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Abstract

This paper examines the Taylor rule in the context of United States monetary policy since 1965, particularly with respect to the zero-lower-bound era of the federal funds rate from 2009 to 2016. A nonlinear Taylor rule is developed which features smooth transitions in the first two moments of the federal funds rate. This flexible specification is found to usefully capture observed nonlinearity, while accounting for the well-documented structural changes in monetary policy formation at the Federal Reserve in the last fifty years, and especially in the recent zero-lower-bound era.

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1 Introduction

The beauty of the Taylor rule and the primary reason for its longevity is its simplicity; the central bank is seen to use a straightforward interest-rate rule when conducting monetary policy. Indeed, the popularity of the rule has survived its well-documented instability. It is an undeniable fact, however, that after the financial crisis of 2007–2009 the United States economy experienced a sharp contraction in economic activity and a prolonged slow recovery. At the same time, the federal funds rate fell and remained at its effective lower bound of zero for a protracted period of time. This period of policy at the zero lower bound calls into question the applicability of a simple linear reaction function linking inflation and economic activity to the short-term interest rate. Consequently, this paper develops and estimates a nonlinear Taylor rule that maintains the simplicity and tractability of the original and also accommodates the nonlinearity induced by the zero-lower bound.

The original Taylor rule (Taylor 1993) represented the target short-term interest rate as a linear function of inflation and output gap. The Taylor rule aligns nicely with the dual mandate of the Federal Reserve (introduced in 1977) which requires that the Board of Governors and the FOMC must act to dampen the business cycle and provide a measure of stability to the economy by promoting “maximum employment, stable prices, and moderate long-term interest rates.” The last point can be achieved by inducing inflation to fall, which is one of the two key explanatory variables in the Taylor rule. The second key explanatory variable (output gap) relates to employment via Okun’s (1962) law.

Several variations to the simple linear Taylor rule have been proposed during the last twenty years. Gonzales and Vera (2002), Nikolsko-Rzhevskyy, Papell and Prodan (2014), and Creel and Hubert (2015) replace the output gap variable with the negative unemployment gap. Other papers vary the linear Taylor rule by way of the assumed time horizon used by the monetary authority; they may use current data, lagged (backward-looking) data, or expected (forward-looking) data. Others have experimented with including explanatory variables in addition to the contemporaneous values of the inflation and output gap. For instance, Clarida, Galí and Gertler (1998), Taylor (1999b, 2001), Casellina and Uberti (2008), Aizenman, Hutchison and Noy (2011), and Caporale, Helmi, Çatık, Ali and Akdeniz (2018) all use the real exchange rate in the estimation of their Taylor rule models. By contrast, the Taylor rule papers by Fuhrer and Tootell (2008), Bauducco,

Bulř and Čihák (2008), Alcidi, Flamini and Fracasso (2011), Kasai and Naraidoo (2012), Lee and Son (2013), and Roskelley (2016) all feature some variable representing activity in the financial markets.

A wealth of literature on the Taylor rule has found that the historically unusual high values of the federal funds rate between 1978 and 1983, coinciding with the Federal Reserve Chair transitions from Burns to Miller and from Miller to Volcker, pose serious parameter instability problems for the estimation of the linear Taylor rule in this era. A variety of nonlinear Taylor rule model specifications, which include smooth transitions, thresholds, Markov-switching and time-varying parameters, have been previously proposed to accommodate the apparent structural instabilities (Kim, Osborn and Sensier 2005, Boivin 2006, Petersen 2007, Koustas and Lamarche 2012).

As already mentioned, the approach taken in this paper is to propose a nonlinear model for the interest rate that explicitly accounts for the era spent in the proximity of the zero lower bound after 2008, where the Taylor rule relationship 'breaks down' (Martin and Milas 2013, Bhar and Malliaris 2016). The majority of the Taylor rule literature avoids this problematic era entirely by restricting estimation to time periods ending in or before 2007. In this paper, the smooth transition regression zero-lower-bound (STR-ZLB) model for the Taylor rule is introduced. Specifically, this smooth transition specification also extends to the conditional variance in the federal funds rate, permitting the error variance to adjust in line with the persistence observed during the period in which the federal funds rate was at the zero lower bound, see Figure 1. This feature not only adjusts the the Taylor rule in the presence of the zero lower bound but it also permits modelling of recovery from the lower bound, a feature that is consistent with recent observed behaviour.

2 Models for the Taylor rule

The traditional rule for target short-term interest rate in the United States introduced by Taylor (1993) is given by

$$i_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t,$$

in which y_t is the output gap, measured as the difference between the real GDP and the estimated potential real GDP, and π_t is year-on-year percentage change in aggregate

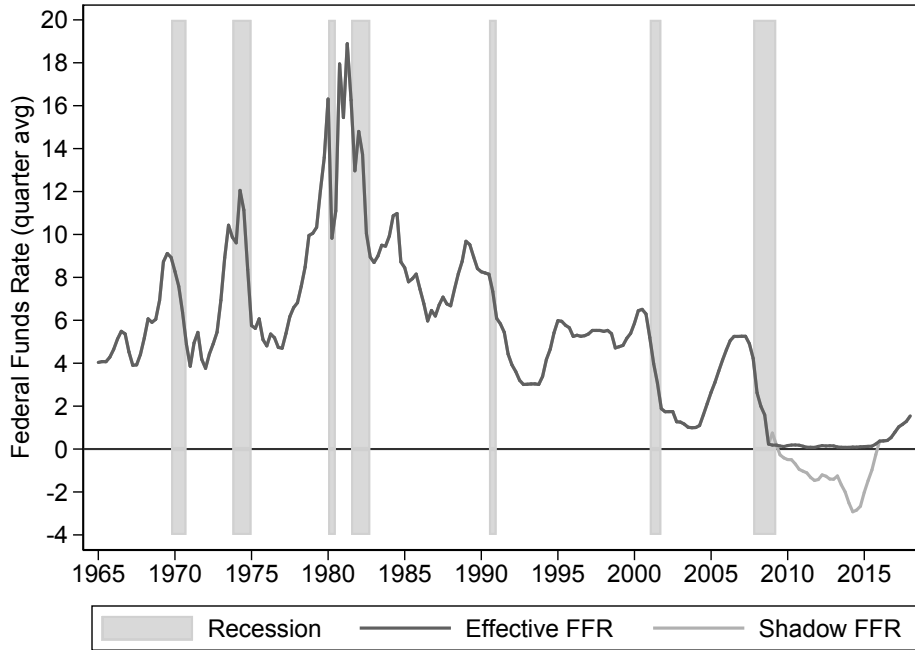


Figure 1: The quarterly-averaged United States federal funds rate from 1965:Q1 to 2018:Q1. The shadow rate plotted when the effective federal funds rate is at the zero lower bound is that of Wu and Xia (2015).

prices (actual inflation). For the years from 1987 to 1992, Taylor proposed that the output and inflation gaps entered this reaction function with equal weights of 0.5, and that the equilibrium real interest rate and the inflation target were identically equal to 2%, yielding parameter values $\alpha_0 = 1$, $\alpha_1 = 1.5$, and $\alpha_2 = 0.5$.

The Taylor rule can be interpreted to mean that the Federal Reserve raises the federal funds rate target for the short-term nominal interest rate when inflation rises above its desired level and/or the output level rises above trend. Of course, the target output deviation from its natural rate is zero because output cannot exceed potential output in the long run. The Taylor rule assumes a small positive inflation rate because deflation is considered a worse outcome for the economy than controlled inflation. For discussion, see Nikolsko-Rzhevskyy and Papell (2013).

A static Taylor rule model would have a poor fit to the data because of the well-documented persistence in short-term interest rates. Accordingly, it is usual to assume that central banks do not adjust interest rates abruptly but rather smooth them. See Goodfriend (1991), and Rudebusch (2002, 2006) for an extended discussion of monetary

policy inertia and interest rate smoothing. Clarida et al. (1998) suggested an interest rate smoothing whereby the standard rule is augmented by a lag of the interest rate,

$$i_t = (1 - \alpha_3)(\bar{\alpha}_0 + \bar{\alpha}_1\pi_t + \bar{\alpha}_2y_t) + \alpha_3i_{t-1} = \alpha_0 + \alpha_1\pi_t + \alpha_2y_t + \alpha_3i_{t-1}, \quad (1)$$

where $\bar{\alpha}_0$, $\bar{\alpha}_1$, and $\bar{\alpha}_2$ are the long-run coefficient values. Fuhrer (1997) suggested an even more dynamic decision rule with lags on inflation and output gap as well.

Even the augmented version of the Taylor rule, however, cannot be wholly operative in the case where the interest rate approaches the zero lower bound.

Reichschneider and Williams (2000) were among the first to point out that the original Taylor rule is inferior to nonlinear specifications when it comes to stabilization capability in the presence of the zero lower bound. In the vicinity of the zero lower bound, the original formulation of the Taylor rule is no longer applicable due to its failure to account for the implied positive constraint on the interest rate. Liquidity traps were largely considered to be a theoretical textbook phenomenon until February 12, 1999, when the Bank of Japan announced the lowering of overnight interest rates to be ‘as low as possible’ in an attempt to avoid a downward deflationary spiral, see Jung, Teranishi and Watanabe (2005). This situation brought about heightened research interest in monetary policy at the zero lower bound.¹

As an example, consider the following nonlinear specification of the interest rate due to Ruge-Murcia (2006). Let the nominal interest rate, i_t , take the form of a limited dependent variable generated by an underlying unobserved latent index, i_t^* , as follows:

$$i_t = \begin{cases} i_t^* & \text{if } i_t^* > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The unobserved latent index, i_t^* , is called the shadow rate. Now let the dynamics of i_t^* be generated by

$$i_t^* = \alpha + \sum_{j=1}^p \phi_j i_{t-j} + \beta' \mathbf{x}_t + \varepsilon_t, \quad (3)$$

where $\alpha \geq 0$, and \mathbf{x}_t is a vector of explanatory variables. Combining (2) and (3) means that i_t^* is censored and that the behaviour of i_t is described by a Tobit model

¹The European Central Bank introduced negative interest rates in 2014, and the Japanese call rate was set negative for the first time in early 2016.

Amemiya (1984). The error ε_t has mean zero and the conditional error variance follows a GARCH(1,1) model. Consequently, $E\varepsilon_t\varepsilon_{t-j} = 0$ for $j \neq 0$. Although Ruge-Murcia (2006) uses this model for bond pricing, it can easily be employed as a nonlinear Taylor rule.

Taylor and Williams (2011) suggested a specification to accommodate the zero lower bound within the framework in the Taylor rule, given by,

$$i_t = \max\{0, E[(1 - \alpha_3)(\bar{\alpha}_0 + \bar{\alpha}_1\pi_t + \bar{\alpha}_2y_t) + \alpha_3i_{t-1}^*]\} \quad (4)$$

where i_{t-1}^* denotes the lagged level of the interest rate that would occur in the absence of the zero lower bound. It is in fact the unconstrained shadow rate and replaces the actual rate in (1).

Left-censoring of monetary policy reaction functions has been previously estimated using a Tobit approach by authors such as Kato and Nishiyama (2005) and Huang (2015). Equation (4) is not an econometric model, but its structure is similar to the model of Ruge-Murcia (2006), in which the observable interest rate is prevented from becoming negative, so that the zero-lower-bound constraint is binding.

While these nonlinear specifications based on censoring provide easy-to-estimate alternatives to the simple linear Taylor rule, they are not developed any further here. Rather, our approach is to develop a dynamic model of the interest rate at the zero lower bound that describes the nonlinearity imposed by this constraint but also recognises that the behaviour of the error variance must be adapted to account for the existence of the bound.

3 A zero-lower-bound smooth transition model

The model proposed here is based on the same idea as the smooth transition autoregressive target zone (STARTZ) model due to Lundbergh and Teräsvirta (2006). The STARTZ model characterises the behaviour of an exchange rate that is restricted to fluctuate within a band and is a discrete-time version of the continuous time model of Krugman (1991). The dynamic behaviour of the exchange rate is very different when it approaches a boundary it cannot breach from what it is further away from these bounds. The present situation is somewhat simpler than that of the target zone as the interest rate only has a

(zero) lower bound. The conditional mean component of the model embeds both a Taylor rule regime and a zero-lower-bound regime and is defined as follows:

$$\begin{aligned}
i_t &= (\alpha_0 + \alpha_1\pi_t + \alpha_2y_t + \alpha_3i_{t-1})G_m(i_{t-1}) + (\alpha_4 + \alpha_5i_{t-1})(1 - G_m(i_{t-1})) + \varepsilon_t \\
&= (\alpha_0^* + \alpha_1\pi_t + \alpha_2y_t + \alpha_3^*i_{t-1})G_m(i_{t-1}) + \alpha_4 + \alpha_5i_{t-1} + \varepsilon_t \\
&= i_t^\circ G_m(i_{t-1}) + \alpha_4 + \alpha_5i_{t-1} + \varepsilon_t,
\end{aligned} \tag{5}$$

where ε_t is white noise, $E\varepsilon_t = 0$ and $E\varepsilon_t^2 = \sigma^2$. The conditional variance of ε_t will be considered later. In the lower bound regime, the linear Taylor rule breaks down and inflation and the output no longer predict the interest rate, hence their absence from that regime. The parameters $\alpha_0^* = \alpha_0 - \alpha_4$ and $\alpha_3^* = \alpha_3 - \alpha_5$ have been constructed for the ease of estimation. If needed, the original parameters α_0 and α_3 can be recovered from these relations.

The transition function $G_m(\cdot)$ of the STARTZ model is a generalized logistic function because the smooth pasting assumption is essential to the modelling of an exchange rate in the target zone. Since this assumption is not needed here and transition variable $i_t \geq 0$ almost surely, we follow Lanne and Saikkonen (2005) who suggested the use of the function $G(i_{t-1}): (0, \infty) \rightarrow [0, 1]$. Their suggestion of using the cumulative distribution function of the single parameter gamma distribution as the transition function is adopted here. Consequently the transition function in (5) is given by

$$G_m(i_{t-1}) = G_m(i_{t-1}; \gamma_m) = \frac{1}{\Gamma(\gamma_m)} \int_0^{i_{t-1}} e^{-s} s^{\gamma_m-1} ds, \tag{6}$$

where γ_m is the shape parameter and the implied scale parameter has been set equal to one.

This form of the transition function (6) is properly known as a regularized lower incomplete gamma function, and is a bounded, monotonically increasing function of i_{t-1} . It does not have a closed-form representation, so its values have to be computed numerically.² When $G_m(i_{t-1}) = 1$, the behaviour of the interest rate is determined by the linear dynamic Taylor rule, whereas when $G_m(i_{t-1}) \rightarrow 0$, which happens when the interest rate approaches zero, the Taylor rule is no longer operational. More generally, the interest rate for a given

²In the application the two-parameter gamma c.d.f. was also tried, but using it barely increased the maximum value of the likelihood and substantially slowed down convergence.

$i_{t-1} \geq 0$ is a convex combination of the Taylor rule equation and a simple AR(1) model for i_{t-1} .

In order to model an implied persistent regime for the interest rate at the zero lower bound, the variance of ε_t also has to be time-varying and appropriately parameterized. The conditional variance is decomposed as in the well known ARCH model Engle (1982). Thus $\varepsilon_t = z_t h_t^{1/2}$, where $z_t \sim \text{iid}\mathcal{N}(0, 1)$. The conditional variance h_t is a function of i_{t-1} given by

$$h_t = \delta + (d - \delta)G_v(i_{t-1}), \quad (7)$$

where

$$G_v(i_{t-1}) = G_v(i_{t-1}; \gamma_v) = \frac{1}{\Gamma(\gamma_v)} \int_0^{i_{t-1}} e^{-s} s^{\gamma_v-1} ds. \quad (8)$$

Both d and δ are positive parameters, with δ very close to zero, and, consequently, $d - \delta > 0$. When the interest rate is close to zero, $G_v(i_{t-1})$ is also expected to either equal zero or be very close to zero, in which case the error variance is small. In this sense, the zero-lower-bound constraint is not strictly binding, but by design is unlikely to be crossed without the action of a sufficient exogenous shock.³ The error variance increases with i_{t-1} and approaches d when the interest rate is sufficiently large. The model defined by (5), (6), (7) and (8) may be called the smooth transition regression zero-lower-bound (STR-ZLB) Taylor rule model.

4 Log-likelihood, score and the information matrix

The log-likelihood function of the STR-ZLB model for observation t has the form

$$\ell_t = k - \frac{1}{2} \ln h_t - \frac{\varepsilon_t^2}{2h_t}, \quad (9)$$

where

$$\varepsilon_t = i_t - i_t^\circ G_m(i_{t-1}) - \alpha_4 - \alpha_5 i_{t-1}$$

for $t = 1, \dots, T$. Let the mean parameter vector be $\theta_m = (\alpha_0^*, \alpha_1, \alpha_2, \alpha_3^*, \alpha_4, \alpha_5, \gamma_m)'$ and the variance parameter vector be $\theta_v = (\delta, d, \gamma_v)'$ and further set $\theta = (\theta_m', \theta_v')'$. Then the

³The model does permit very small negative interest rates to occur and so could feasibly be used outside of the United States context.

score for observation t equals

$$\frac{\partial \ell_t}{\partial \theta} = \left(\frac{\partial \ell_t}{\partial \theta'_m}, \frac{\partial \ell_t}{\partial \theta'_v} \right)' \quad (10)$$

In (10), the partial derivatives w.r.t. the mean parameters are

$$\begin{aligned} \frac{\partial \ell_t}{\partial \theta_m} &= \frac{\varepsilon_t}{h_t} (G_m(i_{t-1}), \pi_t G_m(i_{t-1}), y_t G_m(i_{t-1}), i_{t-1} G_m(i_{t-1}), 1, i_{t-1}, i_t^\circ \frac{\partial G_m(i_{t-1})}{\partial \gamma_m})' \\ &= \frac{\varepsilon_t \mathbf{s}_{mt}}{h_t}, \end{aligned}$$

with

$$\begin{aligned} \frac{\partial G_m(i_{t-1})}{\partial \gamma_m} &= -\psi(\gamma_m) G_m(i_{t-1}) + \frac{1}{\Gamma(\gamma_m)} \int_0^{i_{t-1}} e^{-s} s^{\gamma_m-1} (\ln s) ds \\ &= \frac{1}{\Gamma(\gamma_m)} \int_0^{i_{t-1}} e^{-s} s^{\gamma_m-1} (\ln s - \psi(\gamma_m)) ds, \end{aligned} \quad (11)$$

where $\psi(\gamma_m)$ is the digamma function. The partial derivatives w.r.t. the variance parameters are

$$\begin{aligned} \frac{\partial \ell_t}{\partial \theta_v} &= \frac{1}{2h_t} \left(\frac{\varepsilon_t^2}{h_t} - 1 \right) (1 - G_v(i_{t-1}), G_v(i_{t-1}), (d - \delta) \frac{\partial G_v(i_{t-1})}{\partial \gamma_v})' \\ &= \frac{1}{2h_t} \left(\frac{\varepsilon_t^2}{h_t} - 1 \right) \mathbf{s}_{vt}, \end{aligned} \quad (12)$$

where, analogous to (11),

$$\frac{\partial G_v(i_{t-1})}{\partial \gamma_v} = -\psi(\gamma_v) G_v(i_{t-1}) + \frac{1}{\Gamma(\gamma_v)} \int_0^{i_{t-1}} e^{-s} s^{\gamma_v-1} (\ln s) ds. \quad (13)$$

The information matrix for observation t is given by

$$\mathbf{I}(\theta^0) = \begin{bmatrix} \mathbb{E} \frac{\partial \ell_t}{\partial \theta_m} \frac{\partial \ell_t}{\partial \theta'_m} \big|_{\theta_m = \theta_m^0} & \mathbf{0} \\ \mathbf{0}' & \mathbb{E} \frac{\partial \ell_t}{\partial \theta_v} \frac{\partial \ell_t}{\partial \theta'_v} \big|_{\theta_v = \theta_v^0} \end{bmatrix},$$

where θ_m^0 and θ_v^0 are true parameter vectors,

$$\mathbb{E} \frac{\partial \ell_t}{\partial \theta_m} \frac{\partial \ell_t}{\partial \theta'_m} \big|_{\theta_m = \theta_m^0} = \mathbb{E} \frac{1}{h_t^0} \mathbf{s}_{mt}^0 (\mathbf{s}_{mt}^0)' \quad (14)$$

and

$$\mathbb{E} \frac{\partial \ell_t}{\partial \theta_v} \frac{\partial \ell_t}{\partial \theta'_v} \big|_{\theta_v = \theta_v^0} = \frac{1}{2} \mathbb{E} \frac{1}{h_t^0} \mathbf{s}_{vt}^0 (\mathbf{s}_{vt}^0)', \quad (15)$$

where \mathbf{s}_{mt}^0 and \mathbf{s}_{vt}^0 are the true values (true parameter values under H_0) of \mathbf{s}_{mt} and \mathbf{s}_{vt} , respectively, and h_t^0 is the true value of the conditional variance h_t . There are no closed form expressions for the expectations (14) and (15).

5 Estimation of the STR–ZLB Taylor rule

The data used here comprise quarterly United States time series for the period 1965:Q1 to 2018:Q1 for a total of 213 observations. The dependent variable is the quarterly averaged effective federal funds rate, derived from the monthly time series found in the FRED databases and expressed in basis points. For the contemporaneous output gap measure, real-time real GDP data obtained from the Philadelphia Federal Reserve are used and the gap is constructed as the difference between GDP and potential GDP, where the latter is constructed using a quadratic filter of actual GDP.⁴ Finally, inflation is calculated as the percentage annual change in the real-time GDP deflator, also obtained from the Philadelphia Federal Reserve.

Before estimating the STR–ZLB Taylor rule over the full timespan of the dataset, it is interesting to see the characteristics of the dynamic linear Taylor rule before the federal funds rate was reduced towards the zero lower bound in 2008. The estimation results over the 170 observations from 1965:Q1–2007:Q3 are

$$i_t = 0.094 + 0.131\pi_t + 0.055y_t + 0.900i_{t-1} + \hat{\varepsilon}_t, \quad (16)$$

(0.234) (0.053) (0.029) (0.039)

with standard errors in parentheses. Before the onset of the zero lower bound, it is clear that the response coefficients to inflation and the output gap are both positive and reasonably significant. All the magnitudes of the parameters are in line with those reported in previous studies, see for example Rudebusch (2006) and Martin and Milas (2010).

It is equally instructive to confirm that the dynamic linear Taylor rule breaks down after the onset of the zero lower bound. The estimation results over the 33 observations

⁴A number of different filters have been used to generate potential output. These include a linear and quadratic filters and the Hodrick-Prescott (HP) filter. The linear filter produces an output gap that is almost exclusively positive and is thus discarded. The real-time quadratic and HP output gaps, however, are much more commonly used in the literature Boinet and Martin (2008). As pointed out in Nikolsko-Rzhevskyy et al. (2014), however, the HP output gap tends to empirically understate the output gap during periods of peak unemployment in the 1970s and 1980s. More generally, Hamilton (2017) argues convincingly that one should not apply the HP filter. As a consequence, the quadratic output gap is used in this work.

from 2007:Q4–2015:Q4 are

$$i_t = \underset{(0.197)}{0.039} + \underset{(0.084)}{0.002}\pi_t + \underset{(0.055)}{0.010}y_t + \underset{(0.114)}{0.712}i_{t-1} + \hat{\varepsilon}_t, \quad (17)$$

where, as before, standard errors are in parentheses.

The difference is stark. Here the short run response to inflation and the output is almost zero, and the equation is practically autoregressive of order one. A pure AR(1) model fitted to this period has the following form:

$$i_t = \underset{(0.436)}{0.013} + \underset{(0.034)}{0.732}i_{t-1} + \hat{\varepsilon}_t. \quad (18)$$

These results are consistent with the conclusions drawn in other studies such as Jung et al. (2005) and Zhu and Chen (2017).

The STR–ZLB model (5), (6), (7) and (8) contains the pre-zero-lower-bound dynamic Taylor rule and the zero-lower-bound regime as extreme cases. Its parameters are estimated by maximum likelihood. Estimation is started from several initial values to increase the probability that the highest local maximum found is a global maximum. The scale parameters γ_m and γ_v in (6) and (8) are replaced by $\eta_m = \ln \gamma_m$ and $\eta_v = \ln \gamma_v$, respectively. This transformation has the advantage that the estimation becomes computationally easier when the scale parameters γ_m and γ_v are small (the value of the function increases rapidly with its argument). In addition, the estimate of the very small positive parameter δ in (7) is kept positive by setting $\delta = k^2$.

The estimated conditional mean of the STR–ZLB model is given by (with standard error estimates in parentheses)

$$i_t = \underset{(0.299)}{(0.013 + 0.102\pi_t + 0.037y_t + 0.936i_{t-1})} \underset{(1.487)}{\hat{G}_m(i_{t-1})} + \underset{(0.090)}{(0.051 + 0.815i_{t-1})} (1 - \underset{(0.330)}{\hat{G}_m(i_{t-1})}) + \hat{\varepsilon}_t, \quad (19)$$

where

$$\hat{G}_m(i_{t-1}) = \frac{1}{\Gamma(1.809)} \int_0^{i_{t-1}} e^{-s} s^{0.809} ds.$$

The estimated conditional variance is

$$\hat{h}_t = \underset{(0.045)}{0.150} + \underset{(3.525)}{(7.302 - 0.150)} \underset{(0.045)}{\hat{G}_v(i_{t-1})}, \quad (20)$$

with

$$\hat{G}_v(i_{t-1}) = \frac{1}{\Gamma(11.244)} \int_0^{i_{t-1}} e^{-s} s^{10.244} ds.$$

Plots of the residuals $\hat{\varepsilon}_t$ and the standardised residuals \hat{z}_t can be found in Figures 2a and 2b, respectively. The effect of the STR-ZLB model having captured most of the time-variation in the error variance is apparent. It appears though that the residuals contain autocorrelation, which counters the assumption that the standardised errors $\varepsilon_t/h_t^{1/2}$ of the model are not serially correlated. To test this assumption, the Godfrey-Breusch test of no error autocorrelation was applied. The test is based on

$$z_t = \phi_1 z_{t-1} + \dots + \phi_p z_{t-p} + \zeta_t = \phi' \mathbf{z}_{t-1} + \zeta_t,$$

where $\mathbf{z}_t = (z_t, z_{t-1}, \dots, z_{t-p+1})'$, $\phi = (\phi_1, \dots, \phi_p)'$, and $\zeta_t \sim \text{iid}\mathcal{N}(0, \sigma_\zeta^2)$. The null hypothesis is $H_0 : \phi = \mathbf{0}$. The robust version of the test is carried out in the TR^2 form by the following steps, see Godfrey (1978) and Wooldridge (1990):

1. Estimate the STR-ZLB model, compute the residuals \tilde{z}_t and the residual sum of squares $SSR_0 = \sum_{t=1}^T \tilde{z}_t^2$.
2. Regress \tilde{z}_t on $\tilde{h}_t^{-1/2} \tilde{\mathbf{s}}_{mt}$ and $\tilde{\mathbf{z}}_{t-1}$, where the tilde implies that the elements have been estimated under H_0 , and compute the sum of squared residuals SSR_1 .
3. Compute the test statistic $LM_{ac}^{TR} = T(1 - SSR_1/SSR_0)$.

Under H_0 and the assumption that the conditional variance is correctly specified, LM_{ac}^{TR} has an asymptotic χ^2 null distribution with p degrees of freedom.

This test results in a strong rejection of the null hypothesis over the entire sample period as well as in subperiods, which agrees with the visual inspection of the residuals. This result is not surprising given the simple dynamic structure of the STR-ZLB model and characteristic of all Taylor rule models based on simple interest rate smoothing. Consequently, care must be exercised in making statistical inferences and, wherever possible, robust standard errors should be employed.

6 Interpretation

Figures 3a and 3c plot the estimated transition functions on the conditional mean and variance respectively, as a function of the transition variable i_{t-1} . The features of the

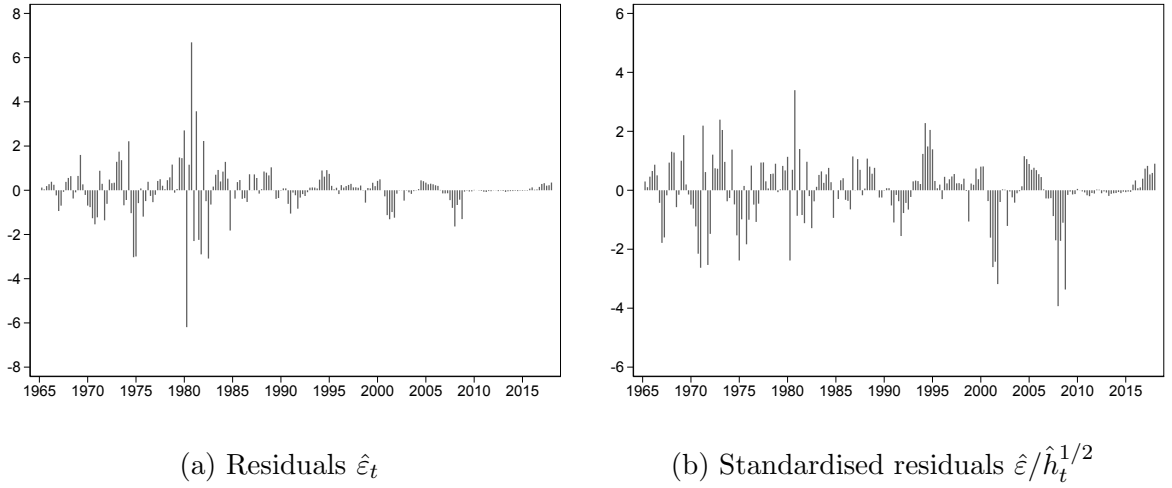


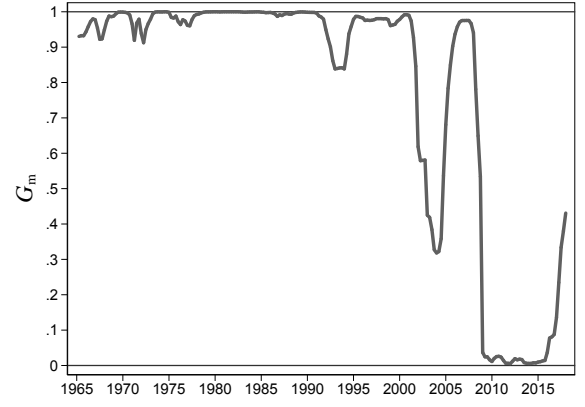
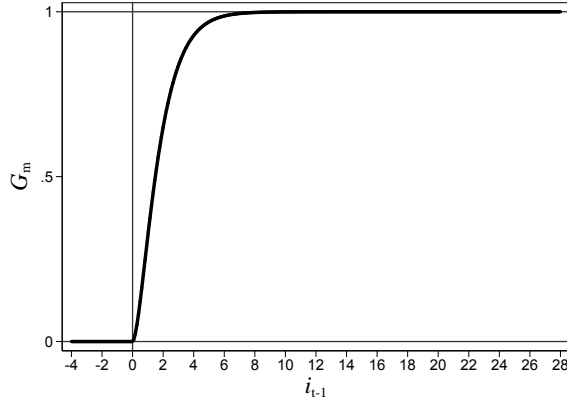
Figure 2: Residuals and standardised residuals from the estimated STR-ZLB Taylor rule from 1965:Q1 to 2018:Q1.

gamma transition are clear with the zero-lower-bound regime occurring for interest rates near or below zero, and the ‘regular’ or ‘Taylor’ regime occurring for interest rates sufficiently above zero. Therefore the model can also be used to accommodate small negative interest rates. An important property of these gamma transition functions is that the model fully arrives at the zero-lower-bound regime ($G_m = 0$) when $i_{t-1} \downarrow 0$.

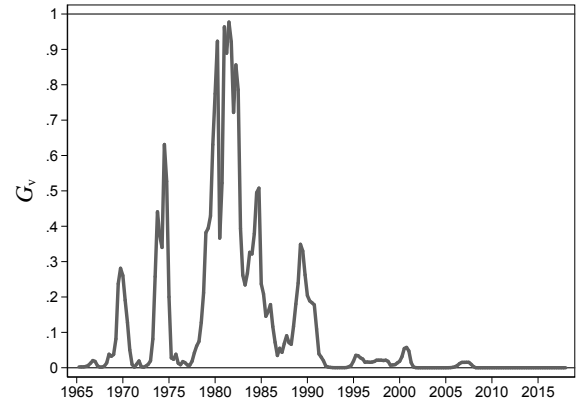
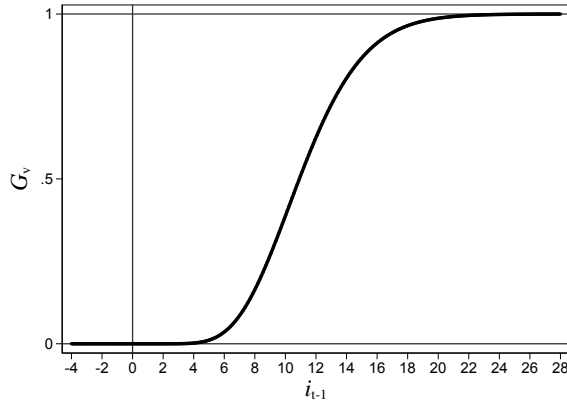
As may be expected, the estimates for the “Taylor rule component” in (19) are close to the ones in (16), whereas the AR(1) component closely resembles (18). One of the advantages of the STR-ZLB model is that not only does it parameterise the two extremes, it also describes the (infinitely many) states between these extremes.

The corresponding Figures 3b and 3d respectively show the estimated mean and variance transitions, plotted over the time span of the dataset. The zero-lower-bound era from 2008 to 2016 is represented by a clear regime shift in the conditional mean. The conditional mean does not completely reach the zero-lower-bound regime, as G_m does not fully reach zero, but the mean value of G_m during the centre of the zero-lower-bound era (2009:Q1–2014:Q4) is found to be 0.0152. The mean value of the quarterly averaged federal funds rate for the same time period is $i_t = 0.128\%$.

On the other hand, from Figure 3d it is clear that the transition function in the conditional variance reflects the high degree of variability induced by the monetary policy experiments around 1979–1982. Numerous studies in the United States monetary policy



(a) Transition function in the conditional mean as a function of i_{t-1} (b) Transition function in the conditional mean as a function of time



(c) Transition function in the conditional variance as a function of i_{t-1} (d) Transition function in the conditional variance as a function of time

Figure 3: Estimation of the STR–ZLB Taylor rule from 1965:Q1 to 2018:Q1.

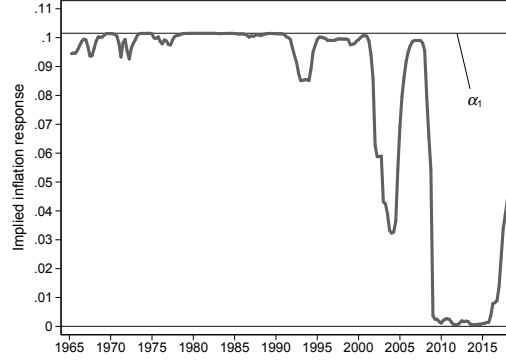
literature have found structural breaks in the data during this time period, e.g. Taylor (1999a), Clarida, Galí and Gertler (2000), Orphanides (2004), Yoon (2011), and Yao (2014). In these papers the error variance is assumed constant. In the present work, however, the linear Taylor rule representation is retained for this period, but the time-varying variance accommodates possible inadequacies in the linear rule by allowing the conditional variance to increase during this period. Another interesting feature of the estimation results, visible in Figure 3d, is that the conditional error variance becomes small in the 1990s, long before the beginning of the zero-lower-bound era. This result indicates that the ‘regular’ dynamic Taylor rule fits the data quite well during this period.

There is an alternative approach to interpreting the estimated conditional mean in equation (19), namely, by viewing it as a linear form Taylor rule with time-varying coefficients given by

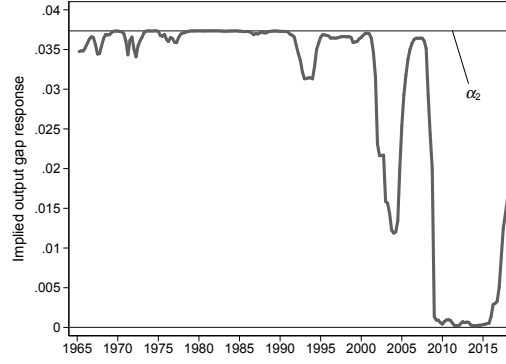
$$i_t = \hat{\beta}_0(i_{t-1}) + \hat{\beta}_1(i_{t-1})\pi_t + \hat{\beta}_2(i_{t-1})y_t + \hat{\beta}_3(i_{t-1})i_{t-1} + \hat{\varepsilon}_t. \quad (21)$$

The estimated intercept is $\hat{\beta}_0(i_{t-1}) = \hat{\alpha}_0^* \hat{G}_m(i_{t-1}) + \hat{\alpha}_4 = \hat{\alpha}_0 \hat{G}_m(i_{t-1}) + \hat{\alpha}_4(1 - \hat{G}_m(i_{t-1}))$, the response to inflation is $\hat{\beta}_1(i_{t-1}) = \hat{\alpha}_1 \hat{G}_m(i_{t-1})$, the response to the output gap is $\hat{\beta}_2(i_{t-1}) = \hat{\alpha}_2 \hat{G}_m(i_{t-1})$, and the implied interest smoothing coefficient is given by $\hat{\beta}_3(i_{t-1}) = \hat{\alpha}_3^* \hat{G}_m(i_{t-1}) + \hat{\alpha}_5 = \hat{\alpha}_3 \hat{G}_m(i_{t-1}) + \hat{\alpha}_5(1 - \hat{G}_m(i_{t-1}))$.

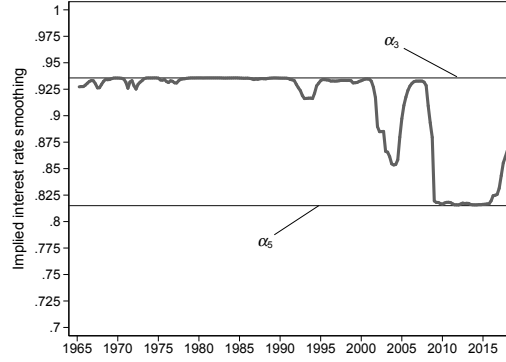
The estimated time-varying coefficients $\hat{\beta}_1(i_{t-1})$, $\hat{\beta}_2(i_{t-1})$, and $\hat{\beta}_3(i_{t-1})$ are plotted against time in Figures 4a to 4c. In Figure 4a, the horizontal line corresponds to the value of $\hat{\alpha}_1$ which is the coefficient on inflation during the linear regime. There is a sharp transition in the value of this coefficient to the lower horizontal line at zero during the zero-lower-bound era. This pattern is repeated in Figure 4b for the output gap parameter $\hat{\alpha}_2$. The coefficient on the lagged interest rate in Figure 4c, however, retains its magnitude over the whole period of observation as the two estimates shown by the horizontal lines $\hat{\alpha}_3$ and $\hat{\alpha}_5$ are not significantly different. It is very interesting to note that after 2015, the estimated time-varying coefficients on inflation and the output gap both increase as the gamma smooth transition is engaged. This feature demonstrates that the model not only captures the transition into the lower-bound period but it is also effective in modelling the recovery as the economy begins to emerge from the zero-lower-bound era.



(a) $\hat{\beta}_1(i_{t-1})$ over time



(b) $\hat{\beta}_2(i_{t-1})$ over time



(c) $\hat{\beta}_3(i_{t-1})$ over time

Figure 4: Shifts in the estimated time-varying Taylor rule parameters $\beta_1(i_{t-1})$ (the response to inflation), $\beta_2(i_{t-1})$ (the response to the output gap), and $\beta_3(i_{t-1})$ (the interest smoothing coefficient) from the STR-ZLB Taylor rule in the vicinity of the zero lower bound, as shown in equation (21).

7 Conclusion

In contrast to existing nonlinear specifications of the Taylor rule, the STR-ZLB model incorporates smooth transitions in both the conditional mean and conditional variance of the federal funds rate. The specification of the variance is a novel addition to the model and is crucial in ensuring that the short-term interest rate is able to remain at the lower bound. By design, the linear Taylor rule is nested within the STR-ZLB model and completely characterises interest rate behaviour when the lower bound is not binding. In the vicinity of the zero lower bound, the STR-ZLB model resembles an autoregressive process of order one for the interest rate with a vanishing variance. In this way, both conventional and unconventional monetary policy regimes, and the transition between them, are captured by a single model. The model is also applicable outside of the United States because it does not entirely preclude small negative interest rates.

The log-likelihood function of the model is straightforward to derive and maximum likelihood estimates of the model parameters are relatively straightforward to obtain. The estimates of the linear Taylor rule component are very much as expected. The estimates of the transition functions indicate a gradual but clear shift in the conditional mean and conditional variance during the zero-lower bound era. The transitions in the variance also suggest that the linear dynamic Taylor rule does not explain developments of the Volcker period particularly well. The simple dynamic specification of the model retains the essential structure of the Taylor rule but at the cost of serially correlated errors. Care must therefore be exercised to ensure correct inference.

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