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Abstract

Several OECD countries have recently implemented an automatic link between the statutory retirement age and life expectancy for the total population to insure sustainability in their pension systems when life expectancy is increasing. Significant mortality differentials are observed across socio-economic groups and future changes in these differentials will determine whether some socio-economic groups drive increases in the retirement age leaving other groups with fewer years in receipt of pensions. We forecast life expectancy by socio-economic groups and compare the forecast performance of competing models using Danish mortality data and find that the most accurate model assumes a common mortality trend. Life expectancy forecasts are used to analyse the consequences of a pension system where the statutory retirement age is increased when total life expectancy is increasing.

Keywords: Compositional data, forecasting, longevity, pension, socioeconomic groups.

JEL Codes: C22, C23, C53, I12.

1 Introduction

Recently, several OECD countries have established an automatic link between their pension systems and increases in life expectancy: for example in Finland, Denmark, Portugal, Italy, the Netherlands, the Slovak Republic, and Sweden (OECD, 2017). The link is either between pension payments and life expectancy, as in Sweden and Italy, or between the statutory retirement age and life expectancy, as in Denmark and the Netherlands (OECD, 2012) which are rated to have the best pension systems (ACFS, 2018). Denmark and the Netherlands In the latter case, life expectancy for the total population is used to regulate the pension system, such that the statutory retirement age will be increased if life expectancy for the whole population increases. Thus, the pace by which life expectancies for different socio-economic groups will have implications for the number of pensionable years. This article analyses the consequences of such a pension system on socio-economic inequalities by forecasting life expectancy by socio-economic status. The aim is to identify the most accurate model for forecasting mortality by socio-economic status by comparing different models, basing the selection on their inclusion of dependence among the socio-economic groups. As an example, forecasts are used to measure the implication of the current pension scheme in Denmark in terms of expected years with pension. Our empirical setting focuses on Denmark but as mortality differs significantly by socio-economic status for almost all developed countries (Mackenbach et al., 2003) the outline of the results presented are relevant for all countries which link life expectancy changes with their pension systems. The relation between income, wealth, and mortality is a topic of general interest and often studied, e.g. by Snyder and Evans (2006) and Evans and Moore (2012).

A successful forecast of mortality differentials between socio-economic sub-populations relies on the model's ability to capture different aspects of the differentials. Mortality differences can be separated into two parts (Villegas and Haberman, 2014): mortality differential in the average level of mortality by age which can be assumed constant over time and the part due to mortality changes over time. Villegas and Haberman (2014) showed that changes in mortality differentials can be modelled successfully using multi-population mortality models and the analysis presented in this article uses this approach. Multi-population models aim at coherently modelling and forecasting mortality data from several populations or sub-populations and estimating both the level and the changing part of the differentials.

Mortality forecasts are, currently, almost exclusively performed using models which decompose age-specific mortality rates into age, period, and sometimes also cohort effects, inspired by the Lee and Carter (1992) (LC) model, which is the most popular mortality forecasting model in countries with data of high quality (Booth, 2006; Cairns et al., 2009; Enchev et al., 2017). One major limitation with the LC type of models is that they generally underestimate improvements in mortality as a result of assuming constant age-specific and relative improvements (Bergeron-Boucher et al., 2017a; Booth and Tickle, 2008). As the LC model is fitted to historical data the model gives a high weight to improvements in mortality for relatively young ages when fitted to data from developed countries as mortality has declined most in these age groups. When this pattern is imposed in forecasts the model fails to capture improvements at higher ages.

Oeppen (2008) and later Bergeron-Boucher et al. (2017a) suggested using life table deaths to forecast mortality based on compositional data analysis (CoDa) and alleviate this limitation in the LC models by an adaptation of the LC model to a framework using life table deaths. CoDa mortality models shift deaths from younger ages towards older ages. A redistribution of the density of death is captured in line with the shifting and compressing patterns of mortality we see today in most developed countries. CoDa mortality models are especially useful when forecasting mortality for populations where the mortality patterns are changing e.g. if life expectancy has been stagnating and then begins to experience improving mortality again. This is the case for the Danish population because Denmark in the 1980's and first part of the 1990's experienced a stagnation in the improvement of mortality and from 1995 and onwards experienced relatively large improvements in mortality (Jarner et al., 2008). Other countries experienced a similar stagnation periods, for example, the Netherlands and the U.S. from around from 1984 to 2000 (Meslé and Vallin, 2006). The LC type of models do not capture the shifting patterns which leads to less accurate life expectancy forecasts. In contrast CoDa models allow for more interactive dynamics in the observed mortality trajectories, and more accurate forecasts are often found when these models are fitted to data (Bergeron-Boucher et al., 2017a). Thus, CoDa models offer an attractive alternative to LC type models when forecasting mortality by socio-economic groups. A few CoDa mortality models have been suggested (Oeppen, 2008; Bergeron-Boucher et al., 2017a, 2018; Kjærgaard et al., 2019) and this paper discusses the suitability of using CoDa models to forecast mortality in socio-economic groups by comparing existing models as well as proposing a new model. The analysis could have included other multi-population models formulated on death rates such as the models presented in Villegas and Haberman (2014) or alternative models using the probability of death such as Cairns et al. (2006) or Cairns et al. (2009). We choose not to do this as the number of analysed models already is high and because these models have the same tendency to produce low life expectancy forecasts because they also assume constant age-specific improvements without any inter-generational transfer of deaths as in the CoDa mortality models (Bergeron-Boucher et al., 2017b). We refer to Bergeron-Boucher et al. (2017b) for further details and discussions. Although Cairns et al. (2006) and Cairns et al. (2009) are highly relevant and more sophisticated than models analysed in Bergeron-Boucher et al. (2017b) it is outside the scope of this article to analyse this in further detail and we limit the number of cross comparisons to the Lee-Carter and Li-Lee models as they are the most widely used models for developed countries.

For the total population in a country, an independent modelling of sub-populations might be reasonable, but for sub-populations within the same country it is likely that factors such as health care, public policy, technology, etc. affect all sub-populations. Factors which affect all sub-populations could be incorrectly specified by the independent model and could lead to implausible forecasts with possibly unbounded divergence among the sub-groups without any boundaries (Villegas and Haberman, 2014). Hence, modelling the dependence between socio-economic groups is important but it is not straightforward to determine how and to what extent dependence should be included to obtain the most accurate forecasts. A key contribution of this article is in the selection of models which include models with different levels of dependence, imposed both in the time and age structures of mortality. The models used in the analysis are selected based on their inclusion of time and age dependent parameters so that different degrees of flexibility with respect to the time- and age-dimensions are allowed for. The inclusion of dependence spans from an one side an independent treatment (including 6 parameter vectors both for the time and the age dimension), that is the Independent-CoDa (Inde-CoDa) and Lee-Carter (LC) models. On the other side, we include models with one parameter vector for both the time and age dimension, that is the Relative-CoDa (Rela-CoDa), Three Dimensional CoDa (3D-CoDa), and Lee-Li (LL) models. The LL model is an extension of the LC model to multiple populations. The new model suggested in this article, Dynamic Factor CoDa (Dynam-CoDa), places itself in between these extremes by having one age dimension and multiple time dimensions. Further, by comparing CoDa mortality and models formulated on death rates (LC and LL) we are checking whether it is necessary to include the intergenerational dynamics which are imposed in the CoDa models. The CoDa models were developed into a multi-population framework but their performance has not been tested in a socio-economic setting.

We find that, out of six different models, CoDa mortality models that model mortality changes for each socio-economic group proportional to a common trend provide the most accurate life expectancy forecasting for Danish males and females. Thus, models allowing for multiple trends and an individual treatment of the socio-economic groups are less suitable when forecasting, indicating a high degree of homogeneity among the socio-economic specific mortality trends. Further, models based on death rates (the LC and LL models) provided in general the lowest life expectancy forecasts and less accurate forecasts compared to the CoDa models assuming a common trend. Hence, these models were not able to capture the mortality development observed in a developed country with shifting mortality patterns. By measuring life expectancy forecasts at the statutory retirement age from 2016 to 2030 we find large socio-economic differences in the number of pensionable years. These differences are expected to persist until 2030.

2 Danish pension system and socio-economic groups

In 2007, Denmark implemented a pension scheme that gradually increases the pension age in line with the increase in life expectancy, targeting receipt of pension for 14.5 years. The scheme was implemented to finance an increasing number of retirees from large birth cohorts while also taking into account increasing life time in the entire population. The exact rules are complicated but basically the pension age will be increased if life expectancy exceeds 14.5 years at the statutory retirement age. Increases are notified 15 years ahead assuming a 0.6 years increase in the current life expectancy with a ceiling of maximum one year increase in the pension age over a five year period (Finansministeriet, 2017). Hence, the pension age will increase if life expectancy increases regardless of the population subgroups experiencing mortality improvements. Widening life expectancy differentials across socio-economic groups will therefore not only imply larger inequality in life span but also a larger difference in the number of years people can expect to receive a public pension. For example, if the highest socio-economic group is experiencing a decline in mortality and the other groups experience no change, the pension age will increase leaving the lower socio-economic groups with fewer expected years with a pension. Analysing and forecasting life expectancy across socio-economic status is therefore highly relevant when studying the distributional consequences of a pension system.

Socio-economic status is, in this study, based on an individual's income and wealth and not education status, as is more traditional, because we want to capture mortality trends that measure the underlying life span inequality in the population over time. This is not well captured by analysing mortality by education if the population experiences large changes in the educational level of the population as well as compositional chances over recent decades (Brønnum-Hansen and Baadsgaard, 2012). Mortality trends by education thus include a selection effect where in particular people with very low or no education consist of a small selected group in addition to mortality differences caused by different health conditions (Colardyn and Baltzer, 2008). This selection effect is referred to as the downward bias in mortality from education (Hendi, 2015).

Using income and wealth for measuring socio-economic status, groups of approximately equal size can be found because income and wealth are continuous variables, constituting an individual ranking basis. Time consistent mortality trends can thereby be calculated. Cairns et al. (2019) show that it is important to consider both income and wealth as both high income and high wealth are associated with low mortality. An individual person can be well off with a medium level of income if he enjoys a sufficiently high accumulation of wealth. The contrary is also true for low income or low wealth (Cairns et al., 2019). We delve more into the details of socio-economic classification in the next section.

2.1 Data for Danish socio–economic groups

The Danish population was, for each sex, divided into five (almost) equally large socio-economic groups based on an affluence index following the procedure suggested by Cairns et al. (2019), as it is found to produce a consistent and relevant classification of socio-economic groups in relation to life expectancy in each sub-group. Cairns et al. (2019) define socio-economic groups weighting individual gross annual income with a factor of 15, compared to their net wealth, that is $A = W + K \times Y$, where A is the affluence index, W is net wealth, $K = 15$ is the weighting factor, and Y is gross annual income. Sensitivity tests for the K value are provided in Cairns et al. (2019). Gross annual income consists of all incomes subject to tax such as wages, income from self-employment, income from social benefits and pension-related incomes (Cairns et al., 2019). Net wealth is the difference between total assets such as real estate, bonds, deposits etc. and total liabilities such as mortgage, financial loans, unpaid tax, etc. Cairns et al. (2019) find that migration between socio-economic groups should be allowed until age 67 after which the groups are fixed. Age 67 was selected because it was the statutory retirement age for most of the data period until it was reduced to 65 years in 2004. Even though data are from the central Danish register, income and wealth are missing for a very small fraction of the population these missing values were imputed by an average of the income and wealth, respectively, for each individual.

Individual information about income, wealth and marital status was obtained from the Danish central registers by merging information from the Population Register and the Income and Tax Register. Data are only available from 1985 because reliable information about income at an individual level does not exist for the whole study population prior to 1985. Further details about the socio-economic measure and data can be found in Cairns et al. (2019). This study considers only five socio-economic groups so that each group is of a large enough size, enabling the models we consider in the next section to be estimated.

The socio-economic data are available from age 50 and grouped at 100+. Mortality is measured by the death rates (m_x), life table deaths distributions (d_x), or life expectancy (e_x), all calculated using standard life table techniques following Preston et al. (2001). All variables are measured by single age $x = (50, ..., X)$, single year $t = 1985, ..., T$, and $g = (1, ..., G)$ is used to denote a socio-economic group constituting a sub-population. The life table death distributions, $d_{t,x}$ are ungrouped when fitting the CoDa models to avoid a problem of artificial compression in the life table forecasts (Bergeron-Boucher et al., 2017a). We use a penalized composite link model for ungrouping as suggested by Rizzi et al. (2015) and ungroup until age 105. Ungrouping to age 105 gives a smooth and realistically downward sloping $d_{t,x}$ at higher ages and only affects the life expectancy of the first age group minimally. [1]

The socio-economic groups are labelled G1 to G5 with G1 having the lowest income and wealth and G5 the highest.

Table 1: Summary table of Danish life expectancies

Year	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
	Males at 50 years					Females at 50 years				
1985	20.70	24.22	26.17	27.21	27.83	25.92	28.84	30.96	31.73	32.98
1995	22.03	24.63	25.98	27.30	28.66	27.56	28.41	29.69	30.65	31.99
2005	24.65	26.10	28.53	30.04	31.77	29.54	29.94	31.81	33.08	34.21
2015	26.97	27.90	30.56	32.32	33.92	31.40	32.02	33.43	34.97	36.43
	Males at 65 years					Females at 65 years				
1985	11.27	13.55	14.67	15.22	15.72	14.43	17.93	18.77	19.03	19.87
1995	12.09	13.42	14.17	15.00	15.99	15.95	16.63	17.50	17.98	19.04
2005	14.17	14.79	15.98	16.90	18.27	17.70	17.73	18.68	19.70	20.74
2015	16.04	16.54	17.75	18.79	20.06	19.29	19.50	20.09	21.32	22.33

Other studies have, similar to Cairns et al. (2019), used income to define socio-economic groups, for example: Tarkiainen et al. (2012). Tarkiainen et al. (2012) used taxable income (similar to the income variable used in this

[1] As a sensitivity analysis we compared life expectancies when socio-economic groups were defined using disposable income instead of gross annual income. Using disposable income produces similar mortality trends but smaller gaps between the lowest socio-economic groups for both males and females. Given that the main objective of this study is to model and forecast mortality by socio-economic groups, it is beyond the scope of this paper to analyse the definition of socio-economic group in further detail. Further sensitivity tests of the affluence index procedure used can be found in Cairns et al. (2019) for Danish males.

study) and divide the Finnish population into five groups. For the period 1988 to 2007 widening life expectancy differentials are found for the lowest socio-economic group compared with the others but differentials remain constant between the other groups in the Finnish population. Mortality trends by socio-economic groups, similar to those presented in this study, are found for Danish males and females by Brønnum-Hansen and Baadsgaard (2012). A steep increase for the lowest socio-economic group during the first part of the data period are found by Brønnum-Hansen and Baadsgaard (2012), suggesting that the large improvements for the lowest group are due to changing conditions in the labour market and the flexibility model of the Danish labour market. The socio-economic classification used by Brønnum-Hansen and Baadsgaard (2012) differs from the one suggested by Cairns et al. (2019) by using disposable income, missing inclusion of any wealth measure, and by the number of groups as Brønnum-Hansen and Baadsgaard (2012) divide the Danish population into four groups.

3 Methods

The LC model is included in the analysis as a benchmark model and compared to the proposed CoDa models. For comparison, all models use similar notation for the time and age index.

3.1 The Lee-Carter model (LC)

The LC model is a single population model and when applied to sub-populations treats them independently. The model decomposes age-specific mortality rates $m_{t,x,g}$ by Singular Value Decomposition (SVD), using only the first rank, after having subtracted the average level of mortality. That is,

$$log(m_{t,x,g}) = \alpha_{x,g} + \beta_{x,g} k_{t,g} + \epsilon_{t,x,g}, \tag{1}$$

where $\alpha_{x,g} = \frac{1}{T}\sum_t log(m_{t,x,g})$ is the time arithmetic average measuring the general mortality age pattern, $k_{t,g}$ an index over time of the general level of mortality, $\beta_{x,g}$ age-specific responses to the index, and $\epsilon_{t,x,g}$ the iid. error term. Mortality forecasts are calculated with the LC model by extrapolating $k_{t,g}$ using an ARIMA model to produce mortality rate forecasts.

3.2 The Li-Lee model (LL)

The CoDa models are also compared with the multi-population extension of the LC model, the Li-Lee (LL) model. The LL model (Li and Lee, 2005) estimates a common factor term for the average population by applying an LC model to the average death rates. Next, the common factor is subtracted from each socio-economic sub-population and SVD is used to decompose deviations by sub-population specific age and time parameters. The

LL thereby extends the LC model by modelling dependence among the sub-populations. The LL model can be written as,

$$log(m_{t,x,g}) = \alpha_{x,g} + B_x K_t + \beta_{x,g} k_{t,g} + \epsilon_{t,x,g}, \tag{2}$$

where K_t is an index of the general level of mortality for the average population, B_x is the age specific response to changes in the index, and $\epsilon_{t,x,g}$ the iid. error term. The group-specific parameters are interpreted similar to the parameters in the LC model but relative to changes in the common factor term. Following Li and Lee (2005), we assume that $k_{t,g}$ follow an AR(1) model which is used to calculate forecasts and changes in sub-population specific mortality thus converge to the national level described by the common factor.

3.3 Compositional Data Methods

We present and analyse four different CoDa models and forecast mortality for each sub-population. Three of the models have already been used to forecast mortality in different settings whereas the Dynamic Factor Coda model is proposed for the first time. The CoDa models differ from the traditional LC modelling by using life table death distributions instead of death rates and by applying compositional data analysis techniques to introduce redistribution of deaths.

Compositional data are defined as a composition with only positive entries summing to a fixed constant and life table deaths are densities summing to 1 in each year, if rescaled to the life table radix, and therefore only contain relative information. Aitchison (1986) showed that traditional decomposition methods, for example (SVD), do not apply to compositional data as data coordinates cannot vary freely but are constrained to vary between 0 and a constant. Instead, it is necessary to transform the data so it can vary freely and back-transform after the decomposition has been carried out (Pawlowsky-Glahn and Buccianti, 2011).

The covariance structure in compositional data is utilised when analysing life table deaths with CoDa. For example, a decreasing number of deaths at young ages implies that more deaths occur at older ages. Thus, the use of life table deaths has an advantage, compared to models that use mortality rates, as the shifting and compression process observed in mortality data is modelled directly.

The analysis presented in this paper uses the centered log-ratio transformation (clr) transformation which is the log ratio of the composition divided by its geometric mean (g_t). That is,

$$clr(d_{t,x}) = ln\left(\frac{d_{t,x}}{g_t}\right). \tag{3}$$

All the models except the Inde-CoDa and the LC models account for dependence among sub-populations but differ in their assumptions on the nature of dependence. The Dynam-CoDa model restricts the age dimension but allows for different mortality time trends and by including the Dynam-CoDa model in the analysis we explore whether flexibility in the time dimension is important when forecasting mortality by socio-economic groups. The Rela-CoDa model constraint both age and time dimensions to a national trend for all sub-populations by modelling deviations from a national common trend. The 3D-CoDa also restricts the sub-groups to follow the same common age and time factors but instead of modelling the residuals, as with the Rela-CoDa model, a third dimension, related to the population-specific pace of mortality, is introduced. Summing up, we analyse whether dependence among socio-economic groups is most useful for forecasting when it is introduced in the time dimension (Dynam-CoDa), by a common national trend (Rela-CoDa), or by the structure in the age and time dimensions (3D-CoDa). It is important to analyse different levels of flexibility in order to determine the most suitable forecasting model. A very complex model might provide a very accurate fit of the observed mortality but could lack ability to forecast mortality due to bias-variance trade off. By analysing several models that provide different forecast methods we test for alternative ways that mortality by socio-economic status can be related. Further, because models using either death rates or life table deaths are included we explore which of these two indicators is most suitable when forecasting life expectancy for different socio-economic groups.

All the CoDa models ensure that deaths not occurring at a certain age are moved to another age where they are likely to occur. All models capture level differentials by calculating sub-population specific means ($a_{x,g}$) but mortality improvement differentials are captured differently in the models; in Dynam-CoDa, g mortality time indexes measuring time changes ($k_{t,g}$); in Rela-CoDa by both age specific dynamics ($b_{x,g}$) and $k_{t,g}$; and in 3D-CoDa by an additional third dimensional parameter vector related to population-specific pace of mortality.

3.4 Independent (Inde-CoDa)

The simplest way to analyse socio-economic sub-populations is to treat each sub-population independently, similar to the LC model but in a CoDa setting. Using CoDa, that means applying the model suggested by Oeppen (2008) to each sub-population. The Inde-CoDa model centres life table deaths for each sub-population ($d_{t,x,g}$) by differencing out the age-specific geometric mean. The model uses the operator $\ominus$ which is the subtraction operator in CoDa. The centred $d_{t,x,g}$ are approximated by SVD. That is,

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = b^1_{x,g}k^1_{t,g} + ... + b^p_{x,g}k^p_{t,g} + \epsilon_{t,x,g}, \quad (4)$$

where $k^p_{t,g}$ is an index representing the overall mortality development over time for rank-p approximation, $b^p_{x,g}$ the age-specific response changes in $k^p_{t,g}$, and $\epsilon_{t,x,g}$ the iid. error term. The changes in mortality are thereby decomposed into an age and time dimension. β_x displays how deaths are redistributed in the forecasts, fulfilling the restriction that the total number of deaths needs to be maintained. For positive k_t values, which is the case for all forecast years, deaths are redistributed from ages with negative β_x values towards those with positive β_x values. Mortality forecasts are calculated by extrapolating $k^p_{t,g}$ using ARIMA models. Estimates and forecasts of $d_{t,x,g}$ are transformed back using the inverse of the clr transformation and estimates of $\alpha_{x,g}$ are added. The inverse procedure of *clr* ensures that the initial life table constraint is fulfilled so deaths sum to the radix in each year. For more details see Oeppen (2008). See Figure 1 for graphical representation.

3.5 Dynamic Factor Coda model (Dynam-CoDa)

One way to incorporate dependence among sub-populations is to estimate k_t time trends for each sub-group and forecast these in a system estimating dependence among time trends but use the same β_x all sub-groups. To do so, we propose to stack the life table deaths for each sub-population vertically and perform an analysis similar to the Inde-CoDa. A matrix of size $(T \cdot G) \times X$ of life table deaths is first centred and transformed and from an SVD, g group-specific time parameter vectors and one age parameter vector are calculated. That is,

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = b_x^1 k_{t,g}^1 + ... + b_x^p k_{t,g}^p + \epsilon_{t,x,g} \tag{5}$$

The Dynam-CoDa model is thereby assuming g time indexes describing the time dimension and one age parameter vector common to all sub-populations measuring the age dimension, for each rank approximation. $\epsilon_{t,x,g}$ is the iid. error term

If the sub-population mortality trends move together, it is possible that they share common time trends which can be modelled together. We use a multi-level dynamic factor model here to model $k_{t,g}$ jointly. The multi-level dynamic factor model determines a factor common to all sub-groups and factors which are only shared by one or more of the sub-groups. More specifically $k_{t,g}^1$ are factorized using:

$$k_{t,g}^1 = \gamma_g' P_t + \lambda_g' R_t + \epsilon_{t,g}, \qquad g = 1, \ldots, G, \tag{6}$$

where P_t is the vector of factors that pervade all groups, R_t is the vector of factors that pervade only a subset of groups, and γ_g and λ_g are the corresponding loadings. Using the selection method suggested by Hallin and Liska (2007) one P_t factor is estimated for both Danish males and females, two R_t factors for Danish females, and one R_t factors for Danish males. The Dynam-CoDa is thereby incorporating dependence between sub-groups by estimating a common factor for all groups but also factors (R_t) which are shared only by some of the sub-groups. Details about the multi level dynamic factor model are presented in the supplementary material section A.

Forecasts of P_t and R_t are calculated using similar ARIMA selection procedures to the Inde-CoDa models. Finally, $k_{t,g}^1$ forecasts are perturbed on β_x and forecasts of the life table deaths are calculated by back-transforming and centre the life table deaths forecast, similar to the procedure in Inde-CoDa. See Figure 1 for graphical representation.

3.6 Relative model (Rela-CoDa)

The third model forecasts the mortality for each sub-population in relation to the national mortality. The model is a variation of the model suggested by Bergeron-Boucher et al. (2017a).

Rela-CoDa is estimated in two steps. In step one, a simple CoDa mortality model, using rank-1 SVD, is fitted to the national life table deaths and national forecasts of age specific responses and a mortality index are produced. The subscript g is left out in the first step and a superscript N added when denoting the national mortality, $d^N_{t,x}$.

$$clr(d^N_{t,x} \ominus \alpha_x) = B_x K_t + \epsilon_{t,x}, \tag{7}$$

In a second step each sub-population is considered after subtracting the geometrical means of national deaths from the sub-population specific geometrical means using CoDa perturbation. Both the sub-population specific and national life table deaths are first rescaled so each row sums to one. A rank-1 SVD is used to calculate sub-population specific estimates of b_x, k_t, and a $\epsilon_{t,x,g}$ iid. error term. That is,

$$clr(d_{t,x,g} \ominus \alpha_{x,g} \ominus d^N_{t,x}) = b_{x,g} k_{t,g} + \epsilon_{t,x,g}, \tag{8}$$

The Rela-CoDa model is thereby a variation of the Li and Lee (2005) model within a CoDA framework. Forecasts of K_t and $k_{t,g}$ are calculated with an ARIMA model and an ARMA model, respectively, and the specific AR and MA terms selected using the AIC. Rela-CoDa thus assumes stationary $k_{t,g}$ parameters such that the change in mortality will converge for the sub-populations towards the national level, similar to what is normally assumed in the Li and Lee (2005) model. Forecasts of the life table deaths in each sub-population are calculated by the inverse perturbation of the national and sub-population specific mortality indexes including the age specific responses. The inverse perturbation is defined as the element-wise division followed by a rescaling of the variables according to the initial data constraint. Thus, the sub-population specific deaths are measured as the ratio of the national death density. Life table forecasts are calculated as,

$$\tilde{d}_{t,x,g} = \alpha_{x,g} \oplus clr^{-1}(B_x \tilde{K}_t + b_x \tilde{k}_t). \tag{9}$$

where tilde indicates forecasts of the parameter vectors. Rela-CoDa is a variation of the model suggested by Bergeron-Boucher et al. (2017a). The difference between the two models is in step two were Bergeron-Boucher et al. (2017) use $B_x K_t$ estimates from an average population instead of the national life table deaths. Arguably in our approach, more of the national variation is accounted for due to the use of national deaths. See Figure 1 for graphical representation.

3.7 Three dimensional model (3D-CoDa)

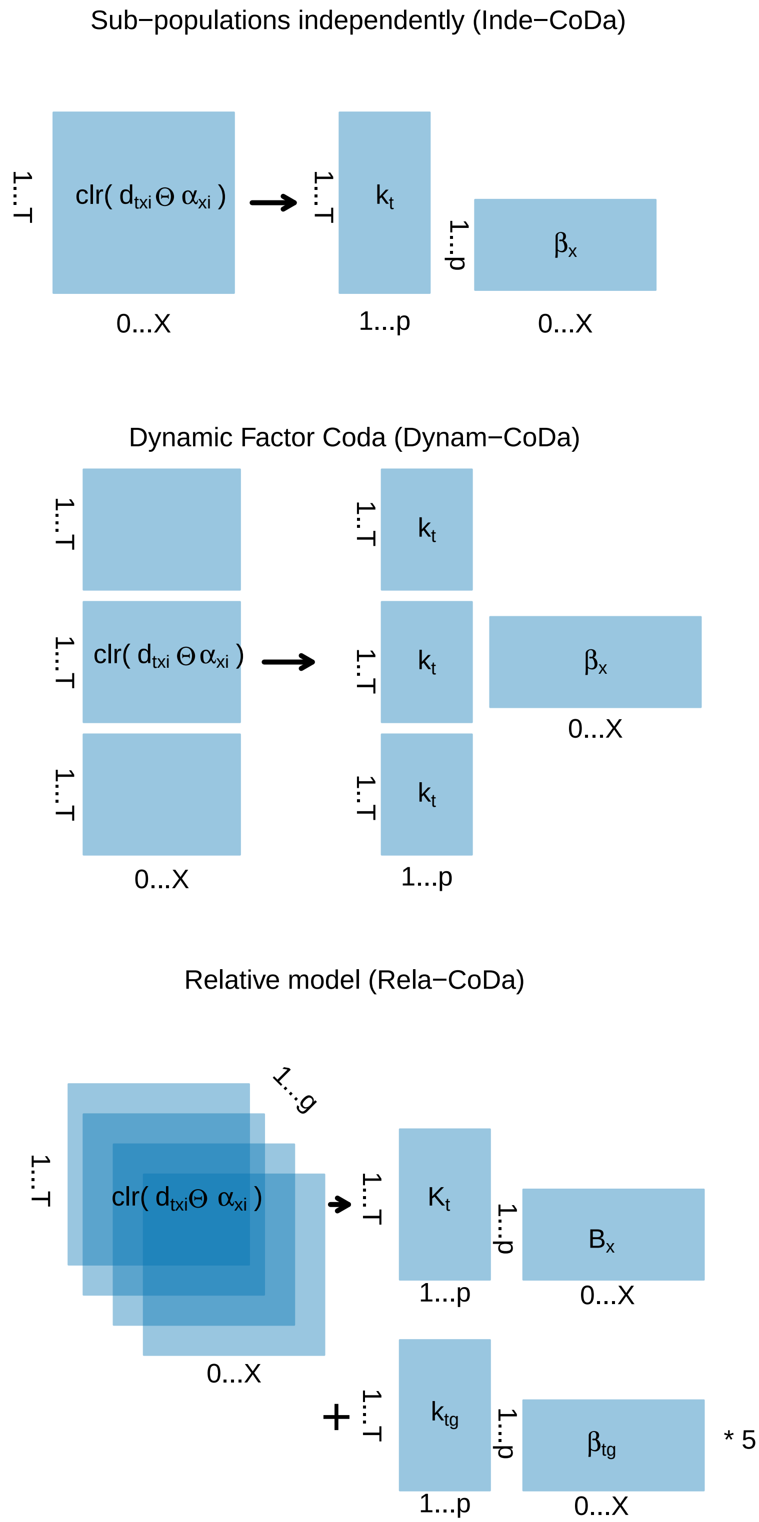
The fourth CoDa model we consider introduces a third dimension to capture sub-population differentials. The model is suggested by Bergeron-Boucher et al. (2018) and applied to Canadian regions. Bergeron-Boucher et al. (2018) suggest, similar to the other CoDa models, first to centre and transform the life table deaths but instead of an SVD approximation, the 3D-CoDa model uses a three-way principal component analysis - the Tucker3 method (Tucker, 1966). The model can be written as,

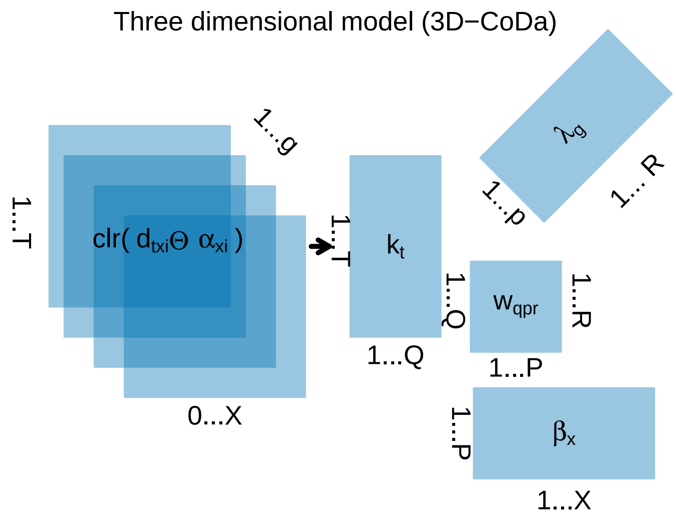
$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{r=1}^{R} w_{qpr}(k_{t,q}\beta_{x,p}\gamma_{g,r}) + \epsilon_{t,x,g}, \tag{10}$$

where w_{qpr} are elements in a weighting array containing weights between the loading matrices $\boldsymbol{\beta}$, $\mathbf{k}$, and $\boldsymbol{\gamma}$

Thus, the model assumes that all sub-populations, for each rank, share the same mortality index $k_{t,g}$ and the same age responses $b_{x,p}$ but that each sub-population experiences changes in mortality at a different pace measured by the parameter vector $\gamma_{g,r}$. $\epsilon_{t,x,g}$ is an iid. error term. A high degree of similarity in the time and age structures of the mortality development in each sub-populations is therefore assumed. In line with the analysis by Bergeron-Boucher et al. (2018) we only consider equal elements of q,p, and r for the 3D-Coda model but unequal elements are analysed with the more general Population Value Decomposition (PVD) which is described in the supplementary material Section C together with comparison results. The PVD model did not provide more accurate forecasts than the 3D-CoDa model. Figure 1 presents a graphical representation of the CoDa models.

Figure 1: Graphical representation of the CoDa models





None of the models include any covariates such as health indexes, GDP, smoking prevalences, or other relevant factors. The main problem of including covariates is that many of these covariates are often harder to forecast than the mortality patterns themselves. Thus, forecasts including covariates are often found to be less reliable when forecasting mortality (Cairns et al., 2011). A few studies have included covariates in the LC model, for example French and O'Hare (2014), but it is beyond the scope of this paper to include covariates in the CoDa mortality model framework. These extensions are left for further research.

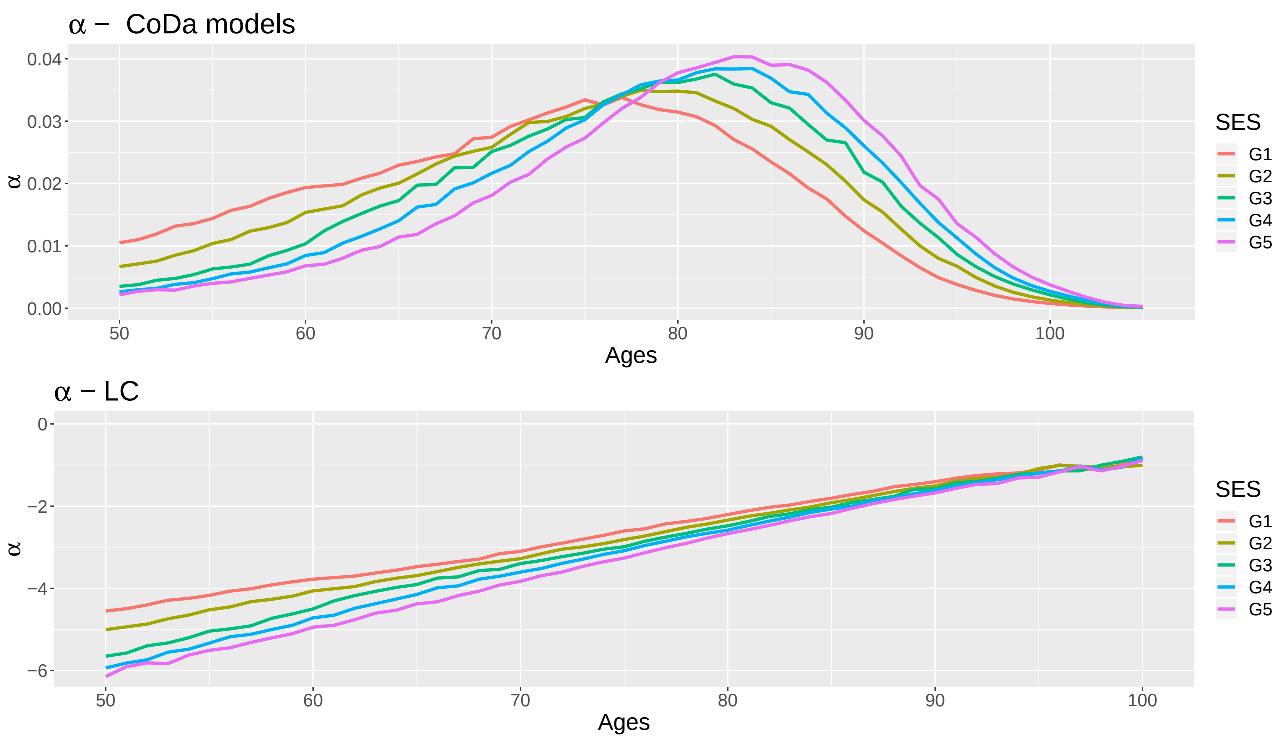
4 Results

4.1 Estimation findings

We show parameter estimates for the first rank SVD in this section, fitting the models to the whole data period for Danish males (Parameter estimates for Danish females are also shown in the supplementary material in Figures A1 to A5.). The first rank accounts for over 65% of the variation in the centred life table deaths for all Danish male socio-economic groups in all models. The Inde-CoDa and 3D-CoDa (p=r=q=2), models are estimated using two ranks and parameter estimates for the second rank are shown in the supplementary material Figures A6 and A7. A rank-1 approximation was found to be sufficient for the Dynam-CoDa model as higher rank approximations did not improve the forecast accuracy for this model. Mortality forecasts are calculated by extrapolating $k^p_{t,g}$ using ARIMA models selected using the Akaike information criterion (AIC) (Akaike, 1974) and the Augmented Dickey Fuller test for determination of the order of integration (Dickey and Fuller, 1979). A random walk with drift is generally used as a suitable model similar to other extrapolative mortality models as for example the LC model (Booth and Tickle, 2008).

A correction for a possible mismatch between observed and fitted mortality rates, in the last observed year, is often applied in the mortality forecasting literature to correct for what is known as the jump-off bias (Lee and Miller, 2001). The correction adds the distance between the fitted and the observed mortality indicator in the final year (death rates or life table deaths) to the forecast and adjusts forecasts so that the jump-off bias is reduced. Hyndman et al. (2013) argue that this correction can introduce a forecast bias and thus a trade-off exists between correcting the jump-off bias and introducing a forecast bias. The analysis presented in this article makes use of the jump-off correction for all models as, because of data limitations, we only make short to medium term forecasts in the out-of-sample comparison. Accurate forecasts in the beginning of the forecast period are therefore important and the jump-off error would dominate the results if not corrected. Similarly, a correction between actual and fitted values is applied for the multi-level dynamic factor model in the Dynam-CoDa model.

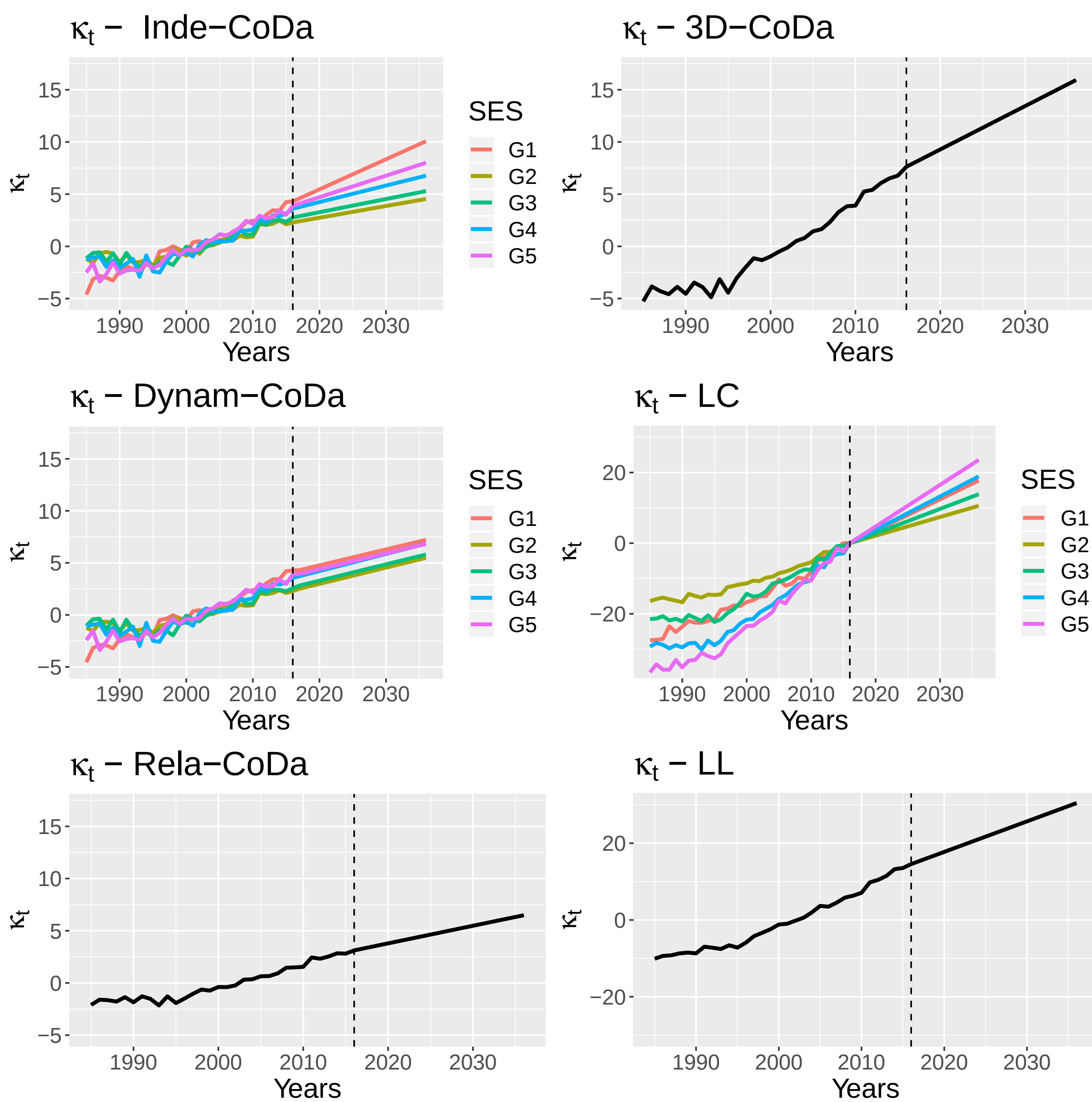
Figure 2: Alpha estimates for the CoDa and LC models for Danish males



Note: α_x estimates are the same for all CoDa models and the same for the LC and LL models. Thus, the estimates are only shown once.

Figure 2 shows estimates for α_x for the CoDa models and for the LC/LL models. α_x for the CoDa models is bell shaped as it describes the general death distribution whereas α_x is increasing for LC/LL as these models use log death rates for modelling, where the rate of mortality is increasing with age. Differences in α_x by socio-economic status follow the ordering of the socio-economic groups with a higher mortality at lower ages for G1 compared to G5. Hence, the part of the mortality differentials between the socio-economic groups that is related to differences in the level of mortality follows the ordering of the socio-economic groups. Further, because α_x is assumed to be stable over time the ordering in the level differentials component will persist in the forecasts (Villegas and Haberman, 2014).

Figure 3: First rank kappa estimates and forecasts using different models for Danish males



Note: The LC and LL parameter estimates are plotted on a different scale compared to the CoDa models for visibility reasons.

Figure 3 shows k_t^1 estimates and forecasts for all the models which capture the overall mortality development over time in an index. The time indexes are increasing for the CoDa models and for LC and the LL models meaning that mortality has been declining over time from 1985 to 2016 for all models. Parameters for the LC and the LL models are plotted on a different scale for visibility reasons. The scale differences are a consequent of the LC and LL models being based on death rates, as with the β_x estimates.

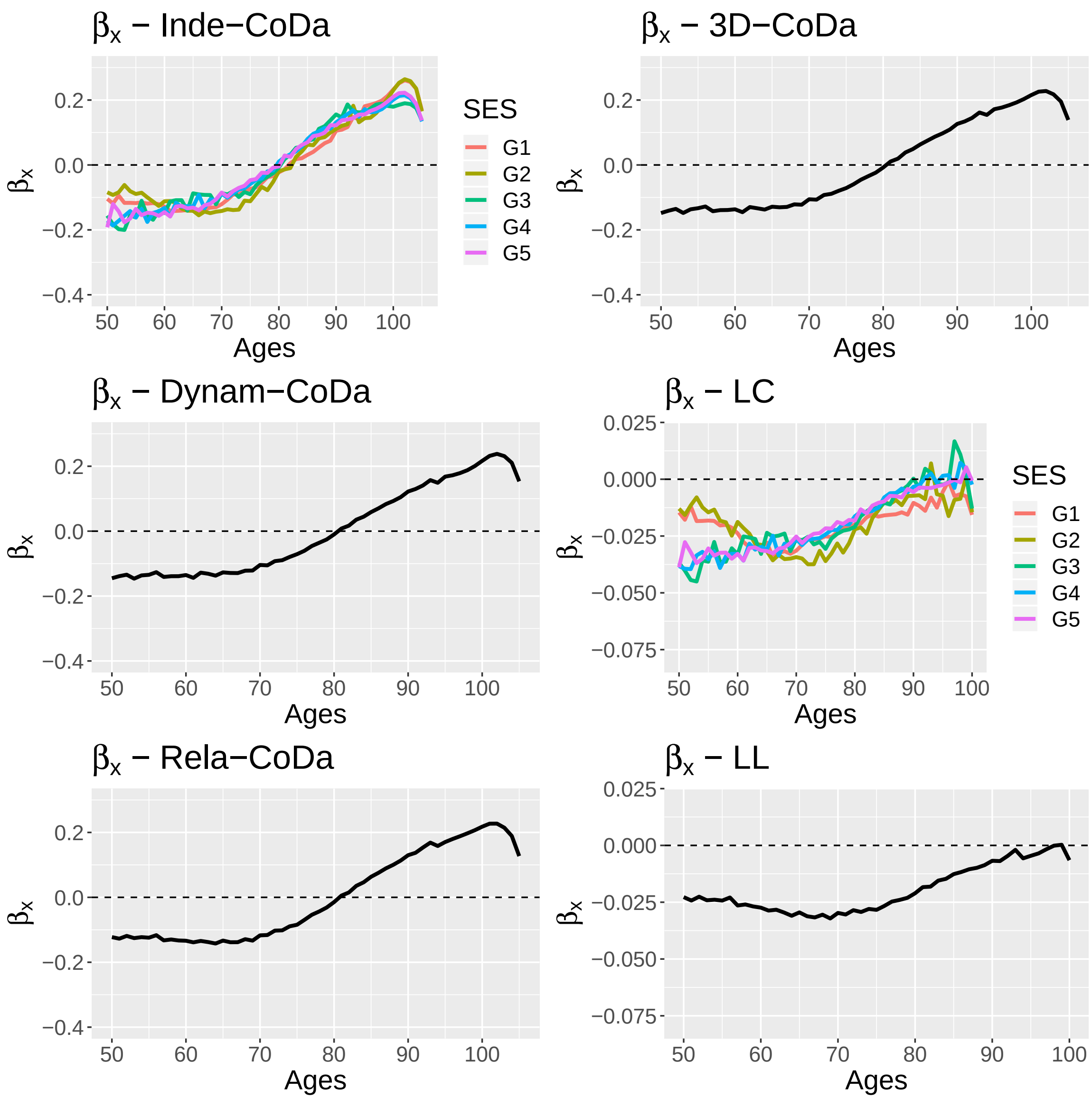
The Inde-CoDa model estimates for k_t show differences across socio-economic groups with the steepest increase for the lowest socio-economic groups meaning that this group experiences the largest improvements in mortality. The second lowest socio-economic group in the Inde-CoDa model is predicted to have the lowest future improvements in mortality. Similar k_t^1 are estimated with the Dynam-CoDa model, but different forecasts are produced when dependence among sub-groups is taken into account by the multi level dynamic factor model. The multi level dynamic factor model estimates a significant common factor and, thus, identifies dependency among the socio-economic groups in the mortality index k_t^1. The five socio-economic groups are predicted to have a similar increase in k_t^1 meaning that they also are predicted to have similar experience progress in mortality improvements. The common factor is thereby dominating the Dynam-CoDa forecasts. The CoDa models with common factor terms, 3D-CoDa and Rela-CoDa, both identify an upward sloping time trend in both estimates and forecasts.

β_x estimates for the models are showed in Figure 4. All the CoDa-models follow a similar pattern which can be described by looking at the β_x estimates from the Inde-CoDa model. Over age, the generally increasing pattern in β_x means that when $k_{t,g}$ is increasing and becomes positive, deaths are shifted from ages with negative β_x towards older ages where β_x is positive. For the Inde-CoDa model β_x is, for G1 and G2, decreasing at ages 50 to 70 years and increasing over age until around age 102 followed by a decrease. For G3 to G5 no decrease is found for ages 50 to 70. G1 and G2 have the highest β_x estimates for the ages 50 to 60 but lowest from age 60 to 95 compared to other groups. The relatively high β_x values at ages 50 to 60 and subsequent low values at ages 60 to 96 for the G1 and G2 groups imply that less deaths are transferred to higher ages for the same shift in $k_{t,g}$ values: meaning that these groups will experience a lower improvement in life expectancy for the same increase in $k_{t,g}$ compared to the other groups.

β_x parameter estimates for models assuming a common β_x vector generally follow patterns observed for the groups G3 to G5. Assuming a common β_x parameter, thus, provides a better fit for these groups than for G1 and G2. Similar patterns are found for the LC/LL models with relative high values for G1 and G2 at ages 50 to 60. Note that these models do not imply transfer of deaths as with the CoDa-models.

Figure 5 shows the group specific parameter estimates for the Rela-Coda and LL models, that is the group specific adjustments to the common trend identified in the Rela-CoDa and LL models. Because the group specific $k_{t,g}$ terms are assumed to be stationary, and thus mean reverting, all socio-economic groups are assumed to follow the same long-run mortality trend dominated by the common K_t term. No particular trending pattern is found for $k_{t,g}$ parameters in the Rela-CoDa model meaning that the stationarity and thereby mean reverting pattern fits the Rela-CoDa model well. The $k_{t,g}$ parameters in the LL model display more upwards and downwards trending patterns making it harder to forecast and less suitable to assume a mean reverting pattern. This makes the Rela-CoDa formulation more attractive for forecasting as the patterns are easier to predict. The different patterns in the Rela-CoDa and the LL models are a consequence of Rela-CoDa subtracting $d_{t,g}^N$ instead of parameter estimates $K_t B_x$ as in the LL model. Further, Coda mortality models have a tendency to implicitly

Figure 4: First rank beta estimates and forecasts using different models for Danish males



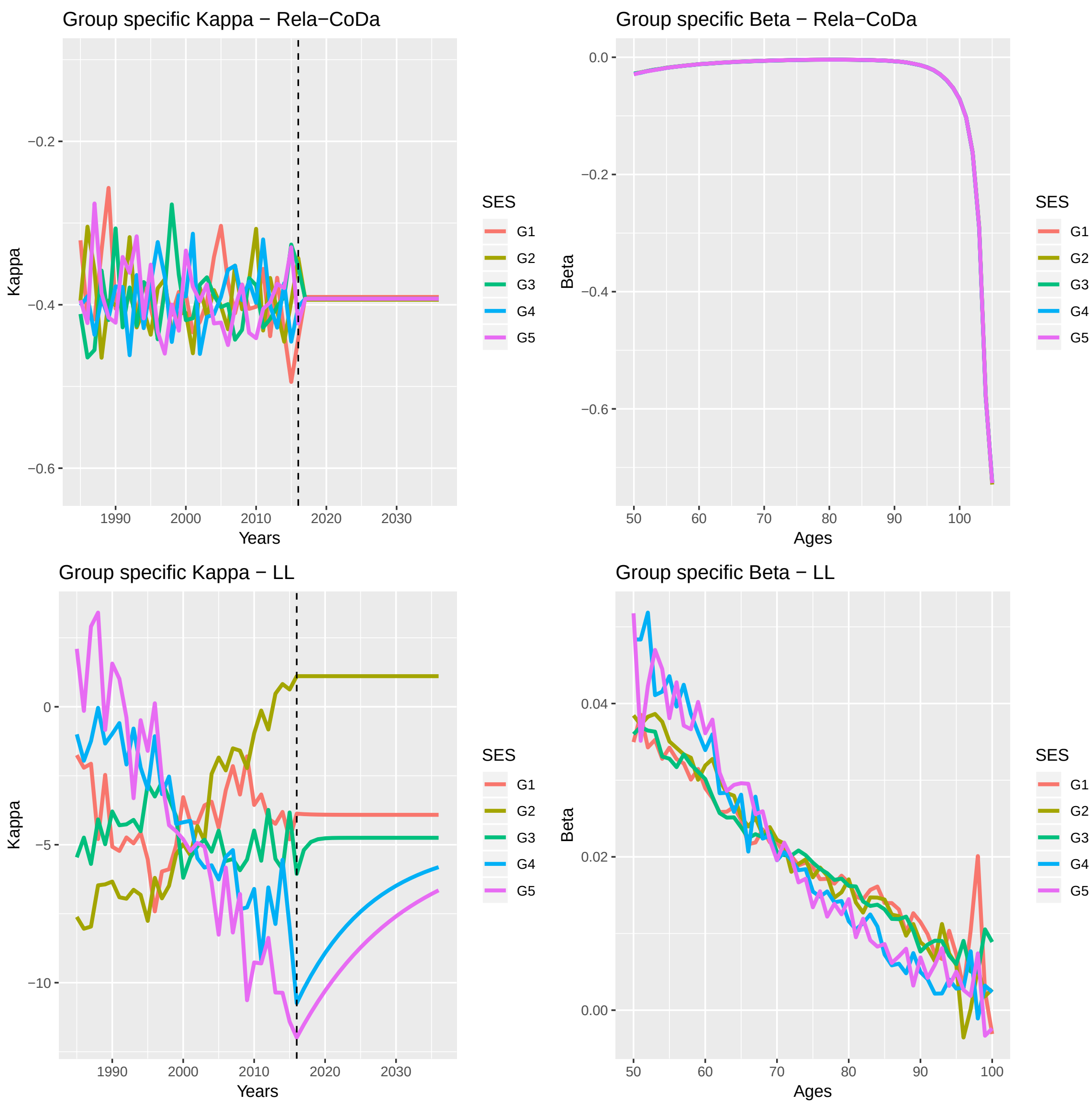
Note: The LC and LL parameter estimates are plotted on a different scale compared to the CoDa models for visibility reasons.

weight older ages more than younger because they are formulated on d_x where intergenerational dependence is modelled though the *clr* transformation (Bergeron-Boucher et al., 2017a). The age- and group-specific age responses $\beta_{x,g}$ are similar for all age groups and describe how each age group responds to changes in $k_{t,g}$.

The γ_g estimates for the 3D-Coda model show how the common factor terms are scaled for each socio-economic group (Figure 6). Higher γ_g estimates are found for the lower socio-economic groups meaning that, in the view of the 3D-Coda model, mortality is decreasing more slowly for those groups compared to higher socio-economic groups. The only exception is G1 where the fastest decline is found.

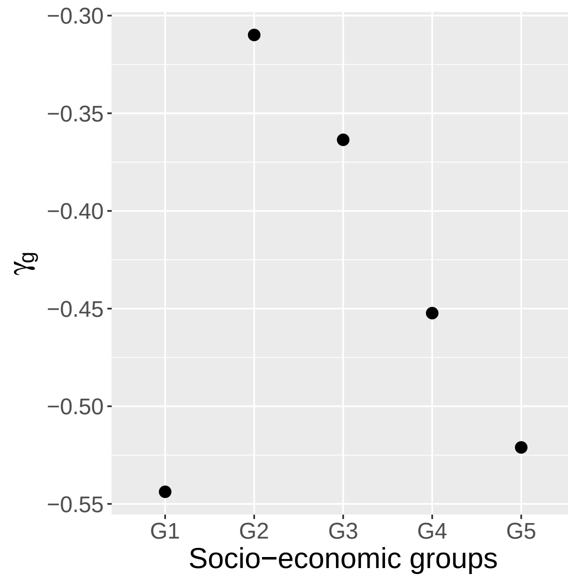
Figures A10 to A14 in the supplementary material show standardized residuals for all the studied models across

Figure 5: Group specific kappa and beta estimates for the LL and Rela-CoDa models for Danish males



all socio-economic groups. None of the residuals shows any particular pattern and thus we do not include specific cohort terms in the models. Cohort terms can be included in the models but cannot be identified uniquely because of the exact relation between age, calender year and cohort birth year (cohort = calender year - age). Thus a specific form needs to be assumed for identification and a linear identification form is often used for mortality models: for example by Haberman and Renshaw (2009).

Figure 6: Gamma estimates for Danish males using the 3D-CoDa model

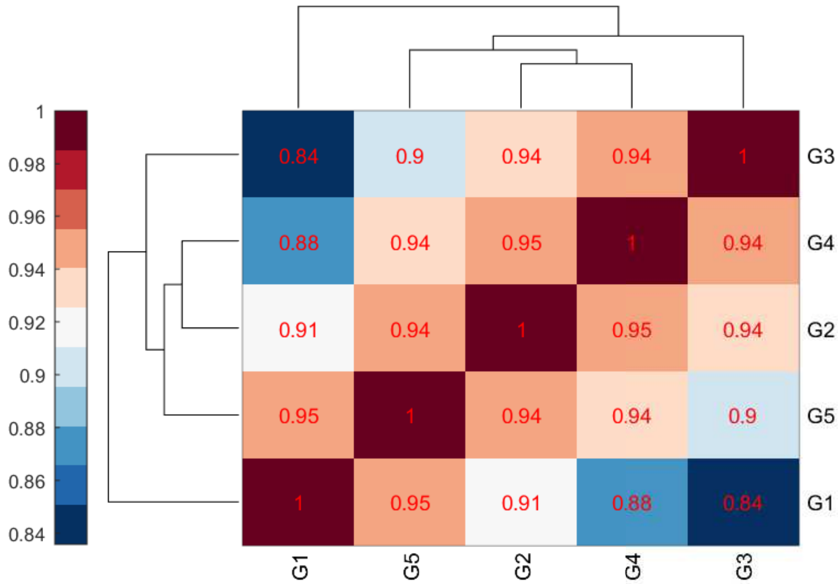


5 Cluster analysis of the main mortality trends

To illustrate the high degree of similarity between the mortality time trends observed for each socio-economic groups we provide a cluster analysis of $k_{t,g}$ obtained from the Dynam-Coda model where multiple time trends are allowed. In Figure 7, we present the cluster analysis for males and results for females are shown in the supplementary material Figure A9. The cluster analysis is performed using the *Clustergram* algorithm in MATLAB.

The cluster analysis presented in Figure 7 shows a high correlation between $k_{t,g}$ for each group with correlations above 0.84 for all groups. The group G1 is least related to the other group with the lowest correlation coefficient relative to the other groups. The G1 also clusters in its own subgroup as indicated by solid lines in Figure 7 meaning that G2 to G5 have a similar pattern whereas G1 to some degree follows its own pattern. More specifically G2 to G5 cluster in one group with G2 and G4 being most similar and G1 in its own cluster. These clustering pattern do have implications for the different forecasting models as models with one common k_t will fit the mortality time trends better for G2 to G5 compared to G1. This is also seen in the results in Table 1 where more accurate forecasts are found for G1 with Inde-CoDa using an independent fit compared to Rela-CoDa and 3D-CoDa which assume one common trend. Contrary, more accurate forecasts are found for the other groups using models with a common trend. Because $k_{t,g}$ were subject to unit roots of order one, as argued in Section 3.4, we also test for co-integration between the variables to check whether the correlations are spurious. Using the Johansen trace test (Johansen, 1991) co-integration is found between the variables suggesting that equilibrium relationships exist and hence the correlations are real. Results of the co-integration analysis are presented in supplementary material Section B.

Figure 7: Cluster analysis of the main mortality time trend ($k_{t,g}$) for Danish males



5.1 Out-of-sample comparison and selection of forecasting model

To determine which mortality model is most suitable for forecasting mortality we compare the out-of-sample forecast performance of the different models. Data are available from 1985 to 2016. To have a sufficient number of years for fitting the models, we consider forecasts with a length of 5 to 11 years calculated by rolling the onset of the forecasts. That is, for the first forecast we use a fitting period from 1985 to 2005 and the period from 2005 to 2016 for validation. For the second forecast, one year is added to the fitting period by reducing the validation period by one year. From this a minimum of 2/3 of the data period is used to fit the models and 1/3 for validation. Forecast errors are measured by the root-mean-square error (RMSE) comparing observed and forecast life expectancy at age 50 which is the youngest age group in the data set,

$$RMSE = \sqrt{\frac{\sum_{h=1}^{H}(e_{h,50} - \tilde{e}_{h,50})^2}{H}} \tag{11}$$

where $h \in (1, 2..., H)$ is the number of forecast years and e_{50} the observed life expectancy at age 50 and $\tilde{e}_{50}$ the corresponding forecast. The average RMSE is calculated by averaging over the different forecast horizons and used to compare the models. We use life expectancy at age for comparison as it is calculated using mortality information for all the considered age groups. Alternatively one could evaluate at a higher age which would exclude data at younger age-groups. A good in-sample fit can be achieved by introducing a high number of parameters but this will not guarantee a good forecast. Because forecasting is the objective of this paper, only the out-of-sample performance of the models is considered when selecting the most suitable model.

RMSE's are shown in Table 2 and 3 for Danish males and females, respectively. In-sample fit measures can be found in the supplementary material Section F and show that the models which fit each sub-group independently fit more accurately. This is a consequence of the higher number of parameters.

Table 2: RMSE of e_{50}, average over 7 forecast horizons for Danish males

	SES1	SES2	SES3	SES4	SES5	Avr.
Inde-CoDA	**0.4511**	0.8159	0.8970	0.8657	0.5136	0.7087
Dynam-CoDa	0.7190	0.6016	0.7286	0.8333	0.6419	0.7049
Rela-CoDa	0.7088	**0.1980**	**0.3615**	0.5626	0.5884	**0.4838**
3D-CoDa	0.7200	0.5040	0.4641	**0.5117**	**0.4110**	0.5221
LC	0.6637	0.5284	0.5487	0.6442	0.4878	0.5746
LL	0.7459	0.3212	0.5617	0.8514	0.9750	0.6910

Note: Lowest RMSE forecast error is indicated with bold font

Table 3: RMSE of e_{50}, average over 7 forecast horizons for Danish females

	SES1	SES2	SES3	SES4	SES5	Avr.
Indt-CoDA	0.3243	0.8037	0.8056	0.8551	1.2521	0.8081
Dynam-CoDa	0.8396	0.8327	0.7752	0.9767	0.9456	0.8740
Rela-CoDa	0.6475	**0.6366**	**0.5778**	**0.7384**	0.8240	0.6848
3D-CoDa	**0.2797**	0.7305	0.7581	0.8308	**0.7846**	**0.6767**
LC	0.3460	0.8033	0.7827	0.8298	0.9123	0.7348
LL	0.7968	0.6709	0.6927	0.8805	0.9745	0.8030

Note: Lowest RMSE forecast error is indicated with bold font

For the Danish males, the Rela-CoDa, 3D-Coda, and Inde-CoDa models provide the lowest forecast errors for the different socio-economic groups. For the Danish females, the Rela-CoDa and 3D-CoDa models are the most accurate. The Rela-CoDa model provides the lowest forecast error on average for all the groups for Danish males and thus we conclude that this model is the most accurate model for forecasting mortality by socio-economic status. Similarly, the 3D-CoDa provides the lowest forecast error for females, but the accuracy is just slightly higher for the Rela-CoDa model. A good fit for the Rela-CoDa and 3D-CoDa models indicates a high degree of homogeneity in the Danish mortality trends across socio-economic status and a model based on a national trend is thus useful when forecasting. Further, it also shows the usefulness of modelling the dependence between the sub-group specific mortality trends. As the Dynam-CoDa model did not provide better forecasts, in general, it does not seem relevant to allow for different patterns in the time trends. It is sufficient to have the same time and age pattern for each group and adjust by modelling the residuals from a national pattern as in the Rela-CoDa model or by allowing for a different pace as in the 3D-CoDa model. A model based on death rates (LC or LL) did not provide the lowest forecast error for any of the groups. Hence, better forecasts are obtained for Danish mortality data using life table deaths when forecasting life expectancy. [1]

Figure 8: Observed life expectancy (Obs) and 20 years life expectancy forecasts using different models for Danish males

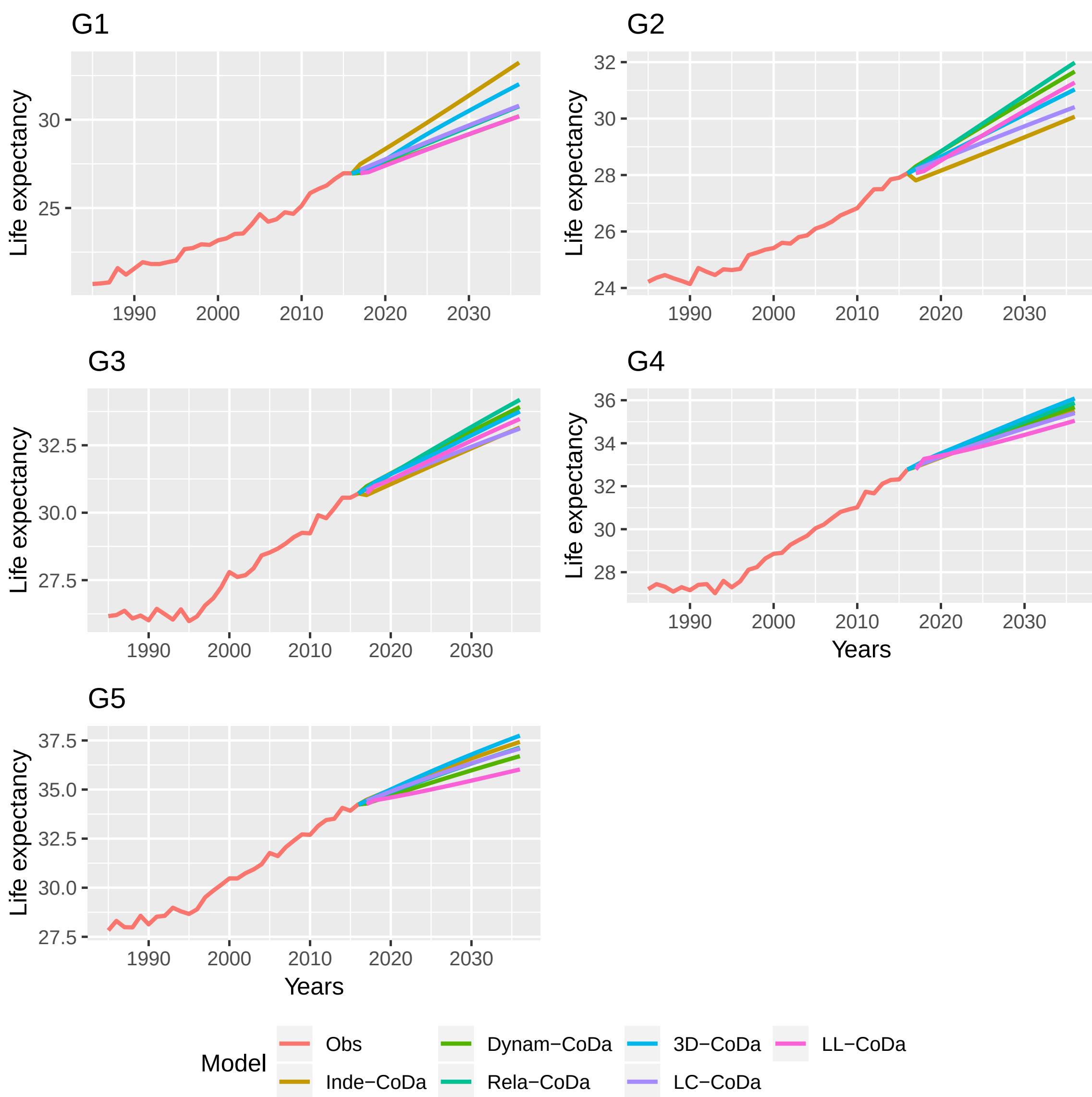


Figure 8 shows 20 year life expectancy forecasts for Danish males at age 50 using the different models. The models show relatively large differences between the forecasts. Forecasts from the different models are most aligned for G3 and G4 whereas larger differences are found for the other socio-economic groups. Hence, the selection of model is relatively more important for these socio-economic groups.

The Rela-CoDa model has, compared to the other models, an optimistic life expectancy forecast for G2 and G3

[1] We test the significance of forecast performance differences of the seven models using a Clark-West test (Clark and West, 2006) in the medium horizon. Details about the Clark-West test are presented in the supplementary material Section D. We test the forecast differences for each socio-economic group towards the best performing model for each sex and find that a large majority of the forecasts are significantly different. Results of the Clark-West test are presented in the supplementary material Table A3.

but intermediate forecasts for G1, G4, and G5. This is a consequence of the common time trend in the Rela-CoDa model forcing the socio-economic groups towards the same common time trend. Groups with a mortality trend which is increasing faster than the common trend, as G5, will tend to have a lower life expectancy forecast compared to the Inde-CoDa model and the opposite for groups with a mortality trend increase below the common trend. As anticipated in the theoretical comparison, the LC and LL forecasts are among the lowest for most of the socio-economic groups; only the LL model gives a medium life expectancy forecast for G2. The low life expectancy forecasts are a consequence of the assumption that $\beta_{x,g}$ is constant over time and in contrast to the CoDa models no dynamics are modifying the assumption (Bergeron-Boucher et al., 2017a). The downward bias in LC and LL models are not necessarily that pronounced for short forecasts horizons but it is easy to detect in a 20 year forecast as in Figure 8.

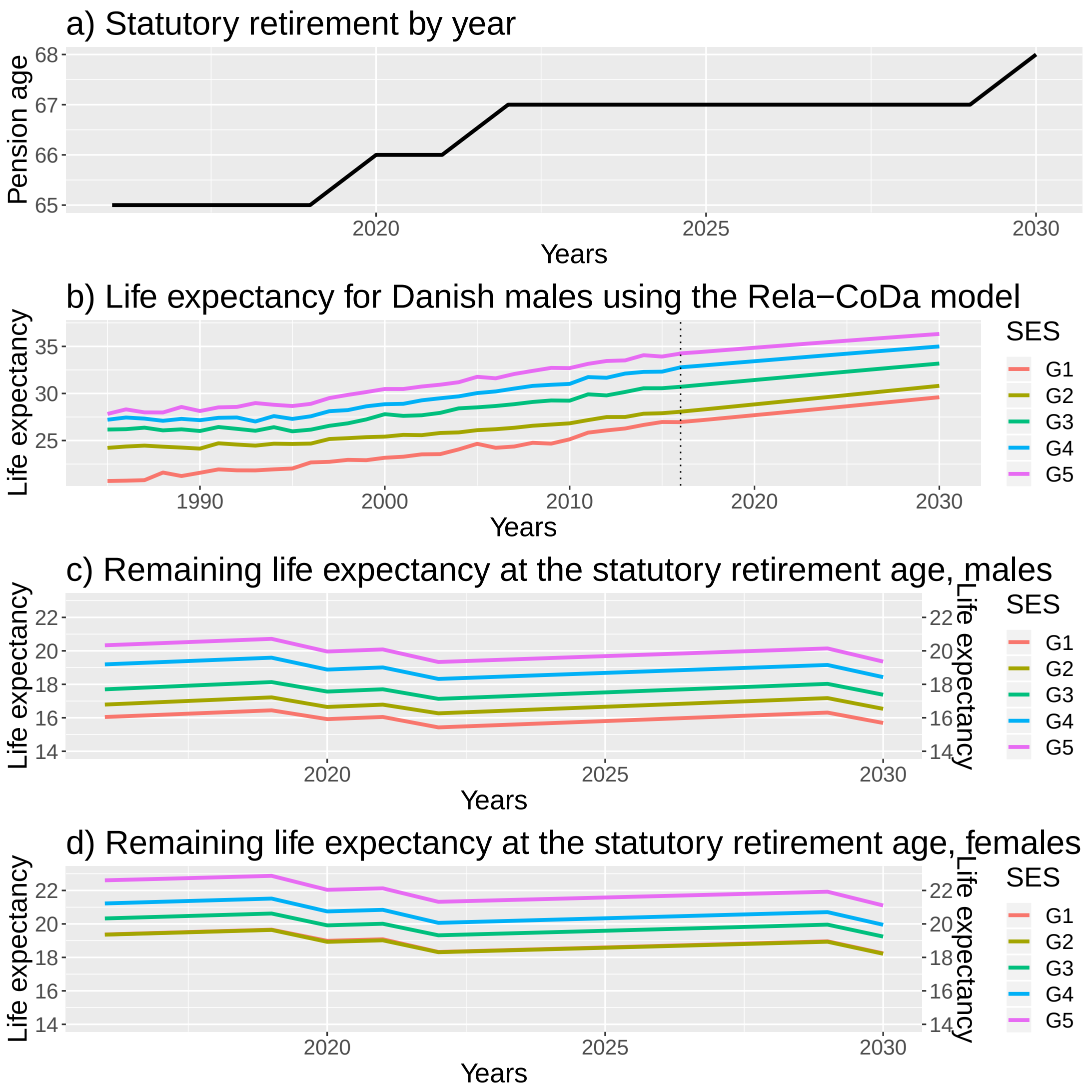
6 Implication for the pension age and its developments

Having identified the CoDa models with a common trend as the most suitable models for forecasting life expectancy for Danish socio-economic groups, these forecasts and remaining life expectancy at the pension age are reported for both Danish males and females in Figure 9c and 9d until 2030. Life expectancy forecasts are calculated using the Rela-CoDa model for both sexes as this model provided the lowest RMSE forecast error for males and just slightly higher forecast errors for females compared to the 3D-CoDa model. This ensures coherence in the analysis of the consequences for the pension system as the same model is used for both sexes. Figure 9a shows the statutory retirement age in Denmark until 2030 and finally, Figure 9b shows life expectancy forecasts for Danish males at age 50 for completeness.

At age 50 life expectancy for the lowest socio-economic group is converging towards the other groups at a decreasing pace from 1985 to 2026. Mortality differentials for the other groups stay roughly the same during the data period. Similar trends are observed for Danish females and shown in the supplementary material.

In Figure 9c, Danish males in the lowest socio-economic group would have a remaining life expectancy at pension age of around 16 years in 2016 which is 4.5 years less than the highest group. The other groups fall between with remaining life expectancy from around 17 to 19 years in 2016. Danish males are, for all groups, forecast to have slightly falling life expectancy at pension age meaning that pension age is expected to increase faster than life expectancy until 2030. For Danish females in Figure 9d, G1 and G2 have the same remaining life expectancy at pension age at around 19 years in 2016 and the similarity remains in the forecast. The other socio-economic groups follow with the highest life expectancy for G5 at around 23 years. All groups, for both males and females, will have a life expectancy more than 14.5 years, which is the desired long term goal in the current pension scheme, despite the social inequality of pensionable years. Thus, all socio-economic groups can expect to receive public pension in more years than the political desired number of years if they retire at the statutory retirement age.

Figure 9: Threshold ages for statutory retirement age and remaining life expectancy at the statutory retirement age using the Rela-CoDa model for forecasting



Note: the vertical dotted line in panel b) indicates the start of the forecast.

Future mortality differential by socio-economic groups are highly relevant for determining the consequences of the current pension system in Denmark. Life expectancy forecasts show that relatively large differentials are expected 14 years ahead between socio-economic groups. As a strong common mortality trend was found across socio-economic groups, increases in the pension age are predicted to have a similar effect across socio-economic groups. No particular group is expected to drive future changes in the total life expectancy. Another implication of the strong common trend is that mortality differentials between the groups persist.

7 Concluding remarks

This study provides life expectancy forecasts for the Danish population by socio-economic status. Socio-economic status was measured with an affluence index constructed by weighting income and wealth. Two models using death rates and five models using life table deaths were compared and, based on the models' out-of-sample performance, the Rela-CoDa and 3D-CoDa models were found to most accurately forecast life expectancy for males and females, respectively. Both models use a common trend for all sub-groups to forecast mortality. The six models differed by their ways of including dependence among the socio-economic groups: the Inde-CoDa and LC models treat the socio-economic groups independently, the LL and Rela-CoDa in relation to a common mortality level, the Dynam-CoDa model by relating multiple mortality trends, and the 3D-CoDa by scaling common age and time structures. That the Rela-CoDa and 3D-CoDa models provided the most accurate forecast indicates the existence of a high degree of homogeneity in the mortality trends between the Danish socio-economic groups, so common k_t and b_x parameters could be assumed for all socio-economic groups when forecasting. Despite the similarity in the trend by which mortality changes, large mortality differentials were observed throughout the whole data period. Models formulated on death rates did not provide the most accurate forecast indicating that these models did not capture changes in life expectancy in the validation period.

As several countries in the OECD have linked changes in their pension system to changes in life expectancy for the whole population, the consequences of future mortality differentials are even larger today than before. Several studies measure socio-economic status by education, e.g. Jasilionis and Shkolnikov (2016). Education is problematic because of compositional changes in the educational levels where a smaller and smaller fraction of a birth cohort has limited or no education. This leads to a downward bias in the mortality differentials across educational groups because the lower educated group becomes more and more selected.

The relatively large mortality differentials also constitute a distributional issue because the lower socio-economic groups have less private pension and lower capital income from saving making them more dependent on the public pension (Pensionskommissionen, 2015). The lower groups are thereby affected more when the statutory retirement age is increased because they do not have sufficient wealth to retire before the statutory retirement age. Forecasts of the mortality differentials are not only relevant from an individual and public point of view but also for private pension companies that could experience a mismatch in the composition of socio-economic groups between their insured population and the national population. Forecasts of the mortality differential could be used to inform pension companies about their actual longevity risk when taking differences in mortality by socio-economic status into account.

References

ACFS 2018. Melbourne mercer global pension index 2018 (mmgpi), Mercer and the Australian Centre for Financial Services (ACFS), Melbourne.

Aitchison, J. 1986. *The Statistical Analysis of Compositional Data.* Chapman & Hall, Ltd.

Akaike, H. 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6):716–723.

Bergeron-Boucher, M.-P., Canudas-Romo, V., Oeppen, J., and Vaupel, J. W. 2017a. Coherent forecasts of mortality with compositional data analysis. *Demographic Research*, 37(17):527–566.

Bergeron-Boucher, M.-P., Kjærgaard, S., Oeppen, J., and Vaupel, J. W. 2017b. What to forecast? an exploratory study of the implication of using different indicators to forecast mortality. In *IUSSP 2017 International Population Conference*.

Bergeron-Boucher, M.-P., Simonacci, V., Oeppen, J., and Gallo, M. 2018. Coherent Modeling and Forecasting of Mortality Patterns for Subpopulations Using Multiway Analysis of Compositions: An Application to Canadian Provinces and Territories. *North American Actuarial Journal*, 22(1):92–118.

Booth, H. 2006. Demographic forecasting, 1980 to 2005 in review. *International Journal of Forecasting*, 22(3):547 – 581. Twenty five years of forecasting.

Booth, H. and Tickle, L. 2008. Mortality modelling and forecasting: a review of methods. *Annals of Actuarial Science*, 3(1-2):3 – 43.

Brønnum-Hansen, H. and Baadsgaard, M. 2012. Widening social inequality in life expectancy in Denmark. A register-based study on social composition and mortality trends for the Danish population. *BMC Public Health*, 12(1):994.

Cairns, A., Blake, D., Dowd, K., D.Coughlan, G., Epstein, D., and Khalaf-Allah, M. 2011. Mortality density forecasts: An analysis of six stochastic mortality models. *Insurance: Mathematics and Economics*, 48(3):355 – 367.

Cairns, A. J., Kallestrup-Lamb, M., Rosenskjold, C. P. T., Blake, D., and Dowd, K. 2019. Modelling Socio-Economic Differences in the Mortality of Danish Males Using a New Affluence Index. *ASTIN Bulletin*.

Cairns, A. J. G., Blake, D., and Dowd, K. 2006. A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration. *Journal of Risk and Insurance*, 73:687 – 718.

Cairns, A. J. G., Blake, D., Down, K., Coughland, G. D., Epstein, D., Ong, A., and Balevich, I. 2009. A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. *North American Actuarial Journal*, 13:1–35.

Clark, T. E. and West, K. D. 2006. Using out-of-sample mean squared prediction errors to test the martingale difference hypothesis. *Journal of Econometrics*, 135(1):155–186.

Colardyn, D. and Baltzer, K. 2008. Case study: Denmark, in teaching, learning and assessment for adults: Improving foundation skills. *OECD Publishing*.

Dickey, D. A. and Fuller, W. A. 1979. Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association*, 74(366):427–431.

Enchev, V., Kleinow, T., and Cairns, A. J. G. 2017. Multi-population mortality models: fitting, forecasting and comparisons. *Scandinavian Actuarial Journal*, 2017(4):319–342.

Evans, W. N. and Moore, T. J. 2012. Liquidity, economic activity, and mortality. *The Review of Economics and Statistics*, 94(2):400–418.

Finansministeriet 2017. Det danske pensionssystem nu og i fremtiden, The Danish Ministry of Finance, Copenhagen.

French, D. and O'Hare, C. 2014. Forecasting death rates using exogenous determinants. *Journal of Forecasting*, 33(8):640 – 650.

Haberman, S. and Renshaw, A. E. 2009. On age-period-cohort parametric mortality rate projections. *Insurance: Mathematics and Economics*, 45(2):255–270.

Hallin, M. and Liska, R. 2007. Determining the Number of Factors in the General Dynamic Factor Model. *Journal of the American Statistical Association*, 102(478):603–617.

Hendi, A. S. 2015. Trends in U.S. life expectancy gradients: the role of changing educational composition. *International Journal of Epidemiology*, 44(3):946–955.

Hyndman, R. J., Booth, H., and Yasmeen, F. 2013. Coherent Mortality Forecasting: The Product-Ratio Method With Functional Time Series Models. *Demography*, 50:261–283.

Jarner, S. F., Kryger, E. M., and Dengsøe, C. 2008. The evolution of death rates and life expectancy in Denmark. *Scandinavian Actuarial Journal*, 2008(2-3):147–173.

Jasilionis, D. and Shkolnikov, V. 2016. Longevity and education: A demographic perspective. *Gerontology*, 62(3):253–262.

Johansen, S. 1991. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, 59(6):1551–1580.

Kjærgaard, S., Ergemen, Y. E., Kallestrup-Lamb, M., Oeppen, J., and Lindahl-Jacobsen, R. 2019. Forecasting Causes of Death using Compositional Data Analysis: the Case of Cancer Deaths. *Forthcoming in Journal of the Royal Statistical Society: Series C.*

Lee, R. D. and Carter, L. R. 1992. Modeling and Forecasting U. S. Mortality. *Journal of the American Statistical Association*, 87(419):659–671.

Lee, R. D. and Miller, T. 2001. Evaluating the performance of the Lee-Carter method for forecasting mortality. *Demography*, 38(4):537–549.

Li, N. and Lee, R. D. 2005. Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. *Demography*, 42:575–594.

Mackenbach, J. P., Bos, V., Andersen, O., Cardano, M., Costa, G., Harding, S., Reid, A., Hemström, Ö., Valkonen, T., and Kunst, A. E. 2003. Widening socioeconomic inequalities in mortality in six Western European countries. *International Journal of Epidemiology*, 32(5):830–837.

Meslé, F. and Vallin, J. 2006. Diverging Trends in Female Old-Age Mortality: The United States and the Netherlandsversus France and Japan. *Population and Development Review*, 32(1):123–145.

OECD 2012. *OECD Pensions Outlook 2012, OECD Publishing.*

OECD 2017. *Pensions at a Glance 2017, OECD Publishing.*

Oeppen, J. 2008. Coherent forecasting of multiple-decrement life tables: a test using Japanese cause of death data. In *European Population Conference 2008, European Association for Population Studies.*

Pawlowsky-Glahn, V. and Buccianti, A. 2011. *Compositional Data Analysis: Theory and Applications.* John Wiley and Sons, Ltd.

Pensionskommissionen 2015. The Danish pension system - internationally praised but not without problems (Det danske pensionssystem - international anerkendt, men ikke problemfrit). Technical report, Pensionskommissionen, Copenhagen.

Preston, S., Heuveline, P., and Guillot, M. 2001. *Demography, Measuring and Modeling Population Processes.* Oxford: Blackwell Publishers.

Rizzi, S., Gampe, J., and Eilers, P. H. C. 2015. Efficient estimation of smooth distributions from coarsely grouped data. *American Journal of Epidemiology*, 182(2):138–147.

Snyder, S. E. and Evans, W. N. 2006. The effect of income on mortality: Evidence from the social security notch. *The Review of Economics and Statistics*, 88(3):482–495.

Tarkiainen, L., Martikainen, P., Laaksonen, M., and Valkonen, T. 2012. Trends in life expectancy by income from 1988 to 2007: decomposition by age and cause of death. *Journal of Epidemiology & Community Health*, 66(7):573–578.

Tucker, L. R. 1966. Some mathematical notes on three-mode factor analysis. *Psychometrika*, 31(3):279–311.

Villegas, A. M. and Haberman, S. 2014. On the Modeling and Forecasting of Socioeconomic Mortality Differentials: An Application to Deprivation and Mortality in England. *North American Actuarial Journal*, 18(1):168–193.

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