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Abstract: We suggest a bivariate component GARCH model that simultaneously obtains factor betas' long- and short-run components. We apply this new model to industry portfolios using market, small-minus-big, and high-minus-low portfolios as risk factors and find that the cross-sectional average and dispersion of the betas' short-run component increase in bad states of the economy. Our analysis of the risk premium highlights the importance of decomposing risk across horizons: The risk premium associated with the short-run market beta is significantly positive. This is robust to the portfolio-set choice.

Keywords: long-run betas; short-run betas; risk premia; component GARCH model; MIDAS

JEL Classifications: G12; C58

1. Introduction

A vast literature is devoted to investigating the cross-sectional relationship between expected returns and risk (see, e.g., Engle, Bollerslev, and Wooldridge, 1988; Harvey, 1989; Schwert and Seguin, 1990; Ang, Hocrick, Xing, and Zhang, 2006; Bali, 2008). However, the empirical evidence is inconclusive and inference is sensitive to model specification and estimation procedure (see, e.g., Jagannathan and Wang, 1996; Lewellen and Nagel, 2006; Lewellen, Nagel, and Shanken, 2010).

In this paper, we suggest a new conditional asset-pricing model by using the mixed-data-sampling approach in a component bivariate GARCH model. This new framework allows us to simultaneously obtain long- and short-run factor-beta components. We apply the new asset-pricing model to a widely used data set of industry portfolios to evaluate its ability to explain the cross-sectional differences in expected equity returns.

An important motivation for an asset-pricing model with a component structure is the information flow in financial markets. Andersen and Bollerslev (1997) and Calvet and Fisher (2007) show that information in financial markets arrives at various frequencies and has different degrees of persistence. Therefore, information affects the return dynamics differently at different frequencies. The risk premia reflect the compensation for exposure to the shocks driven by information arriving at different frequencies and with heterogeneous degrees of persistence. Ignoring this may result in poor estimates of systematic risk and the related risk premia.

A number of studies show that the choice of frequency is important for obtaining a correct measure of risk and capturing the risk–return relationship. For example, Gilbert, Hrdlicka, Kalodimos, and Siegel (2014) show that there are large differences between high-frequency (daily) and low-frequency (quarterly) stock betas. They show that opacity makes high-frequency betas unusable.

According to Lewellen and Nagel (2006), compounding ensures that the betas vary across different frequencies. Engle and Lee (1999) and Engle and Rangel (2008) show that models with both low- and high-frequency volatility and correlation components capture the dynamics of equity returns better than single-frequency models. Adrian and Rosenberg (2008) explore cross-sectional pricing of risk by decomposing equity-market volatility into short-run (capturing market-skewness risk) and long-run components (closely related to business-cycle risk). Cenesizoglu and Reeves (2015) use a nonparametric approach and measure market beta with short-, medium-, and long-run components. The short- and medium-run components are estimated from daily returns over one- and five-year periods and the long-run component is estimated from monthly returns over a 10-year period. Boons and Tamoni (2016) show that dividing risk into long- and short-run components helps uncover a link between risk premia and the macro economy.

Several studies have used the mixed data sampling (MIDAS) approach to estimate systematic risk. Gonzalez, Nave, and Rubio (2012) use a weighted average of daily returns to estimate monthly betas. Gonzalez, Nave, and Rubio (2016) define the conditional beta with two additive components, a transitory component estimated from daily returns and a long-run beta based on macroeconomic state variables. Baele and Londono (2013) use Colacito, Engle, and Ghysels's (2011) dynamic conditional correlation (DCC) MIDAS model to obtain long-run betas. They find that DCC-MIDAS betas are superior to ordinary betas in limiting the downside risk and ex-post market exposure for the minimum-variance strategy. Ghysels, Santa-Clara, and Valkanov (2005) use MIDAS volatilities to analyze the risk–return trade-off. They investigate the effects of changing the frequency of the returns in the MIDAS risk–return trade-off regressions and find that using high-frequency returns (above monthly) provides excessively noisy estimates. Therefore, they conclude that monthly returns are preferable. Ghysels, Guérin, and Marcellino (2014) continue this analysis by combining regime switching with MIDAS and consider variations across horizons.

To the best of our knowledge, this is the first conditional asset-pricing model that uses a component model to decompose betas and risk premia into long- and short-run components. Our bivariate component GARCH model enables us to simultaneously decompose total variances and covariances into long- and short-run variances and covariances and thereby to estimate the corresponding components of the asset betas. The new component GARCH model builds upon Engle and Lee (1999). Theoretical and empirical evidence suggests that deterministic changes in long-run returns can generate long-memory behavior, which may be interpreted as changes in the unconditional variance of long time series (Mikosch and Stărică, 2004; Amado and Teräsvirta, 2013, 2014, 2015). This motivates us to model the long-run variances and covariances based on the unconditional variance and covariance of past long-run returns—i.e., the monthly returns—while the short-run variances and covariances are based on data with different frequencies (weekly or monthly). The decomposition can be used both on mixed-frequency data (as a MIDAS approach) and on single-frequency data.

For our methodological contribution, we employ a different framework from the traditional decomposition approach of the GARCH–MIDAS model with a multiplicative form. That approach works well with a univariate model, but it does not work well in a bivariate setting to estimate variance and covariance at the same time: We may have negative covariances. Colacito, Engle, and Ghysels's (2011) two-step DCC-MIDAS approach has a multiplicative form in the univariate model of the first step and an additive form in the second step. Our new decomposition method with an additive form estimates the variance and covariance components simultaneously.

We investigate the dynamics and determinants of industry betas based on the three Fama and French (1993) factors: the market portfolio, the small-minus-big portfolio (SMB), and the high-minus-low portfolio (HML). We apply our component GARCH model to each factor and an asset (industry portfolio) to estimate long- and short-run variances and covariances. From these we calculate long- and short-run betas. The estimated long- and short-run betas are subsequently used in cross-sectional regressions to estimate the long- and short-run risk premia associated with each factor. The main analysis is based on the 30 industry portfolios from Fama and French (1993, 1997), but we also estimate our new model with 17 and 49 industry portfolios as well as with 25 portfolios sorted based on size and book-to-market value.

Our paper is also closely related to the literature exploring the impact of macroeconomic variables on market betas across portfolios. Many studies show that business-cycle exposure is heterogeneous across industries and provide models for industries' and equity portfolios' heterogeneous reactions to business-cycle conditions (see, e.g., Berk, Green, and Naik, 1999; Gomes, Yaron, and Zhang, 2003; Lewellen and Nagel, 2006; Baele and Londono, 2013; Gonzalez, Nave, and Rubio 2016). In particular, Baele and Londono (2013) show that industry betas display substantial heterogeneity with respect to their business-cycle exposure. This is consistent with Berk, Green, and Naik's (1999) and Gomes, Yaron, and Zhang's (2003) models. Moreover, Baele and Londono (2013) also show that the cross-sectional dispersion on industry betas is larger during recessions, supporting Gomes, Yaron, and Zhang's (2003) theoretical predictions. We investigate the macro-economic conditions that can explain the fluctuations of factor betas around their long-run paths. Furthermore, we analyze how macro-economic conditions affect the cross-sectional dispersions of short-run betas.

We find that the short-run component of betas for all three factors increase on average in contractionary states of the economy, where industries related to necessities are less affected by fluctuations in economic conditions. In line with earlier results, the cross-sectional dispersion in short-run betas increases in these economic conditions. Moreover, we find that the data frequency matters for estimation of the risk premium: None of the risk premia estimated at weekly frequencies is significant, which is in contrast to the risk premia obtained at the monthly frequency. This implies that risk premia estimated more frequently than monthly are noisy. At the monthly frequency, our analysis of the risk premium highlights the importance of decomposing risk across horizons. Although the risk premia associated with both the long- and short-run SMB betas are significant, only the risk premium associated with the short-run market beta is significantly positive. This is robust to the choice of test portfolio. By excluding recessions from our sample, the risk premia for all the betas, except for the long-run market beta, become significantly positive. It seems that investors demand compensation only when the market beta increases from its long-run level.

The rest of the paper is structured as follows. First, we introduce the econometric framework. Second, we introduce the data before we discuss the empirical results. Finally, we conclude. The appendix contains various technical details.

2. The component asset-pricing model

In this section, we present the new component GARCH model. The empirical analysis follows a two-step estimation procedure similar to Fama and MacBeth (1973). The first step entails time-series regressions to obtain total, long-, and short-run betas. In the second step, we estimate the corresponding risk premia.

2.1 First step: Bivariate component GARCH model

Within the component GARCH models, there are two general approaches to distinguish short-run from long-run movements, an additive approach, proposed by Engle and Lee (1999) and a multiplicative GARCH–MIDAS approach, proposed by Engle, Ghysels, and Sohn (2013). The multiplicative approach, despite working well in a univariate GARCH model, cannot be applied to bivariate models to simultaneously estimate variances and covariances, as we may have negative covariances. We use an additive decomposition approach and extend Asgharian and Hansson's (2000) and Bali's (2008) bivariate GARCH model to a bivariate component GARCH model to decompose the total variance and covariance to a long-run, persistent, component and a short-run, transitory, component.

We work with both single-frequency and mixed-frequency models. We use the subscripts *s* and *t* to keep track of the periods. In the single frequency model, *s* and *t* are identical; in the mixed-frequency model, *s* and *t* denote periods corresponding to the weekly and monthly frequency, respectively.[1]

We assume that the mean equations for the excess returns for portfolio i ($r_{i,s,t}$) and the state variable x ($r_{x,s,t}$) follow a simple form where they are equal to a constant plus an error term ($\varepsilon_{i,s,t}$ and $\varepsilon_{x,s,t}$, respectively):

$$\begin{aligned} r_{i,s,t} &= \gamma_{0i} + \varepsilon_{i,s,t} \\ r_{x,s,t} &= \gamma_{0x} + \varepsilon_{x,s,t}. \end{aligned} \tag{1}$$

[1] We also estimate the model with the monthly–daily combination. The results are similar to those with the monthly–weekly combination. For the sake of brevity, those results are not reported.

The error terms are assumed to follow normal distributions with mean zero and time dependent variances, $q_{i,s,t}$ and $q_{x,s,t}$. The error terms have a time dependent covariance, $q_{ix,s,t}$.

Engle and Lee's (1999) univariate additive component GARCH model defines the total conditional variance of an asset as the sum of a long-run (permanent) component and a short-run (transitory) component where the total variance follows a GARCH(1,1). Engle and Lee (1999) replace the unconditional variance by the long-run time-varying variance. The idea of an unconditional time-varying variance is also presented in, for example, Amado and Teräsvirta's (2014) model. We extend the idea above to a bivariate GARCH(1,1) model to estimate portfolios' conditional variances as well as their conditional covariances with the common factors.

In the parameterization of the GARCH equation, we use the BEKK specification to reduce the number of parameters. The formulation of the intercept follows de Santis and Gerard (1997), while the unconditional moments (the τ's) are time varying. The total variances and covariance are modeled as

$$\begin{aligned} q_{i,s,t} &= \tau_{i,t}(1 - a_i^2 - b_i^2) + a_i^2 \varepsilon_{i,s-1,t}^2 + b_i^2 q_{i,s-1,t} \\ q_{x,s,t} &= \tau_{x,t}(1 - a_x^2 - b_x^2) + a_x^2 \varepsilon_{x,s-1,t}^2 + b_x^2 q_{x,s-1,t} \\ q_{ix,s,t} &= \tau_{ix,t}(1 - a_i a_x - b_i b_x) + a_i a_x \varepsilon_{ix,s-1,t}^2 + b_i b_x q_{ix,s-1,t}, \end{aligned} \tag{2}$$

where $\tau_{i,t}$, $\tau_{x,t}$, and $\tau_{ix,s,t}$ are the long-run variances and covariance.

We use the weighted moving average of the past observations to estimate the long-run variances and covariances. This is similar to Colacito, Engle, and Ghysels's (2011) approach to estimating long-run correlation in the DCC-MIDAS model:

$$\tau_{i,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{i,t-k}$$
$$\tau_{x,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{x,t-k} \quad (3)$$
$$\tau_{ix,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{ix,t-k},$$

where

$$V_{i,t} = (r_{i,t} - \mu_{i,t})^2$$
$$V_{x,t} = (r_{x,t} - \mu_{x,t})^2 \quad (4)$$
$$V_{ix,t} = (r_{i,t} - \mu_{i,t})(r_{x,t} - \mu_{x,t}),$$

$\mu_{i,t}$ and $\mu_{x,t}$ are the means of the monthly returns for *i* and *x* over five-year historical data before each period *t*, and *K* is the number of periods within the five years.[2] The long-run component is the average of the squared deviations of the monthly returns from their mean to follow the conventional approach to estimating beta.[3] The weighting scheme used in equation (3) is described by a beta-lag polynomial:

$$\varphi_k(w_1, w_2) = \frac{\left(k/K\right)^{w_1-1}(1 - k/K)^{w_2-1}}{\sum_{j=1}^{k}(j/K)^{w_1-1}(1 - k/K)^{w_2-1}}. \quad (5)$$

The short-run component of the variance and covariance is the difference between the total and long-run components:

$$g_{i,s,t} \equiv q_{i,s,t} - \tau_{i,t} = a_i^2\left(\varepsilon_{i,s-1,t}^2 - \tau_{i,t}\right) + b_i^2(q_{i,s-1,t} - \tau_{i,t})$$
$$g_{x,s,t} \equiv q_{x,s,t} - \tau_{x,t} = a_x^2\left(\varepsilon_{x,s-1,t}^2 - \tau_{x,t}\right) + b_x^2(q_{x,s-1,t} - \tau_{x,t}) \quad (6)$$

[2] The five-year window with monthly returns is conventional for estimating unconditional betas (see, e.g., Fama and French, 1993).

[3] In the GARCH–MIDAS model, the long-run component is calculated as the weighted sum of the realized variances and covariance. We also estimate the model with realized moments based on daily data and exponential weights. The conclusions remain unaltered.

$$g_{ix,s,t} \equiv q_{ix,s,t} - \tau_{ix,t} = a_{ix}^2\left(\varepsilon_{ix,s-1,t}^2 - \tau_{ix,t}\right) + b_{ix}^2\left(q_{ix,s-1,t} - \tau_{ix,t}\right).$$

The total betas are calculated from the total covariance and variance, and the long-run betas from the long-run covariance and variance:

$$\hat{\beta}_{x,i,s,t}^{total} = \frac{\hat{q}_{ix,s,t}}{\hat{q}_{x,s,t}} \qquad \hat{\beta}_{x,i,s,t}^{long} = \frac{\hat{\tau}_{ix,s,t}}{\hat{\tau}_{x,s,t}}. \tag{7}$$

The short-run betas are the differences between the total and long-run betas.[4]

Several restrictions have been applied to ensure that the conditional variance–covariance matrix is positive definite at each *s* and *t*. The details are in the appendix, where we also discuss identification and stationarity of the model. The log-likelihood function for model estimation is also given in the appendix.

2.2 Second step: Cross-sectional regressions

In our setting, the expected returns depend on both long- and short-run components of three risk premia, market-, SMB- and HML-based risk.

The second step concerns the Fama and MacBeth (1973) cross-sectional regressions, where we investigate the sign and significance of the risk premia of the state variables. There is one cross-sectional regression for each period *s*. When we consider the total betas, it reads as follows.

$$R_{i,s,t} = C_{0s,t}^{\text{total}} + C_{1s,t}^{\text{total}}\beta_{M,i,s,t}^{\text{total}} + C_{2s,t}^{\text{total}}\beta_{SMB,i,s,t}^{\text{total}} + C_{3s,t}^{\text{total}}\beta_{HML,i,s,t}^{\text{total}} + \varepsilon_{i,s,t}, \text{for } i = 1, \dots, N \tag{8}$$

We also do cross-sectional regressions with both short- and long-run betas and thereby obtain long- and short-run risk premia. This is new to the literature.

[4] We also calculate the short-run betas from the short-run covariance and variance defined in equation (6). This approach is noisy as it results in extreme values when the short-run factor variance is very small.

$$R_{i,s,t} = c_{0s,t} + c_{1,s,t}^{\text{long}}\beta_{M,i,s,t}^{\text{long}} + c_{1,s,t}^{\text{short}}\beta_{M,i,s,t}^{\text{short}} + c_{2,s,t}^{\text{long}}\beta_{SMB,i,s,t}^{\text{long}} + c_{2,s,t}^{\text{short}}\beta_{SMB,i,s,t}^{\text{short}} + c_{3,s,t}^{\text{long}}\beta_{HML,i,s,t}^{\text{long}} + c_{3,s,t}^{\text{short}}\beta_{HML,i,s,t}^{\text{short}} + \varepsilon_{i,s,t}, \text{for } i = 1, \dots, N \quad (9)$$

The risk premia are the average of the estimated coefficients, the c's. We use the time series of the estimated parameters for the factors to investigate such properties of the factor risk premia as whether the average coefficients are significant and, if so, whether they are positive or negative. This corresponds to traditional analysis of risk premia. We use Newey and West's (1987) correction for the standard errors.

3. Data

Our analysis is based on the value-weighted excess returns for 30 industry portfolios at weekly and monthly frequencies. We use market, SMB, and HML risk factors as state variables (Fama and French, 1993, 1997). We gratefully obtain the data from Kenneth French's online data library. The sample covers the period from 1945 to 2015 and includes several business cycles such as the dotcom bubble and the recent financial crisis. For robustness, we also use 17- and 49-value weighted industry portfolios and 25 size and book-to-market double-sorted portfolios, which are also available from the same website.

Table 1 shows descriptive statistics for the monthly excess returns of the 30 industry portfolios. The mean return is significantly positive and varies from 6.0% per year ("Other") to 11.5% per year ("Smoke"). The standard deviations are relatively large, ranging from 13.3% per year ("Utilities") to 32.4% per year ("Coal"). For all industry portfolios, we observe negative skewness and positive excess kurtosis, revealing extreme negative returns.

We also use a set of macroeconomic explanatory variables measured at the monthly frequency including industrial-production growth (*IP*) and five variables from Goyal and Welch's (2008) data set: T-bill rate (*TBL*), term spread (*TMS*), inflation (*INFL*), default-yield spread (*DFY*), and default-return spread (*DFR*). We use their first two principle components (PC1 and PC2) to condense the information about the state of the macro economy. Table 2 shows the correlation of the PCs with the individual variables. PC1 loads strongly negatively on the T-bill rate and the inflation rate and strongly positively on the term spread. A large positive value of PC1 amounts to a contractionary state of the economy. PC2 loads strongly positively on the industrial production growth and strongly negatively on the term spread and the default-yield spread. A large positive value of PC2 amounts to an expansionary state of the economy.

4. Empirical results

In this section, we show the empirical results. First, we show the results regarding estimations of betas and the risk premia. Then, we discuss how the risk premia are related to the state of the economy. At the end, we investigate the robustness of the results to using other data sets.

4.1 Estimation of the bivariate component GARCH model

We use different frequency pairs (s, t) to decompose the long- and short-run components. The long-run component, t, is at the monthly frequency and the short-run component, s, varies from weekly to monthly frequency. Our default model is based on the monthly frequency. That is, the returns in equation (1), the short-run variance and covariance in equation (2), and the long-run variances and covariance in equation (3) are all based on monthly returns (hereafter denoted monthly–monthly or M–M). This is a GARCH specification with time-varying unconditional moments. We use the monthly–monthly approach as the base case to be able to compare our results with earlier studies since it is conventional to use a five-year moving window with monthly returns to estimate betas and also a monthly frequency to estimate the cross-sectional regression. We also use an alternative specification of the component GARCH model in which we keep the long-run moments in equation (3) at the monthly frequency while changing the frequency of the bivariate variance and covariance in equation (2) and the returns in equation (1) to weekly (denoted monthly–weekly or M–W). This is a MIDAS specification.

The results presented in the paper are based on fixed weights in equation (3) with $w_1 = w_2 = 1$, which implies equal weights for all observations. In this case, $\tau_{i,t}$, $\tau_{x,t}$, and $\tau_{ix,t}$ are the moving averages over the past five years of $V_{i,t}$, $V_{x,t}$, and $V_{ix,t}$, respectively. We have also used exponentially weighted moving average by setting $w_1 = 1$ and estimating w_2. Since the estimation of w_2 converges to 1 in most cases, we only report the results with the fixed weights. The advantage is that our estimated long-run betas are equal to the conventional estimate of the unconditional beta. This facilitates straightforward comparisons with earlier studies.

Table 3 shows the means and the standard deviations of the parameter estimates of the bivariate component GARCH models for the 30 industry portfolios and for each of the three factors, both for the monthly–weekly and monthly–monthly specifications. The parameter estimates show that the volatilities are persistent, because all the *b*s are much greater than the corresponding *a*s. The related standard deviations are very small, indicating that the volatility persistence should hold for most of the industries. As expected, the estimated mean returns (the γs) are larger in the monthly–monthly specification than in the monthly–weekly specification.

Figure 1 shows the average and standard deviation of the total betas estimated from the bivariate component GARCH models for the monthly–monthly and monthly–weekly combinations for each of the 30 industry portfolios. In general, the cross-sectional rank of the betas is similar across the two frequencies. The average market betas are similar across frequencies, while the SMB and HML betas are typically lower for the monthly–weekly frequency than for the monthly–monthly frequency. The standard deviations of the market and HML betas are larger for weekly returns, indicating a larger variation of these betas over time.

To illustrate the estimated betas over time, we use the financial industry as an example. Figure 2 shows the time series of the total and long-run betas for the monthly–monthly and monthly–weekly frequencies for this industry. The long-run betas are smoother than the total betas, especially when we use the monthly–weekly frequency instead of the monthly–monthly frequency. The market betas are less variable than the SMB and HML betas at both frequencies. As expected, the estimated betas are, in general, very large during the recent financial crises, which supports the large contribution of the financial industry to the systematic risks during this period.

4.2 Variations in betas across the state of the economy

Understanding how the market exposures of different industry betas change over the states of economy is essential for investors' portfolio choice and risk-management strategies. Many papers have identified the heterogeneous reaction of equity portfolios to the business cycle and the general economic conditions. Boudoukh, Richardson, and Whitelaw (1994) show that different industries possess different cyclical tendencies with the overall economy. Some industries are highly cyclical, while others are less dependent on the state of the economy. Petersen and Strauss (1991) find that industries producing durable goods tend to exhibit much more cyclical investment behavior than industries producing nondurable goods. This is because the cash flow is more procyclical in the durable-goods sector than in the nondurable-goods sector. Berk, Green, and Naik (1999) argue that the cross-sectional variations in betas depend on firms' investment opportunities, which vary across industries. Gomes, Yaron, and Zhang (2003) link the cyclical behavior of betas to different size and growth opportunities determining firm-specific reactions to aggregated productivity. Finally, Gonzalez, Nave, and Rubio (2016) conclude that value, small, low momentum, and low-long-reversal stocks have countercyclical betas, while growth, big, high momentum, and high-long-reversal stocks have procyclical betas.

We analyze how short-run betas are related to economic variables to see if the fluctuations of factor betas around their long-run paths can be explained by the macro-economic conditions.[5] Table 4 reports the results of regressing the cross-sectional averages of the monthly–monthly short-run betas on one-month lagged macro-economic variables. In Panel A, we use the individual macroeconomic variables; in Panel B, we use the first two principle components calculated from these macro-economic variables.

From Table 4, we see that the short-run market beta depends positively on the T-bill rate, the term spread, the inflation rate, and the default yield spread. Similarly, the short-run market beta depends positively on PC1, which is positively correlated with the contractionary state. The short-run SMB beta depends negatively on industrial-production growth, the default return spread, and PC2, which is positively correlated with the expansionary state. The short-run HML beta depends positively on the T-bill rate, the term spread, the inflation rate, and the default yield spread, and negatively on the default return spread. It depends positively on PC1 and negatively on PC2. All in all, we can conclude that the short-run component of all factor betas increase when the economy is in a bad state.

Table 5 shows the results from regressing the short-run beta of each industry portfolio on the PC1 and PC2. In line with Petersen and Strauss (1991), Boudoukh, Richardson, and Whitelaw (1994), and Baele and Londono (2013), we find that industries related to the necessities (such as "Beer," "Smoke," and "Clothes") are less affected by fluctuations in economic conditions. We also observe

[5] We do not analyze the relation of the long-run betas to the state of the economy because they are estimated over longer periods; matching the period of the macroeconomic variables with the long-run betas is not straightforward.

a positive relation between the economic conditions and market betas for industries such as "Chemical," "Telecommunications," "Business equipment," "Papers," and "Retail." However, in contrast with the results reported from previous studies, we find a significant increase in short-run market beta during recessions for such nondurable industries as "Food," "Games," and "Household."

The link between the cross-sectional dispersion of industry betas and the state of the economy has been examined previously. Gomes, Yaron, and Zhang (2003) find that the heterogeneity of betas across firms increases during recessions leading to increasing beta dispersion. This effect is reinforced by the countercyclical behavior of dispersion of the firms' characteristics, which is in line with the findings of Chan and Chen (1988). Baele and Londono (2013) find that the empirical cross-sectional dispersions in industry betas increase during recessions.

In this paper, we examine the link between the cross-sectional dispersions of short-run betas and the economic conditions. We use the method in Baele and Londono (2013) to calculate the cross-sectional dispersion of the betas for each month. Then, we regress the dispersion coefficients on PC1 and PC2. The regression results are reported in Table 6. The cross-sectional dispersion of the estimated short-run SMB and HML betas depend positively on PC1 and negatively on PC2, which, in line with Baele and Londono (2013) and Gomes, Yaron, and Zhang (2003) shows that cross-sectional dispersion of betas increases in contractionary economic conditions.

4.3 Cross-sectional regressions

To evaluate our suggested component GARCH model, we compare its pricing ability with that of two alternative models for estimating beta: the traditional rolling-window OLS regressions (unconditional betas) and the bivariate GARCH model. For these comparisons, we use both weekly and monthly returns.

Table 7 shows the total risk premia obtained from various models, the mean of the estimated time-series coefficients from the cross-sectional regressions in equation (8). First, the table shows the estimated risk premia associated with the unconditional betas. The market and HML risk premia are not significant. The SMB risk premium is significantly positive at the monthly frequency, which is in accordance with earlier findings, whereas it is insignificant at the weekly frequency. Table 7 then shows the risk premia obtained from the conventional bivariate GARCH model. None of the risk premiums are significant irrespectively of data frequency. Finally, the table shows the total risk premia related to the component GARCH model. Here the total risk premia are qualitatively similar to the unconditional risk premia, namely, that only the SMB risk premium is significant, and only so at the monthly–monthly frequency. So, if we were only interested in total risk premia, the component GARCH model provides the same information as the traditional model.

Now we move on to the cross-sectional regressions in equation (9) with long- and short-run betas from the component GARCH mode. Table 8 shows the risk premia of the long- and short-run components of beta. At the monthly–weekly frequency, none of the risk premia in Tables 7 and 8 is significant. This indicates that risk premia based on the weekly frequency is too noisy. For the monthly–monthly frequency, several of the risk premia are significantly positive. The risk premia associated with both the long- and short-run SMB betas are significant. Interestingly, the risk premium associated with the short-run market beta is also significantly positive. This is, to some extent, consistent with Cenesizoglu and Reeves (2015), who show that the overall performance of the three-component beta model (short, medium, and long) is mostly due to short and medium term betas, while the long-run beta contributes little.

To investigate if the significance of the risk premium of the short-run market beta is robust to the choice of the test assets, we estimate our model for some alternative portfolios. First, we use 25 doubled-sorted Fama and French (1993) book-to-market and size portfolios. Second, we use a finer division into industries (49 industry portfolios) and broader division into industries (17 industry portfolios). Table 9 shows the variations in the long- and short-run risk premia for the four data sets. The risk premia related to the short-run market beta are significantly positive for all four data sets. So, this finding is not specific to the 30-industry data set. It seems that investors demand compensation only when the market beta increases from its long-run level.

4.4 Risk premia across the business cycle

In Table 10, we relate the risk premia to the state of the economy as measured by NBER recessions. More specifically, we calculate the average of the estimated parameters of the cross-sectional regressions in recessions and normal periods. The table shows the risk premia for the entire sample period (analogous to Tables 7 and 8), only for normal periods, and only for recessions for the monthly–monthly frequency. Panel A is concerned with total risk premia and panel B with short- and long-run risk premia. For the unconditional model, the risk premia during normal periods are similar to those for the entire sample period. The values are very different in recessions, where the market risk premium is significantly negative, showing the large average ex-post realized return for risky firms, firms with high market betas. For the bivariate GARCH model, the risk premia of all the factors are insignificant for all the subsamples, except the market risk premium which is significantly negative in recession. The total betas from the component GARCH model also give significant risk premia for SMB and HML (only at the 10% level for the latter). Overall, none of these estimations gives a significantly positive risk premium for the market beta, which is consistent with findings from the previous literature.

For the component GARCH model (Panel B of Table 10), the short- and long-run SMB risk premia are significantly positive and slightly larger in normal periods than for the entire sample period.

The short-run market risk premium is significantly positive in the entire sample period and during normal periods. The short-run market risk premium is larger in normal periods than for the entire sample period, which is caused by the negative (and insignificant) risk premium in recessions. The negative short-run market risk premium in recessions is similar to the negative unconditional risk premium. Risk premia for both short- and long-run HML for nonrecession periods are positive and significant at the 10% level. The insignificance of the HML factor for the total period is due to the fact that our period of study has some important recession periods that cause a large negative realized mean return and result in an insignificant risk premium. In general, excluding recessions from our sample makes the risk premia for all the betas, except for the long-run market beta, significant and with the expected sign.

5. Conclusion

This paper proposes a new model for decomposing systematic risk into long- and short-run components and provides an important empirical application. The new bivariate component GARCH model enables us to simultaneously decompose total variances and covariances into long- and short-run variances and covariances and thereby to estimate the corresponding components of the asset betas. We model the long-run variances and covariances based on the unconditional variance and covariance of past long-run monthly returns, while the short-run variances and covariances are based on data with different frequencies (weekly or monthly).

The main analysis is based on Fama and French's (1993) 30 industry portfolios. We investigate the dynamics and determinants of market, SMB, and HML industry betas (Fama and French, 1993)We apply our component GARCH model to each factor and an industry portfolio to estimate long- and short-run variances and covariances. From these, we calculate long- and short-run betas and use them in cross-sectional regressions to estimate the long- and short-run risk premia associated with each factor.

We find that the short-run component of betas for all three factors increase on average in contractionary states of the economy, where industries related to necessities are less affected by fluctuating economic conditions. We also find that the cross-sectional dispersion in short-run betas increases in contractionary economic conditions. Moreover, we find that the data frequency matters for estimation of the risk premium: None of the risk premia estimated at weekly frequency is significant. At the monthly frequency, our analysis of the risk premia highlights the importance of decomposing risk across horizons. Although, the risk premia associated with both the long- and short-run SMB betas are significant, only the risk premium associated with the short-run market beta is significantly positive. The results appear to be robust to choice of data set, at least for different divisions into industry portfolios and for portfolios based on size and book-to-market. In the future, it could be interesting to see if the empirical results also hold for other than the US stock market. Moreover, it would be interesting to put the component GARCH model to use in other settings.

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Appendix

This appendix contains technical details about the component GARCH model.

A.1. Likelihood function

The bivariate component GARCH model written in matrix form is as

$$\begin{pmatrix} r_{i,s,t} \\ r_{x,s,t} \end{pmatrix} = \begin{pmatrix} \gamma_{0i} \\ \gamma_{0x} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,s,t} \\ \varepsilon_{x,s,t} \end{pmatrix} \sim Q_t^{1/2} \zeta_t \tag{A.1.1}$$

$$Q_t = \begin{pmatrix} q_{i,s,t} & q_{ix,s,t} \\ q_{ix,s,t} & q_{x,s,t} \end{pmatrix}$$

$$\begin{aligned} &= \begin{pmatrix} \tau_{i,t}(1-\alpha_i^2-b_i^2) & \tau_{ix,t}(1-\alpha_i\alpha_x-b_ib_x) \\ \tau_{ix,t}(1-\alpha_i\alpha_x-b_ib_x) & \tau_{x,t}(1-\alpha_x^2-b_x^2) \end{pmatrix} \\ &+ \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \begin{pmatrix} \varepsilon_{i,s-1,t}^2 & \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} \\ \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} & \varepsilon_{x,s-1,t}^2 \end{pmatrix} \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \\ &- \begin{pmatrix} b_i & 0 \\ 0 & \alpha b_x \end{pmatrix} \begin{pmatrix} q_{i,s-1,t} & q_{ix,s-1,t} \\ q_{ix,s-1,t} & q_{x,s-1,t} \end{pmatrix} \begin{pmatrix} b_i & 0 \\ 0 & b_x \end{pmatrix}. \end{aligned} \tag{A.1.2}$$

The error terms in the return equation are assumed to be bivariate normally distributed with Q_t as the conditional variance–covariance matrix and ζ_t is an IID vector process such that $\mathrm{E}(\zeta_t\zeta_t') = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix.

The log likelihood function is

$$L(\Theta) = -\frac{1}{2}\sum_{i=1}^{T}[\ln(2\pi) + ln|Q_t|] + \varepsilon_{st}' Q_t^{-1} \varepsilon_{st}. \tag{A.1.3}$$

A.2. Positive definiteness

Here, we discuss the necessary restrictions on the parameters to ensure the positive definiteness of the conditional variance–covariance matrix, $Q_{s,t}$.

Recall that the residuals from the return equations of the bivariate component GARCH model are assumed joint normal:

$$\boldsymbol{\varepsilon}_{s,t}|\mathcal{F}_{s-1,t} \sim \mathbb{N}\,(0, \boldsymbol{Q}_{s,t}), \tag{A.2.1}$$

where, without loss of generality, s and t denote periods corresponding to the higher and lower frequency, respectively, $t = 1, \dots, T$ and $s = 1, \dots, t, \dots, 2t, \dots, S$, $S = T \times m$, m is the block size. [.]denotes a floor function of the quotient. Clearly, $t = [s/m] + 1$ (note that in practice the block size, m, might be different, one might not have T full blocks of data.).

Denote returns $R_t = (r_{it}, r_{xt})^T$ as an exogenous p covariate, where i and x denote portfolios and state variables, respectively. Assume also that R_t are component-wise stationary, ergodic strongly mixing processes with mixing coefficient $\sum_{n=1}^{\infty} \alpha_n^{1-2/\gamma} < \infty\, (\gamma > 2)$. Q_t is parameterized as measurable to $\mathcal{F}_{s-1,t}$ and exogenous variable R_t. $\boldsymbol{\varepsilon}_{s,t}$ is a $d \times 1$ and $\boldsymbol{Q}_{s,t}$ is a $d \times d$ matrix. Without loss of generality, $\boldsymbol{\varepsilon}_{s,t} = \boldsymbol{Q}_{s,t}^{1/2} \eta_{s,t}$, and $\eta_{s,t}$ is assumed to be iid bivariate Gaussian $\sim \mathbb{N}(0, I_d)$, and is independent of

$$\mathcal{F}_{s-1,t} = \sigma(\varepsilon_{s-1,t}, \varepsilon_{s-2,t}, \dots, \varepsilon_{s-1,t}, \varepsilon_{s-2,t}, \dots).$$

The bivariate component GARCH model in matrix form is

$$\boldsymbol{Q}_{s,t} = \boldsymbol{\tau}_t - \boldsymbol{A}'\tau_t\boldsymbol{A} - \boldsymbol{B}'\tau_t\boldsymbol{B} + \boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,t}, \boldsymbol{\varepsilon}'_{s-1,t}\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{Q}_{s-1,t}\boldsymbol{B}, \tag{A.2.2}$$

where $\boldsymbol{\tau}_t$ is a $d \times d$ random variable (long-run exogenous matrix), $\boldsymbol{A}$, $\boldsymbol{B}$ are $d \times d$ coefficient matrices and are assumed to be real matrices. In particular, $A = (a_i, 0; 0, a_x)$, $B = (b_i, 0; 0, b_x)$ as in equation (2).

Remark The term $\boldsymbol{\tau}_t - \boldsymbol{A}\tau_t\boldsymbol{A} - \boldsymbol{B}'\tau_t\boldsymbol{B}$ resembles the variance-targeting constant term in Pedersen and Rahbek (2014). However, it is worth noting that the constant term is time varying in our case and is driven by low-frequency variables.

$\boldsymbol{\tau}_t$ is defined to be $\boldsymbol{V}_{tK}\mathrm{diag}(\boldsymbol{\omega})\boldsymbol{V}'_{tK} = \sum_{k=1}^{K} \boldsymbol{\omega}_k \boldsymbol{V}_{tk}\boldsymbol{V}'_{tk}$, where $\boldsymbol{V}_{tK}$ is a $d \times K$ matrix of (low-frequency) exogenous shocks, $\boldsymbol{\omega}$ is a $K \times 1$ vector, and $\mathrm{diag}(\boldsymbol{\omega})$ is a $K \times K$ matrix with diagonal element as ω . In our case, d = 2 and $\omega = (\omega_1, \omega_2, \dots, \omega_K)^T$ is set to be $(\phi_1(\omega_1, \omega_2), \phi_2(\omega_1, \omega_2), \dots, \phi_K(\omega_1, \omega_2))^T$ defined in equation (3), and $\boldsymbol{V}_{tK} = (V_{t1}, \dots, V_{tk})$ with $\boldsymbol{V}_{tk} = (r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_2)^T$.

To ensure $\boldsymbol{Q}_{s,t}$ is positive definite at each s, t, we first need to impose a condition to guarantee that $\boldsymbol{C}_{rt} \stackrel{\text{def}}{=} \boldsymbol{\tau}_t - \boldsymbol{A}'\boldsymbol{\tau}_t\boldsymbol{A} - \boldsymbol{B}'\boldsymbol{\tau}_t\boldsymbol{B}$ is positive definite. We define first the matrix $\boldsymbol{C} = 1 - \boldsymbol{A}'1\boldsymbol{A} - \boldsymbol{B}'1\boldsymbol{B}$, where 1 is an all-one 2 x 2 matrix. Namely $\boldsymbol{C} = (1 - a_i^2 - b_i^2, 1 - a_i a_x - b_i b_x; 1 - a_i a_x - b_i b_x, 1 - a_x^2 - b_x^2) \stackrel{\text{def}}{=} (c_i, c_{ix}; c_{ix}, c_x)$.

Proposition 1. *(Positive definiteness of $\boldsymbol{C}_{rt}$) If $c_i > 0, c_x > 0, c_i c_x - c_{ix}^2 > 0$, then the matrix $\boldsymbol{C}$ is positive definite, and $\boldsymbol{C}_{rt}$ is positive definite almost surely.*

Proof. Because $\boldsymbol{\tau}_t = \boldsymbol{V}_{tK}\text{diag}(\boldsymbol{\omega})\boldsymbol{V}_{tK}' = \sum_{k=1}^K \omega_k V_{tk} V_{tk}'$, we can define the matrix $\boldsymbol{\tau}_t = (\widetilde{\omega}_1^2, \widetilde{\omega}_{12};\ \widetilde{\omega}_{12}, \widetilde{\omega}_2^2)$ with $\widetilde{\omega}_1^2 = \sum_{k=1}^K \omega_k (r_{i,t-k} - \mu_i)^2$, $\widetilde{\omega}_2^2 = \sum_{k=1}^K \omega_k (r_{x,t-k} - \mu_x)^2$ and $\widetilde{\omega}_{12} = \sum_{k=1}^K \omega_k (r_{x,t-k} - \mu_x)(r_{i,t-k} - \mu_i)$. Now, we can write that $\boldsymbol{C}_{rt} = (c_i \widetilde{\omega}_1^2, c_{ix}\widetilde{\omega}_{12}; c_{ix}\widetilde{\omega}_{12}, c_x \widetilde{\omega}_2^2)$. We now calculate the two eigenvalues of $\boldsymbol{C}_{rt}$. Letting $T_i = c_i \widetilde{\omega}_1^2 + c_x \widetilde{\omega}_2^2$ and $D_i = c_i \widetilde{\omega}_1^2 c_x \widetilde{\omega}_2^2 - c_{ix}^2 \widetilde{\omega}_{12}^2$,

$$\lambda_1(C_{rt}) = \left(\frac{T_i}{2}\right) + \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}, \lambda_2(C_{rt}) = \left(\frac{T_i}{2}\right) + \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}.$$

As $c_i, c_x > 0$, $T_i > 0$ because $\frac{T_i^2}{4} - D_i = \frac{(c_i \widetilde{\omega}_1^2 + c_x \widetilde{\omega}_2^2)^2}{4} - c_i \widetilde{\omega}_1^2 c_x \widetilde{\omega}_2^2 + c_{ix}^2 \widetilde{\omega}_{12}^2 = \frac{(c_i \widetilde{\omega}_1^2 + c_x \widetilde{\omega}_2^2)^2}{4} + c_{ix}^2 \widetilde{\omega}_{12}^2 \geq 0$. Further, by the Cauchy-Schwarz inequality $\widetilde{\omega}_{12}^2 \leq \widetilde{\omega}_1^2 \widetilde{\omega}_2^2$, we have $D_i \geq (c_i c_x - c_{ix}^2 \widetilde{)\omega}_1^2 \widetilde{\omega}_2^2$. Therefore, by $c_i c_x - c_{ix}^2 > 0$, $D_i \geq 0$. This would lead to $\lambda_1(\boldsymbol{C}_{rt}), \lambda_2(\boldsymbol{C}_{rt}) \geq 0$. Moreover $\lambda_2(\boldsymbol{C}_{rt}) = 0$ if and only if $\widetilde{\omega}_1 = 0$ or $\widetilde{\omega}_2 = 0$. As weights ω are positive and returns $r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_x$ are continuously distributed, $\widetilde{\omega}_1 = 0$ and $\widetilde{\omega}_2 = 0$ with probability 0.

Remark If one would like to extend the model to a multidimensional case, this result can also be proved by considering $\boldsymbol{C}_{rt} = \boldsymbol{C} \circ \boldsymbol{r}_t$, where $\circ$ denotes the elementwise (Hadamard) product of two matrices. As $\boldsymbol{r}_t s$ is a weighted sum of almost surely positive-definite matrices $\boldsymbol{V}_{tk}\boldsymbol{V}'_{tk}$ (symmetric and real), then by the Weyl's inequality in matrix theory, the smallest eigenvalue of $\boldsymbol{r}_t$ is almost surely positive as well. Also, $\boldsymbol{C}$ is positive definite according to our conditions. Therefore, we have by the Schur product theory for the Hadamard product, as $\boldsymbol{c}_{rt}$ is the Hadamard product of $\boldsymbol{C}$ and $\boldsymbol{\tau}_t$, $\boldsymbol{C}_{rt}$ is almost surely positive definite.

Proposition 2. (Positive definiteness of $\boldsymbol{Q}_{s.t}$) Suppose that diagonal element $b_x \neq 0$ and $a_i > 0$, $b_i > 0$, $\boldsymbol{Q}_0$ is a positive definite matrix and conditions in proposition 1 hold, then $\boldsymbol{Q}_{s,t}$ is positive definite for all s.

Proof. If $\boldsymbol{C}_{rt}$ is almost surely positive definite, $\boldsymbol{B}'\boldsymbol{Q}_{s-1,t}\boldsymbol{B}$ is positive definite, and $\boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}'_{s-1,t}\boldsymbol{A}$ is semipositive definite, we have $\boldsymbol{Q}_{s,t}$ to be positive definite. The positive definiteness of $\boldsymbol{C}_{rt}$is addressed by proposition 1. Since $\boldsymbol{Q}_0$ is positive definite, $\boldsymbol{B}'\boldsymbol{Q}_0\boldsymbol{B}$ is positive definite, so the positive definiteness of $\boldsymbol{B}'\boldsymbol{Q}_{s,t}\boldsymbol{B}$ follows by iteration. As it can be seen that $\text{rank}\left(\boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,i}\boldsymbol{\varepsilon}'_{s-1,i}\boldsymbol{A}\right) = 1$, $\boldsymbol{Q}_{s,t}$ is positive definite.

A.3. Stationarity

Here, we show the identifiability and stationarity results. We rewrite the model in vector form as

$$\text{vec}\{\boldsymbol{Q}_{s,t}\} = (I - \boldsymbol{A}' \otimes \boldsymbol{A}' - \boldsymbol{B}' \otimes \boldsymbol{B}')\text{vec}\{\boldsymbol{r}_t\} + \boldsymbol{A}' \otimes \boldsymbol{A}'\text{vec}\{\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}'_{s-1,t}\} + \boldsymbol{B}' \otimes \boldsymbol{B}'\text{vec}\{\boldsymbol{Q}_{s-1,t}\}. \quad \text{(A.2.3)}$$

As we would not need all the elements of a matrix if it is symmetric, we write (A.2.2) in terms of the vech operator. The vech form of the bivariate component model specified in equation (A.2.2) can be derived as

$$\text{vech}\{\boldsymbol{Q}_{s,t}\} = \widetilde{\boldsymbol{C}}\text{vech}\{\boldsymbol{r}_t\} + \widetilde{\boldsymbol{A}}\text{vech}\{\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}'_{s-1,t}\} + \widetilde{\boldsymbol{B}}\text{vech}\{\boldsymbol{Q}_{s-1,t}\}, \tag{A.2.4}$$

where the operator vech denotes vectorize the lower diagonal elements of a symmetric matrix. $\widetilde{\boldsymbol{A}} = \text{diag}(a_i^2, a_i a_x, a_x^2)$, $\widetilde{\boldsymbol{B}} = \text{diag}(b_i^2, b_i b_x, b_x^2)$, and $\widetilde{\boldsymbol{C}} = \boldsymbol{I}_{d(d+1)/2} - \tilde{A} - \widetilde{\boldsymbol{B}}$.

Proposition 3. *(Identifiability) Suppose that* $a_i > 0$ *and* $b_i > 0$. *Then the parameters in equation (4) are identifiable.*

Proof. In equation (2), the coefficients attached to $\varepsilon_{s-1,t}^2$ are a_i^2, which is identified up to its sign, as are b_i. The coefficient associated with $\varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t}$ is $a_i a_x$. Since a_i is identified, a_x would be identified as well. Similarly, b_x is identified. ■

The stationarity of the BEKK model is studied in Boussama, Fuchs, and Stelzer (2011). Next, we prove that we need to ensure the spectral radius of $\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}}$ is less than one for the stationarity of our model. In particular, this is equivalent to $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

Proposition 4. *(Covariance Stationarity) If* $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$, *the model is covariance stationary, and the stationary covariance* Σ *is of the form* $\text{vech}\{\Sigma\} = (I - \widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{B}})^{-1}\widetilde{\boldsymbol{C}}_{\tau_\infty}$.

The stationary solution of equation (2) is

$$\text{vech}\{\boldsymbol{Q}_{s,t}\} = \sum_{l=1}^{\infty} \widetilde{\boldsymbol{B}}^{l-1}\widetilde{\boldsymbol{A}}\text{vech}\{\boldsymbol{\varepsilon}_{s-l,i}\boldsymbol{\varepsilon}'_{s-l,t}\} + \sum_{l=1}^{\infty} \widetilde{\boldsymbol{B}}^{l-1}\widetilde{\boldsymbol{C}}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l)/m \rfloor+1,t}\}. \tag{A.2.5}$$

Proof. *As in the proof of proposition 2.7 in Engle and Kroner (1995), denote by* $\mathbb{E}_t$ *the conditional expectation* $\mathbb{E}(\cdot\,|\mathcal{F}_t)$, *conditioning on the information set* $\mathcal{F}_t$.

$$\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\varepsilon}_{s,t}\boldsymbol{\varepsilon}'_{s,t}\} = \sum_{l=2}^{L}(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2}\widetilde{\boldsymbol{C}}\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}\} + (\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{L-1}\text{vech}\{\boldsymbol{Q}_{s-L+1}\} \tag{A.2.6}$$

As $L \to \infty$, $(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{L-1} \to 0$ *if* $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

As we have assumed that $\{\boldsymbol{R}_{\mathrm{t}}\}$ are element-wise strong mixing processes, the elements in $\boldsymbol{\tau}_t$ are the weighted sum of functions relating to $\{\boldsymbol{R}_t\}$. Mixing series are measure preserving. It can be seen that for the blocks $b = 1, 2, \ldots, \lfloor L/m \rfloor, \sum_{l=(b-1)m+1}^{bm} (\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2} \widetilde{\boldsymbol{C}} \mathbb{E}_{s-L} \text{vech} \boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}$ will be mixing. Note that within block b, as $\text{vech}\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}$ does not vary with respect to s, therefore the value $\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}$ stays the same within a block. As long as $L/m \rightarrow \infty$, it is not hard to see that $\lim_{L\to\infty} \sum_{l=2}^{L} (\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2} \widetilde{\boldsymbol{C}} \mathbb{E}_{s-L} \text{vech} \boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t} \underset{p}{\rightarrow} (I - \widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{B}})^{-1} \widetilde{\boldsymbol{C}}_{\tau_\infty}$, where $\boldsymbol{\tau}_\infty = \mathbb{E}\text{vech}\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1}$.

Table 1: Summary statistics for excess returns of 30 industry portfolios

The table shows the yearly means, standard deviations, excess kurtosis, and skewness of the excess returns in percentage for the 30 industrial portfolios. The monthly sample covers the period from 1945 to 2015. The data are from Kenneth French's online data library. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

	Mean	St. dev	Excess kurtosis	Skewness
Food	8.610***	14.179	2.479***	−0.056
Beer	9.767***	18.303	4.868***	0.412***
Smoke	11.512***	19.633	2.870***	−0.065
Games	9.370***	23.862	2.519***	−0.186**
Books	7.641***	19.509	2.412***	−0.025
Hshld	8.179***	16.119	1.503***	−0.302***
Clths	8.400***	20.611	3.115***	−0.083
Hlth	10.165***	16.960	1.903***	0.066
Chems	8.006***	18.456	2.121***	−0.096
Txtls	8.865***	23.308	9.418***	0.492***
Cnstr	7.811***	19.776	2.431***	−0.210**
Steel	6.138***	23.904	2.335***	−0.240***
FabPr	7.751***	20.155	2.513***	−0.384***
ElcEq	9.906***	20.931	1.443***	−0.160*
Autos	7.768***	22.176	5.893***	0.209**
Carry	9.737***	21.213	1.360***	−0.260***
Mines	6.400***	23.920	2.220***	−0.173**
Coal	9.475***	32.400	2.714***	0.143*
Oil	9.232***	18.199	1.062***	−0.005
Util	7.096***	13.343	1.098***	−0.201**
Telcm	6.486***	14.695	1.837***	−0.174**
Servs	9.935***	21.490	1.514***	−0.151*
BusEq	9.681***	22.163	2.051***	−0.311***
Paper	8.612***	17.269	2.098***	−0.169**
Trans	7.584***	19.354	1.279***	−0.200**
Whlsl	8.294***	18.602	2.283***	−0.298***
Rtail	9.001***	17.528	2.409***	−0.222***
Meals	10.068***	20.450	2.478***	−0.405***
Fin	8.523***	17.779	1.808***	−0.405***
Other	5.959***	19.152	1.752***	−0.388***

Table 2: Correlation matrix between principal components and macro variables

This table shows the correlation between the first two principle components (PC1 and PC2) and the underlying macro variables: industrial production growth (IP), T-bill rate (TBL), term spread (TMS), inflation (INFL), default-yield spread (DFY), and default-return spread (DFR). The monthly sample covers the period from 1945 to 2015.

	PC1	PC2	IP	TBL	TMS	INFL	DFY	DFR
PC1	1.000							
PC2	0.000	1.000						
IP	0.143	0.598	1.000					
TBL	−0.885	−0.071	−0.038	1.000				
TMS	0.619	−0.558	−0.037	−0.440	1.000			
INFL	−0.762	0.023	−0.044	0.499	−0.280	1.000		
DFY	−0.283	−0.827	−0.281	0.334	0.268	0.101	1.000	
DFR	0.068	−0.277	−0.021	−0.039	0.085	0.005	0.077	1.000

Table 3: Parameter estimates of the component GARCH model

The table shows the means and standard deviations of the parameter estimates from the bivariate component GARCH model in equations (1)–(5) for the monthly–weekly (M–W) and monthly–monthly (M–M) frequencies for the 30 industry portfolios and the market, small-minus-bi, and high-minus-low factors. The sample covers the period from 1945 to 2015.

		γ_i		γ_x		a_i		a_x		b_i		b_x	
		Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
M–W	Market	0.208	0.036	0.190	0.011	0.241	0.015	0.265	0.016	0.963	0.005	0.954	0.007
	SMB	0.201	0.038	0.012	0.005	0.290	0.047	0.262	0.015	0.942	0.021	0.949	0.008
	HML	0.209	0.032	0.062	0.004	0.251	0.023	0.272	0.006	0.959	0.008	0.958	0.002
M–M	Market	0.731	0.146	0.662	0.057	0.282	0.020	0.284	0.026	0.941	0.014	0.943	0.011
	SMB	0.693	0.150	0.008	0.031	0.284	0.030	0.319	0.021	0.939	0.020	0.880	0.037
	HML	0.803	0.192	0.305	0.057	0.289	0.028	0.313	0.016	0.927	0.028	0.939	0.014

Table 4: Macroeconomic influences on cross-sectional average of short-run betas

The table shows the coefficients and *t*-values from univariate (Panel A) and multivariate (Panel B) regressions of the cross-sectional averages of the monthly–monthly short-run betas on the individual macroeconomic variables and the principle components, respectively. The intercepts of the univariate regressions are not tabulated. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Panel A. Univariate regressions

	Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
IP	0.006	1.518	−0.141***	−6.457	−0.023	−0.919
TBL	0.006***	4.870	−0.009	−1.187	0.040***	4.717
TMS	0.006***	4.870	−0.009	−1.187	0.040***	4.717
INFL	0.006***	4.870	−0.009	−1.187	0.040***	4.717
DFY	0.006***	4.870	−0.009	−1.187	0.040***	4.717
DFR	0.000	−0.135	−0.044***	−5.281	−0.020**	−2.104

Panel B. Multivariate regressions

	Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Intercept	0.006	1.518	−0.141***	−6.457	−0.023	−0.919
PC1	0.006***	4.870	−0.009	−1.187	0.040***	4.717
PC2	0.000	−0.135	−0.044***	−5.281	−0.020**	−2.104

Table 5: Macroeconomic influences on individual short-run betas

The table shows the coefficients and *t*-values from regressions of the individual monthly–monthly short-run betas on the first two principle components. Intercepts are included in all regressions but are not tabulated. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

	Market		SMB		HML	
	PC1	*PC2*	*PC1*	*PC2*	*PC1*	*PC2*
Food	0.020***	−0.014***	−0.008	−0.042***	0.007	0.014
Beer	−0.004	−0.015***	−0.003	−0.062***	0.058***	−0.024**
Smoke	0.038***	−0.012**	−0.006	−0.001	0.050***	0.035***
Games	0.014***	0.002	0.010	−0.067***	0.046***	−0.069***
Books	0.003	−0.017***	−0.019*	−0.084***	0.035***	−0.050***
Hshld	0.021***	0.000	0.002	−0.054***	0.025**	−0.027**
Clths	0.007	0.000	−0.012	−0.054***	0.029**	−0.020
Hlth	0.019***	−0.014***	0.005	−0.026***	0.028***	0.007
Chems	0.009***	0.001	−0.010	−0.043***	0.049***	0.001
Txtls	0.006	−0.034***	−0.019	−0.122***	0.014	−0.036**
Cnstr	−0.007***	0.001	−0.019*	−0.049***	0.061***	−0.023*
Steel	0.005	0.027***	−0.030***	−0.056***	0.057***	0.002
FabPr	0.003	0.000	0.005	−0.052***	0.056***	−0.051***
ElcEq	−0.003	0.006*	−0.003	−0.055***	0.066***	−0.038***
Autos	0.009**	−0.011**	−0.023**	−0.075***	0.047***	−0.037***
Carry	0.009**	0.008**	0.018*	−0.056***	0.054***	−0.055***
Mines	0.002	0.028***	−0.017	−0.033**	0.046***	−0.009
Coal	−0.014**	0.038***	−0.008	−0.021	0.071***	0.007
Oil	−0.013***	0.007*	−0.001	−0.006	0.038***	−0.009
Util	0.010***	−0.017***	−0.003	−0.022***	0.016**	0.015**
Telcm	0.020***	−0.014***	−0.010*	−0.024***	0.007	0.018**
Servs	−0.007**	0.022***	−0.022***	0.000	0.055***	−0.026*
BusEq	0.010***	0.006	−0.016	−0.055***	0.038***	−0.040***
Paper	0.008***	−0.004	−0.012	−0.048***	0.031***	−0.021*
Trans	0.000	0.013***	−0.018**	−0.037***	0.054***	−0.035***
Whlsl	0.001	0.013***	−0.009	−0.012	0.053***	−0.038***
Rtail	0.006**	−0.011***	−0.018**	−0.051***	0.016*	−0.033***
Meals	0.016***	0.018***	0.003	−0.025**	0.031***	−0.036***
Fin	0.004	−0.021***	−0.012	−0.070***	0.023**	−0.016
Other	0.001	−0.011***	−0.010	−0.032***	0.047***	−0.017

Table 6: Dispersion regressions

The table shows the coefficients and *t*-values from regressions of the cross-sectional dispersion in short-run betas on the first two principle components constructed on macro variables. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

	Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Intercept	0.134***	43.618	0.212***	33.695	0.245***	31.142
PC1	0.001	0.956	0.017***	7.867	0.007***	2.567
PC2	0.000	0.371	−0.011***	−4.515	−0.012***	−3.992

Table 7: Total risk premia

The table shows the risk premia estimated using unconditional betas, betas from a conventional bivariate GARCH model for weekly and monthly frequencies, and the total betas from the bivariate component GARCH model for the monthly–weekly (M–W) and monthly–monthly (M–M) frequencies. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

		Intercept		Market		SMB		HML	
		Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Unconditional	*Weekly*	0.175***	4.11	−0.010	−0.12	0.034	0.90	0.026	0.65
	Monthly	0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71
Bivariate GARCH	*Weekly*	0.147***	4.21	0.001	0.02	−0.008	−0.27	−0.040	−1.10
	Monthly	0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04
Component GARCH	*M–W*	0.156***	4.24	0.019	0.26	0.005	0.16	−0.013	−0.34
	M–M	0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23

Table 8: Long- and short-run risk premia

The table shows the risk premia for long- and short-run betas from the bivariate component GARCH model with monthly–weekly (M–W) and monthly–monthly (M–M) frequencies. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

			Long						Short					
	Intercept		Market		SMB		HML		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
M–W	0.150***	3.32	0.018	0.20	0.013	0.30	0.021	0.42	0.050	0.58	0.039	0.90	−0.019	−0.43
M–M	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29

Table 9: Variations in long- and short-run risk premia across data sets

The table shows the risk premia estimated using long- and short-run betas from the bivariate component GARCH model for the monthly–monthly frequency using four data sets. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

			Long						Short					
	Intercept		Market		SMB		HML		M		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
25 BM–Size	0.876***	4.29	−0.306	−1.05	0.371***	2.55	0.356**	2.01	0.318*	1.76	0.054	0.32	0.102	0.65
30 Industries	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29
49 industries	0.467***	3.35	0.202	0.83	0.350**	2.41	0.284*	1.72	0.690***	2.60	0.470**	2.31	0.105	0.61
17 Industries	1.032***	4.72	−0.513	−1.44	0.411*	1.74	−0.004	−0.02	1.169***	3.79	0.038	0.12	−0.353	−0.65

Table 10: Recession and risk premia

The table shows the risk premia for the entire sample, for NBER normal periods, and NBER recessions. Panel A shows the risk premia from the monthly total betas for the unconditional model, the bivariate GARCH model, and the component GARCH model with monthly–monthly frequency. Panel B shows the short- and long-run risk premia from the component GARCH model for the monthly–monthly frequency. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Panel A. Risk premia for total betas

		Intercept		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.
Unconditional	*Entire*	0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71
	Normal	0.834***	4.378	−0.159	−0.516	0.427**	2.493	0.231	1.382
	Recession	1.004**	2.128	−2.335***	−3.070	0.876**	2.063	−0.604	−1.459
Bivariate GARCH	*Entire*	0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04
	Normal	0.557***	3.231	0.294	1.054	0.132	0.692	0.119	0.634
	Recession	1.174***	2.751	−1.522**	−2.206	0.375	0.795	−0.681	−1.468
Component GARCH	*Entire*	0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23
	Normal	0.651***	3.876	0.134	0.501	0.433**	2.404	0.316*	1.782
	Recession	0.720*	1.730	−0.964	−1.452	0.647	1.451	−0.389	−0.887

Panel B. Risk premia for component GARCH betas

			Long						Short					
	Intercept		Market		SMB		HML		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Entire	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29
Normal	0.777***	4.253	−0.011	−0.033	0.668***	3.148	0.443*	1.935	1.025**	2.369	0.753**	2.073	0.490*	1.642
Recession	1.259***	2.783	−1.897**	−2.395	0.507	0.966	−0.431	−0.760	−0.450	−0.420	0.156	0.174	−0.137	−0.185

Figure 1: Averages and standard deviations of total betas from the component GARCH model

The graphs show the time-series average and standard deviations of the estimated total betas using the bivariate component GARCH model at the monthly–monthly (M–M) and monthly–weekly (M–W) frequency. The industries are sorted with respect to the size of the total beta estimated with monthly–monthly frequency. SMB = small-minus-big, HML = high-minus-low. The sample covers the period from 1945 to 2015.

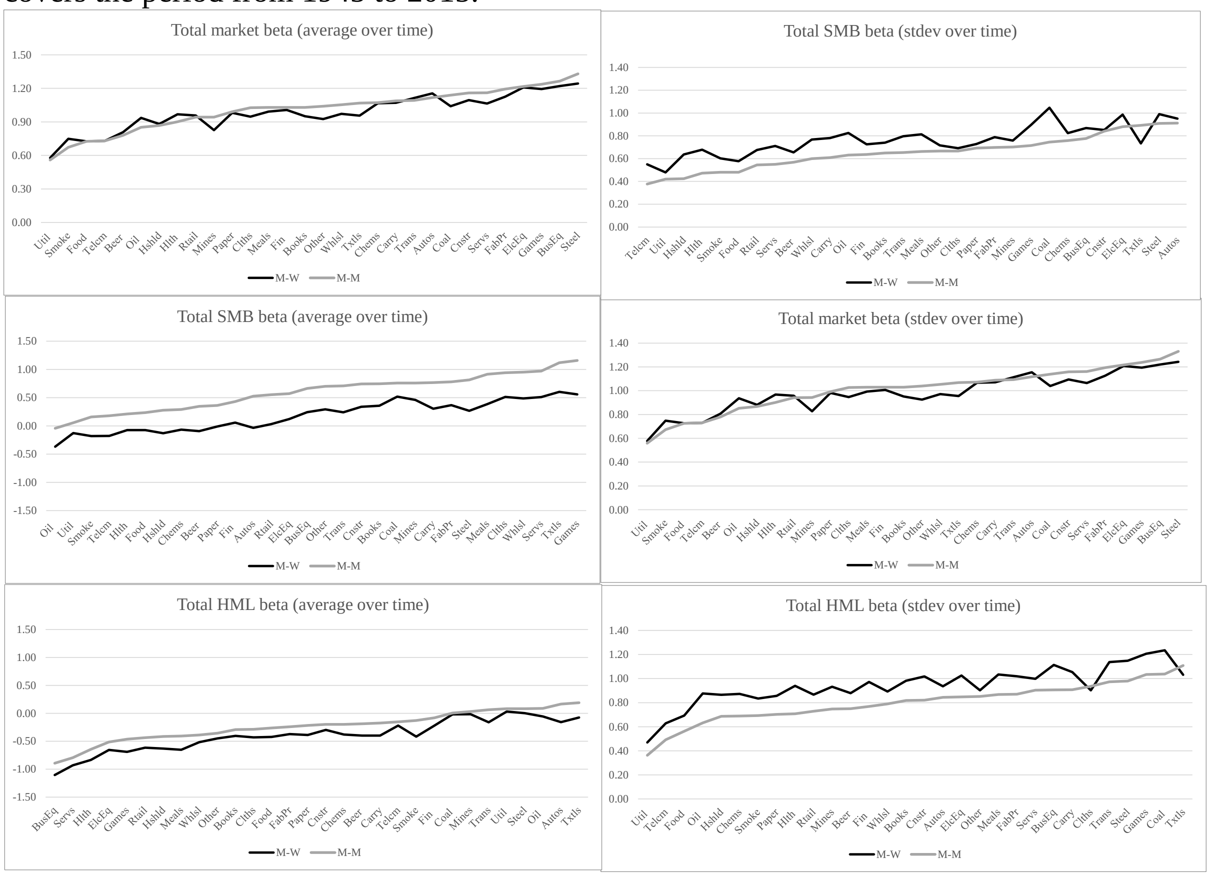
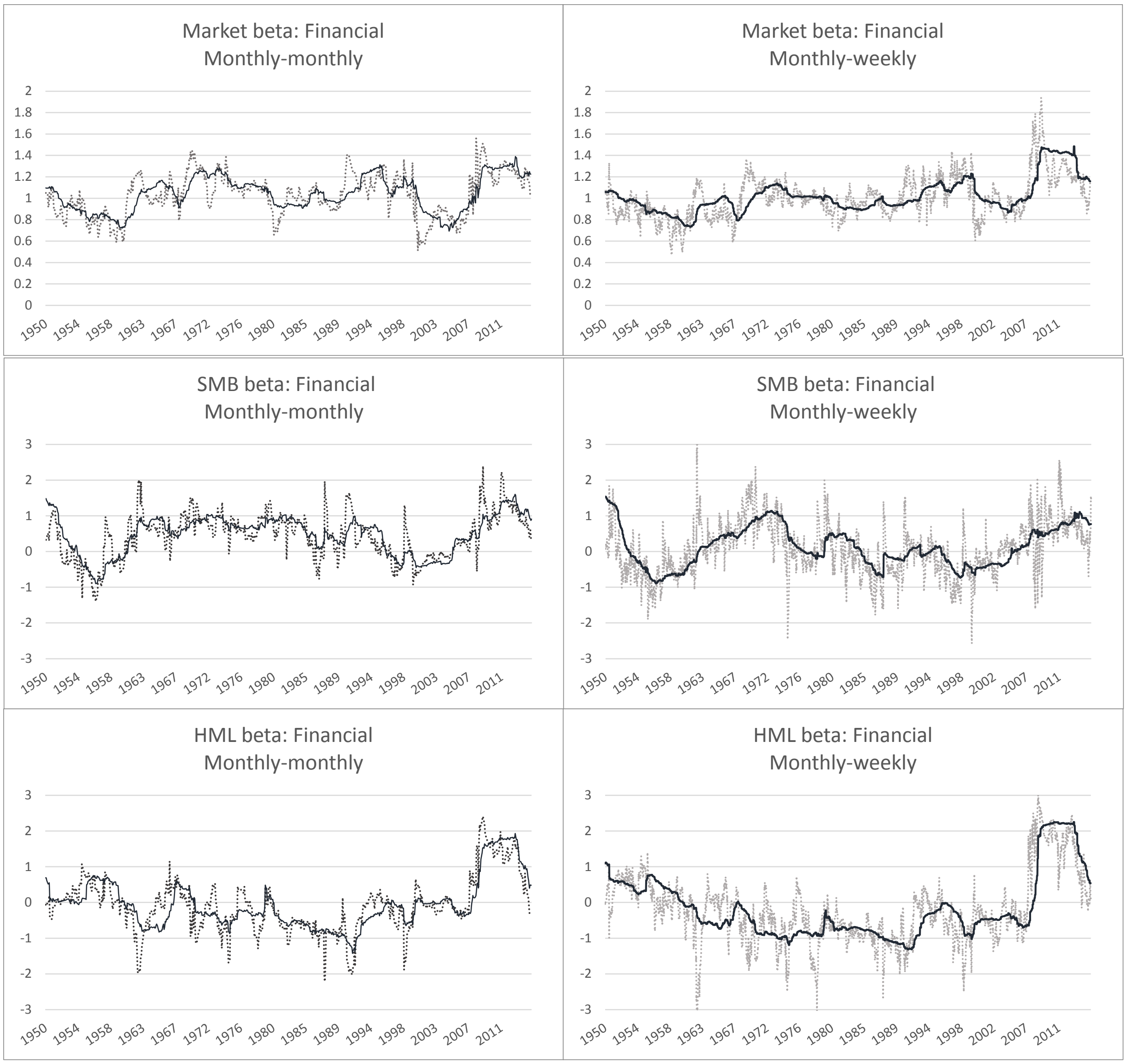


Figure 2: Factor betas estimated by the component GARCH model

The graphs plot the total (dotted line) and long-run (solid line) market, small-minus-big (SMB), and high-minus-low (HML) betas estimated by the component GARCH model at monthly–monthly and monthly–weekly frequencies for the financial industry portfolio. The sample covers the period from 1945 to 2015.



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