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# The move towards riskier pensions: The importance of mortality*

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**Abstract**

This paper models the impact of unanticipated changes in forecasted life expectancies on guaranteed and unguaranteed pension products. We study a unique data set containing individuals offered the opportunity to substitute a guaranteed pension product with relatively low levels of risk to an unguaranteed product with a higher degree of financial and longevity risk. The complexity of the products and the increase in the level of financial literacy required by the individual to make such a decision motivate the need to properly model the most important drivers that characterize the differences between guaranteed and unguaranteed pension products. This is done within the standard Merton, Black and Scholes framework and we find a clear tradeoff between financial risk and longevity risk in terms of their effect on future pension payments. We find that unguaranteed pension products allow for more financial risk-taking and thus higher expected returns. However, unexpected longevity shocks can reduce pension payments in unguaranteed pension products to a lower level relative to guaranteed products.

**Keywords:** Macro longevity risk; Variable annuities; Guarantee; Unguarantee
**JEL codes:** J32; J11; J17; G22

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# 1 Introduction

Pension systems around the world are challenged by unexpected increases in life expectancy, historical low interest levels, and increased Solvency II capital requirements. Consequently, the pension industry witnesses a move from guaranteed pension products to unguaranteed products. The result is a significant risk transfer, from risk accruing to the pension provider to risk accruing to the individual pension holder.[1] Even in countries like The Netherlands and Denmark, which have consistently been rated "the best pension systems in the world", we observe a move towards more uncertainty.[2] In Denmark, to take just one example, around 10% of pension contributions went to unguaranteed pension products in 2005. In 2017, around 75% of pension contributions went to unguaranteed products (FSA, 2017).

The rapid move towards more risk started by enabling pension providers to offer not only risk-free guarantees or fixed annuities, but also variable annuities in which individuals are allowed to invest in risky assets. The shift in risk is currently taking on an additional layer of risk in the form of mortality risk as it potentially affects the individual's payout profile in the decumulation phase. The contribution of this paper is twofold. Firstly, we analyze which type of individual voluntarily chooses to opt for more risk despite the complexity and opaqueness of the products. Secondly, in order to fully understand the consequences of the risk transfer, we model the most important drivers that characterize the differences between guaranteed and unguaranteed pension products with a particular focus on longevity risk.

When investigating the effect of longevity, it is important to distinguish between micro longevity risk and macro longevity risk. Micro longevity risk is the risk that an *individual* might live longer (or shorter) than the longevity forecast. This is an idiosyncratic risk that can be shared among individuals. Macro longevity risk, on the other hand, is the risk that the life expectancy of the population as a whole increases. This is a systematic risk. When the pool of individuals is large enough, micro longevity risk can be shared among the individuals in the pool. This means that the expected path of pensions will be unaffected by the existence of micro longevity risk – it cancels out in expectations. However, systematic macro-wide changes in longevity cannot be shared as it affects everybody in the pool. It is these types of unforeseen changes in life expectancies that we allow to affect the pensions in the decumulation phase of the unguaranteed product.

We define a pension guarantee as a pension product that guarantees a minimum annual

[1] When this paper discusses "risky pension products", it is implicitly understood as referring to risks for the pension holder. A shift from a guaranteed pension product to an unguaranteed product typically lowers the risk of insolvency of the pension provider, but then, as a consequence, increases the risk for the pension holder, as returns on pension savings become riskier. We refer to the latter effect when discussing "increases in risk".

[2] 2018 Melbourne Mercer Global Pension Index. https://www.mercer.com/our-thinking/mmgpi.html.

return on remaining pension savings. Thus, the guaranteed product incorporates a floor whereas the unguaranteed product is a pure variable annuity which implies, for instance, that there is no zero lower bound as is sometimes the case in the accumulation phase when speaking about non-guarantees empirically. Besides financial risk, unforeseen changes in longevity thus affect pensions in the decumulation phase of unguaranteed products, while less financial risk and no longevity risk are carried by the pension holders with a guaranteed product.

Historically, the literature on pension choices has mainly investigated the puzzle of the unexplainable low demand for annuitization, see Beshears et al. (2014), Brown (2001), and Bütler and Teppa (2007). However, we investigate the choice between two annuity products rather than a lump sum. To the best of our knowledge, we are the first to provide empirical evidence on real transition decisions between annuity products. We address the question of what characterizes individuals who voluntarily decide to switch from a guaranteed pension product to a non-guaranteed product by making use of a unique data set from a Danish pension fund that offered its customers to switch pension product. Defined-contribution products in Denmark have traditionally included a return guarantee that was fixed and life-long, i.e., contrary to the unguaranteed product. Thus after the creation of a pension contract, the guarantee did not depend on subsequent interest rate movements, contribution rates, or developments in longevity. We find that being single, male, young, and having moderate levels of pension wealth will increase the probability that the individual switches to an unguaranteed product. We also find that the higher the guaranteed return, the lower the probability of switching. This is interesting because a higher guarantee means that a larger fraction of pension savings must be allocated to the risk-free asset in order to secure the guarantee. We conclude that it is the built-in longevity hedge that works in favor of the guaranteed product.

The risk transfer of financial and longevity risk significantly adds to the complexity of the pension product. This leads to an increase in the required degree of financial literacy of the individual in order to optimally plan and prepare for retirement. It is well documented that individuals consistently make errors with regard to financial decisions, see Mitchell (1988) and Van Rooij et al. (2011). It also raises the question of how to correctly inform pension holders about the multiple risk factors they face when they move away from a guaranteed product with a relatively low level of risk to an unguaranteed product. These features motivate a new modeling framework that characterizes the differences between guaranteed and unguaranteed pension products while accounting for both financial risk, micro, and in particular macro longevity risk.

The literature on modeling pension products has traditionally focused on the implications of financial risks, see Døskeland and Nordahl (2008), Jørgensen and Linnemann (2012), Balter and Werker (2019), and Guillén et al. (2013). However, a number of papers analyze the effect of micro longevity risk. Chen et al. (2015), for instance, build upon the work of Døskeland and

Nordahl (2008) and explicitly add micro longevity risk to investigate the attractiveness of different pension products. They show that an expected utility maximizing investor, who is sufficiently risk averse, prefers the guarantee when the investor is faced with the possibility of pre-mature death. Donnelly et al. (2013) investigate pooled annuity funds in which the individuals share investment risk in a Black-Scholes setting and mortality risk by a deterministic force of mortality. They compare this with a mortality-linked fund where the mortality risk is, at some costs, borne by the insurer. Since all individuals are assumed to be independent copies of one another, the sharing aspect boils down to the individual perspective we adopt in this paper, where the micro longevity risk is borne by the insurer.

Recent developments in longevity and associated risks (Oeppen and Vaupel, 2002; Blake and Morales, 2017; Pascariu et al., 2018) demonstrate that there is a need to understand how unexpected increases in life expectancy influence pensions, too. Qiao and Sherris (2013) investigate macro longevity risk and highlight its importance as benefit payments can be expected to decrease, and the volatility of payments expected to increase, over time. Maurer et al. (2013) show that many households with constant relative risk aversion prefer deferred annuities that vary due to unexpected mortality developments over deferred guaranteed annuities as the latter are expensive given that the fair price includes longevity risk. Macro longevity risk in pooled funds is investigated by Piggott et al. (2005), who update the expectations of the individuals within group self annuitization in which macro longevity risk is not allowed to be shared among cohorts. De Waegenaere et al. (2017), De Waegenaere et al. (2018), and Broeders et al. (2019) explicitly investigate different sharing rules for different types of macro longevity risk. Our main contribution is that we model the impact of macro longevity risk between different annuity products in a way that differs significantly from what has previously been reported in the literature. In particular, we analyze the effect of longevity risk in the decumulation phase in a fashion that resembles practice by focusing on the impact of changes in expectations due to updated mortality forecasts.

More generally, we extend the variable annuity class of products in which financial risk is present, to explicitly incorporate unforeseen deviations in life expectancies. We model guarantees and unguarantees where the latter is obtained by investing freely in the risky asset while the guarantee needs a separation of the exposure to the bonds opposed to dynamic rebalancing. The guaranteed product consists of a “floor” that can be built in by allocating part of the pension wealth to bonds with maturities equal to the horizon of the pension payments. For our analysis, we assume a Black-Scholes economy, and we divide the total pension wealth such that the pension payments are ex ante neither expected to decrease nor increase. However, as we unexpectedly might live longer than expected, future pension payments for unguaranteed products can still deviate in terms of updated expectations, i.e. after new mortality tables (also called life tables or actuarial tables) have been published.

We find that – in the absence of macro longevity risk – the unguaranteed product generates a higher expected return compared to the guaranteed product, but at the cost of more financial risk (which is driven by equity risk). The reason is that the guarantee requires that a larger fraction of wealth is invested in the risk-free asset, in order to secure the guarantee. Consequently, there is less room for taking on risk. We find that unforeseen increases in macro longevity forecasts make the guaranteed product relatively more attractive. The guarantee secures against such unforeseen changes. Situations might even arise where reductions in longevity forecasts make pension payments during the decumulation phase lower in the unguaranteed product than in the guaranteed. Accounting for systematic longevity risk is thus important when comparing guaranteed and unguaranteed pension products.

We organize the paper as follows. In Section 2 we analyze which type of pension holders switch from guaranteed to unguaranteed pensions. In Section 3 we investigate between which two products the pension holders actually had to make a choice. We model the main characteristics of the guaranteed and unguaranteed products based on realistic industry practices of using (updated) mortality tables. And in Section 4 we conclude.

## 2 Empirical study

We have access to unique data from a Danish pension fund that offered its members to voluntarily switch from a guaranteed pension product to an unguaranteed product. The data covers individual level member's data for the Danish pension fund, Juristernes og Økonomernes Pensionskasse (JØP). This is a fund for lawyers and economists. It is fully funded and owned by its more than 60,000 members. The amount of assets managed by JØP amounted to DKK 78 billion (approximately €10.5 billion) in 2018. We use the data to investigate which type of individual would voluntary switch from a guaranteed to an unguaranteed product.

Until the mid 2000s the fund has only offered guaranteed average interest rate products. These products ensured the pension holder a minimum annual return on his/her pension savings. The level of the guaranteed returns was determined at the start of the contract, i.e. when the individual became a member of the pension fund. The higher the prevailing interest rate on the market when a member entered the pension contract, the higher the guarantee. As interest rates have been falling during the last decades, the guarantees offered by the fund have been falling, too. The members have thus been offered different guarantees depending on the date of admission, ranging from 4.25% to 0.00%. This means that the fund has undergone a transition during the last decade or so, from guaranteed products (with different levels of guarantees) to unguaranteed products.

Pension holders with a guaranteed interest rate above zero are grouped into a separate division within the pension fund called Division 1. In May 2007, the 31,497 pension holders in Division 1 – who at the time had a guaranteed product – were offered a choice to voluntarily give up the guarantee in return for an investment strategy that enables more risky investments and thus higher expected returns. It is important to stress that this transition was an individual voluntary choice. I.e., each member (with a guarantee) chose individually whether to switch from a guaranteed product to an unguaranteed or not. Moreover, members were informed that future expected Solvency II capital requirements would most likely lower expected returns for individuals with a guaranteed product. Finally, the pension holders were to some extent informed about the consequences of the effect of longevity risk. However, this was presented as an unlikely event. If a pension holder gave up his/her guarantee, the pension holder received a compensation equal to 20% of the present value of his/her accumulated pension wealth.[3]

In Appendix A we investigate the transition by a probit model. We see that males have a slightly higher probability to give up the guarantee. Marital status has virtually no effect on the decision whereas being retired decreases the probability of relinquishing the guarantee. Compared to being young (between the ages of 20 and 29), the older you are, the more likely you are to remain in your current contract. Being above the age of 50 decreases the probability substantially. We observe regional differences as pension holders in Copenhagen are more likely to abolish their guarantee. We find strong significant effects that the higher the level of guarantee, the less likely you are to give it up. Finally, moderate levels of pension wealth increase the probability of giving up your guarantee. All in all, this case study shows that demographic characteristics influence the decision to switch from a guaranteed to an unguaranteed pension product. In particular, men, living in Copenhagen, with low guarantees, and moderate pension wealth, were more likely to give up their guarantee.

We find it particularly interesting that individuals with a high guarantee are more likely to keep on their guarantee, i.e. not switch to the unguaranteed pension product. There are two effects at play. On the one hand, as we show in the next section, the higher the guarantee, the lower is the fraction of pension wealth that can be invested freely, and thus earn the expected risk premium. I.e., the higher the guarantee, the lower the expected pension payments, ceteris paribus. On the other hand, about a third (see Table 2 in Appendix A) of the guarantees were close to the prevailing risk-free rate at the time the pension holders were offered the choice to switch product.[4] This could persuade some pension holders to stick to their relatively high guarantee.

[3]Source: The general information guide "Pensionsvalg 2007" that was attached to the letter and individual pension overview that the pension holders in Division 1 received.

[4]The average one to twenty year maturity euro spot rate ranges from 4.0% to 4.5% based on daily observations throughout the year 2007 (https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html).

Moreover, the implicit longevity hedge built into the guaranteed product was potentially very valuable, more so than the economic compensation offered to switch.

The risk transfer of both financial risk and longevity risk significantly adds to the complexity of the pension products. The guaranteed product offered certainty in both the accumulation and decumulation phase, but a lower expected return. The unguaranteed product offered higher expected return, but with multiple risk factors adding significantly to the uncertainty in both the accumulation and decumulation phase. The material supplied to the pension holders about potential future outcomes under the unguaranteed product was vague and they were never offered e.g. a lower and upper bound on their future expected pensions. This increased the required degree of financial literacy of the individual in order to optimally plan and prepare for retirement. Moreover, understanding the main mechanism behind these products also adds insights to the funds themselves as well as to the supervisory authorities. All these factors serve as a motivation for properly modeling the differences between guaranteed and unguaranteed pension products when accounting for both financial risk and longevity risk.

## 3 Guaranteed and unguaranteed pension products

Consider a pension holder who enters retirement with total wealth $W_t$ at time $t$ and needs to finance a life-long stream of annual pension payments. The pension payment at each horizon $h$ has to be financed from the initial total pension wealth $W_t$ where $0 \leq t \leq h$.[5] We divide the total pension wealth $W_t$ into separate buckets, where each bucket ($W_t(h)$) is used to finance the pension payment in one future year ($h$). The fraction of wealth that is reserved for each pension payment is determined by the control variable that is called the "assumed interest rate". This assumed interest rate (AIR) determines how much of the total wealth is allocated to the different buckets and thus determines the level of the expected pension payment in the different years ahead. Based on all information available, the expected pension payments are based on the expected return on the financial market and the mortality forecasts that are often summarized into what is known as the "mortality table". See Balter and Werker (2019) for more details on the modeling of variable annuities via these buckets (and the assumed interest rate). We follow the same approach in a Merton economy to which we add mortality and longevity risk. Moreover, we also explicitly add built-in guarantees.

We investigate the risk that the pension holders carry when they have the guaranteed or unguaranteed product. For the guaranteed product, deviations in expectations due to updates of mortality tables are paid by the insurer, leaving only the financial risk to the individual. Whereas in the unguaranteed product, both the investment risk and unforeseen changes in life

[5]The maximum attainable age is generally set at 110 after which survival probabilities are zero.

expectancies are carried by the pension holders. In both cases micro longevity risk is faced by the insurer and can be diversified for large enough pools. First, we describe the model setup by extending Balter and Werker (2019) to mortality and longevity risk, and we introduce the building blocks of both products. Thereafter we explicitly model the guaranteed and unguaranteed pension products.

## 3.1 Model setup and building blocks

Let $W_t(h)$ be the unique market-consistent value, in a complete and arbitrage-free market, of the wealth allocated at year $t$ for the pension payment in year $h$. The budget constraint implies

$$W_t = \sum_{h \geq t} W_t(h). \tag{1}$$

Thus, at time $t$ we consider an amount of wealth $W_t(h)$ that is available to finance the pension payment at time $h$, where $h \geq t$. The actual pension payment will, of course, depend on the investment strategy, the financial market returns, and the number of survivors.

For the financial market, we consider the standard Merton (1969)/Black and Scholes (1973) setting. This implies that there is a risk-free asset, also referred to as the money market or bank account, $M_t$ that pays a constant risk-free rate $r_f$. The dynamics of the money market is described by

$$\mathrm{d}M_t = r_f M_t \mathrm{d}t. \tag{2}$$

The dynamics of the risky asset $S_t$ are described by the geometric Brownian motion

$$\begin{aligned} \mathrm{d}S_t &= \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t & (3)\\ &= \left(r_f + \lambda\sigma\right) S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t, & (4) \end{aligned}$$

where $\mu$ stands for the expected return, $\sigma$ is the stock volatility, $\lambda = \left(\mu - r_f\right)/\sigma$ is the Sharpe ratio, and $Z$ is a standard Brownian motion on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

We now consider what happens in case each of the buckets is partly invested in the risky asset $S_t$ and the remainder on the risk-free bank account. That is, we invest each $W_t(h)$ in a continuously rebalanced portfolio with stock market exposure $w_t$. Standard calculations reveal that wealth $W_t(h)$, for the pension payment at time $h$, evolves as

$$\mathrm{d}W_t(h) = \left(r_f + w_t \lambda\sigma\right) W_t(h)\,\mathrm{d}t + w_t \sigma W_t(h)\,\mathrm{d}Z_t. \tag{5}$$

As a result, pension payments follow a log-normal distribution, and risk in the pension payment can be determined by calculating the volatility and quantiles of the payment.

For ease of exposition we assume that the risk exposure $w_t$ is constant[6], as this is optimal in a classical Merton setting under preferences with constant relative risk aversion (CRRA) (Karatzas and Shreve, 1998). Note that we do not optimize investment decisions within pension products as in Huang et al. (2008), nor do we investigate the demand for different products as in Chen et al. (2015). We focus instead on the differences between the two products and the effects due to changes in riskiness, especially longevity risk. The guaranteed pension product is suboptimal for a CRRA investor, which is inherent to constraint optimization problems. See Chen et al. (2018) for a comparison of different guarantees in a collective setting in the same Merton world. The comparison that pension holders, and in particular Danish pension holders, have to make is, however, more asymmetric due to the difference in who is liable for the longevity risk.

Biometric return, also called mortality credit or survivor credit (Donnelly et al., 2013), is the return due to the incorporation of mortality risk. The associated micro longevity risk – the risk that some individuals live longer or shorter than the average – is carried by the insurer and decreases rapidly with the size of the pool. Thus, without any risk, the positive biometric return is received by the pension holders. This is the return due to the release of the wealth of the pension holders who pass away. Including survival probabilities implies that an individual can allocate less wealth to each bucket because there is a probability that he might not survive. We incorporate this by defining the biometric return as

$$e^{(h-t)r^b_{x,t}(h)} = \frac{1}{{}_{h-t}p_{x,t}}, \tag{6}$$

where ${}_{h-t}p_{x,t}$ is the probability that a person aged $x$ in year $t$ survives at least to year $h$. Thus, in case the pension payment in year $h$ only needs to be made with probability ${}_{h-t}p_{x,t}$ independent of the evolution of financial markets, a fraction $1 - {}_{h-t}p_{x,t}$ less can be allocated to each bucket $W_t(h)$. Consequently, the development of the reservations is as follows: if a person aged $x$ allocates $W_t(h)$ to the bucket that delivers his pension in the year $h$, this accumulates with the expected return depending on the market exposure and the biometric return.

For an investment exposure of $w$, the expected pension payment at horizon $h$ is given in the following proposition. The risk is based on $z_\alpha$ denoting the corresponding quantile of the standard normal distribution.

**Proposition 1** (Expected return)**.** *When reserving and investing $W_t(h)$ at time $t$ to the pension payment for year $h$, its expectation and $\alpha$-quantile with respect to stock market risk including the*

[6] Without loss of generality the results hold for deterministic exposure by replacing $(h-t)\,w$ by $\int_t^h w_u \mathrm{d}u$.

*biometric return are given by*

$$\mathbb{E}_t\left[W_h(h)\right] = \frac{W_t(h)\, e^{(h-t)\left(r_f+w\lambda\sigma\right)}}{{}_{h-t}p_{x,t}} = W_t(h)\, e^{(h-t)\left(r_f+w\lambda\sigma+r^b_{x,t}(h)\right)}, \tag{7}$$

*and*

$$\begin{aligned} Q_t^{(\alpha)}\left(W_h(h)\right) &= \frac{W_t(h)\, e^{(h-t)\left(r_f+w\lambda\sigma-\frac{1}{2}w^2\sigma^2\right)+z_\alpha\sqrt{h-t}\,w\sigma}}{{}_{h-t}p_{x,t}} \\ &= W_t(h)\, e^{(h-t)\left(r_f+w\lambda\sigma+r^b_{x,t}(h)-\frac{1}{2}w^2\sigma^2\right)+z_\alpha\sqrt{h-t}\,w\sigma}. \end{aligned} \tag{8}$$

We assume that wealth is allocated over the different horizons such that the expected pension payments are constant[7] with respect to $h$, i.e. $\mathbb{E}_t\left[W_h(h)\right] = W_t(t)$. The assumed interest rate $a_t(h)$ determines the allocation of wealth over time, defined by

$$\frac{W_t(h)}{W_t} = \frac{e^{-(h-t)a_t(h)}}{\sum_{k\geq t} e^{-(k-t)a_t(k)}}, \tag{9}$$

and in particular $W_t(t) = \frac{W_t}{\sum_{k\geq t}\exp(-(k-t)a_t(k))}$. Without mortality, constant expected pension payments are obtained by an assumed interest rate equal to the risk-free rate for fixed annuities, while for variable annuities the assumed interest rate has to be equal to the expected return as given by Proposition 4.1 of Balter and Werker (2019). They analyze optimal withdrawals and associated utility losses. We fix the allocation to the one that leads to constant expected pension payments. Balter and Werker (2019) found that the associated utility loss is small. We add mortality to the variable annuities[8] and, in particular, we investigate the impact of realized updates of longevity forecasts, i.e. mortality tables. The following building block is a variable annuity that includes survival probabilities, which is an important aspect of the unguaranteed pension product.

**Proposition 2** (Variable annuity)**.** *A life-long variable annuity with an investment exposure of $w$ and a prevailing risk-free rate of $r$, pays out the following constant expected pension payments*

$$\mathbb{E}_t\left[W_h(h)\right] = W_t^{r,w}(t), \tag{10}$$

[7] This resembles the daily practice of pension providers when they communicate the expected pension payments to be received by the pension holder. In some countries, pension holders are protected against consuming too much pension in the early years of retirement, which is accomplished by prohibiting AIRs above the one that ensures constant expectations.

[8] Variable annuities are products in which the pension payments are not fixed. The riskiness and thus variability is usually driven by exposure to the financial market. Fixed annuities are the opposite as the future pension payments are completely certain at the moment of conversion. We refer to the variable annuities as an *unguaranteed product* that incorporates an additional layer of variability due to mortality risk.

*for all $h > t$, equal to*

$$W_t^{r,w}(t) = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)(r+w\lambda\sigma)}\,{}_{k-t}p_{x,t}} = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)\left(r+w\lambda\sigma+r^b_{x,t}(k)\right)}}. \tag{11}$$

*Proof.* By (9) we can rewrite $W_t(h)$ as

$$W_t(h) = W_t(t)e^{-(h-t)a_t(h)}. \tag{12}$$

Plugging this into (7) gives

$$\mathbb{E}_t\left[W_h(h)\right] = \frac{W_t(t)e^{-(h-t)a_t(h)}}{{}_{h-t}p_{x,t}}e^{(h-t)(r+w\lambda\sigma)} \tag{13}$$

$$= W_t(t)e^{(h-t)(r+w\lambda\sigma-a_t(h))-\ln\left({}_{h-t}p_{x,t}\right)}. \tag{14}$$

Let the assumed interest rate be

$$a_t(h) = r+w\lambda\sigma-\frac{1}{h-t}\ln\left({}_{h-t}p_{x,t}\right) = r+w\lambda\sigma+r^b_{x,t}(h), \tag{15}$$

then we obtain constant expectations as given in (11). □

Including the fact that the life time is uncertain within the variable annuity makes the assumed interest rate horizon-, time- and age-dependent in order to obtain constant expectations.

In constructing the difference in guaranteed and unguaranteed products, we make use of this variable annuity which is equivalent to an unguaranteed product if we ignore longevity risk.

Since fixed annuities are an important building block of guarantees, we can straightforwardly derive the yearly pension payments from Proposition 2 as given in the following lemma. A fixed annuity is embedded as a special case, i.e. when $w = 0$, of the variable annuity.

**Lemma 1** (Fixed annuity)**.** *A life-long fixed annuity with a prevailing risk-free rate of $r$, pays out the following constant pension payments*

$$W_h(h) = W_t^{r,0}(t), \tag{16}$$

*for all $h > t$ equal to*

$$W_t^{r,0}(t) = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)r}\,{}_{k-t}p_{x,t}} = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)\left(r+r^b_{x,t}(k)\right)}}. \tag{17}$$

## 3.2 Guaranteed pension product

Before the transition, most Danish pension holders owned a guaranteed pension product. At the moment of retirement, the minimum yearly constant pension payment is calculated based on the promised guaranteed rate $r_g < r_f$ that was specified at the moment when the individual entered the pension fund. Since the future pension payments depend on the survival probabilities/mortality table and no micro or macro longevity risk is carried by the pension holder, the survival expectations prevailing at the moment of retirement determine the minimum level that is guaranteed. This level is obtained by plugging in $r_g$ for the risk-free rate in Lemma 1. We denote this guaranteed pension payment by $W_t(t)^{r_g,0}$.

To ensure this floor $W_t(t)^{r_g,0}$ for each horizon $h$, a specific fraction of $W_t(h)$ has to be invested in the money market account, and the remainder can be invested in a diversified risky return portfolio (possibly partly invested without risk too). No rebalancing between these two subaccounts is allowed. The part of wealth that is allocated to the fixed annuity (by investing in the money market (2)) ensures the floor, while the remainder can freely be invested in the variable annuity (5). This division between accounts is known as splitting the investment into a hedge demand and a speculative demand. The speculative demand should generally still be partially invested in bonds as they offer a risk-return tradeoff and thus lead to diversification benefits. However, theoretically continuous rebalancing can imply that the account value of the risky process goes to zero. Therefore, we explicitly disentangle the two subaccounts. Moreover, the hedge that ensures the floor is model-free if each bucket is invested in the bond with maturities equal to the horizon of the buckets.[9] The exposure division between the fixed and variable annuities is similar to what is known as the constant proportion portfolio insurance (CPPI) investment strategy (Black and Perold, 1992). The part that is invested in the variable annuity, i.e. the part that goes to the risky subaccount, is known as the "cushion".

Let $v_t$ be the fraction of wealth that is invested in the risk-free asset, which we call the hedge demand.[10] Then $1 - v_t$ is the remainder that is used to buy the variable annuity, which we call the speculative demand (the "cushion").

**Proposition 3** (Guaranteed pension product)**.** *If $v_t W_t$ is invested in the money market, and on a separate account $(1 - v_t)W_t$ is invested in a rebalanced portfolio, then the pension payments are at*

[9] The division of wealth into buckets naturally embeds a model-free investigation. For our analysis, we assume a Black-Scholes economy, but the assumed interest rate construction allows for general financial market specifications. It is the expected return that is the main driver of the comparison. E.g., investigating interest rate risk by factor models does not change the characteristics of the guaranteed product studied in this paper. Prevailing market prices at time $t$ of bonds with maturity $h$ determine the term structure of interest rates. The guarantee can be locked in by allocating a proportion of each bucket to the bond with the maturity equal to the horizon of the bucket.

[10] In the Merton/Black-Scholes economy it is assumed that there is a money market with a constant risk-free rate, effectively simplifying $v_t = v$. To highlight that the division does depend on the available mortality tables at time $t$, we however use the notation $v_t$ throughout.

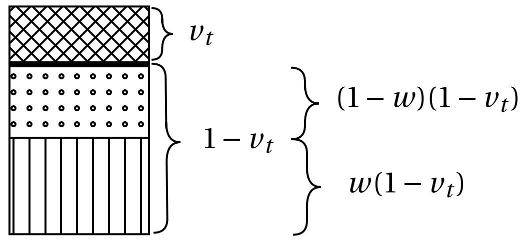



Figure 1: Exposures. The sum of the weights add up to one. The crossed section represents the fraction of the total wealth that is invested in the money market to ensure the guarantee, and the dotted and vertical line-sections represent the part that is invested as a variable annuity of which the vertical line part is invested in the risky asset while the dotted section is invested in the money market. The thick black line indicates that there is no continuous rebalancing between those sections.

*least equal to the guaranteed payments based on $r_g$, when*

$$v_t = \frac{W_t^{r_g,0}(t)}{W_t^{r_f,0}(t)} = \frac{\sum_{k\geq t} e^{-(k-t)r_f} {}_{k-t}p_{x,t}}{\sum_{k\geq t} e^{-(k-t)r_g} {}_{k-t}p_{x,t}} = \frac{\sum_{k\geq t} e^{-(k-t)\left(r_f + r^b_{x,t}(k)\right)}}{\sum_{k\geq t} e^{-(k-t)\left(r_g + r^b_{x,t}(k)\right)}}. \tag{18}$$

*Proof.* If $W_t$ is invested in the money market where the prevailing risk-free rate is $r = r_f$, then the yearly pension payments are all equal to (17)

$$W_t^{r_f,0}(t) = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)r_f} {}_{k-t}p_{x,t}}. \tag{19}$$

Thus if $v_t W_t$ is invested in the money market, then the yearly pension payments are equal to the guaranteed payments since

$$v_t W_t^{r_f,0}(t) = \frac{\sum_{k\geq t} e^{-(k-t)r_f} {}_{k-t}p_{x,t}}{\sum_{k\geq t} e^{-(k-t)r_g} {}_{k-t}p_{x,t}} W_t^{r_f,0}(t) = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)r_g} {}_{k-t}p_{x,t}} = W_t^{r_g,0}. \tag{20}$$

□

Figure 1 shows how the hedge and speculative demand are split. Since $r_g < r_f$, it follows that $v_t < 1$. Therefore some positive part of the total wealth is left over and can serve as a sort of bonus that is paid out on top of the guarantee. Of this part $(1 - v_t)\, W_t$, a fraction $w$ is invested in the risky asset, and the remainder on the bank account. Note that rebalancing does take place to ensure this division $w$ to hold through time, though that is not the case with respect to the division $v_t$. Moreover, the exposure towards risk is reduced because less than the total wealth is invested with an exposure of $w$. Therefore the effective risk exposure of the guaranteed product equals $(1 - v_t)\, w$.

The remaining wealth that is left to invest freely after isolating the part that ensures the guarantee, can freely be distributed over the horizons and invested in equity and thus be individual specific. In Balter and Werker (2019) the optimal assumed interest rate is derived for an investor exhibiting CRRA preferences. For the figures throughout, we impose that the pension payments that are added on top of the guarantee are constant in expectation.[11]

Only guarantees that satisfy the assumption that $r_g < r_f$ are feasible and sustainable. A guaranteed rate above the risk-free rate would imply arbitrage. From an empirical viewpoint, we see that the drop in risk-free rates has caused previously promised guarantees (Grosen and Jørgensen, 2002) to become unsustainable. This partly explains the need for the transition to unguaranteed products that we see in many pension systems and which is the focus of our paper.

For all figures we assume the following parameter settings: we assume that the risk-free rate is $r_f = 2\%$, the guaranteed return is $r_g = 1\%$, the expected return on the risky stock is $\mu = 6\%$, and the volatility $\sigma = 20\%$. For empirical validation of our choice of the equity premiums, see Dimson et al. (2008) and Mehra and Prescott (1985), among others. The mortality table forecasts are based on Danish data for males from 1978 to 2007 unless stated differently. And the initial pension wealth is normalized to $W_t = €100,000$.

**Example 1** (Guaranteed pension product)**.** *In Figure 2, we show the* $1\%$ *guarantee by the green dash-dotted line, the expected value of the pension payments by the solid green line, and the* $5\%$ *and* $95\%$ *confidence intervals by the green dashed and dotted lines, respectively. The Danish survival expectations are based on the Lee-Carter forecast with data from 1978 to 2007. Men aged 67 in 2007 had a remaining life expectancy of 15 years and 4 months. At a* $1\%$ *fixed rate, the yearly payments would be* $€7,559$ *based on the Lee-Carter forecast in 2008. This is the amount that could be paid with certainty if the risk-free rate was* $1\%$*. However, the risk-free rate is* $r_f = 2\%$*. Thus, the fixed annuity in the market would pay out* $€8,200$*. In order to secure the ability of paying the guarantee, the fund has to invest* $v_t = \frac{€7{,}559}{€8{,}200} = 92.18\%$ *in the fixed annuity. The rest constitutes the bonus that can be freely invested with risk exposure* $w$*. We set this* $w$ *to* $100\%$ *such that the net risk exposure of* $7.82\%$ *causes a visible add-on effect.*

### 3.3 Unguaranteed pension product

Because of the decrease in interest rates and increase in survival rates, the guaranteed pension product decreases the sustainability of insurance companies and pension funds. The industry consequently created unguaranteed pension products in which there is no guaranteed return

[11] And thus equal to $(1-v_t)W_t^{r_f,w}(t) = W_t \frac{\sum_{k\geq t}\left(e^{-(k-t)r_f} - e^{-(k-t)r_g}\right)_{k-t}p_{x,t}}{\left(\sum_{k\geq t} e^{-(k-t)r_g}\,_{k-t}p_{x,t}\right)\left(\sum_{k\geq t} e^{-(k-t)\left(r_f+w\lambda\sigma\right)}\,_{k-t}p_{x,t}\right)}$.

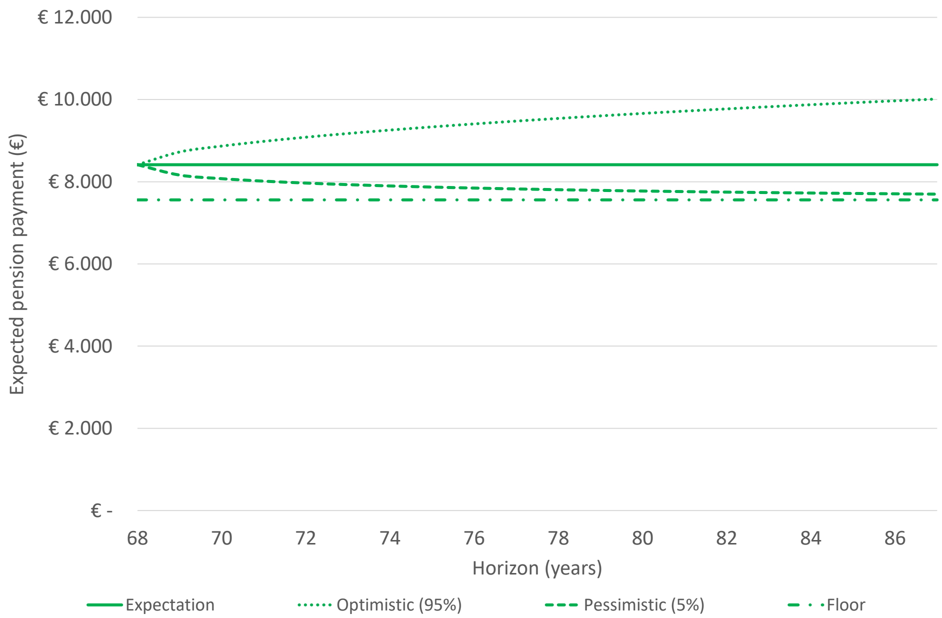



Figure 2: Guaranteed pension product. The expected pension payment of a guaranteed pension product with a risk exposure $w = 100\%$ in the speculative part is shown by the solid line, and the optimistic and pessimistic financial quantiles as well as the minimum guaranteed pension payments are shown by the dotted and dashed lines, respectively.

anymore, nor are changes in mortality forecasts subsidized by the fund. At first, one might myopically believe that this leads to lower expected pension payments. However, if only feasible products are allowed, then being exempted from building in a floor at some predetermined level below the risk-free rate leaves more freedom to invest in a diversified portfolio. This would increase expected pension payments. At the same time, the possibility of changes in these expectations occurs after mortality table updates. However, in *expectation*, mortality changes do not occur because the current forecasts include all information available. What does change is the riskiness. More exposure to risky assets increases the risk, but this is compensated by the higher expected financial return. But the increase in risk due to unforeseen changes in mortality forecasts, i.e. macro longevity risk, is now carried by the pension holder. As macro longevity risk refers to the risk of unexpected deviations from the mortality forecasts (Hari et al., 2008), it follows that in expectation these deviations are zero, but its quantiles measure some level of riskiness. Richards et al. (2014) investigate micro, macro, and model risk of mortality rates. We do not model the likelihood of such unforeseen macro shocks here. Richards et al. (2014) simulate new data based on old calibrations to measure trend risk. In contrast, we focus on mortality table updates based on the acquisition of data on realized deaths. Micro longevity risk is, on the other hand, not changing compared to the guaranteed product, and thus it is still carried by the insurer whereas the positive biometric return is also now received by the pension holder.

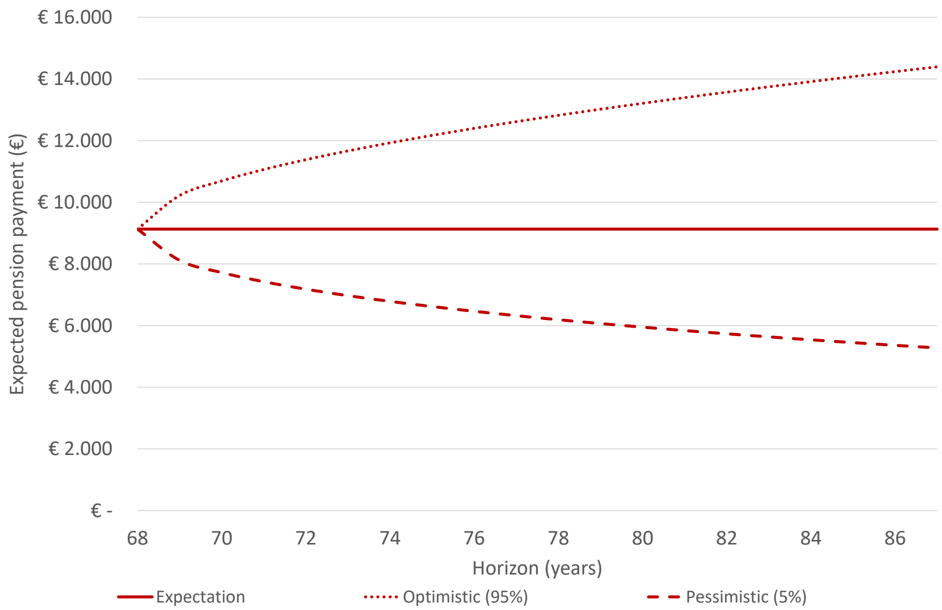



Figure 3: Variable annuity. Unguaranteed pension product without longevity risk. The expected pension payment of a variable annuity with a risk exposure of $w = 35\%$ is shown by the solid line, and the optimistic and pessimistic financial quantiles by the dotted and dashed lines, respectively.

Since in expectations there will not be any mortality table update, the unguaranteed pension product equals the variable annuity with survival rates as given by Proposition 2.

**Example 2** (Variable annuity)**.** *In Figure 3, we show in red the decumulation phase of the unguaranteed product without macro longevity risk. Hence, this coincides with the variable annuity product. The solid line reflects the expected pension payments without unforeseen deviations, the dotted line the* 95% *quantile for financial risk, and the dashed line the* 5% *quantile for financial risk, all based on the pure variable annuity described by Propositions 1 and 2. The unguaranteed product has a (net) risk exposure of* 35%.

However, when allowing for unforeseen deviations in survival forecasts, the inclusion of this longevity risk causes unanticipated jumps in the expected pension payments. Imagine that the mortality forecasts are updated at some time $t$, and thus the new mortality table consists of expected one-year survival rates for each age $x$ in the years from $t$ onwards. We denote updated survival rates by ${}_{h-t}\tilde{p}_{x,t}$. An update of mortality forecasts to ${}_{h-t}\tilde{p}_{x,t}$ would simply mean that the expected pension payments change to

$$\mathbb{E}_t[W_h(h)] \quad = \quad W_t^{r_f,w}(t) = \frac{W_t}{\sum_{k\geq t} e^{-(k-t)(r_f+w\lambda\sigma)}\,{}_{k-t}\tilde{p}_{x,t}}. \tag{21}$$

The ratio of the expected pension payment under the old rates divided by the equation above is similar to the changed expectation adjustment (CEA) factor in group self annuitization products (GSA) as given by Piggott et al. (2005). We model an unforeseen deviation rate of ${}_{k-t}d_{x,t}$ as follows

$$ {}_{k-t}d_{x,t} = \ln\left(\frac{{}_{k-t}\tilde{p}_{x,t}}{{}_{k-t}p_{x,t}}\right), \tag{22} $$

for an individual aged $x$ in year $t$ surviving to at least age $x+k-t$ in year $k$. Thus, the average deviation is the change in the biometric return

$$ \frac{{}_{k-t}d_{x,t}}{k-t} = r^b_{x,t}(k) - \tilde{r}^b_{x,t}(k), \tag{23} $$

where $\tilde{r}^b_{x,t}(k)$ represent the biometric return after the update. If we are expected to live longer under the new survival forecasts, then the deviation rate is positive. Note, the fact that the life expectancies are rising is already included in the current mortality table and pension. It is however the unforeseen changes in these forecasts, i.e. the possibility that we are expected to live even longer than the already forecasted increase, that we model here.

**Proposition 4** (Unguaranteed pension product)**.** *An update of survival forecasts from ${}_{k-t}p_{x,t}$ to ${}_{k-t}\tilde{p}_{x,t}$ for all $k \geq t$, $x$ and $t$ changes the expectations and quantiles of the pension payments by a shock equal to*

$$ \frac{\sum_{k\geq t} e^{-(k-t)(r_f+w\lambda\sigma)}\,{}_{k-t}p_{x,t}}{\sum_{k\geq t} e^{-(k-t)(r_f+w\lambda\sigma)}\,{}_{k-t}\tilde{p}_{x,t}} = \frac{\sum_{k\geq t} e^{-(k-t)(r_f+w\lambda\sigma)}\,{}_{k-t}p_{x,t}}{\sum_{k\geq t} e^{-(k-t)\left(r_f+w\lambda\sigma-\frac{{}_{k-t}d_{x,t}}{k-t}\right)}\,{}_{k-t}p_{x,t}} = \frac{\sum_{k\geq t} e^{-(k-t)\left(r_f+w\lambda\sigma+r^b_{x,t}(k)\right)}}{\sum_{k\geq t} e^{-(k-t)\left(r_f+w\lambda\sigma+\tilde{r}^b_{x,t}(k)\right)}}. \tag{24} $$

This ratio is smaller than one and thus reduces the payments if ${}_{k-t}\tilde{p}_{x,t} > {}_{k-t}p_{x,t}$, i.e. if we are expected to live longer. For an empirical investigation of the quantity of unforeseen longevity deviations[12], see Appendix B. We investigate changes in forecasted survival rates based on a rolling window of death rates. We forecast the survival rates for several countries using the Lee and Carter (1992) model. Using a rolling window of 30 years of observations, we analyze three updates each after 3 years. We compare (1) forecasts starting in 2011 based on data from 1978 to 2007 with forecasts based on data from 1981 to 2010, (2) we compare the latter (the forecasts based on data from 1981 to 2010) from 2014 onwards with the forecasts based on data from 1984 to 2013 and, (3) lastly we compare the latter (the forecasts based on data from 1984 to 2013) from 2017 onwards with the forecasts based on data from 1987 to 2016. In Figure 6 in Appendix B,

[12]Since $h$-year survival rates are obtained by multiplication of one-year survival rates, only one-year survival rates and their improvements are needed, where ${}_{h-t}d_{x,t} = {}_1d_{x,t} + {}_1d_{x+1,t+1} + \ldots + {}_1d_{x+h-1,t+h-1}$.

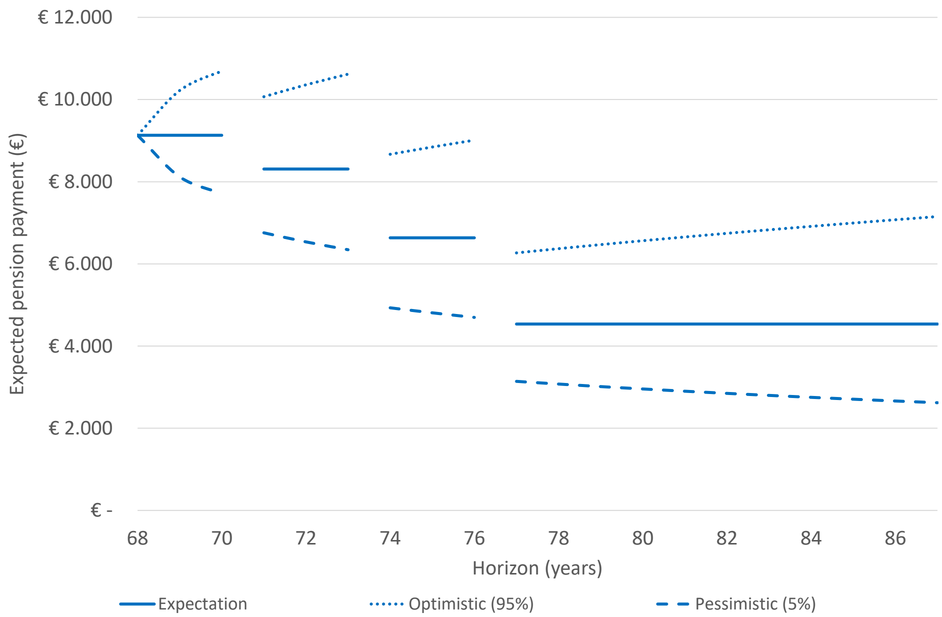
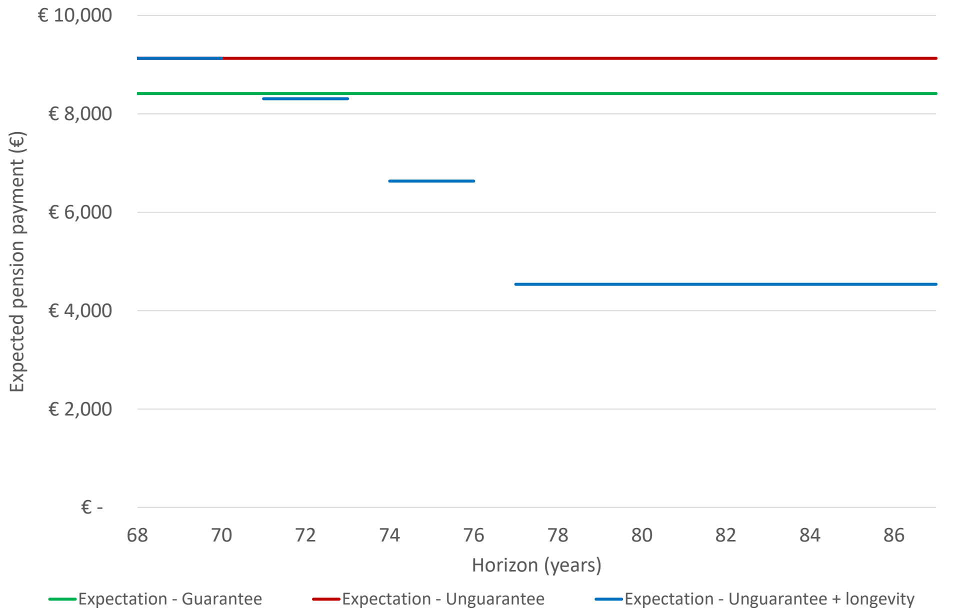



Figure 4: Unguaranteed pension product. The expected pension payment of an unguaranteed pension product with a risk exposure $w = 100\%$ in the speculative part is shown by the solid line, and the optimistic and pessimistic financial quantiles by the dotted and dashed lines, respectively. The expected pension payments for the first three years of the unguaranteed product are based on the mortality table forecasts with data from 1978 to 2007, while the three updates are based on a rolling window of 3 years ahead, each based on Danish data for males.

we show the deviations over time averaged over all individuals between ages 67 and 100, and in Figure 7 we show the deviations per age averaged over the forecast horizon up to 2100.

**Example 3** (Unguaranteed pension product)**.** *Building upon Example 2 we now incorporate the effect that new longevity forecasts have on the expected pension payments. Since living longer than anticipated is independent of financial market returns, unforeseen longevity deviation can occur at any quantile level $\alpha$ of the stock returns. In Figure 4, we show (in blue) the impact of longevity after an update of survival rates every* 3 *years in the optimistic, average, and pessimistic financial scenarios. The individuals were asked to switch in 2007, and thus we depict here the expected pension payments based on the mortality forecasts including the historic data for Danish males up to* 2007*. We update the mortality table every* 3 *years. After a* 3 *year update of the survival rates, an unforeseen deviation leads to a change in the expected pension payments from that point onwards until the next update.*

*Table 1 shows the remaining life expectancy for an individual who was 67 years old in 2007, based on data up to 2007, based on data up to 2010, based on data up to 2013, and based on data up to 2016, each conditional on the individual to be alive until the year 2008, 2011, 2014, and 2017 while reaching ages 67, 70, 73, or 76, respectively. Thus a Danish male who is 67 in 2007 is expected to live another 15 years and 4 months based on the Lee-Carter forecasts with data from*



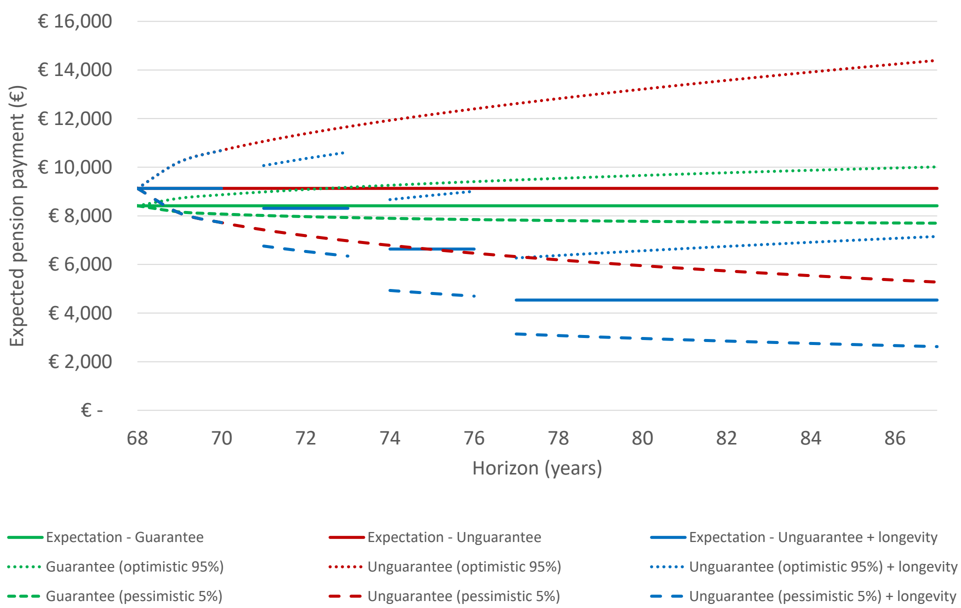




Figure 5: Guaranteed versus unguaranteed pension product.

*1978 to 2007. Conditional on that person to survive to the age of 70 in the year 2010, he is expected to live 12 years and 5 months based on the forecasts with data from 1978 to 2007. However, if the updates forecasts are used, i.e. based on data from 1981 to 2010, the Danish male of 70 year in 2010 is expected to live 13 years and 8 months. The unforeseen deviations in longevity lead thus to an unexpected increase of 1 year and 3 months in remaining life expectancy. The table is read analogously for the other two updates. We observe that each update leads to an increase in forecasted survival rates and thus remaining life expectancy.*

*If longevity increases unexpectedly, average pension payments decrease, as do the associated optimistic and pessimistic financial scenarios. If there is a positive unforeseen improvement in longevity at the next longevity update, expected pension might be reduced even further. Unforeseen longevity shocks can happen after each update of survival rate expectations. Interestingly, the negative effect of unforeseen longevity shocks can dominate the positive effect of the unrestricted risk exposure, which thus increases the likelihood that pension income is lower in the unguaranteed product than in the guaranteed product.*

| data\conditional | 2008 | 2011 | 2014 | 2017 |
|---|---|---|---|---|
| 2007 | 15y4m | 12y5m | 9y8m | 7y3m |
| 2010 | | 13y8m | 10y8m | 8y2m |
| 2013 | | | 12y3m | 9y4m |
| 2016 | | | | 10y7m |

Table 1: Unforeseen increases in life expectancy

When combining Figures 2, 3, and 4 as done in Figure 5, we note that the expected pension payments are higher in the unguaranteed product during the early years of retirement. If unforeseen increases in longevity do not occur, expected pension payments continue to be higher in the unguaranteed product. Unexpected increases in longevity can reduce pension payments in the unguaranteed product, however. Multiple and large increases in mortality tables can lower expected pension payments in the unguaranteed product so much that they become lower than expected pension payments in the guaranteed product. In our numerical example in Figure 5, pension payments are lower already after the first update in mortality tables.

The analyses in this section show that there is a built-in longevity hedge in the guaranteed pension product that protects the pension holder from unforeseen longevity increases, but at the same time this guarantee restricts the risk exposure and lowers the expected returns, resulting in lower expected pension payments during retirement. In the unguaranteed product, unforeseen longevity shocks can have large impacts on future pension payments. Potentially, they can even lead to lower pension payments than under the guaranteed product. In our numerical example, the negative effect from longevity increases will not even be outweighed in the optimistic financial scenario, as illustrated by the dotted blue lines in Figure 5.

## 4 Conclusion

This paper models expected pension payments in guaranteed and unguaranteed pension products. We focus in particular on the consequences of unforeseen increases in longevity. We find that the higher the guaranteed rate of return, the larger the fraction of pension savings that needs to be invested in the risk-free asset. This causes – all things being equal – expected pension payments to be lower when guarantees are higher. However, as there is no hedge against unforeseen increases in longevity in unguaranteed pension products, expected pension payments in unguaranteed pension products will be negatively affected by macro longevity improvements. Expected pensions might even be lower in unguaranteed products when there are frequent and large improvements in longevity.

A shift from guaranteed pension products to unguaranteed products typically lowers the risk of insolvency of the pension provider, but the transfer of financial risk and longevity risk significantly adds to the complexity of the pension product. This raises the question of how to correctly inform pension holders about the multiple risk factors they face when they move away from a guaranteed product with a relatively low level of risk to an unguaranteed product. When informing pension holders about risk, it is important to show both the quantiles of financial risks and especially to inform explicitly about uncertainty with respect to longevity. As shown in Figure 5, there seems to be good reasons to include information about longevity risk.

Furthermore, as the longevity risk is transferred to the pension holder in the unguaranteed product, a natural demand for hedging this risk arises. This calls for the creation of a liquid and transparent market for individualistic longevity products.

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# A Empirical study

We have information about various personal characteristics as well as financial information. Table 2 contains descriptive statistics and reports the mean and the standard errors of the explanatory variables. The dependent variable "Election Outcome" is a dummy variable that takes the value 1 if a pension holder in Division 1 voluntarily chose to opt out of the current guaranteed contract. We see from Table 2 that 18% made that choice. The male dummy variable for gender shows that more than half (56%) of the policy holders are male which corresponds well with the higher labor force participation rates from men. The majority of pension holders are married (65%), but only 6% of the pension holders are currently in retirement. Age is divided into seven age categories. We clearly see that the pension fund is relatively young as almost 70% of the pension holders are under the age of 50. The level of education corresponds for almost all members to a university degree at the Bachelor level or higher. Education is divided into five field categories: economics, political science, law, business economics, and others. Around 29% of the pension holders have a degree in economics or business economics, 20% a degree in political science, and 33% in law. Regarding the geographical location, we distinguish between nine different regions in Denmark. Almost 60% of the members live in Copenhagen and Greater Copenhagen, 11% in Central Jutland (including the second largest city, Aarhus), and the remaining 30% are distributed around the country. In terms of the levels of the guaranteed return, 30% of the members in Division 1 had the highest level of 4.25%, 26% had a 3.7% level of guarantee, 12% a 3% level of guarantee, and 32% a 2% level of guarantee. Finally, we have information about the size of pension wealth. This is divided into five different categories. From Table 2, we see that almost 50% of the policy holders have pension wealth below DKK 400,000 (approximately €55,000), whereas only 8% have pension wealth above DKK 2,000,000 (approximately €270,000).

Let $Y$ be the "Election Outcome" for an individual. This is a binary variable equal to one if the individual has switched to the unguarantee and zero if the individual kept the guaranteed product. The probability of an individual switching is described by the probit model

$$P(Y = 1|X) = \Phi\left(\beta_0 + \beta X\right)$$

where $\Phi$ is the cumulative distribution function of the standard normal distribution, $X$ is a vector of explanatory variables, $\beta$ are the corresponding coefficients, and $\beta_0$ is the intercept. Coefficient estimates are obtained by maximizing the likelihood function, and standard errors are based on

| Variable | Mean | Std. Dev. |
|---|---|---|
| Election Outcome | 17.9% | 38.3% |
| **General** | | |
| Male | 56.2% | 49.6% |
| Married | 65.4% | 47.6% |
| Retired | 5.7% | 23.2% |
| **Age** | | |
| Between 20 and 29 | 3.7% | 18.9% |
| Between 30 and 39 | 38.4% | 48.6% |
| Between 40 and 49 | 27.5% | 44.7% |
| Between 50 and 59 | 18.1% | 38.5% |
| Between 60 and 65 | 8.3% | 27.7% |
| Between 66 and 69 | 2.0% | 14.0% |
| Between 70 and 100 | 2.0% | 14.0% |
| **Education** | | |
| Economics | 15.7% | 36.4% |
| Political Science | 19.3% | 39.5% |
| Law | 33.8% | 47.3% |
| Business Economics | 12.9% | 33.5% |
| Other Education | 18.3% | 38.6% |
| **Region** | | |
| Copenhagen | 50.6% | 50.0% |
| Greater Copenhagen | 9.0% | 28.6% |
| Zealand & Falster | 8.5% | 27.9% |
| Funen & Islands | 4.0% | 19.6% |
| South Jutland | 3.5% | 18.3% |
| West Jutland | 3.0% | 17.0% |
| Central Jutland | 11.2% | 31.6% |
| North Jutland | 4.4% | 20.6% |
| Other Region | 5.8% | 23.5% |
| **Level of Guarantee** | | |
| 2% | 31.9% | 46.6% |
| 3% | 11.8% | 32.3% |
| 3.70% | 25.8% | 43.8% |
| 4.25% | 30.5% | 46.0% |
| **Level of Pension Wealth** | | |
| Less than 100,000 | 16.4% | 37.0% |
| Between 100,001 and 400,000 | 32.1% | 46.7% |
| Between 400,001 and 800,000 | 22.1% | 41.5% |
| Between 800,001 and 2,000,000 | 21.2% | 40.9% |
| Greater than 2,000,000 | 8.1% | 27.3% |

Table 2: Summary statistics.

| | | | | |
|---|---|---|---|---|
| Number of obs | 31,497 | | | |
| Pseudo R2 | 16.96% | | | |
| | dy/dx | Std.Err. | z | P>\|z\| |
| **General** | | | | |
| Male | 2.72% | 0.41% | 6.58 | 0.0% |
| Married | -0.87% | 0.43% | -2.04 | 4.2% |
| Retired | -11.35% | 4.14% | -2.74 | 0.0% |
| **Age** | | | | |
| Between 30 and 39 | -6.05% | 0.91% | -6.62 | 0.0% |
| Between 40 and 49 | -15.31% | 1.02% | -14.99 | 0.0% |
| Between 50 and 59 | -30.64% | 1.33% | -23.01 | 0.0% |
| Between 60 and 65 | -39.21% | 2.26% | -17.38 | 0.0% |
| Between 66 and 69 | -33.26% | 4.07% | -8.17 | 0.0% |
| Between 70 and 100 | -38.50% | 7.82% | -4.92 | 0.0% |
| **Education** | | | | |
| Political Science | -1.40% | 0.64% | -2.19 | 2.9% |
| Law | -4.85% | 0.63% | -7.74 | 0.0% |
| Business Economics | -5.79% | 0.72% | -8.07 | 0.0% |
| Other Education | -4.75% | 0.71% | -6.7 | 0.0% |
| **Region** | | | | |
| Greater Copenhagen | -1.73% | 0.76% | -2.29 | 2.2% |
| Zealand & Falster | -3.34% | 0.78% | -4.26 | 0.0% |
| Funen & Islands | -1.29% | 1.06% | -1.22 | 22.3% |
| South Jutland | -3.29% | 1.19% | -2.76 | 0.6% |
| West Jutland | -4.41% | 1.25% | -3.53 | 0.0% |
| Central Jutland | 0.17% | 0.65% | 0.26 | 79.1% |
| North Jutland | -3.80% | 1.01% | -3.75 | 0.0% |
| Other Region | -6.33% | 0.92% | -6.85 | 0.0% |
| **Level of Guarantee** | | | | |
| 3% | -5.49% | 0.64% | -8.61 | 0.0% |
| 3.70% | -8.13% | 0.68% | -11.91 | 0.0% |
| 4.25% | -12.40% | 1.15% | -10.82 | 0.0% |
| **Level of Pension Wealth** | | | | |
| Between 100,000 and 400,000 | 6.27% | 0.58% | 10.85 | 0.0% |
| Between 400,001 and 800,000 | 8.35% | 0.76% | 11.05 | 0.0% |
| Between 800,001 and 2,000,000 | 6.28% | 1.05% | 5.99 | 0.0% |
| Greater than 2,000,000 | 0.69% | 2.19% | 0.32 | 75.2% |

Table 3: Results from probit estimation.

computation of the information matrix. The average marginal effects are given by

$$\frac{\partial P\left(Y=1|X\right)}{\partial X_i}=\phi\left(\beta_0+\beta X\right)\beta_i$$

where $\phi$ is the standard normal density $\phi\left(\beta_0+\beta X\right)=(2\pi)^{-1/2}\exp\left(-\left(\beta_0+\beta X\right)^2/2\right)$ that scales $\beta_i$. The reference group consists of single females, aged 20 and 29, not retired, educated in economics, living in Copenhagen, having a pension wealth of less than 100,000 DKK (approximately €13,500), and having a guarantee of 2%. Table 3 presents marginal effects from the probit estimation exploring the relationship between the election outcome and the set of independent variables listed above.

All except three variables (pension wealth greater than 2,000,000 DKK (approximately €270,000), Central Jutland, and Funen & Islands) are individually significant at a 5% level. Further, the likelihood ratio test is jointly significant with a $\chi^2(28)$ of 5023.68. We see that males have a slightly higher probability (2.7%) to give up the guarantee. Marital status has virtually no effect on the decision, whereas being retired decreases the probability of relinquishing the guarantee by 11.3%. Compared to being young (between the ages of 20 and 29), the older you are, the more likely you are to remain in your current contract. Being above the age of 50 decreases the probability between 30-39%. We observe regional differences as pension holders in Copenhagen (used as the reference group) are more likely to abolish their guarantee. We find strong significant effects that the higher the level of guarantee, the less likely you are to give it up. Compared to a 2% level of guarantee an individual with a 4.25% level of guarantee is 12.4% less likely to relinquish it. Finally, higher levels of pension wealth increase the probability of giving up your guarantee by 6-8%. All in all, this case study shows that demographic characteristics influence the decision to switch from a guaranteed to an unguaranteed pension product. In particular, men, living in Copenhagen, with low guarantees and moderate pension wealth, are more likely to give up their guarantee.

# B Unforeseen longevity deviation

To quantify unforeseen longevity deviations $d_{x,t}$ we investigate changes in forecasted survival rates based on a rolling window of death rates. For several countries, we forecast the survival rates via the Lee-Carter model. The historic observations determine the central death rate, i.e. force of mortality, $m_{x,t}$ defined by $m_{x,t}=\frac{D_{x,t}}{E_{x,t}}$, where $E_{x,t}$ is the exposure and $D_{x,t}$ the number of deaths. Expected future mortality rates are obtained using theLee and Carter (1992) model

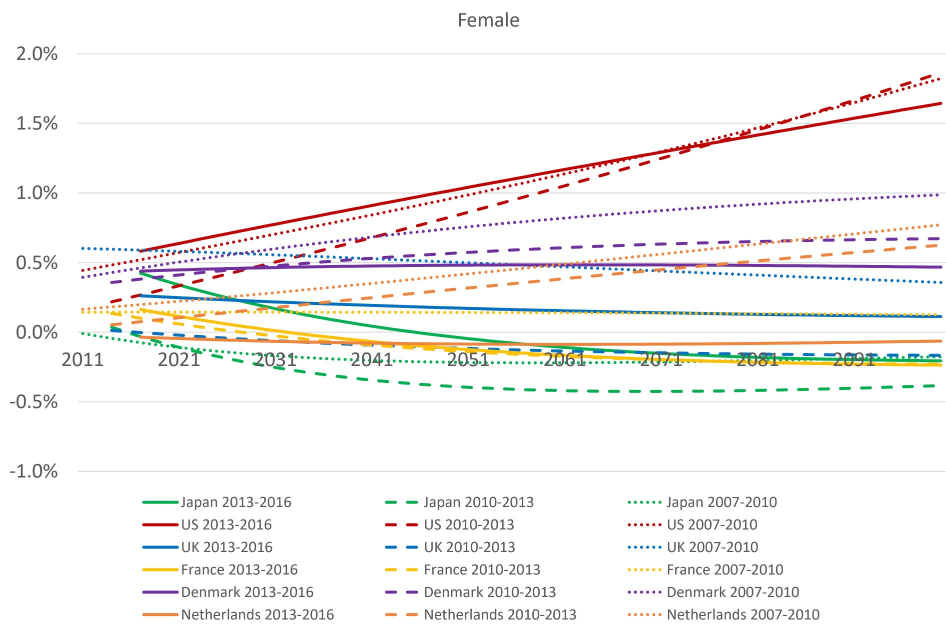
$$\ln\left(m_{x,t}\right) = a_x+b_xk_t+\epsilon_{x,t}, \tag{25}$$

where $\epsilon_{x,t}\ N\left(0,\sigma_\epsilon^2\right)$ and with the restrictions that $\sum_x b_x = 1$ and $\sum_t k_t = 0$. We estimate $a_x$ by averaging log-rates over time, and $b_x$ and $k_t$ are estimated by the singular value decomposition. Subsequently, we obtain the future survival rates by forecasting the time series of $k$, for which we assume a simple random walk with drift $k_t = k_{t-1} + \theta + \xi_t$. The $h-t$-year survival rate ${}_{h-t}p_{x,t}$ equals ${}_{h-t}p_{x,t} = \exp\left(-\hat{m}_{x+h,t+h}\right)$, where $\hat{m}_{x+h,t+h}$ is the forecasted survival rate $\hat{m}_{x+h,t+h} = \hat{\hat{a}}_{x+h} + \hat{b}_x\hat{k}_{t+h} = \hat{\hat{a}}_{x+h} + \hat{b}_x\left(k_t + h\theta\right)$.[13] The one-year deviations in expected survival rates are obtained by ${}_1d_{x,t} = -\ln\left(\frac{{}_1p_{x,t}}{{}_1\tilde{p}_{x,t}}\right)$, where ${}_1p_{x,t}$ are the forecasts rates based on data set $D$, and ${}_1\tilde{p}_{x,t}$ are the forecast rates based on data set $\tilde{D}$. We investigate improvements based on adding one year of new death observations to the data set and keep the amount of data equal. Hence, we model a rolling window with $T$ years of historic data, and different start and end years.

We model the Lee-Carter forecasts for the following countries; Denmark, the Netherlands, France, the U.K., Japan, and the U.S. based on historic data from mortality.org. We model a rolling window with 30 years of historic data with data up to 2016, hence $D = \Big\{[1978,2007], [1979,2008], ..., [1987,2016]\Big\}$.

In Figure 6 we show the average of the deviations across all individuals between ages 67 and 100, i.e. $\frac{1}{34}\sum_{x=67}^{100}{}_1d_{x,t}$ over the forecast horizon $t = \{(2011,...,2100), (2014,...,2100), (2017,...,2100)\}$. Thus, we compare the survival forecasts based on observations $[1978,2007]$ with $[1981,2010]$ generating deviations starting from 2011, and similarly for $[1981,2010]-[1984,2013]$ and for $[1984,2013]-[1987,2016]$. Between the rolling windows, the deviations differ significantly, mainly in parallel shifts. The deviations are also age dependent as can be seen in Figure 7, where $\frac{1}{90}\sum_{t=2011}^{2100} d_{x,t}$, $\frac{1}{87}\sum_{t=2014}^{2100}{}_1d_{x,t}$ or $\frac{1}{84}\sum_{t=2017}^{2100}{}_1d_{x,t}$ are plotted for the three updates. We observe that longevity risk plays a bigger role at older ages in terms of relative deviations.

[13] To reduce the bias of the forecasts, we use the last observed (let this be time $t$) log age-specific death rate to approximate $\hat{a}_x = \ln m_{x,t} - \hat{b}_x\hat{k}_t$ as suggested by Lee and Miller (2001).



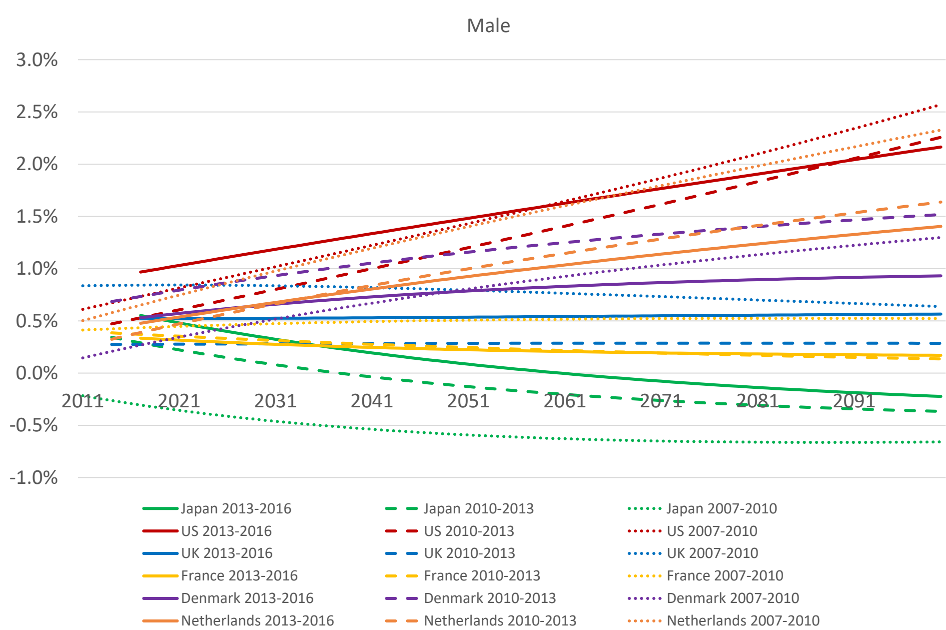
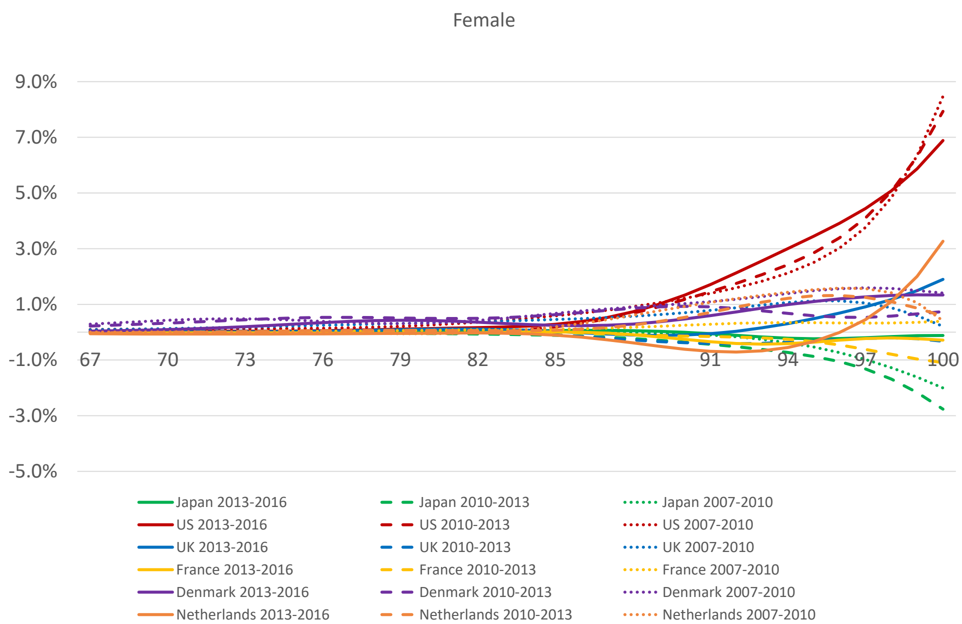




Figure 6: Deviations across forecast horizon.



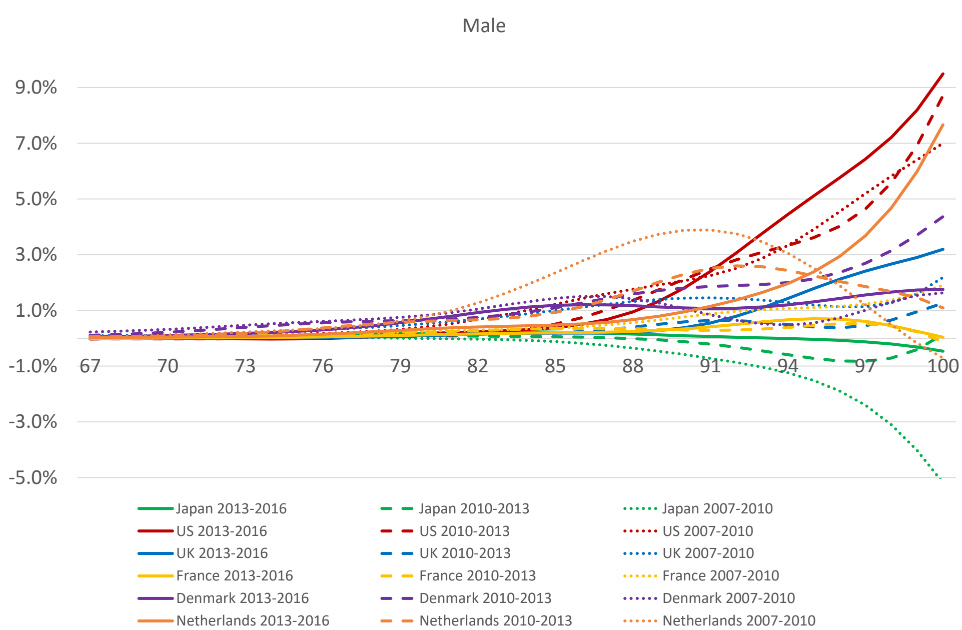




Figure 7: Deviations across age.

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