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Abstract

This paper provides new evidence on bond risk premia by conditioning the classic Campbell-Shiller regressions on the business cycle. In expansions, we find mostly positive intercepts and negative regression slopes, but the results are completely reversed in recessions with *negative* intercepts and *positive* regression slopes. We reproduce these coefficients in a term structure model with business cycle dependent loadings in the market price of risk. This model also predicts excess returns in the right direction during expansions and recessions, whereas the Gaussian affine term structure model predicts excess returns for medium- and long-term bonds with the wrong sign during recessions.

Keywords: Bond return predictability, Business cycle variation in excess returns, Market price of risk, Zero-lower bound, Unspanned macroeconomic risk.

JEL: E43, E44, G12.

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1 Introduction

The expectations hypothesis of the term structure specifies that any long-term interest rate is given by the average of expected future short rates and that excess returns on any long-term bond therefore are unpredictable. Despite its appealing intuition, the expectations hypothesis is often rejected empirically, perhaps most forcefully in the seminal study of Campbell and Shiller (1991). This paper provides new evidence on bond risk premia by conditioning the classic Campbell-Shiller regressions on the business cycle through separate intercepts and slope coefficients in expansions and recessions. This extension is motivated by previous research showing that interest rates are more persistent in expansions than in recessions and that two-state models describe interest rate dynamics much better than single-state models, see Hamilton (1988), Gray (1996), Ang and Bekaert (2002), Bansal and Zhou (2002), among others.

Our modified Campbell-Shiller regressions reveal a unique business cycle dependent pattern in the relation between the slope of the yield curve (i.e., the yield spread) and subsequent yield changes. In expansions, we find mostly positive intercepts and negative regression slopes that decrease with maturity, similar to what typically is reported for ordinary Campbell-Shiller regressions, i.e. without conditioning on the business cycle (see, e.g., Bekaert, Hodrick and Marshall (1997) and Dai and Singleton (2002)). By contrast, in recessions, we obtain *negative* intercepts and *positive* slope coefficients that generally increase with maturity. This switch in the regression coefficients is significant using both asymptotic and bootstrapped Wald tests. Given that the slope coefficients are closer to the expectations hypothesis in recessions than in expansions, this novel result suggests that bond risk premia, as measured by excess returns, are more predictable by the yield spread during expansions than during recessions. We also show that the evidence of asymmetric return predictability extends to other prominent yield-based predictors such as the forward spread (Fama and Bliss (1987)) and the return forecasting factor introduced by Cochrane and Piazzesi (2005).¹

To provide an explanation of these new empirical Öndings, we propose a dynamic term structure model (DTSM) with business cycle dependent loadings in the market price of risk. Contrary to previous DTSMs with regime switching, we discipline our model to have observed regimes of either expansions or recessions but include time-varying physical transition probabilities between

 $1¹$ Hence, forecasting bond returns during recessions from macro variables as in Ludvigson and Ng (2009) seems more promising, although Duffee (2013) and Bauer and Hamilton (2015) question the ability of macro variables to predict bond returns beyond the information contained in the yield curve when accounting for small-sample distortions.

regimes, which may depend on the business cycle and the current yield curve. To ensure fast and reliable inference of our model, we extend the sequential regression (SR) approach of Andreasen and Christensen (2015) to estimate all parameters in our regime-dependent market price of risk in closed form by a modified OLS regression. This implies that our regime-switching model can be estimated in a few minutes with no additional computational costs compared to the Gaussian ATSM.

The main results from estimating our regime-switching model with three pricing factors on monthly U.S. data (1961:6 to 2013:12) are as follows. First, our model is able to match the empirical intercepts and slope coefficients in ordinary Campbell-Shiller regressions and in our modified Campbell-Shiller regressions conditioning on the business cycle. As a consequence, model-implied expected excess returns correlate positively with realized excess returns during both expansions and recessions. In contrast, the inability of the Gaussian ATSM to match the switch in the Campbell-Shiller regression slopes implies that this model predicts excess returns for medium- and long-term bonds with the wrong sign during recessions. Second, our model also replicates the observed asymmetry in return predictability, i.e. that the yield spread, the forward spread, and the Cochrane-Piazzesi factor have strong predictive power in expansions but not in recessions. Third, to account for these asymmetric patterns in the yield curve across the business cycle, our model generates a negative relation between the short rate and excess returns in expansions (as in the Gaussian ATSM), but a positive relation in recessions. This means that our model displays a tendency for expected excess returns to peak at the start of a recession and then mean-revert during the middle and last part of a recession when the Federal Reserve starts to lower its policy rate. Thus, our model suggests that the Federal Reserve is able to remove some of the risks attached to recessions, as accommodating monetary policy reduces the required risk compensation in the bond market. This effect of monetary policy is not present in the Gaussian ATSM, which generally predicts that expected excess returns increase throughout the entire recession. We finally show that our fully flexible regimeswitching model is robust to accounting for the zero lower bound and may be simplified to only have two parameters in the market price of risk that switch between expansions and recessions.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on bond risk premia from modified Campbell-Shiller and return regressions. We then introduce a new DTSM with regime-switching in Section 3 and explain how to estimate this model by the SR approach. Section 4 explores the ability of this model to replicate the empirical findings from Section 2, while Section 5 examines the robustness of our model and relates it to the existing literature. Section 6 concludes, while Appendix A contains some econometric details related to our analysis.²

2 New Empirical Evidence on Bond Risk Premia

This section presents new evidence on the business cycle properties of bond risk premia. We present our main empirical Önding in Sections 2.1 and 2.2 and explore its robustness in Section 2.3. The main implication of our new empirical results for DTSMs are discussed in Section 2.4.

2.1 Modified Campbell-Shiller Regressions

To motivate our new empirical Önding, consider the ordinary Campbell and Shiller (1991) regression

$$
y_{t+m,k-m} - y_{t,k} = \alpha_k + \beta_k \frac{m}{k-m} (y_{t,k} - y_{t,m}) + u_{t+m,k},
$$
\n(1)

where $y_{t,k}$ refers to the k-period bond yield in period t. As is common in the literature, we set one period equal to one month and implement (1) by running the regressions with $m = 3$ for $k = 6, 9, 12, ..., 120$, implying a total of 39 regressions. That is, $y_{t,m}$ in (1) corresponds to the three month interest rate. The upper part of Figure 1 shows the results of estimating (1) from 1961:6 to 2013:12, with intercepts on the left and slope estimates on the right. The full and dashed lines are based on Fama and Bliss (1987) and Gürkaynak, Sack and Wright (2007) bond yields, respectively.³ According to the expectations hypothesis $y_{t,k}^{EH} = \frac{1}{k}$ $\frac{1}{k} \sum_{i=0}^{k-1} \mathbb{E}_t [y_{t+i,1}] + c_k$, we should see $\beta_k = 1$ for all k. However, as in Campbell and Shiller (1991), the slope estimates are negative and decreasing with maturity, constituting a clear violation of the expectations hypothesis.

Now consider what happens when conditioning on the state of business cycle. That is, we run the Campbell-Shiller regressions interacted with business cycle dummies, i.e.

$$
y_{t+m,k-m} - y_{t,k} = \alpha_k^{EXP} 1_{\{z_t \ge c\}} + \beta_k^{EXP} \frac{m}{k-m} 1_{\{z_t \ge c\}} (y_{t,k} - y_{t,m})
$$

+ $\alpha_k^{REC} (1 - 1_{\{z_t \ge c\}}) + \beta_k^{REC} \frac{m}{k-m} (1 - 1_{\{z_t \ge c\}}) (y_{t,k} - y_{t,m}) + \tilde{u}_{t+m,k},$ (2)

²In addition, detailed data descriptions, robustness checks, and all model derivations are provided in an Online Appendix, which is available from the authors' homepages or upon request.

³See Appendix A.1 for further details on the data.

where $1_{\{z_t\geq c\}}$ is the indicator function with a value of one for expansions when $z_t \geq c$ and zero otherwise. We measure recessions by letting z_t refer to the Purchasing Managers' Index (PMI) and use the threshold value $c = 44.5$ from Berge and Jordà (2011) to identify recessions and expansions. The PMI is a widely watched indicator of business cycle activity and has the advantage of being available in real time without publication lags or subsequent data revisions. Furthermore, Christiansen, Eriksen and Møller (2014) provide empirical evidence that the PMI is the single best recession indicator among a large panel of economic variables. As shown below, our results are robust to using standard NBER recession dates, although they are subject to publication lags and therefore not our preferred recession indicator.

The lower part of Figure 1 summarizes the results of estimating (2). In expansions, the estimated slopes β_k^{EXP} and intercepts α_k^{EXP} are broadly similar to those obtained in the ordinary Campbell-Shiller regressions. In recessions, however, the slope coefficients β_k^{REC} are *positive* and mostly increasing with maturity. At long maturities they are even above one. In addition, the intercepts α_k^{REC} are negative at all maturities which under the expectations hypothesis is consistent with an upward sloping yield curve.⁴ Hence, in recessions the slope of the yield curve predicts future long-term bond yields in the direction implied by the expectations hypothesis, whereas the opposite holds during expansions.

Although recessions typically are short lived, allowing for the sign switch has a large effect on the goodness-of-Öt in the Campbell-Shiller regressions. This is illustrated in Figure 2, which shows that the R^2 statistic increases from around 3% in the ordinary Campbell-Shiller regressions to between 5% and 6% when conditioning on the business cycle.

Having outlined economically interesting differences in the Campbell-Shiller loadings across recessions and expansions, a natural next step is to test whether these differences are statistically significant. Table 1 shows Wald statistics that test for joint equality between expansion and recession coefficients across maturities. We report both asymptotic and bootstrap p -values. Appendices A.2 to A.4 describe the construction of the Wald test and the bootstrap procedure. To also explore robustness, we report results for the full sample and two subsamples: 1973:1-2013:12 (the panel of bond yields is balanced starting from 1973:1) and 1983:1-2013:12 (the post-Volcker period). To

⁴The derivation is provided in the Online Appendix. The pure version of the expectations hypothesis, i.e. $y_{t,k}^{pure\ EH} = \frac{1}{k} \sum_{i=0}^{k-1} \mathbb{E}_t [y_{t+i,1}],$ also restricts the intercept to zero, so that expected bond returns across all maturities are equal, i.e. $\alpha_k = 0$ for all k. However, the literature typically only focuses on whether bond risk premia are constant over time, i.e. whether $\beta_k = 1$ for all k.

identify recessions and expansions, we consider both the PMI and the NBER recession dates. The results of the Wald tests are clear-cut. The null hypothesis that expansion and recession slopes coefficients are equal, i.e. $\beta_k^{REC} = \beta_k^{EXP}$ for all k, is strongly rejected at any conventional significance level. Similarly, a Wald test also rejects the null that $\alpha_k^{REC} = \alpha_k^{EXP}$ for all k. Hence, the observed change in the intercepts and the slope coefficients across expansions and recessions are statistically significant. The Online Appendix also provides Wald tests of the expectations hypothesis, which is rejected in both regimes.⁵

2.2 Modified Return Regressions

Our modified Campbell-Shiller regressions in (2) show that the slope coefficients during recessions generally are closer to the expectations hypothesis than the corresponding loadings in expansions. As excess returns are unpredictable under the expectations hypothesis, we therefore find higher return predictability by the yield spread during expansions than in recessions.⁶ But does this pattern also hold for other classic yield-based predictors of bond returns such as the forward spread (Fama (1976); Fama and Bliss (1987)) and the Cochrane and Piazzesi (2005) (CP) return forecasting factor?

We address this question by running standard univariate return regressions for three-months ahead excess returns

$$
xhpr_{t+m,k} = \mu_k + \theta_k x_{t,k} + e_{t+m,k},\tag{3}
$$

and its modified version conditioning on the business cycle, i.e.

$$
xhpr_{t+m,k} = \mu_k^{EXP} 1_{\{z_t \ge c\}} + \theta_k^{EXP} 1_{\{z_t \ge c\}} x_{t,k} + \mu_k^{REC} (1 - 1_{\{z_t \ge c\}}) + \theta_k^{REC} (1 - 1_{\{z_t \ge c\}}) x_{t,k} + \tilde{e}_{t+m,k}.
$$
\n(4)

Here, $x h p r_{t+m,k} \equiv h p r_{t+m,k} - \frac{m}{12} y_{t,m}$ and the holding period return $h p r_{t+m,n} \equiv -\frac{k-m}{12} y_{t+m,k-m} +$ $\frac{k}{12}y_{t,k}$, whereas $x_{t,k}$ may either refer to the yield spread $y_{t,k} - y_{t,m}$, the forward spread, or the CP factor.⁷

 5 Althought the slope coefficients are positive and closer to one in recessions, their values are substantially larger than one at long maturities and this explains why the expectations hypothesis also is rejected in recessions.

⁶Here, we exploit the one-to-one relation between Campbell-Shiller regressions and regressing excess returns on the yield spread. The exact expression for this relation is provided in our Online Appendix.

⁷The forward spread is $f_t^{(k-m,k)} - \frac{m}{12} y_{t,m}$, where $f_t^{(k-m,k)} \equiv \frac{k}{12} y_{t,k} - \frac{k-m}{12} y_{t,k-m}$ is the forward rate between time $t + k - m$ and $t + k$. The CP factor is given by $c p_t = \hat{\gamma}' F_t$, where $F_t \equiv \left[1, y_{t,12}, f_t^{(12,24)}, f_t^{(24,36)}, f_t^{(36,48)}, f_t^{(48,60)}\right]'$

The upper panel of Figure 3 uses the yield spread in (3) and (4) and shows the expected pattern in bond return predictability across the business cycle. That is, θ_k^{EXP} is positive in expansions and generally increases with maturity, whereas θ_k^{REC} is negative in recessions and generally decreases with maturity. The middle and lower part of Figure 3 show that the forward spread and the CP factor, respectively, also imply different degrees of predictability for bond returns across the business cycle, with regression slopes mostly having the 'wrong' sign in recessions.

To summarize, the return regressions reveal a striking pattern in the slope coefficients across the business cycle. In expansions we find the typical pattern of positive slopes on the yield spread, the forward spread, and the CP factor, while in recessions the slopes are either close to zero or even negative.

2.3 Robustness

2.3.1 Extending the Sample and the Maturity Range

Since our empirical findings are new, we have done a wide range of robustness checks. First of all, we extend the sample back to 1926:1 in the top part of Table 2 to increase the number of observations, where the economy is in recession. The considered return series is for a 10-year government bond provided by Global Financial Data. Regimes are here identified from the NBER recessions dates, as the PMI indicator is unavailable over this longer sample. For the period 1926:1-2013:12, $y_{t,k} - y_{t,m}$ is statistically significant and explains 5.3% of the variation in excess bond returns. The R^2 statistic increases to 7.7% by allowing the regression coefficients to change across expansions and recessions. In expansions, the slope coefficient is positive and strongly statistically significant $(t\text{-statistic of})$ 5.0), whereas the slope coefficient in recessions is close to zero and insignificant. Results from different subsamples confirm a significant positive relation between the slope of the yield curve and subsequent excess bond returns in expansions, while in recessions the relation is insignificant.

Second, we next extend the maturity range to 20 years by using long-term government bond returns from Ibbotson and a related measure of the 20-year yield spread. The second part of Table 2 reveals the same pattern as above, with strong predictability in expansions but not in recessions. Hence, the systematic difference in bond predictability across the business cycle is not limited to the 10-year maturity range, although this is the main focus of our paper.

and $\hat{\gamma}$ is obtained by regressing one-year ahead excess bond returns on F_t , i.e. $\frac{1}{4} \sum_{i=2}^{5} rx_{t+12,i12} = \gamma' F_t + \overline{e}_{t+12}$.

2.3.2 Speed of Transition, Out-of-Sample Evidence, and Small-Sample Biases

We have also investigated whether the switch in (2) and (4) between recessions and expansions is best modeled using a smoothly changing function or the binary specification based on dummy variables used above. In smooth transition regressions with the logistic function, we find that the transition is practically instant. In addition, out-of-sample regressions show that all three yieldbased predictors (i.e., the yield spread, the forward spread, and the CP factor) only imply significant bond return predictability in expansions. Finally, we have checked that our results are robust to small-sample biases using a stationary bootstrap. The details of all these robustness checks are delegated to our Online Appendix.

2.4 Implication for DTSMs

There is a strong conclusion to draw from the above empirical evidence: Bond market asymmetries across the business cycle are substantial and economically important. In particular, the predictive power of the yield spread for future bond yields and bond returns crucially depends on the state of business cycle. To reproduce the negative regression loadings from the ordinary Campbell-Shiller regressions in (1), most DTSMs require a negative correlation between the short rate and excess returns according to Dai and Singleton (2002). Thus, to explain the sign switch in regression loadings from our modified Campbell-Shiller regressions in (2), a successful model should generate a negative relation between the short rate and excess returns in expansions, but a positive relation in recessions.

3 A DTSM with a Regime-Dependent Market Price of Risk

We next explore whether a DTSM can reproduce the documented asymmetries in the U.S. yield curve across the business cycle, while at the same time matching properties of bond risk premia unrelated to the business cycle as given by (1) and (3) . Dai and Singleton (2002) show that the Gaussian ATSM can reproduce loadings from ordinary Campbell-Shiller regressions, and this model therefore serves as a natural starting point. We proceed by motivating our extension of the Gaussian ATSM in Section 3.1, before formally presenting our proposed model in Section 3.2. Estimation of this model by the SR approach is briefly described in Section 3.3. We finally relate our proposed DTSM to the existing literature in Section 3.4.

3.1 Motivation

This section explores whether a further generalization of the market price of risk in the Gaussian ATSM has the potential to reproduce the documented asymmetries in the U.S. yield curve. To introduce our notation for this model, the short rate is given by

$$
r_t = \alpha + \beta' \mathbf{x}_t,\tag{5}
$$

where α is a scalar and β is an $n_x \times 1$ vector. The dynamics of the n_x pricing factors under the risk-neutral measure $\mathbb Q$ is specified as

$$
\mathbf{x}_{t+1} = (\mathbf{I} - \mathbf{\Phi}) \mathbf{x}_t + \Sigma_{\mathbf{x}} \varepsilon_{\mathbf{x},t+1}^{\mathbb{Q}} \tag{6}
$$

with $\epsilon_{\mathbf{x},t+1}^{\mathbb{Q}} \sim \mathcal{NID}(\mathbf{0},\mathbf{I}).^8$ The \mathbb{P} dynamics follow from an essential affine market price of risk as in Duffee (2002), i.e. $\lambda_t = \Sigma_{\mathbf{x}}^{-1} (\lambda_0 + \lambda_{\mathbf{x}} \mathbf{x}_t)$. In the absence of arbitrage, the price of a zero-coupon bond in period t with maturity k is then given by $P_{t,k} = \exp\{A_k + \mathbf{B}_k'\mathbf{x}_t\}$, where the recursive expressions for A_k and B_k are easily derived. Provided that the $\mathbb P$ distribution for $\mathbf x_t$ is stationary, the ordinary Campbell-Shiller coefficients in the Gaussian ATSM are then given by (with $m = 1$):⁹

$$
\beta_k = 1 + \frac{\left(\frac{1}{k}\mathbf{B}_k' + \boldsymbol{\beta}'\right) \mathbb{V}\left[\mathbf{x}_t\right] \boldsymbol{\lambda}_\mathbf{x}' \mathbf{B}_{k-1}}{\left(\frac{1}{k}\mathbf{B}_k' + \boldsymbol{\beta}'\right) \mathbb{V}\left[\mathbf{x}_t\right] \left(\frac{1}{k}\mathbf{B}_k + \boldsymbol{\beta}\right)}
$$
(7)

$$
\alpha_k = -\frac{1}{k-1} A_{k-1} + \frac{1}{k} A_k - \hat{\beta}_k \mathbb{E}\left[\frac{y_{t,k} - y_{t,1}}{k-1}\right] -\frac{1}{k-1} \left(\mathbf{B}'_{k-1} \left(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_\mathbf{x} \mathbb{E}\left[\mathbf{x}_t\right]\right) + \left(\frac{1}{k} \mathbf{B}'_k + \boldsymbol{\beta}'\right) \mathbb{E}\left[\mathbf{x}_t\right]\right).
$$
\n(8)

Now suppose the average U.S. bond investor re-prices risk across the business cycle in such a way that $\lambda_{\mathbf{x}}$ has different values in expansions and recessions. According to (7), this modification has the potential to generate negative loadings of $\hat{\beta}_k$ in expansions and positive loadings in recessions. Equation (8) shows that a switch in λ_x between expansions and recessions also affects the intercept

⁸The intercept in the $\mathbb Q$ distribution for $\mathbf x_t$ is normalized to zero.

⁹The derivations are provided in our Online Appendix.

 $\hat{\alpha}_k$ even if λ_0 is constant. However, it may be necessary to also let λ_0 switch between recessions and expansions to match the observed difference in $\hat{\alpha}_k^{REC}$ and $\hat{\alpha}_k^{EXP}$.

Thus, considering a regime-dependent market price of risk based on economic activity has the potential to explain the observed asymmetries in our modified Campbell-Shiller regressions or, equivalently, the asymmetric behavior of bond risk premia across the business cycle.

3.2 Model Description

We next formally present a DTSM with a regime-dependent market price of risk based on economic activity. As in the Gaussian ATSM, the dynamics of the short rate under $\mathbb Q$ is given by (5) and (6), implying that zero-coupon bond prices and the yield curve have the same expression as in the Gaussian ATSM, i.e.

$$
y_{t,k} = \tilde{A}_k + \tilde{\mathbf{B}}'_k \mathbf{x}_t,\tag{9}
$$

with $\tilde{A}_k \equiv \frac{-1}{k} A_k$ and $\tilde{\mathbf{B}}_k \equiv \frac{-1}{k} \mathbf{B}_k$ for $k = 1, 2, ..., K$.

We now deviate from the Gaussian ATSM by assuming that the market price of risk is piece-wise affine in the price factors x_t , with loadings depending on whether the economy is in expansion or recession. That is, we let

$$
\boldsymbol{\lambda}_t = 1_{\{z_t \ge c\}} \boldsymbol{\Sigma}_{\mathbf{x}}^{-1} \left(\boldsymbol{\lambda}_0^{(1)} + \boldsymbol{\lambda}_{\mathbf{x}}^{(1)} \mathbf{x}_t \right) + \left(1 - 1_{\{z_t \ge c\}} \right) \boldsymbol{\Sigma}_{\mathbf{x}}^{-1} \left(\boldsymbol{\lambda}_0^{(2)} + \boldsymbol{\lambda}_{\mathbf{x}}^{(2)} \mathbf{x}_t \right). \tag{10}
$$

To discipline our model, the regimes are taken to be observed, whereas previous work mainly considers unobserved regimes. As in Section 2, z_t refers to the PMI and recessions are identified when z_t is below its threshold value of $c = 44.5$. Given that our model remains conditional Gaussian, a simple change of measure gives the following \mathbb{P} dynamics

$$
\mathbf{x}_{t+1} = 1_{\{z_t \ge c\}} \lambda_0^{(1)} + (1 - 1_{\{z_t \ge c\}}) \lambda_0^{(2)} + \left(\mathbf{I} - \mathbf{\Phi} + 1_{\{z_t \ge c\}} \lambda_{\mathbf{x}}^{(1)} + (1 - 1_{\{z_t \ge c\}}) \lambda_{\mathbf{x}}^{(2)}\right) \mathbf{x}_t + \mathbf{\Sigma}_{\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{x},t+1}^{\mathbb{P}}.
$$
\n(11)

The model is closed by letting the $\mathbb P$ dynamics of z_t evolve as

$$
z_{t+1} = \gamma_0 + \gamma_z z_t + \gamma'_x \mathbf{x}_t + \Sigma_{zz} \epsilon_{z,t+1}^{\mathbb{P}}, \tag{12}
$$

where $\epsilon_{z,t+1}^{\mathbb{P}} \sim \text{NID}(0,1)$ and is independent of $\epsilon_{\mathbf{x},t+1}^{\mathbb{P}}$. That is, z_t may depend on its own lag if $\gamma_z \neq 0$ and lagged values of the pricing factors if $\gamma_x \neq 0_{n_x \times 1}$. The latter implies that yield curve dynamics affect economic activity and hence introduce a feedback effect from financial markets to the real economy. If $\gamma_x = \beta \times \kappa$, where κ is some scalar, then (12) reduces to $z_{t+1} =$ $(\gamma_0 - \kappa \alpha) + \gamma_z z_t + \kappa r_t + \sum_{z} \varepsilon_{z,t+1}^{\mathbb{P}},$ implying that the short rate is a sufficient statistic for economic activity, as assumed in the standard New Keynesian model, see Woodford (2003). In the general case where $\gamma_{\mathbf{x}} \neq 0_{n_x \times 1}$, all yield curve dynamics as captured by the pricing factors \mathbf{x}_t may matter for economic activity.¹⁰ For instance, changes in long-term bond yields may have an independent effect on economic activity beyond variation in the short rate, as considered in some of the recent macroeconomic literature (see, for instance, Andrés, López-Salido and Nelson (2004) and Gerler and Karadi (2013)). Finally, if $\gamma_x = 0_{n_x \times 1}$, then yield curve dynamics do not affect economic activity as assumed in Ang and Piazzesi (2003).

We also note from (11) that the switch in λ_t between recessions and expansions generates an instantaneous switch in the pricing factors and hence in the yield curve. This property of our model is thus consistent with our modified Campbell-Shiller and excess return regressions in Sections 2.1 and 2.2.

An interesting aspect of our model relates to the fact that z_t does not enter in the $\mathbb Q$ distribution of the short rate and hence as a pricing factor in (9) . This means that z_t is a 'hidden' factor as in Duffee (2011) and that our model displays unspanned macroeconomic risk similar to the work of Joslin, Priebsch and Singleton (2014). Hence, our macro variable z_t only affects yield curve dynamics indirectly by improving the forecast distribution of the pricing factors. Such improved predictions may be useful when forecasting the yield curve but also when computing expected excess returns or other measures of bond risk premia. Joslin, Priebsch and Singleton (2014) consider a setup where the macro variables enter linearly in the law of motion for the pricing factors under the $\mathbb P$ measure, implying that their model may be characterized as displaying 'linear' unspanned macroeconomic risk. Our setup differs slightly from the one considered by Joslin, Priebsch and Singleton (2014), because the macro variable z_t in our model has a non-linear effect on the $\mathbb P$ distribution for the pricing factors, meaning that our model may be described as having 'nonlinear' unspanned macroeconomic risk.

Finally, given that the Q distribution of the short rate is identical to the one in the Gaussian

 10 A similar specification is adopted in Diebold, Rudebusch and Aruoba (2006).

ATSM, our model implies the same identifying assumptions as the Gaussian ATSM with latent pricing factors. We require i) $\beta = 1$, ii) Φ to be diagonal with increasing eigenvalues, and iii) $\Sigma_{\rm x}$ to be triangular.¹¹ This identification scheme constrains the Q dynamics for the pricing factors whereas the P dynamics are unrestricted to simplify estimation by the SR approach.

3.3 Model Estimation by the SR Approach

It is well known that DTSMs may be challenging to estimate, mainly because the parameters describing the market prices of risk can be hard to identify with highly persistent pricing factors. One may therefore encounter numerical instability and problems with local optima when estimating DTSMs (see, for instance, Duffee (2002)). Compared to the Gaussian ATSM, our model has potentially twice the number of parameters for the market prices of risk, suggesting that the proposed model may be quite demanding to estimate. To overcome this limitation and avoid problems with numerical instability, we draw on recent innovations in estimation methods for DTSMs, starting with the pioneering work of Joslin, Singleton and Zhu (2011). More specifically, we extend the SR approach of Andreasen and Christensen (2015) to estimate all parameters in our regime-dependent market price of risk by a modified OLS regression, even when allowing for measurement errors on all bond yields, as recommended by Hamilton and Wu (2014). In other words, all parameters in the market price of risk are obtained instantaneously within the SR approach, meaning that our extension of the Gaussian ATSM comes at no additional computational costs.¹²

We next describe the SR approach when adopted to our proposed DTSM with a regime-dependent market price of risk. In the interest of space, we only present the three steps in the SR approach and refer to Andreasen and Christensen (2015) for technical details and how to obtain standard errors.

Step 1: The first step of the SR approach estimates the latent pricing factors x_t and the risk-neutral coefficients by a sequence of cross-section regressions. More formally, the risk-neutral coefficients are denoted by $\boldsymbol{\theta}_1 \equiv \begin{bmatrix} \boldsymbol{\theta}'_{11} & \boldsymbol{\theta}'_{12} \end{bmatrix}'$, where $\boldsymbol{\theta}_{11} \equiv \begin{bmatrix} \alpha & vec(\boldsymbol{\Phi})' \end{bmatrix}'$ and $\boldsymbol{\theta}_{12} \equiv vech(\boldsymbol{\Sigma}_{\mathbf{x}})$. Selecting

 11 There exist other normalization schemes, for instance the one recently suggested by Joslin, Singleton and Zhu (2011).

 12 If some linear combinations of bonds yields are assumed to be perfectly priced by the model, the linear regression methods of Joslin, Singleton and Zhu (2011) and Hamilton and Wu (2012) may also be modified to estimate our regime-switching model with no additional computational costs compared to the Gaussian ATSM. However, we prefer the adopted approach because it also enables us to estimate the shadow rate extension of our regime-switching model considered below in Section 5.1, whereas the estimators of Joslin, Singleton and Zhu (2011) and Hamilton and Wu (2012) do not apply to nonlinear DTSMs.

 n_y maturities along the yield curve and accounting for measurement errors $v_{t,k}$, we then express (9) in stacked form as

$$
\mathbf{y}_t = \tilde{\mathbf{A}}_k \left(\boldsymbol{\theta}_1 \right) + \tilde{\mathbf{B}}_k \left(\boldsymbol{\theta}_1 \right) \mathbf{x}_t + \mathbf{v}_t, \tag{13}
$$

where $\mathbf{y}_t \equiv \begin{bmatrix} y_{t,1} & y_{t,2} & \dots & y_{t,n_y} \end{bmatrix}$ and similarly for $\mathbf{\tilde{A}}_k(\theta_1)$, $\mathbf{\tilde{B}}_k(\theta_1)$, and \mathbf{v}_t .

When n_y is large relative to the number of pricing factors, the SR approach estimates x_t by minimizing the squared distance between observed and model-implied bond yields, i.e. by regressing $\mathbf{y}_t - \tilde{\mathbf{A}}_k (\boldsymbol{\theta}_1)$ on $\tilde{\mathbf{B}}_k (\boldsymbol{\theta}_1)$. For our ATSM in (13), this implies

$$
\hat{\mathbf{x}}_{t}(\theta_{1}) = \left(\tilde{\mathbf{B}}_{k}(\theta_{1})'\tilde{\mathbf{B}}_{k}(\theta_{1})\right)^{-1}\tilde{\mathbf{B}}_{k}(\theta_{1})'\left(\mathbf{y}_{t} - \tilde{\mathbf{A}}_{k}(\theta_{1})\right)
$$
(14)

for $t = 1, 2, ..., T$. The estimated factors are denoted $\{\hat{\mathbf{x}}_t(\theta_1)\}_{t=1}^T$ because they are computed for a given θ_1 . We then estimate θ_1 by pooling all squared residuals from these cross-section regressions and minimizing their sum with respect to θ_1 , i.e.

$$
\hat{\boldsymbol{\theta}}_1^{step1} = \arg \min_{\boldsymbol{\theta}_1 \in \Theta_1} \frac{1}{T n_y} \sum_{t=1}^T \left\| \mathbf{y}_t - \tilde{\mathbf{A}}_k \left(\boldsymbol{\theta}_1 \right) - \tilde{\mathbf{B}}_k \left(\boldsymbol{\theta}_1 \right) \hat{\mathbf{x}}_t \left(\boldsymbol{\theta}_1 \right) \right\|^2,
$$

where Θ_1 denotes the feasible domain of θ_1 and $\|\mathbf{a}\| \equiv \sqrt{\sum_{i=1}^n a_i^2}$ for any $\mathbf{a} \in \mathbb{R}^n$.

Step 2: The second step estimates the \mathbb{P} dynamics of z_t and \mathbf{x}_t . To describe the procedure, let

$$
\boldsymbol{\theta}_{2}^{\mathbf{x}} \equiv \left[\begin{array}{cc} \left(\boldsymbol{\lambda}_{0}^{(1)} \right)^{\prime} & \left(\boldsymbol{\lambda}_{0}^{(2)} \right)^{\prime} & vec \left(\boldsymbol{\lambda}_{\mathbf{x}}^{(1)} \right)^{\prime} & vec \left(\boldsymbol{\lambda}_{\mathbf{x}}^{(2)} \right)^{\prime} & vec \left(\boldsymbol{\lambda}_{\mathbf{x}}^{(2)} \right)^{\prime} \end{array} \right]^{T}
$$

and $\boldsymbol{\theta}_2^{\mathbf{z}} \equiv$ $\left[\begin{array}{cc} \gamma_0 & \gamma_z & \gamma'_x & \Sigma_{zz} \end{array}\right]'$ contain all the parameters governing the $\mathbb P$ dynamics of $\mathbf x_t$ and $\mathbf z_t$, respectively. Replacing the unobserved \mathbf{x}_t in (11) by $\hat{\mathbf{x}}_t \left(\hat{\boldsymbol{\theta}}_1^{step1} \right)$ 1 from the first step, we then estimate $\theta_2^{\mathbf{x}}$ by extending the SR approach of Andreasen and Christensen (2015) to $\mathbb P$ dynamics with regime-switching. That is, we run a modified regression based on (11) that accounts for estimation uncertainty in $\hat{\mathbf{x}}_t \left(\hat{\boldsymbol{\theta}}_1^{step1} \right)$ 1), as described in Appendix A.5. The elements in $\theta_2^{\mathbf{z}}$ are obtained in a similar fashion based on (12) using the regression provided in Andreasen and Christensen (2015). Importantly, both $\left(\hat{\theta}^{\mathbf{x}}_2\right)$ 2 \int^{step2} and $\left(\hat{\theta}^{\mathbf{z}}_{2}\right)$ 2 s^{step2} are given in closed form, meaning that all of the parameters in the $\mathbb P$ distribution of z_t and x_t are obtained instantaneously, including all coefficients in the market price of risk.

To ensure stationarity of $y_{t,k}$ we require \mathbf{x}_t to be stationary under the \mathbb{P} measure. This condition

holds if the loading matrices on x_t in (11) are stable in recessions and in expansions, i.e. if all eigenvalues of the matrices $I - \Phi + \lambda_{\mathbf{x}}^{(i)}$ for $i = \{1, 2\}$ are inside the unit circle.¹³ If one of these conditions are not satisfied, then we downscale $\mathbf{I} - \mathbf{\Phi} + \mathbf{\lambda}_{\mathbf{x}}^{(i)}$ by δ_i for $i = \{1, 2\}$ using the data-driven procedure of Andreasen and Meldrum (2014), which we describe in Appendix A.6.

Step 3: The matrix Σ_x is estimated in both the first and second step. As noted by Andreasen and Christensen (2015), $\hat{\Sigma}_{\mathbf{x}}^{step1}$ is estimated very inaccurately compared to $\hat{\Sigma}_{\mathbf{x}}^{step2}$, which is therefore the preferred estimate.¹⁴ Given this more efficient estimate of $\Sigma_{\mathbf{x}}$, we then condition on the value of $\hat{\mathbf{\Sigma}}_{\mathbf{x}}^{step2}$ and re-estimate $\boldsymbol{\theta}_{11}$, i.e.

$$
\hat{\boldsymbol{\theta}}_{11}^{step3} = \arg \min_{\boldsymbol{\theta}_{11} \in \Theta_{11}} \frac{1}{Tn_y} \sum_{t=1}^{T} \left\| \mathbf{y}_t - \tilde{\mathbf{A}}_k \left(\boldsymbol{\theta}_{11}; \hat{\boldsymbol{\Sigma}}_{\mathbf{x}}^{step2} \right) - \tilde{\mathbf{B}}_k \left(\boldsymbol{\theta}_{11}; \hat{\boldsymbol{\Sigma}}_{\mathbf{x}}^{step2} \right) \hat{\mathbf{x}}_t \left(\boldsymbol{\theta}_{11} \right) \right\|^2.
$$

Given the estimated factors $\left\{ \hat{\mathbf{x}}_t \left(\hat{\boldsymbol{\theta}}_{11}^{step3}, \hat{\mathbf{\Sigma}}^{step2} \right) \right\}^T$ $\sum_{t=1}^{\infty}$, we finally update our estimates of $\theta_2^{\mathbf{x}}$ and $\theta_2^{\mathbf{z}}$ by re-running step 2.

3.4 Comparing to Existing DTSMs with Regime-Switching

When formulating DTSMs with regime-switching there is an inherent trade-off between the richness of a given model and the computational complexity related to bond pricing and estimation. We have therefore chosen to consider the most parsimonious model capable of matching the asymmetric properties of bond yields documented in Section 2. This implies that we omit regime-switching in $\Sigma_{\rm x}$ as in Ang, Bekaert and Wei (2008) or variation in $\Sigma_{\rm x}$ within regimes as in Bansal and Zhou (2002), because time-varying second moments are not required to reproduce the new properties of bond risk premia provided in Section 2^{15} We also restrict the flexibility of our model by having observed regimes of either expansions or recessions, whereas most DTSMs with regime-switching consider unobserved regimes, although their estimated values often are closely related to the business cycle as in Bansal and Zhou (2002) and Dai, Singleton and Yang (2007). Similar to these two papers, the market price of factor risk in our model is allowed to change freely across regimes, whereas Ang, Bekaert and Wei (2008) consider a somewhat more restricted formulation. As in Dai, Singleton and Yang (2007), we also accommodate time-varying transition probabilities between regimes under the

 $13A$ formal proof of this result is provided in Theorem 6.12 of Pötscher and Prucha (1997).

 14 See Andreasen and Christensen (2015) for how to combine the two estimates in an optimal way.

¹⁵ Furthermore, a direct extension of our estimator for Σ_x in (19) to accommodate regime-switching reveals only minor changes in the estimated value of $\Sigma_{\mathbf{x}}$ between recessions and expansions.

P measure when $\gamma_z \neq 0$ or $\gamma_x \neq 0$, whereas these probabilities are constant in Bansal and Zhou (2002) and Ang, Bekaert and Wei (2008).

4 Empirical Findings

This section estimates our DTSM with regime-switching in the case of three pricing factors $(n_x = 3)$, given a total of four state variables when including z_t . Here, we use the same Fama-Bliss dataset as applied in Section 2. We first discuss the estimated coefficients in Section 4.1, before studying the ability of our regime-switching model to match the observed asymmetry in the U.S. yield curve in Section 4.2. The implied estimates of excess returns and term premia from our regime switching model and the Gaussian ATSM are finally compared in Section 4.3.

4.1 Estimation Results

Our proposed ATSM with regime switching in the market price of risk is indexed by $\mathcal{M}_{\lambda_0,\lambda_x}^{ATSM}$, where subscripts indicate whether λ_0 and/or λ_x are allowed to switch between expansions and recessions. Table 3 shows that the first factor is very persistent under the $\mathbb Q$ measure with $\Phi(1,1) \approx 1.2 \times 10^{-8}$ in all models and may therefore be interpreted as a level factor. The low value of $\Phi(1,1)$ implies that the intercept in the short rate is unidentified and we therefore let $\alpha = 0$. The second and the third factor display less persistence under Q in all considered models and may therefore be interpreted as a slow and fast decaying factors. Unlike $\hat{\Phi}$, the estimates of $\Sigma_{\mathbf{x}}$ are somewhat affected by regime switching in the market price of risk, although the sign of the off-diagonal elements in Σ_x is similar across all models. Table 3 also provides the estimated dynamics for economic activity, which displays moderate persistence with $\gamma_z = 0.93$ across all specifications of λ_t . We also find that each of the pricing factors has a negative effect on economic activity. Importantly, the null hypothesis that $\gamma_x(1,1) = \gamma_x(2,1) = \gamma_x(3,1)$ is clearly rejected using a Wald test with a p-value of 0.000 for all models, implying that the policy rate is not a sufficient statistic for economic activity according to our model.

\langle Table 3 about here $>$

The estimates of the market price of risk are provided in Table 4. For the Gaussian ATSM \mathcal{M}^{ATSM} all elements of λ_0 are significantly different from zero, meaning that investors on average require compensation for exposure to each of the pricing factors. We also find the familiar result that time-variation in level risk through $\lambda_{\mathbf{x}}(1, :), \lambda_{\mathbf{x}}(2, 1)$, and $\lambda_{\mathbf{x}}(3, 1)$ is significant and controls much of the variability in the market price of risk and hence excess returns (see, for instance, Cochrane and Piazzesi (2008)).

We next allow the constant λ_0 in the market price of risk to switch in $\mathcal{M}_{\lambda_0}^{ATSM}$. During recessions λ_0 (2, 1) has a higher value than in expansions, whereas the opposite holds for λ_0 (3, 1). These differences are significant as we reject the null hypothesis that $\lambda_0^{(1)} = \lambda_0^{(2)}$ $_0^{(2)}$ using a Wald test (*p*value of 0.0124). Suppose instead that only λ_x is allowed to switch in the market price of risk, i.e. λ_0 is constant as in the Gaussian ATSM. For $\mathcal{M}^{ATSM}_{\lambda_x}$ in Table 4 we find that time-variation in the second factor through λ_x (2, 2) now carries a negative price, although most predominately in recessions, whereas variation in this second factor is priced positively in \mathcal{M}^{ATSM}_{AB} and $\mathcal{M}^{ATSM}_{\lambda_0}$. We also observe that $\lambda_{\mathbf{x}}(3,2)$ and $\lambda_{\mathbf{x}}(3,3)$ have opposite signs in recessions and expansions. Despite the somewhat wide standard errors for $\lambda_{\mathbf{x}}^{(2)}$, the null hypothesis that $\lambda_{\mathbf{x}}^{(1)} = \lambda_{\mathbf{x}}^{(2)}$ is clearly rejected $(p$ -value of $0.0000)$.

We finally consider the full model $\mathcal{M}_{\lambda_0,\lambda_x}^{ATSM}$, where both λ_0 and λ_x are allowed to switch. We once again find that the value of $\lambda_0 (2, 1)$ changes between recessions and expansions, and that time-variation in the second factor through $\lambda_x (2, 2)$ carries a positive price in expansions but a negative price in recessions. The latter implies that the pricing factors and hence interest rates are more persistent in expansions than in recessions when measured by the largest eigenvalue of $\mathbf{I} - \mathbf{\Phi} + \mathbf{\lambda}_{\mathbf{x}}^{(i)}$, which is 0.9957 in expansions and 0.9605 in recessions. This finding is thus similar to Bansal and Zhou (2002). However, most elements in $\lambda_0^{(2)}$ $\lambda_{\mathbf{x}}^{(2)}$ and $\lambda_{\mathbf{x}}^{(2)}$ for the recession regime are estimated somewhat imprecisely, and we are therefore unable to reject the null hypothesis of no regime switching in λ_t (p-value of 0.26), although we rejected it for both $\mathcal{M}_{\lambda_0}^{ATSM}$ and $\mathcal{M}_{\lambda_x}^{ATSM}$. This indicates that our general specification for regime-switching in λ_0 and λ_x may be simplified without affecting the model's ability to fit the data. We return to this possibility below in Section 5.2.

 $<$ Table 4 about here $>$

4.2 Explaining Asymmetries in the Yield Curve

We next examine whether the full model with switching in λ_0 and λ_x can match the asymmetries in the yield curve from Section 2. The model-implied moments are here obtained by running the regressions in (1) to (4) on a simulated sample of 100; 000 observations using the estimates of $\mathcal{M}_{\lambda_0,\lambda_x}^{ATSM}$ reported above.

The top part of Figure 4 shows that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ convincingly matches both intercepts and slope coefficients in ordinary Campbell-Shiller regressions and thus preserves the ability of the Gaussian ATSM in matching these unconditional properties of bond risk premia. The next question is whether $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ meets the challenge of replicating the switch in the Campbell-Shiller regressions across expansions and recessions. The bottom part of Figure 4 shows that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ reproduces the negative slope estimates in expansions and the *positive* slope coefficients in recessions, although the latter are somewhat higher than the empirical moments. In addition, $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ matches almost perfectly the switch in intercepts.

We next explore whether $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ also replicates the predictability in bond returns implied by i) the yield spread, $ii)$ the forward spread, and $iii)$ the CP factor. The left part of Figure 5 shows that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ matches the slope coefficients from ordinary return regressions using each of these predictors. The ability of our model to reproduce the predictability of the CP factor suggests that the behavior of this factor to a large extent is spanned by the three canonical pricing factors in our model.¹⁶ The right part of Figure 5 further shows that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ also generates much of the observed asymmetry in return predictability across business cycles for all three predictors. That is, our model implies strong predictability of bond returns during expansions, whereas recessions are characterized by weak predictability and often with the 'wrong' sign.

\langle Figure 4 and 5 about here \rangle

Combining the results from Figure 4 and 5, we conclude that our regime switching model $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ to a large extent is capable of explaining the asymmetric behavior of bond yields from Section 2.¹⁷ Thus, the switch in the Campbell-Shiller and return regressions across expansions and recessions

 $16A$ similar finding is reported in Dai, Singleton and Yang (2004).

¹⁷We have also analyzed the ability of $\mathcal{M}_{\lambda_0}^{ATSM}$ and $\mathcal{M}_{\lambda_x}^{ATSM}$ to match the observed asymmetric patterns in the data. $\mathcal{M}_{\lambda_0}^{ATSM}$ matches the sign switch in intercepts but not in the slope coefficients of our modified Campbell-Shiller regression and it fails to generate low return predictability in recessions. $\mathcal{M}^{ATSM}_{\lambda_x}$ does not match neither intercepts nor slope coefficients in the modified Campbell-Shiller regressions and it generates too high return predictability in expansions.

may be rationalized by a re-pricing of risk among bond investors when the U.S. economy enters recessions.

4.3 Model-implied Excess Returns and Term Premia

The presence of regime-switching in the market price of risk has profound implications for bond risk premia. Figure 6 therefore plots expected excess returns from $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ and \mathcal{M}^{ATSM} at the 10-year maturity, with NBER recessions indicated by the shaded regions. The two models provide very similar dynamics for expected excess returns in expansions. However, before and at the start of recessions, we generally find that expected excess returns in $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ increase more than in \mathcal{M}^{ATSM} . The prime examples are the two recessions in the early 1980s. This effect arises because $\mathcal{M}^{ATSM}_{\lambda_0\lambda_x}$ assigns an increasing fraction of the forecast distribution to the recession regime, where all bond yields are predicted to be substantially lower than in expansions. This in turn raises bond prices and hence expected excess returns. This *recession amplifier* disappears towards the middle and the end of recessions where excess returns fall, after which excess returns in $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ approach the level implied by \mathcal{M}^{ATSM} .

\langle Figure 6 about here \rangle

Any estimate of bond risk premia is model-dependent, and it may therefore in general be challenging to argue in favor of one set of estimates compared to another. However, the regression evidence we provide in Figures 4 and 5 clearly indicates that our regime-switching model matches important properties of bond returns unlike the Gaussian ATSM, suggesting that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ gives more accurate estimates of bond risk premia compared to \mathcal{M}^{ATSM} . Another way to illustrate this is to run Mincer-Zarnowitz regressions of realized excess returns on model-implied expected excess returns $\mathbb{E}_t [x h p r_{t+m,k}]$. Conditioning on the business cycle as in Section 2, we consider

$$
xhpr_{t+m,k} = \alpha_{0,k}^{EXP} \left(1 - 1_{\{NBER\}}\right) + \alpha_{1,k}^{EXP} \left(1 - 1_{\{NBER\}}\right) \mathbb{E}_{t} \left[xhpr_{t+m,k} \right] + \alpha_{0,k}^{REC} 1_{\{NBER\}} + \alpha_{1,k}^{REC} 1_{\{NBER\}} \mathbb{E}_{t} \left[xhpr_{t+m,k} \right] + u_{t,k},
$$
\n(15)

with $m = 3$, where recessions and expansions are identified using the NBER recessions dates $1_{\{NBER\}}$ for comparability with Figure 6. The slope coefficients $\alpha_{1,k}^{EXP}$ and $\alpha_{1,k}^{REC}$ are reported in Table 5 when pooling excess returns in bins of two years along the maturity range. Consistent with

the regression evidence in Figure 4 and 5, \mathcal{M}^{ATSM} performs well in expansions with $\alpha_{1,k}^{EXP}$ close to the desired value of one, but the model struggles during recessions, where $\alpha_{1,k}^{REC}$ is substantially below one and even negative for bond yields with six or more years to maturity. That is, the Gaussian ATSM predicts excess returns for medium- and long-term bonds in the wrong direction during recessions. Our regime switching model largely alleviates this shortcoming with both $\alpha_{1,k}^{EXP}$ and $\alpha_{1,k}^{REC}$ in the range of 0.75 to 0.54, meaning that $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ predicts excess returns of all bond yields in the right direction during expansions and recessions.

 $<$ Table 5 about here $>$

The lower part of Figure 6 reports model-implied term premia, defined as $TP_{t,k} \equiv y_{t,k} - \frac{1}{k}$ $\frac{1}{k} \sum_{i=0}^{k-1} \mathbb{E}_t [r_{t+i}],$ which is another commonly used measure of bond risk premia. In expansions, we find term premia to be very similar for \mathcal{M}^{ATSM} and $\mathcal{M}^{ATSM}_{\lambda_0\lambda_x}$, but substantial differences appear before and at the start of recessions, where term premia in $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ tend to increase more than in \mathcal{M}^{ATSM} . That is, term premia is more counter-cyclical in our regime-switching model compared to the standard Gaussian ATSM.

5 Additional Analysis

This section studies in greater detail the ability of regime-switching in the market price of risk to explain the documented asymmetries in the U.S. yield curve from Section 2. We first consider the implications of extending $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ to enforce the zero lower bound (ZLB) in Section 5.1. Given our findings in Section 4.1, we then present a simplified version of $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ in Section 5.2, which we use to provide an economic explanation for the observed switch in the Campbell-Shiller and return regressions across expansions and recessions. The ability of DTSMs with linear unspanned and spanned macroeconomic risk to match loadings from our modified Campbell-Shiller regressions are finally explored in Section 5.3.

5.1 Accounting for the Zero Lower Bound

An important feature of our regime-switching model is to generate a larger fall in expected bond yields just before and during recessions than implied by the Gaussian ATSM. Given that short rates typically are low during recessions and may be constrained by the ZLB, it seems natural to explore whether the better performance of our model is robust to enforcing the ZLB. We address this question by presenting a shadow rate extension of our regime-switching model where the short rate is given by $r_t = \max\{0, \alpha + \beta' \mathbf{x}_t\},$ but the model is otherwise identical to the one described in Section 3.2. Hence, the proposed shadow rate model (SRM) has Gaussian pricing factors under the Q measure and bond prices may therefore be computed by the second-order approximation advocated in Priebsch (2013) when formulated in discrete time. This SRM with regime-switching $M_{\lambda_0\lambda_x}^{SRM}$ is estimated as described in Section 3.3, except (14) is replaced by nonlinear cross-section regressions to extract the pricing factors. The estimates are provided in Tables 6 and 7.

Figure 7 shows that $\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$ provides an extremely close fit to intercepts in ordinary Campbell-Shiller regressions and even improves upon the slope coefficients in these regressions for mediumand long-term bond yields compared to $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$. This improvement is further seen to carry over to the slope coefficients during expansions, whereas $\mathcal{M}^{SRM}_{\lambda_0\lambda_x}$ largely generates the same intercepts and slope coefficients for the recession regime as found for $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$. Although not reported below, we also find that $\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$ matches the return regressions just as well as seen for $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$. Finally, $M_{\lambda_0\lambda_x}^{SRM}$ also forecasts realized excess returns in the correct direction during both expansions and recessions, as shown in Table 5.

$$
<
$$
 Table 6-7, Figure 7 about here $>$

Thus, our proposed explanation for the documented asymmetries in the U.S. yield curve, i.e. that investors re-price risk during recessions, is robust to accounting for the ZLB.

5.2 A Simplified Regime-Switching Model

To provide a simplified version of $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ with a clear economic interpretation, it seems beneficial to first impose more structure on the factor loadings than implied by our fully flexible regimeswitching model $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$. We therefore first note that the null hypothesis of $\Phi(1,1) = 0$ and $\Phi(2,2) = \Phi(3,3)$ is not rejected using a Wald test (p-value of 0.6406). Given these restrictions, it is straightforward to show that the factor loadings in $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ simplify to those in the no-arbitrage Nelson-Siegel (AFNS) model of Christensen, Diebold and Rudebusch (2011) when using its discretetime formulation in Fontaine and Garcia (2012). That is, we let $\beta = \begin{bmatrix} 1 & \frac{1-e^{-\tilde{\lambda}}}{\tilde{\lambda}} & \frac{1-e^{-\tilde{\lambda}}}{\tilde{\lambda}} - e^{-\tilde{\lambda}} \end{bmatrix}'$ and impose $\Phi(2,2) = \Phi(3,3) = 1 - e^{-\tilde{\lambda}}$ with $\Phi(2,3) = -\tilde{\lambda}e^{-\tilde{\lambda}}$, whereas all the remaining elements

of Φ are zero. Given this specification, the pricing factors now have the familiar interpretation as representing the level, slope, and curvature of the yield curve.

The results from Section 4.1 indicate that not all elements in λ_0 and λ_x switch between expansions and recessions, and we therefore search for the most parsimonious specification of λ_t capable of matching the switch in the Campbell-Shiller regressions across the business cycle. Inspired by the return regressions with the yield spread in Section 2.2, the proposed model displays only switching in λ_0 (2, 1) and λ_x (2, 2), both related to the slope factor at position two in our state vector. This restricted model is denoted $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ and has only two additional coefficients in the market price of risk compared to the Gaussian ATSM. The top part of Figure 8 shows that $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ captures the overall pattern in intercepts and slope coefficients from ordinary Campbell-Shiller regressions. More surprising, perhaps, is the ability of this very parsimonious regime-switching specification to also capture the switch in both intercepts and slope coefficients in the Campbell-Shiller regressions between expansions and recessions. Thus, simply allowing for regime-switching in the dynamics of the slope factor is sufficient to reproduce the loadings in our modified Campbell-Shiller regressions.

To see what drives these results for $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$, we first note from Table 7 that $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ has a lower value of $\lambda_0 (2, 1)$ in recessions than in expansions and that $\lambda_x (2, 2)$ switches sign between the two regimes. The latter arises because $1 - \Phi(2, 2) + \lambda_x^{(1)}(2, 2) = 0.97$ in expansions, whereas the corresponding estimate in recessions is 0.73. Given that $\Phi(2,2) = 0.0381$ in $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$, this change in the persistence of the slope factor then generates a switch in the market price of risk from a insignificant positive value of $\lambda_x^{(1)}(2,2) = 0.0081$ in expansions to a significant negative value of $\lambda_x^{(2)}(2,2) = -0.2363$ in recessions.

To analyze the effects of these estimates for quarterly excess returns, as considered in our implementation of the Campbell-Shiller regressions in (1) and (2) , it is useful to first understand their impact on monthly excess returns which are available in closed form and proportional to $\mathbf{B}_{k-1}'\boldsymbol{\lambda}_t$.¹⁸ Given that $B_{k-1}(2,1) < 0$, a more negative value of $\lambda_0(2,1)$ in recessions therefore increases monthly excess returns, as investors require a larger compensation for slope risk in this regime. On the other hand, a negative value of $\lambda_x^{(2)}(2,2)$ in recessions has a positive effect on $\lambda_x^{(2)} \mathbf{x}_t$ with an upward sloping yield curve, because the slope factor with the AFNS loadings is defined as the short rate *minus* the long rate (see Diebold and Li (2006)), meaning that $\lambda_x^{(2)}(2,2)$ in total

 18 Here, we ignore a minor convexity term in the expression for monthly excess returns. The exact expression is provided in our Online Appendix.

has a negative impact on monthly excess returns when accounting for $B_{k-1} (2, 1) < 0$. Figure 9 shows that these partial effects carry over to the quarterly excess return for a 10-year bond which we compute by Monte Carlo integration. Note also how fears of recessions on its own may affect expected quarterly excess returns (as in the mid 1990s). We finally emphasize that $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ predicts realized excess returns with the right sign during expansions and recessions according to the Mincer-Zarnowitz regressions in Table 5, despite $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ only having two parameters that switch between regimes.

Thus, the narrative implied by our model is as follows. When the U.S. economy enters a recession, bond investors immediately re-price the risks attached to economic activity with a switch in λ_0 (2, 1) and therefore require a higher compensation for exposure to the slope of the yield curve. At the start of a recession, this effect generally dominates the one from a switch in $\lambda_x (2, 2)$ according to Figure 9, as the yield curve here tends to be fairly flat, and this increases expected excess returns. However, as monetary policy becomes more accommodating during the course of a recession, the lower short rate generates a steepening of the yield curve and reduces the risks attached to low future economic activity. This in turn strengthens the effect from the switch in λ_x (2, 2), which generally has a negative impact on excess returns according to Figure 9. Thus, our model suggests that the Federal Reserve is able to remove some of the risks attached to recessions, as accommodating monetary policy reduces the required risk compensation in the bond market. Importantly, the switch in $\lambda_x(2, 2)$ from positive in expansions to negative in recessions implies that excess returns and the short rate both fall during this phase of a recession and hence become positively correlated, as required to match the positive slope coefficients in the Campbell-Shiller regressions during recessions. The Gaussian ATSM does not allow for a switch in λ_x (2, 2), meaning that this model does not imply that accommodating monetary policy is able to remove some of the risks in the bond market during recessions. As a result, excess returns tend to increase throughout recessions in the Gaussian ATSM, as also seen in Figure 6. After a recession, $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ reverts back to the familiar setting described in Dai and Singleton (2002), where excess returns fall when the short rate increases, and vice versa. That is, excess returns and the short rate are once again negatively correlated as required to match the negative slope in the Campbell-Shiller regressions during expansions.

 \langle Figure 8 about here \rangle

5.3 Models with Linear Unspanned and Spanned Macroeconomic Risk

Our proposed mechanism to explain the documented asymmetries in U.S. bond yields modifies the essentially affine specification for the market price of risk such that it only holds locally for expansions and recessions. Hence, λ_t depends nonlinearly on the business cycle, as captured by the state variable z_t for PMI. To explore whether this nonlinearity is necessary to match the switch in the Campbell-Shiller regressions, we next follow Joslin, Priebsch and Singleton (2014) and briefly study the case where λ_t only depends linearly on z_t . That is, we now let the market price of risk be essentially affine in all four state variables. As above, z_t remains unspanned by the current yield curve, i.e. $\beta(4,1) = 0$, given our specification with z_t appearing last in the state vector. The estimates for this model \mathcal{M}^{Linear} are provided in Tables 6 and 7.

Figure 10 shows that \mathcal{M}^{Linear} provides a very close fit to both intercepts and slope coefficients in ordinary Campbell-Shiller regressions. This is in line with previous findings in the literature, as this model nests the standard three-factor Gaussian ATSM. The lower part of Figure 10 shows that \mathcal{M}^{Linear} also matches the regime-dependent intercepts in the Campbell-Shiller regressions but only the negative slope coefficients in expansions. That is, \mathcal{M}^{Linear} cannot generate positive slope coefficients during recessions. Table 5 further shows that \mathcal{M}^{Linear} only has predictive power for realized excess returns in expansions but not in recessions. Hence, it must be the nonlinear effect of z_t on the market price of risk that allows $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$, $\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$, and $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ to match the slope coefficients in our modified Campbell-Shiller regression and predict excess returns with the right sign during expansions and recessions.

Following the work of Bauer and Rudebusch (2016), we finally explore whether these results for M^{Linear} are robust to letting z_t be spanned by the current yield curve. That is, we once again let the market price of risk be essentially affine in all four state variables but omit the zero restriction for $\beta(4,1)$. Figure 10 shows that this model $\mathcal{M}^{Spanned}$ displays broadly the same performance as \mathcal{M}^{Linear} in terms of matching intercepts and slope coefficients from ordinary and modified Campbell-Shiller regressions. As a result, the Mincer-Zarnowitz regressions for excess returns in Table 5 are also very similar for the two models. Hence, the spanning assumption of macroeconomic risk, as measured by the PMI, does not seem essential for the aspects of U.S. bond yields studied in this paper.

6 Conclusion

By conditioning the classic Campbell-Shiller (1991) regressions on the business cycle, we identify a strong asymmetric pattern in the relation between yield spreads and subsequent yield changes. When the economy is expanding, we find the familiar pattern of mostly positive intercepts and negative regression slopes that decrease with maturity. However, when the economy is contracting, we observe negative intercepts and positive regression slopes that mostly increase with maturity, i.e. the complete opposite pattern. We also show that this asymmetric effect has profound implications for bond return predictability, as the classic yield-based predictions have strong forecasting power for excess returns in expansions but not in recessions.

To explain these new empirical Öndings, the Gaussian ATSM is extended with business cycle dependent loadings in the market price of risk. We show that this model reproduces the empirical intercepts and slope coefficients from ordinary Campbell-Shiller regressions and, in addition, matches evidence from our modified Campbell-Shiller regressions conditioning on the business cycle. We also show that our model predicts realized excess returns with the right sign during both expansions and recessions unlike the Gaussian ATSM, which predicts excess returns for medium- and long-term bonds in the wrong direction during recessions. Our model also replicates the observed asymmetry in return predictability, i.e. that the classic yield-based predictors have strong forecasting power in expansions but not in recessions. To account for these asymmetric patterns in the yield curve across the business cycle, our model generates a *negative* relation between the short rate and excess returns in expansions (as in the Gaussian ATSM), but a positive relation in recessions. Thus, our model suggests that the Federal Reserve is able to remove some of the risks attached to recessions, as accommodating monetary policy reduces the required risk compensation in the bond market. This effect of monetary policy is not present in the Gaussian ATSM, which therefore provides less accurate estimates of bond risk premia and expected future short rates compared to our regime-switching model. Accordingly, our model suggests that the positive slope coefficients in the Campbell-Shiller regressions emerge because accommodating monetary policy is able to eleminate some of the risks in the bond market during recessions.

Our simple, yet powerful, approach of conditioning the study of bond markets on the business cycle obviously goes beyond what is covered in the present paper. Ongoing research by Andreasen, Møller and Sander (2016) shows that bond markets in several other countries display a similar switch in the Campell-Shiller regressions as found in the present paper, both when conditioning on local recessions or U.S. recessions. Andreasen, Møller and Sander (2016) further show that this finding has important implications for exchange rate dynamics, as the slope coefficients in uncovered interest parity (UIP) regressions also switch between expansions and regressions. Thus, recessions may not only have profound effects for the local U.S. bond investor but also for other bond markets and related exchange rate markets.

A Appendix

A.1 Data

We use monthly bond yields from an unsmoothed Fama and Bliss (1987) dataset running from 1961:06-2013:12. We limit the analysis to maturities from $m/12$ to 10 years with m-month increments where $m = 3$. A similar panel of yields is used in Adrian, Crump and Moench (2013) to obtain their pricing factors. At the very long end of the yield curve, we do not observe all increments and, hence, we interpolate between bond yields of the two nearest maturities.¹⁹ We also report results for the Gürkaynak, Sack and Wright (2007) dataset, which is based on a static Svensson model for computing smooth yield curves.

A.2 Wald Tests

We use Wald tests to examine whether the coefficients in our modified Campbell-Shiller regressions in (2) change across expansions and recessions. In particular, we test the following joint hypotheses:

$$
H_0: \alpha_k^{REC} = \alpha_k^{EXP}, \quad k = 6, 9, 12, ..., 120
$$
\n(16)

$$
H_0: \beta_k^{REC} = \beta_k^{EXP}, \quad k = 6, 9, 12, ..., 120 \tag{17}
$$

The tests are carried out by setting up the regressions in a GMM framework, where all parameters are collected in

$$
\bm{\Theta}' = \begin{bmatrix} \bm{\theta}'_6, \ \bm{\theta}'_9, \ \ ..., \ \bm{\theta}'_{120} \end{bmatrix},
$$

with $\boldsymbol{\theta}'_k = \left[\alpha_k^{EXP}, \beta_k^{EXP}, \alpha_k^{REC}, \beta_k^{REC}\right]$. The sample moments for the system of linear regressions are

$$
\mathbf{g}_T(\mathbf{\Theta})^{\prime} = \left[\mathbf{g}^{\prime}_{T,6}, \mathbf{g}^{\prime}_{T,9}, ..., \mathbf{g}^{\prime}_{T,120} \right],
$$

where $\mathbf{g}_{T,k} = \frac{1}{T}$ $\frac{1}{T} \sum_{t=1}^{T} \mathbf{f}_{t,k}, \, \mathbf{f}_{t,k} = \tilde{u}_{t+m,k} \mathbf{z}_{t,k},$ and

$$
\mathbf{z}_{t,k}^{'} = \left[1_{\{z_t \geq c\}}, \frac{m}{k-m} 1_{\{z_t \geq c\}} (y_{t,k} - y_{t,m}), (1 - 1_{\{z_t \geq c\}}), \frac{m}{k-m} (1 - 1_{\{z_t \geq c\}}) (y_{t,k} - y_{t,m})\right].
$$

Expressing our null hypothesis as $\mathbf{R}\Theta = \mathbf{c}$, a Wald test is then given by

$$
W = \left(\mathbf{R}\widehat{\mathbf{\Theta}} - \mathbf{c}\right)' \left(\mathbf{R}\left(\mathbf{D}_T^{\prime} \mathbf{S}_T^{-1} \mathbf{D}_T\right)^{-1} \mathbf{R}^{\prime}/T\right)^{-1} \left(\mathbf{R}\widehat{\mathbf{\Theta}} - \mathbf{c}\right) \sim \chi_K^2,
$$

where \mathbf{D}_T is the Jacobian of the moment conditions and \mathbf{S}_T is the spectral density matrix obtained by the Newey-West estimator using $m + 1$ lags. For the hypothesis in (16), we have

$$
\mathbf{R} = \mathbf{ID}_K \otimes [1, 0, -1, 0],
$$

where ID_K is the identity matrix with dimension K and $c = 0$. K is the number of maturities we

 19 Fama-Bliss forward rates are known to be somewhat rough for longer maturities. We therefore smooth them using a 12-month equal-weighted window.

examine and is equal to 39 for the full panel. For the hypothesis in (17), we have

$$
\mathbf{R}=\mathbf{ID}_K\otimes[0,1,0,-1],
$$

and $\mathbf{c} = \mathbf{0}$.

A.3 Lynch-Wachter Estimator

Conducting the tests for the full sample period involves making some implementation choices as the Fama-Bliss panel is unbalanced. We use the adjusted moments estimator of Lynch and Wachter (2013) , which is an efficient way of exploiting the long sample. For simplicity we partition the moments in two parts: One part where we have yields for full the period from 1961:06-2013:12 and one part where we only have yields starting from 1971:11. The first group contains moments on maturities up to 66 months, while the second group contains those with maturities from 69 to 120 months. In principle, we could make many partitions based on the available observations for each moment, but this will heavily complicate the computations. We refer to Lynch and Wachter (2013) and our Online Appendix for further details.

A.4 Bootstrap

We address potential small sample distortions in the inference by a non-parametric stationary bootstrap procedure in order to account for possible time series dependencies and to preserve any cross-sectional dependencies in the data. The bootstrap procedure resamples the data in blocks of consecutive observations of the left and right hand side of the Campbell-Shiller regressions. We determine the optimal average block size using the Politis and White (2004) approach. When drawing the blocks, we make sure to preserve cross-sectional correlations by resampling from the same time points for all maturities. For each of the 100; 000 bootstrap samples, we estimate the coefficients in the Campbell-Shiller regressions.²⁰ Based on a matrix of the $100,000$ simulated coefficient estimates, we can then compute the variances and covariances of the estimates across maturities.

A.5 Step 2 of the SR approach: Regime-Switching in the Time Series Regression

This subsection describes how to estimate (11) by GMM when accounting for measurement errors in the estimated pricing factors. For notational convenience, we express (11) as

$$
\mathbf{x}_{t+1} = \mathbf{1}_{\{z_t \ge c\}} \mathbf{h}_0^{(1)} + \left(1 - \mathbf{1}_{\{z_t \ge c\}}\right) \mathbf{h}_0^{(2)} + \mathbf{h}_{\mathbf{x}}^{(1)} \mathbf{x}_t^{(1)} + \mathbf{h}_{\mathbf{x}}^{(2)} \mathbf{x}_t^{(2)} + \mathbf{w}_{t+1},
$$

where $\mathbf{h}_0^{(i)} \equiv \boldsymbol{\lambda}_0^{(i)}$ $\mathbf{b}_0^{(i)}$, $\mathbf{h}_{\mathbf{x}}^{(i)} \equiv \mathbf{I} - \mathbf{\Phi} + \boldsymbol{\lambda}_{\mathbf{x}}^{(i)}$, $\mathbf{x}_t^{(1)} \equiv 1_{\{z_t \geq c\}} \mathbf{x}_t$, $\mathbf{x}_t^{(2)} \equiv (1 - 1_{\{z_t \geq c\}}) \mathbf{x}_t$, and $\mathbf{w}_{t+1} \equiv \boldsymbol{\Sigma}_{\mathbf{x}} \boldsymbol{\epsilon}_{t+1}$ for $i = \{1, 2\}$. Using the same procedure as in Andreasen and Christensen (2015), we estimate θ_2^*

 20 We require at least 15 recession observations in each simulated sample.

based on

$$
\begin{bmatrix}\nE\left[\hat{\mathbf{w}}_{t+1}\left(1-1_{\{z_t\geq c\}}\right)\right] \\
E\left[\hat{\mathbf{w}}_{t+1}\left(\hat{\mathbf{x}}_t^{(1)}\right)'\right] \\
E\left[\hat{\mathbf{w}}_{t+1}\left(\hat{\mathbf{x}}_t^{(2)}\right)'\right] \\
E\left[\hat{\mathbf{w}}_{t+1}\left(\hat{\mathbf{x}}_t^{(2)}\right)'\right]\n\end{bmatrix} = \begin{bmatrix}\n0 \\
0 \\
Cov\left(\mathbf{u}_{t+1}^{(1)}, \mathbf{u}_t^{(1)}\right) - \mathbf{h}_{\mathbf{x}}^{(1)}Var\left(\mathbf{u}_t^{(1)}\right) \\
Cov\left(\mathbf{u}_{t+1}^{(2)}, \mathbf{u}_t^{(2)}\right) - \mathbf{h}_{\mathbf{x}}^{(2)}Var\left(\mathbf{u}_t^{(2)}\right) \\
Var\left(\hat{\mathbf{w}}_{t+1}\right)\n\end{bmatrix},\n\tag{18}
$$

where

$$
\mathbf{\Omega}_{t+1} = Var(\mathbf{u}_{t+1}) + \mathbf{h}_{\mathbf{x}}^{(1)} Var(\mathbf{u}_{t}^{(1)}) (\mathbf{h}_{\mathbf{x}}^{(1)})' + \mathbf{h}_{\mathbf{x}}^{(2)} Var(\mathbf{u}_{t}^{(2)}) (\mathbf{h}_{\mathbf{x}}^{(2)})' - Cov(\mathbf{u}_{t+1}^{(1)}, \mathbf{u}_{t}^{(1)}) (\mathbf{h}_{\mathbf{x}}^{(1)})' - \mathbf{h}_{\mathbf{x}}^{(1)} Cov(\mathbf{u}_{t}^{(1)}, \mathbf{u}_{t+1}^{(1)}) - Cov(\mathbf{u}_{t+1}^{(2)}, \mathbf{u}_{t}^{(2)}) (\mathbf{h}_{\mathbf{x}}^{(2)})' - \mathbf{h}_{\mathbf{x}}^{(2)} Cov(\mathbf{u}_{t}^{(2)}, \mathbf{u}_{t+1}^{(2)})
$$

Here, \mathbf{u}_t refers to the estimation uncertainty in the estimated pricing factors, i.e. $\hat{\mathbf{x}}_t = \mathbf{x}_t + \mathbf{u}_t$, where $\mathbf{u}_t^{(1)} \equiv 1_{\{z_t \geq c\}} \mathbf{u}_t$ and $\mathbf{u}_t^{(2)} \equiv (1 - 1_{\{z_t \geq c\}}) \mathbf{u}_t$. As in Andreasen and Christensen (2015), all required moments of \mathbf{u}_t follow from the first step of the SR approach. The solution to the first four moments conditions in (18) is given in closed form by

$$
\begin{bmatrix}\n\widehat{\mathbf{h}}_0^{(1)} & \widehat{\mathbf{h}}_0^{(2)} & \widehat{\mathbf{h}}_{\mathbf{x}}^{(1)} & \widehat{\mathbf{h}}_{\mathbf{x}}^{(2)}\n\end{bmatrix} = \left(\sum_{t=1}^{T-1} \widehat{\mathbf{x}}_{t+1} \mathbf{a}_t' - \sum_{t=1}^{T-1} \widehat{\mathbf{A}}_{t+1}\right) \left(\sum_{t=1}^{T-1} \mathbf{a}_t \mathbf{a}_t' - \sum_{t=1}^{T-1} \widehat{Var}(\mathbf{u}_{\mathbf{a},t})\right)^{-1},
$$

where

$$
\mathbf{a}_t \equiv \left[\begin{array}{c}1_{[z_t \geq c]} \\ 1 - 1_{[z_t \geq c]} \\ \hat{\mathbf{x}}_t^{(1)} \\ \hat{\mathbf{x}}_t^{(2)}\end{array}\right] \qquad \mathbf{u}_{\mathbf{a},t} \equiv \left[\begin{array}{c}0 \\ 0 \\ \mathbf{u}_t^{(1)} \\ \mathbf{u}_t^{(2)}\end{array}\right]
$$

and $\mathbf{\hat{A}}_{t+1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \widehat{Cov} \end{bmatrix} \mathbf{u}_{t+1}^{(1)}, \mathbf{u}_t^{(1)}$ t $\widehat{Cov}\left(\mathbf{u}_{t+1}^{(2)}, \mathbf{u}_t^{(2)}\right)$ $\begin{pmatrix} 2 \ t \end{pmatrix}$. The solution to the last moment condition in (18) is also given in closed form and implies the following estimator

$$
\widehat{Var}(\mathbf{w}_{t+1}) = \frac{1}{T - 1 - 2(n_x + 1)} \sum_{t=1}^{T-1} \widehat{\mathbf{w}}_{t+1} \widehat{\mathbf{w}}'_{t+1} - \frac{1}{T - 1} \sum_{t=1}^{T-1} \widehat{\mathbf{\Omega}}_{t+1},
$$
\n(19)

where

$$
\hat{\mathbf{\Omega}}_{t+1} = \widehat{Var}(\mathbf{u}_{t+1}) + \hat{\mathbf{h}}_{\mathbf{x}}^{(1)} \widehat{Var}(\mathbf{u}_{t}^{(1)}) (\hat{\mathbf{h}}_{\mathbf{x}}^{(1)})' + \hat{\mathbf{h}}_{\mathbf{x}}^{(2)} \widehat{Var}(\mathbf{u}_{t}^{(2)}) (\hat{\mathbf{h}}_{\mathbf{x}}^{(2)})'
$$
\n
$$
-\widehat{Cov}(\mathbf{u}_{t+1}^{(1)}, \mathbf{u}_{t}^{(1)}) (\hat{\mathbf{h}}_{\mathbf{x}}^{(1)})' - \hat{\mathbf{h}}_{\mathbf{x}}^{(1)} \widehat{Cov}(\mathbf{u}_{t}^{(1)}, \mathbf{u}_{t+1}^{(1)})
$$
\n
$$
-\widehat{Cov}(\mathbf{u}_{t+1}^{(2)}, \mathbf{u}_{t}^{(2)}) (\hat{\mathbf{h}}_{\mathbf{x}}^{(2)})' - \hat{\mathbf{h}}_{\mathbf{x}}^{(2)} \widehat{Cov}(\mathbf{u}_{t}^{(2)}, \mathbf{u}_{t+1}^{(2)})
$$

 $\text{with} \ \ \widehat{\hat{\mathbf{w}}}_{t+1} \equiv \hat{\mathbf{x}}_{t+1} - \mathbb{1}_{\{z_t \geq c\}} \hat{\mathbf{h}}_0^{(1)} - \big(1 - \mathbb{1}_{\{z_t \geq c\}}\big) \hat{\mathbf{h}}_0^{(2)} - \hat{\mathbf{h}}_{\mathbf{x}}^{(1)} \hat{\mathbf{x}}_t^{(1)} - \hat{\mathbf{h}}_{\mathbf{x}}^{(2)} \hat{\mathbf{x}}_t^{(2)}$ $t^{(2)}$. Note that we adopt a standard degree of freedom correction to the first term in (19) because we estimate $2(n_x + 1)$ unknown parameters per equation in the model. The asymptotic distribution of $\theta_2^{\mathbf{x}}$ for $T \longrightarrow \infty$ follows from Hansen (1982) when applied on the moment conditions in (18). The estimated loadings in the market prices of risk are then given by $\hat{\lambda}_0^{(i)} = \hat{\mathbf{h}}_0^{(i)}$ and $\hat{\lambda}_{\mathbf{x}}^{(i)} = \hat{\mathbf{h}}_{\mathbf{x}}^{(i)}$ - $(I - \hat{\Phi})$ for $i = \{1, 2\}.$ Finally, the standard errors for $\hat{\lambda}_0^{(i)}$ and $\hat{\lambda}_{\mathbf{x}}^{(i)}$ are identical to those for $\hat{\mathbf{h}}_0^{(i)}$ and $\hat{\mathbf{h}}_{\mathbf{x}}^{(i)}$, respectively. That is, we omit uncertainty about $\hat{\Phi}$, because this estimator uses $T n_y$ observations and therefore tends faster to infinity than $\hat{\theta}_2^{\mathbf{x}}$ when also $n_y \longrightarrow \infty$, as noted in Andreasen and Christensen (2015).

A.6 Step 2 of the SR Approach: Inducing Stationarity

If the stability condition for \mathbf{x}_t is not satisfied, we then downscale $\mathbf{I} - \mathbf{\Phi} + \mathbf{\lambda}_{\mathbf{x}}^{(i)}$ by δ_i for $i = \{1, 2\}$ if the eigenvalues of $I - \Phi + \lambda_{\mathbf{x}}^{(i)}$ are greater than or equal to one. The values of δ_1 and δ_2 are determined as in Andreasen and Meldrum (2014), i.e. by

$$
(\delta_1, \delta_2) = \arg \min_{\{\delta_{lower} \leq \delta_i < 1\}_{i=1}^2} \sum_{i=1}^{n_x} \left(\frac{\sigma_{i, model}^2 (\delta_1, \delta_2) - \sigma_{i, sample}^2}{\sigma_{i, sample}^2} \right)^2.
$$

We follow Andreasen and Meldrum (2014) and estimate the unconditional variance of the ith pricing factor in the sample from $\{\hat{x}_{i,t}\}_{i=1}^T$ using

$$
\hat{\sigma}_{i,sample}^{2} = \frac{1}{T-1} \sum_{t=1}^{T} (\hat{x}_{i,t} - \hat{E} [\hat{x}_{i}])^{2} - \frac{1}{T} \sum_{t=1}^{T} \widehat{Var} (u_{i,t}),
$$

where $\hat{E} [\hat{x}_{i,t}] = 1/T \sum_{t=1}^T \hat{x}_{i,t}$ and $\widehat{Var}(u_{i,t})$ refers to the estimated variance of $\hat{x}_{i,t}$. The value of the unconditional variance of $x_{i,t}$ in the model is computed by simulation, using

$$
\mathbf{x}_{t+1} = 1_{[z_t \geq c]} \lambda_0^{(1)} + (1 - 1_{[z_t \geq c]}) \lambda_0^{(2)} + (\delta_1 1_{[z_t \geq c]} \left\{ \mathbf{I} - \mathbf{\Phi} + \lambda_{\mathbf{x}}^{(1)} \right\} + \delta_2 (1 - 1_{[z_t \geq c]}) \left\{ \mathbf{I} - \mathbf{\Phi} + \lambda_{\mathbf{x}}^{(2)} \right\} \right) \mathbf{x}_t + \mathbf{\Sigma}_{\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{x},t+1}^{\mathbb{P}}
$$

and (12).

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Table 1: Wald tests: Campbell-Shiller Regressions

The table reports asymptotic and bootstrap Wald statistics denoted W_{as} and W_{boot} , respectively. Asymptotic p-values are in parentheses and bootstrap p-values in brackets. The dataset is unsmoothed Fama-Bliss interest rates. Panel A reports results for the full sample period from 1961:06 to 2013:12. Here we use the Lynch and Wachter (2013) adjusted moments estimator as the panel of bond yields is unbalanced. For the two subperiods in panels B and C, we have observations on all maturities and use standard GMM estimation. For these subperiods, we both report results using a standard asymptotic GMM covariance matrix (Newey-West) and a small sample covariance matrix. The small sample estimates are based on a stationary bootstrap where we resample blocks of the left and right hand side of the Campbell-Shiller regressions with optimal average block size using the Politis and White (2004) approach. When drawing the blocks, we make sure to preserve cross-sectional correlations by resampling from the same time points for all maturities. The covariance matrix is computed using a matrix of 100,000 simulated coefficient estimates. Further details are provided in the Online Appendix.

Table 2: Return Predictability by the Yield Spread: Robustness

Panel A shows the results of regressing excess returns on a 10-year Treasury bond from Global Financial Data on the 10-year yield spread, whereas Panel B shows the results of regressing excess returns on a 20-year Treasury bond from Ibbotson on the 20-year yield spread obtained from the Goyal and Welch dataset. Excess returns are in both panels computed for the next three months with recessions and expansions identified using the NBER recession dates. Newey-West standard errors with 4 lags are provided in parentheses. T is the total number of observations and T_{BEC} is the number of recession months.

μ paremeness. The che covar number of observations and $I_{\tilde{R}\tilde{L}\tilde{U}}$ Sample period									
				Recessions vs. expansions					
	μ	θ		μ_{EXP}	μ_{REC}	θ_{EXP}	θ_{REC}	$\overline{R^2}$	
1053	-0.006	0.70	5.3%	-0.010	0.012	0.83	0.05	7.7%	
(199)	(0.003)	(0.16)		(0.003)	(0.008)	(0.17)	(0.56)		
422	-0.002	0.37	4.5%	-0.007	0.014	0.58	-0.24	14.5%	
(116)	(0.002)	(0.14)		(0.002)	(0.004)	(0.14)	(0.29)		
628	-0.007	0.83	5.8%	-0.010	0.012	0.91	0.29	7.5%	
(83)	(0.004)	(0.22)		(0.004)	(0.016)	(0.22)	(1.01)		
1053	-0.007	0.73	5.0%	-0.014	0.021	0.98	-0.42	9.2%	
(199)	(0.003)	(0.18)		(0.003)	(0.009)	(0.19)	(0.50)		
422	-0.001	0.36	2.3%	-0.008	0.019	0.61	-0.36	11.8%	
								9.2%	
(83)	(0.005)	(0.23)		(0.004)	(0.016)	(0.22)	(0.76)		
	T (T_{REC}) Panel A: 10-year bond returns Panel B: 20-year bond returns (116) 628	(0.003) -0.009	(0.19) 0.85	No switch $\overline{R^2}$ 5.8%	(0.003) -0.016	(0.005) 0.024	(0.19) 1.08	(0.37) -0.45	

Table 3: Estimation Results: Risk-Neutral Coefficients and Dynamics of PMI Given the low estimates of $\Phi(1,1)$, the value of α is unidentified and set to 0 for all models. Asymptotic standard errors for $\hat{\Phi}$ in brackets are robust to the measurement errors $v_{t,k}$ displaying heteroskedasticity in the time series dimension, cross-sectional correlation, and autocorrelation. We use $w_D = 5$ and $w_T = 10$ in the provided estimator of Andreasen and Christensen (2015). The asymptotic standard errors for $\hat{\Sigma}_{\mathbf{x}}$ and the parameters for PMI are computed as described in Appendix A.5 and reported in parentheses. Significance at the 10 and 5 percent level is denoted by $*$ and $**$, respectively. $=$

	\sim \sim \sim \sim \sim \mathcal{M}^{ATSM}	\sim \sim \sim \sim \sim $\mathcal{M}_{\lambda_0}^{\overline{ATSM}}$	$,$ \ldots $,$ \ldots $,$ \mathcal{M}^{ATSM}_χ	$\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$
$\Phi\left(1,1\right)$	1.18×10^{-8}	1.54×10^{-8}	1.19×10^{-8}	1.28×10^{-8}
	(4.38×10^{-4})	(4.37×10^{-4})	(7.41×10^{-4})	(4.37×10^{-4})
$\Phi(2,2)$	$0.0352**$	$0.0353**$	$0.0309**$	$0.0352**$
	(0.0046)	(0.0046)	(0.0054)	(0.0046)
$\Phi(3,3)$	$0.0416**$	$0.0416**$	$0.0350**$	$0.0416**$
	(0.0044)	(0.0044)	(0.0047)	(0.0044)
$\Sigma_{\mathbf{x}}(1,1)$	$3.64 \times 10^{-4**}$	$3.64 \times 10^{-4**}$	$5.39 \times 10^{-4**}$	$3.65 \times 10^{-4**}$
	(1.97×10^{-5})	(1.96×10^{-5})	(2.46×10^{-5})	(1.94×10^{-5})
$\Sigma_{\mathbf{x}}(2,1)$	$-0.0034**$	$-0.0034**$	$-0.0049**$	$-0.0034**$
	(3.61×10^{-4})	(3.63×10^{-4})	(5.32×10^{-4})	(3.57×10^{-4})
$\Sigma_{\mathbf{x}}(2,2)$	$0.0042**$	$0.0042**$	$0.0068**$	$0.0041**$
	(1.69×10^{-4})	(1.71×10^{-4})	(2.51×10^{-4})	(1.68×10^{-4})
$\Sigma_{\mathbf{x}}(3,1)$	$0.0031**$	$0.0031**$	$0.0045**$	$0.0031**$
	(3.45×10^{-4})	(3.48×10^{-4})	(5.09×10^{-4})	(3.42×10^{-4})
$\Sigma_{\mathbf{x}}(3,2)$	$-0.0041**$	$-0.0041**$	$-0.0066**$	$-0.0041**$
	$(1.71{\times}10^{-4})$	(1.72×10^{-4})	(2.49×10^{-4})	(1.69×10^{-4})
$\Sigma_{\mathbf{x}}\left(3,3\right)$	$3.63 \times 10^{-4**}$	$3.51 \times 10^{-4**}$	$4.06 \times 10^{-4**}$	$3.40 \times 10^{-4**}$
	(4.05×10^{-5})	(3.66×10^{-5})	(1.98×10^{-5})	(2.24×10^{-5})
PMI				
γ_0		$0.0094**$ (0.0043)	$0.0095**$ (0.0044)	$0.0094**$ (0.0043)
γ_z		$0.9338**$ (0.0180)	$0.9338**$ (0.0180)	$0.9338**$ (0.0180)
$\gamma_x(1,1)$		$-1.4022**$ (0.6955)	$-1.3459**$ (0.6775)	$-1.4023**$ (0.6955)
$\gamma_x(2,1)$		$-3.1301**$ (0.6795)	$-2.9007**$ (0.6413)	$-3.1288**$ (0.6793)
$\gamma_x(3,1)$		$-3.4533**$ (0.7357)	$-3.1354**$ (0.6791)	$-3.4552**$ (0.7361)
Σ_{zz}		$0.0222**$ $(8.87{\times}10^{-4})$	$0.0222**$ $(8.87{\times}10^{-4})$	$0.0222**$ (8.87×10^{-4})

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Estimates of λ_0 are common across expansions (EXP) and recessions (REC) in \mathcal{M}_x^{ATSM} , while estimates of λ_x are common across expansions and \mathcal{M}_x^{ATSM} , while estimates of λ_x are common across expansions Estimates of λ_0 are common across expansions (EXP) and recessions (REC) in $\mathcal{M}^{ATSM}_{\lambda_x}$, while estimates of λ_x are common across expansions and Table 4: Estimation Results: The Market Prices of Risk

Table 5: Slope Coefficients in Modified Mincer-Zarnowitz Regressions This table reports the regression slopes of modified Mincer-Zarnowitz regressions of realized three months excess returns on a constant and model-implied expected excess returns when conditioning on the state of the economy by using the NBER recession dates. Excess returns are pooled within bins of two years for these regressions, given a total of eight excess returns per reported maturity range. Newey-West standard errors with 6 lags are provided in parentheses.

---a-	Expansions				Recessions					
	Maturity range in years:				Maturity range in years:					
	$2 - 4$ $0-2$ $4-6$ $6 - 8$ $8 - 10$				$0 - 2$	$2 - 4$	$4-6$	$6 - 8$	$8-10$	
$\mathcal{\bar{M}}^{ATSM}$	0.976	1.163	1.176	1.120	0.997	0.624	0.390	0.097	-0.093	-0.050
	(0.115)	(0.094)	(0.098)	(0.106)	(0.100)	(0.775)	(0.565)	(0.526)	(0.551)	(0.487)
$\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$	0.648	0.760	0.759	0.695	0.612	0.754	0.667	0.656	0.612	0.544
	(0.090)	(0.069)	(0.073)	(0.078)	(0.076)	(0.103)	(0.074)	(0.085)	(0.100)	(0.102)
$\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$	0.709	0.836	0.812	0.735	0.650	0.799	0.668	0.630	0.593	0.525
	(0.099)	(0.074)	(0.075)	(0.079)	(0.078)	(0.109)	(0.081)	(0.092)	(0.105)	(0.104)
$\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$	0.775	0.860	0.812	0.682	0.578	0.710	0.707	0.737	0.731	0.732
	(0.103)	(0.081)	(0.083)	(0.084)	(0.081)	(0.138)	(0.099)	(0.108)	(0.132)	(0.130)
\mathcal{M}^{Linear}	0.937	1.139	$1.263\,$	1.204	1.072	0.306	0.089	-0.063	-0.159	-0.159
	(0.088)	(0.082)	(0.090)	(0.099)	(0.093)	(0.404)	(0.389)	(0.394)	(0.428)	(0.391)
${\cal M}^{Spanned}$	0.927	1.145	1.275	1.222	1.118	0.326	0.109	-0.036	-0.119	-0.117
	(0.087)	(0.082)	(0.090)	(0.099)	(0.095)	(0.392)	(0.381)	(0.387)	(0.416)	(0.380)

Table 6: Additional Model Estimates: Risk-neutral Coefficients and Dynamics of PMI Given the low estimates of $\Phi(1,1)$, the value of α is unidentified and set to 0 for all models. Asymptotic standard errors for $\hat{\Phi}$ in brackets are robust to the measurement errors $v_{t,k}$ displaying heteroskedasticity in the time series dimension, cross-sectional correlation, and autocorrelation. We use $w_D = 5$ and $w_T = 10$ in the provided estimator of Andreasen and Christensen (2015). The asymptotic standard errors for $\hat{\Sigma}_{\mathbf{x}}$ and the parameters for PMI are computed as described in Appendix A.5 and reported in parentheses. For $\mathcal{M}^{Spanned}$, we let $\mu(4, 1) = \Phi(4, i) = \Phi(i, 4) = 0$ for $i = \{1, 2, 3\}$ as the fit of bond yields is completely unaffected by these coefficients. Significance at the 10 and 5 percent level is denoted by $*$ and $**$, respectively. $=$

	$\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$	$\overline{\mathcal{M}^{AFNS}_\cdot}$ $\lambda_0(2,1)\lambda_x(2,2)$	$\overline{\mathcal{M}^{Linear}}$	$\overline{\mathcal{M}^{Spanned}}$
$\beta(4,1)$				$0.1283**$ (0.0204)
$\tilde{\lambda}$		$0.0388**$ (0.0021)		
$\Phi(1,1)$	1.00×10^{-8} (3.64×10^{-4})		1.09×10^{-8} (4.44×10^{-4})	1.11×10^{-8} (6.92×10^{-4})
$\Phi(2,2)$	$0.0351**$ (0.0061)		$0.0352**$ (0.0046)	0.0430 (0.1243)
$\Phi(3,3)$	$0.0536**$ (0.0053)		$0.0416**$ (0.0044)	0.0447 (0.1281)
$\Phi(4,4)$				$0.0513**$ (0.0171)
$\Sigma_{\mathbf{x}}(1,1)$	$3.44 \times 10^{-4**}$ (1.67×10^{-5})	$3.67 \times 10^{-4**}$ (1.98×10^{-5})	$3.65 \times 10^{-4**}$ (3.06×10^{-5})	$3.16 \times 10^{-4**}$ (2.31×10^{-5})
$\Sigma_{\mathbf{x}}(2,1)$	$-0.0012**$ (1.36×10^{-4})	$-0.0003**$ (2.66×10^{-5})	$-0.0034**$ (5.38×10^{-4})	$-0.0091**$ (0.0018)
$\Sigma_{\mathbf{x}}(2,2)$	$0.0019**$ (7.17×10^{-5})	$3.56 \times 10^{-4**}$ (2.60×10^{-5})	$0.0042**$ (2.01×10^{-4})	$0.0194**$ (0.0010)
$\Sigma_{\mathbf{x}}(3,1)$	$9.68 \times 10^{-4**}$ (1.25×10^{-4})	$-0.0006**$ (6.06×10^{-5})	$0.0031**$ (5.12×10^{-4})	$0.0088**$ (0.0019)
$\Sigma_{\mathbf{x}}(3,2)$	$-0.0018**$ (7.55×10^{-5})	$7.92 \times 10^{-5*}$ (4.34×10^{-5})	$-0.0041**$ (2.01×10^{-4})	$-0.0210**$ (0.0011)
$\Sigma_{\mathbf{x}}(3,3)$	$3.69 \times 10^{-4**}$ (2.19×10^{-5})	$7.05 \times 10^{-4**}$ (2.97×10^{-5})	$3.53 \times 10^{-4**}$ (6.19×10^{-5})	$0.0023**$ (1.07×10^{-4})
$\Sigma_{\mathbf{x}}(4,1)$			6.62×10^{-4} $(9.02{\times}10^{-4})$	2.71×10^{-4} (8.72×10^{-4})
$\Sigma_{\mathbf{x}}(4,2)$			$0.0040**$ (0.0012)	$0.0134**$ (0.0013)
$\Sigma_{\mathbf{x}}\left(4,3\right)$			$0.0051**$ (0.0019)	$-0.0176**$ (9.03×10^{-4})
$\Sigma_{\mathbf{x}}(4,4)$			$0.0213**$ (9.81×10^{-4})	$0.0027**$ (4.51×10^{-4})
PMI				
γ_0	$0.0101**$ (0.0042)	$0.0094**$ (0.0042)		
γ_z	$0.9320**$ (0.0180)	$0.9338**$ (0.0180)		
$\gamma_x(1,1)$	$-1.4882**$ (0.6842)	$-1.4000**$ (0.6948)		
$\gamma_x(2,1)$	$-3.1238**$ (0.6649)	$-3.2161**$ (0.6911)		
$\gamma_x(3,1)$	$-3.8735***$ (0.8001)	1.8307** (0.5536)		
Σ_{zz}	$0.0222**$ (8.88×10^{-4})	$0.0221**$ (8.90×10^{-4})		

	significance at the 10 and 9 percent level is denoted by $\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$		anu $\overline{\mathcal{M}^{AFNS}}$, respectively. $\lambda_0(2,1)\lambda_x(2,2)$	$\overline{\mathcal{M}}^{Linear}$		$M^{Spanned}$		
	EXP	REC	EXP	REC	EXP	REC	EXP	REC	
Intercepts:									
$\lambda_0(1,1)$	$2.33 \times 10^{-4**}$ (5.23×10^{-5})	$5.84 \times 10^{-4*}$ (3.40×10^{-4})		$3.17 \times 10^{-4**}$ (5.84×10^{-5})	$2.36 \times 10^{-4**}$ (7.24×10^{-5})		$1.96 \times 10^{-4**}$ (6.43×10^{-5})		
$\lambda_0(2,1)$	$-0.0012**$ (3.50×10^{-4})	-0.0014 (0.0017)	$-0.0002**$ $-0.0007**$ (7.56×10^{-5}) (2.67×10^{-4})		$-0.0030**$ (0.0010)		-0.0064 (0.0040)		
$\lambda_0(3,1)$	$0.0011**$ $(3.41{\times}10^{-4})$	6.30×10^{-4} (0.0016)		$-0.0005**$ (1.56×10^{-4})	$0.0027**$ (9.75×10^{-4})		0.0052 (0.0044)		
$\lambda_0(4,1)$					$0.0094**$ (0.0044)		0.0070 (0.0047)		
Slopes:									
$\lambda_{\mathbf{x}}(1,1)$	$-0.0338**$ (0.0093)	$-0.0728*$ (0.0408)		$-0.0468**$ (0.0097)	$-0.0315**$ (0.0113)		$-0.0237**$ (0.0099)		
$\lambda_{\mathbf{x}}(1,2)$	$0.0206**$ (0.0085)	0.0376 (0.0457)		$0.0200**$ (0.0093)		$0.0224**$ (0.0098)		$0.0230**$ (0.0085)	
$\lambda_{\mathbf{x}}(1,3)$	0.0156 (0.0103)	0.0183 (0.0414)	$0.0214**$ (0.0101)		$0.0188*$ (0.0105)		$0.0224**$ (0.0087)		
$\lambda_{\mathbf{x}}(1,4)$					6.44×10^{-6} (1.75×10^{-4})		$0.0026**$	(0.0012)	
$\lambda_{\mathbf{x}}(2,1)$	$0.2008**$ (0.0666)	0.1186 (0.2103)		$0.0326**$ (0.0146)	$0.4888**$ (0.1479)		$1.1312**$	(0.5562)	
$\lambda_{\mathbf{x}}(2,2)$	0.0889 (0.0723)	-0.2956 (0.2527)	0.0081 (0.0138)	$-0.2363**$ (0.1183)	0.0734 (0.1846)		-0.8757 (0.7308)		
$\lambda_{\mathbf{x}}(2,3)$	$0.1766**$ (0.0854)	-0.2380 (0.2903)		$-0.0404**$ (0.0156)	0.1550 (0.1960)		-0.8365 (0.7401)		
$\lambda_{\mathbf{x}}(2,4)$					-0.0024 (0.0031)		-0.1060 (0.1024)		
$\lambda_{\mathbf{x}}(3,1)$	$-0.1801**$ (0.0674)	-0.0880 (0.1972)		$0.0853**$ (0.0279)	$-0.4658**$ (0.1452)		-0.9329 (0.6010)		
$\lambda_{\mathbf{x}}(3,2)$	-0.0821 (0.0722)	-0.0039 (0.2155)		0.0179 (0.0305)	-0.0731 (0.1781)		$1.3262*$ (0.7677)		
$\lambda_{\mathbf{x}}(3,3)$	$-0.1516*$ (0.0853)	-0.0769 (0.2431)		$-0.0925**$ (0.0264)	-0.1469 (0.1894)		$1.3000*$ (0.7773)		
$\lambda_{\mathbf{x}}(3,4)$					0.0037 (0.0030)		0.1738 (0.1074)		
$\lambda_{\mathbf{x}}(4,1)$					$-1.4020**$ (0.6978)		$-1.4433**$ (0.7166)		
$\lambda_{\mathbf{x}}(4,2)$					$-3.1287**$ (0.6690)		$-3.4870**$ (0.7216)		
$\lambda_{\mathbf{x}}(4,3)$					$-3.4530**$ (0.7086)		$-3.5736**$ (0.7326)		
$\lambda_{\mathbf{x}}(4,4)$					$-0.0662**$ (0.0164)		$-0.5074**$ (0.0981)		

Table 7: Additional Model Estimates: The Market Prices of Risk
The asymptotic standard errors are computed as described in Appendix A.5 and reported in parentheses. Significance at the 10 and 5 percent level is denoted by $*$ and $**$, respectively.

Figure 1: Ordinary and Modified Campbell-Shiller Regressions

Estimation results from ordinary and modified Campbell-Shiller regressions are provided in the upper and lower part of the figure, respectively, using monthly data from 1961:6 to 2013:12. The dataset is unsmoothed Fama-Bliss (FB) interest rates or the data companying Gürkaynak, Sack and Wright (2007), denoted GSW. Intercepts are multiplied by 100.

Figure 2: Goodness-of-fit in Ordinary and Modified Campbell-Shiller Regressions The goodness of fit as measured by the R^2 statistic (in percent) for the ordinary and modified Campbell-Shiller regressions using monthly data from 1973:1 to 2013:12. The dataset is unsmoothed Fama-Bliss (FB) interest rates or the data companying Gürkaynak, Sack and Wright (2007), denoted GSW.

Figure 3: Ordinary and Modified Return Regressions

These charts report the slope coefficients in return regressions where excess bond returns three months ahead are regressed on either the yield spread, the forward spread, or the CP factor (with a constant) using monthly data from 1961:6 to 2013:12. The dataset is unsmoothed Fama-Bliss (FB) interest rates or the data companying Gürkaynak, Sack and Wright (2007), denoted GSW. Given the well-known reliance of the CP factor on FB forward rates, the CP factor for the GSW dataset is computed as $cp_t^{GSW} = (\hat{\gamma}^{GSW})' F_t$, where $F_t \equiv \left[1, y_{t,12}, f_t^{(12,24)}, f_t^{(24,36)}, f_t^{(36,48)}, f_t^{(48,60)}\right]'$ are FB forward rates but $\hat{\gamma}^{GSW}$ is obtained by regressing GSW returns on FB forward rates.

Figure 4: Model Evaluation: Ordinary and Modified Campbell-Shiller Regressions The empirical intercepts and slope coefficients in ordinary and modified Campbell-Shiller regressions are computed based on monthly unsmoothed Fama-Bliss interest rates from 1961:6 to 2013:12. The model-implied regression loadings are computed at the estimated parameters for $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ using a simulated sample path of 100,000 observations. Intercepts are multiplied by 100.

Figure 5: Model Evaluation: Ordinary and Modified return Regressions The empirical slope coefficients in ordinary and modified return regressions are computed based on monthly unsmoothed Fama-Bliss interest rates from 1961:6 to 2013:12. The model-implied regression loadings are computed at the estimated parameters for $\mathcal{M}_{\lambda_0\lambda_x}^{ATSM}$ using a simulated sample path of 100,000 observations.

Figure 6: Excess Returns and Term Premia: The Gaussian ATSM vs. Regime-Dependent ATSM
The top chart reports quarterly expected excess returns for the 10-year bond when expressed in annualized percent. Term premia (in perce The top chart reports quarterly expected excess returns for the 10-year bond when expressed in annualized percent. Term premia (in percent) at $\left(\hat{\boldsymbol{\theta}}^{step3}_{11}, \hat{\boldsymbol{\Sigma}}^{step2}\right)$ With regime switching, expected excess returns and term premia are computed by Monte Carlo integration using 10; 000 draws. Shaded areas Figure 6: Excess Returns and Term Premia: The Gaussian ATSM vs. Regime-Dependent ATSM
t reports quarterly expected excess returns for the 10-year bond when expressed in annualized percent. Term premia (in p]. Both measures of bond risk premia are computed at the estimated states, i.e. $\hat{\mathbf{x}}_t$ \mathbb{E}_t $[r_{t+i}$ $\frac{1}{k}\sum_{i=0}^{k-1}$ $y_{t,k}$ denote NBER recessions. denote NBER recessions. is deÖned as یمه maturity

Figure 7: SRM with Regime-Switching: Ordinary and Modified Campbell-Shiller Regressions

The empirical intercepts and slope coefficients in ordinary and modified Campbell-Shiller regressions are computed based on monthly unsmoothed Fama-Bliss interest rates from 1961:6 to 2013:12. The model-implied regression loadings are computed at the estimated parameters for $\mathcal{M}_{\lambda_0\lambda_x}^{SRM}$ using a simulated sample path of 100,000 observations. Intercepts are multiplied by 100.

Figure 8: AFNS Model with Regime-Switching: Ordinary and Modified Campbell-Shiller Regressions

The empirical intercepts and slope coefficients in ordinary and modified Campbell-Shiller regressions are computed based on monthly unsmoothed Fama-Bliss interest rates from 1961:6 to 2013:12. The model-implied regression loadings are computed at the estimated parameters for $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ using a simulated sample path of 100,000 observations. Intercepts are multiplied by 100.

Figure 9: AFNS Model with Regime-Switching: Excess Returns We report annualized quarterly excess return for the 10-year bond in $\mathcal{M}_{\lambda_0(2,1)\lambda_x(2,2)}^{AFNS}$ computed by the Monte Carlo method using 10,000 draws. The corresponding excess return in the data is only available from the start of the 1970s due to the availability of the 10-year bond yield. The partial effect of a switch in $\lambda_0(2,1)$ is the difference in excess returns between $\mathcal{M}_{\lambda_0(2,1)}^{AFNS}$, where $\lambda_x^{(2)}(2,2) = \lambda_x^{(1)}(2,2)$, and \mathcal{M}^{AFNS} , where $\lambda_0^{(2)}(2,1) = \lambda_0^{(1)}(2,1)$ and $\lambda_x^{(2)}(2,2) = \lambda_x^{(1)}(2,2)$, and similarly for the effect of a switch in $\lambda_x(2,2)$. All returns and interest rates are in percent. Shaded regions denote NBER recessions.

Figure 10: Linear and Spanned Models: Ordinary and Modified Campbell-Shiller Regressions

The empirical intercepts and slope coefficients in ordinary and modified Campbell-Shiller regressions are computed based on monthly unsmoothed Fama-Bliss interest rates from 1961:6 to 2013:12. The model-implied regression loadings are computed at the estimated parameters for M_{Linear} and $M_{Spanned}$ using a simulated sample path of 100,000 observations. Intercepts are multiplied by 100.

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