danish females in the lowest socio-economic group have approached those in G5. This is reflected in the increasingly overlapping areas of the distributions of lifespans, which in 1985 comprised 73% and 84% in 2015. For males, the overlapping area between G1 and G5 groups remained similar over time. Despite the large shared area between distributions, we can observe noticeable differences between them.

The distribution of lifespans for the G5 group is, in all years, shifted to the right and its tail is much heavier, compared to the distribution of the lowest socio-economic group. This means that a greater number of individuals form the highest socio-economic group outlived those in the lowest socio-economic group. The shape of the distribution of lifespans for males also differs considerably in comparison to that of females. These discrepancies in the distribution of lifespans result in different longevity outcomes depending on the profile of each socio-economic group. Such inequalities are examined in Section 4.

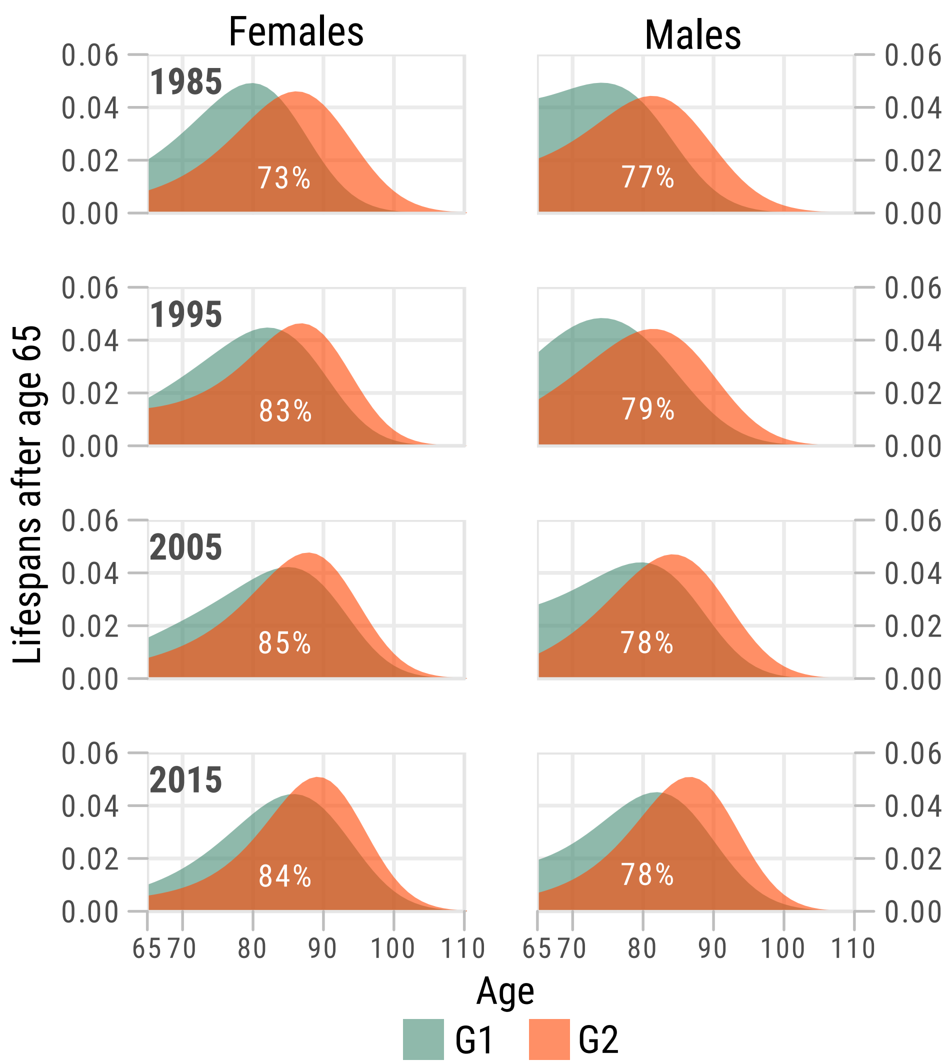

Figure 7: *Distribution of life table deaths conditioned on survival to age 65 for the highest (G5) and the lowest (G1) socio-economic groups in Denmark. Both sexes, selected years.*

B Alternative expression of $\bar{H}(x,\delta)$

The present value of a life annuity at age x (Bowers et al., 1997) is defined as

$$\bar{a}(x) = \frac{\int_x^\infty l(y)e^{-\delta y}dy}{l(x)e^{-\delta x}}, \tag{8}$$

where $l(x)$ is the survivorship function at age x and δ represents the force of interest. Haberman et al. (2011) defined the entropy of a life annuity as

$$\bar{H}(x,\delta) = \frac{-\int_0^\infty \ln(l(y))l(y)e^{-\delta y}dy}{\bar{a}(x)}, \tag{9}$$

given that $\ln(l(y)) = -\int_0^y \mu(x)dx$, where $\mu(x)$ denotes the force of mortality and using reverse integration as in Vaupel (1986), we can re-express the numerator of Equation 9 as a measure of heterogeneity such that

$$\begin{aligned}
-\int_0^\infty \ln(l(y))l(y)e^{-\delta y}dy &= \int_0^\infty l(y)e^{-\delta y}\int_0^y \mu(x)dxdy \\
&= \int_0^\infty \mu(x)\int_x^\infty l(y)e^{-\delta y}dydx \\
&= \int_0^\infty \mu(x)l(x)e^{-\delta x}\bar{a}(x)dx \\
&= \int_0^\infty d(x)e^{-\delta x}\bar{a}(x)dx
\end{aligned}$$

Thus,

$$\bar{H}(x,\delta) = \frac{\int_0^\infty d(x)e^{-\delta x}\bar{a}(x)dx}{\bar{a}(x)} = \frac{\bar{a}^\dagger(x)}{\bar{a}(x)}. \tag{10}$$

C Decomposition of differences over time in $\bar{H}(\delta)$ between two populations

We denote the derivative with respect to time by placing a dot on top of the measure of interest. For example, the derivative of the life annuity factor is denoted as $\dot{\bar{a}}(x)$. As in Fernandez and Beltrán-Sánchez (2015), it is straightforward to show that the relative derivative of $\bar{H}(\delta)$ with respect to time is:

$$\frac{\dot{\bar{H}}(x,\delta)}{\bar{H}(x,\delta)} = \frac{\dot{\bar{a}}^{\dagger}(x)}{\bar{a}^{\dagger}(x)} - \frac{\dot{\bar{a}}(x)}{\bar{a}(x)} \tag{11}$$

Thus, Equation 11 implies that the relative derivative of $\bar{H}(x,\delta)$ with respect to time can be expressed as the difference between the relative derivative of $\bar{a}^{\dagger}(x)$ and $\bar{a}(x)$.

Assuming that we want to compare two populations A and B with entropies $\bar{H}_A(x,\delta)$ and $\bar{H}_B(x,\delta)$, by Equations 10 and 11 we can decompose the difference over time between $\bar{H}_A(x,\delta)$ and $\bar{H}_B(x,\delta)$ such that:

$$\frac{\dot{\bar{H}}_A(x,\delta)}{\bar{H}_A(x,\delta)} - \frac{\dot{\bar{H}}_B(x,\delta)}{\bar{H}_B(x,\delta)} = \underbrace{\frac{\dot{\bar{a}}_A^{\dagger}(x)}{\bar{a}_A^{\dagger}(x)} - \frac{\dot{\bar{a}}_B^{\dagger}(x)}{\bar{a}_B^{\dagger}(x)}}_{dispersion} + \underbrace{\frac{\dot{\bar{a}}_B(x)}{\bar{a}_B(x)} - \frac{\dot{\bar{a}}_A(x)}{\bar{a}_A(x)}}_{translation} \tag{12}$$

The *translation* component determines the difference due to changes in the mean value of the life annuity $\bar{a}(x)$ and the *dispersion* component denotes differences due to changes in absolute lifespan inequality denoted by $\bar{a}^{\dagger}(x)$. Positive values in any of these effects contribute to increasing the gap in $\bar{H}(x,\delta)$ between socio-economic groups, while negative values contribute to reducing it. We performed the decomposition at the current and the target retirement ages and the results are shown in Figure 9. We selected the highest and lowest socio-economic groups as an illustration.

Figure 9 shows that the dispersion component contributes positively to the socio-economic gap in $\bar{H}(x,\delta)$ for both sexes and at both retirement ages (orange bars in all Panels of Figure 9). This means that the gap in $\bar{H}(x,\delta)$ between the highest and the lowest socio-economic groups is largely explained by differences in absolute lifespan inequality measured by $\bar{a}^{\dagger}(x)$. This component has remained similar over time, meaning that the gap in absolute lifespan inequality between the lowest and highest socio-economic groups has remained unchanged during the period of observation.

However, we also observe that the translation component (green bars in all Panels of Figure 9) plays an important role in the reduction of the gap for both sexes, but particularly for females (Panels A.1, A.3, B.1 and B.3 of Figure 9). The translation component is related to the cost of life annuities (or life expectancy in Panels A.1 and B.1, since $\bar{a}(x) = e(x)$ at $\delta = 0\%$). Thus, this finding entails that cost of life annuities for females in the lowest socio-economic group has decreased over time in comparison to the life annuity cost for the highest socio-economic groups. For males (Panels A.2 and B.2), the translation component has a smaller effect in reducing the gap between socio-economic groups than the one observed in females.

Figure 8: *Relative differences by socio-economic groups in life annuities (Panles A and B) and in life annuity entropies (Panels C and D) by sex assuming constant force of mortality of 5%. Both sexes, 1985-2016.*

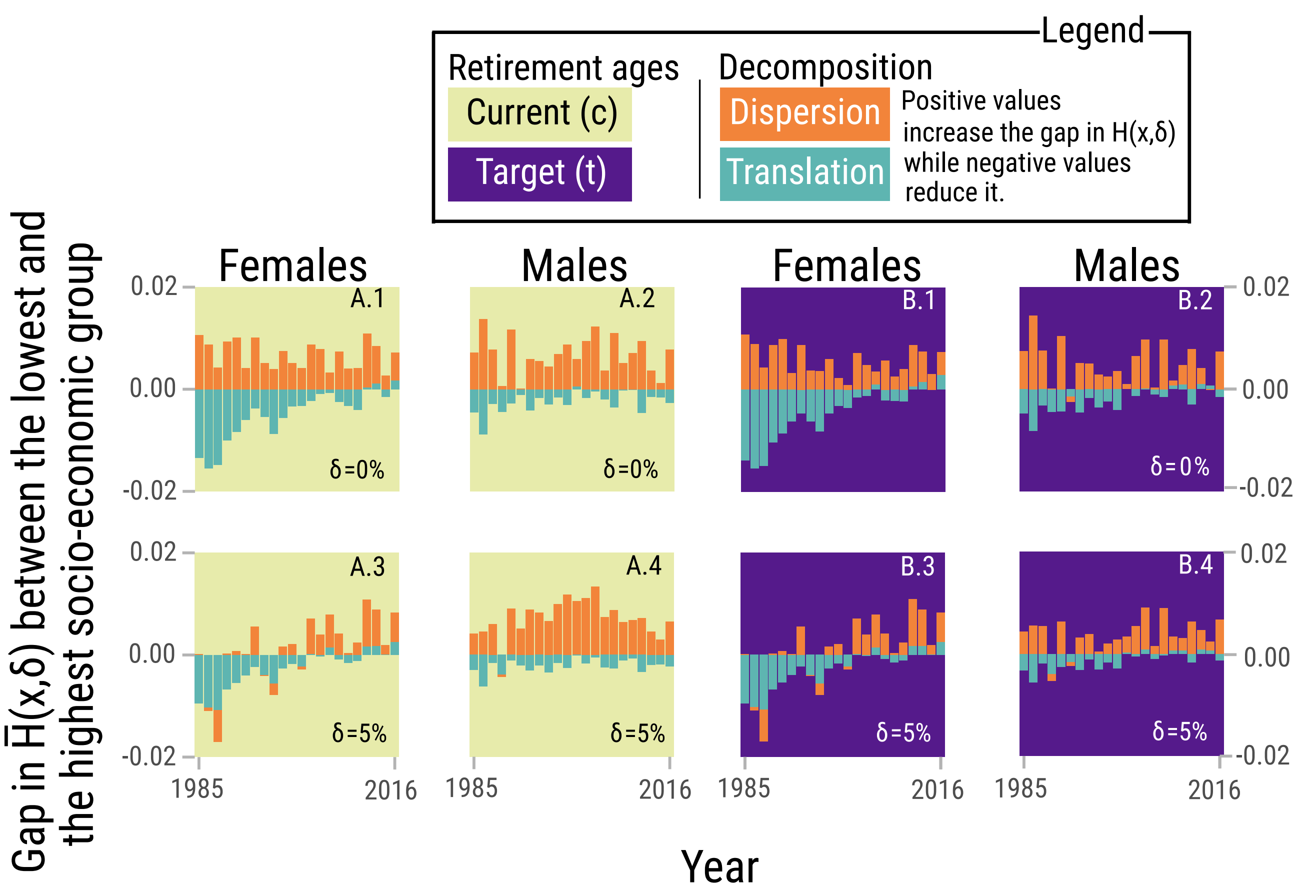

Figure 9: *Decomposition of differences in the relative change over time in the entropy of a life annuity by socio-economic group. Both sexes, 1985-2016.*

For an interest rate level of 5%, the dispersion and translation components reduce as the difference in survival matters less for the gap in the cost of the annuity. For males, the dispersion component remains the same when $\delta = 5\%$, since the absolute lifespan inequality weighs more in $\bar{H}(x,\delta)$ than the financial factor $e^{-\delta}$ (see Equation 9). A δ higher than 5% would be necessary to reduce the dispersion component in males.

In conclusion, Figure 9 depicts the demographic processes behind the socio-economic gap in $\bar{H}(x,\delta)$ and how these processes react to different interest rates. Figure 9 shows that the translation effect played an important role in reducing the socio-economic gap in $\bar{H}(x,\delta)$ for females. Hence, improving survival for low socio-economic females not only reduced the gap in life expectancy and in the cost of life annuities, but also reduced the uncertainty related to the cost of the annuity, making it easier to hedge their micro-longevity risk. We also show that the socio-economic gap in $\bar{H}(x,\delta)$ is similar under both the current and the target retirement schemes.

D Target retirement age from a cohort perspective

To quantify the impact of using cohort measures instead of period ones, we first forecasted period mortality rates by socio-economic groups using the method developed by Li and Lee (2005). Then, we reorganized such rates by cohort and computed target retirement age and life annuity factors by socio-economic groups. The rule to calculate the target retirement age in a cohort perspective is the same as the one used in this study with period data (i.e. the value of the variable t_c that solves the equation $e(t_c) = 14.5$, where tc is the target retirement age by cohort). To compare the cohort trend (t_c) against the target retirement age in period basis, we calculate the corresponding year at which each cohort will be eligible to retire such that Year = Cohort $+t_c$. Note that by calculating cohort trends from forecasted data we introduce uncertainty regarding the validity of assumptions used to project future demographic trends (Ayuso et al., 2020).

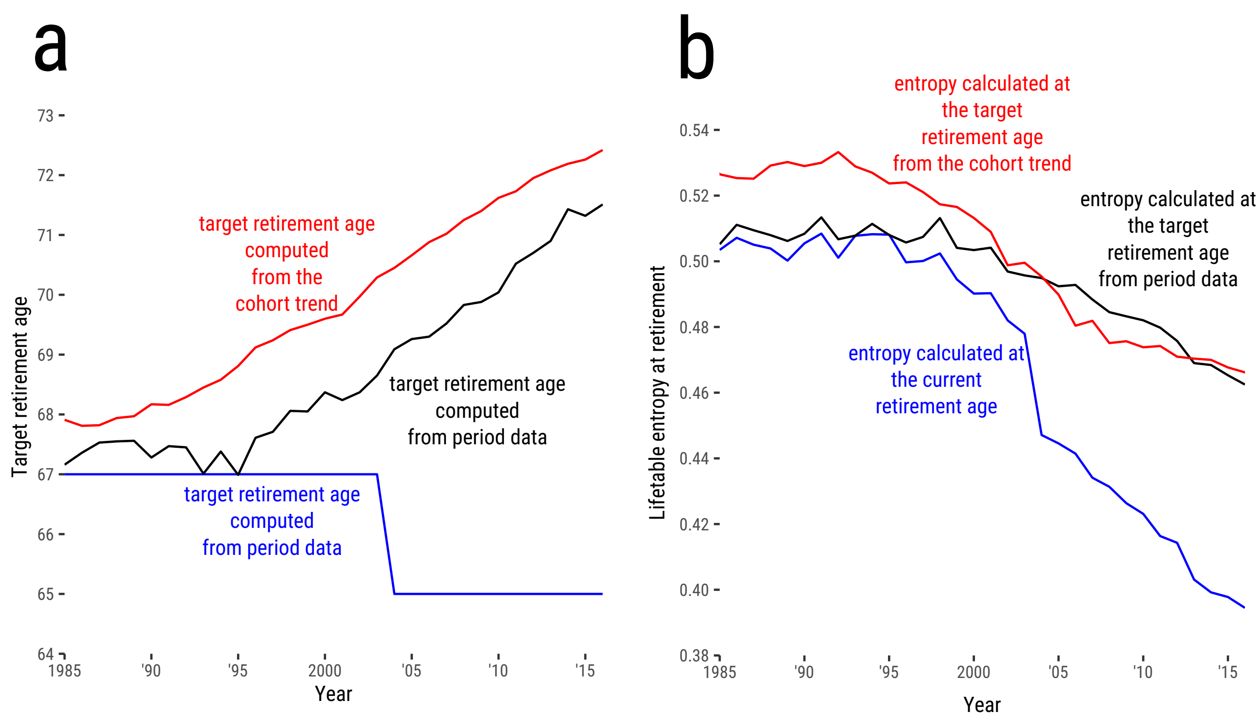

Figure 10: *Comparison of retirement ages and entropies using cohort data. 1985-2016.*

In panel A of Figure 10, we observe that the target retirement age in the cohort basis (t_c, in red) is higher than the one calculated from period data (in black). From 1985 to 1990, the gap was less than one year. After 1990, both retirement ages increased at the same pace and the gap remained constant around one year. While it is true that there are discrepancies between both target retirement ages, the magnitude of the gap is not overwhelmingly large, as also observed in other populations (e.g. the USA, see (Goldstein and Wachter, 2006)). Next, we computed life table entropy at the cohort retirement age (panel B of Figure 10) and compared it with the entropy calculated with period data. We found that both entropies (period and cohort) are remarkable similar. This shows that, in terms of lifespan inequality after retirement, our results are practically the same under period or cohort perspectives.

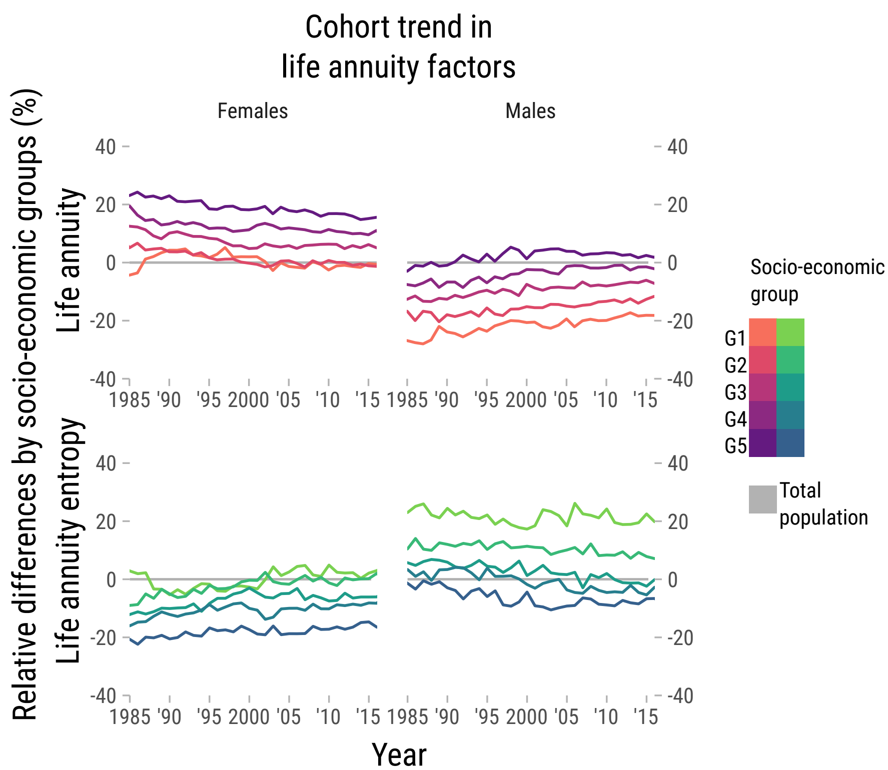

Figure 11: *Relative differences in the financial cost of pensions by socio-economic group with respect to the total population. Calculations performed using a cohort trend. 1985-2016*

In Figure 11, we show the relative difference in the financial cost of pensions (life annuities) across socio-economic groups, by assuming interest rates of zero percent (which equals life expectancy) and five percent (as in Figure 5 in the manuscript). We observe that the socio-economic gap under the cohort perspective is almost identical to the gap in the period perspective. Similarly, when increasing interest rates, the socio-economic gap reduces and converges towards the total population. Figure 11 shows that the socio-economic gap in life annuities remains the same, regardless of using period or cohort data. Hence, this shows that our results capture well the existing socio-economic gap in life annuities. Such gap remains similar when looking at either the cohort and period perspective.

E Socio-economic disparities in longevity and interaction adequacy of pensions

Replacement rates (including the old age pension and mandatory defined contribution schemes) are high in Denmark. The OECD (2019b) reports an average gross replacement ratio of 74.4% from public and private pension schemes, which is among the highest in the OECD countries (the OECD average is about 49%). When looking at socio-economic differences, the OECD (2019b) reports a gross replacement rate of 113% for the lowest income group and 64% for the highest income group. We assert that linking retirement age to life expectancy could affect these replacement rates and pension adequacy across the different socio-economic groups. Under the new retirement setting, individuals with higher life expectancies (i.e. higher socio-economic groups) will work for longer periods and accumulate more pension wealth. Conversely, those in lower socio-economic groups, while they might also save for a longer time, will not benefit from it, because they still die earlier. Thus, replacement rates reported by the OECD (2019b) are likely to be affected by the retirement policy.

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