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Abstract

The fairly new VIX ETPs have been promoted for providing effective and easily accessible diversification. We examine the economic value of using VIX ETPs for diversification of stock-bond portfolios. We consider seven different investment strategies based on short-sales constrained and unconstrained investors who use four different investment styles for their optimization strategy. Our analysis begins in 2009, when the first VIX ETPs are introduced, and therefore only considers the period after the recent financial crisis. For investors prohibited from short selling, the diversification benefits of the VIX ETPs do not offset the negative returns on the VIX ETPs. Hence there is a negative economic value of including VIX ETPs in stock-bond portfolios. This applies to all investment styles. It even applies when adjusting for a simulated market crash. For investors who are not constrained from selling assets short, the results are mixed as the economic value of VIX ETPs vary with respect to investment style and product.

Keywords: VIX; VIX ETPs; Portfolio diversification; Realized volatility; Mean-variance analysis

JEL Classification: G11; G15; G23.

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I. Introduction

In this paper, we use the framework of Fleming et al. (2001) to quantify the economic value stemming from investing in portfolios that not only consist of the benchmark assets (stocks and bonds) but also relies on the new and increasingly popular VIX exchange-traded products (ETPs) for diversification. We extend the previous usage of economic value from evaluating trading on futures based on intra-daily realized variances to its usage for investing in VIX ETPs.

In 2009 the first exchange traded volatility products, *iPath S&P 500 VIX Short-Term Futures Exchange Traded Note* (VXX) and *S&P 500 VIX Mid-Term Futures Exchange Traded Note* (VXZ) are launched by Barclays Capital. The introduction of VIX ETPs makes volatility exposure available to retail investors who are typically too small or unsophisticated to trade in the futures market and to institutions such as pension funds and endowment funds who may be restricted from trading derivatives. Today, 13 VIX ETPs are listed, which differ in terms and format, e.g., some provide inverse and leveraged exposure to volatility. As other asset classes like stocks, bonds, and commodities tend to become near perfectly correlated in times of severe distress, volatility exposure is a desirable portfolio component as it diversifies and protects portfolios when it is needed the most. However, this exposure comes at a cost in terms of negative expected returns of long positions in VIX ETPs during normal times (see e.g., Alexander and Korovilas (2013), Alexander et al. (2015), and Eraker and Wu (2017)).

We use the concept of economic value to look further into the VIX ETPs. Our study is highly timely as the first VIX ETPs, VXX and VXZ, have expired on January 30, 2019, 10 years after their inception. New versions have been launched subsequently. This prompts the question of whether the economic value from investing in these products offer diversification benefits that are sufficient to compensate for the quite substantial negative returns that they have generated so far. The purpose of this paper is to provide an answer.

Our paper makes two main contributions. First, we make use of the popular concept of economic value from Fleming et al. (2001) to measure the advantage of adding a third asset class to the traditional stock-bond portfolios. This allows us to quantify the portfolio performance in an economic sense. This is an improvement compared to basing the analysis on the classical portfolio performance measures such as the Sharpe ratio. Second, we investigate in detail the new and understudied VIX ETPs. Here, we use an up-to-date sample which is longer than in previous studies. Furthermore, we use intra-daily data for estimating the conditional covariance matrix, which gives us much more efficient estimates than using only daily data as in the previous literature. A third minor contribution is that we use the intra-daily quote and trade data to estimate the transaction costs of trading in VIX ETPs.[1]

The prior literature studies the benefits of volatility investing. Both Dash and Moran (2005) and Daigler and Rossi (2006) find diversification benefits of adding variance swaps to a portfolio of stocks and portfolios of hedge funds, respectively. Szado (2009) finds positive diversification benefits based on arbitrary allocations between VIX futures and other assets in a sample focused on the period around the recent financial crisis. Brière et al. (2010) find that a long stock investor who is mean value-at-risk optimizing increases the risk-adjusted return by adding a combination of long VIX futures and short variance swaps to the portfolio, in a sample running from 2004 to 2008. In a sample ending in 2008, Chen et al. (2011) perform a mean-variance spanning test on four US stock portfolios and find that VIX futures enlarge the investment opportunity set. Warren (2012) analyses a base portfolio which includes US stocks, fixed income, and real estate exposure, finding that only a short position in the prompt month in the VIX futures enhances the Sharpe ratio. Hancock (2013) constructs different portfolios of the S&P 500 index, VIX ETPs, and VIX futures using two different hedge strategies in a sample starting in 2009. She finds increased performance for a portfolio hedged with short-term VIX futures, however this is not robust to the choice of hedging strategy. Whaley (2013) investigates how the indices which the VIX ETPs are tracking perform as an asset class on their own. He shows that from December 2005 to March 2012,

[1]The VIX ETP bid-ask spreads obtainable from data vendors such as Bloomberg and Thomson Reuters are missing or flawed for many observation points.

investors in VIX ETPs (excluding inverse products) had lost about $3.89bn.

A recent study is Alexander et al. (2016), who consider three investor types that allocate capital between stocks, bonds, and VIX ETPs on a monthly basis using different optimization methods. They also introduce the concept of the optimal diversification threshold, which is the minimum expected return for VIX ETPs in order to be included in the optimal portfolio. They find that diversification with VIX ETPs is frequently ex-ante optimal, however, the apparent benefits are never realized in the ex-post performance due to the high roll costs for these products. They only find diversification benefits of the VIX ETPs (constructing these synthetic prior to 2009) during the banking crisis of 2008.[2] Caloiero and Guidolin (2017) use the same approach where they back-test different portfolios with exposure to either a short-term VIX ETP or the VIX index (not investable) on a sample running from 2010 to 2016. In some cases, depending on the allocation strategy, they find benefits (measured by certainty equivalents) of including the VIX in a portfolio but never the VIX ETP. By means of a simple regression analysis Bordonado et al. (2017) determine weights in the VIX ETP to fully hedge a position in the S&P 500 index. In an in-sample analysis, the performance is then compared with an un-hedged position. On a sample running from 2006 to 2013, they find that inclusion of the VIX ETP would have improved the Sharpe ratio marginally.[3] This result, however, is sensitive to the choice of re-balancing frequency and when the impact of the financial crisis in 2008 is filtered out, the inclusion of the VIX ETP offers no improvement on the Sharpe ratio. One of the newest studies on the subject is Berkowitz and DeLisle (2018) who use a five-factor model to do a performance evaluation of portfolios comprised of a broad stock index and VIX ETPs on a sample beginning at the inception of the first VIX ETPs. They find that the VIX ETPs are too expensive and that they underperform the pure stock portfolio.

In this study, we consider seven different investment strategies. For each investment

[2] As VIX ETPs were not available during the 2008 banking crisis they construct synthetic returns for this period using VIX futures.

[3] Prior to the inception of the first VIX ETPs, they use returns of the index that the products track.

strategy, we examine three different portfolios. The first portfolio serves as the benchmark and contains only stocks and bonds. The second and third portfolios are extended (if optimal) with a VIX ETP, either a short- or a medium-term VIX ETP. The investors allocate capital between the assets in the portfolios on a monthly basis. The portfolio weights depend on the investor's optimization problem, i.e., the investment style.

For each investment strategy, the realized out-of-sample performance of the two portfolios containing VIX ETPs is compared to the benchmark portfolio. For the performance evaluation, we apply the concept of economic value introduced in Fleming et al. (2001). In our context, the economic value is interpreted as the performance fee that a mean-variance optimizing investor will be willing to pay to include VIX ETPs in the portfolio. We find that for short-sales constrained investors, the value of protection during times of market stress is quickly vaporized due to the roll costs associated with the rebalancing strategy of the VIX ETPs. Except for one period, the economic value of diversifying with VIX ETPs is negative, hence a short-sales constrained mean-variance optimizing investor should be willing to pay for not including VIX ETPs in her portfolio. However, inclusion of these products increases the skewness of portfolio returns, which could suggest that these negative economic values are skewness premiums that investors who consider higher-order moments could be willing to pay. For investors that are unrestricted from short-sales, our conclusion varies with the investment strategy and product.

Like Alexander et al. (2016) and Caloiero and Guidolin (2017), our approach is back-testing in nature. Concerns may be raised that such results are influenced by the validity of the inputs used in the optimization. Hence, in order to control for this, we model one of these inputs, namely the covariance matrix, in a more forward-looking and realistic manner, relying on the theory of realized measures. We control for the validity of the other input, the expected asset returns, by also considering investment strategies that are independent of the expected asset returns.

The literature described above can be divided into three categories based on the sample period considered. These are (i) before and during the recent financial crisis, (ii) before, during, and after the financial crisis, and (iii) after the financial crisis. Our study belongs to the latter category because we only want to use traded data of the VIX ETPs, and thus the sample period must start when the VIX ETPs actually started trading in 2009. As our analysis leaves out the financial crisis, our analysis partly takes the perspective of investors who have used VIX ETPs since their inception in anticipation that these products will shield portfolio performance during times of market stress. Hence, our study can be seen as a test of whether or not the market volatility in the period after the launch of VIX ETPs has been sufficiently high for these products to offer positive economic value. As we find that this is not the case during our sample period, we make a simple simulation of a new market crash of the same magnitude as the recent financial crisis in 2008. Hereby, we investigate if the simulated crash would enable investors who have held long positions in VIX ETPs to catch up with the benchmark portfolio of stocks and bonds. Even accounting for the simulated market crash is not enough for the VIX ETPs to add economic value to the investor.

The paper is organized into six additional sections. Section II reviews the methodology of our analysis. Section III provides a thorough description of the VIX ETPs, and Section IV describes the data that we use. Section V documents the economic value of investing in VIX ETPs, while Section VI discusses the empirical portfolio allocations for the seven different investment strategies. Finally, section VII contains the conclusions.

II. Methodology

The aim of our paper is to consider the economic value of portfolio diversification using the new financial product, VIX ETPs, relative to the benchmark portfolio, which only diversifies between stocks and bonds. We evaluate portfolios consisting of VIX ETPs as well as stocks and bonds by considering their economic value compared to the benchmark portfolios. The economic value calculations follow Fleming et al. (2001). Previously, the concept of economic value has been used in relation to realized volatility of futures contracts based on intra-daily data and in relation to forecasting, so we extend the economic value literature.

First, in Section II.A, we provide details about the three portfolios that investors use and seven different investment strategies for their investment decisions. Second, in Section II.A, we describe the economic value of these investment strategies. Third, in Section II.C, we describe how we calculate returns and covariances.

II.A. Investment strategies

We consider two different VIX ETPs that only differ with respect to their maturity (short- and medium-term). The two VIX ETPs are described in detail in Section III. It is of course possible to use our framework to access the economic value of adding any other asset than VIX ETPs to the traditional stock-bond portfolio.

We consider different portfolios that contain the VIX ETP and measure their performance against the benchmark portfolio. The benchmark portfolio (P-bench) only contains US stocks and bonds. The second portfolio (P-short) is comprised of stocks, bonds, and a short-term VIX ETP, while the third portfolio (P-mid) is comprised of a mid-term VIX ETP instead. Hence, the portfolios that we analyze are:

- P-bench: US stocks and bonds.
- P-short: US stocks, bonds, and short-term VIX ETP.
- P-mid: US stocks, bonds, and mid-term VIX ETP.

P-short and P-mid will also be referred to as the "VIX portfolios".

The investor re-balances her portfolio at a monthly frequency (at month end) and at the re-balance date, the investor allocates her funds across assets according to her asset allocation strategy.

We distinguish between two types of investors. The first type is a short-sales constrained (“constrained” in the following) investor (e.g., a pension fund or a retail investor) whose portfolio can only be composed of long positions and who cannot apply leverage. The second type of investor is an unconstrained investor (e.g., a hedge fund) who can apply leverage and hold short positions.

We consider four different allocation styles. We label these investment styles constant (constant weights), MinVar (minimum variance optimizing), CE (certainty equivalent optimizing), and MD (maximum diversification), respectively.

Within the constrained and unconstrained investors, we have four and three different investment styles. This gives us seven investment strategies in total.[4]

We fix notation first. Let $\mathbf{R}_{t+1}$ denote an $N \times 1$ vector of risky asset returns. $\boldsymbol{\mu}_t \equiv \mathrm{E}_t[\mathbf{R}_{t+1}]$ denotes the conditional expected value of $\mathbf{R}_{t+1}$. $\boldsymbol{\Sigma}_t \equiv \mathrm{E}_t[(\mathbf{R}_{t+1}\text{-}\boldsymbol{\mu}_t)(\mathbf{R}_{t+1}\text{-}\boldsymbol{\mu}_t)']$ is the conditional covariance matrix of $\mathbf{R}_{t+1}$. R_f is the return on the risk-free asset. $\mathbf{w}_t$ is an $N \times 1$ vector of portfolio weights on the risky assets.

The first allocation style, *constant*, is simply a static portfolio where the weights to each asset are constant through time at conventional levels using weights similar to Szado (2009). For the benchmark portfolio, $\mathbf{w}_t' = (60\%, 40\%)$ where the first and second element is the stock and bond weight, respectively. This is commonly referred to as the 60/40 rule by the financial media and has historically been used as a rule of thumb by financial planners and advisers. For the two VIX portfolios, $\mathbf{w}_t' = (60\%, 30\%, 10\%)$ where the first, second, and third elements are the stock, bond, and VIX ETP weight respectively.

The second allocation style, *MinVar*, is to minimize the portfolio variance for a pre-specified target portfolio return μ_p. Then at each re-balancing date t, the minimum variance strategy allocates across assets by solving the quadratic program:

$$
\begin{aligned}
&\min_{\mathbf{w}_t} \mathbf{w}_t' \Sigma_t \mathbf{w}_t, \\
&\text{s.t. } \mathbf{w}_t' \boldsymbol{\mu}_t + (1 - \mathbf{w_t}'\mathbf{1}) R_f = \mu_p, \\
&\mathbf{w}_t \geq 0 \;\forall t, \qquad (1)
\end{aligned}
$$

[4]For the constant investment style we only consider short-sales constrained investors.

for the constrained investor and:

$$\begin{aligned}
&\min_{\mathbf{w}_t} \mathbf{w}_t' \Sigma_t \mathbf{w}_t, \\
&\text{s.t. } \mathbf{w}_t' \boldsymbol{\mu}_t + (1 - \mathbf{w_t}' \mathbf{1}) R_f = \mu_p,
\end{aligned} \tag{2}$$

for the unconstrained investor. We use a target expected return, μ_p, of 10%, for the MinVar optimization.

The third allocation style, *CE*, is to maximize the expected utility and the certainty equivalent. For a quadratic utility function the investor's realized utility in period $t+1$ can be written as:

$$U(W_{t+1}) = W_t R_{p,t+1} - \frac{aW_t^2}{2} R_{p,t+1}^2 \tag{3}$$

where W_{t+1} is the investor's wealth at $t+1$, a is her absolute risk aversion parameter, and $R_{p,t+1} = R_f + \mathbf{w}' R_{t+1}$ is the period $t+1$ return on her portfolio. The utility function can also be expressed in terms of the certainty equivalent given as:

$$CE \approx \mathrm{E}[R_{p,t+1}] + \frac{1}{2} \frac{U''(\mathrm{E}[R_{p,t+1}])}{U'(\mathrm{E}[R_{p,t+1}])} \mathrm{Var}[R_{p,t+1}], \tag{4}$$

This implies that the investor maximizes utility by maximizing the certainty equivalent. Then from Equations (3) and (4), we get that the certainty equivalent (CE) maximizing strategy is obtained by solving:

$$\begin{aligned}
&\max_{\mathbf{w}_t} \mathbf{w}_t' (\boldsymbol{\mu}_t - R_f \mathbf{1}) - \frac{a}{1 - a\mathbf{w}_t' (\boldsymbol{\mu}_t - R_f \mathbf{1})} \mathbf{w}_t' \Sigma_t \mathbf{w}_t, \\
&\text{s.t. } \mathbf{w}_t' \mathbf{1} \leq 1, \\
&\mathbf{w}_t \geq 0 \ \forall t,
\end{aligned} \tag{5}$$

for the constrained investor and:

$$\max_{\mathbf{w}_t} \mathbf{w}_t'(\boldsymbol{\mu}_t - R_f\mathbf{1}) - \frac{a}{1 - a\mathbf{w}_t'(\boldsymbol{\mu}_t - R_f\mathbf{1})}\mathbf{w}_t'\Sigma_t\mathbf{w}_t, \tag{6}$$

for the unconstrained investor, respectively. We set the absolute risk aversion parameter, a, to 4 as in Alexander et al. (2016).

The fourth allocation style, *MD*, is to maximize the diversification of the portfolio. As in Yves and Coignard (2008), the maximum diversification is obtained by applying an objective function that maximizes the ratio of the weighted average asset volatilities to portfolio volatility. The objective is to solve:

$$\begin{aligned} &\max_{\mathbf{w}_t} \frac{\mathbf{w}_t'\boldsymbol{\sigma}}{\sqrt{\mathbf{w}_t'\Sigma_t\mathbf{w}_t}}, \\ &\text{s.t. } \mathbf{w}_t'\mathbf{1} = 1, \\ &\mathbf{w}_t \geq 0 \ \forall t, \end{aligned} \tag{7}$$

for the constrained investor and:

$$\begin{aligned} &\max_{\mathbf{w}_t} \frac{\mathbf{w}_t'\boldsymbol{\sigma}}{\sqrt{\mathbf{w}_t'\Sigma_t\mathbf{w}_t}}, \\ &\text{s.t. } \mathbf{w}_t'\mathbf{1} = 1, \end{aligned} \tag{8}$$

for the unconstrained investor, respectively, where $\boldsymbol{\sigma}$ is an $N\times$ 1 vector of asset volatilities, the square root of the diagonal terms of $\boldsymbol{\Sigma}_t$. The objective function in equation (7) and (8) has the form of a Sharpe ratio, where the asset volatility vector, $\boldsymbol{\sigma}$, replaces the expected returns vector.

II.B. Economic value

To asses the economic diversification benefits of adding the VIX ETPs to the benchmark portfolio we follow Fleming et al. (2001) and calculate the economic value of portfolio diversification with the short- and mid-term VIX ETPs.

In (3) we hold aW_t constant. This is equivalent to setting the investor's relative risk aversion, $\gamma_t = -U''/U'W_t = aW_t/(1-aW_t)$ equal to some fixed value γ. The average realized utility can then be used to estimate the expected utility generated by a given level of initial wealth W_0, as follows:

$$\overline{U}(\cdot) = W_0 \left(\sum_{t=0}^{T-1} R_{p,t+1} - \frac{\gamma}{2(1+\gamma)} R^2_{p,t+1} \right) \tag{9}$$

We estimate the economic value of VIX products by equating the average utilities for two alternative portfolios P-bench and P-short (or P-mid) as follows:

$$\sum_{t=0}^{T-1} (R^{short/mid}_{p,t+1} - \Delta) - \frac{\gamma}{2(1+\gamma)} \left(R^{short/mid}_{p,t+1} - \Delta \right)^2 = \sum_{t=0}^{T-1} R^{bench}_{p,t+1} - \frac{\gamma}{2(1+\gamma)} \left(R^{bench}_{p,t+1} \right)^2 \tag{10}$$

where $R^{short/mid}_{p,t+1}$ and $R^{bench}_{p,t+1}$ are the portfolio returns from the portfolio holding a VIX ETP and the benchmark portfolio, respectively. We can interpret Δ as the maximum performance fee that the investor would be willing to pay for switching from the benchmark portfolio to the portfolio with a VIX product. Δ is thereby the economic value of investing in the portfolio also containing the VIX ETP.

We report our estimates of Δ as annualized fees in basis points using two different values of γ, 1 and 10.[5]

[5] According to Gandelman and Hernàndez-Murillo (2015) the most commonly accepted measures of the coefficient of relative risk aversion lie between 1 and 3 hence our chosen levels should indeed test the most extreme cases.

II.C. Expected returns and conditional covariances

The optimization problems faced by our different investment strategies all rely on estimates of the expected asset returns, $\hat{\mu}_t$ and the conditional covariance matrix $\hat{\Sigma}_t$.

We use the monthly average realized returns as a measure of the conditional expected returns.[6] Obviously, this approach lacks sophistication and might seem unrealistic, hence concerns may be raised that our results could be partly driven by incorrect modelling of expected returns. However, the application of the two allocation styles constant and maximum diversification which do not depend on the expected returns, provides evidence that our conclusions are not driven by how we model $\hat{\mu}_t$.

For estimating the conditional covariance matrix, we rely on the theory of realized measures. The work of Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) suggests that we can use intra-daily returns to construct volatility estimators that are more efficient than those based on daily returns. By a standard no-arbitrage condition, we assume that log-prices are semimartingales. Then as shown in Andersen et al. (2001) we can think of the quadratic covariation as an unbiased estimator of the conditional covariance matrix, where the quadratic covariation between asset j and k is defined as:

$$\mathbf{\Sigma}_{t,t+1}(j,k) = \int_t^{t+1} \sigma_{j,k}(s)ds. \tag{11}$$

The quadratic covariation may be approximated directly from high-frequency return data. Suppose we divide the time interval t to $t+1$ into m sub-periods of length $h = 1/m$ and let $\mathbf{r_{t+ih}}$ denote the $n \times 1$ vector of continuously compounded returns that starts at time $t + (i-1)h$ and ends at time $t + ih$, $i = 1, \ldots, m$. We then define the realized covariance matrix as:

$$\mathbf{V}_{t,t+1} = \sum_{i=1}^{m} \mathbf{r}_{t+ih}\mathbf{r}'_{t+ih}. \tag{12}$$

[6] We have also applied the approach in Fleming et al. (2001) who estimate the expected returns using the unconditional average return for the entire sample. This approach results in zero allocations to both of the VIX ETPs portfolios.

Then, Andersen et al. (2001) show that under weak regularity conditions:

$$plim_{m\to\infty}\mathbf{V}_{t,t+1} = \mathbf{\Sigma}_{t,t+1}, \tag{13}$$

where $\mathbf{\Sigma}_{t,t+1} = (\mathbf{\Sigma}_{t,t+1}(j,k))_{j,k=1,\ldots,n}$. Hence, for a sufficiently large m, the realized covariance provides a good approximation to the quadratic covariation, which in turn is an unbiased estimator of the conditional covariance matrix. So by using intra-day returns, we can construct non-parametric and consistent estimates of the quadratic covariance matrix.

Several studies (see e.g., Renò (2003), Griffin and Oomen (2011), and Barndorff-Nielsen et al. (2011)) confirm a bias towards zero for realized covariances computed over a short fixed time period due to non-synchronous trading. This phenomenon is often referred to as the Epps effect. However, as long as the price series is fairly liquid, this effect will be small or even negligible (see Renò (2003) and Zhang (2011)). Another potential issue with intradaily returns is the lack of observations when markets are closed overnight, which causes a downward bias in the realized covariance matrix. To mitigate this, we include the overnight returns when constructing $\mathbf{V}_t$.

We denote P_t as the upper triangular components of $\mathbf{V}_t$ and let $X_t = vech(P_t)$ be the $n(n+1)/2 \times 1$ vector obtained by stacking P_t. We then estimate the expected covariance matrix by applying an AR(1) structure such that future values of X_t are estimated by:

$$X_{t+1} = \beta_0 + \beta_1 X_t + u_{t+1}, \tag{14}$$

where the β parameters are estimated by OLS. The regressor, X_t is scaled to match the frequency of the left-hand side. Hence if we want to forecast the monthly covariance matrix, $X_t = \sum_{i=0}^{20} X_{t-i}$. From the estimates of X_{t+1}, we can easily construct our estimate of the expected covariance matrix $\hat{\mathbf{\Sigma}}_{t+1}$. By estimating the β parameters using the full dataset, we could potentially be introducing a look-ahead bias into our results. However, as in Fleming et al. (2003), this is not an empirical problem since the estimate implied by the minimum

MSE criterion is different from the one which maximizes the economic value.[7]

III. Introduction to VIX ETPs

Although the VIX index itself is not a tradeable product, the Chicago Futures Exchange launched futures contracts on the VIX index in March 2004. A vital property of VIX ETPs is that they are linked to VIX futures and not the VIX index itself. S&P computes four constant maturity VIX futures indexes and all VIX ETPs track one of these.

The index tracked by most products is the *S&P 500 VIX Short-Term Futures Index* (SPVXSP) which tracks a strategy of holding long positions in the nearby and second nearby VIX futures contracts in proportions such that the average maturity is kept constant at 30 days at the close of trading. On the following day, a fraction of the nearby contract is sold and the same amount is invested in the second nearby contract. This rolling strategy continues until the nearby contract expires at which point the position is fully invested in the second nearby contract and the cycle repeats. Hence losses and gains are realized on a daily basis.

Consider the following example: On October 30, 2018, the nearby and second nearby contract expire on 11/21/2018 and 12/19/2018, respectively. In order to have an average maturity of 30 days, the VIX futures position is comprised of 75% of the November contract and 25% of the December contract. On the next day, the fraction held in the November contract is reduced to 70% and increased for the December contract to 30%.[8] For hedging the exposure, the VIX ETPs must follow a similar strategy.[9]

The main problem with this strategy becomes obvious when looking at the VIX futures term structure which is in contango (upward sloping), as depicted in Figure I, more often than not. This implies that at each re-balancing point there is a small but positive roll

[7] E.g., Changing our β_1-estimate by -.1 gives better economic results of using VIX ETPs.

[8] For further elaboration on how these fractions are calculated please see the index methodology by S&P which is available at https://us.spindices.com/indices/strategy/sp-500-vix-short-term-index-mcap.

[9] Note that most of the ETPs are ETNs that are not required to hold the underlying futures contracts. Hence the issuer of these products can hedge themselves in other products or simply choose not to hedge.

cost generated by selling the lower priced nearest contract and buying the higher priced next-to-nearest contract. This small daily roll cost creates highly negative long-run returns for products with long positions in the VIX futures.

[Insert Figure I About Here.]

We pay special attention to two VIX ETPS, namely VXX and VXZ. Figure II shows the price development of VXX and VXZ from inception date until September 2018. VXX is the largest VIX ETP, and it is benchmarked to SPVIXSTR with a leverage factor of one, which means that its performance is benchmarked to one times the daily index return (less management fees and expenses). Because of the roll cost, a long position in VXX, from inception date until 09/14/2018 would have lost 99.9 % of the initial investment.

[Insert Figure II About Here.]

VXZ is a longer-dated product that rolls the 4th-month futures position into the 7th-month to maintain a constant average maturity of five months. Since the curve is typically not as steep for longer maturities, the price difference between the contracts sold and bought is smaller, yielding a smaller roll cost. As a consequence, the price deterioration has been less severe than for VXX.

When volatility is high and the VIX index spikes, the futures curve inverts and becomes downward sloping (backwardation), depicted in Figure III. Short-dated products will benefit more from this than longer-dated products due to the steepness at the short end of the curve. So when volatility spikes, the returns of the short-dated products tend to be highest. Hence, an investor who wants to insure against corrections in the equity market faces the trade-off between paying higher premiums in terms of negative returns and then getting larger payouts in times of market distress versus paying lower premiums but also getting lower potential payouts from the longer dated products.[10]

[10] See also Alexander et al. (2015) and Bollen et al. (2017) for descriptive statistics, trading volume and size on different VIX ETPs.

[Insert Figure III About Here.]

With a return profile as depicted in Figure II, it seems obvious that these VIX ETPs are not suitable for buy-and-hold strategies which is also stated in their prospectus. So for the existence of the products to be justified, they must provide portfolio benefits from increased diversification. Otherwise, they should be characterized as easy access to volatility speculation, which is typically not advisable for retail investors or within the mandate of endowment funds and pension funds.

IV. The data set

IV.A. Sample period

Our sample period is from January 30, 2009 (inception date of the VXX and VXZ) to June 29, 2018. Hence, although our sample is after the financial crisis, it contains several episodes of turbulence such as May 2010 (Flash Crash), June 2010 (Greek debt crisis), August 2011 (S&P downgrade of the US credit rating), August 2015 (Renminbi devaluation), and February 2018 (Volmageddon).

In our sample, we have 2,369 trading days, and, assuming 21 trading days per month, we have 112 monthly re-balancing points.

IV.B. Data sources

For our empirical analysis, we use the products VXX and VXZ for our short-term (P-short) and mid-term (P-mid) VIX portfolios. These are the first VIX ETPs issued, hence using these gives us the longest sample. Furthermore, VXX has the largest market value and is the most liquid of all the VIX ETPs (across both leverage and term).

As a proxy for the US stock and bond components of the portfolios, we use two market-wide indexes traded as exchange-traded funds (ETFs) namely SPY tracking the S&P 500 index and AGG tracking the Bloomberg Barclays U.S. Aggregate Bond Index.[11]

For calculating the realized covariance matrix and the expected returns, we use intra-daily observations. The data are extracted from the NYSE Trade and Quote (TAQ) database and include both trade and quote data for the official trading hours from 9:30 to 16:00 local New York time. For cleaning the data, we follow the routines proposed in Christensen et al. (2014) and Barndorff-Nielsen et al. (2009).

For the risk-free interest rate, we use the 1-month US Treasury bill, collected from Kenneth French's webpage.[12]

IV.C. Realized returns

Very high frequent data are contaminated with microstructure noise (e.g., bid-ask bounce and price discreteness) which will make our realized covariance estimator diverge. To mitigate this, we use sparse sampling at five-minute intervals, which is the standard frequency in the literature. This is done by constructing a grid of five-minute intervals that spans the trading day. Next, we identify and take the log of the last traded price at each grid point and then take the first differences of these prices. This sampling method gives us 78 prices during the trading day (the first sampling is 9.35) hence 77 log-returns. With the addition of the overnight return we finally get a total of 78 log returns for each trading day.

Figure IVa plots the realized returns for each instrument. We note the very severe spikes for both VIX ETPs on February 5, 2018, where the VIX complex blew up, also referred to as the "Volmageddon" event. On this day, the VIX made a one-day move from 17.31 to 37.32, and as a consequence, the VIX futures term structure went into steep backwardation. This yielded a very large one-day return for both VXX and VXZ of 28.9% and 15.1%, respectively. We also note severe spikes in June 2016 when stocks tumbled due to weakening activity data in the US and China and around August 2011 where the US credit rating was downgraded.

[Insert Figure IV About Here.]

[11] The index measures the performance of the total US investment-grade bond market.
[12] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Table I provides descriptive statistics for the realized returns. Panel A shows that the VIX ETPs have very poor average returns which are more negative than reported in previous studies (e.g, Alexander et al. (2015) and Alexander and Korovilas (2013)). This is, of course, no surprise given the fact that the VIX futures term structure has mainly been in contango during our sample period. The standard deviations indicate that the VIX ETPs are far more volatile than both stocks and bonds. Finally, we note that contrary to SPY and AGG, the return skewness for both VIX ETPs is positive. This indicates a potential for increasing skewness in a stock-bond portfolio.

[Insert Table I About Here.]

IV.D. Realized volatilities and correlations

All instruments that we use are fairly liquid, so we do not expect any issues related to the potential Epps effect, cf. the discussion in Section II.

The average realized volatility estimates are reported as $\overline{\sigma}_t$ in Panel A of Table I and are generally consistent with the standard deviation of the realized returns. The entire series of realized volatilities are plotted in IVb. We see that for both the VXX and VXZ, volatility varies considerably over the entire sample with pronounced spikes in May 2010, August 2015, and February 2018. Furthermore, the estimates for both VIX ETPs are far above those for stocks and bonds for the entire sample.

Figure IVc plots the daily realized correlations. Panel B of Table I provides the correlation matrix for the different products, where the reported correlations are the average realized correlations over the sample period.

As expected and in line with the results reported by Alexander et al. (2015), both the VXX and VXZ correlate very negatively with the SPY. The SPY-VXX correlation is between -0.7 and -0.9 with a maximum in absolute terms of -0.98. However on December 13, 2016, the SPY, VXX, and VXZ all have a positive realized return and the correlation dropped, in absolute terms, to -0.02, and the correlation between SPY and VXZ actually became positive.

For the entire sample period, VXX is more negatively correlated with the SPY than VXZ, which is no surprise given the different structures of the products.

IV.E. Transaction costs

For assessing the out-of-sample portfolio performance, transaction costs are taken into account. At each portfolio re-balancing point, a cost equal to the product of the bid-ask spread in basis points and the absolute change in weights, summed over all assets, is subtracted from the portfolio return.

We estimate the daily bid-ask spread via quote data from TAQ as the size weighted median spread of all the quotes during the trading day. The mid-price is computed as the sum of the size weighted average bid and ask, divided by two.[13]

V. Empirical economic value of VIX ETPs

In this section, we estimate the economic value of holding the VIX ETPs in the portfolios.

Throughout the sample period, the portfolios are marked to market each day. That is, the ex post daily return for each portfolio is computed by multiplying the portfolio weights by the observed next-day returns on the components. At each re-balancing day, the weights are changed according to the optimization problem for the particular investment strategy. We subtract the transaction cost from the portfolio return.

V.A. Constrained investors

Starting with the constrained strategies, Figure V depicts the evolution of $100 invested in each of the portfolios for both the constrained MinVar, CE, MD, and constant strategies.

[Insert Figure V About Here.]

[13] The bid-ask spread in basis points is the spread divided by mid price.

The overall performances of the constrained VIX portfolios are clearly worse than the benchmark portfolio, irrespective of the investment strategy. For the constrained minimum-variance optimizing (MinVar) strategy, the total return of the benchmark is 21.4% against -5.7% for P-short and 1.1% for P-mid. For the constrained certainty equivalence optimizing (CE) strategy, the total returns of the portfolio are 47.2% (P-bench), -22.6%(P-short), and -31.2% (P-mid). Portfolio returns for the constrained maximum diversification optimizing (MD) investor are 29.7% (P-bench), 5.3% (P-short), and 13.3% (P-mid). Finally, the constant weights (constant) strategy yields total returns of 136.9% (P-bench), 75.7% (P-short), and 107.9% (P-mid).

For the constrained MinVar strategy, Figure Va shows that from the end of 2012, the performance of P-short and P-mid are consistently below that of P-bench. Prior to that, the portfolio values are much at the same level.

Considering then the portfolios of the constrained CE strategy, we see from Figure Vb that there are several periods where the values of P-short and P-mid are above the benchmark. Specifically, P-mid has significant spikes in May 2010, a month that was characterized by market turmoil due to the "Flash Crash," the downgrade of Spain's credit, and monetary tightening in China. Through this month, P-mid is substantially weighted in VXZ (see Section VI.A on the allocations for each constrained portfolio) and has a monthly return of 22.15%. During the same month, the benchmark portfolio drops 6.1% in value. P-short has an allocation of 85% in SPY and 15% in VXX and only loses 1.42%. Hence the exposure to volatility shields, to some extent, the performance against the large drawdowns in stocks. P-short increases sharply in value at the beginning of August 2011, when the US credit is downgraded. Through this month it has a 15.2% increase in value. However, as the VIX index starts to collapse in value from November 2011, the values of P-short and P-mid deplete and from June 2012 none of the VIX portfolios plot above P-bench. From this point, the gap in values continues to widen for the rest of the sample.

Panel A of Table II reports the performance results for the constrained MinVar strategy.

For the entire sample, the two VIX portfolios yield mean returns of -0.44% (P-short) and 0.25% (P-mid). The sample volatilities are 6.04% (P-short) and 5.32% (P-mid).[14] The benchmark portfolio clearly beats this performance with a higher mean return of 2.20% and lower volatility of 4.92%.

The differences in performance translate into economic values which are negative for both VIX portfolios. For P-short, a constrained MinVar investment strategy would be willing to pay between 267.6 and 270.7 annual basis points, depending on the relative risk aversion, to switch to the benchmark portfolio. For P-mid, the switching fee is between 196.3 and 196.9. Hence, a constrained MinVar investment strategy would be willing to pay for <u>not</u> having either of the VIX products in the portfolio.

[Insert Table II About here.]

The performance of all three portfolios is also broken down by two-year sub-periods. For only one period (2009-2010), would the MinVar investment strategy be willing to pay a small fee for having VXZ. For all other periods, both VIX ETPs have negative economic value.

Panel B of Table II shows that the findings for the constrained CE strategy are the same, however the fees that the investor is willing to pay for not holding the VIX ETPs are more substantial. The investor would pay between 663.8 and 685.1 annual basis points for not holding VXX in the portfolio and between 791.7 and 810.9 for not holding VXZ. Furthermore, for all but one sub period, holding either VIX product in a portfolio has a negative economic value. However, for the sub period 2009-2010, the value of holding both VIX products is actually positive. This is the period where high allocations coincided with turmoil in May 2010.

The results for the constrained MD and constant investment strategies are reported in Panel C and D of Table II and show the same picture as for the MinVar and CE investment strategies, namely that the economic value for both products is negative for the entire sample period and all sub-periods.

[14] We refrain from comparing Sharpe ratios as it is negative for P-short.

An interesting observation from table II is that the skewness of P-short and P-mid tends to lie below, in absolute terms, the skewness of the benchmark portfolio. So by the inclusion of VIX products, an investor is able to counteract the typical negative skewness of stock and bond portfolios. This could suggest that the negative returns investors in general pay for holding long positions in VIX ETPs, can be viewed as a skewness premium, in order to reduce negative skewness of common portfolio components.

As the overall performance of the VIX portfolios yield negative economic value, the question is then when does it add value to hold VIX ETPs for constrained investors? Intuitively, we expect VIX portfolios to have positive value when market uncertainty is high and the VIX index spikes.

Considering the constrained MD strategy, Figure VIa plots the three-month rolling economic values for P-short and P-mid, together with movements in the VIX index. The overall correlation between the three-month rolling economic values and the VIX is 0.31 for P-short and 0.40 for P-mid.

[Insert Figure VI About Here.]

Figure VIb plots the 3-month rolling Sharpe ratios against the VIX index. The correlations between the Sharpe ratios and the VIX are 0.30 and 0.15 for P-short and P-mid, respectively. As expected, there is a clear connection between changes in the VIX index and the value of holding VIX products. This is clearly visible during the pronounced VIX spikes in 2010, 2011, and 2018 where the economic values of both VIX portfolios increase sharply. However, as the VIX then reverts, so does the economic value of both portfolios, and it becomes negative for lower levels of the VIX index. This suggests that throughout our sample, spikes in volatility have been too rare and too short-lived for these products to have any sustainable longterm value as a component in a long-only portfolio.

V.B. *Financial crisis scenario*

A potential weakness of our analysis is that our sample period does not contain the recent financial crisis of 2008 (the 2008 crash in the following), an event during which the previous literature has reported benefits of holding some kind of volatility instrument (variance swaps, VIX futures, or VIX ETPs).

For this reason, we extend our results with a simple what-if analysis. What would have happened to the value of the portfolios if a scenario like the 2008-crash had occurred at the end of the sample period? Would the distress in asset prices during such a crash be sufficient for the VIX portfolios to have caught up with the benchmark portfolio?

We take the perspective of an investor who has held a position in VIX ETPs in expectation of protection during market crashes. Now, after having endured a long period of suppressed volatility, we simulate that a 2008 crash occurs. The question is whether this gives sufficient reason for having held VIX ETPs in the portfolio and thereby having suffered long periods of inferior returns compared to the benchmark.

We consider the level of the VIX index during the height of the 2008 crash. We assume that during the simulated new crash, the VIX index will follow the same path as during the actual 2008 crash. The return paths of each portfolio asset (SPY, AGG, VXX, and VXZ) in the crash scenario are predicted from regressions of the asset returns on the returns of the VIX index. The regression equation is:

$$R_{i,t} = \alpha_i + \beta_i R_{VIX,t} + \epsilon_{i,t}, \tag{15}$$

where $R_{i,t}$ denotes the return of the i^{th} asset on day t and $R_{VIX,t}$ is the return of the VIX index on day t. We run the regression using returns from September 2, 2008, to December 31, 2008, which encapsulates the 2008 crash (e.g. it contains the two historic highs of the VIX at 80.06 and 80.86 in October and November). As the VXX and VXZ did not exist in 2008, we proxy these by the returns of their underlying futures indices (SPVIXSTR and SPVIXMTR).

This is also done in other studies (see e.g., Whaley (2013) and Bordonado et al. (2017)).

The regression results are provided in Table III.[15] From the estimates we get a sense of the change in the asset returns given changes in the VIX index. For example, the SPY would drop 0.23% given a 1% increase in the VIX index.

[Insert Table III About Here.]

We apply the parameter estimates from Table III together with the returns series of the VIX index for the period September 2, 2008, to December 31, 2008, to simulate the return path of each asset during the crash scenario. Table IV reports the annualized mean return and volatility for each asset. For each portfolio, we use the average weights from Section V.A, together with the estimated asset returns to calculate the portfolio returns during the crash scenario.

[Insert Table IV About Here.]

Figure VII depicts the value development of each portfolio in the crash scenario. We see that across investment styles, in terms of portfolio value, none of the VIX portfolios have caught up with their respective benchmark at the end of the crash period.

[Insert Figure VII About Here.]

However, consulting Table V for the performance statistics, it is clear that the allocations to the VIX ETPs have to some extent cushioned the portfolio returns through the crash period. All the VIX portfolios have performed much better than the benchmark, which translates into large economic values.

[Insert Table V About Here.]

[15]The regression results only document correlation and are not intended to express causality.

The question is then what the overall picture looks like if we extend the actual portfolio returns with the returns from the crash scenario. Table VI reports the performance results. Compared with the results based on the actual sample period (cf. Table II), we see that adding the crash scenario period has increased the value of holding VIX ETPs in the portfolios. However, as the economic values are still negative across investment strategies, the overall conclusion stays the same.

[Insert Table VI About Here.]

Even the occurrence of a market crash of the same magnitude as the 2008 crash is not enough for the VIX ETPs to add positive economic value. For this to happen either the crash should have been even more severe or the VIX portfolios should have larger weights in VIX ETPs. The latter premise would imply a very lucky investor or an investor who possesses the rare ability to predict market crashes. So for investors who consider holding VIX ETPs to protect portfolios in harsh times, the conclusion is clear. It has been way too expensive for it to have been optimal to hold these products over a long calm period and not even a crash of similar magnitude as the 2008 crash would have been enough to break even.

V.C. Unconstrained investors

Focusing then on the unconstrained investors who can go short, Figure VIII shows the value development of a $100 investment in the unconstrained versions of P-bench, P-short and P-mid.

Consider first the unconstrained MinVar investment strategy. The results are mixed as only one of the VIX portfolios performs better than the benchmark. The total return for the different portfolios are -7.4% for P-bench, -5.6% for P-short and -11.0% for P-mid. At the beginning of the sample, the values are quite synchronous but diverge at the end of 2009 where the benchmark portfolio has a significant drop, due to a short position in the SPY (see Section VI.B on the allocations for each unconstrained portfolio), which has a positive realized return. From this point to the end of the sample, P-short is above the other portfolios except for a few brief periods in 2012 and 2013.

Figure VIIIb depicts the portfolio evolution for the unconstrained CE investment strategy. For the main part of the first year, the portfolios follow the same development with the two VIX portfolios slightly above the benchmark due to the contango carry earned by being short in the VIX ETPs. However, from the end of November 2009, the values of P-short and P-mid diverge as the return on VXZ becomes positive yielding a loss on the short position that P-mid holds in this instrument. From June 2010, the gap between P-short and P-mid continues to widen further. P-short tends to hold quite large short positions in VXX, and as a consequence, there are several larger drawdowns in value occurring in periods of high volatility. This is especially pronounced in October 2011, November 2012, August 2015, and February 2018.

[Insert Figure VIII About Here.]

Panel A of Table VII reports the performance results for the unconstrained MinVar investment strategy. For the entire sample, the two VIX portfolios have mean daily returns of -0.58% (P-short) and -1.21% (P-mid) and sample volatilities of 2.48% (P-short) and 2.40% (P-mid). The benchmark portfolio has mean return and sample volatility of -0.74% and 4.17%, respectively. As for the economic values, the results are mixed as they suggest that an unconstrained MinVar investment strategy would be willing to pay between 18.8 and 21.1 annual basis points for having access to positions in VXX but would require between 41.8 and 44.0 in the case of VXZ. However, for P-short, there are several sub-periods with negative economic value, most significant for 2017-2018 which contains the “Volmageddon” event.

[Insert Table VII About Here.]

For the unconstrained CE investment strategy, the results, reported in Panel B of Table VII, are also blurred. For the entire sample, P-short has a less negative mean return than the benchmark, which translates into an economic value between 58.1 and 115.5 depending on the relative risk aversion. However, there are two sub-periods where the economic value is very negative. For P-mid, the economic value is negative for the entire sample, and only positive for two sub-periods.

Panel C of Table VII reports the results for the unconstrained MD investment strategy, which suggests that there is no economic value of holding the VIX ETPs.

Regarding the question of when the VIX portfolios have economic value for unconstrained investors, consider Figure VII, which shows that the connection with the VIX index is not as clear as for the constrained investors. Considering the constrained CE investment strategy, the correlation between the three-month rolling economic values and the VIX index is 0.13 (P-short) and -0.04 (P-mid). The correlations between the Sharpe ratios and the VIX index is 0.08 (P-short) and -0.08 (P-mid).

[Insert Figure IX About Here.]

Overall, the results do not provide clear evidence of the economic value of VIX products to the unconstrained investment strategies, as the economic value varies with the VIX ETPs and with the investment strategy.

VI. Empirical portfolio allocations

Here we examine when and how often the optimal portfolios for the seven different investment strategies contain the VIX ETPs. Moreover, we investigate the changes in the VIX ETP portfolio weights.

VI.A. Constrained investors

Table VIII reports the proportion of re-balancing points when diversification into VIX ETPs is optimal. Panel A shows the results for the constrained investors. We do not show the constrained constant investment strategy as the VIX ETPs are always included in the P-short and P-mid.

[Insert Table VIII About Here.]

The constrained minimum-variance optimizing (MinVar) investment strategy frequently (around 84% of the months) allocates capital to a VIX ETP with only a little difference between short-term VXX in P-short and the mid-term VXZ in P-mid.

The constrained certainty equivalence optimizing (CE) investment strategy diversifies less frequently with VIX products, and there are only small differences between the two types of VIX ETPs. These results are much in line with the diversification frequencies reported in Alexander et al. (2016).

Allocating capital using the constrained maximum diversification (MD) strategy results in an allocation to both VIX products at every re-balancing.

Now consider the question of how much capital is allocated to the VIX ETPs. Figure Xa and Xb show the weights held in VXX and VXZ, respectively.

[Insert Figure X About Here.]

For both products, the constrained CE investment strategy allocates more capital to the VIX ETPs than the constrained MinVar and the constrained MD investment strategies. The average allocations of the constrained strategies to the VXX are 8.1% (CE), 2.6% (MinVar), and 6.9% (MD), respectively.

Conditioning on the weights being greater than zero, the average allocations are 25.1% (CE), 3.1% (MinVar), and 6.9% (MD). The constrained MinVar and MD strategies have fairly constant weights, whereas the constrained CE strategy have more volatile weights.

With a target return portfolio return of 10%, the average sum of weights to all risky assets in the portfolio is 50% for the constrained MinVar strategy, whereas the constrained CE strategy on average allocates 89% of the capital to risky assets. Weights not summing to 1 means that the allocation will be completed with the residual invested in the risk-free rate. The constrained CE strategy is further out on the efficient frontier in order to maximize expected utility than what is necessary for the constrained MinVar strategy in order to obtain the target expected return of 10%. This also explains the more pronounced changes in the weights for the constrained CE strategy as the allocation between the risky assets relies heavily on changes in the expected returns.

All the constrained investment strategies tend to allocate more capital to the medium-term VXZ than to the short-term VXX, with average allocations of 14.4%, 3.4%, and 7.9% for the CE, MinVar, and MD investor, respectively. Also, in this case, the MinVar strategy has more stable allocations, and the means conditional on them being positive are 41.3% for CE and 3.9% for MinVar.

VI.B. Unconstrained investors

Then focusing on the investment strategies of the unconstrained investors who can hold short positions, Panel B in Table VIII shows that long positions in VIX ETPs occur much less frequently for both the MinVar and CE strategies compared to the similar unconstrained strategies. Now there are larger differences between the two VIX ETPs. Investors more frequently hold long positions in VXZ than in VXX and more often take short rather than long positions.

For VXX there is no difference between the unconstrained MinVar and CE strategies, and for VXZ the difference is minor. For the unconstrained MD strategy, however, most positions are still long in both products.

Figure XI displays the allocations to the VIX ETPs for the unconstrained strategies. As in the constrained version, the CE strategy invests more extremely into VIX ETPs both in the medium and short direction. In unreported results, we find for VXX that the average CE weights are -18%, -2.2% for MinVar, and 5.1% for MD. Conditional on being short, the corresponding average weights are -50%, -4.1%, and -36.0%. Finally, conditional on being long, the average allocations are 34.2% for the CE strategy against 2.8% for the MinVar and 15.7% for the MD strategy.

[Insert Figure XI About Here.]

For the unconstrained CE strategy, the sum of the portfolio weights is on average -10.2%. Disregarding the bond allocation the weight to the stock component is on average -26%, and conditional on the weight to VXX being negative, the average is -104%. Hence for the unconstrained CE strategy, short positions in volatility are often accompanied by even shorter positions in stocks. This suggests that by combining short positions in volatility and stocks, a mean-variance optimizing investor can obtain positions that are more efficient than holding stocks alone.

A short position in VIX ETPs generates a positive carry due to the contango effect of the VIX futures term structure, and the risk of increases in the VIX index is partly offset by the short position in stocks. This fits well in line with the findings of Brière et al. (2010) and Chen et al. (2011). The weights in VXZ display similar patterns. The allocations to VXZ are more extreme for the unconstrained CE strategy with an unconditional mean of -18.3% against -1.6% (MinVar) and 3.4% (MD). Conditional on being short, the average is -63.1% (CE), -5.4% (MinVar), and -79.2%(MD). Conditional on long, produces an average of 43.7% (CE), 3.9% (MinVar), and 10.6% (MD).

VII. Conclusion

In this paper, we use the concept of economic value of Fleming et al. (2001) to evaluate the portfolio performance of investment strategies that include VIX ETPs in addition to stocks and bonds.

With the proliferation of VIX ETPs ten years ago, retail and restricted institutional investors have gained access to volatility trading and the potential of improving portfolio diversification in periods of severe market turmoil. The VIX ETPs have become quite popular, as the number of listed products and the combined market value has increased significantly over the decade. The purpose of this paper has been to quantify the diversification benefits that these products offer in terms of economic value.

Our empirical study considers in total seven different investment strategies. We consider constrained investors (no short positions) who pursue four different investment styles, namely constant weights, minimum-variance, certainty equivalence, and maximum diversification optimization, and unconstrained investors who pursue the latter of the three styles.

We employ high-frequency data using the longest possible sample, namely from 2009 when VIX ETPs started trading to 2018 and we apply a more forward-looking estimate of the expected covariance matrix than in the prior literature. Our analysis is after the recent financial crisis, which implies that it is a period where VIX ETPs have been less valuable to investors than during crisis periods.

Evaluating the ex-post performance for the short-sales constrained investors, representing, e.g., a retail investor or mutual fund, both portfolios with positions in VIX ETPs perform worse than the benchmark equity-bond portfolio. In economic terms, the value added of these products is negative, implying that an investor would be willing to pay for not diversifying with any of the considered VIX ETPs. A simple what-if simulation analysis shows that not even the occurrence of a market crash of similar magnitude as the recent financial crisis in 2008, will imply an overall economic value of the VIX ETPs. Hence, the costs of holding these products in a portfolio clearly outweigh the diversification benefits in periods of market turmoil. However, return skewness was in general higher for the VIX portfolios, which could suggest that the negative economic value can be regarded as a skewness premium.

For the unconstrained strategies, the results are mixed and dependent on the specific VIX ETP. For two investment strategies, the short-term product VXX, has positive economic value measured over the entire sample, however with sub-periods of significant negative value. For the mid-term product, VXZ, the value is negative for across investor types.

We estimate how often these investment strategies allocate capital to either a short-term or a mid-term VIX ETP and if so, how much capital is allocated. We find that diversification of a stock-bond portfolio with a VIX ETP is often perceived ex-ante optimal. The frequency and portfolio weights vary with the product and investment strategy. The minimum-variance and maximum diversification strategies, more frequently hold positions in a VIX product and the weights vary less than for the utility-maximizing strategy, who on average holds the largest positions in absolute terms.

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Table I: Summary Statistics for SPY, AGG, VXX and VXZ returns

This table provides summary statistics for daily returns (computed as sum of intraday log-returns) on SPY, AGG, VXX and VXZ.omputed as the sum of the intraday log-returns. Panel A reports the mean returns (μ), standard deviations (σ), and mean realized volatilities($\overline{\sigma}_t$). These values are annualized using 252 trading days per year. Panel B reports the mean realized correlations based on our covariance matrix estimated by the procedure described in II.C.

Panel A: Annualized Mean Return, Standard Deviation, Mean Realized Volatility and Skew				
Ticker	μ	σ	$\overline{\sigma}_t$	Skew
SPY	12.34	16.22	13.08	-0.28
AGG	0.46	3.67	3.50	-0.33
VXX	-84.32	62.11	52.52	0.87
VXZ	-32.34	30.27	28.81	0.66
Panel B: Correlation Matrix				
Ticker	SPY	AGG	VXX	VXZ
SPY	1.00	-0.22	-0.77	-0.51
AGG	-0.22	1.00	0.19	0.15
VXX	-0.77	0.19	1.00	0.59
VXZ	-0.51	0.15	0.59	1.00

Table II: Ex Post Performance of Short-Sales Constrained Portfolios

The table summarizes the ex-post performance for a constrained investor. Performance is shown for the benchmark portfolio P-bench and the two portfolios with access to VIX products, P-short and P-mid. Panel A report the results for the MinVar investor with a target expected return, μ_p of 10%. Panel B reports the results for the CE investor with an absolute risk aversion, a, equal to 4. Panel C reports the results for the MD investor, and Panel D for the investor using a constant allocation. For each portfolio, we report the annualized mean return (μ), the annualized volatility (σ), the Sharpe Ratio (SR), and Skewness of the returns. Furthermore, for P-short and P-mid, we report the economic value (Δ) over the sample period in annual BP. The economic value we report for a level of relative risk aversion (γ) of 1 and 10. The sample period is January 30, 2009, through June 2018. The first month of data is withheld for estimating the expected return and conditional covariance matrix for the first re-balancing date. We also report results for each two-year subsample.

	P-bench				P-short						P-mid					
Period	μ	σ	SR	Skew	μ	σ	SR	Skew	Δ_1	Δ_{10}	μ	σ	SR	Skew	Δ_1	Δ_{10}
Panel A: MinVar																
Entire sample	2.20	4.92	0.402	-0.21	-0.44	6.04	-0.11	-0.11	-267.6	-270.7	0.25	5.32	0.060	-1.13	-196.3	-196.9
2009-2010	3.96	3.38	1.132	2.36	3.29	3.29	0.957	2.65	-64.0	-68.9	3.72	3.07	1.161	3.24	-24.61	-29.53
2011-2012	5.35	5.12	1.042	1.61	2.45	6.25	0.389	0.84	-292.9	-296.5	1.82	5.20	0.346	1.58	-354.4	-354.4
2013-2014	1.80	4.71	0.381	-0.03	-4.19	4.57	-0.917	-1.33	-599.2	-598.6	0.31	4.84	0.065	0.06	-148.9	-147.7
2015-2016	-1.64	6.95	-0.257	-1.15	-3.87	9.77	-0.407	-0.24	-235.6	-246.1	-4.64	8.24	-0.577	-1.88	-305.8	-310.1
2017-2018	1.53	2.78	0.171	-0.56	0.71	2.59	-0.132	-0.51	-81.2	-83.7	0.43	2.62	-0.236	-0.50	-108.3	-108.3
Panel B: CE																
Entire sample	4.69	10.43	0.428	-0.46	-1.69	14.54	-0.132	0.13	-663.8	-685.1	-2.99	14.25	-0.226	-0.19	-791.7	-810.9
2009-2010	11.08	13.54	0.807	-0.04	14.97	15.50	0.956	-0.01	374.1	362.99	17.65	17.39	1.006	0.15	627.5	605.4
2011-2012	8.05	9.90	0.811	-0.55	1.81	16.40	0.109	-0.04	-664.5	-701.4	-4.72	15.52	-0.302	-0.81	-1311.7	-1342.4
2013-2014	4.00	9.37	0.426	-0.47	-11.10	14.24	-0.780	0.05	-1536.9	-1560.2	-6.96	12.74	-0.546	-0.01	-1114.8	-1127.1
2015-2016	-2.98	9.23	-0.334	-0.853	-8.58	14.3	-0.608	0.15	-589.4	-614.0	-12.62	13.55	-0.939	-0.09	-989.3	-1007.8
2017-2018	2.91	9.03	0.206	-1.57	-1.40	11.81	-0.207	1.01	-443.0	-456.5	-2.42	11.24	-0.389	-0.18	-546.3	-553.7
Panel C: MD																
Entire sample	2.85	3.42	0.767	0.05	0.60	3.29	0.115	0.49	-224.1	-226.4	1.38	3.03	0.383	0.14	-145.7	-142.7
2009-2010	7.13	4.56	1.530	0.33	1.86	3.93	0.435	0.60	-526.6	-525.4	5.73	3.97	1.406	0.36	-139.0	-118.1
2011-2012	3.18	2.96	1.068	0.06	0.59	3.16	0.180	0.25	-259.6	-258.4	0.66	2.7	0.238	0.25	-246.1	-251.0
2013-2014	2.57	3.17	0.810	-0.32	2.37	2.92	0.813	-0.16	-19.7	-19.7	1.27	2.76	0.065	-0.41	-130.4	-128.6
2015-2016	-0.09	3.36	-0.059	-0.33	-3.02	3.33	-0.938	0.78	-290.4	-290.4	-0.81	2.97	-0.310	-0.10	-72.0	-68.9
2017-2018	1.44	2.67	0.146	-0.48	1.57	3.02	0.172	0.80	14.8	9.8	0.08	2.44	-0.400	-0.20	-132.9	-135.4
Panel D: Constant																
Entire sample	9.68	9.24	2.763	-0.07	6.29	7.01	1.184	0.49	-329.46	-321.9	8.18	8.15	2.619	0.12	-144.0	-140.7
2009-2010	21.23	12.21	4.620	0.40	15.01	10.15	3.785	0.77	-610.3	-600.5	19.83	11.18	4.959	0.52	134.1	-129.2
2011-2012	5.80	10.64	1.956	-0.42	2.97	8.08	0.934	-0.09	-270.7	-260.9	3.83	9.33	1.409	-0.29	-189.5	-184.6
2013-2014	11.24	6.74	3.539	-0.35	8.49	4.74	2.909	-0.25	-268.2	-264.6	9.53	5.81	3.455	-0.37	-177.2	-167.3
2015-2016	2.79	8.45	0.797	-0.29	0.02	5.80	-0.025	0.25	-268.2	-258.4	2.10	7.16	-0.667	-0.15	-65.2	-60.3
2017-2018	7.73	6.49	2.508	-1.29	5.42	4.21	1.448	-0.16	-225.18	-220.3	5.97	5.50	2.016	-0.97	-172.3	-169.8

Table III: Financial crisis scenario - Regression output

The table shows the output from the regression in Equation (15). The regression is run on data from September 2, 2008, to December 31, 2008, yielding 85 observations. The reported t-stats are calculated using Newey-West standard errors.

Ticker	Adj. R^2	α	t(α)	β	t(β)
SPY	0.73	0.001	0.48	-0.33	-8.74
AGG	-0.002	0.000	0.66	-0.01	-0.85
SPVIXSTR	0.77	0.008	2.37	0.45	13.60
SPVIXMTR	0.72	0.004	1.77	0.25	11.25

Table IV: Financial crisis scenario - Summary statistics

The table shows the annualized mean return (μ) and volatility (σ) for each asset in the crash scenario analysis. The return series are estimated using the returns of the VIX index for the period September 2, 2008, to December 31, 2008, and the regression coefficients provided in Table III.

Ticker	μ	σ
SPY	-84.3	56.1
AGG	11.9	1.9
VXX	350.6	77.3
VXZ	188	43.6

Table V: Financial crisis scenario - Portfolio performance

The table summarizes the performance of each constrained portfolio throughout the crash scenario. For each portfolio, we report the annualized mean return (μ), and the annualized volatility (σ). For P-short and P-mid, we report the economic value (Δ) in annual BP for a level of relative risk aversion (γ) of 1 and 10.

Portfolio	μ	σ	Δ_1	Δ_{10}
Panel A: MinVar				
P-bench	-14.5	12.9		
P-short	-7.9	11.6	669.4	671.1
P-mid	-10.0	11.9	452.8	472.5
Panel B: CE				
P-bench	-47.0	33.9		
P-short	-23.6	30.0	2441.3	2451.1
P-mid	-21.5	27.6	2648.0	2726.7
Panel C: MD				
P-bench	-3.5	10.6		
P-short	1.4	15.5	462.7	393.6
P-mid	2.3	11.6	570.9	551.3
Panel D: Constant				
P-bench	-45.8	34.4		
P-short	-28.9	30.5	1762.0	1811.3
P-mid	-37.0	32.2	920.4	945.0

Table VI:
Portfolio performance: Entire sample extended with financial crisis scenario

The table summarizes the performance of each constrained portfolio for the entire sample with the addition of the crash scenario. For each portfolio, we report the annualized mean return (μ), the annualized volatility (σ), the Sharpe Ratio (SR), and the Skewness of returns. For P-short and P-mid, we report the economic value (Δ) in annual BP for a level of relative risk aversion (γ) of 1 and 10.

Portfolio	μ	σ	SR	Skew	Δ_1	Δ_{10}
Panel A: MinVar						
P-bench	1.62	5.40	0.250	-0.56		
P-short	-0.7	6.31	-0.154	-0.25	-235.3	-236.3
P-mid	-0.11	5.67	-0.066	-1.14	-172.3	-173.5
Panel B: CE						
P-bench	2.88	12.05	0.217	-0.98		
P-short	-2.46	15.33	-0.178	-0.08	-556.8	-574.6
P-mid	-3.64	14.91	-0.262	-0.32	-671.8	-687.2
Panel C: MD						
P-bench	2.63	3.89	0.605	-0.48		
P-short	0.63	4.33	0.084	-0.80	-200.3	-201.2
P-mid	1.42	3.67	0.313	-0.61	-118.1	-120.3
Panel D: Constant						
P-bench	7.74	11.13	0.671	-0.98		
P-short	5.06	8.93	0.537	-0.93	-256.6	-247.3
P-mid	6.60	10.0	0.633	-0.95	-107.5	-102.4

Table VII: Ex Post Performance of Optimal Portfolios for Unconstrained Investors

The table summarizes the ex post performance for an unconstrained investor. Performance is shown for the benchmark portfolio P-bench and the two portfolios with access to VIX products P-short and P-mid. Panel A reports the results for the MinVar investor with a target expected return, μ_p, of 10%. Panel B reports the results for the CE investor with an absolute risk aversion, a, equal to 4. Panel C reports the results for the MD investor. For each portfolio, we report the annualized mean return (μ), the annualized volatility (σ), the Sharpe Ratio (SR), and Skewness of the returns. Furthermore, for P-short and P-mid, we report the economic value (Δ) over the sample period in annual BP. The economic value we report for a level of relative risk aversion (γ) of 1 and 10. The sample period is January 30, 2009, through June, 2018. The first month of data is withheld for estimating the expected return and conditional covariance matrix for the first re-balancing date. We also report results for each two-year subsample.

	P-bench				P-short						P-mid					
Period	μ	σ	SR	Skew	μ	σ	SR	Skew	Δ_1	Δ_{10}	μ	σ	SR	Skew	Δ_1	Δ_{10}
Panel A: MinVar																
Entire sample	-0.74	4.17	-0.231	-0.94	-0.58	2.48	-0.326	-0.86	18.8	21.1	-1.21	2.40	-0.597	-0.51	-44.0	-41.8
2009-2010	-1.26	3.36	-0.419	-0.27	0.94	1.88	0.421	-0.10	221.5	223.9	0.28	2.15	0.060	0.40	155.7	156.3
2011-2012	1.94	3.85	0.500	-0.18	1.67	2.31	0.712	0.06	-24.6	-22.15	-0.89	2.14	-0.423	-0.43	-275.6	-280.5
2013-2014	-2.97	3.27	-0.907	-0.88	-3.94	2.88	-1.370	-0.86	-97.2	-98.4	-0.76	2.76	-0.273	-0.49	221.9	222.1
2015-2016	-1.81	6.34	-0.418	-1.03	1.71	2.72	0.613	-0.74	360.5	369.1	-2.39	2.84	-0.766	-0.70	-49.2	-44.3
2017-2018	0.78	2.61	-0.104	-0.09	-4.06	2.38	-2.148	-2.39	-483.6	-482.3	-2.49	1.80	-1.965	-1.43	-324.8	-334.7
Panel B: CE																
Entire sample	-5.93	20.77	-0.296	-0.35	-4.07	26.69	-0.161	-1.06	115.5	58.1	-12.85	22.94	-0.570	-0.86	-716.1	-735.5
2009-2010	-9.12	24.87	-0.381	-0.25	16.8	23.95	1.586	-0.33	2602.4	2611.1	0.15	23.58	0.414	-0.23	940.1	955.5
2011-2012	4.87	21.15	0.387	-0.22	13.45	24.77	0.969	-0.20	807.2	782.6	-11.02	26.03	-0.225	-1.17	-1646.4	-1653.7
2013-2014	-17.06	19.59	-0.871	-0.18	-20.13	23.25	-0.866	-0.32	-344.5	-376.5	-12.69	20.09	-0.632	-4.78	433.1	428.2
2015-2016	-1.97	18.84	-0.111	-0.32	4.34	29.11	0.146	-1.23	509.4	403.6	-19.39	23.01	-0.847	-0.51	-1785.4	-1820.5
2017-2018	-6.69	18.67	-0.415	-1.16	-42.94	32.56	-1.351	-1.93	-3802.1	-3947.3	-22.70	21.22	-1.119	-2.61	-1626.7	-1647.6
Panel C: MD																
Entire sample	6.43	6.87	0.904	-0.35	-5.60	14.36	-0.406	1.02	-1242.9	-1275.4	-1.23	9.16	-0.159	-5.09	-775.20	-782.6
2009-2010	10.59	7.64	1.367	-0.15	-30.95	19.10	-1.628	-0.34	-4227.9	-4292.5	1.61	17.74	0.083	-3.62	-962.2	-1013.9
2011-2012	5.06	8.53	0.591	-0.33	6.61	19.04	0.346	2.63	83.7	24.6	0.77	7.69	0.098	0.91	-425.7	-422.7
2013-2014	9.05	5.28	1.713	-0.34	1.78	7.40	0.240	-1.70	-733.4	-738.3	-2.44	3.06	-0.795	-0.37	-1144.3	-1140.6
2015-2016	2.17	6.77	0.305	-0.29	-8.67	12.01	-0.731	-0.08	-1109.3	-1129.6	-3.96	5.29	-0.769	-1.43	-609.1	-605.4
2017-2018	5.36	5.22	0.825	-1.22	3.75	9.21	0.294	-0.55	-172.3	-187.0	-2.20	2.79	-1.164	-0.86	-753.0	-745.7

Table VIII: Frequency of optimal equity-bond diversification

The proportion of re-balancing periods when the ex-ante optimal portfolio weight to VIX products is greater than 0 (Constrained investor) and different from 0 (Unconstrained investor).

	Panel A: Constrained		Panel B: Unconstrained			
	P-short	P-mid	P-short		P-mid	
Investor	>0	>0	<0	>0	<0	>0
MinVar	83.04%	85.71%	67.86%	32.14%	58.93%	41.07%
CE	32.14%	34.82%	67.86%	32.14%	58.04%	41.96%
MD	100%	100%	20.54%	79.46%	8.04%	91.96%

Figure I: VIX Futures Price Curve on March 18, 2015

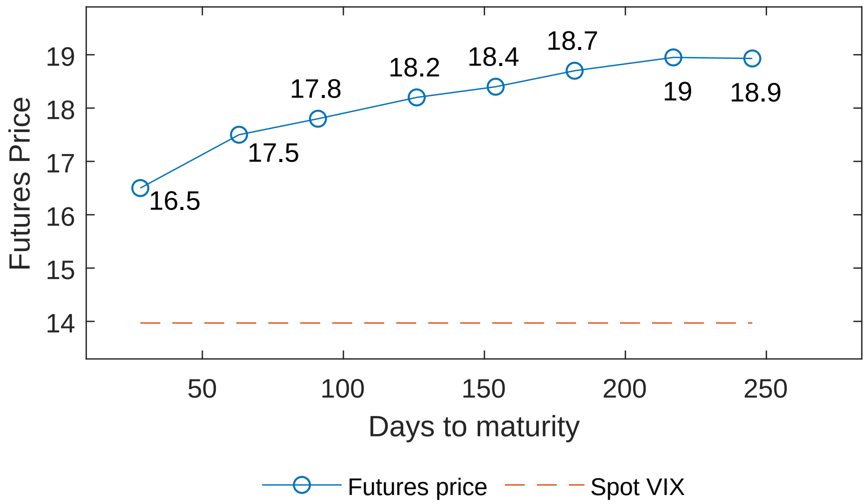


Figure II: Price development of VXX and VXZ

Over the lifetime of the products, the value of VXX has been severely eroded and the issuer has made no less than five 1-for-4 reverse splits (only one for VXZ). The depicted price development has been adjusted for these hence the magnitude of the left-hand y-axis.

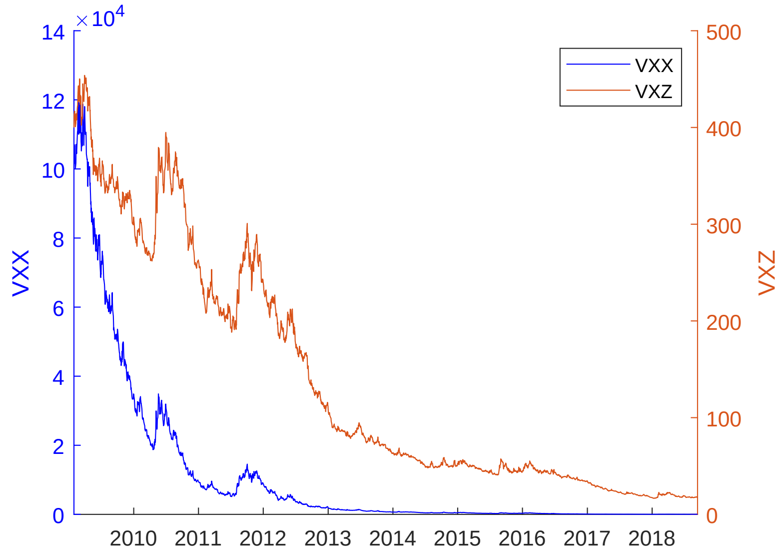


Figure III: VIX Futures Price Curve on February 5, 2018

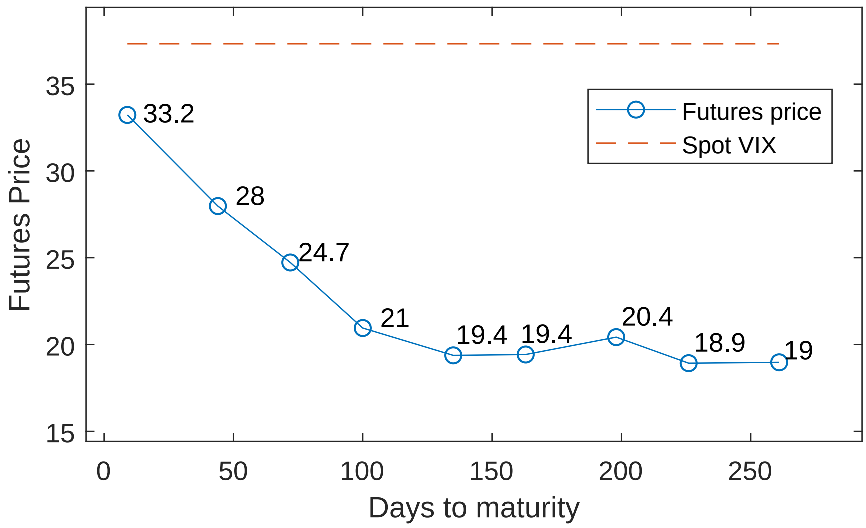
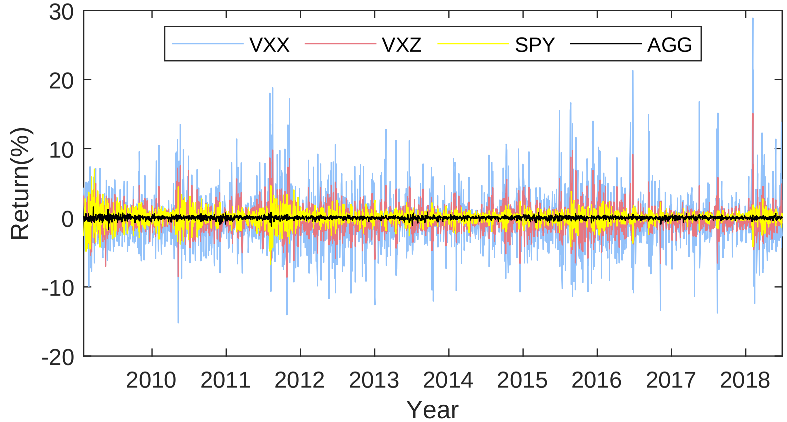
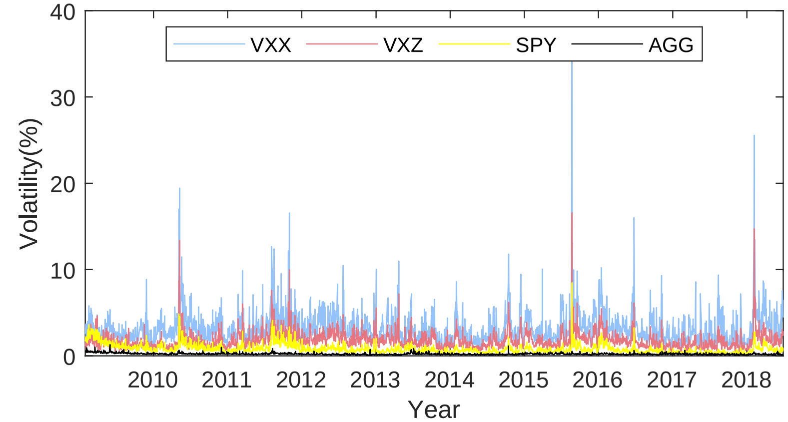


Figure IV: Daily realized returns, realized volatilities and realized correlations

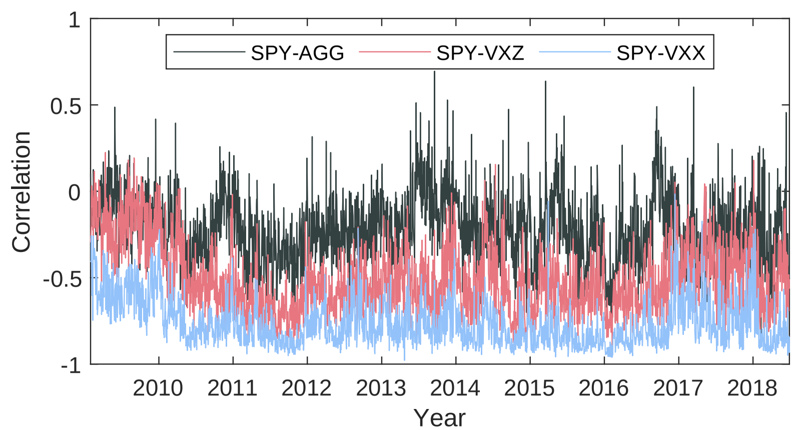
Panel a and b show the daily realized returns and realized volatilities, respectively, for SPY, AGG, VXX, and VXZ. The realized returns are computed as the sum of intraday log-returns. The values are not annualized. Panel c shows the daily realized cross-market correlations between SPY and the three other tickers used for constructing portfolios. The realized correlations are based on our covariance matrix estimated by the procedure described in II.C.



(a) Realized returns



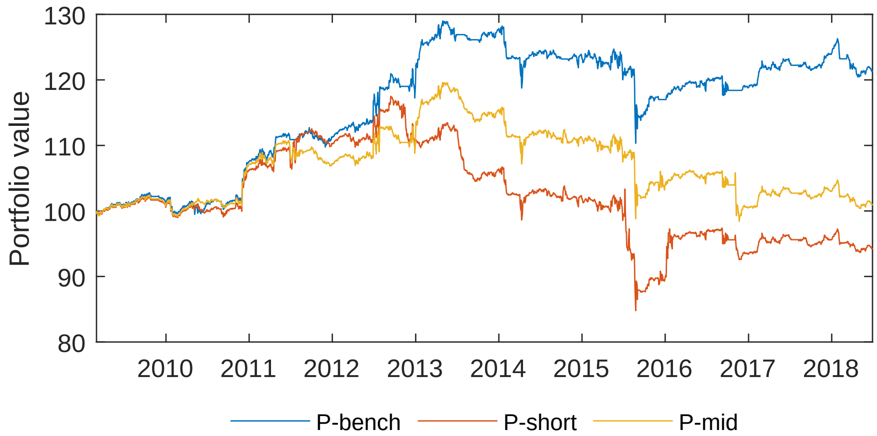
(b) Realized volatilities



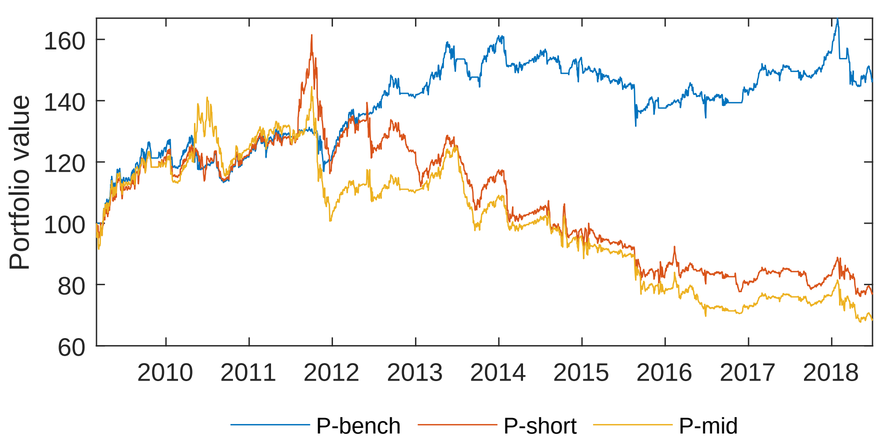
(c) Realized correlations

Figure V: Ex Post Performance of Constrained Portfolios

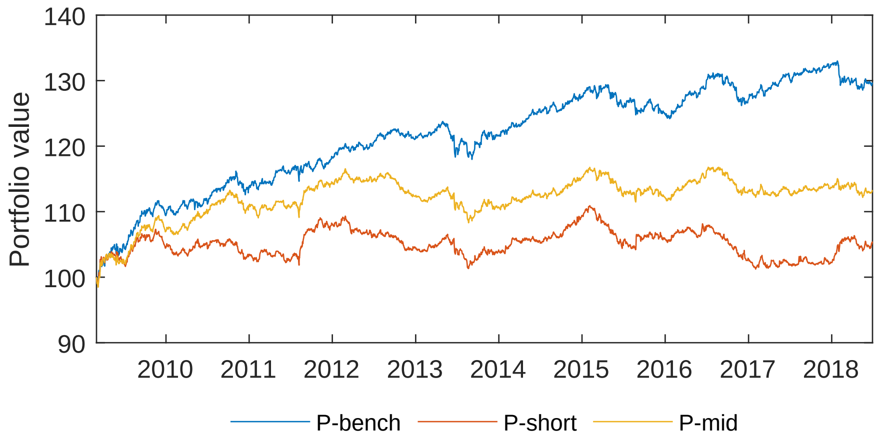
Panel a, b, c, and d show the ex-post performance of the constrained portfolios for the MinVar, CE, MD, and Constant portfolios, respectively.



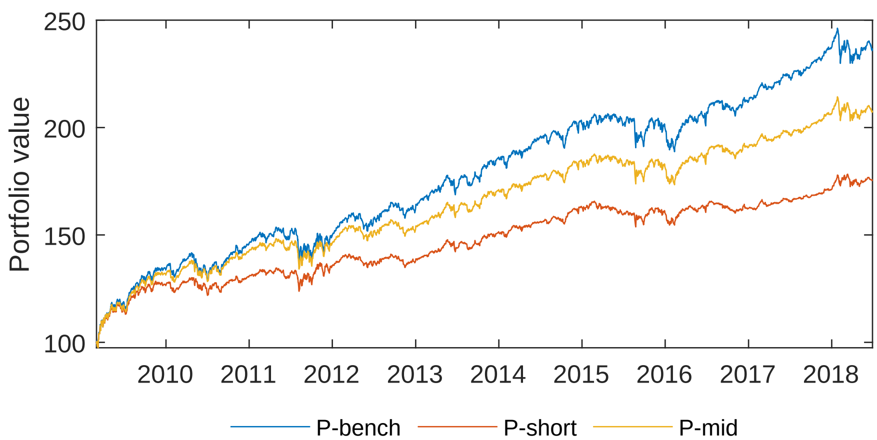
(a) MinVar investor



(b) CE investor



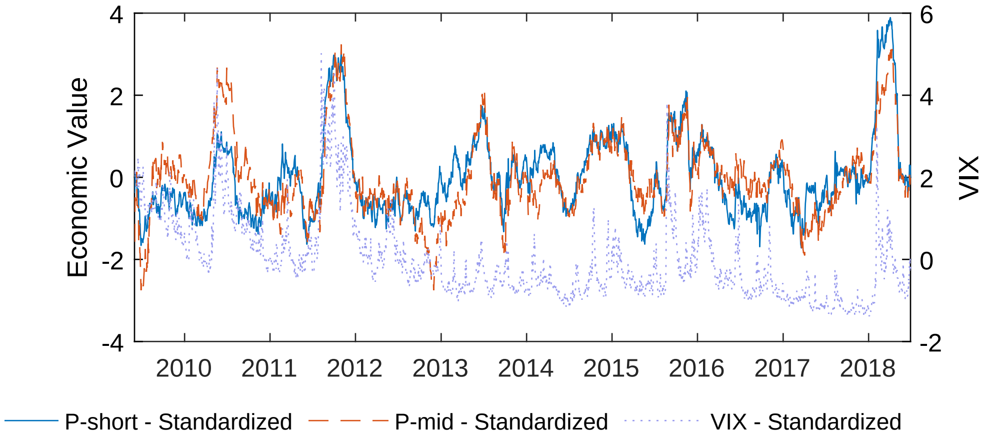
(c) MD investor



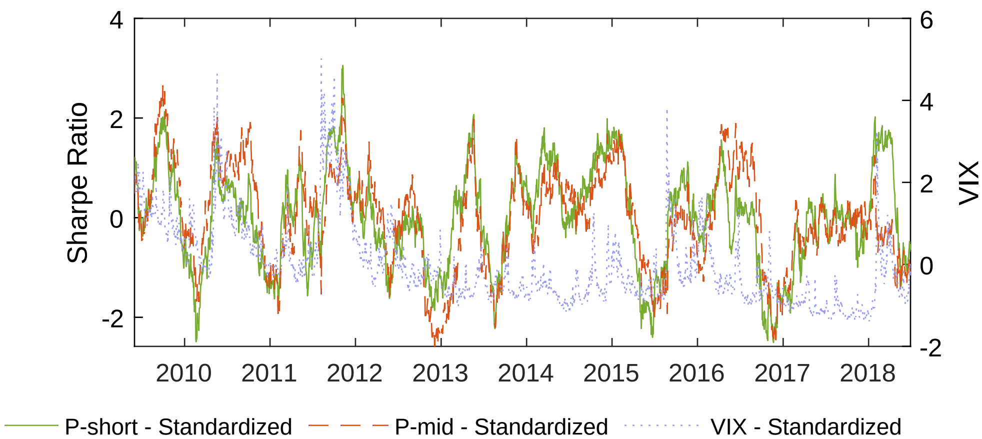
(d) Constant investor

Figure VI: 3-month rolling performance and the VIX level - constrained

Figure VIa shows the three-month rolling economic value of portfolios P-short and P-mid against the level of the VIX index. The economic value is for a relative risk aversion(γ) equal to 1. Figure VIb shows the three-month rolling Sharpe ratios of portfolios P-short and P-mid against the level of the VIX index. To facilitate a comparison, the time series are standardized to have a 0 mean and unit standard deviation. These values are for an MD investor. The correlation between economic value and the VIX is 0.31 (P-short) and 0.40 (P-mid). The correlation between Sharpe ratios and the VIX is 0.30 (P-short) and 0.25 (P-mid). All values are for constrained portfolios.



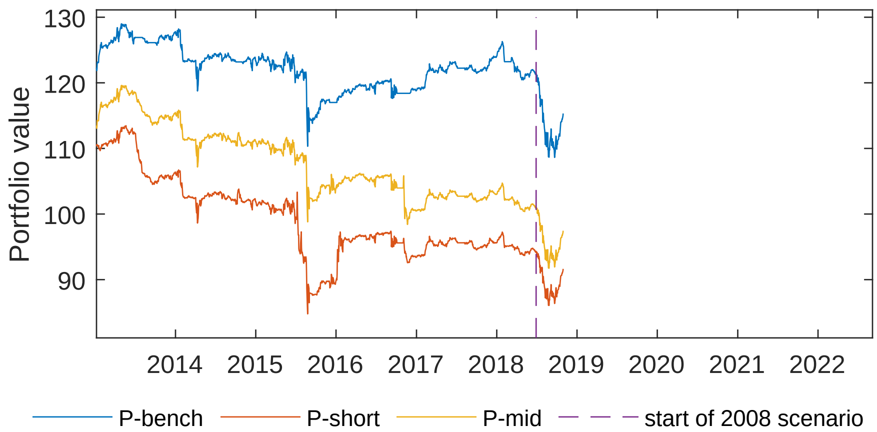
(a) Economic value



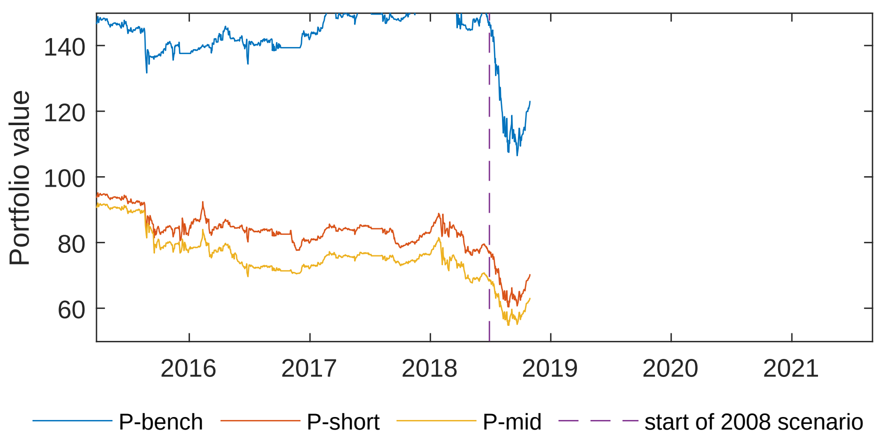
(b) Sharpe ratio

Figure VII: Financial crisis scenario - Portfolio performance

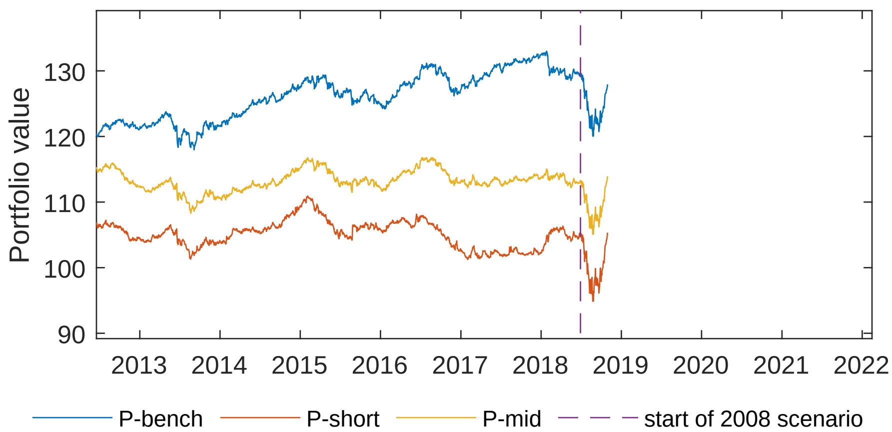
Panel a, b, c, and d show the ex-post performance of the constrained portfolios for the MinVar, CE, MD and Constant portfolios, respectively, throughout the crash scenario.



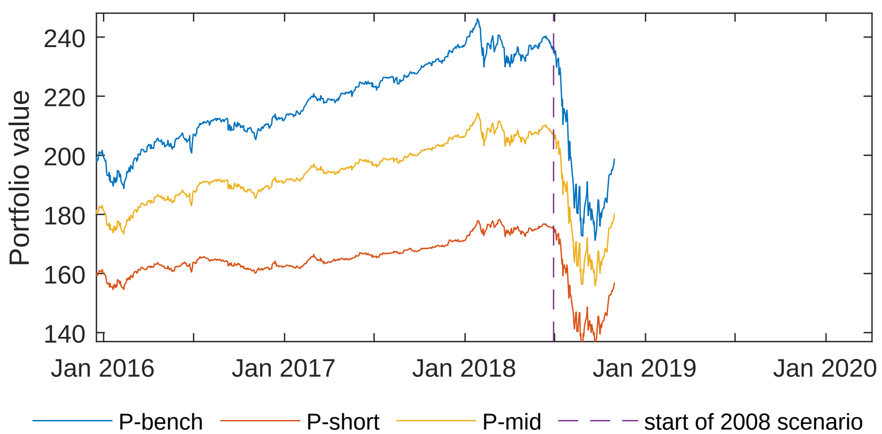
(a) MinVar investor



(b) CE investor



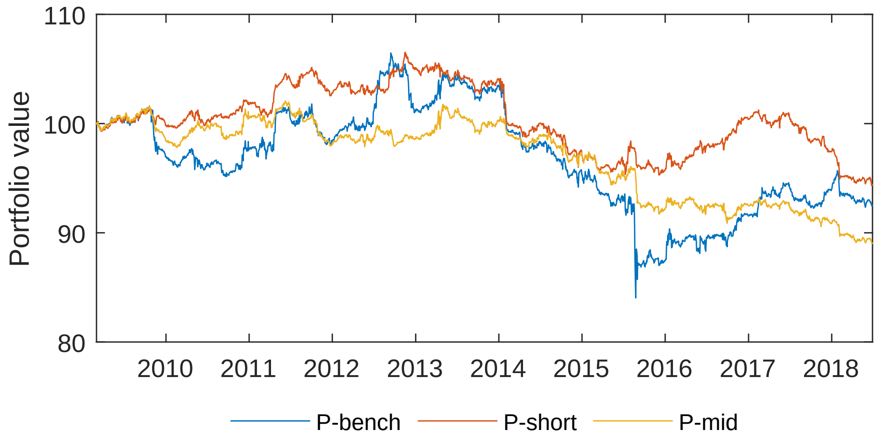
(c) MD investor



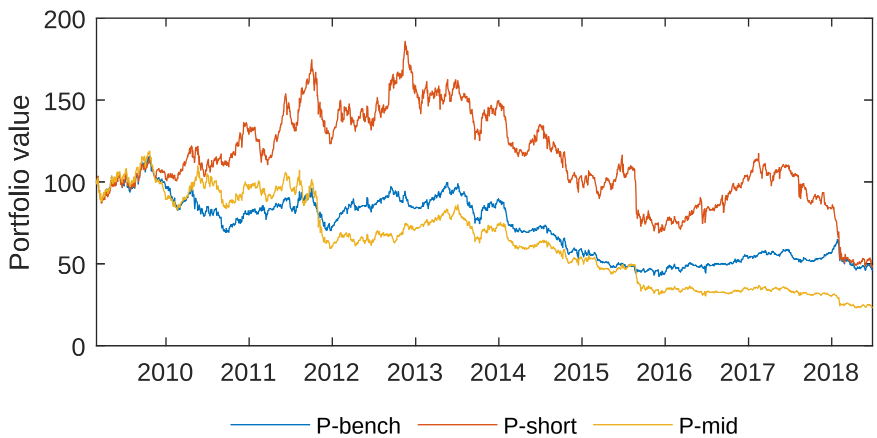
(d) Constant investor

Figure VIII: Ex Post Performance of Unconstrained Portfolios

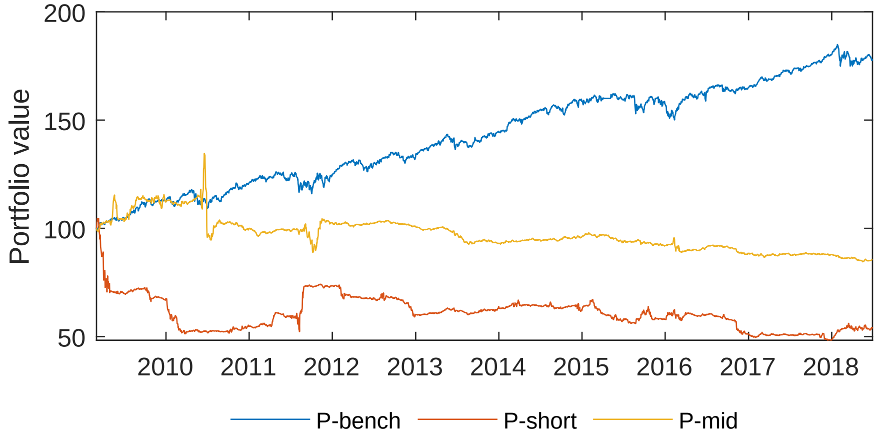
Panel a, b, and c show the ex-post performance of the unconstrained portfolios for the MinVar, CE, and MD respectively.



(a) MinVar investor



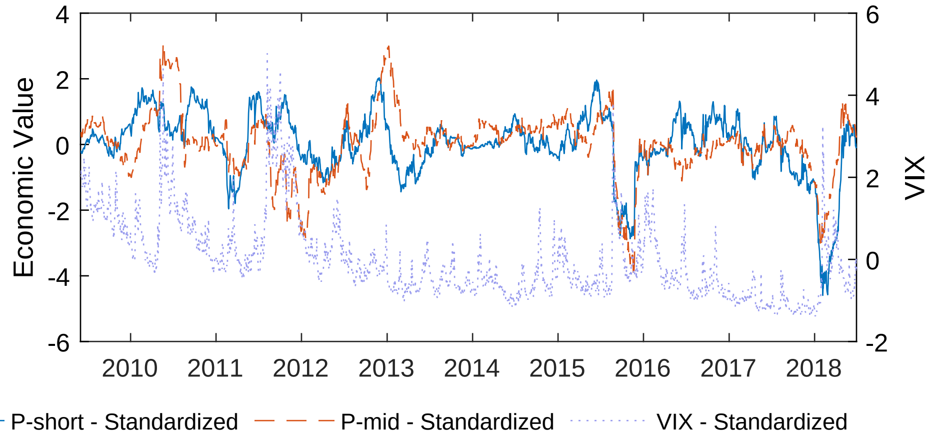
(b) CE investor



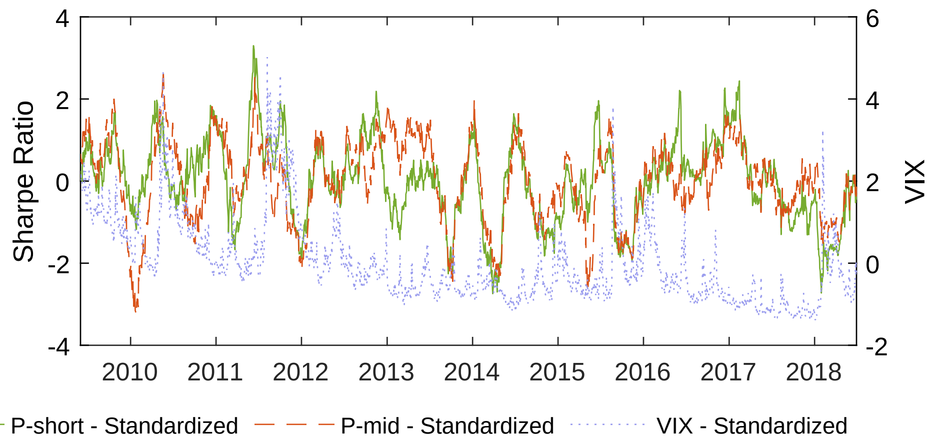
(c) MD investor

Figure IX: 3-month Sharpe ratios and the VIX level-unconstrained

Figure IXa shows the three-month rolling economic value of portfolios P-short and P-mid against the level of the VIX index. The economic value is for a relative risk aversion(γ) equal to 1. Figure VIb shows the three-month rolling Sharpe ratios of portfolios P-short and P-mid against the level of the VIX index. To facilitate a comparison, the time series are standardized to have a 0 mean and unit standard deviation. These values are for a CE investor. The correlation between economic value and the VIX is 0.13 (P-short) and -0.04 (P-mid). The correlation between Sharpe ratios and the VIX is 0.08 (P-short) and -0.08 (P-mid). All values are for unconstrained portfolios.



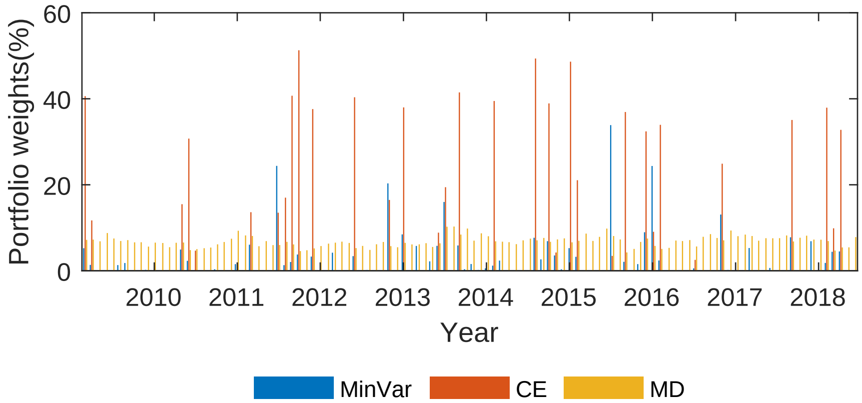
(a) Economic value



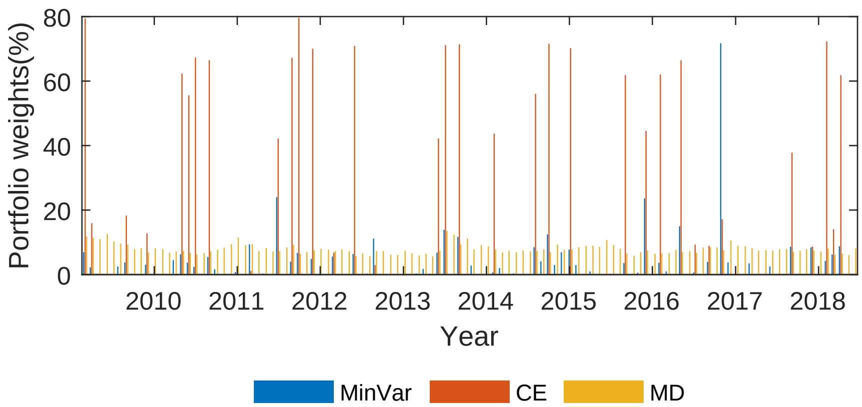
(b) Sharpe ratio

Figure X: Short-sales constrained allocations to VXX and VXZ

Figure a and b show the optimal allocation to VXX and VXZ, respectively, for a constrained investor. Weights are shown both for MinVar, CE, and MD investors. The MinVar investor has a target expected return, μ_p of 10% and the CE investor has an absolute risk aversion, a, equal to 4.



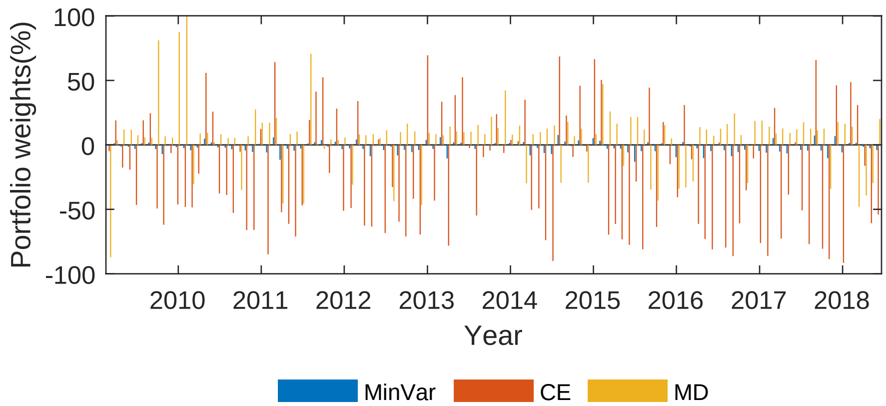
(a) Allocation to VXX



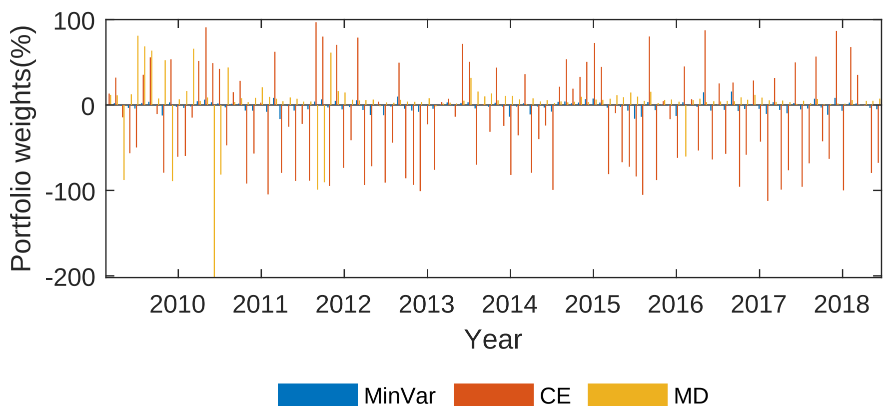
(b) Allocation to VXZ

Figure XI: Unconstrained allocations to VXX and VXZ

Figure a and b show the optimal allocation to VXX and VXZ, respectively, for an unconstrained investor. Weights are shown both for MinVar, CE, and MD investors. The MinVar investor has a target expected return, μ_p of 10% and the CE investor has an absolute risk aversion, a, equal to 4.



(a) Allocation to VXX



(b) Allocation to VXZ

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