The TIPS Liquidity Premium

Martin M. Andreasen, Jens H.E. Christensen and Simon Riddell

CREATES Research Paper 2017-27

Department of Economics and Business Economics
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 8716 5515

The TIPS Liquidity Premium

Martin M. Andreasen[†]

Jens H. E. Christensen[‡]

Simon Riddell[*]

Abstract

We introduce an arbitrage-free term structure model of nominal and real yields that accounts for liquidity risk in Treasury inflation-protected securities (TIPS). The novel feature of our model is to identify liquidity risk from individual TIPS prices by accounting for the tendency that TIPS, like most fixed-income securities, go into buy-and-hold investors' portfolios as time passes. We find a sizable and countercyclical TIPS liquidity premium, which greatly helps our model in matching TIPS prices. Accounting for liquidity risk also improves the model's ability to forecast inflation and match inflation surveys, although none of these series are included in the estimation.

JEL Classification: E43, E47, G12, G13

Keywords: term structure modeling, liquidity risk, financial market frictions

---

We thank participants at the 9th Annual SoFiE Conference 2016, the Financial Econometrics and Empirical Asset Pricing Conference in Lancaster, the 2016 NBER Summer Institute, the Vienna-Copenhagen Conference on Financial Econometrics, and the IBEFA Summer Meeting 2017, including our discussant Azamat Abdymomunov, for helpful comments. We also thank seminar participants at the National Bank of Belgium, the Debt Management Office of the U.S. Treasury Department, the Federal Reserve Board, the Office of Financial Research, CREATES at Aarhus University, the Bank of Canada, the Federal Reserve Bank of San Francisco, and the IMF for helpful comments. Furthermore, we are grateful to Jean-Sébastien Fontaine, Jose Lopez, and Thomas Mertens for helpful comments and suggestions on earlier drafts of the paper. Finally, Kevin Cook deserves a special acknowledgement for outstanding research assistance during the initial phase of the project. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

[†]Department of Economics and Business Economics, Aarhus University and CREATES, Denmark, phone: +45-87165982; e-mail: mandreasen@econ.au.dk.

[‡]Corresponding author: Federal Reserve Bank of San Francisco, 101 Market Street MS 1130, San Francisco, CA 94105, USA; phone: 1-415-974-3115; e-mail: jens.christensen@sf.frb.org.

[*]Amazon; e-mail: simonruw@gmail.com.

This version: August 11, 2017.

1 Introduction

In 1997, the U.S. Treasury started to issue inflation-indexed bonds, which are now commonly known as Treasury inflation-protected securities (TIPS). The market for TIPS has steadily expanded since then and had a total outstanding notional amount of $973 billion, or 8.2 percent of all marketable debt issued by the Treasury, by the end of 2013.

Despite the large size of the TIPS market, an overwhelming amount of research suggests that TIPS are less liquid than Treasury securities without inflation indexation—commonly referred to simply as Treasuries. Fleming and Krishnan (2012) report market characteristics of TIPS that indicate smaller trading volume, longer turnaround time, and wider bid-ask spreads than observed in Treasuries (see also Sack and Elsasser (2004), Campbell et al. (2009), Dudley et al. (2009), and Gürkaynak et al. (2010) among others). These factors are likely to raise the implied yields from TIPS because investors generally require compensation for carrying liquidity risk. However, the size of this TIPS liquidity premium remains a topic of debate because it cannot be directly observed.

At least three identification schemes have been considered in the literature to estimate the TIPS liquidity premium so far. The work of Fleckenstein et al. (2014) uses market prices on TIPS and inflation swaps to document systematic mispricing of TIPS relative to Treasuries, which may be interpreted as a liquidity premium in TIPS. Their approach relies on a liquid market for inflation swaps, but this assumption seems questionable given that U.S. inflation swaps have low trading volumes and wide bid-ask spreads (see Fleming and Sporn (2013) and Christensen and Gillan (2012)). The second identification scheme approximates inflation expectations in the TIPS market by those from surveys (e.g., D'Amico et al. (2014)). But inflation expectations from surveys are unavailable in real time and may easily differ from the desired expectations of the marginal investor in the TIPS market. The final identification scheme relies on a set of observable characteristics for the TIPS market (e.g., market volume) as noisy proxies for liquidity risk (see, e.g., Abrahams et al. (2016) and Pflueger and Viceira (2016)). The accuracy of this approach is clearly dependent on having good proxies for liquidity risk, which in general is hard to ensure.

The present paper introduces a new identification scheme for the TIPS liquidity premium within a dynamic affine term structure model (ATSM) for nominal and real yields. Our model identifies liquidity risk directly from individual TIPS prices by accounting for the typical market phenomenon that many TIPS go into buy-and-hold investors' portfolios as time passes. This in turn limits the number of securities available for trading and hence increases the liquidity risk. We formally account for this effect by pricing each TIPS using a stochastic discount factor with a unique bond-specific term that reflects the added compensation investors demand for buying a bond with low expected future liquidity. A key implication of the proposed model is that liquidity risk is identified from the implied price differential of otherwise identical principal and coupon payments. Individual TIPS prices are therefore sufficient to estimate the TIPS liquidity premium within our model, meaning that we avoid the limitations associated with the existing identification schemes in the literature. The proposed identification scheme is thus related to the approach taken in Fontaine and Garcia (2012), as they also exploit the relative price differences of very similar coupon bonds to estimate a liquidity premium in Treasuries, although our model and its application differ along other dimensions from the analysis in Fontaine and Garcia (2012).

The proposed model is estimated based on TIPS prices and a standard sample of nominal Treasury yields from Gürkaynak et al. (2007), both covering the period from mid-1997 through the end of 2013. To get a clean read of the liquidity factor, we also account for the deflation protection option embedded in TIPS during the estimation using formulas provided in Christensen et al. (2012). Our main analysis uses TIPS prices and Treasury yields within the commonly considered ten-year maturity spectrum, where our key findings are as follows. First, the average liquidity premium for TIPS is sizable and fairly volatile, with a mean of 38 basis points and a standard deviation of 34 basis points. To support the proposed identification scheme, we also show that the estimated liquidity premium is highly correlated with well-known observable proxies for liquidity risk such as the VIX options-implied volatility index, the on-the-run spread on Treasuries, and the TIPS mean absolute fitted errors from Gürkaynak et al. (2010). Second, we find a large improvement in the ability of our ATSM to fit individual TIPS prices by accounting for liquidity risk. The root mean-squared error of the fitted TIPS prices converted into yields to maturity falls from 14.6 basis points to just 4.9 basis points when the liquidity factor is included, meaning that TIPS pricing errors are at the same low level as found for nominal yields. Third, by accounting for liquidity risk, the proposed model avoids the well-known positive bias in real yields, and hence the negative bias in breakeven inflation, i.e., the difference between nominal and real yields of the same maturity. This implies that the proposed model does not predict spells of deflation fears during our sample, contrary to the results obtained when ignoring liquidity risk. Fourth, the model-implied forecasts of one-year CPI inflation are greatly improved by correcting for liquidity risk in TIPS, and so is the ability of the model to match inflation expectations from surveys. We emphasize that the improved ability of the proposed model to forecast inflation and match inflation surveys is obtained without including any of these series in the estimation.

The remainder of the paper is structured as follows. Section 2 provides reduced-form evidence on the liquidity risk of TIPS, while Section 3 introduces the general ATSM framework and the specific Gaussian model we use to account for the liquidity disadvantage of TIPS. The model is estimated in Section 4, while Section 5 studies the estimated liquidity premium in the TIPS market. Its robustness is explored in Section 6, while Section 7 studies the liquidity-adjusted real yield curve and the implied inflation forecasts from the proposed model. Section 8 concludes and offers directions for future research. Additional technical details are provided in two appendices at the end of the paper and a supplementary Online Appendix.

2 The Dynamics of TIPS Liquidity Risk

Building on the work of Amihud and Mendelson (1986), we define liquidity risk as the cost of immediate execution. For instance, if a bond holder is forced to liquidate his position prematurely at a disadvantageous price compared with the mid-market quote, then this price differential reflects the liquidity cost. Given this definition, it is well-established that liquidity risk in Treasuries is decreasing with (i) high market volume (Garbade and Silber (1976)), (ii) high competition among market makers (Tinic and West (1972)), and (iii) high market debt (Goldreich et al. (2005)).

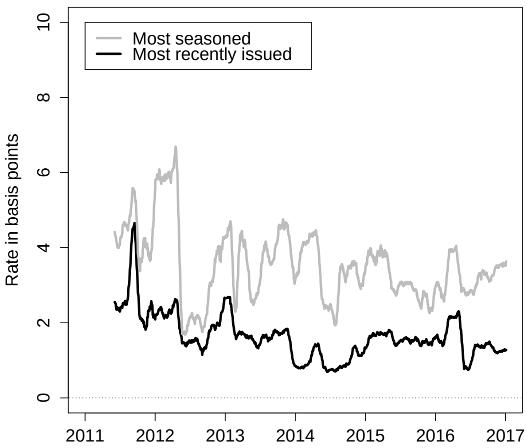
A commonly used observable proxy for liquidity risk is the implied yield spread from the bid and ask prices. These spreads are reported in Figure 1 for each of the four TIPS categories issued in the U.S. The spreads from Bloomberg appear unreliable before the spring of 2011, and we therefore restrict our analysis in this section to a weekly sample from May 2011 to December 2016. The top row in Figure 1 reports the bid-ask spreads for the most recently issued (on-the-run) five- and ten-year TIPS and for the corresponding most seasoned TIPS with at least two years to maturity. We highlight two results from these charts. First, the bid-ask spreads for seasoned five- and ten-year TIPS are systematically *above* those of newly issued TIPS. Second, the bid-ask spreads on seasoned TIPS are around 4 basis points and hence of economic significance. In comparison, the bid-ask spreads for the ten-year Treasuries issued between 2011 and 2016 have an average of only 0.4 basis points, i.e., a factor 10 smaller than the corresponding spread in the TIPS market.[1] The bottom part of Figure 1 reveals that we generally see the same pattern for twenty- and thirty-year TIPS, although the bid-ask spreads for newly issued securities here are somewhat noisy due to the few traded bonds in this part of the maturity spectrum. Figure 1 therefore reveals that TIPS carry sizable liquidity risk, which is higher for seasoned TIPS than for newly issued securities.

We next test for the statistical significance of the positive relationship between liquidity risk and the age of a bond. The considered panel regression is given by

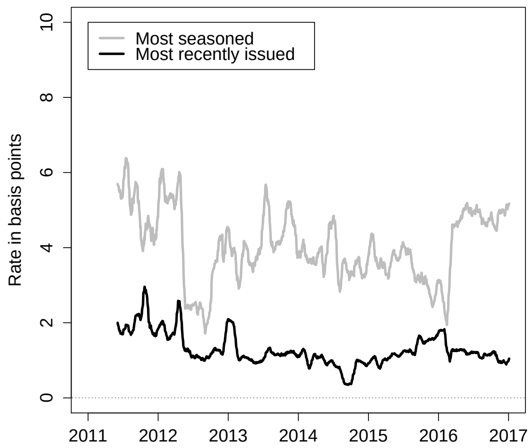
$$Spread_t^i = \tau_t + \alpha_i + \beta_1 Notional_t^i + \beta_2 Age_t^i + \varepsilon_t^i, \tag{1}$$

where the bid-ask spread for the ith TIPS in period t is denoted $Spread_t^i$. To control for unobserved heterogeneity, we allow for both time fixed effects τ_t and bond-specific fixed effects α_i in equation (1). As argued by Garbade and Silber (1976), securities with large outstanding notional amounts often have greater trading volumes, and we therefore also include the notional value of each security $Notional_t^i$, which for TIPS grows over time with CPI inflation. Finally, Age_t^i measures the time since issuance of the ith TIPS and ε_t^i is a zero-mean error term. Using all available securities, we then estimate the panel regression

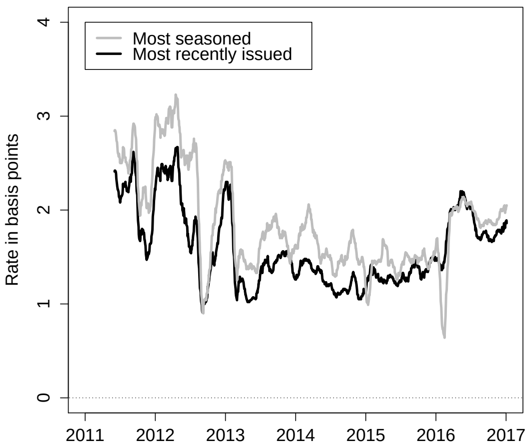
[1] The average bid-ask spreads for the individual ten-year Treasury notes issued in January of 2011, 2012, 2013, 2014, 2015, and 2016 are 0.52, 0.42, 0.38, 0.28, 0.26, and 0.21 basis points, respectively.



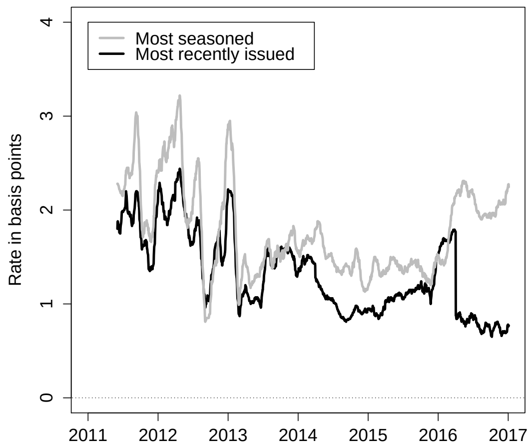
(a) Five-year TIPS



(b) Ten-year TIPS



(c) Twenty-year TIPS



(d) Thirty-year TIPS

Figure 1: **TIPS Bid-Ask Spreads**
For the five- and ten-year TIPS, the bid-ask spreads are computed using the most recently issued TIPS and the corresponding most seasoned TIPS with at least two years to maturity. For the twenty-year TIPS, the series are obtained by tracking the bid-ask spreads of the same two twenty-year TIPS over the period due to the few issuances at this maturity. For the thirty-year TIPS, the bid-ask spread for the most seasoned TIPS is that of the first thirty-year TIPS issued back in 1998, while the most recently issued series tracks the bid-ask spread of the newest thirty-year TIPS. All series (measured in basis points) are weekly covering the period from May 31, 2011, to December 30, 2016, and smoothed by a four-week moving average to facilitate the plotting.

in equation (1) by OLS separately within each of the four TIPS categories. Table 1 shows that TIPS with a larger outstanding notional value have significantly lower bid-ask spreads and, more importantly, that the age of a security has a significant positive effect on the bid-ask spread, as also suggested by Figure 1. The latter result is obviously very similar to the

	n_{TIPS}	N	$Notional_t^i \times 1,000$	Age_t^i	No. of parameters	$adj\ R^2$
5-year TIPS	9	$1,140$	-0.042^{**} (0.0037)	3.125^{**} (0.259)	296	0.77
10-year TIPS	28	$5,111$	-0.020^{**} (0.0025)	0.997^{**} (0.046)	315	0.70
20-year TIPS	5	$1,435$	-0.308^{**} (0.053)	1.933^{**} (0.264)	292	0.74
30-year TIPS	10	$2,149$	-0.0053^{*} (0.0022)	0.194^{**} (0.008)	297	0.75

Table 1: **Panel Regression: The Bid-Ask Spread in the TIPS Market**
This table reports the results of separately estimating equation (1) by OLS for each of the four categories of TIPS. The estimated loadings for the time and bond-specific fixed effects are not provided. The variable $Notional_t^i$ is measured in millions of dollars and Age_t^i in years since issuance. White's heteroscedastic standard errors are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The adjusted R^2 is computed based on the demeaned variation in dependent variable. The data used are weekly covering the period from May 31, 2011, to December 30, 2016.

well-known finding in the Treasury market, where newly issued securities also are more liquid than existing bonds (see, for instance, Krishnamurthy (2002), Gurkaynak et al. (2007), and Fontaine and Garcia (2012) among many others). However, these spreads are much wider for TIPS compared with Treasuries. Therefore, unlike the Treasury market, it is crucial to account for this dynamic pattern in TIPS liquidity to fully understand the price dynamics in the TIPS market.

We draw two conclusions from these reduced-form regressions. First, current liquidity in the TIPS market exhibit notable variation over time, and liquidity therefore represents a risk factor to bond investors in this market, as also emphasized in the work of Gürkaynak et al. (2010), Fleming and Krishnan (2012), and Fleckenstein et al. (2014) among others. Second, seasoned TIPS are less liquid than more recently issued securities within the same maturity category. Although equation (1) does not provide an explanation for this dynamic pattern in liquidity, anecdotal evidence suggests that it most likely arises because increasing amounts of the securities get locked up in buy-and-hold investors' portfolios as time passes and becomes unavailable for trading.[2] The objective in the present paper is not to provide a more detailed explanation for this dynamic pattern in liquidity but instead to examine its asset pricing implications. The effect we want to explore is based on the assumption that rational and forward-looking investors are aware of this dynamic pattern in liquidity and therefore demand compensation for holding bonds with low future liquidity. The ATSM we propose in the next section formalizes this effect and quantifies how *current* TIPS prices are affected by expected *future* TIPS liquidity.

[2]See, for instance, the evidence provided in Sack and Elsasser (2004), which indicates that the primary participants in the TIPS market are large institutional investors (e.g., pension funds and insurance companies) with long-term real liability risks that they want to hedge.

3 An ATSM of Nominal and Real Yields with Liquidity Risk

This section introduces a general class of ATSMs of nominal and real bond prices that accounts for liquidity risk. We formally present the model framework in Section 3.1 and describe a Gaussian version of it in Section 3.2. The proposed identification strategy of liquidity risk is then compared with the existing literature in Section 3.3.

3.1 A Canonical ATSM with Liquidity Risk

As commonly assumed, the instantaneous nominal short rate r_t^N is given by

$$r_t^N = \rho_0^N + \left(\rho_x^N\right)' X_t,$$

where ρ_0^N is a scalar and ρ_x^N is an $N \times 1$ vector. The dynamics of the N pricing factors in X_t with dimension $N \times 1$ evolve as

$$dX_t = \mathcal{K}_x^{\mathbb{Q}} \left(\theta_x^{\mathbb{Q}} - X_t\right) dt + \Sigma_x \sqrt{S_{x,t}} d\bar{W}_t^{\mathbb{Q}}, \tag{2}$$

where $\bar{W}_t^{\mathbb{Q}}$ is a standard Wiener process in $\mathbb{R}^N$ under the risk-neutral measure $\mathbb{Q}$ and S_t is an N-dimensional diagonal matrix. Its elements are given by $[S_{x,t}]_{k,k} = [\delta_0]_k + \delta_{x,k}' X_t$ for $k = 1, 2, ..., N$, where $[\delta_0]_k$ denotes the kth entry of δ_0 with dimension $N \times 1$. Hence, $\theta^{\mathbb{Q}}$ and $\delta_{x,k}$ are $N \times 1$ vectors, whereas $\mathcal{K}^{\mathbb{Q}}$ and Σ_x have dimensions $N \times N$. Absence of arbitrage implies that the price of a nominal zero-coupon bond maturing at time $t + \tau$ is given by

$$P_t^N(\tau) = \exp\left\{A^N(\tau) + B^N(\tau)' X_t\right\}, \tag{3}$$

where the functions $A^N(\tau)$ and $B^N(\tau)$ satisfy well-known ordinary differential equations (ODEs); see, for instance, Dai and Singleton (2000).

The price of bonds with payments indexed to inflation (i.e., real bonds) may in principle be obtained in a similar manner by letting the instantaneous real short rate be affine in the pricing factors, as done in Adrian and Wu (2010) and Joyce et al. (2010) among others. An implicit assumption within this classic asset pricing framework is that bonds are trading in a frictionless market without any supply- or demand-related constraints. This is often a reasonable assumption for Treasuries due to the large size of this market and its low bid-ask spreads.[3] This assumption is much more debatable for TIPS, as seen from the wide bid-ask spreads in Section 2.

The main innovation of the present paper is to relax the assumption of a frictionless market for real bonds in ATSMs and explicitly account for the dynamic pattern in TIPS

[3] A minor exception relates to the small liquidity spread between newly issued Treasuries that are "on-the-run" and somewhat older "off-the-run" bonds; see, for instance, Krishnamurthy (2002), Gürkaynak et al. (2007), and Fontaine and Garcia (2012).

liquidity documented in Section 2. Inspired by the work of Amihud and Mendelson (1986), our contribution is to price TIPS by a real rate that accounts for liquidation costs, which we specify for the ith TIPS as $h(t-t_0;i)\,X_t^{liq}$. The first term $h(t-t_0;i)$ is a deterministic function of time since issuance $t-t_0$ of the ith TIPS and serves to capture the empirical regularity from Section 2 that liquidation costs (i.e. the bid-ask spread) increase as the bond approaches maturity. Here, we initially only assume that $h(t-t_0;i)$ is bounded, nonnegative, and increasing in $t-t_0$. The second term in our specification of liquidation costs is a latent factor X_t^{liq}, which is included to capture the cyclical variation in these costs, as is evident from the bid-ask spreads in Figure 1. Hence, we suggest accounting for liquidity risk by discounting future cash flows from the ith TIPS using a liquidity-adjusted instantaneous real short rate of the form

$$r_t^{R,i} = \underbrace{\rho_0^R + \left(\rho_x^R\right)' X_t}_{\text{frictionless real rate}} + \underbrace{h\left(t-t_0;i\right) X_t^{liq}}_{\text{liquidity adjustment}}, \tag{4}$$

where ρ_0^R is a scalar and ρ_x^R is an $N \times 1$ vector. The first term in $r_t^{R,i}$ is the traditional affine specification for the frictionless part of the real rate, which is common to all TIPS, whereas the liquidity adjustment varies across securities. The latter implies that we will price TIPS using a bond-specific instantaneous real short rate or, equivalently, a bond-specific stochastic discount factor when combining equation (4) with a distribution for the market prices of risk.

Letting $Z_t \equiv \left[\begin{array}{cc} X_t' & X_t^{liq} \end{array}\right]'$, the dynamics of this extended state vector is assumed to be

$$dZ_t = \mathcal{K}_z^{\mathbb{Q}} \left(\theta_z^{\mathbb{Q}} - Z_t\right) dt + \Sigma_z \sqrt{S_{z,t}} dW_t^{\mathbb{Q}}, \tag{5}$$

where $W_t^{\mathbb{Q}}$ is a standard Wiener process in $\mathbb{R}^{N+1}$. Similarly, $\mathcal{K}_z^{\mathbb{Q}}$, $\theta_z^{\mathbb{Q}}$, $S_{z,t}$, and Σ_z are appropriate extensions of the corresponding matrices related to equation (2). Thus, our specification in equation (5) accommodates the case where the liquidity factor is restricted to only attain nonnegative values, as assumed in Abrahams et al. (2016), by letting X_t^{liq} follow a square-root process that enters in $S_{z,t}$ to determine the conditional volatility in Z_t. Another and less restrictive specification is to omit X_t^{liq} in $S_{z,t}$ and allow the liquidity factor to occasionally attain negative values and hence accommodate episodes with negative liquidity risk.[4] For this second specification, the estimated time series of X_t^{liq} may serve as an indirect test of the model's ability to capture liquidity risk, as we predominantly expect X_t^{liq} to be positive.

From the Feynman-Kac theorem and equations (4) and (5), it follows that the price at time t of a real zero-coupon bond maturing at time T is given by

$$P^{R,i}\left(t_0,t,T\right) = \exp\left\{A^{R,i}\left(t_0,t,T\right) + B^{R,i}\left(t_0,t,T\right)' Z_t\right\}, \tag{6}$$

[4]This corresponds to periods when an investor pays to hold liquidity risk. This may happen when a bond helps an investor (e.g., a pension fund) to hedge some of his liabilities.

when discounting cash flows related to the ith TIPS. The functions $A^{R,i}(t_0,t,T)$ and $B^{R,i}(t_0,t,T)$ with dimensions $(N+1)\times 1$ satisfy the ODEs

$$\frac{\partial A^{R,i}}{\partial t}(t_0,t,T) = \delta_0^R - \left(\mathcal{K}_z^{\mathbb{Q}}\theta_z^{\mathbb{Q}}\right)' B^{R,i}(t_0,t,T) - \frac{1}{2}\sum_{k=1}^{N+1}\left[\Sigma_z' B^{R,i}(t_0,t,T)\right]_k^2 \delta_{0,k}, \tag{7}$$

$$\frac{\partial B^{R,i}}{\partial t}(t_0,t,T) = \begin{bmatrix} \delta_x^R \\ h(t-t_0;i) \end{bmatrix} + \left(\mathcal{K}_z^{\mathbb{Q}}\right)' B^{R,i}(t_0,t,T) - \frac{1}{2}\sum_{k=1}^{N+1}\left[\Sigma_z' B^{R,i}(t_0,t,T)\right]_k^2 \delta_{z,k} \tag{8}$$

with the terminal conditions $A^{R,i}(t_0,T,T)=0$ and $B^{R,i}(t_0,T,T)=0$. Here, $[a]_k^2$ denotes the squared kth element of vector a and $\delta_z \equiv \begin{bmatrix} \delta_x' & \delta_{x^{liq}} \end{bmatrix}'$ with dimensions $(N+1)\times 1$. Thus, the price of a real zero-coupon bond is exponentially affine in Z_t even when accounting for liquidity risk by using the modified real short rate in equation (4). The implied breakeven inflation rate from equations (3) and (6) is given by

$$y_t^N(\tau) - y_t^{R,i}(t_0,t,\tau) = \frac{A^{R,i}(t_0,t,t+\tau)}{\tau} - \frac{A(\tau)}{\tau} - \frac{B(\tau)'}{\tau}X_t + \frac{B^{R,i}(t_0,t,t+\tau)'}{\tau}\begin{bmatrix} X_t \\ X_t^{liq} \end{bmatrix},$$

where $y_t^N(\tau) \equiv -\frac{1}{\tau}\log P_t^N(\tau)$ and $y_t^{R,i}(t_0,t,\tau) \equiv -\frac{1}{\tau}\log P_t^{R,i}(t_0,t,t+\tau)$ denote the yield to maturity from nominal and real bonds, respectively, with $T \equiv t+\tau$. Hence, X_t^{liq} can also be viewed as capturing the relative liquidity difference between Treasuries and TIPS. In this respect, our model is similar to the work of D'Amico et al. (2014) and Abrahams et al. (2016), who also use a single factor to capture the relative liquidity differential of TIPS compared with Treasuries.

We also note that the bond prices in equation (6) depend on the calender time t, which enters as a state variable in our model to determine the time since issuance $t-t_0$ of a given security and hence its liquidity adjustment. This property of our model is similar to the class of calibration-based term structure models dating back to Ho and Lee (1986) and Hull and White (1990), where the drift is a deterministic function of calendar time and repeatedly recalibrated to perfectly match the current yield curve. These calibration-based models are known to be time-inconsistent, as the future drift at $t+\tau$ is repeatedly modified until reaching time $t+\tau$. Our model does not suffer from the same shortcoming because we only use calender time t to determine the liquidity adjustment and not to change any dynamic model parameters.

The model is closed by adopting the extended affine specification for the market prices of risk Γ_t, as described by Cheridito et al. (2007).

We summarize our model presentation by extending the classification scheme of Dai and Singleton (2000) to our ATSM, which is referred to as $\mathbb{A}_m^L(N+1)$. That is, we consider N frictionless pricing factors and one factor for the liquidity risk of TIPS, as indicated by the superscript L. Among the $N+1$ pricing factors, we allow for m variance-influencing factors and impose the same restrictions for the model to be admissible (i.e., well-defined) as in Dai and Singleton (2000).[5]

3.2 A Gaussian ATSM with Liquidity Risk

We next analyze a particular Gaussian version of our model with closed-form expressions for liquidity-adjusted real bond prices. Beyond providing useful intuition on the liquidity adjustment, we also note that this special case of our model should be particularly interesting given the well-known success of Gaussian models in matching yields and risk premia, as also exploited in D'Amico et al. (2014) and Abrahams et al. (2016). To facilitate the interpretation of our Gaussian model, we consider the familiar case where factor loadings for nominal yields and the frictionless part of real yields represent level, slope, and curvature components. This is done by using the parameterization in Christensen et al. (2010), which represents an extension of the arbitrage-free Nelson-Siegel specification derived in Christensen et al. (2011) to a joint model for nominal and real yields.

Starting with the nominal short rate, it is defined as

$$r_t^N = L_t^N + S_t, \tag{9}$$

where L_t^N is the level factor of nominal yields and S_t is the slope factor. The parameterization of the liquidity-adjusted real short rate for the ith TIPS is given by

$$r_t^{R,i} = L_t^R + \alpha^R S_t + \beta^i (1 - e^{-\lambda^{L,i}(t-t_0)}) X_t^{liq}. \tag{10}$$

The first part $L_t^R + \alpha^R S_t$ constitutes the frictionless real rate using the specification adopted in Christensen et al. (2010). The variable L_t^R represents the level factor of real yields and is absent in the expression for nominal yields. This specification is consistent with nominal yields containing a hidden factor that is observable from real yields and inflation expectations (see Chernov and Mueller (2012)). Note also that, for simplicity, we define the real slope factor as $\alpha^R S_t$ with $\alpha^R \in \mathbb{R}$ based on the empirical evidence in Christensen et al. (2010). The adopted functional form for $h\,(t - t_0; i)$ controlling the liquidity adjustment is given by the parsimonious specification $\beta^i(1 - e^{-\lambda^{L,i}(t-t_0)})$, where $\beta^i \geq 0$ and $\lambda^{L,i} \geq 0$. To provide some interpretation of β^i and λ^i, it is useful to think of the trading activity in the ith TIPS as taking place in two phases. The first phase may be characterized by a large supply of bonds just after bond issuance, but also strong demand pressure from buy-and-hold investors, who gradually purchase a large fraction of the outstanding securities. The second phase then starts when buy-and-hold investors have acquired their share of the ith TIPS and the number of securities available for trading has become relatively scarce. Given this categorization of the trading cycle, the value of $\lambda^{L,i}$ determines the length of the first phase, where exposure to

[5] It is straightforward to verify that the proposed specification to account for liquidity risk can be extended to nonlinear dynamic term structure models. Section 6.5 provides one illustration of such an extension by incorporating the zero lower bound on nominal interest rates into the model.

X_t^{liq} is fairly low. That is, a low value of $\lambda^{L,i}$ implies that this first phase of bond trading is fairly long, whereas a high value of $\lambda^{L,i}$ means that this first phase of bond trading is much shorter.[6] The value of β^i determines the maximal exposure of the ith TIPS to the liquidity factor X_t^{liq} in the second phase, which appears when $e^{-\lambda^{L,i}(t-t_0)} \approx 0$. It is obvious that more sophisticated specifications of $h(t - t_0; i)$ may be considered, as opposed to the one used in equation (10), although such extensions are not explored in the current paper.

Letting $Z_t \equiv \begin{bmatrix} L_t^N & S_t & C_t & L_t^R & X_t^{liq} \end{bmatrix}'$, we consider $\mathbb{Q}$ dynamics of the form

$$dZ_t = \underbrace{\begin{bmatrix} \mathcal{K}_x^{\mathbb{Q}} & 0_{4\times 1} \\ 0_{1\times 4} & \kappa_{liq}^{\mathbb{Q}} \end{bmatrix}}_{\mathcal{K}_z^{\mathbb{Q}}} \left(\underbrace{\begin{bmatrix} \theta_x^{\mathbb{Q}} \\ \theta_{liq}^{Q} \end{bmatrix}}_{\theta_z^{\mathbb{Q}}} - Z_t \right) + \Sigma_z dW_t^{\mathbb{Q}}, \tag{11}$$

where $\theta_x^{\mathbb{Q}} = 0_{4\times 1}$ due to the adopted normalization scheme. Following Christensen et al. (2010), we let $\left[\mathcal{K}_x^{\mathbb{Q}}\right]_{2,2} = \left[\mathcal{K}_x^{\mathbb{Q}}\right]_{3,3} = \lambda$ and $\left[\mathcal{K}_x^{\mathbb{Q}}\right]_{2,3} = -\lambda$ for $\lambda > 0$, with all remaining elements of the 4×4 matrix $\mathcal{K}_x^{\mathbb{Q}}$ equal to zero. This ensures that the factor loadings represent level, slope, and curvature components in the nominal and real yield curves provided $\left[\mathcal{K}_z^{\mathbb{Q}}\right]_{5,i} = 0$ for $i = \{1, 2, 3, 4\}$. The next restrictions $\left[\mathcal{K}_z^{\mathbb{Q}}\right]_{i,5} = 0$ for $i = \{1, 2, 3, 4\}$ imply that X_t^{liq} either operates as a level or slope factor depending on the value of $\kappa_{liq}^{\mathbb{Q}}$, although these restrictions could be relaxed without altering the interpretation of the four frictionless factors. That is, our parameterization does not accommodate a curvature structure for X_t^{liq}, which is consistent with our reduced-form evidence in Section 2 that older TIPS are more affected by liquidity risk than are newly issued securities.

Using equations (9) and (11), the yield at time t for a nominal zero-coupon bond maturing at $t + \tau$ is easily shown to have the well-known structure from the static model of Nelson and Siegel (1987)

$$y_t^N(\tau) = L_t^N + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^N(\tau)}{\tau}, \tag{12}$$

where $A^N(\tau)$ is an additional convexity adjustment provided in Christensen et al. (2011). The price for the ith real zero-coupon bond maturing at time T is given by equation (6) with the closed-form expression for $A^{R,i}(t_0, t, T)$ and $B^{R,i}(t_0, t, T)$ provided in Appendix A. To facilitate the interpretation of this solution, consider the implied yield to maturity on the ith

[6] For instance, a short initial trading phase may coincide with the bond ceasing to be the most recently issued TIPS within its maturity range and hence is no longer "on-the-run."

real zero-coupon bond, which we write as

$$
\begin{aligned}
y_t^{R,i}(t_0,t,\tau) \;=\; & \underbrace{L_t^R + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)S_t + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)C_t}_{\text{frictionless loadings}} \\
& \underbrace{+\beta^i\left(\frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}\tau}}{\kappa_{liq}^{\mathbb{Q}}\tau} - e^{-\lambda^{L,i}(t-t_0)}\frac{1-e^{-\left(\kappa_{liq}^{\mathbb{Q}}+\lambda^{L,i}\right)\tau}}{\left(\kappa_{liq}^{\mathbb{Q}}+\lambda^{L,i}\right)\tau}\right)X_t^{liq}}_{\text{liquidity adjustment}} \\
& - \underbrace{\frac{A^{R,i}(t_0,t,\tau)}{\tau}}_{\text{convexity adjustment}},
\end{aligned} \tag{13}
$$

where $\tau = T - t$. The first three terms in $y_t^{R,i}(t_0,t,\tau)$ capture the frictionless part of real yields, where the factor loadings have the same familiar interpretation as in equation (12) due to the imposed structure on $r_t^{R,i}$ and its dynamics under $\mathbb{Q}$. The next term in equation (13) represents an adjustment for liquidity risk. Its first term $\left(1-\exp\left\{-\kappa_{liq}^{\mathbb{Q}}\tau\right\}\right)/\left(\kappa_{liq}^{\mathbb{Q}}\tau\right)$ describes the maximal effect of liquidity risk, which is obtained when $e^{-\lambda^{L,i}(t-t_0)} \approx 0$ and the ith bond has full exposure to variation in X_t^{liq}. This upper limit for liquidity risk clearly operates as a traditional slope factor for $\kappa_{liq}^{\mathbb{Q}} > 0$, where the size of the liquidity adjustment is decreasing in τ and hence increasing in t as the bond approaches maturity. We have the opposite pattern when $\kappa_{liq}^{\mathbb{Q}} < 0$, whereas the upper limit for liquidity risk is constant in t with $\kappa_{liq}^{\mathbb{Q}} \longrightarrow 0$, i.e., a level factor. In other words, the $\mathbb{Q}$ dynamics of X_t^{liq} determines the term structure for the maximal effect of liquidity risk.

The second term in the liquidity adjustment in equation (13) serves as a negative correction to $\left(1-\exp\left\{-\kappa_{liq}^{\mathbb{Q}}\tau\right\}\right)/\left(\kappa_{liq}^{\mathbb{Q}}\tau\right)$ during the initial phase with large trading volume, where buy-and-hold investors have not acquired a large proportion of the ith bond. The term $\beta^i e^{-\lambda^{L,i}(t-t_0)}$ is clearly decreasing in t, whereas the remaining term is similar to the one for the maximal effect of liquidity risk (except with decay parameter $\kappa_{liq}^{\mathbb{Q}} + \lambda^{L,i}$), and hence typically increasing in t.

As a result, the liquidity adjustment in equation (13) may either increase or decrease in t, depending on the relative values of $\kappa_{liq}^{\mathbb{Q}}$ and $\lambda^{L,i}$. Focusing on the most plausible parameterization with $\kappa_{liq}^{\mathbb{Q}} > 0$, the combined loading on X_t^{liq} is clearly positive, meaning that liquidity risk increases the real yield whenever $X_t^{liq} > 0$.[7] Hence, our model captures the effect that forward-looking investors require compensation for carrying the risk that low *future* liquidity reduces the price of the bond if sold before maturity. This in turn reduces the *current* bond price or, equivalently, raises the *current* yield. An effect which is documented empirically for Treasuries in Goldreich et al. (2005). On the other hand, liquidity risk is absent if $\beta^i = 0$ and

[7] It is also straightforward to show that the combined loading on X_t^{liq} remains positive even if $\kappa_{liq}^{\mathbb{Q}} < 0$, provided $0 \leq \lambda^{L,i} < -\kappa_{liq}^{\mathbb{Q}}$.

real yields in equation (13) therefore simplify to those in the frictionless model of Christensen et al. (2010).

Finally, in Gaussian models the extended affine specification for the market prices of risk reduces to the essential affine parameterization of Duffee (2002). Hence, we have

$$\Gamma_t = \Sigma_z^{-1} \left(\gamma_0 + \gamma_z Z_t \right), \tag{14}$$

where γ_0 and γ_z have dimensions $(N+1) \times 1$ and $(N+1) \times (N+1)$, respectively.

We denote this Gaussian version of our model as the $G^L(5)$ model. It is the focus of the remaining part of the present paper and will be compared extensively with the model of Christensen et al. (2010), denoted the $G(4)$ model, which has the same frictionless dynamic factor structure, but omits accounting for TIPS liquidity risk.

3.3 Identification of Liquidity Risk and the Existing Literature

As described above, the proposed model discounts coupon and principal payments from TIPS using bond-specific real short rates, which only differ in their loadings on the common liquidity factor X_t^{liq}. This implies that liquidity risk in TIPS is identified from the implied price differential of otherwise identical cash flow payments—or equivalently, the degree of "mispricing" based on the frictionless part of our ATSM. This is a very direct measure of liquidity risk that only requires a panel of market prices for TIPS, which is readily available. The proposed identification scheme based on market prices is therefore closely related to the one by Fleckenstein et al. (2014), who use market prices on TIPS and inflation swaps to document systematic mispricing of TIPS relative to Treasuries, which may be interpreted as reflecting liquidity premiums in TIPS. The approach of Fleckenstein et al. (2014) relies on a liquid market for inflation swaps, but as argued by Abrahams et al. (2016) this assumption is questionable for the U.S. given the low trading volumes and wide bid-ask spreads in the U.S. inflation swap market (see Fleming and Sporn (2013) and Christensen and Gillan (2012)). The identification strategy we propose does not rely on a well-functioning and liquid market for inflation swaps but instead uses an ATSM to identify liquidity risk solely from TIPS market prices and a standard panel of Treasury yields.

Another commonly adopted procedure to identify liquidity risk is to regress breakeven inflation from TIPS on inflation expectations from surveys and various proxies for liquidity risk (see Gürkaynak et al. (2010) and Pflueger and Viceira (2016) among others). In contrast to our identification strategy, such reduced-form estimates do not account for the inflation risk premia in breakeven inflation, which often is sizable and quite volatile (see for instance D'Amico et al. (2014) and Abrahams et al. (2016)).

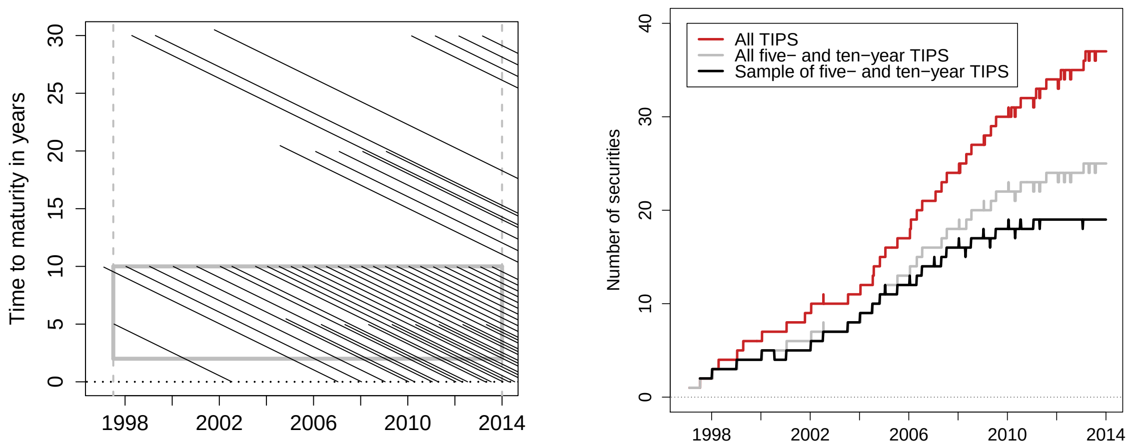
Obviously, the idea of relating liquidity risk to a limited supply of certain bonds is not unique to our paper. For instance, Amihud and Mendelson (1991) consider the case where an increasing fraction of Treasury notes are locked away in investors' portfolios to explain the yield differential in Treasury notes and bills of the same maturity. Another example is provided by Keane (1996), who uses a similar explanation for the repo specialness of Treasuries. From this perspective, our main contributions are to incorporate effects of a limited bond supply in an arbitrage-free ATSM and to show how this effect may explain the liquidity disadvantage of TIPS.

Our model is also related to the ATSM of D'Amico et al. (2014), where real bonds are discounted with a modified real rate common to all TIPS, contrary to our specification in equation (4) where each TIPS is priced using its own unique real rate. We also note that the model of D'Amico et al. (2014) beyond nominal and real zero-coupon yields requires time series dynamics of CPI inflation and especially its expected future level from surveys to properly identify liquidity risk. But inflation expectations from surveys are unavailable in real time and may differ from the expectations of the marginal investor in the TIPS market. The identification scheme we propose avoids these well-known limitations of inflation surveys by solely identifying the TIPS liquidity premium from individual TIPS prices.

Another closely related paper is the one by Abrahams et al. (2016), which also relies on an ATSM to estimate liquidity risk in TIPS. They take the liquidity factor to be observed and constructed from (i) the TIPS mean absolute fitted errors from Gürkaynak et al. (2010) and (ii) a measure of the relative transaction volume between Treasuries and TIPS. This alternative and somewhat more indirect approach to identify the TIPS liquidity premium relies heavily on having good observable proxies for liquidity risk, which in general is hard to ensure. The identification scheme of Abrahams et al. (2016), however, is similar to ours by not relying on CPI inflation or inflation expectations from surveys to estimate the TIPS liquidity premium.

An important similarity between our approach and the ones considered in D'Amico et al. (2014) and Abrahams et al. (2016) is to include the liquidity factor in the set of pricing factors when deriving TIPS prices based on no-arbitrage conditions. Fontaine and Garcia (2012) adopt the opposite approach when studying the off-the-run liquidity spread in Treasuries, as they omit the liquidity factor from the set of pricing factors in their ATSM. This implies that the model of Fontaine and Garcia (2012) allows for arbitrage within bond prices, where liquidity risk is identified from the relative price differences of maturing coupon bonds. We acknowledge that market participants may not always be able to exploit all arbitrage opportunities due to financial frictions, as emphasized by Fontaine and Garcia (2012), but we nevertheless find it useful to discipline our model by imposing no-arbitrage requirements, as typically done in the literature.

Overall, our model offers a new and very direct way to identify liquidity risk in ATSMs without including additional information from inflation swaps, CPI inflation, or inflation



(a) Distribution of TIPS

(b) Number of TIPS

Figure 2: **Overview of the TIPS Data**
Panel (a) shows the maturity distribution of all TIPS issued since 1997. The solid grey rectangle indicates the sample used in our benchmark analysis, where the sample is restricted to start on July 11, 1997, and limited to TIPS prices with less than ten years to maturity at issuance and more than two years to maturity after issuance. Panel (b) reports the number of outstanding TIPS at a given point in time for various samples.

surveys.[8]

4 Empirical Findings

As mentioned above, the proposed model is constructed for a sample of TIPS market prices in addition to a standard panel of Treasury yields. Given that ATSMs are rarely estimated directly on market prices for coupon bonds, we first describe our data set and estimation procedure in Section 4.1 and 4.2, respectively, before presenting the estimation results in Section 4.3.

4.1 Data

TIPS have been available in the five- to thirty-year maturity range since 1997, although only ten-year TIPS have been issued regularly. Panel (a) in Figure 2 shows the remaining time to maturity of all TIPS at a given date for all 50 bonds in our sample ending in 2013. Our empirical application is primarily devoted to the ten-year maturity spectrum as in D'Amico et al. (2014) and Abrahams et al. (2016), except for the robustness analysis in Section 6.3. This reduces the considered number of TIPS to $n_{TIPS} = 38$. The evolution in the number of

[8]We stress for completeness that our model and the subsequent estimation approach in Section 4 is sufficiently general to include such additional information if desired.

outstanding TIPS is shown in Panel (b) of Figure 2 for all maturities (the red line) and for the ten-year maturity spectrum (the grey line). Given that TIPS prices near maturity tend to exhibit erratic behavior due to seasonal variation in CPI inflation, we exclude TIPS from our sample when they have less than two years to maturity.[9] Using this cutoff reduces the number of TIPS in our sample further, as shown by the grey rectangle in Panel (a) and the solid black line in Panel (b) of Figure 2. We use the clean mid-market TIPS prices as reported each Friday by Bloomberg.[10] Given that our model has two pricing factors specific to TIPS, reliable identification of these factors requires at least two TIPS prices. This dictates the start of our weekly sample on July 11, 1997, when the second ever TIPS (with five years to maturity) becomes available. Our sample ends on December 27, 2013.

Finally, the considered panel of nominal zero-coupon yields are taken from Gürkaynak et al. (2007), where we include the following $n_y = 12$ maturities: 3-month, 6-month, 1-year, 2-year, . . . , 10-year. We use weekly data and limit our sample to the same period as considered for TIPS.

4.2 Estimation Methodology

We estimate the $G^L(5)$ model using the conventional likelihood-based approach, where we extract the latent pricing factors from the observables, which in our case are nominal zero-coupon yields and TIPS market prices. The functional form for nominal yields is provided in equation (12), whereas the expression for the price of the ith TIPS is more evolved and given by

$$\begin{aligned} P^{R,i}(t_0,t,T) \;=\; & \frac{C}{2}\frac{(t_1-t)}{1/2}\exp\Big\{-(t_1-t)y_t^{R,i}(t_0,t,t_1)\Big\} \\ & +\sum_{k=2}^{n}\frac{C}{2}\exp\Big\{-(t_k-t)y_t^{R,i}(t_0,t,t_k)\Big\} \\ & +\exp\Big\{-(T-t)y_t^{R,i}(t_0,t,T)\Big\}+DOV\left[Z_t;T,\frac{\Pi_t}{\Pi_{t_0}}\right], \end{aligned} \tag{15}$$

where Π_t/Π_{t_0} is the accrued CPI inflation compensation since issuance of the ith TIPS. That is, at time t we use the liquidity-adjusted real yields in equation (13) to discount the coupon payments attached to the ith bond.[11] The last term in equation (15) accounts for the deflation option value (DOV) embedded in TIPS, meaning that the principal at maturity is only adjusted for inflation if accumulated inflation since issuance of the bond is positive. We

[9] A similar procedure is used in Gürkaynak et al. (2010), who omit TIPS with 18 months to maturity and linearly downweight TIPS with 18 to 24 months to maturity. Section 6.2 explores the sensitivity of our results to gradually including more observations for each TIPS as it approaches maturity.

[10] If prices are unavailable on a particular Friday, we use the reported price on the last trading day before this Friday.

[11] The implementation here is greatly simplified by the continuous-time formulation of our model. For discrete-time models with one period exceeding one day (say, a week or a month), standard interpolation schemes may be used to price the coupon payments related to the ith bond at time t.

compute the value of this option as outlined in Christensen et al. (2012).[12] Following Joslin et al. (2011), all nominal yields in equation (12) have independent Gaussian measurement errors $\varepsilon^i_{y,t}$ with zero mean and a common standard deviation σ_y, denoted $\varepsilon^i_{y,t} \sim \mathcal{NID}\left(0, \sigma^2_y\right)$ for $i = 1, 2, \ldots, n_y$. We also account for measurement errors in the price of each TIPS through $\varepsilon^i_{TIPS,t}$, where $\varepsilon^i_{TIPS,t} \sim \mathcal{NID}\left(0, \sigma^2_{TIPS}\right)$ for $i = 1, 2, \ldots, n_{TIPS}$. To ensure that the TIPS measurement errors are comparable across maturities and of the same magnitude as the errors for nominal yields, we use the procedure in Gürkaynak et al. (2007) and scale both empirical and model-implied TIPS prices by duration to convert the related pricing errors into the same units as zero-coupon yields. Here, we use the standard Macaulay duration, as it allows us to obtain a model-free measure of duration from TIPS market prices and their implied yield to maturity, which is also available from Bloomberg.[13]

Combining equations (11) and (14), the state transition dynamics for Z_t under the physical measure $\mathbb{P}$ is easily shown to be

$$dZ_t = \mathcal{K}^{\mathbb{P}}_z \left(\theta^{\mathbb{P}}_z - Z_t\right) dt + \Sigma_z dW^{\mathbb{P}}_t,$$

where $\theta^{\mathbb{P}}_z$ and $\mathcal{K}^{\mathbb{P}}_z$ are free parameters with dimensions 5×1 and 5×5, respectively.

Due to the nonlinearities in equation (15) with respect to Z_t when pricing TIPS, we cannot apply the standard Kalman filter for the model estimation. Instead, the extended Kalman filter (EKF) is used to obtain an approximated log-likelihood function $\mathcal{L}^{EKF}$, which serves as the basis for the well-known quasi-maximum likelihood (QML) approach, as also used in Duan and Simonato (1999) and Kim and Singleton (2012) among many others.[14] To facilitate the estimation process, Appendix B provides a tailored expectation-maximization (EM) algorithm that efficiently deals with optimizing the quasi log-likelihood function across the relatively large number of bond-specific parameters in our model.

It is obvious from equation (10) that the level of X^{liq}_t and all the loadings $\left\{\beta^i\right\}^{n_{TIPS}}_{i=1}$ are not jointly identified, although the level of $r^{R,i}_t$ and the related TIPS liquidity premium (defined below in Section 5.1) are identified in the proposed model. The model of Fontaine and Garcia (2012) displays the same feature and we therefore follow their suggestion and normalize the loading on a given bond. In our case, the loading on the first bond in our sample is fixed to one (i.e. $\beta^1 = 1$), which is the ten-year TIPS issued in January 1997. This implies

[12]We do not account for the approximately 2.5 month lag in the CPI indexation of TIPS, given that Grishchenko and Huang (2013) and D'Amico et al. (2014) find that this adjustment normally is within a few basis points for the implied yield on TIPS and hence very small.

[13]For robustness, we have also estimated the $G^L(5)$ model using the yields to maturity for each TIPS and got very similar results. However, this alternative implementation is extremely time consuming as the yield to maturity is defined as an implicit fix-point problem that must be solved numerically for each observation.

[14]The details for implementing the EKF are provided in our Online Appendix. Using the more accurate central difference Kalman filter of Norgaard et al. (2000) gives basically identical values for the quasi log-likelihood function compared with the values implied by the EKF. For instance, the difference is 0.25 log points at the optimum for our benchmark model presented in Section 4.3. This suggests that the nonlinearity in equation (15) with respect to Z_t are very small and that the efficiency loss from using a QML approach as opposed to the infeasible maximum likelihood approach is likely to be very small in our case.

Maturity in months	$G(4)$		$G^L(5)$	
	Mean	RMSE	Mean	RMSE
3	-0.97	7.52	-1.14	7.44
6	-0.94	2.67	-0.87	2.70
12	0.82	7.13	1.14	7.04
24	2.48	6.30	2.75	6.17
36	1.36	3.73	1.38	3.63
48	-0.37	2.98	-0.56	2.96
60	-1.61	3.64	-1.88	3.65
72	-2.02	3.83	-2.25	3.80
84	-1.63	3.28	-1.76	3.16
96	-0.61	2.56	-0.60	2.36
108	0.82	3.10	0.98	3.03
120	2.50	5.16	2.76	5.26
All maturities	-0.01	4.64	0.00	4.59

Table 2: **Pricing Errors of Nominal Yields**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of nominal yields in the $G(4)$ and $G^L(5)$ models estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z. All errors are computed using the posterior state estimates in the EKF and reported in basis points.

that all remaining loadings for liquidity risk are expressed relative to this particular bond. Preliminary estimation shows that the value of $\lambda^{L,i}$ is badly identified when it is close to zero or attains large values, and we therefore impose $\lambda^{L,i} \in [0.01, 10]$ for $i = 1, 2, \ldots, n_{TIPS}$, which are without any practical consequences for our results. Finally, to ensure numerical stability of our estimation routine, we also impose the restrictions $\beta^i \in [0, 80]$ for $i = 2, 3, \ldots, n_{TIPS}$, although they are not binding at the optimum.

4.3 Estimation Results

This section presents our benchmark estimation results. In the interest of simplicity, in this section we focus on a version of the $G^L(5)$ model where $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z are diagonal matrices. As shown in Section 6.4, these restrictions have hardly any effects on the estimated liquidity premium for each TIPS, because it is identified from the model's $\mathbb{Q}$ dynamics, which are independent of $\mathcal{K}_z^{\mathbb{P}}$ and only display a weak link to Σ_z through the small convexity-adjustment in yields.

Given that the $G^L(5)$ model includes Treasury yields, it seems natural to first explore how well it fits nominal yields. Table 2 documents that it provides a very satisfying fit to all nominal yields, where the overall root mean-squared error (RMSE) is just 4.59 basis points. The corresponding version of this model without a liquidity factor is denoted the $G(4)$ model and gives broadly the same fit to nominal yields with an overall RMSE of 4.61 basis points.[15]

[15]Unreported results further show that omitting TIPS prices in the estimation gives basically the same satisfying fit of nominal yields with an overall RMSE of 4.41 basis points.

Thus, accounting for the liquidity disadvantage of TIPS does not affect the ability of the $G^L(5)$ model to match nominal yields.

The impact of accounting for liquidity risk is, however, much more apparent in the TIPS market. The first two columns in Table 3 show that the TIPS pricing errors produced by the $G(4)$ model are fairly large, with an overall RMSE of 14.58 basis points. The following two columns reveal a substantial improvement in the pricing errors when correcting for liquidity risk, as the $G^L(5)$ model has a very low overall RMSE of just 4.87 basis points. Hence, accounting for liquidity risk leads to a significant improvement in the ability of our model to explain TIPS market prices. Accordingly, the pricing errors of these securities within the $G^L(5)$ model are at the same low level as found for nominal yields in Table 2.

The final columns of Table 3 report the estimates of the specific parameters attached to each TIPS. Except for bond number 37, all bonds in our sample are exposed to liquidity risk, as β^i are significantly different from zero at the conventional 5 percent level. An inspection of $\lambda^{L,i}$ in Table 3 reveals that all five-year TIPS issued before the financial crisis in 2008 have very high values of $\lambda^{L,i}$, meaning that the first phase with active buy-and-hold investors is very short for these bonds. For the remaining TIPS, we generally find somewhat lower values of $\lambda^{L,i}$ and hence somewhat longer initial trading phases, where these bonds are not fully exposed to variation in the liquidity factor. As explained in Section 3.2, the impact of liquidity risk on real yields at various maturities is ambiguous, and Figure 3 therefore plots the liquidity adjustment in equation (13) as a function of time t for each of the 38 bonds in our sample. For the five-year TIPS in panel (a), this term structure of liquidity risk displays notable variation across securities due to the bond-specific estimates of $\lambda^{L,i}$. The corresponding loadings for ten-year TIPS are shown in panel (b), where we also find that the liquidity adjustment is increasing in t due to the strong mean-reversion in X_t^{liq} under the $\mathbb{Q}$ measure ($\kappa_{liq}^{\mathbb{Q}} = 0.90$ according to Table 4). Thus, liquidity risk operates as a traditional slope factor within the $G^L(5)$ model, although its steepness varies across the universe of TIPS.

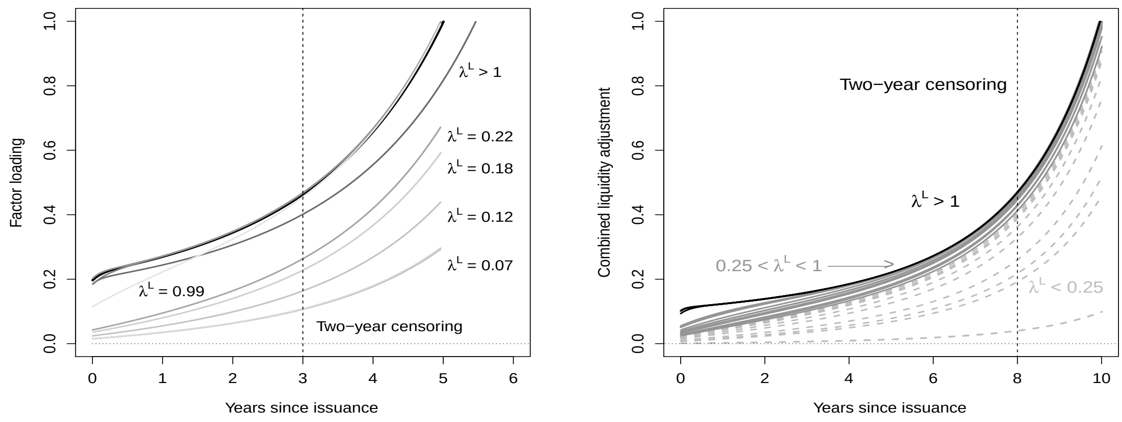
The remaining estimated model parameters are provided in Table 4, which shows that the dynamics of the four frictionless factors are very similar across the $G(4)$ and $G^L(5)$ models, both under the $\mathbb{P}$ and the $\mathbb{Q}$ measure. We draw the same conclusion from Figure 4, which plots the estimated factors in the two models. The only noticeable difference appears for the real level factor L_t^R, which in the $G(4)$ model generally exceeds the real level factor in the $G^L(5)$ model. This difference is most pronounced from 2001 to 2002 following the 9/11 attacks and around the financial crisis in 2008. The frictionless instantaneous real rate $r_t^{R,FL} = L_t^R + \alpha^R S_t$ therefore has a higher level in the $G(4)$ model, which in turn implies that this model has a much *lower* level for the instantaneous inflation rate $r_t - r_t^{R,FL}$ compared with the $G^L(5)$ model (see panel (f) of Figure 4). Finally, panel (e) shows the estimated liquidity factor X_t^{liq}, which is unique to the $G^L(5)$ model. As expected, this factor attains mostly positive values and peaks during the same episodes where the real level factor in the $G(4)$ model exceeds

TIPS security	Pricing errors				Estimated parameters			
	$G(4)$		$G^L(5)$		$G^L(5)$			
	Mean	RMSE	Mean	RMSE	β^i	SE	$\lambda^{L,i}$	SE
(1) 3.375% 1/15/2007 TIPS	-3.33	10.17	2.43	4.80	1	n.a.	0.79	0.35
(2) 3.625% 7/15/2002 TIPS*	1.04	10.58	3.26	4.04	0.84	0.13	7.90	1.98
(3) 3.625% 1/15/2008 TIPS	-1.33	10.44	2.15	4.38	2.72	0.68	0.10	0.07
(4) 3.875% 1/15/2009 TIPS	1.12	9.40	1.36	2.69	3.99	1.27	0.07	0.06
(5) 4.25% 1/15/2010 TIPS	2.41	9.38	0.86	3.04	2.25	0.28	0.22	0.07
(6) 3.5% 1/15/2011 TIPS	3.50	21.00	-0.22	4.27	2.56	0.34	0.21	0.06
(7) 3.375% 1/15/2012 TIPS	3.79	11.33	-0.10	5.19	2.59	0.30	0.24	0.06
(8) 3% 7/15/2012 TIPS	1.47	10.12	-0.36	4.99	2.57	0.27	0.26	0.06
(9) 1.875% 7/15/2013 TIPS	-0.48	14.66	-0.98	6.52	3.63	0.60	0.14	0.07
(10) 2% 1/15/2014 TIPS	5.54	12.25	0.39	3.72	7.32	1.63	0.06	0.03
(11) 2% 7/15/2014 TIPS	3.41	13.80	-0.09	4.49	2.81	0.19	0.31	0.06
(12) 0.875% 4/15/2010 TIPS*	1.46	9.79	2.36	4.46	2.13	0.08	10	n.a.
(13) 1.625% 1/15/2015 TIPS	8.33	14.99	0.90	4.34	3.87	0.32	0.18	0.02
(14) 1.875% 7/15/2015 TIPS	1.66	11.20	0.22	4.50	2.32	0.11	0.90	0.20
(15) 2% 1/15/2016 TIPS	4.27	9.40	1.11	4.77	2.79	0.14	0.38	0.03
(16) 2.375% 4/15/2011 TIPS*	15.05	33.31	4.76	12.08	2.06	0.11	4.64	2.10
(17) 2.5% 7/15/2016 TIPS	-3.51	10.13	-0.47	5.55	2.09	0.09	9.81	2.99
(18) 2.375% 1/15/2017 TIPS	-0.72	8.21	1.95	4.46	2.12	0.09	10	n.a.
(19) 2% 4/15/2012 TIPS*	19.46	37.86	5.59	11.19	1.98	0.10	10	n.a.
(20) 2.625% 7/15/2017 TIPS	-8.43	16.60	0.55	3.76	1.77	0.06	10	n.a.
(21) 1.625% 1/15/2018 TIPS	-7.33	18.05	0.48	3.73	2.16	0.08	0.48	0.04
(22) 0.625% 4/15/2013 TIPS*	-0.14	16.03	0.32	11.35	4.36	1.90	0.22	0.07
(23) 1.375% 7/15/2018 TIPS	-15.52	26.57	0.29	4.58	1.53	0.07	0.90	0.18
(24) 2.125% 1/15/2019 TIPS	-7.80	20.35	-0.09	3.23	32.0	1.06	0.01	n.a.
(25) 1.25% 4/15/2014 TIPS*	0.73	10.98	0.25	4.27	15.2	1.61	0.07	0.002
(26) 1.875% 7/15/2019 TIPS	-7.96	14.01	0.00	2.29	1.77	0.10	0.47	0.07
(27) 1.375% 1/15/2020 TIPS	0.93	8.14	-0.65	3.62	35.7	1.31	0.01	n.a.
(28) 0.5% 4/15/2015 TIPS*	8.73	15.06	0.54	3.23	9.56	1.81	0.12	0.004
(29) 1.25% 7/15/2020 TIPS	0.13	8.27	-0.28	2.67	2.26	0.18	0.41	0.07
(30) 1.125% 1/15/2021 TIPS	9.23	11.61	-0.50	3.79	3.62	0.48	0.28	0.07
(31) 0.125% 4/15/2016 TIPS*	5.53	8.70	-0.14	3.53	6.85	1.69	0.18	0.02
(32) 0.625% 7/15/2021 TIPS	5.77	8.33	0.11	2.58	2.81	0.15	0.57	0.15
(33) 0.125% 1/15/2022 TIPS	14.26	15.58	0.12	2.32	4.32	0.29	0.37	0.08
(34) 0.125% 4/15/2017 TIPS*	1.51	5.20	-0.01	2.53	15.8	1.97	0.07	0.007
(35) 0.125% 7/15/2022 TIPS	10.88	11.87	0.34	3.40	2.89	0.12	4.71	2.40
(36) 0.125% 1/15/2023 TIPS	19.59	20.46	0.12	5.30	3.60	0.17	10	n.a.
(37) 0.125% 4/15/2018 TIPS*	5.47	6.28	-0.21	3.08	3.46	3.93	0.99	0.007
(38) 0.375% 7/15/2023 TIPS	9.80	10.19	0.54	2.62	2.56	0.15	10	n.a.
All TIPS yields	1.20	14.58	0.66	4.87	-	-	-	-
Max $\mathcal{L}^{EKF}$	109,593.5		118,945.5		-		-	

Table 3: **Pricing Errors of TIPS and Estimated Parameters for Liquidity Risk**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of TIPS in the $G(4)$ and $G^L(5)$ models estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z. The errors are computed as the difference between the TIPS market price expressed as yield to maturity and the corresponding model-implied yield. All errors are computed using the posterior state estimates in the EKF and reported in basis points. The asterisk * denotes five-year TIPS. Standard errors (SE) are not available (n.a.) for the normalized value of β^1 or parameters close to their boundary. The SE are computed by pre- and post-multiplying the variance of the score by the inverse of the Hessian matrix, which we compute as outlined in Harvey (1989).

the value of L_t^R in the $G^L(5)$ model.

Accordingly, when estimating L_t^R and the frictionless instantaneous real rate from Treasury yields and TIPS market prices, it is essential to account for the liquidity disadvantage of



(a) Five-year TIPS (b) Ten-year TIPS

Figure 3: **The Term Structure of Liquidity Risk**
This figure shows the term structure of liquidity risk, where β^i is omitted to facilitate the comparison. That is, we report $\frac{\left(1-\exp\left\{-\kappa^{\mathbb{Q}}_{liq}(T-t)\right\}\right)}{\kappa^{\mathbb{Q}}_{liq}(T-t)} - \exp\left\{-\lambda^{L,i}\left(t-t_0\right)\right\}\frac{1-\exp\left\{-\left(\kappa^{\mathbb{Q}}_{liq}+\lambda^{L,i}\right)(T-t)\right\}}{\left(\kappa^{\mathbb{Q}}_{liq}+\lambda^{L,i}\right)(T-t)}$ for the yield related to the ith TIPS as implied by the estimated version of the $G^L(5)$ model with a diagonal specification of $\mathcal{K}^{\mathbb{P}}_z$ and Σ_z.

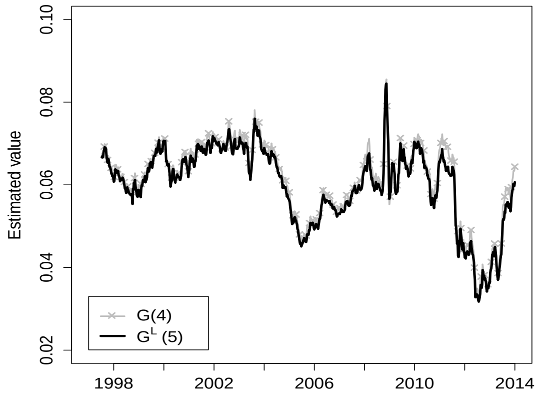
TIPS to avoid a positive bias in the estimated instantaneous real rate, which automatically generates a negative bias in the instantaneous inflation rate—particularly during periods of market turmoil.

5 The TIPS Liquidity Premium

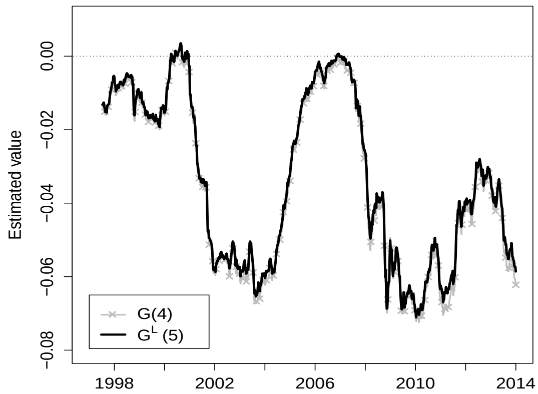
This section studies the TIPS liquidity premium implied by the estimated $G^L(5)$ model described in the previous section. Section 5.1 formally defines the TIPS liquidity premium and studies its historical evolution. The estimated liquidity premium is then related to existing measures of liquidity risk in Section 5.2, while a comprehensive robustness analysis is deferred to Section 6.

5.1 The Estimated TIPS Liquidity Premium

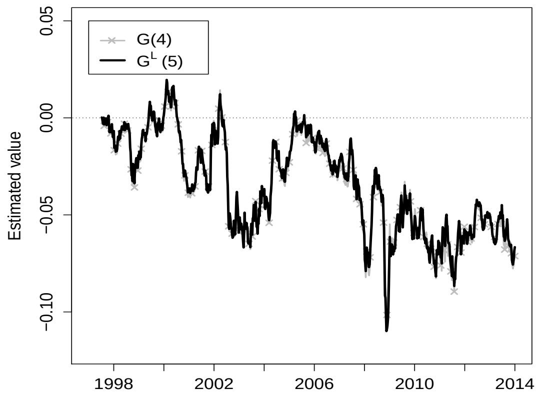
We now use the estimated $G^L(5)$ model to extract the liquidity premium in the TIPS market. To compute this premium we first use the estimated parameters and the filtered states $\left\{Z_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted TIPS prices $\left\{\hat{P}_t^{TIPS,i}\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving



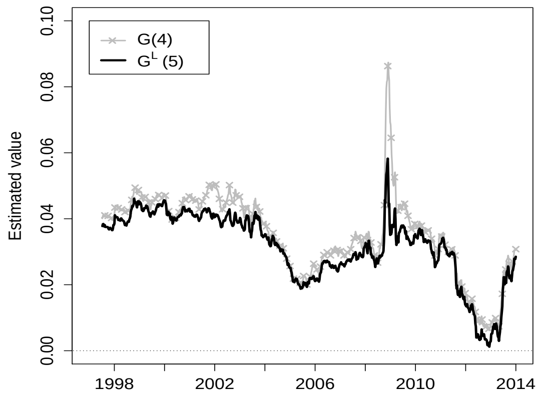
(a) L_t^N: The nominal level factor



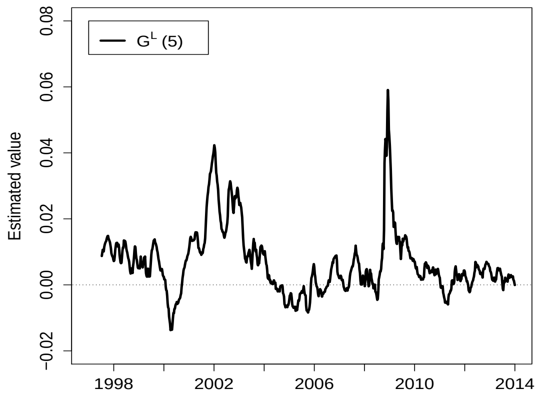
(b) S_t: The common slope factor



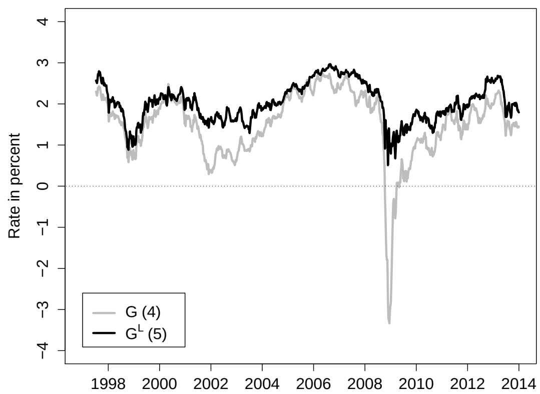
(c) C_t: The common curvature factor



(d) L_t^R: The real level factor



(e) X_t^{liq}: The liquidity factor



(f) The instantaneous inflation rate

Figure 4: **Estimated State Variables and Instantaneous Inflation**
This figure shows the posterior state estimates in the EKF and the instantaneous inflation for the $G\,(4)$ and $G^L\,(5)$ models estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z.

Parameter	$G\,(4)$		$G^L\,(5)$	
	Est.	SE	Est.	SE
$\kappa_{11}^{\mathbb{P}}$	0.2796	0.1956	0.2348	0.2138
$\kappa_{22}^{\mathbb{P}}$	0.0976	0.0656	0.0873	0.0656
$\kappa_{33}^{\mathbb{P}}$	0.5156	0.2779	0.4069	0.2617
$\kappa_{44}^{\mathbb{P}}$	0.4407	0.3208	0.2585	0.1942
$\kappa_{55}^{\mathbb{P}}$	-	-	0.7244	0.3700
σ_{11}	0.0071	0.0004	0.0060	0.0007
σ_{22}	0.0101	0.0005	0.0099	0.0005
σ_{33}	0.0255	0.0014	0.0249	0.0014
σ_{44}	0.0082	0.0005	0.0072	0.0004
σ_{55}	-	-	0.0124	0.0013
$\theta_1^{\mathbb{P}}$	0.0633	0.0051	0.0612	0.0059
$\theta_2^{\mathbb{P}}$	-0.0336	0.0148	-0.0294	0.0140
$\theta_3^{\mathbb{P}}$	-0.0336	0.0122	-0.0323	0.0131
$\theta_4^{\mathbb{P}}$	0.0362	0.0046	0.0342	0.0038
$\theta_5^{\mathbb{P}}$	-	-	0.0074	0.0032
λ	0.4228	0.0056	0.4442	0.0053
α^R	0.6931	0.0129	0.7584	0.0117
$\kappa_{liq}^{\mathbb{Q}}$	-	-	0.9004	0.0902
$\theta_{liq}^{\mathbb{Q}}$	-	-	0.0014	0.0002
σ_y	0.0005	9.34×10^{-6}	0.0005	8.70×10^{-6}
σ_{TIPS}	0.0015	7.37×10^{-5}	0.0005	2.43×10^{-5}

Table 4: **Estimated Dynamic Parameters**
The table shows the estimated dynamic parameters for the $G\,(4)$ and $G^L\,(5)$ models estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z. The reported standard errors (SE) are computed by pre- and post-multiplying the variance of the score by the inverse of the Hessian matrix, which we compute as outlined in Harvey (1989).

the fixed-point problem

$$\begin{aligned} \hat{P}_{t=1}^{TIPS,i} &= \frac{C}{2}\frac{(t_1 - t)}{1/2}\exp\left\{-(t_1 - t)\hat{y}_t^{c,i}\right\} \\ &\quad + \sum_{k=2}^{n}\frac{C}{2}\exp\left\{-(t_k - t)\hat{y}_t^{c,i}\right\} \\ &\quad + \exp\left\{-(T - t)\hat{y}_t^{c,i}\right\} + DOV\left[Z_{t|t}; T, \frac{\Pi_t}{\Pi_{t_0}}\right], \end{aligned} \tag{16}$$

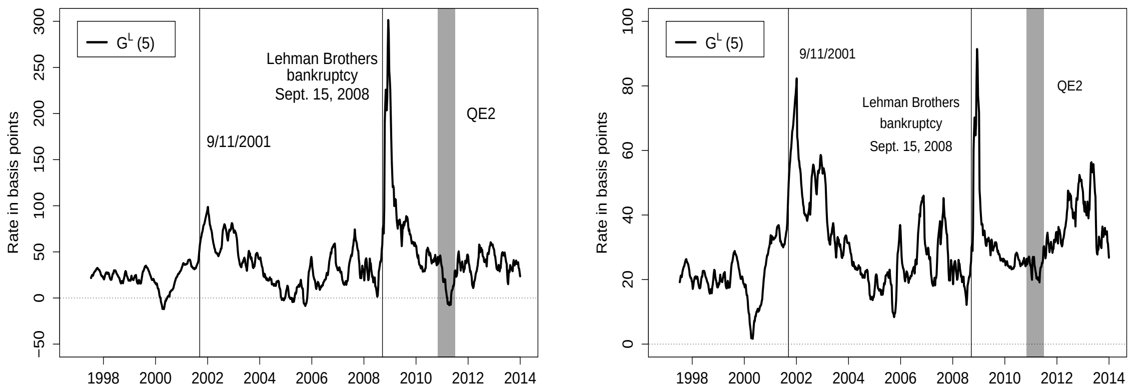
for $i = 1, 2, ..., n_{TIPS}$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the real rate of return on the ith TIPS if held until maturity (see Sack and Elsasser (2004)). To obtain the corresponding yields without correcting for liquidity risk, a new set of model-implied bond prices are computed from the estimated $G^L\,(5)$ model but using only its frictionless part, i.e., with the constraints that $X_{t|t}^{liq} = 0$ for all t and $\sigma_{55} = 0$. These prices are denoted $\left\{\tilde{P}_t^{TIPS,i}\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using (16). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the ith TIPS is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{17}$$

Panel (a) in Figure 5 shows the average liquidity premium $\bar{\Psi}_t$ across the outstanding TIPS at a given point in time. This premium starts at around 25 basis points in July 1997 and falls steadily to just below zero in the beginning of 2000, when the U.S. economy displayed strong economic growth. Thus, there is only a modest liquidity premium in the ten-year maturity spectrum of the TIPS market from 1997 to 2000 according to our model. This finding does not seem too surprising, as the few outstanding TIPS in this period allow the frictionless pricing factors in our model to explain most of the variation in TIPS prices and thereby reduce the reliance on the liquidity correction (see Figure 2). The slowdown in economic activity during 2000 and the following recession coincide with a steady increase in the average liquidity premium, which peaks at about 100 basis points shortly after the 9/11 attacks in 2001. Liquidity generally improves in 2002 and the following years, meaning that $\bar{\Psi}_t$ is close to zero during much of 2005. The Treasuries reaffirmed commitment to the TIPS program in February 2002 has most likely contributed to this downward trend in $\bar{\Psi}_t$, as it seems likely to have raised expectations about the future supply of TIPS. Two other factors contributing to the improved liquidity are the increase in the number of outstanding TIPS after 2003 (Dudley et al. (2009)), and that several dealers expanded their TIPS market-making activities around 2003 (Sack and Elsasser (2004)). Liquidity once again deteriorates in 2008 with the bankruptcy of Lehman Brothers and the financial crisis, where the average liquidity premium peaks at about 300 basis points. Market conditions normalize in 2009 and liquidity improves temporarily during the second round of quantitative easing (QE2) from November 2010 to June 2011 (see Christensen and Gillan (2017) for a detailed analysis).

The average liquidity premium studied so far is computed from the outstanding TIPS at each point in time, meaning that its maturity varies with the composition of securities in the market. Some of the variation in $\bar{\Psi}_t$ therefore reflects the fact that old and somewhat illiquid bonds mature and are replaced by new and more liquid securities. Although the average liquidity premium is of great interest on its own, it may nevertheless also be useful to examine the liquidity premium at a fixed maturity. This is done in panel (b) of Figure 5, where we report the liquidity premium for the most recently issued ten-year TIPS Ψ_t^{10y}, i.e., the ten-year TIPS which is "on-the-run."[16] We first note that the most recently issued ten-year TIPS is more liquid than the average security in the TIPS market, although $\bar{\Psi}_t$ and Ψ_t^{10y} are closely correlated (77%). We also find that the mean of Ψ_t^{10y} is 30 basis points and its standard deviation is 13 basis points, whereas the corresponding figures for $\bar{\Psi}_t$ are 38 and 34 basis points, respectively. It is also worth noticing that the liquidity in the ten-

[16] See also Christensen et al. (2017), who explore the presence of an "on-the-run" liquidity premium in the TIPS market.



(a) The average TIPS liquidity premium

(b) The ten-year on-the-run TIPS liquidity premium

Figure 5: **The Estimated TIPS Liquidity Premium**
This figure shows the TIPS liquidity premium implied by the estimated version of the $G^L(5)$ model described in Section 4.3.

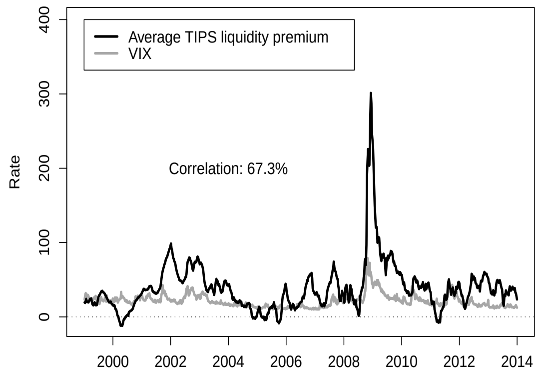
year TIPS is less severely affected by the financial crisis in 2008 compared with the average liquidity premium. This suggests that a large proportion of the elevated level for $\bar{\Psi}_t$ during the financial crisis is due to poor liquidity in the outstanding old securities. Finally, we also note that QE2 hardly affects the liquidity premium in the most recently issued ten-year TIPS, meaning that the impact of QE2 on TIPS liquidity derives mostly from its effect on the liquidity of older securities.

5.2 Observable Proxies for Liquidity Risk

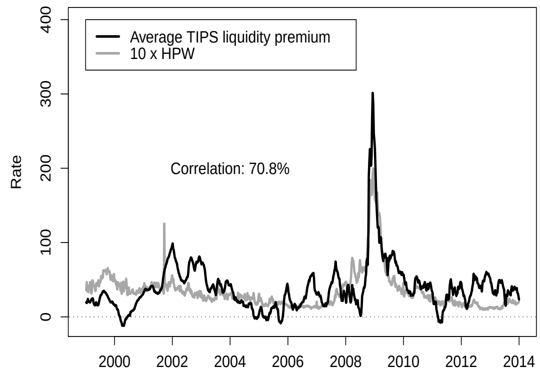
Having demonstrated the close relationship between the evolution of the U.S. economy and our model-implied liquidity premium in the TIPS market, we next show that this liquidity premium is strongly related to several other observable proxies for liquidity risk. Given our interest in understanding the overall evolution in TIPS liquidity, we focus on the average liquidity premium throughout this section.

The first variable we consider is the VIX options-implied volatility index, which represents near-term uncertainty in the Standard & Poor's 500 stock market index. Panel (a) of Figure 6 shows the expected positive correlation (67%) between the VIX and the TIPS liquidity premium, as high uncertainty tends to increase the risk attached to the future resale price of any security and therefore also the required liquidity premium.[17] Our second observable proxy for liquidity risk is the measure suggested by Hu et al. (2013), henceforth HPW, based on deviations in the prices of Treasuries from a fitted yield curve. They argue that this measure reflects limited availability of arbitrage capital and therefore constitutes an economy-wide proxy for illiquidity. Panel (b) in Figure 6 shows that the average TIPS liquidity premium

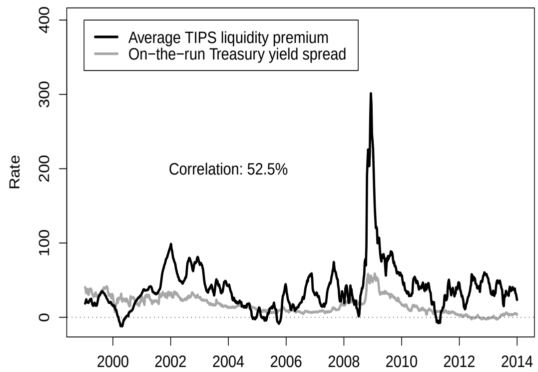
[17]See also Duffie et al (2007) for a model of the positive relationship between uncertainty and liquidity.



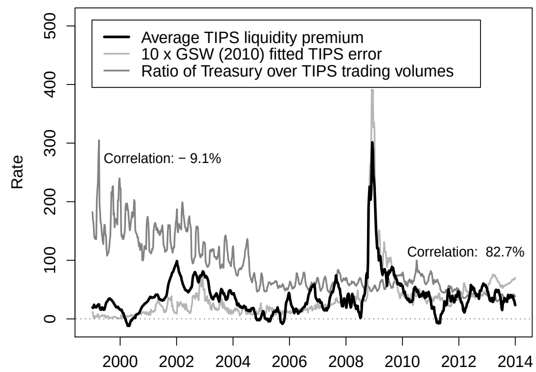
(a) The VIX options-implied volatility index



(b) The HPW illiquidity measure



(c) The on-the-run Treasury par-yield spread



(d) GSW fitting errors and TIPS trading volumes

Figure 6: **Variables Explaining the Average TIPS Liquidity Premium**
In panel (a) the VIX for the S&P 500 is expressed in percentage, in panel (b) the HPW series is scaled by ten, in panel (c) the yield spread is the difference between the ten-year off-the-run Treasury par yield from Gürkaynak et al. (2007) and the ten-year on-the-run Treasury par yield from the H.15 series at the Board of Governors, and in panel (d) we use the ratio of the weekly average of daily trading volume in the secondary market for Treasury coupon bonds over the weekly average of daily trading volume in the secondary market for TIPS, where both series are measured as the eight-week moving average.

from our model is also closely related to the HPW measure with a positive correlation of 71%.

Our third variable is the yield difference between the seasoned (off-the-run) ten-year Treasury as provided by Gürkaynak et al. (2007) and the most recently issued (on-the-run) Treasury of the same maturity from the H.15 series at the Board of Governors. This spread represents the on-the-run liquidity premium in Treasuries, which also correlates positively (53%) with the average TIPS liquidity premium from our model, as seen from panel (c) in Figure 6. The last two proxies for liquidity risk are taken from Abrahams et al. (2016), which use (i) the TIPS mean absolute fitting errors from Gürkaynak et al. (2010) and (ii) the rela-

	Correlations in Percent: VIX	HPW	On-the-run spread	Ratio of trading vol	GSW TIPS errors	Regression: Average TIPS liquidity premium	
Constant						−5.21 (5.36)	−4.30 (4.05)
VIX	100	75	69	18	55	0.85** (0.20)	0.76 ** (0.21)
HPW		100	79	16	69	−2.00 (1.08)	−
On-the-run spread			100	57	34	0.73* (0.29)	0.49 * (0.20)
Ratio of Trading vol				100	−34	0.0001 (0.06)	−
GSW TIPS errors					100	6.06** (0.47)	5.53 ** (0.34)
Adjusted R^2						0.769	0.765

Table 5: **Observable Proxies for Liquidity Risk**
The first part of the table reports correlations between each of the observable proxies for liquidity risk, where GSW TIPS errors refers to the mean absolute fitted errors from Gürkaynak et al. (2010). The second part reports the results of regressing the average TIPS liquidity from the estimated $G^L(5)$ model described in Section 4.3 on these observable proxies for liquidity risk. Standard errors computed by the Newey-West estimator (with 12 lags) are provided in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The data are weekly covering the period from January 8, 1999, to December 27, 2013, a total of 782 observations.

tive TIPS transaction volume, which is measured by the ratio of primary dealers' transaction volume in Treasuries over the corresponding transaction volume in TIPS.[18] Both of these series are plotted in panel (d), which shows a very high correlation of 83% between the average liquidity premium in our model and the TIPS mean absolute fitted errors from Gürkaynak et al. (2010). On the other hand, our model-implied liquidity premium is negatively correlated with the relative TIPS transaction volume (−9%). However, Table 5 reveals that this proxy for liquidity is only weakly correlated with the VIX (18%) and the HPW measure (16%), and even negatively correlated (−34%) with the TIPS fitted errors from Gürkaynak et al. (2010). This suggests that the relative TIPS transaction volume may be an imprecise proxy for liquidity risk in the TIPS market, and this is likely to explain our finding in panel (d).

The second part of Table 5 reports the results of regressing the average liquidity premium from our model on the five observable proxies for liquidity risk. We find a significant effect from the VIX, the on-the-run spread, and the TIPS fitted errors from Gürkaynak et al. (2010), which all have the expected positive sign. On the other hand, the HPW measure and the relative TIPS transaction volume are both insignificant at the 5% level and hardly affect the adjusted R^2 of 0.77. The lack of significance for the HPW measure is explained by its close similarity to the off-the-run spread and the TIPS fitted errors from Gürkaynak et al. (2010), whereas the insignificance of the relative TIPS transaction volume is to be expected given the

[18]The two series for transaction volumes represent weekly averages of the daily trading volumes in the secondary markets for Treasuries and TIPS as reported by the Federal Reserve Bank of New York, where we use an eight-week moving average of both series to smooth out short-term volatility.

imprecise nature of this liquidity proxy.[19]

Thus, our model-implied measure of liquidity risk is highly correlated with other commonly used proxies for liquidity risk, which we interpret as confirming evidence for the new identification strategy of TIPS liquidity we propose.

6 Robustness Analysis

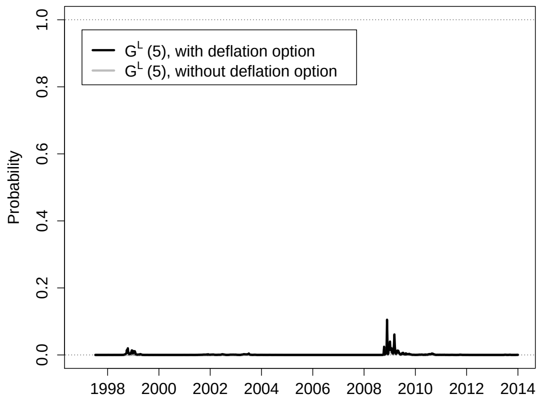
This section examines the robustness of the average liquidity premium reported in Section 5 to some of the main assumptions imposed so far. In particular, Section 6.1 analyzes the effects of accounting for the deflation option in TIPS, Section 6.2 changes the cutoff criterion for maturing bonds, Section 6.3 extends the analysis to the full thirty-year maturity spectrum, Section 6.4 considers a fully flexible specification for the factor dynamics under the $\mathbb{P}$ measure, Section 6.5 modifies the $G^L(5)$ model to accommodate the zero lower bound (ZLB) for nominal yields, and Section 6.6 presents a more flexible form for frictionless real yields than considered in the $G^L(5)$ model. The overall message from this robustness analysis is that the average liquidity premium from our benchmark model in Section 5 is remarkably robust to each of these alternative specifications.

6.1 The Effects of Accounting for the Deflation Option in TIPS

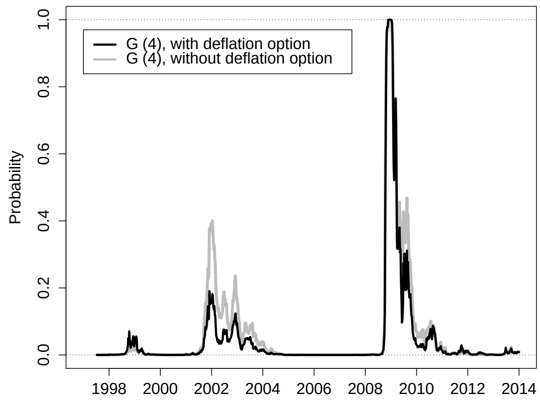
When we estimated the $G^L(5)$ model in Section 4, we accounted for the deflation option in TIPS to isolate its effect from the liquidity adjustment. To evaluate the importance of the deflation option, we next estimate the $G^L(5)$ model *without* correcting for this option effect. We proceed by first reporting the estimated deflation probabilities to locate episodes when the deflation option matters. We then analyze the effect of the deflation option on the TIPS liquidity premium.

The top row of Figure 7 reports one-year implied deflation probabilities under the $\mathbb{P}$ measure from the $G^L(5)$ and the $G(4)$ model, respectively. Panel (a) shows that the option adjustment hardly has any effect in the $G^L(5)$ model, because the model generates very low $\mathbb{P}$ probabilities of deflation during the Russian sovereign debt crisis in late 1998, the bust of the dot-com bubble in 2001, and the financial crisis from 2008 to 2009. These findings seem consistent with the view that the Federal Reserve assigned low probabilities to deflation in the U.S. both in 2001 and during the financial crisis (see, for instance, Bernanke (2002) and Ezer et al. (2008)). To compare these results to the existing literature such as Grishchenko

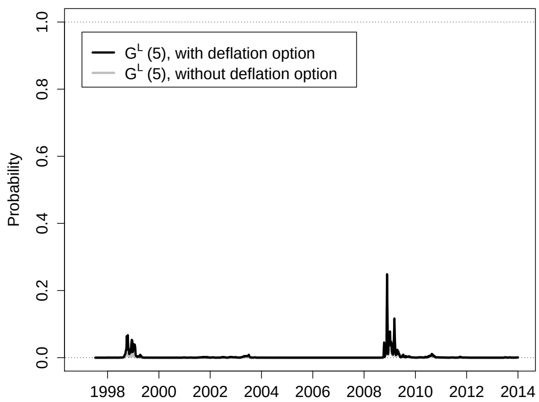
[19]Given that all variables in this regression are very persistent, we have also estimated it in first-differences as a robustness check. That is, the weekly change in the average liquidity premium is regressed on the weekly change in the VIX, the on-the-run spread, the TIPS fitted errors from Gürkaynak et al. (2010), and a constant. The OLS estimates and related Newey-West standard errors (using three lags) for the first three regressors are 0.104 (0.081), 0.591 (0.303), and 3.179 (0.563), respectively. Thus, the on-the-run spread has a P-value of 5.16% and the TIPS fitted errors from Gürkaynak et al. (2010) have a P-value of 0.00%, whereas the VIX becomes insignificant with a P-value of 20.3% although this regressor has the expected sign.



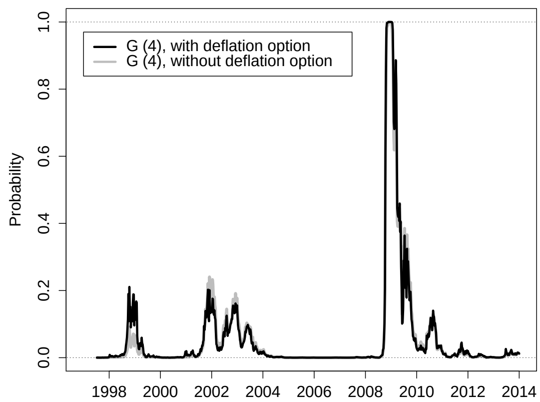
(a) $\mathbb{P}$-probabilities of deflation



(b) $\mathbb{P}$-probabilities of deflation



(c) $\mathbb{Q}$-probabilities of deflation



(d) $\mathbb{Q}$-probabilities of deflation

Figure 7: **Estimated One-Year Deflation Probabilities**
Panels (a) and (b) show the one-year deflation probabilities under the objective probability measure $\mathbb{P}$ from the $G^L(5)$ and $G(4)$ models estimated with and without the deflation option adjustment. Panels (c) and (d) show the corresponding estimated one-year deflation probabilities under the risk-neutral probability measure $\mathbb{Q}$. All models are estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z.

et al. (2016) and Fleckenstein et al. (2017), we also report the corresponding $\mathbb{P}$ probabilities from the $G(4)$ model in Panel (b), where we see clear spells of deflation fears around 2001 and 2008, both with and without the deflation option. To understand these differences, recall from Section 4.3 that the inability of the $G(4)$ model to account for liquidity risk in TIPS generates a downward bias in its instantaneous inflation rate compared with the $G^L(5)$ model. This in turn shifts the entire inflation probability distribution under the $\mathbb{P}$ measure downwards and makes episodes of deflation much more likely in the $G(4)$ model compared with the $G^L(5)$ model.

The bottom row in Figure 7 shows that the results under the $\mathbb{P}$ measure carries over to the corresponding $\mathbb{Q}$ probabilities, which are used to evaluate the deflation option in TIPS.

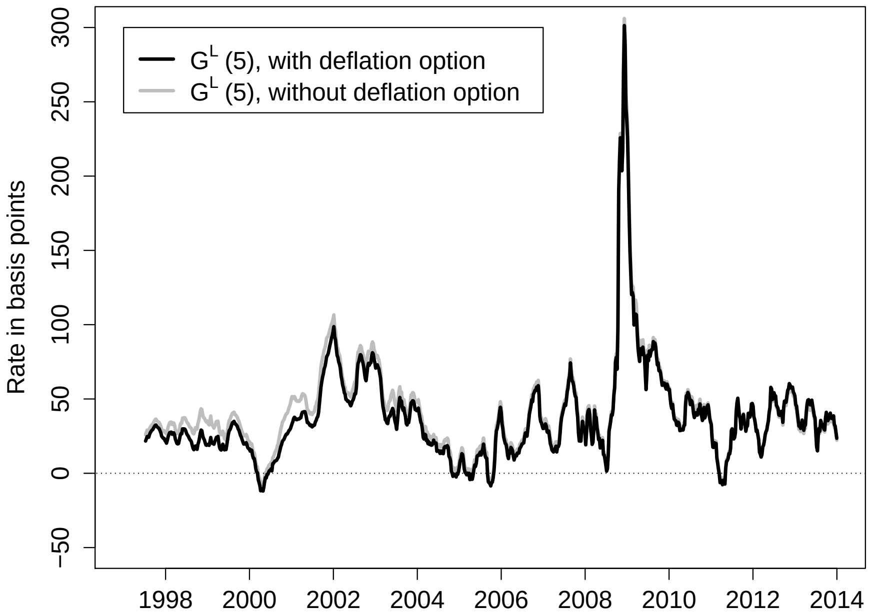


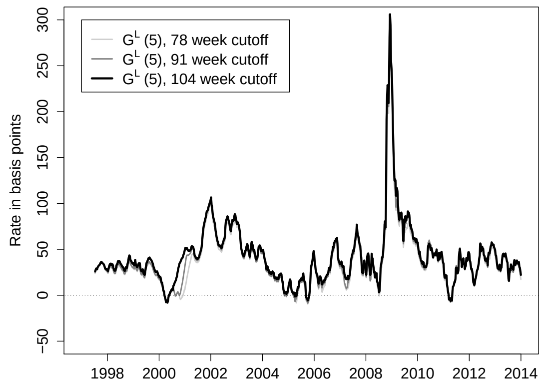
Figure 8: **The Average TIPS Liquidity Premium: The Deflation Option**
This figure shows the average TIPS liquidity premium from the estimated version of the $G^L(5)$ model described in Section 4.3 and from a re-estimated version of the $G^L(5)$ model without the deflation option. Both versions of the $G^L(5)$ model are estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z.

Thus, the probability of the deflation option being in-the-money within the $G^L(5)$ model is very small, and the effect of the deflation option on the average TIPS liquidity premium is therefore hardly visible in Figure 8.[20] Based on this finding and the large computational costs required to include the deflation option in the estimation, we will not account for it in the remaining robustness analysis.

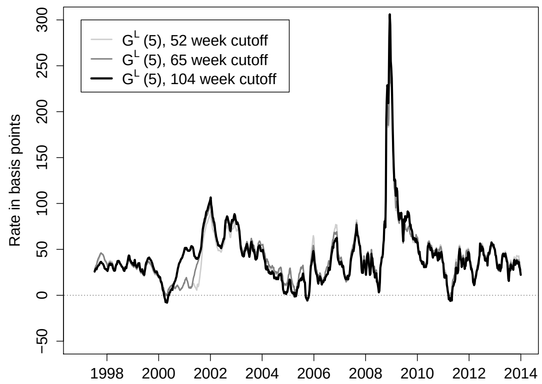
6.2 The Cutoff Criterion for Maturing TIPS

The analysis in Section 4 excluded TIPS with less than two years to maturity from our sample to avoid the erratic behavior in their market prices from seasonal variation in CPI inflation. We next examine the effects of this cutoff criterion by re-estimating the $G^L(5)$ model while gradually reducing this cutoff point. The top row in Figure 9 shows that the average TIPS liquidity premium is nearly unaffected by lowering the cutoff to one year, whereas $\bar{\Psi}_t$ becomes somewhat more volatile around our benchmark estimate when including TIPS with less than one year to maturity (see the bottom row of Figure 9). Thus, our estimated liquidity premium

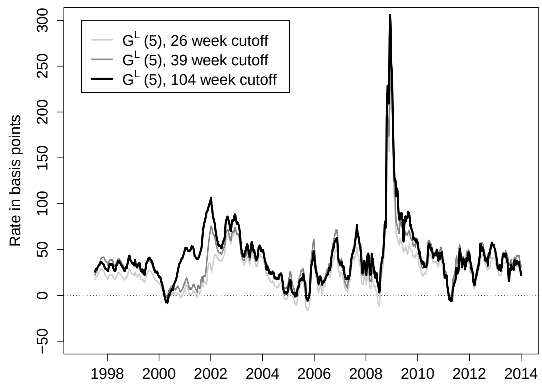
[20]When expressing the size of the deflation option in terms of the yield spread, unreported results reveal that the deflation option in the $G^L(5)$ model is typically between 5 and 10 basis points for newly issued TIPS, except for a few securities around 1999 and 2008 where the effect is around 20 basis points for a brief period.



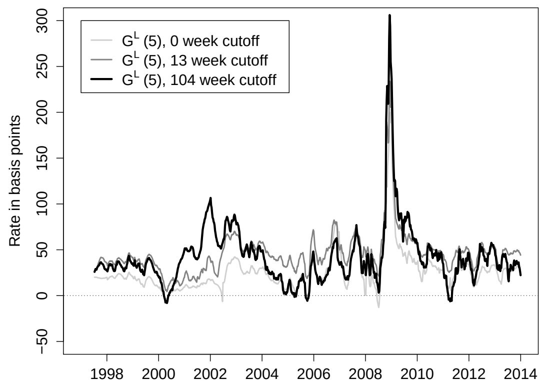
(a) Panel: Truncation at 1.5 years



(b) Panel: Truncation at 1 year



(c) Panel: Truncation at 0.5 year



(d) Panel: No Truncation

Figure 9: **The Average TIPS Liquidity Premium: The Cutoff Criterion**
This figure reports the average TIPS liquidity premium from the $G^L(5)$ model when re-estimated with various cutoff values for excluding maturing TIPS. All reported models are estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z and without the deflation option.

is fully robust to lowering the cutoff point to one year but becomes more noisy when including TIPS with just 26 or even 13 weeks to maturity.

6.3 Extending the Analysis to the Thirty-Year Maturity Range

Our analysis has so far been restricted to the ten-year maturity range for Treasury yields and TIPS. We next study the implications of extending the analysis to the full thirty-year maturity spectrum for Treasury yields and TIPS. That is, we now include twenty- and thirty-year TIPS whenever present and apply the same two-year cutoff criterion for maturing TIPS as in Section 4. To adequately represent the full thirty-year Treasury yield curve, we also expand our panel of nominal yields with the 11-year, 12-year, ... , 30-year maturities from Gürkaynak et al. (2007). Figure 10 shows that the average liquidity premium during the

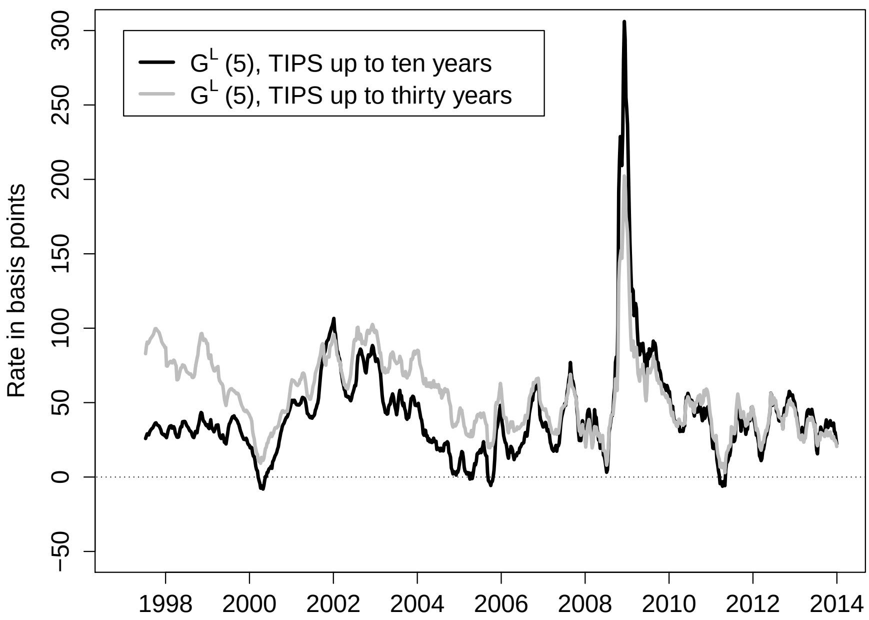


Figure 10: **The Average TIPS Liquidity Premium: The Full Thirty-Year Maturity Range**
This figure shows the average TIPS liquidity premium from the benchmark version of the $G^L(5)$ model and a re-estimated version of the $G^L(5)$ model using TIPS and nominal yields covering the full thirty-year maturity spectrum. Both versions of the $G^L(5)$ model are estimated with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z and without accounting for the deflation option.

late 1990s increases from about 25 basis points to between 75 and 100 basis points when including the full thirty-year maturity range. During this early phase of the TIPS market, this corresponds to adding two thirty-year TIPS to our existing sample, consisting only of two ten-year and one five-year TIPS (see Figure 2). This sizable effect on the average liquidity premium of including the two thirty-year TIPS indicates a considerable degree of mispricing based on the frictionless part of our model. In comparison, the three bonds in the ten-year maturity range contain only a low liquidity premium, and hence low mispricing based on the frictionless part of the model. These findings therefore suggest that the two thirty-year TIPS were somewhat detached from the remaining market in the late 1990s. The effect of including the twenty- and thirty-year TIPS diminishes from 2000 to 2002, but increases again from 2003 to 2006, possibly due to the introduction of the three twenty-year TIPS in this period (see Figure 2). From 2006 onwards, we see generally small effects of including the twenty- and thirty-year TIPS, which may indicate that the TIPS market at the end of our sample is much less segmented than in the late 1990s.

Overall, the mean of the average TIPS liquidity premium increases from 38 to 55 ba-

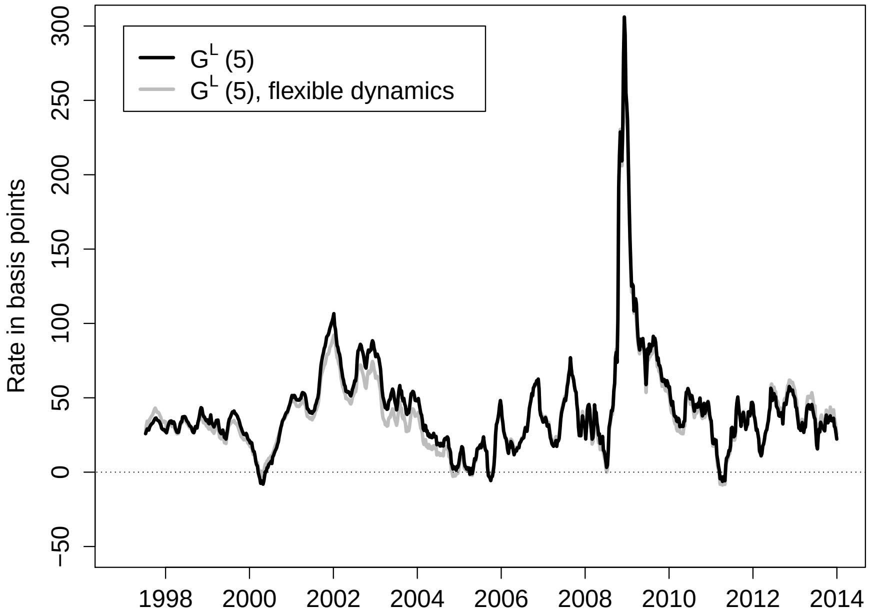


Figure 11: **The Average TIPS Liquidity Premium: Fully Flexible $\mathbb{P}$ Dynamics**
This figure shows the average TIPS liquidity premium from the benchmark version of the $G^L(5)$ model with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z and a version of the $G^L(5)$ model with a fully flexible specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z. Both versions of the $G^L(5)$ model are estimated without accounting for the deflation option.

sis points when extending the analysis to the thirty-year maturity spectrum, whereas the standard deviation falls from 34 to 27 basis points.

6.4 Fully Flexible Factor Dynamics under the Physical Measure

In the interest of simplicity, we have so far studied a restricted version of the $G^L(5)$ model where $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z are diagonal matrices. To explore the impact of these restrictions for the estimated liquidity premium, we momentarily consider the $G^L(5)$ model with a fully flexible specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z. Figure 11 shows that the average liquidity premium in this more flexible version of the $G^L(5)$ model only occasionally exceeds the benchmark estimate by 5 to 10 basis points from 1997 to 2002. That is, the $\mathbb{P}$ dynamics have hardly any effect on the average liquidity premium, as claimed in Section 4.3. This may at first appear somewhat surprising, but this result arises because the liquidity premium is identified from the $\mathbb{Q}$ dynamics in the $G^L(5)$ model, which are independent of $\mathcal{K}_z^{\mathbb{P}}$ and only display a weak link to Σ_z through the small convexity-adjustment in yields.

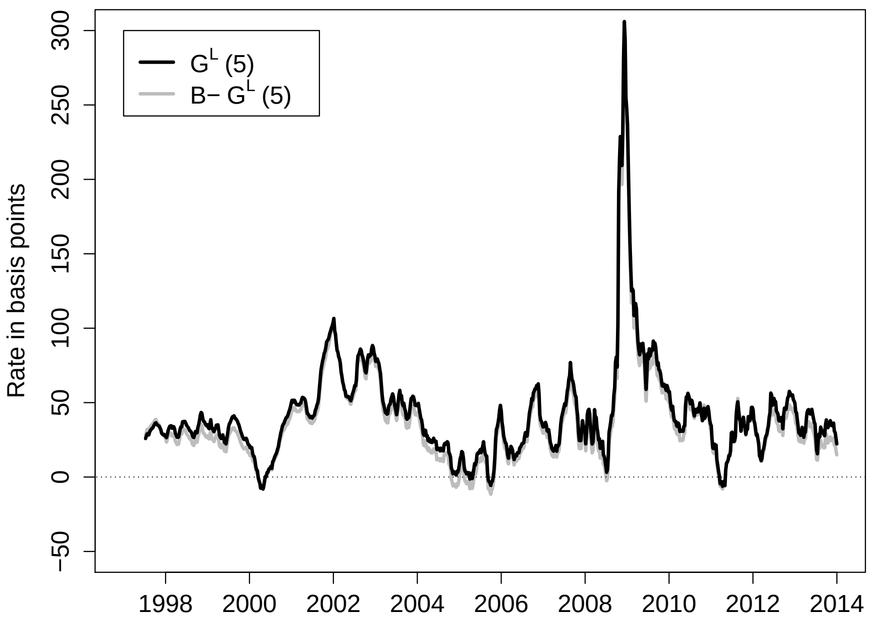


Figure 12: **The Average TIPS Liquidity Premium: Accounting for the ZLB**
This figure shows the average TIPS liquidity premium from the benchmark version of the $G^L\,(5)$ model with a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z and the corresponding shadow rate extension, denoted the B-$G^L\,(5)$ model, which respects the zero lower bound for nominal yields. Both models are estimated without accounting for the deflation option.

6.5 Accounting for the Zero Lower Bound in Nominal Yields

We have so far adopted the standard affine specification for the nominal short rate, which does not enforce the ZLB. However, a large fraction of our sample (since January 2009) is at the ZLB, and we therefore briefly explore whether our estimated liquidity premium is robust to accounting for the ZLB through a shadow rate extension of the $G^L\,(5)$ model. We adopt an approach inspired by Black (1995) and replace r_t^N in equation (9) by $r_t^N = \max\left(L_t^N + S_t, 0\right)$ and solve for nominal yields using the approximation in Christensen and Rudebusch (2015), but the model is otherwise identical to the one presented in Section 3.2. Figure 12 shows that the liquidity premium from this shadow rate extension of our model, denoted the B-$G^L\,(5)$ model, is almost identical to our benchmark estimate from the $G^L\,(5)$ model. Thus, the presence of the ZLB does not affect the extracted liquidity premium in the $G^L\,(5)$ model.

6.6 The Frictionless Real Yield Curve

All the models considered so far explain TIPS prices using a real level factor L_t^R and a liquidity factor X^{liq}, in addition to a common slope and curvature factor for Treasuries and

TIPS. Given that the liquidity factor X^{liq} mostly operates as a real slope factor according to Figure 3, it seems obvious to explore whether X^{liq} mainly captures insufficient variability in the real slope factor $\alpha^R S_t$ due to its tight link to the nominal slope factor S_t in the $G^L(5)$ model. We therefore briefly replace equation (10) with

$$r_t^{R,i} = L_t^R + S_t^R + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0)})X_t^{liq},$$

where S_t^R is a separate real slope factor. Letting $\tilde{Z}_t \equiv \begin{bmatrix} L_t & S_t & C_t & L_t^R & S_t^R & X_t^{liq} \end{bmatrix}'$, the considered $\mathbb{Q}$ dynamics has the form

$$d\tilde{Z}_t = \begin{bmatrix} \mathcal{K}_x^{\mathbb{Q}} & 0_{4\times 1} & 0_{4\times 1} \\ 0_{1\times 4} & \lambda^R & 0 \\ 0_{1\times 4} & 0 & \kappa_{liq}^{\mathbb{Q}} \end{bmatrix} \left(\begin{bmatrix} 0_{4\times 1} \\ 0 \\ \theta_{liq}^{Q} \end{bmatrix} - \tilde{Z}_t \right) + \Sigma_{\tilde{z}} dW_t^{\mathbb{Q}},$$

where $\lambda^R \geq 0$. Nominal yields remain given by equation (12), whereas the expression for the real yield linked to the ith TIPS now reads

$$\begin{aligned} y_t^{R,i}(t_0, t, \tau) &= L_t^R + \left(\frac{1 - e^{-\lambda^R \tau}}{\lambda^R \tau} \right) S_t^R \\ &+ \beta^i \left(\frac{1 - e^{-\kappa_{liq}^{\mathbb{Q}} \tau}}{\kappa_{liq}^{\mathbb{Q}} \tau} - e^{-\lambda^{L,i}(t-t_0)} \frac{1 - e^{-\left(\kappa_{liq}^{\mathbb{Q}} + \lambda^{L,i}\right)\tau}}{\left(\kappa_{liq}^{\mathbb{Q}} + \lambda^{L,i}\right)\tau} \right) X_t^{liq} - \frac{\tilde{A}^{R,i}(t_0, t, \tau)}{\tau}, \end{aligned}$$

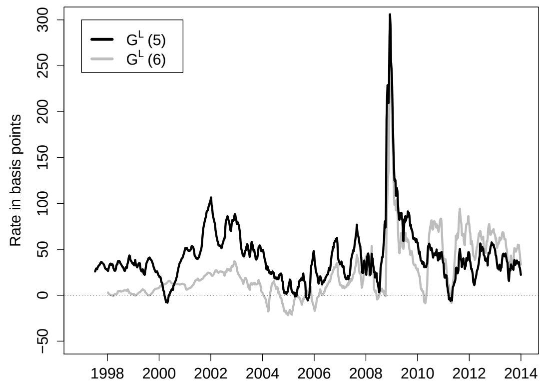
where $\tilde{A}^{R,i}(t_0, t, \tau)$ is a convexity term.[21] An affine specification for the market price of risk implies that the $\mathbb{P}$ dynamics are given by

$$d\tilde{Z}_t = \mathcal{K}_{\tilde{z}}^{\mathbb{P}} \left(\theta_{\tilde{z}}^{\mathbb{P}} - \tilde{Z}_t \right) dt + \Sigma_{\tilde{z}} dW_t^{\mathbb{P}},$$

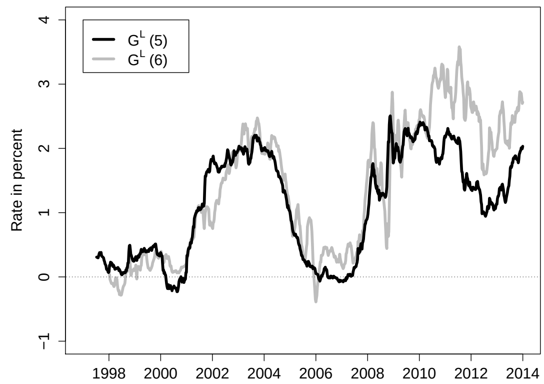
where $\mathcal{K}_{\tilde{z}}^{\mathbb{P}}$ and $\theta_{\tilde{z}}^{\mathbb{P}}$ are free parameters. For comparability with our benchmark model, we let both $\mathcal{K}_{\tilde{z}}^{\mathbb{P}}$ and $\Sigma_{\tilde{z}}$ be diagonal matrices.

Panel (a) in Figure 13 shows that this six-factor $G^L(6)$ model gives a somewhat lower average TIPS liquidity premium before 2005 compared with our benchmark $G^L(5)$ model, but that the liquidity premium in the two models largely coincides after 2005 when more TIPS are available. From panel (b) in Figure 13 we also note that the real slope factor S_t^R in the $G^L(6)$ model is somewhat more noisy than the real slope factor $\alpha^R S_t$ in the $G^L(5)$ model, but that the two factors otherwise are very similar with a correlation of 85.2%. These findings suggest that the link between the nominal and real slope factor imposed in the $G^L(5)$ model is supported by the data, and that this restriction does not materially alter the extracted TIPS liquidity premium.

[21] Note that this version of our model has three factors for Treasury yields, two frictionless factors for TIPS yields, and one liquidity factor, making the number of factors similar to the specification adopted in Abrahams et al. (2016).



(a) Panel: The Average Liquidity Premium



(b) Panel: The Real Slope Factor

Figure 13: **A Separate Real Slope Factor**
This figure reports the average estimated TIPS liquidity premium in the $G^L(5)$ and $G^L(6)$ models, where both models are estimated with a diagonal specification of $\mathcal{K}_m^{\mathbb{P}}$ and Σ_m for $m = \{z, \tilde{z}\}$ and without the deflation option.

7 Economic Applications

This section provides two economic applications of the proposed model. First, we compute a liquidity-adjusted real yield curve and analyze some of its properties in Section 7.1. Second, we explore the ability of the proposed model to forecast inflation in Section 7.2 and study the credibility of the current monetary policy regime based on the model-implied inflation expectations.

7.1 A Liquidity-Adjusted U.S. Real Yield Curve

One of the main motivations for introducing the TIPS market in the late 1990s was to provide assets that allow investors to eliminate inflation risk and hence lock in a real rate of return (Sack and Elsasser (2004)). These assets also allow for the construction of a real yield curve in the U.S., similar to what has been available in the U.K. since the mid-1980s (Evans (1998)). However, the TIPS market contains a non-negligible liquidity risk premium as shown above, and the implied real yield curve from TIPS therefore overstates the level of real rates in the U.S. Much caution is therefore needed when using these real yields, for instance to evaluate if consumption-based asset pricing models should generate an upward or downward sloping real yield curve (see Piazzesi and Schneider (2007), Beeler and Campbell (2012), and Swanson (2016) among others).

The ATSM we introduce allows us to address this problem by stripping out the liquidity premium from real yields. Because it is based on the $G^L(5)$ model, we can easily obtain a liquidity-adjusted real yield curve from equation (13) with $X_{t|t}^{liq} = 0$ for all t and a corre-

	$G^L(5)$		GSW	
	Mean	Std	Mean	Std
1-week	0.305	1,907	-	-
4-week	0.304	1,906	-	-
12-week	0.306	1,902	-	-
1-year	0.364	1,875	-	-
2-year	0.517	1,817	0.869	1.749
3-year	0.707	1,742	1.121	1.689
4-year	0.902	1,664	1.321	1.642
5-year	1.086	1,588	1,485	1,587
6-year	1.252	1,519	1,622	1,525
7-year	1.397	1,459	1,739	1,461
8-year	1.523	1,407	1,839	1,399
9-year	1.629	1,363	1,924	1,341
10-year	1.717	1,326	1,998	1,287

Table 6: **Moments for the U.S. Real Yield Curve**
This table reports mean and standard deviations from liquidity-adjusted real yields based on the benchmark $G^L(5)$ model and real yields from Gürkaynak et al. (2010), denoted GSW. All moments are computed from January 8, 1999, to December 27, 2013, as the yields from Gürkaynak et al. (2010) are unavailable prior to this date.

sponding modification of the convexity-adjustment to reflect this constraint.[22] Table 6 reports means and standard deviations for some of the commonly considered real yields in the ten-year maturity spectrum. We compare these moments with those from the "unadjusted" real curves of Gürkaynak et al. (2010), which are only reliable for maturities greater than two years and only available starting in January 1999. All standard deviations in Table 6 are not materially affected by the liquidity adjustment, and we therefore focus on the sample means. The two-year real yield from Gürkaynak et al. (2010) is seen to have an average of 0.87%, which is 35 basis points higher than the mean in the corresponding liquidity-adjusted yield. The size of this liquidity tends to fall with maturity and is 28 basis points for the ten-year yield. This implies that the average slope of the two- to ten-year segment of the real yield curve is 120 basis points with our liquidity adjustment, compared with 113 basis points in Gürkaynak et al. (2010). The full ten-year slope of the real yield curve (i.e., the ten-year yield minus the short rate) is 141 basis points with our liquidity adjustment, whereas the corresponding estimate is unavailable from Gürkaynak et al. (2010). We therefore conclude that the real yield curve in the U.S. is strongly upward sloping on average, at least from 1999 to the end of 2013.

[22]That is, the convexity adjustment equals the one from the $G(4)$ model but we evaluate it using the estimated parameters from the $G^L(5)$ model. Note that the adopted procedure to compute liquidity-adjusted real yields also removes any effects related to the deflation option in TIPS, although this effect is fairly small within our sample (see Section 6.1).

7.2 Model-Implied Inflation Expectations

Another reason for introducing the TIPS market was to allow market participants and policy-makers to obtain real-time estimates of inflation expectations (Sack and Elsasser (2004)). These forecasts would serve as a useful supplement to existing measures of inflation expectations from surveys, which have the limitation of being unavailable in real time and may differ from the expectations of the marginal investor. Most practitioners compute these market-based forecasts as the difference between nominal and real yields of the same maturity (i.e., breakeven inflation), but this is likely to give a distorted measure of inflation expectations for three reasons. First, nominal yields contain an inflation risk premium because investors require compensation for carrying the uncertainty attached to future inflation, making the real pay-off from Treasuries unknown. Second, the low liquidity in TIPS adds a liquidity premium to real yields as described above. Third, the implied yield from TIPS contains the value of the deflation option, which also increases the implied real yield from TIPS if the option is in-the-money (see equation (16)). The ATSM we propose allows us to correct for all three distortions and hence obtain a more reliable measure of inflation expectations than implied by breakeven inflation (BEI).

To evaluate the model-implied inflation process, we first explore in Table 7 how well our model forecasts headline CPI inflation one year ahead, i.e., $\pi_{t+1y} \equiv \Pi_{t+1y}/\Pi_t$, from 1997 to 2013. The model-implied forecasts are here benchmarked to the random walk, the median of forecasts in the Blue Chip Financial Forecasts survey, and the one-year inflation swap rate (only available after 2005). Table 7 shows that the $G^L(5)$ model improves substantially upon the performance of the corresponding $G(4)$ model without a liquidity factor, both when using the mean absolute forecasting errors (MAE) and the classic root mean-squared forecasting errors (RMSE). Both models are here estimated using the benchmark specification, where $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z are diagonal matrices, given that the $\mathbb{P}$ dynamics hardly affect the TIPS liquidity premium (see Section 6.4). The expected inflation rate, on the other hand, is more dependent on the $\mathbb{P}$ dynamics, and particularly $\mathcal{K}_z^{\mathbb{P}}$, and we therefore also report the forecasting performance of the $G^L(5)$ model when estimated with a fully flexible $\mathcal{K}_z^{\mathbb{P}}$ matrix. Table 7 shows that this version of the $G^L(5)$ model delivers even better forecasts of CPI inflation, as our model now outperforms the Blue Chip survey from 2005 to 2013, both in terms of MAE (113.28 vs. 111.17) and in terms of RMSE (150.93 vs. 143.01). These improvements are sufficiently large to ensure that our model also outperforms the Blue Chip survey for the entire sample, although the survey does slightly better than our model from 1997 to 2004. We also find that inflation expectations from both versions of the $G^L(5)$ model clearly outperform the random walk and the one-year inflation swap rate, suggesting that inflation expectations can be extracted reliably from Treasuries and TIPS when accounting for the relative liquidity differential between the two markets. Thus, correcting for liquidity risk in the $G^L(5)$ model gives a substantially better measure of CPI inflation compared with the corresponding $G(4)$

	1997-2004		2005-2013		1997-2013	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
Random walk	91.50	108.04	176.17	230.51	137.68	185.17
Blue Chip survey	73.43	86.44	113.28	150.93	95.17	125.79
One-year inflation swap rate	n.a.	n.a.	142.24	198.07	n.a.	n.a.
$G\,(4)$ model: $\mathbb{E}_t^{\mathbb{P}}\left[\pi_{t+1y}\right]$	118.06	134.67	127.15	164.91	123.02	151.91
$G^L\,(5)$ model: $\mathbb{E}_t^{\mathbb{P}}\left[\pi_{t+1y}\right]$	91.12	105.30	111.17	143.01	102.05	127.26
$G^L\,(5)$ model with full $\mathcal{K}_z^{\mathbb{P}}$: $\mathbb{E}_t^{\mathbb{P}}\left[\pi_{t+1y}\right]$	76.09	90.06	109.94	140.86	94.55	120.46

Table 7: **Comparison of CPI Inflation Forecasts**
The table reports the mean absolute errors (MAE) and the root mean-squared errors (RMSE) for forecasting headline CPI inflation one year ahead. The Blue Chip forecasts are mapped to the end of each month from July 1997 to December 2013, a total of 198 monthly forecasts. The comparable model forecasts are generated on the nearest available business day prior to the end of each month. The subsequent CPI realizations are year-over-year changes starting at the end of the month before the survey month, implying that the random walk forecasts equal the past year-over-year change in the CPI series as of the beginning of the survey month. The one-year inflation swap rates are not available (n.a.) prior to 2005. The model-implied forecasts account for the deflation option in TIPS and impose a diagonal specification of $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z, unless stated otherwise.

model without a liquidity adjustment.

To understand what drives these results, consider Figure 14, which shows one-year expected inflation from the $G\,(4)$ and $G^L\,(5)$ models and the realized CPI inflation. This figure shows that expected inflation from the $G\,(4)$ model is generally well below realized inflation. The fitted value of breakeven inflation from the $G\,(4)$ model omits the inflation risk premium but is nevertheless close to its expected inflation rate, showing that the inflation risk premium is fairly small at the one-year horizon. On the other hand, accounting for liquidity risk in TIPS through the $G^L\,(5)$ model generates one-year inflation forecasts that are notably closer to the Blue Chip survey and to the actual inflation outcomes. Hence, the satisfying forecasting performance of the $G^L\,(5)$ model is explained by the liquidity adjustment and not by the well-known correction for inflation risk.

These results are obviously very encouraging as they show how the proposed liquidity correction substantially improves the model's ability to predict future inflation. We acknowledge that these results cover the same period as used for the model estimation, but we emphasize that no data on inflation—neither realized CPI inflation nor surveys—have been included in the estimation. In this sense, the results presented in Table 7 and Figure 14 are not driven by in-sample overfitting of inflation or a related survey series.

The proposed model may also be used to extract long-term inflation expectations from financial markets and hence evaluate the credibility of the current monetary policy regime. We illustrate this property of our model in Figure 15, where we decompose nominal and real yields at the ten-year maturity using the following procedure. We first use real yields from Gürkaynak et al. (2010) to compute the observed ten-year breakeven inflation rate, as

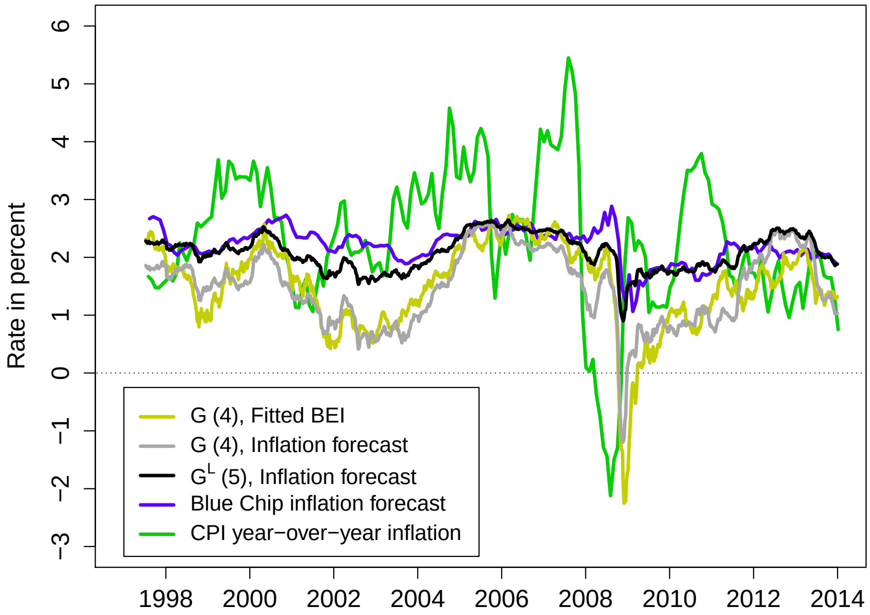


Figure 14: **Inflation and One-Year Inflation Expectations**
This figure reports realized one-year headline CPI inflation and various estimates of one-year inflation expectations. Inflation expectations and breakeven inflation from the $G(4)$ model are obtained for the benchmark specification where $\mathcal{K}_z^{\mathbb{P}}$ and Σ_z are diagonal. The inflation expectations from the $G^L(5)$ model are obtained for a fully flexible $\mathcal{K}_z^{\mathbb{P}}$ matrix. Both models account for the deflation option in TIPS. These data cover the period from July 11, 1997, to December 27, 2013.

shown with a solid black line.[23] From the $G^L(5)$ model with a fully flexible $\mathcal{K}_z^{\mathbb{P}}$ matrix, we obtain a frictionless estimate of breakeven inflation from the difference between the fitted ten-year nominal and real frictionless yields, which is shown with the gray line in Figure 15. The spread between the frictionless and observed BEI represents an estimate of the TIPS liquidity premium at the ten-year maturity and is marked with the light green shading in Figure 15. Finally, the estimated model dynamics under the $\mathbb{P}$ measure allow us to decompose the frictionless ten-year BEI into inflation expectations and an inflation risk premium, shown with red and green solid lines, respectively. We emphasize two results from this decomposition. First, long-term inflation expectations implied by Treasuries and TIPS are remarkably stable around the 2 percent inflation target of the Federal Reserve and close to the long-term inflation expectations from the Blue Chip survey.[24] This suggests that long-term inflation expectations have been well anchored in the U.S. and that bond markets view the current monetary policy regime as fully credible when pricing securities.

[23]Note that this series is only available from January 1999 on, and this explains why Figure 15 omits the first one and a half years of our sample.

[24]We thank Richard Crump for sharing his electronic version of the Blue Chip survey.

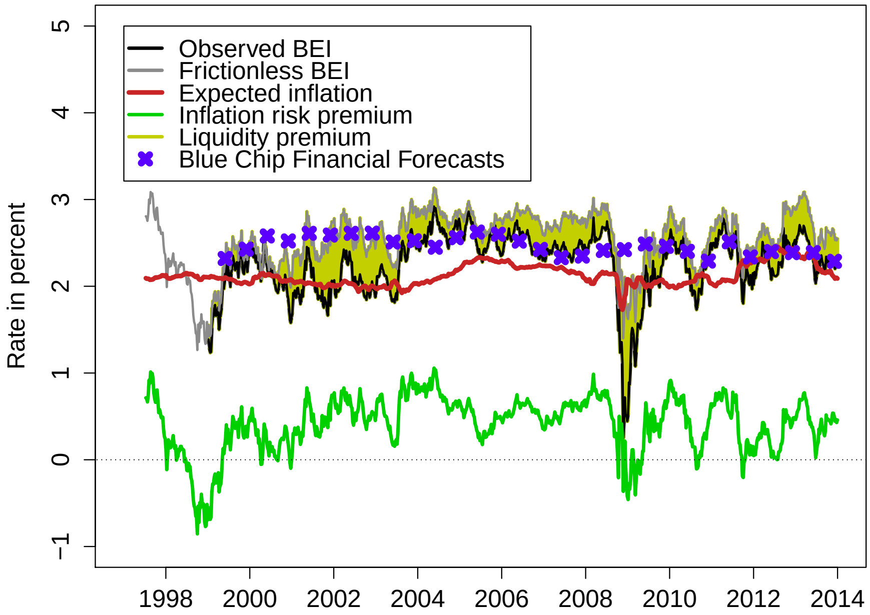


Figure 15: **Ten-Year BEI Decomposition**
This figure decomposes the ten-year BEI based on the $G^L(5)$ model with a fully flexible $\mathcal{K}_z^{\mathbb{P}}$ matrix. Observed breakeven inflation (BEI) is obtained as the difference between nominal and real ten-year yields provided by Gürkaynak et al. (2007) and Gürkaynak et al. (2010), respectively. The corresponding frictionless BEI is obtained from the $G^L(5)$ model with $X_{t|t}^{liq} = 0$ for all t. The difference between frictionless and observed BEI represents the TIPS liquidity premium and is marked with the light green shading. The plotted CPI inflation expectations over the next 10 years are the semiannual consensus expectations from Blue Chip Financial Forecasts. The data cover the period from July 11, 1997, to December 27, 2013.

Second, the stable long-term inflation expectations imply that our model assigns nearly all variation in the frictionless ten-year BEI to the inflation risk premium. The work of D'Amico et al. (2014) and Abrahams et al. (2016) draw the same conclusion, although both papers supplement the information from Treasuries and TIPS with other data sources and identify liquidity risk slightly differently compared with our model (see Section 3.3). This similarity may be interpreted as supporting evidence for the proposed identification scheme in the present paper, and it may also serve to illustrate that additional information beyond what is implied by Treasuries and TIPS is not needed to identify the liquidity premium in TIPS.

8 Conclusion

This paper proposes a new arbitrage-free term structure model for nominal and real yields to estimate the liquidity disadvantage of TIPS. The proposed model relies on a new and very direct way to identify liquidity risk, which we obtain from the implied price difference of identical principal and coupon payments related to TIPS. As a result, only a regular panel of TIPS prices and nominal yields are needed to identify the TIPS liquidity premium. Estimation results document a large improvement in the ability of our ATSM to fit individual TIPS prices when accounting for liquidity risk, as the root mean-squared errors for duration-scaled TIPS prices fall from 14.6 basis points to just 4.9 basis points when correcting for liquidity. Our results also reveal that the average liquidity premium is sizable and highly correlated with well-known observable proxies for liquidity risk such as the VIX options-implied volatility index, the on-the-run spread on Treasuries, and the TIPS fitted errors from Gürkaynak et al. (2010). We also document a substantial improvement in the ability of the proposed model to forecast inflation and match inflation expectations when correcting for liquidity risk.

The proposed identification scheme of the TIPS liquidity premium is obviously applicable to other fixed-income markets. In particular, the suggested approach may be applied to sovereign bond markets in Europe, where market liquidity is considered an issue. Another obvious application is to study liquidity risk within the U.S. Treasury market based on the proposed identification scheme. We leave these and other applications for future research.

Appendix

A The Real Discount Function in the $G^L(5)$ Model

We conjecture that the solution is given by

$$\begin{aligned} P^{R,i}(t_0,t,T) &= \exp\{B_1^i(t_0,t,T)L_t + B_2^i(t_0,t,T)S_t + B_3^i(t_0,t,T)C_t + B_4^i(t_0,t,T)L_t^R \\ &\quad + B_5^i(t_0,t,T)X_t^{Liq} + A^i(t_0,t,T)\}, \end{aligned}$$

with the general boundary conditions $P^{R,i}(t_0,T,T) = e^{(\overline{B}^i)'X_T + \overline{A}^i}$ as represented by $\overline{B}^i$ and $\overline{A}^i$. Clearly, $P^{R,i}(t_0,T,T) = 1$ implies that $\overline{B}^i = 0$ and $\overline{A}^i = 0$. In the absence of stochastic volatility, the expression for $B^i(t_0,t,T)$ in equation (8) is given

$$\frac{dB^i(t_0,t,T)}{dt} = \rho^i(t_0,t) + (\mathcal{K}_z^{\mathbb{Q}})'B^i(t_0,t,T), \tag{18}$$

where we let $\rho^i(t_0^i,t) \equiv L_t^R + \alpha^R S_t + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0)})X_t^{liq}$ to simplify the notation. From equation (18), it follows that

$$\frac{d}{dt}\left[e^{(\mathcal{K}_z^{\mathbb{Q}})'(T-t)}B^i(t_0,t,T)\right] = e^{(\mathcal{K}_z^{\mathbb{Q}})'(T-t)}\rho^i(t_0,t).$$

Integrating both sides of this equation from t to T we get

$$B^i(t_0,t,T) = e^{(\mathcal{K}_z^{\mathbb{Q}})'(T-t)}\overline{B}^i - e^{(\mathcal{K}_z^{\mathbb{Q}})'(T-t)}\int_t^T e^{(\mathcal{K}_z^{\mathbb{Q}})'(T-s)}\rho^i(t_0,s)ds.$$

Exploiting the structure of $(\mathcal{K}_z^{\mathbb{Q}})'$ and $\rho^i(t_0,t)$, simple algebra then implies that

$$B^i(t_0,t,T) = \begin{bmatrix} \overline{B}_1^i \\ e^{-\lambda(T-t)}\overline{B}_2^i \\ \lambda(T-t)e^{-\lambda(T-t)}\overline{B}_2^i + e^{-\lambda(T-t)}\overline{B}_3^i \\ \overline{B}_4^i \\ e^{-\kappa_{Liq}^Q(T-t)}\overline{B}_5^i \end{bmatrix}$$

$$- \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & e^{-\lambda(T-t)} & 0 & 0 & 0 \\ 0 & \lambda(T-t)e^{-\lambda(T-t)} & e^{-\lambda(T-t)} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & e^{-\kappa_{Liq}^Q(T-t)} \end{bmatrix} \begin{bmatrix} \int_t^T 0ds \\ \int_t^T \alpha^R e^{\lambda(T-s)}ds \\ \int_t^T -\alpha^R\lambda(T-s)e^{\lambda(T-s)}ds \\ \int_t^T 1ds \\ \int_t^T e^{\kappa_{Liq}^Q(T-s)}\beta^i(1-e^{-\lambda^{L,i}(s-t_0^i)})ds \end{bmatrix}.$$

We clearly have $\int_t^T 0ds = 0$ and $\int_t^T \alpha^R e^{\lambda(T-s)}ds = -\alpha^R\frac{1-e^{\lambda(T-t)}}{\lambda}$. It is also easy to see that $\int_t^T -\alpha^R\lambda(T-s)e^{\lambda(T-s)}ds = -\alpha^R(T-t)e^{\lambda(T-t)} - \alpha^R\frac{1-e^{\lambda(T-t)}}{\lambda}$ and $\int_t^T 1ds = T-t$.

Finally,

$$\begin{aligned} &\beta^i \int_t^T e^{\kappa_{Liq}^Q(T-s)}(1-e^{-\lambda^{L,i}(s-t_0)})ds = \beta^i \int_t^T e^{\kappa_{Liq}^Q(T-s)}ds - \beta^i e^{\kappa_{Liq}^Q T}e^{\lambda^{L,i}t_0}\int_t^T e^{-(\kappa_{Liq}^Q+\lambda^{L,i})s}ds \\ &= \beta^i\left[-\frac{1}{\kappa_{Liq}^Q}e^{\kappa_{Liq}^Q(T-s)}\right]_t^T - \beta^i e^{\kappa_{Liq}^Q T}e^{\lambda^{L,i}t_0}\left[-\frac{1}{\kappa_{Liq}^Q+\lambda^{L,i}}e^{-(\kappa_{Liq}^Q+\lambda^{L,i})s}\right]_t^T \\ &= -\beta^i\frac{1-e^{\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} + \beta^i e^{-\lambda^{L,i}(T-t_0)}\frac{1-e^{(\kappa_{Liq}^Q+\lambda^{L,i})(T-t)}}{\kappa_{Liq}^Q+\lambda^{L,i}}. \end{aligned}$$

Simple algebra then implies that the solution is given by

$$B^i(t_0,t,T) = \begin{bmatrix} \overline{B}_1^i \\ e^{-\lambda(T-t)}\overline{B}_2^i - \alpha^R \frac{1-e^{-\lambda(T-t)}}{\lambda} \\ \lambda(T-t)e^{-\lambda(T-t)}\overline{B}_2^i + \overline{B}_3^i e^{-\lambda(T-t)} + \alpha^R \left[(T-t)e^{-\lambda(T-t)} - \frac{1-e^{-\lambda(T-t)}}{\lambda}\right] \\ \overline{B}_4^i - (T-t) \\ e^{-\kappa_{Liq}^Q(T-t)}\overline{B}_5^i - \beta^i \frac{1-e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} + \beta^i e^{-\lambda^{L,i}(t-t_0)} \frac{1-e^{-(\kappa_{Liq}^Q+\lambda^{L,i})(T-t)}}{\kappa_{Liq}^Q+\lambda^{L,i}} \end{bmatrix},$$

which completes the proof of $B^i(t_0,t,T)$, given $\overline{B}^i = 0$. Finally, in the absence of stochastic volatility, the expression for $A^i(t_0,t,T)$ in equation (7) reduces to

$$A^i(t_0,t,T) = \overline{A}^i + \frac{1}{2}\int_t^T \sum_{j=1}^{5} \left[\Sigma_z' B(t_0,s,T)B(t_0,s,T)'\Sigma_z\right]_{j,j} ds,$$

which is easily computed analytically by symbolic integration, for instance in Matlab. In the case of a diagonal specification of Σ_z, the solution takes the following form:

$$\begin{aligned} A^i(t_0,t,T) \quad = \quad & \overline{A}^i - \beta^i\theta_{liq}^Q(T-t) + \theta_{liq}^Q\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i \frac{e^{-\lambda^{L,i}(T-t_0)}}{\kappa_{Liq}^Q + \lambda^{L,i}}\Big](1 - e^{-\kappa_{Liq}^Q(T-t)}) \\ & +\beta^i \frac{\kappa_{liq}^Q\theta_{liq}^Q}{\kappa_{Liq}^Q + \lambda^{L,i}} \frac{e^{-\lambda^{L,i}(t-t_0)} - e^{-\lambda^{L,i}(T-t_0)}}{\lambda^{L,i}} + \frac{\sigma_{11}^2}{2}(\overline{B}_1^i)^2(T-t) \\ & +\sigma_{22}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) - \alpha^R\frac{(\alpha^R + \lambda\overline{B}_2^i)}{\lambda^3}[1 - e^{-\lambda(T-t)}] + \frac{(\alpha^R + \lambda\overline{B}_2^i)^2}{4\lambda^3}[1 - e^{-2\lambda(T-t)}]\Big] \\ & +\sigma_{33}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) + \alpha^R\frac{\alpha^R + \lambda\overline{B}_2^i}{\lambda^2}(T-t)e^{-\lambda(T-t)} - \frac{(\alpha^R + \lambda\overline{B}_2^i)^2}{4\lambda}(T-t)^2 e^{-2\lambda(T-t)}\Big] \\ & -\sigma_{33}^2\frac{(\alpha^R + \lambda\overline{B}_2^i)(3\alpha^R + \lambda\overline{B}_2^i + 2\lambda\overline{B}_3^i)}{4\lambda^2}(T-t)e^{-2\lambda(T-t)} \\ & +\sigma_{33}^2\frac{(2\alpha^R + \lambda\overline{B}_2^i + \lambda\overline{B}_3^i)^2 + (\alpha^R + \lambda\overline{B}_3^i)^2}{8\lambda^3}[1 - e^{-2\lambda(T-t)}] - \sigma_{33}^2\alpha^R\frac{2\alpha^R + \lambda\overline{B}_2^i + \lambda\overline{B}_3^i}{\lambda^3}[1 - e^{-\lambda(T-t)}] \\ & +\frac{\sigma_{44}^2}{6}[(\overline{B}_4^i)^3 - (\overline{B}_4^i - (T-t))^3] \\ & +\frac{\sigma_{55}^2}{2}\Bigg[\frac{(\beta^i)^2}{(\kappa_{Liq}^Q)^2}(T-t) + \Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0)}}{\kappa_{Liq}^Q + \lambda^{L,i}}\Big]^2\frac{1 - e^{-2\kappa_{Liq}^Q(T-t)}}{2\kappa_{Liq}^Q} \\ & \quad +\frac{(\beta^i)^2}{(\kappa_{Liq}^Q + \lambda^{L,i})^2}\frac{e^{-2\lambda^{L,i}(t-t_0)} - e^{-2\lambda^{L,i}(T-t_0)}}{2\lambda^{L,i}} - 2\frac{\beta^i}{\kappa_{Liq}^Q}\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0)}}{\kappa_{Liq}^Q + \lambda^{L,i}}\Big]\frac{1 - e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} \\ & \quad -2(\beta^i)^2\frac{1}{\kappa_{Liq}^Q}\frac{1}{\kappa_{Liq}^Q + \lambda^{L,i}}\frac{e^{-\lambda^{L,i}(t-t_0)} - e^{-\lambda^{L,i}(T-t_0)}}{\lambda^{L,i}} \\ & \quad +2\beta^i\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0)}}{\kappa_{Liq}^Q + \lambda^{L,i}}\Big]e^{-\lambda^{L,i}(t-t_0^i)}\frac{e^{-\lambda^{L,i}(T-t)} - e^{-\kappa_{Liq}^Q(T-t)}}{(\kappa_{Liq}^Q)^2 - (\lambda^{L,i})^2}\Bigg]. \end{aligned}$$

B An EM Algorithm

It is unfortunately numerically demanding to evaluate the deflation option value DOV_t for all t. Hence, to avoid continuously evaluating DOV_t during the estimation process, we next outline an EM algorithm that is computationally much more efficient than a brute force optimization of the quasi log-likelihood function. To describe our EM algorithm, let the vector ψ contain all the model parameters and let the initial value of $DOV_t = 0$ for all t. We denote these values by $\left\{DOV_t^{(0)}\right\}_{t=1}^{T}$ and refer to the starting value of ψ as $\psi^{(0)}$. Given $i = 1$, the steps in our EM algorithm are:

Step 1: Condition on $\left\{DOV_t^{(i-1)}\right\}_{t=1}^{T}$ and optimize $\mathcal{L}^{EKF}$ across ψ to obtain $\hat{\psi}^{(i)}$.

Step 2: Use $\hat{\psi}^{(i)}$ and the filtered states $\left\{Z_{t|t}\left(\hat{\psi}^{(i)}\right)\right\}_{t=1}^{T}$ to compute $\left\{DOV_t^{(i)}\right\}_{t=1}^{T}$.

Step 3: If $\left|\mathcal{L}^{EKF}\left(\hat{\psi}^{(i)}; \left\{DOV_t^{(i)}\right\}_{t=1}^{T}\right) - \mathcal{L}^{EKF}\left(\hat{\psi}^{(i-1)}; \left\{DOV_t^{(i-1)}\right\}_{t=1}^{T}\right)\right| < \epsilon$ then stop, otherwise $i = i + 1$ and go to Step 1

The value of ϵ is here some small number, e.g. 0.01. Although one evaluation of $\mathcal{L}^{EKF}$ in Step 1 can be obtained in a few seconds on a standard desktop, the optimization in Step 1 is nevertheless numerically demanding given the large number of bond specific parameters, i.e. $\psi^{BS} \equiv \left(\lambda^{L,1}, \left\{\beta^i, \lambda^{L,i}\right\}_{i=2}^{n_{TIPS}}\right)$, in ψ. Fortunately, the bond specific parameters display weak mutual dependence and have only a small impact on the parameters describing the frictionless part of the $G^L(5)$ model, which we denote by ψ^{FL}. That is, $\psi \equiv \left(\psi^{FL}, \psi^{BS}\right)$. We exploit this property of our model to suggest a sub EM algorithm to obtain near excellent starting values with respect to ψ for the full optimization in Step 1. To describe this sub EM algorithm, let $\psi^{FL,(0)}$ and $\psi^{BS,(0)}$ denote the initial values for the frictionless part and the bond specific part of the model, respectively. For instance, the values of $\psi^{FL,(0)}$ may be obtained from Christensen et al. (2010). Given $j = 1$, the steps in this sub EM algorithm are:

Step A: Condition on $\psi^{FL,(j-1)}$, then optimize $\mathcal{L}^{EKF}$ for one pair of bond specific parameters related to the kth bond (i.e. β^k and $\lambda^{L,k}$) at the time, while conditioning on all the remaining bond specific parameters. In total, we optimize on a small scale n_{TIPS} times to get preliminary estimates of all bond specific parameters, denoted $\psi^{BS,(j)}$.

Step B: Condition on $\psi^{BS,(j)}$, and optimize $\mathcal{L}^{EKF}$ across ψ^{FL} to get $\psi^{FL,(j)}$.

Step C: If $\left|\mathcal{L}^{EKF}\left(\hat{\psi}^{FL,(j)}, \hat{\psi}^{BS,(j)}\right) - \mathcal{L}^{EKF}\left(\hat{\psi}^{FL,(j-1)}, \hat{\psi}^{BS,(j-1)}\right)\right| < \epsilon$ then stop, otherwise $j = j + 1$ and go to Step A.

References

Abrahams, Michael, Tobias Adrian, Richard K. Crump, Emanuel Moench, and Rui Yu, 2016, "Decomposing Real and Nominal Yield Curves," *Journal of Monetary Economics*, Vol. 84, 182-200.

Adrian, Tobias and Hao Wu, 2010, "The Term Structure of Inflation Expectations," Federal Reserve Bank of New York Staff Reports, No. 362.

Amihud, Yakov and Haim Mendelson, 1986, "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics*, Vol. 17, No. 2, 223-249.

Amihud, Yakov and Haim Mendelson, 1991, "Liquidity, Maturity, and the Yields on U.S. Treasury Securities," *Journal of Finance*, Vol. 46, No. 4, 1411-1425.

Black, Fisher, 1995, "Interest Rates as Options," *Journal of Finance*, Vol. 50, No. 7, 1371-1376.

Beeler, Jason and John Y. Campbell, 2012, "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review,* Vol 1, No. 1, 141-182.

Bernanke, Ben S, 2002, "Deflation: Making Sure "It" Doesn't Happen Here,"speech at the National Economists Club, Washington, D.C., November 21.

Campbell, John Y., Robert J. Shiller, and Luis M. Viceira, 2009, "Understanding Inflation-Indexed Bond Markets," *Brookings Papers on Economic Activity*, Spring, 79-120.

Cheridito, Patrick, Damir Filipović, and Robert L. Kimmel, 2007, "Market Price of Risk Specifications for Affine Models: Theory and Evidence," *Journal of Financial Economics*, Vol. 83, No. 1, 123-170.

Chernov, Mikhail and Philippe Mueller, 2012, "The Term Structure of Inflation Expectations," *Journal of Financial Economics*, Vol. 106, No. 2, 367-394.

Christensen, Jens H. E., Francis X. Diebold, and Glenn D. Rudebusch, 2011, "The Affine Arbitrage-Free Class of Nelson-Siegel Term Structure Models," *Journal of Econometrics*, Vol. 164, No. 1, 4-20.

Christensen, Jens H. E. and James M. Gillan, 2012, "Could the U.S. Treasury Benefit from Issuing More TIPS?," Working Paper 2011-16, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E. and James M. Gillan, 2017, "Does Quantitative Easing Affect Market Liquidity?," Working Paper 2013-26, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2010, "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," *Journal of Money, Credit and Banking*, Supplement to Vol. 42, No. 6, 143-178.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2012, "Extracting Deflation Probability Forecasts from Treasury Yields," *International Journal of Central Banking*, Vol. 8, No. 4, 21-60.

Christensen, Jens H. E., Jose A. Lopez, and Patrick Shultz, 2017, "Is There an On-the-Run Premium in TIPS?," Working Paper 2017-10, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2015, "Estimating Shadow-Rate Term Structure Models with Near-Zero Yields," *Journal of Financial Econometrics*, Vol. 13, No. 2, 226-259.

Dai, Qiang and Kenneth J. Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, Vol. 55, No. 5, 1943-1978.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2014, "Tips from TIPS: the Informational Content of Treasury Inflation-Protected Security Prices," Finance and Economics Discussion Series Working Paper 2014-24, Board of Governors of the Federal Reserve System.

Duan, Jin-Chuan and Jean-Guy Simonato, 1999, "Estimating and Testing Exponential-Affine Term Structure Models by Kalman Filter," *Review of Quantitative Finance and Accounting*, Vol. 13, 111-135.

Dudley, William C., Jennifer Roush, and Michelle Steinberg Ezer, 2009, "The Case for TIPS: An Examination of the Costs and Benefits," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 15, No. 1, 1-17.

Duffee, Gregory R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, Vol. 57, No. 1, 405-443.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, 2007, "Valuation in Over-the-Counter Markets," *Review of Financial Studies*, Vol 20, No. 5, 1865-1900.

Ezer, Michelle. S., Michael Fleming, Simon Potter, Tony Rodrigues, Jennifer Roush, Wilbert Van Der Klaauw, 2008, "Assessing Inflation Expectations and the Risk of Deflation," Authorized for public release by the FOMC Secretariat on 03/07/2014.

Evans, Martin D. D., 1998, "Real Rates, Expected Inflation, and Inflation Risk Premia," *Journal of Finance*, Vol. 53, No. 1, 187-218.

Fleckenstein, Mathias, Francis A. Longstaff, and Hanno Lustig, 2014, "The TIPS-Treasury Bond Puzzle," *Journal of Finance*, Vol. 69, No. 5, 2151-2197.

Fleckenstein, Mathias, Francis A. Longstaff, and Hanno Lustig, 2017, "Deflation Risk," forthcoming *Journal of Financial and Quantitative Analysis.*

Fleming, Michael J. and Neel Krishnan, 2012, "The Microstructure of the TIPS Market," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 18, No. 1, 27-45.

Fleming, Michael J. and John R. Sporn, 2013, "Trading Activity and Price Transparency in the Inflation Swap Market," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 19, No. 1, 45-57.

Fontaine, Jean-Sébastien and René Garcia, 2012, "Bond Liquidity Premia," *Review of Financial Studies*, Vol. 25, No. 4, 1207-1254.

Garbade, Kenneth and William L. Silber, 1976, "Price Dispersion in the Government Securities Market," *Journal of Political Economy*, Vol. 84, No. 4, 721-740.

Goldreich, David, Bernd Hanke, and Purnendu Nath, 2005, "The Price of Future Liquidity: Time-Varying Liquidity in the U.S. Treasury Market," *Review of Finance*, Vol. 9, 1-32.

Grishchenko, Olesya V. and Jing-Zhi Huang, 2013, "Inflation Risk Premium: Evidence from the TIPS Market," *Journal of Fixed Income*, Vol. 22, No. 4, 5-30.

Grishchenko, Olesya V., Joel Vanden, and Jianing Zhang, 2016, "The Informational Content of the Embedded Deflation Option in TIPS," *Journal of Banking and Finance*, Vol. 65, 1-26.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, "The U.S. Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics*, Vol. 54, No. 8, 2291-2304.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2010, "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, Vol. 2, No. 1, 70-92.

Harvey, A.C., 1989, *Forecasting, structural time series models and the Kalman filter,* Cambridge: Cambridge University Press.

Ho, T.S.Y. and S.-B. Lee, 1986, "Term Structure Movements and Pricing Interest Rate Contingent Claims," *Journal of Finance*, Vol. 41, No. 5, 1011-1029.

Hu, Grace Xing, Jun Pan, and Jiang Wang, 2013, "Noise as Information for Illiquidity," *Journal of Finance*, Vol. 68, No. 6, 2341-2382.

Hull, John and A. White, 1990, "Pricing Interest-Rate-Derivative Securities," *Review of Financial Studies*, Vol. 3, No. 4, 573-592.

Joslin, Scott, Kenneth Singleton, and Haoxiang Zhu, 2011, "A New Perspective on Gaussian Dynamic Term Structure Models," *Review of Financial Studies*, Vol. 24, No. 3, 926-970.

Joyce, Michael, Peter Lildholdt, and Steffen Sørensen, 2010, "Extracting Inflation Expectations and Inflation Risk Premia from the Term Structure: A Joint Model of the U.K. Nominal and Real Yield Curves," *Journal of Banking and Finance*, Vol. 34, 281-294.

Keane, Frank, 1996, "Repo Rate Patterns for New Treasury Notes," *Current Issues in Economics and Finance*, Vol. 2, No. 10, 1-6.

Kim, Don H. and Kenneth J. Singleton, 2012, "Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields," *Journal of Econometrics*, Vol. 170, No. 1, 32-49.

Krishnamurthy, Arvind, 2002, "The Bond/Old-Bond Spreads," *Journal of Financial Economics*, Vol. 66, 463-506.

Nelson, Charles R. and Andrew F. Siegel, 1987, "Parsimonious Modeling of Yield Curves," *Journal of Business*, Vol. 60, No. 4, 473-489.

Norgaard, C.R., N.K. Poulsen, and O. Ravn, 2000, "Advances in Derivative-Free State Estimation for Nonlinear Systems," *Automatica*, Vol. 36, No. 11, 1627-1638.

Piazzesi, Monica and Martin Schneider, 2007, "Equilibrium Yield Curves," in NBER Macroeconomics, Annual 2006, 389-439.

Pflueger, Carolin E. and Luis M. Viceira, 2016, "Return Predictability in the Treasury Market: Real Rates, Inflation, and Liquidity," in Handbook of Fixed-Income Securities, edited by P. Veronesi, Wiley, Hoboken, N.J., pp. 191-209 (Chapter 10).

Sack, Brian and Robert Elsasser, 2004, "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 10, No. 1, 47-63.

Swanson, Eric, 2016, "A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt," Working Paper, University of California, Irvine.

Tinic, Seha M. and Richard R. West, 1972, "Competition and the Pricing of Dealer Services in the Over-The-Counter-Market," *Journal of Financial and Quantitative Analysis*, Vol. 8, 1707-1727.

Research Papers 2017

CREATES
Center for Research in Econometric Analysis of Time Series