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Abstract

We propose a novel way to assess information processing in a complex environment of market fragmentation. We take a different angle from the price discovery literature, and investigate information processing in the stochastic process driving stock's volatility (volatility discovery). We show that our volatility discovery framework successfully identifies the leading market in the volatility process, whereas price discovery measures are unable to capture the dynamics of the market-specific volatilities. We compute volatility discovery for 30 stocks and find significant differences in how exchanges impound information into the efficient volatility, as ARCA and NYSE are more important than NASDAQ. Interestingly, price discovery measures suggest different results for nearly half the sample.

JEL classification: C32, C51, C52, G12, G14

Keywords: fragmented markets, information processing, volatility persistency, market microstructure, price discovery, high-frequency data.

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I. Introduction

The proliferation of trading venues within many developed security markets has been a new trend in the financial equity market. The U.S., for instance, features eleven exchanges, eighty-five alternative trading systems (ATS) and more than two-hundred dealers.[1] The market fragmentation phenomenon experienced in the U.S. highlights the importance of the microstructure of trading venues, as traditional listing exchanges have created markets within markets, changed markets' pricing structures, and set up specialized microstructures to attract specific trading clienteles (Menkveld (2014), O'Hara (2015), and Menkveld (2016)). Retail brokers have responded to this new market setting by routing their orders to multiple trading platforms (Battalio, Corwin, and Jennings (2016)). Consequently, the way traders and markets learn and disseminate information is also dispersed across markets, as O'Hara (2015) points out: "*Even more important is to recognize that these data must be looked at across markets, and not just within individual markets. High-frequency algorithms operate across markets, and if order books are linked, then so, too, must be order flows and price behavior*".

In this context of a decentralized system in which there are multiple prices for one homogenous security, the study of information processing among trading venues becomes highly complex and, at the same time, of great significance for both market participants and regulators. To date, the literature has focused on the stochastic process that drives a stock's price (price discovery). In this paper, we take a different angle and investigate information processing in the stochastic process driving stock's volatility.[2] We call this analysis 'volatility discovery'.

The price discovery literature has identified the unobserved efficient price by extracting

[1] https://www.sec.gov/news/statement/us-equity-market-structure.html

[2] The information contained in the volatility process has compelled traders to use derivative markets to trade volatility (Ni, Pan, and Poteshman (2008) study equity volatility information trading), and the finance and econometrics literature has long viewed volatility as a separate stochastic process (see the excellent surveys on stochastic volatility models in Ghysels, Harvey, and Renault (1996) and Shephard and Andersen (2009)).

commonalities within the transaction prices of a homogeneous security. In this context, prices are cointegrated, and the vector error correction (VEC) model has become the workhorse of price discovery analyses. We rely on a similar intuition and exploit evidence that the realized variances are long memory processes and cointegrated in order to identify the latent efficient stochastic volatility using the fractionally cointegrated vector autoregressive (FC-VAR) model of Johansen and Nielsen (2012).[3] We then investigate how different markets impound information into the efficient stochastic volatility. We evaluate volatility discovery by examining the adjustment coefficients of the FCVAR. Our first result comes from an illustration of our theoretical framework. We use an example of a price process designed so that price and volatility discovery processes occur in distinct markets. Our volatility discovery framework successfully identifies the leading market in the volatility process, whereas price discovery measures are unable to capture the dynamics of the market-specific volatilities.

With this novel methodology in hand, we investigate the volatility discovery mechanism for 30 of the most actively traded stocks in the U.S. The tick-by-tick quotes sample consists of firms from different industries that are listed on the NYSE or the NASDAQ and also traded on the ARCA from January 2007 to December 2013. Our results reveal that trading venues incorporate new information into the stochastic volatility process in an individual manner. We find that the NYSE is more important than the NASDAQ for 68% of the assets and contributes, on average, 60% to the efficient volatility, while the NASDAQ is responsible for the remaining 40%. When using stocks traded on the ARCA and NASDAQ, the ARCA also appears to be more important than the NASDAQ, as it leads in 68% of the stocks, making a 54% contribution to volatility discovery, in contrast to the 46% contributed by the NASDAQ. Interestingly, when we compute price discovery measures, we find distinct market leaders. The NASDAQ appears to be more important for 74% of the assets when compared to the NYSE and for 79% of the assets when compared to the ARCA. Our methodology and

[3] Well-documented evidence is found in the literature to show that estimates of integrated variance (e.g., realized variance) depict long memory features, i.e., these estimates are characterized as highly persistent and presenting an autocorrelation function that decays at a hyperbolic rate (Andersen and Bollerslev (1997), Andersen, Bollerslev, Diebold, and Labys (2003a), Corsi (2009), among others).

results represent a significant step toward the understanding of information processing in a fragmented market context.

Our study relates to different strands in the literature. For instance, multivariate models of volatility have considered the role that information arrival plays in determining the dynamics of the volatility process. Tauchen and Pitts (1983) and Andersen (1996)'s bivariate mixture models use trading volume to identify the effect of information arrival on the persistence of the volatility process. Liesenfeld (2001) generalizes the previous bivariate mixture models to allow two latent processes within the volatility dynamics: information arrival and the market participant's sensitivity to new information. More recently, Berger, Chaboud, and Hjalmarsson (2009) show that both information flow and market sensitivity to information are long memory processes and that the latter is at least as relevant as the former in explaining the persistence of volatility. We relate the volatility and price discovery mechanisms to this framework and show that the price discovery mechanism is related only to the portion of information that permanently affects asset prices (information arrival), while volatility discovery is also be able to capture market sentiment towards information.

A more recent strand of literature (Andersen, Bollerslev, Diebold, and Vega (2003b) and Andersen, Bollerslev, Diebold, and Vega (2007)) investigates the role played by public news announcements and macroeconomic variables in explaining high-frequency returns, jumps and realized measures (RMs) of integrated volatility. A well-established body of literature relates news to high-frequency returns and jumps; however, results considering the specific case of RMs of volatility have only recently begun to emerge. These recent results show that public and firm-specific news, measures of disagreement in belief between market participants, and macroeconomic uncertainty help to explain the dynamics of the volatility process. Specifically, Paye (2012) finds that macroeconomic uncertainty Granger-causes volatility, while Engle, Hansen, and Lunde (2012) reinforce the role of private information processing in explaining volatility variations. Finally, Bollerslev, Li, and Xue (2016) document that intraday volume-volatility elasticity is lower at times of high uncertainty and of disagreement among market participants. Our study relates to this literature because it addresses the issue of how financial markets process the information that drives stochastic volatility processes in the context of fragmented markets, and it ultimately provides answers on the direction of Granger causality among markets.

The remainder of the paper proceeds as follows. Section II presents the theoretical setting and Section III explains the methodology. Section IV details the high-frequency intraday data and the RMs used in the empirical analysis. Section V presents the empirical analysis of the volatility discovery process. Section VI relates the volatility discovery measure with price discovery and presents empirical results for price discovery. Section VII offers concluding remarks. Appendix A contains a simulation study that provides evidence on the effectiveness of the volatility discovery framework, Appendix B provides additional tables that complement our empirical analysis, and Appendix C provides technical details regarding simulations, a review of the estimation of FCVAR models, and technical background on the efficient volatility representation.

II. Theoretical Setting

In a financial setting where markets are fragmented, a single firm's stocks are traded in multiple venues and incorporate new information into prices in a distinct fashion. These market prices should not drift apart since they reflect the intrinsic value of the firm. In econometric terms, they are cointegrated and share a stochastic trend, which is seen as the efficient price. Suppose that the continuous efficient price follows a *Brownian semimartingale* process:

$$m_t = \int_0^t a_u du + \int_0^t \sigma_u dW_u + m_0,$$

where a is a predictable locally bounded drift, σ is a càdlàg volatility process denoted as the efficient stochastic volatility process and, without loss of generality, W is a Brownian motion. Transaction prices of a homogenous security in S markets share the efficient price

m_t and possess market-specific stochastic processes, σ_s and W_s, for $s = 1, 2, ..., S$, as

$$p_{s,t} = \int_0^t a_{s,u} du + \int_0^t \sigma_{s,u} dW_{s,u} + p_{s,0}, \quad s = 1, 2, ...S, \tag{1}$$

The difference between m_t and $p_{s,t}$ has two components: the well-known market microstructure noise plus any lagged adjustment to new information and price smoothing (see partial adjustment models, as in Hasbrouck and Ho (1987)). Notably, m_t is driven by two stochastic processes: W and σ.[4] The latter is related to the integrated variance (IV), defined as

$$IV_m = \int_0^t \sigma_u^2 du.$$

The IV is central to the pricing of financial instruments, portfolio allocation, and risk management. In a fragmented market context, not only market prices but also their stochastic volatilities are expected to gather information at different speeds, following market characteristics such as market design, trading costs, liquidity, and the presence of informed traders, among other things. Similarly to transaction prices that share an efficient price, we expect these market-specific stochastic volatility processes not to diverge over time and hence to share the efficient stochastic volatility. Therefore, we extend the price discovery concept to the stochastic volatility processes and investigate the contribution of different markets to the dynamics of the efficient price's IV.

To exemplify in a pricing model how markets can be affected by two sources of information, we design a example in which markets adjust to the efficient price and volatility. Therefore, there are two channels of information flow: the usual price discovery process and the novel volatility discovery mechanism. Consider a homogenous asset that is traded in two markets; hence, these prices cointegrate and share the efficient price. Assume that market one is the equivalent of a random walk process in discrete time and market two tracks the first

[4]The exact parametric functional form of σ is not relevant to the present analysis. We stress only that σ is driven by a Brownian motion different from W and that σ and W may be correlated.

market in an error correction fashion (see the zero coefficients in the drift term of (2)). Also suppose that the stochastic volatilities in the two markets are tied to a long-run equilibrium and hence do not diverge.[5] These volatilities are generated by a stationary error correction model in which market one does not Granger-cause market two (see the zero coefficient in the drift term of (3)). In turn, while investors in market one impound all relevant information to the efficient price, market two is exclusively responsible for determining the efficient stochastic volatility process. It follows that price discovery occurs exclusively in market one, whereas volatility discovery takes place only in market two. The price process reads

$$\mathrm{d}p(t) = \begin{bmatrix} 0 & 0 \\ -\pi & \pi \end{bmatrix} \Big(\mu_p - p(t)\Big)\mathrm{d}t + \begin{bmatrix} \sigma_1(t) & 0 \\ 0 & \sigma_2(t) \end{bmatrix} \mathrm{d}W(t) \tag{2}$$

$$\mathrm{d}V(t) = \begin{bmatrix} -\theta_1 & \theta_1 \\ 0 & -\theta_2 \end{bmatrix} \Big(\mu_V - V(t)\Big)\mathrm{d}t + C\mathrm{d}B(t), \tag{3}$$

where $p_t = (p_{1,t}, p_{2,t})'$ and $V(t) = (V_1(t), V_2(t))'$ are 2×1 vectors containing the observed log prices and the logarithm of the instantaneous stochastic volatilities, respectively; μ_p and μ_V are 2×1 vectors of mean parameters; C is 2×2 matrix such that the instantaneous covariance matrix of the log-volatility process is given by $\Lambda = CC'$; $\sigma_s(t) = \exp[\varphi_0 + \varphi_1 V_s(t)]$ with $s = 1, 2$; and W and B are 2×1 vectors of Brownian motions with $\mathrm{Corr}(\mathrm{d}W_s(t)\mathrm{d}B_\ell(t)) = \rho$ for $s, \ell = 1, 2$. Without loss of generality, the off-diagonal elements of the spot volatility matrix in (2) are set to zero. In Section III, we revisit the above example and document that the volatility discovery measure we introduce correctly identifies market two as the leader in the volatility process, whereas the price discovery measure is able to capture information only from market one. We refer the reader to Appendix A for the full analysis of the simulation study conducted using the price process above.

[5]We follow the standard practice in the RM literature and model the stochastic volatilities in (3) as a stationary OrnsteinUhlenbeck (OU) process.

III. Methodology

A. *Econometric Models for Price and Volatility Discovery*

The price discovery literature typically adopts the VEC models to approximate the dynamics of cointegrated prices observed on different markets and to identify the latent efficient price. The general VEC model then reads

$$\Delta p_{t_i} = \gamma\beta' p_{t_{i-1}} + \sum_{j=1}^{q} \Upsilon_j \Delta p_{t_{i-j}} + e_{t_i}, \tag{4}$$

where t_i is the intraday observation i at day t, with $i = 0, 1, ..., N$ and $t = 1, 2, ..., T$, N and T denote the number of intraday observations and the total number of days, respectively, p_{t_i} is a $S \times 1$ vector that collects the log prices on the S different trading venues, γ and β are the speed of adjustment coefficients and the cointegrating vector, respectively, and e_{t_i} is a sequence of uncorrelated innovations with a covariance matrix Σ. Price discovery measures are then based on the estimates of the parameters in (4) and Σ (see further details in Sections III.B and VI.A).

To investigate the dynamics of the market IVs, we first need to define feasible estimates of these latent variables. We generically denote these estimates as RMs, which are consistent estimators computed with ultra-high-frequency data. Second, unlike security prices, which are known to be integrated of order one, $I(1)$, processes, there is well-documented evidence that RMs are persistent and characterized by a long memory. The long memory feature of RMs follows from early work on the conditional volatility of daily financial returns (Ding, Granger, and Engle (1993), Baillie, Bollerslev, and Mikkelsen (1996)) and more recently on modeling RMs (Andersen and Bollerslev (1997), Andersen, Bollerslev, Diebold, and Labys (2001), Andersen et al. (2003a), Corsi (2009)). In particular, Andersen et al. (2003a) note that "*The slow hyperbolic autocorrelation decay symptomatic of long memory is evident ...*". This stylized fact of RMs implies that these series are fractionally integrated of some order

d.[6] One way to accommodate this characteristic is to approximate the dynamics of RMs using the fractionally cointegrated vector autoregressive (FCVAR) model of Johansen (2008) and Johansen and Nielsen (2012),

$$\Delta^d R_t = \alpha\beta' \Delta^{d-b} L_b R_t + \sum_{j=1}^{\kappa} \Gamma_j \Delta^d L_b^j R_t + \varepsilon_t, \quad t = 1, ..., T, \tag{5}$$

where R_t is a $S \times 1$ vector that concatenates the daily market-specific log RMs, $L_b = 1 - (1 - L)^b$ is the usual lag operator for fractional processes, d and b are the fractional order of the RMs and the degree of fractional cointegration, respectively, and ε_t is independently and identically distributed (i.i.d.) with a mean zero and variance Ω.[7] It is relevant to note two important aspects of the approximation in (5). First, we expect the rank of $\alpha\beta'$ to be equal to $S - 1$ in that the RMs share a common fractional stochastic trend: the efficient stochastic volatility. This structural feature follows from the fact that the efficient stochastic volatility is a property of the asset, not the trading venue. Second, we expect deviations between the RMs in the different markets to be transient, i.e., a short memory (covariance stationary) process driven by the trading-venue-specific characteristics, such as cost structure, different degrees of transparency, the speed of order execution and different trader groups, among other features. This stylized fact is accommodated by assuming $d = b$ in (5), which implies that $\beta' R_t$ is an $I(0)$ process. Finally, we allow the RMs to present a fractional order in the range of $0.5 \leq d < 1$, implying that these measures may be nonstationary but mean reverting.[8]

[6] A process z_t is said to be fractionally integrated of order d if $(1 - L)^d z_t = \eta_t$, where η_t is an integrated of order zero (stationary) process. It follows that $\Delta^d := (1 - L)^d$ is the fractional difference operator defined in terms of the binomial expansion $\Delta^d = \sum_{i=0}^{\infty} \binom{d}{i} (-L)^i = \sum_{i=0}^{\infty} \zeta_{i,d} L^d$.

[7] The parameters in the FCVAR framework have the same common interpretation as the parameters in the standard VEC model in that α contains the loading coefficients that correspond to the speed of adjustment and β is the cointegrating vector so that $\beta' R_t$ is integrated of order $d - b$ and represents the long-run equilibrium relation. Moreover, the FCVAR framework is general enough to nest the standard VEC model when $d = b = 1$.

[8] This is consistent with previous empirical findings, such as those of Rossi and de Magistris (2014), among others, who find estimates of d greater than 0.5.

B. Price and Volatility Decompositions

Multivariate random walk decompositions are central to constructing any price discovery measure, as they decompose observed prices into two components: the $I(1)$ efficient price and an $I(0)$ process that is associated with the portion of information that has no permanent impact on prices. Among the several different decompositions, the Granger representation theorem has the advantage of ensuring that the common efficient price is a martingale (Hansen and Lunde (2006)),

$$p_{t_i} = \beta_\perp \left[\gamma_\perp' \left(I - \sum_{j=1}^{q} \Upsilon_j \right) \beta_\perp' \right]^{-1} \gamma_\perp' \sum_{j=1}^{i} e_{t_j} + \sum_{j=0}^{\infty} \Xi_j e_{t_{i-j}} + p_{t_0}, \tag{6}$$

where $\beta_\perp = (1, 1, ..., 1)'$ in the price discovery context, and $\gamma_\perp$ is the orthogonal projection of the speed of adjustment parameter that satisfies $\gamma' \gamma_\perp = 0$. Because the first term of (6) is seen as the common efficient price, the elements of $\gamma_\perp$ are seen as the weights at which market innovations affect the efficient price. This concept has been widely applied to price discovery analysis (Booth, So, and Tseh (1999), Chu, Hsieh, and Tse (1999), Harris, McInish, and Wood (2002), Figuerola-Ferretti and Gonzalo (2010), among others), where the market associated with the highest element of $\gamma_\perp$ is the most important in the price discovery process (see Jong (2002) for a precise discussion of $\gamma_\perp$ and its relation with price discovery measures).

In the same way that the random walk decomposition constitutes the basic building block of price discovery analysis, the fractional counterpart of the Granger representation theorem serves as a building block for volatility discovery analysis. By decomposing the RMs into a long memory term common to all markets and market-specific $I(0)$ terms, we can identify the efficient stochastic volatility process and investigate how different venues incorporate information and adjust to the long-run equilibrium. Johansen (2008) and Johansen and Nielsen (2012) provide the fractional counterpart of the Granger representation theorem and decompose the process R_t into $I(d)$ and $I(0)$ components:

$$R_t = \Psi \Delta_+^{-d} \varepsilon_t + X_t, \quad t = 1, ..., T, \tag{7}$$

where $\Psi = \beta_\perp \left[\alpha'_\perp \left(I_S - \sum_{j=1}^{\kappa} \Gamma_j \right) \beta_\perp \right]^{-1} \alpha'_\perp$, Δ_+^{-d} is a truncated version of the fractional difference operator of order d; $X_t = Y_t + \mu_t$ is an $I(0)$ stochastic term; and $\alpha_\perp$ and $\beta_\perp$ are the orthogonal projections of α and β, respectively, with $\alpha' \alpha_\perp = 0$, $\alpha'_\perp \iota_S = 1$, and ι_S denoting a S-dimensional vector of ones. As discussed in Hasbrouck (1995), in the price discovery case, if $\beta = (I_{S-1}, \iota_{S-1})'$ and $\text{rk}(\alpha\beta') = S - 1$, the matrix Ψ has common rows. Section V.B provides strong empirical support for the cointegrating vector akin to $\beta = (I_{S-1}, \iota_{S-1})'$ in the FCVAR setup. Hence, a similar interpretation as that for the price discovery analysis holds in the volatility discovery setup, implying that R_t shares a single fractional stochastic trend that is integrated of order d, given by

$$R_{m,t} = \psi \Delta_+^{-d} \varepsilon_t = \left[\alpha'_\perp \left(I_S - \sum_{j=1}^{\kappa} \Gamma_j \right) \beta_\perp \right]^{-1} \alpha'_\perp \Delta_+^{-d} \varepsilon_t, \tag{8}$$

where ψ accounts for the common row of Ψ. Notably, $R_{m,t}$ possesses the long-term persistence and slow hyperbolic decay discussed in Andersen et al. (2003a), and it can be seen as the natural efficient stochastic volatility. Given that the efficient volatility can be recovered by $\psi \Delta_+^{-d} \varepsilon_t$, we can investigate how the efficient stochastic volatility is tied to innovations to the RMs of the different markets. In this setting, $\alpha_\perp$ plays a key role, and we define it as a measure of volatility discovery. The elements of $\alpha_\perp$ show how innovations from the market-specific RMs contribute to the efficient volatility. As in the price discovery analysis, as the elements of $\alpha_\perp$ for a given market increase, the importance of this market for the volatility discovery process increases. We carry out simulations using the price process in (2) and (3). The results confirm that $\alpha_\perp$ is a valid measure and successfully identifies the volatility discovery mechanism, whereas $\gamma_\perp$ is able to capture only information from the price discovery channel (see Tables A.1 and A.2 in the Appendix A). Specifically, we first confirm that the

FCVAR model approximates well the dynamics of the RMs and thus opens room to assess the validity of the volatility discovery framework developed in this section. Furthermore, we find that the RMs are characterized by nonstationary and mean reverting long memory, which is in line with the usual hyperbolic decay of the empirical autocorrelation function associated with RMs. Secondly, we document that the FCVAR model is able to correctly identify the structural features associated with the volatility discovery dynamics. The estimates of the speed of adjustment parameters verify the Granger causality structure in (3), i.e., α_1 is negative and significantly different from zero (RM in market one adjusts to changes in the RM of market two), whereas α_2 is not statistically different from zero, implying that the RM in the first market does not Granger-cause the RM in market two. Estimates of α are also fairly stable across all replications. Therefore, the use of the FCVAR model to approximate the dynamics of the RMs and the use of the orthogonal projection of the speed of adjustment parameters to quantify how the different markets impound information to the fractional common stochastic trend (efficient volatility) enable inference about the volatility discovery mechanism.

IV. Data

Our data consist of 30 of the most actively traded assets in the U.S. extracted from the TAQ database. All stocks are simultaneously traded in at least two of the three markets covered in this analysis (the NASDAQ, ARCA and NYSE) and represent a broad set of industries. Although these assets are also traded in other exchanges, we restrict our focus to the listed venues (NASDAQ and NYSE) and ARCA. The ARCA is particularly relevant because it is a trading platform with high levels of liquidity and contains practically all the assets in our sample. We use quotes from a sample period of 7 years, from January 2007 to December 2013, which captures the implementation of the National Market System regulation (Reg NMS) and the market fragmentation phenomenon.[9] The use of quotes instead of transaction prices has two relevant advantages. It adds a significant number of data points to our analysis and provides further information that is not present at the transaction level of data. Market makers update their quotes based on the available information in the market, and this may not be fully realized in the transaction price at every single point in time.

Before computing the RMs, it is necessary to filter the raw data for outliers. We implement the filter rules suggested in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009) for data on TAQ high-frequency quotes. In particular, we discard observations with a time stamp outside the main trading hours (9:30am-4:00pm), and we delete the observations where either the bid or ask is equal to zero or the bid-ask spread is negative; we also discard any entries for which either the spread is more than 50 times the median spread on the day or the mid-quote deviates by more than 10 mean absolute deviations from a rolling centered median of 50 observations. For the cases with a large number of entries with the same time stamp (second), we substitute these observations with their median bid and ask prices. These cleaning steps considerably reduce the sample size primarily because of multiple bid and ask prices in the same second. Table I details the cleaning process and the final number of observations. We begin with a database that contains 19.9 billion entries, and this decreases to 1.9 billion entries after the cleaning process, corresponding to an average of 14.7 thousand observations per day per exchange (one quote every 1.6 seconds on average), which is enough to guarantee good finite sample estimates of the market-specific IV measures.

[Place Table I about here]

We compute the daily RMs using Barndorff-Nielsen, Hansen, Lunde, and Shephard (2011)'s univariate realized kernel estimator because it provides consistent estimates of the market IV measures under rather mild assumptions. More specifically, the realized kernel

[9]The Reg NMS in 2007 allows the entry of new trading venues that are linked together and compete for order flow, liquidity, and trades.

estimator has a number of benefits that suit our analyses particularly well. First, the realized kernel consistently estimates the IV measure in the presence of time-dependent and endogenous market microstructure noise; this type of noise is usually found in tick-by-tick data, as discussed in Hansen and Lunde (2006) and Aït-Sahalia and Yu (2009), and is more pronounced for mid-quotes (Barndorff-Nielsen et al. (2011)).[10] Second, it is robust to accommodating irregularly spaced data, which is the case of the data that we adopt. Third, it can use all the available tick-by-tick data, helping to obtain more precise estimates in the finite sample.[11] Finally, as discussed in Barndorff-Nielsen et al. (2008) and Barndorff-Nielsen et al. (2011), the realized kernel estimator remains consistent even in the presence of jumps, provided that some regularity conditions hold. The use of mid-quotes (the average between the bid and ask prices) observed at a tick-by-tick frequency is crucial to our volatility discovery analysis because we want to gather all the possible information about the evolution of IV variance among the different markets. Hence, by using the realized kernel estimator, we do not lose any information that is embedded in the bid and ask prices.

For the price discovery analyses, we use mid-quotes aggregated at 10 seconds to balance out the negative effect of market microstructure noise (measurement error) on the model parameters' estimates and the positive effect of having information at the highest possible frequency. Because a typical trading day lasts for 6.5 hours, sampling at a 10-second frequency yields 2340 observations per day and a total of 4.12 million observations for a sample of 1762 days (typical stock in our sample). Finally, Figure 1 displays a summary of the time series properties of high-frequency prices and realized kernel estimates of the market-specific

[10] By time-dependent and endogenous market microstructure noise, we mean that the noise is autocorrelated and correlated with the efficient price, respectively (see Assumption **U** in Barndorff-Nielsen et al. (2011)).

[11] The rate of convergence of the realized kernel estimator, $N^{1/5}$, is slower than those of the kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008) and the two time scale estimators of Aït-Sahalia, Mykland, and Zhang (2005), $N^{1/4}$. The latter two estimators, however, require stronger assumptions that do not allow the use of all the available tick-by-tick data. Hence, as identified in Barndorff-Nielsen et al., 2011, p. 150, a much larger number of observations compensates for the slower convergence rate, implying that $390^{1/4} < 14000^{1/5}$, where 390 is the typical number of observations obtained when sampling at a 1-minute frequency, and 14000 is the average of the daily number of quotes in our sample.

integrated variances for AA (Alcoa) stock.[12] Both time series evolution and scatter plots confirm that log prices and log RMs from different trading venues do not drift apart, which is in line with both price and volatility discovery frameworks that postulate that observed prices and RMs in fragmented markets should track the efficient price and integrated variance, respectively. Finally, the sample autocorrelation function of the RMs (the last column of Figure 1) is highly persistent and presents a hyperbolic decay, which are stylized facts associated with long memory processes.

[**Place Figure 1 about here**]

V. Empirical Results

A. Model Specification

We use the log RMs to estimate bivariate FCVAR models with a level parameter, which imposes a restricted constant and accommodates a non-zero starting point for the RMs (see the discussion in Johansen and Nielsen (2015)).[13] We consider three market combinations: NASDAQ-ARCA, NASDAQ-NYSE, and ARCA-NYSE. This choice accommodates assets that are not simultaneously traded on the three exchanges (11 of the 30 firms in our sample) and ultimately yields a richer and broader sample, which is particularly important for handling firms that are listed on the NASDAQ but are not traded on the NYSE. A robustness exercise also considers the three markets jointly. We choose the lag length, κ, as the minimum value that makes the LM test for serial correlation on the residuals at the first 10 lags nonsignificant at the 5% significance level (Table B.1). For all the assets in our sample but four, the optimal lag length chosen is zero, which eliminates the identification problem

[12] To conserve space, we report only plots for AA. Similar figures for the remaining 29 stocks are available upon request.

[13] Estimation results for the FCVAR models are obtained using the computer program by Nielsen and Popiel (2014).

raised in Carlini and de Magistris (2017).[14]

To evaluate whether the RMs at different trading venues are fractionally cointegrated, we implement the sequential likelihood ratio (LR) test for the cofractional rank of Johansen and Nielsen (2012), which is discussed in Appendix C.2. For each two-market combination, the RMs are fractionally cointegrated and share a single common fractional stochastic trend if $\text{rk}(\alpha\beta') = 1$. Table II presents the p-values associated with the LR cofractional rank test when the null hypothesis is $\text{rk}(\alpha\beta') \leq 1$.[15] The null is not rejected in all but four stocks, suggesting that the RMs are fractionally cointegrated and should share the efficient stochastic volatility as a common fractional stochastic trend. Moreover, in line with the price discovery literature, Table II provides evidence that $\widehat{\beta} = (1, -1)'$ holds for all assets and market combinations, indicating that the market stochastic volatilities are expected to be equal in equilibrium.

[Place Table II about here]

Regarding the long memory feature of the RMs, Table III shows that $\widehat{d}$ is highly significant and usually greater than 0.5, confirming the RMs' long memory characteristic (nonstationary and mean reverting). The $\widehat{d}$ estimates are remarkably similar across the different market combinations, which suggests that the fractional trend's degree of long memory is virtually the same. Hence, the results in Table III further demonstrates that the estimates of $R_{m,t}$ are robust across the different market combinations, following their very similar long memory degree. In summary, Tables II and III provide the necessary preliminary results to ensure that the volatility discovery framework formulated in Section III suits the data well and approximates the dynamics of the RMs of a homogenous security traded on multiple markets.

[Place Table III about here]

[14]In addition to the requirement that residuals should be a white noise process, we choose κ so that the roots of the characteristic polynomials lie outside the transformed unit circle, $\mathbb{C}_{\widehat{b}}$ (see Johansen (2008) for a theoretical discussion on this identification). We set $\kappa = 0$ for the NOK asset in both NASDAQ-NYSE and ARCA-NYSE systems because the roots of the characteristic polynomial lie inside the transformed unit circle for any $\kappa > 0$.

[15]We do not report the p-values when the null hypothesis is $\text{rk}(\alpha\beta') = 0$ because we strongly reject the null (p-values are zero, even considering three decimal places) for the 30 assets in all markets.

B. Volatility Adjustment

We now turn our attention to the parameter estimates that determine the volatility discovery mechanism. The speed of adjustment parameters reflects the adjustment implemented in each market such that the RMs do not deviate from the latent $R_{m,t}$. Hence, as α_s approaches zero, the extent to which market s must adjust to $R_{m,t}$ decreases. In the limit, when $\alpha_s = 0$, market s does not need to adjust to shocks in the $R_{m,t}$, implying that it is the efficient market. Table B.3 in Appendix B reports that the adjustment parameter associated with one of the markets is statistically significant for most assets, implying that market leadership in the volatility discovery mechanism can be inferred.[16] We also conclude that the FCVAR model fits the data well, since their residuals are serially uncorrelated (see Table B.2 in Appendix B).

To investigate market leadership in the volatility discovery mechanism, it is easier to look at the estimates of the orthogonal projection of the α parameters, $\widehat{\alpha}_{\perp}$. The market with the highest $\widehat{\alpha}_{\perp}$ has the lowest need to adjust towards the efficient stochastic volatility, and hence, it is the one that leads the volatility discovery process. This measure also implies the proportion of each market's importance to the volatility discovery process, given that $\widehat{\alpha}_{\perp,1}+\widehat{\alpha}_{\perp,2} = 1$. Furthermore, the use of $\alpha_{\perp}$ has an important benefit: by means of an LR test, it allows us to test whether a given market contributes to the volatility discovery mechanism. Testing zero restrictions on the elements of $\alpha_{\perp}$ is equivalent to testing for the weak exogeneity of one of the RMs with respect to the cointegrating vector β. More importantly, when $\kappa = 0$ in (5), testing for weak exogeneity implies testing for Granger causality (strong exogeneity). Because we find $\kappa = 0$ for most stocks in our empirical analysis, the hypothesis tests below can be most often interpreted as Granger causality tests.[17] A linear hypothesis for $\alpha_{\perp}$ can be tested directly on either $\alpha_{\perp}$ or α (Dolatabadi, Nielsen, and Xu (2015)). Inspired by these

[16] Estimating α is particularly challenging within the FCVAR framework, and high standard errors are expected.

[17] Notably, for all stocks but IBM, KO, NOK and XOM, results from the weak exogeneity tests can be understood as Granger causality tests.

authors, we formulate the following null hypotheses:

H.1 $\mathcal{H}_0$: $\alpha_1 = 0$ or, equivalently, $\alpha_{\perp,2} = 0$. Volatility discovery occurs exclusively in market 1;

H.2 $\mathcal{H}_0$: $\alpha_2 = 0$ or, equivalently, $\alpha_{\perp,1} = 0$. Volatility discovery occurs exclusively in market 2.

The rejection of both null hypotheses suggests that volatility discovery occurs mutually in the two markets. The rejection of only one null hypothesis (either H.1 or H.2) indicates that volatility discovery occurs exclusively in one market. If this is the case – say, we reject H.1 and do not reject H.2 – then $R_{2,t}$ is said to be weakly exogenous, and hence, the second market appears to be the sole driver of the volatility discovery process. Moreover, if $\kappa = 0$, these results would also suggest that $R_{1,t}$ does not Granger-cause $R_{2,t}$. Finally, the non-rejection of both null hypotheses does not allow us to conclude whether any market is the single driver of the volatility discovery mechanism.

[Place Table IV about here]

Table IV presents the estimates of $\alpha_\perp$ for each market combination and displays the p-values associated with the tests for weak exogeneity, H.1 and H.2. Considering the first market combination (NASDAQ-ARCA), pointwise analyses show that $\widehat{\alpha}_{\perp,1} < \widehat{\alpha}_{\perp,2}$ holds for 68% of the assets (where 1 stands for the NASDAQ and 2 for the ARCA). This suggests that the ARCA is more important than the NASDAQ for the volatility discovery process. Additionally, the ARCA has average importance for the volatility discovery process of 54% compared to 46% for the NASDAQ, suggesting that on average, the ARCA contributes more to the volatility discovery process than the NASDAQ does. A more precise analysis is obtained when considering the hypothesis tests outlined above. Inferences based on the hypothesis tests suggest that the volatility discovery mechanism occurs exclusively in the ARCA for 11 of the 19 assets for which the ARCA leads; hence, the NASDAQ should not Granger-cause the ARCA (i.e., we reject H.1, ($\alpha_{\perp,2} = 0$), and do not reject H.2, ($\alpha_{\perp,1} = 0$) at 10% significance level).[18] Similar analyses offer statistical evidence that the NASDAQ is

[18]The ticker symbols are BRKB, F, GE, GM, GOOG, HPQ, JCP, MRVL, PFE, WMT, and YHOO.

the single driver for 5 assets and that volatility discovery occurs in both the NASDAQ and the ARCA for 4 assets.[19] Overall, our results support the conclusion that the ARCA is more informative than the NASDAQ in the volatility discovery process.

Results from the second market combination (NASDAQ-NYSE) also suggest that the NASDAQ is less important. We find that 68% of the assets present $\widehat{\alpha}_{\perp,1} < \widehat{\alpha}_{\perp,2}$ (where 1 stands for the NASDAQ and 2 for the NYSE), indicating that the NYSE leads the volatility discovery process. In terms of the markets' relative importance, the NYSE contributes, on average, to 60% of the volatility discovery mechanism, whereas the NASDAQ contributes only 40%. Using the null hypotheses H.1 and H.2, we find that the NASDAQ does not Granger-cause the NYSE for 9 assets, which reinforces the possible secondary role this exchange plays in the volatility discovery process.[20]

Finally, when comparing the results obtained from the ARCA-NYSE market combination (the last four columns in Table IV), there are mixed results considering market leadership and Granger causality. The NYSE and ARCA lead for the same number of assets, although, on average across stocks, the NYSE contributes 58% to the volatility process, compared to the ARCA's 42%. Using the hypothesis tests H.1 and H.2 does not provide strong evidence regarding overall market leadership. More precisely, we document that the NYSE does not Granger-cause the ARCA for 6 assets, whereas the ARCA does not Granger-cause the NYSE for 8 assets.[21] Note that although the ARCA and NYSE are part of the same holding (Intercontinental Exchange, Inc. - ICE), they possess very different characteristics in terms of their regulatory, operational and fee/rebate structures. In particular, the NYSE is a listing exchange, whereas the ARCA is a fully electronic trading platform that makes use of competing market makers and a smart order-routing algorithm. The latter communicates to

[19]The ticker symbols for which the NASDAQ is the single driver of the volatility discovery mechanism are CSCO, JPM, MSFT, ORCL, and PG. The ticker symbols for which volatility discovery occurs in both markets are JNJ, MO, MRK, and VZ.

[20]The ticker symbols are AA, HPQ, JCP, JNJ, KO, MO, MRK, PFE, and VZ.

[21]The ticker symbols for which the NYSE does not Granger-cause the ARCA are BAC, C, F, GE, GM, and JPM. The ticker symbols for which the ARCA does not Granger-cause the NYSE are HPQ, JCP, JNJ, MO, MRK, PFE, PG, and VZ.

alternative trading platforms (the NASDAQ, regional exchanges and Electronic Communication Networks (ECNs)), providing displayed and dark liquidity. Considering market quality measures such as the average quoted spread and the NBBO (national best bid and offer), the ARCA is ranked first with respect to its competitors.[22] The combination of the usual market quality measures with the volatility discovery results suggests that the NYSE may capture a different attribute of market quality that may not be reflected in the standard measures: information on the efficient stochastic volatility. This finding is consistent with the results from our simulation study, which demonstrates that the usual price discovery framework cannot identify the volatility discovery channel, whereas the volatility discovery machinery does so successfully.

C. Robustness

As a robustness exercise, we estimate α in (5) with d and b as free parameters. The main theoretical implication from relaxing $d = b$ is that deviations from the common fractional stochastic trend are now allowed to be a long memory process of order $d - b$. Table B.4 reports estimates of both $\alpha_\perp$ and $\widetilde{\alpha}_\perp$, where the latter denotes the orthogonal projection of the α parameters obtained when d and b are treated as free parameters. We find that our results are robust to this more flexible specification. In particular, the volatility discovery measures in the NASDAQ-ARCA market combination remain virtually unchanged. With regard to the other two market specifications, we observe that despite a nominal change in $\widetilde{\alpha}_\perp$ compared to $\alpha_\perp$, the market leadership hierarchy remains the same.

Economic intuition demands that the volatility discovery measures presented in Table IV are transitive. Transitiveness holds if, for example, the following pattern is observed: the ARCA is more important than the NASDAQ; the NASDAQ is more important than the NYSE; and the ARCA is more important than the NYSE. By comparing the results in Table IV across the different market combinations, we find that transitivity holds in 18

[22] All data are as of March 2015 and available on https://www.nyse.com/markets/nyse-arca.

of the 19 assets that are simultaneously traded on the NASDAQ, ARCA and NYSE. It is possible, however, to estimate the volatility discovery measures from a higher dimensional FCVAR model that contains the three markets. Estimating a three-dimensional FCVAR model is, in fact, the most appropriate approach to assessing whether transitivity holds. Table B.5 presents the results of this exercise. To be in accordance with our volatility discovery framework, we expect to find two cointegrating vectors so that the three RMs share a single fractional stochastic trend. Results support our theoretical framework in that there are two cointegrating vectors, estimates of d are highly significant and most often greater than 0.5, and residuals are uncorrelated.[23] Furthermore, because α is a 3×2 matrix, $\alpha_\perp$ is a 3×1 vector, implying that, as in the bivariate case, the elements of $\alpha_\perp$ are the volatility discovery measures. We find that market leadership is consistent with the results in Table IV for 17 of the 19 assets.

VI. Relation to previous literature

A. *Information Share of Prices and Volatility*

In addition to $\gamma_\perp$ presented in Section III.B, another prominent way to measure price discovery is given by Hasbrouck's 1995's Information Share (IS) measure. To verify how the IS measure is related to the volatility discovery mechanism, we slightly modify the IS to account for the stochastic volatility. Consider a version of the standard IS measure in which the covariance matrix of the VEC residuals is replaced by the realized covariance matrix, $\overline{R}_t$. This change allows us to write the IS as function of the RMs and the FCVAR parameters.[24] For simplicity of exposition, assume that $\overline{R}_t$ is a diagonal matrix that contains the elements of R_t. By assuming that the correlation among markets is zero, we avoid the issue of dealing with the upper and lower bounds associated with the factorization of $\overline{R}_t$. The IS measure of

[23]The complete set of results is available upon request.

[24]Formulating the IS in terms of RMs also delivers a price discovery measure in continuous time (Dias, Fernandes, and Scherrer (2016)).

market $s \in (1, 2, ..., S)$ then reads

$$IS_{s,t} = \frac{\vartheta_s^2 R_{s,t}}{\vartheta \overline{R}_t \vartheta'}, \quad t = 1, ..., T, \tag{9}$$

where ϑ_s denotes the sth element of $\vartheta = \left[\gamma_\perp' \left(I - \sum_{j=1}^{q} \Upsilon_j\right) \beta_\perp'\right]^{-1} \gamma_\perp'$. From (7) and (8), we can write the market-specific RMs in terms of $R_{m,t}$ (common to all RMs) and X_t (market-specific short memory terms) such that $R_t = \beta_\perp R_{m,t} + X_t$, and the IS measure of market s reads

$$IS_{s,t} = \frac{\vartheta_s^2 \left(R_{m,t} + X_{s,t}\right)}{\sum_{s=1}^{S} \vartheta_s^2 \left(R_{m,t} + X_{s,t}\right)}, \quad t = 1, 2, ..., T. \tag{10}$$

The $\text{IS}_{s,t}$ measure depends on three components: ϑ, $R_{m,t}$, and $X_{s,t}$. Specifically, ϑ captures the permanent effect of market innovations on prices and originates from the VEC parameters; hence, it is not affected by the volatility discovery mechanism. The $R_{m,t}$ term plays the role of a normalization factor because it is not informative to identify how the innovations to the stochastic market volatilities affect $R_{m,t}$, i.e., the $\alpha_\perp$ parameter does not load on this term. The market-specific terms, $X_{s,t}$ for $s \in (1, 2, ..., S)$, summarize the transitory effects of shocks in the volatility processes and have no effect on the efficient stochastic volatility. These terms can be seen as a source of contamination of the IS measure because they do not carry relevant information for either the price or the volatility discovery mechanisms. Overall, two conclusions emerge from (10). First, taking into account stochastic volatility processes contaminates the IS measure. Second and most important, the IS measure cannot answer questions regarding the volatility discovery mechanism because it cannot separate the contribution of each market innovation to the market-specific stochastic volatilities on $R_{m,t}$. Simulations carried out using the price process in (2) and (3) corroborate these conclusions, as although the IS correctly identifies market one as the unique contributor to the price discovery process, it remains uninformative about the volatility discovery mechanism (see Appendix A).

B. Information Arrival and Sensitivity

A further step in our analyses is to formally relate the price and the volatility discovery mechanisms to the extant literature that associates the persistence of the volatility process with the rate of information arrival and market participants' sensitivity to information. The so-called Liesenfeld's 2001 generalized bivariate mixture model relates the volatility and volume dynamics to two latent variables (information arrivals and market participants' sensitivity to information) and ultimately allows the volatility and volume processes to respond differently to distinct pieces of information (see Tauchen and Pitts (1983) and Andersen (1996) for earlier work on the bivariate mixture model and Epps and Epps (1976) for a first attempt to formally relate volatility and information flow). On the basis of these earlier works, Berger et al. (2009) associate returns and variance in prices to information flow and the market's sensitivity to information, respectively, such that we can write

$$\Delta m_{t_i} = \xi_t of_{t_i}, \tag{11}$$

$$\ln\left(RV_{m,t}\right) = \ln\left(\sum_{i=1}^{N} of_{t_i}^2\right) + 2\ln\left(\xi_t\right), \tag{12}$$

where m_{t_i} is the latent efficient price; $RV_{m,t}$ is the realized variance defined as the sum of the squared intraday returns; ξ_t is interpreted as the sensitivity of the price to the information flow (market sensitivity); and of_{t_i} is the detrended order flow that proxies the information flow. Equation (12) has $\ln\left(RV_{m,t}\right)$ as a function of $\sum_{i=1}^{N} of_{t_i}^2$ and ξ_t, where the former reflects the information that is permanently impounded to prices, while the latter captures how the market participants process and react to new information (market sensitivity).[25] Berger et al. (2009) find that the degree of persistence of the two elements on the right-hand side of (12) is characterized by a long memory, with d ranging from 0.31 to 0.60.

To relate the price and volatility discovery mechanisms to this literature, we write $RV_{m,t}$

[25] This is also consistent with the work of Engle et al. (2012), which delivers the concept of the private processing of public information loading significantly on changes in volatility and helping to explain the volatility's persistence.

in (12) as a function of the VEC and FCVAR parameters in (4) and (5), respectively, so it reads

$$RV_{m,t} = \alpha'_{\perp}\Big\{\iota_S \sum_{i=1}^{N}\left(\gamma'_{\perp}e_{t_i}e'_{t_i}\gamma_{\perp}\right)\Big\} + \alpha'_{\perp}\sum_{i=1}^{N}A_i + \textit{initial value}, \tag{13}$$

where $A_i = \left(\left[2\gamma'_{\perp}e_{t_i}A_{1,i} + A^2_{1,i}\right], ..., \left[2\gamma'_{\perp}e_{t_i}A_{S,i} + A^2_{S,i}\right]\right)'$ is a $(S \times 1)$ vector with $A_{s,i} = \sum_{j=0}^{\infty}\Xi_{s,j}\Delta e_{t_{i-j}}$, $s = 1, 2, ..., S$, and $\Xi_{s,j}$ denoting the sth row of the Ξ_j parameter matrix from the $I(0)$ component of the Granger representation theorem in (6).[26]

As the efficient volatility in (13) contains two long memory terms, it is natural to relate them to market sensitivity and order flow as in (12). The first component of $RV_{m,t}$, $\sum_{i=1}^{N}\left(\gamma'_{\perp}e_{t_i}e'_{t_i}\gamma_{\perp}\right)$, is the denominator in Hasbrouck's 1995's IS measure (see Section VI.A) and therefore delivers a straightforward association with this price discovery measure. We relate this term to the order flow in (12), since $\left(\sum_{i=1}^{N} of^2_{t_i}\right)$ proxies the rate of information arrival that affects the efficient price. The relationship between price discovery and order flow, trading volume and liquidity is consistent with the empirical price discovery literature (see Eun and Sabherwal (2003), Frijns, Gilbert, and Tourani-Rad (2015)). Specifically, Figuerola-Ferretti and Gonzalo (2010) formulate an equilibrium model in which $\gamma_{\perp}$ is associated with the number of market participants (proxied by trading volume). We relate the second component in (13), $\alpha'_{\perp}\sum_{i=1}^{N}A_i$, to the market's sensitivity to information because the A_i term contains the terms associated with short-run deviations from equilibrium. It should be noted that as the deviations from the equilibrium become larger, the impact on efficient volatility strengthens, ultimately reflecting how market participants agree on incoming news. Finally, (13) also allows us to understand why $\alpha_{\perp}$ is a good choice of measure to evaluate volatility discovery, as it loads on both components of $RV_{m,t}$, unlike $\gamma_{\perp}$.

[26]The steps to obtain (13) are detailed Appendix C.3. The representation in (13) can be readily extended to the case in which the log RV are approximated by a FCVAR model so that $\ln(RV_{m,t})$ assumes the form

$$\ln(RV_{m,t}) = \alpha'_{\perp}\ln\left(\iota_S\sum_{i=1}^{N}\left(\gamma'_{\perp}e_{t_i}e'_{t_i}\gamma_{\perp}\right) + \sum_{i=1}^{N}A_i\right) + \text{initial value}.$$

C. Price Discovery vs. Volatility Discovery

Our theoretical setting shows that the price and volatility discovery mechanisms gather two distinct sources of information. Having documented that the NASDAQ is usually less important for the volatility discovery mechanism, we next examine whether the ARCA and NYSE are also the leading markets in the price discovery analysis. For this purpose, we follow the price discovery literature and adopt the VEC model in (4) as a discrete approximation of the observed prices in the three market combinations discussed previously. We find one cointegrating vector and estimate the adjustment coefficients γ in (4) using the OLS estimator, as it yields consistent and asymptotically normally distributed estimates when β is assumed to be known.[27] Our preferred choice of price discovery measure is the orthogonal projection of γ in (4), $\gamma_{\perp}$, which is normalized such that $\gamma_{\perp,1} + \gamma_{\perp,2} = 1$. Hence, as in the volatility discovery analysis, $\gamma_{\perp,1} > \gamma_{\perp,2}$ implies that market 1 is more important for the price discovery process. This approach is convenient because the competing IS measure is found to be virtually equal to 0.5 for all stocks in our sample even at a frequency as high as 10 seconds. Unfortunately, this feature of the IS measure does not come as a surprise because there is mounting empirical evidence that markets have become faster and more tightly inter-connected, which ultimately increases the correlation among them (see Menkveld (2014), O'Hara (2015), and Menkveld (2016)).

Table V reports the results for the price discovery analysis. In general, the results suggest that the NASDAQ leads the price discovery mechanism when compared to both the ARCA and NYSE trading venues. Considering the NASDAQ-ARCA market combination, we find that $\widehat{\gamma}_{\perp,1} > \widehat{\gamma}_{\perp,2}$ for 79% of the assets in our sample, suggesting that the NASDAQ dominates the ARCA in terms of price discovery.[28] Comparing the NASDAQ to the NYSE, we find

[27] We fix $\beta = (1, -1)'$ as is the standard practice in the literature and choose the lag length in the VEC specification in (4) as the minimum value that makes the LM test for serial correlation on the residuals at lag 15 not statistically significant at the 5% level. The choice of $\mathrm{rk}(\gamma\beta') = 1$ follows the results from the Johansen cointegration rank test. The p-values associated with the Johansen and serial correlation tests are available upon request.

[28] The ticker symbols are AA, AAPL, BAC, BRKB, CSCO, F, GE, GM, GOOG, HPQ, JNJ, JPM, KO, MRK, MRVL, MS, MSFT, ORCL, PFE, VZ, WMT, and YHOO.

that the NASDAQ leads the price discovery process in 74% of the cases.[29] Next, the ARCA-NYSE market combination shows results slightly in favor of the ARCA, with leadership in 62% of the stocks.[30] It is important to highlight that we reject, at a 5% significance level, both the H.1 and H.2 null hypotheses for the vast majority of assets in the three market combinations. When both null hypotheses are simultaneously rejected, there is evidence that price discovery occurs simultaneously in both markets.

Having separately analyzed the volatility discovery and price discovery measures in the three markets combinations, we now make a more direct comparison of the results in Tables IV and V. In general, evidence suggests that distinct trading venues lead the price and volatility discovery processes. For the NASDAQ-ARCA case, the price and volatility discovery processes occur in different markets in 54% of the stocks in our sample. For instance, in the cases of BRKB, F, GE, GM, GOOG, HPQ, JNJ, KO, MRK, MRVL, PFE, VZ, WMT, and YHOO, the NASDAQ is more important for the price discovery process, whereas the ARCA leads the volatility discovery mechanism. Interestingly, the NASDAQ presents higher quoting activity than the ARCA for all these stocks.[31] Similarly, our results for the NASDAQ-NYSE combination suggest that the NASDAQ is more important for the price discovery process whereas the NYSE is more important for the volatility discovery process in half of our sample (AA, HPQ, JNJ, KO, MO, MRK, VZ, WMT, and XOM). Again here, the NASDAQ shows higher quoting activity than the NYSE for virtually all these stocks. Overall, the leading market in the price discovery analysis is often also the one with the highest quoting activity (75% and 63% of the stocks for ARCA-NASDAQ and NYSE-NASDAQ market combinations, respectively). This result is in line with the link between price discovery, order flow and the rate of information arrival developed in Section VI.B, as well as with previous studies that associate price discovery with different measures of liquidity (Eun and Sabherwal (2003) and Frijns et al. (2015)). For the volatility discovery

[29] The ticker symbols are AA, BAC, F, GE, GM, HPQ, JNJ, JPM, KO, MO, MRK, VZ, WMT, and XOM.

[30] The ticker symbols are AA, BAC, C, F, GE, GM, HPQ, JPM, KO, MO, MRK, WMT, and XOM.

[31] We define quoting activity as the daily average of quotes. It can be computed by dividing the number of quotes before any cleaning filter by the total number of days in Table I.

measure, these ratios are much lower, 29% and 42%, indicating that, possibly, liquidity more strongly affects price discovery than it does volatility discovery. Finally, the ARCA-NYSE combination shows generally more balanced results. We observe a difference in the venue where price discovery and volatility discovery mechanisms occur for only 32% of the stocks.

To offer a more precise analysis of the relationship between quoting intensity and price and volatility discovery, we perform a simple probit regression analysis. We regress a binary variable that takes the value of 1 if a given market leads the price (volatility) discovery process and 0 otherwise on quote intensity, volatility (price) discovery measures and a dummy variable that returns 1 if the stock is listed in a given market. Consistent with our previous explanation, it appears that price discovery is more influenced by quoting activity than volatility discovery is, as the probability of leading the price discovery process is positively (statistically significant) related with quote intensity and the volatility discovery measure (see Table B.6).

Overall, our results suggest that the volatility and price discovery mechanisms do not necessarily occur in the same trading venue. This finding reinforces our theoretical motivation that the price and volatility discovery measures capture how markets incorporate news into two distinct efficient stochastic processes: the efficient price and the efficient stochastic volatility. These results suggest that market quality should be analyzed using broader measures that consider how information is incorporated into prices and risk.

VII. Conclusions

In the current context of market fragmentation, we develop a novel theoretical framework to investigate how distinct markets contribute to the efficient stochastic volatility process: the volatility discovery mechanism. The economic rationale supporting the volatility discovery analysis rests on the premise that the efficient volatility process is a separate stochastic process and that trading venues possess different characteristics and market designs and hence are populated by different trader groups or clientele. The price discovery literature has examined how different markets impound new information into the latent efficient price. Because economic agents may also differ in terms of their risk assessment, it is natural to investigate how markets impound information into the efficient stochastic volatility process.

The volatility discovery framework builds on evidence that the RMs of a homogenous asset cointegrate and share a common factor: the efficient stochastic volatility. We exploit RMs' long memory feature and adopt the recently developed fractionally cointegrated vector autoregressive (FCVAR) model of Johansen and Nielsen (2012) to estimate how innovations to market volatilities contribute to the efficient volatility process.

We compute volatility discovery for 30 of the most actively traded stocks in three U.S. markets: the NASDAQ, ARCA and NYSE. We find that markets indeed incorporate new information into the stochastic volatility process in a distinct way. In particular, our results suggest that the NYSE and the ARCA are the most efficient venues, i.e., they incorporate changes into the efficient volatility most quickly. We confirm this less prominent role of the NASDAQ in the volatility discovery process using Granger causality tests. Comparing price and volatility discovery, our results indicate that these mechanisms do not necessarily occur in the same trading venue, suggesting significant differences in how exchanges impound information into the efficient price and volatility processes.

We further investigate the differences between the price and volatility discovery measures and formally show that the IS measure cannot identify market leadership in the volatility process and becomes contaminated when stochastic volatility is allowed in the price process. This finding is confirmed in a simple example in which we show that while the standard price discovery framework fails to identify the volatility discovery channel, the methodology put forward in this article successfully identifies this information channel. Berger et al. (2009) highlight the role of market participants' sensitivity to information in driving volatility persistence and show that it is at least as important as the rate of information arrival. We write the efficient price volatility as a function of the parameters that evaluate price and volatility discovery and hint that price discovery summarizes only information regarding the rate of information arrival, whereas the volatility discovery measure loads on both information arrival and market sensitivity.

Finally, we believe that future research will study the drivers of volatility discovery and their relation to market characteristics and structure. The methodology developed in this article allows room to test a number of empirical questions that involve the volatility dynamics in fragmented markets. Further empirical findings on the volatility process of a homogeneous asset in the context of multiple markets should also be expected.

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Figure 1. Prices and Realized Measures at Nasdaq, Arca and Nyse markets: AA (Alcoa)

The figure consists of three panels with plots of intraday log-prices and daily log-RMs at different market combinations for AA (Alcoa). The first row presents plots for the Nasdaq-Arca market combination, while the second and third panels refer to the Nasdaq-Nyse and Arca-Nyse market combinations. The first column displays plots for the intraday log-prices in the different market combinations. The second column presents scatter plots around the 45 degrees line of intraday log-prices. Specifically, we denote Nasdaq as 'T', Arca as 'P', and Nyse as 'N'. The third column presents plots of daily logarithmic estimates of integrated variance using the realized kernel estimator of Barndorff-Nielsen et al. (2011). The fourth column shows scatter plots around the 45 degrees line of daily logarithmic RMs. Finally, the fifth column displays three plots with sample autocorrelation function of the logarithmic realized kernel estimates for Nasdaq, Nyse, and Arca.

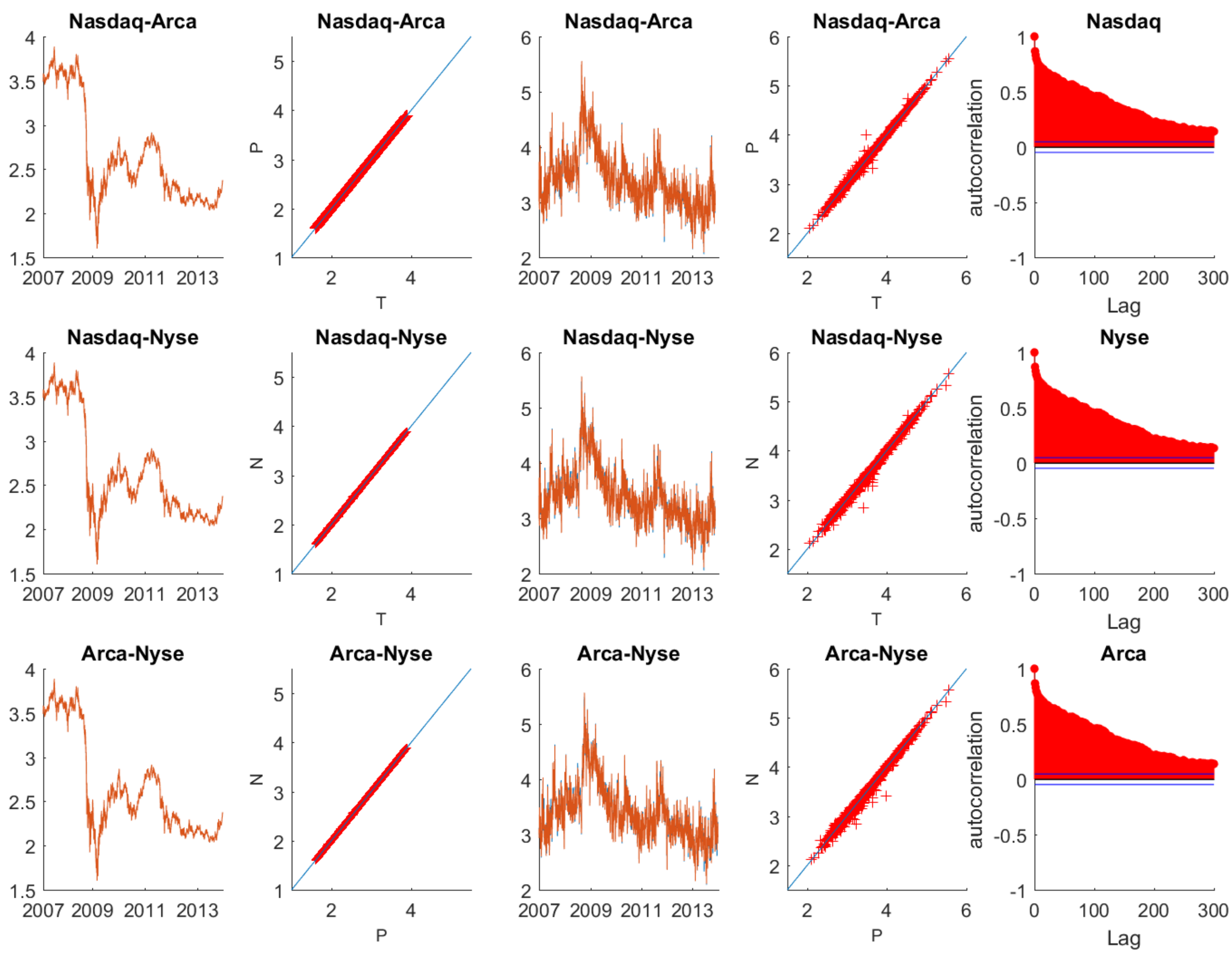

Table I Data Description

We report summary statistics for raw and cleaned data considering Nasdaq, Arca, and Nyse. The first three columns present the number of quotes (in millions) for each stock before any cleaning filter (raw data). The following three columns (Clean obs) display the total number of quotes (in millions) after the implementation of the cleaning procedure. The following three columns (Avg obs per day) stand for the daily average (in thousands) of quotes. The last three columns report the total number of days we have for each stock for the time span 01/01/2007 to 31/12/2013.

	Initial Obs (Million)			Clean Obs (Million)			Avg obs per day (Thousand)			Number of days		
	Nasdaq	Arca	Nyse	Nasdaq	Arca	Nyse	Nasdaq	Arca	Nyse	Nasdaq	Arca	Nyse
AA	237	138	247	25	23	27	14.32	13.27	15.14	1735	1762	1762
AAPL	459	256	-	31	30	-	17.50	17.24	-	1762	1762	-
BAC	523	292	503	31	30	34	17.85	17.28	19.05	1735	1762	1762
BRKB	98	68	-	11	10	-	9.12	8.35	-	1183	1142	-
C	-	319	549	-	30	32	-	16.93	18.39	-	1762	1762
CSCO	210	67	-	14	11	-	17.72	14.79	-	777	777	-
F	137	70	172	13	12	15	13.24	11.77	14.91	982	1009	1009
GE	363	214	427	29	27	31	16.53	15.58	17.78	1735	1762	1762
GM	202	90	174	16	15	19	11.81	10.83	13.34	1364	1391	1391
GOOG	149	124	-	19	19	-	10.77	10.79	-	1762	1762	-
HPQ	326	167	277	26	24	28	14.84	13.67	15.86	1735	1762	1762
IBM	122	102	149	21	20	25	11.96	11.50	14.13	1735	1762	1762
JCP	175	113	149	20	20	22	11.55	11.14	12.26	1735	1762	1762
JNJ	280	137	251	25	22	27	18.37	15.84	19.76	1364	1391	1391
JPM	696	345	542	32	31	33	18.43	17.52	18.64	1735	1762	1762
KO	244	123	205	23	21	25	13.27	11.90	14.44	1735	1762	1762
MO	178	95	204	22	19	26	12.40	10.83	14.90	1735	1762	1762
MRK	271	151	244	25	23	27	14.19	13.01	15.47	1735	1762	1762
MRVL	252	101	-	24	19	-	13.46	11.03	-	1762	1762	-
MS	233	416	-	28	27	-	16.03	15.40	-	1735	1762	-
MSFT	669	228	-	33	28	-	18.59	16.13	-	1762	1762	-
NOK	199	120	187	22	21	23	12.46	11.70	12.99	1735	1762	1762
ORCL	494	181	-	30	26	-	17.15	14.49	-	1762	1762	-
PFE	309	159	342	27	25	30	15.29	13.92	17.12	1735	1762	1762
PG	283	136	198	25	22	26	14.41	12.54	14.77	1735	1762	1762
VZ	264	141	257	25	23	29	14.44	13.03	16.19	1735	1762	1762
WFC	-	305	427	-	29	31	-	16.67	17.81	-	1762	1762
WMT	269	144	251	25	23	28	14.40	12.98	16.01	1735	1762	1762
XOM	503	337	417	31	31	33	18.10	17.56	18.84	1735	1762	1762
YHOO	367	121	-	26	22	-	14.87	12.27	-	1762	1761	-

Table II Cofractional Rank Test and Estimates of the Cointegrating vector

We report results for 30 assets considering the Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. For each set of results, the first column ($\text{rk}(\alpha\beta')$) displays the number of cointegrating relations used in the estimation of the FCVAR model. The second column has the p-value of the likelihood-ratio (LR) cofractional rank test when the null hypothesis is $\text{rk}(\alpha\beta') \leq 1$. Finally, the last two columns display the estimated cointegrating vector, $\widehat{\beta}' = (\widehat{\beta}_1, \widehat{\beta}_2)$. There are few assets where the null of $\text{rk}(\alpha\beta') \leq 1$ in the LR cofractional rank test is rejected. The FCVAR parameters are computed using the MLE estimator of Johansen and Nielsen (2012) where κ is chosen according to Table B.1, and $d = b$. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca				Nasdaq-Nyse				Arca-Nyse			
	$\text{rk}(\alpha\beta')$	LR	$\widehat{\beta}'$		$\text{rk}(\alpha\beta')$	LR	$\widehat{\beta}'$		$\text{rk}(\alpha\beta')$	LR	$\widehat{\beta}'$	
AA	1	0.38	1.00	-1.00	1	0.36	1.00	-1.00	1	0.42	1.00	-1.00
AAPL	1	0.99	1.00	-1.00	-	-	-	-	-	-	-	-
BAC	1	0.70	1.00	-1.00	1	0.59	1.00	-1.00	1	0.71	1.00	-1.00
BRKB	1	0.04	1.00	-1.01	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	1	0.94	1.00	-1.00
CSCO	1	0.72	1.00	-1.00	-	-	-	-	-	-	-	-
F	1	0.83	1.00	-1.00	1	0.80	1.00	-1.01	1	0.80	1.00	-1.01
GE	1	0.51	1.00	-1.00	1	0.26	1.00	-1.00	1	0.22	1.00	-1.00
GM	1	0.71	1.00	-1.00	1	0.73	1.00	-1.01	1	0.60	1.00	-1.01
GOOG	1	0.71	1.00	-1.00	-	-	-	-	-	-	-	-
HPQ	1	0.88	1.00	-1.00	1	0.89	1.00	-1.00	1	0.96	1.00	-1.00
IBM	1	0.99	1.00	-1.00	1	0.95	1.00	-1.00	1	0.95	1.00	-1.01
JCP	1	0.02	1.00	-1.01	1	0.01	1.00	-1.00	1	0.01	1.00	-0.99
JNJ	1	0.70	1.00	-1.00	1	0.57	1.00	-1.01	1	0.46	1.00	-1.01
JPM	1	0.76	1.00	-1.00	1	0.74	1.00	-1.00	1	0.81	1.00	-1.00
KO	1	0.85	1.00	-0.99	1	0.87	1.00	-1.01	1	0.85	1.00	-1.01
MO	1	0.16	1.00	-1.00	1	0.14	1.00	-1.02	1	0.04	1.00	-1.02
MRK	1	0.72	1.00	-1.00	1	0.57	1.00	-1.02	1	0.56	1.00	-1.01
MRVL	1	0.00	1.00	-1.00	-	-	-	-	-	-	-	-
MS	1	0.86	1.00	-1.00	-	-	-	-	-	-	-	-
MSFT	1	0.92	1.00	-1.00	-	-	-	-	-	-	-	-
NOK	1	0.64	1.00	-1.00	1	0.34	1.00	-1.00	1	0.69	1.00	-1.00
ORCL	1	0.58	1.00	-1.00	-	-	-	-	-	-	-	-
PFE	1	0.53	1.00	-1.00	1	0.48	1.00	-1.00	1	0.49	1.00	-1.00
PG	1	0.57	1.00	-1.00	1	0.62	1.00	-1.00	1	0.47	1.00	-1.00
VZ	1	0.44	1.00	-1.00	1	0.39	1.00	-1.01	1	0.24	1.00	-1.00
WFC	-	-	-	-	-	-	-	-	1	0.58	1.00	-1.00
WMT	1	0.04	1.00	-1.00	1	0.06	1.00	-1.00	1	0.01	1.00	-1.00
XOM	1	0.45	1.00	-1.00	1	0.44	1.00	-1.00	1	0.89	1.00	-1.00
YHOO	1	0.15	1.00	-1.00	-	-	-	-	-	-	-	-

Table III Long Memory Estimates

We report the long memory estimates, $\widehat{d}$, for 30 assets considering the Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. The long memory parameter, d, belongs to the set of free parameters in the FCVAR model, which are computed using the MLE estimator of Johansen and Nielsen (2012) where $\text{rk}\,(\alpha\beta') = 1$, κ is chosen according to Table B.1, and $d = b$. For each set of market combinations, the first column displays the estimates of the the long memory degree, $\widehat{d}$, while the second column presents its standard errors in parentheses. The symbols *, ** and *** denote rejection at the 10%, 5% and 1% levels of the null hypothesis of $\widehat{d} = 0$. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca		Nasdaq-Nyse		Arca-Nyse	
AA	0.52***	(0.02)	0.52***	(0.02)	0.53***	(0.02)
AAPL	0.52***	(0.02)	-	-	-	-
BAC	0.60***	(0.02)	0.60***	(0.02)	0.59***	(0.02)
BRKB	0.52***	(0.02)	-	-	-	-
C	-	-	-	-	0.59**	(0.02)
CSCO	0.54***	(0.02)	-	-	-	-
F	0.56***	(0.02)	0.56***	(0.02)	0.56***	(0.02)
GE	0.55***	(0.02)	0.54***	(0.02)	0.53***	(0.02)
GM	0.54***	(0.02)	0.54***	(0.02)	0.54***	(0.02)
GOOG	0.51***	(0.02)	-	-	-	-
HPQ	0.51***	(0.02)	0.51***	(0.02)	0.51***	(0.02)
IBM	0.62***	(0.03)	0.60***	(0.03)	0.64***	(0.04)
JCP	0.49***	(0.02)	0.49***	(0.02)	0.47***	(0.02)
JNJ	0.53***	(0.02)	0.52***	(0.02)	0.52***	(0.02)
JPM	0.61***	(0.02)	0.60***	(0.02)	0.59***	(0.02)
KO	0.67***	(0.04)	0.51***	(0.02)	0.51***	(0.02)
MO	0.49***	(0.02)	0.49***	(0.02)	0.48***	(0.02)
MRK	0.51***	(0.02)	0.51***	(0.02)	0.51***	(0.02)
MRVL	0.44***	(0.02)	-	-	-	-
MS	0.61***	(0.02)	-	-	-	-
MSFT	0.51***	(0.02)	-	-	-	-
NOK	0.62***	(0.03)	0.51***	(0.02)	0.51***	(0.02)
ORCL	0.51***	(0.02)	-	-	-	-
PFE	0.52***	(0.02)	0.51***	(0.02)	0.51***	(0.02)
PG	0.51***	(0.02)	0.52***	(0.02)	0.51***	(0.02)
VZ	0.51***	(0.02)	0.52***	(0.02)	0.51***	(0.02)
WFC	-	-	-	-	0.59***	(0.02)
WMT	0.48***	(0.02)	0.49***	(0.02)	0.48**	(0.02)
XOM	0.73***	(0.05)	0.74***	(0.05)	0.71***	(0.06)
YHOO	0.47***	(0.02)	-	-	-	-

Table IV Volatility Discovery Measures

We report the volatility discovery measures for 30 assets considering the Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. The FCVAR parameters are computed using the MLE estimator of Johansen and Nielsen (2012) where $\text{rk}(\alpha\beta') = 1$, κ is chosen according to Table B.1, and $d = b$. For each set of results, the first column shows the estimate of the first element of the orthogonal projection of α, $\widehat{\alpha}_{\perp,1}$, while the second column (value inside the square brackets) displays the p-value associated with the likelihood-ratio (LR) test with the null hypothesis defined in H.2, $\widehat{\alpha}_{\perp,1} = 0$. The third column presents $\widehat{\alpha}_{\perp,2}$, while the fourth column (value inside the square brackets) reports the p-value associated with H.1, ($\widehat{\alpha}_{\perp,2} = 0$). "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca				Nasdaq-Nyse				Arca-Nyse			
	$\widehat{\alpha}_{\perp,1}$		$\widehat{\alpha}_{\perp,2}$		$\widehat{\alpha}_{\perp,1}$		$\widehat{\alpha}_{\perp,2}$		$\widehat{\alpha}_{\perp,1}$		$\widehat{\alpha}_{\perp,2}$	
AA	0.57	[0.13]	0.43	[0.25]	0.45	[0.15]	0.55	[0.09]	0.52	[0.06]	0.48	[0.09]
AAPL	0.85	[0.11]	0.15	[0.78]	-	-	-	-	-	-	-	-
BAC	0.74	[0.11]	0.26	[0.58]	0.88	[0.01]	0.12	[0.74]	0.73	[0.06]	0.27	[0.49]
BRKB	0.34	[0.25]	0.66	[0.02]	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	1.20	[0.00]	-0.20	[0.61]
CSCO	0.68	[0.01]	0.32	[0.22]	-	-	-	-	-	-	-	-
F	-0.10	[0.87]	1.10	[0.07]	1.08	[0.01]	-0.08	[0.85]	1.43	[0.00]	-0.43	[0.32]
GE	0.00	[0.99]	1.00	[0.06]	0.87	[0.03]	0.13	[0.74]	1.27	[0.01]	-0.27	[0.57]
GM	-0.11	[0.81]	1.11	[0.01]	0.64	[0.02]	0.36	[0.19]	0.91	[0.01]	0.09	[0.80]
GOOG	0.27	[0.48]	0.73	[0.05]	-	-	-	-	-	-	-	-
HPQ	-0.09	[0.83]	1.09	[0.01]	0.06	[0.86]	0.94	[0.00]	0.36	[0.28]	0.64	[0.06]
IBM	0.43	[0.24]	0.57	[0.12]	0.45	[0.33]	0.55	[0.24]	0.76	[0.13]	0.24	[0.65]
JCP	0.41	[0.15]	0.59	[0.04]	0.14	[0.65]	0.86	[0.01]	0.24	[0.47]	0.76	[0.03]
JNJ	0.45	[0.06]	0.55	[0.02]	0.29	[0.20]	0.71	[0.00]	0.24	[0.35]	0.76	[0.00]
JPM	0.66	[0.08]	0.34	[0.36]	0.83	[0.01]	0.17	[0.56]	0.65	[0.09]	0.35	[0.35]
KO	0.18	[0.77]	0.82	[0.17]	0.26	[0.24]	0.74	[0.00]	0.20	[0.37]	0.80	[0.00]
MO	0.44	[0.09]	0.56	[0.03]	0.27	[0.28]	0.73	[0.00]	0.24	[0.37]	0.76	[0.00]
MRK	0.44	[0.07]	0.56	[0.02]	0.02	[0.94]	0.98	[0.00]	0.04	[0.86]	0.96	[0.00]
MRVL	0.22	[0.56]	0.78	[0.03]	-	-	-	-	-	-	-	-
MS	0.63	[0.15]	0.37	[0.39]	-	-	-	-	-	-	-	-
MSFT	1.15	[0.01]	-0.15	[0.76]	-	-	-	-	-	-	-	-
NOK	-0.28	[0.75]	1.28	[0.14]	0.57	[0.06]	0.43	[0.16]	0.58	[0.19]	0.42	[0.35]
ORCL	0.93	[0.00]	0.07	[0.77]	-	-	-	-	-	-	-	-
PFE	0.13	[0.75]	0.87	[0.03]	0.31	[0.29]	0.69	[0.02]	0.35	[0.26]	0.65	[0.03]
PG	0.69	[0.02]	0.31	[0.27]	0.37	[0.07]	0.63	[0.00]	0.25	[0.27]	0.75	[0.00]
VZ	0.40	[0.08]	0.60	[0.01]	0.30	[0.18]	0.70	[0.00]	0.24	[0.34]	0.76	[0.00]
WFC	-	-	-	-	-	-	-	-	0.47	[0.06]	0.53	[0.04]
WMT	0.29	[0.39]	0.71	[0.04]	0.49	[0.08]	0.51	[0.07]	0.63	[0.03]	0.37	[0.20]
XOM	-1.64	[0.06]	2.64	[0.48]	0.03	[0.96]	0.97	[0.14]	1.77	[0.06]	-0.77	[0.43]
YHOO	-0.01	[0.98]	1.01	[0.01]	-	-	-	-	-	-	-	-

Table V Price Discovery Measures

We report price discovery results for 30 assets considering Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. The VEC parameters are computed using the OLS estimator where rk $(\gamma\beta') = 1$, $\beta = (1, -1)'$, and the lag length is chosen as the minimum value which makes the LM test for serial correlation on the residuals at lag 15 to be insignificant at the 5% significance level. For each set of results, the first column shows the estimate of the first element of the orthogonal projection of γ, $\widehat{\gamma}_{\perp,1}$, while the second column (value inside the square brackets) displays the p-value associated with null hypothesis $\widehat{\gamma}_{\perp,1} = 0$. The third column presents $\widehat{\gamma}_{\perp,2}$, while the fourth column (value inside the square brackets) reports the p-value associated with the null hypothesis $\widehat{\alpha}_{\perp,2} = 0$. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca				Nasdaq-Nyse				Arca-Nyse			
	$\widehat{\gamma}_{\perp,1}$		$\widehat{\gamma}_{\perp,2}$		$\widehat{\gamma}_{\perp,1}$		$\widehat{\gamma}_{\perp,2}$		$\widehat{\gamma}_{\perp,1}$		$\widehat{\gamma}_{\perp,2}$	
AA	0.57	[0.00]	0.43	[0.00]	0.60	[0.00]	0.40	[0.00]	0.54	[0.00]	0.46	[0.00]
AAPL	0.51	[0.00]	0.49	[0.00]	-	-	-	-	-	-	-	-
BAC	0.63	[0.00]	0.37	[0.00]	0.67	[0.00]	0.33	[0.00]	0.62	[0.00]	0.38	[0.00]
BRKB	0.71	[0.00]	0.29	[0.00]	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	0.53	[0.00]	0.47	[0.00]
CSCO	0.67	[0.00]	0.33	[0.00]	-	-	-	-	-	-	-	-
F	0.55	[0.00]	0.45	[0.00]	0.69	[0.00]	0.31	[0.01]	0.65	[0.00]	0.35	[0.00]
GE	0.70	[0.00]	0.30	[0.00]	0.71	[0.00]	0.29	[0.00]	0.60	[0.00]	0.40	[0.00]
GM	0.50	[0.00]	0.50	[0.00]	0.53	[0.00]	0.47	[0.00]	0.53	[0.00]	0.47	[0.00]
GOOG	0.68	[0.00]	0.32	[0.00]	-	-	-	-	-	-	-	-
HPQ	0.53	[0.00]	0.47	[0.00]	0.60	[0.00]	0.40	[0.00]	0.59	[0.00]	0.41	[0.00]
IBM	0.37	[0.02]	0.63	[0.00]	0.37	[0.04]	0.63	[0.01]	0.49	[0.00]	0.51	[0.00]
JCP	0.45	[0.00]	0.55	[0.00]	0.44	[0.00]	0.56	[0.00]	0.48	[0.00]	0.52	[0.00]
JNJ	0.69	[0.00]	0.31	[0.00]	0.58	[0.00]	0.42	[0.00]	0.40	[0.00]	0.60	[0.00]
JPM	0.59	[0.00]	0.41	[0.00]	0.75	[0.00]	0.25	[0.00]	0.73	[0.00]	0.27	[0.00]
KO	0.52	[0.00]	0.48	[0.00]	0.59	[0.00]	0.41	[0.00]	0.57	[0.00]	0.43	[0.00]
MO	0.50	[0.00]	0.50	[0.00]	0.53	[0.00]	0.47	[0.00]	0.53	[0.00]	0.47	[0.00]
MRK	0.56	[0.00]	0.44	[0.00]	0.63	[0.00]	0.37	[0.00]	0.59	[0.00]	0.41	[0.00]
MRVL	0.60	[0.00]	0.40	[0.00]	-	-	-	-	-	-	-	-
MS	0.51	[0.00]	0.49	[0.00]	-	-	-	-	-	-	-	-
MSFT	0.56	[0.00]	0.44	[0.00]	-	-	-	-	-	-	-	-
NOK	0.42	[0.00]	0.58	[0.00]	0.41	[0.00]	0.59	[0.00]	0.39	[0.00]	0.61	[0.00]
ORCL	0.57	[0.00]	0.43	[0.00]	-	-	-	-	-	-	-	-
PFE	0.51	[0.00]	0.49	[0.00]	0.42	[0.00]	0.58	[0.00]	0.38	[0.00]	0.62	[0.00]
PG	0.22	[0.13]	0.78	[0.04]	0.47	[0.00]	0.53	[0.02]	0.46	[0.00]	0.54	[0.00]
VZ	0.70	[0.00]	0.30	[0.00]	0.63	[0.00]	0.37	[0.00]	0.40	[0.00]	0.60	[0.00]
WFC	-	-	-	-	-	-	-	-	0.44	[0.00]	0.56	[0.00]
WMT	0.64	[0.00]	0.36	[0.00]	0.64	[0.00]	0.36	[0.00]	0.52	[0.00]	0.48	[0.00]
XOM	0.45	[0.05]	0.55	[0.03]	0.67	[0.02]	0.33	[0.01]	0.73	[0.01]	0.27	[0.01]
YHOO	0.60	[0.00]	0.40	[0.00]	-	-	-	-	-	-	-	-

Appendices

A. Simulation

To illustrate our theoretical setting and to verify whether the price and volatility discovery measures correctly identify market leadership in both price and volatility discovery processes, we revisit the price process in (2) and (3). Specifically, we simulate (2) and (3) for 1,700 days. We choose 1,700 days (approximately 7 years of data) because it is similar to the sample size for the heavily traded stocks we consider in Section V. Recall that our simple price and volatility discovery model implies that market one is the sole contributor to the price discovery process, whereas market two contributes completely to the volatility discovery mechanism. Our first goal is to assess the performance of the usual price discovery measure in identifying market one as the leading market in the price discovery process. We compute the mean and standard deviation of daily $\widehat{\gamma}$, $\widehat{\gamma}_{\perp}$, and IS estimates across 1,000 replications (see Appendix C.1 for details).

[Place Table A.1 about here]

Results in the upper panel of Table A.1 confirm that the speed of adjustment parameter associated with the first market, γ_1, is zero, which implies that the first market does not adjust to changes in the second market. The lower panel in Table A.1 reports the mean and standard deviations of daily estimates of $\gamma_{\perp}$ and the Hasbrouck's 1995 IS measure. Because $\gamma_{\perp} = (1, 0)'$ in our example, the source of contamination $X_{s,t}$ in (10) cancels out in that estimates of the IS measure are virtually equal to $\gamma_{\perp}$, as should be in the case of constant volatilities. While both measures confirm that the price discovery mechanism occurs exclusively in the first market, they are unable to recognize that market two is the sole contributor to the efficient stochastic volatility process.

To assess the validity of the volatility discovery framework, we use the discrete prices simulated from (2) and (3) and compute daily consistent estimates of the integrated variance with the realized variance estimator.[32] Each replication yields 1,700 daily realized variances. We then fit a FCVAR model as in (5) and record estimates of d, α, β and $\alpha_\perp$.

[Place Table A.2 about here]

Table A.2 presents the mean and the standard deviations of these estimates across all replications. The theoretical framework developed in this section successfully identifies the volatility discovery channel embedded in Example 1, and we highlight several results. First, we confirm that the FCVAR model approximates well the dynamics of the RMs and thus allows us to assess the validity of the volatility discovery framework. Additionally, we find that the RMs are characterized by nonstationary and mean-reverting long memory, which is in line with the usual hyperbolic decay of the empirical autocorrelation function associated with RMs. Second, we document estimates of α_1 that are negative and significantly different from zero (the RM in market one adjusts to changes in the RM of market two), whereas estimates of α_2 are not statistically different from zero, implying that the RM in the first market does not Granger-cause the RM in market two. The estimates are also fairly stable across all replications and the cointegrating vector corroborates the equilibrium relationship implied by the error correction model used to generate the volatility factors because β_2 is not statistically different from -1. Therefore, we conclude that RMs are cointegrated with cointegrating vector $\beta = (1,\ -1)'$. When considering the accuracy of the volatility discovery measures, the results in the lower panel in Table A.2 confirm that the volatility discovery process occurs exclusively in market two, as the 5%, mean and 95% percentiles of $\alpha_{\perp,2}$ computed across all replications are 0.8626, 0.9941 and 1.1582, respectively. Therefore, the use of the FCVAR model to approximate the dynamics of the RMs and the use of the orthogonal projection of the speed of adjustment parameters to quantify how the different markets impound information into the fractional common stochastic trend (efficient volatility) enable us to draw inferences about the volatility discovery mechanism.

[32]Notably, the stationary multivariate Ornstein-Uhlenbeck (OU) model used to simulate the log-stochastic volatilities is a special case of the single-factor log-linear stochastic volatility model commonly used in the RMs literature (see Huang and Tauchen (2005), Barndorff-Nielsen et al. (2008), among others).

Table A.1 Simulation Results: Example - Price Discovery

We report the mean and standard deviations (in brackets) of the estimates of the VEC parameters (γ and β) and the price discovery measures ($\gamma_\perp$) and IS computed across 1,000 replications from models simulated using the data generation process in Example 1 ((2) and (3)). For each replication, intraday prices and stochastic volatilities are simulated via an Euler scheme over the unit interval $t \in [0,1]$ with steps of size 1/2,340 which corresponds to 10 second frequency. In turn, the interval $t \in [0,1]$ contains 6.5 hours. We simulate 1,700 days (about 7 years). Appendix C.1 presents a detailed explanation of the simulation design. Because (2) is a continuous time VEC model (cointegrated multivariate OU process), the Euler discretization yields true parameters that are comparable with the estimates of the discrete VEC model. It follows that $\gamma = (0, 0.10)'$, $\beta = (1, -1)'$ and $\gamma_\perp = (1, 0)'$.

VEC approximation of high-frequency log-prices

$$\Delta\left(\begin{bmatrix} p_{1,t_i} \\ p_{2,t_i} \end{bmatrix}\right) = \underbrace{\begin{bmatrix} \underset{(1.57\times 10^4)}{-0.00} \\ \underset{(1.54\times 10^4)}{0.10} \end{bmatrix}}_{\widehat{\gamma}} \underbrace{\begin{bmatrix} 1 & -1 \end{bmatrix}}_{\beta'} \begin{bmatrix} p_{1,t_{i-1}} \\ p_{2,t_{i-1}} \end{bmatrix} + \begin{bmatrix} e_{1,t_i} \\ e_{2,t_i} \end{bmatrix}$$

Price discovery measures

$$\widehat{\gamma}_\perp = \left(\underset{(16\times 10^4)}{1.00}, \quad \underset{(16\times 10^4)}{0.00}\right)'$$

$$IS = \left(\underset{(1.82\times 10^4)}{1.00}, \quad \underset{(1.82\times 10^4)}{0.00}\right)'$$

Table A.2 Simulation Results: Example - Volatility Discovery

We report the mean and standard deviations (in brackets) of the estimates of the FCVAR parameters (d, b, α and β) and the volatility discovery measures ($\alpha_\perp$) computed across 1,000 replications from models simulated using the data generation process in Example 1 ((2) and (3)). For each replication, intraday prices and stochastic volatilities are simulated via an Euler scheme over the unit interval $t \in [0,1]$ with steps of size 1/2,340 which corresponds to 10 second frequency. In turn, the interval $t \in [0,1]$ contains 6.5 hours. We simulate 1,700 days (about 7 years) and the daily realized measures are computed using the realized variance estimator defined as the sum of the squared intraday returns. Appendix C.1 presents a detailed explanation of the simulation design.

FCVAR approximation of daily realized measures

$$\Delta^{\underset{(0.05)}{0.88}}\left(\begin{bmatrix} R_{1,t_i} \\ R_{2,t_i} \end{bmatrix}\right) = \underbrace{\begin{bmatrix} \underset{(0.01)}{-0.11} \\ \underset{(0.01)}{0.01} \end{bmatrix}}_{\widehat{\alpha}} \underbrace{\begin{bmatrix} 1 & \underset{(0.05)}{-1.04} \end{bmatrix}}_{\widehat{\beta}'} \begin{bmatrix} R_{1,t_{i-1}} \\ R_{2,t_{i-1}} \end{bmatrix} + \sum_{j=1}^{\kappa} \widehat{\Gamma}_j \Delta^{\underset{(0.05)}{0.88}} L^j_{\underset{(0.05)}{0.88}} R_t + \begin{bmatrix} \varepsilon_{1,t_i} \\ \varepsilon_{2,t_i} \end{bmatrix}$$

Volatility discovery measures

$$\widehat{\alpha}_\perp = \left(\underset{(0.09)}{0.01}, \ \underset{(0.09)}{0.99} \right)'$$

B. Auxiliary Tables

Table B.1 Model Specification: Lag Length Selection

We report results for 30 stocks and the three market combinations discussed in Section V, (Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse). For each set of results, κ accounts for the selected lag length in the FCVAR model; and LM_1 and LM_2 bring the p-values associated with the heteroskedastic robust LM test for serial autocorrelation at lag 10 for the residuals in the first and second equations in the FCVAR model, respectively. The null hypothesis of the LM test is that the process is serially uncorrelated. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca			Nasdaq-Nyse			Arca-Nyse		
	κ	LM_1	LM_2	κ	LM_1	LM_2	κ	LM_1	LM_2
AA	0	0.87	0.83	0	0.90	0.97	0	0.76	0.94
AAPL	0	0.18	0.19	-	-	-	-	-	-
BAC	0	0.48	0.48	0	0.48	0.56	0	0.63	0.69
BRKB	0	0.29	0.23	-	-	-	-	-	-
C	-	-	-	-	-	-	0	0.21	0.30
CSCO	0	0.33	0.18	-	-	-	-	-	-
F	0	0.29	0.15	0	0.24	0.30	0	0.23	0.45
GE	0	0.23	0.19	0	0.24	0.29	0	0.18	0.25
GM	0	0.19	0.12	0	0.20	0.10	0	0.24	0.18
GOOG	0	0.01	0.01	-	-	-	-	-	-
HPQ	0	0.16	0.10	0	0.15	0.19	0	0.10	0.23
IBM	1	0.15	0.12	1	0.13	0.13	2	0.20	0.20
JCP	0	0.43	0.27	0	0.44	0.39	0	0.22	0.36
JNJ	0	0.71	0.56	0	0.77	0.68	0	0.75	0.72
JPM	0	0.91	0.82	0	0.91	0.84	0	0.80	0.81
KO	3	0.31	0.14	0	0.14	0.24	0	0.02	0.15
MO	0	0.59	0.30	0	0.44	0.42	0	0.25	0.34
MRK	0	0.84	0.57	0	0.77	0.85	0	0.57	0.87
MRVL	0	0.30	0.23	-	-	-	-	-	-
MS	0	0.41	0.31	-	-	-	-	-	-
MSFT	0	0.44	0.38	-	-	-	-	-	-
NOK	2	0.25	0.18	0	0.06	0.03	0	0.03	0.03
ORCL	0	0.33	0.32	-	-	-	-	-	-
PFE	0	0.17	0.14	0	0.28	0.26	0	0.35	0.25
PG	0	0.21	0.15	0	0.22	0.23	0	0.17	0.25
VZ	0	0.37	0.35	0	0.41	0.42	0	0.39	0.43
WFC	0	0.43	0.40	-	-	-	-	-	-
WMT	0	0.30	0.16	0	0.27	0.28	0	0.18	0.29
XOM	3	0.16	0.16	3	0.12	0.16	3	0.09	0.10
YHOO	0	0.73	0.64	-	-	-	-	-	-

Table B.2 Model Specification: Residuals Analysis

We report the residuals analysis for 30 stocks and the three market combinations discussed in Section V, (Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse). For each set of results, LM_1 and LM_2 bring the p-values associated with the heteroskedastic robust LM test for serial autocorrelation at lag 10 for the residuals in the first and second equations in the FCVAR model, respectively. The null hypothesis of the LM test is that the process is serially uncorrelated. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca		Nasdaq-Nyse		Arca-Nyse	
	LM_1	LM_2	LM_1	LM_2	LM_1	LM_2
AA	0.80	0.73	0.84	0.95	0.65	0.91
AAPL	0.03	0.01	-	-	-	-
BAC	0.47	0.47	0.49	0.56	0.63	0.71
BRKB	0.26	0.25	-	-	-	-
C	-	-	-	-	0.22	0.30
CSCO	0.28	0.14	-	-	-	-
F	0.26	0.13	0.20	0.27	0.20	0.42
GE	0.19	0.15	0.20	0.24	0.13	0.19
GM	0.17	0.11	0.18	0.09	0.21	0.16
GOOG	0.01	0.01	-	-	-	-
HPQ	0.16	0.10	0.15	0.19	0.10	0.23
IBM	0.15	0.13	0.13	0.12	0.21	0.20
JCP	0.33	0.20	0.39	0.30	0.20	0.28
JNJ	0.64	0.47	0.70	0.59	0.62	0.61
JPM	0.90	0.81	0.90	0.84	0.78	0.81
KO	0.33	0.16	0.10	0.18	0.01	0.11
MO	0.48	0.19	0.31	0.33	0.12	0.27
MRK	0.51	0.25	0.49	0.61	0.24	0.59
MRVL	0.11	0.07	-	-	-	-
MS	0.39	0.30	-	-	-	-
MSFT	0.46	0.46	-	-	-	-
NOK	0.22	0.15	0.04	0.02	0.02	0.02
ORCL	0.24	0.22	-	-	-	-
PFE	0.12	0.09	0.13	0.12	0.12	0.14
PG	0.15	0.10	0.16	0.18	0.11	0.19
VZ	0.28	0.25	0.30	0.33	0.27	0.36
WFC	-	-	-	-	0.51	0.42
WMT	0.11	0.05	0.10	0.12	0.06	0.13
XOM	0.12	0.13	0.10	0.12	0.07	0.07
YHOO	0.58	0.47	-	-	-	-

Table B.3 Speed of Adjustment

We report the speed of adjustments coefficients for 30 assets considering the Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. The FCVAR parameters are computed using the MLE estimator of Johansen and Nielsen (2012) where $\mathrm{rk}(\alpha\beta') = 1$, κ is chosen according to Table B.1, and $d = b$. For each set of of market combinations, we report the estimates of the adjustment parameters $\alpha' = (\alpha_1, \alpha_2)$ and their standard errors. More specifically, for each set of results, the first column presents the point estimate of α_1, while the second column displays the standard errors in parentheses. The third column displays the point estimate of α_2 while the fourth column brings its respective standard errors in parentheses. The symbols *, ** and *** denote rejection at the 10%, 5% and 1% levels of the null hypothesis of $\widehat{\alpha}_i' = 0$, for $i = 1, 2$. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca				Nasdaq-Nyse				Arca-Nyse			
	$\widehat{\alpha}_1$		$\widehat{\alpha}_2$		$\widehat{\alpha}_1$		$\widehat{\alpha}_2$		$\widehat{\alpha}_1$		$\widehat{\alpha}_2$	
AA	-0.42	(0.36)	0.55	(0.36)	-0.46^{*}	(0.27)	0.38	(0.26)	-0.43^{*}	(0.25)	0.47^{*}	(0.25)
AAPL	-0.13	(0.47)	0.75	(0.47)	-	-	-	-	-	-	-	
BAC	-0.26	(0.46)	0.73	(0.46)	-0.10	(0.31)	0.76^{**}	(0.31)	-0.23	(0.33)	0.63^{*}	(0.33)
BRKB	-0.57^{**}	(0.24)	0.29	(0.25)	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	0.19	(0.38)	1.16^{**}	(0.38)
CSCO	-0.32	(0.26)	0.68^{**}	(0.26)	-	-	-	-	-	-	-	-
F	-1.07^{*}	(0.60)	-0.10	(0.60)	0.08	(0.40)	1.02^{**}	(0.39)	0.40	(0.40)	1.34^{**}	(0.40)
GE	-0.92^{*}	(0.49)	0.00	(0.49)	-0.10	(0.30)	0.66^{**}	(0.30)	0.18	(0.31)	0.83^{**}	(0.31)
GM	-1.12^{**}	(0.46)	-0.11	(0.46)	-0.35	(0.27)	0.62^{**}	(0.26)	-0.07	(0.28)	0.75^{**}	(0.28)
GOOG	-0.57^{*}	(0.30)	0.21	(0.30)	-	-	-	-	-	-	-	-
HPQ	-1.03^{**}	(0.38)	-0.08	(0.37)	-0.77^{**}	(0.25)	0.05	(0.26)	-0.51^{*}	(0.27)	0.29	(0.27)
IBM	-0.53	(0.34)	0.40	(0.34)	-0.39	(0.33)	0.32	(0.33)	-0.17	(0.37)	0.55	(0.37)
JCP	-0.58^{**}	(0.28)	0.41	(0.28)	-0.73^{**}	(0.26)	0.12	(0.26)	-0.58^{**}	(0.26)	0.19	(0.26)
JNJ	-0.54^{**}	(0.24)	0.45^{*}	(0.24)	-0.54^{**}	(0.17)	0.22	(0.17)	-0.53^{**}	(0.18)	0.16	(0.18)
JPM	-0.34	(0.36)	0.65^{*}	(0.37)	-0.15	(0.26)	0.74^{**}	(0.26)	-0.26	(0.28)	0.48^{*}	(0.28)
KO	-0.65	(0.49)	0.14	(0.49)	-0.61^{**}	(0.18)	0.21	(0.18)	-0.58^{**}	(0.16)	0.14	(0.16)
MO	-0.54^{**}	(0.24)	0.42^{*}	(0.24)	-0.60^{**}	(0.21)	0.22	(0.21)	-0.60^{**}	(0.20)	0.18	(0.20)
MRK	-0.56^{**}	(0.25)	0.45^{*}	(0.25)	-0.91^{**}	(0.20)	0.01	(0.19)	-0.84^{**}	(0.19)	0.03	(0.19)
MRVL	-0.78^{**}	(0.36)	0.21	(0.36)	-	-	-	-	-	-	-	-
MS	-0.35	(0.41)	0.59	(0.41)	-	-	-	-	-	-	-	-
MSFT	0.14	(0.46)	1.12^{**}	(0.46)	-	-	-	-	-	-	-	-
NOK	-1.18	(0.79)	-0.25	(0.80)	-0.42	(0.30)	0.57^{*}	(0.31)	-0.37	(0.40)	0.52	(0.40)
ORCL	-0.07	(0.24)	0.95^{**}	(0.24)	-	-	-	-	-	-	-	-
PFE	-0.87^{**}	(0.40)	0.13	(0.40)	-0.57^{**}	(0.23)	0.25	(0.23)	-0.51^{**}	(0.24)	0.27	(0.24)
PG	-0.29	(0.26)	0.63^{**}	(0.26)	-0.59^{**}	(0.19)	0.35^{*}	(0.19)	-0.63^{**}	(0.19)	0.21	(0.19)
VZ	-0.61^{**}	(0.23)	0.40^{*}	(0.23)	-0.63^{**}	(0.20)	0.27	(0.20)	-0.64^{**}	(0.21)	0.20	(0.21)
WFC	-	-	-	-	-	-	-	-	-0.44^{**}	(0.21)	0.39^{*}	(0.21)
WMT	-0.68^{**}	(0.33)	0.29	(0.33)	-0.49^{*}	(0.27)	0.47^{*}	(0.27)	-0.33	(0.26)	0.57^{**}	(0.26)
XOM	-2.45^{**}	(0.86)	-1.52^{*}	(0.86)	-0.77	(0.52)	0.03	(0.52)	0.49	(0.63)	1.12^{*}	(0.61)
YHOO	-1.01^{**}	(0.41)	-0.01	(0.41)	-	-	-	-	-	-	-	-

Table B.4 Robustness: Volatility Discovery Measures with d and b as Free Parameters

We report results for 30 assets considering Nasdaq and Arca, Nasdaq and Nyse, and Arca and Nyse market combinations. The FCVAR parameters are computed using the MLE estimator of Johansen and Nielsen (2012) where $\text{rk}(\alpha\beta') = 1$ and κ is chosen according to Table B.1. For each set of results, the first two columns show the elements of the orthogonal projection of α, $\alpha'_\perp = (\alpha_{\perp,1}, \alpha_{\perp,2})$, obtained when the restriction $d = b$ is imposed on the MLE estimator. The third and fourth columns $(\widetilde{\alpha}_\perp)$ display the elements of the orthogonal projection of $\widetilde{\alpha}'$, $\widetilde{\alpha}'_\perp = (\widetilde{\alpha}_{\perp,1}, \widetilde{\alpha}_{\perp,2})$, obtained when the d and b are assumed to be free parameters in the MLE estimation. "-" implies that the stock is not traded in at least one of the two trading venues during the entire sample.

	Nasdaq-Arca				Nasdaq-Nyse				Arca-Nyse			
	$\alpha'_\perp$		$\widetilde{\alpha}'_\perp$		$\alpha'_\perp$		$\widetilde{\alpha}'_\perp$		$\alpha'_\perp$		$\widetilde{\alpha}'_\perp$	
AA	0.57	0.43	0.56	0.44	0.45	0.55	0.16	0.84	0.52	0.48	0.30	0.70
AAPL	0.85	0.15	0.85	0.15	-	-	-	-	-	-	-	-
BAC	0.74	0.26	0.75	0.25	0.88	0.12	0.76	0.24	0.73	0.27	0.48	0.52
BRKB	0.34	0.66	0.33	0.67	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	1.20	-0.20	1.07	-0.07
CSCO	0.68	0.32	0.68	0.32	-	-	-	-				
F	-0.10	1.10	-0.11	1.11	1.08	-0.08	1.02	-0.02	1.43	-0.43	1.32	-0.32
GE	0.00	1.00	0.10	0.90	0.87	0.13	0.55	0.45	1.27	-0.27	0.71	0.29
GM	-0.11	1.11	-0.11	1.11	0.64	0.36	0.61	0.39	0.91	0.09	0.81	0.19
GOOG	0.27	0.73	0.34	0.66	-	-	-	-	-	-	-	-
HPQ	-0.09	1.09	-0.10	1.10	0.06	0.94	0.08	0.92	0.36	0.64	0.34	0.66
IBM	0.43	0.57	0.43	0.57	0.45	0.55	0.18	0.82	0.76	0.24	-2.86	3.86
JCP	0.41	0.59	0.44	0.56	0.14	0.86	0.06	0.94	0.24	0.76	0.08	0.92
JNJ	0.45	0.55	0.48	0.52	0.29	0.71	0.20	0.80	0.24	0.76	0.14	0.86
JPM	0.66	0.34	0.65	0.35	0.83	0.17	0.68	0.32	0.65	0.35	0.40	0.60
KO	0.49	0.51	0.49	0.51	0.26	0.74	0.25	0.75	0.20	0.80	0.18	0.82
MO	0.44	0.56	0.46	0.54	0.27	0.73	0.16	0.84	0.24	0.76	0.05	0.95
MRK	0.44	0.56	0.43	0.57	0.02	0.98	0.01	0.99	0.04	0.96	0.01	0.99
MRVL	0.22	0.78	0.24	0.76	-	-	-	-	-	-	-	-
MS	0.63	0.37	0.62	0.38	-	-	-	-	-	-	-	-
MSFT	1.15	-0.15	1.14	-0.14	-	-	-	-	-	-	-	-
NOK	0.53	0.47	0.53	0.47	0.57	0.43	0.58	0.42	0.58	0.42	0.60	0.40
ORCL	0.93	0.07	0.93	0.07	-	-	-	-	-	-	-	-
PFE	0.13	0.87	0.01	0.99	0.31	0.69	0.28	0.72	0.35	0.65	0.27	0.73
PG	0.69	0.31	0.77	0.23	0.37	0.63	0.33	0.67	0.25	0.75	0.15	0.85
VZ	0.40	0.60	0.40	0.60	0.31	0.69	0.25	0.75	0.24	0.76	0.24	0.76
WFC	0.47	0.53	0.24	0.76	-	-	-	-	-	-	-	-
WMT	0.30	0.70	0.34	0.66	0.49	0.51	0.42	0.58	0.63	0.37	0.54	0.46
XOM	-1.64	2.64	-1.64	2.64	0.03	0.97	-0.36	1.36	1.77	-0.77	0.99	0.01
YHOO	-0.01	1.01	0.02	0.98	-	-	-	-	-	-	-	-

Table B.5 Volatility Discovery Measures from Nasdaq-Arca-Nyse

We report the volatility discovery measures for 18 assets that are simultaneously traded at the Nasdaq, Arca and Nyse markets. The volatility discovery measures are obtained by fitting an FCVAR model to the three markets with $\text{rk}(\alpha\beta') = 2$. The free parameters are estimated using the MLE estimator of Johansen and Nielsen (2012). The first column displays the estimates of the long memory parameter, d, while the second column presents its standard errors in parentheses. The third, fourth and fifth columns display the volatility discovery measures for Nasdaq, Arca and Nyse, respectively. More precisely, these measures are the estimates of the elements of the orthogonal projection of α, $\widehat{\alpha}_\perp = (\widehat{\alpha}_{\perp,1}, \widehat{\alpha}_{\perp,2}, \widehat{\alpha}_{\perp,3})'$.

	Nasdaq-Arca-Nyse				
	$\widehat{d}$		$\widehat{\alpha}_{\perp,1}$	$\widehat{\alpha}_{\perp,2}$	$\widehat{\alpha}_{\perp,3}$
AA	0.52	(0.02)	0.25	0.29	0.47
BAC	0.59	(0.02)	0.72	0.24	0.04
F	0.56	(0.02)	0.09	1.34	-0.43
GE	0.54	(0.02)	0.07	1.29	-0.36
GM	0.54	(0.02)	-0.12	1.08	0.05
HPQ	0.50	(0.02)	-0.31	0.51	0.80
IBM	0.53	(0.02)	0.20	0.28	0.52
JCP	0.48	(0.02)	0.05	0.21	0.74
JNJ	0.52	(0.02)	0.20	0.15	0.65
JPM	0.60	(0.02)	0.66	0.32	0.03
KO	0.50	(0.02)	0.10	0.20	0.70
MO	0.49	(0.02)	0.13	0.20	0.67
MRK	0.50	(0.02)	-0.04	0.08	0.96
NOK	0.71	(0.03)	-0.39	1.08	0.31
PFE	0.51	(0.02)	-0.06	0.51	0.56
PG	0.51	(0.02)	0.37	0.00	0.63
VZ	0.51	(0.02)	0.17	0.24	0.59
WMT	0.49	(0.02)	0.14	0.57	0.28
XOM	0.57	(0.02)	0.00	-0.25	1.25

Table B.6 Price (Volatility) Discovery Measures and Quoting Intensity

We report results from probit regressions relating price and volatility discovery measures, Panels A and B, respectively, to quoting intensity and control variables. We consider two market combinations: Nasdaq-Arca and Nyse-Nasdaq. We omit the third market combination, as transitivity would violate the i.i.d. assumption required for the estimation of probit models. The dependent variable in Panel A is an indicator variable that takes the value of 1 if the price discovery measure associated with the first market is greater than 0.5, and 0 otherwise. There are three independent variables: 'Quote Intensity ratio' is the ratio of the daily average of quotes in the first market over the sum of daily averages from both markets; 'Listed Stock' is a dummy variable that takes the value of 1 if the stock is listed in the first market, and 0 otherwise; and 'Volatility Discovery' is the element of $\alpha_\perp$ associated with the the first market. The dependent variable in Panel B is an indicator variable that takes the value of 1 if the volatility discovery measure associated with the first market is greater than 0.5, and 0 otherwise. There are three independent variables: 'Quote Intensity ratio', 'Listed Stock', and 'Price Discovery', that is the element of $\gamma_\perp$ associated with the the first market. The regression in column 1 uses only the first independent variable, 'Quote Intensity ratio', the regression in the second column uses 'Quote Intensity ratio' and 'Listed Stock', and the regression in the third column uses all the independent variables. The robust standard errors are in parentheses below the coefficient estimates and ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively. Wald-test gives the p-value of the Wald statistic of the joint significance of the parameters associated with the independent variables; Log Likelihood is the value of the log likelihood function attained at the parameter estimates; R^2 is the pseudo R^2 goodness-of-fit-statistic in the context of probit models; and No. of obs. is the total number of observations used in the regression analysis.

Panel A. Price Discovery

Explanatory Variables	1	2	3
Quote Intensity ratio	6.48***	5.88**	6.20**
	(2.23)	(2.82)	(2.79)
Listed Stock		−0.20	−0.42
		(0.54)	(0.57)
Volatility Discovery: $\alpha_{\perp,1}$			0.75*
			(0.44)
Wald-test	0.00	0.01	0.01
Log Likelihood	-26.39	-26.30	-25.17
pseudo-R^2	0.18	0.18	0.21
No. of obs.	47	47	47

Panel B. Volatility Discovery

Explanatory Variables	1	2	3
Quote Intensity ratio	−3.52*	−1.01	−1.14
	(1.93)	(2.32)	(2.55)
Listed Stock		0.96**	0.96**
		(0.47)	(0.47)
Price Discovery: $\gamma_{\perp,1}$			0.26
			(1.97)
Wald-test	0.07	0.02	0.05
Log Likelihood	-30.60	-28.37	-28.36
pseudo-R^2	0.06	0.13	0.13
No. of obs.	47	47	47

C. Technical Appendix:

This technical appendix provides a full description of the simulation study conducted in Section A, covers the econometric theory supporting the FCVAR model adopted in Sections III and V (Appendix C.2), and details the $R_{m,t}$ decomposition studied in Section VI.A (Appendix C.3).

1. *Simulation Design*

We simulate prices from the joint continuous time model for price and volatility discovery in (2) and (3) for $T = 1,700$ trading days (approximately 7 years). Specifically, each day is simulated over the unit interval $t \in [0, 1]$. We normalize 10 seconds to be 1/2,340 so that the interval $[0, 1]$ contains 6.5 hours. In turn, we discretize $[0, 1]$ into $N = 2,340$ intervals with size $\delta = 1/2,340$. The discretized prices are thus comparable to observed prices sampled at 10-second frequencies. While the bivariate price process is simulated via the Euler scheme, $V(t)$ is obtained using the exact discretization of the OU process. It follows that discrete prices and stochastic volatilities are obtained using the following iterative scheme:

$$p_{t_{i+1}} = p_{t_i} + \begin{bmatrix} 0 & 0 \\ -\pi & \pi \end{bmatrix} \left(\mu_p - p_{t_i}\right)\delta + \begin{bmatrix} \sigma_{1,t_i} & 0 \\ 0 & \sigma_{2,t_i} \end{bmatrix} \sqrt{\delta}\, \epsilon^{W}_{t_{i+1}}, \tag{C.1.1}$$

$$\sigma_{s,t_{i+1}} = \exp\left[\varphi_0 + \varphi_1 V_{s,t_{i+1}}\right], \qquad s = 1, 2 \tag{C.1.2}$$

$$V_{t_{i+1}} = \mu^*_V + \text{expm}\left(\Theta\delta\right) V_{t_i} + C_\delta \epsilon^{B}_{t_{i+1}}, \tag{C.1.3}$$

where the matrix exponential is defined as $\text{expm}(A) = \sum_{j=0}^{\infty} A^j \left(\frac{1}{j!}\right)$, $\Theta = \begin{bmatrix} -\theta_1 & \theta_1 \\ 0 & -\theta_2 \end{bmatrix}$,
$\mu_V^* = (I_2 - \text{expm}(\Theta\delta))\,\mu_V$,

$$\begin{pmatrix} \epsilon_{t_i}^W \\ \epsilon_{t_{i+1}}^B \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \bullet & \bullet & \bullet \\ 0 & 1 & \bullet & \bullet \\ \rho & \rho & 1 & \bullet \\ \rho & \rho & 0 & 1 \end{pmatrix}\right), \tag{C.1.4}$$

and $\Lambda_\delta = C_\delta C_\delta'$ with $\Lambda_\delta = \int_0^\delta \text{expm}(u\Theta)\,\Lambda\,\text{expm}(u\Theta')\,du$.

Finally, we choose the parameters in (2) and (3) to be in accordance with the RM literature (Huang and Tauchen (2005), Barndorff-Nielsen et al. (2008), Barndorff-Nielsen et al. (2011), among others), as the stationary error correction model in (3) is a special case of the single-factor log-linear stochastic volatility model. Furthermore, the parameter restrictions in the error correction model are imposed such that the equilibrium relationship between the stochastic volatilities in the two markets is given by $\mathbb{E}(V_1(t)) = \mathbb{E}(V_2(t))$.

Specifically, we set the free parameters in (2) and (3) as follows:

$$\begin{pmatrix} \mu_p \\ \pi \\ \varphi_0 \\ \varphi_1 \\ \mu_V \\ \theta_1 \\ \theta_2 \\ \Lambda_{1,1} \\ \Lambda_{1,2} \\ \Lambda_{2,2} \\ \rho \end{pmatrix} = \begin{pmatrix} 0.003 \\ 223 \\ 0 \\ 0.125 \\ 0 \\ 0.150 \\ 0.025 \\ 1 \\ 0 \\ 1 \\ -0.30 \end{pmatrix}, \tag{C.1.5}$$

where $\Lambda_{1,1}$, $\Lambda_{1,2}$ and $\Lambda_{2,2}$ are the elements of Λ. Notably, values assigned to μ_p, μ_V, φ_1, θ_2 Λ and ρ follow from Barndorff-Nielsen et al. (2008) and Barndorff-Nielsen et al. (2011), while $\varphi_0 = 0$ follows from Huang and Tauchen (2005). We choose θ_1 so that the volatility process in the first market (satellite market) is less persistent than the volatility process associated with the leading market. It follows that because the satellite market does not contribute to the volatility discovery, a less persistent stochastic volatility process increases the speed of adjustment to the efficient stochastic volatility given by the second market. For each replication, VEC and FCVAR models are fitted to intraday prices and daily realized variances, respectively, so that price and volatility discovery measures are recorded. We report the mean and standard deviations computed across 1,000 replications.

2. *Estimation and Inference*

This section summarizes the theoretical results in Johansen and Nielsen (2012) relative to the estimation and inference of the FCVAR model. Our main focus is discussing the consistency and the asymptotic normality of the maximum likelihood estimator and the way in which these results can be used to make inferences on the parameter estimates in our baseline model

$$\Delta^d R_t = \alpha\beta'\Delta^{d-b} L_b R_t + \sum_{j=1}^{\kappa} \Gamma_j \Delta^d L_b^j R_t + \varepsilon_t, \quad t = 1, ..., T, \tag{C.2.1}$$

where ε_t in (5) is an i.i.d. process with $\mathbb{E}(\varepsilon_t) = 0$, $Var(\varepsilon_t) = \Omega$ and $\mathbb{E}|\varepsilon_t|^8 < \infty$. Johansen (2008) and Johansen and Nielsen (2015) define the properties of the solution of (C.2.1) in terms of the characteristic polynomials

$$\Pi(z) = (1-z)^d I - \alpha\beta'\left(1-(1-z)^b\right)(1-z)^{d-b} - \sum_{j=1}^{\kappa} \Gamma_j \left(1-(1-z)^b\right)^j (1-z)^d \tag{C.2.2}$$

$$\Psi(u) = \alpha\beta' u - (1-u)\sum_{j=0}^{\kappa} \psi_j (1-u) u^j, \tag{C.2.3}$$

where $u = 1-(1-z)^b$, $\sum_{j=1}^{\kappa} \psi_j = I$, $\psi_0 = I - \sum_{j=1}^{\kappa} \Gamma_j$, $\psi_\kappa = (-1)^{\kappa+1}\Gamma_\kappa$ and $\Pi(z) = (1-z)^{d-b}\Psi(u) = (1-z)^{d-b}\Psi\left(1-(1-z)^b\right)$ for $|z| \le 1$. Using (C.2.2) and (C.2.3), the FCVAR model in (C.2.1) can be written as

$$\Pi(L) R_t = \Delta^{d-b}\Psi(L_b) R_t = \varepsilon_t, \tag{C.2.4}$$

such that $\Delta^{d-b}\Psi(L_b) R_t$ satisfies a VAR in the lag operator L_b. It follows that the conditional log-likelihood function is given by

$$L_T(\lambda) = -\frac{T}{2}\left[\log\det(\Omega) + \text{tr}\left(\Omega^{-1}T^{-1}\sum_{t=1}^{T}\varepsilon_t(\lambda)\varepsilon_t(\lambda)'\right)\right], \tag{C.2.5}$$

where $\lambda = (d, b, \alpha, \beta, \Gamma_1, ..., \Gamma_\kappa, \Omega)'$ concatenates the free parameters in (C.2.1), and $\varepsilon_t(\lambda)$ is defined as in (C.2.2) conditioned to a set of initial values so that $\varepsilon_t(\lambda) = \Pi(L) R_t$.[33] As discussed in Johansen and Nielsen (2012), for a fixed $\lambda^* = (d, b)'$, the conditional MLE estimator based on (C.2.5) reduces to a reduced rank regression similar to the standard VEC case. Following this, the set of parameters $\lambda^\star = (\alpha, \beta, \Gamma_1, ..., \Gamma_\kappa, \Omega)'$ can be concentrated out of the likelihood function, and the fractional parameters in λ^* can be estimated through the numerical optimization of the profile likelihood function.

Johansen and Nielsen (2012) formulate the asymptotic theory for the conditioned MLE estimator considering two different cases: $0 < b < 1/2$ and $b > 1/2$. Both results hold under some technical conditions.[34] In particular, it is assumed that the initial values are uniformly bounded with $R_{-t} \neq 0$ for $0 \leq t < \bar{n}$ and $R_{-t} = 0$ for $t \geq \bar{n}$, where $n \geq T^\nu$ and $\nu < 1/2$. If $0 < b < 1/2$ and $\mathbb{E}\left|\varepsilon_t\right|^8 < \infty$, then the MLE estimate of λ is consistent and asymptotically normally distributed. When $b > 1/2$, which happens to be the general case found in Section V, the MLE is a consistent estimator for λ but has a nonstandard distribution. In fact, if $\mathbb{E}\left|\varepsilon_t\right|^{\bar{q}} < \infty$ with $\bar{q} > (b - 1/2)^{-1}$, the asymptotic distribution of $\left(\widehat{d}, \widehat{b}, \widehat{\alpha}, \widehat{\Gamma_1}, ..., \widehat{\Gamma}_\kappa\right)'$ is asymptotically normally distributed, while $\widehat{\beta}$ is asymptotically mixed normal.

The limiting distribution of the MLE estimator for λ determines the asymptotic distribution underlining the LR-based cointegration rank test, which is used to determine the number of cointegrating vectors in the FCVAR model. Johansen and Nielsen (2012) show that in the case where $0 < b < 1/2$, the LR cointegration rank test has the usual χ^2 distribution, whereas its distribution is not standard when $b > 1/2$. With regard to the inference on the α estimates, a standard LR-based hypothesis test can be adopted. Because the MLE estimates of α are asymptotically normally distributed for all $b > 0$, LR hypothesis tests can be conducted in the usual way. It follows that the LR test statistics will have the standard asymptotic χ^2 distribution.

[33] Johansen and Nielsen (2012) define the log-likelihood function depending on $\lambda = (d, b, \alpha, \beta, \psi_1, ..., \psi_{\kappa-1}, \Omega)'$ where $\psi_\kappa = I - \sum_{j=0}^{\kappa-1} \psi_j$. Using this definition, the residuals are defined in a similar manner as in (C.2.3) (see Johansen and Nielsen, 2012, p. 2681)

[34] See Johansen and Nielsen, 2012, pp. 2678 and 2696 for more details

3. Technical Background on the Efficient Volatility Representation

In the absence of market microstructure noise, Andersen et al. (2003a) show that

$$RV_{s,t} = \sum_{i=1}^{N} \left(p_{s,t_i} - p_{s,t_{i-1}}\right)^2, \quad s = 1, 2, ..., S \tag{C.3.1}$$

is a consistent estimator of the integrated variance of market s as $N \to \infty$. Assume that the dynamics of the S-dimensional processes p_{t_i} and RV_t are approximated by a VEC(q) and a FCVAR(κ) model, respectively. Without loss of generality, further assume that $q = 0$, $\kappa = 0$, and $\beta = (I_{S-1}, -\iota_{S-1})'$ in that $\beta_\perp = \iota_S$. The VEC and FCVAR models then read

$$\Delta p_{t_i} = \gamma\beta' p_{t_{i-1}} + e_{t_i}, \quad i = 1, 2, ..., N, \tag{C.3.2}$$

$$\Delta^d RV_t = \alpha\beta' L_b RV_t + \varepsilon_t, \quad t = 1, ..., T. \tag{C.3.3}$$

From the Granger representation theorem in (6), returns from market s are given by

$$\Delta p_{s,t_i} = \gamma'_\perp e_{t_i} + \sum_{j=0}^{\infty} \Xi_{s,j} \Delta e_{t_{i-j}}, \tag{C.3.4}$$

where $\Xi_{s,j}$ is the sth row of the $(S \times S)$ matrix Ξ_j. Define $A_{s,i} := \sum_{j=0}^{\infty} \Xi_{s,j} \Delta e_{t_{i-j}}$ and combine (C.3.4) with (C.3.1) such that

$$RV_{s,t} = \sum_{i=1}^{N} \left(\gamma'_\perp e_{t_i} e'_{t_i} \gamma_\perp + 2\gamma'_\perp e_{t_i} A_{s,i} + A_{s,i}^2\right). \tag{C.3.5}$$

Next, using the fractional counterpart of the Granger representation theorem as in (7), the s element of the RV_t vector in (C.3.3) reads

$$RV_{s,t} = \left[\left(\alpha'_\perp \beta_\perp\right)^{-1} \alpha'_\perp\right] \Delta_+^{-d} + X_{s,t}. \tag{C.3.6}$$

Recall from (8) that $RV_{m,t} = \left[(\alpha'_{\perp}\beta_{\perp})^{-1} \alpha'_{\perp} \right] \Delta_{+}^{-d} \varepsilon_t$. Using $\alpha'_{\perp}\alpha = 0$, $\Delta_{+}^{-d}\Delta^{d} = 1_{+} + \Delta_{+}^{-d}\Delta_{-}^{d}$ and rearranging terms, we express $RV_{m,t}$ in terms of the market-specific realized variances

$$RV_{m,t} = \alpha'_{\perp} RV_t + \left\{ \alpha'_{\perp} \Delta_{+}^{-d} \Delta_{-}^{d} RV_t \right\}, \tag{C.3.7}$$

where the last term of (C.3.7) is a linear combination of infinitely many initial values. Finally, to write $RV_{m,t}$ as a function of price innovations, e_{t_i}, and the price discovery measures, $\gamma_{\perp}$, substitute (C.3.5) into (C.3.7) so that

$$RV_{m,t} = \alpha'_{\perp} \iota_S \sum_{i=1}^{N} \left(\gamma'_{\perp} e_{t_i} e'_{t_i} \gamma_{\perp} \right) + \alpha'_{\perp} \sum_{i=1}^{N} A_i + \text{initial value}, \tag{C.3.8}$$

where $A_i = \left(\left[2\gamma'_{\perp} e_{t_i} A_{1,i} + A_{1,i}^2 \right], ..., \left[2\gamma'_{\perp} e_{t_i} A_{S,i} + A_{S,i}^2 \right] \right)'$ is a $(S \times 1)$ vector containing the market-specific components that are a function of both the long- and short-term components of the Granger representation theorem of prices.

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