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**Price discovery in a continuous-time setting**

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**Abstract:** We formulate a continuous-time price discovery model and investigate how the standard price discovery measures vary with respect to the sampling frequency. We find that the component share measure is invariant to the sampling frequency, and hence a continuous-time price discovery measure can be identified from discrete sampled prices. Our second contribution consists on proposing an estimation strategy for the continuous-time measure of price discovery, which evolves stochastically on a daily basis. We adopt a kernel-based estimator that compares favourable to the standard daily VECM regression. We compute daily estimates of price discovery and investigate their relationship with trading volume for 10 actively traded stocks in the U.S. from 2007 to 2013.

**JEL classification numbers:** C13, C32, C51, G14

**Keywords:** high-frequency data, price discovery, continuous-time model, sampling frequency, time-varying coefficients

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# 1 Introduction

Equity markets in the U.S. and Europe have experienced an ongoing process of market fragmentation following regulatory policy changes that aimed to increase competition (see Menkveld, 2014; O'Hara, 2015; Menkveld, 2016; among others).[1] The immediate consequence of this process is that the market share of the listing exchanges have decreased dramatically, while the new entrants have captured significant order flow. High-frequency trading has also contributed to scatter quotes across the different exchanges as well as has made markets much faster with time scales of microseconds or even nano-seconds (O'Hara, 2015). In turn, quotes, trades, and information are now dispersed across a variety of exchanges and markets which are populated by players possessing different strategic behaviours. In this context, it becomes relevant to investigate how exchanges and markets impound information to the efficient price of securities in an environment of highly competitive fragmented markets that operate in extremely fast time frames. This paper addresses this issue by formulating a price discovery measure in a continuous-time setting and showing how to identify it from discrete sampled prices.

There are essentially two standard price discovery measures, both in a discrete-time setting. The first comprises any variant of Hasbrouck's (1995) information share that gauges the contribution of each market/venue to the total variation of the efficient price innovation (see, for instance, Grammig, Melvin and Schlag, 2005; Lien and Shrestha, 2009; Fernandes and Scherrer, 2014). The second relies on the permanent-transitory decomposition of Gonzalo and Granger (1995) and Gonzalo and Ng (2001). Applications of this measure to the price discovery analysis are usually named component share and include, among others, Booth, So and Tseh (1999), Chu, Hsieh and Tse (1999) and Figuerola-Ferretti and Gonzalo (2010). The common feature between the information share and the component share is that they both rely on the estimation of the vector error-correction model (VECM). The speed-of-adjustment parameter from the VECM is specifically important for the construction of both measures. Baillie, Booth, Tse and Zabotina (2002), Jong (2002) and Yan and Zivot (2010) provide a formal comparison between component and information shares in a discrete-time setting, showing that they render a similar result, (up to a different normalization), if

[1] Specifically, Regulation ATS (alternative trading systems; RegATS) in 2000, and Regulation National Market System (Reg NMS) in 2007 in the U.S., and Markets in Financial Instruments Directive (MiFiDin) in 2007 in Europe set the foundation to the existence of multiple trading venues linked together and competing for liquidity and trades.

market innovations are contemporaneously uncorrelated and market variances are similar in value.

This paper has two contributions to this literature. First, we examine both component share and information share in a continuous-time setting. In particular, we put forward a $k$-dimensional reduced-rank multivariate Ornstein-Uhlenbeck (OU) process (continuous-time counterpart of the VECM) as the price process for a homogenous asset traded at different venues. Using the exact discretization from our continuous-time price discovery model, we investigate how the component and information shares vary with respect to the sampling frequency. We find that the component share is invariant to the sampling frequency, implying that one may learn about the continuous-time price discovery at lower frequencies. This is in stark contrast with the information share measure, which converges as the sampling frequency decreases to the uninformative value of $1/k$, where $k$ denotes the number of markets in the analysis. Next, we extend the early comparison in discrete-time between component and information share measures (Baillie et al. (2002), Jong (2002) and Yan and Zivot (2010)) to encompass our price process in continuous-time. We show that the required assumption for these two measures to present similar values is unlikely in a continuous-time price process, as discretization (frequency in which prices are sampled) makes the information share to diverge from the component share.

The second contribution concerns the estimation of time-varying price discovery measures in a continuous-time setting. The importance to allow for time variation in the price discovery process has been highlighted in the literature. The standard approach to capture this time-varying nature of price discovery is to estimate daily VEC models (Hasbrouck, 2003; Chakravarty, Gulen and Mayhew, 2004; Hansen and Lunde, 2006; Mizrach and Neely, 2008). Estimating individual daily VECMs essentially boils down to assuming that they are independent across days. Differently, we propose a framework that relaxes this independence assumption to estimate time-varying price discovery measures that evolve smoothly over time and thus exploits the inter-daily information to obtain better finite-sample performance. In particular, we estimate daily speed-of-adjustment parameters in the VECM using Giraitis, Kapetanios and Yates's (2013) kernel methods. This means that we keep the estimates as nonparametric as possible in that our method allows for deterministic or stochastic variation of unknown form in the VECM parameters. This is in sharp contrast with the parametric nature of Ozturk, van der Welv and Dijk's (2017) interesting state-space approach for the estimation of intraday price discovery measures, for instance.

We compare our time-varying price discovery measures with the standard measures based on daily VECM estimates. Through an extensive Monte Carlo exercise we show that our proposed estimation strategy compares favourable to price discovery measures based on daily VECMs. The results indicate that our estimation strategy is able to alleviate most of the noise in the daily VECM estimation, and hence offer a more precise picture of the relative informativeness of each market. We then carry out an empirical application that examines the price informativeness of the New York Stock Exchange (NYSE) relative to the Nasdaq from 2007 to 2013. By entertaining such a long time span, we deviate from the current practice of considering at most one year of data in price discovery analyses at the high frequency level. Using high-frequency midquotes of 10 actively-traded stocks at the NYSE and Nasdaq, we find that there is indeed significant daily variation in the price discovery mechanism. Additionally, to better understand the daily variation in the price discovery mechanism, we study the long-run relationship between price discovery and trading volume. The cointegrating vectors are such that the NYSE contribution to the price discovery increases with the NYSE-Nasdaq volume ratio. We find that, for every stock, the volume ratio between NYSE and Nasdaq seems to respond more significantly to deviations from the long-run equilibrium than the price discovery measure.

The remainder of this paper is organized as follows. Section 2 describes the continuous-time setting for price discovery and discusses how the information share and component share measures are affected by sampling frequency. Section 3 shows how to estimate price discovery measures in a consistent manner accounting for daily stochastic changes in the speed-of-adjustment parameter. Section 4 investigates the finite sample performance of the proposed kernel-based estimator vis-à-vis the daily VECM estimation. Section 5 investigates how the price informativeness of the NYSE relative to the Nasdaq changes over time and the long-run relationship between price discovery and volume. Finally, we summarize our contributions in Section 6.

## 2 Price discovery in continuous time

In this section, we propose a continuous-time model for price discovery and investigate how the component and information share measures change with the sampling frequency. Furthermore, we show how to identify continuous-time price discovery measures from discrete sampled prices.

### 2.1 A continuous-time setting

Let prices for a given asset that trades on multiple venues follow in day $d$ the process

$$\mathrm{d}P_t = \Pi^{(d)} P_t \, \mathrm{d}t + C^{(d)} \, \mathrm{d}W_t, \qquad \text{with} \;\; P_0 = p_0^{(d)}, \tag{1}$$

where $P_t = (p_{1,t}, \ldots, p_{k,t})'$ is a $(k \times 1)$ vector of log-prices with $k$ denoting the number of trading venues, $\Pi^{(d)} = \alpha^{(d)} \beta'$ is a $(k \times k)$ reduced-rank matrix with rank equal to $r = k - 1$, $\alpha^{(d)}$ and $\beta$ are $(k \times r)$ full rank matrices, $W$ is a $k \times 1$ vector of Brownian motions, and $C^{(d)}$ is a $k \times k$ matrix, such that the covariance matrix $\Sigma^{(d)} = C^{(d)} C^{(d)\prime}$ is assumed to be positive definite. Finally, the superscript $(d)$ indicates that $\Pi^{(d)}$ and $C^{(d)}$ may vary on a daily basis.

The reduced-rank Ornstein-Uhlenbeck process in (1) is the continuous-time counterpart of the discrete-time VECM adopted in Hasbrouck (1995). Prices at the different markets should not drift much apart, oscillating around the (latent) efficient price, as they refer to the same asset. Accordingly, there is only one cointegrating relationship ($r = k - 1$), with log-prices sharing the asset's efficient price as a common stochastic trend. We assume without loss of generality that $\beta$ is known and constant across days. In turn, $\alpha^{(d)}$ determines how quickly each market reacts to deviations from the long-run equilibria given by $\beta' P_t$.

We assume prices are observed regularly and equidistantly over $[0, 1]$, which is considered as one trading day (calendar-time sampling, as discussed in Hansen and Lunde, 2006). Denote each interval contained in $[0, 1]$ as $[t_i, t_{i-1}]$, where $i = 1, 2, \ldots, n$ and $n$ is the total number of intervals such that $0 = t_0 < t_1 < \ldots < t_n = 1$. The length of each interval is $\delta = t_i - t_{i-1} = 1/n$, and therefore $\delta$ denotes the frequency of observations in $[0, 1]$. For instance, the usual trading day in the U.S. market lasts for 6.5 hours (23,400 seconds), and so sampling at the 1-minute frequency yields $n = 390$ and $\delta = 1/390$. Next, denote by $\exp(A)$ the matrix exponential of a $(k \times k)$ matrix $A$ such that $\exp(A) = \sum_{\ell=0}^{\infty} \frac{1}{\ell!} A^{\ell}$. The exact discretization of (1) at sampling frequency $\delta$ in day $d$ reads

$$\Delta P_{t_i} = \Pi_\delta^{(d)} P_{t_{i-1}} + \varepsilon_{t_i}^{(d)}, \tag{2}$$

where $\Pi_\delta^{(d)} = \alpha_\delta^{(d)} \beta'$ and $\alpha_\delta^{(d)} = \alpha^{(d)} \left( \beta' \alpha^{(d)} \right)^{-1} \left[ \exp(\delta \beta' \alpha^{(d)}) - I_r \right]$, with $I_r$ denoting a $r$-dimensional identity matrix. The innovation $\varepsilon_{t_i}^{(d)}$ is iid Gaussian with zero mean and covariance matrix given by

$\Sigma_\delta^{(d)} = \int_0^\delta \exp\left(u\Pi^{(d)}\right)\Sigma^{(d)}\exp\left(u\Pi^{(d)\prime}\right)\mathrm{d}u$. It is important to note that temporal aggregation preserves the cointegration rank, i.e., $\operatorname{rank}\Pi_\delta^{(d)} = \operatorname{rank}\Pi^{(d)}$, and that the definition of (co)integration for Ornstein-Uhlenbeck processes in continuous time is consistent with the definition in discrete time (Kessler and Rahbek, 2004). This means that one may conduct inference about rank and cointegrating space using discrete-time procedures and then interpret the results in the continuous-time setting.

Computing price discovery measures requires the Granger representation of (1) and (2). Kessler and Rahbek's (2001) Theorem 1 shows that the Granger representation indeed holds in continuous time, namely,

$$P_t = \Xi^{(d)}\left(C^{(d)}W_t + P_0^{(d)}\right) + \eta_t^{(d)}, \tag{3}$$

where $\beta_\perp$ and $\alpha_\perp^{(d)}$ are $(k \times 1)$ vectors denoting the orthogonal projection of $\beta$ and $\alpha^{(d)}$, respectively, $\Xi^{(d)} = \beta_\perp\left(\alpha_\perp^{(d)\prime}\beta_\perp\right)^{-1}\alpha_\perp^{(d)\prime}$, $P_0^{(d)}$ contains initial values, and $\eta_t^{(d)}$ is a stationary Ornstein-Uhlenbeck process. In turn, the Granger representation in discrete time reads

$$P_{t_i} = \Xi_\delta^{(d)}\sum_{h=1}^{i}\varepsilon_{t_h}^{(d)} + \sum_{h=0}^{\infty}\Upsilon_{\delta,h}^{(d)}\varepsilon_{t_{i-h}} + P_{t_0}^{(d)}, \tag{4}$$

where $\alpha_{\delta,\perp}^{(d)}$ is a $(k \times 1)$ vector denoting the orthogonal projection of $\alpha_\delta^{(d)}$, $\Xi_\delta^{(d)} = \beta_\perp\left(\alpha_{\delta,\perp}^{(d)\prime}\beta_\perp\right)^{-1}\alpha_{\delta,\perp}^{(d)\prime}$, $\sum_{h=0}^{\infty}\Upsilon_{\delta,h}^{(d)}\varepsilon_{t_{i-h}}$ is a stationary process, and $P_{t_0}^{(d)}$ is a vector of initial values. The stochastic common trend given by the first term on the right-hand side of (4) reflects the efficient price of the asset and follows from $\beta_\perp$ being a vector of ones, which implies that $\Xi_\delta^{(d)}$ has common rows. In particular, it is reassuring to observe that the stochastic trend, $\Xi_\delta^{(d)}\sum_{h=1}^{i}\varepsilon_{t_h}^{(d)}$, is a martingale and thus is consistent with the asset pricing theory with regard to non-arbitrage requirements (see discussion in Hansen and Lunde, 2006).

The speed-of-adjustment matrix $\alpha_\delta^{(d)}$ plays a major role in any measure of price discovery. The matrix $\alpha_\delta^{(d)}$ reflects the adjustment that each market implements such that their prices do not deviate from the efficient latent price. Hence, the closer the $\alpha_\delta^{(d)}$ of a given market is to zero, the less it has to adjust to the efficient price. In the limit, $\alpha_{\delta,m}^{(d)} = 0$ means that the price of market $m \in \{1, 2, ..., k\}$ coincides with the efficient price. Accordingly, $\alpha_\delta^{(d)}$ shows that satellite markets have to adjust more strongly to deviations from the long-run equilibrium than leading markets.

### 2.2 The effect of the sampling frequency

We now focus our attention on investigating how the sampling frequency affects the component share and IS measures and how the continuous-time price discovery measure can be identified from prices observed in discrete time, i.e. at different sampling frequencies. Next, we present the continuous-time comparison of the IS and CS; and finally, we illustrate our theoretical findings with an example. For simplicity of exposition, consider an asset traded in two trading venues ($k = 2$).

#### 2.2.1 Component share and the continuous-time measure

The component share relies on the orthogonal projection of $\alpha_{\delta}^{(d)}$, namely, $\alpha_{\delta,\perp}^{(d)}$ such that $\alpha_{\delta,\perp}^{(d)\prime}\alpha_{\delta}^{(d)} = 0$ (see, among others, Booth et al., 1999; Chu et al., 1999; Harris, McInish and Wood, 2002; Hansen and Lunde, 2006). Note that $\alpha_{\delta,\perp}^{(d)}$ is not unique, and hence one typically imposes $\sum_{m=1}^{2} \alpha_{\delta,\perp,m}^{(d)} = 1$.[2] While $\alpha_{\delta}^{(d)}$ corresponds to the stationary direction of the process in (2), $\alpha_{\delta,\perp}^{(d)}$ relates to the nonstationary direction, gauging the amount of information that is incorporated in the common stochastic trend (i.e. the efficient price). This makes $\alpha_{\delta,\perp}^{(d)}$ a natural quantity to assess how the efficient price relates to each market innovation. Unlike $\alpha_{\delta}^{(d)}$, $\alpha_{\delta,\perp}^{(d)}$ is increasing with price informativeness (see, among others, Harris, McInish and Wood, 2002; Jong, 2002; Hansen and Lunde, 2006). The market with the highest $\alpha_{\delta,\perp}^{(d)}$ has the least need of adjustment towards the latent price and, hence, it is the one that leads the price discovery process. Using the normalization $\alpha_{\delta,\perp,1} + \alpha_{\delta,\perp,2} = 1$, the $m$th element of $\alpha_{\delta,\perp}$ reads

$$\alpha_{\delta,\perp,m}^{(d)} = \frac{\alpha_{\delta,\iota}^{(d)}\,(-1)^m}{\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}}, \qquad \iota, m = 1,2 \quad \text{and} \quad \iota \neq m. \tag{5}$$

To formally assess the effect of the sampling frequency on $\alpha_{\delta,\perp}$, we use the exact discretization of the reduced-rank Ornstein-Uhlenbeck process in (2) to re-write (5) as function of the continuous-time parameters and $\delta$:

$$\alpha_{\delta,\perp,m}^{(d)} = \frac{\alpha_{\iota}^{(d)}\,(-1)^m \left\{\left(\beta'\alpha^{(d)}\right)^{-1}\left[\exp(\delta\beta'\alpha^{(d)}) - I_r\right]\right\}}{\left(\alpha_1^{(d)} - \alpha_2^{(d)}\right)\left\{\left(\beta'\alpha^{(d)}\right)^{-1}\left[\exp(\delta\beta'\alpha^{(d)}) - I_r\right]\right\}} = \frac{\alpha_{\iota}^{(d)}\,(-1)^m}{\left(\alpha_1^{(d)} - \alpha_2^{(d)}\right)}, \tag{6}$$

[2] Under the $\sum_{m=1}^{k} \alpha_{\delta,\perp,m}^{(d)} = 1$ normalization, the component share (CS) measure of market $m \in \{1, 2, ..., k\}$, $CS_m = \frac{\alpha_{\delta,\perp,m}^{(d)}}{\sum_{m=1}^{k} \alpha_{\delta,\perp,m}^{(d)}}$, is the orthogonal projection of the $m$-element of the speed-of-adjustment parameter, i.e. $\alpha_{\delta,\perp,m}^{(d)}$.

for $\iota, m = 1,2$ and $\iota \neq m$. The result in (6) has an important implication. Because $\alpha^{(d)}_{\delta,\perp,m}$ for $m \in (1,2)$ is solely a function of the continuous-time speed-of-adjustment parameters and does not depend on $\delta$, $\alpha^{(d)}_{\delta,\perp,m}$ is invariant to the sampling frequency and hence $\alpha^{(d)}_{\delta,\perp,m} = \alpha^{(d)}_{\perp,m}$ for any $0 < \delta < 1$. The intuition behind this result follows naturally from the interpretation of the orthogonal-projection of the speed-of-adjustment parameters: $\alpha^{(d)}_{\delta,\perp,m}$ corresponds to the nonstationary direction of the price process and hence is associated with the amount of information that has a permanent effect on prices. Furthermore, it implies that identification and inference of the continuous-time price discovery measure arises directly from estimating $\alpha^{(d)}_{\delta,\perp}$ for any frequency of observations. Viewed through an empirical perspective, the result in (6) is also informative as it allows to move away from tick-by-tick data to prices sampled at lower frequencies (freer of market microstructure noise) and still learn about the continuous-time price discovery mechanism.

### 2.2.2 Information share

Another popular price discovery measure in the literature is Hasbrouck's (1995) information share (IS) (see, among others, Baillie et al., 2002; Jong, 2002; Grammig, Melvin and Schlag, 2005; and Yan and Zivot, 2010). The IS measure is also based on the discretized version of the Granger representation theorem in (4). In short, the IS measure gives the share of each market contribution to the total variance of the efficient price with $\sum_{m=1}^{2} IS_{\delta,m} = 1$. Using the exact discretization of (1), the IS measure of a given market $m \in \{1,2\}$ for $0 < \delta < 1$ is

$$IS^{(d)}_{\delta,m} = \frac{\left[\xi^{(d)}_{\delta} C^{(d)}_{\delta}\right]^2_m}{\xi^{(d)}_{\delta} \Sigma^{(d)}_{\delta} \xi^{(d)\prime}_{\delta}}, \tag{7}$$

where $\Sigma^{(d)}_{\delta} = C^{(d)}_{\delta} C^{(d)\prime}_{\delta} = \int_0^{\delta} \exp\left(u\Pi^{(d)}\right)\Sigma^{(d)} \exp\left(u\Pi^{(d)\prime}\right) \mathrm{d}u$, $\xi^{(d)}_{\delta}$ is the common row of $\Xi^{(d)}_{\delta}$ in (4) which follows from $\beta_{\perp} = (1,1)'$, and $[.]_m$ denotes the $m$th element of a 2-dimensional vector.

We now investigate the effect of $\delta$ on the IS measure. Considering the two-market case, we can re-write (7) as function of the market-specific variances and correlations. Specifically, let

$$\Sigma_{\delta}^{(d)} = \begin{pmatrix} \sigma_{\delta,1}^2 & \rho_\delta \sigma_{\delta,1}\sigma_{\delta,2} \\ \rho_\delta \sigma_{\delta,1}\sigma_{\delta,2} & \sigma_{\delta,2}^2 \end{pmatrix},$$

$$\underline{C}_{\delta}^{(d)} = \begin{pmatrix} \sigma_{\delta,1} & 0 \\ \rho_\delta \sigma_{\delta,2} & \sigma_{\delta,2}\left(1-\rho_\delta^2\right)^{1/2} \end{pmatrix},$$

$$\overline{C}_{\delta}^{(d)} = \begin{pmatrix} \sigma_{\delta,1}\left(1-\rho_\delta\right)^{1/2} & \rho_\delta C u \sigma_{\delta,1} \\ 0 & \sigma_{\delta,2} \end{pmatrix},$$

where $\underline{C}_{\delta}^{(d)}$ and $\overline{C}_{\delta}^{(d)}$ are the Cholesky decomposition of $\Sigma_{\delta}^{(d)}$ obtained from varying the ordering of the variables such that $\Sigma_{\delta}^{(d)} = \underline{C}_{\delta}^{(d)}\underline{C}_{\delta}^{(d)\prime}$ and $\Sigma_{\delta}^{(d)} = \overline{C}_{\delta}^{(d)}\overline{C}_{\delta}^{(d)}$, $\sigma_{\delta,1}^2$ and $\sigma_{\delta,2}^2$ are the market-specific variances at some frequency of observation $\delta$, and $\rho_\delta$ is the correlation between the two markets for $0 < \delta < 1$.[3] As a matter of easying the notation, we suppress the superscript $(d)$ from the market-specific variances and correlation, but reinforce that these measure also vary on a daily basis. Using $\underline{C}_{\delta}^{(d)}$ and $\overline{C}_{\delta}^{(d)}$, respectively, and using the fact that $\alpha_{\delta,\perp,m}^{(d)} = \alpha_{\perp,m}^{(d)}$ for any $0 < \delta < 1$, the overall IS measure in a given market $m \in \{1, \ldots, k\}$ for $0 < \delta < 1$ reads

$$\overline{IS}_{\delta,m}^{(d)} = \frac{1}{2}\left\{ \frac{\left[\xi_{\delta}^{(d)}\underline{C}_{\delta}^{(d)}\right]_m^2}{\xi_{\delta}^{(d)}\Sigma_{\delta}^{(d)}\xi_{\delta}^{(d)\prime}} + \frac{\left[\xi_{\delta}^{(d)}\overline{C}_{\delta}^{(d)}\right]_m^2}{\xi_{\delta}^{(d)}\Sigma_{\delta}^{(d)}\xi_{\delta}^{(d)\prime}} \right\},$$

$$\overline{IS}_{\delta,m}^{(d)} = \frac{1}{2}\left\{ \frac{\left[\alpha_{\perp,m}\sigma_{\delta,m} + \alpha_{\perp,\iota}\rho_\delta\sigma_{\delta,\iota}\right]^2 + \left[\alpha_{\perp,m}\sigma_{\delta,m}(1-\rho_\delta^2)^{1/2}\right]^2}{\alpha_{\perp,m}^2\sigma_{\delta,m}^2 + \alpha_{\perp,\iota}^2\sigma_{\delta,\iota}^2 + 2\alpha_{\perp,m}\alpha_{\perp,\iota}\rho_\delta\sigma_{\delta,m}\sigma_{\delta,\iota}} \right\}, \quad \iota, m = 1,2 \text{ and } \iota \neq m, \tag{8}$$

where $\overline{IS}_{m}^{(d)}$ denotes the overall IS measure for $m \in (1,2)$. Differently from the component share measure analysed before, the IS measure in (8) is not invariant to the sampling frequency because the market-specific variances and markets correlation in (8) depend on $\delta$. The intuition for this result is straightforward: as $\delta$ increases (the length of the interval between two observations increases), the $\alpha_{\delta}^{(d)}$ parameters transfer the price corrections from lags to the contemporaneous correlation between the two markets, as now more information becomes available within this greater sampling interval. Furthermore, because the exact discretization of $\Sigma_{\delta}^{(d)}$ implies that $|\rho_\delta|$ converges to one as $\delta$ increases, the limiting behavior of $\overline{IS}_{\delta,m}^{(d)}$ as $\delta$ approaches one converges to the uninformative value of $1/2$, $\lim_{\delta\to 1}\overline{IS}_{\delta,m}^{(d)} = 1/2$.

[3] The standard practise in the literature consists of decomposing the covariance matrix in (7) using the Cholesky decomposition. As the Cholesky decomposition is ordering variant, the usual fix is to compute upper and lower bounds for the IS measures considering every possible variables ordering and then averaging them up to obtain the overall IS measure.

There are two empirical implications on the relation between sampling frequency and IS measures. First, a fair comparison involving IS measures computed across different studies, markets or financial instruments has to take into consideration the frequency prices are sampled and the contemporaneous correlation $\rho$ across markets (in continuous time). However, accounting for $\rho$ is not a trivial task, as markets have substantially changed in the past few years with the rise of high-frequency trading (see, among others, Menkveld, 2014; Menkveld, 2016). Specifically, despite the increase in market fragmentation, markets also are more tightly inter-connected with both high-frequency market makers and high-frequency traders using statistical arbitrage across and within markets (see excellent discussion in O'Hara, 2015). In short, faster and more connected markets should lead to higher values of $\rho$, which contributes to blur the price discovery analysis conducted with the information share measures. Second, conducting inference on the continuous-time price discovery mechanism is not obvious, as recovering estimates of $\Sigma^{(d)}$ from estimates of $\Sigma_\delta^{(d)}$ tend to produce poor results in finite samples.[4]

### 2.2.3 A continuous-time comparison of CS and IS

We next extend Baillie et al.'s (2002) and Jong's (2002) comparison between component and information shares from discrete to continuous-time. Using the continuous-time version of the Granger representation theorem in (3), the continuous-time IS measure of a given market $m \in (1,2)$ reads

$$\overline{IS}_m^{(d)} = \frac{1}{2}\left\{\frac{\left[\alpha_{\perp,m}\sigma_m + \alpha_{\perp,\iota}\rho\sigma_\iota\right]^2 + \left[\alpha_{\perp,m}\sigma_m\left(1-\rho^2\right)^{1/2}\right]^2}{\alpha_{\perp,m}^2\sigma_m^2 + \alpha_{\perp,\iota}^2\sigma_\iota^2 + 2\alpha_{\perp,m}\alpha_{\perp,\iota}\rho\sigma_m\sigma_\iota}\right\}, \quad \iota, m = 1,2 \;\text{ and }\; \iota \neq m, \tag{9}$$

where $\xi^{(d)}$ is the common row of $\Xi^{(d)}$, $\sigma_{\delta,1}^2$, $\sigma_{\delta,2}^2$, and $\rho$ are the market-specific variances and the correlation between the two markets in continuous time. The component share and the IS measures are directly related and provide similar results (up to a different normalization) only in the case of $\rho = 0$ and $\sigma_1 = \sigma_2$, as below

$$\overline{IS}_m^{(d)} = \frac{\alpha_{\perp,m}^2\sigma_m^2}{\alpha_{\perp,m}^2\sigma_m^2 + \alpha_{\perp,\iota}^2\sigma_\iota^2}, \quad \iota, m = 1,2 \;\text{ and }\; \iota \neq m. \tag{10}$$

Baillie et al. (2002) and Jong (2002) find the same result as in (10) assuming $\rho_\delta = 0$ and $\sigma_1, \delta = \sigma_2, \delta$. However, from the exact discretization in (2), it follows that the discrete-time contemporaneous

[4] Using the analytical solution of $\Sigma_\delta^{(d)} = C_\delta^{(d)} C_\delta^{(d)\prime} = \int_0^\delta \exp\left(u\Pi^{(d)}\right)\Sigma^{(d)}\exp\left(u\Pi^{(d)\prime}\right)du$ to back up $\Sigma^{(d)}$ as function of $\Sigma_\delta^{(d)}$ generally produces negative semi-definite estimates of $\Sigma^{(d)}$ for prices sampled at frequencies lower than 10 seconds.

correlation $\rho_\delta$ is not invariant to the sampling frequency $\delta$ and $|\rho_\delta|$ converges to one as $\delta$ increases. This implies that assuming $\rho_\delta = 0$ becomes an unlikely assumption if one bears a continuous-time data generation process such as (1) in mind. In other words, assuming that there is a continuous time process, the result in (2) is implausible, which highlights the importance of working in a continuous-time setting.

### 2.2.4 A numerical illustration

We illustrate our theoretical findings using a continuous-time version of Hasbrouck's (1995) example. Suppose that a homogenous asset trades on two markets. Market 1 is the leading trading venue, with prices fully reflecting the efficient price, whereas the price on Market 2 reacts to deviations with respect to the (efficient) price on Market 1. Prices cointegrate with $\alpha^{(d)} = (0, \alpha_2^{(d)})'$ and $\beta' = (1, -1)$:

$$\mathrm{d}\begin{pmatrix} p_{1,t} \\ p_{2,t} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_2^{(d)} \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} p_{1,t} \\ p_{2,t} \end{pmatrix} \mathrm{d}t + C^{(d)} \mathrm{d}W(t), \tag{11}$$

where $C^{(d)}$ and $W$ are defined as in (1). The absence of adjustment in Market 1 ($\alpha_1^{(d)} = 0$) implies that $p_{1,t}$ coincides with the stochastic trend, and hence a coherent price discovery measure should identify Market 1 as the sole contributor to the price discovery process, regardless of the sampling frequency. This setup is general enough to nest the case where the component share and IS measures are equal in continuous time, and it is also informative to show how the two price discovery measures diverge when the sampling frequency decreases. In particular, we entertain the exact discretization of the bivariate price process in (11), with $\delta$ ranging from $1/23,400$ (1 second) to $1/13$ (30 minutes), with different contemporaneous correlation across markets, $\rho \in \{0, 0.3, 0.5, 0.7, 0.9\}$, and with $\sigma_1^2 = \sigma_2^2$.

The left panel of Figure 1 displays the continuous- and discrete-time $IS_{\delta,1}^{(d)}$ and $\alpha_{\delta,\perp,1}^{(d)}$ measures, whereas the right panel of Figure 1 plots $\rho$ and $\rho_\delta$ at different $\delta$'s. In line with our theoretical results, the discretization affects the price discovery measures in different manners. The price discovery measure based on $\alpha_{\delta,\perp}^{(d)}$ is completely immune to the sampling frequency in that $\alpha_\perp^{(d)} = \alpha_{\delta,\perp}^{(d)}$ for any $0 < \delta < 1$. In contrast, the IS measure becomes totally uninformative as $\delta$ increases essentially because $|\rho_\delta|$ converges to one. The higher the correlation across markets and/or the absolute value of the speed-of-adjustment parameters, the faster the IS measure converges to $1/2$. This means the

IS measure is unable to identify Market 1 in (11) as the unique contributor to the price discovery process for any $\rho > 0$ and $0 < \delta < 1$. Furthermore, the $\alpha_{\perp}^{(d)}$ and information share measures yield the same result only in continuous time, in the absence of contemporaneous correlation between markets, and when market specific variances are the same, i.e. $\rho = 0$ and $\sigma_1^2 = \sigma_2^2$.

To sum up, the continuous-time setting developed in this section allows a broader understanding of the price discovery measures at different sampling frequencies than earlier comparisons in discrete time (Baillie et al., 2002; Jong, 2002; Yan and Zivot, 2010). Furthermore, we show that inference on the continuous-time $\alpha_{\perp}^{(d)}$ can be directly conducted from discrete sampled prices, whereas the IS measure depends on the frequency prices are sampled.

# 3 Estimation of the time-varying continuous-time price discovery

This section discusses a feasible estimation method which produces consistent and asymptotically normally distributed estimates of the daily speed-of-adjustment parameters and hence of the continuous-time price discovery measure. From (4), the price discovery mechanism is constant within the day, but may change across days. This flexible parametrization is consistent with previous price discovery studies which identify these measures to be time varying (see, for instance, Hasbrouck, 1995; Hasbrouck, 2003; Mizrach and Neely, 2008). The idea is that the ability of a trading venue to impound new information mostly depends on market features (e.g., cost structure, market design, technological infrastructure, and relative presence of high-frequency traders) and market characteristics (e.g., trading intensity, trading volume and volatility). Market features and characteristics definitely change over time, but neither in a continuous nor in a brusk fashion. This is well in line with Eun and Sabherwal (2003) and Frijns, Gilbert and Tourani-Rad (2015), who show that highly persistent variables such as volume and volatility are relevant price discovery drivers. Accordingly, we henceforth assume that $\alpha_{\delta}^{(d)}$ changes in discrete time, at the daily frequency, as a bounded local stable stochastic process.

Consistent estimation of $\alpha_{\delta,\perp}^{(d)}$ requires only a consistent estimator of $\alpha_{\delta}^{(d)}$ for any frequency $0 < \delta < 1$, given that we assume $\beta$ known. Because our sample consists of prices observed intradaily over different days and our estimation method uses the entire sample, it is convenient to adapt our notation as follows. Denote by $T = nD$ the total number of observations, where $n$ is the number of intraday observations and $D$ is the number of trading days. As it is standard in the daily VECM approach, we augment the lag structure in (2) in order to allow the autoregressive parameter matrices $\Gamma_{\delta,j}^{(d)}$ for $j = 1, \ldots, \ell - 1$ also to vary over time:

$$\Delta P_\tau = \alpha_\delta^{(d)} \beta' P_{\tau-1} + \sum_{j=1}^{\ell-1} \Gamma_{\delta,j}^{(d)} \Delta P_{\tau-j} + \varepsilon_\tau, \quad d = 1, 2, \ldots, D \quad \tau = 1, 2, \ldots, T. \tag{12}$$

The literature usually captures the daily variation in price discovery by estimating daily VECM (see, for instance, Hasbrouck, 2003; Chakravarty et al., 2004; Mizrach and Neely, 2008). In contrast, we employ Giraitis et al.'s (2013) kernel-based estimator to retrieve daily estimates of $\alpha_\delta^{(d)}$ and $\Gamma_{\delta,j}^{(d)}$. This means that, as opposed to the daily VECM least-square estimates, our $\alpha_\delta^{(d)}$ estimates are not independent across days. The kernel-based least-square (KLS) estimator exploits the assumption that the daily variations in the alpha and Gamma matrices are persistent processes (either deterministic or stochastic) in order to obtain more efficient estimators.

Because we assume that $\beta$ is known, consistent estimation of the time-varying parameters in (12) follows directly from the results in Giraitis et al. (2013). Rewriting (12) in a more compact notation yields

$$\Delta P_\tau = B^{(d)} X_\tau + \varepsilon_\tau, \tag{13}$$

where $B^{(d)}$ is a $(k \times r + k(\ell - 1))$ matrix collecting the free parameters in (12) and $X_\tau = (P_{1,\tau-1} - P_{k,\tau-1}, \ldots, P_{k-1,\tau-1} - P_{k,\tau-1}, \Delta P_{t_{i-1}}, \ldots, \Delta P_{t_{i-\ell-1}})'$ with dimension $(r + k(\ell - 1) \times 1)$. In the event that the parameters are driven by stochastic processes, Giraitis et al. (2013) requires that $\sup_{d \leq D} \left\| B^{(d)} \right\| = O_p(1)$ and a local stability condition in the form of $\sup_{d^*:|d^*-d| \leq h} \left\| B^{(d)} - B^{(d^*)} \right\|^2 = O_p(h/d)$. For instance, the bounded random walk process we consider in our Monte Carlo study meets these conditions. Alternatively, in the case of deterministic variation in $B^{(d)}$, Robinson (1989) shows that asymptotic normality of the KLS estimator requires the parameters to satisfy a Lipschitz condition.

From Giraitis et al. (2013), the KLS estimator reads

$$\widehat{B}^{(d)} = \left( \sum_{\tau=1}^{T} b_{\tau,d} \Delta P_\tau X_\tau' \right) \left( \sum_{\tau=1}^{T} b_{\tau,d} X_{t_\tau} X_\tau' \right)^{-1}, \tag{14}$$

where $b_{\tau,d} := K\left((nd - \tau)/H\right)$, with $H$ denoting a bandwidth such that, for fixed $n$, $H \to \infty$ and $H = o\left(nD/\ln(nD)\right)$ as $D \to \infty$. The kernel function $K(x)$ is positive for any $x \in \mathbb{R}$ as well as it is continuous and bounded, with a bounded first derivative such that $\int K(x)dx = 1$. Because the regressors in (13) are covariance stationary processes, it readily follows from Giraitis et al. (2013, 2015) that, as $D \to \infty$,

$$\sqrt{H}\left(\mathrm{vec}\widehat{B}^{(d)} - \mathrm{vec}B^{(d)}\right) \xrightarrow{d} N\left(0, \left[I_k \otimes Q^{(d)^{-1}}\right] S^{(d)} \left[I_k \otimes Q^{(d)^{-1}}\right]\right), \tag{15}$$

where $\otimes$ denotes the Kronecker product, $Q^{(d)} = \operatorname*{plim}_{D\to\infty} \frac{1}{H} \sum_{\tau=1}^{T} b_{\tau,d} X_{t_\tau} X_{t_\tau}'$, and $S^{(d)} = \operatorname*{plim}_{D\to\infty} \frac{1}{H} \sum_{\tau=1}^{T} \zeta_t^{(d)} \zeta_t^{(d)\prime}$ for $\zeta_t^{(d)} = (b_{\tau,d} X_\tau \varepsilon_{1,\tau}, \ldots, b_{\tau,d} X_\tau \varepsilon_{k,\tau})'$.

To establish the asymptotic behavior of the continuous-time price discovery measures $\alpha_\perp^{(d)}$, it suffices to employ the delta method. For instance, in the case of $k = 2$ markets, the orthogonal projections of the speed-of-adjustment parameters are

$$\alpha_\perp^{(d)} = \left( \frac{-\alpha_{\delta,2}^{(d)}}{\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}}, \frac{\alpha_{\delta,1}^{(d)}}{\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}} \right)', \tag{16}$$

where $\alpha_\perp^{(d)\prime} \alpha_\delta^{(d)} = 0$. The asymptotic distribution of the KLS estimator $\widehat{\alpha}_\perp^{(d)}$ then reads

$$\sqrt{H}\left(\widehat{\alpha}_\perp^{(d)} - \alpha_\perp^{(d)}\right) \xrightarrow{d} N\left(0, \Lambda_\perp R_\alpha \left[I_k \otimes Q^{(d)^{-1}}\right] S^{(d)} \left[I_k \otimes Q^{(d)^{-1}}\right] R_\alpha' \Lambda_\perp'\right), \tag{17}$$

where $R_\alpha = \left(I_2, 0_{2\times 4(\ell-1)}\right)$ is a deterministic matrix that selects the elements of $\alpha_\delta^{(d)}$ from $\mathrm{vec}\widehat{B}^{(d)}$, and $\Lambda_\perp$ is the matrix of partial derivatives given by

$$\Lambda_\perp \equiv \frac{\partial \alpha_\perp^{(d)\prime}}{\partial \alpha_\delta^{(d)}} = \begin{pmatrix} \frac{\alpha_{\delta,2}^{(d)}}{\left(\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}\right)^2} & -\frac{\alpha_{\delta,1}^{(d)}}{\left(\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}\right)^2} \\ -\frac{\alpha_{\delta,2}^{(d)}}{\left(\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}\right)^2} & \frac{\alpha_{\delta,1}^{(d)}}{\left(\alpha_{\delta,1}^{(d)} - \alpha_{\delta,2}^{(d)}\right)^2} \end{pmatrix}. \tag{18}$$

# 4 Monte Carlo study

We assess the relative performance of the KLS estimator compared to the standard approach in the literature of estimating daily measures of price discovery. Our setup consists of an asset traded at $k = 2$ trading venues. We simulate prices from the exact discretization of the continuous-time process in (1) for $D = 500$ trading days (about 2 years), with contemporaneous correlation $\rho \in \{0, 0.3, 0.5, 0.7, 0.9\}$. We sample prices at fixed intervals of 0.5, 1, 3, and 5 minutes. As a usual trading day entails 23,400 seconds (6.5 hours), our samples have sizes ranging from 39,000 to 390,000 observations. Specifically, the data generation process works as follows. The elements of the speed-of-adjustment vector $\alpha_{\delta}^{(d)} = \left(\alpha_{1,\delta}^{(d)}, \alpha_{2,\delta}^{(d)}\right)'$ follow bounded random walk processes at the 5-minute frequency (lowest frequency prices are sampled), ranging within the following intervals: $\alpha_{1,\delta=1/78}^{(d)} \in [-0.49, -0.01]$ and $\alpha_{2,\delta=1/78}^{(d)} \in [0.01, 0.49]$. Parameterizing the speed-of-adjustment parameters in their lowest frequency suffices to guarantee that the eigenvalues of $\alpha_{\delta}^{(d)}\beta'$ are strictly smaller than one in absolute value for any $0 < \delta \leq 1/78$.

Similarly to Giraitis et al. (2013), we generate the time-varying speed-of-adjustment parameters $\alpha_{\delta=1/78}^{(d)} = \left(\alpha_{1,\delta=1/78}^{(d)}, \alpha_{2,\delta=1/78}^{(d)}\right)'$ as

$$\alpha_{1,\delta=1/78}^{(d)} = \overline{\alpha}\frac{a_1^d}{\max_{0\leq d^*\leq d}\left|a_1^{d^*}\right|} - \overline{\alpha} - 0.01, \tag{19}$$

$$\alpha_{2,\delta=1/78}^{(d)} = \overline{\alpha}\frac{a_2^d}{\max_{0\leq d^*\leq d}\left|a_2^{d^*}\right|} + \overline{\alpha} + 0.01, \tag{20}$$

where $\overline{\alpha} = 0.24$ restricts the time-varying parameters away from the upper and lower boundary points and $a_1^d$ and $a_2^d$ are driftless random walk processes driven by uncorrelated white noise innovations. We then compute the speed-of-adjustment parameters at the continuous-time frequency from extracting the first column of $\Pi^{(d)} = \alpha^{(d)}\beta'$, which follows directly from imposing $\beta = (1, -1)'$. To this end, denote $\log(A)$ the matrix logarithm, such that a logarithm of $A$ is any matrix $Z$ such that $\exp(Z) = A$. It follows that using the exact discretization of the reduced-rank Ornstein-Uhlenbeck process, $\Pi^{(d)}$ reads

$$\Pi^{(d)} = \alpha^{(d)}\beta' = \frac{1}{78}\log\left(\alpha_{\delta=1/78}^{(d)}\beta' + I_k\right). \tag{21}$$

The next step consists of modelling $\Sigma^{(d)}$. The instantaneous covariance matrix $\Sigma^{(d)}$ is set to evolve daily in a stochastic manner. Specifically, the log-variances follow AR(1) processes:

$$\ln\sigma_{m,t}^2 = \varphi_0 + \varphi_1\ln\sigma_{m,t-1}^2 + \varsigma\,\vartheta_{m,t}, \qquad \text{for} \quad m = 1, 2 \tag{22}$$

with $\sigma_{1,t}^2$ and $\sigma_{1,t}^2$ denoting the diagonal elements of $\Sigma^{(d)}$. The volatility innovations $\vartheta_{1,t}$ and $\vartheta_{2,t}$ are standard Gaussian white noises with a correlation of 0.95. The coefficient of variation is given by $\text{var}\left(\sigma_{m,t}^2\right)/\mathbb{E}\left[\sigma_{m,t}^2\right]^2 = \exp\left(\varsigma/(1-\varphi_1^2)\right) - 1$. We calibrate the stochastic volatility models as in Jacquier, Polson and Rossi (1994), namely, we fix the autoregressive parameter to 0.98, the expected annual volatility to 20%, and the coefficient of variation to 0.5. Having specified the free parameters at the continuous-time frequency, we generate prices at the highest frequency (30 seconds) using the exact discretization in (2) and sample accordingly from these prices to obtain the alternative lower frequencies (1, 3, and 5 minutes).

We assume a known cointegrating vector $\beta = (1, -1)'$ and estimate $\alpha_\delta^{(d)}$ both by least squares (LS) as in the daily VECM approach and by KLS using a Epanechnikov kernel, with bandwidth $H \in \{n^{8/10}\sqrt{D}, n^{9/10}\sqrt{D}, n\sqrt{D}\}$, where $n$ and $D$ account for the numbers of intraday and trading days, respectively. We report the root mean squared errors of the LS estimates of the speed-of-adjustment parameters and of $\alpha_{\delta,1,\perp}$ relative to their KLS counterparts. Given the normalization of the orthogonal projections, the relative (root) mean squared errors of the $\alpha_{\delta,1,\perp}$ and $\alpha_{\delta,2,\perp}$ estimates are identical by construction.

Table 1 displays the root mean squared error of the KLS estimator relative to LS over 1,000 replications. The KLS estimates entail much lower root mean squared errors than those based on a daily VECM approach. This holds for every instance we entertain, that is to say, regardless of the contemporaneous correlation between markets, sampling frequency, and bandwidth choice. It turns out, nonetheless, that the difference in performance increases with the correlation across markets, but declines with the sampling frequency and with the bandwidth value. Table 2 report the sample bias of both LS and KLS estimators. Biases are very small for both estimators. There is a clear decreasing pattern with the sampling interval and bandwidth value, whereas the magnitude of the bias does not seem to vary with the correlation across markets. In the absence of significant bias, the lower mean squared error of the KLS estimator seems to be due to a decline of the sampling variance. All in all, the results heavily support the use of the KLS estimator for price discovery analyses.

# 5 Price informativeness: NYSE versus Nasdaq

In this section, we take to data our estimation strategy based on the KLS estimation of the speed-of-adjustment matrix to obtain estimates of the continuous-time price discovery measures. Given that we allow for time variation in the VECM, we use a massive high-frequency data set that ranges from January 2007 to December 2013. Most studies in price discovery consider much shorter periods, ranging from 4 to 12 months (see, for instance, de Jong and Schotman, 2010; Riordan and Storkenmaier, 2012; Ozturk et al., 2014; Benos and Sagade, 2016; Otsubo, 2017). We focus on 10 very actively traded stocks, namely, Bank of America (BAC), General Electric (GE), Hewlett-

Packard (HPQ), International Business Machines (IBM), J.C. Penney Company (JCP), JP Morgan Chase (JPM), Coca-Cola Company (KO), Altria Group (MO), Verizon Communications (VZ), and ExxonMobil (XOM).

We extract quotes data from TAQ for the two most active trading venues, NYSE and Nasdaq. We implement the same cleaning filters as in (Barndorff-Nielsen, Hansen, Lunde and Shephard, 2009), discarding any observation with a zero quote, negative bid-ask spread, or outside the main trading hours (9:30 to 16:00). We also discard any data point either with a bid-ask spread higher than 50 times the median spread on that day or with a midquote deviating by more than 10 mean absolute deviations from a rolling centered median of 50 observations. Finally, we take the median bid and ask quotes at each second in the event that there are multiple ticks taking place at the same second. We synchronize the midquotes from both trading venues by sampling at regularly spaced intervals of 1 minute, as it helps alleviating market microstructure noise (Hupperets and Menkveld, 2002; Grammig, Melvin and Schlag, 2005 among others). Furthermore, sampling at 1-minute frequency yields an average of $n = 390$ intraday observations, which increases the bandwidth parameter, $H$, and hence improves the convergence rates, $\sqrt{H}$, of the KLS estimator. Table 3 details the cleaning process and provides the final number of observations for the 10 stocks we entertain.

We set the bandwidth to $H = n\sqrt{D}$ as in Giraitis et al. (2013). The Monte Carlo study confirms that this bandwidth choice indeed performs very well. We determine the lag structure by minimizing the Bayesian information criterion (BIC). [5] As expected, we find only one cointegrating vector for every pair of stock prices using Johansen's maximum eigenvalue and trace tests at the 1% significance level.[6]

Figure 2 plots the KLS estimates of the continuous-time measure of price discovery, $\hat{\alpha}_{\perp}^{(d)}$, and their respective 95% confidence intervals. As aforementioned, we normalize the orthogonal projections such that $\hat{\alpha}_{N,\perp}^{(d)} + \hat{\alpha}_{T,\perp}^{(d)} = 1$, with subscripts $N$ and $T$ denoting NYSE and Nasdaq, respectively. We obtain confidence intervals for $\hat{\alpha}_{\perp}^{(d)}$ using the asymptotic distribution in (17).

The daily variation in $\hat{\alpha}_{\perp}^{(d)}$ is visible, with relative market informativeness alternating between NYSE and Nasdaq. Results are mixed across stocks in the beginning of the sample, but Nasdaq contribution to the price discovery picks up from 2008 for the majority of the stocks, which might be associated with the accentuated decrease of NYSE market share (see Figure 3). This coincides with

[5] Choosing the lag length by obtaining the most parsimonious specification in which we cannot reject the absence of residual autocorrelation at the 5% significance level as in Hansen and Lunde (2006) does not yield any qualitative changes. Results are available upon request.

[6] Test results are available upon request.

the period of increase in market fragmentation in the U.S. market, and significant gain in market share of the new entrant markets (see, for instance, Menkveld, 2014; O'Hara, 2015; Menkveld, 2016). At the same time, NYSE starts its hybrid market in 2007, with a decline on its floor operation. From mid 2010, NYSE gradually recovers its importance in the price discovery process for most of the stocks in our sample. In particular, NYSE seems to contribute the most to price discovery as from mid 2011 (7 out of 10 stocks). Price discovery contribution typically moves in tandem with relative liquidity and NYSE experiences on average much more volume than Nasdaq. Finally, the standard errors are generally small over the entire sample, allowing a distinction between the price discovery measures in the two markets in the great majority of days. This is mainly due to the use of intraday data (sampling at 1-minute frequency), which alleviates the slow convergence rates associated with the KLS estimator.

To better understand the daily variation in the price discovery mechanism, we estimate a VEC model that links daily estimates of the orthogonal projection of alpha and relative volume at the NYSE.[7] In particular, we measure the latter by the logarithm of the NYSE-Nasdaq volume ratio. Both time series are very persistent, indicating the presence of unit roots. This is not surprising given that we allow the orthogonal projection of alpha to follow a bounded random walk. In addition, standard cointegration tests strongly suggest a long-run positive relationship between price discovery and relative volume.

Table 4 shows that every cointegrating vector is such that the NYSE contribution to the price discovery increases with the NYSE-Nasdaq volume ratio. It is interesting that the latter reacts significantly to deviations from the long-run equilibrium for every stock, whereas the same does not apply to the daily estimates of the orthogonal projection of alpha. In particular, the price discovery measures of the BAC, GE, HPQ and JCP stocks do not respond to deviations from the long-run equilibrium. In addition, changes in the relative volume do not significantly affect future changes in the price discovery measures of BAC, HPQ, JCP, KO, MO, and VZ. Accordingly, we cannot reject at the usual significance levels the absence of Granger causality running from relative volume to the price discovery measures for BAC, HPQ and JCP. Altogether, it seems that relative volume chases more price discovery than the other way around.

[7] Depending on the stock, we need between 5 and 9 lags in the VAR specification to cope with the persistence in the orthogonal projection of alpha and relative volume. To simplify matters, we carry out the cointegration analysis using a VAR(9) specification for every stock, which leads to VEC representations with 8 lags.

# 6 Conclusion

We entertain a continuous-time price discovery model in which the speed-of-adjustment parameters evolve stochastically across days. We show that their orthogonal projection is invariant to the discretization frequency. This implies that inference on the continuous-time price discovery measures can be directly conducted from discrete sampled prices. In contrast, Hasbrouck's (1995) information share converges to $1/k$ as the sampling frequency decreases and hence comparison across different studies, markets or financial instruments should take the sampling frequency and the contemporaneous correlation between the markets into consideration. We then extend Baillie et al.'s (2002) comparison between the component and information share measures and document that these measures are only likely to produce similar results in continuous time. In light of the rising of high-frequency trading, we expect the component share to offer a better picture of the price discovery than the information share, even if the latter arguably has more economic appeal and interpretation.

We also show how to estimate continuous-time price discovery measures by kernel-based least squares (Giraitis et al., 2013) under the assumption that the VECM parameters are persistent over time. By exploiting the inter-dependence across days, the kernel-based approach yields more efficient estimates than we would otherwise obtain by treating the daily variation in the VECM parameters as independent over time. Monte Carlo simulations indeed confirm that the KLS estimator easily outperforms the standard least-square daily VECM estimator under different scenarios.

Empirically, we assess the informativeness of the NYSE relative to Nasdaq for 10 actively-traded stocks. We find strong evidence that market leadership alternates over time, with NYSE leading most of the price discovery as from mid-2011. Finally, by examining the long-run relationship between price discovery and volume, we find that relative volume is more reactive than the component share to deviations from their long-run relationship.

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Table 1: Relative root mean squared errors of daily alpha and component share estimates

We document the performance of the LS and the KLS estimators of $\alpha_{\delta,1}$ and $\alpha_{\delta,2}$ as well as of the orthogonal projection $\alpha_{\delta,1,\perp}$ for $D = 500$ days. In particular, we report the root mean squared error of the KLS estimator relative to LS, so that figures below one imply better performance of the KLS estimator. Note that we do not report the results for $\alpha_{\delta,2,\perp}$ because they are by construction identical to the ones for $\alpha_{\delta,\perp,1}$. The instantaneous correlation between markets $\rho$ ranges from 0 to 0.90, whereas the sampling frequency ranges from 30 seconds to 5 minutes. More specifically, we sample data at the 0.5-, 1-, 2-, 3- and 5-minute frequencies, yielding $\delta = 1/780, 1/390, 1/195, 1/130, 1/78$, respectively, and $n = 780, 390, 195, 130, 78$ intraday observations. We compute the KLS estimator using three bandwidths: $H = n^{8/10}\sqrt{D}$, $H = n^{9/10}\sqrt{D}$ and $H = n\sqrt{D}$.

| | | $\delta = 1/780$ | | | $\delta = 1/390$ | | | $\delta = 1/195$ | | | $\delta = 1/130$ | | | $\delta = 1/78$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ |
| $H = n^{8/10}\sqrt{D}$ | 0.00 | 0.72 | 0.72 | 0.68 | 0.57 | 0.56 | 0.56 | 0.43 | 0.44 | 0.42 | 0.37 | 0.37 | 0.37 | 0.30 | 0.30 | 0.29 |
| | 0.30 | 0.62 | 0.61 | 0.40 | 0.49 | 0.49 | 0.45 | 0.38 | 0.39 | 0.36 | 0.33 | 0.33 | 0.33 | 0.28 | 0.28 | 0.27 |
| | 0.50 | 0.54 | 0.54 | 0.45 | 0.44 | 0.44 | 0.40 | 0.35 | 0.35 | 0.33 | 0.31 | 0.31 | 0.31 | 0.26 | 0.27 | 0.26 |
| | 0.70 | 0.46 | 0.46 | 0.42 | 0.39 | 0.38 | 0.35 | 0.32 | 0.32 | 0.30 | 0.29 | 0.29 | 0.28 | 0.25 | 0.25 | 0.25 |
| | 0.90 | 0.37 | 0.37 | 0.34 | 0.32 | 0.32 | 0.27 | 0.28 | 0.28 | 0.26 | 0.26 | 0.26 | 0.26 | 0.24 | 0.24 | 0.23 |
| $H = n^{9/10}\sqrt{D}$ | 0.00 | 0.89 | 0.88 | 0.82 | 0.65 | 0.64 | 0.63 | 0.45 | 0.46 | 0.44 | 0.37 | 0.37 | 0.37 | 0.28 | 0.28 | 0.27 |
| | 0.30 | 0.73 | 0.72 | 0.48 | 0.54 | 0.53 | 0.49 | 0.39 | 0.39 | 0.36 | 0.32 | 0.32 | 0.31 | 0.26 | 0.26 | 0.25 |
| | 0.50 | 0.62 | 0.61 | 0.50 | 0.46 | 0.45 | 0.42 | 0.34 | 0.34 | 0.32 | 0.29 | 0.29 | 0.28 | 0.23 | 0.24 | 0.23 |
| | 0.70 | 0.49 | 0.49 | 0.43 | 0.38 | 0.37 | 0.34 | 0.29 | 0.29 | 0.28 | 0.25 | 0.25 | 0.25 | 0.22 | 0.22 | 0.21 |
| | 0.90 | 0.34 | 0.34 | 0.30 | 0.28 | 0.28 | 0.24 | 0.24 | 0.24 | 0.22 | 0.22 | 0.22 | 0.21 | 0.19 | 0.19 | 0.19 |
| $H = n\sqrt{D}$ | 0.00 | 1.16 | 1.14 | 1.02 | 0.79 | 0.78 | 0.74 | 0.51 | 0.52 | 0.49 | 0.39 | 0.39 | 0.39 | 0.28 | 0.28 | 0.27 |
| | 0.30 | 0.93 | 0.91 | 0.57 | 0.63 | 0.62 | 0.59 | 0.41 | 0.41 | 0.40 | 0.32 | 0.32 | 0.32 | 0.24 | 0.24 | 0.23 |
| | 0.50 | 0.77 | 0.75 | 0.60 | 0.52 | 0.52 | 0.48 | 0.35 | 0.35 | 0.34 | 0.28 | 0.28 | 0.28 | 0.22 | 0.22 | 0.21 |
| | 0.70 | 0.60 | 0.59 | 0.50 | 0.41 | 0.41 | 0.37 | 0.29 | 0.29 | 0.28 | 0.24 | 0.24 | 0.23 | 0.19 | 0.19 | 0.19 |
| | 0.90 | 0.37 | 0.36 | 0.31 | 0.28 | 0.27 | 0.23 | 0.21 | 0.21 | 0.19 | 0.19 | 0.19 | 0.18 | 0.16 | 0.16 | 0.16 |

Table 2: Bias of daily alpha and component share estimates

We document the bias ($\times 100$) of the LS and KLS estimators of $\alpha_{\delta,1}$ and $\alpha_{\delta,2}$ as well as of the orthogonal projection $\alpha_{\delta,1,\perp}$ for $D = 500$ days. As before, we do not report the results for $\alpha_{\delta,2,\perp}$ because they are symmetrical to the bias in the estimation of $\alpha_{\delta,\perp,1}$ by construction. The instantaneous correlation between markets $\rho$ ranges from 0 to 0.90, whereas the sampling frequency ranges from 0.5, 1, 2, 3 and 5 minutes, yielding $\delta = 1/780, 1/390, 1/195, 1/130, 1/78$, respectively. We compute the KLS estimator using a bandwidth $H = n^{b/10}\sqrt{D}$, with $b = 8, 9, 10$.

| | | $\delta = 1/780$ | | | $\delta = 1/390$ | | | $\delta = 1/195$ | | | $\delta = 1/130$ | | | $\delta = 1/78$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ | $\alpha_{\delta,1}$ | $\alpha_{\delta,2}$ | $\alpha_{\delta,1,\perp}$ |
| least squares | 0.00 | 0.21 | -0.22 | -0.01 | 0.35 | -0.35 | -0.03 | 0.60 | -0.54 | 0.06 | 0.84 | -0.75 | 0.01 | 1.35 | -1.12 | 0.12 |
| | 0.30 | 0.23 | -0.23 | -0.25 | 0.38 | -0.37 | -0.05 | 0.62 | -0.59 | -0.02 | 0.90 | -0.76 | 0.06 | 1.41 | -1.15 | 0.05 |
| | 0.50 | 0.23 | -0.24 | 0.03 | 0.38 | -0.38 | -0.04 | 0.66 | -0.57 | 0.04 | 0.91 | -0.79 | 0.02 | 1.45 | -1.14 | 0.06 |
| | 0.70 | 0.24 | -0.25 | -0.03 | 0.39 | -0.40 | -0.08 | 0.68 | -0.58 | 0.06 | 0.90 | -0.84 | -0.04 | 1.42 | -1.20 | 0.01 |
| | 0.90 | 0.23 | -0.26 | -0.05 | 0.40 | -0.41 | -0.32 | 0.76 | -0.52 | 0.13 | 1.02 | -0.74 | 0.14 | 1.61 | -1.04 | 0.17 |
| $H = n^{8/10}\sqrt{D}$ | 0.00 | -0.25 | 0.23 | -0.02 | -0.31 | 0.30 | -0.01 | -0.33 | 0.36 | 0.00 | -0.33 | 0.36 | 0.00 | -0.28 | 0.36 | 0.03 |
| | 0.30 | -0.25 | 0.24 | -0.02 | -0.31 | 0.31 | -0.01 | -0.35 | 0.36 | -0.01 | -0.31 | 0.40 | 0.04 | -0.27 | 0.39 | 0.05 |
| | 0.50 | -0.25 | 0.23 | -0.03 | -0.31 | 0.30 | -0.02 | -0.32 | 0.38 | 0.02 | -0.31 | 0.38 | 0.02 | -0.25 | 0.39 | 0.06 |
| | 0.70 | -0.25 | 0.23 | -0.04 | -0.32 | 0.30 | -0.03 | -0.32 | 0.37 | 0.01 | -0.33 | 0.33 | -0.02 | -0.30 | 0.34 | 0.00 |
| | 0.90 | -0.26 | 0.23 | -0.07 | -0.31 | 0.30 | -0.03 | -0.25 | 0.45 | 0.08 | -0.23 | 0.45 | 0.09 | -0.13 | 0.50 | 0.16 |
| $H = n^{9/10}\sqrt{D}$ | 0.00 | -0.44 | 0.42 | -0.02 | -0.52 | 0.51 | -0.01 | -0.52 | 0.55 | 0.00 | -0.49 | 0.53 | 0.00 | -0.41 | 0.48 | 0.03 |
| | 0.30 | -0.44 | 0.42 | -0.03 | -0.53 | 0.52 | -0.02 | -0.55 | 0.55 | -0.02 | -0.49 | 0.57 | 0.02 | -0.41 | 0.52 | 0.05 |
| | 0.50 | -0.44 | 0.42 | -0.03 | -0.51 | 0.51 | -0.01 | -0.50 | 0.57 | 0.02 | -0.47 | 0.55 | 0.03 | -0.37 | 0.52 | 0.07 |
| | 0.70 | -0.44 | 0.41 | -0.05 | -0.52 | 0.50 | -0.03 | -0.51 | 0.56 | 0.00 | -0.48 | 0.51 | -0.01 | -0.45 | 0.43 | -0.03 |
| | 0.90 | -0.44 | 0.41 | -0.07 | -0.52 | 0.50 | -0.03 | -0.45 | 0.62 | 0.06 | -0.40 | 0.61 | 0.08 | -0.29 | 0.60 | 0.13 |
| $H = n\sqrt{D}$ | 0.00 | -0.74 | 0.72 | -0.03 | -0.84 | 0.83 | -0.02 | -0.84 | 0.82 | -0.03 | -0.75 | 0.78 | 0.00 | -0.62 | 0.66 | 0.01 |
| | 0.30 | -0.73 | 0.72 | -0.03 | -0.82 | 0.81 | -0.03 | -0.79 | 0.80 | -0.02 | -0.72 | 0.72 | -0.02 | 1.35 | -1.19 | -0.02 |
| | 0.50 | -0.73 | 0.71 | -0.04 | -0.83 | 0.81 | -0.02 | -0.78 | 0.82 | 0.00 | -0.71 | 0.75 | 0.00 | -0.58 | 0.64 | 0.02 |
| | 0.70 | -0.73 | 0.71 | -0.05 | -0.83 | 0.81 | -0.04 | -0.78 | 0.82 | -0.01 | -0.71 | 0.73 | -0.02 | -0.61 | 0.61 | -0.03 |
| | 0.90 | -0.74 | 0.71 | -0.08 | -0.82 | 0.82 | -0.04 | -0.72 | 0.89 | 0.05 | -0.65 | 0.82 | 0.05 | -0.49 | 0.73 | 0.08 |

Table 3: Data description

We report summary statistics for raw and cleaned data for Nasdaq and NYSE. The first two columns present the number of quotes (in millions) for each stock on the two trading venues before any cleaning filter (raw data). The following two columns display the total number of quotes (in millions) after the implementation of the cleaning procedure. The following two columns (obs per day) report the daily average number of quotes (in thousands). The last two columns report the total number of days we have for each stock in the sample period (January 2007 to December 2013).

| | raw ('000,000) | | clean ('000,000) | | obs per day ('000) | | number of days | |
|---|---|---|---|---|---|---|---|---|
| | Nasdaq | NYSE | Nasdaq | NYSE | Nasdaq | NYSE | Nasdaq | NYSE |
| BAC | 523 | 503 | 31 | 34 | 17.85 | 19.05 | 1,735 | 1,762 |
| GE | 363 | 427 | 29 | 31 | 16.53 | 17.78 | 1,735 | 1,762 |
| HPQ | 326 | 277 | 26 | 28 | 14.84 | 15.86 | 1,735 | 1,762 |
| IBM | 122 | 149 | 21 | 25 | 11.96 | 14.13 | 1,735 | 1,762 |
| JCP | 175 | 149 | 20 | 22 | 11.55 | 12.26 | 1,735 | 1,762 |
| JPM | 696 | 542 | 32 | 33 | 18.43 | 18.64 | 1,735 | 1,762 |
| KO | 244 | 205 | 23 | 25 | 13.27 | 14.44 | 1,735 | 1,762 |
| MO | 178 | 204 | 22 | 26 | 12.40 | 14.90 | 1,735 | 1,762 |
| VZ | 264 | 257 | 25 | 29 | 14.44 | 16.19 | 1,735 | 1,762 |
| XOM | 503 | 417 | 31 | 33 | 18.10 | 18.84 | 1,735 | 1,762 |

Table 4: Long-run relationship between price discovery and volume

We report the results of a VECM(8) for the daily price discovery measure and relative volume at the NYSE. We report the cointegrating vector estimates and their standard errors (note that we omit the one related to the price discovery measure as we force its loading to one) as well as the corresponding speeds of adjustment for the price discovery measure and for the relative volume. Finally, the last column displays the adjusted $R^2$ to gauge how much the VECM explains of the overall variation of the daily changes in the price discovery measure.

| | cointegrating vector | | speed of adjustment | | |
|---|---|---|---|---|---|
| | intercept | volume | price discovery measure | volume | $R^2$ |
| BAC | $-0.0749$ | $-1.6909$ | $-7.33 \times 10^{-5}$ | 0.0417 | 0.4772 |
| | (0.1732) | (0.3827) | (0.0002) | (0.0097) | |
| GE | | $-1.0747$ | $-0.0011$ | 0.1242 | 0.4387 |
| | | (0.0825) | (0.0006) | (0.0228) | |
| HPQ | | $-1.9721$ | $-7.16 \times 10^{-5}$ | 0.0482 | 0.2347 |
| | | (0.2823) | (0.0003) | (0.0102) | |
| IBM | | $-0.5962$ | $-0.0049$ | 0.1211 | 0.2520 |
| | | (0.0617) | (0.0014) | (0.0282) | |
| JCP | | $-1.8361$ | $-5.33 \times 10^{-5}$ | 0.0403 | 0.2648 |
| | | (0.2824) | (0.0005) | (0.0091) | |
| JPM | $-0.3339$ | $-1.1122$ | $-0.0010$ | 0.0678 | 0.3263 |
| | (0.0926) | (0.2241) | (0.0004) | (0.0140) | |
| KO | | $-0.6748$ | $-0.0054$ | 0.1499 | 0.2405 |
| | | (0.0501) | (0.0013) | (0.0323) | |
| MO | | $-0.4463$ | $-0.0040$ | 0.1324 | 0.2416 |
| | | (0.0552) | (0.0014) | (0.0362) | |
| VZ | | $-0.9991$ | $-0.0032$ | 0.1224 | 0.1291 |
| | | (0.0864) | (0.0012) | (0.0238) | |
| XOM | | $-1.0343$ | $-0.0011$ | 0.1150 | 0.3262 |
| | | (0.0897) | (0.0007) | (0.0214) | |

Figure 1: Information share and contemporaneous correlation as sampling frequency decreases

The first plot displays the information share and the component share of market 1 in continuous time and for $\delta$ ranging from $1/23,400$ (1 second) to $1/13$ (30 minutes). The second plot depicts the exact correlation across markets in continuous time and at each sampling frequency $\delta$. We consider a range of values for the correlation across markets at continuous time, and then compute discrete-time counterparts using the exact discretization of the reduced-rank Ornstein-Uhlenbeck process as in (2). Specifically, $\alpha = (0, 522)'$ which corresponds to $\alpha_\delta = (0, 0.74)'$ at the 1-minute frequency.

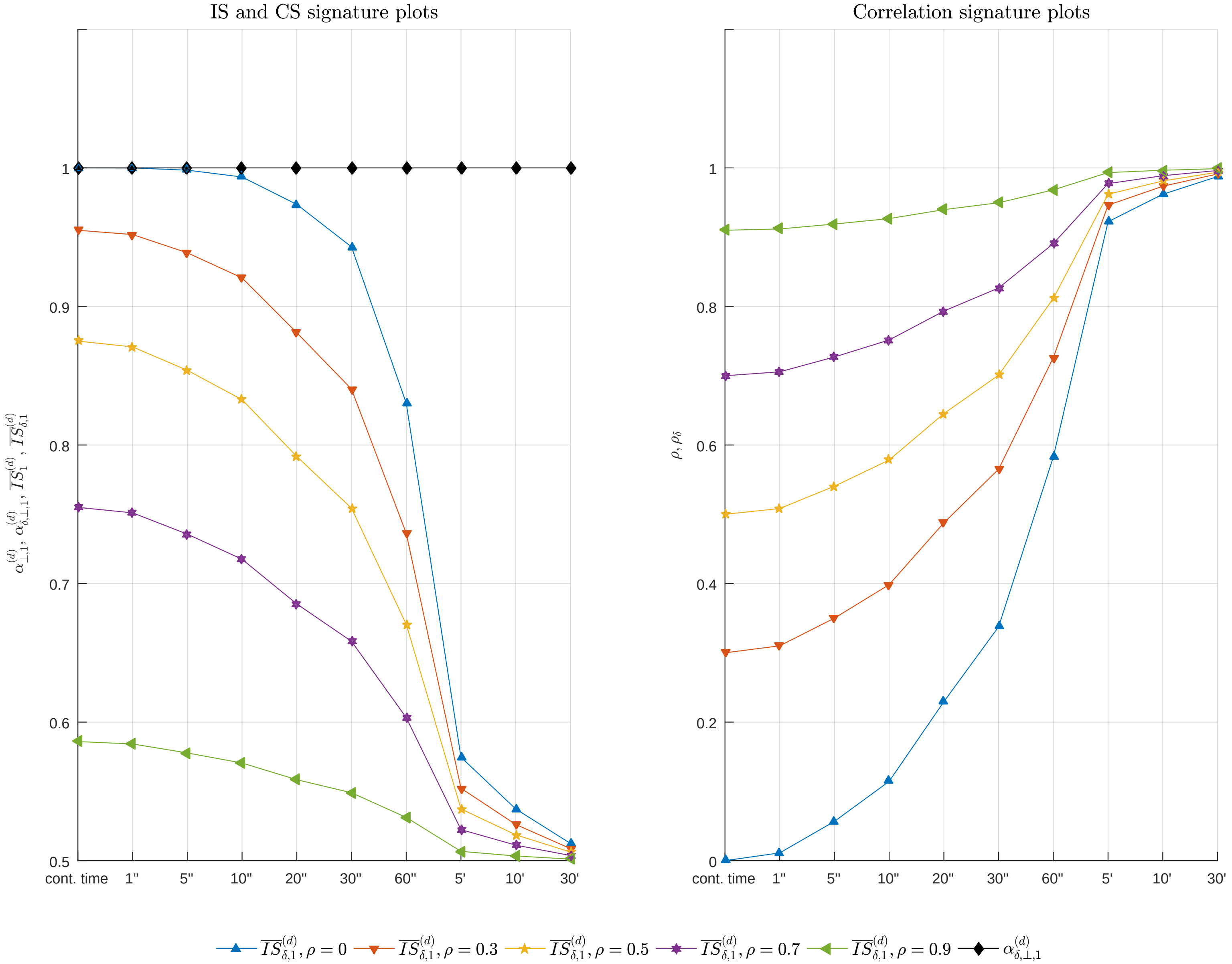


Figure 2: KLS estimates of $\alpha_\perp$, with 95% confidence intervals

The plots portrait the kernel-based LS daily estimates of $\alpha_\perp^{(d)}$ for each stock, with their 95% confidence bands (in shades). We fix the bandwidth at $n\sqrt{D}$, where $n$ is the number of intraday observations (average of 390 observations per day) and $D$ is the number of trading days (1735 days). We normalize the orthogonal projections such that the element wise estimates sum up to one, i.e. $\widehat{\alpha}_{1,\perp}^{(d)} + \widehat{\alpha}_{2,\perp}^{(d)} = 1$.

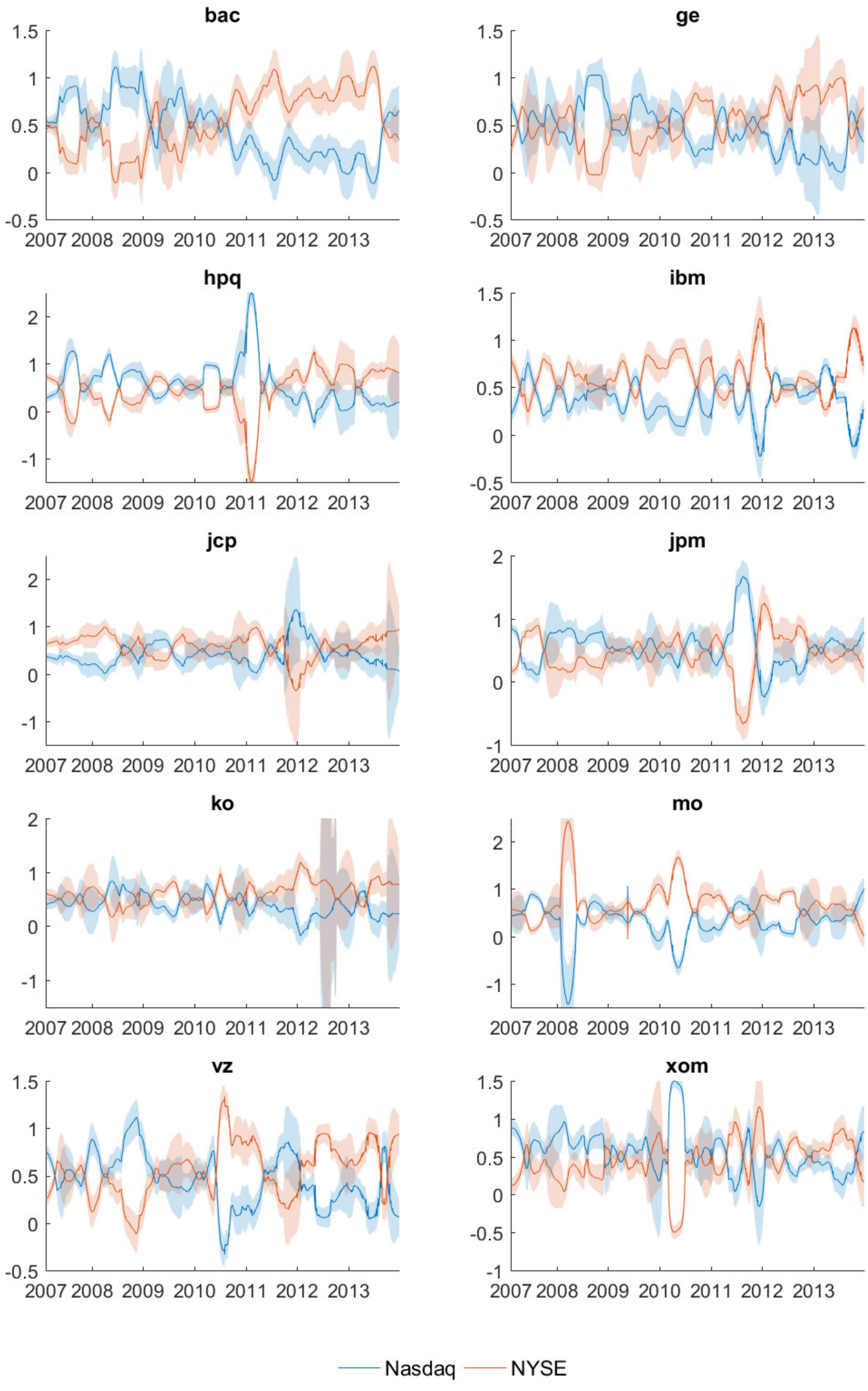
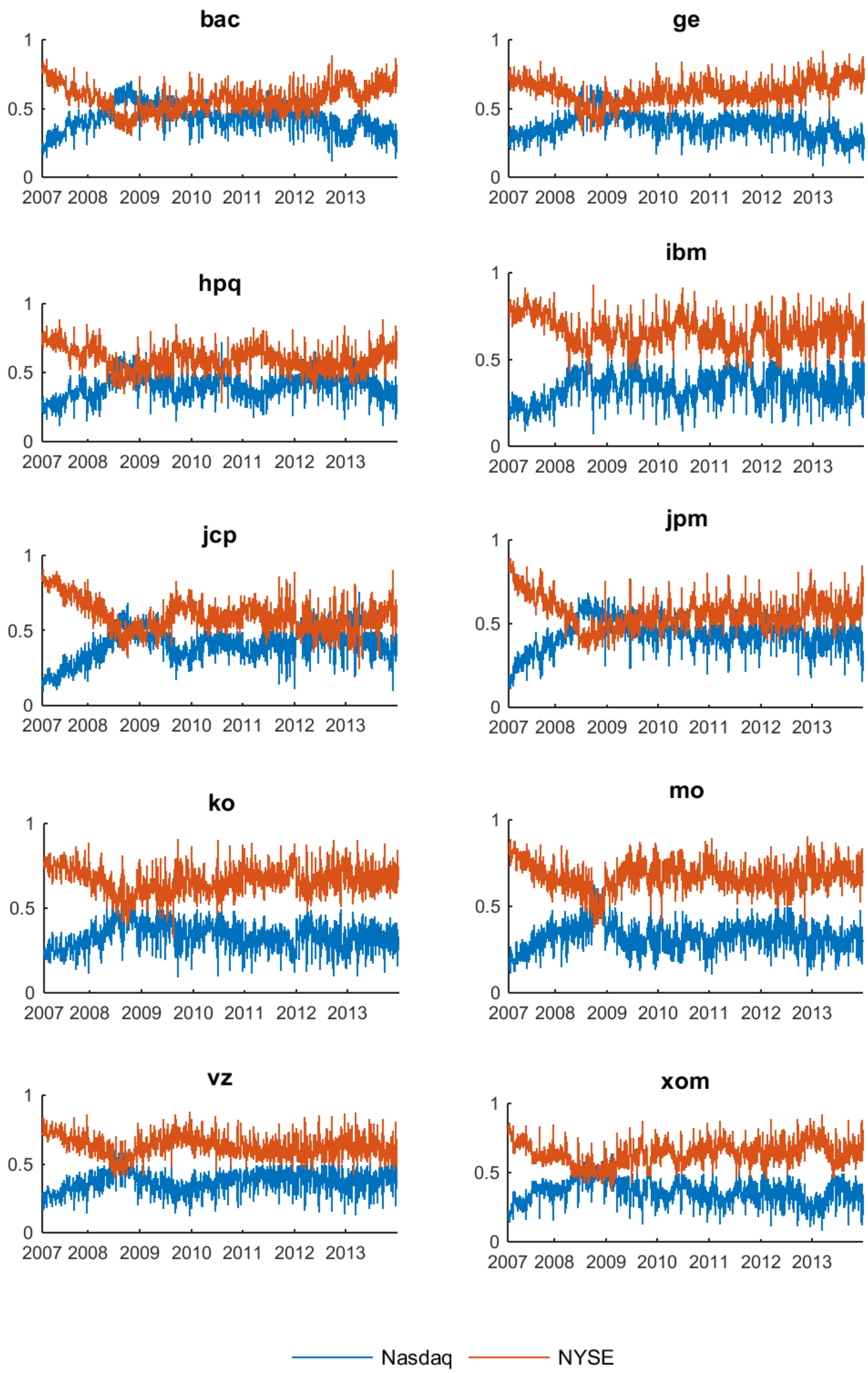


Figure 3: Trading Volume

The plots portrait the volume shares of Nasdaq (T) and NYSE (N) for each stock. For example, the volume share of Nasdaq is computed as the trading volume at Nasdaq over the sum of the trading volume at Nasdaq and NYSE.



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