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Explaining Asset Prices with Low Risk Aversion and Low Intertemporal Substitution[*]

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Abstract

This paper extends the class of Epstein-Zin-Weil preferences with a new utility kernel that disentangles uncertainty about the consumption trend (long-run risk) from short-term variation around this trend (cyclical risk). Our estimation results show that these preferences enable the long-run risk model to explain asset prices with a low relative risk aversion (RRA) of 9.8 *and* a low intertemporal elasticity of substitution (IES) of 0.11. We also show that the proposed preferences allow an otherwise standard New Keynesian model to match the equity premium, the bond premium, and the risk-free rate puzzle with a low IES of 0.07 *and* a low RRA of 5.

Keywords: Bond premium puzzle, Equity premium puzzle, Long-run risk, Perturbation Approximation, Risk-free rate puzzle.

JEL: E44, G12.

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1 Introduction

The seminar work of Mehra and Prescott (1985) and Weil (1989) show that it is challenging to explain the high equity premium and the low risk-free rate in consumption-based models with plausible levels of risk aversion and intertemporal substitution. This has been termed the equity premium puzzle and the risk-free rate puzzle, respectively.

One of the most successful endowment models to explain these puzzles is the long-run risk model of Bansal and Yaron (2004). This model introduces stochastic volatility and a small but very persistent component in consumption growth to match the high equity premium with a low relative risk aversion (RRA) of 10. To avoid the risk-free rate puzzle, Bansal and Yaron (2004) adopt a relatively high intertemporal elasticity of substitution (IES) of 1.5. An IES well above one is generally also required in real business cycle models to match asset prices, as illustrated in Kaltenbrunner and Lochstoer (2010) and Croce (2014) with homoscedastic shocks and in Gourio (2012) with rare disasters.[1] Although the IES is hard to estimate accurately, most reduced-form estimates are typically below one or even close to zero.[2] It therefore seems desirable to extend consumption-based asset pricing models to also match historical data with an IES well below one, as it would serve to robustify the asset pricing channels emphasized in these models.

The contribution of the present paper is to explain the equity premium and the risk-free rate puzzle with low risk aversion and an IES well below one. This is done by introducing a new utility kernel (i.e. periodic utility function) within the class of Epstein-Zin-Weil preferences that disentangles uncertainty about the consumption trend (long-run risk) from short-term variation around this trend (cyclical risk). This utility kernel is derived by combining the ratio-habits of Abel (1990) with positive preference externalities from the level of government infrastructure as in Barro (1981). The utility contribution from long-run risk is given by a power function, whereas the contribution from cyclical consumption risk $\tilde{C}_t$ initially has the general form $u\left(\tilde{C}_t\right)$ where $u'\left(\tilde{C}_t\right) > 0$ and $u''\left(\tilde{C}_t\right) < 0$.

Within the long-run risk model of Bansal and Yaron (2004), we first use a second-order perturbation approximation to show analytically that these preferences may explain the

[1] Jermann (1998) and Boldrin et al. (2001) show that consumption habits with an IES<1 and capital frictions can explain the equity premium and the risk-free rate puzzle but not the low variability in the risk-free rate. Campanale et al. (2010) use disappointment aversion (or high risk aversion) with an IES close to zero to match the equity premium and the risk-free rate puzzle, but this model is also unable to match the low variability in the risk-free rate.

[2] For instance, Barsky et al. (1997) use surveys to estimate an IES between 0.15 and 0.21, whereas Hall (1988) and Yogo (2004) use time series regressions to estimate an IES between 0 and 0.2. The meta-study of Havranek (2015) finds that the IES in micro studies are about 0.2, or between 0.3-0.4, when conditioning on households with asset holdings.

equity premium and the risk-free rate puzzle with a low IES. We next estimate this extension of the long-run risk model by generalized method of moments (GMM) using unconditional first and second moments for the price-dividend ratio, the risk-free rate, the market return, consumption growth, and dividend growth. Our estimates show that disentangling cyclical and long-run risk lowers the IES from 1.5 in Bansal and Yaron (2004) to just above one when $u\left(\tilde{C}_t\right)$ has a power-specification. We also show that the IES can not be reduced below one for this specification of $u\left(\tilde{C}_t\right)$, mainly because long-run risk then reduces the equity premium and raises the risk-free rate. To explore the full potential of disentangling cyclical and long-run risk in the utility kernel, we next consider a semi-parametric version of our model, where $u\left(\tilde{C}_t\right)$ is approximated by a second-order Taylor expansion around the steady state of $\tilde{C}_t$. Our estimation results reveal that asset prices can be matched with a low IES of 0.12 and a RRA equal to 9.67. We then largely reproduce these semi-parametric results by adopting an exponential power-specification for $u\left(\tilde{C}_t\right)$, which gives an IES of 0.11 and a RRA of 9.8. In addition to matching the first and second unconditional moments for the considered five variables in the estimation, we also show that this extension of the long-run risk model matches i) the predictability of the price-dividend ratio for excess market return, ii) the inability of the price-dividend ratio to forecast consumption and dividend growth, and iii) the negative relationship between consumption volatility and the price-dividend ratio.

To provide further support for the proposed preferences disentangling cyclical and long-run risk, we also consider their asset pricing implications in a dynamic stochastic general equilibrium (DSGE) model, where consumption and dividends are determined endogenously. Our GMM estimates reveal that these preferences allow an otherwise standard New Keynesian model to explain the equity premium, the risk-free rate puzzle, and the bond premium (i.e. the level and variability of the 10-year nominal term premium) with a low IES of 0.07 *and* a low RRA of 5. In contrast, most existing New Keynesian models are only able to match asset prices by postulating highly risk-averse households (see Rudebusch and Swanson (2012), Andreasen (2012), Swanson (2015), among others).[3]

The remainder of this paper is organized as follows. Section 2 introduces our new utility kernel within the long-run risk model, and we present our analytical expressions for the unconditional mean of the market return and the risk-free rate. Section 3 estimates our extension of the long-run risk model and study its empirical performance. Section 4 considers a New Keynesian model with our proposed preferences and explore its empirical performance. Concluding comments are provided in Section 5.[4]

[3] The recent paper by Li and Palomino (2014) considers a somewhat low RRA of 16 but at the expense of generating a too low equity premium of 0.96% per annum compared to 7.12% in their data.

[4] All technical derivations and proofs are deferred to an online appendix available from the homepage of

2 A Long-Run Risk Model

This section extends the long-run risk model with a utility kernel that disentangles cyclical and long-run risk. The representative agent is introduced in Section 2.1, and the exogenous processes for consumption and dividends are specified in Section 2.2. We present our new utility kernel in Section 2.3 and discuss its asset pricing implications in Section 2.4 using an analytical second-order perturbation solution.

2.1 The Representative Household

Consider a representative household with recursive preferences as in Epstein and Zin (1989) and Weil (1990). Using the convenient formulation proposed in Rudebusch and Swanson (2012), the value function V_t is given by

$$V_t = \begin{cases} \mathcal{U}_t + \beta \mathbb{E}_t [V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}} & \text{for } \mathcal{U}_t > 0 \\ \mathcal{U}_t - \beta \mathbb{E}_t [(-V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}} & \text{for } \mathcal{U}_t < 0 \end{cases}, \tag{1}$$

where $\mathbb{E}_t$ is the conditional expectation given information in period t.[5] The subjective discount factor is given by $\beta \in (0,1)$ and $\mathcal{U}_t \equiv \mathcal{U}(C_t)$ denotes the utility kernel as a function of consumption C_t. For higher values of $\alpha \in \mathbb{R} \setminus \{1\}$, these preferences generate higher levels of risk aversion if $\mathcal{U}_t$ is always positive, and vice versa for $\mathcal{U}_t < 0$. The formulation in (1) also implies preferences for early resolution of uncertainty if $\alpha > 0$ for $\mathcal{U}_t > 0$, or if $\alpha < 0$ for $\mathcal{U}_t < 0$. The main benefit of considering Epstein-Zin-Weil preferences is to disentangle risk aversion and the IES that otherwise have a perfect inverse relationship when $\alpha = 0$ and (1) simplifies to standard expected utility.

2.2 Consumption and Dividends

The consumption process is specified to be compatible with production economies displaying balanced growth.[6] Hence, we let

$$C_t \equiv Z_t \times \tilde{C}_t, \tag{2}$$

where $Z_t > 0$ is the balanced growth path of technology and $\tilde{C}_t$ refers to cyclical variation in consumption around Z_t. The level of Z_t coincides with long-lasting supply shocks in

the authors or on request.

[5] The specification in (1) is equivalent to the one in Epstein and Zin (1989), i.e. $\hat{V}_t^{\rho} = \hat{\mathcal{U}}_t^{\rho} + \beta \mathbb{E}_t \left[\hat{V}_{t+1}^{\hat{\alpha}} \right]^{\rho/\hat{\alpha}}$, if we let $V = \hat{V}^{\rho}$, $\mathcal{U} = \hat{\mathcal{U}}^{\rho}$, and $\alpha = 1 - \hat{\alpha}/\rho$.

[6] See King et al. (1988) and King and Rebelo (1999) for a detailed exposition.

production economies and is typically specified with deterministic and stochastic trends. Uncertainty about the growth path is therefore captured by Z_t, which allow us to incorporate long-run risk as in Bansal and Yaron (2004). Following Bansal et al. (2010), the second component $\tilde{C}_t$ in (2) introduces cyclical consumption risk, which in production economies originates from demand-related shocks, monetary policy shocks, or short-lived supply shocks (see Smets and Wouters (2007), Justiniano and Primiceri (2008), among others).

Inspired by the work of Bansal and Yaron (2004), we assume that the balanced growth path of technology evolves as

$$\begin{array}{rcl} \log Z_{t+1} & = & \log Z_t + \log \mu_z + x_t + \sigma_z \sigma_t \varepsilon_{z,t+1} \\ x_{t+1} & = & \rho_x x_t + \sigma_x \sigma_t \varepsilon_{x,t+1} \end{array}, \tag{3}$$

with the conditional volatility σ_t for fluctuating economic uncertainty given by

$$\sigma_{t+1}^2 = 1 - \rho_\sigma + \rho_\sigma \sigma_t^2 + \sigma_\sigma \varepsilon_{\sigma,t+1}. \tag{4}$$

Here, $\varepsilon_{i,t+1} \sim \mathcal{NID}(0,1)$ for $i \in (z, x, \sigma)$ with $|\rho_x| < 1$ and $|\rho_\sigma| < 1$.[7] Hence, the balanced growth path of technology has a deterministic trend when $\log \mu_z \neq 0$ and a stochastic trend for $\sigma_z > 0$ or $\sigma_x > 0$. Our specification in (2) leads to the same trends in consumption, where x_t introduces persistent changes in the growth rate of Z_t and captures long-run risk. The innovation $\varepsilon_{z,t}$ does not generate any persistence in the growth rate of Z_t and is therefore referred to as short-run risk.[8] Variation in consumption around its balanced growth path Z_t is specified as in Bansal et al. (2010) by letting

$$\log \tilde{C}_{t+1} = \rho_{\tilde{c}} \log \tilde{C}_t + \sigma_{\tilde{c}} \sigma_t \varepsilon_{\tilde{c},t+1}, \tag{5}$$

where $\varepsilon_{\tilde{c},t} \sim \mathcal{NID}(0,1)$ and $|\rho_{\tilde{c}}| < 1$.

To see how (2) to (5) relate to the existing literature, note that

$$\Delta c_{t+1} = \log \mu_z + x_t + \Delta \tilde{c}_{t+1} + \sigma_z \sigma_t \varepsilon_{z,t+1}, \tag{6}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$, $\tilde{c}_t \equiv \log \tilde{C}_t$, and $\Delta \tilde{c}_{t+1} \equiv \rho_{\tilde{c}} \Delta \tilde{c}_t + \sigma_{\tilde{c}} (\sigma_t \varepsilon_{\tilde{c},t+1} - \sigma_{t-1} \varepsilon_{\tilde{c},t})$. Our specification is thus similar to the one in Bansal et al. (2010) without jumps, which for $\sigma_{\tilde{c}} = 0$

[7] Although (4) does not enforce $\sigma_t^2 \geq 0$, we nevertheless maintain this specification for comparison with Bansal and Yaron (2004) and Bansal et al. (2010). Accounting for the non-negativity constraint on σ_t^2 may be done using a log-normal process for σ_t, as in Schorfheide et al. (2014), or the two specifications mentioned in Andreasen (2010). Asset prices may also for these alternative specifications be computed by the perturbation method as applied below.

[8] Hence, we follow the terminology from the long-run risk model (see for instance Bansal et al. (2010)), although variation in $\varepsilon_{z,t}$ has a permanent effect on the *level* of Z_t.

reduces to the consumption process in Bansal and Yaron (2004) without cyclical risk.

Finally, as in Bansal and Yaron (2004), we let dividends D_t be positively correlated with the balanced growth path of technology in the following way

$$\Delta d_{t+1} = \log \mu_d + \phi x_t + \sigma_d \sigma_t \varepsilon_{d,t+1}, \tag{7}$$

where $d_{t+1} \equiv \log D_{t+1}$, $\varepsilon_{d,t} \sim \mathcal{NID}(0,1)$, and ϕ denotes firm leverage (see Abel (1999)). For completeness, all innovations are assumed to be mutually uncorrelated at all leads and lags.

2.3 A New Utility Kernel

The Epstein-Zin-Weil preferences may be applied to a wide range of utility kernels and are in this sense very general. That is, modifying the utility kernel $\mathcal{U}(C_t)$ induces different preferences and different behavior for the representative household. The single behavioral characteristic which we study in this paper is to disentangle effects from cyclical and long-run risk in the utility kernel as follows. First, the household obtains utility from consumption relative to an exogenous habit stock H_t, i.e. $u(C_t/H_t)$, as in Abel (1990) and Fuhrer (2000) among others. Second, as for instance in Barro (1981) and Guo and Harrison (2008), the marginal utility of consumption may depend positively on the level of government infrastructure G_t but with a diminishing effect for higher values of G_t. That is, we let $\mathcal{U}(C_t) = G_t^{\chi} u\left(\frac{C_t}{Z_t}\right)$ with $0 \leq \chi < 1$, where the level of G_t is considered to be exogenous to the household. To keep our model as simple as possible, we assume that the level of government infrastructure corresponds to the balanced growth path in the economy, i.e. $G_t = Z_t$.[9] Thus, the proposed utility kernel has the form

$$\mathcal{U}(C_t) = Z_t^{\chi} u\left(\frac{C_t}{Z_t}\right). \tag{8}$$

Given that C_t/Z_t is non-trending, tractability of this utility kernel does not restrict the functional form of $u(\cdot)$, where we initially only impose the standard conditions $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The standard power-specification $\mathcal{U}(C_t) = C_t^{\chi}$ is clearly nested by (8) when $u(C_t/Z_t) = (C_t/Z_t)^{\chi}$ and the diminishing marginal effect of public infrastructure and private consumption coincide. Importantly, the utility kernel in (8) allows for the possibility that an increase in the economy's balanced growth path Z_t may not only increase $\mathcal{U}(C_t)$ through higher consumption, as traditionally assumed, but also through a higher level of public infrastructure Z_t^{χ}. In doing so, we disentangle effects of cyclical and long-run risk,

[9] Using the same line of argument as in Rudebusch and Swanson (2012), another way to justify these positive externalities is to consider non-separable preferences for home production C_t^h, and let the production function for C_t^h be proportional to Z_t.

with cyclical risk operating through $u(\cdot)$ and long-run risk through χ. This is in contrast to the standard power-specification $\mathcal{U}(C_t) = C_t^{\chi}$, where χ controls the effects of both cyclical and long-run risk.

The degree of intertemporal substitution in (8) is determined by the partial derivatives of $u(\cdot)$ as

$$\text{IES} = -\frac{\mathcal{U}'(C_{ss})}{\mathcal{U}''(C_{ss})\, C_{ss}} = -\frac{u'(C_{ss}/Z_{ss})}{u''(C_{ss}/Z_{ss})\, C_{ss}/Z_{ss}}, \tag{9}$$

where the subscript ss denotes steady state values. Using the expression in Swanson (2013), the relative risk aversion for (1) and (8) is

$$\text{RRA} = \frac{1}{\text{IES}} + \alpha \frac{u'(C_{ss}/Z_{ss})\, C_{ss}/Z_{ss}}{u(C_{ss}/Z_{ss})} \tag{10}$$

in the steady state. Hence, risk aversion is determined by the IES, the Epstein-Zin-Weil parameter α, and the ratio of $u'(\cdot)$ to $u(\cdot)$.[10] With marginal utility $\mathcal{U}'(C_t) = Z_t^{\chi-1} u'(C_t/Z_t)$, the stochastic discount factor becomes

$$M_{t,t+1} = \beta \left(\frac{\mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}}{V_{t+1}} \right)^{\alpha} \frac{u'(C_{t+1}/Z_{t+1})}{u'(C_t/Z_t)} \frac{Z_{t+1}^{\chi-1}}{Z_t^{\chi-1}}. \tag{11}$$

Although χ does not affect the IES or RRA, it still affects asset prices through the stochastic discount factor. The main asset pricing implications of χ go through three channels: i) the value function, ii) the ratio of marginal utilities, and iii) the conditional covariance between $M_{t,t+1}$ and dividends as both Δc_{t+1} and Δd_{t+1} co-move with the balanced growth path of technology, i.e. Z_t.

2.4 Understanding Asset Prices

To explain the main effects of our new utility kernel, we follow Bansal and Yaron (2004) and consider a simplified version of the long-run risk model without stochastic volatility, i.e. $\sigma_\sigma = 0$. Given the unspecified form of $u(\cdot)$, the stochastic discount factor is not necessarily log-linear and asset prices can not be computed by the standard log-normal method as in Bansal and Yaron (2004) or the approach taken in Hansen et al. (2008). Instead, we use the perturbation method of Judd and Guu (1997) to derive an analytical second-order approximation. This solution captures quite accurately the mean level of asset prices and is

[10] Unreported results show that χ has a small impact on RRA outside of steady state, for which (10) serves as a good approximation.

therefore sufficient to understand how our utility kernel helps to simultaneously resolve the equity premium and the risk-free rate puzzle.[11]

To obtain our analytical approximation, we first solve for the log-transformed value function $v_t \equiv \log V_t$ and the *twisted* log-transformed value function $ev_t \equiv \log \mathbb{E}_t \left[e^{(1-\alpha)\left(v_{t+1}+\chi \log \mu_{z,t+1}\right)} \right]$ where $\mu_{z,t} \equiv Z_t/Z_{t-1}$. These approximations are then used to solve for the net risk-free rate $r_t^f \equiv \log R_t^f$, the price-dividend ratio $pd_t \equiv \log\left(P_t/D_t\right)$, and finally the net expected equity return $r_t^{m,e} \equiv \mathbb{E}_t\left[r_{t+1}^m\right]$ where $r_t^m \equiv \log R_t^m$. In the interest of space, we only provide the solution for the risk-free rate and the expected equity return.

Proposition 1 *The second-order approximation to the risk-free rate* r_t^f *and the expected equity return* $r_t^{m,e}$ *at the steady state are given by*

$$
\begin{aligned}
r_t^f &= r_{ss}^f + r_{\tilde{c}}^f \tilde{c}_t + r_x^f x_t + \frac{1}{2} r_{\tilde{c}\tilde{c}}^f \tilde{c}_t^2 + r_{\tilde{c}x}^f \tilde{c}_t x_t + \frac{1}{2} r_{xx}^f x_t^2 + \frac{1}{2} r_{\sigma\sigma}^f \\
r_t^{m,e} &= r_{ss}^{m,e} + r_{\tilde{c}}^{m,e} \tilde{c}_t + r_x^{m,e} x_t + \frac{1}{2} r_{\tilde{c}\tilde{c}}^{m,e} \tilde{c}_t^2 + r_{\tilde{c}x}^{m,e} \tilde{c}_t x_t + \frac{1}{2} r_{xx}^{m,e} x_t^2 + \frac{1}{2} r_{\sigma\sigma}^{m,e}
\end{aligned}
$$

where

$$
\begin{aligned}
r_{ss}^f &= r_{ss}^{m,e} = -\log\beta - (\chi-1)\log\mu_z \\
r_{\tilde{c}}^f &= r_{\tilde{c}}^{m,e} = (1-\rho_{\tilde{c}}) \frac{u''(1)}{u'(1)} \\
r_x^f &= r_x^{m,e} = 1-\chi \\
r_{\tilde{c}\tilde{c}}^f &= r_{\tilde{c}\tilde{c}}^{m,e} = \left(1-\rho_{\tilde{c}}^2\right)\left(\frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)} - \left(\frac{u''(1)}{u'(1)}\right)^2\right) \\
r_{\tilde{c}x}^f &= r_{\tilde{c}x}^{m,e} = 0 \\
r_{xx}^f &= r_{xx}^{m,e} = 0 \\
r_{\sigma\sigma}^f &= -\alpha v_x^2 \sigma_x^2 - \left[1 + (1-\alpha)\chi(\chi-2)\right]\sigma_z^2 - \sigma_{\tilde{c}}^2 \left(\frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)}\left(1-2\alpha v_{\tilde{c}}\right) + \alpha v_{\tilde{c}}^2\right) \\
r_{\sigma\sigma}^{m,e} &= -(1-\kappa_1) pd_{\sigma\sigma} + \kappa_1\left(pd_{\tilde{c}\tilde{c}} + pd_{\tilde{c}}^2\right)\sigma_{\tilde{c}}^2 + \kappa_1\left(pd_{xx} + pd_x^2\right)\sigma_x^2 + \sigma_d^2
\end{aligned}
$$

and $\kappa_1 = \frac{\beta\mu_z^{(\chi-1)}\mu_d}{1+\beta\mu_z^{(\chi-1)}\mu_d}$. *The derivatives of* v_t *and* pd_t *are provided in Appendix A.*

Proposition 1 shows that the expressions for r_t^f and $r_t^{m,e}$ only differ in their uncertainty corrections $r_{\sigma\sigma}^f$ and $r_{\sigma\sigma}^{m,e}$, which is explained by the fact that all the remaining terms represent a perfect foresight approximation. The steady state value of r_t^f and $r_t^{m,e}$ is directly affected

[11] The analytical perturbation approximation may also be applied to the version of the long-run risk model with stochastic volatility, but its additional state variables complicate the model solution and make the effects of our new utility kernel less transparent.

by the impact of long-run risk in the utility kernel as parametrized by χ, where high values of χ ensure low levels of r_{ss}^{f} and $r_{ss}^{m,e}$. For a standard power utility kernel, i.e.

$$u\left(\frac{C_t}{Z_t}\right)=\frac{1}{1-1/\psi}\left(\frac{C_t}{Z_t}\right)^{1-1/\psi} \quad \text{and} \quad \chi=1-1/\psi, \tag{12}$$

we obtain the well-known result $r_{ss}^{f}=r_{ss}^{m,e}=-\log\beta+\log\mu_z/\psi$, where a high ψ =IES ensures low steady state returns, for instance $\psi=1.5$ as in Bansal and Yaron (2004). Returning to our general utility kernel, the first-order effects from variation in $\tilde{c}_t$ may be re-expressed as $-\left(1-\rho_{\tilde{c}}\right)/\text{IES}$ using (9) and $C_{ss}/Z_{ss}=1$, showing that $r_{\tilde{c}}^{f}=r_{\tilde{c}}^{m,e}<0$ due to the negative auto-correlation in $\Delta\tilde{c}_t$. A similar negative effect on the risk-free rate from cyclical risk is reported in Bansal et al. (2010). We also find the traditional positive effect of long-run risk, i.e. $r_x^{f}=r_x^{m,e}>0$, given our assumption that $\chi<1$. For the standard power kernel in (12), note also that r_x^{f} simplifies to $1/\psi>0$ as in Bansal and Yaron (2004). The uncertainty correction in the risk-free rate depends i) on the size of the shocks, ii) the curvature of $u\left(\cdot\right)$, iii) the Epstein-Zin-Weil parameter α, and iv) how cyclical risk affects the value function through $v_{\tilde{c}}$. The same holds for $r_{\sigma\sigma}^{m,e}$ following an inspection of the approximated expression for pd_t provided in Proposition A.2.

To further explore the implications of our new utility kernel and how it relates to the equity premium and the risk-free rate puzzle, Proposition 2 provides the unconditional mean of the risk-free rate and the ex ante equity premium.

Proposition 2 *The unconditional mean of the risk-free rate and the ex ante equity premium in a second-order approximation at the steady state are given by*

$$\begin{aligned}\mathbb{E}\left[r_t^{f}\right] &= r_{ss}^{f}-\frac{\sigma_x^2}{2}\frac{\alpha\kappa_0^2\chi^2}{\left(1-\kappa_0\rho_x\right)^2}-\frac{\sigma_z^2}{2}\left[1+\left(1-\alpha\right)\chi\left(\chi-2\right)\right]\\ &\quad -\frac{\sigma_{\tilde{c}}^2}{2}\left[\frac{u''\left(1\right)^2}{u'\left(1\right)^2}-2\alpha\frac{u''\left(1\right)}{u'\left(1\right)}\frac{u'\left(1\right)}{u\left(1\right)}\frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}}+\alpha\frac{u'\left(1\right)^2}{u\left(1\right)^2}\frac{\left(1-\kappa_0\right)^2}{\left(1-\kappa_0\rho_{\tilde{c}}\right)^2}\right]\end{aligned}$$

and

$$\mathbb{E}\left[r_{t+1}^{m}-r_t^{f}\right]=\beta_x\sigma_x^2+\beta_{\tilde{c}}\sigma_{\tilde{c}}^2,$$

where

$$\begin{aligned}\beta_x &= \frac{\alpha\kappa_0\kappa_1}{\left(1-\kappa_0\rho_x\right)\left(1-\kappa_1\rho_x\right)}\chi\left(\phi+\chi-1\right)\\ \beta_{\tilde{c}} &= \kappa_1\frac{1-\rho_{\tilde{c}}}{1-\kappa_1\rho_{\tilde{c}}}\frac{u''\left(1\right)^2}{u'\left(1\right)^2}-\alpha\kappa_1\frac{\left(1-\rho_{\tilde{c}}\right)\left(1-\kappa_0\right)}{\left(1-\kappa_0\rho_{\tilde{c}}\right)\left(1-\kappa_1\rho_{\tilde{c}}\right)}\frac{u''\left(1\right)}{u'\left(1\right)}\frac{u'\left(1\right)}{u\left(1\right)}.\end{aligned}$$

The auxiliary parameters are $\kappa_0 = \beta \mu_z{}^{\chi}$ *and* $\kappa_1 = \frac{\beta \mu_z^{(\chi-1)} \mu_d}{1+\beta \mu_z^{(\chi-1)} \mu_d}$.

Proposition 2 shows that the mean risk-free rate is given by its steady level r^f_{ss} and uncertainty corrections for each of the shocks affecting consumption. The first uncertainty correction $-\frac{1}{2}\sigma_x^2 \alpha \kappa_0^2 \chi^2 / (1-\kappa_0 \rho_x)^2$ relates to long-run risk and has a negative impact on $\mathbb{E}\left[r^f_t\right]$, given a positive value function with $\alpha > 0$. Importantly, this correction becomes more negative for larger values of χ and α. The second correction $-\frac{\sigma_z^2}{2}\left[1 + (1-\alpha)\chi(\chi-2)\right]$ relates to short-run risk and is also negative, given $\chi \in [0, 1[$ and $\alpha > 0$. The uncertainty correction attached to cyclical risk has three terms. The first term is given by $-0.5\sigma_{\tilde{c}}^2 \left(u''(1)/u'(1)\right)^2 = -0.5\sigma_{\tilde{c}}^2/\text{IES}^2$ and decreases for lower values of the IES. The second term $\sigma_{\tilde{c}}^2 \alpha \frac{u''(1)}{u'(1)} \frac{u'(1)}{u(1)} \frac{1-\kappa_0}{1-\kappa_0 \rho_{\tilde{c}}}$ is also negative, because $\alpha u(1)$ must be positive to ensure a reasonable level of RRA, whereas $u'(1) > 0$ and $u''(1) < 0$ by assumption. The final term $-\frac{\sigma_{\tilde{c}}^2}{2} \alpha \frac{u'(1)^2}{u(1)^2} \frac{(1-\kappa_0)^2}{(1-\kappa_0 \rho_{\tilde{c}})^2}$ is also negative provided $\alpha > 0$.

The exposure of the equity premium to innovations in x_t is given by β_x which is positive if $\phi + \chi > 1$ and $\alpha > 0$. The size of this exposure is increasing in i) the persistency of x_t determined by ρ_x, ii) the Epstein-Zin-Weil parameter α, iii) firm leverage ϕ, and iv) the impact of long-run risk in the utility kernel through χ. The exposure of the equity premium to cyclical risk is given by $\beta_{\tilde{c}}$, where the first term always is positive, whereas the final term only is positive for $\alpha > 0$. The magnitude of these terms dependent on the curvature of $u(\cdot)$, the Epstein-Zin-Weil parameter α, and the persistency of the cyclical risk $\rho_{\tilde{c}}$.

To summarize our insights from these analytical expressions, recall that existing models tend to generate too low equity premia and too high risk-free rates. We thus require a positive correction in $\mathbb{E}\left[r^m_{t+1} - r^f_t\right]$ and a negative correction in $\mathbb{E}\left[r^f_t\right]$ to simultaneously resolve the equity premium and the risk-free rate puzzle. The utility kernel we propose in (8) does exactly so for high values of α and χ in combination with *low* values of the IES. That is, we break the tight link between χ and the IES in the standard power kernel by disentangling effects of cyclical and long-run risk in $\mathcal{U}(C_t)$. This allow us to control the impact of long-run risk in $\mathcal{U}(C_t)$ without affecting the IES. We also note from our analytical approximation that the proposed preferences are not solely characterized by their RRA and IES. That is, one *can not* eliminate α and derivatives of $u(\cdot)$ from the solution and solely use RRA and IES to describe preferences. It is also important to note that $\alpha > 0$ is required to ensure a high equity premium and a low risk-free rate.

3 Estimation Results: The Long-Run Risk Model

This section studies the ability of our long-run risk model to explain key features of the post-war U.S. stock market. We first describe our model solution and estimation methodology in Section 3.1. The estimation results for the standard long-run risk model are provided in Section 3.2 as a natural benchmark. The following four subsections consider different versions of our long-run risk model disentangling cyclical and long-run in the utility kernel. The ability of these models to match moments not included in the estimation is finally explored in Section 3.7.

3.1 Model Solution and Estimation Methodology

We solve for asset prices using a perturbation approximation, as in Section 2.4, but use a third-order expansion to allow for time-variation in risk premia. This approximation is computed using the algorithm of Binning (2013). The estimation is carried out on quarterly data, as this data frequency strikes a good balance between getting a reasonably long sample and providing reliable measures of consumption and dividend growth.[12] Consistent with the common calibration procedure for the long-run risk model, we let one period in the model correspond to one month and time-aggregate its moments to the considered data frequency.

Our quarterly data set is from 1947Q1 to 2014Q4, where we use the same five variables as in Bansal and Yaron (2004): i) the log-transformed price dividend ratio pd_t, ii) the real risk-free rate r_t^f, iii) the market return r_t^m, iv) consumption growth Δc_t, and v) dividend growth Δd_t.[13] All variables are stored in $\mathbf{data}_t$ with dimension 5×1. We explore whether our model can match the means, the variances, the contemporaneous covariances, and the persistence in these five variables. Hence, we let

$$\mathbf{q}_t \equiv \begin{bmatrix} \mathbf{data}_t \\ vec\left(\mathbf{data}_t \mathbf{data}_t'\right) \\ diag\left(\mathbf{data}_t \mathbf{data}_{t-1}'\right) \end{bmatrix},$$

where $diag\left(\cdot\right)$ denotes the diagonal elements of a matrix. Letting $\boldsymbol{\theta}$ contain the model parameters, the GMM estimator of Hansen (1982) is then given by the value of $\boldsymbol{\theta}$ that minimizes

[12] Although the long-run risk model often is calibrated to moments in annual data, as in Bansal and Yaron (2004), its performance is remarkably robust and carries over to quarterly data (see Bansal et al. (2012b) and Beeler and Campbell (2012)).

[13] Details on the data sources and data construction are provided in the online appendix.

the objective function

$$Q = \left(\frac{1}{T}\sum_{t=1}^{T}\mathbf{q}_t - \mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]\right)' \mathbf{W}_T \left(\frac{1}{T}\sum_{t=1}^{T}\mathbf{q}_t - \mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]\right),$$

where $\frac{1}{T}\sum_{t=1}^{T}\mathbf{q}_t$ denotes the empirical moments. The model-implied moments $\mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]$ are computed in closed form as in Andreasen et al. (2013) using a pruning scheme when constructing the approximated model solution.[14] We adopt the conventional 2-step implementation of GMM and use a diagonal weighting matrix based on the variance of the sample moments in a preliminary first step, before obtaining our final estimate $\hat{\boldsymbol{\theta}}$ using the optimal weighting matrix.[15]

3.2 The Benchmark Model

As a natural benchmark, we first consider the standard long-run risk model without separate effects for cyclical and long-run risk in the utility kernel. That is, we let

$$\mathcal{U}(C_t) = \frac{1}{1-1/\psi}C_t^{1-1/\psi}. \tag{13}$$

For comparability with nearly all calibrations of the long-run risk model, we let RRA $= 10$ and IES $= 1.5$ as in Bansal and Yaron (2004).[16] Table 1 shows that x_t generates a small but very persistent component in consumption growth with $\hat{\sigma}_x = 0.0004$ and $\hat{\rho}_x = 0.9802$. As in the calibration of Bansal et al. (2012a), we also find the conditional volatility to be highly persistent ($\hat{\rho}_\sigma = 0.9942$), as it amplifies the volatility channel in the model. Variation in cyclical risk is much more mean-reverting ($\tilde{\rho}_{\tilde{c}} = 0.17$), but still important given the relative large value of $\hat{\sigma}_{\tilde{c}} = 0.0033$. We also note that the constraint on the effective discount factor $\beta^* \equiv \beta\mu_z^{1-1/\psi} < 1$ for (1) with (13) is binding, suggesting that the standard utility kernel at least along this dimension is constrained in its ability to fit the data.

< Table 1 about here >

Table 2 verifies the common finding in the literature that the standard long-run risk model is very successful in explaining asset prices. The model largely reproduces the mean and standard deviation of the price-dividend ratio, the risk-free rate, and the market return.

[14] Omitting the pruning scheme for the approximated model solution and computing unconditional moments by the Monte Carlo method gives nearly identical results.

[15] The weigthing matrices are in both steps computed by the Newey-West estimator using 15 lags, but our results are robust to using either 10 or 20 lags.

[16] The desired level of risk aversion is obtained by setting α appropriately using (10). Estimating the RRA and the IES give nearly identical results to those provided in Tables 1 and 2.

The only possible exception is the mean market return which is somewhat lower than in the data (5.44% vs. 6.92%), but still well within the 95% confidence interval. Omitting short-run risk by letting $\sigma_{\tilde{c}} = 0$ as in Bansal and Yaron (2004), does not affect the mean of asset prices but lowers the standard deviation of the risk-free rate from 2.21% to 1.66%.[17] The last part of Table 2 shows that our estimated version of the long-run risk model underestimates the persistence in the risk-free rate (0.59 vs. 0.87), overestimates the standard deviation in consumption growth (2.90% vs. 2.04%) as well as its persistence (0.57 vs. 0.31), and generates too high contemporaneous correlations for the price-dividend ratio and the risk-free rate.

< Table 2 about here >

3.3 A New Utility Kernel: a Power Specification

Next, we explore whether disentangling cyclical and long-run risk in the utility kernel allows us to match asset prices with an IES well below one. Given the popularity of the power utility kernel, the most obvious specification is probably to let

$$\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{1-1/\psi}\left(\frac{C_t}{Z_t}\right)^{1-1/\psi}, \tag{14}$$

where χ and ψ are free parameters.[18] Table 1 shows that the GMM estimates are $\hat{\chi} = 0.51$ and $\hat{\psi} = 1.02$. A Wald test clearly rejects the restriction $1 - 1/\psi = \chi$ from the benchmark model in (13) at all conventional significance levels, as $1 - 1/\hat{\psi} = 0.02$ and thus substantially lower than $\hat{\chi}$.

Table 2 further shows that this version of the long-run risk model generally improves the fit of auto-correlations and most contemporaneous correlations compared to the benchmark model. However, this comes at the cost of a slightly worse fit in matching the standard deviations. To evaluate the goodness of the fit, Table 2 also reports the value of the objective function Q^{step2} in step 2 of our GMM estimation and the related P-value for the well-known J-test for model misspecification (see Hansen (1982)). The benchmark model and our extension(s) are not rejected by the data, but we note that the J-test has low power given our relative short sample ($T = 271$). The values of Q^{step2} are unfortunately not comparable across models, because they are computed for model-specific weighting matrices. To facilitate

[17] Similar effects of cyclical risk are reported in Bansal et al. (2012*a*).

[18] For instance, the recent paper by Creal and Wu (2016) considers Epstein-Zin-Weil preferences with ratio-habits when the utility kernel has a power specification.

model comparison, we therefore introduce the following measure for goodness of fit

$$Q^{scaled} = \sum_{i=1}^{n} \left(\frac{m_i^{data} - m_i^{model}}{1 + m_i^{data}} \right)^2, \tag{15}$$

where m_i^{data} and m_i^{model} refer to the scaled moments in the data and the model, respectively, reported in Table 2.[19] Although the moments in (15) are weighted differently than in the estimation, Q^{scaled} may nevertheless serve as a natural summary statistics for model comparison from an economic perspective. We find that the benchmark model has a value of $Q^{scaled} = 1.14$ and therefore marginally dominates an extension based on (14) with $Q^{scaled} = 1.17$. Thus, the better fit of most auto- and contemporaneous correlations in this extension of the long-run risk model does not compensate for its weaker performance in matching standard deviations when using the weights in (15).

These findings suggest that the long-run risk model can be improved along some dimensions by disentangling cyclical and long-run risk in the utility kernel. Given that ψ in (14) corresponds to the IES, this is achieved with a lower IES of 1.02 compared to 1.5 in the benchmark model. Although this is a sizable reduction in the IES, it is still somewhat far from most reduced-form estimates of the IES. To understand why the utility kernel in (14) is unable to reduce the IES below one, recall from our analytical expressions in Section 2.4 that the IES should be as low as possible and $\alpha > 0$ to resolve the equity premium and the risk-free rate puzzle. Our estimates show that we obtain the best fit by having a relative high IES of 1.02, because it ensures $u(1) > 0$ and hence the desired positive value of α to get realistic levels of RRA. To illustrate the sizable effect of an IES less than one, suppose ψ is lowered to 0.95 with all other parameters given as in Table 1. This minor change means that $u(1) < 0$ and therefore $\alpha < 0$. As shown in Section 2.4, a negative Epstein-Zin-Weil parameter implies that many of the uncertainty corrections go in the opposite direction to resolve the equity premium and the risk-free rate puzzle. For instance, long-run risk now has a negative effect on $\mathbb{E}\left[r_{t+1}^m - r_t^f\right]$ and a positive effect on $\mathbb{E}\left[r_t^f\right]$ when IES<1.[20] As a result, lowering the IES to 0.95 increases the mean risk-free rate from 0.98% to 4.35% and reduces the equity return from 8.74% to 1.56%.

3.4 A New Utility Kernel: a Semi-Parametric Specification

An important insight from our analytical expressions in Section 2.4 is that the asset pricing implications of the considered preferences are not solely characterized by their RRA and IES.

[19] The difference $m_i^{data} - m_i^{model}$ in (15) is standardized by $1 + m_i^{data}$, as oppose to just m_i^{data}, to ensure that moments close to zero do not get very large weights.

[20] A similar result is reported in Bansal et al. (2010).

Hence, the inability of (14) to fit asset prices with an IES well below one may be due to the adopted power-specification of $u(\cdot)$. Given that our setup does not restrict the functional form of $u(\cdot)$, we can easily explore this possibility by considering a semi-parametric version of our utility kernel, where $u\left(\frac{C_t}{Z_t}\right)$ is approximated by a second-order expansion at $C_{ss}/Z_{ss} = 1$. Hence, we let

$$\mathcal{U}(C_t) = Z_t^{\chi}\left(u(1) + u'(1)\left(\frac{C_t}{Z_t} - 1\right) + \frac{1}{2}u''(1)\left(\frac{C_t}{Z_t} - 1\right)^2\right). \tag{16}$$

The value of $u(1)$ is not identified and therefore normalized to one, leaving only $u'(1)$ and $u''(1)$ as free parameters.[21] Table 1 shows that the preferred utility kernel is characterized by $\hat{u}'(1) = 5.190 \times 10^{-4}$ and $\hat{u}''(1) = -0.0042$, which imply $\hat{u}'(1)/u(1) = 5.190 \times 10^{-4}$ and $|\hat{u}''(1)/\hat{u}'(1)| = 8.0259$. Thus, these estimates differ substantially from the power kernel in (14) by having a much larger value of $|\hat{u}''(1)/\hat{u}'(1)|$, which corresponds to an IES of 0.12. Table 2 further shows that this low IES is fully consistent with asset prices, as this extension of the long-run risk model matches most of the considered moments, including the mean of the risk-free rate and the market return. Note also, that this version of the long-run risk model has a very low value of $Q^{scaled} = 0.42$, implying that it provides a substantially better overall fit to the data than the benchmark model with $Q^{scaled} = 1.14$.

3.5 A New Utility Kernel: an Exponential Power Specification

Our semi-parametric estimates show that disentangling effects of cyclical and long-run risk *and* considering a sufficiently flexible utility kernel allows us to explain asset prices with an IES well below one. A natural question is what functional form of $u(\cdot)$ is capable of reproducing these semi-parametric estimates? Our proposed functional form is given by

$$\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{\chi}\left(\frac{1 - e^{-\tau(C_t/Z_t)}}{\tau}\right)^{\chi}, \tag{17}$$

which is referred to as the exponential power utility kernel. The impact of long-run risk remains controlled by χ, whereas the effect of cyclical risk is determined by an exponential function indexed by $\tau > 0$. The utility of cyclical risk is raised to the power of χ to ensure that (17) reduces to the standard power utility kernel in (13) when $\tau \to 0$.[22] To illustrate

[21] Unreported results reveal that third- and fourth-order derivatives of $u(\cdot)$ are also not identified in our case, although these terms in principle may affect our third-order perturbation approximation.

[22] Another alternative would be to let $\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{1-1/\psi}\left(\frac{1-e^{-\tau(C_t/Z_t)}}{\tau}\right)^{1-1/\psi}$, but unreported results suggest that χ, ψ, and τ are not jointly identified.

the asset pricing implications of (17), consider the two steady state ratios

$$u'(1)/u(1) = \frac{\chi\tau}{e^{\tau}-1} \quad \text{and} \quad u''(1)/u'(1) = -\tau\left(1+\frac{1-\chi}{e^{\tau}-1}\right). \tag{18}$$

Hence, a higher value of τ lowers $u'(1)/u(1)$ and increases $|u''(1)/u'(1)|$. Given that the IES$= -u'(1)/u''(1)$, the latter corresponds to a lower IES for higher values of τ. These effects are also illustrated in Figure 1, which shows that τ controls the concavity of $u(\cdot)$ and hence the rate of decay in marginal utility. The preference parameter τ therefore also has a significant effect on risk aversion, which is given by

$$\text{RRA} = \frac{1}{\text{IES}} + \alpha\chi\frac{\tau}{e^{\tau}-1}. \tag{19}$$

The first term in (19) coincides with the measure of RRA obtained with standard expected utility, i.e. $\alpha = 0$, and increases in τ. The second term in (19) is due to the presence of Epstein-Zin-Weil preferences and decreases in τ. To understand this effect, recall that the second term of RRA equals $\alpha u'(1)/u(1)$, meaning that higher values of τ generate a larger reduction in $u'(1)$ compared to $u(1)$, which then reduces $u'(1)/u(1)$. This in turn makes the value function less responsive to changes in future consumption paths, which then lowers the required compensation for holding risky assets, i.e. the RRA.

< Figure 1 about here >

The final column of Table 1 shows that $\hat{\tau} = 8.95$, which is statistically different from zero given a standard error of 1.03. This suggests that (17) improves upon the standard power utility kernel in (13). In line with previous calibrations of the long-run risk model, we also find a low relative risk aversion with $\widehat{\text{RRA}} = 9.78$. Given $\hat{\chi} = 0.51$, the two steady state ratios are $u'(1)/u(1) = 0.0006$ and $|u''(1)/u'(1)| = 8.95$ with the exponential power utility kernel, and hence very close to the semi-parametric estimates in Section 3.4. This implies that the IES based on (17) is 0.11 and hence well below one. The last column of Table 2 shows that this extension of the long-run risk model is able to explain the considered moments for asset prices, consumption growth, and dividend growth. This includes matching the mean of the market return and the risk-free rate, implying that we resolve the equity premium and the risk-free rate puzzle with low risk aversion *and* low IES. It is also worth noticing that this fully parametric extension of the long-run risk model gives an $Q^{scaled} = 0.50$ and hence provides a better overall fit to the data than the benchmark model with $Q^{scaled} = 1.14$.

3.6 Analyzing the Exponential Power Utility Kernel

We next explore the main asset pricing implications of the exponential power utility kernel by gradually increasing τ from $\tau \to 0$ to its estimated value. Table 3 shows that a higher value of τ generate a fast reduction in the IES and a substantial increase in the required value of α to ensure a constant RRA. We also see that increasing τ has the desired effects on asset prices as it i) reduces $\mathbb{E}[pd_t]$ and $\mathbb{E}[r_t^f]$, ii) increases $\mathbb{E}[r_t^m]$, and iii) generates more variability in pd_t, r_t^f, and r_t^m. The final column in Table 3 illustrates that we are unable to explain these asset pricing moments by using the standard power utility kernel (i.e. for $\tau \to 0$) with the same low IES$= 0.11$ as implied by the exponential power utility kernel. In particular, $\mathbb{E}[r_t^f]$ increases to 20.4% and the excess market return is now negative $\left(\mathbb{E}[r_t^m] - \mathbb{E}[r_t^f] = -0.139\right)$.

To understand the effects of τ on $\mathbb{E}[r_t^f]$ and $\mathbb{E}[r_t^m]$ in greater detail, Table 3 also decomposes their values based on Proposition 2.[23] Here, we emphasize to main effects. First, lowering the IES through higher values of τ does not affect $r_{ss}^f = r_{ss}^m = -\log\beta - (\chi - 1)\log\mu_z$, which in contrast increases to extreme levels in the standard utility kernel when reducing the IES through large negative values of χ for $\log\mu_z > 0$. Second, a high value of τ lowers $u'\left(\tilde{C}_t\right)/u\left(\tilde{C}_t\right)$ and implies relative large values of the Epstein-Zin-Weil parameter α to make the household sufficiently risk averse, even for RRA≈ 10. To understand the effect of increasing α for a *given* level of RRA, recall that the household is indifferent to resolution of uncertainty when $\alpha = 0$, and all uncertainty corrections are therefore either very small or absent. This case is well-represented by the first column in Table 3 where $\tau \to 0$. Now suppose we increase α to make the household prefer early resolution of uncertainty, but without affecting the RRA. This modification makes the certain cashflow from the one-period risk-free bond more attractive, and a lower risk-free rate is therefore required. This effect explains why we see larger negative corrections for long-run, short-run, and cyclical risk in $\mathbb{E}[r_t^f]$. On the other hand, uncertain future dividends from equity become less attractive for higher values of α due to the presence of long-run risk. A household with strong preferences for early resolution of uncertainty therefore requires a larger compensation for long-run risk (and its stochastic volatility) to hold equity compared to the case of $\alpha = 0$. Table 3 shows that these effects on $\mathbb{E}[r_t^m]$ from higher values of α dominate the larger negative risk corrections from short-run and cyclical risk, which are similar to those for the risk-free rate and hence constitute a pure discounting effect.

< Table 3 about here >

[23] This decomposition exploits the fact that the unconditional mean of any variable in the pruned state-space system with Gaussian shocks is identical at second- and third order (see Andreasen et al. (2013)). The contribution from stochastic volatility is then given by the difference between the unconditional mean at third order and the mean implied by Proposition 2, which omits stochastic volatility.

3.7 Additional Model Implications

In addition to the moments used for the estimation, the long-run risk model is also frequently evaluated on its ability to reproduce several stylized facts for the U.S. stock market. We first study the ability of the price-dividend ratio to predict the excess market return, consumption growth, and dividend growth. The first column of Figure 2 shows that our extension of the long-run risk model with an exponential power utility kernel preserves the good performance of the benchmark model and almost perfectly reproduces the ability of a high price-dividend ratio to forecast low excess market returns at all considered horizons. As found in Beeler and Campbell (2012), the price-dividend ratio does not forecast either consumption or dividend growth in the data, which is also matched when using the exponential power utility kernel in the long-run risk model.

As a supplement to these univariate predictability tests, Bansal et al. (2012*a*) suggest expanding the information set in these forecasts regressions by consumption growth and the risk-free rate. The R-squared for these multivariate regressions are provided in the second column of Figure 2, showing that the long-run risk model with the exponential power utility kernel also in this case reproduces the desired degree of predictability in excess market returns. For consumption and dividend growth, our estimated version of the standard long-run risk model generally produces too much predictability. A similar finding is reported in Beeler and Campbell (2012) for two calibrated versions of this model. On the other hand, our extension of the long-run risk model with an exponential power utility kernel generates too little predictability in consumption and dividend growth.

$<$ Figure 2 about here $>$

Following Beeler and Campbell (2012), we next study the ability of the price-dividend ratio to explain past and future consumption growth. Figure 3 shows that our estimated version of the standard long-run risk model generates too high correlation between past consumption growth and the price-dividend ratio compared to empirical evidence. A similar finding is reported in Beeler and Campbell (2012) for two calibrated versions of this model. On the other hand, our extension of the long-run risk model with an exponential power utility kernel implies that past consumption growth and the price-dividend ratio are completely uncorrelated as seen in the data.

The last two charts in Figure 3 explore the relationship between consumption volatility and the price-dividend ratio. As in Bansal and Yaron (2004), we measure the conditional volatility σ_t by the absolute value of the residual from an AR-model for consumption growth. In line with empirical evidence, our extension of the long-run risk model has the properties that i) a high price-dividend ratio predicts future low volatility and ii) high uncertainty forecasts a low price-dividend ratio (see also Bansal et al. (2005)). Hence, our extension of the long-run risk model with an exponential power utility kernel matches the negative relationship between volatility and the price-dividend ratio with an IES well below one. This is in contrast to the standard long-run risk model, which only reproduces this negative relationship with an IES larger than one, as emphasized in Bansal and Yaron (2004). Our results show that one can avoid this restriction by i) disentangling cyclical and long-run risk in the utility kernel *and* ii) modeling utility of cyclical risk by an exponential power specification.

< Figure 3 about here >

4 A New Keynesian Model

To provide further support for the considered Epstein-Zin-Weil preferences with the exponential power utility kernel in (17), we next show that they also help to explain asset prices in an otherwise standard New Keynesian DSGE model, where consumption and dividends are derived endogenously. In addition to explaining the equity premium and the risk-free rate puzzle, we also show that these preferences enable the New Keynesian model to match the level and the variability of the nominal term premium and hence resolve the bond premium puzzle (see Rudebusch and Swanson (2008)). Our main finding is to match these puzzles with a low IES of 0.07 *and* a low RRA of 5, whereas most existing DSGE models require extreme levels of RRA to explain asset prices.

We proceed by presenting our New Keynesian model in Section 4.1, the adopted estimation routine in Section 4.2, and finally our estimation results in Section 4.3.

4.1 Model Description

The considered model is specified along the lines of Rudebusch and Swanson (2012) and Swanson (2015) for comparability with much of the existing macro-finance literature building on the standard New Keynesian models (see Hordahl et al. (2008), Andreasen (2012), among others).

4.1.1 Household

The representative household is similar to the one considered in Section 2, except for a variable labor supply L_t. To match the persistence in consumption growth, we follow much of the New Keynesian tradition and extend our consumption habits with bC_{t-1}. These modifications are included in the exponential power utility kernel by letting

$$\mathcal{U}\left(C_t, L_t\right)=\frac{Z_t^{\chi}}{\chi}\left(\frac{1-e^{-\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}}{\tau}\right)^{\chi}+Z_t^{\chi}\varphi_0\frac{\left(1-L_t\right)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}, \tag{20}$$

where $\varphi_0>0$ and $\varphi\in\mathbb{R}\setminus\{1\}$. To ensure a balanced growth path, the utility of leisure is scaled by Z_t^{χ}, which may be justified by home production, as shown in Rudebusch and Swanson (2012). The degree of RRA implied by (20) can be obtained by the general formulas in Swanson (2013), which in our case gives

$$\text{RRA}=\frac{\tilde{C}_{ss}}{\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}{\tau\left(\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-\chi\right)}+\varphi\tilde{W}_{ss}\left(1-L_{ss}\right)}+\alpha\frac{\tilde{C}_{ss}\chi}{\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}{\tau}+\frac{\chi\tilde{W}_{ss}\left(1-L_{ss}\right)}{1-\frac{1}{\varphi}}} \tag{21}$$

where $\tilde{C}_{ss}$ and $\tilde{W}_{ss}$ refer to the steady state of consumption and the real wage in the normalized economy without trending variables. The deterministic trend in consumption is given by $\mu_{Z,ss}$ which coincides with the deterministic trend in technology, specified below in (23). Simple inspection of (21) reveals that the first term is undetermined in τ, whereas the second term decreases in τ for the same reason as outlined in Section 3.5. The IES in the steady state is given by

$$\text{IES}=\frac{1}{\tau C_{ss}}\frac{1-\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}}{\chi-\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}}, \tag{22}$$

which converges to the familiar expression $\left(1-\frac{b}{\mu_{Z,ss}}\right)/\left(1-\chi\right)$ when $\tau\rightarrow 0$. A careful inspection of (22) shows that the effect of τ on the IES is undetermined. However, for a given value of $\tilde{C}_{ss}$, the IES is decreasing in τ if

$$\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}{\tau\left[\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}}{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}\right]\left(1-\frac{b}{\mu_{Z,ss}}\right)\tilde{C}_{ss}-1}>1-\chi.$$

That is, the IES may increase in τ for low values of this parameter but will eventually decrease when τ becomes sufficiently high.[24]

Finally, the real budget constraint is given by $\mathbb{E}_t\left[M_{t,t+1}\frac{X_{t+1}}{\pi_{t+1}}\right]+C_t=\frac{X_t}{\pi_t}+W_tL_t+D_t$, where X_t is nominal state-contingent claims, π_t denotes gross inflation, W_t is the real wage, and D_t is real dividend payments from firms.

[24] The assumption that $\tilde{C}_{ss}$ is unaffected by τ is rather weak and satisfied in our version of the New Keynesian model.

4.1.2 Firms

Final output Y_t is produced by a perfectly competitive representative firm, which combines a continuum of differentiated intermediate goods $Y_t(i)$ using the production function $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$ where $\eta > 1$. This implies that the demand for the ith good is $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} Y_t$, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\eta} di\right)^{\frac{1}{1-\eta}}$ denotes the aggregate price level.

The differentiated goods are produced by intermediate firms using the production function $Y_t(i) = Z_t A_t K_{ss}^{\theta} L_t(i)^{1-\theta}$, where K_{ss} and $L_t(i)$ denote capital and labor services at the ith firm, respectively. Technology shocks are allowed to have the traditional stationary component A_t, but also a non-stationary component Z_t to generate long-run risk into the model. For the stationary shocks, we let $\log A_{t+1} = \rho_A \log A_t + \sigma_A \varepsilon_{A,t+1}$, where $|\rho_A| < 1$, $\sigma_A > 0$, and $\varepsilon_{A,t+1} \sim \mathcal{NID}(0,1)$. Similarly for the non-stationary shocks, we introduce $\mu_{Z,t+1} = Z_{t+1}/Z$ and let

$$\log\left(\frac{\mu_{Z,t+1}}{\mu_{Z,ss}}\right) = \rho_Z \log\left(\frac{\mu_{Z,t}}{\mu_{Z,ss}}\right) + \sigma_Z \varepsilon_{Z,t+1}, \tag{23}$$

where $|\rho_Z| < 1$, $\sigma_Z > 0$, and $\varepsilon_{Z,t+1} \sim \mathcal{NID}(0,1)$.[25]

Intermediate firms can freely adjust their labor demand at the given market wage W_t and are therefore able to meet demand in every period. Price stickiness is introduced via Calvo contracts, where a fraction ζ of randomly selected firms can not set the optimal nominal price $P_t(i)$ of the good they produce and instead let $P_t(i) = \pi_{ss} P_{t-1}(i)$.

4.1.3 The Central Bank and Aggregation

The central bank sets the one-period nominal interest rate i_t as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(i_{ss} + \beta_\pi \log\left(\frac{\pi_t}{\pi_{ss}}\right) + \beta_y \log\left(\frac{Y_t}{Z_t Y_{ss}}\right)\right),$$

based on a desire to close the inflation and the output gap with $\beta_n = [0,5]$ for $n = \{\pi, y\}$, subject to smoothing changes in the policy rate with $\rho_i \in [0,1]$. Note that the inflation gap accounts for steady-state inflation π_{ss}, and that the output gap is expressed in deviation from the balanced growth path as in Justiniano and Primiceri (2008), Rudebusch and Swanson (2012), among others.

[25] The specification of long-run risk adopted in the endowment model, i.e. (3), could also be used in the New Keynesian model, but we prefer the more parsimonious specification in (23) for comparability with the existing DSGE literature (see for instance Justiniano and Primiceri (2008), Altig et al. (2011), Swanson (2015), among others)

Summing across the heterogeneous firms implies $Y_t S_{t+1} = Z_t A_t K_{ss}^{\theta} L_t^{1-\theta}$, where $L_t \equiv \int_0^1 L_t(i)\, di$ is aggregated labor demand and S_{t+1} is the price dispersion index. As in Rudebusch and Swanson (2012), $\delta K_{ss} Z_t$ units of output are used to maintain a constant capital stock, meaning that aggregate resource constraint is given by $Y_t = C_t + \delta K_{ss} Z_t$.

4.1.4 Equity and Bond Prices

Equity is defined as a claim on aggregate dividends from firms, i.e. $D_t = Y_t - W_t L_t$, and its real price is therefore $1 = \mathbb{E}_t \left[M_{t,t+1} R_{t+1}^m \right]$ where $R_{t+1}^m = \left(D_{t+1} + P_{t+1}^m \right) / P_t^m$.

The price in period t of a default-free zero-coupon bond $B_t^{(n)}$ maturing in n periods with a face value of one dollar is $B_t^{(n)} = \mathbb{E}_t \left[\frac{M_{t,t+1}}{\pi_{t+1}} B_{t+1}^{(n-1)} \right]$ for $n = 1, ..., N$ with $B_t^{(0)} = 1$. Its yield-to-maturity with continuously compounding is then $i_t^{(n)} = -\frac{1}{n} \log B_t^{(n)}$. Following Rudebusch and Swanson (2012), we define term premia as $\Psi_t^{(n)} = i_t^{(n)} - \widetilde{i}_t^{(n)}$, where $\widetilde{i}_t^{(n)}$ is the yield-to-maturity on a zero-coupon bond $\widetilde{B}_t^{(n)}$ under risk-neutral valuation, i.e. $\widetilde{B}_t^{(n)} = e^{-i_t} \mathbb{E}_t \left[\widetilde{B}_{t+1}^{(n-1)} \right]$ with $\widetilde{B}_t^{(0)} = 1$.

4.2 Model Solution and Estimation Methodology

As in Section 3, we approximate the model solution by a third-order perturbation approximation and estimate the model by GMM using unconditional first and second moments computed as in Andreasen et al. (2013). The selected series describing the macro economy and the bond market are given by Δc_t, π_t, i_t, $i_t^{(40)}$, $\Psi_t^{(40)}$, and $\log L_t$, where one time period in the model corresponds to one quarter. The 10-year nominal interest rate and its term premium (obtained from Adrian et al. (2013)) are available from 1961Q3, leaving us with quarterly data from 1961Q3 to 2014Q4. We include all means, variances, and first-order auto-covariances of these six variables for the estimation, in addition to nine contemporaneous covariances related to the correlations reported at the end of Table 5. To examine whether our New Keynesian model is able to match the equity premium, we also include the mean of the net market return $r_t^m = \log R_t^m$ in the set of moments.[26] Finally, the GMM estimation is implemented using the conventional two-step procedure outlined in Section 3.1.

We estimate all structural parameters in the model except for a few badly identified parameters. That is, we let $\delta = 0.025$ and $\theta = 1/3$ as typically considered for the U.S. economy. We also let $\eta = 6$ to get an average markup of 20% and impose $\varphi = 1/4$ to match a Frisch labor supply elasticity in the neighborhood of 0.5. Finally, we set the ratio of capital to output in the steady state equal to 2.5 as in Rudebusch and Swanson (2012).

[26] Details on the data sources and data construction are provided in the online appendix.

4.3 Estimation Results

We proceed by first considering Epstein-Zin-Weil preferences with a standard power utility kernel in Section 4.3.1, before disentangling effects of cyclical and long-run risk in Section 4.3.2.

4.3.1 A Standard Power Utility Kernel

Given that RRA is hard to estimate accurately in the New Keynesian model, the analysis is conducted by conditioning the estimation on different values of RRA. Following Kaltenbrunner and Lochstoer (2010), we first let RRA $= 5$, which is within the middle range of reasonable values for the RRA suggested by Mehra and Prescott (1985). The estimated coefficients are summarized in Table 4 and are all fairly standard, except for a high steady state inflation ($\hat{\pi}_{ss} = 1.14$) and high curvature in consumption utility ($\hat{\chi} = -13.4$) giving an IES $= 0.034$. Table 5 shows that the model does well in matching all means (including the 10-year term premium and market return), but that this comes at the cost of too much variability in consumption growth (3.31% vs. 1.80%) and labor supply (2.93% vs. 1.61%). These results just iterate the finding in Rudebusch and Swanson (2008) that the standard New Keynesian model with low RRA can not match key asset pricing moments without distorting the macro economy.

< Table 4 about here >

We next follow Swanson (2015) and increase RRA to 60, although such extreme level of risk aversion is hard to justify based on micro-evidence. Table 5 shows that the New Keynesian model now reproduces all means without generating too much variability in the macro economy, except for a slightly elevated standard deviation in the log-transformed labor supply (2.44% vs. 1.61%). High risk aversion also helps in matching most auto- and contemporaneous correlations, and the model therefore has a much better overall fit with $Q^{scaled} = 0.268$ compared to $Q^{scaled} = 1.012$ with RRA$= 5$.

< Table 5 about here >

4.3.2 The Exponential Power Utility Kernel

We finally estimate our proposed utility kernel in (20) when conditioning on a realistic level of risk aversion with RRA $= 5$. Table 4 shows that $\hat{\tau} = 16.9$, which is statistical different from zero, meaning that disentangling cyclical and long-run risk in the utility kernel also helps the New Keynesian model to explain postwar U.S. data. Correcting for consumption habits and $C_{ss} = 0.8$, the scale-adjusted estimate of τ is $\hat{\tau}\left(1-\hat{b}\right)C_{ss} = 6.5$ and hence somewhat similar to $\hat{\tau} = 8.95$ in the long-run risk model - at least when accounting for estimation uncertainty. Using (22), we thus estimate a relative low IES of 0.07.

Table 5 shows that the New Keynesian model now matches all means and standard deviations, except for labor supply that displays the same degree of variability as in the standard New Keynesian model with RRA = 60. Subject to this qualification, the New Keynesian model now explains the equity premium and the risk-free rate puzzle with a low IES = 0.07 *and* a low RRA = 5. We also match the mean and the variability of the 10-year nominal term premia, implying that our New Keynesian model also explains the bond premium puzzle with low IES *and* low RRA. The auto- and contemporaneous correlations are also well matched, and our extension of the New Keynesian model therefore has a slightly better overall fit with $Q^{scaled} = 0.255$ compared to $Q^{scaled} = 0.268$ for the standard New Keynesian model with RRA = 60.

Table 6 illustrates the effects of gradually increasing τ in the New Keynesian model. As for the long-run risk model in Section 3.6, we emphasize two main effects. First, a higher value of τ reduces the IES without affecting returns in the steady state. Second, increasing τ lowers $u'\left(\tilde{C}_t\right)/u\left(\tilde{C}_t\right)$ and imply that relative large values of the Epstein-Zin-Weil parameter α are needed to ensure RRA = 5. The large value of α then amplifies the existing risk corrections and allows the model to explain asset prices with low IES *and* low RRA.

< Table 6 about here >

5 Conclusion

This paper extends the class of Epstein-Zin-Weil preferences with a new utility kernel that disentangles uncertainty about the consumption trend (long-run risk) from short-term variation around this trend (cyclical risk). Adopting a power-specification for cyclical risk within the long-run risk model, we first show that the IES can be lowered to just above one. To reduce the IES further, we then propose an exponential power utility kernel for cyclical risk, enabling us to match asset prices with a low RRA of 9.8 and a low IES of 0.11. In addition to accounting for the equity premium and the risk-free rate puzzle, we also show that our extension of the long-run risk model matches i) the predictability of the price-dividend ratio for excess market return, ii) the inability of the price-dividend ratio to forecast consumption and dividend growth, and iii) the negative relationship between consumption volatility and the price-dividend ratio.

To examine the performance of the proposed preferences in a production-based model, we finally extend an otherwise standard New Keynesian model with the exponential power utility kernel disentangling cyclical and long-run risk. Our GMM estimates reveal that these preferences allow the New Keynesian model to explain the equity premium, the risk-free rate puzzle, and the bond premium with a low IES of 0.07 *and* a low RRA of 5.

A A Perturbation Approximation under Homoscedasticity

Proposition A.1 *The second-order approximated log-transformed value function v_t and the log-transform twisted value function ev_t at the steady state are given by*

$$
\begin{aligned}
v_t &= v_{ss} + v_{\tilde{c}}\tilde{c}_t + v_x x_t + \frac{1}{2}v_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + v_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}v_{xx}x_t^2 + \frac{1}{2}v_{\sigma\sigma}, && (24)\\
ev_t &= ev_{ss} + ev_{\tilde{c}}\tilde{c}_t + ev_x x_t + \frac{1}{2}ev_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + ev_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}ev_{xx}x_t^2 + \frac{1}{2}ev_{\sigma\sigma} && (25)
\end{aligned}
$$

where

$$
\begin{aligned}
v_{ss} &= \log\left(\frac{1_{\{u(1)>0\}}u(1) - 1_{\{u(1)<0\}}u(1)}{1-\kappa_0}\right)\\
v_{\tilde{c}} &= \frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\\
v_x &= \frac{\kappa_0}{1-\rho_x\kappa_0}\chi\\
v_{\tilde{c}\tilde{c}} &= \frac{1-\kappa_0}{1-\rho_{\tilde{c}}^2\kappa_0}\left[\frac{u''(1)}{u(1)} + \frac{u'(1)}{u(1)}\right] - \left[\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\right]^2\\
v_{\tilde{c}x} &= \frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\chi\left[\frac{\rho_{\tilde{c}}\kappa_0}{1-\rho_{\tilde{c}}\rho_x\kappa_0} - \frac{\kappa_0}{1-\rho_x\kappa_0}\right]\\
v_{xx} &= \frac{\kappa_0}{1-\rho_x^2\kappa_0}\frac{1-\kappa_0}{(1-\rho_x\kappa_0)^2}\chi^2\\
v_{\sigma\sigma} &= \frac{\kappa_0}{1-\kappa_0}\left[v_{\tilde{c}\tilde{c}}\sigma_{\tilde{c}}^2 + (1-\alpha)v_{\tilde{c}}^2\sigma_{\tilde{c}}^2 + v_{xx}\sigma_x^2 + (1-\alpha)v_x^2\sigma_x^2 + (1-\alpha)\chi^2\sigma_z^2\right]
\end{aligned}
$$

and

$$
\begin{aligned}
ev_{ss} &= (1-\alpha)\left(\log\left(\frac{1_{\{u(1)>0\}}u(1) - 1_{\{u(1)<0\}}u(1)}{1-\kappa_0}\right) + \chi\log\mu_z\right)\\
ev_{\tilde{c}} &= (1-\alpha)\rho_{\tilde{c}}\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\\
ev_x &= \frac{(1-\alpha)\chi}{1-\rho_x\kappa_0}\\
ev_{\tilde{c}\tilde{c}} &= (1-\alpha)\rho_{\tilde{c}}^2\frac{1-\kappa_0}{1-\rho_{\tilde{c}}^2\kappa_0}\left[\frac{u''(1)}{u(1)} + \frac{u'(1)}{u(1)}\right] - (1-\alpha)\rho_{\tilde{c}}^2\left[\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\right]^2\\
ev_{x\tilde{c}} &= (1-\alpha)\rho_x\rho_{\tilde{c}}v_{x\tilde{c}}\\
ev_{xx} &= (1-\alpha)\rho_x^2\frac{\kappa_0}{1-\rho_x^2\kappa_0}\frac{1-\kappa_0}{(1-\rho_x\kappa_0)^2}\chi^2\\
ev_{\sigma\sigma} &= \frac{1-\alpha}{1-\kappa_0}\left[v_{\tilde{c}\tilde{c}}\sigma_{\tilde{c}}^2 + (1-\alpha)v_{\tilde{c}}^2\sigma_{\tilde{c}}^2 + v_{xx}\sigma_x^2 + (1-\alpha)v_x^2\sigma_x^2 + (1-\alpha)\chi^2\sigma_z^2\right]
\end{aligned}
$$

and $\kappa_0 = \beta\mu_z^{\chi}$.

Proposition A.2 *The second-order approximated log-transformed price-dividend ratio* pd_t *at the steady state is given by*

$$pd_t = pd_{ss} + pd_{\tilde{c}}\tilde{c}_t + pd_x x_t + \frac{1}{2}pd_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + pd_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}pd_{xx}x_t^2 + \frac{1}{2}pd_{\sigma\sigma},$$

where

$$\begin{aligned}
pd_{ss} &= \log\frac{\kappa_1}{1-\kappa_1} \\
pd_{\tilde{c}} &= -\frac{1-\rho_{\tilde{c}}}{1-\kappa_1\rho_{\tilde{c}}}\frac{u''(1)}{u'(1)} \\
pd_x &= \frac{\phi+\chi-1}{1-\kappa_1\rho_x} \\
pd_{\tilde{c}\tilde{c}} &= -pd_{\tilde{c}}^2 - 2\frac{u''(1)}{u'(1)}pd_{\tilde{c}} - \frac{1-\rho_{\tilde{c}}^2}{1-\kappa_1\rho_{\tilde{c}}^2}\left(\frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)}\right) \\
pd_{\tilde{c}x} &= -pd_{\tilde{c}}pd_x + \frac{\kappa_1\rho_{\tilde{c}}pd_{\tilde{c}}(\chi-1+\phi)}{1-\kappa_1\rho_{\tilde{c}}\rho_x} - \frac{(1-\rho_{\tilde{c}})\frac{u''(1)}{u'(1)}(\chi-1+\phi+\rho_x\kappa_1 pd_x)}{1-\kappa_1\rho_{\tilde{c}}\rho_x} \\
pd_{xx} &= -pd_x^2 + \frac{(\chi-1+\phi)^2}{1-\rho_x^2\kappa_1} + 2\kappa_1\rho_x\frac{\chi-1+\phi}{1-\rho_x^2\kappa_1}pd_x \\
pd_{\sigma\sigma} &= \frac{\sigma_d^2}{1-\kappa_1} + \frac{\sigma_z^2}{1-\kappa_1}\left[\alpha + (1-\alpha)(1-\chi)^2\right] \\
&\quad + \frac{\sigma_{\tilde{c}}^2}{1-\kappa_1}\Big[\alpha v_{\tilde{c}}^2 - 2\alpha\kappa_1 pd_{\tilde{c}}v_{\tilde{c}} + \kappa_1 pd_{\tilde{c}\tilde{c}} \\
&\qquad + \kappa_1 pd_{\tilde{c}}^2 - 2\alpha\frac{u''(1)}{u'(1)}v_{\tilde{c}} + 2\kappa_1 pd_{\tilde{c}}\frac{u''(1)}{u'(1)} + \frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)}\Big] \\
&\quad + \frac{\sigma_x^2}{1-\kappa_1}\left[\alpha v_x^2 - 2\alpha\kappa_1 pd_x v_x + \kappa_1 pd_{xx} + \kappa_1 pd_x^2\right]
\end{aligned}$$

and $\kappa_1 = \frac{\beta\mu_z^{(\chi-1)}\mu_d}{1+\beta\mu_z^{(\chi-1)}\mu_d}$. *The expressions for* $v_{\tilde{c}}$ *and* v_x *are provided in Proposition A.1.*

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Table 1: The Long-Run Risk Model: The Structural Parameters

Estimation results using data from 1947Q1 to 2014Q4. The reported estimates are from the second step in GMM with the weigthing matrix estimated by 15 lags in the Newey-West estimator. The model has a monthly time frequency with model-implied moments time-aggregated to a quarterly time frequency. In column (1), the values of RRA and ψ are calibrated and standard errors are therefore not available.

	Benchmark	Cyclical vs. long-run risk		
	(1) Power utility kernel	(2) Power utility kernel	(3) Semi-parametric utility kernel	(4) Exponential power utility kernel
$u'(1)$	–	–	5.19×10^{-4} (0.0002)	–
$u''(1)$	–	–	-0.0042 (0.0019)	–
χ	–	0.5106 (0.0066)	0.4637 (0.9579)	0.5144 (0.4998)
RRA	10	13.515 (2.4618)	9.6716 (3.1934)	9.7809 (0.9846)
ψ	1.5	1.0181 (0.0119)	–	–
τ	–	–	–	8.9505 (1.0252)
β	0.9995[a]	0.9980 (0.0010)	0.9981 (0.0019)	0.9987 (0.0008)
$\rho_{\tilde{c}}$	0.1651 (0.0272)	0.0041 (6.1043)	0.9998 (0.0004)	0.9962 (0.0018)
ρ_x	0.9802 (0.0030)	0.6450 (0.2446)	0.1510 (1.1698)	0.3756 (0.6678)
ρ_σ	0.9942 (0.0008)	0.9933 (0.0020)	0.9889 (0.0094)	0.9941 (0.0016)
μ_z	1.0014[a]	1.0018 (0.0028)	1.0019 (0.0001)	1.0018 (0.0001)
μ_d	1.0006 (0.0001)	1.0015 (0.0011)	1.0015 (0.0001)	1.0014 (0.0006)
ϕ	1.8858 (0.0414)	3.6231 (2.6532)	2.2725 (4.6025)	1.9909 (2.4266)
$\sigma_{\tilde{c}}$	0.0033 (0.0003)	0.0017 (0.0080)	0.0025 (0.0004)	0.0025 (0.0004)
σ_z	0.0021 (0.0004)	0.0024 (0.0006)	0.0008 (0.0018)	0.0008 (0.0017)
σ_d	0.0147 (0.0009)	0.0129 (0.0014)	0.0133 (0.0013)	0.0148 (0.0014)
σ_x	0.0004 (0.0001)	0.0009 (0.0005)	0.0013 (0.0023)	0.0012 (0.0015)
σ_σ	0.0876 (0.0134)	0.0775 (0.0267)	0.1199 (0.0984)	0.1042 (0.0353)
Memo				
IES	1.50	1.02	0.12	0.11
α	28.00	703.42	3,171	1,390

[a] The coefficient is at the boundary of its domain as $\beta\mu_z^{1-1/\psi} < 1$ and its standard error is therefore not available.

Table 2: The Long-Run Risk Model: Fit of Moments

Except for the price-dividend ratio, all means and standard deviations are expressed in annualized percent. Moments are annualized through a multiplication of 400, except for the standard deviation of the market return which is multiplied by 200. All model-implied moments in columns (2) to (5) are from the unconditional distribution, whereas the empirical data moments in column (1) are the sample means. In column (1), figures in parentesis refer to the standard error of the empirical moment, computed based on the Newey-West estimate (with 15 lags) of the co-variance matrix for the considered set of moments.

		Benchmark	Cyclical vs. long-run risk		
	(1) Data	(2) Power utility kernel	(3) Power utility kernel	(4) Semi-parametric utility kernel	(5) Exponential power utility kernel
Means					
pd_t	3.495 (0.095)	3.514	3.505	3.519	3.499
r_t^f	0.831 (0.411)	0.997	0.975	1.393	1.100
r_t^m	6.919 (1.842)	5.440	8.737	8.520	6.001
Δc_t	1.905 (0.209)	1.735	2.189	2.229	2.108
Δd_t	2.391 (0.979)	0.692	1.775	1.836	1.722
Stds					
pd_t	0.421 (0.060)	0.392	0.361	0.430	0.422
r_t^f	2.224 (0.378)	2.206	1.782	1.914	1.986
r_t^m	16.445 (1.201)	16.521	15.351	15.658	16.559
Δc_t	2.035 (0.169)	2.904	1.859	1.791	1.875
Δd_t	9.391 (1.221)	8.710	7.516	7.173	8.040
Persistence					
$corr(pd_t, pd_{t-1})$	0.982 (0.150)	0.980	0.980	0.985	0.983
$corr\left(r_t^f, r_{t-1}^f\right)$	0.866 (0.085)	0.574	0.930	0.909	0.925
$corr\left(r_t^m, r_{t-1}^m\right)$	0.084 (0.058)	0.002	-0.006	0.005	-0.006
$corr\left(\Delta c_t, \Delta c_{t-1}\right)$	0.306 (0.130)	0.571	0.253	0.220	0.248
$corr\left(\Delta d_t, \Delta d_{t-1}\right)$	0.396 (0.092)	0.298	0.207	0.138	0.143

Table 2: Long-Run Risk Model: Fit of Moments (continued)

		Benchmark	Cyclical vs. long-run risk		
	(1) Data	(2) Power utility kernel	(3) Power utility kernel	(4) Semi-parametric utility kernel	(5) Exponential power utility kernel
Correlations					
$corr\left(pd_t, r_t^f\right)$	0.035 (0.250)	0.437	0.952	0.157	0.543
$corr\left(pd_t, r_t^m\right)$	0.058 (0.073)	0.112	0.078	0.058	0.073
$corr\left(pd_t, \Delta c_t\right)$	0.025 (0.093)	0.197	0.007	0.005	0.021
$corr\left(pd_t, \Delta d_t\right)$	-0.017 (0.132)	0.109	0.005	0.000	0.001
$corr\left(r_t^f, r_t^m\right)$	0.023 (0.062)	0.093	0.060	0.052	0.042
$corr\left(r_t^f, \Delta c_t\right)$	0.161 (0.102)	0.362	0.132	0.059	0.055
$corr\left(r_t^f, \Delta d_t\right)$	-0.168 (0.102)	0.303	0.104	0.035	0.036
$corr\left(r_t^m, \Delta c_t\right)$	0.233 (0.065)	0.063	0.032	0.047	0.140
$corr\left(r_t^m, \Delta d_t\right)$	0.104 (0.050)	0.275	0.262	0.265	0.282
$corr\left(\Delta c_t, \Delta d_t\right)$	0.062 (0.062)	0.374	0.274	0.158	0.156
Goodness of fit					
Q^{Step2}	-	0.0623	0.0500	0.0388	0.0366
J-test: P-value	-	0.112	0.195	0.311	0.449
Q^{scaled}	-	1.145	1.171	0.417	0.497

Table 3: The Long-Run Risk Model: Analyzing the Exponential Power Kernel
Moments are computed using a third-order perturbation approximation and represented as in Table 2. Unless stated otherwise, all parameters attain the estimated values from column (4) in Table 1. For decomposing $\mathbb{E}[r_t^f]$ and $\mathbb{E}[r_t^m]$, the contribution from the steady state, long-run risk, short-run risk, and cyclical risk are computed based on Proposition 2, while the contribution from stochastic volatility is given by the difference between the unconditional mean and the sum of these four terms.

	$\chi = 0.51$			$\chi = -8.091$
	$\tau \to 0$	$\tau = 5$	$\tau = \hat{\tau}^{GMM}$	$\tau \to 0$
Means				
pd_t	7.287	6.589	3.499	4.162
r_t^f	2.611	2.276	1.100	20.438
r_t^m	2.540	3.180	6.001	20.299
Stds				
pd_t	0.017	0.131	0.422	0.059
r_t^f	0.563	0.945	1.986	10.548
r_t^m	5.395	6.838	16.559	7.181
Decomposing $\mathbb{E}[r_t^f]$				
r_{ss}^f	2.631	2.631	2.631	20.807
Long-run risk	-0.011	-0.159	-0.809	0.012
Short-run risk	-0.005	-0.080	-0.406	-0.034
Cyclical risk	-0.005	-0.117	-0.317	-0.346
Stochastic volatility	0.000	0.000	0.000	0.000
Decomposing $\mathbb{E}[r_t^m]$				
r_{ss}^m	2.631	2.631	2.631	20.807
Long-run risk	0.013	0.199	1.011	0.004
Short-run risk	-0.005	-0.080	-0.406	-0.034
Cyclical risk	-0.005	-0.116	-0.314	-0.343
Stochastic volatility	-0.095	0.545	3.079	-0.134
Memo				
RRA	9.78	9.78	9.78	9.78
IES	2.06	0.20	0.11	0.11
α	18.07	273.05	$1,389.69$	-0.09

Table 4: The New Keynesian Model: The Structural Parameters

Estimation results using data from 1961Q3 to 2014Q4. The reported estimates are from the second step of GMM with the weigthing matrix estimated by 15 lags in the Newey-West estimator. The estimate of β in column (1) is on the boundary and the standard error is therefore not available.

	Benchmark		Cyclical vs. long-run risk
	(1)	(2)	(3)
	RRA=5	RRA=60	RRA=5
β	0.9999	0.9955	0.9908
		(0.0032)	(0.0019)
b	0.5085	0.6970	0.5157
	(0.0093)	(0.0718)	(0.1499)
χ	-13.3678	-4.0710	0.8148
	(0.9909)	(1.3655)	(0.5563)
τ	$\to 0$	$\to 0$	16.9450
			(6.8596)
ζ	0.5195	0.7300	0.6677
	(0.0164)	(0.0143)	(0.0351)
β_π	1.4232	2.2226	1.1810
	(0.0176)	(0.2670)	(0.0788)
β_y	0.2175	0.7255	0.0190
	(0.0179)	(0.3458)	(0.0422)
μ_Z	1.0040	1.0055	1.0052
	(0.0001)	(0.0004)	(0.0004)
π_{ss}	1.1407	1.0300	1.0431
	(0.0113)	(0.0017)	(0.0065)
L_{ss}	0.3364	0.3355	0.3367
	(0.0005)	(0.0005)	(0.0009)
ρ_r	0.5890	0.8872	0.5975
	(0.0261)	(0.0252)	(0.0605)
ρ_A	0.9953	0.9878	0.9909
	(0.0004)	(0.0006)	(0.0012)
ρ_Z	0.1009	0.4818	0.6272
	(0.0086)	(0.0895)	(0.3229)
σ_A	0.0125	0.0051	0.0082
	(0.0003)	(0.0006)	(0.0010)
σ_Z	0.0130	0.0040	0.0029
	(0.0004)	(0.0002)	(0.0019)
Memo			
IES	0.034	0.061	0.074
$\mathcal{U}_{ss}$	$-274,454.69$	-571.80	0.568
α	-1.31	-32.54	171.62

Table 5: The New Keynesian Model: Fit of Moments

All variables are expressed in annualized terms in percentage, except for the mean of $\log(l_t)$. All model-implied moments in columns (2) to (5) are from the unconditional distribution, whereas the empirical data moments in column (1) are given by the sample means. In column (1), figures in parentesis refer to the standard error of the empirical moment, computed based on the Newey-West estimate (with 15 lags) of the co-variance matrix for the considered set of moments.

		Benchmark		Cyclical vs. long-run risk
	(1) Data	(2) RRA=5	(3) RRA=60	(4) RRA=5
Means				
Δc_t	1.975	1.595	2.182	2.055
	(0.253)			
π_t	3.890	3.391	3.417	3.432
	(0.512)			
i_t	4.999	5.188	5.040	5.061
	(0.682)			
$i_t^{(40)}$	6.497	6.556	6.463	6.507
	(0.617)			
$\Psi_t^{(40)}$	1.663	1.741	1.583	1.678
	(0.251)			
$\log L_t$	−1.081	−1.081	−1.080	−1.081
	(0.003)			
r_t^m	5.527	4.760	5.390	5.419
	(1.786)			
Stds				
Δc_t	1.802	3.313	1.442	1.361
	(0.137)			
π_t	2.716	2.938	2.744	2.696
	(0.342)			
i_t	3.173	2.942	2.615	2.912
	(0.478)			
$i_t^{(40)}$	2.621	2.672	2.308	2.547
	(0.441)			
$\Psi_t^{(40)}$	1.165	1.084	1.092	1.109
	(0.167)			
$\log L_t$	1.619	2.926	2.437	2.476
	(0.162)			
Persistence				
$corr\left(\Delta c_t, \Delta c_{t-1}\right)$	0.529	0.506	0.709	0.777
	(0.082)			
$corr\left(\pi_t, \pi_{t-1}\right)$	0.953	0.777	0.858	0.894
	(0.037)			
$corr\left(i_t, i_{t-1}\right)$	0.949	0.947	0.989	0.981
	(0.044)			
$corr\left(i_t^{(40)}, i_{t-1}^{(40)}\right)$	0.976	0.991	0.981	0.985
	(0.066)			
$corr\left(\Psi_t^{(40)}, \Psi_{t-1}^{(40)}\right)$	0.937	0.995	0.988	0.991
	(0.063)			
$corr\left(\log L_t, \log L_{t-1}\right)$	0.932	0.753	0.942	0.969
	(0.545)			

Table 5: The New Keynesian Model: Fit of Moments (continued)

	(1) Data	Benchmark (2) RRA=5	Benchmark (3) RRA=60	Cyclical vs. long-run risk (4) RRA=5
Correlations				
$corr\left(\Delta c_t, \pi_t\right)$	-0.184 (0.136)	0.275	-0.045	-0.112
$corr\left(\Delta c_t, i_t\right)$	0.021 (0.181)	0.143	-0.039	-0.037
$corr\left(\Delta c_t, \Psi_t^{(40)}\right)$	-0.036 (0.165)	-0.008	-0.022	-0.045
$corr\left(\pi_t, i_t\right)$	0.703 (0.059)	0.932	0.877	0.965
$corr\left(\pi_t, i_t^{(40)}\right)$	0.585 (0.146)	0.822	0.851	0.859
$corr\left(\pi_t, \Psi_t^{(40)}\right)$	0.236 (0.146)	0.442	0.419	0.377
$corr\left(i_t, i_t^{(40)}\right)$	0.900 (0.043)	0.853	0.869	0.878
$corr\left(i_t, \Psi_t^{(40)}\right)$	0.424 (0.222)	0.358	0.381	0.377
$corr\left(i_t^{(40)}, \Psi_t^{(40)}\right)$	0.757 (0.252)	0.698	0.766	0.721
Goodness of fit				
Q^{Step2}	-	0.0609	0.0525	0.0583
J-test: P-value	-	0.6051	0.6724	0.4941
Q^{scaled}	-	1.0116	0.2680	0.2546

Table 6: The New Keynesian Model: Analyzing the Exponential Power Kernel
All moments are computed using a third-order perturbation and represented as in Table 5. Unless stated otherwise, all parameters attain the estimated values from column (3) in Table 4.

	$\chi = 0.815$			$\chi = -5.54$
	$\tau \to 0$	$\tau = 10$	$\tau = \hat{\tau}^{GMM}$	$\tau \to 0$
Means				
Δc_t	2.055	2.055	2.055	2.055
π_t	16.916	14.910	3.432	9.089
i_t	20.979	18.614	5.061	24.806
$i_t^{(40)}$	20.946	18.834	6.507	24.351
$\Psi_t^{(40)}$	0.058	0.427	1.678	-0.185
$\log L_t$	-1.089	-1.083	-1.081	-1.075
r_t^m	4.073	4.217	5.419	15.989
Stds				
Δc_t	4.190	1.955	1.361	1.848
π_t	3.356	4.585	2.696	6.785
i_t	3.231	4.995	2.912	6.746
$i_t^{(40)}$	2.679	4.120	2.547	3.723
$\Psi_t^{(40)}$	0.015	0.204	1.109	0.118
$\log L_t$	1.793	4.902	2.476	5.580
Memo				
RRA	5	5	5	5
IES	2.629	0.124	0.074	0.074
α	0.50	19.85	171.62	-1.56

Figure 1: The Exponential Power Utility Kernel

All plots are done for $\chi = 1/3$.

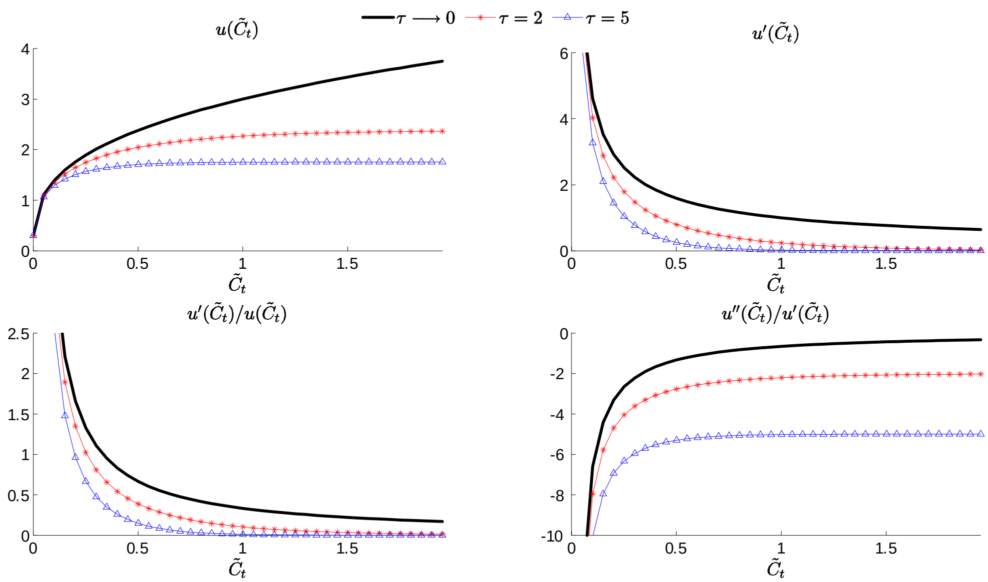


Figure 2: Predictive Regressions

All model-implied moments are computed given the estimated parameters in Table 1 using a simulated sample path of $1,000,000$ observations. The 95 percentage confidence bands are computed using the Newey-West estimator with $2 \times j$ lags for the univariate regressions, and for the multivariate regressions by the block bootstrap using a window of $2 \times j$ observations.

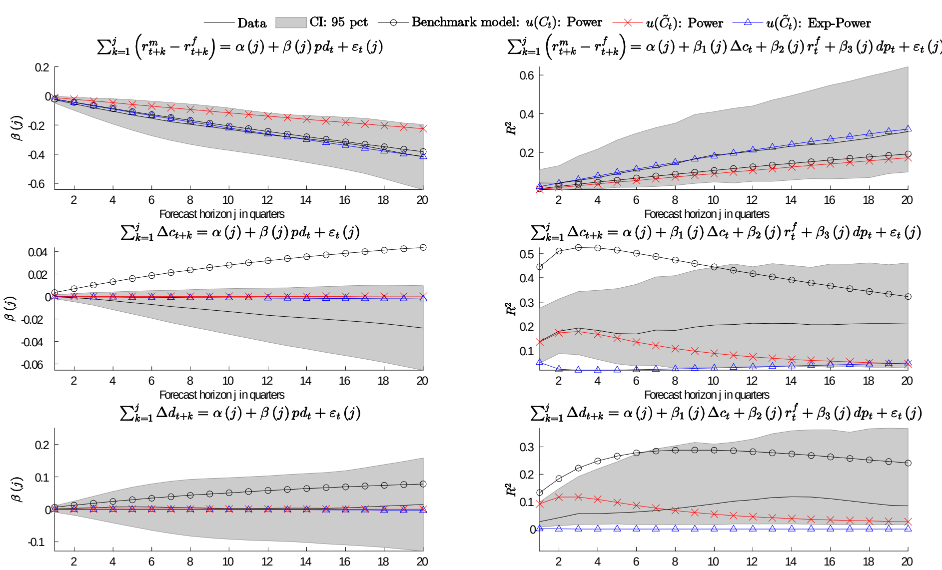
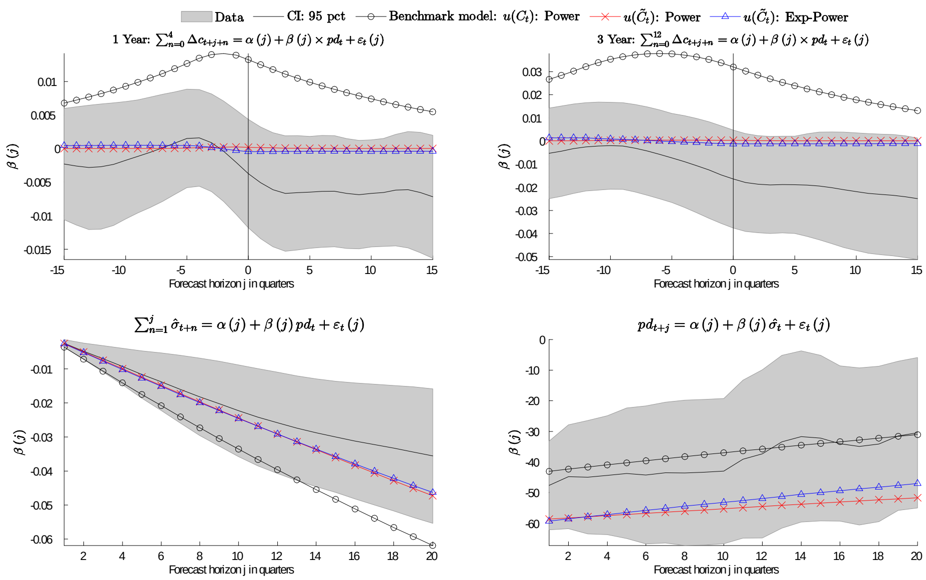


Figure 3: Properties of Consumption Growth and Volatility

All model-implied moments are computed given the estimated parameters in Table 1 using a simulated sample path of $1,000,000$ observations. The conditional volatility $\hat{\sigma}_t$ is estimated by $|\hat{u}_t|$ where $\hat{u}_t$ is the residual from the OLS regression $\Delta c_t = \alpha + \sum_{j=1}^{4} \beta(j)\, \Delta c_{t-j} + u_t$. The 95 percentage confidence bands are computed using the Newey-West estimator with $max(10, 2 \times j)$ lags for the two consumption growth regressions, whereas the lag length in the two volatility regressions are $2 \times j$.



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