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# **Modeling, Forecasting, and Nowcasting U.S. CO<sub>2</sub> Emissions Using Many Macroeconomic Predictors**

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# Modeling, Forecasting, and Nowcasting U.S. CO<sub>2</sub> Emissions Using Many Macroeconomic Predictors\*

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## Abstract

We propose a structural augmented dynamic factor model for U.S. CO<sub>2</sub> emissions. Variable selection techniques applied to a large set of annual macroeconomic time series indicate that CO<sub>2</sub> emissions are best explained by industrial production indices covering manufacturing and residential utilities sectors. We employ a dynamic factor structure to explain, forecast, and nowcast the industrial production indices and thus, by way of the structural equation, emissions. We show that our model has good in-sample properties and out-of-sample performance in comparison with univariate and multivariate competitor models. Based on data through September 2019, our model nowcasts a reduction of about 2.6% in U.S. CO<sub>2</sub> emissions in 2019 compared to 2018 as the result of a reduction in industrial production in residential utilities.

**Keywords:** CO<sub>2</sub> emissions; macroeconomic variables; dynamic factor model; variable selection; forecasting; nowcasting.

**JEL Codes:** C01; C13; C32; C51; C52; C53; C55; C82; Q43; Q47.

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# 1 Introduction

In this paper, we propose a structural augmented dynamic factor model for U.S. CO<sub>2</sub> emissions, where emissions are explained contemporaneously by industrial production (IP) variables, and IP, in turn, is modeled by macroeconomic factors obtained from a large data set.

The literature on modeling the relation between macroeconomic activity and CO<sub>2</sub> emissions discusses a range of effects: the scale effect, by which increased IP increases emissions, changes in input and output mixes, changes in production efficiency, and changes in energy intensity (Stern, 2017, p.10). A large body of literature discusses a possible tipping point effect under the label of an environmental Kuznets curve, or EKC (e.g. Grossman and Krueger, 1991; Arrow et al., 1995; Schmalensee et al., 1998; Millimet et al., 2003; Brock and Taylor, 2005; Wagner, 2008, 2015).

The pertinent models that are commonly considered in climate science, integrated assessment models (IAMs), focus in their macroeconomic modules on highly aggregated measures of economic activity, such as global gross domestic product (GDP) and global population (Blanco et al., 2014; Bosetti et al., 2006; Calvin et al., 2019; Fujimori et al., 2017; Gambhir et al., 2019; Luderer et al., 2015; Messner and Strubegger, 1995; Nordhaus and Sztorc, 2013; Stehfest et al., 2014). The aggregated economic output then implies, by way of an energy intensity of GDP and a greenhouse gas (GHG) intensity of energy, an evolution of emissions, following the Kaya identity (Blanco et al., 2014; Raupach et al., 2007). At this point in time, IAMs are predominantly non-statistic, and parameters are set or calibrated to produce plausible output. In contrast to this, econometric theory for EKC estimation is highly sophisticated (Wagner, 2015).

Given this state of the literature, manifold causal channels with different magnitudes and signs of effects, and increasing availability of macroeconomic and emissions data, there is a need for a statistical model class that can capture the cumulative effects of economic activity on CO<sub>2</sub> emissions and that can be used to solve very practical problems, such as forecasting and nowcasting emissions. These problems arise, for example, in the annual update of the Global Carbon Budget (Le Quéré et al., 2018b).

To arrive at such a statistical model class, we start out by identifying a set of contemporaneous explanatory variables for U.S. CO<sub>2</sub> emissions from a large panel of 226 annual macroeconomic time series, using a range of variable selection techniques. We demonstrate that two variables in particular, the IP index and the IP: Residential Utilities index, explain CO<sub>2</sub> emissions to an extent that further information contained in the macroeconomic data set has only negligible explanatory

power for emissions.

The macroeconomic data set does have explanatory power for the IP indices, however. Therefore, we draw on the extensive literature on macroeconomic forecasting and specify a dynamic factor model (DFM) in order to exploit the forecast power of the macroeconomic data set for the IP indices (Elliott et al., 2006; Elliott and Timmermann, 2013, 2016; Hillebrand and Koopman, 2016). The resulting model is labeled a structural augmented dynamic factor model (SADFM): structural because it contains a contemporaneous relation between CO<sub>2</sub> emissions and IP index time series, augmented because individual IP indices and factors that summarize large amounts of data are modeled jointly, and dynamic factor model because a dynamic state equation for the factors harnesses the forecasting power of many macroeconomic predictors for the IP indices.

We demonstrate that this model describes the data well and has good in-sample properties. We present a pseudo-out-of-sample forecast exercise and show that the model performs better than a set of univariate and multivariate competitors, such as ARMA, vector autoregressions (VAR), structural VAR, principal components regressions, and standard DFMs. Finally, we show how the model can be used in nowcasting problems such as those faced by the Global Carbon Budget initiative.

The remainder of the paper is organized as follows: Section 2 describes the data used in this study. Section 3 presents the SADFM. Section 4 discusses the selection of the contemporaneous explanatory variables for U.S. CO<sub>2</sub> emissions. Section 5 discusses the estimation of the model, the in-sample fit, and an extension with time-varying parameters. Section 6 considers forecasting and nowcasting. Section 7 concludes.

## 2 Data

### 2.1 U.S. CO<sub>2</sub> Emissions Data

Let  $E_t$  denote yearly U.S. CO<sub>2</sub> emissions data at year  $t$ , i.e. the sum of yearly emissions from fossil fuels and cement production in the United States (U.S.), obtained from The Global Carbon Project (Le Quéré et al., 2018b), which in turn is compiled from Boden et al. (2018) and UNFCCC (2018). The variable  $Y_t = E_t/pop_t$ , where  $pop_t$  denotes total U.S. population in year  $t$ , is defined as CO<sub>2</sub> emissions per capita in the U.S., in tons of carbon emitted per capita. and  $y_t = \log Y_t$  is referred to as log-emissions per capita. Population data,  $pop_t$ , are obtained from the World Bank.<sup>1</sup>

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<sup>1</sup><https://data.worldbank.org>, downloaded on May 7, 2019.

Finally,  $\Delta y_t$  is taken as a good approximation of the growth rate of CO<sub>2</sub> emissions per capita.

The variables  $Y_t$  and  $y_t$  are observed at a yearly frequency, from 1960 to 2017. Figure 1 presents these data series as well as their first differences,  $\Delta Y_t$  and  $\Delta y_t$ , respectively. The results of tests for stationarity and for unit roots in these time series are given in Table 1; we conclude that there is evidence for a unit root in both  $Y_t$  and  $y_t$  and that their differences appear to be stationary. Table 2 contains further descriptive statistics of the time series.

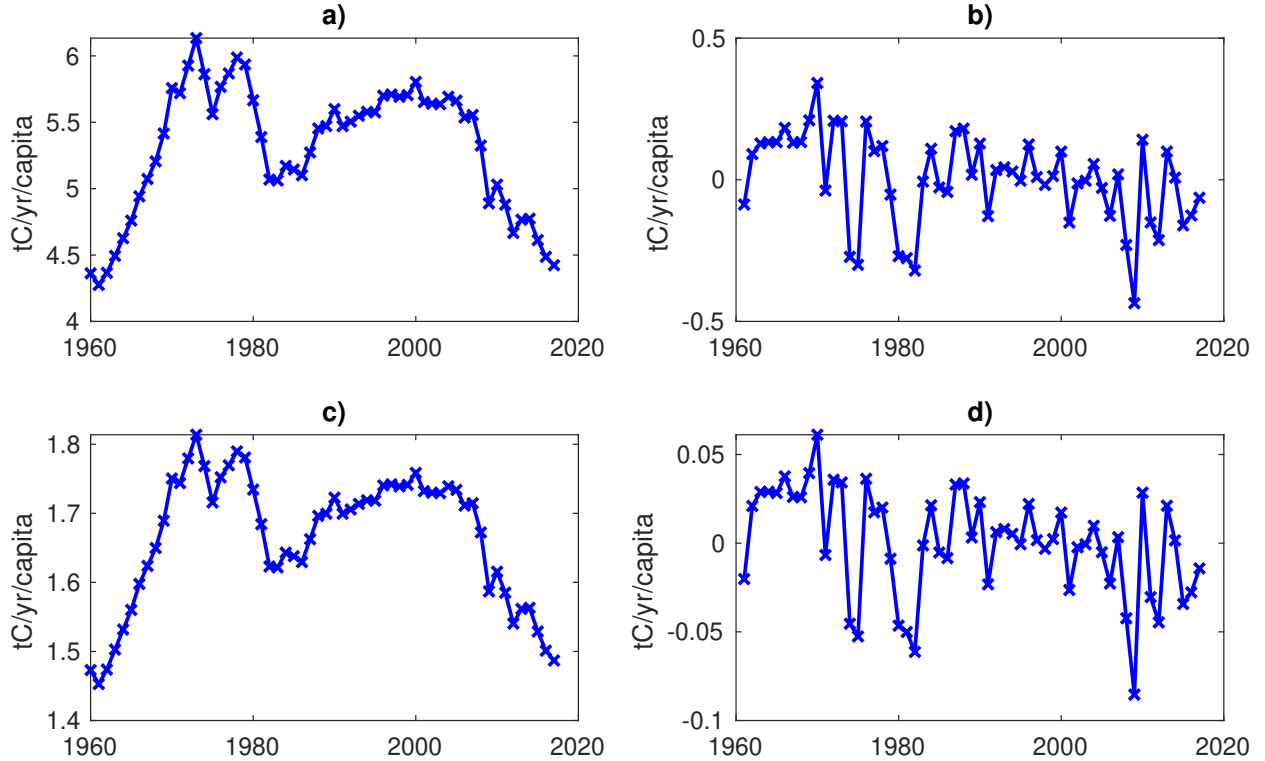


Figure 1: *Per capita U.S. CO<sub>2</sub> emissions from  $t = 1960$  to  $t = 2017$ . a): Levels,  $Y_t$ . b): Differences,  $\Delta Y_t$ . c): Log-levels,  $y_t$ . d): Log-differences (growth rates),  $\Delta y_t$ .*

## 2.2 U.S. Macroeconomic Data

We have collected economic data representative of the U.S. macroeconomy, with a particular emphasis on variables that can plausibly influence the amount of CO<sub>2</sub> emitted to the atmosphere. We consider  $N = 226$  variables in our study, of which 126 are macroeconomic variables from the so-called “FRED-MD” database, compiled and maintained by the Federal Reserve Bank of St.

Table 1: *Tests for stationarity and unit roots: p-values for the KPSS test of the null of stationarity (left) and for the Augmented Dickey-Fuller (ADF) test for the null of a unit root (right). The tests require a choice of the number of autoregressive lags to include in the linear regressions that are part of the tests; we consider 0 to 4 lags as indicated in the top of the table. The alternative in the ADF test is that the series is stationary; results for a trend-stationary alternative are similar and thus not presented here*

	KPSS					ADF				
	0	1	2	3	4	0	1	2	3	4
Number of lags:										
$Y_t =$ per capita emissions										
$Y_t :$	0.01	0.01	0.01	0.01	0.02	0.62	0.62	0.60	0.53	0.51
$\Delta Y_t :$	0.10	0.10	0.10	0.10	0.10	0.00	0.00	0.01	0.00	0.01
$y_t =$ log-per capita emissions										
$y_t :$	0.01	0.01	0.01	0.01	0.02	0.62	0.60	0.65	0.72	0.76
$\Delta y_t :$	0.10	0.10	0.10	0.10	0.10	0.00	0.00	0.01	0.01	0.01

Table 2: *Descriptive statistics and diagnostics of per capita emissions data.  $N$  is the test-statistic from the Jarque-Bera test (Jarque and Bera, 1987): the null hypothesis that the data comes from a Gaussian distribution can be rejected if  $N$  is larger than the 95% critical value of 5.99.  $DW$  is the Durbin-Watson test statistic (Durbin and Watson, 1971): If  $DW < 2$  there is evidence of positive serial correlation in the data; if  $DW > 2$  there is evidence of negative serial correlation in the data; data without serial correlation will have  $DW = 2$ .  $Q$  is the Ljung-Box  $Q$  test statistic (Ljung and Box, 1978) for presence of autocorrelation. The 95% critical value for the  $Q$ -test is 31.41; hence if the test statistic is above 31.41 then the null of no autocorrelation can be rejected at a 5% level.*

	Descriptive statistics					Diagnostics		
	NumObs	Mean	Std.	Skew	Kurt	$N$	$DW$	$Q$
$Y_t =$ per capita emissions								
$Y_t :$	58	5.30	0.48	-0.56	2.21	4.52	0.11	188
$\Delta Y_t :$	57	0.00	0.16	-0.57	2.93	3.10	1.38	25
$y_t =$ log-per capita emissions								
$y_t :$	58	1.67	0.09	-0.68	2.36	5.49	0.10	181
$\Delta y_t :$	57	0.00	0.03	-0.60	2.94	3.43	1.36	27

Louis.<sup>2</sup> The remaining 100 time series variables are related to different sectors in U.S. production: 18 variables represent U.S. agricultural production; 76 variables represent U.S. foreign trade<sup>3</sup>; 2 time series represent U.S. cement production; and 4 represent U.S. transport<sup>4</sup>. We have followed the procedure described in [McCracken and Ng \(2016\)](#) to transform the non-stationary time series to a set of stationary time series and to studentize all resulting time series to mean zero and unit variance. Further details of the data set, including how we collected and transformed each individual data series, are given in [Appendix A](#).

The sample period of the yearly economic data set is from 1960 until 2018. Given that, at the time of writing, the official emissions variable  $E_t$  is only available until 2017<sup>5</sup>, we only consider economic data up to 2017. Many of the time series start in 1961 after first differencing, while some variables start even later. We are therefore faced with an *unbalanced panel* of economic variables. Hence the economic data matrix  $X$ , with  $T = 57$  rows and  $N = 226$  columns, contains a number of missing entries. We impute these missing values using a factor model approach, following [Stock and Watson \(2002\)](#) and [McCracken and Ng \(2016\)](#). As a result, in our empirical study we work with a  $T \times N$  balanced panel of economic data  $X$ . The details of the imputation method are given in [Appendix B](#).

### 3 Statistical Model for Growth in CO<sub>2</sub> Emissions

The variable of interest  $y_t$  in our study is the log-difference (i.e., growth rate) of U.S. CO<sub>2</sub> per-capita emissions. Our analysis is based on the statistical dynamic model

$$y_t = \alpha + \sum_{j=1}^k \beta_j x_t^{(i_j)} + \beta'_f f_t + \gamma' z_t + \epsilon_t^y, \quad (3.1)$$

where  $x_t^{(i_j)}$ , for  $j = 1, \dots, k$ , are pre-selected individual economic variables,  $i_j \in \{1, 2, \dots, N\}$  indicates the column number in the data matrix  $X$ ,  $\beta_j$  is the regression coefficient for the  $i_j$ th variable,  $f_t$  is an  $r \times 1$  vector of economic factors or principal components,  $\beta_f$  is the  $r \times 1$  corresponding loading coefficient vector,  $z_t$  is an  $l \times 1$  vector of exogenous variables not contained

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<sup>2</sup><https://research.stlouisfed.org/econ/mccracken/fred-databases/>, downloaded on May 7, 2019, see [McCracken and Ng \(2016\)](#).

<sup>3</sup>Collected from the World Bank website, <https://data.worldbank.org>, downloaded on September 19, 2019.

<sup>4</sup>Collected from the website of the U.S. Federal Reserve Bank, <https://fred.stlouisfed.org>, downloaded on October 10, 2019.

<sup>5</sup>UNFCCC inventories are available at [https://di.unfccc.int/time\\_series](https://di.unfccc.int/time_series), accessed on November 18, 2019.

in  $X$ , such as dummy variables,  $\gamma$  is the corresponding  $l \times 1$  vector of regression coefficients, and  $\epsilon_t^y$  is the disturbance term. For any disturbance term in our modelling framework, we assume it is an independent and identically distributed (IID) random sequence with mean zero and a positive variance. All  $2 + k + r + l$  unknown coefficients are collected in the parameter vector  $\theta_y = (\alpha, \beta_1, \dots, \beta_k, \beta'_f, \gamma', \sigma_y)'$  with  $\sigma_y^2 = \text{Var}(\epsilon_t^y)$  for all  $t$ . The statistical model (3.1) can be regarded as a standard regression model with the addition of latent dynamic factors in  $f_t$ . A similar modeling framework is considered in many macroeconomic studies, for example, in [Stock and Watson \(2002\)](#), [Giannone et al. \(2008\)](#), and [Stock and Watson \(2010\)](#).

### 3.1 Structural augmented dynamic factor model

The main motivation for model (3.1) is that we have a large number of macroeconomic time series available for our analysis, in our case  $N = 226$ . The formulation of a statistical model with all individual time series present in  $X$  is not feasible, despite the relatively small number of observations, in our case  $T = 57$ . Our model faces this challenge in two ways. First, we make a pre-selection of relevant variables for  $y_t$  and only include the  $k \ll N$  most important variables  $x_t^{(i_1)}, \dots, x_t^{(i_k)}$  as regressors in (3.1). The selection of these  $k$  “most important” variables can be done in different ways, depending on selection strategies and criteria; see the discussion in Section 4. Second, we include  $r$  latent dynamic *common factors*  $f_t$ , with  $r \ll N$ , that are connected with all economic variables in  $X$ . For this purpose, we consider the  $N \times 1$  vector of economic time series  $x_t$ , being the  $t$ -th row of  $X$ , and specify the dynamic factor model as

$$x_t = \Lambda f_t + \epsilon_t^x, \tag{3.2}$$

where  $\Lambda$  is an  $N \times r$  matrix of *factor loadings*, and  $\epsilon_t^x$  is an idiosyncratic disturbance term with  $\text{Var}(\epsilon_t^x) = \Sigma_x$ , an  $N \times N$  matrix. We assume that the mild conditions of [Stock and Watson \(2002\)](#) apply to our setting. The dynamic properties of the common factors are implied by a vector autoregressive process of order one, or VAR(1), for  $f_t$ , as given by

$$f_t = \Phi f_{t-1} + \eta_t, \tag{3.3}$$

where  $\Phi$  is an  $r \times r$  autoregressive coefficient matrix, and  $\eta_t$  is the disturbance vector with  $r \times r$  variance matrix  $\text{Var}(\eta_t) = \Sigma_\eta$ . We assume that the roots of the characteristic polynomial  $|I_r - \Phi z|$  lie outside the unit circle, where  $I_r$  denotes the  $r \times r$  identity matrix. This condition ensures that



the VAR(1) model for  $f_t$  is stable.<sup>6</sup> We further restrict the variance matrix  $\Sigma_\eta$  such that the factors are orthogonal and normalized to have unit variance:  $\text{Var}(f_t) = I_r$ .

In case  $\beta_1 = \beta_2 = \dots = \beta_k = 0$  in equation (3.1), the model for  $y_t$  reduces to the “standard” dynamic factor model (DFM) as studied in Doz et al. (2012), Jungbacker and Koopman (2015), Stock et al. (2016), among others. Stock and Watson (2002) have suggested to improve forecast performance of the DFM by including lags of selected variables,  $x_{t-1}^{(ij)}$ , in the forecast equation for  $y_t$ . In this spirit, we include *contemporaneous* individual variables, the selection  $x_t^{(ij)}$ , for  $j = 1, \dots, k$ , in the equation for  $y_t$ , and we consider  $\beta_j \neq 0$ , for  $j = 1, \dots, k$ , in (3.1). This alternative specification provides a more structural interpretation of the model and hence we refer to the resulting model as a *structural augmented dynamic factor model* (SADFM).<sup>7</sup>

### 3.2 State space model representation

The SADFM model (3.1)–(3.3) can be written jointly in matrix form by

$$B \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} \beta'_f \\ \Lambda \end{pmatrix} f_t + \begin{pmatrix} \gamma' \\ 0 \end{pmatrix} z_t + \begin{pmatrix} \epsilon_t^y \\ \epsilon_t^x \end{pmatrix}, \quad f_{t+1} = \Phi f_t + \eta_{t+1},$$

where  $B$  is the  $(N+1) \times (N+1)$  selection matrix

$$B = \begin{pmatrix} 1 & -\beta^{s'} \\ 0 & I_N \end{pmatrix},$$

where  $\beta^s$  is the  $N \times 1$  vector of zeros except for the entries  $i_1, \dots, i_k$  which are equal to  $\beta_j$ , respectively, for  $j = 1, \dots, k$ . Given that the inverse of  $B$  is

$$B^{-1} = \begin{pmatrix} 1 & \beta^{s'} \\ 0 & I_N \end{pmatrix},$$

it follows that the model (3.1)–(3.3) is equivalent to

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} \beta'_f + \beta^{s'} \Lambda \\ \Lambda \end{pmatrix} f_t + \begin{pmatrix} \gamma' \\ 0 \end{pmatrix} z_t + u_t, \quad f_{t+1} = \Phi f_t + \eta_{t+1}, \quad (3.4)$$

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<sup>6</sup>If a stable VAR process is initialized according to its stationary distribution, then the resulting process is stationary and ergodic.

<sup>7</sup>This model is related to the structural dynamic factor model (SDFM) as studied by Stock et al. (2016) and elsewhere. The lagged dependence of  $x_t^{(ij)}$ , for  $j = 1, \dots, k$ , in the SADFM is implicitly specified through the dynamics of the factors in  $f_t$ .

with

$$u_t = B^{-1} \begin{pmatrix} \epsilon_t^y \\ \epsilon_t^x \end{pmatrix}, \quad \text{Var}(u_t) = \begin{pmatrix} \sigma_y^2 + \beta^{s'} \Sigma_x \beta^s & \beta^{s'} \Sigma_x \\ \Sigma_x \beta^s & \Sigma_x \end{pmatrix}.$$

The  $(N + 1) \times 1$  vector  $(y_t, x_t)'$  is the observed data vector for the complete model, the  $r \times 1$  vector  $f_t$  is a latent dynamic factor, the  $l \times 1$  vector  $z_t$  is exogenous (fixed covariates), and the  $(N + 1) \times 1$  vector  $u_t$  is the disturbance. The system of equations (3.4) represents a linear state space model, with the initial moment conditions  $E(f_1) = 0$  and  $\text{Var}(f_1) = I_r$ ; see [Durbin and Koopman \(2012\)](#) for a complete treatment of this class of models. The model is subject to a number of choices, including the composition of the economic data matrix  $X$ , the selection of indices  $i_1, \dots, i_k$ , and the number of factors  $r$ . In the empirical part of our study we discuss these choices further and show how they have been made for our data set.

### 3.3 Parameter estimation

The unknown parameters in (3.4) are the collection of  $\theta_y$ ,  $\Lambda$ ,  $\Sigma_x$ , and  $\Phi$ .<sup>8</sup> All parameters can be consistently estimated (as  $T, N \rightarrow \infty$ ) in a two-step iterative procedure as proposed in [Doz et al. \(2011, 2012\)](#). It requires the computation of principal components from the data matrix  $X$  as in [Stock and Watson \(2002\)](#), where the principal components (or diffusion indices) are used as regressors for a variable of interest, for the purpose of forecasting. In our analysis we consider the  $r$  principal components from  $X$  that are associated with the  $r$  largest eigenvalues of the sample variance matrix of  $X$ . The principal components are collected in the  $r \times 1$  vector  $F_t$  and are regarded as proxies of  $f_t$ , for  $t = 1, \dots, T$ .

First, we carry out two regressions: one to estimate matrix  $\Phi$  by regressing the principal components  $F_{t+1}$  on  $F_t$  (equation by equation), the other to estimate matrix  $\Lambda$  and diagonal matrix  $\Sigma_x$  by regressing  $x_t$  on  $F_t$ . We obtain the estimate  $\Sigma_\eta$  using its definition  $\Sigma_\eta = I_r - \Phi\Phi'$  and replacing  $\Phi$  by its estimate. Next, we take these estimates to replace their unknown counterparts in (3.4) while the remaining parameters in vector  $\theta_y$  are estimated by the method of maximum likelihood, for which the Kalman filter is used to evaluate the loglikelihood function based on the prediction error decomposition; see [Jungbacker and Koopman \(2015\)](#) for further details in the context of the DFM. After this first stage of our estimation process, we consider (3.4), with the parameter estimates, to signal extract the dynamic factors  $f_t$ , for  $t = 1, \dots, T$ , using the Kalman

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<sup>8</sup>For computational convenience, we restrict  $\Sigma_x$  as a diagonal matrix. The variance matrix  $\Sigma_\eta = I_r - \Phi\Phi'$  is restricted and not explicitly estimated.

filter and the associated smoother (KFS) method; see [Durbin and Koopman \(2012, Chapter 4\)](#). We denote these smoothed estimates by  $\tilde{f}_t$ , for  $t = 1, \dots, T$ .

The smoothed factor estimates  $\tilde{f}_t$  will be different from the principal components  $F_t$ , for  $t = 1, \dots, T$ . Based on these new factor estimates, the procedure described above can be repeated by replacing  $F_t$  by  $\tilde{f}_t$ . The two regressions can be repeated and, based on the resulting new estimates for  $\Phi$ ,  $\Lambda$  and  $\Sigma_x$ , the maximum likelihood estimation for  $\theta_y$  can also be repeated. In turn, based on these new parameter estimates, new smoothed estimates can be obtained for  $f_t$ , for  $t = 1, \dots, T$ , using KFS. This iterative procedure is close to what is described in [Doz et al. \(2012, p. 1018\)](#). Let  $\mathcal{L}_m$  be the loglikelihood function value for the parameter estimates in iteration  $m$ . We define the statistic

$$c_m = \frac{\mathcal{L}_m - \mathcal{L}_{m-1}}{(\mathcal{L}_m + \mathcal{L}_{m-1})/2},$$

and compute it at the end of each iteration. This iterative procedure can stop at iteration  $M$ , where  $M$  is the first integer such that  $c_M < 10^{-4}$ . In our empirical study it turns out that the number of iterations  $M$  is as low as 4 or 5.

### 3.4 Collapsed structural augmented dynamic factor model

From an econometric viewpoint, we can base our empirical analysis on the model formulation (3.4). However, in case the model needs to cope with a high-dimensional  $N \times 1$  data vector  $x_t$ , or with many consecutive estimations in a forecast exercise, various computational steps in the estimation procedure become somewhat cumbersome. In our empirical study below, we consider the analysis on  $N = 226$  economic variables: it requires many regression computations in each iteration. To alleviate the computational burden somewhat, we consider a dimension reduction by employing the *collapsed dynamic factor model*, as proposed by [Bräuning and Koopman \(2014\)](#).

In this collapsed approach, we consider the principal components in  $F_t$  as data proxies of the corresponding latent dynamic factors in  $f_t$ , for  $t = 1, \dots, T$ . In particular, we assume that

$$F_t = f_t + \epsilon_t^f,$$

where  $\epsilon_t^f$  is a disturbance vector with  $\text{Var}(\epsilon_t^f) = \Sigma_f$ . We further treat the principal component vector  $F_t$  as a data vector that can replace the original large data vector  $x_t$ , since the principal components provide a sufficiently accurate description of the data matrix  $X$ . This suggestion leads to a much lower-dimensional model with much smaller number of parameters and a smaller computational burden for estimation. The estimation process is also easier (as a smaller number

of parameters are involved) and may potentially provide more accurate forecasts, given that the model is more parsimonious; see [Bräuning and Koopman \(2014\)](#) and [Hindrayanto et al. \(2016\)](#).

The collapsed SADF<sub>M</sub> is given by

$$\begin{pmatrix} y_t \\ x_t^* \\ F_t \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \beta'_f + \beta^{*'}\Lambda^* \\ \Lambda^* \\ I_r \end{pmatrix} f_t + \begin{pmatrix} \gamma' \\ 0 \\ 0 \end{pmatrix} z_t + u_t^*, \quad f_{t+1} = \Phi f_t + \eta_{t+1}, \quad (3.5)$$

where  $x_t^*$  is the  $k \times 1$  sub-vector  $x_t$  (or is equal to  $x_t^s$  but with all zeros removed),  $\beta^*$  is the corresponding regression coefficient vector for  $x_t^*$ ,  $\Lambda^*$  is the loading matrix only with the rows of  $\Lambda$  corresponding to  $i_1, \dots, i_k$ , and  $u_t^*$  is the same as the disturbance vector  $u_t$  with  $\epsilon_t^x$  replaced by  $\epsilon_t^f$ , for  $t = 1, \dots, T$ . While the SADF<sub>M</sub> has an observation vector of dimension  $N + 1$ , the collapsed SADF<sub>M</sub> has a reduced observation vector of dimension  $k + r + 1$  where both  $k \ll N$  and  $r \ll N$ .

We employ the same iterative procedure for the estimation of the parameters in the collapsed SADF<sub>M</sub>, as described above for the non-collapsed model. First, we carry out two regressions: one to estimate matrix  $\Phi$  by regressing the principal components  $F_{t+1}$  on  $F_t$  (equation by equation); the other to estimate matrix  $\Lambda^*$  by regressing  $x_t^*$  on  $F_t$ . The remaining parameters in vector  $\theta_y$  are estimated by the method of maximum likelihood. After this first stage, we signal extract the dynamic factors  $f_t$ , for  $t = 1, \dots, T$ , using KFS, and we denote these by  $\tilde{f}_t$ , for  $t = 1, \dots, T$ . The iterative process can start by replacing  $F_t$  with  $\tilde{f}_t$ , for  $t = 1, \dots, T$ . The iterations can be stopped on the basis of the same convergence criterion  $c_m$  as suggested above.

## 4 Selection of macroeconomic variables for CO<sub>2</sub> emissions

To select economic variables to be included in the sub-vector  $x_t^*$ , we carry out a preliminary statistical analysis of per-capita CO<sub>2</sub> emission growth  $y_t$  and the economic data matrix consisting of the  $N \times 1$  data vectors  $x_t$ , with  $N = 226$ . The description of the data matrix is provided in [Section 2](#) and in the supplementary material.

First, we run  $N$  static separate regressions for  $y_t$  on each variable in the data vector  $x_t$  in simple univariate regressions with intercept. The resulting  $N$  coefficients of determination ( $R^2$ ) are displayed in [Figure 2](#). It shows that many individual economic variables in  $x_t$  can provide a considerable goodness-of-fit statistic for the growth in CO<sub>2</sub> emissions. The variables producing a particularly high  $R^2$  are macroeconomic variables related to the real economy, such as production

indices ( $i = 1, 2, \dots, 20$ ) and employment ( $i = 21, \dots, 49$ ). Some of the trade-related variables ( $i = 145, \dots, 220$ ) also enjoy a high  $R^2$ , as do the cement-production-related variables ( $i = 221, 222$ ).

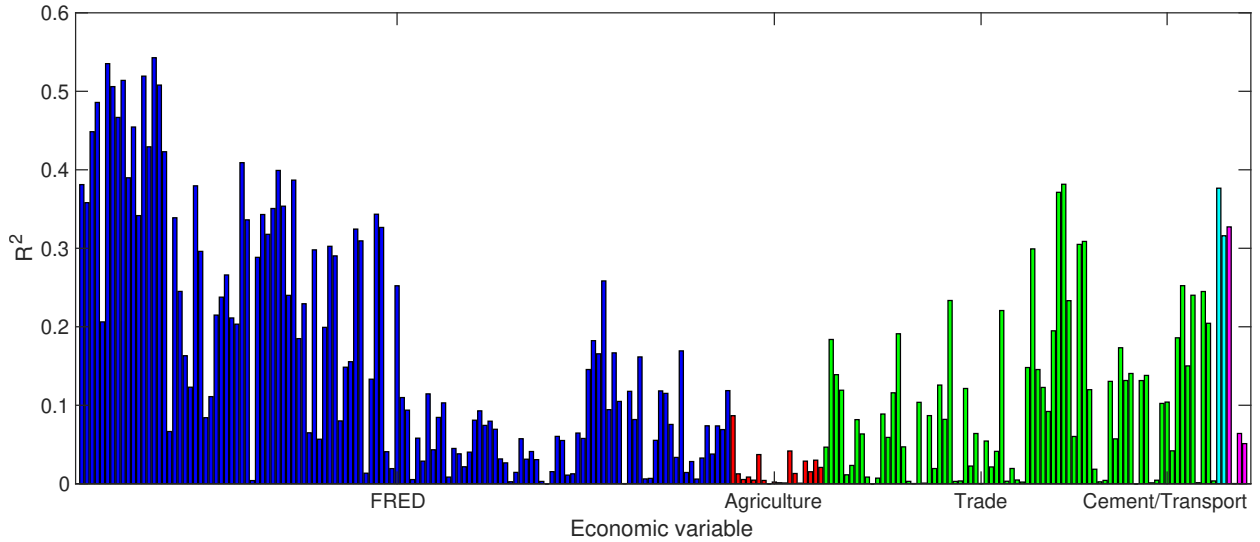


Figure 2: *Coefficient of determination ( $R^2$ ) obtained from single regressions for  $CO_2$  growth, with intercept and a single explanatory economic variable (each at a time). The economics variables are grouped (and coloured) into those coming from the FRED-MD database (blue), agricultural data series (red), trade-related data series (green), cement-related data series (cyan), and transport-related data series (magenta).*

The results so far only show how a *single* economic variable explains growth in  $CO_2$  emissions. We are interested in selecting *multiple* economic variables that provide the best possible fit for  $CO_2$  emissions growth. For this purpose, we run multiple regressions (with intercept) based on a subset of economic variables indexed by a subset of indices  $I \subseteq \{1, 2, \dots, N\}$ , with  $|I|$  being the cardinality of the set  $I$ . For example, in case of  $I = \{1, 5, 10\}$ , the multiple regression contains  $y_t$  ( $CO_2$  growth), the intercept, and the vector of regressors  $(x_t^{(1)}, x_t^{(5)}, x_t^{(10)})'$ . Depending on the size of the permissible set, there is a vast range of possible sets  $I$  with all possible combinations. There are various strategies for selection. Well-known methods are the LASSO of Tibshirani (1996) and the Elastic Net (EN) of Zou and Hastie (2005). For our data set with many series that are highly correlated, we find that both LASSO and EN include implausibly many variables in their preferred sets  $I$ .<sup>9</sup> Another regression selection method is advocated by Doornik and Hendry (2015) and is referred to as “AutoMetrics”; it is an automated “general-to-specific” (GETS) method that chooses

<sup>9</sup>Using a 25-fold cross validation to minimize the mean squared error of the LASSO/Elastic Net regression, they both retain the variables  $I = \{3, 6, 11, 13, 15, 17, 23, 38, 46, 55, 102, 159, 185, 213, 222\}$ .

the best set  $I$  on the basis of multiple criteria including goodness-of-fit, model diagnostics, and exogeneity tests. We have adopted the AutoMetrics implementation in the PcGive module of the OxMetrics software; see [Doornik and Hendry \(2018\)](#). The documentation provides the details of the automated selection method. AutoMetrics has the option to specify a prior belief on the size of  $I$ . We report results based on sizes that are indicated as “Tiny” and “Small”. The AutoMetrics results are presented in the top panel of [Table 3](#). The “Tiny” setting provides  $I = \{6, 17, 48, 51, 56\}$ , while the “Small” setting yields  $I = \{6, 17, 48, 51, 55, 89\}$ . Furthermore, the procedure detects two outliers in  $t = 1970$  and  $t = 1990$ .<sup>10</sup>

Although the selected set  $I$  obtained from the AutoMetrics procedure is smaller than those from the LASSO and EN procedures, the selected set  $I$  is still large. Therefore, we consider an alternative route. Given a pre-set maximum size  $s \geq 1$  for  $I$ , and subject to  $|I| \leq s$ , we determine which variables need to be selected to provide the best goodness-of-fit. [Table 3](#) presents these results for a range of  $s$  values and for a variety of selection procedures. At this stage, all regressions include the intercept and two outlier dummy variables, for the time indices  $t = 1970, 1990$ . When  $s$  is small, it is computationally feasible to search over all possible combinations of variables in  $x_t$  to be included in  $I$ , subject to  $|I| \leq s$ . For each combination, we record the realized values for the  $R^2$ , the maximized loglikelihood value (LogL), the Akaike Information Criterion (AIC), its corrected version (AICc), and the Bayes Information Criterion (BIC). We rank the sets according to each criterion separately. Furthermore, we restrict the LASSO and EN procedures by choosing the sets with the smallest penalizing parameter such that the number of variables retained in  $I$  are  $|I| \leq s$ . The results from these different selection procedures (and for different  $s$ ) are presented in the bottom panel of [Table 3](#).<sup>11</sup>

In case of the complete search, for a given  $s$ , and with the different rankings based on  $R^2$ , LogL, AIC, AICc, and BIC, the selections all agree on the set of variables to include in  $I$ . The LASSO and EN methods choose similar variable sets, although their selections differ from those of the complete search strategy. All methods agree that the variable corresponding to  $i = 6$  is the most important variable to include in the model. This variable represents the general level of industrial production (IP) in the U.S. economy. A regression with this single variable and an intercept,

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<sup>10</sup>An outlier around  $t = 1970$  has been found in similar data; see, for example, [Auffhammer and Steinhauser \(2012\)](#) and [Schmalensee et al. \(1998\)](#). The interpretation of the outlier in  $t = 1990$  is not clear.

<sup>11</sup>We have also carried out the selection procedures with the inclusion of ten principal components,  $F_t$ , from  $X$  that correspond to the ten largest eigenvalues of the sample variance matrix of  $X$ . In this case, the principal components have never been selected as part of the best model.

Table 3: *Regression output from the models chosen by the various selection criteria. The AutoMetrics procedure detects two outliers in  $t = 1970$  and in  $t = 1990$ . These have been included in the other models as well.*

AutoMetrics	<i>Tiny</i>		<i>Small</i>	
	{6, 17, 48, 51, 56}		{6, 17, 48, 51, 55, 89}	
	$s = 1$	$s \leq 2$	$s \leq 3$	$s \leq 4$
$R^2$	{6}	{4, 17}	{4, 17, 185}	{6, 17, 51, 53}
logL	{6}	{4, 17}	{4, 17, 185}	{6, 17, 51, 53}
AIC	{6}	{4, 17}	{4, 17, 185}	{6, 17, 51, 53}
AICc	{6}	{4, 17}	{4, 17, 185}	{6, 17, 51, 53}
BIC	{6}	{4, 17}	{4, 17, 185}	{6, 17, 51, 53}
LASSO	{6}	{6, 9}	{6, 9}	{6, 9, 15, 17}
Elastic Net	{6}	{6, 9}	{6, 9}	{6, 9, 15, 17}

produces an  $R^2$  of 0.54. Many other IP related indices produce the same degree of goodness-of-fit. The variable corresponding to  $i = 17$  is present in most models. This variable is the industrial production index of residential utilities (IP: Residential Utilities). The variable set  $I = \{6, 17\}$  produces an  $R^2$  of 0.71. It is plausible that these two variables explain much of the variation in the growth of CO<sub>2</sub> emissions: variable  $i = 6$  represents the production part of the economy while variable  $i = 17$  represents the residential part of the economy. When more explanatory power is desired, a trade-related variable ( $i = 185$ ) or a housing-related variable ( $i = 51$  or  $i = 53$ ) can be included in the set  $I$ .

The results in Table 3 show that for  $s \leq 2$  and  $s \leq 3$ , the variable  $i = 4$  (real manufacturing and trade industries sales) is preferred over  $i = 6$  (industrial production). However, the correlation between the two variables  $x_t^{(4)}$  and  $x_t^{(6)}$  is 95%. There is little gain from including further variables. Figure 3 plots the  $R^2$  from the regression from the sets  $I$  selected by the various methods. The IP variable  $x_t^{(6)}$  clearly contributes most of the explanatory power with  $R^2 = 0.54$ . When adding the IP: Residential Utilities variable  $x_t^{(17)}$ , we obtain  $R^2 = 0.73$ . The increase in  $R^2$  levels off when further variables are added: we obtain  $R^2 = 0.75$ , for  $s \leq 3$ , and  $R^2 = 0.77$ , for  $s \leq 4$ . We thus prefer a small set  $I$ , set  $k = 2$ , and select the variables  $x_t^{(i_1)}$  and  $x_t^{(i_2)}$  with  $i_1 = 6$  and  $i_2 = 17$  in the following.

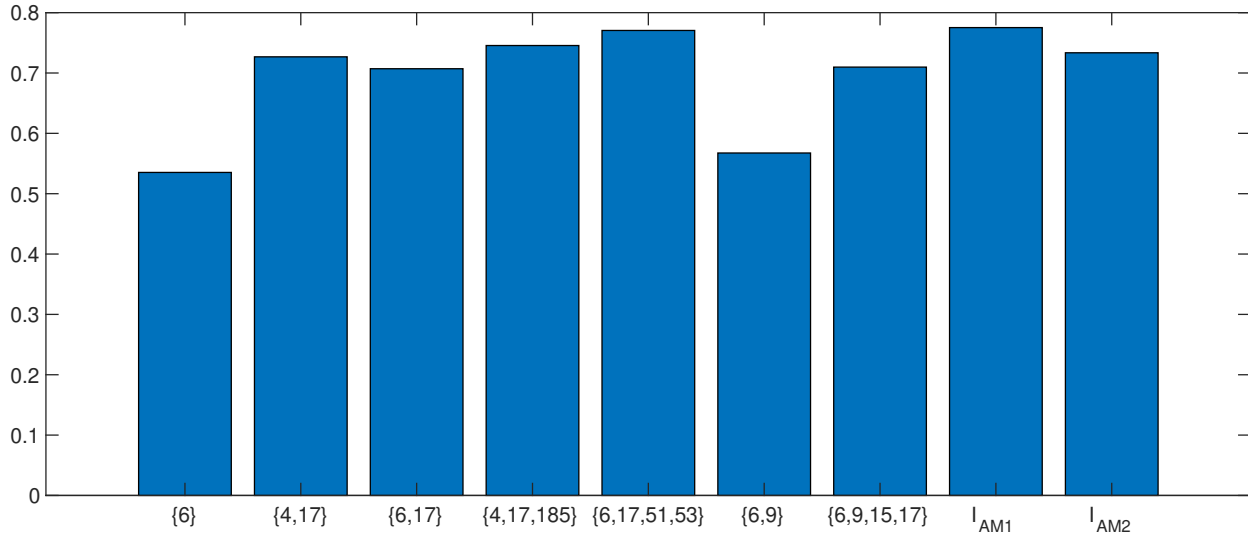


Figure 3: Coefficient of determination ( $R^2$ ) for the regression for different sets  $I$ . The AutoMetrics sets  $I_{AM1} = \{6, 17, 48, 51, 56\}$  and  $I_{AM2} = \{6, 17, 48, 51, 55, 58\}$  are the sets obtained from the options “Tiny” and “Small”, respectively.

## 5 Estimation Results and In-Sample Analysis

In our empirical analysis of U.S. per-capita CO<sub>2</sub> emissions growth, we consider the SADFM where we need to specify  $k$ ,  $l$  and  $r$ , and select variables for  $x_t^*$  and  $z_t$ . As motivated in Section 4, we take  $k = 2$ , with  $i_1 = 6$  and  $i_2 = 17$ , that is  $x_t^* = (x_t^{(6)}, x_t^{(17)})'$ , and  $l = 2$ , with  $z_t$  representing two dummy variables to extract the outliers in per-capita CO<sub>2</sub> emissions growth for 1970 and 1990. The dimension of  $f_t$  is  $r$ , and it is determined by the  $p_2$  criterion of Bai and Ng (2002). In almost all our model settings, we find strong empirical support for  $r = 4$ . We therefore present the results for the SADFM with these settings.

Furthermore, we consider three variants of the SADFM specification:

- (i) DFM: with restriction  $\beta_1 = \beta_2 = 0$ , leading to the standard DFM model;
- (ii) SADFM-U: without any restriction;
- (iii) SADFM-R: with restriction  $\beta_f = 0$ , leading to a model with factors only for  $x_t$ .

These specifications have their own specific and different features. The DFM specification leads to a standard dynamic factor analysis and can be used for forecasting and nowcasting in a regular manner. The SADFM-U provides a powerful intersection of including both the specifically important economic variables and the overall summary of dynamic features in a large data set



of economic variables for the prediction of CO<sub>2</sub> emissions growth. The SADFM-R reduces the equation for  $y_t$  into a basic regression model and with only the equation for  $x_t$  being made subject to the factors. The simplicity of a regression model for  $y_t$  is a nice feature while the practicalities of forecasting  $x_t$ 's, when a forecast of  $y_t$  is computed, are handled by the dynamic factor structure for  $x_t$ . These different specifications with their different impacts on the empirical analyses also illustrate the flexibility of our SADFM framework.

## 5.1 Parameter estimation results

The parameter estimation results are presented in Panel A of Table 4. For all three model specifications, the estimate of the intercept  $\alpha$  is estimated to be statistically indistinguishable from zero, while the two regression coefficients in  $\gamma$ , associated with the two dummy variables for 1970 and 1990 are statistically significant. In the case of the DFM specification,  $y_t$  loads significantly on the first two economic factors, while the other two factors appear to have less significant support for  $y_t$ . In case of the unrestricted model SADFM-U, the regression coefficient for  $x_t^{(17)}$  is highly significant, while the loadings on the first three factors are also statistically significant. The regression coefficient for  $x_t^{(6)}$  is not significant, but we stress that the first principal component in  $F_t$  closely resembles the variable  $x_t^{(6)}$ ; the sample correlation between the two variables is as high as  $-87\%$ . Given that the first principal component is a proxy for the first factor in  $f_t$ , we believe that the insignificance of the coefficient for  $x_t^{(6)}$  can be explained by this resemblance. In this light, and given the findings in Section 4, it is not surprising that in the case of the restricted model SADFM-R, we obtain highly significant regression parameter estimates for both  $x_t^{(6)}$  and  $x_t^{(17)}$ .

## 5.2 In-sample analysis

In Panel B of Table 4 we report the various goodness-of-fit measures for the three SADFM specifications; these measures are introduced in Section 4. In the context of SADFM, the  $R^2$  measures relate only to the fit of  $y_t$ . The likelihood-based measures are all calculated using the full observation vector with  $y_t$  and  $x_t$  in equation (3.4). Overall, the goodness-of-fit for the SADFM is high. A basic regression analysis based on  $x_t^{(6)}$  and  $x_t^{(17)}$  produces the  $R^2$  statistic of 0.73, and with some other regression specifications we may raise it to 0.77; see the discussion in Section 4. The SADFM produces  $R^2$  values of 0.84 and 0.86; these are clearly better fits. The fit measures of LogL and AIC point towards SADFM-U as providing the best fit while AICc and BIC point towards SADFM-R for the best fit. However, the differences are small. We tend towards favoring the SADFM-R

specification, as it is more parsimonious and more convenient for out-of-sample analysis, including forecasting and nowcasting; see the discussions in Section 6.

Panel C of Table 4 reports diagnostic statistics for the standardised one-step ahead prediction residuals, which are obtained from the Kalman filter. If the model is correctly specified, these prediction residuals have properties that can be verified. We focus particularly on the properties that the prediction residuals are serially uncorrelated and normally distributed. We report the Durbin-Watson (DW) and Ljung-Box (LB) tests for serial correlation and the skewness (skew), kurtosis (kurt) and the two combined (N) tests for normality. The reported results for the SADFM specifications can be viewed as follows. The LB test cannot reject the null of no serial correlation in  $\hat{\epsilon}_t^y$  at a 5% significance level. However, the DW test statistics indicate that there is still some positive autocorrelation left in the prediction residuals. The reported normality diagnostics are reasonable. Overall we can conclude that the SADFM provides an adequate model for the data at hand.

The fits of  $y_t$  for both SADFM-U and SADFM-R specifications are presented in Figure 4. The upper graph is showing the fits with the actual CO<sub>2</sub> growth observations and the lower graph is presenting the residuals ( $y_t$  minus the fit). The plots confirm the successful performances of SADFM in our empirical study. It also shows that the overall performance of SADFM-R is somewhat superior. The residual plots reveal that there may still be some subtle (short-term) serial correlation present.

### 5.3 Temporal stability of the estimation results

Given the in-sample analysis so far, we have a slight preference for the SADFM-R specification. We can scrutinize this specification further by verifying the stability of the presented results. For example, we can split the time series sample in two parts: the first part covers the period from 1961 to 1988, and the second period is from 1990 to 2017, resulting in two non-overlapping sample periods. To ensure that both periods have the same number of observations, that is  $T_1 = T_2 = 28$ , the observation for 1989 is discarded. We then re-estimate the parameters for SADFM-R and particularly focus on the parameters in the equation for  $y_t$ , that is  $(\alpha, \beta_1, \beta_2)'$ . We adopt the same maximum likelihood estimation strategy as for the full sample. Figure 5 reports the results for the parameters  $\alpha$ ,  $\beta_1$  and  $\beta_2$ . While it may be plausible that the effect of the various economic processes on CO<sub>2</sub> emissions growth will change over different time periods, we do find that only the intercept appears to be somewhat different for the two sub-periods. The estimates of  $\beta_1$  and

Table 4: *SADFM estimation output*

Panel A: Estimates									
	$\alpha$	$\beta_1$	$\beta_2$	$\beta_{f1}$	$\beta_{f2}$	$\beta_{f3}$	$\beta_{f4}$	$\delta_{1970}$	$\delta_{1990}$
DFM	-0.0020 (-0.79)			-0.0211 (-7.27)	-0.0114 (-4.89)	-0.0026 (-1.39)	-0.0030 (-1.39)	0.0910 (5.58)	0.0391 (2.40)
SADFM-U	-0.0020 (-1.33)	0.0030 (0.38)	0.0136 (7.03)	-0.0145 (-2.07)	-0.0086 (-2.29)	-0.0038 (-2.05)	0.0011 (0.60)	0.0775 (7.19)	0.0505 (4.69)
SADFM-R	-0.0019 (-1.14)	0.0201 (9.11)	0.0115 (6.65)					0.0770 (6.52)	0.0448 (3.79)

Panel B: Diagnostics (fit)						
	$R^2$	$R_a^2$	logL	AIC	AICc	BIC
DFM	0.72	0.68	-12124.57	24261.15	24262.83	24273.41
SADFM-U	0.88	0.86	-12102.06	24220.12	24223.12	24236.47
SADFM-R	0.85	0.84	-12106.58	24221.16	24221.93	24229.34

Panel C: Diagnostics (residuals)						
	std	skew	kurt	N	DW	Q
DFM	0.02	0.16	2.60	0.64	1.56	18.95
SADFM-U	0.01	-0.09	2.32	1.16	1.75	18.34
SADFM-R	0.01	-0.34	2.79	1.17	1.54	19.94

Output from the SADFM estimation procedure.  $t$ -stats in parentheses.  $Q$  is the Ljung-Box (LB) test statistic; the 5% critical value is 31.4104.

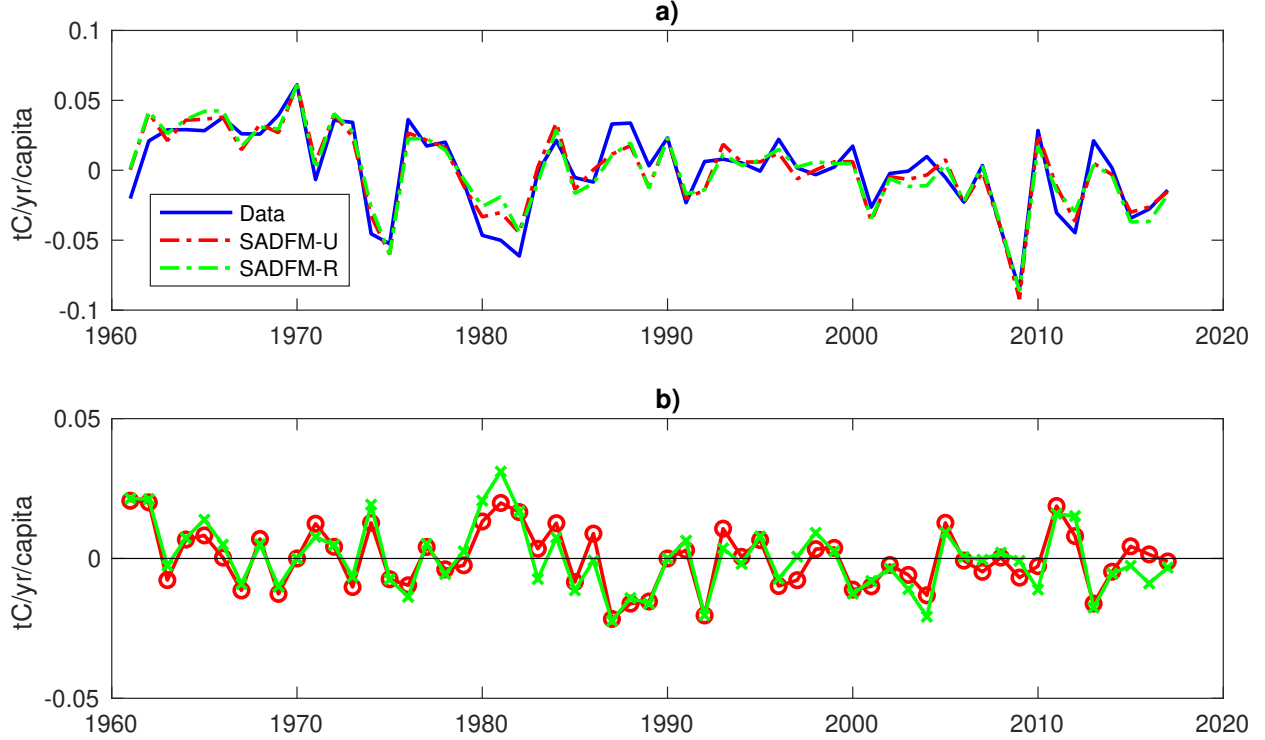


Figure 4: a): *In-sample fit of SADFM.* b): *Residuals from SADFM.*

$\beta_2$  for the two periods are very similar and statistically not different from each other. Hence our results can be viewed as rather stable in this respect. The estimates for the intercept  $\alpha$  appear to be larger in the second period when compared to the first period, providing tentative evidence that the base level of emission growth has increased slightly over time.

#### 5.4 Time-varying parameters for observation equation

Finally, as part of our in-sample analysis, we investigate the possibility of time-varying parameters in the observation equation for our SADFM-R specification, which is

$$y_t = \alpha + \beta_1 x_t^{(6)} + \beta_2 x_t^{(17)} + \gamma' z_t + \epsilon_t^y,$$

for  $t = 1, \dots, T$ . Due to technological changes in the fuel mix, the  $\beta_1$  and  $\beta_2$  coefficients may change over time. Also, technology changes that are not affecting economic variables but are still affecting CO<sub>2</sub> emissions may lead to changes in the intercept  $\alpha$ . To investigate these possible effects in our SADFM-R model, we replace  $\alpha$ ,  $\beta_1$  and  $\beta_2$  by the corresponding elements in the time-varying parameter vector  $\alpha_t^* = (\alpha_t, \beta_{1t}, \beta_{2t})'$ . The time-varying parameter (TVP) version of

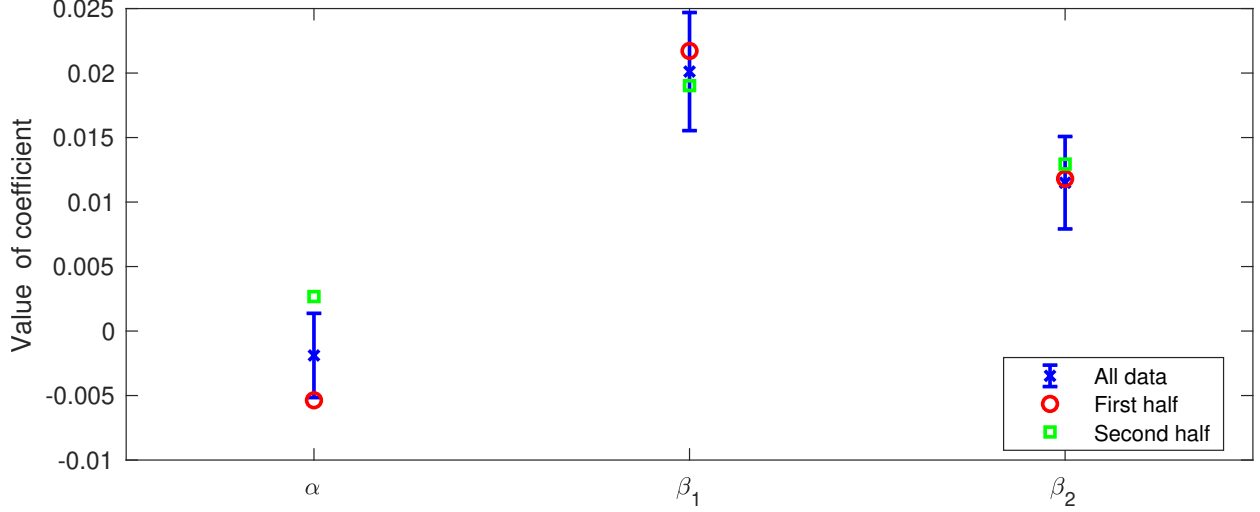


Figure 5: *Estimation of SADFM-R when the data are split in half ( $T_1 = T_2 = 28$  observations in each half). The first coefficient is the intercept, the second coefficient relates to  $x_t^{(6)}$ , and the third coefficient relates to  $x_t^{(17)}$ . The estimates using the full data set, along with 95% confidence bands, are given as blue crosses; the estimates from using only the first half of the data are given in red circles; the estimates from using only the second half of the data are given in green squares.*

the observation equation of the SADFM-R model is then given by the system of equations

$$y_t = (1, x_t^{(6)}, x_t^{(17)})\alpha_t^* + \gamma'z_t + \epsilon_t^y, \quad \alpha_{t+1}^* = \alpha_t^* + \kappa_t, \quad (5.1)$$

where  $\kappa_t$  is a  $3 \times 1$  disturbance vector with mean zero and diagonal variance matrix  $\Sigma_\kappa$ . In effect, we let the time-varying parameters follow independent random walk processes. The resulting TVP model can be viewed as a specific linear state space model with initial moment conditions  $E(\alpha_1) = 0$  and  $\text{Var}(\alpha_1) = d \cdot I_3$  where  $d$  is typically a large positive value, say  $d = 10^7$ ; see the discussion in [Durbin and Koopman \(2012, Chapters 4 and 5\)](#).

The estimation results for the time-varying parameters in the TVP model are presented in Figure 6. The overall fit of the TVP model appears to be highly satisfactory. The right-hand plot of Figure 6 presents the estimated time-varying paths for the three parameters in  $\alpha_t^*$ . We find clear evidence of time-varying behavior in some of the elements of  $\alpha_t^*$ . The intercept  $\alpha_t$  is upward trending. This confirms the earlier finding that the estimated intercept in the second half of the sample is larger than the one in the first half. The estimated time-varying coefficient for the IP index  $x_t^{(6)}$  reduces to a constant, fixed estimate in the full sample. The estimated time-varying coefficient for the residential utility production index  $x_t^{(17)}$  is varying somewhat over time. Its time-

varying path appears to oscillate. The in-sample evidence for time-varying parameter behavior in the SADF-M-R specification is therefore somewhat weak. Overall we may conclude that there is some evidence of a time-varying  $\alpha_t$  but not much evidence for time-varying  $\beta_{1t}$  and  $\beta_{2t}$  coefficients. Apart from the increasing general level of emissions captured by  $\alpha_t$ , technology developments do not seem to have changed the economy-emissions relationship noticeably in our in-sample analysis.

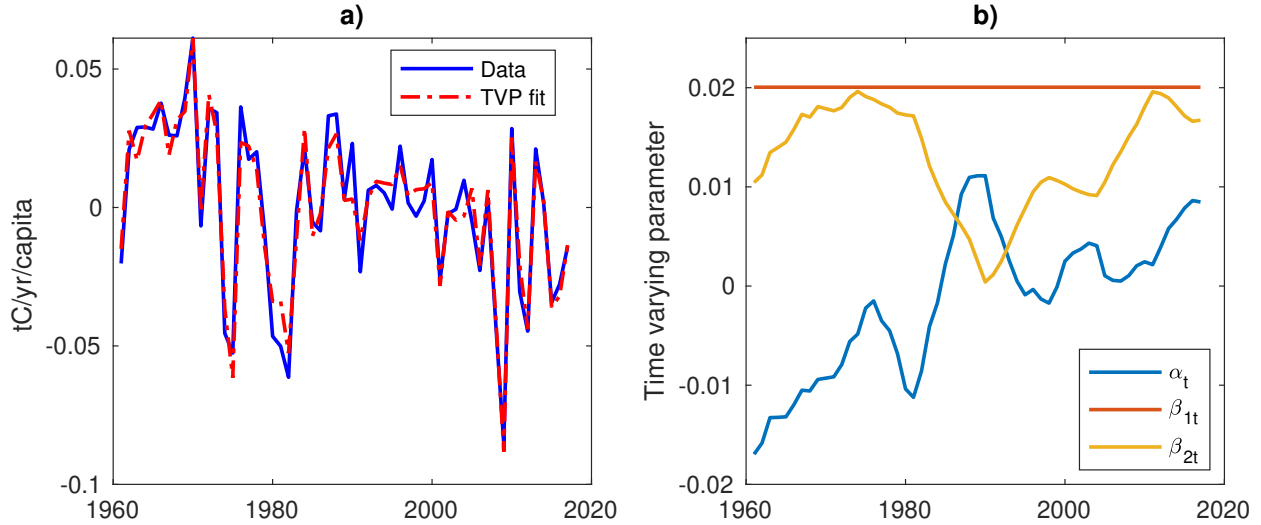


Figure 6: *TVP model. a): Fit of model. b): Smoothed estimates of  $\alpha_t$  and  $\beta_t$ .*

## 6 Out-of-sample Analysis

Dynamic factor models have been successfully applied to many macroeconomic forecasting problems (e.g. Stock and Watson, 2002; Brüning and Koopman, 2014) and nowcasting problems (e.g. Giannone et al., 2008; Hindrayanto et al., 2016). Given the good in-sample fit of the SADFMs when applied to CO<sub>2</sub> emissions data, this section investigates their out-of-sample performance.

Since the sample period is short, containing only  $T = 57$  observations, an out-of-sample experiment must necessarily be limited. Let  $h \geq 0$  denote the forecast horizon in years;  $h = 0$  corresponds to “nowcasting”, while  $h > 0$  corresponds to forecasting. We consider the in-sample period from  $t = 1960$  to  $t = 2001 - h$  and use an expanding window for estimation. That is, we estimate the models using the initial period from 1960 to  $2001 - h$ , containing  $42 - h$  observations, and then nowcast ( $h = 0$ ) or forecast ( $h > 0$ ) per capita emissions growth in year 2001. We then update the in-sample period and estimate the models using data from 1960 to  $2002 - h$  to nowcast/forecast per capita emissions growth in 2002, and so forth. For  $h = 0$  we do not use the emissions data in

the year we are nowcasting, but only the economic data. This setup ensures that the out-of-sample period covers 17  $h$ -step ahead forecasts of per capita CO<sub>2</sub> emissions growth from 2001 to 2017, regardless of the value of  $h$ .

We consider two different loss metrics: root mean squared error (RMSE) and mean absolute error (MAE):

$$RMSE_h = \sqrt{\frac{1}{17} \sum_{t=2001}^{2017} (\hat{y}_{t|t-h} - y_t)^2},$$

$$MAE_h = \frac{1}{17} \sum_{t=2001}^{2017} |\hat{y}_{t|t-h} - y_t|,$$

where  $\hat{y}_{t|t-h}$  denotes the forecast of  $y_t$  using information at time  $t - h$  for any given model. The results for this exercise with  $h = 0, 1, 2$  for the SADFM-R model are shown in Figure 7. The top row in the figure displays the raw losses  $RMSE_h$  and  $MAE_h$ ; the bottom row of the plot displays the corresponding losses as fractions of the corresponding losses from the constant growth model. The constant growth model simply predicts  $h$ -step ahead emissions to be the historical mean of  $y_t$  up to year  $t - h$ , that is  $\hat{y}_{t|t-h} = \frac{1}{t-h-1961+1} \sum_{t=1961}^{t-h} y_t$ . Numbers smaller than one indicate that the SADFM-R outperforms the constant growth model and vice versa for numbers greater than one. Figure 7 shows that for  $h = 0$  and  $h = 1$ , the SADFM-R model outperforms the constant growth model. For  $h = 2$ , the advantage disappears.

The results in Figure 7 indicate that the SADFM-R model is well-suited to nowcasting ( $h = 0$ ) and forecasting one year ahead ( $h = 1$ ). In relative terms, the SADFM-R improves on the constant growth model. It has 64% of the constant model's MAE and 67% of the RMSE when nowcasting and 20% of both measures when forecasting one year ahead. The following two sections take a closer look at the forecasting and nowcasting performance of our proposed models: Section 6.1 conducts an extended forecast exercise with more benchmark models than just the constant growth model; Section 6.2 examines the nowcasts from the various models considered in this paper and also takes a closer look at more recent nowcasts and compares with those from the Global Carbon Project.

## 6.1 Forecasting

Accurate forecasting of the future growth in country-level CO<sub>2</sub> emissions is important, since such forecasts can be used to gauge whether a country is on track to keep their emissions targets, for instance. This section conducts a pseudo-out-of-sample study to assess how the models for

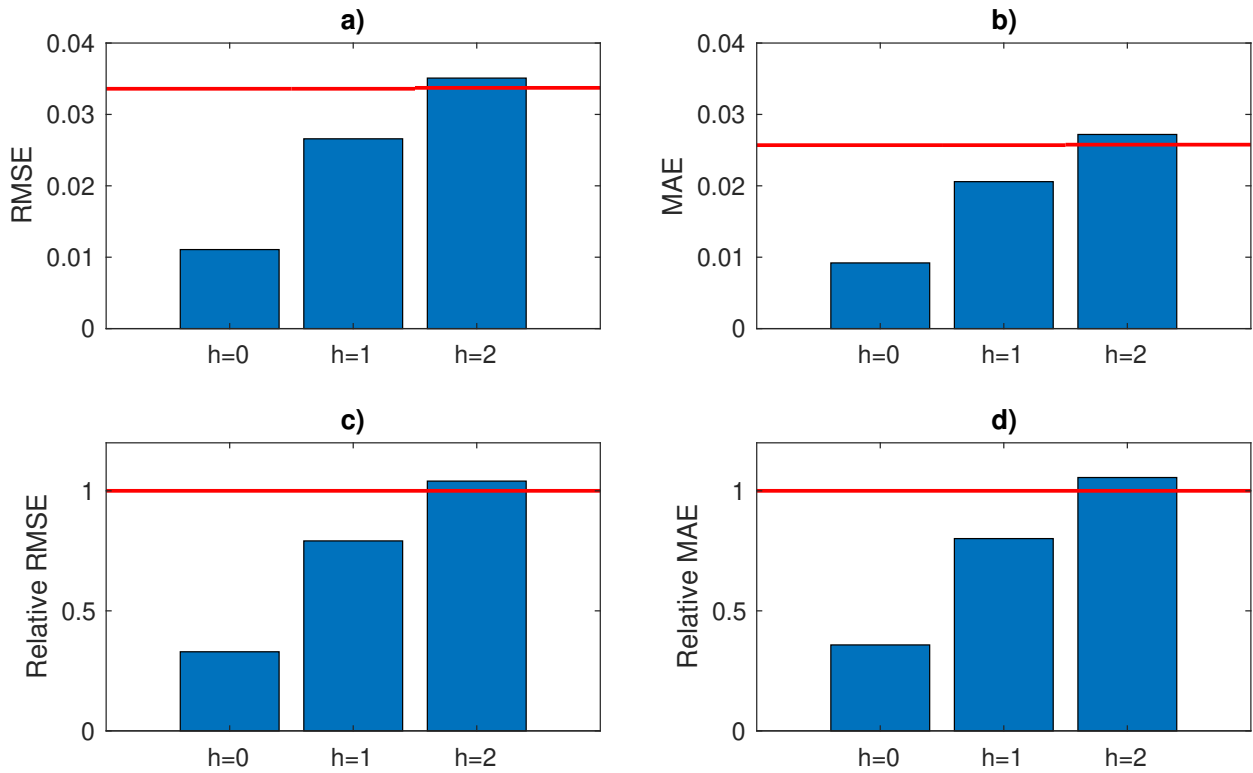


Figure 7: *Out-of-sample forecasting  $h$  years ahead for  $h = 0, 1, 2$ .  $h = 0$  is “nowcasting”. a): Root mean squared error (RMSE) of SADF-M-R. b): Mean absolute error (MAE) of SADF-M-R. c): RMSE of SADF-M-R as fraction of the RMSE of the constant growth model. d): MAE of SADF-M-R as fraction of the RMSE of the constant growth model. The red line shows the performance of the constant growth model.*



forecasting CO<sub>2</sub> emissions growth proposed above compare with standard univariate and multivariate alternatives. The main benchmark is the constant growth model (“Cst”), which predicts the historical mean of  $y_t$  up until the time of the forecast. Other alternative models are:

- RW: A random walk model. This model predicts  $y_{t+h}$  using the current level of CO<sub>2</sub> growth, i.e.  $y_t$ .
- AR1: An autoregressive model of order one. This model assumes  $y_t = a + \phi y_{t-1} + \epsilon_t$ , where  $a, \phi \in \mathbb{R}$  and  $\epsilon_t$  is zero-mean white noise.
- MA1: A moving-average model of order one. This model assumes  $y_t = a + \epsilon_t + \theta \epsilon_{t-1}$ , where  $a, \theta \in \mathbb{R}$  and  $\epsilon_t$  is zero-mean white noise.
- ARMA(1,1): An autoregressive-moving-average model of order (1,1). This model assumes  $y_t = a + \phi y_{t-1} + \epsilon_t + \theta \epsilon_{t-1}$ , where  $a, \phi, \theta \in \mathbb{R}$  and  $\epsilon_t$  is zero-mean white noise.

We also consider two VAR models, i.e.,

$$Y_t = a + \sum_{j=1}^p \Psi_j Y_{t-j} + \epsilon_t, \quad (6.1)$$

where  $Y_t = (y_t, x_t^{(6)}, x_t^{(17)})'$ , and  $x_t^{(6)}, x_t^{(17)}$  are the IP index and the IP: Residential Utilities index, respectively.

- VAR: The 3-dimensional VAR of (6.1).
- SVAR: The structural counterpart of (6.1). That is, here we consider the model  $B_1 Y_t = \Psi Y_{t-1} + \epsilon_t$ , with  $B_1$  similar, mutatis mutandis, to the  $B$  matrix considered in Section 3.

Inspired by the “direct” forecasting approach of [Stock and Watson \(2002\)](#), we also consider the predictive regression

$$y_t = a + b' F_{t-h} + \epsilon_t, \quad (6.2)$$

where  $F_t$  are principal components estimates of the economic factors. The numbers of factors,  $r$ , in  $F_t$  is determined by the  $p_2$  criterion of [Bai and Ng \(2002\)](#). The parameters  $(a, b)'$  can be estimated by OLS and the forecast of  $y_{t+h}$  is  $\hat{y}_{t|t-h} = \hat{a} + \hat{b}' F_{t-h}$ .

- PCA (all): This approach uses all  $r$  factors in the predictive regression (6.2).

- PCA ( $|t| > 1.96$ ): Same as “PCA (all)” but instead of using all  $r$  factors in (6.2), only the factors that have significant  $t$ -statistics at a 5% level are included.

The results from the forecasting experiment for  $h = 1$  are shown in Table 5. We again consider the two loss functions root mean squared error (RMSE) and mean absolute error (MAE). The loss numbers MAE and RMSE are fractions of the loss from the benchmark (constant growth) model. Thus, numbers less than one indicate that a particular model outperforms the benchmark. The factor-based models appear to be superior, improving on the benchmark by 11–21%. There is not much difference between the “direct” approaches and the “indirect” approaches. The large differences between the (S)VAR and the (SA)DFMs indicate that including many macroeconomic variables, summarized by the economic factors, helps forecasting  $x_t^{(i_1)}$  and  $x_t^{(i_2)}$  and, thus, forecasting  $y_t$ .

The asterisks in Table 5 denote whether a particular model, for the specified loss function, is included in the *Model Confidence Set* (MCS) of Hansen et al. (2011). Following Hansen et al. (2011), we consider the  $\alpha = 10\%$  and  $\alpha = 25\%$  MCS. Only the direct PCA ( $|t| > 1.96$ ) and SADF-M-R are in the 25% MCS for both RMSE and MAE, denoted by two asterisks. There are no models in the 10% MCS.

We also ran the forecasting experiment for  $h = 2$ . The results are omitted for brevity. As shown in Figure 7, the models considered here run out of forecasting power at this horizon. This resonates with the macroeconometric literature, where it is often found that economic time series are difficult to forecast far into the future (e.g., Giannone et al., 2008).

Lastly, we forecast 2019 changes in U.S. CO<sub>2</sub> emissions. For this exercise, we updated the economic data set to include 2018 data. Using these, the 2019 forecasts for the series  $x_t^{(6)}$  and  $x_t^{(17)}$  are  $-0.16$  and  $-0.27$ , respectively. These are studentized values; they correspond to an increase of 1.9 percent in IP and an increase in IP: Residential Utilities of 1.8 percent. Together with the estimated parameter vector  $(\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2) = (-.00150, .0200, .0117)$ , this results in a forecasted CO<sub>2</sub> emissions growth of  $-0.008$ , or about minus one percent.

## 6.2 Nowcasting

Estimating a variable of interest in period  $t$  while period  $t$  is still in progress is termed nowcasting in the macroeconomic literature. A common application in macroeconomics is nowcasting quarterly GDP growth, see, e.g., Giannone et al. (2008).

Table 5: *Out-of-sample forecasting experiment*

	Diagnostics (residuals)								
	RMSE	MAE	mean	std	skew	kurt	N	DW	LB
<i>Main benchmark</i>									
Cst	1.00	1.00	0.02	0.03	0.59	3.53	3.86	2.35	13.48
<i>Simple benchmarks</i>									
RW	1.26	1.27	0.00	0.04	-1.11	4.06	14.11	2.79	19.31
AR1	1.04	1.07	0.01	0.03	-0.64	3.53	4.42	2.69	16.46
MA1	1.01	1.08	0.01	0.03	-0.39	3.04	1.39	2.75	14.81
ARMA(1,1)	1.04	1.10	0.01	0.03	-0.54	3.36	3.05	2.76	14.62
<i>VAR methods</i>									
VAR	1.05	1.09	0.00	0.04	-0.52	2.82	2.56	2.71	18.73
SVAR	1.14	1.17	0.00	0.04	-0.73	2.93	5.04	2.79	19.79
<i>Direct methods</i>									
PCA (all)	0.84	0.91	0.01	0.03	-0.12	2.61	0.49	2.01	14.42
PCA ( $ t  > 1.96$ )	0.82**	0.87**	0.02	0.02	-0.08	2.60	0.43	2.39	16.97
<i>Indirect methods</i>									
DFM	0.81**	0.85	0.02	0.02	0.24	2.64	0.85	2.10	18.46
SADFM-U	0.81	0.86	0.02	0.02	0.30	2.77	0.95	2.11	18.46
SADFM-R	<b>0.79**</b>	<b>0.80**</b>	0.02	0.02	0.40	2.91	1.48	2.05	18.92

*Out-of-sample forecast exercise  $h = 1$  period ahead. Residuals are forecast errors. Boldface numbers indicate the models with lowest root mean squared error (RMSE) or mean absolute error (MAE). For RMSE and MAE, two stars indicate that the model is in the 25% MCS. RMSEs (MAEs) are fractions of the RMSE (MAE) of the main benchmarks.*

Table 6: *Out-of-sample nowcasting*

	Diagnostics (residuals)								
	RMSE	MAE	mean	std	skew	kurt	N	DW	LB
<i>Constant growth model</i>									
Cst	1.00	1.00	0.02	0.03	0.59	3.53	3.93	2.35	13.48
<i>Other models</i>									
DFM	0.60	0.63	0.00	0.02	0.05	2.18	1.60	1.94	10.07
SADFM-U	0.54**	0.49**	0.00	0.02	1.34	5.66	33.93	2.16	10.94
SADFM-R	<b>0.33**</b>	<b>0.36**</b>	-0.00	0.01	0.55	2.71	3.05	1.88	14.12
TVP	0.34**	<b>0.34**</b>	-0.00	0.01	1.14	3.26	12.53	1.77	12.58

*Out-of-sample nowcast exercise, i.e.  $h = 0$ . Residuals are forecast errors. Boldface numbers indicate the models with lowest root mean squared error (RMSE) or mean absolute error (MAE). For RMSE and MAE, two stars indicate that the model is in the 25% MCS. RMSEs (MAEs) are fractions of the RMSE (MAE) of the constant growth model.*

In calculating yearly CO<sub>2</sub> emissions, a country tallies consumption of energy carriers – use of coal, oil, gas, etc. – and calculates emissions implied by these statistics. Numbers on energy use in year  $t$  are generally not available until year  $t + 1$  (Le Quéré et al., 2015a, p. 55), introducing a lag in the reporting of country-level CO<sub>2</sub> emissions. Economic data used in this paper are generally published with a shorter lag than CO<sub>2</sub> emissions. We propose using our statistical framework to nowcast growth in CO<sub>2</sub> emissions in year  $t$  as soon as the economic data series  $x_t^{(i_1)}$  and  $x_t^{(i_2)}$  are available.

In Figure 7, we have already indicated the nowcasting performance of the SADFM-R compared to the constant growth model ( $h = 0$ ). Table 6 reports the results for the various models considered in this paper. All models outperform the constant growth model, in particular the SADFM-R and the TVP models appear to be superior. The SADFM-R, the TVP, and the SADFM-U models are in the 25% MCS. None of the other models are in the 10% or 25% MCS.

In the annual research report on the global carbon cycle, published by the Global Carbon Project (GCP, see e.g., Le Quéré et al., 2018b), the authors collect and maintain annual data on sources and sinks of global CO<sub>2</sub> emissions. Because official CO<sub>2</sub> emissions are reported with a lag,

the GCP paper published in year  $t$  contains a nowcast of year  $t$  emissions. These nowcasts are based on emissions estimates made by the U.S. Energy Information Administration (EIA) and on data on cement production from the United States Geological Survey (U.S.GS), see, e.g., [Le Quéré et al. \(2018b\)](#) p. 2168.

The GCP has been supplying nowcasts of U.S. emissions since 2015, so that we only have four data points to compare with. We note that our (SA)DFMs use economic data from the entirety of a given year  $t$ . Consequently, the GCP nowcasts are potentially constructed using less data than the ones from the (SA)DFM, because the GCP (and EIA and U.S.GS) might not have the full year  $t$  data set available for nowcasting. The nowcasting results should be interpreted in this light.

Figure 8 compares the (later) reported realized per capita emissions growth rates from 2015-2017 with the nowcasts made by the GCP, as published in [Le Quéré et al. \(2015b\)](#), [Le Quéré et al. \(2016\)](#), and [Le Quéré et al. \(2018a\)](#), and with those from our two preferred nowcasting models, the SADF-M-R and the TVP model. We adjust the GCP nowcasts using the realized growth rates of population in the U.S. to arrive at nowcasts comparable to our per capita nowcasts.<sup>12</sup> The nowcasts produced by the SADF-M-R and TVP model are very accurate for the three years 2015-2017. They appear to be closer to the eventual true value than the GCP nowcasts for 2015 and 2017, while for 2016, the GCP nowcast was better. These results come with the two caveats explained above: we only have three reliable data points, and our models use *all* economic data from year  $t$  to make the nowcast for year  $t$ , while the GCP nowcasts likely use less data.

We have not been able to obtain an official estimate of U.S. CO<sub>2</sub> emissions for the year 2018, since this number has not been reported on the UNFCCC web site at the time of writing.<sup>13</sup> This illustrates why nowcasting is a useful exercise: at the time of writing towards the end of 2019, the official U.S. estimate of CO<sub>2</sub> emissions in  $t = 2018$  is still not available. Figure 8 shows the GCP nowcast from 2018, published in [Le Quéré et al. \(2018b\)](#), and the nowcasts from our models. For this exercise, we updated the economic data set to include 2018 data. To convert the GCP nowcast into a per capita emissions growth rate, we adjust by the realized growth rate in U.S. population (0.6% in 2018). The GCP nowcasts a 2018 growth of 1.9% in per capita emissions, while the SADF-M-R and TVP model nowcast 2.3% and 4.1%, respectively.

Lastly, we nowcast the current year, 2019. We downloaded the most recent economic data set

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<sup>12</sup>If the GCP nowcast of emissions growth in year  $t$  is  $e_t^*$ , we report  $y_t^* = e_t^* - p_t$ , where  $p_t$  is the population growth in the U.S. in year  $t$ .

<sup>13</sup>See e.g. [https://di.unfccc.int/time\\_series](https://di.unfccc.int/time_series), accessed on November 18, 2019.

from the Federal Reserve Bank of St. Louis, which at the time of writing runs until September 2019.<sup>14</sup> From this data set, we use the two monthly data series on IP and IP: Residential Utilities. We average the values from January to September (9 observations with monthly frequency), to estimate yearly growth in the two annual series  $x_t^{(6)}$  and  $x_t^{(17)}$  in our models. These estimates are (in studentized values)  $-0.4095$  and  $-1.3698$ , respectively. This corresponds to 0.9 percent growth in IP and a 1.8 percent contraction in IP: Residential Utilities. With these, we can nowcast 2019 emissions growth from SADFM-R and the TVP model. The results are shown in Figure 8: With the estimated parameter vector  $(\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2) = (-.00150, .0200, .0117)$ , the SADFM-R model nowcasts that 2019 U.S. CO<sub>2</sub> emissions per capita will decline by approximately 2.6% in 2019; the TVP model nowcasts a decline of 2.2%.<sup>15</sup>

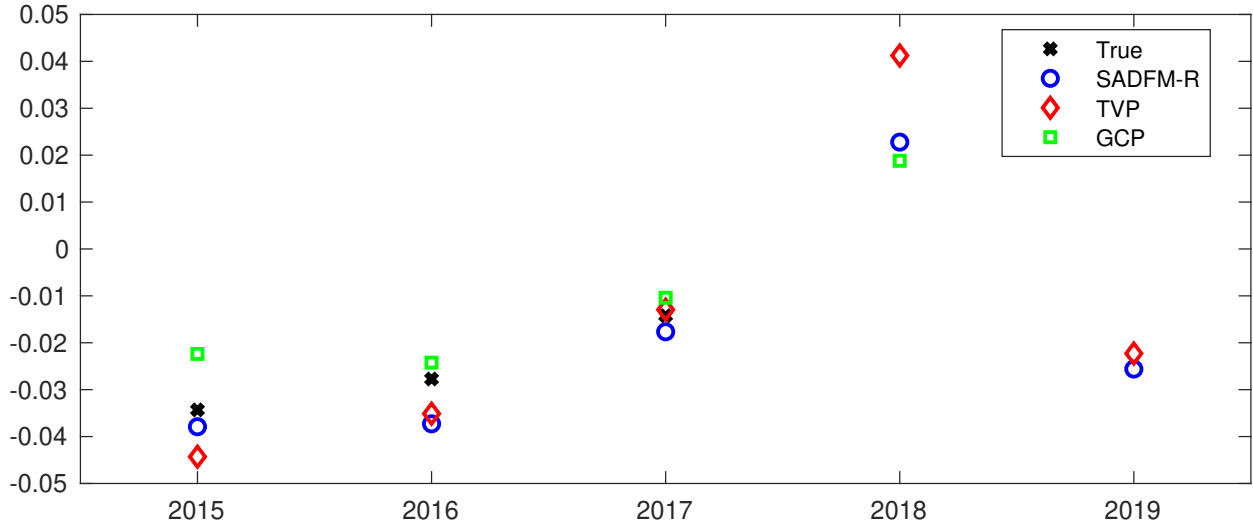


Figure 8: *Nowcasting U.S. CO<sub>2</sub> emissions per capita growth. Black crosses: True growth rates. Blue circles: Nowcast from SADFM-R. Red diamond: Nowcast from TVP model. Green square: Nowcast from the Global Carbon Project. At the time of writing, no official numbers of U.S. CO<sub>2</sub> emissions per capita growth for 2018 and 2019 are available.*

## 7 Conclusion

We proposed a structural augmented dynamic factor model (SADFM) for U.S. CO<sub>2</sub> emissions depending on macroeconomic activity. Emissions are best explained by contemporaneous industrial

<sup>14</sup><https://research.stlouisfed.org/econ/mccracken/fred-databases/>, downloaded on November 18, 2019.

<sup>15</sup>See the supplementary online material for a detailed description of the annualization and stationarity transformations.

production, both in manufacturing and in residential utilities. In order to forecast IP, we deployed a dynamic factor structure using a large set of annual time series on U.S. macroeconomic variables. The model has good in-sample and out-of-sample properties.

It is of course of high interest to extend this model to other countries and macroeconomic areas such as the European Union. We leave this to future research. With regard to the commonly employed GDP time series in explaining U.S. emissions, for example in integrated assessment exercises, we recommend to use industrial production indices that cover manufacturing and residential sectors instead.

Our model can be used for forecasting and nowcasting emissions. Using 2018 economic data (except emissions for 2018, which at the time of writing this paper were not yet available in the UNFCCC inventories), our model forecasts 2019 emissions to decrease by approximately 1%. Using data up to September 2019, the model nowcasts a decrease of 2.6% (2.2% for the model with time-varying parameters).

Other possible future generalizations of this model include allowing for a non-linear dependency of emissions on predictor variables (Auffhammer and Steinhauser, 2012) and using data of mixed frequency to further improve forecasting and nowcasting performance.

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## A Economic data

As explained in Section 2, we consider the  $T \times N$  matrix  $X$  of economic data. In our study, we have  $N = 226$  time series of length  $T = 57$  years. The  $N_{fed} = 126$  first data series are taken from the so-called “FRED-MD” database, compiled and maintained by the Federal Reserve Bank of St. Louis.<sup>16</sup> Detailed information on these data can be found in the Online Appendix to McCracken and Ng (2016).<sup>17</sup> In Table 7 we supply similar information concerning the remaining economic data series considered in this paper, namely  $N_{agri} = 18$  variables represent U.S. agricultural production<sup>18</sup>;  $N_{trade} = 76$  variables represent U.S. foreign trade<sup>19</sup>;  $N_{cement} = 2$  time series represent U.S. cement production<sup>20</sup>; and  $N_{transport} = 4$  represent U.S. transport<sup>21</sup>. Originally, the data set contained even more economic variables, but in a first step, we excluded all data series which contained more than 25% missing values, thus arriving at  $N = 226$  data series. Appendix B explains how we handle the remaining missing values.

The  $N_{agri} + N_{trade} = 18 + 76 = 94$  data series obtained from the World Bank are, like CO<sub>2</sub> emissions, recorded at a yearly frequency. The  $N_{fed} + N_{cement} + N_{transport} = 126 + 2 + 4 = 132$  data series obtained by the St. Louis Fed are originally all recorded at a monthly frequency. Since the emissions data used in this paper are available at a yearly frequency, we “aggregate” these economic data to a yearly frequency, arriving at the  $T = 57$  observations for each time series. The

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<sup>16</sup><https://research.stlouisfed.org/econ/mccracken/fred-databases/>, downloaded on May 7, 2019.

<sup>17</sup>[https://s3.amazonaws.com/files.fred.stlouisfed.org/fred-md/Appendix\\_Tables\\_Update.pdf](https://s3.amazonaws.com/files.fred.stlouisfed.org/fred-md/Appendix_Tables_Update.pdf), accessed on November 22, 2019.

<sup>18</sup>Collected from the World Bank website, <https://data.worldbank.org>, downloaded on September 19, 2019.

<sup>19</sup>Collected from the World Bank website, <https://data.worldbank.org>, downloaded on September 19, 2019.

<sup>20</sup>Collected from the website of the U.S. Federal Reserve Bank, <https://fred.stlouisfed.org>, downloaded on October 10, 2019.

<sup>21</sup>Collected from the website of the U.S. Federal Reserve Bank, <https://fred.stlouisfed.org>, downloaded on October 10, 2019.

method of aggregation might vary from time series to time series and is based on the nature of the series in question: for data in levels, we take the arithmetic mean over the 12 observations from January to December in year  $t$  to arrive at the year  $t$  observation for the data series; for data in monthly growth rates, we sum over the 12 observations from January to December in year  $t$  to arrive at the year  $t$  growth rate; for interest rates we take the geometric mean over the 12 observations from January to December in year  $t$  to arrive at the year  $t$  interest rate.

We then follow the procedure described in [McCracken and Ng \(2016\)](#) to transform the economic time series to a set of stationary variables. The exact method used to transform the a particular data series is based on the statistical properties of the series. If the series is positive, we take the natural logarithm. After this possible transformation, we examine the integration properties of the data to check whether they need to be differenced before they can be considered stationary. To be precise, we test for a unit root using the Augmented Dickey Fuller (ADF) test; if the null of a unit root is rejected, we stop; if the null is not rejected, we take first differences of the time series and perform another unit root test; we proceed until the null is rejected. In [Table 7](#) we give the transformation code (“tcode”) denoting what transformation has been applied to a particular time series. Again we follow the convention of transformation codes of the Online Appendix of [McCracken and Ng \(2016\)](#). For one time series, we differenced the series even though the null of a unit root was rejected by the ADF test; this decision was taken on the basis of graphical inspection of the time series and on our economic intuition. The transformation code for this time series is supplemented with an asterisk.

## B Dealing with missing values

The data matrix  $X$  constructed using the procedure described in the preceding paragraph contains a number of missing values. Of the  $N \cdot T = 226 \cdot 57 = 12882$  data points,  $M = 492$  (3.8%) are missing. There are only very few missing values in the  $N_{fed} = 126$  first economic data series based on the FRED-MD data set ( $M_{fed} = 24$ ); hence, most of the missing values come from the remaining 100 data series. Most of the missing values are located in the beginning of the data set.

Before the analyses described in the paper are performed, we impute the missing values using a variant of the iterative factor-based approach suggested in [Stock and Watson \(2002\)](#) and implemented by [McCracken and Ng \(2016\)](#).<sup>22</sup>

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<sup>22</sup>Our implementation of this procedure use the MATLAB code available at <https://research.stlouisfed.org/econ/mccracken/fred-databases/>, downloaded on February 1, 2019.

Table 7: The column “tcode” denotes the data transformation for a series  $x$  following McCracken and Ng (2016): (1) no transformation; (2)  $\Delta x_t$ ; (3)  $\Delta^2 x_t$ ; (4)  $\log x_t$ ; (5)  $\Delta \log x_t$ ; (6)  $\Delta^2 \log x_t$ ; (7)  $\Delta(x_t/x_{t-1} - 1)$ . The transformation code of one series was set manually by the authors; this is denoted by an asterisk.

Group 9: Agriculture				
id	tcode	Indicator name	description	
1	127	TX.VAL.AGRI.ZS.UN	Agricultural raw materials exports (% of merchandise exports)	
2	128	SP.RUR.TOTL.ZS	Rural population (% of total population)	
3	129	SP.RUR.TOTL.ZG	Rural population growth (annual %)	
4	130	SP.RUR.TOTL	Rural population	
5	131	AG.YLD.CREL.KG	Cereal yield (kg per hectare)	
6	132	AG.SRF.TOTL.K2	Surface area (sq. km)	
7	133	AG.PRD.LVSK.XD	Livestock production index (2004-2006 = 100)	
8	134	AG.PRD.FOOD.XD	Food production index (2004-2006 = 100)	
9	135	AG.PRD.CROP.XD	Crop production index (2004-2006 = 100)	
10	136	AG.PRD.CREL.MT	Cereal production (metric tons)	
11	137	AG.LND.TRAC.ZS	Agricultural machinery, tractors per 100 sq. km of arable land	
12	138	AG.LND.TOTL.K2	Land area (sq. km)	
13	139	AG.LND.CROP.ZS	Permanent cropland (% of land area)	
14	140	AG.LND.CREL.HA	Land under cereal production (hectares)	
15	141	AG.LND.ARBL.ZS	Arable land (% of land area)	
16	142	AG.LND.ARBL.HA.PC	Arable land (hectares per person)	
17	143	AG.LND.ARBL.HA	Arable land (hectares)	
18	144	AG.AGR.TRAC.NO	Agricultural machinery, tractors	

## Group 10: Trade

id	tcode	Indicator name	description
1	145	TX.VAL.TRVL.ZS.WT	Travel services (% of commercial service exports)
2	146	TX.VAL.SERV.CD.WT	Commercial service exports (current USD)
3	147	TX.VAL.OTHR.ZS.WT	Computer, communications and other services (% of commercial service exports)
4	148	TX.VAL.MRCH.WL.CD	Merch. exports by the reporting economy (current USD)
5	149	TX.VAL.MRCH.RS.ZS	Merch. exports by the reporting economy, residual (% of total merch. exports)
6	150	TX.VAL.MRCH.R6.ZS	Merch. exports to low- and middle-income in Sub-Saharan Africa (% of total merch. exports)
7	151	TX.VAL.MRCH.R5.ZS	Merch. exports to low- and middle-income in South Asia (% of total merch. exports)
8	152	TX.VAL.MRCH.R4.ZS	Merch. exports to low- and middle-income in Middle East & North Africa (% of total merch. exports)
9	153	TX.VAL.MRCH.R3.ZS	Merch. exports to low- and middle-income in Latin America & the Caribbean (% of total merch. exports)
10	154	TX.VAL.MRCH.R2.ZS	Merch. exports to low- and middle-income in Europe & Central Asia (% of total merch. exports)
11	155	TX.VAL.MRCH.R1.ZS	Merch. exports to low- and middle-income in East Asia & Pacific (% of total merch. exports)
12	156	TX.VAL.MRCH.OR.ZS	Merch. exports to low- and middle-income outside region (% of total merch. exports)
13	157	TX.VAL.MRCH.HI.ZS	Merch. exports to high-income (% of total merch. exports)
14	158	TX.VAL.MRCH.CD.WT	Merch. exports (current USD)
15	159	TX.VAL.MRCH.AL.ZS	Merch. exports to economies in the Arab World (% of total merch. exports)
16	160	TX.VAL.MRTL.ZS.UN	Ores and metals exports (% of merchandise exports)
17	161	TX.VAL.MANF.ZS.UN	Manufactures exports (% of merch. exports)
18	162	TX.VAL.INSF.ZS.WT	Insurance and financial services (% of commercial service exports)
19	163	TX.VAL.FUEL.ZS.UN	Fuel exports (% of merchandise exports)
20	164	TX.VAL.FOOD.ZS.UN	Food exports (% of merchandise exports)
21	165	TX.VAL.AGRI.ZS.UN	Agricultural raw materials exports (% of merchandise exports)
22	166	TM.VAL.TRVL.ZS.WT	Travel services (% of commercial service imports)
23	167	TM.VAL.SERV.CD.WT	Commercial service imports (current USD)
24	168	TM.VAL.OTHR.ZS.WT	Computer, communications and other services (% of commercial service imports)
25	169	TM.VAL.MRCH.WL.CD	Merch. imports by the reporting economy (current USD)
26	170	TM.VAL.MRCH.RS.ZS	Merch. imports by the reporting economy, residual (% of total merch. imports)
27	171	TM.VAL.MRCH.R6.ZS	Merch. imports from low- and middle-income in Sub-Saharan Africa (% of total merch. imports)
28	172	TM.VAL.MRCH.R5.ZS	Merch. imports from low- and middle-income in South Asia (% of total merch. imports)
29	173	TM.VAL.MRCH.R4.ZS	Merch. imports from low- and middle-income in Middle East & North Africa (% of total merch. imports)
30	174	TM.VAL.MRCH.R3.ZS	Merch. imports from low- and middle-income in Latin America & the Caribbean (% of total merch. imports)
31	175	TM.VAL.MRCH.R2.ZS	Merch. imports from low- and middle-income in Europe & Central Asia (% of total merch. imports)
32	176	TM.VAL.MRCH.R1.ZS	Merch. imports from low- and middle-income in East Asia & Pacific (% of total merch. imports)
33	177	TM.VAL.MRCH.OR.ZS	Merch. imports from low- and middle-income outside region (% of total merch. imports)
34	178	TM.VAL.MRCH.HI.ZS	Merch. imports from high-income economies (% of total merch. imports)
35	179	TM.VAL.MRCH.CD.WT	Merch. imports (current USD)
36	180	TM.VAL.MRCH.AL.ZS	Merch. imports from economies in the Arab World (% of total merch. imports)
37	181	TM.VAL.MANF.ZS.UN	Manufactures imports (% of merchandise imports)
38	182	TM.VAL.INSF.ZS.WT	Insurance and financial services (% of commercial service imports)

## Group 10: Trade (contd.)

id	tcode	Indicator name	description	
39	183	5	TM.VAL.INSF.ZS.WT	Fuel imports (% of merchandise imports)
40	184	5	TG.VAL.TOTL.GD.ZS	Merchandise trade (% of GDP)
41	185	5	NY.EXP.CAPM.KN	Exports as a capacity to import (constant LCU)
42	186	5	NE.TRD.GNFS.ZS	Trade (% of GDP)
43	187	2	NE.RSB.GNFS.ZS	External balance on goods and services (% of GDP)
44	188	2	NE.RSB.GNFS.CD	External balance on goods and services (current USD)
45	189	5	NE.IMP.GNFS.ZS	Imports of goods and services (% of GDP)
46	190	1	NE.IMP.GNFS.KD.ZG	Imports of goods and services (annual % growth)
47	191	5	NE.IMP.GNFS.KD	Imports of goods and services (constant 2010 USD)
48	192	5	NE.IMP.GNFS.CD	Imports of goods and services (current USD)
49	193	5	NE.EXP.GNFS.ZS	Exports of goods and services (% of GDP)
50	194	1	NE.EXP.GNFS.KD.ZG	Exports of goods and services (annual % growth)
51	195	5	NE.EXP.GNFS.KD	Exports of goods and services (constant 2010 USD)
52	196	5	NE.EXP.GNFS.CD	Exports of goods and services (current USD)
53	197	5	MS.MIL.XPRT.KD	Arms exports (SIPRI trend indicator values)
54	198	5	MS.MIL.MPRT.KD	Arms imports (SIPRI trend indicator values)
55	199	5	GC.TAX.IMPT.ZS	Customs and other import duties (% of tax revenue)
56	200	5	EG.IMP.CON.S.ZS	Energy imports, net (% of energy use)
57	201	5	BX.GSR.TRVL.ZS	Travel services (% of service exports, BoP)
58	202	5	BX.GSR.TOTL.CD	Exports of goods, services and primary income (BoP, current USD)
59	203	6	BX.GSR.NFSV.CD	Service exports (BoP, current USD)
60	204	5	BX.GSR.MRCH.CD	Goods exports (BoP, current USD)
61	205	5	BX.GSR.INSF.ZS	Insurance and financial services (% of service exports, BoP)
62	206	5	BX.GSR.GNFS.CD	Exports of goods and services (BoP, current USD)
63	207	5	BX.GSR.CMCP.ZS	Communications, computer, etc. (% of service exports, BoP)
64	208	5	BX.GSR.CCIS.ZS	ICT service exports (% of service exports, BoP)
65	209	5	BX.GSR.CCIS.CD	ICT service exports (BoP, current USD)
66	210	2	BN.GSR.MRCH.CD	Net trade in goods (BoP, current USD)
67	211	2	BN.GSR.GNFS.CD	Net trade in goods and services (BoP, current USD)
68	212	5	BM.GSR.TRVL.ZS	Travel services (% of service imports, BoP)
69	213	5*	BM.GSR.TRAN.ZS	Transport services (% of service imports, BoP)
70	214	5	BM.GSR.TOTL.CD	Imports of goods, services and primary income (BoP, current USD)
71	215	6	BM.GSR.NFSV.CD	Service imports (BoP, current USD)
72	216	5	BM.GSR.MRCH.CD	Goods imports (BoP, current USD)
73	217	5	BM.GSR.INSF.ZS	Insurance and financial services (% of service imports, BoP)
74	218	5	BM.GSR.GNFS.CD	Imports of goods and services (BoP, current USD)
75	219	5	BM.GSR.CMCP.ZS	Communications, computer, etc. (% of service imports, BoP)
76	220	5	BG.GSR.NFSV.GD.ZS	Trade in services (% of GDP)

## Group 11: Cement and transport

id	tcode	Indicator name	description	
1	221	5	IPN32732T9S	Industrial Production: Durable Goods: Concrete and product, Index 2012 = 100
2	222	5	IPN32731S	Industrial Production: Durable Goods: Cement, Index 2012 = 100
3	223	5	CES4300000001	All Employees: Transportation and Warehousing
4	224	5	CPITRNSL	Consumer Price Index: Transportation in U.S. City Average, All Urban Consumers
5	225	5	IPG3364T9S	Industrial Production: Durable manufacturing: Aerospace and miscellaneous transportation equipment
6	226	5	IPN32411AS	Industrial Production: Nondurable Goods: Aviation fuel and kerosene

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