DEPARTMENT OF ECONOMICS AND BUSINESS ECONOMICS
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Department of Economics and Business Economics
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 8716 5515

Long monthly temperature series and the Vector Seasonal Shifting Mean and Covariance Autoregressive model

Changli He*, Jian Kang*, Timo Teräsvirta†‡ and Shuhua Zhang*
*Tianjin University of Finance and Economics
†CREATES, Aarhus University
‡C.A.S.E., Humboldt-Universität zu Berlin

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Abstract

We consider a vector version of the Shifting Seasonal Mean Autoregressive model. The model is used for describing dynamic behaviour of and contemporaneous dependence between a number of long monthly temperature series for 20 cities in Europe, extending from the second half of the 18th century until mid-2010s. The results indicate strong warming in the winter months, February excluded, and cooling followed by warming during the summer months. Error variances are mostly constant over time, but for many series there is systematic decrease between 1820 and 1850 in April. Error correlations are considered by selecting two small sets of series and modelling correlations within these sets. Some correlations do change over time, but a large majority remains constant. Not surprisingly, the correlations generally decrease with the distance between cities, but geography also plays a role.

Keywords. Changing seasonality; nonlinear model; vector smooth transition autoregression;

JEL Classification Codes: C32; C52; Q54

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1 Introduction

Climate scientists have examined variations in the mean and to some extent dispersion in seasonal temperature averages over long periods of time. Variations by season in European temperatures based on station data have been considered, among others, by Luterbacher, Dietrich, Xoplaki, Grosjean and Wanner (2004), Xoplaki, Luterbacher, Paeth, Dietrich, Steiner, Grosjean and Wanner (2005), Casty, Wanner, Luterbacher, Esper and Böhm (2005) and Casty, Raible, Stocker, Wanner and Luterbacher (2007), to name a few. Long monthly temperature series have been analysed as well. Harvey and Mills (2003), Proietti and Hillebrand (2017) and He, Kang, Teräsvirta and Zhang (2019) built models for the well-known central England temperature (CET) series, as did Vogelsang and Franses (2005). Harvey and Mills (2003) modelled quarterly and annual versions of this series. Vogelsang and Franses (2005) also examined the long De Bilt series which is considered in this study as well, albeit in a somewhat shortened form. Among other things, Hillebrand and Proietti (2017) modelled a number of long, mostly European temperature series and built a separate structural time series model for each of them.

In this paper we shall construct a multivariate time series model called the Vector Seasonal Shifting Mean and Covariance Autoregressive (VSSMC-AR) model. The purpose of the model is to make it possible to examine monthly variations in the means, variances and correlations of 20 long monthly European time series extending, with some exceptions, over 250 years from the second half of the 18th century to the second half of the present decade. The motivation for this work is to obtain more detailed information about long-run temperature changes in Europe using these monthly time series.

To this end, we use the VSSMC-AR model to highlight salient features of these long temperature series. They include monthly means and, in particular, changes in them over time, and possible systematic changes in monthly error variances. Furthermore, and this may be new, correlations between the errors will be considered. The correlations represent contemporaneous exogenous effects or lack of them, influencing the series after the effects of changing monthly means and variances have been removed. They are assumed to be potentially time-varying as well, and when the model is estimated, some of them turn out to be just that.

The plan of the paper is as follows. The VSSMC-AR model is introduced in Section 2. The log-likelihood and the score of the model are presented in Section 3. Model specification, estimation and misspecification tests are discussed in Section 4. The dataset is presented in Section 5. Section 6 contains the empirical results. Conclusions can be found in Section 7. Appendix A contains the figures and Appendix B the estimated mean equations.

2 The model

The VSSMC-AR model is a multivariate generalisation of the Shifting Mean Autoregressive model with seasonally time-varying variances developed by He et al. (2019) and the Vector Shifting Mean Autoregressive model by Holt and Teräsvirta (in press). In this work we consider several series simultaneously and are also interested in contemporaneous links between them. The mean component of the N-dimensional VSSMC-AR model for unit or season s is defined as follows:

$$\mathbf{y}_{Sk+s} = \sum_{j=1}^{S} \boldsymbol{\delta}_j(\frac{Sk+j}{SK})D^{(j)}_{Sk+s} + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \mathbf{y}_{Sk+s-i} + \boldsymbol{\varepsilon}_{Sk+s} \tag{1}$$

where $\mathbf{y}_{Sk+s} = (y_{1,Sk+s}, ..., y_{N,Sk+s})'$, $D^{(j)}_{Sk+s} = 1$ when $j = s$, zero otherwise ($D^{(j)}_{Sk+s}$ is the jth seasonal dummy variable), $\boldsymbol{\varepsilon}_{Sk+s}$ is an $N \times 1$ error vector with mean zero (more of it later), and the roots of $|\mathbf{I} - \sum_{i=1}^{p} \boldsymbol{\Phi}_i z^i| = 0$ lie outside the unit circle. The vector $\boldsymbol{\delta}_j(\frac{Sk+j}{SK}) = (\delta_{1j}(\frac{Sk+j}{SK}), ..., \delta_{Nj}(\frac{Sk+j}{SK}))'$ is the vector of seasonal dummy variables. The jth time-varying coefficient $\delta_{nj}(\frac{Sk+j}{SK})$ of equation n equals

$$\delta_{nj}(\frac{Sk+j}{SK}) = \delta_{nj0} + \sum_{i=1}^{q_{nj}} \delta_{nji} g_{nji}(\frac{Sk+j}{SK}; \gamma_{nji}, \mathbf{c}_{nji}) \tag{2}$$

where

$$g_{nji}(\frac{Sk+j}{SK}; \gamma_{nji}, c_{nji}) = (1 + \exp\{-\gamma_{nji}(\frac{Sk+j}{SK} - c_{nji})\})^{-1} \tag{3}$$

or

$$g_{nji}(\frac{Sk+j}{SK};\gamma_{nji},c_{1nji},c_{2nji}) = 1-\exp\{-\gamma_{nji}(\frac{Sk+j}{SK}-c_{nji})^2\} \quad (4)$$

where $\gamma_{nji}>0$, $i=1,...,q_{nj}$. It follows that (1) at time $Sk+s$ may simply be written as

$$\mathbf{y}_{Sk+s} = \boldsymbol{\delta}_s(\frac{Sk+s}{SK}) + \sum_{i=1}^{p}\boldsymbol{\Phi}_i\mathbf{y}_{Sk+s-i} + \boldsymbol{\varepsilon}_{Sk+s}. \quad (5)$$

Assume, for notational simplicity, that the transitions in (2) are of type (3), that is, monotonic logistic functions. To fix notation, let $\boldsymbol{\theta}_{ns} = (\boldsymbol{\delta}'_{ns},\boldsymbol{\gamma}'_{ns},\mathbf{c}'_{ns})'$, where $\boldsymbol{\delta}_{ns} = (\delta_{ns0},\delta_{ns1},...,\delta_{nsr_s})'$ is an $(r_{ns}+1)\times 1$ vector, whereas $\boldsymbol{\gamma}_{ns} = (\gamma_{ns1},...,\gamma_{nsr_s})'$ and $\mathbf{c}_{ns} = (c_{ns1},...,c_{nsr_s})'$ for $n=1,...,N$ are $r_{ns}\times 1$ vectors. Let $\boldsymbol{\theta}_s = (\boldsymbol{\theta}'_{1s},...,\boldsymbol{\theta}'_{Ns})'$. The autoregressive component has the parameter matrix $\boldsymbol{\Phi} = (\boldsymbol{\Phi}_1,...,\boldsymbol{\Phi}_p)$, where $\boldsymbol{\Phi}_i = [\phi_{imn}]$, $m,n=1,...,N$ for $i=1,...,p$. Furthermore, setting $\boldsymbol{\phi} = \mathrm{vec}(\boldsymbol{\Phi})$, $\boldsymbol{\theta}_M = (\boldsymbol{\theta}'_1,...,\boldsymbol{\theta}'_S,\boldsymbol{\phi}')' \in \Theta_M$ is a vector containing all parameters in the mean part of (1). Finally, $\boldsymbol{\theta}^0_M = (\boldsymbol{\theta}^{0\prime}_1,...,\boldsymbol{\theta}^{0\prime}_S,\boldsymbol{\phi}^{0\prime})'$ is the corresponding true parameter vector.

The error process $\{\boldsymbol{\varepsilon}_{Sk+s}\}$ is assumed to have a seasonally time-varying covariance matrix. The error vector is decomposed as

$$\boldsymbol{\varepsilon}_{Sk+s} = \mathbf{V}_{Sk+s}\mathbf{z}_{Sk+s} \quad (6)$$

where $\mathbf{z}_{Sk+s} \sim \mathrm{iid}\mathcal{N}(\mathbf{0},\mathbf{P}_{Sk+s})$ and $\mathbf{P}_{Sk+s}$ is a seasonally time-varying correlation matrix. It follows that the covariance matrix $\mathbf{H}_{Sk+s} = \mathsf{E}\boldsymbol{\varepsilon}_{Sk+s}\boldsymbol{\varepsilon}'_{Sk+s} = \mathbf{V}_{Sk+s}\mathbf{P}_{Sk+s}\mathbf{V}_{Sk+s}$. The deterministic diagonal matrix $\mathbf{V}_{Sk+s} = \mathrm{diag}(\sigma_{1s}(\frac{Sk+s}{SK}), ..., \sigma_{Ns}(\frac{Sk+s}{SK}))$ contains the time-varying error standard deviations. The error variance of the nth equation at season s equals

$$\sigma^2_{ns}(\frac{Sk+s}{SK}) = \sum_{j=1}^{S}\sigma^2_{nj}(\frac{Sk+j}{SK})D^{(j)}_{Sk+s} \quad (7)$$

where

$$\sigma^2_{nj}(\frac{Sk+j}{SK}) = \sigma^2_{nj0} + \sum_{i=1}^{r_{nj}}\omega_{nji}g^{(v)}_{nji}(\frac{Sk+j}{SK};\gamma^{(v)}_{nji},c^{(v)}_{nji}) \quad (8)$$

with

$$g^{(v)}_{nji}(\frac{Sk+j}{SK};\gamma^{(v)}_{nji},c^{(v)}_{nji}) = (1+\exp\{-\gamma^{(v)}_{nji}(\frac{Sk+j}{SK}-c^{(v)}_{nji})\})^{-1} \quad (9)$$

or

$$g^{(v)}_{nji}(\frac{Sk+j}{SK};\gamma^{(v)}_{nji},c^{(v)}_{nji}) = 1-\exp\{-\gamma^{(v)}_{nji}(\frac{Sk+j}{SK}-c^{(v)}_{nji})^2\}. \quad (10)$$

In (9) and (10), $\gamma_{nji}^{(v)} > 0$, $i = 1, ..., r_{nj}$; $j = 1, ..., S$ and $n = 1, ..., N$. To guarantee positivity of each element in (8), $\sigma_{nj0}^2 > 0$ and $\sigma_{nj0}^2 + \sum_{i=1}^{k} \omega_{nji} > 0$ for $k = 1, ..., r_{nj}$, $n = 1, ..., N$, and $j = 1, ..., S$. This definition imposes restrictions on ω_{nji}, $i = 1, ..., r_{nj}$. For the individual season s it conforms to the one in Silvennoinen and Teräsvirta (2016). Let $\boldsymbol{\theta}_s^{(v)} = (\boldsymbol{\theta}_{1s}^{(v)\prime}, ..., \boldsymbol{\theta}_{Ns}^{(v)\prime})'$ be the vector of parameters of season s in the N equations, where $\boldsymbol{\theta}_{ns}^{(v)} = (\sigma_{ns0}^2, \boldsymbol{\omega}_{ns}', \boldsymbol{\gamma}_{ns}^{(v)\prime}, \mathbf{c}_{ns}^{(v)\prime})'$ with $\boldsymbol{\omega}_{ns} = (\omega_{ns1}, ..., \omega_{nsr_s})'$, $\boldsymbol{\gamma}_{ns}^{(v)} = (\gamma_{ns1}^{(v)}, ..., \gamma_{nsr_s}^{(v)})'$, and $\mathbf{c}_{ns}^{(v)} = (c_{1ns1}^{(v)}, ..., c_{1nsr_s}^{(v)})'$, for $n = 1, ..., N$. To complete the notation for the variance component, let $\boldsymbol{\theta}_V = (\boldsymbol{\theta}_1^{(v)\prime}, ..., \boldsymbol{\theta}_S^{(v)\prime})' \in \Theta_V$ contain all parameters in this component and let $\boldsymbol{\theta}_V^0 = (\boldsymbol{\theta}_1^{(v)0\prime}, ..., \boldsymbol{\theta}_S^{(v)0\prime})'$ be the corresponding true parameter vector.

The error correlation matrix for season s has the following form:

$$\mathbf{P}_{Sk+s} = \sum_{j=1}^{S} \{(1 - g_j^{(c)}(\frac{Sk+j}{SK}))\mathbf{P}_{(j1)} + g_j^{(c)}(\frac{Sk+j}{SK})\mathbf{P}_{(j2)}\} D_{Sk+s}^{(j)} \tag{11}$$

where $\mathbf{P}_{(j1)}$ and $\mathbf{P}_{(j2)}$ are $N \times N$ positive definite correlation matrices, $j = 1, ..., S$. Furthermore, in (11),

$$g_j^{(c)}(\frac{Sk+j}{SK}; \gamma_j^{(c)}, c_j^{(c)}) = (1 + \exp\{-\gamma_j^{(c)}(\frac{Sk+j}{SK} - c_j^{(c)})\})^{-1} \tag{12}$$

or

$$g_j^{(c)}(\frac{Sk+j}{SK}; \gamma_j^{(c)}, c_j^{(c)}) = 1 - \exp\{-\gamma_j^{(c)}(\frac{Sk+j}{SK} - c_j^{(c)})^2\}. \tag{13}$$

Since both (12) and (13) are bounded between zero and one, as a convex combination of two positive definite correlation matrices $\mathbf{P}_{Sk+s}$ is positive definite for all $Sk + s$; see, for example, Berben and Jansen (2005) or Silvennoinen and Teräsvirta (2005, 2015).

For further use, denote the lower-half stacked square matrix $\mathbf{A}$ by $\mathrm{vecl}(\mathbf{A})$, and let $\boldsymbol{\rho}_{si} = \mathrm{vecl}(\mathbf{P}_{(si)})$, $i = 1, 2$. Then $\boldsymbol{\theta}_s^{(c)} = (\boldsymbol{\rho}_{s1}', \boldsymbol{\rho}_{s2}', \gamma_s^{(c)}, c_s^{(c)})'$ contains all parameters in the correlation matrix $\mathbf{P}_{Sk+s}$. Finally, denote the vector of all correlation parameters by $\boldsymbol{\theta}_C = (\boldsymbol{\theta}_1^{(c)\prime}, ..., \boldsymbol{\theta}_S^{(c)\prime})' \in \Theta_C$ and the vector of true parameters by $\boldsymbol{\theta}_C^0 = (\boldsymbol{\theta}_1^{(c)0\prime}, ..., \boldsymbol{\theta}_S^{(c)0\prime})'$.

3 Log-likelihood and maximum likelihood estimation

Assuming normal and independent errors $\mathbf{z}_{Sk+s}$, the log-likelihood function of the model (the log conditional density of $\mathbf{z}_t$) for observation $Sk + s$ (ignoring the constant) becomes

$$
\begin{aligned}
&\ell_{Sk+s}(\mathbf{z}_{Sk+s}|\boldsymbol{\theta}_M,\boldsymbol{\theta}_V,\boldsymbol{\theta}_C;\mathcal{F}_{Sk+s-1}) = -(1/2)\ln|\mathbf{V}_{Sk+s}\mathbf{P}_{Sk+s}\mathbf{V}_{Sk+s}| \\
&-(1/2)\boldsymbol{\varepsilon}'_{Sk+s}\{\mathbf{V}_{Sk+s}\mathbf{P}_{Sk+s}\mathbf{V}_{Sk+s}\}^{-1}\boldsymbol{\varepsilon}_{Sk+s} \\
= \; & -\ln|\mathbf{V}_{Sk+s}| - (1/2)\ln|\mathbf{P}_{Sk+s}| - (1/2)\boldsymbol{\varepsilon}'_{Sk+s}\{\mathbf{V}_{Sk+s}\mathbf{P}_{Sk+s}\mathbf{V}_{Sk+s}\}^{-1}\boldsymbol{\varepsilon}_{Sk+s}
\end{aligned}
$$

where $\mathcal{F}_{Sk+s-1}$ contains the conditioning information at $Sk+s$, and $\boldsymbol{\varepsilon}_{Sk+s} = \mathbf{y}_{Sk+s} - \boldsymbol{\delta}_s(\frac{Sk+s}{SK}) - \sum_{i=1}^{p}\boldsymbol{\Phi}_i\mathbf{y}_{Sk+s-i}$. As $\mathbf{V}_{Sk+s}$ is diagonal, one may write

$$
\begin{aligned}
&\ell_{Sk+s}(\mathbf{z}_{Sk+s}|\boldsymbol{\theta}_M,\boldsymbol{\theta}_V,\boldsymbol{\theta}_C;\mathcal{F}_{Sk+s-1}) \\
= \; & -(1/2)\sum_{n=1}^{N}\sum_{i=1}^{r_{nsj}}\ln g_{nsi}^{(v)}(\frac{Sk+s}{SK};\gamma_{nsi}^{(v)},c_{nsi}^{(v)}) - (1/2)\ln|\mathbf{P}_{Sk+s}| \\
&-(1/2)\mathbf{z}'_{Sk+s}\mathbf{P}^{-1}_{Sk+s}\mathbf{z}_{Sk+s}.
\end{aligned}
\tag{14}
$$

Since the mean equations are estimated one by one, we present the score for equation n. In order to do that, we need the partial derivatives of $\boldsymbol{\delta}_{ns}(\frac{Sk+s}{SK})$ in (2). They are given in Lemma 1 of below:

Lemma 1 *(He et al., 2019)The partial derivatives*

$$
\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\boldsymbol{\theta}_{ns}} = (\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\delta_{ns0}},\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\delta_{ns1}},\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\gamma_{ns1}},\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial c_{ns1}})'
$$

are as follows: $\partial\delta_{ns}(\frac{Sk+s}{SK})/\partial\delta_{s0} = 1$, $\partial\delta_{ns}(\frac{Sk+s}{SK})/\partial\delta_{s1} = g_{ns1}$,

$$
\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\gamma_{ns}} = -\delta_{ns1}g_{ns1}(1-g_{ns1})(\frac{Sk+s}{SK}-c_{ns1})
$$

and

$$
\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial c_{ns}} = \delta_{ns1}\gamma_{ns1}g_{ns1}(1-g_{ns1}\}
$$

where $g_{ns1} = g_{ns1}(\frac{Sk+s}{SK};\gamma_{ns1},c_{ns1})$ *for short.*

Furthermore, $\partial\delta_{ns1}(\frac{Sk+s}{SK})/\partial\boldsymbol{\theta}_{nj} = \mathbf{0}$ for any $j \neq s$.

The average score of (14) for the mean parameters of equation n can also be borrowed from He et al. (2019):

Lemma 2 *The* $(4S+p)\times 1$ *average score function* $(1/SK)\partial L_{SK}(\boldsymbol{\theta},\boldsymbol{\varepsilon})/\partial\boldsymbol{\theta}$ *of the mean parameter block of equation* n *of (14) has the following form. The partial derivatives with respect to* $\boldsymbol{\theta}_{ns}$ *and* $\boldsymbol{\phi}_n$ *equal*

$$
\mathbf{s}_K(\boldsymbol{\theta}_{ns}) = \frac{1}{K}\sum_{k=0}^{K-1}\frac{\varepsilon_{n,Sk+s}}{\sigma^2_{ns}(\frac{Sk+s}{SK})}\frac{\partial\delta_{ns}(\frac{Sk+s}{SK})}{\partial\boldsymbol{\theta}_{ns}} \tag{15}
$$

for $s = 1, ..., S$, *and*

$$\mathbf{s}_{SK}(\boldsymbol{\phi}_n) = \frac{1}{SK}\sum_{k=0}^{K-1}\sum_{j=1}^{S}\frac{\varepsilon_{n,Sk+j}}{\sigma^2_{ns}(\frac{Sk+j}{SK})}\mathbf{y}_{Sk+j-1} \tag{16}$$

where $\mathbf{y}_{Sk+j} = (y_{Sk+j}, ..., y_{Sk+j-p+1})'$. *The elements of* $\partial\delta_{ns}(\frac{Sk+s}{SK})/\partial\boldsymbol{\theta}_{ns}$ *are defined in Lemma 1.*

When the mean parameters are estimated for the first time, $\sigma^2_{ns}(\frac{Sk+j}{SK}) = \sigma^2_{ns0}$ for $s = 1, ..., S$. In order to estimate the variance terms, we need the score for the variance parameters of equation n. They have been derived in Amado and Teräsvirta (2013) and are presented in the following lemma:

Lemma 3 *The partial derivatives*

$$\frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\boldsymbol{\theta}^{(v)}_{ns}} = (\frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\sigma^2_{ns0}}, \frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\omega_{ns1}}, \frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\gamma^{(v)}_{ns1}}, \frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial c^{(v)}_{ns1}})'$$

are as follows: $\partial\sigma^2_{ns}(\frac{Sk+s}{SK})/\partial\sigma^2_{s0} = 1$, $\partial\sigma^2_{ns}(\frac{Sk+s}{SK})/\partial\omega_{ns1} = g^{(v)}_{ns1}$,

$$\frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\gamma^{(v)}_{ns1}} = \omega_{ns1}g^{(v)}_{ns1}(1 - g^{(v)}_{ns1})(\frac{Sk+s}{SK} - c^{(v)}_{ns1})$$

and

$$\frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial c^{(v)}_{ns1}} = -\gamma^{(v)}_{ns1}\omega_{ns1}g^{(v)}_{ns1}(1 - g^{(v)}_{ns1}).$$

As in the mean component, $\partial\sigma^2_{ns}(\frac{Sk+s}{SK})/\partial\boldsymbol{\theta}^{(v)}_{nj} = \mathbf{0}$ for $j \neq s$. The average score for the variance component in equation n is given in the following lemma:

Lemma 4 *(He et al., 2019) The sth* 4×1 *block of the average score function* $(1/SK)\partial L_{SK}(\boldsymbol{\theta}_V, \boldsymbol{\varepsilon})/\partial\boldsymbol{\theta}_V$ *of the variance part of (14) for equation* n *assuming uncorrelated errors equals*

$$\mathbf{s}_K(\boldsymbol{\theta}^{(v)}_{ns}) = \frac{1}{2K}\sum_{k=0}^{K-1}(\frac{\varepsilon^2_{n,Sk+s}}{\sigma^2_{ns}(\frac{Sk+s}{SK})} - 1)\frac{1}{\sigma^2_{ns}(\frac{Sk+s}{SK})}\frac{\partial\sigma^2_{ns}(\frac{Sk+s}{SK})}{\partial\boldsymbol{\theta}^{(v)}_{ns}} \tag{17}$$

$s = 1, ..., S$, *where the elements of* $\partial\sigma^2_{ns}(\frac{Sk+s}{SK})/\partial\boldsymbol{\theta}^{(v)}_{ns}$ *are defined in Lemma 3.*

This result is applied when the error variances are estimated the first time. The resulting estimates serve as initial estimates for further iterations. In order to consider the joint average score function of the variance and correlation parameters, let

$$\mathbf{s}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = (\mathbf{s}_t'(\boldsymbol{\rho}_{s1}), \mathbf{s}_t'(\boldsymbol{\rho}_{s2}), \mathbf{s}_t(\gamma_s^{(c)}), \mathbf{s}_t(c_s^{(c)}))'. \tag{18}$$

denote the score vector for observation $Sk+s$ of the correlation block for season s. The form of the sub-blocks in (18) as well as the variance block under correlated errors are given in the following lemma, adopted from Silvennoinen and Teräsvirta (2017):

Lemma 5 *The average score for the sth 4×1 variance block of (14) for equation n has the following representation:*

$$\mathbf{s}_K(\boldsymbol{\theta}_{ns}^{(v)}) = \frac{1}{2K}\sum_{k=0}^{K-1} \frac{1}{\sigma_{ns}^2(\frac{Sk+s}{SK})} \frac{\partial \sigma_{ns}^2(\frac{Sk+s}{SK})}{\partial \boldsymbol{\theta}_{ns}^{(v)}} (\mathbf{z}_{Sk+s}'\mathbf{e}_n\mathbf{e}_n'\mathbf{P}_{Sk+s}^{-1}\mathbf{z}_{Sk+s} - 1) \tag{19}$$

where $\mathbf{e}_i = (\mathbf{0}_{i-1}', 1, \mathbf{0}_{N-i}')'$, $i = 1, ..., N$. The sth $(N(N-1)/2+2) \times 1$ correlation block equals

$$\mathbf{s}_K(\boldsymbol{\theta}_s^{(c)}) = -\frac{1}{2K}\sum_{k=0}^{K-1} \frac{\partial vec(\mathbf{P}_{Sk+s})'}{\partial \boldsymbol{\theta}_s^{(c)}} \{vec(\mathbf{P}_{Sk+s}^{-1}) - (\mathbf{P}_{Sk+s}^{-1} \otimes \mathbf{P}_{Sk+s}^{-1}) vec(\mathbf{z}_{Sk+s}\mathbf{z}_{Sk+s}')\}.$$

Dividing $\partial vec(\mathbf{P}_{Sk+s})/\partial \boldsymbol{\theta}_s^{(c)}$ into four sub-blocks as follows:

$$\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \boldsymbol{\theta}_s^{(c)}} = \left[\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \boldsymbol{\rho}_{s1}'}, \frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \boldsymbol{\rho}_{s2}'}, \frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \gamma_s^{(c)}}, \frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial c_s^{(c)}}\right]$$

the two $N^2 \times N(N-1)/2$ blocks are

$$\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \boldsymbol{\rho}_{s1}'} = (1 - g_s^{(c)}) \frac{\partial vec(\mathbf{P}_{(1)})}{\partial \boldsymbol{\rho}_{s1}'} \tag{20}$$

$$\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \boldsymbol{\rho}_{s2}'} = g_s^{(c)} \frac{\partial vec(\mathbf{P}_{(2)})}{\partial \boldsymbol{\rho}_{s2}'} \tag{21}$$

and the two $N^2 \times 1$ vectors have the form

$$\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial \gamma_s^{(c)}} = g_s^{(c)}(1 - g_s^{(c)})(\frac{Sk+s}{SK} - c_s^{(c)}) vec(\mathbf{P}_{(2)} - \mathbf{P}_{(1)})$$

and

$$\frac{\partial vec(\mathbf{P}_{Sk+s})}{\partial c_s^{(c)}} = -\gamma_s^{(c)} g_s^{(c)}(1 - g_s^{(c)}) vec(\mathbf{P}_{(2)} - \mathbf{P}_{(1)}).$$

The difference between (17) and (19) is that in the latter, the time-varying seasonal correlations $\mathbf{P}_{Sk+s}$ are properly accounted for. The expression (17) is nevertheless needed in the estimation; see Section 4.3.1.

4 Modelling

4.1 General idea

The VSSMC-AR model is nonlinear in the mean, error covariances and correlations. This requires care in specifying the model. It is seen from (2) that the nonlinear seasonal intercept nests a linear (constant) one. This implies that if the intercept has a constant coefficient for equation n and season s, say, the nonlinear model is not identified. Because of this, constancy of the seasonal intercepts has to be tested equation by equation and season by season before fitting a VSSMC-AR model to the data. More generally, the number of transitions in (2) has to be determined before proceeding to estimating the mean component and the same number in (8) before estimating the error variances. Note that these numbers can equal zero. A similar argument applies to correlations.

The parameters in the mean component of (1) are estimated equation by equation, assuming that the errors are iid and that the error covariance matrix is a diagonal matrix. After this has been done, the model is tested for serial correlation in the errors using a test that is robust against heteroskedasticity. This is done equation by equation because the error covariance matrix is at this stage assumed diagonal. If the null hypothesis is rejected, the lag structure of the model has to be respecified and the test performed again. The test is described in Section 4.2.2.

After the mean component has been estimated, constancy of the monthly error variances is tested season by season against (8) with one transition. As seen from (8) and (9) (or (10)), the error variance with one transition is not identified if the transition does not exist. As a result, even this testing situation is nonstandard and considered in more detail in Section 4.2.3. Testing proceeds by adding one transition at a time until the first non-rejection. The error variances are then estimated season by season from the residuals. The monthly components whose square roots form the main diagonal of matrix $\mathbf{V}_{Sk+s}$ in the error decomposition (6) are completely orthogonal to each other.

The error variances are estimated without regard to the correlations. Once this has been done, the next step is to test constancy of the correlation matrix $\mathbf{P}_{Sk+s}$ season by season, that is, for $s = 1, ..., S$. Since this matrix is not identified under the null hypothesis $\mathbf{P}_{Sk+s} = \mathbf{P}_s$, the testing situation is again nonstandard. Testing this null hypothesis is studied in Section 4.2.4. After this has been done, the variances and correlations are estimated jointly. This is discussed in Section 4.3.

4.2 Specifying and testing the VSSMC-AR model

4.2.1 Testing constancy of the intercept and determining the number of transitions

The first step in building the VSSMC-AR model is to test constancy the coefficients of seasonal dummies against the alternative that there is one transition, $q_{nj} = 1$ in (2). This can be done equation by equation and season by season. For equation n and season s, the null hypothesis is $\gamma_{ns1} = 0$. This is a nonstandard testing situation because the model is not identified (δ_{ns1} and c_{ns1} are unidentified nuisance parameters) when the null hypothesis holds. The test is based on approximating the alternative by a polynomial as in Luukkonen, Saikkonen and Teräsvirta (1988). For details, see He et al. (2019). If the null hypothesis is rejected, the choice is made between (3) and (4), and the equation with one transition estimated. This is done for all n equations. Next, the null hypothesis of one transition is tested against two transitions. The test is a slightly generalised version of the previous test, see, for example, Teräsvirta, Tjøstheim and Granger (2010, Chapter 16). The tests are robustified against heteroskedasticity as in Wooldridge (1990). Testing and estimation continue until the null hypothesis of no more transitions is no longer rejected.

4.2.2 Testing the hypothesis of no serial correlation in errors

The test of no error autocorrelation is a standard test for the errors in nonlinear models; see for example Teräsvirta, Tjøstheim and Granger (2010, Chapter 5). A robust version is needed because the errors are heteroskedastic. The test against autocorrelation of order r is carried out for each equation separately. As Wooldridge (1990) showed, it may be carried out in the 'TR^2-form' as follows.

1. Estimate the model (1), save the residual vectors $\widehat{\boldsymbol{\varepsilon}}_{Sk+s} = (\widehat{\varepsilon}_{1,Sk+s}, ..., \widehat{\varepsilon}_{N,Sk+s})'$, $s = 1, ..., S$; $k = 0, 1..., K-1$.

2. For equation n, regress 1 on the residuals $\widehat{\varepsilon}_{n,Sk+s-1}, ..., \widehat{\varepsilon}_{n,Sk+s-r}$ and the gradient vector

$$G_{n,Sk+s}(\boldsymbol{\theta}_{ns}, \boldsymbol{\phi}_n) = (\widehat{\varepsilon}_{n,Sk+s}\frac{\partial\widehat{\delta}_{ns}(\frac{Sk+s}{SK})}{\partial\boldsymbol{\theta}'_{ns}}, \widehat{\varepsilon}_{n,Sk+s}\mathbf{y}'_{n,Sk+s-1})'$$

   (the constant variance can be ignored). Save the residuals $\widehat{\delta}_{n,Sk+s}$ and compute the sum of squared errors $SSR_{n1} = \sum_{k=0}^{K-1}\sum_{s=1}^{S}\widehat{\delta}^2_{n,Sk+s}$.

3. Compute the test statistic $\mathcal{T}_n = SK - SSR_{n1}$.

Under H_0 of no error autocorrelation of order r, the statistic $\mathcal{T}_n$ has an asymptotic χ^2-distribution with r degrees of freedom.

4.2.3 Testing constancy of error variances

After estimating the seasonal means assuming normal and independent errors, constancy of seasonal error variances is tested equation by equation and season by season, assuming that the errors are not seasonally contemporaneously correlated. In order to test constancy in equation n and season s against (at least) one transition in (8) with $j = s$, that is, $r_{ns} = 1$. The null hypothesis is H_0: $\gamma_{ns1}^{(v)} = 0$, so $\delta_{ns1}^{(v)}$, and $c_{ns1}^{(v)}$ are unidentified nuisance parameters when this hypothesis holds. The test is set up along the lines in He et al. (2019). For notational simplicity we consider the case where the transition function equals is logistic, see (9). The logistic function is expanded into a Taylor series around $\gamma_{ns1}^{(v)} = 0$ and reparameterised accordingly. Assuming $\omega_{ns1} \neq 0$ and choosing the third-order polynomial expansion one obtains

$$\sigma_{ns}^2(\frac{Sk+s}{SK}) = \alpha_{ns0}^{(v)} + \alpha_{ns1}^{(v)} \frac{Sk+s}{SK} + \alpha_{ns2}^{(v)}(\frac{Sk+s}{SK})^2 + \alpha_{ns3}^{(v)}(\frac{Sk+s}{SK})^3 + R_{n3,Sk+s} \tag{22}$$

where, under $\mathrm{H}_0, \alpha_{ns0}^{(v)} = \sigma_{ns0}^2$, and $R_{n3,Sk+s}$ is the remainder. Since $\alpha_{nsi}^{(v)} = \gamma_{ns1}^{(v)} \widetilde{\omega}_{nsi}^{(v)}$, where $\widetilde{\omega}_{nsi}^{(v)} \neq 0$, $i = 1, 2, 3$, the new null hypothesis equals H_0': $\alpha_{ns1}^{(v)} = \alpha_{ns2}^{(v)} = \alpha_{ns3}^{(v)} = 0$. Under this hypothesis, $R_{n3,Sk+s} = 0$, and because we are considering a Lagrange multiplier test, the remainder does not affect the inference. If the null hypothesis is rejected, the model with a seasonally time-varying error variance is estimated. Under H_0', the resulting test statistic has an asymptotic χ^2-distribution with three degrees of freedom. The test is repeated for all seasons $s = 1, ..., S$.

If the null hypothesis is rejected and the error variance with one transition estimated, the next step is to test one transition against two. Testing and estimation continue until the first non-rejection of the null hypothesis. Testing r_{ns} transitions against $r_{ns} + 1$ relies on a Taylor series approximation of the $(r_{ns} + 1)$st transition function similar to the one in (22).

4.2.4 Testing constancy of correlations

Following estimation of the error variances in $\mathbf{V}_{Sk+s}^2 = \mathrm{diag}(\sigma_{1s}^2(\frac{Sk+s}{SK}), ..., \sigma_{Ns}^2(\frac{Sk+s}{SK}))$, a constant correlation matrix $\widehat{\mathbf{P}}_s = \mathrm{corr}(\widehat{\mathbf{z}}_{Sk+s})$ is obtained for each $s = 1, ..., S$. The next step is to test constancy of the correlations in (11) using $\widehat{\mathbf{P}}_s$. As discussed in Silvennoinen and Teräsvirta (2005, 2015, 2017), the time-varying correlation model is only identified under the alternative, which invalidates the standard asymptotic inference. As already done in the case of the mean and the variance components, the identification problem is circumvented by approximating the transition function (12) or (13) by its Taylor expansion around the null hypothesis, H_0: $\gamma_s^{(c)} = 0$.

The test can be constructed along the lines presented in Hall, Silvennoinen and Teräsvirta (2019), see also the appendix of Silvennoinen and Teräsvirta (2005). The only difference is that because of orthogonality of the seasonal covariances, the test is carried out separately for each season. To derive the test statistic, consider the first-order Taylor expansion of (13) around $\gamma_s^{(c)} = 0$, assuming that the test is against (11) with (13). It has the following form:

$$\begin{aligned} g_s^{(c)}(\frac{Sk+s}{SK};\gamma_s^{(c)},c_s^{(c)}) &= 1-\exp\{-\gamma_s^{(c)}(\frac{Sk+s}{SK}-c_s^{(c)})^2\} \\ &= \frac{1}{2}(\frac{Sk+s}{SK}-c_s^{(c)})^2\gamma_s^{(c)} + R_1(\frac{Sk+s}{SK};\gamma_s^{(c)}) \end{aligned} \quad (23)$$

where $R_1(\frac{Sk+s}{SK};\gamma_s^{(c)})$ is the remainder. Using (23), (11) becomes

$$\begin{aligned} \mathbf{P}_{Sk+s} &= \mathbf{P}_{(s1)} + \{\frac{1}{2}(\frac{Sk+s}{SK})^2 - c_s^{(c)}\frac{Sk+s}{SK} + \frac{1}{2}(c_s^{(c)})^2\}\gamma_s^{(c)}(\mathbf{P}_{(s2)} - \mathbf{P}_{(s1)}) \\ &= \{\mathbf{P}_{(s1)} + \frac{1}{2}(c_s^{(c)})^2\gamma_s^{(c)}(\mathbf{P}_{(s2)} - \mathbf{P}_{(s1)})\} \\ &\quad +(\frac{1}{2}(\frac{Sk+s}{SK})^2 - c_s^{(c)}\frac{Sk+s}{SK})(\mathbf{P}_{(s2)} - \mathbf{P}_{(s1)})\gamma_s^{(c)} \\ &\quad +(\mathbf{P}_{(s2)} - \mathbf{P}_{(s1)})R_1(\frac{Sk+s}{SK};\gamma_s^{(c)}) \\ &= \mathbf{P}_{(As0)} + (\frac{Sk+s}{SK})\mathbf{P}_{(As1)} + (\frac{Sk+s}{SK})^2\mathbf{P}_{(As2)} \\ &\quad +R_1(\frac{Sk+s}{SK};\gamma_s^{(c)})(\mathbf{P}_{(s2)} - \mathbf{P}_{(s1)}) \end{aligned}$$

where $\mathbf{P}_{(s1)} \neq \mathbf{P}_{(s2)}$. The main diagonals of $\mathbf{P}_{(As1)}$ and $\mathbf{P}_{(As2)}$ consist of zeroes. Setting $\boldsymbol{\rho}_{As} = (\boldsymbol{\rho}'_{As0}, \boldsymbol{\rho}'_{As1}, \boldsymbol{\rho}'_{As2})'$, where $\boldsymbol{\rho}_{Asi} = \mathrm{vecl}(\mathbf{P}_{(Asi)})$, $i = 0, 1, 2$, the new null hypothesis is H'_0: $\boldsymbol{\rho}_{As1} = \boldsymbol{\rho}_{As2} = \mathbf{0}_{N(N-1)/2}$. This is because $\boldsymbol{\rho}_{Asi} = \gamma_s^{(c)}\widetilde{\boldsymbol{\rho}}_{Asi}$, where $\widetilde{\boldsymbol{\rho}}_{Asi} \neq \mathbf{0}$, for $i = 1, 2$. The resulting test statistic has an asymptotic χ^2-distribution with $N(N-1)$ degrees of freedom under H'_0; for details, see Hall et al. (2019). In Section 6.3, tests based on the first-, second- and third-order polynomials are applied.

If none of the S null hypotheses is rejected, it is possible to test the hypothesis that the correlations are the same for $s = 1, ..., S$. The null hypothesis is H_0: $\mathbf{P}_s = \mathbf{P}$, $s = 1, ..., S$. This can for example be done using a likelihood ratio test in which the number of degrees of freedom equals $(S-1)N(N-1)/2$.

4.3 Estimation

4.3.1 Iterations

Since the mean and the covariance components do not contain common parameters, the estimation can be divided into two main steps as suggested by Sargan (1964). First estimate the mean parameters and then, conditionally on the mean, the variance parameters and the correlations. Iterate until convergence. Both the mean, the variance and the correlations are nonlinear and have to be estimated iteratively. In the application, $\mathbf{\Phi}_i$, $i = 1, ..., p$, are assumed diagonal, because for monthly temperature series it is reasonable to exclude 'spillovers' from one weather station to others. The main advantage of this restriction is a reduction in the number of parameters to be estimated. When the mean is estimated the first time, it is done under the assumption that $\mathbf{H}_{Sk+s} = \sigma_s^2 \mathbf{I}_N$. He et al. (2019) showed that under normality and regularity conditions, the maximum likelihood estimators are consistent and asymptotically normal.

After estimating the mean parameters, seasonal covariance components can be estimated separately for each $s = 1, ..., S$. This is the case because the seasonal error covariances are completely orthogonal to each other; see (7) and (11). Estimation is carried out using a simplified version of the algorithm in Silvennoinen and Teräsvirta (2017). In that paper, the error variances also contain conditional heteroskedasticity, which is not a concern here.

For season s, estimation proceeds as follows.

1. Estimate the parameters in $\boldsymbol{\theta}_s^{(v)}$, $s = 1, ..., S$, equation by equation, assuming $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{I}_N$, where $\boldsymbol{\theta}_s^{(c)}$ is the vector of correlation parameters for season s. Denote the estimate $\widehat{\boldsymbol{\theta}}_s^{(v,1,1)} = (\widehat{\boldsymbol{\theta}}_{1s}^{(v,1,1)\prime}, ..., \widehat{\boldsymbol{\theta}}_{Ns}^{(v,1,1)\prime})'$. This means that the deterministic components $g_{ns}^{(v)}(\frac{Sk+s}{SK}; \gamma_{nsi}^{(v)}, c_{nsi}^{(v)})$, $n = 1, ..., N$, have been estimated once.

2. Estimate $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$ given $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,1,1)}$. This requires a separate iteration because $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$ is nonlinear in parameters, see (8) and (11). Denote the estimate $\mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,1,1)})$.

3. Re-estimate $\boldsymbol{\theta}_s^{(v)}$ assuming $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,1,1)})$. This yields $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,1,2)}$. Then re-estimate $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$ given $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,1,2)}$. Iterate

Station	Latitude	Longitude	Elevation, m	Years
Berlin	52°31'0"	13°23'20"	34	1756-2015
Brno-Turany	49°12'	16°37'	237	1772-2015
Budapest	47°29'33"	19°03'5"	102	1780-2015
Copenhagen	55°40'34"	12°34'6"	9	1798-2014
De Bilt	52°10'	5°18'	15	1750-2017
Hohenpeissenberg	47°48'	11°0'	780	1781-2015
Innsbruck	47°16'	11°23'	574	1777-2016
Karlsruhe	49°00'33"	8°24'14"	115	1779-2015
Kremsmünster	48°03'18"	14°7'51"	384	1767-2016
Milan	45°28'18"	9°11'	120	1763-2012
Munich	48°08'	11°34'	520	1781-2015
Paris	48°51'24"	2°21'3"	34	1757-2000
Regensburg	49°1'	12°5'	338	1773-2015
Stockholm	59°19'46"	18°4'7"	15	1756-2015
Stuttgart	48°47'	9°11'	245	1792-2015
Trondheim	63°25'47"	10°23'36"	115	1761-1981
Uppsala	59°51'63"	17°38'44"	15	1756-2017
Vienna	48°12'	16°22'	170	1775-2016
Vilnius	54°41'	25°17'	124	1777-2015
Warsaw	52°14'	21°1'	93	1779-2015

Table 1: Location of stations and time span for the long monthly average temperature series in the sample

until convergence. Let the result after R_1 iterations be $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,1,R_1)}$ and $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,1,R_1)})$. The resulting estimates are maximum likelihood ones.

As already mentioned, this procedure is repeated S times in order to obtain the estimates for all seasons. In the application, however, the series are not equally long. This requires some modifications to the residual series in testing and estimation of correlations because only the overlapping parts of the series can be used for computing them. This complicates things somewhat because for the overlapping period, the sample mean of the standardised residuals differs from zero, which biases the constancy test. The problem can be solved simply by centring the residuals. (It is also possible to re-estimate the (time-varying) error variances for the overlapping period.) The centred residuals are used for testing and estimation. Parameter estimates of the time-varying error variances obtained before considering correlations serve as initial values in joint estimation of error variances and correlations.

4.3.2 Details

When the means and error variances are estimated, the iterations are terminated when the slope parameter, γ_{nji} in (3) or (4), or $\gamma_{nji}^{(v)}$ in (9) or (10), or $\gamma_j^{(c)}$ in (12) or (13) reaches 40. This is done both to save computing time and because it can be assumed that changes in seasonal means, error variances or correlations are not very abrupt. The standard deviations estimated for the other parameter estimates are then slightly too small because they are conditioned on the fact that the slope parameter is fixed.

Another noteworthy detail is that if the transition function to be estimated is a logistic one, it may happen that there is very little information in the data on where the function bends (its first derivative gradually increases from zero or approaches zero). Consequently, the estimation algorithm may not converge. In such situations, the logistic function is replaced by a linear trend:

$$\delta_{nj}^L(\frac{Sk+j}{SK}) = \delta_{nj0} + \delta_{nj1}((Sk+j)/SK).$$

A few such cases can be found among the results; see Tables 9–28. They are restricted to seasonal mean estimates. The corresponding graphs in Figures A4–A15 are linear.

If the transition function is an exponential one, see (4) or (10), a similar situation may occur. When it does, the location parameter c_{nj} is fixed, typically to unity. The estimates of γ_{nj} and δ_{nj1}, however, remain extremely strongly negatively correlated, which means that their joint uncertainty is very large. This manifests itself in the very large standard deviations of the estimates of these parameters, see again Tables 9–28. Buncic (2019) has an illustrative discussion of this case. Analogously to the previous case, one could replace the estimated function by a quadratic trend but this has not been done because the estimation algorithm has nonetheless converged after fixing the value of the location parameter.

5 Data

Our sample comprises 20 long European monthly temperature series, each with about 3000 observations, for a list see Table 1. Many of these time series were examined in Hillebrand and Proietti (2017). Five series may be classified as Northern European (Copenhagen, Stockholm, Trondheim, Uppsala and Vilnius), ten as Central or Eastern European, four as Western European (De

Bilt, Karlsruhe, Paris and Stuttgart), whereas Milan is the sole Southern European (south of the Alps) series, see Figure A1. With two exceptions, Paris, ending 2000, and Trondheim, ending 1981, the series extend to the 2010s. For details of the De Bilt series, see van Engelen and Nellestijn (1995). (De Bilt hosts the headquarters of the Royal Dutch Meteorological Institute.) The daily Uppsala series, constructed by Bergström and Moberg (2002), has been converted to a monthly series and shortened to begin at the same time as the Stockholm one (1756); for this series see Moberg, Bergström, Ruiz Krigsman and Svanered (2002). The series for the Alpine region stations are from HISTALP, see Auer et al. (2007).

The last country in our sample to switch to from the Julian to the Gregorian calendar is Sweden, where this happened in 1753. As both Swedish series start in 1756, none of the weather stations was using the Julian calendar when their recordings began. The De Bilt series actually begins in 1706 but is in this study shortened to start in 1750 in order to make it comparable with the other series. The Berlin series is also shortened to start in this year because of a large amount of missing values in the beginning of the series.

A few other time series contain missing values. They have been replaced with smoothed values obtained by the Kalman filter. The most conspicuous gaps occur in the Brno and Warsaw series. The period with missing values in the former series stretches from November 1939 to March 1949, whereas in the latter the corresponding months are May 1938 and December 1950.

Figures A2 and A3 contain histograms of seasonal densities of monthly temperatures for the 20 weather stations. It is seen that there is more dispersion in the boreal winter than in the summer. Vilnius seems to have the most dispersed winter averages of all series, whereas the opposite is found for Milan.

6 Results

6.1 Mean equations

6.1.1 Definition of shifting mean

We begin by reporting seasonal means for each of the 20 series. As already mentioned, it is assumed that the lag matrices $\mathbf{\Phi}_i$, $i = 1, ..., p$, are diagonal: $\mathbf{\Phi}_i = \text{diag}(\boldsymbol{\phi}_i)$, where $\boldsymbol{\phi}_i = (\phi_{i1}, ..., \phi_{iN})'$, $i = 1, ..., p$. In order to define the mean, write the nth equation of (5) as

$$(1 - \sum_{i=1}^{p} \phi_{ni} \mathsf{L}^i) y_{n,Sk+s} = \delta_{ns}(\frac{Sk+s}{SK}) + \varepsilon_{n,Sk+s}$$

where L is the lag operator, and $\mathbf{y}_{n,Sk+s} = (y_{n,Sk+s}, ..., y_{n,Sk+s-p+1})'$. Then, assuming as before that the roots of the lag polynomial lie outside the unit circle,

$$\mathsf{E}y_{n,Sk+s} = (1 - \sum_{i=1}^{p} \phi_{ni})^{-1} \delta_{ns}(\frac{Sk+s}{SK}) + O(\frac{1}{SK}) \tag{24}$$

see He et al. (2019), where the asymptotically vanishing term $O(1/SK)$ is due to the fact that $\delta_{ns}(\frac{Sk+s}{SK})$ is a nonlinear function of time. The mean can thus vary seasonally and over time within a season.

6.1.2 General observations

Constancy of coefficients of the seasonal dummy variables is tested for each equation and rejected. As already mentioned, the transition function (4) is selected if the test based on the second-order polynomial has a lower p-value than the corresponding test based on the first-order polynomial. Otherwise the standard logistic function (3) is selected. The test results can be found in Table 2. There are plenty of rejections, many of them very strong. However,

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Berlin	**	-	**	**	*	*	**	***	-	*	***	-
Brno	***	-	***	**	-	-	-	***	-	**	***	**
Budapest	**	*	**	**	*	**	**	***	*	-	***	*
Copenhagen	**	*	***	**	*	-	*	***	-	*	***	*
De Bilt	***	-	***	-	-	-	*	*	-	*	***	**
Hohenpeissenbg	**	-	***	*	***	**	**	**	-	*	***	**
Innsbruck	***	-	***	*	***	**	***	***	-	*	***	***
Karlsruhe	***	-	***	**	**	*	***	***	*	***	***	***
Kremsmünster	**	-	**	*	**	**	***	***	-	-	***	**
Milan	***	-	**	-	**	*	**	***	**	**	***	***
Munich	**	-	***	**	***	**	***	***	-	***	***	***
Paris	***	-	***	-	-	-	-	**	-	***	***	***
Regensburg	***	-	***	*	*	-	**	***	-	**	***	***
Stockholm	*	-	***	***	-	***	**	**	-	*	***	**
Stuttgart	***	-	***	*	**	**	**	***	*	***	**	**
Trondheim	*	-	*	-	*	-	-	*	-	***	***	***
Uppsala	**	-	***	***	**	**	*	**	-	*	***	***
Vienna	**	*	**	**	**	***	**	***	-	-	***	*
Vilnius	*	-	-	**	-	**	**	***	-	-	***	***
Warsaw	**	-	***	-	-	***	*	***	-	*	***	***

Table 2: Results on testing constancy of seasonal means by month for the 20 stations. Rejection at the 0.05 level of significance (*), 0.01 level (**), 0.001 level (***)

for February, the three rejections only occur at the significance level 0.05.

When the model with one transition in appropriate places is estimated and the time-varying dummy coefficients tested against another transition, there is not a single p-value equal to or below 0.05. The results of the test appear in Appendix C, Tables 29 and 30.

When this hypothesis is tested, constancy of the intercepts for which there was no rejection are tested again. This results in three rejections, all in September, for Copenhagen, Munich and Regensburg. The corresponding equations are re-estimated and the results given in Appendix B. The hypothesis of no error autocorrelation is tested, and the p-values do not give cause for concern for any of the final equations, see Appendix C, Table 31.

All estimated mean equations with at most one transition per month are reported in Appendix B, Tables 9–28. In illustrating estimation results by graphs the months are arranged by season. The boreal winter consists of December, January and February, the spring of March, April and May, and the summer of June, July and August. The remaining months, September, October and November, belong to the boreal autumn. Casty et al. (2007) examined the period 1766–2000 by season and concluded that the dominant patterns of climate variability for winter, spring, and autumn resembled the North Atlantic Oscillation. They found a distinct positive trend for the period 1960–2000 for winter and spring. For the Alpine region Casty et al. (2005) pointed out that the pre-1900 winter temperatures were generally colder than those of the twentieth century. They detected a strong transition to warm winter conditions between 1890 and 1915.

We begin with the winter temperatures. Luterbacher et al. (2004) pointed out that the North Atlantic Oscillation has a dominant influence over winter temperatures in Europe but added that regional variations may be large. This is more or less what we find. The December mean shifts are plotted in Figure A4. It should be noted that the ordinates in this figure and the ones for the remaining months are not the same. The plots mostly suggest a steady increase in the mean temperature, which is in line with Casty et al. (2005). Only three mean shifts differ from this pattern. No shift can be detected for Berlin, and both Budapest and Vienna display a positive shift that occurs late, from about 1920 to 1960. Shifts in the northern locations are generally larger than the ones in the south. The largest increases, around 2.5^{o}C or slightly over have occurred in Vilnius and Warsaw in the north and Regensburg, Karlsruhe, Paris, Stuttgart and Munich in the west.

The January shifts in Figure A5 suggest a steady increase in the mean temperature throughout the period or growth that accords with what Casty et al. (2005) noted and begins in the 20th century. These shifts are generally larger than the ones in December. There are four cases which are specific to January in that the mean shift is already over by 1900. This group contains three of the four northernmost stations in the sample (Stockholm, Trondheim and Vilnius), completed with Berlin. It may be mentioned that He et al. (2019) found the same phenomenon in the central England temperature series. Even here, Budapest and Vienna surge late. The shift in these cities begins after the first half of the 20th century, and the increase up until the end of the series has been about 1.5°C (Vienna) and 2°C (Budapest). Milan also has a similar shift. The increase over time has been strongest in Warsaw in the north, more than 2°C, Stuttgart, Regensburg and Kremsmünster (2.5°C) in the west, and Innsbruck in the Alps. It has been more modest in the other northern locations than Warsaw.

It is seen from Figure A6 that February is different from the two other winter months. According to the estimated model, in 17 out of 20 locations the mean temperature has not shifted. Again, Budapest and Vienna are different from the rest. In the former there is a positive shift beginning around 1950, whereas the estimates for Vienna suggest that the mean has decreased until 1890 and then gradually increased to its original 1770 level.

Even if March officially counts as a spring month, the mean shifts shown in Figure A7 resemble those estimated for December or January. Accelerating growth in the mean is quite typical for this month. For Berlin and Innsbruck, the rather modest shift is concentrated on the period 1880–1920, whereas the shift for Milan begins after 1920. For a majority of locations, the shift has been close to 2°C or even exceeded it (Stuttgart and Munich).

It is seen from Figure A8 that the mean shifts for April are radically different from the ones estimated for March. The most frequent pattern is the one in which there is a decline ending before 1900 and an increase which continues to the present. Stuttgart has a strong late surge, about 2.5°C, starting only after 1970. Warsaw and Budapest show a steady growth, but the total increases are small, around 1.5°C for the former and less than 1°C for the latter.

Shifts in May, see Figure A9, are rather similar to the ones for April. Berlin appears to show a positive mean shift occurring between 1880 and 1920, but it is less than 0.5°C. . In general, shift patterns in many locations for April and May correspond to European findings by Casty et al. (2007) who reported cooling until 1900 and warming thereafter. However, for six locations, our modelling strategy has found no shifts. Milan has experienced a late surge that exceeds 1.5°C

For June, the patterns change by latitude. For the northern cities, there is either a decline (Uppsala, Stockholm, Vilnius, Berlin and Warsaw) or no shift (Trondheim and Copenhagen, both close to the Atlantic Ocean). Moberg, Alexandersson, Bergström and Jones (2003) argued, however, that the observed summer temperatures in Stockholm and Uppsala series before 1860 or so may have been 0.7–0.8^{o}C too high due to insufficient protection of instruments from radiation. This bias accentuates the estimated downward shift. Then there is a 'mid-category' (De Bilt, Brno, Regensburg and Paris) with no or a minimal positive shift. Moving south, beginning from Karlsruhe, but excepting Paris in the west, the dominant pattern is a late positive shift. For most these cities, this shift has been quite strong, even close to 2^{o}C. The only real exception is Budapest, where a small positive shift is found roughly between 1850 and 1900.

The situation for July, shown in Figure A11, is different. The 19th century cooling, common to most series, is usually over before the end of the century, whereafter it is followed by warming. The only exception is Warsaw, where the minimum occurs around 1915. Besides, the latest estimated July averages for Warsaw have not yet reached their earliest values in the sample. The increases in the mean are less than in the winter, and there is not a single example of a steady increase in the mean over the observation period. For Vilnius, similarly to June the small downward shift in the 19th century is not followed by any warming. On the other hand, Copenhagen shows a late, around 2^{o}C, positive surge.

The 19th century decrease followed by a positive shift is very clear in August, see Figure A12. The increase from the bottom is less pronounced for the northern locations than the rest. Milan and Stuttgart are the only exceptions. A late (post-1950) but rather strong (close to 2^{o}C) surge in the mean is estimated for these two locations. A general observation is that if the mean temperature at the end of the observation period exceeds the values in the beginning, the overall increase remains small.

Moving to the autumn, plots of the September mean shift appear in Figure A13. While there are still some 19th century downward shifts followed by a positive shift, the dominant pattern is 'no change'. Milan and Stuttgart still have a positive shift in the mean, but it begins earlier than in August and is small, less than 1^{o}C.

Figure A14 contains the October patterns where positive shifts dominate. A typical shift begins in the 20th century and often rather late. They are again more modest in the north than in the other locations. Karlsruhe and Stuttgart have the largest (close to 2^{o}C) shifts.

A comparison to December suggests that November, see Figure A15, may already be classified as a boreal winter month. All 20 stations display a positive mean shift, and in a majority of cases they are clearly larger than in October, exceeding 1.5^{o}C. In most cases the shift is gradual and starts already at the end of the 19th century. Vilnius and Munich have the largest shifts.

The estimated shift patterns suggest that the year may be divided into three seasons instead of four. The 'winter' extends from November until March. It contains February which is different from the rest but has to be included because of its position between by January and March. The 'summer' consists of the months from April to August, whereas September and October are the 'autumn' months. September has more in common with August than with November. For October the situation is the opposite, which is what one might expect. It may also be concluded that the positive shifts in northern locations are often, but by no means always, somewhat smaller than the ones elsewhere. Of these stations, Karlsruhe and Stuttgart frequently seem to have shifts whose size exceeds the median.

These results may be compared to the ones in Hillebrand and Proietti (2017). The authors construct monthly models that contain two trends: a linear deterministic trend and a stochastic one. They write that the latter is included to account for an increase in the temperature towards the end of the period. The deterministic trends are mostly positive, often strongly, and the few negative ones fall on June, July and August. This accords with our results. Although the authors do not mention it, sometimes the stochastic trend helps explain the 19th century cooling; see their Figures C2–C5. As already noticed, this cooling is quite prominent in our results for the months from April to August.

6.2 Error variances

After estimating the means, the estimated series are subjected to misspecification tests. They are tested for serial correlation in the errors. The lag structure of each equation is adjusted such that the test of the final equation does not reject the null hypothesis. Two or three lags are the norm, and the roots of the lag polynomial are invariably far from the unit circle; see Tables 9–28.

After this, constancy of error variances is tested against a single transition. The results appear in Table 3. Constancy is rejected only in a few occasions at the 0.05 or a lower level. In those cases the error variance containing one transition is estimated, and the corresponding model (8) with (9) is fitted to the residuals. Typically, if the error variance changes over time, it decreases, sometimes rather smoothly, and not by much. Most rejections occur in the spring (in April and to some extent in May) and winter (December). But then, the null hypothesis is never rejected for February and seldom for the boreal summer months and September.

Figures A16–A18 contain plots of the shifts in variance for December, April and May. The shifts in 16 out of 20 April error variances shown in Figure A17 occur with remarkable regularity during the first half of the 19th

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Berlin	-	-	-	*	*	-	-	-	-	-	-	-
Brno	**	-	-	**	-	-	*	*	-	-	-	-
Budapest	-	-	-	*	-	-	-	-	-	-	-	*
Copenhagen	-	-	-	**	***	-	-	-	-	-	-	-
De Bilt	-	-	-	*	-	*	-	-	-	-	-	-
Hohenpeissenbg	-	-	-	*	-	-	-	-	-	-	*	-
Innsbruck	*	-	-	**	-	-	-	**	-	-	*	**
Karlsruhe	*	-	**	**	-	-	-	-	-	-	-	**
Kremsmünster	*	-	-	**	*	-	-	*	-	-	-	-
Milan	**	-	-	-	-	-	-	-	**	-	-	**
Munich	-	-	-	*	*	-	-	**	-	-	**	*
Paris	-	-	-	-	*	-	-	-	-	*	-	*
Regensburg	-	-	*	**	*	-	-	**	-	*	**	*
Stockholm	-	-	-	*	*	-	-	-	-	*	-	-
Stuttgart	**	-	-	**	-	-	-	-	-	-	-	*
Trondheim	-	-	-	***	*	-	-	-	-	-	-	-
Uppsala	-	-	-	*	*	-	-	-	-	-	-	-
Vienna	-	-	-	**	**	*	-	*	-	*	-	*
Vilnius	-	-	-	-	-	-	*	-	-	-	-	**
Warsaw	-	-	-	-	-	-	*	*	-	-	-	**

Table 3: Results of testing constancy of error variances by month for the 20 stations. Rejection at the 0.05 level of significance (*), 0.01 level (**), 0.001 level (***)

century and are all downward shifts. They may hardly be ascribed to improvements in measurement techniques during that period because the same phenomenon is not observed for any other month. Interestingly, He et al. (2019) found the same pattern for April in their univariate model for the CET series. A simple, but admittedly not very informative, explanation for April shifts is that there is early weather induced variation in April temperatures that the mean component of the model cannot explain.

Shifts in the following month, May (Figure A18) are no longer as frequent, and a majority of them take place during the second half of the 19th century. The remaining May shifts are rather smooth and occur later.

Finally, a majority of shifts in December, see Figure A16, also take place on the second half of the 19th century. The error variance for Vilnius displays a steady decrease during the whole 19th century. The late shift for Vienna may, due to its location, be a statistical artifact. All May and December changes are downward shifts as well.

Both Stockholm and Uppsala have a similar decrease in the error variance in May during the second half of the 19th century. There is, however, none during the summer months. This suggests that the measurement error

Moberg et al. (2003) discussed has indeed been systematic in the sense that it has not affected dispersion around the estimated mean.

6.3 Correlations

Since the temperature series do not cover exactly the same period, the residuals used for computing (constant) correlation estimates are adjusted as discussed in Section 4.3.1. For the adjusted series we compute pairwise correlations and test their constancy as described in Section 4.2.4. In general, the null hypothesis is not rejected at any relevant significance level. For January and October, there is not a single rejection for the 190 pairs; for February there is one. To save space, these results are not reported in detail here. Rejections are most frequent for the summer months June, July and August; see Tables 4, 5 and 6. Compared to the very large number of tests, however, the 5% significance level is not very suggestive, and rejections on that level may not be considered particularly striking.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1 Berlin																				
2 Brno	-																			
3 Budapest	-	-																		
4 Copenhagen	-	-	-																	
5 De Bilt	-	-	-	-																
6 Hohenpeissenberg	-	-	-	-	-															
7 Innsbruck	-	-	-	-	-	-														
8 Karlsruhe	-	-	-	-	-	-	-													
9 Kremsmünster	-	-	-	-	-	-	-	-												
10 Milan	-	-	-	-	-	-	-	-	-											
11 Munich	-	-	-	-	-	-	-	-	-	-										
12 Paris	-	-	-	-	-	-	-	-	-	-	-									
13 Regensburg	-	-	-	-	-	-	-	-	-	-	-	-								
14 Stockholm	-	-	*	-	-	*	*	-	*	*	-	-	*							
15 Stuttgart	-	-	-	-	-	-	-	-	-	-	-	-	-	-						
16 Trondheim	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-					
17 Uppsala	-	-	*	-	-	*	*	*	*	*	*	-	-	-	*	-				
18 Vienna	-	-	-	-	-	-	-	-	-	-	-	-	-	*	-	-	*			
19 vilnius	-	-	-	-	-	-	-	-	*	-	-	-	-	-	-	-	-	-		
20 Warsaw	*	-	-	*	-	-	-	-	*	-	*	-	-	*	*	-	*	-	-	

Table 4: Results of testing constancy of standardised errors by month for the 20 stations for June. Rejection at the 0.05 level of significance (*), 0.01 level (**), 0.001 level (***)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1 Berlin																				
2 Brno	-																			
3 Budapest	-	-																		
4 Copenhagen	*	-	-																	
5 De Bilt	*	-	-	-																
6 Hohenpeissenberg	-	-	-	-	-															
7 Innsbruck	-	-	-	-	-	-														
8 Karlsruhe	-	-	-	*	-	-	-													
9 Kremsmünster	*	-	-	-	-	-	-	-												
10 Milan	*	-	-	**	-	-	-	-	-											
11 Munich	-	-	-	-	-	-	-	-	-	-										
12 Paris	-	-	-	-	-	-	-	-	-	-	-									
13 Regensburg	-	-	-	-	-	-	-	-	-	-	-	-								
14 Stockholm	-	-	-	-	-	-	-	-	-	-	-	-	-							
15 Stuttgart	-	-	-	*	-	-	-	-	-	-	-	-	-	-						
16 Trondheim	-	*	-	-	-	*	-	-	*	-	*	-	*	-	-					
17 Uppsala	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-				
18 Vienna	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-			
19 Vilnius	*	*	-	-	*	-	-	-	-	*	-	*	-	-	-	-	-	-		
20 Warsaw	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

Table 5: Results of testing constancy of standardised errors by month for the 20 stations for July. Rejection at the 0.05 level of significance (*), 0.01 level (**), 0.001 level (***)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1 Berlin																				
2 Brno	**																			
3 Budapest	-	*																		
4 Copenhagen	-	**	**																	
5 De Bilt	-	**	*	-																
6 Hohenpeissenberg	-	**	*	*	-															
7 Innsbruck	*	**	-	*	-	-														
8 Karlsruhe	-	**	*	*	-	-	-													
9 Kremsmünster	-	*	-	*	-	-	-	-												
10 Milan	*	**	-	*	-	-	-	-	-											
11 Munich	-	**	-	*	-	-	*	-	-	-										
12 Paris	-	*	*	-	-	-	-	-	-	-	-									
13 Regensburg	-	**	-	-	-	*	*	-	-	*	*	-								
14 Stockholm	-	*	*	-	-	**	**	-	-	**	*	-	-							
15 Stuttgart	-	**	*	-	-	-	-	-	-	-	-	-	-	-						
16 Trondheim	-	-	-	*	-	-	*	*	-	-	*	*	*	-	-					
17 Uppsala	-	*	*	-	-	*	*	-	-	*	*	-	-	-	-	-				
18 Vienna	-	*	-	**	*	*	-	-	-	*	*	-	*	*	-	-	-			
19 Vilnius	-	*	-	-	-	-	-	-	-	*	-	-	-	-	-	-	-	-		
20 Warsaw	-	-	-	*	-	*	*	*	-	**	*	-	-	-	*	-	-	-	-	

Table 6: Results of testing constancy of error correlations by month for the 20 stations for August. Rejection at the 0.05 level of significance (*), 0.01 level (**), 0.001 level (***)

Evidence on shifts in correlations seem to be most widespread in August in Table 6. The rejections seem to concentrate on a small number of locations. Surprisingly, the null hypothesis is rejected for 17 out of the 19 pairs for Brno, often even at the 0.01 level, but nothing similar is observed for the other months. Other locations with many August rejections include Copenhagen, Innsbruck, Milan, Stockholm and Trondheim.

It is possible to discern patterns. For example, for Copenhagen constancy is often rejected when the other location lies in the Alpine Region or south of the Alps (Milan), and the same is true for Stockholm and Uppsala. For these two, the results are similar in the sense that the other member of the pair when the null hypothesis is rejected is the same for both. This is not surprising, given the proximity of these two locations. Rejections in June seem to be concentrated on Stockholm and Uppsala as well, whereas for July, a similar pattern is not found. Most of the June rejections (only at the 0.05 level) for these two locations occur when the other member of the pair is a location in the Alpine region.

These test results already give an idea of whether the correlations may or may not change over time. Nevertheless, it may be more interesting to consider more than two series at the same time. One can in principle test the correlation matrix of all 20 series, but such a test would require 380 degrees of freedom, and the results might not be very informative. Besides, even if the null hypothesis were rejected, assuming that a single transition function would control the change in all series simultaneously would be rather unrealistic.

For this reason we focus on lower-dimensional vectors of standardised residual series. Their potentially time-varying correlations we estimate by month as discussed in Section 4.3.1.

We shall begin by considering four series: Hohenpeissenberg, Innsbruck, Milan and Stockholm. The first two Alpine series are selected as a check because they are close to each other, and the (error) correlations between them, time-varying or not, should therefore be high. In pairwise tests the null hypothesis is often rejected for pairs that involve Stockholm and an Alpine location, which is why Stockholm is selected. Besides, Stockholm represents the North, whereas Milan is the only city in our sample located south of the Alps. Stockholm and Milan may therefore be expected to display the lowest correlations of all.

Results of constancy tests for this quartet can be found in Table 7. For eight of the twelve months, constancy is not rejected, while the strongest rejections of the null hypothesis appear in November and December. Time-varying correlations are estimated for these four months: April, August, November and December. Since it cannot be assumed that all correlations

Month	1	2	3
Jan	0.379	0.157	0.378
Feb	0.111	0.381	0.648
Mar	0.191	0.174	0.139
Apr	0.133	0.008	0.026
May	0.890	0.680	0.810
Jun	0.863	0.393	0.703
Jul	0.240	0.155	0.082
Aug	0.015	0.075	0.151
Sep	0.572	0.690	0.936
Oct	0.214	0.403	0.488
Nov	0.018	0.007	0.003
Dec	0.019	0.047	0.000

Table 7: p-values of tests of constant error correlations for the 12 months for correlations between Innsbruck, Hohenpeissenberg, Stockholm and Milan. Column headings indicate the order of the polynomial used in the test

are changing over time, we search estimates that look similar in $\mathbf{P}_{(s1)}$ to their counterparts in $\mathbf{P}_{(s2)}$. If such pairs of estimates are found, variances and correlations are re-estimated assuming that these correlations are equal, and a likelihood ratio test is carried out to test this restriction. If the null hypothesis is not rejected, the restrictions are retained and the correlations in the null model reported.

The results are summed up in Figure A19. Stockholm is involved in blue but not in red correlations, and it is seen that the correlations between the standardised errors of this location and the rest are much weaker than correlations between the other series. Not surprisingly, the Hohenpeissenberg-Innsbruck correlation is the highest of all, often exceeding 0.9. The Stockholm-Milan correlation for April seems to be an anomaly. The best fit is obtained by characterising this correlation instead of the exponential function (13) by a double logistic transition function; see Jansen and Teräsvirta (1996). It is high in the beginning of the series when this is not the case for any other month, remains low for most of the time, and increases again towards the end of the period.

Interestingly, all changes except this particular one are pure downward shifts. The shifts in the Stockholm correlations for August agree with results of the pairwise tests in Table 6. By 1900, the changing correlations reach zero, and one of them becomes even slightly negative. The other conspicuous shifts occur in November and December. In November, all shifts involve Milan,

Month	1	2	3
Jan	0.094	0.058	0.104
Feb	0.614	0.920	0.983
Mar	0.372	0.827	0.711
Apr	0.653	0.427	0.028
May	0.426	0.343	0.503
Jun	0.630	0.583	0.833
Jul	0.028	0.079	0.116
Aug	0.447	0.248	0.174
Sep	0.275	0.506	0.627
Oct	0.780	0.909	0.839
Nov	0.591	0.529	0.623
Dec	0.302	0.416	0.251

Table 8: p-values of tests of constant correlations for the 12 months for correlations between De Bilt, Stuttgart, Vienna and Vilnius. Column headings indicate the order of the polynomial used in the test

and in December the only correlation that does not change is the already low Stockholm-Milan one. The November shifts occur somewhat earlier but later than the August ones and are slightly sharper than the ones in December. Since the model does not make use of any extraneous information, causes for these fading correlations remain unclear. Warming does not occur in tandem in the north and the south, but the purpose of the mean component of the model has been to take care of this difference. Perhaps this has not been a complete success for November or December.

In order to complete this analysis, we conduct another four-station investigation, this time in the east-west direction. The stations are Vilnius in the north-east, Vienna just east of the Alpine region and De Bilt and Stuttgart in the west, the former being close to the Atlantic Ocean. The test results in Table 8 show that for ten months, constancy of correlations is not rejected. April and July are the only exceptions, and even for these two locations the evidence in favour of shifts is not very strong.

Plots of the estimated correlations can be found in Figure A20. The error correlations between Vilnius and the other locations are systematically the lowest ones, the one between Vilnius and Vienna being somewhat higher than the other two. This agrees with the previous results in that the northernmost location is different from the rest. These correlations are generally higher from October to March than they are from May to September. April is an intermediate month in that Stuttgart-Vilnius and Vienna-Vilnius error correlations lie on the 'winter or October-March level' until around 1950 and then descend to the 'summer level'. There is a similar decrease in the Stuttgart-Vienna correlation. For July, there is an inexplicable late surge in correlations between Vilnius, and De Bilt and Vienna, respectively.

The highest correlations can be found between Stuttgart and Vienna (except July after 1950), and the lowest 'non-Vilnius' ones are the correlations between the Atlantic De Bilt and continental Vienna. They do not change with the month as much as the correlations involving Vilnius.

7 Conclusions

In this work we develop a multivariate seasonal time series model called the VSSMC-AR model and design a systematic modelling structure for it. We fit the model to twenty about 250 years long monthly European temperature series. The purpose of the exercise is to characterise main features of monthly temperature changes in various locations in Europe and have a look at possible correlations between the errors. Our results agree with the general conclusion that winters have been warming up more quickly than the summers, but due to the monthly observation frequency, the information obtained is more detailed than that. For example, we are able to show that warming (or lack of it) in February is strikingly different from that found for the adjacent months, both January and March. We also find some systematic behaviour in error variances in cases where they vary over time.

The multivariate model also gives an opportunity to learn about correlations between the (standardised) errors. Within a month they are mostly constant over time, but the ones between a northern and a southern location appear lower in the summer than in the winter. The two examples seem to suggest that the north-south distance between the locations matters more than the east-west one. It seems difficult to explain changes over time without extraneous information.

The VSSMC-AR model provides a framework for studying multivariate seasonal time series in situations where seasonality is suspected to vary over time. Some climate scientists may prefer quarterly temperature series to monthly ones, and the model is of course applicable to quarterly data as well, at least if the series are not too short. Seasonal time series can also be found in economics, and the model, including the modelling structure presented here, can also be applied to economic series. Such applications and further developments of the model will be left for further research.

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A Figures

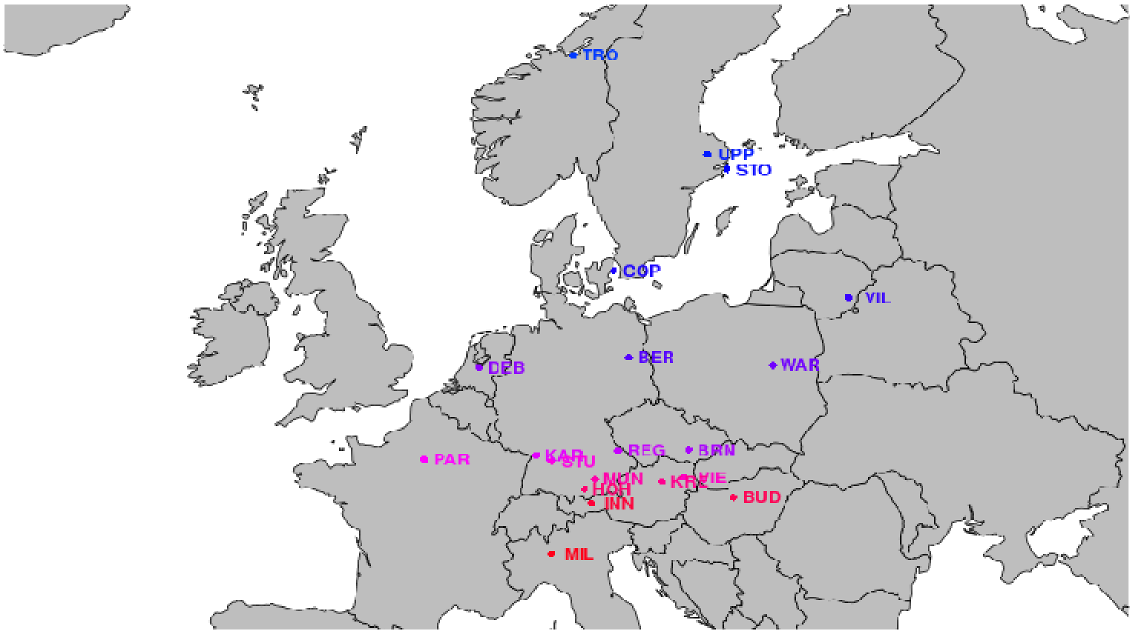

Figure A1. Locations of the 20 weather stations, from Trondheim in the north to Milan in the south, and from Paris in the west to Vilnius in the east

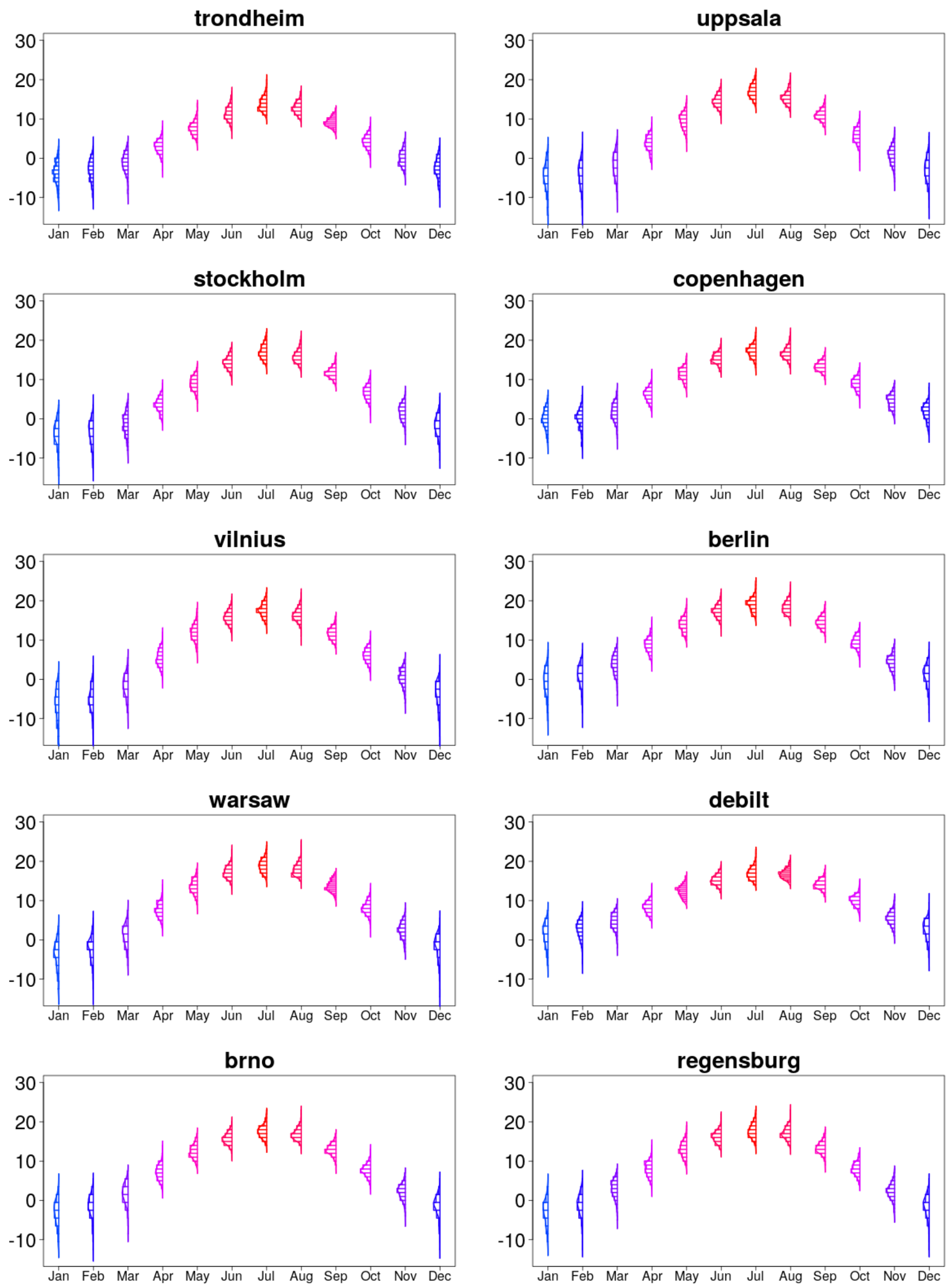

Figure A2. Histograms of monthly temperatures for the first ten locations from north to south, Trondheim to Regensburg

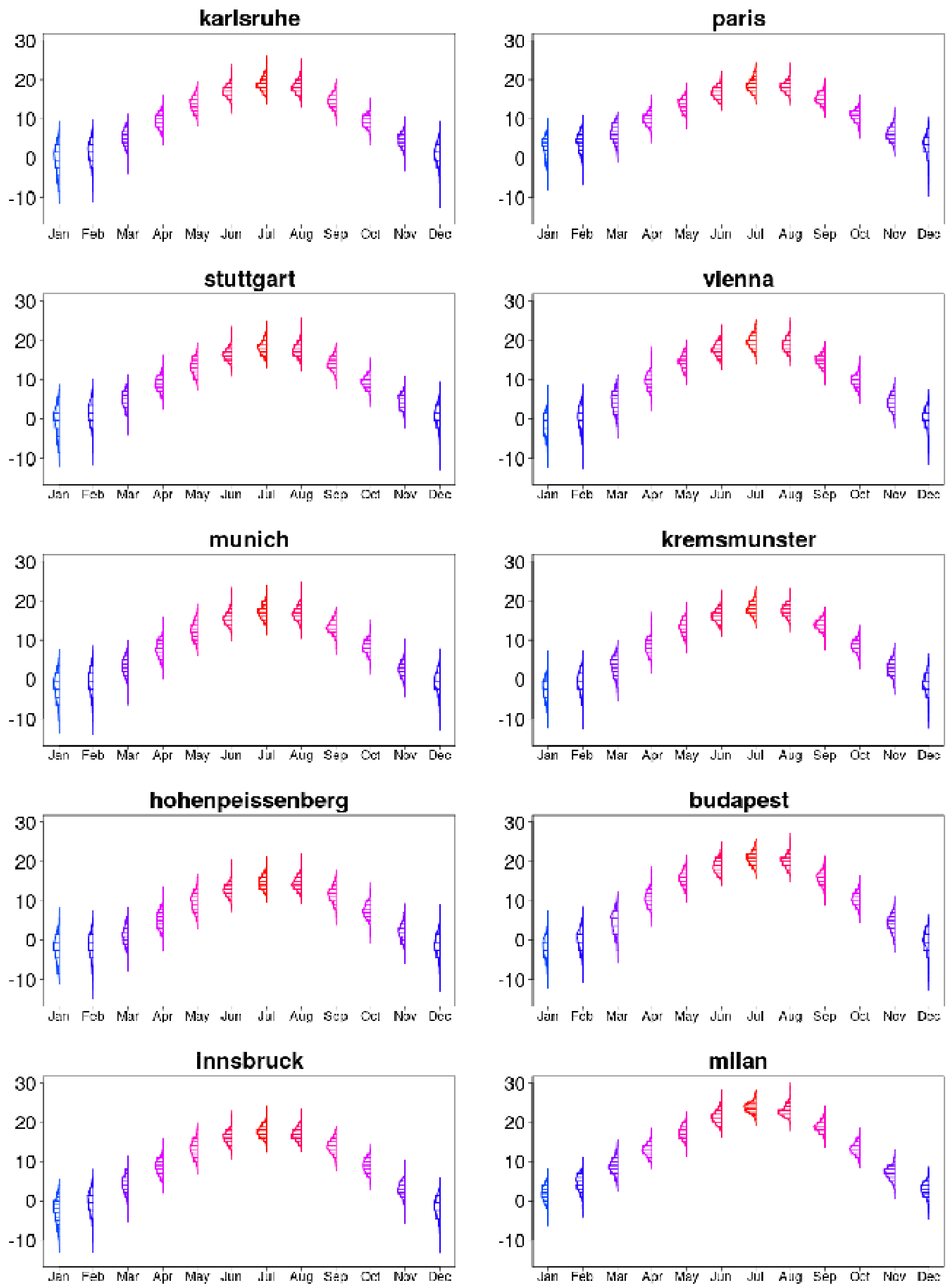

Figure A3. Histograms of monthly temperatures for the last ten locations from north to south, Karlsruhe to Milan

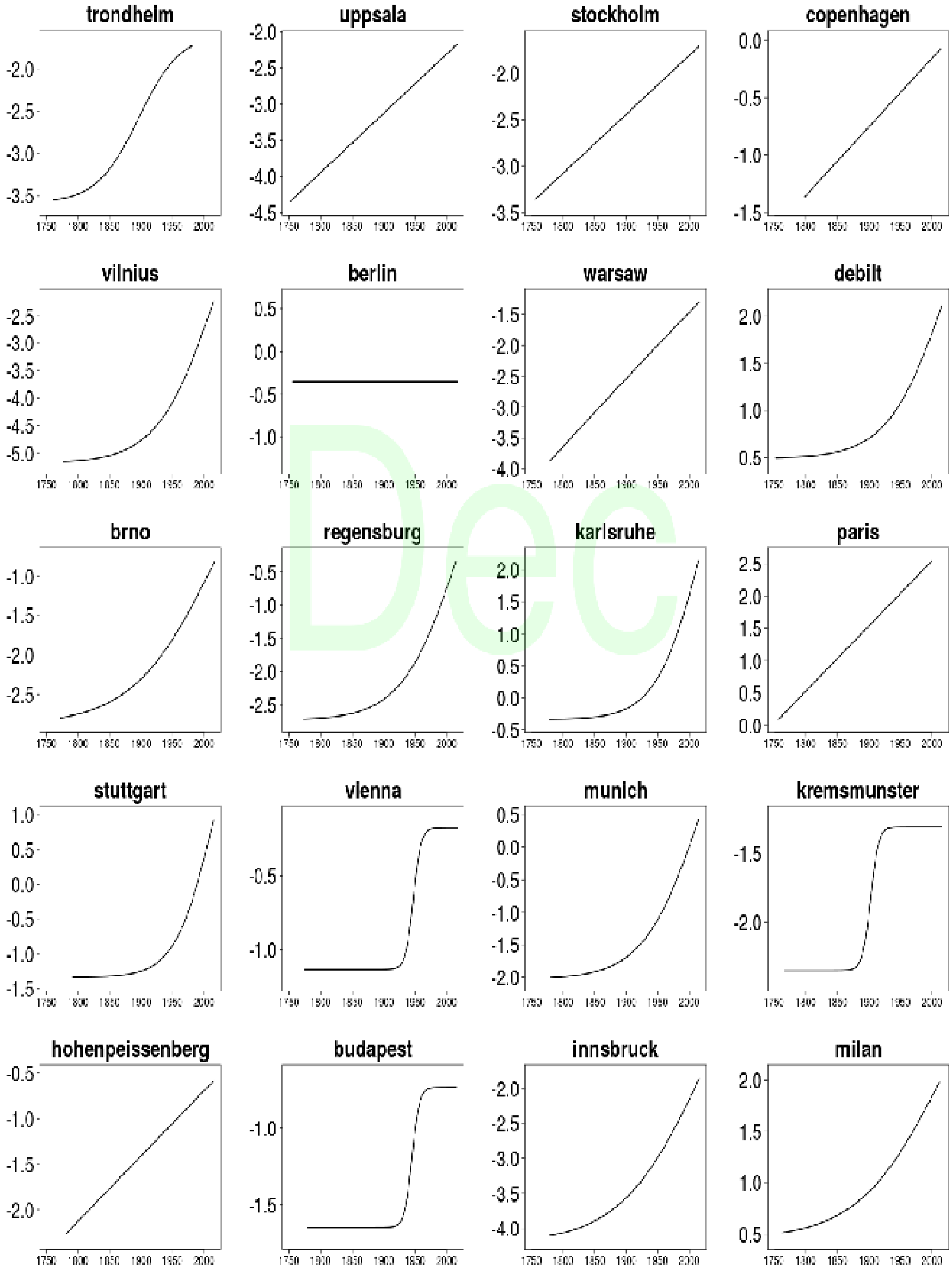

Figure A4. Shifting means of December for the 20 weather stations (from north to south) over more than 250 years of monthly observations

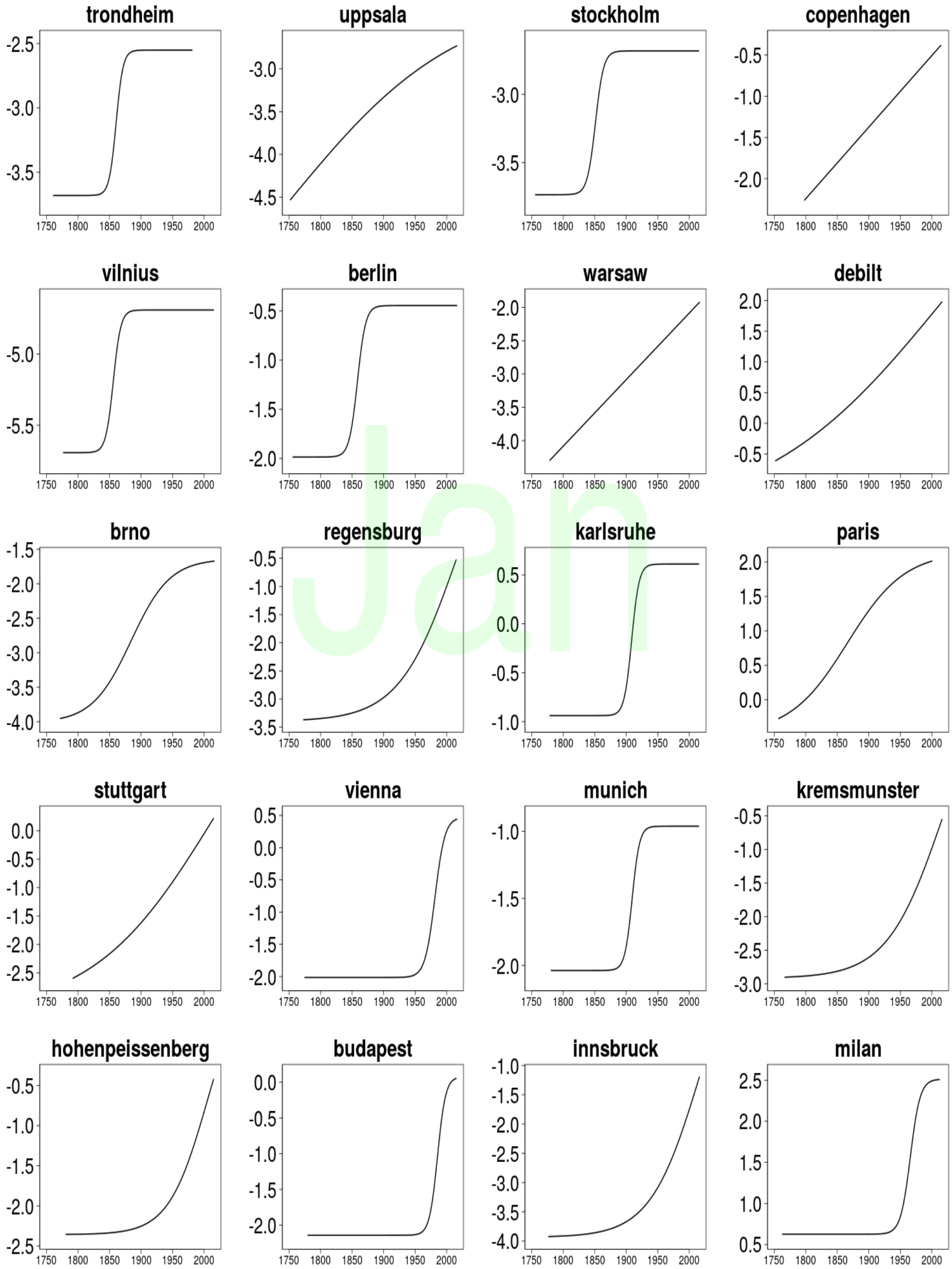

Figure A5. Shifting means of January for the 20 weather stations (from north to south) over more than 250 years of monthly observations

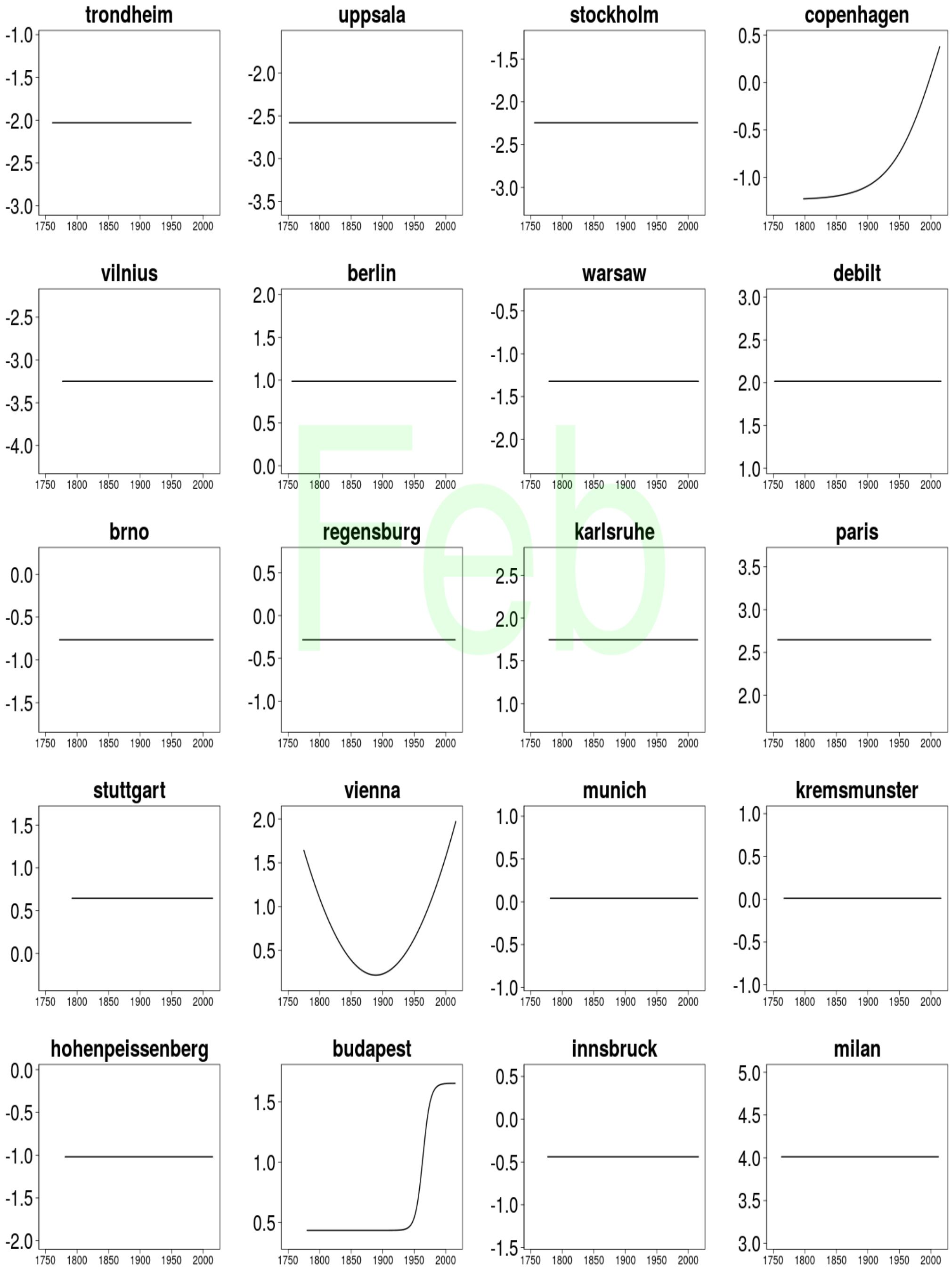

Figure A6. Shifting means of February for the 20 weather stations (from north to south) over more than 250 years of monthly observations

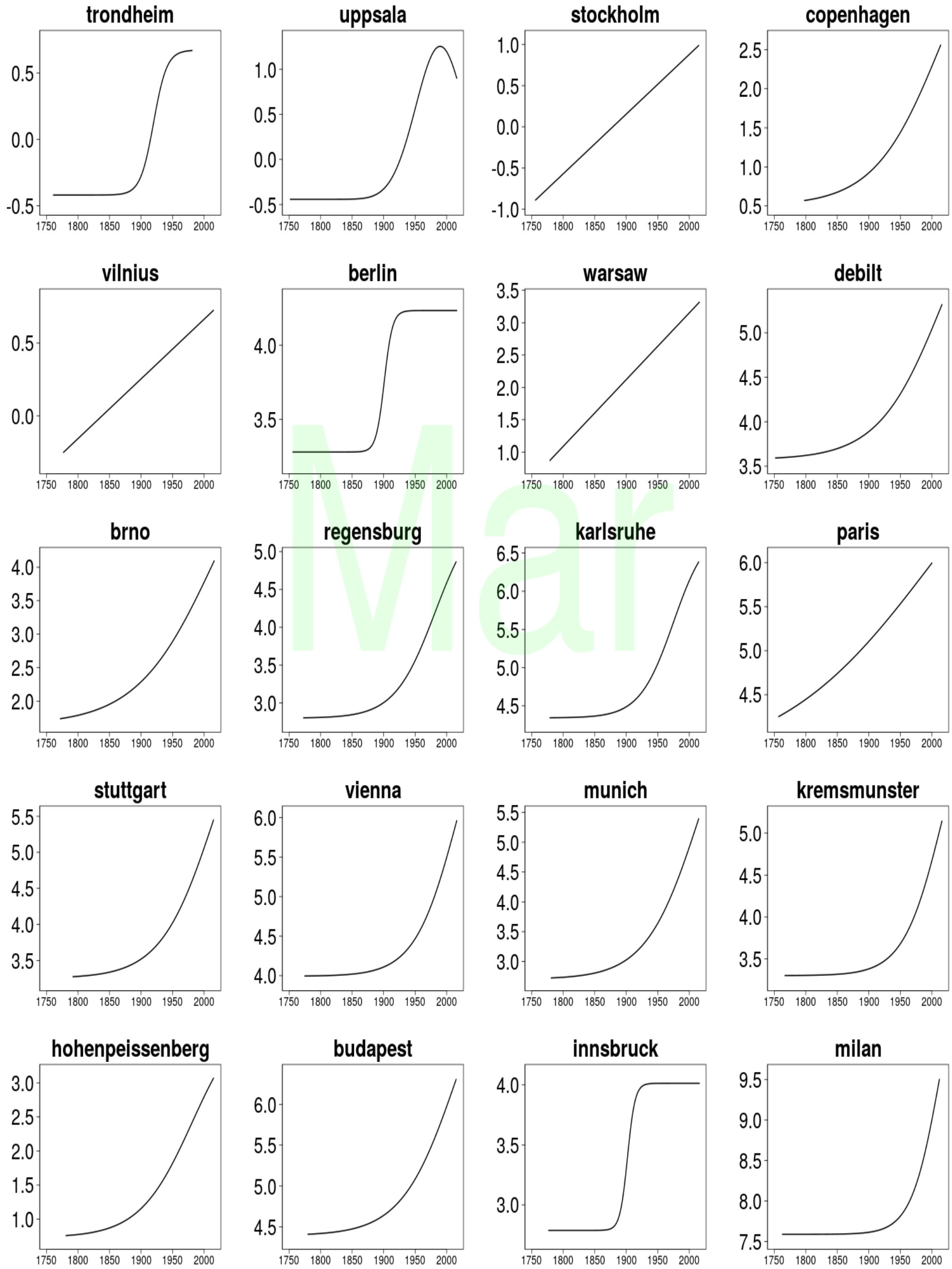

Figure A7. Shifting means of March for the 20 weather stations (from north to south) over more than 250 years of monthly observations

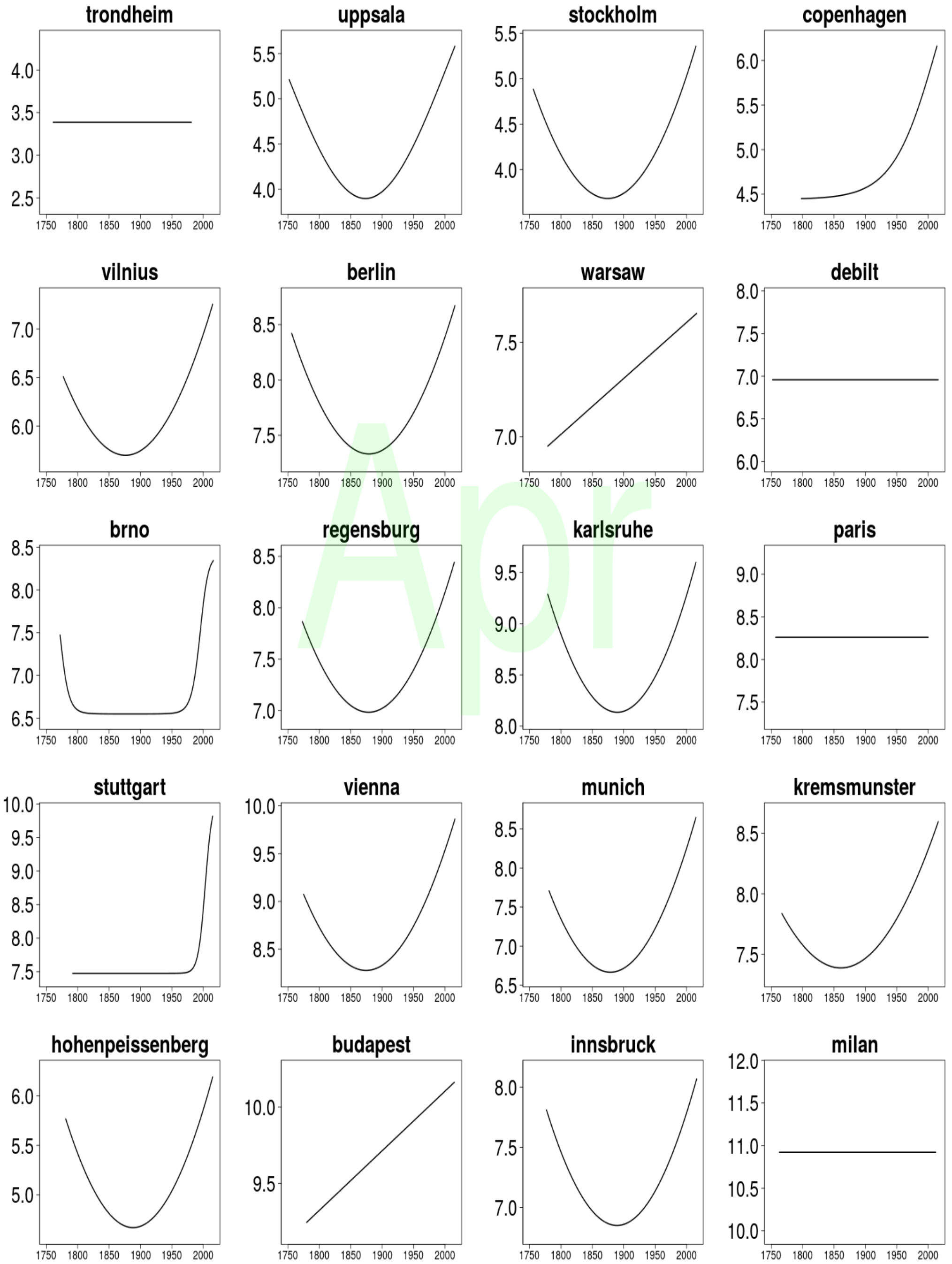

Figure A8. Shifting means of April for the 20 weather stations (from north to south) over more than 250 years of monthly observations

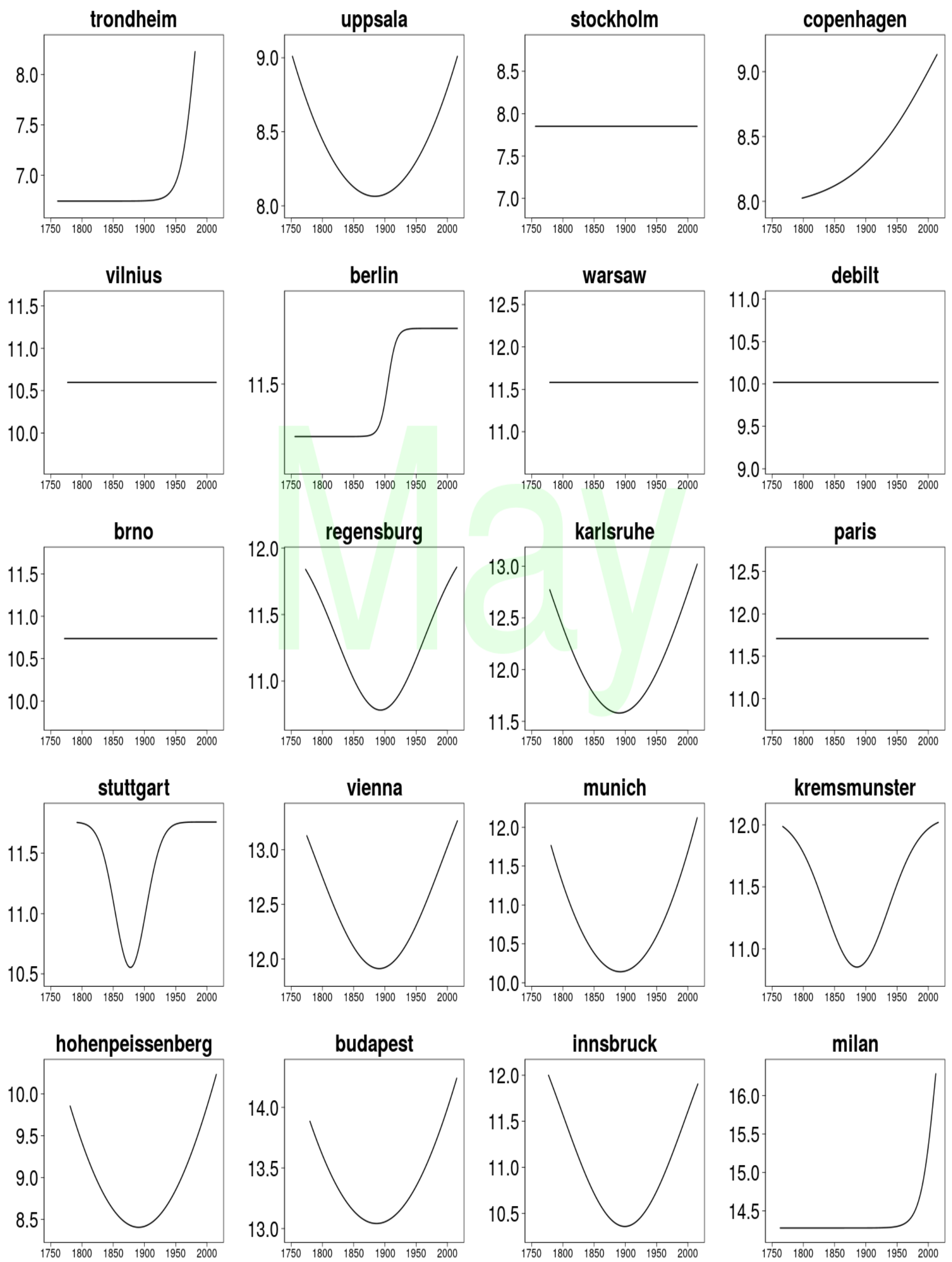

Figure A9. Shifting means of May for the 20 weather stations (from north to south) over more than 250 years of monthly observations

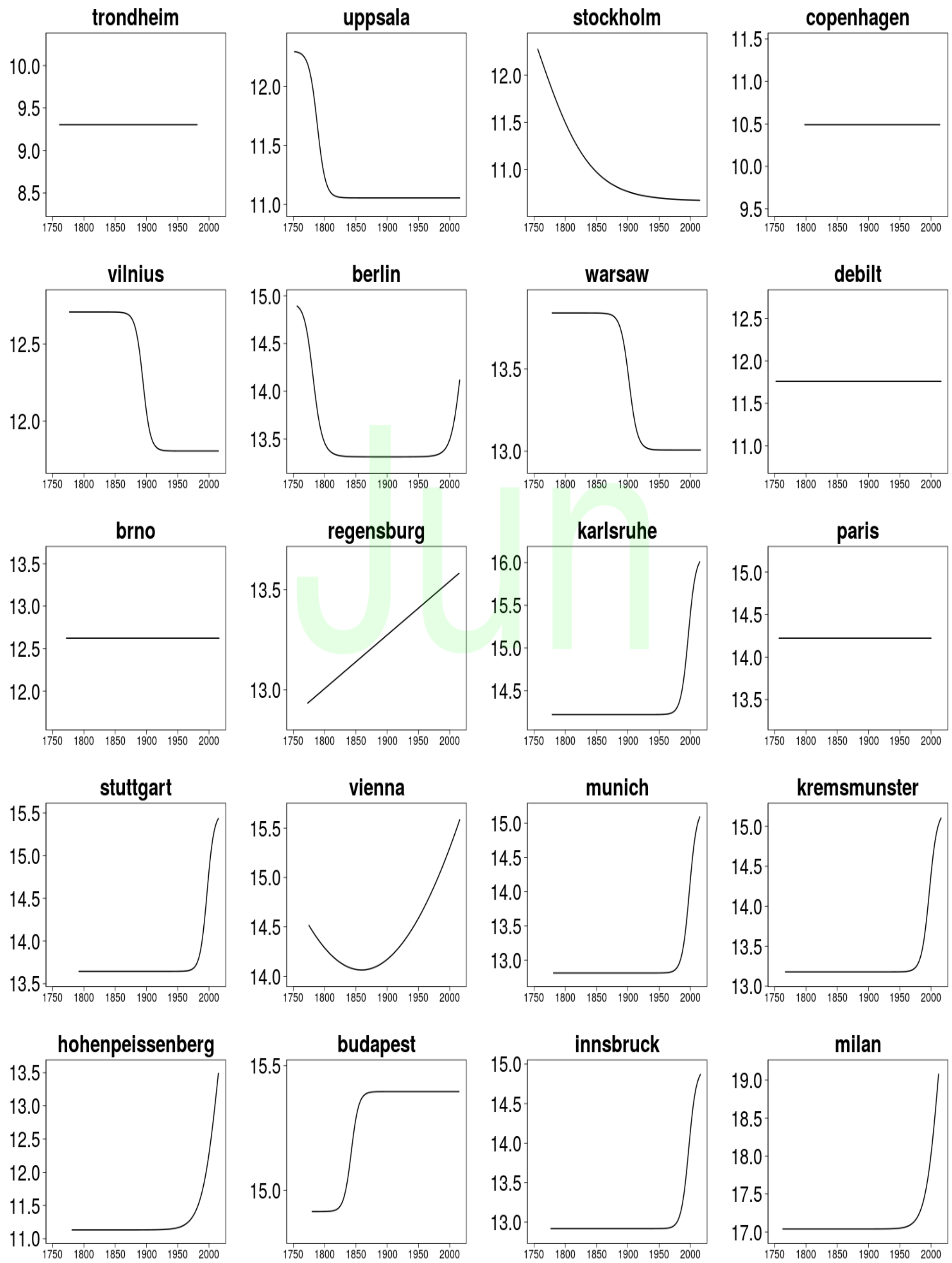

Figure A10. Shifting means of June for the 20 weather stations (from north to south) over more than 250 years of monthly observations

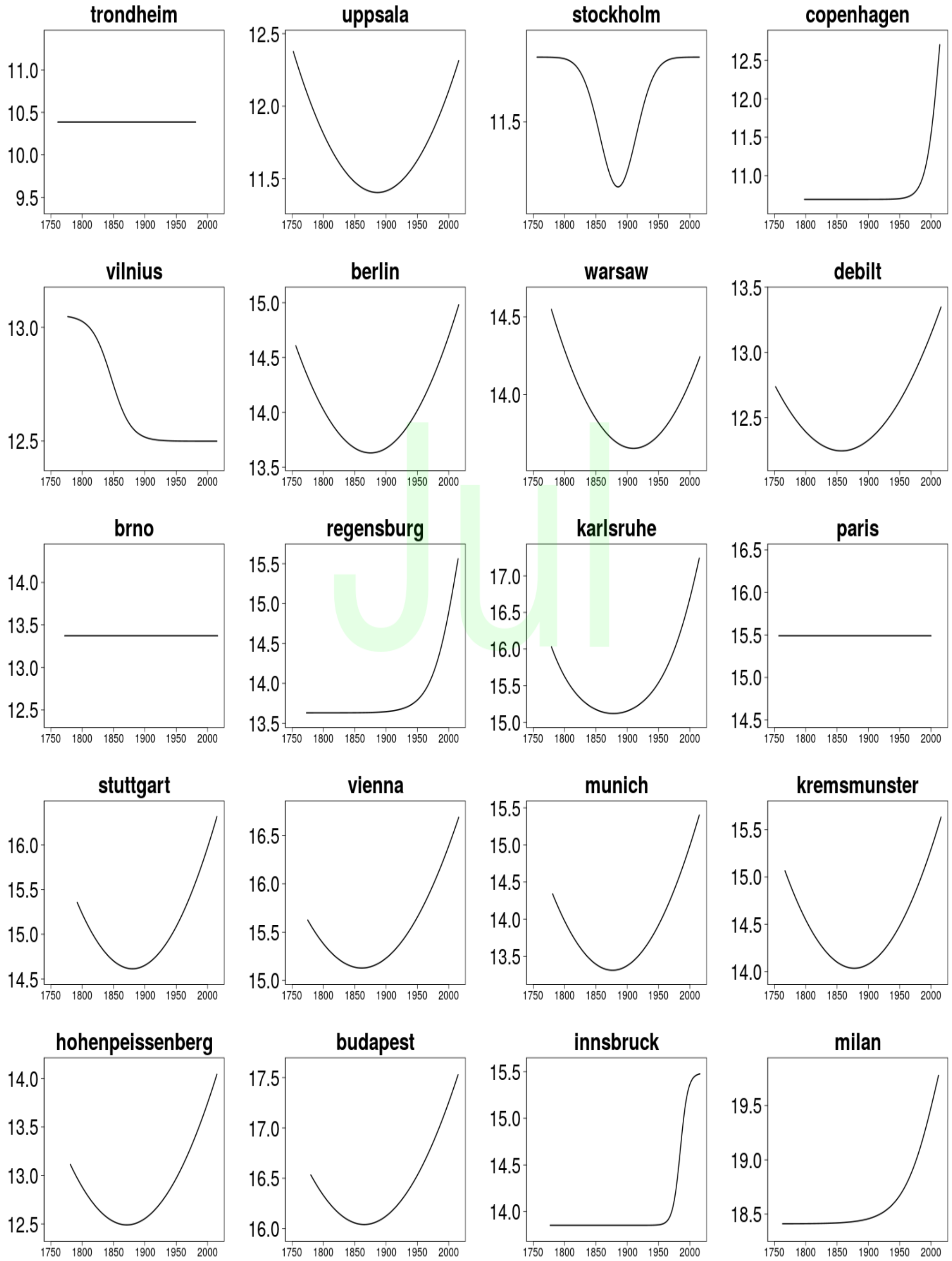

Figure A11. Shifting means of July for the 20 weather stations (from north to south) over more than 250 years of monthly observations

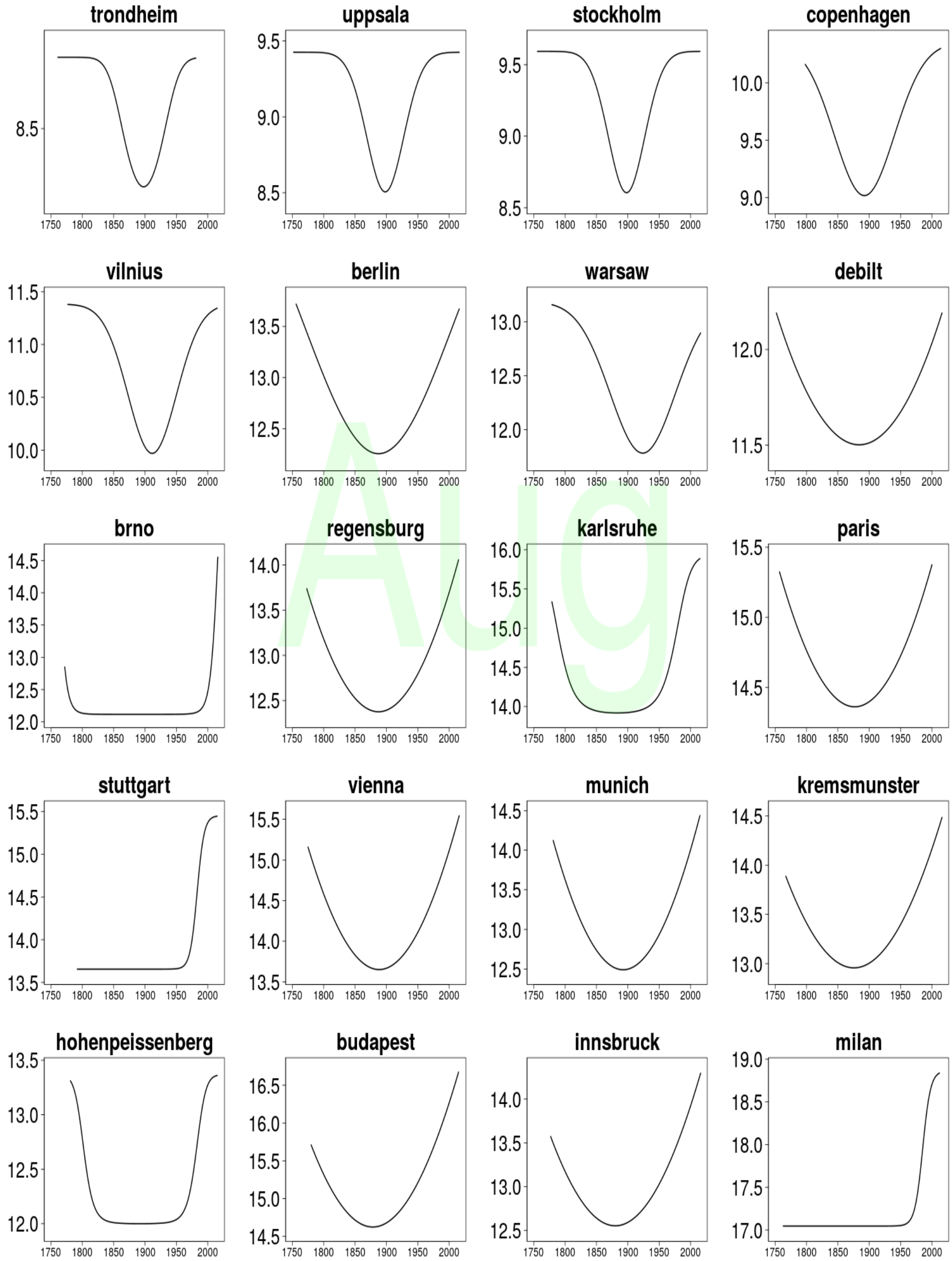

Figure A12. Shifting means of August for the 20 weather stations (from north to south) over more than 250 years of monthly observations

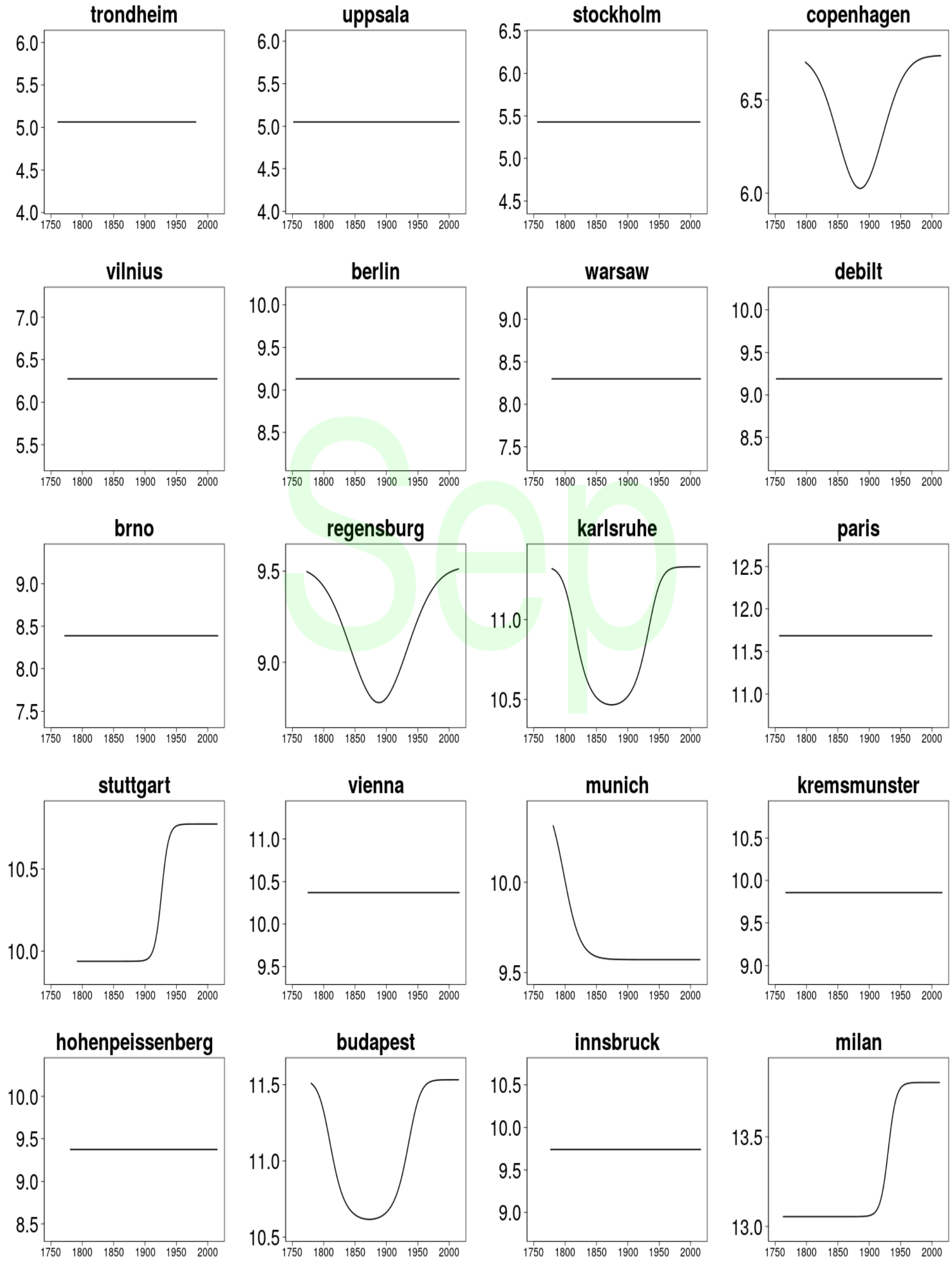

Figure A13. Shifting means of September for the 20 weather stations (from north to south) over more than 250 years of monthly observations

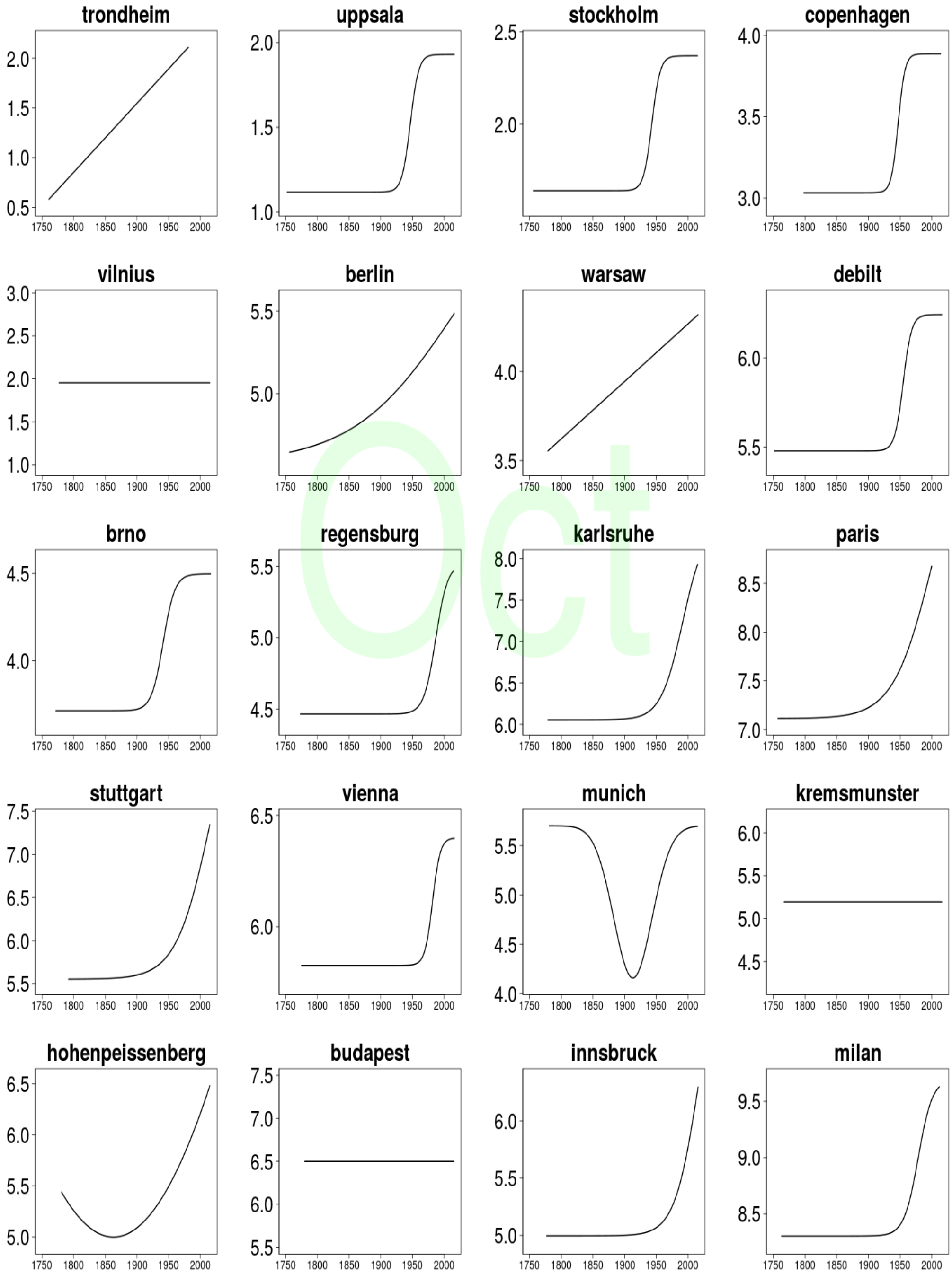

Figure A14. Shifting means of October for the 20 weather stations (from north to south) over more than 250 years of monthly observations

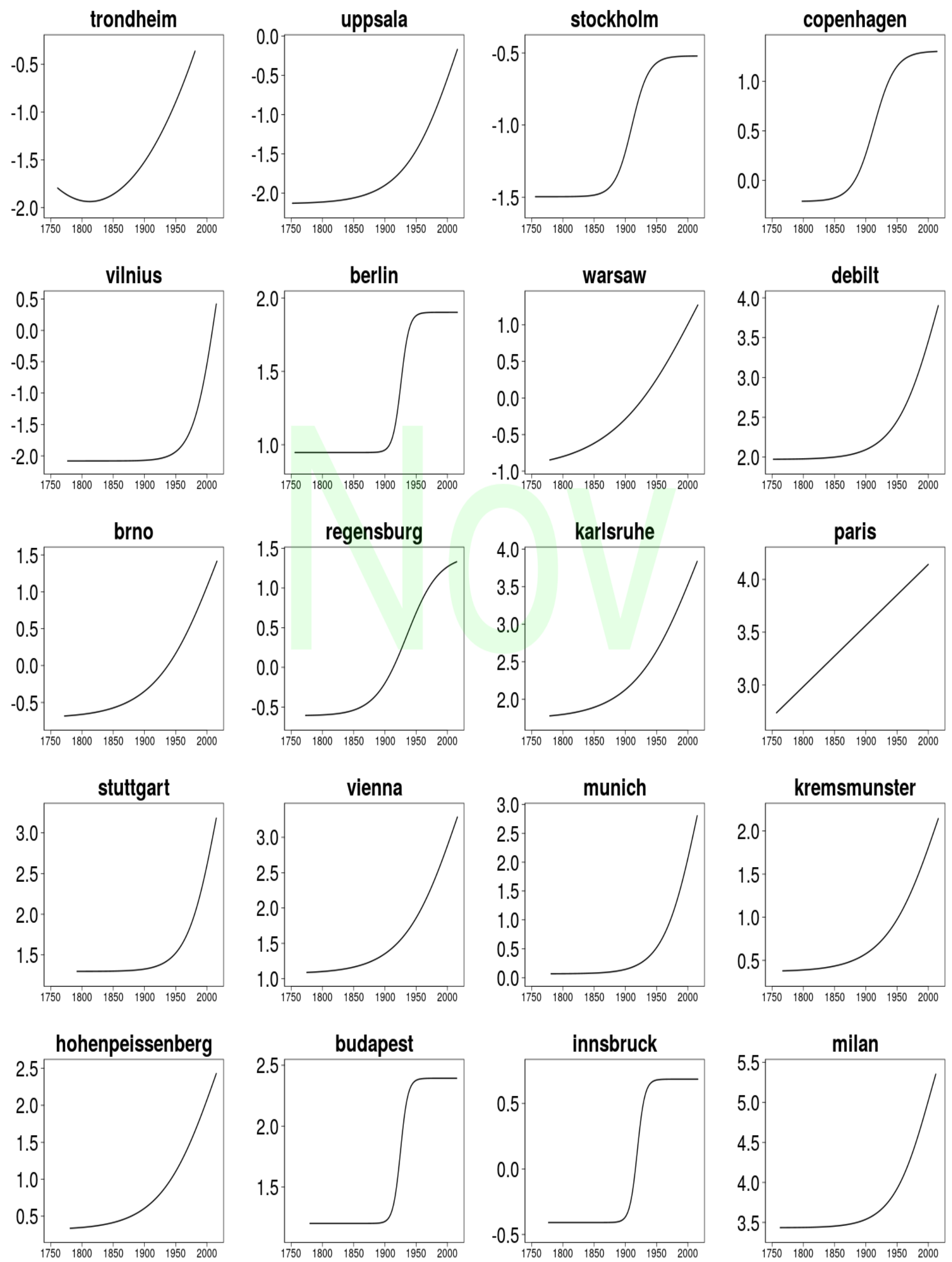

Figure A15. Shifting means of November for the 20 weather stations (from north to south) over more than 250 years of monthly observations

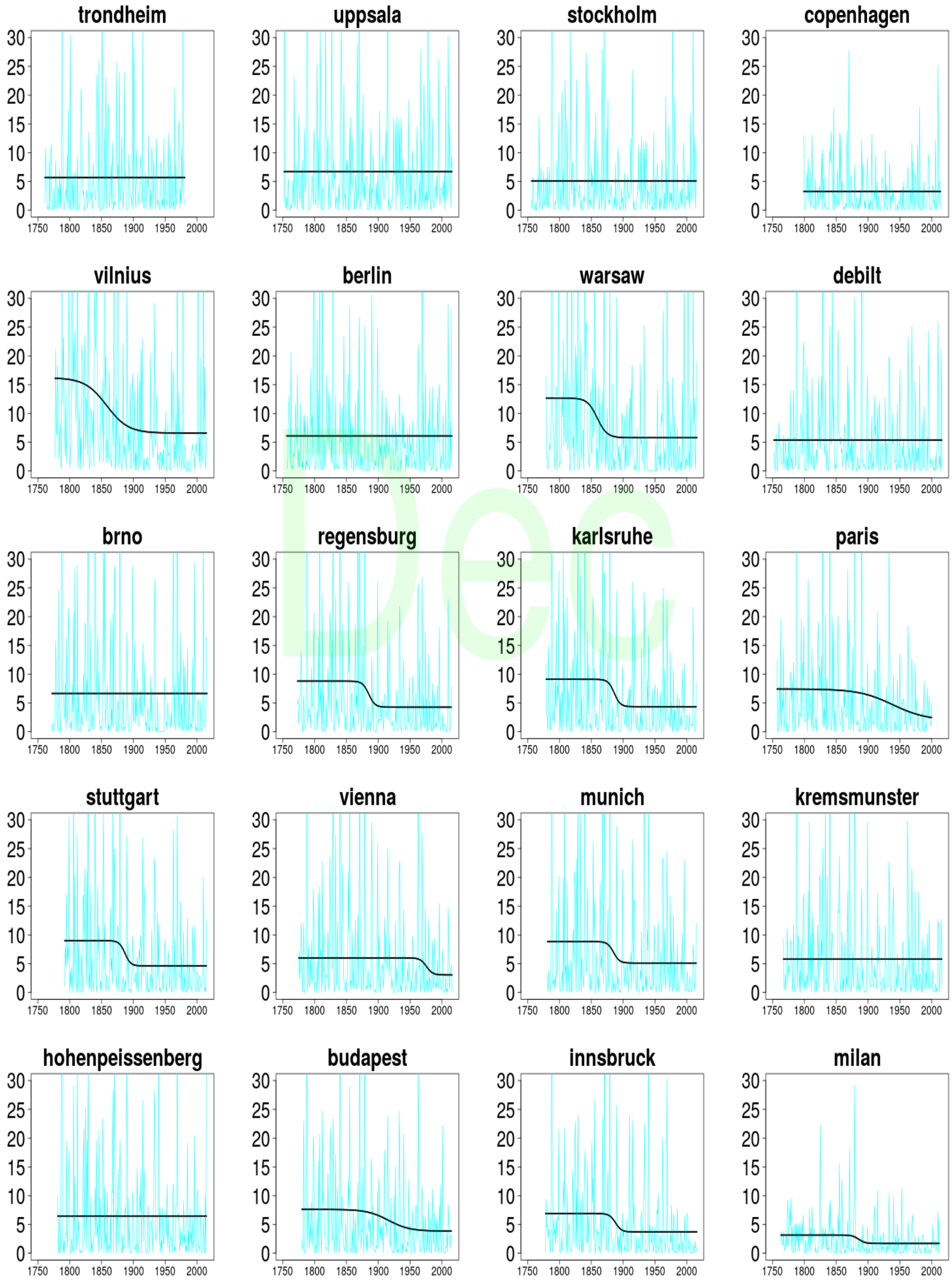

Figure A16. Estimated error variances (green) and the shifting variance (black) component for December in the 20 locations (from north to south)

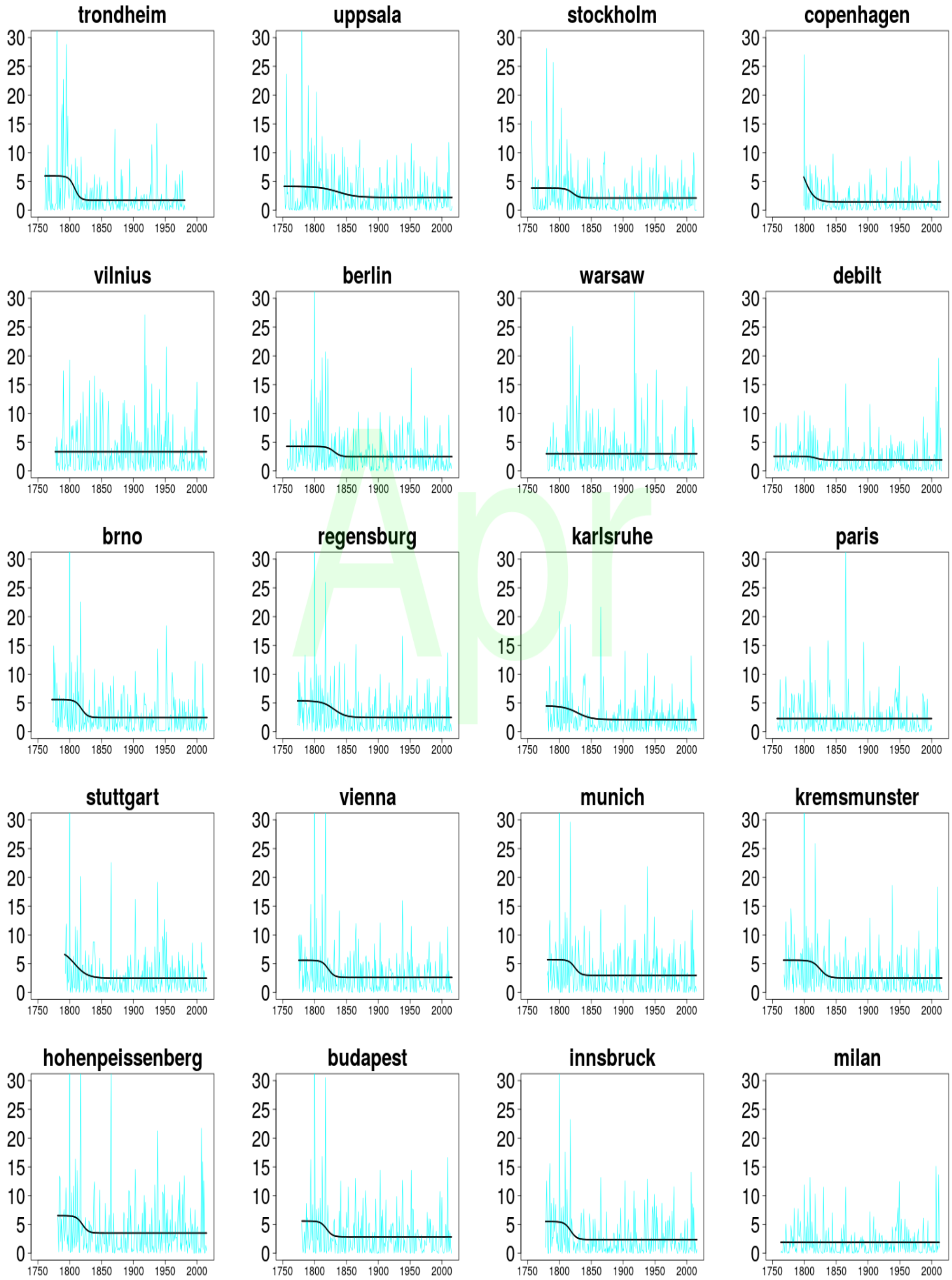

Figure A17. Estimated error variances (green) and the shifting variance (black) component for April in the 20 locations from north to south

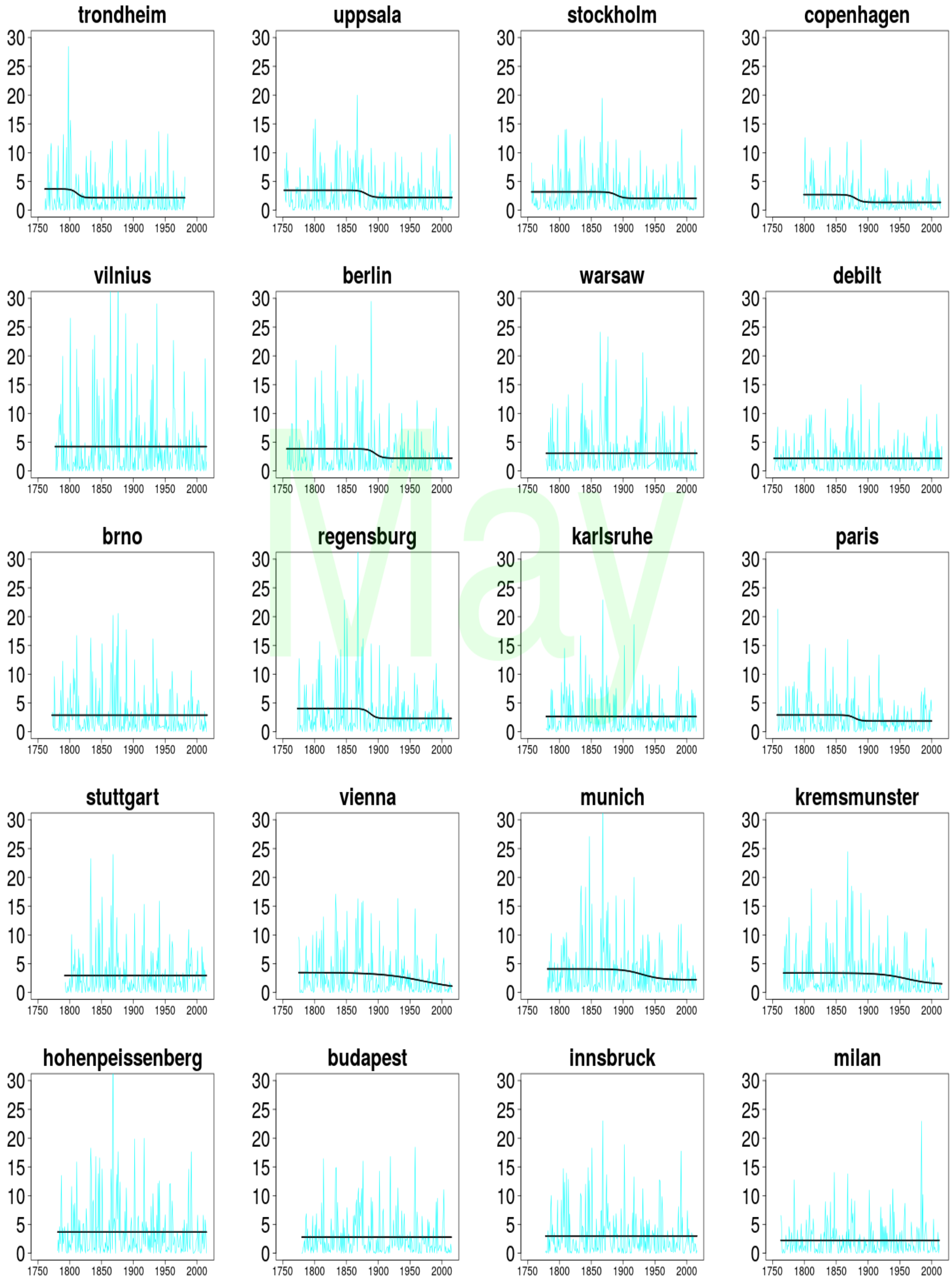

Figure A18. Estimated error variances (green) and the shifting variance (black) component for May in the 20 locations from north to south

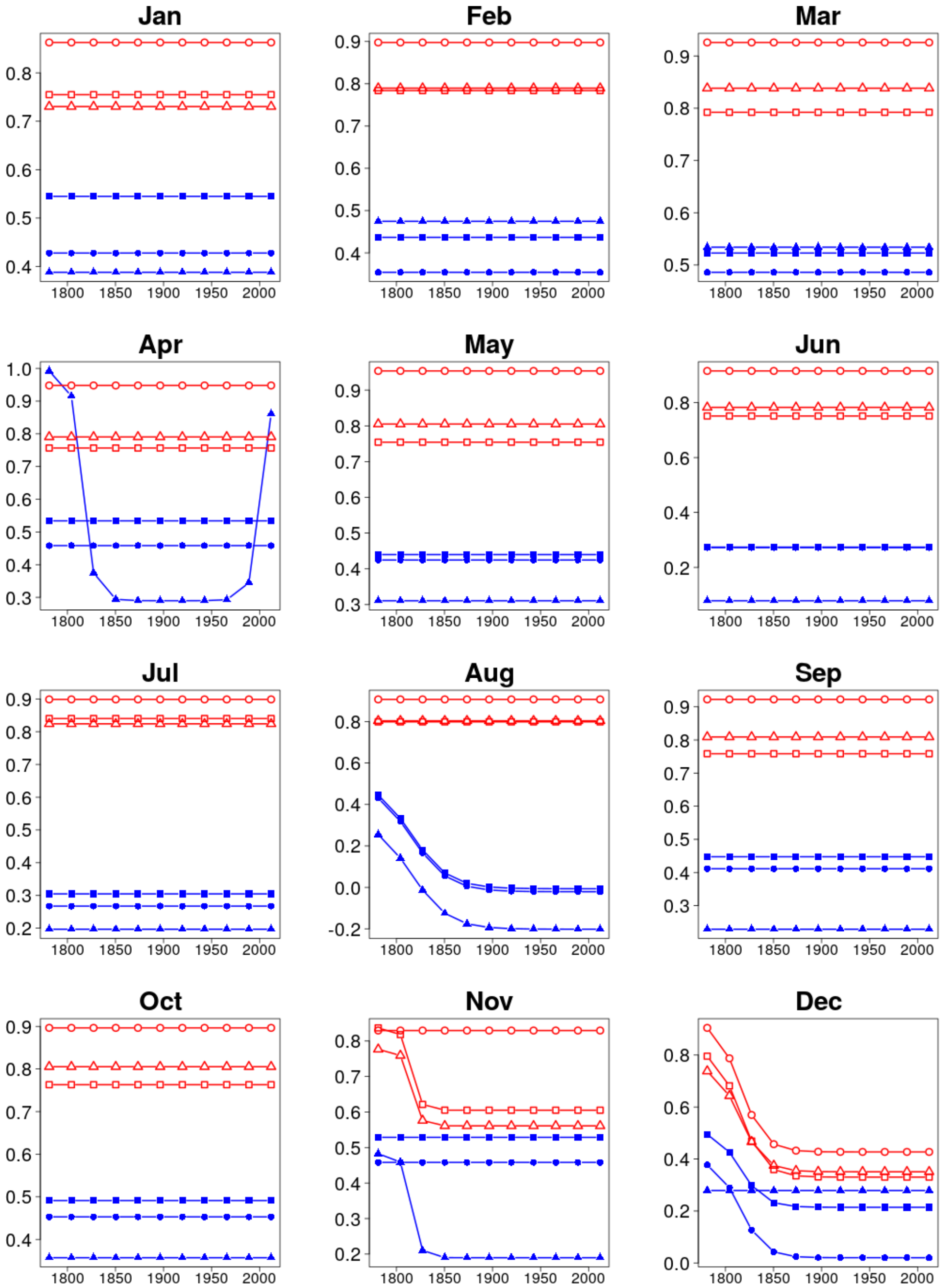

Figure A19. Time-varying correlations for four stations: red circles, Inns-Hpb; red squares, Inns-Mil; red triangles, Hpb-Mil; blue circles, Sto-Inns; blue squares, Sto-Hpb; blue triangles, Sto-Mil

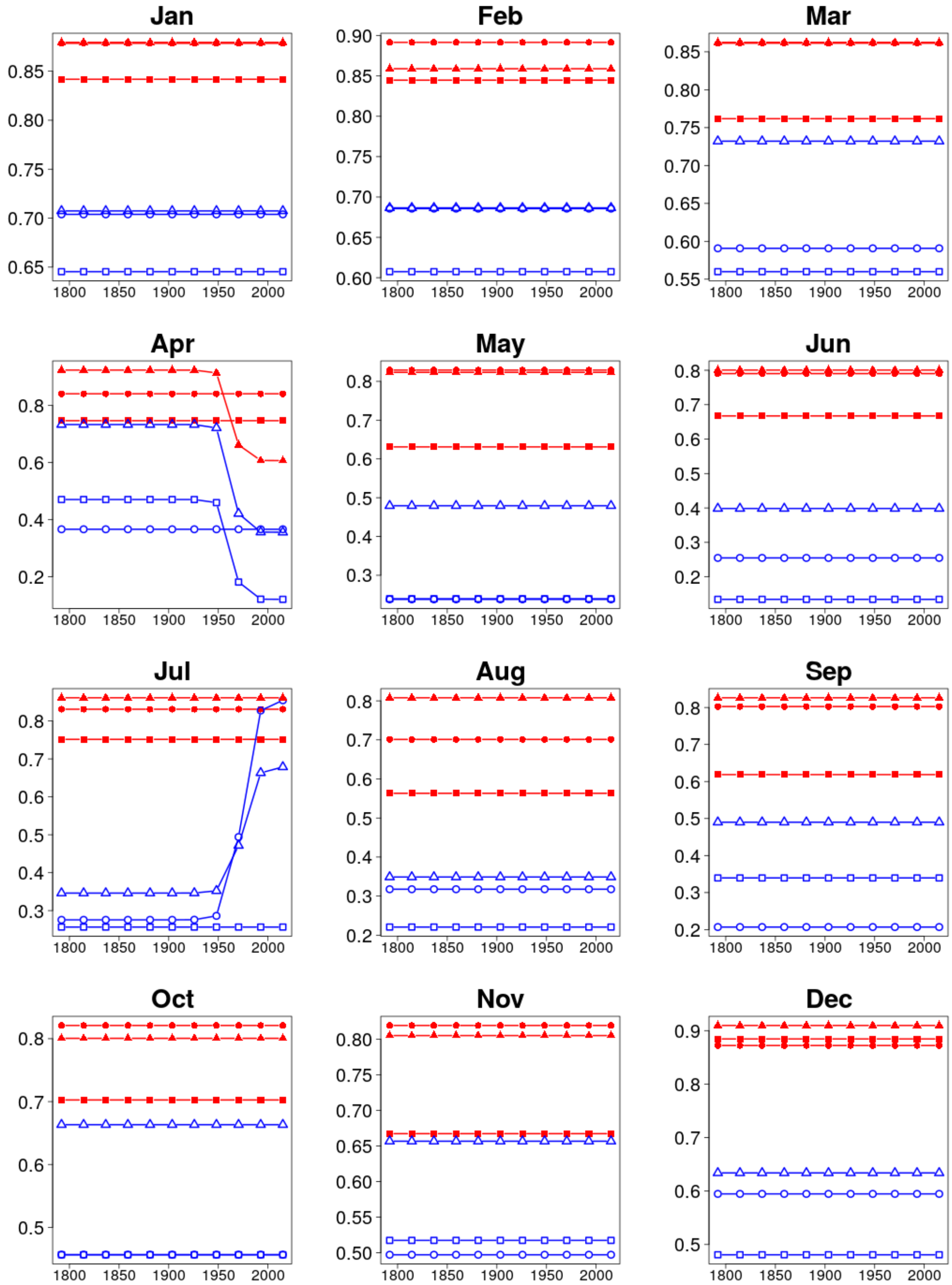

Figure A20. Time-varying correlations for four stations: blue circles, DeB-Vil; blue squares, Stu-Vil; blue triangles, Vie-Vil; red circles, DeB-Stu; red squares, DeB-Vie; red triangles, Stu-Vie

B Estimated equations

This appendix contains the estimated seasonal mean equations of the 20 temperature series, arranged according to latitude from north to south. Three remarks are in order. First, sometimes it has not been numerically possible to estimate the logistic transition function as there has not been enough information in the data about the two tails of the function. It has then been replaced by a linear trend. These cases are distinguished in the table by the fact that the estimates for both the slope parameter γ_j and location c_j are missing and replaced by dashes. The two parameter estimates, $\widehat{\delta}_{j0}$ and $\widehat{\delta}_{j1}$, are the intercept and the coefficient of the linear trend t/T. When there is no shift for month j, i.e., $\delta_{j1} = 0$, even this nonexisting estimate is replaced by a dash.

Second, in some cases the transition is rapid, and the slope parameter γ_{nj} has been fixed to the upper bound equalling 40. This is why the 'estimate' lacks a standard deviation estimate. Third, in the estimation of an exponential transition function, c_j has sometimes been fixed to a value, typically zero or unity. At the same time, the standard deviations of $\widehat{\gamma}_{nj}$ and $\widehat{\delta}_{nj1}$ are very large because the two estimates are strongly negative correlated and their joint uncertainty is large. As discussed in Section 4.3.2, the reason for this is that there has not been enough information in the data to cover the part of the exponential transition function where it begins to approach its limiting value unity. As a result, it has not been numerically possible to obtain estimates for these three parameters because the estimation algorithm has not converged properly.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.269	0.019	$\delta_{7,0}$	10.387	0.262
ϕ_2	0.040	0.008	$\delta_{7,1}$	-	-
$\delta_{1,0}$	-3.680	0.214	γ_7	-	-
$\delta_{1,1}$	1.128	0.272	c_7	-	-
γ_1	40.000	-	$\delta_{8,0}$	7.572	3.430
c_1	0.452	0.047	$\delta_{8,1}$	1.281	3.355
$\delta_{2,0}$	-2.030	0.148	γ_8	40.000	-
$\delta_{2,1}$	-	-	$c_{8,1}$	0.624	30.298
γ_2	-	-	$c_{8,2}$	0.620	30.298
c_2	-	-	$\delta_{9,0}$	5.064	0.333
$\delta_{3,0}$	-0.419	0.195	$\delta_{9,1}$	-	-
$\delta_{3,1}$	1.090	0.444	γ_9	-	-
γ_3	20.920	30.794	c_9	-	-
c_3	0.722	0.082	$\delta_{10,0}$	0.581	0.366
$\delta_{4,0}$	3.386	0.140	$\delta_{10,1}$	1.528	0.442
$\delta_{4,1}$	-	-	γ_{10}	-	-
γ_4	-	-	c_{10}	-	-
c_4	-	-	$\delta_{11,0}$	-7.716	188.303
$\delta_{5,0}$	6.742	0.161	$\delta_{11,1}$	14.711	375.964
$\delta_{5,1}$	2.973	1.645	γ_{11}	0.741	19.377
γ_5	19.248	15.556	$c_{11,1}$	1.000	-
c_5	1.000	-	$c_{11,2}$	-0.533	0.470
$\delta_{6,0}$	9.303	0.191	$\delta_{12,0}$	-3.584	0.446
$\delta_{6,1}$	-	-	$\delta_{12,1}$	2.004	1.229
γ_6	-	-	γ_{12}	6.616	7.111
c_6	-	-	c_{12}	0.615	0.165

Table 9: Estimates of parameters of the seasonal mean equation for Trondheim. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.332	0.018	c_6	0.137	0.050
ϕ_2	0.047	0.010	$\delta_{7,0}$	11.406	0.341
$\delta_{1,0}$	-6.904	1.775	$\delta_{7,1}$	23.757	2068.478
$\delta_{1,1}$	4.750	3.793	γ_7	0.161	14.290
γ_1	1.977	2.599	c_7	0.509	0.060
c_1	0.000	-	$\delta_{8,0}$	8.506	0.425
$\delta_{2,0}$	-2.581	0.149	$\delta_{8,1}$	0.920	0.371
$\delta_{2,1}$	-	-	γ_8	40.000	-
γ_2	-	-	c_8	0.553	0.050
c_2	-	-	$\delta_{9,0}$	5.050	0.351
$\delta_{3,0}$	1.255	0.334	$\delta_{9,1}$	-	-
$\delta_{3,1}$	-1.698	0.355	γ_9	-	-
γ_3	23.358	20.797	c_9	-	-
c_3	0.900	0.063	$\delta_{10,0}$	1.117	0.313
$\delta_{4,0}$	3.897	0.253	$\delta_{10,1}$	0.813	0.321
$\delta_{4,1}$	3.906	12.633	γ_{10}	40.000	-
γ_4	1.937	7.789	c_{10}	0.737	0.068
c_4	0.460	0.041	$\delta_{11,0}$	-2.137	0.297
$\delta_{5,0}$	-0.528	1550.554	$\delta_{11,1}$	3.930	0.991
$\delta_{5,1}$	19.076	3100.486	γ_{11}	6.263	2.950
γ_5	0.796	130.407	c_{11}	1.000	-
$c_{5,2}$	1.000	-	$\delta_{12,0}$	-4.342	0.273
$c_{5,1}$	0.000	-	$\delta_{12,1}$	2.163	0.443
$\delta_{6,0}$	12.301	0.495	γ_{12}	-	-
$\delta_{6,1}$	-1.247	0.484	c_{12}	-	-
γ_6	40.000	-			

Table 10: Estimates of parameters of the seasonal mean equation for Uppsala. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.350	0.018	c_6	0.000	-
ϕ_2	0.043	0.008	$\delta_{7,0}$	11.176	0.363
$\delta_{1,0}$	-3.736	0.226	$\delta_{7,1}$	0.646	0.349
$\delta_{1,1}$	1.054	0.268	γ_7	40.000	-
γ_1	40.000	-	c_7	0.499	0.066
c_1	0.367	0.048	$\delta_{8,0}$	8.604	0.409
$\delta_{2,0}$	-2.246	0.135	$\delta_{8,1}$	0.989	0.349
$\delta_{2,1}$	-	-	γ_8	40.000	-
γ_2	-	-	c_8	0.548	0.043
c_2	-	-	$\delta_{9,0}$	5.428	0.350
$\delta_{3,0}$	-0.888	0.256	$\delta_{9,1}$	-	-
$\delta_{3,1}$	1.875	0.419	γ_9	-	-
γ_3	-	-	c_9	-	-
c_3	-	-	$\delta_{10,0}$	1.638	0.317
$\delta_{4,0}$	3.682	0.228	$\delta_{10,1}$	0.732	0.295
$\delta_{4,1}$	25.343	784.258	γ_{10}	40.000	-
γ_4	0.232	7.379	c_{10}	0.722	0.071
c_4	0.457	0.040	$\delta_{11,0}$	-1.497	0.275
$\delta_{5,0}$	7.850	0.142	$\delta_{11,1}$	0.976	0.341
$\delta_{5,1}$	-	-	γ_{11}	19.227	27.307
γ_5	-	-	c_{11}	0.596	0.085
c_5	-	-	$\delta_{12,0}$	-3.356	0.263
$\delta_{6,0}$	13.879	0.935	$\delta_{12,1}$	1.641	0.418
$\delta_{6,1}$	-3.212	0.933	γ_{12}	-	-
γ_6	6.216	3.390	c_{12}	-	-

Table 11: Estimates of parameters of the seasonal mean equation for Stockholm. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.371	0.020	c_6	-	-
ϕ_2	0.043	0.013	$\delta_{7,0}$	10.692	0.316
ϕ_8	0.041	0.012	$\delta_{7,1}$	4.023	1.449
$\delta_{1,0}$	-2.261	0.282	γ_7	20.363	10.482
$\delta_{1,1}$	1.876	0.370	c_7	1.000	-
γ_1	-	-	$\delta_{8,0}$	9.017	0.411
c_1	-	-	$\delta_{8,1}$	1.335	0.459
$\delta_{2,0}$	-1.235	0.339	γ_8	10.185	9.061
$\delta_{2,1}$	3.222	0.807	c_8	0.437	0.041
γ_2	5.766	2.788	$\delta_{9,0}$	6.025	0.430
c_2	1.000	-	$\delta_{9,1}$	0.712	0.306
$\delta_{3,0}$	0.504	0.447	γ_9	18.171	21.677
$\delta_{3,1}$	4.110	0.767	c_9	0.405	0.064
γ_3	4.118	1.824	$\delta_{10,0}$	3.033	0.348
c_3	1.000	-	$\delta_{10,1}$	0.854	0.248
$\delta_{4,0}$	4.443	0.354	γ_{10}	40.000	-
$\delta_{4,1}$	3.437	0.831	c_{10}	0.690	0.053
γ_4	6.169	2.756	$\delta_{11,0}$	-0.214	0.331
c_4	1.000	-	$\delta_{11,1}$	1.521	0.367
$\delta_{5,0}$	7.939	0.568	γ_{11}	12.749	9.701
$\delta_{5,1}$	2.386	0.920	c_{11}	0.533	0.066
γ_5	3.292	3.107	$\delta_{12,0}$	-1.366	0.268
c_5	1.000	-	$\delta_{12,1}$	1.295	0.371
$\delta_{6,0}$	10.490	0.272	γ_{12}	-	-
$\delta_{6,1}$	-	-	c_{12}	-	-
γ_6	-	-			

Table 12: Estimates of parameters of the seasonal mean equation for Copenhagen. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.268	0.019	c_6	0.493	0.070
ϕ_2	0.060	0.010	$\delta_{7,0}$	13.054	0.640
$\delta_{1,0}$	-5.693	0.302	$\delta_{7,1}$	-0.555	0.641
$\delta_{1,1}$	1.001	0.342	γ_7	15.077	50.089
γ_1	40.000	-	c_7	0.292	0.260
c_1	0.332	0.063	$\delta_{8,0}$	9.969	0.506
$\delta_{2,0}$	-3.249	0.189	$\delta_{8,1}$	1.412	0.431
$\delta_{2,1}$	-	-	γ_8	18.984	16.528
γ_2	-	-	c_8	0.565	0.045
c_2	-	-	$\delta_{9,0}$	6.273	0.407
$\delta_{3,0}$	-0.251	0.334	$\delta_{9,1}$	-	-
$\delta_{3,1}$	0.977	0.521	γ_9	-	-
γ_3	-	-	c_9	-	-
c_3	-	-	$\delta_{10,0}$	1.954	0.354
$\delta_{4,0}$	5.699	0.282	$\delta_{10,1}$	-	-
$\delta_{4,1}$	20.547	715.656	γ_{10}	-	-
γ_4	0.232	8.308	c_{10}	-	-
c_4	0.417	0.070	$\delta_{11,0}$	-2.082	0.285
$\delta_{5,0}$	10.597	0.187	$\delta_{11,1}$	5.003	1.602
$\delta_{5,1}$	-	-	γ_{11}	12.794	6.193
γ_5	-	-	c_{11}	1.000	-
c_5	-	-	$\delta_{12,0}$	-5.184	0.339
$\delta_{6,0}$	12.707	0.310	$\delta_{12,1}$	5.894	1.086
$\delta_{6,1}$	-0.899	0.315	γ_{12}	5.357	2.026
γ_6	40.000	-	c_{12}	1.000	-

Table 13: Estimates of parameters of the seasonal mean equation for Vilnius. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.254	0.018	$c_{6,1}$	0.100	0.044
ϕ_2	0.043	0.009	$\delta_{7,0}$	13.629	0.402
$\delta_{1,0}$	-1.985	0.225	$\delta_{7,1}$	23.727	1086.442
$\delta_{1,1}$	1.541	0.268	γ_7	0.200	9.372
γ_1	40.000	-	c_7	0.459	0.051
c_1	0.394	0.034	$\delta_{8,0}$	12.256	0.459
$\delta_{2,0}$	0.986	0.126	$\delta_{8,1}$	3.129	8.824
$\delta_{2,1}$	-	-	γ_8	2.461	9.053
γ_2	-	-	c_8	0.506	0.039
c_2	-	-	$\delta_{9,0}$	9.130	0.424
$\delta_{3,0}$	3.279	0.178	$\delta_{9,1}$	-	-
$\delta_{3,1}$	0.954	0.264	γ_9	-	-
γ_3	40.000	-	c_9	-	-
c_3	0.558	0.054	$\delta_{10,0}$	4.580	0.719
$\delta_{4,0}$	7.332	0.237	$\delta_{10,1}$	1.810	1.082
$\delta_{4,1}$	16.298	506.258	γ_{10}	3.254	4.790
γ_4	0.309	9.925	c_{10}	1.000	-
c_4	0.473	0.047	$\delta_{11,0}$	0.946	0.313
$\delta_{5,0}$	11.329	0.228	$\delta_{11,1}$	0.955	0.277
$\delta_{5,1}$	0.352	0.264	γ_{11}	40.000	-
γ_5	40.000	-	c_{11}	0.653	0.055
c_5	0.573	0.147	$\delta_{12,0}$	-0.349	0.207
$\delta_{6,0}$	13.313	0.288	$\delta_{12,1}$	-	-
$\delta_{6,1}$	1.607	0.615	γ_{12}	-	-
γ_6	40.000	-	c_{12}	-	-
$c_{6,2}$	1.000	-			

Table 14: Estimates of parameters of the seasonal mean equation for Berlin. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.257	0.019	c_6	0.519	0.073
ϕ_2	0.037	0.008	$\delta_{7,0}$	13.653	0.426
$\delta_{1,0}$	-4.294	0.280	$\delta_{7,1}$	24.697	2979.495
$\delta_{1,1}$	2.370	0.472	γ_7	0.121	14.787
γ_1	-	-	c_7	0.553	0.091
c_1	-	-	$\delta_{8,0}$	11.782	0.490
$\delta_{2,0}$	-1.322	0.149	$\delta_{8,1}$	1.404	0.494
$\delta_{2,1}$	-	-	γ_8	10.390	9.637
γ_2	-	-	c_8	0.611	0.050
c_2	-	-	$\delta_{9,0}$	8.299	0.434
$\delta_{3,0}$	0.872	0.286	$\delta_{9,1}$	-	-
$\delta_{3,1}$	2.445	0.472	γ_9	-	-
γ_3	-	-	c_9	-	-
c_3	-	-	$\delta_{10,0}$	-8.315	8056.581
$\delta_{4,0}$	6.951	0.276	$\delta_{10,1}$	25.267	16112.652
$\delta_{4,1}$	0.701	0.471	γ_{10}	0.121	77.442
γ_4	-	-	c_{10}	1.000	-
c_4	-	-	$\delta_{11,0}$	-0.983	0.638
$\delta_{5,0}$	11.581	0.192	$\delta_{11,1}$	4.509	1.091
$\delta_{5,1}$	-	-	γ_{11}	3.475	2.092
γ_5	-	-	c_{11}	1.000	-
c_5	-	-	$\delta_{12,0}$	-14.479	118.714
$\delta_{6,0}$	13.841	0.326	$\delta_{12,1}$	21.220	237.903
$\delta_{6,1}$	-0.833	0.312	γ_{12}	0.494	5.741
γ_6	35.295	79.452	c_{12}	0.000	-

Table 15: Estimates of parameters of the seasonal mean equation for Warsaw. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.250	0.018	$\delta_{7,0}$	12.246	0.346
ϕ_2	0.048	0.012	$\delta_{7,1}$	20.654	1003.719
$\delta_{1,0}$	-1.557	1.960	γ_7	0.152	7.520
$\delta_{1,1}$	7.069	3.594	c_7	0.398	0.077
γ_1	1.866	1.501	$\delta_{8,0}$	3.147	2787.025
c_1	1.000	-	$\delta_{8,1}$	18.089	5573.516
$\delta_{2,0}$	2.016	0.114	γ_8	0.611	189.009
$\delta_{2,1}$	-	-	$c_{8,2}$	1.000	-
γ_2	-	-	$c_{8,1}$	0.000	-
c_2	-	-	$\delta_{9,0}$	9.189	0.372
$\delta_{3,0}$	3.575	0.232	$\delta_{9,1}$	-	-
$\delta_{3,1}$	3.483	0.768	γ_9	-	-
γ_3	5.254	2.377	c_9	-	-
c_3	1.000	-	$\delta_{10,0}$	5.479	0.351
$\delta_{4,0}$	6.958	0.133	$\delta_{10,1}$	0.764	0.277
$\delta_{4,1}$	-	-	γ_{10}	40.000	-
γ_4	-	-	c_{10}	0.769	0.060
c_4	-	-	$\delta_{11,0}$	1.971	0.308
$\delta_{5,0}$	10.017	0.179	$\delta_{11,1}$	3.860	0.885
$\delta_{5,1}$	-	-	γ_{11}	7.784	3.019
γ_5	-	-	c_{11}	1.000	-
c_5	-	-	$\delta_{12,0}$	0.492	0.261
$\delta_{6,0}$	11.757	0.248	$\delta_{12,1}$	3.243	0.794
$\delta_{6,1}$	-	-	γ_{12}	6.069	2.815
γ_6	-	-	c_{12}	1.000	-
c_6	-	-			

Table 16: Estimates of parameters of the seasonal mean equation for De Bilt. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.227	0.018	c_6	-	-
ϕ_2	0.045	0.009	$\delta_{7,0}$	13.373	0.344
$\delta_{1,0}$	-4.016	0.504	$\delta_{7,1}$	-	-
$\delta_{1,1}$	2.375	0.768	γ_7	-	-
γ_1	7.909	5.586	c_7	-	-
c_1	0.460	0.086	$\delta_{8,0}$	12.116	0.397
$\delta_{2,0}$	-0.765	0.135	$\delta_{8,1}$	4.869	1.871
$\delta_{2,1}$	-	-	γ_8	40.000	-
γ_2	-	-	$c_{8,1}$	1.000	-
c_2	-	-	$c_{8,2}$	-0.043	0.044
$\delta_{3,0}$	1.649	0.420	$\delta_{9,0}$	8.387	0.407
$\delta_{3,1}$	4.889	0.913	$\delta_{9,1}$	-	-
γ_3	3.984	1.802	γ_9	-	-
c_3	1.000	-	c_9	-	-
$\delta_{4,0}$	6.548	0.148	$\delta_{10,0}$	3.714	0.378
$\delta_{4,1}$	1.849	0.703	$\delta_{10,1}$	0.784	0.359
γ_4	40.000	-	γ_{10}	26.413	54.206
$c_{4,1}$	0.000	-	c_{10}	0.694	0.090
$c_{4,2}$	0.912	0.040	$\delta_{11,0}$	-0.712	0.381
$\delta_{5,0}$	10.736	0.178	$\delta_{11,1}$	4.253	0.889
$\delta_{5,1}$	-	-	γ_{11}	4.983	2.237
γ_5	-	-	c_{11}	1.000	-
c_5	-	-	$\delta_{12,0}$	-2.889	0.466
$\delta_{6,0}$	12.623	0.265	$\delta_{12,1}$	4.133	0.925
$\delta_{6,1}$	-	-	γ_{12}	3.798	2.097
γ_6	-	-	c_{12}	1.000	-

Table 17: Estimates of parameters of the seasonal mean equation for Brno. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.203	0.019	c_6	-	-
ϕ_2	0.042	0.011	$\delta_{7,0}$	13.631	0.374
$\delta_{1,0}$	-3.396	0.276	$\delta_{7,1}$	3.862	1.314
$\delta_{1,1}$	5.734	0.934	γ_7	11.671	6.011
γ_1	5.331	1.763	c_7	1.000	-
c_1	1.000	-	$\delta_{8,0}$	12.376	0.454
$\delta_{2,0}$	-0.284	0.136	$\delta_{8,1}$	21.201	548.502
$\delta_{2,1}$	-	-	γ_8	0.297	7.932
γ_2	-	-	c_8	0.473	0.038
c_2	-	-	$\delta_{9,0}$	8.778	0.489
$\delta_{3,0}$	2.800	0.260	$\delta_{9,1}$	0.749	0.417
$\delta_{3,1}$	2.796	3.663	γ_9	14.411	23.316
γ_3	7.529	8.568	c_9	0.476	0.077
c_3	0.862	0.361	$\delta_{10,0}$	4.465	0.383
$\delta_{4,0}$	6.985	0.232	$\delta_{10,1}$	1.052	1.112
$\delta_{4,1}$	20.849	719.309	γ_{10}	25.865	57.372
γ_4	0.228	8.069	c_{10}	0.883	0.124
c_4	0.436	0.055	$\delta_{11,0}$	-0.610	0.367
$\delta_{5,0}$	10.782	0.289	$\delta_{11,1}$	2.027	0.801
$\delta_{5,1}$	1.387	1.628	γ_{11}	9.436	7.836
γ_5	5.858	12.788	c_{11}	0.670	0.104
c_5	0.496	0.055	$\delta_{12,0}$	-2.736	0.296
$\delta_{6,0}$	12.934	0.351	$\delta_{12,1}$	4.799	0.928
$\delta_{6,1}$	0.648	0.434	γ_{12}	5.591	2.155
γ_6	-	-	c_{12}	1.000	-

Table 18: Estimates of parameters of the seasonal mean equation for Regensburg. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.171	0.019	$\delta_{7,1}$	5.857	3.818
ϕ_2	0.032	0.010	γ_7	5.434	7.056
$\delta_{1,0}$	-0.939	0.193	$c_{7,1}$	1.000	-
$\delta_{1,1}$	1.550	0.266	$c_{7,2}$	-0.162	0.094
γ_1	40.000	-	$\delta_{8,0}$	13.873	0.534
c_1	0.549	0.034	$\delta_{8,1}$	2.063	0.945
$\delta_{2,0}$	1.746	0.126	γ_8	24.076	22.439
$\delta_{2,1}$	-	-	$c_{8,1}$	0.044	0.057
γ_2	-	-	$c_{8,2}$	0.837	0.059
c_2	-	-	$\delta_{9,0}$	10.364	0.604
$\delta_{3,0}$	4.342	0.224	$\delta_{9,1}$	0.966	0.418
$\delta_{3,1}$	2.500	2.330	γ_9	40.000	-
γ_3	8.794	9.045	$c_{9,1}$	0.173	0.114
c_3	0.830	0.235	$c_{9,2}$	0.633	0.114
$\delta_{4,0}$	8.135	0.245	$\delta_{10,0}$	6.053	0.423
$\delta_{4,1}$	24.731	1007.202	$\delta_{10,1}$	2.357	2.988
γ_4	0.217	9.032	γ_{10}	13.650	17.194
c_4	0.469	0.044	c_{10}	0.901	0.213
$\delta_{5,0}$	11.580	0.294	$\delta_{11,0}$	1.740	0.436
$\delta_{5,1}$	3.898	17.955	$\delta_{11,1}$	4.193	0.881
γ_5	1.646	9.094	γ_{11}	4.689	2.201
c_5	0.471	0.044	c_{11}	1.000	-
$\delta_{6,0}$	14.222	0.312	$\delta_{12,0}$	-0.337	0.272
$\delta_{6,1}$	1.869	0.826	$\delta_{12,1}$	4.954	1.000
γ_6	40.000	-	γ_{12}	6.850	2.462
c_6	0.923	0.041	c_{12}	1.000	-
$\delta_{7,0}$	14.313	2.260			

Table 19: Estimates of parameters of the seasonal mean equation for Karlsruhe. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.146	0.018	c_6	-	-
ϕ_2	0.037	0.012	$\delta_{7,0}$	15.490	0.379
ϕ_6	0.047	0.012	$\delta_{7,1}$	-	-
$\delta_{1,0}$	-0.493	1.016	γ_7	-	-
$\delta_{1,1}$	2.639	1.524	c_7	-	-
γ_1	5.326	4.934	$\delta_{8,0}$	14.363	0.472
c_1	0.449	0.138	$\delta_{8,1}$	19.437	1076.928
$\delta_{2,0}$	2.647	0.347	γ_8	0.208	11.810
$\delta_{2,1}$	-	-	c_8	0.493	0.050
γ_2	-	-	$\delta_{9,0}$	11.684	0.456
c_2	-	-	$\delta_{9,1}$	-	-
$\delta_{3,0}$	3.695	1.836	γ_9	-	-
$\delta_{3,1}$	4.603	3.294	c_9	-	-
γ_3	1.984	2.348	$\delta_{10,0}$	7.112	0.455
c_3	1.000	-	$\delta_{10,1}$	3.126	0.972
$\delta_{4,0}$	8.261	0.250	γ_{10}	7.980	4.159
$\delta_{4,1}$	-	-	c_{10}	1.000	-
γ_4	-	-	$\delta_{11,0}$	2.737	0.438
c_4	-	-	$\delta_{11,1}$	1.406	0.389
$\delta_{5,0}$	11.708	0.247	γ_{11}	-	-
$\delta_{5,1}$	-	-	c_{11}	-	-
γ_5	-	-	$\delta_{12,0}$	0.095	0.412
c_5	-	-	$\delta_{12,1}$	2.448	0.390
$\delta_{6,0}$	14.222	0.307	γ_{12}	-	-
$\delta_{6,1}$	-	-	c_{12}	-	-
γ_6	-	-			

Table 20: Estimates of parameters of the seasonal mean equation for Paris. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.167	0.019	c_6	0.919	0.042
ϕ_2	0.036	0.006	$\delta_{7,0}$	14.614	0.427
ϕ_6	0.053	0.006	$\delta_{7,1}$	20.936	526.002
$\delta_{1,0}$	-3.200	1.445	γ_7	0.232	6.012
$\delta_{1,1}$	6.835	2.436	c_7	0.395	0.063
γ_1	2.327	1.591	$\delta_{8,0}$	13.655	0.445
c_1	1.000	-	$\delta_{8,1}$	1.796	0.474
$\delta_{2,0}$	0.644	0.360	γ_8	40.000	-
$\delta_{2,1}$	-	-	c_8	0.857	0.035
γ_2	-	-	$\delta_{9,0}$	9.938	0.469
c_2	-	-	$\delta_{9,1}$	0.837	0.285
$\delta_{3,0}$	3.254	0.386	γ_9	40.000	-
$\delta_{3,1}$	4.389	0.970	c_9	0.603	0.066
γ_3	5.322	2.391	$\delta_{10,0}$	5.552	0.451
c_3	1.000	-	$\delta_{10,1}$	3.588	1.153
$\delta_{4,0}$	7.476	0.235	γ_{10}	8.316	4.395
$\delta_{4,1}$	2.619	1.398	c_{10}	1.000	-
γ_4	40.000	-	$\delta_{11,0}$	1.297	0.409
c_4	0.947	0.039	$\delta_{11,1}$	3.768	1.216
$\delta_{5,0}$	10.553	0.347	γ_{11}	9.447	4.858
$\delta_{5,1}$	1.206	0.379	c_{11}	1.000	-
γ_5	40.000	-	$\delta_{12,0}$	-1.342	0.417
c_5	0.384	0.038	$\delta_{12,1}$	4.323	12.275
$\delta_{6,0}$	13.644	0.310	γ_{12}	7.625	9.793
$\delta_{6,1}$	1.861	0.822	c_{12}	0.987	0.683
γ_6	40.000	-			

Table 21: Estimates of parameters of the seasonal mean equation for Stuttgart. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.203	0.019	c_6	0.349	0.090
ϕ_2	0.037	0.005	$\delta_{7,0}$	15.128	0.431
$\delta_{1,0}$	-2.014	0.161	$\delta_{7,1}$	20.328	563.428
$\delta_{1,1}$	2.485	0.704	γ_7	0.194	5.542
γ_1	30.640	25.555	c_7	0.358	0.083
c_1	0.857	0.034	$\delta_{8,0}$	13.651	0.492
$\delta_{2,0}$	0.218	0.233	$\delta_{8,1}$	20.948	413.541
$\delta_{2,1}$	25.270	727.353	γ_8	0.338	6.922
γ_2	0.259	7.665	c_8	0.470	0.034
c_2	0.473	0.036	$\delta_{9,0}$	10.369	0.469
$\delta_{3,0}$	3.993	0.203	$\delta_{9,1}$	-	-
$\delta_{3,1}$	3.933	1.051	γ_9	-	-
γ_3	7.257	3.316	c_9	-	-
c_3	1.000	-	$\delta_{10,0}$	5.826	0.420
$\delta_{4,0}$	8.278	0.231	$\delta_{10,1}$	0.572	0.450
$\delta_{4,1}$	20.750	581.519	γ_{10}	40.000	-
γ_4	0.230	6.642	c_{10}	0.858	0.103
c_4	0.413	0.058	$\delta_{11,0}$	1.071	0.394
$\delta_{5,0}$	11.912	0.296	$\delta_{11,1}$	4.432	0.916
$\delta_{5,1}$	2.551	6.566	γ_{11}	5.591	2.309
γ_5	2.803	9.797	c_{11}	1.000	-
c_5	0.481	0.045	$\delta_{12,0}$	-1.133	0.235
$\delta_{6,0}$	14.064	0.355	$\delta_{12,1}$	0.957	0.300
$\delta_{6,1}$	18.937	516.216	γ_{12}	40.000	-
γ_6	0.198	5.574	c_{12}	0.714	0.056

Table 22: Estimates of parameters of the seasonal mean equation for Vienna. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.184	0.019	c_6	0.929	0.035
ϕ_2	0.040	0.006	$\delta_{7,0}$	13.312	0.405
$\delta_{1,0}$	-2.038	0.198	$\delta_{7,1}$	21.294	364.924
$\delta_{1,1}$	1.077	0.284	γ_7	0.296	5.268
γ_1	40.000	-	c_7	0.409	0.048
c_1	0.549	0.052	$\delta_{8,0}$	12.490	0.459
$\delta_{2,0}$	0.040	0.136	$\delta_{8,1}$	20.663	400.603
$\delta_{2,1}$	-	-	γ_8	0.362	7.296
γ_2	-	-	c_8	0.477	0.034
c_2	-	-	$\delta_{9,0}$	10.515	3.822
$\delta_{3,0}$	2.706	0.271	$\delta_{9,1}$	-0.944	3.855
$\delta_{3,1}$	5.372	1.000	γ_9	17.960	69.069
γ_3	5.607	2.057	c_9	0.073	0.522
c_3	1.000	-	$\delta_{10,0}$	4.157	0.494
$\delta_{4,0}$	6.666	0.242	$\delta_{10,1}$	1.543	0.379
$\delta_{4,1}$	22.828	477.313	γ_{10}	29.924	19.867
γ_4	0.268	5.795	c_{10}	0.565	0.033
c_4	0.418	0.048	$\delta_{11,0}$	0.066	0.331
$\delta_{5,0}$	10.141	0.282	$\delta_{11,1}$	5.473	1.186
$\delta_{5,1}$	20.781	395.159	γ_{11}	8.649	3.065
γ_5	0.362	7.166	c_{11}	1.000	-
c_5	0.474	0.034	$\delta_{12,0}$	-2.030	0.333
$\delta_{6,0}$	12.815	0.282	$\delta_{12,1}$	4.909	0.965
$\delta_{6,1}$	2.410	0.981	γ_{12}	5.287	2.150
γ_6	40.000	-	c_{12}	1.000	-

Table 23: Estimates of parameters of the seasonal mean equation for Munich. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.196	0.018	c_6	0.925	0.037
ϕ_2	0.035	0.007	$\delta_{7,0}$	14.036	0.396
$\delta_{1,0}$	-2.919	0.245	$\delta_{7,1}$	24.214	767.447
$\delta_{1,1}$	4.728	0.905	γ_7	0.221	7.177
γ_1	5.696	2.127	c_7	0.444	0.046
c_1	1.000	-	$\delta_{8,0}$	12.957	0.444
$\delta_{2,0}$	0.010	0.127	$\delta_{8,1}$	20.255	577.705
$\delta_{2,1}$	-	-	γ_8	0.247	7.254
γ_2	-	-	c_8	0.436	0.050
c_2	-	-	$\delta_{9,0}$	9.856	0.426
$\delta_{3,0}$	3.300	0.186	$\delta_{9,1}$	-	-
$\delta_{3,1}$	3.687	1.065	γ_9	-	-
γ_3	8.168	3.867	c_9	-	-
c_3	1.000	-	$\delta_{10,0}$	5.195	0.383
$\delta_{4,0}$	7.387	0.216	$\delta_{10,1}$	-	-
$\delta_{4,1}$	20.711	958.811	γ_{10}	-	-
γ_4	0.155	7.329	c_{10}	-	-
c_4	0.377	0.092	$\delta_{11,0}$	0.369	0.351
$\delta_{5,0}$	10.854	0.299	$\delta_{11,1}$	3.550	0.900
$\delta_{5,1}$	1.210	0.467	γ_{11}	5.966	2.907
γ_5	12.051	12.938	c_{11}	1.000	-
c_5	0.477	0.047	$\delta_{12,0}$	-2.350	0.229
$\delta_{6,0}$	13.180	0.283	$\delta_{12,1}$	1.041	0.252
$\delta_{6,1}$	2.021	0.819	γ_{12}	40.000	-
γ_6	40.000	-	c_{12}	0.545	0.048

Table 24: Estimates of parameters of the seasonal mean equation for Kremsmünster. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.132	0.019	$\delta_{7,0}$	12.490	0.363
ϕ_2	0.025	0.013	$\delta_{7,1}$	20.559	638.147
$\delta_{1,0}$	-2.356	0.255	γ_7	0.208	6.656
$\delta_{1,1}$	3.311	9.580	c_7	0.385	0.076
γ_1	7.670	11.791	$\delta_{8,0}$	11.996	0.384
c_1	0.956	0.717	$\delta_{8,1}$	1.373	0.453
$\delta_{2,0}$	-1.020	0.143	γ_8	40.000	-
$\delta_{2,1}$	-	-	$c_{8,1}$	0.859	0.056
γ_2	-	-	$c_{8,2}$	0.091	0.055
c_2	-	-	$\delta_{9,0}$	9.374	0.390
$\delta_{3,0}$	0.731	0.421	$\delta_{9,1}$	-	-
$\delta_{3,1}$	3.373	5.259	γ_9	-	-
γ_3	5.603	7.149	c_9	-	-
c_3	0.855	0.535	$\delta_{10,0}$	5.000	0.398
$\delta_{4,0}$	4.670	0.253	$\delta_{10,1}$	23.962	965.279
$\delta_{4,1}$	23.546	932.475	γ_{10}	0.151	6.249
γ_4	0.227	9.229	c_{10}	0.350	0.102
c_4	0.458	0.050	$\delta_{11,0}$	0.316	0.380
$\delta_{5,0}$	8.404	0.270	$\delta_{11,1}$	4.229	0.990
$\delta_{5,1}$	10.726	118.754	γ_{11}	5.302	2.564
γ_5	0.661	7.876	c_{11}	1.000	-
c_5	0.469	0.039	$\delta_{12,0}$	-2.264	0.301
$\delta_{6,0}$	11.132	0.248	$\delta_{12,1}$	1.678	0.474
$\delta_{6,1}$	4.717	1.768	γ_{12}	-	-
γ_6	17.713	9.516	c_{12}	-	-
c_6	1.000	-			

Table 25: Estimates of parameters of the seasonal mean equation for Hohenpeissenberg. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.208	0.019	$\delta_{7,0}$	16.042	0.459
ϕ_2	0.032	0.013	$\delta_{7,1}$	20.332	623.966
$\delta_{1,0}$	-2.142	0.163	γ_7	0.187	5.909
$\delta_{1,1}$	2.218	0.720	c_7	0.362	0.084
γ_1	36.724	36.716	$\delta_{8,0}$	14.624	0.516
c_1	0.875	0.034	$\delta_{8,1}$	20.797	340.986
$\delta_{2,0}$	0.434	0.152	γ_8	0.306	5.223
$\delta_{2,1}$	1.218	0.347	c_8	0.418	0.043
γ_2	40.000	-	$\delta_{9,0}$	10.542	0.588
c_2	0.782	0.045	$\delta_{9,1}$	0.991	0.362
$\delta_{3,0}$	4.392	0.262	γ_9	40.000	-
$\delta_{3,1}$	3.826	0.930	$c_{9,1}$	0.144	0.098
γ_3	5.463	2.658	$c_{9,2}$	0.646	0.098
c_3	1.000	-	$\delta_{10,0}$	6.498	0.447
$\delta_{4,0}$	9.244	0.259	$\delta_{10,1}$	-	-
$\delta_{4,1}$	0.918	0.432	γ_{10}	-	-
γ_4	-	-	c_{10}	-	-
c_4	-	-	$\delta_{11,0}$	1.200	0.358
$\delta_{5,0}$	13.041	0.301	$\delta_{11,1}$	1.193	0.272
$\delta_{5,1}$	9.110	190.809	γ_{11}	40.000	-
γ_5	0.472	10.438	c_{11}	0.617	0.044
c_5	0.454	0.059	$\delta_{12,0}$	-1.647	0.247
$\delta_{6,0}$	14.915	0.423	$\delta_{12,1}$	0.915	0.296
$\delta_{6,1}$	0.481	0.310	γ_{12}	40.000	-
γ_6	40.000	-	c_{12}	0.702	0.058
c_6	0.267	0.112			

Table 26: Estimates of parameters of the seasonal mean equation for Budapest. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.177	0.019	c_6	0.922	0.036
ϕ_2	0.063	0.013	$\delta_{7,0}$	13.853	0.372
ϕ_7	0.055	0.013	$\delta_{7,1}$	1.632	0.472
$\delta_{1,0}$	-3.935	0.376	γ_7	40.000	-
$\delta_{1,1}$	5.469	0.944	c_7	0.870	0.036
γ_1	6.136	1.965	$\delta_{8,0}$	12.554	0.455
c_1	1.000	-	$\delta_{8,1}$	20.633	459.707
$\delta_{2,0}$	-0.440	0.352	γ_8	0.272	6.264
$\delta_{2,1}$	-	-	c_8	0.431	0.046
γ_2	-	-	$\delta_{9,0}$	9.740	0.425
c_2	-	-	$\delta_{9,1}$	-	-
$\delta_{3,0}$	2.787	0.362	γ_9	-	-
$\delta_{3,1}$	1.226	0.255	c_9	-	-
γ_3	40.000	-	$\delta_{10,0}$	4.996	0.390
c_3	0.523	0.041	$\delta_{10,1}$	2.601	1.301
$\delta_{4,0}$	6.851	0.336	γ_{10}	12.909	9.688
$\delta_{4,1}$	22.093	1128.961	c_{10}	1.000	-
γ_4	0.201	10.500	$\delta_{11,0}$	-0.409	0.345
c_4	0.470	0.051	$\delta_{11,1}$	1.095	0.259
$\delta_{5,0}$	10.358	0.315	γ_{11}	40.000	-
$\delta_{5,1}$	3.074	5.964	c_{11}	0.595	0.046
γ_5	2.928	7.803	$\delta_{12,0}$	-4.213	0.537
c_5	0.511	0.035	$\delta_{12,1}$	4.699	0.896
$\delta_{6,0}$	12.917	0.292	γ_{12}	3.776	1.787
$\delta_{6,1}$	2.037	0.791	c_{12}	1.000	-
γ_6	40.000	-			

Table 27: Estimates of parameters of the seasonal mean equation for Innsbruck. Note: see the explanations in the beginning of Appendix B.

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_1	0.220	0.018	c_6	1.000	-
ϕ_2	0.022	0.011	$\delta_{7,0}$	18.412	0.457
$\delta_{1,0}$	0.626	0.165	$\delta_{7,1}$	2.732	0.867
$\delta_{1,1}$	1.887	0.376	γ_7	9.045	4.590
γ_1	32.236	24.264	c_7	1.000	-
c_1	0.815	0.028	$\delta_{8,0}$	17.045	0.514
$\delta_{2,0}$	4.011	0.108	$\delta_{8,1}$	1.817	0.435
$\delta_{2,1}$	-	-	γ_8	40.000	-
γ_2	-	-	c_8	0.893	0.027
c_2	-	-	$\delta_{9,0}$	13.055	0.533
$\delta_{3,0}$	7.589	0.142	$\delta_{9,1}$	0.747	0.215
$\delta_{3,1}$	3.831	0.976	γ_9	40.000	-
γ_3	11.150	4.293	c_9	0.673	0.053
c_3	1.000	-	$\delta_{10,0}$	8.303	0.482
$\delta_{4,0}$	10.921	0.187	$\delta_{10,1}$	1.395	0.803
$\delta_{4,1}$	-	-	γ_{10}	22.223	26.233
γ_4	-	-	c_{10}	0.866	0.077
c_4	-	-	$\delta_{11,0}$	3.433	0.398
$\delta_{5,0}$	14.276	0.270	$\delta_{11,1}$	3.345	6.774
$\delta_{5,1}$	4.019	1.332	γ_{11}	8.289	8.689
γ_5	21.146	9.893	c_{11}	0.964	0.465
c_5	1.000	-	$\delta_{12,0}$	0.454	0.412
$\delta_{6,0}$	17.038	0.360	$\delta_{12,1}$	3.053	0.690
$\delta_{6,1}$	4.079	1.399	γ_{12}	3.858	2.145
γ_6	23.680	11.409	c_{12}	1.000	-

Table 28: Estimates of parameters of the seasonal mean equation for Milan. Note: see the explanations in the beginning of Appendix B.

C Misspecification tests

This appendix contains results of the test of one transition against at least two and the test of no error correlation, discussed in Section 4.2.2. The former test is based on the third-order polynomial approximation of the alternative. The test based on the first-order approximation yields similar results, so they are not reported here.

	Jan	Feb	Mar	Apr	May	Jun
Trondheim	0.782	0.851	0.956	0.959	0.509	0.140
Uppsala	0.784	0.143	0.726	0.958	0.162	0.862
Stockholm	0.274	0.455	0.360	1.000	0.071	1.000
Copenhagen	0.763	0.956	0.989	0.996	0.507	0.753
Vilnius	0.053	0.215	0.131	0.759	0.817	0.167
Berlin	0.193	0.190	0.193	0.984	0.119	0.578
Warsaw	0.774	0.372	0.989	0.134	0.615	0.921
De Bilt	1.000	0.440	0.989	0.063	0.135	0.447
Brno	0.804	0.288	1.000	0.750	0.065	0.377
Regensburg	0.983	0.218	0.530	0.409	0.900	0.146
Karlsruhe	0.400	0.271	0.997	0.730	1.000	0.950
Paris	0.972	0.781	0.946	0.211	0.935	0.106
Stuttgart	0.997	0.182	0.988	0.702	0.321	0.689
Vienna	0.745	0.858	0.774	0.406	0.725	0.341
Munich	0.265	0.369	0.920	0.712	0.873	0.690
Kremsmünster	0.998	0.111	0.782	0.109	0.266	0.825
Hohenpeissenberg	0.596	0.448	0.939	0.683	0.914	0.976
Budapest	0.908	0.668	0.995	0.653	0.960	0.578
Innsbruck	0.809	0.173	0.371	0.311	0.636	0.966
Milan	0.712	0.088	0.854	0.421	0.532	0.543

Table 29: p-values of the test of one transition against (at least) two based on the third-order polynomial expansion for the 20 stations, months January–June

	Jul	Aug	Sep	Oct	Nov	Dec
Trondheim	0.184	0.883	0.600	0.443	0.053	0.975
Uppsala	0.926	0.371	0.062	0.840	0.819	0.905
Stockholm	0.147	0.519	0.212	0.610	0.652	0.950
Copenhagen	0.314	0.908	0.900	0.326	0.690	0.981
Vilnius	0.298	0.727	0.089	0.455	0.457	0.917
Berlin	0.974	0.935	0.355	0.984	0.139	0.346
Warsaw	0.293	0.795	0.271	0.721	0.688	0.972
De Bilt	0.509	0.219	0.181	0.655	0.954	0.987
Brno	0.072	0.202	0.383	0.244	0.801	1.000
Regensburg	0.957	0.371	0.968	0.177	0.757	1.000
Karlsruhe	0.990	0.999	0.947	0.807	0.609	1.000
Paris	0.501	0.741	0.250	0.175	0.584	0.400
Stuttgart	0.990	0.122	0.790	0.129	0.609	0.853
Vienna	0.309	0.203	0.163	0.203	0.536	0.972
Munich	0.584	0.272	0.188	0.117	0.210	1.000
Kremsmünster	0.917	0.075	0.179	0.051	0.930	0.209
Hohenpeissenberg	0.827	0.489	0.259	0.341	0.997	0.567
Budapest	0.533	0.729	0.993	0.057	0.456	0.957
Innsbruck	0.952	0.686	0.306	0.562	0.378	0.990
Milan	1.000	0.651	0.997	0.472	0.273	0.986

Table 30: p-values of the test of one transition against (at least) two based on the third-order polynomial expansion for the 20 stations, months July–December

	pAR(1)	pAR(2)	pAR(3)	pAR(6)	pAR(12)
Trondheim	0.911	0.992	0.999	0.943	0.886
Uppsala	0.924	0.807	0.931	0.295	0.186
Stockholm	0.785	0.946	0.985	0.395	0.371
Copenhagen	0.908	0.709	0.873	0.153	0.170
Vilnius	0.728	0.668	0.766	0.610	0.117
Berlin	0.802	0.854	0.883	0.504	0.116
Warsaw	0.729	0.760	0.492	0.444	0.282
De Bilt	0.773	0.659	0.639	0.188	0.255
Brno	0.648	0.565	0.334	0.328	0.063
Regensburg	0.749	0.837	0.845	0.409	0.383
Karlsruhe	0.787	0.820	0.756	0.274	0.101
Paris	0.686	0.836	0.702	0.135	0.140
Stuttgart	0.781	0.809	0.708	0.512	0.162
Vienna	0.738	0.811	0.182	0.417	0.301
Munich	0.719	0.625	0.637	0.234	0.223
Kremsmünster	0.642	0.778	0.668	0.921	0.572
Hohenpeissenberg	0.791	0.890	0.748	0.643	0.501
Budapest	0.772	0.776	0.427	0.581	0.610
Innsbruck	0.565	0.551	0.300	0.127	0.195
Milan	0.614	0.797	0.809	0.575	0.678

Table 31: p-values of the test of no error autocorrelation for the 20 estimated seasonal mean equations

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