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Common long-range dependence in a panel of hourly Nord Pool electricity prices and loads

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Abstract

Equilibrium electricity spot prices and loads are often determined simultaneously in a day-ahead auction market for each hour of the subsequent day. Hence daily observations of hourly prices take the form of a periodic panel rather than a time series of hourly observations. We consider novel panel data approaches to analyse the time series and the cross-sectional dependence of hourly Nord Pool electricity spot prices and loads for the period 2000-2013. Hourly electricity prices and loads data are characterized by strong serial long-range dependence in the time series dimension in addition to strong seasonal periodicity, and along the cross-sectional dimension, i.e. the hours of the day, there is a strong dependence which necessarily has to be accounted for in order to avoid spurious inference when focusing on the time series dependence alone. The long-range dependence is modelled in terms of a fractionally integrated panel data model and it is shown that both prices and loads consist of common factors with long memory and with loadings that vary considerably during the day. Due to the competitiveness of the Nordic power market the aggregate supply curve approximates well the marginal costs of the underlying production technology and because the demand is more volatile than the supply, equilibrium prices and loads are argued to identify the periodic power supply curve. The estimated supply elasticities are estimated from fractionally co-integrated relations and range between 0.5 and 1.17 with the largest elasticities being estimated during morning and evening peak hours.

JEL codes: C33, C38, Q4, Q41.

Keywords: Electricity prices and loads, panel data models, fractional integration, long memory.

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1 Introduction

For the past 15 years, electricity power markets have been subject to deregulation and liberation to ensure competitive price determination driven by the market forces of demand and supply. Electricity markets differ from most other commodity markets due to the non-storability of electricity and clearly this has a major impact on the way such markets function. In particular, it has consequences for the price behaviour with characteristics such as excessive electricity price volatility, price spikes, strong intra-day, weekly, and yearly seasonality, long memory features, possible negative prices and many other features, see e.g. Knittel and Roberts (2005) and Longstaff and Wang (2004) and the review article by Weron (2014). Such characteristics can lead to significant price risk faced by market participants which has consequences for the financial electricity markets where future and forward contracts traded on electricity address delivery over an extended period of time rather than for a particular moment in time. Thus, it is of great importance to understand the dynamic nature of the price behaviour for forecasting, derivative pricing, and risk management more generally in such markets.

The literature on econometric electricity market modelling is expanding these years. Many papers focus on dynamic models of electricity prices for forecasting purposes, see e.g. the review by Weron (2014). Weron and Misiorek (2008) compare the forecasting performance for a range of linear and non-linear time series models such as autoregressive models, jump-diffusion models, and regime-switching models. Several papers address the predictive content from covariates such as fuel prices, the level of demand, and temperature level information amongst other things, see Karakatsani and Bunn (2008) and Huurman et al. (2012). Often the models focus on price volatility or prediction of the occasional extreme spikes observed in electricity prices, see e.g. Christensen et al. (2012). In Koopman et al. (2007), Haldrup and Nielsen (2006b), and Haldrup et al. (2010) the focus is on the feature of electricity prices that their autocorrelation function decays at a hyperbolic rate and suggest the series to be modelled as fractionally integrated processes. In these papers, area prices within the Nord Pool Spot Exchange are modelled and account for the possibility of regime dependent long-memory dynamics subject to the presence or absence of transmission congestion across neighbour areas.

Most electricity price models focus on the modelling of the daily average price which plays a key role in electricity markets. The daily price acts as a proxy for the spot price of electricity and is a reference price for forward and futures contracts in addition to many other derivative products traded in the financial electricity market. In most electricity markets, including the Nord Pool Spot Exchange considered in the present paper, the daily average price is established in a day-ahead market. Prices are determined in a double auction where the participants trade power for delivery during the 24 hours of the subsequent day. At noon before the delivery day all buy and sell orders are gathered and the market price, called the system price, is determined as the intersection of the aggregated demand and supply curves. Based on the 24 hourly prices the daily average system price is calculated.

Even though the daily average price may be of primary focus, it is of interest to understand the price dynamics for the single hours constituting the daily average calculation. For instance Raviv et al. (2015) show that the daily average of the disaggregated hourly forecasts contain useful predictive information for the daily average price in the Nord Pool market. Hence it is important to properly model the hourly prices for gaining insights about the underlying electricity price dynamics.

The modelling of hourly electricity prices as well as the loads has reached some attention in the literature. Often the single hourly prices or loads for each day are modelled as univariate autoregressive time series processes, see e.g. Ramanathan et al. (1997), Cuaresma et al. (2004), Soares and Medeiros (2008), and Kristiansen (2012). However, an important consideration which is frequently overlooked when modelling the hourly (intra-day) prices is that the vector of 24 hourly prices is determined simultaneously in the day-ahead market. Hence the proper form of the data set is a panel of prices and naturally it is to be expected that there is a considerable amount of cross-sectional dependence across the panel. The panel data set is somewhat special as the cross-section dimension has a natural ordering, and hence we may refer to this particular type of panel as a *periodic* panel. Given this observation it is not appropriate to model the hourly data series as a single time series since consecutive prices are determined simultaneously and ignoring important cross-sectional dependence may potentially lead to spurious inference. Only few other papers consider the hourly electricity prices as a panel; exceptions include Huisman et al. (2007), Härdle and Trück (2010), and Raviv et al. (2015). In principle, it could be considered to model the panel as a (possibly co-fractional) VAR model, Johansen and Nielsen (2012), but the size of the cross-sectional dimension would be too large for practical implementation in this case.

The present paper considers a panel of hourly Nord Pool system prices and loads for 14 years of daily observations covering the years 2000-2013. The prices and loads for the single hours exhibit a considerable degree of long memory but generally this feature is ignored in the vast majority of papers analysing the dynamics of electricity markets. We use a novel panel data framework for fractionally integrated panels with fixed effects and cross-section dependence which is particularly well suited for the present type of high dimensional data. The analysis makes it possible to determine the fractional integration orders of the single (hourly) panel data elements and their likely fractional cointegration features. We consider two different approaches. The first approach consists in the estimation of a fractionally integrated panel data model with fixed effects, proposed by Ergemen and Velasco (2015), where appropriately defined common factors are treated as covariates in the individual models for prices and loads. The second approach employs the fractionally integrated panel data system with fixed effects proposed by Ergemen (2015) to allow for feedback effects between electricity prices and loads, which is introduced through the allowance for contemporaneous correlation in the model. Both of these approaches account for long memory, seasonality, and spikes that make them well suited to our analysis given the stylized facts of electricity data. The methodology makes it possible to examine the time series and cross-sectional dynamics of both prices and loads as well as their common dynamics that we argue help identify the periodic supply elasticity of the underlying power production technology.

When analyzing the panel of electricity prices and loads we find that fractional cointegration is present for all 24 hours of the day. This means that both series are driven by a common (non-stationary and mean-reverting) fractional trend component. In fact, these results are much in line with the dynamics of the common factors that can be extracted from the factor analysis.

Rather than considering conditioning on common factors used as input from factor analysis, the second approach considers a panel data model where the loads are used as a covariate variable. We find that fractional cointegration between prices and loads primarily exists in the day hours from 7 a.m. to 7 p.m. which is consistent with the analysis of prices and loads when they are modelled separately. Because the main variability in loads is due to changing demand within the day, and across the week and the year, the price-load scheme is likely to identify the supply curve of the power market technology. The supply elasticity is shown to be periodically varying with the hour of the day in the interval 0.50-1.17. Not surprisingly, the elasticity is highest during the peak hours in the mornings and evenings and lowest during the night.

The paper is organized as follows. In section 2, some background information is provided regarding the functioning of the Nord Pool power market with particular focus on the day-ahead spot market. Section 3 presents the data and discusses time series properties of the daily observations of hourly data. This includes the seasonal and periodic features of the data and its long memory properties. The panel feature of the data is used to conduct common factor analysis of the price and load data where the extracted factors will be used in the subsequent panel data analysis. In section 4 the novel fractional panel data analysis of Ergemen and Velasco (2015) is conducted separately for the price and load series, and finally the joint estimation using the approach of Ergemen (2015) is used to examine the possible fractional cointegration between prices and loads and to provide estimates of the periodically varying supply elasticity. The final section concludes.

2 The operation of the Nord Pool power market

The data to be used in this paper is from the Nordic power exchange, Nord Pool Spot, which is owned by the transmission system operators within the Nordic and Baltic countries. For reviews of the functioning of the Nord Pool market, see e.g. NordPool (2013), NordPool (2015), Weron (2007), and Weron (2014). There are almost 400 companies from 20 countries trading in the Nord Pool Spot markets and includes producers and large consumers. Nord Pool has a number of different auction markets targeted for different purposes along different time scales. Elspot is a day-ahead auction market which determines spot prices and loads for each hour of the subsequent day. The market participants act in a double auction and submit their supply and demand orders for each individual hour of the next day through the on-line trading system. Orders are placed between 8 a.m. and 12 a.m. Buy and sell orders are then gathered into demand and supply curves for each delivery hour. The equilibrium price, called the system price per megawatt hour (MWh), is determined by the intersection of the demand and supply curves where also the transmission capacity of the power system is accounted for. The hourly prices and the loads are announced to the market at 12.42 p.m. and the trades are invoiced the following couple of hours. Note that the system prices announced is a 24 dimensional price vector which is determined simultaneously.

The system price is important because it serves as the Nordic reference price for the trading and clearing of most financial contracts and hence is crucial for derivative pricing and risk management. It should be noted that the system price is the (unconstrained) equilibrium price for the entire Nordic region in case all power could be transmitted smoothly without any capacity constraints. In practice, however, the system price does not clear all areas within the Nordic market and hence the Elspot market is divided into several bidding areas. The transmission capacity available in the different regions may vary and potentially congest the flow of electricity power between the bidding areas and hence different price areas will be established. For neighbouring regions where there is no congestion, the prices will be the same whereas congestion results in different area prices.

We only consider the system prices in the present paper, but area prices may be equally important. In fact, some financial contracts are made for an area rather than the system prices. In addition to the day-ahead market, Nord Pool has a real time intraday auction market which serves as a balancing market (called Elbas) to support the day-ahead market and to refine physical positions before the final balancing measures are taken by the local transmission system operators (TSO). These markets operate at a 15, 30, (for TSOs) and 60 minutes, (for Elbas) horizon to offer the flexibility needed for each market.

The annual average power generation in the Nordic and Baltic countries is about 420 Twh and about half of the production is from hydro power plants. Norway has almost all of its power generated from hydro power whereas Sweden and Finland produce from a mix of hydro, nuclear and thermal power plants. Denmark, Estonia, and Lithuania produce predominantly thermal power, but renewable energy in the form of wind power plays an increasing role, particularly in Denmark. During years where the water reservoirs in Norway are low, the countries are dependent upon import from Russia, the Netherlands, Poland, and Germany.

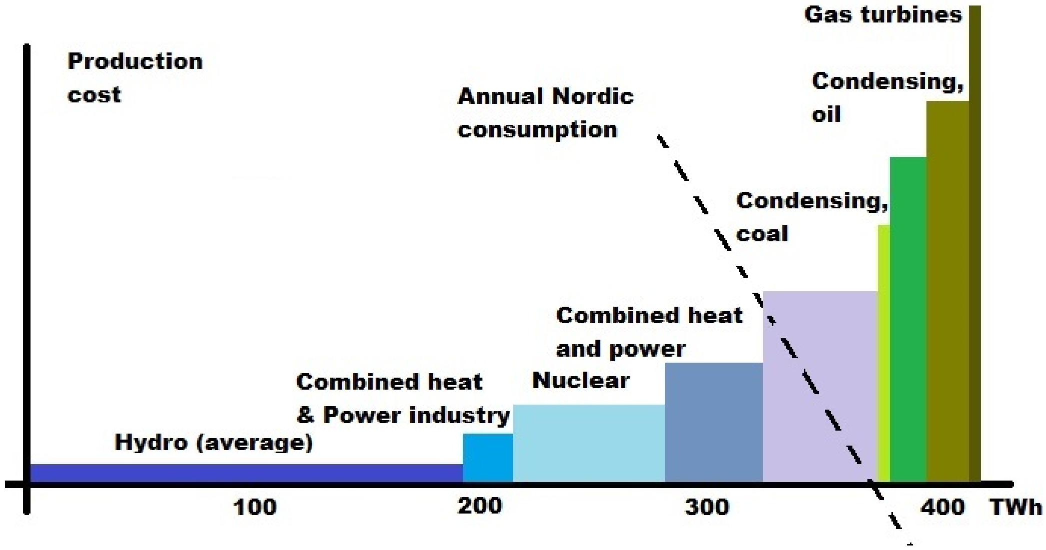

Figure 1: The production costs and the typical annual Nordic consumption for various sources of power. Source: NordPool (2015).

The production costs vary to a considerable degree and Figure 1 shows the typical production costs of the various power sources for the annual Nordic consumption. Hydro power (and wind) have relatively low marginal costs and hence the market is generally much dependent on rainfall and water reservoir levels over a yearly cycle.

For a given hour the system price is determined from the bids of supply and demand orders. When the demand for power is low the equilibrium price will be low and the plants with the lowest marginal costs will primarily produce the power needed. On the other hand, when the demand is high the marginal costs will be high and so will be the prices. With intra-day electricity demand that varies considerably and because both supply and particular demand depend upon seasonal variation over the year, the shape of the marginal cost (supply) curve of electricity may result in much volatility and spikes in power prices, see e.g. Weron (2007) and Kirschen (2003). Figure 2 shows the typical shape of the marginal cost (supply) curve of power and it is clear that even small changes in demand may result in large price changes depending on the scale of production. Even though equilibrium prices are determined in a double auction the major variability in production is due to changes in demand. Because of deregulation and liberalization Nord Pool Spot can be considered a highly competitive market and we would expect prices to clear close to the short-run marginal cost, especially because of a substantial amount of forward contracting in the market, see Bunn (2000) and Kirschen (2003). From an empirical point of view we would thus anticipate that the equilibrium prices and loads will identify the marginal cost or supply curve as the demand curve shifts.

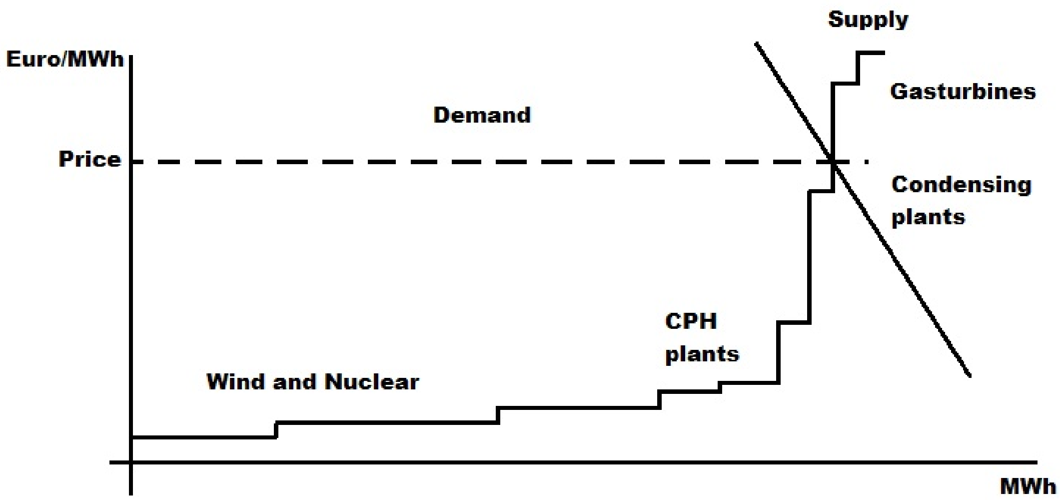

Figure 2: Electricity power supply and demand and the source of power reflecting the increasing marginal costs of production. Source: NordPool (2015).

3 The data

3.1 Time series features

The data set under consideration is a balanced (periodic) panel consisting of 24 hourly prices and loads for each day for the period 1 January 2000 to 31 December 2013, yielding a total of 5114 daily (or 122,736 hourly) observations. The series are downloaded from the Nord Pool ftp server. The prices are denominated in euros per Mwh of load. Both panels are shown in Figures 3 and 4 after log transformation. The descriptive statistics of the price and load series are reported in Table 1.

It is clear from Figures 3 and 4 that electricity system prices and loads vary considerably over the years and especially the price series show several spikes. Both series exhibit a strong seasonal variation. Figures 5-8 show the average hourly prices and loads across the day of week and month of year. The shape of the graphs demonstrates that prices and loads co-vary positively with a similar annual seasonal and weekly periodic pattern. Average prices and loads are highest during the winter season and lowest during the summer. Over the week the average prices and loads are rather similar during the work days Monday through Thursday. On Friday afternoons prices and loads gradually change towards the weekend level where the demand and prices of Saturdays and Sundays are significantly lower than for the rest of the week. Note that the intra-day variation for Saturday and Sunday is still rather different.

Even though the data has a panel structure, we consider initially the individual panel elements as time series, that is, for each variable twenty-four separate time series are treated individually. From Table 1 it is seen that both prices and loads have considerable variation across the hours of the day (as measured by the standard deviation). Skewness and kurtosis is rather constant during the day except for the morning hours (5 a.m.-8 a.m.) where especially the price series show excess kurtosis.

In addition to weekday periodicity and annual seasonality there are also other (deterministic) factors that need to be accounted for prior to the analysis. A number of events in the delineation and market infrastructure of Nord Pool has changed in the sample period which may have an impact on the price and load behaviour. We have identified seven such events: i) 02/01/2002 when Nord Pool's spot market activities were organized in a separate company, ii-iii) 01/01/2004 and 02/04/2007 when Eastern and Western Denmark joined the Elbas market, iv) 01/01/2006 when Elbas was launched in Germany, v) 01/04/2010 when Nord Pool Spot opened a bidding area in Estonia, vi) 01/01/2011 when Elbas began operation in the intraday market in the Netherlands and Belgium, and vii) 26/06/2012 when Nord Pool Spot opened a new bidding area in Lithuania.

Given the observations above we suggest to seasonally adjust and detrend each panel element of both series by the least-squares filtering,

$$y_{it} = \alpha_{i0} + \alpha_{i1}\ t + \alpha_{i2}\ D_t + \mathbb{B}_t' A_i + \alpha_{i3}\ \cos(\tfrac{2\pi t}{365}) + \alpha_{i4}\ \cos(\tfrac{2\pi t}{7}) + \alpha_{i5}\ \cos(\tfrac{2\pi t}{3.5}) + y_{it}^*, \quad (1)$$

where $\mathbb{B}_t$ is a vector of shift dummies which captures level changes caused by structural breaks. D_t is a dummy variable for holidays that takes the value of 1 if any of the countries participating in the Nord Pool system suspend or reduce normal business activities by custom or law, and 0 otherwise. The data for non-working days in each of the countries of the Nord Pool System is extracted from the Bloomberg platform, which is then incorporated into the analysis due to the strong effect of holidays in the electricity market, see Koopman et al. (2007).

In the filtering of the data we accommodate a yearly (365 day) cycle in the data together with a weekly and a two-cycles per week periodicity.[1] The series y_{it} is the unadjusted log price or log load series, and y_{it}^* is the resulting adjusted series. For the rest of the analysis in this paper we use the filtered series, y_{it}^* to represent each panel element. For notational economy we shall continue denoting the filtered series y_{it}.

To see the effect of data filtering Figures 9 and 10 show the scatter plot of non-filtered and filtered log prices and loads respectively, where the data has been pooled across all hours in the panel. Hence each figure contains more than 122,000 data points. The graphs clearly demonstrate how deterministic components (breaks and seasonal cycles) account for a significant variation in the data. The graphs also separate observation points into working hours and non-working hours and as can be seen there is a clear pattern in working hours mainly contributing to the hours with high demand and

[1]The remaining cycles associated with the harmonic weekly frequencies were insignificant in estimations.

hence high prices. This reflects the relatively high marginal costs of production when the demand is high.

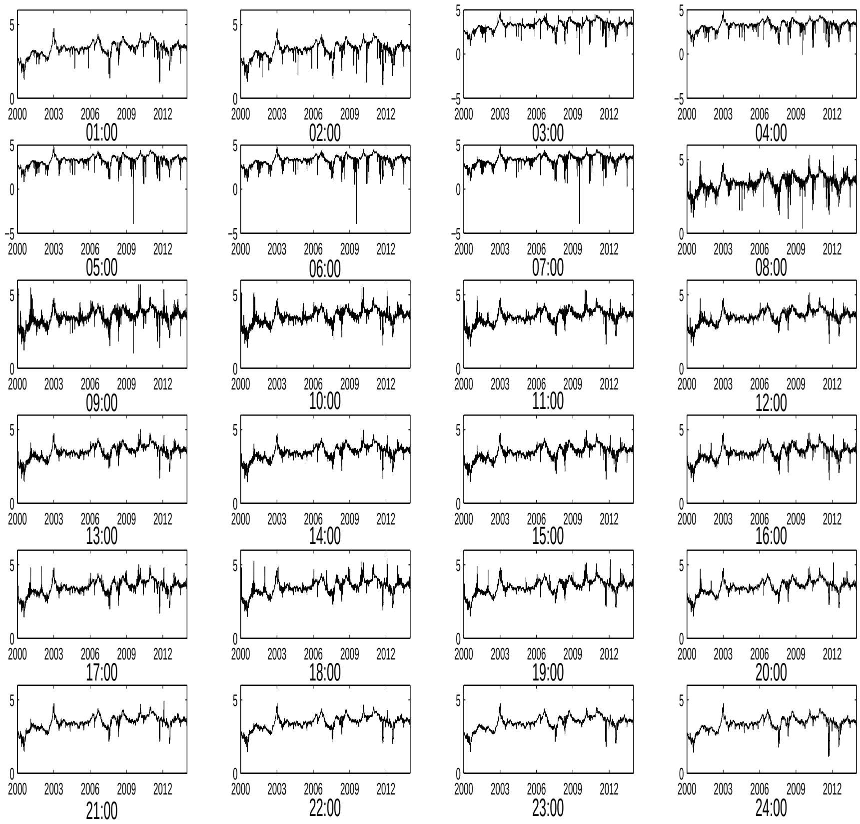

Figure 3: Hourly electricity system prices (in logs) within the Nord Pool area, 1 January 2000 to 31 December 2013.

Table 1: Descriptive statistics for the log prices and log loads for each hour of the day.

Log electricity prices	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00	12:00
Min	1.03	0.90	-0.04	-0.04	-3.91	-3.91	-3.91	0.34	1.02	1.41	1.49	1.50
Mean	3.37	3.33	3.29	3.27	3.27	3.32	3.39	3.47	3.52	3.52	3.53	3.52
Median	3.43	3.39	3.37	3.35	3.36	3.40	3.46	3.51	3.55	3.55	3.56	3.55
Max	4.74	4.72	4.70	4.70	4.72	4.76	4.76	5.30	5.70	5.67	5.34	5.13
Std.dev	0.49	0.52	0.54	0.56	0.57	0.55	0.53	0.50	0.48	0.46	0.45	0.44
Skewness	-0.82	-0.94	-1.01	-1.06	-1.40	-1.47	-1.53	-0.81	-0.51	-0.50	-0.54	-0.55
Kurtosis	4.52	4.80	5.02	5.10	9.62	10.78	12.28	4.93	4.53	4.16	3.95	3.91
	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00	21:00	22:00	23:00	24:00
Min	1.48	1.46	1.47	1.46	1.46	1.48	1.52	1.49	1.48	1.46	1.47	1.14
Mean	3.50	3.49	3.47	3.47	3.47	3.50	3.50	3.49	3.47	3.46	3.44	3.39
Median	3.53	3.52	3.51	3.50	3.50	3.52	3.53	3.52	3.51	3.50	3.48	3.45
Max	5.02	4.97	4.95	4.78	5.01	5.42	5.35	5.14	4.91	4.77	4.75	4.75
Std.dev	0.44	0.45	0.45	0.46	0.47	0.48	0.47	0.46	0.45	0.45	0.45	0.47
Skewness	-0.59	-0.61	-0.62	-0.64	-0.58	-0.47	-0.54	-0.65	-0.69	-0.69	-0.66	-0.75
Kurtosis	3.89	3.91	3.94	3.99	4.03	4.14	4.07	4.11	4.14	4.15	4.01	4.29

Log loads	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00	12:00
Min	8.79	8.76	8.75	8.72	8.68	8.70	8.70	8.74	8.82	8.84	8.86	8.87
Mean	9.89	9.86	9.85	9.84	9.85	9.89	9.96	10.03	10.06	10.07	10.08	10.07
Median	10.09	10.06	10.04	10.03	10.03	10.06	10.12	10.19	10.24	10.27	10.28	10.29
Max	10.77	10.72	11.11	10.71	10.73	10.80	10.91	10.96	10.97	10.96	10.97	10.96
Std.dev	0.51	0.51	0.51	0.51	0.51	0.51	0.53	0.54	0.54	0.54	0.53	0.54
Skewness	-0.44	-0.44	-0.43	-0.43	-0.43	-0.42	-0.40	-0.38	-0.40	-0.41	-0.41	-0.41
Kurtosis	1.83	1.85	1.88	1.88	1.90	1.92	1.91	1.90	1.88	1.84	1.81	1.79
	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00	21:00	22:00	23:00	24:00
Min	8.88	8.87	8.89	8.89	8.90	8.90	8.91	8.91	8.90	8.90	8.89	8.88
Mean	10.06	10.05	10.05	10.04	10.05	10.06	10.06	10.05	10.03	10.02	9.99	9.93
Median	10.28	10.27	10.26	10.25	10.24	10.25	10.25	10.24	10.23	10.22	10.20	10.15
Max	10.96	10.95	10.95	10.95	10.96	10.97	10.97	10.96	10.93	10.90	10.87	10.80
Std.dev	0.54	0.54	0.54	0.54	0.54	0.55	0.55	0.55	0.54	0.53	0.52	0.52
Skewness	-0.42	-0.41	-0.41	-0.41	-0.41	-0.41	-0.41	-0.41	-0.42	-0.43	-0.44	-0.44
Kurtosis	1.79	1.79	1.79	1.81	1.84	1.86	1.87	1.87	1.85	1.82	1.79	1.80

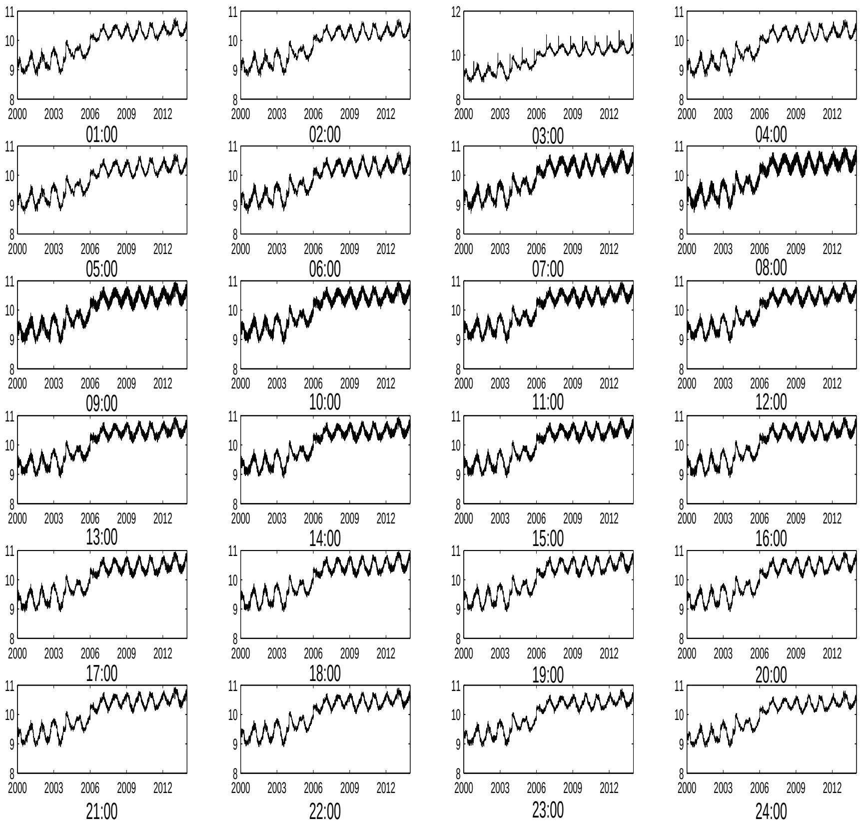

Figure 4: Hourly electricity system loads (in logs) within the Nord Pool area, 1 January 2000 to 31 December 2013.

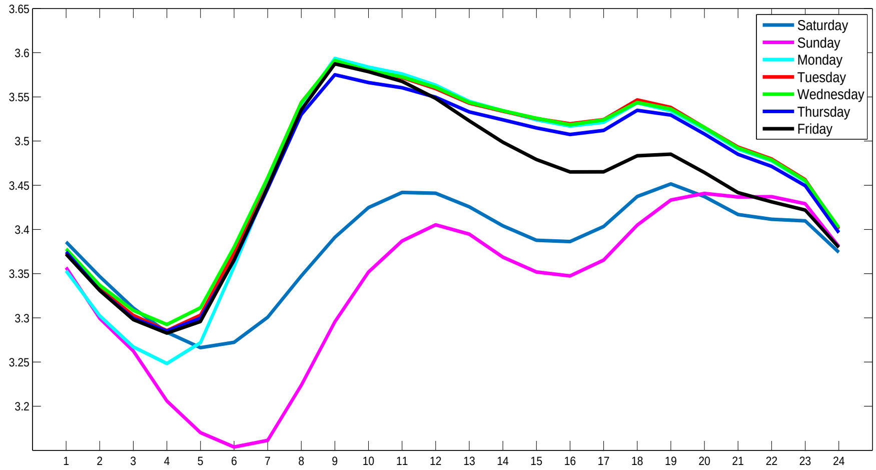

Figure 5: Average hourly prices across the day of a week.

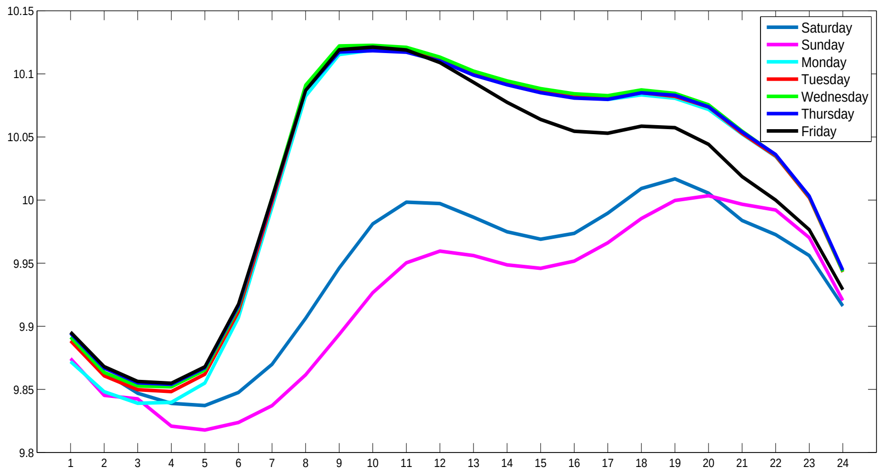

Figure 6: Average hourly loads across the day of a week.

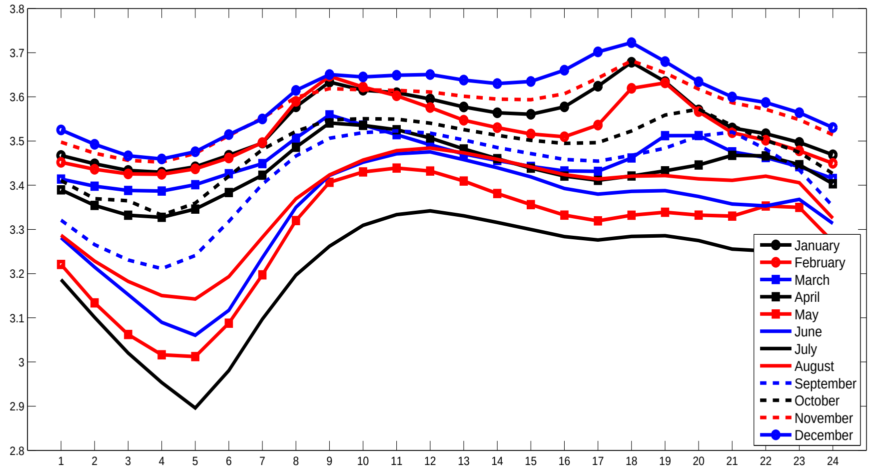

Figure 7: Average hourly prices across the month of a week.

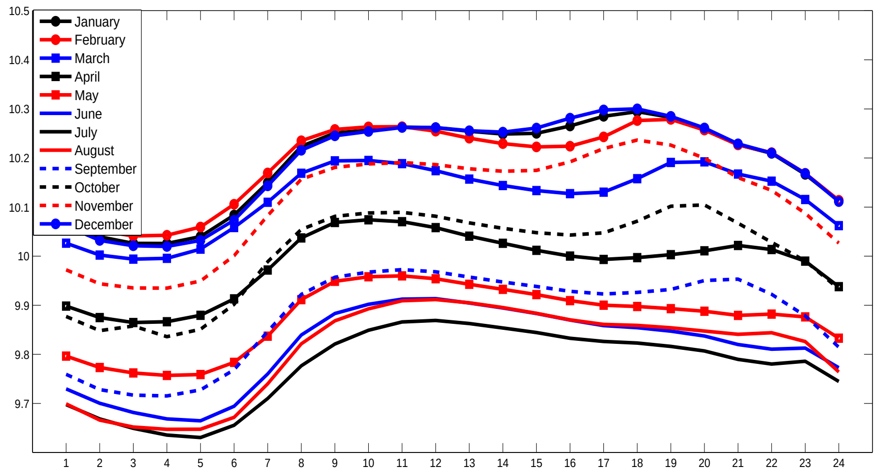

Figure 8: Average hourly loads across the month of a week.

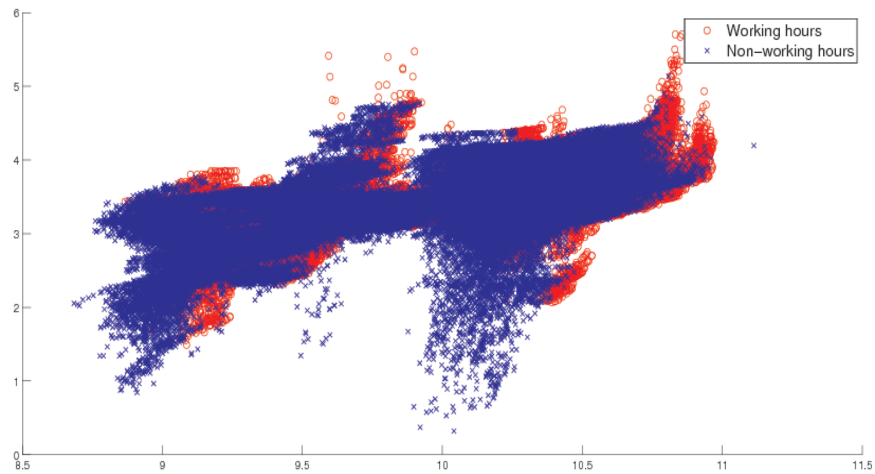

Figure 9: Scatter plot of non-filtered log prices and loads.

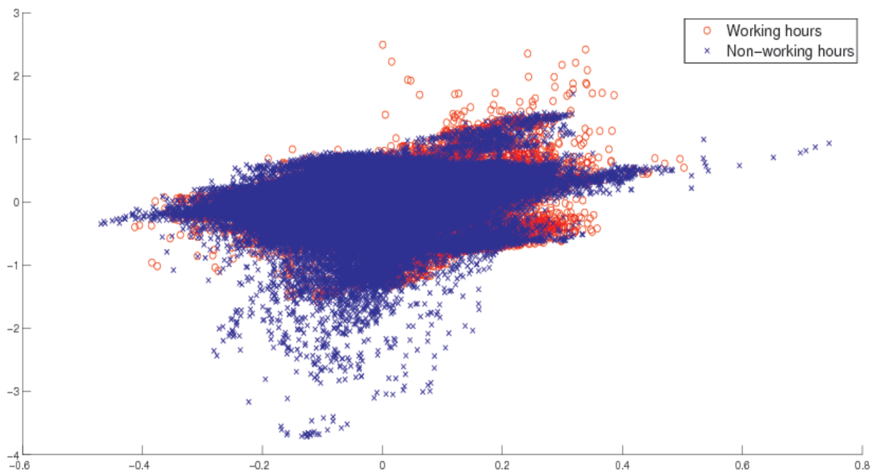

Figure 10: Scatter plot of filtered log prices and loads.

3.2 Long memory

A time series x_t is said to be a type-II fractionally integrated process of order d if

$$x_t = \Delta_{t+1}^{-d}\varepsilon_t,$$

where ε_t corresponds to $I(0)$ short-memory innovations. Throughout the paper, the subscript at the fractional differencing operator attached to a vector or scalar ε_t has the meaning

$$\Delta_{t+1}^{-d}\varepsilon_t = \Delta^{-d}\varepsilon_t 1(t \geq 0) = \sum_{j=0}^{t} \pi_j(-d)\varepsilon_{t-j},$$

$$\pi_j(-d) = \frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)},$$

where $1(\cdot)$ is the indicator function, and $\Gamma(\cdot)$ denotes the gamma function such that $\Gamma(d) = \infty$ for $d = 0, -1, -2, \ldots$, but $\Gamma(0)/\Gamma(0) = 1$. The parameter d bestows possible stationary long memory when $0 < d < 0.5$ and non-stationary long memory $d \geq 0.5$ upon x_t. The use of the truncated filter, Δ_{t+1}^{-d}, is motivated by a desire to allow for $d \geq 0.5$ when the untruncated filter, Δ^{-d}, does not converge, see e.g. Davidson and Hashimzade (2009).

In this subsection we also treat the panel elements as daily time series of hourly observations and hence abstract from possible panel data dependence. Several papers in the literature have addressed the issue of long-range dependence in electricity prices and loads, see e.g. Haldrup and Nielsen (2006a) and Koopman et al. (2007). Justifying this claim, Figure 11 reports the fractional integration parameter estimates of the hourly electricity prices and loads based on univariate local Whittle estimation (Kuensch (1987) and Robinson (1995)), an extended local Whittle estimation (Abadir et al. (2007)) and an extended multivariate local Whittle method (Nielsen (2011)). Table 2 collects the numerical estimation results of the memory parameters along with their asymptotic standard deviations.

As seen from Figure 11 and Table 2, there is a semi-cyclical variation in the memory index estimates of prices while the memory for loads seems to be rather constant. Given the asymptotic standard errors the difference between the highest memory estimates (late evenings) and the smallest memory (early mornings) for the price series seems to be significant. The results further indicate that both electricity prices and loads exhibit non-stationarity and mean-reversion throughout the day since the parameter estimates $\widehat{d_i} \in [0.5, 1)$.

3.3 Factor analysis of the panel observations

So far we have discussed the time series properties of the single elements of the panel data series. It is obvious that the time series behaviour across the single hours for prices

Table 2: Univariate and multivariate estimates of fractional integration orders of hourly electricity prices and loads

Fractional Memory Estimates													
	Prices			Loads				Prices			Loads		
	LW	EXLW	EMLW	LW	EXLW	EMLW		LW	EXLW	EMLW	LW	EXLW	EMLW
01:00	0.69	0.70	0.51	0.69	0.69	0.55	13:00	0.77	0.78	0.57	0.68	0.68	0.63
			(0.01)			(0.01)				(0.01)			(0.01)
02:00	0.65	0.66	0.50	0.70	0.70	0.55	14:00	0.78	0.78	0.57	0.68	0.68	0.63
			(0.01)			(0.01)				(0.01)			(0.01)
03:00	0.62	0.63	0.50	0.69	0.69	0.55	15:00	0.78	0.79	0.57	0.68	0.68	0.62
			(0.01)			(0.01)				(0.01)			(0.01)
04:00	0.61	0.62	0.52	0.70	0.70	0.55	16:00	0.77	0.78	0.58	0.69	0.70	0.62
			(0.01)			(0.01)				(0.01)			(0.01)
05:00	0.61	0.62	0.51	0.70	0.70	0.55	17:00	0.74	0.75	0.61	0.70	0.70	0.62
			(0.01)			(0.01)				(0.01)			(0.01)
06:00	0.66	0.66	0.50	0.71	0.71	0.56	18:00	0.69	0.70	0.61	0.71	0.71	0.64
			(0.01)			(0.01)				(0.01)			(0.01)
07:00	0.71	0.72	0.50	0.70	0.70	0.61	19:00	0.70	0.71	0.62	0.71	0.71	0.62
			(0.01)			(0.01)				(0.01)			(0.01)
08:00	0.69	0.69	0.54	0.68	0.68	0.64	20:00	0.78	0.79	0.61	0.70	0.70	0.60
			(0.01)			(0.01)				(0.01)			(0.01)
09:00	0.63	0.64	0.54	0.69	0.69	0.64	21:00	0.82	0.83	0.62	0.70	0.70	0.58
			(0.01)			(0.01)				(0.01)			(0.01)
10:00	0.68	0.68	0.56	0.69	0.69	0.65	22:00	0.83	0.84	0.60	0.70	0.71	0.57
			(0.01)			(0.01)				(0.01)			(0.01)
11:00	0.70	0.71	0.56	0.69	0.69	0.64	23:00	0.80	0.81	0.57	0.70	0.70	0.56
			(0.01)			(0.01)				(0.01)			(0.01)
12:00	0.74	0.75	0.57	0.69	0.69	0.64	00:00	0.72	0.73	0.53	0.71	0.71	0.54
			(0.01)			(0.01)				(0.01)			(0.01)

Note. LW: Local Whittle, EXLW: Extended Local Whittle, EMLW: Extended Multivariate Local Whittle. Standard errors are 0.0312 for the univariate estimates with bandwidth $m = T^{0.65}$. Standard errors of the multivariate estimates are in parentheses.

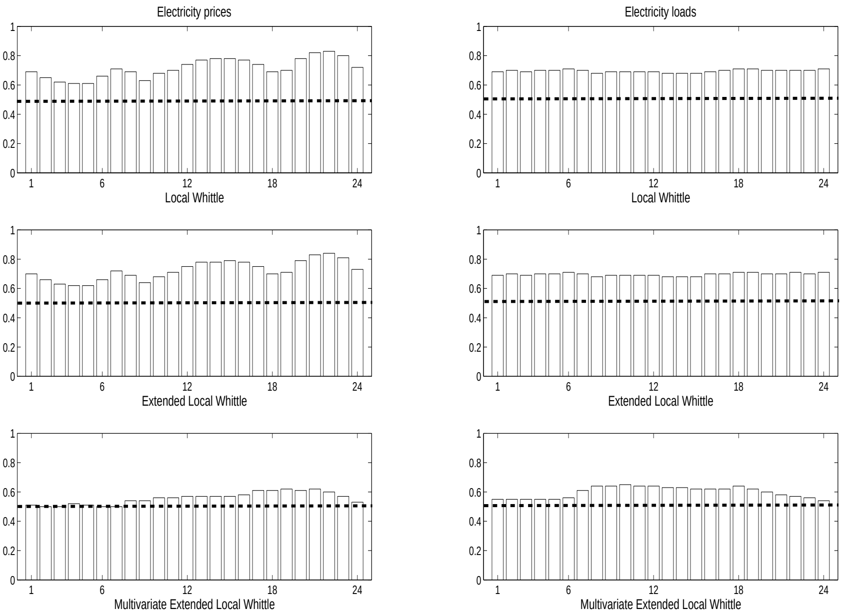

Figure 11: Fractional integration parameter estimates for electricity prices (left panel) and loads (right panel) corresponding to each hour of the day. Estimations are carried out on filtered data. The number of Fourier frequencies used is $m = T^{0.65}$ with $T = 5,114$ corresponding to $m = 257$. The standard error of the univariate estimates is 0.0312 while that of multivariate estimates lies between 0.007 and 0.009. See Table 2 for details.

and loads tends to be rather similar and the question naturally arises as to whether the panel variation can be described in terms of a limited number of (dynamic) factors. The literature on modelling electricity prices and loads has mainly used information at an aggregate, typically daily, level. However, the cross-sectional dependence of the disaggregated series may be of separate interest for the analysis. For instance, one may ask whether one should forecast the aggregated electricity price series or the single elements of the disaggregated series, see e.g. Raviv et al. (2015). Although there is a vast literature on the modelling of intra-day prices using disaggregated data, the complex dependence structure of hourly electricity prices or loads has not been extensively considered; exceptions include Raviv et al. (2015), Huisman et al. (2007) and Härdle and Trück (2010). Figure 12 displays a correlation heat map of hourly prices and loads before and after pre-filtering and shows that the series are highly correlated across the hours of the day, and hence stresses the necessity to jointly model the data as a panel. It seems natural to consider a factor model as a dimension reduction device under these circumstances, see e.g. Bai and Ng (2008).

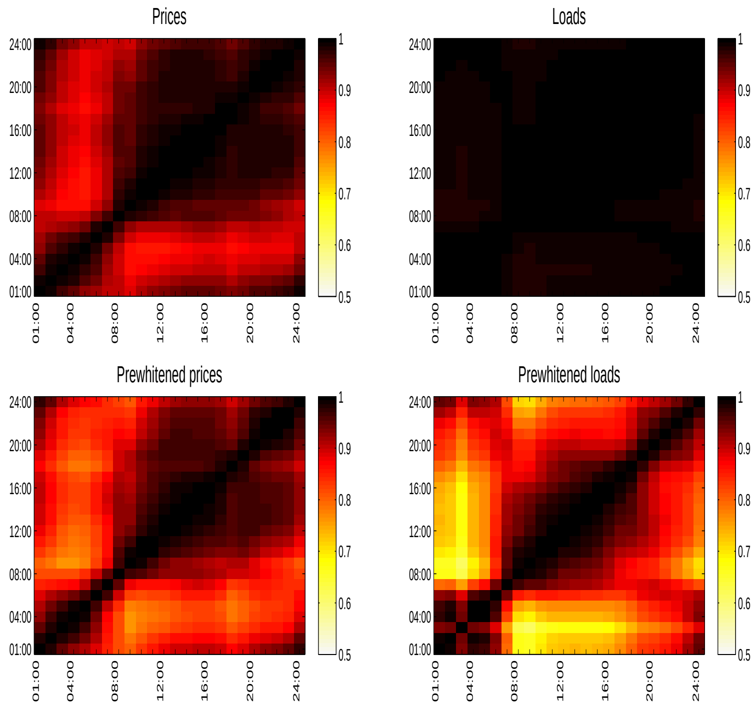

Figure 12: Correlation heat maps of the electricity prices (left panel) and loads (right panel) of the Nord Pool area across each hour of the day. Top panels show the correlation maps of the raw log series. Bottom panels show the correlation maps of the filtered series.

To model the common properties of hourly electricity prices, a dynamic factor model can be employed, with L denoting the lag operator such that $Lz_t = z_{t-1}$, as in

$$x_{it} = \lambda'(L) f_t + e_{it}, \tag{2}$$

where $\lambda_i(L) = (1 - \lambda_{i1}L - \cdots - \lambda_{is}L^s)$ are dynamic factor loading polynomials of order s, the factors f_t are assumed to evolve according to $f_t = C(L)\epsilon_t$ where ϵ_t are $q \times 1$ i.i.d. disturbances. The premise of the model in (2) is that f_t drives the commonalities of a high-dimensional vector of time series while $\lambda_i(L)$ shows how much each cross-section unit i is affected by f_t. The dynamic factor model in (2) can be represented as a static factor model with r static factors as long as $r = q\,(s+1) \geq q$. However, in practice it is not possible to verify whether this constraint holds, and as a consequence it is desirable to allow $r < q(s+1)$ by holding r and q fixed and letting s vary, see Forni et al. (2009). Empirically, the static and dynamic factor models have a similar predictive ability but the static setup is clearly more parsimonious.

To conduct a factor analysis of the price and load series we require the series to be stationary. We previously found that both series have non-stationary long memory and hence there is a need for pre-differencing prior to estimating the number of factors. The pre-differenced series (p^*_{it}, l^*_{it}) are obtained by fractionally differencing the filtered series as

$$p^*_{it} \equiv \Delta^{\delta^*_p} p_{it} \qquad \text{and} \qquad l^*_{it} \equiv \Delta^{\delta^*_l} l_{it}, \tag{3}$$

where

$$\delta^*_p \; = \; \max_i \, \widehat{d}_{p_i} \qquad \text{and} \qquad \delta^*_l \; = \; \max_i \, \widehat{d}_{l_i} \; ,$$

for $i = 1, 2, ..., 24$, and $\widehat{d}_{p_i}$ and $\widehat{d}_{l_i}$ are the fractional differencing parameters of electricity prices and loads, respectively. Based on the maximum values of the extended local Whittle estimates with bandwidth $m = T^{0.65}$ Fourier frequencies we have $\delta^*_p = 0.84$ and $\delta^*_l = 0.71$, according to Table 2.

We estimate a generalized dynamic factor model (GDFM), proposed by Forni et al. (2005). The model assumes that the stochastic process x_{it} in model (2) is stationary as satisfied by the construction of p^*_{it} and l^*_{it}. Estimation of the common factor structure then requires determining the number of dynamic factors, q, and the number of static factors, r. There are some criteria to determine the number of static or dynamic factors such as analyzing a scree plot of the shared behaviour of the estimated number of factors and the empirical variance. Another commonly used method is based on the percentage of the total variability successfully explained by the common factors, which is compared against a level determined by the econometrician. We consider both methods in the present analysis.

First, we use a criterion initially proposed by Bai and Ng (2002) and later modified by Alessi et al. (2010) (information criteria IC^*_1) which has improved finite sample properties by introducing a tuning multiplicative constant in the penalty function to fix r. Second, we use the information criterion IC_1 proposed by Hallin and Liška (2007) to compute the number of dynamic factors, q. Scree plots suggest $r = 3$ and $q = 3$ in both cases. However, when examining the total variance explained by factors, we choose only two static factors due to the third common factor explaining only 6% and 4% respectively of the variation which we consider to be relatively small. With two static factors, we explain 82% of the variation in the panel of electricity prices and 88% of the variation of the panel of electricity loads.

Once we fix the number of static (r) and dynamic (q) factors, we estimate (2) in two steps. In the first step, the covariance matrices of common and idiosyncratic components are obtained in order to produce generalized principal components of the contemporaneous variables p^*_{it} and l^*_{it}, depending on the case, with minimum idiosyncratic common variance ratio, see Forni et al. (2005).

After estimating the GDFM, we integrate back the r static factors of electricity prices using the original filters δ^*_p and δ^*_l, respectively. The factor loadings are displayed in Figures 13 and 15 and the common factors are displayed in Figures 14 and 16.

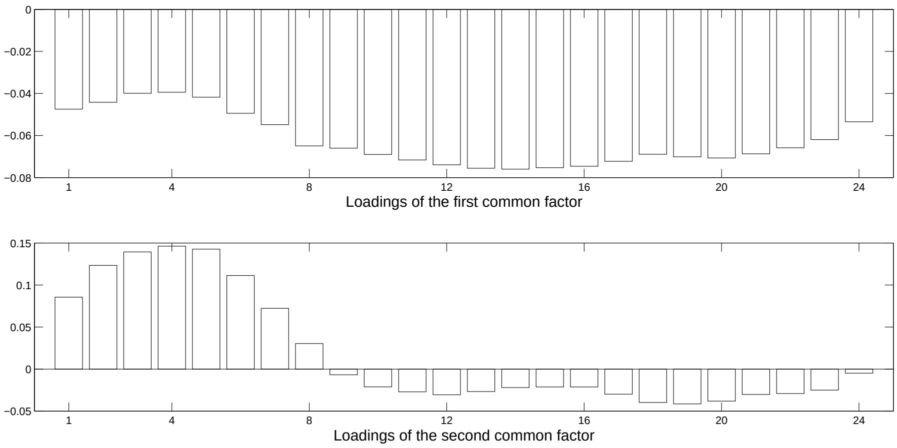

Figure 13: Factor loadings of electricity prices.

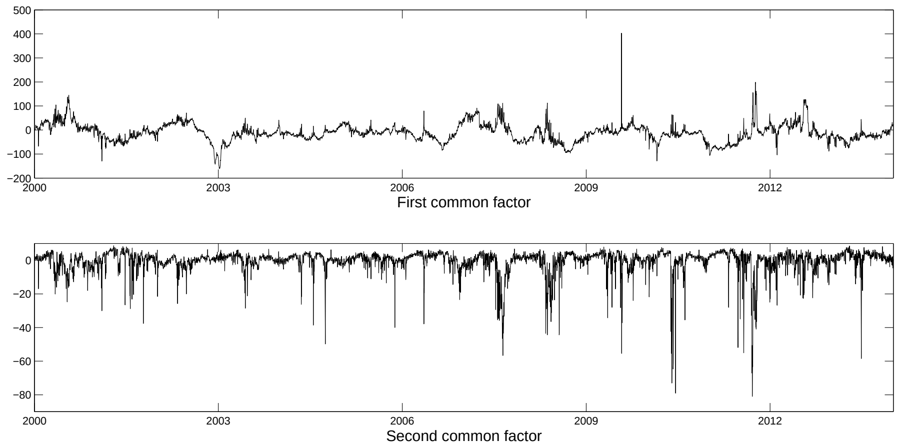

Figure 14: Common factors of electricity prices. The first and second factors explain 68% and 14% of the total variation, respectively.

The factor loadings corresponding to both first common factors are all negative, see Figures 13 and 15, with larger absolute magnitudes during work hours. In turn, factor loadings of both second common factors are positive and larger (in absolute terms) than those of both first common factors during night hours. This indicates that both second common factors play a key role from 1 a.m.-6 a.m. whereas the first factor predominantly affects the remaining hours of the day.

At a first glance, Figures 14 and 16 show that the estimated common factors of electricity prices and loads exhibit volatility clustering and the factors associated with

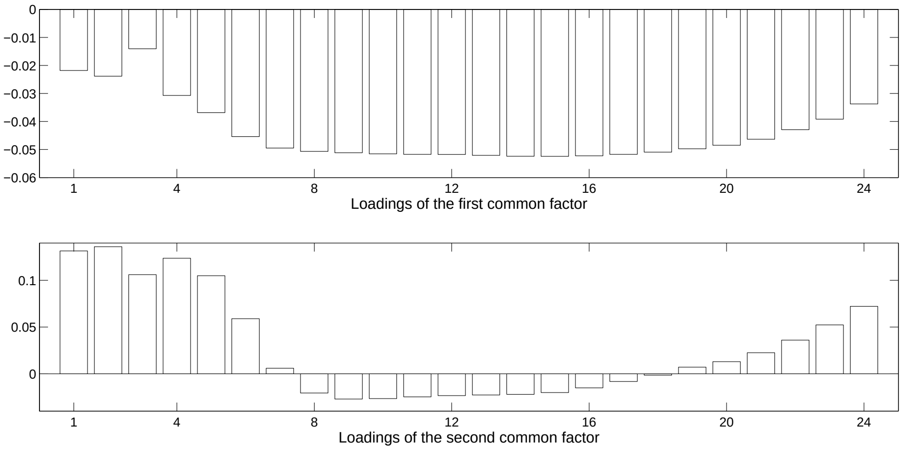

Figure 15: Factor loadings of electricity loads.

loads also show some annual periodicity. A similar result was found employing the periodic seasonal ARFIMA-GARCH model by Koopman et al. (2007) who also considered electricity prices within Nord Pool area.

A further inspection on Figures 14 and 16 indicates differences in the persistence level between the first and second common factors of electricity prices while the same cannot be concluded for electricity loads. As we have previously discussed, the electricity prices are characterized by having several price spikes. As seen from Figure 14 it appears that the second factor for this series manages to include many of these spikes and hence may explain the reduced memory of this factor. Both common factors of the electricity loads seem to capture a seasonally varying component.

To support the visual impression we estimate the fractional integration orders of the common factors. All factors display mean-reversion with memory estimates less than unity. However, as expected there are significant differences between price common factors in terms of persistence: the first common factor is non-stationary with estimated memory equal to 0.76, whereas the second factor is stationary with memory 0.41. On the other hand, there is no significant difference between the memory estimates of the common factors for loads which are both estimated to have memory around 0.70.

4 Fractional panel data analysis

4.1 Preliminary discussion

Our preliminary analysis suggests that electricity spot prices and loads exhibit considerable long-range dependence. Moreover, it is clear from Figures 3 and 4 that hourly

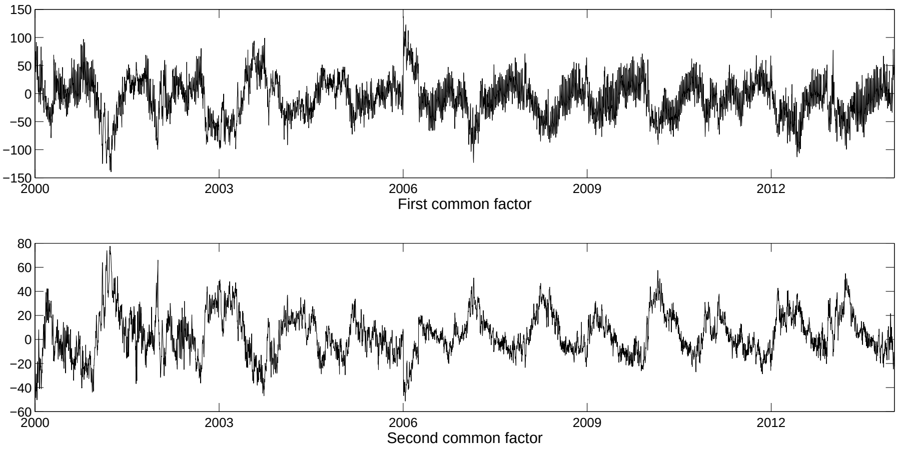

Figure 16: Common factors of electricity loads. The first and second factors explain 78% and 10% of the total variation, respectively.

electricity system prices and loads are heterogeneous. It is well known that time-series or cross-sectional models *per se* cannot accommodate heterogeneity very flexibly. Alternatively, panel data analyses lead to a more robust inference thanks to their ability to control for individual heterogeneity and interactive fixed effects while also allowing for the study of further complex dynamics.

As we have seen, both intraday electricity prices and loads have commonalities that can be modelled as a cross-sectionally dependent panel employing a parametric common-factor structure. This way, for example, the effect of specific actions or general electricity market regulations and other factors that provoke common effects in hourly prices can be isolated. Bearing this fact in mind, we consider two different approaches in turn. The first approach consists in the estimation of a fractionally integrated panel data model with fixed effects proposed by Ergemen and Velasco (2015), in that the common factors extracted in Section 3.3 are treated as covariates in individual models for prices and loads. The second approach employs the fractionally integrated panel data system with fixed effects, proposed by Ergemen (2015) to allow for feedback effects between electricity prices and loads, which is introduced through the allowance for contemporaneous correlation in the model. Both of these approaches account for long memory and seasonality that make them well suited for our analysis given the stylized facts of the data.

Ergemen and Velasco (2015) and Ergemen (2015) propose a conditional-sum-of-squares approach to estimate the slope and long-range dependence parameters. The estimation procedure is based on defactored series which are computed after projecting the prewhitened model on its cross-section average. First, fixed effects are removed by taking first differences. The projection of defactored series is then performed by prewhitening the model variables to stationarity thus guaranteeing the asymptotic removal of projection errors. In our case, taking first differences is enough. The estimates are then obtained in the least-squares sense, which are consistent and asymptotically normally distributed under the regularity conditions that we also assume to hold throughout the paper.

In order to assess the main source of persistence in the data, we first estimate the basic model proposed by Ergemen and Velasco (2015) which includes a common memory parameter for the entire panel and no covariates while allowing for individual and interactive fixed effects:

$$y_{it} = \alpha_i + \lambda_i \mathbb{F}_t + \Delta_{t+1}^{-d} u_{it}. \tag{4}$$

Estimating (4) yields a common memory parameter estimate of 0.47 for electricity prices and 0.61 for electricity loads, and for both estimates the standard error is 0.0022. These estimation results show evidence that the common factor is the main source of persistence when compared to the results in Table 2, especially for the price series. While this is readily established based on a homogeneous memory parameter, this restriction still leaves something to be desired. In particular, there may be an interest in allowing for heterogeneity in the fractional integration parameter to separately examine the behaviour of the series and allowing for explicit interdependencies between the hourly cross-sectional elements.

4.2 Fractional panel cointegration analysis

We now study a more general framework adopting the general model proposed by Ergemen and Velasco (2015) that includes covariates and heterogeneous parameters.

We are primarily interested in understanding the common dynamics of the hourly electricity prices and loads, each treated in separation. We consider the common factors estimated in Section 3.3 as covariates for each of the panels. The aim of this analysis is twofold; first the behaviour of hourly electricity prices and loads is investigated, and then possible cointegrating relationships between hourly electricity prices (or loads) and their respective common factors are analysed with the goal of identifying the supply curve.

We consider the following two-factor augmented panel data models for the price and loads series:

$$p_{it} = \alpha_i + \lambda_{1_i} \hat{f}_{1_{t_{prices}}} + \lambda_{2_i} \hat{f}_{2_{t_{prices}}} + \phi_i \mathbb{F}_{t_{prices}} + \Delta_{t+1}^{-d_i} \epsilon_{it}, \tag{5}$$

and

$$l_{it} = \mu_i + \kappa_{1_i} \hat{f}_{1_{t_{loads}}} + \kappa_{2_i} \hat{f}_{2_{t_{loads}}} + \varphi_i \mathbb{F}_{t_{loads}} + \Delta_{t+1}^{-\delta_i} \varepsilon_{it}, \tag{6}$$

where p_{it} and l_{it} are the filtered series. $\hat{f}_{1t}$ and $\hat{f}_{2t}$ are the respective common factors previously estimated in Section 3.3 for prices and loads. As we discussed in the previous section, common factors exhibit long-range dependence, in particular $\hat{f}_{1_{t_{prices}}} \sim I(\vartheta_{p1})$, $\hat{f}_{2_{t_{prices}}} \sim I(\vartheta_{p2})$, $\hat{f}_{1_{t_{loads}}} \sim I(\vartheta_{l1})$, and $\hat{f}_{2_{t_{loads}}} \sim I(\vartheta_{l2})$. $\mathbb{F}_t$ is incorporated to capture remaining unobservable common factors, which may also be persistent, $\mathbb{F}_{t_{prices}} \sim I(\varsigma_p)$ and $\mathbb{F}_{t_{loads}} \sim I(\varsigma_l)$.

Ergemen and Velasco (2015) present the regularity conditions of this estimation method and show consistency and asymptotic normality of estimates. Under their set of assumptions, we study (5) and (6) where the common unobservable component, $\phi_i \mathbb{F}_t$, along with the other model parameters, is identified cf. Ergemen and Velasco (2015), Pesaran (2006), and Bai (2009).

In the estimation, first-differencing removes the unobserved heterogeneity terms. Then, the unobserved common factor structure, $\phi_i \mathbb{F}_t$, is proxied by projecting the pre-whitened data on its cross-sectional average. Tables 3 and 4 present the estimation results for (5) and (6), respectively. Figure 17 presents an overview of the long-memory parameter estimates.

The main interest in this analysis is to shed light on the relationship between the hourly electricity prices (or loads) and the common factors that drive them. Under certain conditions the hourly prices (or loads) will exhibit intraday fractional cointegration. This happens when the covariates, $\hat{f}_{1_{t.}}$ and $\hat{f}_{2_{t.}}$, exhibit more persistence than the idiosyncratic shocks, for example for prices when $\vartheta_{p1}, \vartheta_{p2} > d_i$, and for loads when $\vartheta_{l1}, \vartheta_{l2} > \delta_i$ for some i.

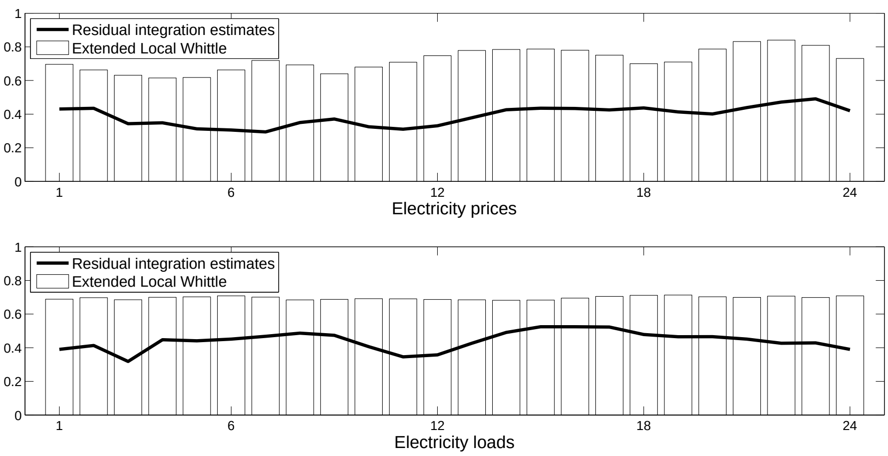

Figure 17: Comparison of the residual integration order estimates of models in (5) and (6) with the original memory estimates of electricity prices and loads. Standard error of the estimates is 0.011.

In this regard, a fractional cointegrating relationship in the panel of electricity

Table 3: Residual integration order estimates and estimated slope parameters of the model in (5) (Prices).

	$\hat{d}_i$	$\hat{\lambda}_{1i}$	$\hat{\lambda}_{2i}$		$\hat{d}_i$	$\hat{\lambda}_{1i}$	$\hat{\lambda}_{2i}$
01:00	0.43	-0.007***	0.013***	13:00	0.38	-0.008***	-0.004***
		(0.001)	(0.002)			(0.000)	(0.000)
02:00	0.43	-0.007***	0.019***	14:00	0.43	-0.008***	-0.003***
		(0.000)	(0.001)			(0.000)	(0.000)
03:00	0.34	-0.007***	0.025***	15:00	0.44	-0.008***	-0.002***
		(0.000)	(0.001)			(0.0004)	(0.001)
04:00	0.35	-0.007***	0.028***	16:00	0.43	-0.008***	-0.002***
		(0.001)	(0.001)			(0.000)	(0.001)
05:00	0.31	-0.007***	0.028***	17:00	0.43	-0.008***	-0.002***
		(0.001)	(0.001)			(0.000)	(0.001)
06:00	0.31	-0.007***	0.025***	18:00	0.44	-0.008***	-0.003***
		(0.000)	(0.002)			(0.000)	(0.001)
07:00	0.29	-0.007***	0.016***	19:00	0.41	-0.008***	-0.003***
		(0.001)	(0.003)			(0.000)	(0.001)
08:00	0.35	-0.008***	0.006***	20:00	0.40	-0.008***	-0.003***
		(0.000)	(0.001)			(0.000)	(0.001)
09:00	0.37	-0.009***	0.002*	21:00	0.44	-0.008***	-0.002***
		(0.001)	(0.001)			(0.000)	(0.001)
10:00	0.33	-0.008***	-0.002*	22:00	0.47	-0.007***	-0.002*
		(0.001)	(0.007)			(0.000)	(0.001)
11:00	0.31	-0.008***	-0.004***	23:00	0.49	-0.007***	0.002*
		(0.000)	(0.001)			(0.000)	(0.001)
12:00	0.33	-0.008***	-0.005***	00:00	0.42	-0.007***	0.008***
		(0.000)	(0.001)			(0.000)	(0.001)

Note: Residual integration order estimates. Standard error of the memory estimates is 0.011 and robust Newey-West standard errors are given in parentheses. Asterisks (*,** and ***) indicate significance at the 10%, 5% and 1% levels, respectively. Fitted seasonal models in (1) were subtracted from log series prior to estimation of the models.

Table 4: Residual integration order estimates and estimated slope parameters of the model in (6) (Loads).

	$\hat{\delta}_i$	$\hat{\kappa}_{1i}$	$\hat{\kappa}_{2i}$		$\hat{\delta}_i$	$\hat{\kappa}_{1i}$	$\hat{\kappa}_{2i}$
01:00	0.39	-0.001***	0.005***	13:00	0.43	-0.003***	-0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
02:00	0.41	-0.001***	0.005***	14:00	0.49	-0.003***	-0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
03:00	0.32	-0.001***	0.005***	15:00	0.52	-0.003***	-0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
04:00	0.45	-0.001***	0.005***	16:00	0.52	-0.003***	-0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
05:00	0.44	-0.001***	0.004***	17:00	0.52	-0.003***	-0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
06:00	0.45	-0.002***	0.003***	18:00	0.48	-0.003***	-0.001**
		(0.000)	(0.000)			(0.000)	(0.000)
07:00	0.47	-0.004***	0.001	19:00	0.47	-0.002***	0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
08:00	0.49	-0.005***	-0.002***	20:00	0.47	-0.002***	0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
09:00	0.47	-0.005***	-0.003***	21:00	0.45	-0.002***	0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
10:00	0.41	-0.004***	-0.002***	22:00	0.43	-0.002***	0.001***
		(0.000)	(0.000)			(0.000)	(0.000)
11:00	0.35	-0.004***	-0.002***	23:00	0.43	-0.001***	0.002***
		(0.000)	(0.000)			(0.000)	(0.000)
12:00	0.36	-0.003***	-0.001***	00:00	0.39	-0.001***	0.003***
		(0.000)	(0.000)			(0.000)	(0.000)

Note: Residual integration order estimates. Standard error of the memory estimates is 0.011 and robust Newey-West standard errors are given in parentheses. Asterisks (*,** and ***) indicate significance at the 10%, 5% and 1% levels, respectively. Fitted seasonal models in (1) were subtracted from log series prior to estimation of the models.

prices is found for every hour and hence we may consider the electricity prices to be driven by the same fractional trend. We find the same conclusion for electricity loads.

While the single price and load series are non-stationary of fractional orders for all hours, the residual integration order estimates are in the stationary range throughout the day for both series. Moreover, the slope parameters $(\lambda_{ji}, \kappa_{ji})$ of both series have the same sign as the loading factors estimated in Section 3.3 making our estimation results consistent with the results previously presented.

5 Fractional panel cointegration analysis of the electricity prices and loads relationship

In this section, we address the supply curve relationship between electricity loads and prices. As discussed in Section 2, the power demand curve varies more than the supply curve which represents the marginal cost curve of the power production technology due to the competitive environment of the electricity market. Hence equilibrium prices and loads are believed to trace the power supply curve, see Bunn (2000).

In the previous section, we considered the general model proposed by Ergemen and Velasco (2015) which assumes independence of the idiosyncratic shocks in the system. This first attempt, in which we estimate equations (5) and (6), allows us to see whether neglected feedback effects between electricity prices and loads play an important role while also providing interpretable results that may be used for policy decisions. To contrast our findings, we next consider the panel data model proposed by Ergemen (2015) which allows for long-range dependence and contemporaneous correlation among idiosyncratic shocks. The model considered can be written as

$$\begin{aligned} p_{it} &= \alpha_i + \beta_i l_{it} + \phi_{1,i}\mathbb{F}_t + \Delta_{t+1}^{-d_i}\epsilon_{1it}, \\ l_{it} &= \nu_i + \phi_{2,i}\mathbb{F}_t + \Delta_{t+1}^{-\vartheta_i}\epsilon_{2it}, \end{aligned} \tag{7}$$

where the dependent variable p_{it} is the filtered electricity prices, and l_{it} is the filtered electricity loads that are treated as a covariate. In (7), $p_{it} \sim I(\max\{d_i, \vartheta_i, \delta\})$, $l_{it} \sim I(\max\{\vartheta_i, \delta\})$ and $\mathbb{F}_t \sim I(\delta)$ for $i = 1, \cdots, 24$ and $t = 1, \cdots, T$ where the true integration orders are unknown. Since both idiosyncratic shocks can be contemporaneously correlated, we take $\varepsilon_{it} = (\epsilon_{1it}, \epsilon_{2it})'$ to be a bivariate covariance stationary process that is assumed to be governed by a first order vector autoregressive process (VAR(1)). Furthermore, GARCH effects in the common factor can be incorporated as discussed by Ergemen (2015), with the projection details readily following from the arguments made therein.

The model in (7) is similar to the model in (5) due to the allowance for cross-sectional dependence through the unobservable common component, and heterogeneity through the unobservable fixed effects α_i and ν_i, and the unobservable factor loadings

$\phi_{1,i}$ and $\phi_{2,i}$. However, the system in (7), unlike (5), conditions p_{it} on the observable series l_{it} with the purpose of identifying the supply curve. In (7), the (observed) factor estimates used in (5) are implicitly contained by $\mathbb{F}_t$ as though they were unobserved since otherwise parameter identification would be impossible. By using (7), possible long-run fractional cointegrating relationship can be disclosed between the idiosyncratic components of the observable series p_{it} and l_{it}, which departs from using (5) that only considers single-equation estimation. Note that failure of accounting for the cross-sectional factor $\mathbb{F}_t$ would potentially lead to spurious regression in a time series context.

In the estimation, the fixed effects are removed by taking first differences in (7),

$$\begin{aligned} \Delta p_{it} &= \beta_i \Delta l_{it} + \phi_{1,i} \Delta \mathbb{F}_t + \Delta_{t+1}^{1-d_i} \epsilon_{1it}, \\ \Delta l_{it} &= \phi_{2,i} \Delta \mathbb{F}_t + \Delta_{t+1}^{1-\vartheta_i} \epsilon_{2it}, \end{aligned} \tag{8}$$

and the model is estimated cf. Ergemen (2015). The estimation results are displayed in Table 5 from which we can check for cointegrating relationships between electricity prices and loads for the hours that satisfy $\hat{\vartheta}_i > \hat{d}_i$. Statistical significance of $\hat{\beta}_i$ for all i in Table 5 implies that the cointegrating relationships that exist are non-trivial. Based on these facts, a long-run equilibrium relationship is confirmed between electricity prices and electricity loads from 7 a.m. to 8 p.m., and this finding is mostly pronounced during work hours.

Figure 18 presents a comparison of the residual integration order estimates of model (7), the Extended Local Whittle estimates of the electricity prices in Table 2 and of the cross-sectionally averaged time-stacked series constituting the maximum memory of the unobserved common factor. As we showed earlier, the common factors are the main source of persistence but nevertheless Extended Local Whittle estimates show some variation (around the blue line in Figure 18). This can simply be explained by the fact that factor loadings can vary across hours and depending on their magnitude, the series can be affected from common factors differently at each hour. Furthermore, Figure 18 visualises significant non-trivial long-run equilibrium relationships between electricity prices and loads from (defining the power supply curve) 7 a.m. to 8 p.m. that can be verified employing a t-test constructed as $t = (\hat{\vartheta}_i - \hat{d}_i)/s.e.(\hat{\vartheta}_i - \hat{d}_i)$ in the direction $\hat{\vartheta}_i > \hat{d}_i$.

Note that the estimates β_i represent the periodically varying power supply elasticities. As Kirschen (2003) shows, a typical supply curve in an electricity market in which prices do not vary significantly for most of the capacity offered (cheap generation capacity), prices do increase abruptly during peak-load conditions due to the steepness of the supply curve demonstrated also in Figure 2. This means that prices vary over time depending on loads conditions, i.e. depending on the time of the day in our case. This is consistent with the supply estimates presented in Table 5 and Figure 19. As seen, the supply elasticities vary between 0.50 and 1.17 with the largest elasticities during peak hours in the mornings and evenings and the lowest during night hours.

Table 5: Residual integration order estimates and estimated slope parameters of the model in (8) (System model).

	$\hat{d}_i$	$\hat{\vartheta}_i$	$\hat{\beta}_i$		$\hat{d}_i$	$\hat{\vartheta}_i$	$\hat{\beta}_i$
01:00	0.17***	0.15***	0.62***	13:00	0.03***	0.27***	0.77***
	(0.00)	(0.00)	(0.07)		(0.00)	(0.00)	(0.05)
02:00	0.17***	0.22***	0.73***	14:00	0.01***	0.28***	0.84***
	(0.00)	(0.00)	(0.1)		(0.00)	(0.00)	(0.05)
03:00	0.18***	0.21***	0.91***	15:00	0.04***	0.24***	0.9***
	(0.00)	(0.00)	(0.05)		(0.00)	(0.00)	(0.05)
04:00	0.16***	0.21***	0.93***	16:00	0.08***	0.28***	0.94***
	(0.00)	(0.00)	(0.13)		(0.00)	(0.00)	(0.05)
05:00	0.15***	0.17***	0.97***	17:00	0.1***	0.27***	0.95***
	(0.00)	(0.00)	(0.13)		(0.00)	(0.00)	(0.05)
06:00	0.08***	0.11***	1.17***	18:00	0.04***	0.25***	0.95***
	(0.00)	(0.00)	(0.12)		(0.00)	(0.00)	(0.05)
07:00	0.08***	0.15***	1.11***	19:00	0.02***	0.21***	0.89***
	(0.00)	(0.00)	(0.09)		(0.00)	(0.00)	(0.05)
08:00	0.06***	0.28***	1.03***	20:00	0.08***	0.13***	0.8***
	(0.00)	(0.00)	(0.06)		(0.00)	(0.00)	(0.05)
09:00	0.07***	0.31***	1.01***	21:00	0.14***	0.16***	0.72***
	(0.00)	(0.00)	(0.05)		(0.00)	(0.00)	(0.04)
10:00	0.09***	0.31***	0.92***	22:00	0.14***	0.21***	0.63***
	(0.00)	(0.00)	(0.04)		(0.00)	(0.00)	(0.03)
11:00	0.05***	0.3***	0.84***	23:00	0.17***	0.27***	0.55***
	(0.00)	(0.00)	(0.05)		(0.00)	(0.00)	(0.04)
12:00	0.02***	0.28***	0.78***	00:00	0.16***	0.28***	0.50***
	(0.00)	(0.00)	(0.05)		(0.00)	(0.00)	(0.05)

Note: Estimation results of the individual slope and memory parameters of the model in (7). Robust Newey-West standard errors are given in parentheses. Asterisks (*,** and ***) indicate significance at the 10%, 5% and 1% levels, respectively.

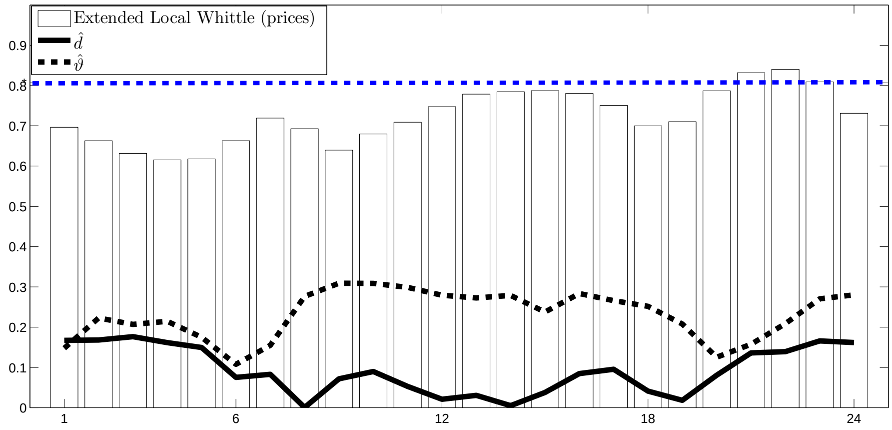

Figure 18: Comparison of the memory estimates of the model in (7) with the Extended Local Whittle estimates of the electricity prices in Table 2 (white bars). Black solid line is the residual integration order estimates of model in (7) whereas the black dot line indicates the values of ϑ_i. The flat dotted (blue) line shows the memory estimate of cross-sectionally averaged stacked series that pertains to the integration order of the unobservable common factor. Standard errors of the Extended Local Whittle estimates are 0.053.

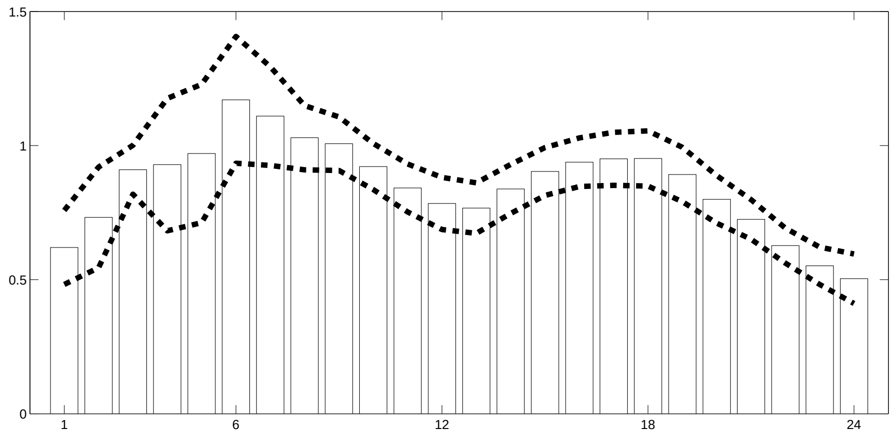

Figure 19: Estimated periodic supply elasticities from the model (7). Dot lines indicate confidence intervals at 95%.

6 Conclusions

We have analysed the complex dynamics of Nord Pool electricity prices and loads in a large periodic panel of hourly observations. Traditionally, hourly electricity spot prices have been modelled in terms of univariate time series for the single hours or by treating the observations as a time series where the information set is updated by moving from one observation to the next over time. However, this assumption is invalid for the electricity spot market due to the particular market microstructure which does not allow continuous trading. Prices and loads are determined in a day-ahead market where the hourly prices of the next day are determined simultaneously in an auction. The hours can thus be seen as a cross section that vary over the day, and hence the panel data structure. The paper is novel because it takes advantage of this particular characteristic of the data and accounts for the stylized features of electricity data such as seasonal variation, spikes, as well as long-range dependence. Both prices and loads contain strong common long-memory components in the cross section of hourly observations as well as across the series themselves. The methodology adopted accounts for unobserved heterogeneity and cross-section dependence and shows that both prices and loads themselves share common fractional trends for all 24 hours of the day. It is also found that by conditioning on the loads it is possible to empirically identify the hourly supply curves of the Nord Pool power production technology. This relation is particularly strong during day-hours.

The analysis provides important insights into the dynamics of the Nord Pool power market. Future research will address how such complex systems can be used from a forecasting perspective. The model class is not directly applicable for forecasting purposes due to the curse of dimensionality problem and the presence of unobservable common factors. It is likely that data reduction methods should be considered under these circumstances such as factor-augmented VARs or similar.

After all, the paper shows that cross-sectional dependence is a significant element for the dynamics of electricity prices and loads.

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