DEPARTMENT OF ECONOMICS AND BUSINESS ECONOMICS
AARHUS UNIVERSITY

CREATES
Center for Research in Econometric Analysis of Time Series

Estimating the Price Markup in the New Keynesian Model

Martin M. Andreasen and Mads Dang

CREATES Research Paper 2019-3

Department of Economics and Business Economics
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 8716 5515

Estimating the Price Markup in the New Keynesian Model*

Martin M. Andreasen†
Aarhus University, CREATES, and The Danish Finance Institute

Mads Dang‡
Aarhus University and CREATES

March 1, 2019

Abstract

This paper shows that the price demand elasticity can be estimated reliably in a standard log-linearized version of the New Keynesian model when including firm profit as an observable in the estimation. Using this identification strategy for the post-war US economy, we find an estimated price demand elasticity of 2.58 with a tight standard error of 0.31. This corresponds to an average price markup of 63% with a 95% confidence interval of $[39\%, 88\%]$. We also show that a calibrated markup of 20%, as commonly used in the literature, is rejected by the data, because it generates too much variability in firm profit.

Keywords: Aggregate supply curve, Identification, Likelihood inference, New Keynesian model, Price markup.

JEL: C10, E12.

*We thank Torben M. Andersen, Jesús Fernandez-Villaverde, Jens Iversen, Svend Erik Hougaard Jensen, Giovanni Pellegrino, and Johannes Pfeifer for useful comments and discussions. Martin M. Andreasen acknowledges funding from the Independent Research Fund Denmark, project number 7024-00020B.

†Fuglesangs Allé 4, 8210 Aarhus V, Denmark, email: mandreasen@econ.au.dk, telephone +45 87165982.

‡Fuglesangs Allé 4, 8210 Aarhus V, Denmark, e-mail: madsdang@econ.au.dk, telephone +45 87166017.

1 Introduction

An essential feature of the New Keynesian model is to accommodate monopolistic competition in the goods market, because it gives firms market power and hence an ability to set prices. The degree of market power is most often determined by the substitution elasticity between competing goods, which therefore becomes a key parameter for controlling firms' markups and pricing decisions in the New Keynesian model. This implies that the demand elasticity is an essential parameter for determining the slope of the aggregate supply curve when price stickiness is specified as in Rotemberg (1982). Unfortunately, the demand elasticity is not well-identified when the New Keynesian model is estimated using a standard set of aggregate quantities, inflation, and some measure of the monetary policy rate. Hence, this parameter is almost exclusively calibrated in the literature, both when using a log-linear solution and higher order perturbation approximations (see Ireland (2001), Smets and Wouters (2007), Rudebusch and Swanson (2012), Christiano et al. (2014), Fernández-Villaverde et al. (2015) among others).

The contribution of the present paper is to show that the demand elasticity can be estimated reliably in a standard log-linearized version of the New Keynesian model when the variability of firm profit is included in the estimation. Note that our use of a log-linear solution constitutes a conservative choice, as more accurate approximations include higher-order terms that further help to identify structural parameters (see An and Schorfheide (2007) and Ruge-Murcia (2012)). The economic mechanism behind our identification result is as follows. Suppose the demand elasticity between goods in the economy is low and firms therefore have high market power and face steep demand curves. This allows firms to charge high markups and generate stable profits, because price changes lead to relatively small changes in demand and hence profits. On the other hand, firm profits are more volatile when the demand elasticity is high and firms face relatively flat demand curves, because even small changes in prices lead to large changes in demand and hence profits. That is, the variability in firm profit contains information about the demand elasticity. We show in a Monte Carlo study that using firm profit enables us to accurately estimate the demand elasticity in samples of the same length as typically used in empirical applications. Here, we follow the common practice in the literature and estimate the New Keynesian model around its trend by using detrended measures of aggregate quantities and firm profit.[1]

Given this new identification strategy for the demand elasticity, an empirical application estimates the New Keynesian model by maximum likelihood on post-war US data from 1960

[1] This also implies that we do not rely on the level of firm profit to identify the demand elasticity, although the level contains information about this parameter, because a low demand elasticity generates a high level of firm profit, and vice versa.

Q1 to 2007 Q4. Our main finding is an estimated demand elasticity of 2.58 with a tight standard error of 0.31. This fairly low estimate of the demand elasticity implies a high average markup of 63%. In contrast, most New Keynesian models use a calibrated markup of 20%, but we show that such a low markup (i.e. high demand elasticity) is rejected by the data because it generates too much variability in firm profit. A low demand elasticity is also consistent with recent micro evidence, as Loecker and Eeckhout (2017) show in a recent NBER working paper that the US economy has experienced a gradual increase in the average markup from about 20% in 1980 to around 70% in 2014. Thus, the new identification strategy proposed in this paper suggests that a substantially lower benchmark value for the demand elasticity should be used in the New Keynesian model.

The paper is organized as follows. Section 2 presents the considered New Keynesian model, while Section 3 provides analytical identification results for the demand elasticity in a log-linearized version of the model. We illustrate these results numerically in Section 4 for likelihood inference using the Kalman filter. Section 5 presents our empirical application for the post-war US economy, while Section 6 concludes.

2 A New Keynesian Model

This section presents a fairly standard New Keynesian model with monopolistic competition in the goods market, nominal price rigidities as in Rotemberg (1982), endogenous labor and capital supply, and quadratic investment adjustment costs. Note that our choice of specifying nominal price rigidities as in Rotemberg (1982) is equivalent to the approach taken in Calvo (1983) when using a log-linear solution to the model without trend inflation (see Keen and Wang (2007)). We proceed by presenting the decision problem of the households and the firms in Section 2.1 and 2.2, respectively. The behavior of the government is outlined in Section 2.3, while aggregation is performed in Section 2.4 and the model solution is stated in Section 2.5.

2.1 Households

We consider a representative household with preferences for consumption c_t and labor l_t, i.e.

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left\{ \frac{c_{t+s}^{1-\omega}}{1-\omega} - \psi n_{t+s} \frac{l_{t+s}^{1+\upsilon}}{1+\upsilon} \right\}. \tag{1}$$

Here, $\mathbb{E}_t$ is the conditional expectation operator given information in period t, β is the household's subjective discount factor, and υ is the inverse of the Frisch labor supply elasticity.

The variable d_t captures preference shocks which evolve as $\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}$, where $\varepsilon_{d,t}$ is normally and independently distributed across time with zero mean and unit variance, denoted $\varepsilon_{d,t} \sim \mathcal{NID}\left(0,1\right)$. The variable n_t represents labor supply shocks and evolves as $\log n_t = \rho_n \log n_{t-1} + \sigma_n \varepsilon_{n,t}$ with $\varepsilon_{n,t} \sim \mathcal{NID}\left(0,1\right)$.

The household's budget constraint is given by

$$c_t + i_t + b_t = w_t l_t + r_{k,t} k_{t-1} + b_{t-1} \frac{R_{t-1}}{\pi_t} - T_t + f_t. \tag{2}$$

That is, the household's wealth is allocated to consumption, investment i_t, and one-period government bonds b_t. The right-hand side of (2) describes the household's wealth and consists of real labor income $w_t l_t$, income from capital supplied to firms $r_{k,t} k_{t-1}$, government bond holdings $b_{t-1} \frac{R_{t-1}}{\pi_t}$, lump-sum taxes T_t, and profits received from firms f_t. Here, $r_{k,t}$ is the rate of return on supplied capital, R_{t-1} the gross nominal interest rate on a one-period government bond in period $t-1$, and π_t is the gross inflation rate.

The capital stock k_t evolves according to

$$k_t = \left(1-\delta\right) k_{t-1} + \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) i_t, \tag{3}$$

where $\delta \geq 0$ is the depreciation rate and $\kappa \geq 0$ controls investment adjustment costs.

The objective of the representative household is to maximize (1) with respect to c_t, b_t, l_t, i_t, and k_t subject to (2) and (3).

2.2 Firms

Production has the standard two-layered structure. That is, the final output y_t is produced by a perfectly competitive representative firm, which combines a continuum of differentiated intermediate goods $y_{i,t}$ indexed by $i \in [0,1]$. This is done using the production function $y_t = \left(\int_0^1 y_{i,t}^{(\epsilon_p - 1)/\epsilon_p} di\right)^{\epsilon_p/(\epsilon_p - 1)}$, where $\epsilon_p > 1$ describes the demand elasticity for the ith good and is the main focus of the present paper. The demand for the ith good is therefore given by $y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_p} y_t$, where $P_t \equiv \left(\int_0^1 P_{i,t}^{1-\eta} di\right)^{\frac{1}{1-\eta}}$ denotes the aggregate price level and $P_{i,t}$ is the price of the ith good.

Intermediate firms produce slightly differentiated goods using the production function $y_{i,t} = a_t k_{i,t-1}^{\alpha} l_{i,t}^{1-\alpha}$ with $\alpha \in [0,1]$, where $k_{i,t-1}$ and $l_{i,t}$ are the rented capital and hired labor by firm i, respectively. The variable a_t denotes stationary productivity shocks, i.e. $\log a_t = \rho_a \log a_{t-1} + \sigma_a \varepsilon_{a,t}$ with $\varepsilon_{a,t} \sim \mathcal{NID}(0,1)$. Each intermediate firm can freely adjust its labor demand at the given market wage w_t and is therefore able to meet demand in every period. Price stickiness is introduced as in Rotemberg (1982), where $\phi_p \geq 0$ controls the size of firms' real cost $AC^p_{i,t} = \frac{\phi_p}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - \pi_{ss}\right)^2 y_{i,t}$ when changing the nominal price $P_{i,t}$ of the good they produce. Here, π_{ss} is the inflation level in the deterministic steady state (ss). The objective of the ith intermediate firm is to maximize the discounted sum of all future profits with respect to $k_{i,t-1}$, $l_{i,t}$, and $P_{i,t}$, subject to satisfying demand as given by $y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_p} y_t$.

2.3 The Government

The government consists of a monetary and fiscal authority. The monetary authority sets the nominal interest rate based on a desire to smooth changes in the interest rate as well as closing the inflation gap $\log(\pi_t/\pi_{ss})$ and the output gap $\log(y_t/y_{ss})$. Thus, the considered Taylor rule is given by

$$\log\left(\frac{R_t}{R_{ss}}\right) = \phi_R \log\left(\frac{R_{t-1}}{R_{ss}}\right) + (1-\phi_R)\left[\gamma_\pi \log\left(\frac{\pi_t}{\pi_{ss}}\right) + \gamma_y \log\left(\frac{y_t}{y_{ss}}\right)\right] + \sigma_R \varepsilon_{R,t},$$

where $\varepsilon_{R,t} \sim \mathcal{NID}(0,1)$ captures monetary policy shocks.

The government expenditures G_t are specified through the ratio $g_t \equiv G_t/y_t$, where we assume that

$$\log g_t = \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t},$$

with $\varepsilon_{g,t} \sim \mathcal{NID}(0,1)$ capturing fiscal policy shocks. All government expenditures are financed by lump-sum taxes, implying that $T_t = G_t$.

2.4 Model Aggregation

In equilibrium, all intermediate good producing firms face the same decision problem and therefore set the same prices, implying that $P_{i,t} = P_t$. Hence, the aggregate profit from firms to households is given by

$$f_t = y_t - w_t l_t - r_{k,t} k_{t-1} - \frac{\phi_p}{2}(\pi_t - \pi_{ss})^2 y_t. \tag{4}$$

Combining (4), the budget constraint in (2), and the fact that bonds are in zero net supply, we obtain the aggregate resource constraint

$$c_t + i_t + g_t y_t = y_t - \frac{\phi_p}{2}(\pi_t - \pi_{ss})^2 y_t. \tag{5}$$

2.5 Model Solution

To describe the model solution, let $\mathbf{x}_t$ contain all the state variables, and let $\mathbf{y}_t$ contain the control variables in the model. Collecting all the structural parameters in the vector $\boldsymbol{\theta}$, the exact solution is given by (see Schmitt-Grohé and Uribe (2004))

$$\begin{aligned}\mathbf{y}_t &= \mathbf{g}\left(\mathbf{x}_t;\boldsymbol{\theta}\right)\\ \mathbf{x}_{t+1} &= \mathbf{h}\left(\mathbf{x}_t;\boldsymbol{\theta}\right)+\boldsymbol{\eta}\boldsymbol{\varepsilon}_{t+1},\end{aligned}$$

where the matrix $\boldsymbol{\eta}$ contains the standard deviations to the structural shocks in $\boldsymbol{\varepsilon}_{t+1}$. The $\mathbf{g}$- and $\mathbf{h}$-functions are determined by the equilibrium conditions and the rational expectation formation in the New Keynesian model and are rarely available in closed form. We therefore follow the existing literature and consider a standard log-linearized solution, implying that

$$\begin{aligned}\mathbf{y}_t &= \mathbf{y}_{ss}\left(\boldsymbol{\theta}\right)+\mathbf{g}_{\mathbf{x}}\left(\boldsymbol{\theta}\right)\left(\mathbf{x}_t-\mathbf{x}_{ss}\left(\boldsymbol{\theta}\right)\right) \qquad (6)\\ \mathbf{x}_{t+1} &= \mathbf{x}_{ss}\left(\boldsymbol{\theta}\right)+\mathbf{h}_{\mathbf{x}}\left(\boldsymbol{\theta}\right)\left(\mathbf{x}_t-\mathbf{x}_{ss}\left(\boldsymbol{\theta}\right)\right)+\boldsymbol{\eta}\boldsymbol{\varepsilon}_{t+1}.\end{aligned}$$

3 Identification of the Demand Elasticity

The demand elasticity for intermediate goods ϵ_p controls the average price markup $\epsilon_p/\left(\epsilon_p-1\right)$ and is a key parameter for determining the slope of the aggregate supply curve in the New Keynesian model. Unfortunately, ϵ_p is hard to identify and this parameter is therefore nearly always calibrated in the literature (see Smets and Wouters (2007), Christiano et al. (2014), Fernández-Villaverde et al. (2015) among others). This section presents a new identification strategy for ϵ_p that enables us to estimate this parameter reliably in a standard log-linear solution. We proceed by presenting overall identification conditions for estimating the structural parameters in Section 3.1 and 3.2, which we use in Section 3.3 to understand why ϵ_p is only weakly identified in the New Keynesian model. Our new identification strategy for the demand elasticity is then presented in Section 3.4

3.1 A Sufficient Identification Condition

We study identification of the structural parameters $\boldsymbol{\theta}$ within the general class of extremum estimators, which are estimators that maximize some scalar objective function $Q_T\left(\boldsymbol{\theta}\right)$ with respect to $\boldsymbol{\theta}$, subject to $\boldsymbol{\theta}$ being in the parameter space $\boldsymbol{\Theta}$ (see Hayashi (2000)). The subscript T on the objective function indicates that its value depends on the considered sample of length T. It is easy to show that extremum estimators are sufficiently general to include some of the most widely used estimators for the New Keynesian model, such as generalized method of moments (GMM), maximum likelihood, and matching impulse response functions.

To state the identification condition for the extremum estimator, let $\boldsymbol{\theta}_o$ denote the true value of $\boldsymbol{\theta}$. Also, let $Q_o(\boldsymbol{\theta})$ refer to the population value of the objective function, which is the objective function in an infinitely long sample, i.e. $Q_o(\boldsymbol{\theta}) = \lim_{T\to\infty} Q_T(\boldsymbol{\theta})$. A sufficient condition for identification of $\boldsymbol{\theta}$ by the extremum estimator is (see Hayashi (2000)):

Condition 1 *$Q_o(\boldsymbol{\theta})$ is uniquely maximized on $\boldsymbol{\Theta}$ at $\boldsymbol{\theta}_o \in \boldsymbol{\Theta}$.*

That is, identification of $\boldsymbol{\theta}$ by the extremum estimator requires that the objective function in the population only has one maximum, and that this optimum is at the true value of the structural parameters. To fully understand this requirement, let us consider cases where Condition 1 does not hold. One obvious violation of Condition 1 is when the New Keynesian model is misspecified and $Q_o(\boldsymbol{\theta})$ attains a unique optimum at a different point than $\boldsymbol{\theta}_o$. Another violation of Condition 1 is when $Q_o(\boldsymbol{\theta})$ has several optima. This may happen when a subset of $\boldsymbol{\theta}$ does not affect the model solution and the objective function therefore is unaffected by this subset of $\boldsymbol{\theta}$. For instance, if only the ratio of two parameters are identified, meaning that all combinations of the two parameters with the same ratio give the same model solution and hence the same objective function. Another situation which may generate infinitely many optima in $Q_o(\boldsymbol{\theta})$ is when a subset of $\boldsymbol{\theta}$ affects the model solution but not the objective function. In the case of maximum likelihood, this situation may happen when the considered score function simply is uninformative about this subset of $\boldsymbol{\theta}$, for instance because too few variables are included in the estimation.

3.2 A Necessary Identification Condition

It follows from Section 3.1 that a necessary condition for identification of the structural parameters is that $\boldsymbol{\theta}$ affects the model solution. This section formalizes this requirement for a standard log-linear solution, which we use below to analyze identification of ϵ_p. The use of a log-linear approximation constitutes a conservative choice, as more accurate approximations also include higher-order terms in the model solution, which further helps to identify the structural parameters as shown in An and Schorfheide (2007) and Ruge-Murcia (2012).

The steady state solution for the states $\mathbf{x}_{ss}(\boldsymbol{\theta})$ and the controls $\mathbf{y}_{ss}(\boldsymbol{\theta})$ in (6) are available in closed-form for the New Keynesian model in Section 2 and provided in Appendix A together with the log-linearized version of the model. However, the New Keynesian model is almost exclusively used to explain short-term business cycles and therefore estimated on detrended data for aggregate quantities with a mean of zero. This implies that we will not rely on $\mathbf{x}_{ss}(\boldsymbol{\theta})$ or $\mathbf{y}_{ss}(\boldsymbol{\theta})$ for identifying the demand elasticity.[2] The first-order loadings $\mathbf{g}_{\mathbf{x}}(\boldsymbol{\theta})$ and $\mathbf{h}_{\mathbf{x}}(\boldsymbol{\theta})$ in (6) are therefore more suitable, because they affect all second moments in the New Keynesian model and hence the variability around the trend. It is well-known that $\mathbf{g}_{\mathbf{x}}(\boldsymbol{\theta})$ and $\mathbf{h}_{\mathbf{x}}(\boldsymbol{\theta})$ solve the difference equation

$$\mathbf{A}(\boldsymbol{\theta})\,\mathbb{E}_t[\mathbf{z}_{t+1}] = \mathbf{B}(\boldsymbol{\theta})\,\mathbf{z}_t,$$

where $\mathbf{z}_t \equiv \begin{bmatrix} \mathbf{y}_t' & \mathbf{x}_t' \end{bmatrix}'$ contains all the model variables (see, for instance, Klein (2000)). The elements in $\mathbf{A}(\boldsymbol{\theta})$ and $\mathbf{B}(\boldsymbol{\theta})$ are referred to as the reduced-form loadings and determine the interaction between the model variables. We collect these reduced-form loadings in the vector $\boldsymbol{\gamma}(\boldsymbol{\theta}) \equiv \begin{bmatrix} vec(\mathbf{A}(\boldsymbol{\theta}))' & vec(\mathbf{B}(\boldsymbol{\theta}))' \end{bmatrix}'$. Given this notation, it is informative to write $\mathbf{g}_{\mathbf{x}}(\boldsymbol{\theta})$ as the convoluted function $\mathbf{g}_{\mathbf{x}}(\boldsymbol{\theta}) = \tilde{\mathbf{g}}_{\mathbf{x}}(\boldsymbol{\gamma}(\boldsymbol{\theta}))$, where $\boldsymbol{\gamma}(\boldsymbol{\theta})$ maps the structural parameters into the reduced-form loadings and $\tilde{\mathbf{g}}_{\mathbf{x}}$ maps the reduced-form loadings into the model solution, and similarly for the $\mathbf{h}$-function. A necessary condition for identification of $\boldsymbol{\theta}$ is therefore that we can recover all the elements in $\boldsymbol{\theta}$ from $\boldsymbol{\gamma}(\boldsymbol{\theta})$. That is, we must have a one-to-one mapping between the structural parameters and the reduced-form loadings.

3.3 Weak Identification

This section uses the necessary condition in Section 3.2 to understand why the demand elasticity is only weakly identified in the New Keynesian model. We carry out the analysis under the assumption that the steady state level of i) labor l_{ss}, ii) inflation π_{ss}, and iii) the ratio of government spending to output g_{ss} are either calibrated using the sample means of l_t, π_t, and g_t or estimated jointly with the remaining structural parameters in the model (perhaps using these sample means). That is, the analysis is conditioned on l_{ss}, π_{ss}, and g_{ss} being identified, which is a very weak assumption.

The considered New Keynesian model has 11 structural parameters, beyond $(l_{ss}, \pi_{ss}, g_{ss})$ and the parameters characterizing the five structural shocks. It is easy to see from Appendix A that the nine parameters $(\beta, \omega, \nu, \kappa, \alpha, \delta, \phi_R, \gamma_\pi, \gamma_y)$ can be recovered from $\boldsymbol{\gamma}(\boldsymbol{\theta})$. Identification of the two remaining parameters (ϵ_p, ϕ_p) is less obvious, mainly because the demand elasticity ϵ_p always enters jointly with other parameters. The log-linearized version of the model in Appendix A reveals that ϵ_p is present in two equations, when ignoring firm profit f_t as typically done in the literature. The first is the aggregate supply (AS) relation, where the first-order condition for the optimal price of the intermediate good implies the familiar

[2] For instance, the steady state level of w_t and the marginal production cost mc_t are affected by ϵ_p, meaning that the level of w_t and mc_t contain information about ϵ_p if these moments were included in the estimation.

expression

$$\hat{\pi}_t = \frac{\epsilon_p - 1}{\phi_p \pi_{ss}^2} \hat{mc}_t + \beta \mathbb{E}_t \left[\hat{\pi}_{t+1} \right], \tag{7}$$

where mc_t denotes marginal production cost. Here, all variables are in deviation from the steady state, as indicated by the "hat", e.g. $\hat{\pi}_t \equiv \log \pi_t - \log \pi_{ss}$. The second equation where ϵ_p enters is in the aggregate resource constraint, which in our case reads

$$\delta\alpha \frac{\epsilon_p - 1}{\epsilon_p} \left(\hat{\imath}_t - \hat{c}_t \right) = \left(\frac{1}{\beta} - (1 - \delta) \right) \left[(1 - g_{ss}) \left(\hat{y}_t - \hat{c}_t \right) - g_{ss} \hat{g}_t \right]. \tag{8}$$

This expression is obtained from a straightforward log-linearization of (5), which gives $\frac{c_{ss}}{y_{ss}} \hat{c}_t + \frac{i_{ss}}{y_{ss}} \hat{\imath}_t = \hat{y}_t - g_{ss} \left(\hat{y}_t + \hat{g}_t \right)$, or $\left(1 - g_{ss} - i_{ss}/y_{ss} \right) \hat{c}_t + \frac{i_{ss}}{y_{ss}} \hat{\imath}_t = \hat{y}_t \left(1 - g_{ss} \right) - g_{ss} \hat{g}_t$, because $c_{ss}/y_{ss} = 1 - g_{ss} - i_{ss}/y_{ss}$. But, we also have that $i_{ss}/y_{ss} = \delta\alpha \frac{\epsilon_p - 1}{\epsilon_p} \left(\frac{1}{\beta} - (1 - \delta) \right)^{-1}$, which then leads to (8).

Ordering $\boldsymbol{\gamma}(\boldsymbol{\theta})$ such that the reduced-form coefficients involving ϵ_p appear first, we then get the following three equations:

$$\frac{\epsilon_p - 1}{\phi_p \pi_{ss}^2} = \gamma_1 \tag{9}$$

$$\delta\alpha \frac{\epsilon_p - 1}{\epsilon_p} = \gamma_2 \tag{10}$$

$$\left(\frac{1}{\beta} - (1 - \delta) \right) (1 - g_{ss}) - \delta\alpha \frac{\epsilon_p - 1}{\epsilon_p} = \gamma_3 \tag{11}$$

Here, (10) is the loading for i_t in (8) and (11) is the loading for $\hat{c}_t$ in (8). The parameters δ, α, and β are easily recovered from the other elements in $\boldsymbol{\gamma}(\boldsymbol{\theta})$, and it therefore follows that (9) to (11) allow us to recover ϵ_p and ϕ_p, provided $\delta > 0$ and $\alpha > 0$. Thus, one way to satisfy the necessary identification condition in Section 3.2 is to include capital with depreciation in the New Keynesian model.

However, the demand elasticity always enters in (10) and (11) through the inverse of the markup, which is multiplied by the small scalar $\delta\alpha$. For instance, $\delta\alpha = 0.0075$ when $\alpha = 0.3$ and $\delta = 0.025$ as typically considered in the literature. Hence, variation in ϵ_p has a fairly small impact on the reduced-form coefficients for the resource constraint, and this observation explains why the demand elasticity is hard to identify in the New Keynesian model, as confirmed in our Monte Carlo study below. Another way to draw the same conclusion is to consider the two limiting cases for δ and α. First, suppose $\delta = 0$ and capital does not depreciate. This implies that variation in ϵ_p is not represented in the aggregate resource constraint, because it simplifies to $\hat{g}_t \frac{g_{ss}}{1 - g_{ss}} = \hat{y}_t - \hat{c}_t$ in this case. Hence, we are left with only (9) to recover the two parameters ϵ_p and ϕ_p, showing that we cannot identify

ϵ_p without capital depreciation. Second, suppose $\alpha = 0$ and capital is not present in the production function. This also implies that the aggregate resource constraint reduces to $\hat{g}_t \frac{g_{ss}}{1-g_{ss}} = \hat{y}_t - \hat{c}_t$, showing that we are unable to identify ϵ_p without capital, as highlighted in An and Schorfheide (2007).

3.4 A New Identification Strategy

The analysis in Section 3.3 suggests that one way to overcome the weak identification of the demand elasticity is to introduce another variable in the New Keynesian model where variation in ϵ_p has a large impact on the reduced-form coefficients. The solution we propose is to include detrended firm profit $\hat{f}_t$, because its dynamics is strongly affected by ϵ_p. This follows from the log-linear expression of (4)

$$\hat{f}_t = \hat{y}_t - (\epsilon_p - 1)\left[(1-\alpha)\left(\hat{w}_t + \hat{l}_t\right) - \alpha\left(\hat{r}_{k,t} + \hat{k}_{t-1}\right)\right], \tag{12}$$

where the reduced-form loadings of $\hat{f}_t$ on the factor prices $(\hat{w}_t,\ \hat{r}_{k,t})$ and the factor inputs $(\hat{l}_t,\ \hat{k}_{t-1})$ depend directly on the demand elasticity. To see where this effect comes from, note that a log-linearization of (4) gives $\frac{f_{ss}}{y_{ss}}\hat{f}_t = \hat{y}_t - \frac{w_{ss}l_{ss}}{y_{ss}}\left(\hat{w}_t + \hat{l}_t\right) - \frac{r_{k,ss}k_{ss}}{y_{ss}}\left(\hat{r}_{k,t} + \hat{k}_{t-1}\right)$. But the steady state implies $\frac{f_{ss}}{y_{ss}} = 1/\epsilon_p$, showing that the ratio of firm profit to output decreases for a higher demand elasticity. We also have that $\frac{w_{ss}l_{ss}}{y_{ss}} = (1-\alpha)(\epsilon_p - 1)/\epsilon_p$ and $\frac{r_{k,ss}k_{ss}}{y_{ss}} = \alpha(\epsilon_p - 1)/\epsilon_p$, which then leads to (12).

The economic intuition behind this direct effect of the demand elasticity for firm profit is as follows. Suppose the demand elasticity between goods in the economy is low and firms therefore have high market power and face steep demand curves $P_{i,t} = P_t\left(\frac{y_{i,t}}{y_t}\right)^{-1/\epsilon_p}$. This allows firms to charge high markups and generate stable profits, because price changes lead to relatively small changes in demand and hence profits. On the other hand, firm profits are more volatile when the demand elasticity is high and firms face relatively flat demand curves, because even small changes in prices lead to large changes in demand and hence profits. Therefore, the size of the demand elasticity affects the variability of firm profit. This implies that variation in ϕ_p *and* ϵ_p lead to notable changes in the reduced-form loadings in (7) and (12), which help to identify the demand elasticity.

4 Simulation Evidence

This section explores the usefulness of the new identification strategy proposed in Section 3.4 when considering a standard calibration of the New Keynesian model. We present the setup for our numerical experiments in Section 4.1, and explore identification in the population in Section 4.2. A Monte Carlo study is presented in Section 4.3 to examine identification in finite samples.

4.1 The Setup

We consider a calibrated version of the New Keynesian model to quarterly post-war US data. The calibration is fairly standard and summarized in Table 1. That is, we allow for trend inflation with $\pi_{ss} = 1.01$ and let $\beta = 0.998$ to get a realistic level for the short rate ($4 \log R_{ss} = 4.8\%$). For the household, we consider log-preferences for consumption ($\omega = 1$) and a unit Frisch labor supply elasticity ($\nu = 1$). In the goods market, we let $\epsilon_p = 6$ to get an average price markup of 20% as commonly considered in the literature. For the degree of price stickiness, we set $\phi_p = 60$ which gives the same slope of the AS curve as implied by Calvo-pricing with an average price duration of about 4 quarters. The monetary policy rule displays a moderate degree of interest rate smoothing ($\phi_R = 0.80$) and assigns more weight to stabilizing inflation than output with $\gamma_\pi = 2$ and $\gamma_y = 0.25$.

Table 1: The Calibrated Parameters

l_{ss}	=	0.33	κ	=	2.00	ρ_d	=	0.95
π_{ss}	=	1.01	δ	=	0.025	ρ_g	=	0.90
g_{ss}	=	0.20	ϕ_p	=	60	ρ_n	=	0.90
β	=	0.9980	ϵ_p	=	6	σ_a	=	0.005
ω	=	1	ϕ_R	=	0.80	σ_d	=	0.015
ν	=	1	γ_π	=	2.00	σ_g	=	0.03
α	=	0.30	γ_y	=	0.25	σ_n	=	0.01
			ρ_a	=	0.95	σ_R	=	0.0025

We study identification of the demand elasticity when using $\hat{c}_t$, $\hat{\imath}_t$, $\hat{\pi}_t$,$\hat{R}_t$, $\hat{l}_t$, and potentially $\hat{f}_t$ to estimate the New Keynesian model by maximum likelihood. The observed variables are stored in $\mathbf{y}_t^{obs}$, where all time series are measured in deviation from the deterministic steady state. This implies that the level of these variables only enter in the estimation through the calibration of the structural parameters. The five structural shocks are considered to be unobserved, and we therefore evaluate the log-likelihood function $\mathcal{L}_T$ by the Kalman filter. To avoid issues related to stochastic singularity, we follow An and Schorfheide (2007) and introduce measurement errors in $\mathbf{y}_t^{obs}$ corresponding to 20% of the variation in each of the observed series.

4.2 Population Results

To study identification of ϵ_p and ϕ_p in the population, we consider a long simulated sample of $T = 5,000$ observations and approximate the population objective function by $\mathcal{L}_o = \frac{1}{T}\mathcal{L}_T$. The joint identification of ϵ_p and ϕ_p is then explored by plotting $\mathcal{L}_o$ for different values of ϵ_p when changing ϕ_p appropriately to ensure that the slope of the AS curve $(\epsilon_p - 1) / \left(\phi_p \pi_{ss}^2\right)$ remains constant. In this way, we remove the impact of changing ϵ_p on the slope of the AS curve (which is determined by ϕ_p) to isolate the pure effects of varying ϵ_p for the dynamics of the New Keynesian model.

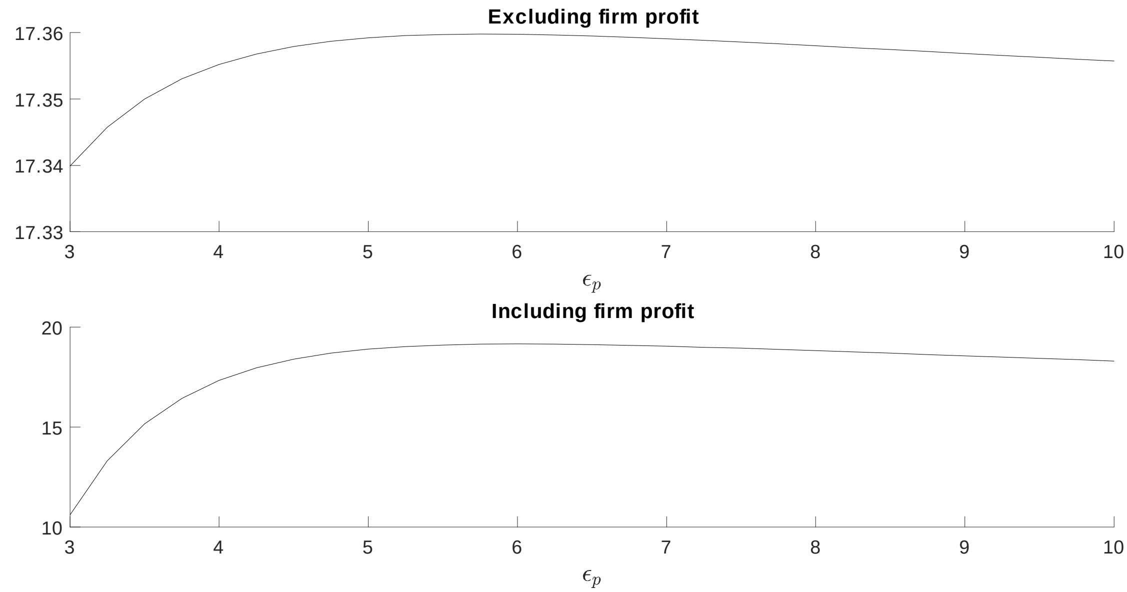
The top chart in Figure 1 plots $\mathcal{L}_o$ when the log-likelihood function is evaluated without firm profit. The log-likelihood function is extremely flat in ϵ_p, as $\mathcal{L}_o$ only changes between 17.34 and 17.36 when varying ϵ_p between 3 to 10. This shows that the demand elasticity is weakly identified, because ϵ_p has a very small effect on the resource constraint as noted in Section 3.3. This finding also implies that the dynamics of the five variables $(\hat{c}_t, \hat{\imath}_t, \hat{\pi}_t, \hat{R}_t, \hat{l}_t)$ are basically unaffected by the demand elasticity when conditioning on a given slope of the AS curve. Unreported results show that the weak identification of ϵ_p is robust to i) reducing the size of the measurement errors, ii) varying the structural parameters and iii) including nonlinear terms by solving the model with a third-order perturbation approximation.[3] Note also that the asymptotic efficiency of maximum likelihood implies that the weak identification of ϵ_p shown in the top chart of Figure 1 will be even more pronounced when using less efficient estimators such as GMM or matching impulse response functions.

The bottom chart in Figure 1 examines the usefulness of our proposed identification strategy by including firm profit in the likelihood function. The very encouraging finding is that the log-likelihood function now is very curved, as $\mathcal{L}_o$ ranges between 10 and 20 when varying ϵ_p from 3 to 10. This shows that firm profit is highly informative about the demand elasticity and greatly facilitates identification of ϵ_p.

[3] When using the third-order perturbation solution, the log-likelihood function is approximated by the central difference Kalman filter of Norgaard et al. (2000), which Andreasen (2013) shows may be a more accurate (and faster) approximation to the infeasible likelihood function than using a particle filter when the structural shocks are Gaussian.

Figure 1: Plot of the Log-Likelihood Function

For the calibration in Table 1, this figure plots $\mathcal{L}_o$ for different values of ϵ_p when changing ϕ_p appropriately to ensure that the slope of the aggregate supply curve $(\epsilon_p - 1) / (\phi_p \pi_{ss}^2)$ remains constant. Here, $\mathcal{L}_o$ is computed on a simulated sample of $T = 5,000$ observations by $\mathcal{L}_o = \frac{1}{T}\mathcal{L}_T$. The top chart considers the case when firm profit is excluded from $\mathcal{L}_o$, while firm profit is included in the bottom chart.



4.3 Finite Samples

It is well-known that identification in the population does not always carry over to finite samples, because the considered moments may only be weakly informative about the structural parameters in shorter samples (see Canova and Sala (2009)). We therefore conduct a Monte Carlo study in this section to further explore the ability of the proposed identification strategy for the demand elasticity. This Monte Carlo study is carried out using the calibration in Table 1 as the data generating process (DGP). Given these parameters, we then simulate 1,000 samples with $T = 250$ observations. This sample size corresponds to using roughly 60 years of quarterly data and is thus representative of most empirical applications for the post-war US economy. To make the Monte Carlo study manageable, we condition the simulations on the structural parameters in the first column of Table 1 and estimate the remaining 16 parameters by maximum likelihood.

The results from this Monte Carlo study are summarized in Table 2, where Panel A explores the performance of estimating the structural parameters without including firm profit in the likelihood function. Focusing on ϕ_p and ϵ_p, we first note that the sticky price parameter ϕ_p has a large positive bias of 47.02 and is estimated very imprecisely with a true

Table 2: The Monte Carlo Study: Results

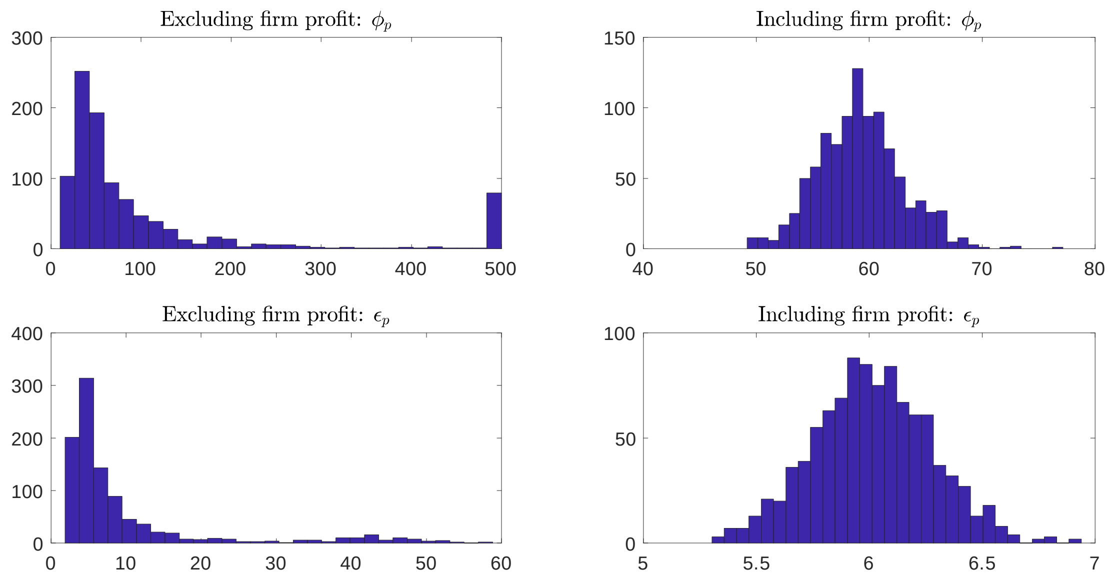
This table reports the results from a Monte Carlo study of maximum likelihood when using $1,000$ repetitions for samples of $T = 250$ observations. The data generating process (DGP) is the calibration stated in Table 1. The columns report the following: i) 'Level bias' refers to the difference between the mean of the sampling distribution and the true value, ii) 'True SE' is the standard deviation in the sampling distribution, and iii) 'Type I: 5 pct.' reports the rejection probabilities of usinga t-test with the null hypothesis that the estimated parameter equals its true value. Panel A excludes firm profit from $\mathcal{L}_T$, while firm profit is included in Panel B.

		Panel A: Excluding firm profit			Panel B: Including firm profit		
	DGP	Level bias	True SE	Type I: 5%	Level bias	True SE	Type I: 5%
κ	2.000	0.093	0.530	0.061	0.034	0.431	0.071
δ	0.025	0.000	0.005	0.088	0.000	0.003	0.055
ϕ_p	60.000	47.022	131.432	0.159	-0.789	3.784	0.059
ϵ_p	6.000	4.126	11.391	0.153	0.014	0.264	0.045
ϕ_R	0.800	0.008	0.053	0.047	0.003	0.047	0.054
γ_π	2.000	0.326	0.959	0.075	0.193	0.682	0.086
γ_y	0.250	0.075	0.228	0.090	0.048	0.171	0.083
ρ_a	0.950	-0.013	0.036	0.122	-0.002	0.009	0.047
ρ_d	0.950	-0.005	0.018	0.099	-0.002	0.008	0.059
ρ_g	0.900	-0.010	0.028	0.083	-0.007	0.020	0.052
ρ_n	0.900	-0.037	0.057	0.094	-0.006	0.022	0.061
σ_a	0.005	0.000	0.001	0.056	0.000	0.001	0.042
σ_d	0.015	0.000	0.002	0.102	0.000	0.002	0.055
σ_g	0.030	0.000	0.003	0.029	0.000	0.002	0.034
σ_n	0.010	0.001	0.002	0.044	0.000	0.001	0.031
σ_R	0.003	0.000	0.000	0.034	0.000	0.000	0.015

standard error of 131. The results for the demand elasticity are equally discouraging, as ϵ_p has a large positive bias of 4.13 and a large true standard error of 11.4. These findings are thus consistent with the results reported in Section 4.2 for $\mathcal{L}_o$ and show that ϵ_p and ϕ_p cannot be estimated jointly when using a standard set of macro variables.

Panel B in Table 2 adds firm profit to the estimation to explore if the satisfying results from Section 4.2 carry over to finite samples. For the sticky price parameter ϕ_p we only observe a small level bias of -0.79, and this parameter is also estimated very accurately with a true standard error of 3.78. The results are equally encouraging for the demand elasticity, which is basically unbiased (level bias of 0.01) and displays a small standard error of 0.26. To evaluate the inference from the asymptotic distribution, Table 2 also reports the rejection probabilities at a 5% significance level (Type I: 5%) from t-tests that a given estimate equals its true value. These type I errors are very close to the desired level of 5% for ϕ_p and ϵ_p, whereas they exceed 15% in Panel A where the information from firm profit is not included in the likelihood function.

Figure 2: The Monte Carlo Study: Selected Sampling Distributions
This figure reports the sampling distribution of ϕ_p and ϵ_p in the Monte Carlo study when excluding and including firm profit in the estimation. The high proportion of observations for ϕ_p around 500 in the top left chart reflects the fact that the estimates hit the imposed upper bound of 500 for ϕ_p.



This notable improvement of using firm profit in the estimation is also evident from Figure 2, showing the distributions of the $1,000$ estimates for ϕ_p and ϵ_p in the Monte Carlo study when excluding and including firm profit. The distributions in the left of Figure 2 do not exploit the information from firm profit and are very wide and non-Gaussian. This is particularly the case for ϕ_p, where several estimates hit the imposed upper bound of 500. This implies that raising the level of this upper bound would further exacerbate the positive level bias for ϕ_p. In contrast, the sampling distributions for ϕ_p and ϵ_p are both tightly bell-shaped around the true value when including firm profit in the estimation.

Thus, the clear message from this Monte Carlo study is that firm profit enables the demand elasticity ϵ_p as well as the sticky price parameter ϕ_p to be reliably estimated in samples of the length typically used in empirical applications.

5 An Empirical Application

This section estimates the New Keynesian model on post-war US data using the proposed identification strategy for the demand elasticity. We proceed by describing the data in Section 5.1, before presenting the estimation results in Section 5.2.

5.1 Data

The post-war US economy is represented by quarterly data from 1960 Q1 to 2007 Q4, where the end point of our sample is chosen to avoid issues related to the interest rate reaching the zero lower bound in 2008. We consider the same six variables for the estimation as used in Section 4. That is, we include i) real consumption per capita, ii) real investment per capita, iii) CPI inflation, iv) the three-month Treasury bill rate, v) the average weekly working hours and vi) tax-adjusted corporate profits per capita. All six data series are stored in $\mathbf{y}_t^{obs}$ and downloaded from the FRED database with detailed descriptions provided in Appendix B. The series for consumption, investment, and firm profit are log-transformed and detrended using the regression procedure in Hamilton (2018) to avoid key shortcomings of the HP filter. The series for inflation, the interest rate, and log-transformed hours are not detrended but simply expressed in deviation from their respective sample mean.

5.2 Estimation Results

We preserve the same split between the calibrated and estimated parameters as considered in our Monte Carlo study. Hence, we let $\pi_{ss} = 1.0106$ to match the sample mean of 4.23% for annual inflation, and we set $\beta = 0.9968$ to fit an annual mean interest rate of 5.51%. For the labor supply, we let $l_{ss} = 0.34$ to match the mean of our series for log-transformed hours. The values for g_{ss}, ω, ν, and α are similar to those provided in Table 1. We also allow for measurement errors in $\mathbf{y}_t^{obs}$ corresponding to 20% of the variation in each of the six series in $\mathbf{y}_t^{obs}$.

As a natural benchmark, we first estimate the New Keynesian model without including firm profit and with $\epsilon_p = 6$ to get an average price markup of 20% as commonly assumed in the literature. A preliminary estimation reveals that the sticky price parameter ϕ_p is badly identified in this case and hits the imposed upper bound of 500 for this parameter. We therefore let $\phi_p = 60$, which gives the same slope of the AS curve as implied by Calvo-pricing with an average price duration of about 4 quarters. The maximum likelihood estimates for this version of the New Keynesian model are reported in the first column of Table 3. These estimates are fairly representative of the typical findings in the literature, as we find sizable investment adjustment costs ($\hat{\kappa} = 0.52$), evidence of interest rate smoothing ($\hat{\phi}_R = 0.74$), and a central bank that assigns a larger weight to stabilizing inflation than output ($\hat{\gamma}_\pi = 1.13$ vs. $\hat{\gamma}_y = 0.12$).

The second column in Table 3 includes firm profit in the estimation. Our main finding is an estimated demand elasticity of 2.58, which has a tight standard error of 0.31. This implies an average price markup of 63% with a 95% confidence interval of $[39\%, 88\%]$ when

Table 3: Maximum Likelihood: The Estimated Parameters

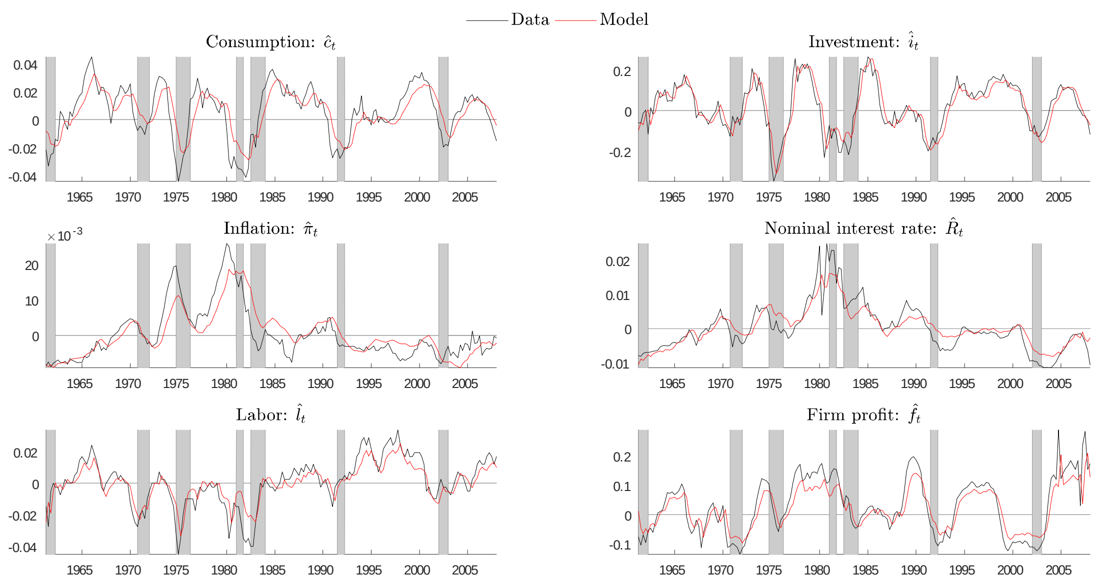
This table reports the estimated parameters for the New Keynesian model when estimated by maximum likelihood from 1960 Q1 to 2007 Q4, with the first four observations reserved for the initialization of the Kalman filter. Asymptotic standard errors are computed from the variance of the score function. In Panel B, $\mathcal{M}^{\text{Profit}}_{\epsilon_p=6}$ refers to the restricted case where firm profit is included in the estimation and the demand elasticity is equal to 6.

	Panel A: Excluding firm profit	Panel B: Including firm profit	
	(1)	(2)	(3)
	$\mathcal{M}_0$	$\mathcal{M}^{\text{Profit}}$	$\mathcal{M}^{\text{Profit}}_{\epsilon_p=6}$
δ	0.0016 (0.0003)	0.0021 (0.0007)	0.0012 (0.0004)
κ	0.5186 (0.1173)	0.2913 (0.0702)	0.1765 (0.0452)
ϕ_p	60	233.964 (68.334)	363.226 (150.442)
ϵ_p	6	2.5764 (0.3085)	6
ϕ_R	0.7439 (0.0476)	0.8493 (0.1171)	0.6107 (0.0987)
γ_π	1.1264 (0.1194)	1.445 (0.5824)	1.0823 (0.0812)
γ_y	0.1238 (0.0655)	0.4518 (0.5369)	0.0736 (0.0594)
ρ_a	0.9742 (0.0087)	0.9969 (0.001)	0.9958 (0.0013)
ρ_d	0.8674 (0.0482)	0.9126 (0.0425)	0.9012 (0.0394)
ρ_g	0.9824 (0.0051)	0.9906 (0.0033)	0.9929 (0.0025)
ρ_n	0.7999 (0.0532)	0.7858 (0.0357)	0.8331 (0.037)
σ_a	0.0017 (0.0004)	0.0015 (0.0004)	0.0015 (0.0003)
σ_d	0.0044 (0.0006)	0.003 (0.0005)	0.0028 (0.0004)
σ_g	0.0271 (0.0033)	0.0228 (0.0035)	0.0212 (0.0035)
σ_n	0.0146 (0.003)	0.0279 (0.0062)	0.0124 (0.0013)
σ_R	0.0020 (0.0002)	0.0016 (0.0002)	0.0014 (0.0002)
Memo			
$\mathcal{L}_T$	$3,194.9$	$3,405.5$	$3,350.8$

using the delta-method. Thus, the commonly used calibrated markup of 20% is not in this 95% confidence interval and hence rejected by the data at a 5% significance level. We also note that our estimated demand elasticity is consistent with the recent micro evidence of Loecker and Eeckhout (2017), showing that the US has experienced a gradual increase in the average markup from about 20% in 1980 to around 70% in 2014. Table 3 further shows that the inclusion of firm profit allow us to estimate the sticky price parameter ϕ_p. The point estimate is $\hat{\phi}_p = 234$, which gives a fairly flat slope for the AS curve of 0.007.[4] We also note that the estimated shock distributions with firm profit included in the estimation are very similar to those obtained in our benchmark specification, except possibly for the persistence in technology shocks (ρ_a increases from 0.974 to 0.997) and the size of labor supply shocks (σ_n increases from 0.015 to 0.028).

Figure 3: In-Sample Fit: One-Step Ahead Predictions

This figure resports the one-step ahead predictions of the New Keynesian model using the estimates in column (2) of Table 3. The shaded gray bars denote NBER recessions. All variables are reported in deviations from the steady state, including inflation and the interest rate shown in quarterly terms.



To evaluate how well the New Keynesian model explains the data, Figure 3 shows the one-step ahead forecasts of the observables in $\mathbf{y}_t^{obs}$, which are used in the Kalman filter to compute the likelihood function. That is, for the estimated states in period $t-1$ conditioned on $\left\{\mathbf{y}_i^{obs}\right\}_{i=1}^{t-1}$, we report the expected value of $\mathbf{y}_t^{obs}$ in the model and the realized value of $\mathbf{y}_t^{obs}$ in the data. Figure 3 shows that the model generally does well in forecasting all six variables in $\mathbf{y}_t^{obs}$, also around NBER recessions as indicated by shaded gray bars.

To understand why the data prefers a fairly low demand elasticity, it is useful to consider the estimates in the second column of Table 3, except with $\epsilon_p = 6$ as commonly assumed in the literature. The standard deviation of detrended firm profit is then 0.242 in the model,

[4]The same slope for the AS curve corresponds to an average price duration of about 13 quarters in the stylized model of Calvo (1983) with homogenous capital. However, it is well-known from Altig et al. (2011) that accounting for firm-specific capital makes a flat slope for the AS curve consistent with much shorter price durations in a setting with Calvo-pricing, as also illustrated in Castelnuovo and Pellegrino (2018).

whereas it is only 0.085 when using the estimated value of $\epsilon_p = 2.58$. The corresponding standard deviation in the data is 0.095. Thus, our maximum likelihood estimates prefer a low demand elasticity because it helps to generate the desired low variability in firm profit.

The final column in Table 3 explores the effects of estimating the New Keynesian model when restricting $\epsilon_p = 6$, although firm profit is included in the estimation. The estimates are reported in the third column of Table 3 and imply a large reduction in the log-likelihood function of 54.7 when compared to the unrestricted model in column two. As a result, the restriction $\epsilon_p = 6$ is clearly rejected by a standard likelihood-ratio test, which has a P-value of zero. We also note that the restricted model in the third column of Table 3 imply a standard deviation for firm profit of 0.134, showing that estimating the model conditioned on an average markup of 20% still generates too much variability in firm profit.

6 Conclusion

This paper proposes to identify the demand elasticity in the New Keynesian model from the variability in firm profit. We show analytically that this alleviates the weak identification of this parameter because the variability of firm profit depends directly on the demand elasticity. A Monte Carlo study demonstrates that this identification strategy enables us to accurately estimate the demand elasticity in samples of the same length as typically used in the literature. In an empirical application for the post-war US economy, we find an estimated demand elasticity of 2.58 with a tight standard error of 0.31. This fairly low estimate of the demand elasticity implies a high average markup of 63%. In contrast, most New Keynesian models use a calibrated markup of 20%, but we show that such a low markup (i.e. high demand elasticity) is rejected by the data because it generates too much variability in firm profit. Thus, the new identification strategy proposed in this paper suggests that a substantially lower benchmark value for the demand elasticity should be used in the New Keynesian model.

A Appendix: The Log-Linearized System

The log-linearized representation of the model in Section 2 is summarized below, except for the five log-linear relations for the shocks which we omit for simplicity.

1. $\omega\left(\mathbb{E}_t\left[\hat{c}_{t+1}\right]-\hat{c}_t\right)=\mathbb{E}_t\left[\hat{d}_{t+1}\right]-\hat{d}_t+\hat{R}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]$
2. $\hat{d}_t+\upsilon\hat{l}_t+\hat{n}_t=\hat{d}_t-\omega\hat{c}_t+\hat{w}_t$
3. $\hat{q}_t=\kappa\left[\hat{i}_t-\hat{i}_{t-1}-\beta\left(\mathbb{E}_t\left[\hat{i}_{t+1}\right]-\hat{i}_t\right)\right]$
4. $\hat{q}_t=\omega\left[\mathbb{E}_t\left[\hat{d}_{t+1}\right]-\hat{d}_t-\left(\mathbb{E}_t\left[\hat{c}_{t+1}\right]-\hat{c}_t\right)\right]+\mathbb{E}\left[r_{k,t+1}\right]+\beta\left(1-\delta\right)\mathbb{E}\left[\hat{q}_{t+1}-r_{k,t+1}\right]$
5. $\hat{w}_t=\hat{k}_{t-1}-\hat{l}_t+\hat{r}_{k,t}$
6. $\hat{mc}_t=\left(1-\alpha\right)\hat{w}_t+\alpha\hat{r}_{k,t}-\hat{a}_t$
7. $\hat{\pi}_t=\frac{\epsilon_p-1}{\phi_p\pi_{ss}^2}\hat{mc}_t+\beta\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]$
8. $\hat{y}_t=\hat{a}_t+\alpha\hat{k}_{t-1}+\left(1-\alpha\right)\hat{l}_t$
9. $\hat{R}_t=\phi_R\hat{R}_{t-1}+\left(1-\phi_R\right)\gamma_\pi\hat{\pi}_t+\left(1-\phi_R\right)\gamma_y\hat{y}_t+\sigma_R\hat{\varepsilon}_{R,t}$
10. $\hat{k}_t=\left(1-\delta\right)\hat{k}_{t-1}+\delta\hat{i}_t$
11. $\delta\alpha\frac{\epsilon_p-1}{\epsilon_p}\left(\hat{\imath}_t-\hat{c}_t\right)=\left(\frac{1}{\beta}-\left(1-\delta\right)\right)\left[\left(1-g_{ss}\right)\left(\hat{y}_t-\hat{c}_t\right)-g_{ss}\hat{g}_t\right]$
12. $\hat{f}_t=\hat{y}_t-\left(\epsilon_p-1\right)\left[\left(1-\alpha\right)\left(\hat{w}_t+\hat{l}_t\right)-\alpha\left(\hat{r}_{k,t}+\hat{k}_{t-1}\right)\right]$

Given that l_{ss}, π_{ss}, and g_{ss} are known, the steady state is given by $R_{ss}=\pi_{ss}/\beta$, $mc_{ss}=\left(\epsilon_p-1\right)/\epsilon_p$, $q_{ss}=1$, $r_{k,ss}=1/\beta-(1-\delta)$, $w_{ss}=\left(mc_{ss}/\left(\frac{1}{(1-\alpha)^{1-\alpha}}\frac{1}{\alpha^\alpha}r_{k,ss}^\alpha\right)\right)^{\frac{1}{1-\alpha}}$, $k_{ss}=\frac{\alpha}{1-\alpha}\frac{w_{ss}}{r_{k,ss}}l_{ss}$, $i_{ss}=\delta k_{ss}$, $y_{ss}=k_{ss}^\alpha l_{ss}^{1-\alpha}$, and $\psi=\frac{\varepsilon_w-1}{\varepsilon_w}\frac{w_{ss}}{c_{ss}^\omega l_{ss}^\upsilon}$.

B Data Description

Consumption is measured by real personal consumption expenditures per capita on non-durables and services. Investment is obtained from real gross private domestic investment divided by the US population. Inflation is measured by the year-on-year percentage change in the consumer price index for all urban consumers. The nominal interest rate is represented by the three-month Treasury bill rate in the secondary market. Hours is measured by the average weekly hours of production and nonsupervisory employees in the manufacturing sector, which we normalize by the constant 24×5. Finally, firm profit per capita is computed as the after tax corporate profits with capital consumption adjustment and inventory valuation adjustment divided by the US population.

References

Altig, D., Christiano, L. J., Eichenbaum, M. and Linde, J. (2011), 'Firm-Specific Capital, Nominal Rigidities and the Business Cycle', *Review of Economic Dynamics* **14**, 225–247.

An, S. and Schorfheide, F. (2007), 'Bayesian analysis of DSGE models', *Econometric Review* **26(2-4)**, 113–172.

Andreasen, M. M. (2013), 'Non-linear DSGE models and the central difference kalman filter', *Journal of Applied Econometrics* **28**, 929–955.

Calvo, G. A. (1983), 'Staggered Prices in a Utility-Maximizing Framework', *Journal of Monetary Economics* **12**(3), 383–398.

Canova, F. and Sala, L. (2009), 'Back to Square One: Identification Issues in DSGE Models', *Journal of Monetary Economics* **56**(4), 431–449.

Castelnuovo, E. and Pellegrino, G. (2018), 'Uncertainty-Dependent Effects of Monetary Policy Shocks: A New-Keynesian Interpretation', *Journal of Economics Dynamic and Control* **93**, 277–296.

Christiano, L. J., Motto, R. and Rostagno, M. (2014), 'Risk Shocks', *American Economic Review* **104**(1), 27–65.

Fernández-Villaverde, J., Guerron-Quintana, P., Kuester, K. and Rubio-Ramirez, J. (2015), 'Fiscal Volatility Shocks and Economic Activity', *American Economic Review* **105**(11), 3352–3384.

Hamilton, J. D. (2018), 'Why You Should Never Use the Hodrick-Prescott Filter', *The Review of Economics and Statistics* **100**(5), 831–843.

Hayashi, F. (2000), *Econometrics*, Princeton University Press, New Jersey.

Ireland, P. N. (2001), 'Sticky-Price Models of the Business Cycle: Specification and Stability', *Journal of Monetary Economics* **47**(1), 3–18.

Keen, B. and Wang, Y. (2007), 'What is a realistic value for price adjustment costs in new keynesian models?', *Applied Economics Letters* **14**, 789–793.

Klein, P. (2000), 'Using the generalized schur form to solve a multivariate linear rational expectations model', *Journal of Economic Dynamic and Control* **24**, 1405–1423.

Loecker, J. D. and Eeckhout, J. (2017), 'The Rise of Market Power and the Macroeconomic Implications', *NBER Working Paper No. 23687* .

Norgaard, M., Poulsen, N. K. and Ravn, O. (2000), 'Advances in derivative-free state estimation for nonlinear systems', *Automatica* **36:11**, 1627–1638.

Rotemberg, J. J. (1982), 'Monopolistic Price Adjustment and Aggregate Output', *Review of Economic Studies* **49**, 517–531.

Rudebusch, G. D. and Swanson, E. T. (2012), 'The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks', *American Economic Journal: Macroeconomics* **4**(1), 1–43.

Ruge-Murcia, F. (2012), 'Estimating Nonlinear DSGE Models by the Simulated Method of Moments: With an Application to Business Cycles', *Journal of Economics Dynamic and Control* **36**(6), 914–938.

Schmitt-Grohé, S. and Uribe, M. (2004), 'Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function', *Journal of Economic Dynamics and Control* **28**(4), 755–775.

Smets, F. and Wouters, R. (2007), 'Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach', *American Economic Review* **97**(3), 586–606.

Research Papers 2019

CREATES
Center for Research in Econometric Analysis of Time Series

2018-23:	Emilio Zanetti Chini: Forecasters' utility and forecast coherence
2018-24:	Tom Engsted and Thomas Q. Pedersen: Disappearing money illusion
2018-25:	Erik Christian Montes Schütte: In Search of a Job: Forecasting Employment Growth in the US using Google Trends
2018-26:	Maxime Morariu-Patrichi and Mikko Pakkanen: State-dependent Hawkes processes and their application to limit order book modelling
2018-27:	Tue Gørgens and Allan H. Würtz: Threshold regression with endogeneity for short panels
2018-28:	Mark Podolskij, Bezirgen Veliyev and Nakahiro Yoshida: Edgeworth expansion for Euler approximation of continuous diffusion processes
2018-29:	Isabel Casas, Jiti Gao and Shangyu Xie: Modelling Time-Varying Income Elasticities of Health Care Expenditure for the OECD
2018-30:	Yukai Yang and Luc Bauwens: State-Space Models on the Stiefel Manifold with A New Approach to Nonlinear Filtering
2018-31:	Stan Hurn, Nicholas Johnson, Annastiina Silvennoinen and Timo Teräsvirta: Transition from the Taylor rule to the zero lower bound
2018-32:	Sebastian Ankargren, Måns Unosson and Yukai Yang: A mixed-frequency Bayesian vector autoregression with a steady-state prior
2018-33:	Carlos Vladimir Rodríguez-Caballero and Massimiliano Caporin: A multilevel factor approach for the analysis of CDS commonality and risk contribution
2018-34:	James G. MacKinnon, Morten Ørregaard Nielsen, David Roodman and Matthew D. Webb: Fast and Wild: Bootstrap Inference in Stata Using boottest
2018-35:	Sepideh Dolatabadim, Paresh Kumar Narayan, Morten Ørregaard Nielsen and Ke Xu: Economic significance of commodity return forecasts from the fractionally cointegrated VAR model
2018-36:	Charlotte Christiansen, Niels S. Grønborg and Ole L. Nielsen: Mutual Fund Selection for Realistically Short Samples
2018-37:	Niels S. Grønborg, Asger Lunde, Kasper V. Olesen and Harry Vander Elst: Realizing Correlations Across Asset Classes
2018-38:	Riccardo Borghi, Eric Hillebrand, Jakob Mikkelsen and Giovanni Urga: The dynamics of factor loadings in the cross-section of returns
2019-01:	Andrea Gatto and Francesco Busato: Defining, measuring and ranking energy vulnerability
2019-02:	Federico Carlini and Paolo Santucci de Magistris: Resuscitating the co-fractional model of Granger (1986)
2019-03:	Martin M. Andreasen and Mads Dang: Estimating the Price Markup in the New Keynesian Model