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Realizing Correlations Across Asset Classes*

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Realizing Correlations Across Asset Classes

Abstract

We introduce a simple and intuitive approach of modeling and forecasting correlations for use in portfolio optimization. The model is composite in nature and consists of elements based on a bivariate realized volatility model. Importantly, our framework allows for volatility spill-overs between assets which provide an edge compared to competing models when forming portfolios. We apply the model to high-frequency data for commodity markets and demonstrate significant economic gains for an investor basing portfolio decisions on our modeling framework. This gain is significant in economic terms, even after imposing realistic constraints on short selling and portfolio turnover.

Keywords: Commodities, futures markets, portfolio selection, Realized Beta GARCH.

JEL Classification: C58, G11, G17.

1. Introduction

The traded volume in commodity markets has increased significantly in recent years and these markets provide exciting new possibilities for portfolio diversification. Diversification has been a keyword in the financial literature since the concept of portfolio selection was formalized by Markowitz (1952), but the diversification benefits from including commodities might be different than people think. There is, for example, a widespread belief in the popular press and among hosts of financial TV shows that adding gold to a portfolio will dramatically reduce the risk of the portfolio.[1] There are certainly cases where a large reduction in portfolio variance can be achieved by adding gold to the portfolio, but in general the benefits will depend on the correlation between gold and the portfolio as well, as how it is added to the portfolio, i.e. does the investor choose a long or a short position in gold. Furthermore, diversification benefits from adding gold to a portfolio are time-varying as correlations vary over time.

In this paper we consider an investor who holds an equity portfolio as represented by S&P 500 futures, and we explore the potential diversification benefits from including selected commodity futures contracts in the portfolio. The lesson from Markowitz (1952) is that correlations alone determine the optimal portfolio composition, therefore it is important for the investor to know the correlations between the returns of all assets in her portfolio. In case the correlations vary over time, she will need a model to forecast how the correlations change in order to select portfolio weights. We illustrate the impact of correlation dynamics on the optimal portfolio composition and we start out by presenting two simple examples, which serve to illustrate that correlations do

[1] In an interview with CNBC (December 30, 2014), Jim Cramer, the host of the TV show MAD Money, put it this way: "I consider gold as an insurance policy." and the article explains that gold is attractive because "it tends to go up when everything else goes down".

indeed change over time.

The top panel of Figure 1 presents three different measures of the correlation between the return of gold futures and the return of the S&P 500 E-mini futures contract over the period from early 2007 to the end of 2014. The blue line represents the unconditional correlation. If it is assumed that the correlation is constant, the correlation can be estimated by the sample correlation of daily returns of the two contracts. Note, that the unconditional correlation is low and slightly positive over this sample indicating that gold futures, contrary to popular beliefs, actually move in the same direction as the S&P 500 futures and that diversification benefits may be smaller than expected.[2] The assumption of constant correlation may be too restrictive. The green line presents the estimates of the correlation obtained from a rolling window analysis, where the correlation is estimated by the sample correlation of 60 observations on daily frequency. The result varies a lot over time. This analysis suggests that the correlations are not only time-varying, but also, that correlations change sign multiple times over the sample. We observe that the correlation can change from positive to negative several times within a single year and that changes can be quite dramatic. In 2008-2009 the correlation changes from -0.5 to 0.5 in a matter of months. This could have huge implications for portfolio choice and thus diversification benefits. Finally, the red line represents the realized correlation of the two contracts. High-frequency data has become available for many commodity futures contracts and based on intradaily observations we can obtain a very accurate estimate of the true, unknown, correlation between the gold and S&P 500 futures contracts. The line is much more volatile than the green line. The question is whether this represents a more informative estimate

[2] In a simple portfolio with long positions in two assets, the portfolio variance is increasing in the correlation between the assets. If the investor expects a negative correlation and the actual correlation is positive, the portfolio variance will be larger than expected.

and whether this will translate into larger diversification benefits for the investor?

[Figure 1 about here.]

Gold is often mentioned because of its diversification potential, but other commodities are equally interesting. The price of crude oil is often used as an indicator of the state of the world economy and could represent a big potential for diversification. As a second example, the bottom panel of Figure 1 presents the three different measures of correlation between prices of crude oil futures and the S&P 500 E-mini futures prices. First of all, we note that the unconditional correlation is much higher than for gold, indicating that crude oil is in general higher correlated with S&P 500 than gold is. Secondly, we notice that there is significant time-variation in the correlation, both when estimated on a rolling window and when we consider the realized correlation. Finally we note that, as it was the case with gold, the realized correlation has the potential to be more informative than the estimate based on the daily observations.

In this paper we will go beyond the two commodities presented in Figure 1 and consider a portfolio of S&P 500 E-mini futures and six different commodities futures in order to mimic an investor who seeks diversification benefits from the inclusion of commodities into a equity portfolio.

Empirically, correlations are not constant and portfolio selection requires models for forecasting the time-varying correlations. The availability of high-frequency data has spurred a large interest in the academic literature of modeling and forecasting financial time series. On the one hand, these forecasting models might prove valuable for investors who seek to choose portfolio weights in order to maximize expected return in a manner which suits their respective risk preferences, but on the other hand, the high-frequency models might require investors to rebalance their portfolios too frequently. In real life, the investors face transaction costs as well as potential short-selling and turnover constraints in this process and might not benefit from the use of high-frequency models. Additionally, if many assets are considered, it can result in very complex problems with a large number of correlations, which can make forecasting very challenging.

This paper makes several interesting contributions. First, we propose a composite modeling approach, that allows us to model and forecast the covariance matrix of many assets. Secondly, we propose to use the Realized Beta GARCH model of Hansen *et al.* (2014) to incorporate the information from high-frequency return data. Third, we propose an intuitive graphical device to asses the performance of the portfolios over time. Finally, we demonstrate economically significant gains from using our proposed approach.

The phrase MacGyver style modeling is popularized by Engle (2009), it refers to the practice of building a multivariate model from a set of bivariate models, which is pioneered by Ledoit *et al.* (2003). We apply this idea by constructing a large covariance matrix for all the assets in our analysis based on the estimates of the pairwise correlations between the assets. Certain regularity conditions have to be satisfied and these are discussed in Hautsch *et al.* (2012) and Lunde *et al.* (2016).

We suggest to model the pairwise correlations using the Realized Beta GARCH model of Hansen *et al.* (2014), which allows us to obtain precise volatility estimates based on the available high-frequency data and to model volatility spillover between assets. High-frequency data presents not only exciting possibilities, but also challenges in the form of microstructure effects. We rely on the methodologies presented in Barndorff-Nielsen & Shephard (2007) and Barndorff-Nielsen *et al.* (2011) to deal with these challenges.

We document impressive diversification benefits from including commodities in a portfolio exercise. When forming the minimum variance portfolio based on our model framework, the volatility is reduced by more than 20% compared to a portfolio consisting only of S&P 500 futures. This effect is not purely from including more assets, but depends highly on the ability to produce good forecasts of the covariance matrix. Portfolios based on the Realized Beta GARCH model are shown to have lower volatilities than portfolios based on existing models, as for example the Dynamic Conditional Correlation GARCH of Engle (2002a) and the multivariate high-frequency-based volatility models of Noureldin *et al.* (2012). These results are robust to different assumptions regarding constraints on short selling and portfolio turnover. The Realized Beta GARCH model combined with our framework is also shown to perform very well when we consider a momentum strategy. We show that among many alternatives, this combination results in the portfolio, which is most desirable to the investor, both in terms of returns based measures and in terms of utility based measures. The Realized Beta GARCH model's ability to capture volatility spillover is shown to be particularly important, when we consider the momentum strategy. Finally, we document that the McGyver style modeling alone is not responsible to the good results as the chosen bivariate model has great impact on the results.

The remainder of this paper is organized as follows. Section 2 presents the data set used in the analysis. Section 3 introduces the Realized Beta GARCH model and the McGyver style modeling approach. Section 4 presents a comprehensive and realistic analysis of the economic gains from using our proposed framework. Finally Section 5 concludes.

2. High-Frequency Futures Data

The high-frequency futures data used in this study are acquired from Tick Data Inc., and includes three metal commodity futures, three energy commodities, and one equity index. Time series of futures prices are constructed using software provided by

Tick Data. As futures contracts eventually expire, each time series consists of data for multiple futures contracts. The roll-over from one contract to the next is set to occur when the next contract becomes the most traded. For more details see Christoffersen *et al.* (2018). The sample period starts at January 3, 2007 and ends December 31, 2014.

We consider S&P 500 E-minis, copper, gold, silver, heating oil, light crude oil, and natural gas futures in this analysis. Table 1 presents contract specific information for the futures considered in this analysis. All seven contracts have traded for a long period of time. Silver started trading in 1983 whereas the S&P 500 E-minis is the most recently introduced contract and started trading in 1997. All contracts are traded on the exchanges in New York and prices are in U.S. dollars. On each trading day there is a break in the trading. For the seven contracts, the breaks are relatively short and completely overlapping. Table 1 presents average numbers of daily trades for each of the futures calculated for four different years. These numbers illustrate the massive inflow of capital into the seven futures contracts over the last decade focusing only on trades in the most active contract. S&P 500 E-minis have been traded heavily throughout the sample. In 2005 there were on average more than 1,000 daily trades in each of the commodity contracts, and in 2007 this number was at least doubled for all the contracts. Crude oil in particular experienced a remarkable capital inflow and the average number of daily trades increased from 3,962 to 50,425.[3] The average numbers of daily trades increased again for all commodities from 2007 to 2010 and from 2010 to 2013 the numbers increased again for five of the six commodities. The average number of trades for crude oil was approximately 130,000 for both 2010 and 2013, a noteworthy amount considering that the underlying of a futures contract is 1,000 barrels of crude

[3] 2006 was the inception year for many large energy ETFs, such as the United States Oil Fund, which explains a large part of this development.

oil and that each trade can consist of more than one futures contract.[4]

[Table 1 about here.]

Selection of the six commodities is based on two criteria. First, relatively short and overlapping trading breaks, which means that we have many trades for the different contracts occurring at roughly the same time.[5] This is very useful when considering the intraday correlations. Second, a high number of trades that allows for very precise estimation of the volatilities and the correlations between the contracts. We choose 2007 as the starting point for our analysis as a high number of intradaily observations are available for all contracts. To include more than six commodities would be helpful for diversification, but other contracts suffer from long trading breaks and low volume, which limits the applicability and benefits of the high-frequency methodology. In Section 4, we demonstrate that the use of high-frequency methods leads to impressive diversification benefits relative to models based on daily observations, therefore we limit our attention to the six commodities presented above.

Daily transaction data for the selected commodities is cleaned following Barndorff-Nielsen *et al.* (2009). Further details are in Appendix A.

3. Modeling and Forecasting Correlations

Modeling volatilities and correlations is an important task, not only for portfolio selection, but also for many other purposes in financial economics. The class of GARCH models has been extensively studied in the literature and many specifications both in the univariate and multivariate setup exist.

[4] We do not report trading volumes, but the information is available from Tick Data Inc.

[5] The fact that the trading breaks are short and overlapping means that we can ignore them when constructing the realized measures.

GARCH models can be divided in two categories depending on the information set, $\mathcal{F}_t = \sigma(\mathcal{X}_t, \mathcal{X}_{t-1}, \ldots)$, they rely on. Classical GARCH models rely on a low-frequency information set where, for modeling the returns $r_{i,t}$ of n assets, $\mathcal{X}_t = (r_{0,t}, r_{1,t}, ..., r_{n,t})'$. As pointed out by Andersen *et al.* (2003), classical GARCH models tend to react slowly to changes in volatility and correlations. A new generation of GARCH models, studied by Engle (2002b), rely on a richer information set spanned by $\mathcal{X}_t = (r_{0,t}, x_{0,t}, r_{1,t}, x_{1,t}, ..., r_{n,t}, x_{n,t})'$, where $x_{i,t}$ denotes a set of realized measures computed with high-frequency data for asset i. Multivariate models that include measures for the realized correlation between asset i and asset j, $y_{i,j,t}$, have also been developed. Realized measures of volatilities and correlations computed from high-frequency data can potentially provide more accurate measurements of the latent volatility and correlations. This greatly enriches the information set of GARCH models and allows for faster detection of large changes in volatility and correlations.

3.1 REALIZED VOLATILITY MEASURES

Correlations and covariances are key elements in portfolio theory, realized covariances are therefore of immense importance in this paper. Realized covariances have been studied in great detail by Barndorff-Nielsen & Shephard (2004). We follow the original implementation of the estimators closely and provide only a general overview, while referring the reader to the original paper for a detailed presentation.

It is well known that high-frequency transactions are not recorded over a homogeneous grid of time coordinates, and synchronization is required to make the estimation techniques considered here feasible. To compute realized covariances, a homogeneous grid of evenly spaced prices is created using previous-tick interpolations, which is considered by Dacorogna *et al.* (2001).

Two issues make the measurement of daily integrated covariances from high-frequency prices challenging. On the one hand asynchronous trading generates microstructure effects at high-frequencies which, depending on the chosen estimator, can lead to downward biases in covariances as pointed out by Epps (1979), but on the other, a set of trading imperfections generate noise in observed prices, which modifies the properties of realized measures and particularly those of realized volatilities. We deal with the microstructure issues in two ways.

The first way relies on results from studies of equities, which show that the impact of microstructure effects on the realized measures are immaterial when sparse sampling is applied, see e.g. Barndorff-Nielsen & Shephard (2007). Microstructure effects in commodity markets are relatively sparsely documented, which leads us to rely on a conservative choice of 15 minutes sampling with 15 seconds sub-sampling. We can reasonably expect this to yield noisy but unbiased measures of the integrated covariance matrix.

The second way relies on multivariate realized kernel introduced by Barndorff-Nielsen *et al.* (2011), which is a class of estimators that are robust to measurement errors and microstructure effects induced by asynchronous trading. Specifically, non-flat-top realized kernels are used to ensure positive semi-definiteness. Further details are presented in Appendix A.

All the realized measures are computed using the transactions data described in the previous section. We do not consider jump robust estimators in this study. For more details on this see e.g. Christensen *et al.* (2014) and Vander Elst & Veredas (2017).

3.2 REALIZED BETA GARCH

We model all the pairwise correlations between the assets in the analysis. Pairwise correlations are later used to construct a larger covariance matrix for all the assets as explained in Section 3.3. To incorporate realized measures of volatility and co-volatility, the bivariate Realized Beta GARCH, which we will refer to as *RBG*, of Hansen *et al.* (2014) is used. The *RBG* is a dynamic model that models the conditional covariance matrix of returns similar to other GARCH models. However, the information set is richer and includes both the realized volatilities of assets i and j on day t, $x_{i,t}$ and $x_{j,t}$, and the realized correlation between the two on day t, $y_{i,j,t}$. As the marginal models which are used for assets i and j are identical, we provide details for asset i only. Let $h_{i,t}$ be the conditional variance of asset i, and let $\tilde{h}_{i,t} := \log h_{i,t}$, and $\tilde{x}_{i,t} := \log x_{i,t}$. Returns for asset i, $r_{i,t}$ are modeled with the following univariate Realized GARCH model

$$\begin{aligned} r_{i,t} &= \mu_i + e^{\tilde{h}_{i,t}/2} z_{i,t}, \\ \tilde{h}_{i,t} &= a_i + b_i \tilde{h}_{i,t-1} + c_i \tilde{x}_{i,t-1} + d_i \tilde{h}_{j,t} + \tau_{i,1} z_{i,t-1} + \tau_{i,2} \left(z_{i,t-1}^2 - 1 \right) \\ \tilde{x}_{i,t} &= \xi_i + \varphi_i \tilde{h}_{i,t} + \delta_{i,1} z_{i,t} + \delta_{i,2} \left(z_{i,t}^2 - 1 \right) + u_{i,t}, \end{aligned} \tag{1}$$

where $z_{i,t} \sim i.i.d.N\,(0,1)$ and $u_{i,t} \sim i.i.d.N\,(0, \sigma_{u_i})$ are mutually independent, and $\theta_i := (\mu_i, a_i, b_i, c_i, d_i\ , \tau_{i,1}, \tau_{i,2}, \xi_i, \varphi_i, \delta_{i,1}, \delta_{i,2}, h_{i,1})'$ is the vector of parameters in the model. In the following we also consider a restricted version of the *RBG* model, where $d_i = 0$ for all assets. We refer to this model as Restricted Realized Beta GARCH, *RBG.r*. For details on the univariate Realized GARCH see Hansen *et al.* (2012).

To model the dynamics of the correlations we consider

$$\begin{aligned} \digamma \left(\rho_{j,i,t} \right) &= a_{ji} + b_{ji} \digamma \left(\rho_{j,i,t-1} \right) + c_{ji} \digamma \left(y_{j,i,t-1} \right) \\ \digamma \left(y_{j,i,t} \right) &= \xi_{ji} + \varphi_{ji} \digamma \left(\rho_{j,i,t} \right) + \nu_{j,t}, \end{aligned} \tag{2}$$

where $\digamma\,(\rho) := \frac{1}{2} \log \frac{1+\rho}{1-\rho}$ denotes the Fisher transform, $\theta_{j,i} := (a_{ji}, b_{ji}, c_{ji}\ , \xi_{ji}, \varphi_{ji}, \rho_{j,i,1})'$ is the vector of parameters in the joint model, and $\nu_{j,t} \sim i.i.d \left(0, \sigma_{\nu_j} \right)$. The last two equations in (1) and (2) are measurement equations required for the specification of the conditional density $f\left(x_{j,t}, y_{j,i,t} \left| r_{j,t}, r_{i,t}, x_{i,t}, \mathcal{F}_{t-1}\right.\right)$. The measurement errors $u_{j,t}$ and $v_{j,t}$ are assumed independent of $z_{i,t}$ and $z_{j,t}$ but allowed to be mutually correlated

$$\boldsymbol{\Sigma} = \mathrm{Var}\begin{bmatrix} u_{i,t} \\ u_{j,t} \\ v_{j,t} \end{bmatrix} = \begin{bmatrix} \sigma^2_{u_i} & \sigma_{u_i,u_j} & \sigma_{u_i,v_j} \\ \bullet & \sigma^2_{u_j} & \sigma_{u_j,v_j} \\ \bullet & \bullet & \sigma^2_{v_j} \end{bmatrix}. \tag{3}$$

Estimation follows Hansen *et al.* (2014) and relies on maximum likelihood estimation. Forecasting is also outlined in Hansen *et al.* (2014) and we can obtain k-step ahead forecasts of conditional variances of assets i and j and their conditional correlation, these are denoted $h_{i,t+k|t}$, $h_{j,t+k|t}$ and $\rho_{i,j,t+k|t}$, respectively. Details for estimation and forecasting are presented in Appendix B.

3.3 BUILDING THE COVARIANCE MATRIX

Individual commodity correlations are of limited interest when forming a portfolio. This leads us to combine the forecasts of all the pairwise correlations into a forecast of the full covariance matrix, $H_{t+k|t}$, for the n assets. We construct this matrix using the techniques presented in Lunde *et al.* (2016). This composite covariance matrix has the typical element

$$H_{i,j,t+k|t} = \rho_{i,j,t+k|t}\sqrt{h_{i,t+k|t}h_{j,t+k|t}}.$$

Several methods can be applied to ensure that $H_{i,j,t+k|t}$ is positive definite. Hautsch *et al.* (2012) and Lunde *et al.* (2016) rely on eigenvalue cleaning. We choose to solve the problem by applying a short-selling constraint, as discussed in Jagannathan & Ma (2003).

The elementwise construction of the covariance matrix helps us avoiding the challenges arising from the estimation of the full covariance matrix, see e.g. Hayashi & Yoshida (2005), Aït-Sahalia *et al.* (2010), Christensen *et al.* (2010), Zhang (2011), Fan *et al.* (2012), Bibinger *et al.* (2014), and Engle *et al.* (2017).

3.4 ALTERNATIVES TO THE RBG MODEL

We produce forecasts of the covariance matrix based on the *RBG* model and we are interested in the model's implications for portfolio selection. We will compare the performance of the resulting portfolio to the performances of portfolios based on nine other forecasting models of the covariance matrix.

The competing models can be classified into two groups, where models in the first group rely on daily information and models in the second group, like the *RBG* model, rely on daily information as well as realized measures.

3.4.a Models with Daily Information Only

The first model is very simple and heavily used in the industry. We call the model $RW.r^2$. The forecast of the covariance matrix is simply the covariance matrix of the daily returns calculated based on a rolling 60-day window. The $RW.r^2$ model places the same weight on observations from the previous day as on observations from 60 days ago. This might be too restrictive. Therefore, we also consider a 60-day rolling window model, where the observations are weighted according to the RiskMetrics model, see Mina & Xiao (2001). We call this model $RWRM.r^2$ and choose an exponential decay rate of 0.97. The third model is the Dynamic Conditional Correlation GARCH of Engle (2002a), referred to as the *DCC* model in this paper. We also consider a version of the *DCC* model, which is based on the McGyver approach, where the covariance matrix is constructed from covariance forecasts from bivariate models. This model is referred to as *DCC*.2.

3.4.b Models with Daily Information and Realized Measures

The first two models we consider are very simple and based exclusively on the multivariate realized kernel. The first, $RW.RK$, is based on the realized kernel based on a 60-day rolling window. Likewise, the $RWRM.RK$ is based on a 60-day rolling window, where observations are weighted with an exponentially decaying function. We also consider the scalar and the diagonal versions of the $HEAVY$ models of Noureldin *et al.* (2012). We denote the scalar version $HEAVY.sc$, and the diagonal version $HEAVY.dg$, respectively.

As mentioned in Section 3.2 we also consider a restricted version of RBG, which we call $RBG.r$. The two $HEAVY$ models, $HEAVY.sc$ and $HEAVY.dg$ rely on the same methodology to construct a composite covariance matrix from pairwise correlations as the RBG and $RBG.r$ models.

4. Performance Evaluation and Economic Gains

The true test of our forecasts of the covariance matrix is whether this forecast will allow an investor to make better decisions with respect to portfolio selection. We consider an investor who chooses $n = 7$ portfolio weights for a portfolio consisting of six different commodity futures and the S&P 500 E-mini futures. She follows a dynamic trading strategy, where she obtains a forecast of the covariance matrix for the next day, $H_{t+1|t}$, and chooses portfolio weights, w_t, accordingly to rebalance her portfolio every day.[6] We assume that the investor has to obey very general restrictions on short selling and

[6]Weekly or monthly rebalancing could also be considered but the daily evaluation allows for the longest evaluation period. Results for weekly rebalancing are presented in Appendix C.

is limited to a certain amount of turnover. The investor faces the following problem.

$$\min_{w_t} w_t' H_{t+1|t} w_t$$

$$s.t. \quad w_t' \iota = 1, \tag{4}$$

$$|w_t'| \, \iota \leq 1 + 2s, \tag{5}$$

$$|w_{kt}| \leq \bar{w}, \quad k = 1, \ldots, n \tag{6}$$

$$w_t' \mu_t \geq \mu_0, \tag{7}$$

$$TO_t \leq \delta, \tag{8}$$

where ι is an $n \times 1$ vector of ones. (4) ensures that portfolio weights sum to one. (5) is a short-selling constraint, where s determines the percentage of short positions allowed as presented in Lunde *et al.* (2016). If $s = 0$, no short selling is allowed. (6) ensures that no single asset gets too large a weight in the portfolio. (7) specifies a minimum required return of the portfolio, given the vector of expected returns, μ_t. (8) is constraining the turnover in the portfolio, where turnover is defined as

$$TO_t = \sum_{i=1}^{n} \left| w_{i,t+1} - w_{i,t} \left(\frac{1 + r_{i,t+1}}{1 + r_{p,t+1}} \right) \right|. \tag{9}$$

Alternatively, the investor might choose an equal weighted portfolio, also called a $1/n$ portfolio. This portfolio has been documented to have good performance, see DeMiguel *et al.* (2009), and we include it as a benchmark in our analysis.

The portfolio exercise is repeated following a rolling window scheme, where the models are estimated based on an estimation window and the portfolio is formed out of sample. Estimation is based on 750 days, meaning that the first portfolio return is realized on December 17, 2009 and the last on December 31, 2014. This leaves us with 1294 daily portfolio returns in our evaluation period.

4.1 THE MINIMUM VARIANCE PORTFOLIO

The global minimum variance portfolio is often considered for economic evaluation in the literature. As argued by Ledoit & Wolf (2018) and Engle *et al.* (2017) it presents a clean problem, in the sense that the performance of a model for the covariance matrix is not influenced by the estimation of expected returns. In addition to the portfolio volatility, the out-of-sample Sharpe ratio of the minimum variance portfolio has been analysed by Haugen & Baker (1991), Jagannathan & Ma (2003), Nielsen & Aylursubraminian (2008), Ledoit & Wolf (2018) and Engle *et al.* (2017). Finally, Ledoit & Wolf (2018) and Engle *et al.* (2017) note that mutual funds are now offering global minimum variance products to their investors. In this analysis we do not consider the global minimum variance portfolio in its strictest sense, as we need to impose the short-selling constraint in (5) to ensure invertibility of the covariance matrix as discussed in Jagannathan & Ma (2003). As the investor in the classical minimum variance problem is not constrained in terms of short selling or turnover, we choose $\bar{w}$ and δ very high to make sure that (6) and (8) are non-binding and we ignore the constraint in (7).

For each day in the out-of-sample evaluation period, portfolio weights, w_t, are constructed based on the covariance forecasts from each model. We obtain time series of portfolio returns for all the models, and we base the evaluation of the models on the realized volatility, $\sigma_{p,t+1} = \sqrt{w_t' RC_{t:t+1} w_t}$, of the returns over the evaluation period. We present the average realized portfolio volatility, $\bar{\sigma}_p$, as well as the ratio of the average of the relalized portfolio volatility to the average of the realized volatility of a portfolio consisting exclusively of S&P 500 futures, $VR = \bar{\sigma}_p / \bar{\sigma}_{S\&P}$.

Comparing means of realized volatilities for eleven different models is problematic as it involves a multiple comparison problem. We suggest to use the Model Confidence Set, MCS, of Hansen *et al.* (2011) to compare the time series of realized volatilities from the different models. The MCS will provide us with a set of models, which includes the best model (the lowest realized volatilities) with a given probability. The size of this set is data dependent and it could contain all the models or any other (non-empty) subset of the models, even just a single model. The MCS results are in the form of a set of p-values, one for each model. Low p-values, say below 0.1, indicate that the corresponding model can be excluded from the set of the best models. We denote the p-values estimated based on realized volatilities as $p_{MCS}(\sigma_{p,t})$.

Table 2 presents the average annualized realized portfolio volatilities $\bar{\sigma}_p$ along with the MCS p-values, $p_{MCS}(\sigma_{p,t})$, for the eleven different models. Results are presented for three different short-selling constraints, $s = 0\%$, $s = 25\%$, and $s = 50\%$. These portfolios are not based on any turnover constraints.

The results in Table 2 shows that the RBG and $RBG.r$ models perform very well. The two models lead to the lowest realized portfolio volatilities and for no short-selling ($s = 0\%$) they are both included in the MCS as the only models. When short selling is allowed ($s = 25\%$ and $s = 50\%$), the $RBG.r$ model is the only model in the MCS based on a 10% level. For all models, except the $1/n$ portfolio and for all three levels of short selling, the variance ratios are below one indicating that there are benefits from diversification compared to a portfolio consisting only of S&P 500. The DCC model is the best performer of the models, which rely on daily information only. Finally, we see that the $1/n$ portfolio leads to the worst performance in this case, where we only consider the variance. The variance ratio is higher than one, which illustrates that simply adding commodities is not enough. In order to obtain diversification benefits the covariance matrix must be estimated. The results also highlight that the McGyver style modeling alone is not driving the results. The $HEAVY.sc$, $HEAVY.dg$, and $DCC.2$ models all apply the McGyver style modeling approach, but underperform compared to the RBG and $RBG.r$ models. The fact that the RBG and

RBG.r models deliver similar results indicate that modeling the volatility spillover is not very important in this particular exercise.

[Table 2 about here.]

4.1.a Short Selling and Turnover

The results in Table 2 are not subject to any turnover constraints.[7] In Table 3 we present the MCS p-values corresponding to the RBG model and investigate effects of varying short-selling constraints, s, and turnover constraints, δ.

Table 2 shows that the *RBG* model performs very well under strict short-selling constraints. When $s = 0\%$ the *RBG* model is always included in the MCS and is the last model in the set. For low values of δ, the *RBG* model remains in the MCS as the amount of short selling allowed increases. For liberal short-selling and turnover constraints, the *RBG* model is excluded from the MCS based on a 10% level. In all these cases, the *RBG.r* model is the last model in the MCS (p-values are not reported here).

[Table 3 about here.]

4.1.b Time-Variation of Relative Performance of Different Models

In the very influential paper of Welch & Goyal (2008), plots of cumulative errors are used to analyze performance of different return prediction models and to pin point periods in which a particular model performs very well or very poorly. We suggest to use a very similar graphical device to compare the performance of portfolios based on forecasts from different models. We also suggest using this approach to compare performance of a portfolio, when considering different short-selling constraints. We

[7] $\delta = 100$ such that (8) is non-binding.

plot the cumulative realized volatility of a portfolio, $\sigma_{p,t,cum} = \sum_{\tau=1}^{t} \sigma_{p,\tau}$, over time. The cumulative realized portfolio volatility is not an intuitive number in its own right, but it allows us to compare the performance of different portfolios. We suggest to augment the graphical approach of Welch & Goyal (2008) with a second plot, which highlights performance within a particular year.

In Figure 2 we present the cumulated volatility for the minimum variance portfolio. The figure presents results for the minimum variance portfolio based on three different models, the *RBG* model, the *DCC* model, and the *HEAVY.dg* model, the bottom panel additionally includes a portfolio consisting exclusively of S&P 500 futures contracts. The *DCC* model is chosen as it results in the lowest volatility of all the models using daily data. The *HEAVY.dg* model is chosen as it results in the lowest volatility of all the existing high-frequency models. The portfolios are formed without short-selling constraints and are not subject to any turnover constraints. The bottom plot highlights the relative performance of the portfolios within the individual years by resetting the cumulative volatility every year.

The top panel of Figure 2 shows that a lower portfolio volatility can be obtained by using the *RBG* model. The cumulated volatility of the portfolio based on the *RBG* model increases more slowly than the volatilities of the portfolios based on the competing models throughout the sample. The bottom panel shows that the good performance of the *RBG* model is not caused by any particular subperiod with good performance, but that it consistently leads to lower volatility year after year. We can also see that including the commodities in the portfolio leads to a lower volatility than for a portfolio consisting only of S&P 500 futures. The diversification benefits are clear in every year in our evaluation sample and in 2010 and 2011 the improvements are quite dramatic.

[Figure 2 about here.]

The results in Figure 2 are based on an assumption of no limits to short-selling. In Figure 3 we investigate the effects of short-selling constraints on the volatility of the portfolio based on the *RBG* model.

The top panel of Figure 3 shows that the diversification benefits are largest when the investor has some flexibility in terms of taking short positions. Interestingly we see that the portfolio volatility is essentially identical for $s = 25\%$ and $s = 50\%$ indicating that the constraint is non-binding for a large part of the evaluation period. In the bottom panel we see that the effects of allowing short selling are largest in the first three years of the evaluation period, whereas the three volatilities are essentially the same in the last two years.

[Figure 3 about here.]

We conclude, that if the objective of the investor is to minimize the volatility of the portfolio, then commodities should be included in the portfolio. The covariance matrix, which is used to find the portfolio weights, should be based on either the *RBG* or the *RBGr* model. These results are robust to various constraints regarding short selling and turnover.

4.1.c Squared Returns

The results in Table 2, Table 3, Figure 2, and Figure 3 are all based on realized portfolio volatility, $\sigma_{p,t+1} = \sqrt{\omega_t' RC_{t:t+1} \omega_t}$. In Appendix C we present the results of Table 2, where the portfolio variance is estimated by daily squared returns.

In general we find that the models using daily data perform best when the portfolio is evaluated based on daily returns, and that the high-frequency models perform best when evaluation is based on high-frequency returns.

4.2 MOMENTUM STRATEGY

To implement any strategy, where (7) is considered, one must specify a model for the expected return vector, μ_t, and specify a target return μ_0. Following Ledoit & Wolf (2018) and Engle *et al.* (2017) one can use the momentum factor of Jagadeesh & Titman (1993) to specify μ_t and choose a value for μ_0.[8] We choose the target return to be the average of the momentum for all the assets, if this quantity is positive, that is $\mu_0 = \max(0, \bar{\mu})$.

4.2.a Financial Evaluation

For each model we solve the portfolio problem for different values of s and δ and obtain daily returns for the momentum portfolio. Following Ledoit & Wolf (2018) and Engle *et al.* (2017) we present, for each model, the out-of-sample Sharpe Ratio, SR, calculated as the ratio of the the average out-of-sample return and the out-of-sample standard deviation of the returns.[9] We also compute the Sharpe Ratio of a portfolio consisting exclusively of S&P 500 futures, $SR_{S\&P}$, and present the ratio of $SR/SR_{S\&P}$. The limitations of the Sharpe ratio, as presented in e.g. Marquering & Verbeek (2004), Colacito & Engle (2006), and King *et al.* (2010) are well understood and an economic evaluation of model performance cannot be based on this ratio alone.

In order to quantify the difference to the $1/n$ portfolio we present the mean absolute deviation from equal-weighted portfolio weights, MAD, which is computed as

$$MAD = \frac{1}{n} \sum_{i=1}^{n} \left| w_i - \frac{1}{n} \right| .$$

[8]At time t we define momentum for asset i as the geometric average of the previous 12 monthly returns on the asset, but exclude the most recent month.

[9]This measure is called the Information Ratio (IR) in Engle *et al.* (2017).

To asses the economic significane of our proposed framework we follow the tradition of West *et al.* (1993), Fleming *et al.* (2001), Rime *et al.* (2010), and Karstanje *et al.* (2013) and assume quadratic utility, which will allow us to further evaluate the economic performance of each model. Assume that the wealth, W, of the investor evolves according to the following

$$W_{t+1} = W_t \left(1 + r_{p,t+1}\right),$$

then, under the assumption of quadratic utility, the utility of the investor at the end of period $t+1$ is

$$U\left(W_{t+1}\right) = W_{t+1} - \frac{\rho}{2} W_{t+1}^2 = W_t \left(1 + r_{p,t+1}\right) - \frac{\rho}{2} W_t \left(1 + r_{p,t+1}\right)^2,$$

where ρ is a parameter controlling the risk preferences of the investor. The amount of wealth invested each period is assumed to be constant meaning the ρ relates to the relative risk aversion, γ, as $\gamma = \frac{\rho W}{1-\rho W}$. The initial wealth, W_0, is set to 1. For this investor the average realized utility is

$$U = \frac{1}{T} \sum_{t=0}^{T-1} U_{t+1} = \frac{1}{T} \sum_{t=0}^{T-1} \left(1 + r_{p,t+1} - \frac{\gamma}{2(1+\gamma)} \left(1 + r_{p,t+1}\right)^2\right)$$

which is shown by West *et al.* (1993) to provide a consistent estimate of expected utility.

Methods based on realized measures often lead to very high turnover in the portfolio. For each forecasting method we present the average turnover, TO, defined as the average over time of the expression in (9). While turnover is interesting in its own right, it is also very important in the context of transaction costs. Transaction costs have been analyzed by Marquering & Verbeek (2004), Han (2006), King *et al.* (2010), and Karstanje *et al.* (2013). In the presence of fixed proportional transaction costs, τ, the average realized utility can be expressed as

$$\frac{1}{T}\sum_{t=0}^{T-1}\left(1+r_{p,t+1}-\tau TO_{t+1}-\frac{\gamma}{2(1+\gamma)}\left(1+r_{p,t+1}-\tau TO_{t+1}\right)^2\right).$$

Specifically, we can find τ^{be}, which is the break-even transaction cost. That is, τ^{be} is the proportional cost per period, which eliminates any utility from using a specific model to construct portfolio weights. If transaction costs are higher than τ^{be}, the strategy is too expensive and should not be undertaken.

We suggest to compare the realized utility from the different forecasting models by applying the MCS. We estimate the MCS based on the time series of realized utility from each model, and denote the corresponding p-values $p_{MCS}(U_t)$.

In Table 4 we present SR, $SR/SR_{S\&P}$, MAD, TO, τ^{be}, and $p_{MCS}(U_t)$ for momentum strategies based on the eleven different models. When forming the portfolios we are allowing for some degree of short selling, $s = 0.5$, and we do not impose any turnover restrictions.

The first column of Table 4 shows that using the RBG model leads to a much higher annualized Sharpe Ratio than any of the other models. The second column shows that only the RBG model, the DCC model, and the $RM.r^2$ model lead to higher Sharpe Ratios than a portfolio consisting of only S&P 500 futures. The equal weighted portfolio performs worst of all the models in this exercise. The large difference in performance between the RBG model and the $RBGr$ model indicates that the volatility spillover effect is particularly important in this context. The question is, whether the good performance comes at the price of too high turnover in the portfolio. The fourth column shows that turnover is indeed highest for the RBG model. The fifth column indicates, however, that the average realized utility from using the RBG model will remain positive for relatively high levels of trading costs. Finally, the sixth column shows that the

RBG model leads to the highest realized utility. When realized utilities are compared using the MCS, the *RBG* model is the only model in the set.

[Table 4 about here.]

4.2.b Short Selling and Turnover

The results in Table 4 are based on the assumption that $s = 0.5$, indicating that some degree of short selling is allowed. The results further assume that the turnover constraint is not binding, δ is high. In Table 5 we investigate the effects of these assumptions and consider the out of sample Sharpe Ratio of the momentum portfolio based on the *RBG* model for different combinations of s and δ.

The results in Table 5 show that in order to benefit from a better model for the covariance matrix, the investor must be allowed a certain turnover in the portfolio and also some degree of short selling. Not surprisingly, we find the best performance, when turnover and short selling constraints are quite liberal. Interestingly the *RBG* model can perform decently, outperforming the S&P 500 portfolio (average annual SR of 0.82) even when no short-selling is allowed, as long as limits to turnover are not too strict. In fact turnover restrictions seem to matter more than short-selling restrictions as the model is struggling to perform for high values of s, when δ is low.

[Table 5 about here.]

4.2.c Performance Fee

Finally, we can use the average realized utility to compare two models, say m_1 and m_2. This approach is also considered by Karstanje *et al.* (2013). We assume that we include fixed costs in every period equal to ϕ in the average realized utility for the returns based on m_2. By equating the average realized utility of the two models, we can evaluate how much the investor is willing to pay to switch from m_1 to m_2. We interpret ϕ as a performance fee and choose it to ensure that

$$\sum_{t=0}^{T-1}\left(1+r_{p,t+1}^{m_1}-\frac{\gamma}{2(1+\gamma)}\left(1+r_{p,t+1}^{m_1}\right)^2\right)$$
$$=\sum_{t=0}^{T-1}\left(1+r_{p,t+1}^{m_2}-\phi-\frac{\gamma}{2(1+\gamma)}\left(1+r_{p,t+1}^{m_2}-\phi\right)^2\right), \tag{10}$$

where $r_{p,t+1}^{m_1}$ and $r_{p,t+1}^{m_2}$ denote the returns at time $t+1$ from the portfolio based on forecasts from models m_1 and m_2, respectively. A positive value of ϕ will mean that an investor will pay to use m_2 in stead of m_1. A negative value of ϕ indicates that an investor requires compensation to use m_2 in stead of m_1.

Table 6 presents the performance fees, which an investor, with preferences specified as above, would be willing to pay to switch from a model in the rows to a model in the columns. These results are based on the momentum strategy where some short selling is allowed, $s = 0.5$, and there are no turnover constraints.

The first row of Table 6 contains only negative numbers. This means that an investor with the stated preferences would have to be compensated to use any other model than the *RBG* model. The compensation ranges from 4 to 8 bps per day and thus represents a significant quantity in an economic sense. Again we find evidence that the $1/n$ model is inferior to all other models as the last column contains only negative numbers.

[Table 6 about here.]

4.2.d Portfolio Value

The results for the momentum portfolio presented above are largely based on time averages over the evaluation period, but it is also interesting to investigate how the value of the portfolio has evolved over time. Based on the vector of daily returns for the futures contracts, r_{t+1}, and the portfolio weights at time t, ω_t, we can calculate the portfolio return at time $t+1$ as $r_{p,t+1} = \omega_t' r_{t+1}$. The value of the momentum portfolio is analysed in Figure 4, where the cumulated portfolio value, $r_{p,t,cum} = \sum_{\tau=1}^{t} r_{p,\tau}$, is plotted. The sub-plots illustrate the relative performance within a given year. The plot includes values of portfolios based on three different models, the *RBG*, the *DCC* model, and the *HEAVY.dg* model, along with a portfolio consisting exclusively of S&P 500 futures. The *DCC* and *HEAVY.dg* models are chosen as they are the best performing alternative models based on daily returns and high-frequency returns, respectively.

The top panel of Figure 4 clearly shows that the *RBG* model outperforms the existing models during our entire evaluation period. Interestingly, the *HEAVY.dg* and *DCC* models struggle to outperform the S&P 500 portfolio, indicating that good covariance forecasts are extremely important for managing portfolios and obtaining benefits from diversification. In the bottom panel we see that the performance of the *RBG* model compared to the S&P 500 portfolio is particularly strong in 2011. There is evidence of diversification benefits in all years except 2013, where an investor would have been better off by investing only in S&P 500 futures.

[Figure 4 about here.]

5. Conclusion

In this paper we have applied a McGyver style modeling approach based on the Realized Beta GARCH model. We have demonstrated that an investor can obtain significant diversification benefits from including commodities in an equity portfolio if she models and forecasts the covariance matrix using our proposed methodology. Results are robust to various specifications of short-selling and turnover constraints. If the investor considers a momentum strategy we show that using our method leads to the most valuable portfolio as long as the investor is allowed a certain turnover in the portfolio. The results are robust to the introduction of trading costs. Interestingly, our approach clearly outperforms an equal-weighted portfolio in all exercises considered here. This means that diversification benefits are not simply a results of including more assets, but are also heavily affected by our ability to forecast covariances. The Realized Beta GARCH model outperforms other models using the McGyver style modeling approach, meaning that this approach alone is not sufficient to achieve the economically meaningful gains, which we document. Finally, we find that it is very important to allow for volatility spillover between assets when considering a momentum strategy, but less important when the only objective is to minimize portfolio volatility.

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A. Realized measures

Let $p^v_{t,i}$ be the intraday arbitrage-free log-price recorded at time t on day v for asset i and define the corresponding intraday log-returns as

$$r^v_{t_m,i} = p^v_{t_m,i} - p^v_{t_{m-1},i}, \qquad m = 1, ..., n^v_i. \tag{11}$$

Returns for asset i are based on the n^v_i intraday prices recorded on a trading period ranging from after 5:15PM on day $v-1$ until just before 5:15PM on day v, the period of trading considered. Time coordinates are scaled to evolve in the unit interval $[0, 1]$ associated with one period of trading and for each asset we define a partition, $\pi_{n^v_i} = [0 = t^i_0 < t^i_1 < ... < t^i_{n^v_i} = 1]$. If $\forall i, j = 1, ..., d : \pi_{n^v_i} = \pi_{n^v_j}$, for the d assets, the baseline realized covariance among the elements of $r_{v,t} = \{r^v_{t,i}\}_{i=1,...,d}$ over $[0, 1]$ is defined as

$$RC_v := \sum_{m=1}^{n^v} r_{v,t_m} r'_{v,t_m}, \qquad \text{where } n^v = n^v_1 = n^v_2 = ... = n^v_d. \tag{12}$$

High-frequency transactions are not recorded over a homogeneous grid of time coordinates, and synchronization is required to make our selected estimation techniques feasible. To compute realized covariances, a homogeneous grid of evenly spaced prices is created using previous-tick interpolations as introduced in Dacorogna *et al.* (2001). Denoting a fixed grid of times containing n_δ points by $G_\delta = [\delta < ... < m\delta < ... < n_\delta \delta \leq 1]$, where δ denotes the calendar time sampling frequency, synchronous prices are constructed along the n_δ points of the grid as $p^v_{m\delta,i} = p^v_{t_j,i}$, where $j = \max(j' \mid t_{j'} \leq m\delta)$ and $t_j \leq m\delta < t_{j+1}$. Realized covariances are then computed using the returns obtained from the synchronized grid of high-frequency prices and are denoted by $RC^{(\delta)}_v$.

Microstructure biases are treated using two approaches. Firstly, sparse sampling of the realized covariances, $\delta = 15$ minutes, is applied with sub-sampling every 15 second. At this frequency the impact of microstructure noise on realized measures is known to be immaterial (see e.g. Barndorff-Nielsen & Shephard (2007) for details). To the best of our knowledge, microstructure effects induced on realized measures have not been thoroughly analyzed in the literature for commodities and justifies the conservative choice of $RC^{(15\,min)}$, which is very likely to be a noisy but unbiased measure of the integrated covariance matrix. Secondly, the multivariate realized kernel (MRK) introduced by Barndorff-Nielsen *et al.* (2011) is used. MRK is a class of estimators that is robust to measurement errors and microstructure effects induced by asynchronous trading. The MRK is based on a homogeneous series of high-frequency prices that are constructed using refresh time (see Barndorff-Nielsen *et al.* (2011) for details). Specifically, non-flat-top realized kernels are used to ensure positive semi-definiteness,

$$K_v(p) = \sum_{h=-H}^{H} k\left(\frac{h}{H+1}\right)\Gamma_{v,h}, \qquad \Gamma_{v,h} = \sum_{j=|h|+1}^{n^v} r_{v,t_j} r'_{v,t_{j-h}}, \tag{13}$$

where $k(x)$ is the non-stochastic Parzen kernel function, $\Gamma_{v,h}$ is the realized auto covariance function, $H = c^\star \xi^{4/5} n^{3/5}$, $c^\star = \left\{\frac{k''(0)^2}{k_\bullet^{0,0}}\right\}^{1/5}$, $k_\bullet^{0,0} = \int_0^\infty k(x)^2 dx$, and $\xi^2 = \frac{\omega^2}{\sqrt{T\int_0^1 \sigma_u^4 du}}$ which is estimated as in Barndorff-Nielsen *et al.* (2011).

B. Estimation and forecasting

B.1 ESTIMATION

Following Hansen *et al.* (2014) we adopt Gaussian specifications for the marginal and conditional densities implying that the maximum likelihood estimators of the variance-covariance parameters are given by

$$\hat{\sigma}_{u_i}^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_{i,t}^2, \quad \hat{\sigma}_{u_j,u_i} = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_{i,t}\hat{u}_{j,t}, \quad \hat{\sigma}_{v_j,u_i} = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_{i,t}\hat{v}_{j,t}, \tag{14}$$

and

$$\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\hat{U}_{j,t}\hat{U}_{j,t}', \qquad \text{where} \quad \hat{U}_{j,t} = \begin{pmatrix} \hat{u}_{j,t} \\ \hat{v}_{j,t} \end{pmatrix} - \begin{pmatrix} \hat{\sigma}_{u_j,u_i}/\hat{\sigma}_{u_i}^2 \\ \hat{\sigma}_{v_j,u_i}/\hat{\sigma}_{u_i}^2 \end{pmatrix}\hat{u}_{i,t}. \tag{15}$$

The parameters are then estimated by maximizing

$$\ell(\theta) = -\frac{1}{2}\left(\ell_{z_i}(\theta_i) + \ell_{u_i}(\theta_i) + \ell_{z_j|z_i}(\theta) + \ell_{u_j,v_j|u_i}(\theta)\right), \tag{16}$$

where $\theta = \left(\theta_i', \theta_j', \theta_{j,i}'\right)'$ and $\ell_{z_i}(\theta_i) = \sum_{t=1}^{T}\left[\tilde{h}_{i,t}(\theta_i) + z_{i,t}^2(\theta_i)\right]$, $\ell_{u_i}(\theta_i) = T\left[\log\hat{\sigma}_{u_i}^2(\theta_i) + 1\right]$, $\ell_{u_j,v_j|u_i}(\theta) = T\left[\log\det\hat{\Omega}(\theta) + 2\right]$, and

$$\ell_{z_j|z_i}(\theta) = \left(\sum_{t=1}^{T}\log\left\{\left[1-\rho_{j,i,t}^2(\theta)\right]h_{j,t}(\theta)\right\} + \frac{\left(z_{j,t}(\theta) - \rho_{j,i,t}(\theta)\,z_{i,t}(\theta)\right)^2}{1-\rho_{j,i,t}^2(\theta)}\right), \tag{17}$$

as we can compute $\rho_{j,i,t} = F^{-1}\left\{a_{ji} + b_{ji}F\left(\rho_{j,i,t-1}\right) + c_{ji}F\left(y_{j,i,t-1}\right)\right\}$ independently of $h_{j,t}$ and $z_{j,t}$ recursively for $t = 2,\ldots,T$. For details on the multivariate Realized Beta GARCH see Hansen *et al.* (2014).

B.2 FORECASTS

The exposition in the main text presents only one-step ahead forecasts, but point forecasts are readily computable for different steps in the framework of the RBG model. The system of equations follows directly from the previous section

$$
\begin{aligned}
\tilde{h}_{i,t+1} &= a_i + c_i\xi_i + (b_i + c_i\varphi_i)\tilde{h}_{i,t} + c_i\delta_i(z_{i,t}) + \tau_i(z_{i,t}) + c_i u_{i,t} \\
\tilde{h}_{j,t+1} &= a_j + c_j\xi_j + (b_j + c_j\varphi_j)\tilde{h}_{j,t} + c_j\delta_j(z_{j,t}) + \tau_j(z_{j,t}) + c_j u_{j,t} \\
\tilde{\rho}_{j,i,t+1} &= a_{ji} + c_{ji}\xi_{ji} + (b_{ji} + c_{ji}\varphi_{ji})\tilde{\rho}_{j,i,t} + c_{ji}\nu_{j,t},
\end{aligned}
\tag{18}
$$

where $\tilde{\rho}_{j,i,t} := F(\rho_{j,i,t})$, $\delta_k(z_{k,t}) := \delta_{k,1}z_{k,t} + \delta_{k,2}(z_{k,t}^2 - 1)$, $\tau_k(z_{k,t}) := \tau_{k,1}z_{k,t} + \tau_{k,2}(z_{k,t}^2 - 1)$ for $k = i, j$. As pointed out by Hansen *et al.* (2014), the system of equations in (18) is seen to have a VARMA(1,1) representation by writing $\tilde{V}_{t+1} = \left(\tilde{h}_{i,t}, \tilde{h}_{j,t}, \tilde{\rho}_{j,i,t}\right)'$. This implies that

$$
\tilde{V}_{t+1} = C + A\tilde{V}_t + B\epsilon_t, \tag{19}
$$

where $\epsilon_t = (\delta_i(z_{i,t}), \tau_i(z_{i,t}), \delta_j(z_{j,t}), \tau_j(z_{j,t}), u_{i,t}, u_{j,t}, \nu_{j,t})'$ and

$$
C = \begin{bmatrix} a_i + c_i\xi_i \\ a_j + c_j\xi_j \\ a_{ji} + c_{ji}\xi_{ji} \end{bmatrix}, \quad A = \begin{bmatrix} b_i + c_i\varphi_i & 0 & 0 \\ 0 & b_j + c_j\varphi_j & 0 \\ 0 & 0 & b_{ji} + c_{ji}\varphi_{ji} \end{bmatrix}
$$

$$
B = \begin{bmatrix} c_i & 1 & 0 & 0 & c_i & 0 & 0 \\ 0 & 0 & c_j & 1 & 0 & c_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{ji} \end{bmatrix}.
$$

The k-step ahead forecasts can be computed as $\mathrm{E}(\tilde{V}_{t+k}|\mathcal{F}_t) = \tilde{V}_{t+k|t} = A^k\tilde{V}_t + \sum_{j=0}^{k-1} A^j C$. The *RBG* model is expressed using non-linear functions of the objects of interest implying for instance that $\mathrm{E}[F^{-1}(\rho_{j,i,t+k})|\mathcal{F}_t] \neq F^{-1}(\mathrm{E}[\rho_{j,i,t+k}|\mathcal{F}_t])$. This require us to base forecasts of volatilities and correlations on simulation methods or alternatively on a bootstrapping procedure. We apply the simulation based approach on which more details are provided in the section below.

B.2.a Simulation Approach

Let $V_t = \left(h_{j,t}, h_{i,t}, \rho_{i,j,t}\right)'$ denote the vector of non-transformed variables, the function $f : \mathbb{R}^2 \times [0,1] \curvearrowright \mathbb{R}^3$ such that $f^{-1}(\tilde{V}_t) = V_t$, and start from the VARMA(1,1) specification in Eq. (19). From this, one can recursively construct point forecasts as

$$\tilde{V}_{t+k} = C + A\tilde{V}_{t+k-1} + B\epsilon_{t+k-1}. \tag{20}$$

The one-step ahead forecast $V_{t+1|t}$ does not require simulation since it is $\mathcal{F}_t$-measurable. For $k > 1$, $V_{t+k|t}$ is computed based on simulations as $\frac{1}{S}\sum_{s=1}^{S} f^{-1}(\tilde{V}^s_{t+k|t})$, where $\tilde{V}^s_{t+k|t}$ is obtained from $\tilde{V}^s_{t+k} = C + A\tilde{V}_{t+k-1} + B\epsilon^s_{t+k-1}$. ϵ^s_{t+k-1} is constructed by sampling the residuals of the RBG model from a conditional Gaussian distribution

$$\zeta_{t+k} := \begin{pmatrix} z_{i,t+k} \\ z_{j,t+k} \\ u_{i,t+k} \\ u_{j,t+k} \\ \nu_{j,t+k} \end{pmatrix} \sim N_5 \left(0, \begin{bmatrix} I_2 & 0 \\ 0 & \Sigma \end{bmatrix}\right), \qquad t = 1, ..., N. \tag{21}$$

The simulation is performed using an estimate of the matrix Σ. Residuals for commodity j, which are correlated with commodity i, can be computed using the law of motion of correlation in Eq. (2) together with the simulated values for $\nu_{j,t+k}$ and defining

$$z_{j,t+k} := \rho_{j,i,t+k} z_{i,t+k} + \sqrt{1 - \rho^2_{j,i,t+k}}\breve{z}_{j,t+k}. \tag{22}$$

The assumptions regarding the distribution of ζ_t might be called into question and a parametric bootstrap may be preferable in some instances. The empirical properties of financial returns on equities standardized by realized measures do, however, have an empirical density close to normal (see e.g. Andersen *et al.* (2003)). We assume that this is also the case for commodities.

C. Additional Results

C.1 SQUARED RETURNS

Table 7 presents results for the minimum variance portfolio, where the portfolio is evaluated based on daily squared returns.

[Table 7 about here.]

C.2 WEEKLY REBALANCING

Table 8 presents results for the minimum variance portfolio based on realized volatility, when the portfolio is rebalanced every week.

[Table 8 about here.]

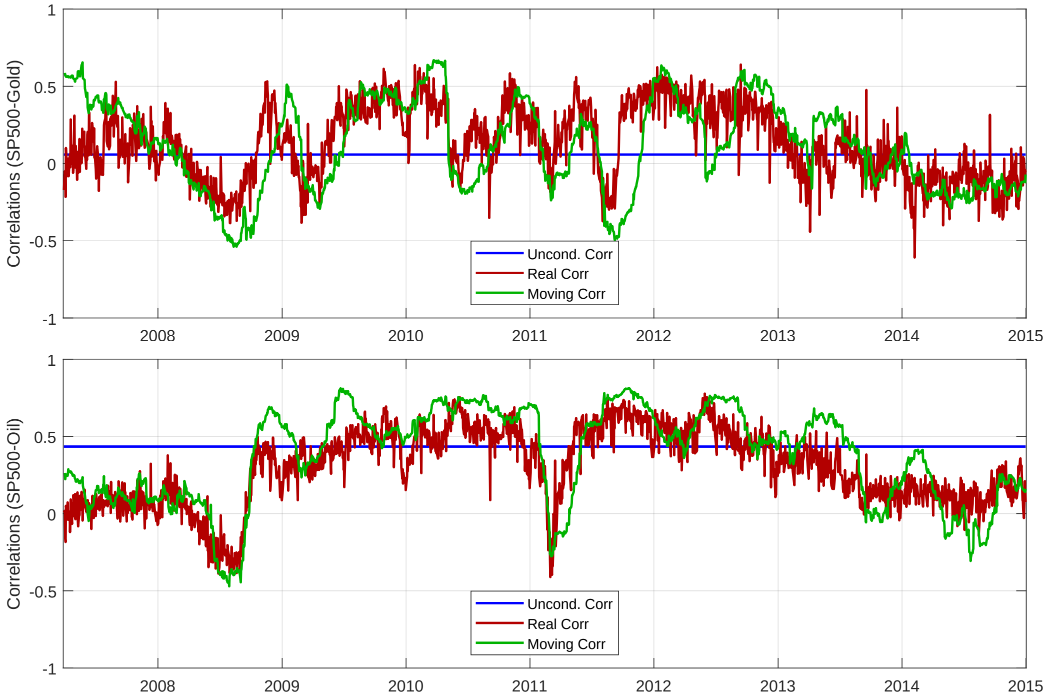

Figure 1: Time-varying correlations. The figure contains plots of correlations between the S&P 500 E-mini futures prices and prices for two different commodities futures prices, gold in the top panel and oil in the bottom panel, respectively. Each panel presents three different measures of the correlation between the S&P 500 futures price and the commodity futures price. The blue line represents the sample correlation based on the full sample of daily observations. The green line represents the sample correlation calculated on a moving window of 60 daily observations. Finally, the red line represents the realized correlation based on the realized kernel and relying on the full sample of high-frequency data.

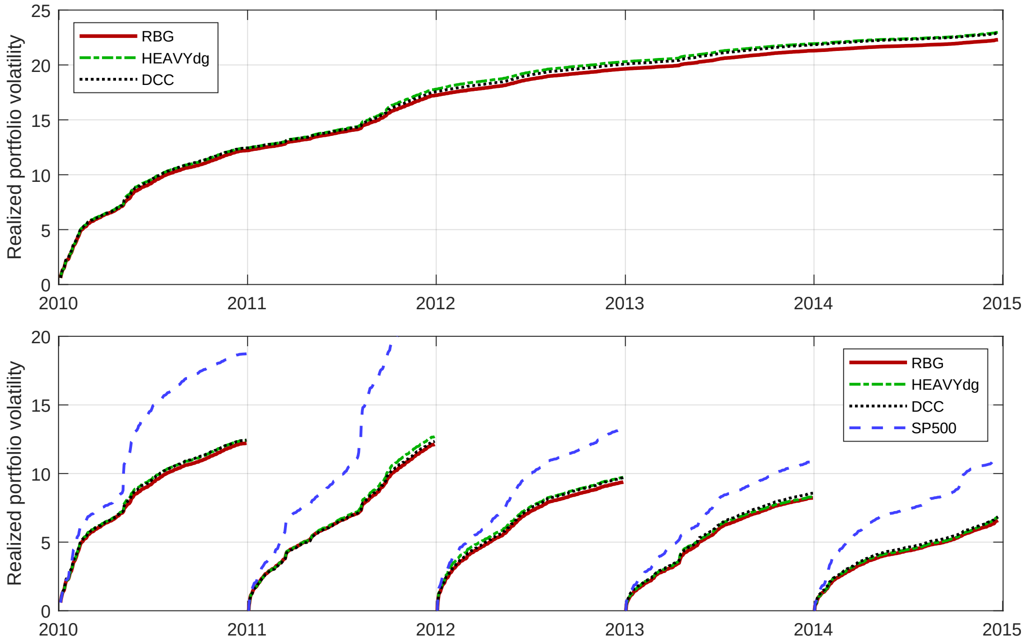

Figure 2: The figure compares the cumulated volatilities of the minimum variance portfolios based on forecasts from three different models. The cumulated volatility, $\sigma_{p,t,cum} = \sum_{\tau=1}^{t} \sigma_{p,\tau}$, is based on the realized portfolio volatility, $\sigma_{p,t+1} = \sqrt{\omega_t' RC_{t:t+1} \omega_t}$. Portfolio volatilities are based on forecasts for the RBG model, the DCC model, and the $HEAVYdg$ model. The figure also presents the cumulated volatility for a portfolio consisting of S&P 500 futures. The portfolios are not subject to any short-selling or turnover constraints. The bottom plot presents results of the analysis of the main plot, when carried out on a yearly basis.

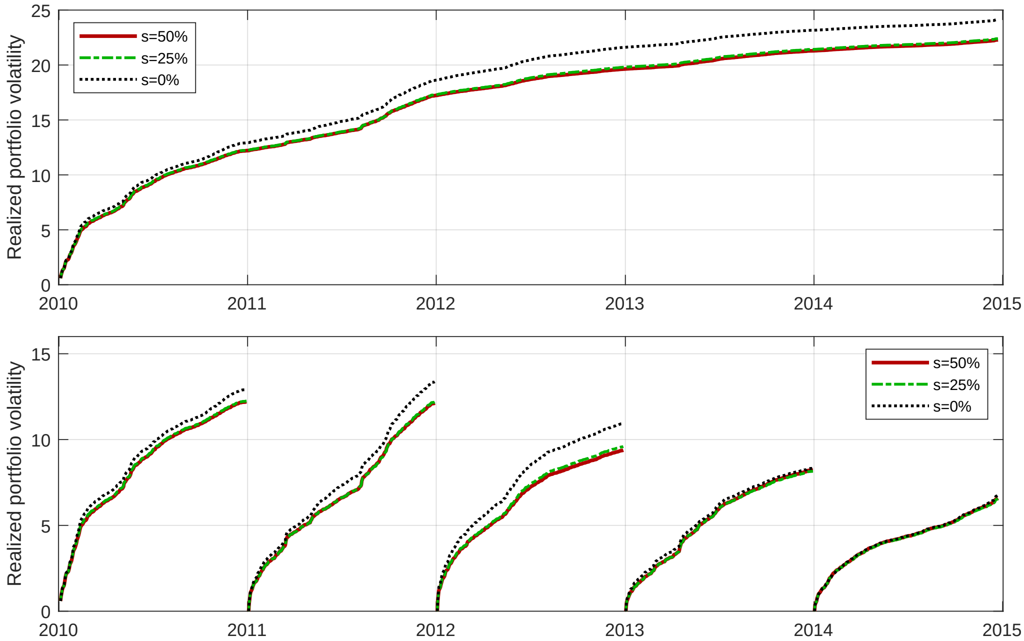

Figure 3: The figure compares the cumulated volatility of the minimum variance portfolios based on forecasts from the *RBG* model for different short-selling constraints. The cumulated volatility, $\sigma_{p,t,cum} = \sum_{\tau=1}^{t} \sigma_{p,\tau}$, is based on the realized portfolio volatility, $\sigma_{p,t+1} = \sqrt{\omega_t' RC_{t:t+1} \omega_t}$. Portfolio volatilities are based on forecasts for the *RBG* model, where the different lines represent different short-selling constraints, and the results are not subject to any turnover constraints. The bottom panel presents results of the analysis of the main plot, when carried out on a yearly basis.

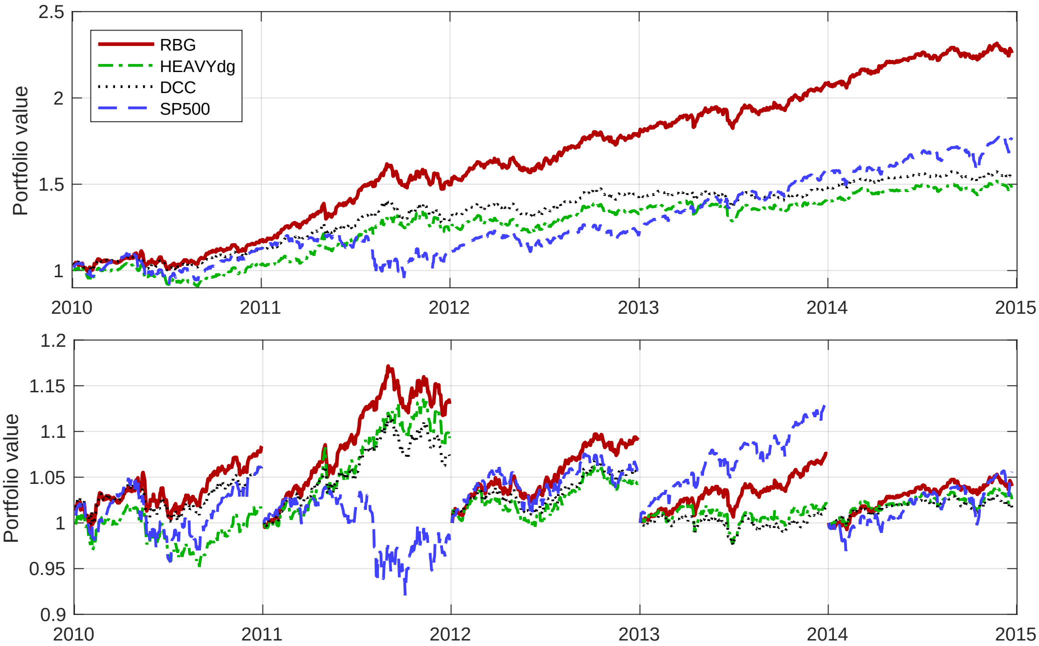

Figure 4: The figure compares the cumulated portfolio value of the momentum portfolio based on forecasts from three different models. The cumulated portfolio value, $r_{p,t,cum} = \sum_{\tau=1}^{t} r_{p,\tau}$, is based on daily portfolio value, $r_{p,t+1} = \omega_t' r_{t+1}$. Portfolio weights are based on the forecasts for the RBG model, the DCC model, and the $HEAVY.dg$ model. The figure also presents the cumulated value for a portfolio consisting of S&P 500 futures. A certain degree of short-selling is allowed, $s = 0.5$, but the results are not subject to any turnover constraints. The sub-plots presents results of the analysis of the main plot, when carried out on a yearly basis.

Table 1: Futures contracts.

Contract	First Trade	Exchange	Break	Average Daily Trades				Realized Covariance
				2005	2007	2010	2013	S&P 500
Equity Index								
S&P 500 (ES)	1997	COMEX	5.15-6.00PM	100,289	112,620	447,970	394,755	1.00
Metal								
Copper (HC)	1989	COMEX	5.15-6.00PM	1,313	4,097	19,513	34,511	0.86
Gold (GC)	1984	COMEX	5.15-6.00PM	4,687	19,892	65,384	103,725	0.21
Silver (SI)	1983	COMEX	5.15-6.00PM	2,900	6,817	22,400	32,341	0.53
Energy								
Heating Oil (HO)	1984	NYMEX	5.15-6.00PM	1,525	10,691	17,401	21,576	0.69
Light Crude (CL)	1987	NYMEX	5.15-6.00PM	3,962	50,425	135,008	129,535	0.98
Natural Gas (NG)	1993	NYMEX	5.15-6.00PM	2,290	15,500	41,064	54,539	0.31

The table presents contract specific information for seven futures contracts. The first column divides the contracts into three groups and specifies the underlying asset and the corresponding symbol in parenthesis. NYMEX and COMEX are both part of the CME Group. All times are eastern standard times. Average daily trade numbers are calculated over the respective calendar year from all intraday trades. Average realized covariances with S&P 500 E-minis are calculated based on realized close-close returns for the entire sample period.

Table 2: Realized volatility of the minimum variance portfolio for different short-selling constraints.

	$s = 0\%$			$s = 25\%$			$s = 50\%$		
	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$
RBG	**10.74**	0.77	[1.0000]	10.04	0.72	[0.0766]	10.03	0.72	[0.0332]
RBG.r	**10.74**	0.77	[0.9998]	**10.01**	0.72	[1.0000]	**10.00**	0.72	[1.0000]
HEAVY.sc	11.49	0.82	[0.0000]	10.70	0.77	[0.0000]	10.72	0.77	[0.0000]
HEAVY.dg	11.42	0.82	[0.0000]	10.61	0.76	[0.0000]	10.62	0.76	[0.0000]
MW.RK	11.23	0.80	[0.0000]	10.54	0.75	[0.0000]	10.53	0.75	[0.0008]
RM.RK	11.11	0.79	[0.0000]	10.39	0.74	[0.0000]	10.38	0.74	[0.0008]
DCC	11.06	0.79	[0.0000]	10.39	0.74	[0.0000]	10.37	0.74	[0.0000]
DCC.2	11.09	0.79	[0.0000]	10.46	0.75	[0.0000]	10.46	0.75	[0.0000]
$MW.r^2$	11.38	0.81	[0.0000]	10.78	0.77	[0.0000]	10.77	0.77	[0.0000]
$RM.r^2$	11.24	0.80	[0.0000]	10.61	0.76	[0.0000]	10.60	0.76	[0.0000]
$1/n$	15.43	1.10	[0.0000]	15.43	1.10	[0.0000]	15.43	1.10	[0.0000]

The table presents the average volatility of the minimum variance portfolio based on forecast for 11 different models. The volatility estimate at time $t+1$, $\sigma_{p,t+1}$, is based on the portfolio weights obtained from the forecast of the covariance matrix for each model at time t and the realized covariance matrix of futures prices, $RC_{t:t+1}$, such that $\sigma_{p,t+1} = \sqrt{\omega_t' RC_{t:t+1} \omega_t}$. In the table we present the average of $\sigma_{p,t}$ taken over the full sample, and denoted $\bar{\sigma}_p$. VR denotes the ratio of $\bar{\sigma}_p$ to the average realized volatility of a portfolio consisting exclusively of S&P 500 futures. The table also presents the p-values, $p_{MCS}(\sigma_{p,t})$, of the Model Confidence Set, which is based on the time series of realized portfolio volatility. The columns present results for three different short-selling constraints. None of the results are subject to turnover constraints.

Table 3: MCS p-values for portfolio volatility for different short-selling and turnover constraints.

	$s = 0\%$	$s = 25\%$	$s = 50\%$
$\delta = 0.01$	**1.0000**	**0.5210**	**0.1072**
$\delta = 0.05$	**1.0000**	**1.0000**	**1.0000**
$\delta = 0.1$	**1.0000**	**1.0000**	**1.0000**
$\delta = 0.5$	**1.0000**	0.0810	0.0314
$\delta = 1$	**1.0000**	0.0818	0.0342
$\delta = 100$	**1.0000**	0.0766	0.0332

p-values for the RBG model. For each model the volatility estimate at time $t+1$, $\sigma_{p,t+1}$, is based on the portfolio weights obtained from the forecast of the covariance matrix at time t and the realized covariance matrix of futures prices, $RC_{t:t+1}$, such that $\sigma_{p,t+1} = \sqrt{\omega_t' RC_{t:t+1} \omega_t}$. The time series for realized portfolio volatility resulting from the different models are compared using the Model Confidence Set. The columns present results for three different short-selling constraints while the rows contain different turnover constraints.

Table 4: Financial performance measures for the momentum portfolio for all models, $s = 0.5$, $\delta = 100$.

	SR	$SR/SR_{S\&P}$	MAD	TO	τ^{be}	$p_{MCS}(U_t)$
RBG	1.735	2.108	0.191	0.157	0.336	1.000
$RBG.r$	0.795	0.965	0.190	0.156	0.132	0.000
$HEAVY.sc$	0.642	0.780	0.209	0.085	0.179	0.000
$HEAVY.dg$	0.627	0.761	0.208	0.091	0.159	0.000
$MW.RK$	0.394	0.478	0.171	0.046	0.099	0.000
$RM.RK$	0.441	0.536	0.173	0.047	0.142	0.000
DCC	0.877	1.065	0.191	0.125	0.179	0.000
$DCC.2$	0.800	0.972	0.193	0.152	0.128	0.000
$MW.r^2$	0.814	0.989	0.183	0.091	0.236	0.001
$RM.r^2$	0.980	1.191	0.185	0.095	0.282	0.001
$1/n$	0.193	0.234	0.000	0.009	-1.510	0.000

The table presents financial measures of the performances of the momentum portfolios, where portfolio weights are based on the forecasts for 11 different models. The column labeled SR contains the out of sample Sharpe ratio of the portfolio, calculated as the ratio of average daily portfolio returns and the corresponding standard deviation. The column labeled $SR/SR_{S\&P}$ presents the ratio of the Sharpe Ratio of a portfolio to the Sharpe Ration of a portfolio consisting exclusively of S&P 500 futures. MAD is the mean absolute deviation of the portfolio weights from a $1/n$ portfolio. The column labeled τ^{be} presents the minimum average proportional trading costs, which will eliminate any gains from forming a portfolio based on forecasts from a particular model. Finally, $p_{MCS}(U_t)$ presents the p-values of the Model Confidence Set estimated based on the time series of realized utility from the momentum portfolios based on the different models. A certain degree of short selling is allowed, $s = 0.5$, and the results are not subject to any turnover constraints.

Table 5: Sharpe Ratios for the momentum portfolio for different short selling and turnover constraints.

	$s = 0\%$	$s = 25\%$	$s = 50\%$
$\delta = 0.01$	0.4986	0.5183	0.4980
$\delta = 0.05$	0.4815	0.4215	0.3537
$\delta = 0.1$	0.5697	0.6000	0.5730
$\delta = 0.5$	0.8733	1.5025	1.5694
$\delta = 1$	0.9105	1.6419	1.7333
$\delta = 100$	0.9154	1.6415	1.7354

The table presents out of sample Sharpe Ratios of the momentum portfolios, where portfolio weights are based on the forecasts from the *RBG* model. The Sharpe Ratio of the portfolio, calculated as the ratio of average daily portfolio returns and the corresponding standard deviation. The columns present results for three different short-selling constraints while the rows contain different turnover constraints.

Table 6: Performance fee for the momentum portfolio for all models, $s = 0.5$, $\delta = 100$.

	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}
m_1, *RBG*	-0.053	-0.055	-0.067	-0.064	-0.054	-0.049	-0.048	-0.041	-0.061	-0.079
m_2, *RBG.r*		-0.002	-0.014	-0.011	-0.001	0.004	0.005	0.012	-0.008	-0.026
m_3, *HEAVY.sc*			-0.013	-0.010	0.001	0.006	0.007	0.014	-0.006	-0.024
m_4, *HEAVY.dg*				0.003	0.013	0.018	0.019	0.026	0.006	-0.012
m_5, *MW.RK*					0.011	0.016	0.016	0.023	0.003	-0.015
m_6, *RM.RK*						0.005	0.006	0.013	-0.007	-0.025
m_7, *DCC*							0.001	0.008	-0.012	-0.030
m_8, *DCC.2*								0.007	-0.013	-0.031
m_9, $MW.r^2$									-0.020	-0.038
m_{10}, $RM.r^2$										-0.018
m_{11}, $1/n$										

The table presents the average daily performance fee (multiplied by 100) that eliminates the utility gains of one model compared to another model. It is the amount an investor would be willing to pay to switch from the model in the row to the model in the column. A certain degree of short selling is allowed, $s = 0.5$, and the results are not subject to any turnover constraints.

Table 7: Volatility of the minimum variance portfolio for different short-selling constraints.

	$s = 0\%$			$s = 25\%$			$s = 50\%$		
	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$
RBG	10.42	0.65	[0.0000]	9.34	0.58	[0.0000]	9.28	0.58	[0.0000]
RBG.r	10.70	0.66	[0.0000]	9.70	0.60	[0.0000]	9.66	0.60	[0.0000]
HEAVY.sc	11.50	0.71	[0.0000]	10.61	0.66	[0.0000]	10.62	0.66	[0.0000]
HEAVY.dg	11.43	0.71	[0.0000]	10.52	0.65	[0.0000]	10.52	0.65	[0.0000]
MW.RK	11.39	0.71	[0.0000]	10.64	0.66	[0.0000]	10.61	0.66	[0.0000]
RM.RK	11.27	0.70	[0.0000]	10.49	0.65	[0.0000]	10.47	0.65	[0.0000]
DCC	10.07	0.63	[0.0000]	8.77	0.54	[0.0000]	8.66	0.54	[0.0000]
DCC.2	**9.82**	0.61	[1.0000]	**8.48**	0.53	[1.0000]	**8.34**	0.52	[1.0000]
$MW.r^2$	10.97	0.68	[0.0000]	9.87	0.61	[0.0000]	9.82	0.61	[0.0000]
$RM.r^2$	10.54	0.65	[0.0000]	9.28	0.58	[0.0000]	9.19	0.57	[0.0000]
$1/n$	16.37	1.02	[0.0000]	16.37	1.02	[0.0000]	16.37	1.02	[0.0000]

The table presents the annualized volatility of the minimum variance portfolio based on forecast for 11 different models. The volatility estimate is based on squared daily returns. VR denotes the ratio of $\bar{\sigma}_p$ to the average volatility of a portfolio consisting exclusively of S&P 500 futures. The table also presents the p-values, $p_{MCS}(\sigma_{p,t})$, of the Model Confidence Set, which is based on the time series of squared portfolio returns. The columns present results for three different short-selling constraints. None of the results are subject to turnover constraints.

Table 8: Realized volatility of the minimum variance portfolio for different short-selling constraints.

	$s = 0\%$			$s = 25\%$			$s = 50\%$		
	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$	$\bar{\sigma}_p$	VR	$p_{MCS}(\sigma_{p,t})$
RBG	**12.35**	0.78	[1.0000]	**11.60**	0.73	[0.6584]	**11.60**	0.73	[0.4532]
RBG.r	12.38	0.78	[0.0888]	**11.59**	0.73	[1.0000]	**11.58**	0.73	[1.0000]
HEAVY.sc	12.91	0.81	[0.0066]	12.04	0.76	[0.0024]	12.05	0.76	[0.0034]
HEAVY.dg	12.83	0.81	[0.0066]	11.92	0.75	[0.0094]	11.92	0.75	[0.0144]
MW.RK	12.66	0.80	[0.0066]	11.90	0.75	[0.0218]	11.88	0.75	[0.0502]
RM.RK	12.57	0.79	[0.0066]	**11.79**	0.74	[0.1766]	11.77	0.74	[0.0502]
DCC	12.51	0.79	[0.0066]	11.74	0.74	[0.0218]	11.72	0.74	[0.0502]
DCC.2	12.54	0.79	[0.0066]	11.81	0.74	[0.0094]	11.80	0.74	[0.0144]
$MW.r^2$	12.80	0.81	[0.0066]	12.12	0.76	[0.0002]	12.11	0.76	[0.0000]
$RM.r^2$	12.68	0.80	[0.0066]	11.98	0.76	[0.0024]	11.97	0.75	[0.0034]
$1/n$	17.14	1.08	[0.0000]	17.14	1.08	[0.0000]	17.14	1.08	[0.0000]

The table presents the average annualized volatility of the minimum variance portfolio based on forecast for 11 different models. The volatility estimate at time $t+5$, $\sigma_{p,t+5}$, is based on the portfolio weights obtained from the forecast of the covariance matrix for each model at time t and the realized covariance matrix of futures prices, $RC_{t:t+5}$, such that $\sigma_{p,t+5} = \sqrt{\omega_t' RC_{t:t+5} \omega_t}$. In the table we present the average of $\sigma_{p,t}$ taken over the full sample, and denoted $\bar{\sigma}_p$. VR denotes the ratio of $\bar{\sigma}_p$ to the average realized volatility of a portfolio consisting exclusively of S&P 500 futures. The table also presents the p-values, $p_{MCS}(\sigma_{p,t})$, of the Model Confidence Set, which is based on the time series of realized portfolio volatility. The columns present results for three different short-selling constraints. None of the results are subject to turnover constraints.

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