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Testing constancy of unconditional variance in volatility models by misspecification and specification tests

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Abstract

The topic of this paper is testing the hypothesis of constant unconditional variance in GARCH models against the alternative that the unconditional variance changes deterministically over time. Tests of this hypothesis have previously been performed as misspecification tests after fitting a GARCH model to the original series. It is found by simulation that the positive size distortion present in these tests is a function of the kurtosis of the GARCH process. Adjusting the size by numerical methods is considered. The possibility of testing the constancy of the unconditional variance before fitting a GARCH model to the data is discussed. The power of the ensuing test is vastly superior to that of the misspecification test and the size distortion minimal. The test has reasonable power already in very short time series. It would thus serve as a test of constant variance in conditional mean models. An application to exchange rate returns is included.

Keywords: autoregressive conditional heteroskedasticity, modelling volatility, testing parameter constancy, time-varying GARCH

JEL Classification Code: C32, C52

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1 Introduction

Testing constancy of the unconditional variance in GARCH processes is an important step in building useful GARCH models for forecasting. There is ample evidence of the fact that for sufficiently long daily return series, be it exchange rates or individual stock or index returns, the unconditional variance of the underlying process does not remain constant over time. This has consequences for modelling. It is possible to accommodate this feature into GARCH by using long-memory models such as the Fractionally Integrated GARCH process in which nonstationarity depends on the value of the fractionality coefficient, see for example Teräsvirta (2009) and references therein. Another possibility, to be considered here, is to explicitly model the nonstationarity by extending the GARCH framework in a suitable way. There is an expanding literature on the topic beginning with Feng (2004) and van Bellegem and von Sachs (2004), see also Engle and Rangel (2008), Brownlees and Gallo (2010), Osiewalski and Pajor (2009) and Mazur and Pipień (2012). For combining long memory and changes in the unconditional variance, see Baillie and Morana (2009).

In this paper, we consider the Time-Varying GARCH (TV–GARCH) model in which the deterministic component is parametric. This model was introduced by Amado and Teräsvirta (2008) and further discussed and applied in Amado and Teräsvirta (2013, 2014, in press). In this framework it is possible to test constancy, that is, the standard stationary GARCH model against TV–GARCH model using a Lagrange multiplier (LM-) type test and standard asymptotic statistical inference. This is discussed and the resulting test applied in Amado and Teräsvirta (in press), see also Amado and Teräsvirta (2014). But then, it is also possible to test constancy of the unconditional variance without specifying the heteroskedasticity using a similar LM-type test. This alternative strategy will be investigated here. While the former test offers a way of testing the adequacy of an estimated GARCH model and is thus a typical misspecification test, the latter one is rather a tool for specifying a volatility model, i.e., a specification test.

Power properties of these two kinds of test will be compared. Both are consistent. Simulations show that the power of the basic misspecification test considered in Amado and Teräsvirta (in press) is weaker than that of the specification test. When the sample size is sufficiently large, the differences in power are bound to vanish because both tests are consistent.

The 1997 Asian Financial crisis aligns with a change in the volatility dynamics in the local exchange rates. The aforementioned tests will be applied to a set of currency returns (Indonesia, Korea, and Taiwan), also studied in Davidson (2004). The drastic differences in results between the specification and misspecification tests are highlighted.

The plan of the paper is as follows. The TV–GARCH model is presented in

Section 2 and the two tests in Section 3. A simulation experiment is described and results reported in Section 4. The application to exchange rates is presented in Section 5. The conclusions appear in Section 6.

2 The model

The first-order multiplicative TV–GARCH model considered by Amado and Teräsvirta (2008, 2013, 2014, in press) is used for describing the common situation in modelling volatility in which the unconditional variance of the process such as daily returns of an index or a single asset is not constant over time. To define the model, assume that a return sequence $\{y_t\}$ has the form

$$y_t = \mathsf{E}(y_t|\mathcal{F}_{t-1}) + \varepsilon_t \tag{1}$$

where $\mathcal{F}_{t-1}$ contains the historical information available at time $t-1$. For simplicity, set $\mathsf{E}(y_t|\mathcal{F}_{t-1}) = 0$. The innovation sequence $\{\varepsilon_t\}$ has a conditional mean $\mathsf{E}(\varepsilon_t|\mathcal{F}_{t-1}) = 0$ and variance σ_t^2. Each ε_t is decomposed as follows:

$$\varepsilon_t = \zeta_t \sigma_t \tag{2}$$

where the variance σ_t^2 is further decomposed as

$$\sigma_t^2 = h_t g_t. \tag{3}$$

In (2), $\{\zeta_t\} \sim \text{iid}(0, 1)$, $\mathsf{E}\zeta_t^3 = 0$, and $\mathsf{E}|\zeta_t^2|^{2+\phi} < \infty$, $\phi > 0$. The function h_t describes the short-run dynamics of the variance of the returns, whereas g_t is a positive-valued deterministic component. Specifically, h_t is modelled as a GARCH(p, q) process of Bollerslev (1986) and Taylor (1986):

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \phi_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{4}$$

where $\phi_t = \varepsilon_t / g_t^{1/2}$. Equation (4) is assumed to satisfy a set of conditions for positivity and weak stationarity of the conditional variance of ϕ_t, see Bollerslev (1986) and Nelson and Cao (1992).

The standard GARCH(p, q) model is nested in (3) when $g_t \equiv 1$. When $g_t \not\equiv 1$, the unconditional variance component g_t is smooth and time-varying, making σ_t^2 nonstationary. It is a positive-valued linear combination of bounded transition functions defined as follows:

$$g(\boldsymbol{\theta}_1, t/T) = g_t = \delta_0 + \sum_{l=1}^{r} \delta_l G_l(t/T; \gamma_l, \boldsymbol{c}_l) \tag{5}$$

where T is the number of observations, and $\boldsymbol{\theta}_1 = (\boldsymbol{\delta}', \boldsymbol{\gamma}', \boldsymbol{c}_1', ..., \boldsymbol{c}_r')' \in \Theta_1 = (\Delta \times \Gamma \times C)$, with $\boldsymbol{\delta} = (\delta_0, \delta_1, ..., \delta_r)'$, $\boldsymbol{\gamma} = (\gamma_1, ..., \gamma_r)'$, $\boldsymbol{c}_l' = (c_{l1}, ..., c_{lK_l})'$, $l = 1, ..., r$, is an element of the parameter space of g_t. For identification reasons, $\delta_0 = 1$ (a known constant). The transition function is the general logistic transition function:

$$G_l(t/T; \gamma_l, \boldsymbol{c}_l) = \left(1 + \exp\left\{-\gamma_l \prod_{k=1}^{K_l} (t/T - c_{lk})\right\}\right)^{-1}, \ \gamma_l > 0. \tag{6}$$

The function (6) is a continuous and non-negative function bounded between zero and one. When $\gamma_l \to \infty$, the transitions or shifts around c_{lk}, $k = 1, .., K_l$, become abrupt.

3 Testing constancy of the unconditional variance

3.1 Testing constancy in the GARCH framework

As discussed for example in Amado and Teräsvirta (in press), the first step in generalising a GARCH model into a TV–GARCH model is to test stability of the unconditional variance over time. This is important because the model defined by (5) and (6) is not identified when the null hypothesis holds, that is, when $g_t \equiv 1$. In what follows, for simplicity we consider the first-order GARCH model, $p = q = 1$ in (4), assume that $r = 1$ under the alternative in (4) and choose $\gamma_1 = 0$ as our null hypothesis. With this choice of H_0, δ_1 and $\boldsymbol{c}_1$ are unidentified nuisance parameters under H_0. This being the case, standard asymptotic inference is invalid. We circumvent this identification problem as in Luukkonen, Saikkonen and Teräsvirta (1988) and approximate the transition function by a third-order Taylor expansion around $\gamma_1 = 0$. After merging terms and reparameterising this yields

$$G_1(t/T; \gamma_1, \boldsymbol{c}_1) = \theta_0 + \theta_1 t/T + \theta_2 (t/T)^2 + \theta_3 (t/T)^3 + R_{3t} = \boldsymbol{\theta}' \boldsymbol{t}^* + R_{3t} \tag{7}$$

where R_{3t} is the remainder and $\theta_j = \gamma_1^j \overline{\delta}_j$ with $\overline{\delta}_j \neq 0$, $j = 0, 1, 2, 3$. The new null hypothesis is thus H_0': $\boldsymbol{\theta} = \mathbf{0}$.

Constructing an LM-type test for testing H_0' has the advantage that the model is only estimated under the null hypothesis, and then $R_{3t} = 0$. This leads to standard asymptotic inference: the LM statistic has an asymptotic χ^2-distribution with four degrees of freedom when the null hypothesis holds.

The approximate log-likelihood for observation t has the form

$$\ell_t = k - (1/2) \ln h_t - (1/2) \ln g_t^* - \frac{\varepsilon_t^2}{2 h_t g_t^*} \tag{8}$$

where $g_t^* = 1 + \boldsymbol{\theta}'\boldsymbol{t}^*$. Set $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \beta_1)'$ and denote $\widehat{\boldsymbol{r}}_{1t} = \partial\widehat{\ell}_t/\partial\boldsymbol{\alpha} = (\widehat{\zeta}_t^2 - 1)\widehat{h}_t^{-1}\partial\widehat{h}_t/\partial\boldsymbol{\alpha}$, where $\widehat{\ell}_t$ is ℓ_t evaluated under H_0', $\widehat{h}_t$ equals h_t estimated under H_0', $\widehat{\zeta}_t^2 = \varepsilon_t^2/\widehat{h}_t$, and $\partial\widehat{h}_t/\partial\boldsymbol{\alpha} = \widehat{\boldsymbol{m}}_{t-1} + \widehat{\beta}_1\partial\widehat{h}_{t-1}/\partial\boldsymbol{\alpha}$ is $\partial h_t/\partial\boldsymbol{\alpha}$ evaluated under H_0' with $\widehat{\boldsymbol{m}}_t = (1, \varepsilon_t^2, \widehat{h}_t)$. Furthermore, let $\widehat{\boldsymbol{r}}_{2t} = \partial\widehat{\ell}_t/\partial\boldsymbol{\theta} = (\widehat{\zeta}_t^2 - 1)\widehat{g}_t^{*-1}\partial g_t^*/\partial\boldsymbol{\theta} = (\widehat{\zeta}_t^2 - 1)\boldsymbol{t}^*$, where $\widehat{g}_t^* = 1$.

The test can be carried out in stages:

1. Estimate the GARCH(1,1) model, save the squared standardised residuals $\widehat{\zeta}_t^2$ and construct the 'residual sum of squares' $SSR_0 = \sum_{t=1}^{T}(\widehat{\zeta}_t^2 - 1)^2$.

2. Regress $\widehat{\zeta}_t^2 - 1$ on $\widehat{\boldsymbol{r}}_{1t}$ and $\widehat{\boldsymbol{r}}_{2t}$ and form the residual sum of squares SSR_1.

3. Compute the test statistic

$$LM = T\frac{SSR_0 - SSR_1}{SSR_0}. \tag{9}$$

   Under H_0, LM has an asymptotic χ^2-distribution with four degrees of freedom.

The value of the test statistic can also be computed from the conventional quadratic form of the χ^2-statistic. Our simulations suggest, however, that the TR^2 form is numerically more stable when the sum $\alpha_1 + \beta_1$ in (4) assuming $p = q = 1$ is close to (but below) one. The simulation results we report are based on the TR^2 form of the statistic.

3.2 Testing constancy without specifying the conditional variance component

Another way of testing the constancy of the unconditional variance is to do it *before* specifying the conditional variance component. The LM-type test would thus serve as a specification tool and not as a misspecification test as it was introduced in Amado and Teräsvirta (in press) and presented in the preceding section. This implies setting $h_t = 1$ in (8). The transition function is defined as

$$g_t = \delta_0 + \delta_1 G_1(t/T; \gamma_1, \boldsymbol{c}_1) \tag{10}$$

where $G_1(t/T; \gamma_1, \boldsymbol{c}_1)$ is defined as in (6) and $\delta_0 > 0$ is a free parameter because $h_t = 1$. The null hypothesis is $\gamma_1 = 0$, and in testing (10) is approximated by (7). Now, since δ_0 is a free parameter, the null hypothesis is H_0'': $\theta_1 = \theta_2 = \theta_3 = 0$ in (7). The approximate log-likelihood for observation t equals (8) with $h_t = 1$. The maximum likelihood estimator of the free intercept δ_0 equals $\widehat{\delta}_0 = T^{-1}\sum_{t=1}^{T}\varepsilon_t^2$.

Then $\widehat{\boldsymbol{r}}_{1t} = \partial\widehat{\ell}_t/\partial\theta_0 = (\varepsilon_t^2/\widehat{\delta}_0 - 1)\widehat{\delta}_0^{-1}$ and $\widehat{\boldsymbol{r}}_{2t} = \partial\widehat{\ell}_t/\partial\boldsymbol{\theta} = (\varepsilon_t^2/\widehat{\delta}_0 - 1)g_t^{*-1}\partial g_t^*/\partial\boldsymbol{\theta}$ $= (\varepsilon_t^2/\widehat{\delta}_0 - 1)\widehat{\delta}_0^{-1}\boldsymbol{t}^*$, where $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)'$ and $\boldsymbol{t}^* = (t/T, (t/T)^2, (t/T)^3)'$. Stages 1 and 2 of the LM-type test are as follows:

1. Estimate the model (2) and (3) with $h_t \equiv 1$ and $g_t = \delta_0$, save the standardised squared residuals $\widehat{\phi}_t^2 = \varepsilon_t^2/\widehat{\delta}_0$ and construct the 'residual sum of squares' $SSR_0 = \sum_{t=1}^{T}(\widehat{\phi}_t^2 - 1)^2$.

2. Regress $\widehat{\phi}_t^2 - 1$ on $\widehat{\boldsymbol{r}}_{2t}$ and form the residual sum of squares SSR_1.

The final stage consists of computing the TR^2 form of the statistic as in (9). The test statistic has an asymptotic χ^2-distribution with three degrees of freedom when the null hypothesis holds. The assumption of independence of ε_t is likely to be violated in practice, however, because $\{\varepsilon_t\}$ typically contains conditional heteroskedasticity. If this is the case, the relevant critical values of the null distribution of the test statistic have to be found by simulation. This will be discussed in Section 4.3.

4 Simulations

4.1 Critical values for the LM-type test

In this section we address the size discrepancy of the LM-type test of Amado and Teräsvirta (in press). In the simulations reported in that paper it turned out that the test was somewhat oversized even in relatively large samples. This concerns the test the authors called non-robust, which is the one we are going to consider. In order to correct the size of the test, we compute the relevant critical values for the test statistic by simulation. The standard way of doing this is as follows:

1. Generate T observations from the GARCH(1,1) model we are simulating, estimate the parameters using these observations and compute the value of the test statistic.

2. Draw T variables $z_t^{(1)}$, $t = 1, ..., T$, with replacement from the population consisting of the estimated residuals $\widehat{z}_t$ and use them and the estimated conditional variances $\widehat{h}_t$ to obtain a new set of observations $\varepsilon_t^{(1)} = z_t^{(1)}\widehat{h}_t^{1/2}$, $t = 1, ..., T$.

3. Fit the GARCH(1,1) model to this series and the compute the value of the test statistic. Repeat step 2 and this step B times. This yields one estimate of the critical value(s).

4. Repeat steps 1–3 K times and compute the critical value of interest as the mean of the values resulting from these K replications. We set $K = 5000$.

This method is time-consuming because it requires estimating KB GARCH models. Besides, adjusting the size is just a prelude to power simulations, which are our main object of interest. In order to save time, we adopt the warp-speed bootstrap by Giacomini, Politis and White (2013). It differs from the ordinary bootstrap in one important respect. Instead of B bootstraps for each replication, only one is performed ($B = 1$). The authors show that the warp-speed bootstrap is practically as efficient as the standard one.

In the ensuing power simulations we generate 1000 replications for $T = 1000$, 2500 and 5000. Because estimation of GARCH models is numerically difficult (estimates unreliable) in small samples, a few realizations have to be discarded when $T = 1000$. The simulated critical values of the LM-type test, a χ^2-test with four degrees of freedom, for the three sample sizes and the significance levels $\alpha = 0.01$ and 0.05, are computed separately for each experiment and can be found in Table 1. It is seen that the misspecification LM-type test is indeed size distorted and that the problem becomes worse when the kurtosis of the GARCH process increases. This suggests that distinguishing between GARCH and changes in the unconditional variance can become quite difficult when the GARCH process generates clusters with a large amplitude, when the changes present are rather modest, and when the number of observations is not very large. Results of power simulations in the next section seem to support this conclusion.

4.2 Power simulations 1: the misspecification test

A central assumption of the TV–GARCH model is that the changes in the unconditional variance can be smooth. It is of interest to consider the behaviour of tests of constant unconditional variance for different degrees of smoothness measured by the slope parameter γ. It is equally interesting to consider the effect of the size of the switch or switches, measured by δ_1, on the power of the test. In addition, the effect of the GARCH parameters α_1 and β_1 (and thus the kurtosis of ϕ_t) on the power of the test should be investigated as well.

Two sets of parameters in (4) are considered. First, the parameters $\alpha_0 = 0.05$, $\alpha_1 = 0.09$, and $\beta_1 = 0.9$ form a 'big GARCH': kurtosis of ϕ_t equals 16.1. Second, the parameters $\alpha_0 = 0.05$, $\alpha_1 = 0.05$, and $\beta_1 = 0.9$ define an intermediate or 'mild GARCH': kurtosis of ϕ_t equals 3.16. The errors are standard normal, and the GARCH(1,1) model is tested against the approximate alternative (7). The true alternative is (5) with $r = 1$ and $K_1 = 1$ in (6), i.e.,

$$g_t = 1 + \delta_1 \left(1 + \exp\{-\gamma_1 (t/T - c_1)\}\right)^{-1}, \; \gamma_1 > 0.$$

The test of H_0': $\boldsymbol{\theta} = \mathbf{0}$ in (7) is carried out using the TR^2 form described in Section 3.1.

The adjusted critical values for the two cases appear in Table 1. It is seen that the size distortion is an increasing function of the kurtosis of the GARCH process. 'Big GARCH' five and one per cent critical values are considerably larger than those of 'mild GARCH'. This leads one to expect that the size-adjusted power of LM-type test is lower for 'big GARCH' than it is for 'mild GARCH'.

Since the test appears to be consistent, it is of interest to find out what happens to the power in finite samples when the parameters δ_1 and $\gamma_1 = e^{\eta}$ in the alternative are varied. (We reparameterise γ_1 following Goodwin, Holt and Prestemon (2011) and Hurn, Silvennoinen and Teräsvirta (in press) and vary η.) The size-adjusted power of the test at significance levels 0.05 and 0.01 for various values of η and δ_1 when $h_t = 1$ and $T = 2500, 5000$, will be reported for both designs, see Tables 2 and 3. The results for $T = 1000$ can be found at `www. ...` [web address to additional material].

The results for 'big GARCH' for $T = 2500$ and $c_1 = 0.5$ in the top panel of Table 2 indicate that when $\delta_1 \leq 2$, the slope parameter η has little effect on the power of the test. This means that for small shifts, smoothness of the shift is not an important factor. Tables 4 and 5 show that for these values of δ_1, the GARCH parameters are well estimated on average. For larger values of δ_1, $\widehat{\alpha}_1 + \widehat{\beta}_1 \approx 1$. (See Hillebrand (2005, Table 1) for similar results when $\eta = \infty$.) At the same time, for a given $\delta_1 \geq 2$ smooth shifts become easier to detect than abrupt ones. This is particularly clear at $\alpha = 0.01$. In other words, it is more difficult to find evidence for a sudden change in the amplitude of the clusters if the change is abrupt than it is when this change is gradual.

The location of the shift matters. It is seen from the mid-panel of Table 2 that when the mid-point of the positive shift is located early ($c_1 = 0.2$), a shift in the conditional variance is easier to detect than if a similar positive shift occurs halfway through the sample or towards the end; results for $c_1 = 0.8$ can be found in the bottom panel of Table 2. A smooth change, $\eta = 1$, constitutes an exception. When the shift has an early location, the power as a function of η is nonmonotonic. It increases from $\eta = 1$ to $\eta = 2$ and decays thereafter. This phenomenon cannot be seen in the top or the bottom panel of the table. Furthermore, when $c_1 = 0.2$, the decrease in power of the test as a function of η is not fully monotonic. This is particularly clear when $\delta = 2^5$ and $\alpha = 0.01$. There the decay does not begin before $\eta = 3$.

The bottom panel of Table 2 also shows that a late positive shift ($c_1 = 0.8$) is generally difficult to detect unless it happens to be large but at the same time quite smooth ($\eta = 1$). This may be explained by the fact that a smooth change can begin quite early, although its mid-point is located late in the sample. Evidence about the change thus stretches over a large part of the sample. But then, an analogous argument does not fully apply to the case $c_1 = 0.2$. As already pointed out, the results in the mid-panel of Table 2 indicate that the smoothest changes are not as easy to detect as the ones with $\eta > 1$.

It should be pointed out, however, that these results are not invariant to the direction of the shift. In the reported simulations, the variance component changes from 1 to $1 + \delta_1$, where $\delta_1 > 0$, so the shift is positive. If the shift is negative and the change is from $1 + \delta_1$ to 1, the conclusions drawn for $c_1 = 0.8$ are valid for $c_1 = 0.2$ and vice versa. Simulation results not reported here support this claim. In our simulations the shift is a monotonic function of the transition variable t/T. Nonmonotonic shifts are of course possible but are not considered here.

Table 3 contains results for the same designs for $T = 5000$. As may be expected, the powers are higher than for $T = 2500$, but the patterns visible in Table 2 repeat themselves here.

Next consider 'mild GARCH'. Results of the power simulations when $T = 2500$ can be found in Table 6. A comparison with the 'big GARCH' results indicates that the decrease in kurtosis from 16.1 to 3.18 has a strongly positive effect on power. The power patterns found in the two tables are similar, however, in that even in Table 6 the power of the test decreases with increasing η (decreasing smoothness). The decrease is not very strong at $\alpha = 0.05$ because the power is generally quite high but is more clearly visible at the 1% level of significance.

The results in the mid- and bottom panels of Table 6 show that shifts occurring early ($c_1 = 0.2$) are more difficult to find than the ones with their mid-point located late in the sample ($c_1 = 0.8$) when the shift is small. When δ_1 is sufficiently large, these differences even out. The differences in power are more visible at $\alpha = 0.01$. An interesting detail in the bottom panel of the table ($c_1 = 0.2$) is that the power is nonmonotonic in δ_1 when $\eta = 1$. This is most clearly seen for $\alpha = 0.01$. Furthermore, for fixed $\eta \geq 3$ and $c = 0.2$, the power first increases, then decreases and then begins to increase again when δ_1 increases. This pattern is also best seen when $\alpha = 0.01$. With few exceptions when $\alpha = 0.05$, the power is highest for $\delta_1 = 2^5$ for all three values of c_1.

When $T = 5000$, the power of the test is very close to one for all designs, and because of this Table 7 only contains results for $\alpha = 0.01$. The previous patterns are now somewhat harder to see because the power of the tests is considerably higher and for large values of δ_1 quite close to one. Differences between the empirical powers reported in this table and the corresponding values in Table 3 are quite remarkable. It seems that volatility in the GARCH component measured by the kurtosis of ϕ_t has a considerable effect on the size-adjusted power of the test. High volatility makes it difficult to detect fluctuations in the unconditional variance.

Table 8 contains the averages of the GARCH parameter estimates for 'mild GARCH' for $T = 2500$ and Table 9 reports the same averages for $T = 5000$.

While α_1 is reasonably well estimated for small shifts ($\delta_1 \leq 2$), the persistence parameter β_1 is consistently overestimated. The average of the sum $\widehat{\alpha}_1 + \widehat{\beta}_1$ reaches unity for $\delta_1 \geq 2^3$ for $c_1 = 0.2, 0.5$, and remains slightly below one for $c_1 = 0.8$. An increase in the sample size from 2500 to 5000 has little effect on these values.

In practice, it is not possible to accurately adjust the size because the size distortion varies according to the (unknown) parameters of the null model. Previous experience suggests that after parameterising the unconditional variance and estimating the TV–GARCH model, the sum $\widehat{\alpha}_1 + \widehat{\beta}_1$ lies clearly below one. A rule of thumb would be to use the critical values determined for 'mild GARCH', but the size correction would in that case be only approximate.

Two solutions to this problem are available. It is possible to use the robustified LM-type test as defined by Amado and Teräsvirta (in press). Simulations in that paper suggest that it is clearly less size distorted than its nonrobust counterpart. Another alternative would be to test constancy of the unconditional variance *before* estimating the GARCH component. Doing so would prevent the GARCH parameter estimates from absorbing nonstationarity due to nonconstant unconditional variance. This possibility will be considered in the next section.

4.3 Power simulations 2: specification test

We simulate a model in which the conditional heteroskedasticity is generated using the 'big GARCH' parameters: $\alpha_0 = 0.05$, $\alpha_1 = 0.09$ and $\beta_1 = 0.9$. However, in testing constancy of the unconditional variance, the GARCH component is ignored, i.e., it is assumed that $h_t = 1$. The alternative to $\varepsilon_t = z_t \delta_0^{1/2}$ is thus assumed to be $\varepsilon_t = z_t g_t^{1/2}$ where g_t is defined in (5) with $r = 1$ and (6) with $K_1 = 1$.

Since the sequence $\{\varepsilon_t\}$ contains (neglected) conditional heteroskedasticity, the test assuming iid observations under H_0 may be oversized. Even here, the size is corrected using the warp-speed bootstrap, so only a single bootstrap replication is performed for each experiment. The following four steps are needed:

1. Generate T observations from the GARCH model we are simulating, estimate the intercept δ_0 using these observations and compute the value of the test statistic.

2. Draw T independent variables $\varepsilon_t^{(1)} = z_t^{(1)} \widehat{\delta}_0^{1/2}$, $t = 1, ..., T$, using the standard normal distribution for $z_t^{(1)}$.

3. Estimate the intercept δ_0 from this series and the compute the value of the test statistic. This yields one estimate of the critical value(s).

4. Repeat the steps 1–3 $K = 5000$ times and compute the critical value of interest as the mean of the values resulting from these K replications.

The critical values thus obtained can be found in Table 1. It is seen that the test is hardly size distorted, which means that the asymptotic critical values can be used for the sample sizes considered here.[1] When conditional heteroskedasticity is present, estimating the GARCH component before testing constancy of the unconditional variance seems to be the single most important cause of the size distortion observed in the misspecification test.

The simulation results for $T = 2500$ and $c_1 = 0.5$ show that the power is very close to one for all combinations of η and δ_1, the lowest value being equal to 0.977 for $\eta \geq 4$ and $\delta_1 = 1$ at the 1% significance level. The situation hardly changes for $c_1 = 0.2$ or $c_1 = 0.8$. For this reason, no tables for these results are provided. We are able to conclude that it would be preferable to test constancy of the unconditional variance *before* modelling the GARCH component instead of doing it thereafter. The LM-type test would thus be a specification tool and not a misspecification test. One might even suggest that the whole deterministic unconditional variance component (up to the intercept) defined in (5) should be specified by sequential testing as in Amado and Teräsvirta (in press) but before modelling the conditional variance. This possibility will be investigated elsewhere.

Power simulations of this section suggest that the test could work well in much smaller samples and could therefore be used for testing constancy of the variance in models for the conditional mean. In order to consider this idea we repeated the previous simulations for $T = 50$ but generated the data without any conditional heteroskedasticity. In that case the asymptotic theory is valid and no size correction is necessary. Table TABLE:sigval shows that this is still true for $T = 50$. The results appear in Table 10. The power is seems largely independent of η for $\eta \geq 2$. It does increase monotonically with δ_1 and c_1. This means that shifts occurring late are easier to detect than ones occurring early in the sample. This is true for a positive shift: for a negative one the conclusion is reversed. In the last two cases, $c_1 = 0.5$ and $c_1 = 0.8$, the power is already quite reasonable when the shift is sufficiently large. Doubling the length of the series ($T = 100$; results not shown here) considerably increases the power. Summing up, the LM-type test considered here appears a promising tool in testing constancy of the variance in conditional mean models estimated from short time series. A further study of the LM-type test in this context would seem worthwhile but lies beyond the scope of this work.

5 A brief application

To illustrate the behaviour of the tests we apply them to daily exchange rate returns of the Indonesian rupiah, Korean won and Taiwanese dollar from 2 January

[1]For completeness, the experiment is repeated using 'mild GARCH' and the resulting critical values can be found in Table 1.

1994, to 31 December 2012, 6939 observations in total. They are extended and modified versions of the series Davidson (2004) considered and are graphed in Figure 1.[2] Modelling these time series would require more information about the changes in the currency regimes during this period, but here the series only illustrate properties of the tests.

It is seen that there is a sharp increase in volatility around 1997–98 in all three series. Davidson ascribes this to a creeping peg, at least for the rupiah and the won that, after the outburst of the 1997 Asian Financial crisis, was abandoned in favour of a free float. Bouts of turbulence on top of normal clustering appear in all series even thereafter. Since the test statistics can be affected by extreme outliers, see the Indonesian and Korean series in particular, we truncate them to equal ±6 standard deviations of the original series. The estimates of the GARCH(1,1)-parameters α_1 and β_1 can be found in Table 11. The sum $\widehat{\alpha}_1 + \widehat{\beta}_1$ either equals (Indonesia and Korea) or is very close to one (Taiwan).

The values of the χ^2-statistic also appear in Table 11. They are much larger for the specification than for the misspecification test. If the 'mild GARCH' critical values for $T = 5000$ are employed, the misspecification test applied to the residuals of the Taiwanese GARCH model does not even reject the null hypothesis at the 1% level. Even stranger results emerge if the period is restricted to be the same as Davidson's, ending 15 June 2000, so that the single turbulent period following the distinct shift visible in the series dominates. Despite this large shift around late 1997 and early 1998, the misspecification test only rejects the null hypothesis for the won when $\alpha = 0.05$. If the 1% significance level is applied, none of the three null hypotheses is rejected. The specification test, however, strongly rejects constancy of the unconditional variance for returns of all three currencies. These results support the conclusion that the constancy hypothesis should be tested before fitting any GARCH models to return series under consideration.

6 Conclusions

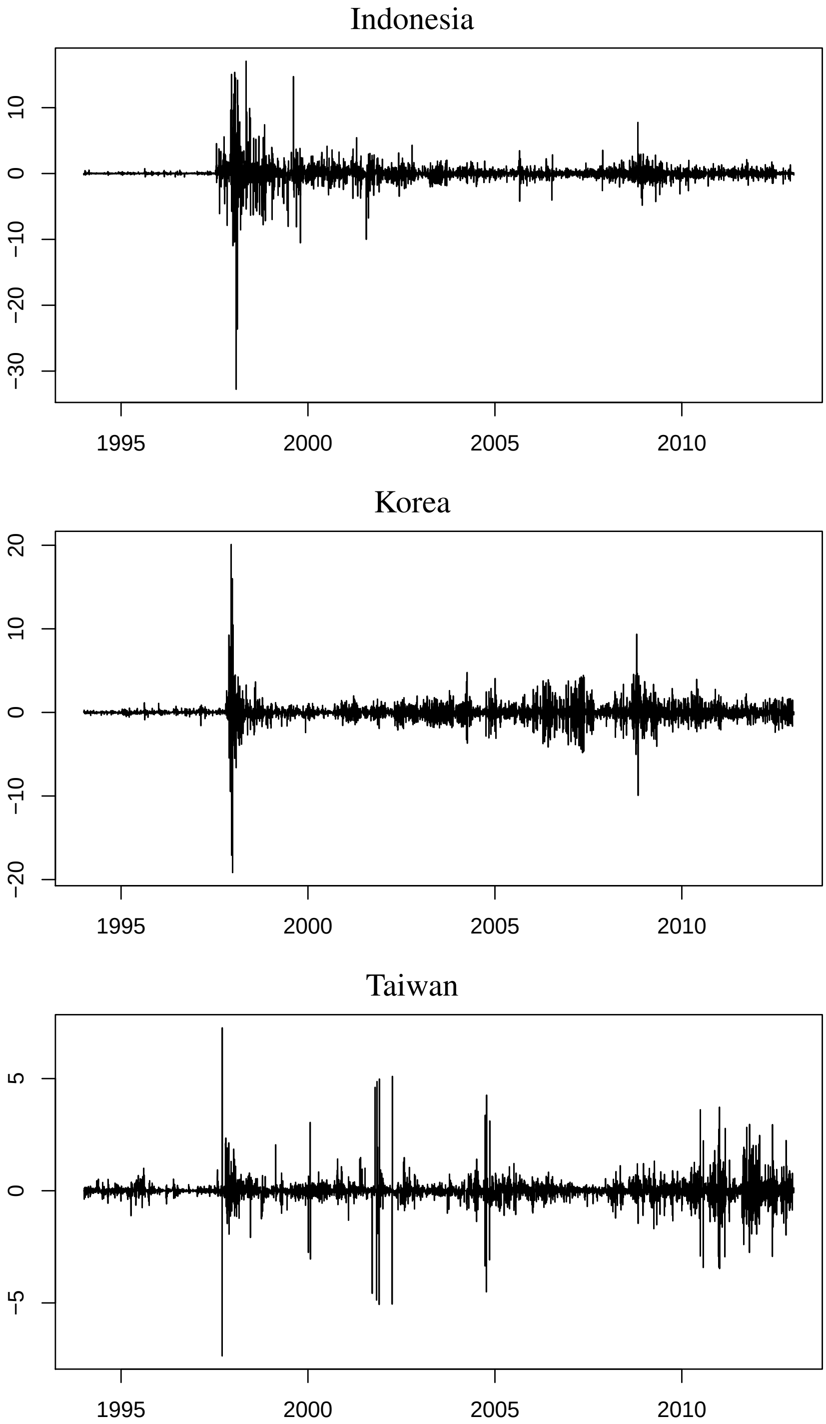
In this work we consider testing the hypothesis of constant unconditional variance in GARCH models. The alternative is that the unconditional variance changes deterministically over time. Such tests have so far been performed as misspecification tests, that is, after fitting a GARCH model to the original series. Previous research has already demonstrated that some of these tests are positively size distorted. We find that size distortion is a function of the kurtosis of the GARCH process. High kurtosis means strong size distortion. Since the null model is unknown in practice, adjusting the size for each application becomes difficult.

[2] Our series are not exactly the same as the ones in Davidson (2004). The difference is that Saturday and Sunday returns are included in our series but not in Davidson's.

This is one reason for considering the possibility of testing constancy of the unconditional variance before fitting a GARCH model to the data. It turns out to be a very useful idea. The power of the test is vastly superior to that of the misspecification test and the size distortion does not seem to be a problem. This suggests rethinking the whole GARCH modelling strategy presented in Amado and Teräsvirta (in press). Instead of fitting the GARCH model to the series first and testing constancy and specifying the unconditional variance component thereafter, one could reverse the order of things. One would then not only test constancy but (if rejected) even specify the whole unconditional variance component of the GARCH process before fitting a GARCH model to the rescaled series. Exploring this suggestion is left for further research.

7 Tables and Figures

Figure 1: Exchange rate returns for Indonesia (IDR/USD), Korea (KRW/USD), and Taiwan (TWD/USD), from 2 January 1994 until 31 December 2012.



Misspecification test

	$T = 1000$		$T = 2500$		$T = 5000$		χ^2_4	
	5%	1%	5%	1%	5%	1%	5%	1%
'big GARCH'	15.547	25.090	13.712	21.596	12.116	19.654	9.488	13.277
'mild GARCH'	12.898	18.556	10.816	15.339	10.182	15.763		

Specification test

	$T = 50$						χ^2_3	
	5%	1%					5%	1%
no heterosk.	7.972	11.301					7.815	11.345

	$T = 1000$		$T = 2500$		$T = 5000$		χ^2_3	
	5%	1%	5%	1%	5%	1%	5%	1%
'big GARCH'	8.018	11.758	7.809	11.633	7.592	11.125	7.815	11.345
'mild GARCH'	8.149	11.777	7.844	11.632	7.636	11.208		

Table 1: Simulated critical values from the 'big GARCH' and 'mild GARCH' experiments (top panel), as well as from the specification test experiments where no heteroskedasticity is present (middle panel) and where the heteroskedasticity is ignored (bottom panel). For reference, the critical values from the theoretical distributions of the test statistics are reported in the two rightmost columns.

$c_1 = 0.5$

$\alpha = 0.05$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.141	0.291	0.483	0.682	0.831	0.928
2	0.168	0.303	0.450	0.610	0.796	0.936
3	0.164	0.268	0.393	0.534	0.741	0.909
4	0.157	0.258	0.356	0.510	0.730	0.903
5	0.156	0.257	0.353	0.505	0.721	0.894
6	0.156	0.255	0.351	0.493	0.703	0.864
7	0.155	0.254	0.356	0.498	0.678	0.813
8	0.155	0.254	0.352	0.496	0.660	0.768
9	0.155	0.254	0.352	0.495	0.653	0.759
10	0.155	0.254	0.352	0.495	0.653	0.757

$\alpha = 0.01$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.030	0.044	0.091	0.188	0.330	0.531
2	0.031	0.049	0.065	0.120	0.241	0.515
3	0.029	0.044	0.046	0.071	0.179	0.446
4	0.027	0.040	0.038	0.066	0.150	0.400
5	0.026	0.039	0.034	0.058	0.142	0.349
6	0.026	0.038	0.035	0.052	0.126	0.321
7	0.026	0.037	0.035	0.051	0.111	0.264
8	0.026	0.037	0.035	0.051	0.108	0.231
9	0.026	0.036	0.035	0.050	0.108	0.218
10	0.026	0.036	0.035	0.050	0.108	0.217

$c_1 = 0.2$

$\alpha = 0.05$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.107	0.188	0.314	0.459	0.570	0.627
2	0.154	0.333	0.581	0.784	0.905	0.958
3	0.169	0.326	0.554	0.741	0.888	0.969
4	0.168	0.308	0.503	0.686	0.863	0.960
5	0.166	0.300	0.476	0.667	0.853	0.958
6	0.164	0.301	0.476	0.662	0.835	0.954
7	0.165	0.302	0.479	0.652	0.819	0.922
8	0.165	0.303	0.481	0.654	0.809	0.890
9	0.165	0.302	0.477	0.659	0.809	0.887
10	0.165	0.302	0.477	0.660	0.809	0.886

$\alpha = 0.01$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.022	0.028	0.051	0.091	0.118	0.155
2	0.025	0.058	0.129	0.261	0.410	0.616
3	0.027	0.054	0.130	0.215	0.374	0.657
4	0.026	0.045	0.108	0.166	0.313	0.616
5	0.026	0.042	0.099	0.149	0.279	0.577
6	0.025	0.042	0.096	0.141	0.262	0.531
7	0.025	0.041	0.097	0.135	0.242	0.451
8	0.024	0.041	0.099	0.132	0.238	0.393
9	0.024	0.041	0.099	0.137	0.235	0.386
10	0.024	0.041	0.099	0.138	0.234	0.386

$c_1 = 0.8$

$\alpha = 0.05$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.103	0.168	0.299	0.485	0.675	0.861
2	0.105	0.158	0.245	0.371	0.534	0.764
3	0.104	0.146	0.203	0.295	0.439	0.659
4	0.107	0.136	0.199	0.265	0.404	0.624
5	0.106	0.135	0.197	0.255	0.391	0.601
6	0.104	0.135	0.195	0.257	0.391	0.578
7	0.104	0.134	0.197	0.253	0.376	0.539
8	0.105	0.134	0.194	0.258	0.358	0.483
9	0.105	0.134	0.194	0.258	0.357	0.482
10	0.105	0.134	0.194	0.258	0.358	0.482

$\alpha = 0.01$

	δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.033	0.036	0.052	0.092	0.170	0.345
2	0.033	0.036	0.040	0.066	0.108	0.235
3	0.035	0.036	0.034	0.046	0.071	0.161
4	0.035	0.036	0.037	0.043	0.052	0.130
5	0.034	0.035	0.033	0.036	0.043	0.115
6	0.033	0.035	0.032	0.036	0.041	0.101
7	0.033	0.035	0.032	0.037	0.041	0.091
8	0.032	0.036	0.032	0.037	0.040	0.085
9	0.032	0.036	0.032	0.039	0.040	0.083
10	0.032	0.036	0.032	0.039	0.040	0.083

Table 2: Estimated power of the LM-type statistic for testing the 'big GARCH' model against time-varying unconditional variance with $T = 2500$, locations $c_1 = 0.5, 0.2$, and 0.8, and significance levels $\alpha = 0.05$ and 0.01.

$c_1 = 0.5$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.306	0.619	0.874	0.949	0.986	0.998	1	0.072	0.201	0.415	0.604	0.777	0.916
2	0.383	0.685	0.865	0.946	0.993	0.999	2	0.094	0.228	0.365	0.509	0.724	0.909
3	0.369	0.648	0.798	0.910	0.983	0.999	3	0.088	0.202	0.285	0.410	0.642	0.857
4	0.361	0.627	0.777	0.898	0.975	0.996	4	0.083	0.179	0.254	0.376	0.611	0.837
5	0.359	0.618	0.766	0.891	0.975	0.996	5	0.083	0.175	0.245	0.375	0.597	0.823
6	0.358	0.617	0.763	0.894	0.977	0.996	6	0.083	0.173	0.245	0.371	0.592	0.820
7	0.357	0.622	0.769	0.899	0.973	0.996	7	0.083	0.174	0.244	0.372	0.584	0.800
8	0.356	0.618	0.770	0.897	0.968	0.989	8	0.082	0.175	0.248	0.373	0.568	0.755
9	0.356	0.617	0.771	0.895	0.967	0.979	9	0.082	0.174	0.247	0.371	0.563	0.725
10	0.356	0.617	0.771	0.896	0.965	0.979	10	0.082	0.174	0.248	0.372	0.557	0.721

$c_1 = 0.2$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.212	0.403	0.635	0.790	0.874	0.907	1	0.045	0.109	0.244	0.389	0.516	0.575
2	0.340	0.652	0.887	0.967	0.994	0.998	2	0.079	0.277	0.529	0.734	0.881	0.954
3	0.364	0.669	0.865	0.956	0.992	1.000	3	0.089	0.281	0.464	0.647	0.850	0.960
4	0.366	0.656	0.836	0.950	0.990	1.000	4	0.091	0.253	0.406	0.589	0.823	0.957
5	0.367	0.644	0.828	0.945	0.991	1.000	5	0.087	0.239	0.380	0.560	0.816	0.957
6	0.364	0.643	0.825	0.945	0.990	1.000	6	0.086	0.239	0.376	0.560	0.813	0.957
7	0.365	0.643	0.825	0.942	0.989	1.000	7	0.086	0.241	0.377	0.560	0.802	0.946
8	0.364	0.642	0.826	0.941	0.987	0.997	8	0.087	0.239	0.377	0.555	0.780	0.903
9	0.364	0.642	0.828	0.940	0.986	0.991	9	0.087	0.238	0.378	0.545	0.767	0.874
10	0.364	0.642	0.827	0.940	0.986	0.992	10	0.087	0.237	0.379	0.549	0.767	0.878

$c_1 = 0.8$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.203	0.379	0.671	0.873	0.962	0.992	1	0.051	0.092	0.228	0.385	0.592	0.796
2	0.210	0.376	0.608	0.766	0.888	0.970	2	0.051	0.092	0.165	0.263	0.406	0.624
3	0.205	0.357	0.533	0.681	0.824	0.926	3	0.052	0.082	0.118	0.192	0.305	0.466
4	0.201	0.336	0.490	0.636	0.789	0.893	4	0.052	0.081	0.111	0.157	0.264	0.417
5	0.198	0.333	0.483	0.628	0.779	0.896	5	0.049	0.074	0.109	0.151	0.250	0.400
6	0.199	0.336	0.487	0.632	0.780	0.895	6	0.050	0.077	0.113	0.155	0.248	0.394
7	0.198	0.337	0.484	0.638	0.783	0.887	7	0.051	0.077	0.112	0.154	0.252	0.386
8	0.198	0.338	0.487	0.643	0.779	0.858	8	0.051	0.078	0.112	0.155	0.248	0.362
9	0.198	0.338	0.485	0.644	0.772	0.851	9	0.051	0.078	0.109	0.155	0.242	0.344
10	0.198	0.338	0.486	0.640	0.772	0.846	10	0.051	0.078	0.109	0.155	0.241	0.344

Table 3: Estimated power of the LM-type statistic for testing the ‘big GARCH’ model against time-varying unconditional variance with $T = 5000$, locations $c_1 = 0.5, 0.2$, and 0.8, and significance levels $\alpha = 0.05$ and 0.01.

$c_1 = 0.5$

$\widehat{\alpha}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.092	0.092	0.094	0.095	0.097	0.099
2	0.092	0.093	0.093	0.094	0.097	0.102
3	0.092	0.093	0.094	0.095	0.098	0.104
4	0.092	0.094	0.095	0.097	0.101	0.107
5	0.092	0.094	0.096	0.098	0.103	0.110

$\widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.899	0.902	0.904	0.905	0.903	0.902
2	0.900	0.903	0.906	0.906	0.903	0.898
3	0.900	0.904	0.906	0.905	0.902	0.896
4	0.901	0.903	0.905	0.903	0.899	0.893
5	0.900	0.903	0.904	0.902	0.897	0.890

$\alpha_1 + \widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.991	0.994	0.998	1.000	1.000	1.000
2	0.992	0.996	0.999	1.000	1.000	1.000
3	0.992	0.996	0.999	1.000	1.000	1.000
4	0.992	0.997	0.999	1.000	1.000	1.000
5	0.992	0.997	0.999	1.000	1.000	1.000

$c_1 = 0.2$

$\widehat{\alpha}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.092	0.093	0.094	0.095	0.096	0.096
2	0.092	0.094	0.095	0.097	0.098	0.101
3	0.092	0.094	0.095	0.096	0.098	0.103
4	0.092	0.094	0.096	0.097	0.099	0.105
5	0.093	0.095	0.097	0.098	0.101	0.108

$\widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.897	0.898	0.899	0.900	0.900	0.900
2	0.898	0.900	0.902	0.902	0.901	0.899
3	0.899	0.901	0.903	0.904	0.902	0.897
4	0.899	0.901	0.903	0.903	0.901	0.895
5	0.898	0.901	0.902	0.902	0.899	0.892

$\alpha_1 + \widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.989	0.991	0.993	0.995	0.996	0.997
2	0.990	0.993	0.997	0.999	1.000	1.000
3	0.991	0.995	0.998	1.000	1.000	1.000
4	0.991	0.995	0.999	1.000	1.000	1.000
5	0.991	0.995	0.999	1.000	1.000	1.000

$c_1 = 0.8$

$\widehat{\alpha}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.091	0.092	0.093	0.094	0.095	0.097
2	0.091	0.092	0.093	0.095	0.097	0.100
3	0.092	0.092	0.093	0.096	0.099	0.103
4	0.092	0.093	0.095	0.098	0.101	0.106
5	0.092	0.093	0.096	0.099	0.104	0.111

$\widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.898	0.901	0.903	0.905	0.905	0.903
2	0.899	0.901	0.903	0.904	0.903	0.900
3	0.899	0.901	0.903	0.902	0.901	0.897
4	0.899	0.901	0.902	0.900	0.898	0.893
5	0.899	0.900	0.901	0.899	0.895	0.889

$\alpha_1 + \widehat{\beta}_1$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.990	0.992	0.996	0.999	1.000	1.000
2	0.990	0.993	0.996	0.999	1.000	1.000
3	0.991	0.993	0.996	0.998	0.999	1.000
4	0.991	0.993	0.996	0.998	0.999	1.000
5	0.991	0.993	0.996	0.998	0.999	1.000

Table 4: Averages of GARCH parameter estimates $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ from the 'big GARCH' experiment, $T = 2500$, and $c_1 = 0.5$, 0.2, and 0.8. Results for $\eta > 5$ are similar to the ones for $\eta = 5$ and not reported.

$c_1 = 0.5$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.091	0.092	0.092	0.092	0.092	0.092	0.901	0.904	0.906	0.908	0.908	0.908	0.992	0.995	0.999	1.000	1.000	1.000
2	0.091	0.092	0.092	0.091	0.091	0.093	0.902	0.905	0.908	0.909	0.909	0.907	0.993	0.997	1.000	1.000	1.000	1.000
3	0.091	0.092	0.092	0.091	0.092	0.095	0.902	0.906	0.908	0.909	0.908	0.905	0.994	0.997	1.000	1.000	1.000	1.000
4	0.091	0.092	0.092	0.092	0.093	0.097	0.902	0.906	0.908	0.908	0.907	0.903	0.994	0.997	1.000	1.000	1.000	1.000
5	0.091	0.092	0.092	0.092	0.095	0.098	0.902	0.906	0.908	0.908	0.905	0.902	0.994	0.997	1.000	1.000	1.000	1.000

$c_1 = 0.2$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.091	0.092	0.092	0.093	0.093	0.093	0.899	0.901	0.902	0.903	0.904	0.905	0.990	0.992	0.994	0.996	0.997	0.998
2	0.091	0.092	0.092	0.092	0.091	0.092	0.900	0.903	0.906	0.908	0.909	0.908	0.992	0.995	0.998	1.000	1.000	1.000
3	0.091	0.092	0.092	0.090	0.090	0.093	0.901	0.904	0.908	0.910	0.910	0.907	0.992	0.996	0.999	1.000	1.000	1.000
4	0.091	0.092	0.092	0.090	0.090	0.093	0.901	0.904	0.908	0.910	0.910	0.907	0.993	0.997	1.000	1.000	1.000	1.000
5	0.092	0.092	0.093	0.091	0.091	0.094	0.901	0.904	0.907	0.909	0.909	0.906	0.993	0.997	1.000	1.000	1.000	1.000

$c_1 = 0.8$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.091	0.091	0.092	0.092	0.092	0.093	0.900	0.902	0.905	0.907	0.908	0.907	0.991	0.993	0.996	0.999	1.000	1.000
2	0.091	0.091	0.092	0.092	0.093	0.095	0.900	0.903	0.905	0.906	0.906	0.905	0.991	0.994	0.997	0.999	1.000	1.000
3	0.091	0.091	0.092	0.093	0.094	0.096	0.901	0.903	0.905	0.905	0.905	0.904	0.992	0.994	0.996	0.998	0.999	1.000
4	0.091	0.092	0.092	0.093	0.095	0.098	0.901	0.903	0.904	0.905	0.904	0.902	0.992	0.994	0.996	0.998	0.999	1.000
5	0.091	0.092	0.093	0.094	0.097	0.100	0.901	0.903	0.904	0.904	0.902	0.900	0.992	0.994	0.996	0.998	0.999	0.999

Table 5: Averages of GARCH parameter estimates $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ from the 'big GARCH' experiment, $T = 5000$, and $c_1 = 0.5$, 0.2, and 0.8. Results for $\eta > 5$ are similar to the ones for $\eta = 5$ and not reported.

$c_1 = 0.5$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.908	0.994	0.995	1.000	1.000	1.000	1	0.722	0.934	0.960	0.982	0.999	1.000
2	0.943	0.970	0.990	0.998	1.000	1.000	2	0.765	0.840	0.829	0.906	0.992	1.000
3	0.919	0.932	0.949	0.992	1.000	1.000	3	0.681	0.680	0.634	0.768	0.963	0.999
4	0.908	0.926	0.936	0.984	0.999	1.000	4	0.637	0.638	0.584	0.705	0.917	0.998
5	0.909	0.932	0.939	0.982	0.999	1.000	5	0.638	0.639	0.577	0.689	0.897	0.991
6	0.909	0.934	0.944	0.980	0.998	0.997	6	0.639	0.642	0.583	0.692	0.875	0.966
7	0.910	0.934	0.945	0.980	0.994	0.987	7	0.639	0.644	0.584	0.696	0.849	0.912
8	0.910	0.936	0.945	0.979	0.991	0.974	8	0.638	0.645	0.586	0.694	0.840	0.868
9	0.910	0.936	0.946	0.979	0.990	0.974	9	0.638	0.645	0.587	0.695	0.837	0.868
10	0.910	0.936	0.946	0.979	0.990	0.974	10	0.638	0.645	0.587	0.696	0.837	0.869

$c_1 = 0.2$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.601	0.940	0.998	1.000	1.000	1.000	1	0.297	0.746	0.960	0.991	0.995	0.997
2	0.854	0.994	0.999	1.000	1.000	1.000	2	0.566	0.936	0.969	0.983	0.997	1.000
3	0.867	0.972	0.979	0.999	1.000	1.000	3	0.570	0.808	0.787	0.876	0.995	1.000
4	0.851	0.947	0.951	0.992	1.000	1.000	4	0.532	0.706	0.619	0.734	0.967	1.000
5	0.850	0.944	0.934	0.982	1.000	1.000	5	0.523	0.681	0.575	0.668	0.929	0.999
6	0.849	0.942	0.930	0.976	1.000	0.999	6	0.522	0.676	0.566	0.629	0.895	0.979
7	0.849	0.942	0.933	0.976	0.995	0.990	7	0.521	0.675	0.568	0.626	0.874	0.942
8	0.848	0.942	0.935	0.976	0.993	0.984	8	0.522	0.677	0.573	0.638	0.856	0.911
9	0.848	0.942	0.935	0.976	0.994	0.983	9	0.522	0.677	0.574	0.640	0.859	0.909
10	0.848	0.942	0.935	0.976	0.994	0.983	10	0.522	0.677	0.574	0.640	0.859	0.909

$c_1 = 0.8$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.757	0.975	0.995	1.000	1.000	1.000	1	0.455	0.848	0.946	0.982	0.999	1.000
2	0.812	0.959	0.980	0.998	1.000	1.000	2	0.517	0.767	0.802	0.870	0.966	0.999
3	0.771	0.912	0.947	0.981	0.999	1.000	3	0.466	0.624	0.659	0.753	0.899	0.989
4	0.748	0.896	0.936	0.975	0.996	1.000	4	0.427	0.581	0.613	0.727	0.886	0.979
5	0.743	0.898	0.944	0.979	0.997	0.999	5	0.422	0.576	0.628	0.753	0.899	0.977
6	0.743	0.901	0.948	0.982	0.996	0.997	6	0.422	0.584	0.648	0.773	0.904	0.963
7	0.744	0.901	0.951	0.981	0.995	0.994	7	0.422	0.584	0.652	0.778	0.897	0.947
8	0.746	0.901	0.951	0.982	0.993	0.990	8	0.423	0.585	0.653	0.779	0.892	0.915
9	0.746	0.901	0.950	0.982	0.993	0.988	9	0.423	0.585	0.652	0.778	0.892	0.913
10	0.746	0.901	0.950	0.982	0.993	0.988	10	0.423	0.585	0.652	0.778	0.892	0.913

Table 6: Estimated power of the LM-type statistic for testing the 'mild GARCH' model against time-varying unconditional variance with $T = 2500$, locations $c_1 = 0.5, 0.2$, and 0.8, and significance levels $\alpha = 0.05$ and 0.01.

$c_1 = 0.5$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	1.000	1.000	1.000	1.000	1.000	1.000	1	0.993	0.999	0.999	1.000	1.000	1.000
2	0.999	1.000	1.000	1.000	1.000	1.000	2	0.994	0.993	0.995	1.000	1.000	1.000
3	0.999	0.999	1.000	1.000	1.000	1.000	3	0.984	0.979	0.969	0.992	1.000	1.000
4	1.000	0.999	1.000	1.000	1.000	1.000	4	0.978	0.963	0.943	0.980	0.999	1.000
5	1.000	0.998	1.000	1.000	1.000	1.000	5	0.977	0.963	0.949	0.982	0.999	1.000
6	1.000	0.998	1.000	1.000	1.000	1.000	6	0.977	0.963	0.959	0.982	0.999	1.000
7	1.000	0.998	1.000	1.000	1.000	1.000	7	0.977	0.962	0.959	0.982	0.998	0.998
8	1.000	0.998	1.000	1.000	1.000	0.998	8	0.977	0.962	0.961	0.984	0.997	0.994
9	1.000	0.998	1.000	1.000	1.000	0.998	9	0.977	0.962	0.961	0.984	0.997	0.984
10	1.000	0.998	1.000	1.000	1.000	0.998	10	0.977	0.962	0.961	0.984	0.997	0.983

$c_1 = 0.2$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.943	0.999	1.000	1.000	1.000	1.000	1	0.738	0.993	1.000	1.000	1.000	1.000
2	0.997	1.000	1.000	1.000	1.000	1.000	2	0.961	0.997	0.997	1.000	1.000	1.000
3	0.997	0.999	1.000	1.000	1.000	1.000	3	0.959	0.978	0.970	0.993	1.000	1.000
4	0.996	0.997	0.998	1.000	1.000	1.000	4	0.944	0.951	0.893	0.974	1.000	1.000
5	0.994	0.997	0.998	1.000	1.000	1.000	5	0.942	0.933	0.867	0.944	1.000	1.000
6	0.994	0.997	0.998	1.000	1.000	1.000	6	0.940	0.931	0.865	0.937	1.000	1.000
7	0.994	0.997	0.998	1.000	1.000	1.000	7	0.940	0.930	0.867	0.936	0.998	1.000
8	0.994	0.997	0.998	1.000	1.000	0.999	8	0.940	0.931	0.873	0.937	0.997	0.991
9	0.994	0.997	0.998	1.000	1.000	0.998	9	0.940	0.931	0.874	0.936	0.996	0.985
10	0.994	0.997	0.998	1.000	1.000	0.998	10	0.940	0.931	0.874	0.938	0.996	0.986

$c_1 = 0.8$

	$\alpha = 0.05$							$\alpha = 0.01$					
	δ_1							δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	η	2^0	2^1	2^2	2^3	2^4	2^5
1	0.989	1.000	1.000	1.000	1.000	1.000	1	0.895	1.000	1.000	1.000	1.000	1.000
2	0.999	1.000	1.000	1.000	1.000	1.000	2	0.952	0.997	0.997	0.998	1.000	1.000
3	0.998	0.999	1.000	1.000	1.000	1.000	3	0.928	0.984	0.986	0.991	0.999	1.000
4	0.996	1.000	1.000	1.000	1.000	1.000	4	0.894	0.972	0.978	0.992	0.999	1.000
5	0.997	1.000	1.000	1.000	1.000	1.000	5	0.889	0.971	0.981	0.995	1.000	1.000
6	0.996	1.000	1.000	1.000	1.000	1.000	6	0.889	0.974	0.983	0.998	1.000	1.000
7	0.996	1.000	1.000	1.000	1.000	1.000	7	0.888	0.975	0.984	0.998	1.000	1.000
8	0.996	1.000	1.000	1.000	1.000	1.000	8	0.888	0.974	0.985	0.998	1.000	0.996
9	0.996	1.000	1.000	1.000	1.000	0.999	9	0.888	0.974	0.985	0.998	0.998	0.990
10	0.996	1.000	1.000	1.000	1.000	0.999	10	0.888	0.974	0.985	0.998	0.998	0.990

Table 7: Estimated power of the LM-type statistic for testing the 'mild GARCH' model against time-varying unconditional variance with $T = 5000$, locations $c_1 = 0.5, 0.2$, and 0.8, and significance levels $\alpha = 0.05$ and 0.01.

$c_1 = 0.5$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.048	0.046	0.047	0.053	0.059	0.065	0.931	0.947	0.951	0.947	0.941	0.935	0.979	0.993	0.998	1.000	1.000	1.000
2	0.047	0.045	0.048	0.055	0.062	0.071	0.939	0.951	0.951	0.945	0.938	0.929	0.986	0.996	0.999	1.000	1.000	1.000
3	0.047	0.046	0.050	0.057	0.065	0.074	0.941	0.950	0.949	0.943	0.935	0.926	0.987	0.996	0.999	1.000	1.000	1.000
4	0.047	0.047	0.052	0.060	0.069	0.078	0.940	0.949	0.947	0.940	0.931	0.922	0.987	0.996	0.999	1.000	1.000	1.000
5	0.048	0.048	0.054	0.063	0.073	0.083	0.940	0.947	0.945	0.937	0.928	0.917	0.987	0.996	0.999	1.000	1.000	1.000

$c_1 = 0.2$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.051	0.050	0.049	0.050	0.052	0.053	0.912	0.927	0.938	0.943	0.944	0.944	0.963	0.977	0.988	0.993	0.995	0.997
2	0.050	0.049	0.052	0.058	0.064	0.071	0.924	0.942	0.946	0.942	0.936	0.929	0.974	0.990	0.998	1.000	1.000	1.000
3	0.050	0.049	0.053	0.059	0.067	0.076	0.929	0.945	0.946	0.941	0.933	0.924	0.979	0.994	0.999	1.000	1.000	1.000
4	0.050	0.050	0.055	0.062	0.070	0.079	0.930	0.945	0.945	0.938	0.930	0.921	0.980	0.995	1.000	1.000	1.000	1.000
5	0.050	0.051	0.057	0.065	0.073	0.083	0.930	0.944	0.943	0.935	0.927	0.917	0.980	0.995	1.000	1.000	1.000	1.000

$c_1 = 0.8$

η	$\widehat{\alpha}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5	$\widehat{\alpha}_1 + \widehat{\beta}_1$, $\delta_1 = 2^0$	2^1	2^2	2^3	2^4	2^5
1	0.050	0.047	0.045	0.048	0.053	0.061	0.920	0.940	0.950	0.951	0.946	0.939	0.970	0.987	0.995	0.998	1.000	1.000
2	0.049	0.047	0.046	0.050	0.056	0.064	0.927	0.943	0.949	0.948	0.944	0.936	0.976	0.989	0.995	0.998	0.999	1.000
3	0.049	0.048	0.049	0.053	0.060	0.068	0.929	0.942	0.946	0.944	0.938	0.930	0.978	0.990	0.995	0.997	0.998	0.999
4	0.050	0.049	0.052	0.057	0.065	0.074	0.928	0.940	0.942	0.939	0.932	0.924	0.978	0.989	0.994	0.996	0.997	0.998
5	0.050	0.050	0.054	0.061	0.069	0.080	0.928	0.938	0.939	0.935	0.927	0.917	0.978	0.989	0.993	0.995	0.996	0.997

Table 8: Averages of GARCH parameter estimates $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ from the 'mild GARCH' experiment, $T = 2500$, and $c_1 = 0.5$, 0.2, and 0.8. Results for $\eta > 5$ are similar to the ones for $\eta = 5$ and not reported.

$c_1 = 0.5$

	$\widehat{\alpha}_1$						$\widehat{\beta}_1$						$\widehat{\alpha}_1 + \widehat{\beta}_1$					
	δ_1						δ_1						δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5
1	0.048	0.044	0.043	0.046	0.050	0.054	0.934	0.950	0.955	0.954	0.950	0.946	0.981	0.993	0.998	1.000	1.000	1.000
2	0.046	0.042	0.044	0.047	0.051	0.058	0.941	0.954	0.955	0.953	0.949	0.942	0.987	0.996	0.999	1.000	1.000	1.000
3	0.045	0.043	0.044	0.048	0.053	0.059	0.943	0.954	0.955	0.952	0.946	0.941	0.989	0.996	0.999	1.000	1.000	1.000
4	0.046	0.043	0.045	0.050	0.056	0.062	0.943	0.953	0.954	0.950	0.944	0.938	0.989	0.997	0.999	1.000	1.000	1.000
5	0.046	0.044	0.047	0.052	0.058	0.065	0.943	0.952	0.952	0.948	0.942	0.935	0.989	0.996	0.999	1.000	1.000	1.000

$c_1 = 0.2$

	$\widehat{\alpha}_1$						$\widehat{\beta}_1$						$\widehat{\alpha}_1 + \widehat{\beta}_1$					
	δ_1						δ_1						δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5
1	0.050	0.048	0.047	0.046	0.046	0.047	0.917	0.932	0.944	0.949	0.950	0.951	0.967	0.981	0.990	0.995	0.997	0.998
2	0.049	0.045	0.045	0.049	0.053	0.058	0.929	0.947	0.953	0.951	0.947	0.942	0.977	0.992	0.998	1.000	1.000	1.000
3	0.048	0.044	0.046	0.049	0.055	0.060	0.934	0.951	0.954	0.951	0.945	0.940	0.982	0.995	1.000	1.000	1.000	1.000
4	0.048	0.045	0.047	0.051	0.056	0.062	0.936	0.951	0.953	0.949	0.944	0.938	0.983	0.996	1.000	1.000	1.000	1.000
5	0.048	0.045	0.048	0.052	0.058	0.064	0.936	0.951	0.951	0.948	0.942	0.936	0.984	0.996	1.000	1.000	1.000	1.000

$c_1 = 0.8$

	$\widehat{\alpha}_1$						$\widehat{\beta}_1$						$\widehat{\alpha}_1 + \widehat{\beta}_1$					
	δ_1						δ_1						δ_1					
η	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5	2^0	2^1	2^2	2^3	2^4	2^5
1	0.049	0.046	0.043	0.043	0.047	0.051	0.923	0.941	0.952	0.955	0.953	0.949	0.972	0.987	0.995	0.998	1.000	1.000
2	0.048	0.045	0.044	0.044	0.048	0.053	0.929	0.945	0.952	0.953	0.951	0.947	0.978	0.990	0.995	0.998	0.999	0.999
3	0.048	0.045	0.045	0.046	0.050	0.055	0.931	0.945	0.950	0.951	0.948	0.943	0.980	0.990	0.995	0.997	0.998	0.998
4	0.048	0.046	0.046	0.048	0.053	0.059	0.932	0.944	0.949	0.948	0.944	0.939	0.980	0.990	0.995	0.996	0.997	0.998
5	0.048	0.047	0.048	0.051	0.056	0.062	0.932	0.943	0.947	0.945	0.941	0.935	0.980	0.990	0.994	0.996	0.997	0.997

Table 9: Averages of GARCH parameter estimates $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ from the 'mild GARCH' experiment, $T = 5000$, and $c_1 = 0.5$, 0.2, and 0.8. Results for $\eta > 5$ are similar to the ones for $\eta = 5$ and not reported.

$c_1 = 0.5$

$\alpha = 0.05$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.103	0.207	0.354	0.509	0.626	0.684
2	0.129	0.255	0.483	0.682	0.793	0.835
3	0.135	0.264	0.496	0.683	0.775	0.823
4	0.134	0.271	0.494	0.671	0.769	0.806
5	0.135	0.271	0.493	0.670	0.769	0.804
6	0.135	0.271	0.493	0.670	0.769	0.804
7	0.135	0.271	0.493	0.670	0.769	0.804
8	0.135	0.271	0.493	0.670	0.769	0.804
9	0.135	0.271	0.493	0.670	0.769	0.804
10	0.135	0.271	0.493	0.670	0.769	0.804

$\alpha = 0.01$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.029	0.064	0.133	0.202	0.283	0.319
2	0.036	0.083	0.192	0.314	0.409	0.469
3	0.037	0.086	0.199	0.323	0.410	0.466
4	0.036	0.085	0.196	0.320	0.401	0.452
5	0.035	0.085	0.197	0.319	0.400	0.447
6	0.035	0.085	0.197	0.319	0.400	0.447
7	0.035	0.085	0.197	0.319	0.400	0.447
8	0.035	0.085	0.197	0.319	0.400	0.447
9	0.035	0.085	0.197	0.319	0.400	0.447
10	0.035	0.085	0.197	0.319	0.400	0.447

$c_1 = 0.2$

$\alpha = 0.05$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.054	0.069	0.088	0.095	0.107	0.112
2	0.051	0.074	0.096	0.117	0.131	0.143
3	0.053	0.078	0.099	0.126	0.148	0.167
4	0.053	0.076	0.100	0.132	0.153	0.166
5	0.053	0.075	0.100	0.132	0.153	0.166
6	0.053	0.075	0.100	0.132	0.153	0.166
7	0.053	0.075	0.100	0.132	0.153	0.166
8	0.053	0.075	0.100	0.132	0.153	0.166
9	0.053	0.075	0.100	0.132	0.153	0.166
10	0.053	0.075	0.100	0.132	0.153	0.166

$\alpha = 0.01$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.012	0.012	0.016	0.020	0.022	0.024
2	0.010	0.014	0.019	0.023	0.027	0.029
3	0.011	0.014	0.017	0.025	0.027	0.030
4	0.011	0.014	0.016	0.025	0.026	0.029
5	0.011	0.014	0.016	0.025	0.025	0.029
6	0.011	0.014	0.016	0.025	0.025	0.029
7	0.011	0.014	0.016	0.025	0.025	0.029
8	0.011	0.014	0.016	0.025	0.025	0.029
9	0.011	0.014	0.016	0.025	0.025	0.029
10	0.011	0.014	0.016	0.025	0.025	0.029

$c_1 = 0.8$

$\alpha = 0.05$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.127	0.273	0.473	0.726	0.876	0.924
2	0.176	0.354	0.607	0.829	0.920	0.949
3	0.187	0.360	0.625	0.808	0.884	0.924
4	0.192	0.363	0.621	0.794	0.869	0.904
5	0.192	0.364	0.620	0.793	0.867	0.904
6	0.192	0.364	0.620	0.793	0.867	0.904
7	0.192	0.364	0.620	0.793	0.867	0.904
8	0.192	0.364	0.620	0.793	0.867	0.904
9	0.192	0.364	0.620	0.793	0.867	0.904
10	0.192	0.364	0.620	0.793	0.867	0.904

$\alpha = 0.01$

η	δ_1: 2^0	2^1	2^2	2^3	2^4	2^5
1	0.048	0.107	0.262	0.444	0.618	0.725
2	0.065	0.186	0.371	0.593	0.737	0.795
3	0.073	0.204	0.376	0.596	0.713	0.759
4	0.072	0.203	0.372	0.580	0.692	0.742
5	0.073	0.202	0.370	0.576	0.690	0.742
6	0.073	0.202	0.370	0.576	0.690	0.742
7	0.073	0.202	0.370	0.576	0.690	0.742
8	0.073	0.202	0.370	0.576	0.690	0.742
9	0.073	0.202	0.370	0.576	0.690	0.742
10	0.073	0.202	0.370	0.576	0.690	0.742

Table 10: Estimated power of the LM-type statistic for the specification test with $T = 50$, locations $c_1 = 0.5$, 0.2, and 0.8, and significance levels $\alpha = 0.05$ and 0.01.

2 January 1994 – 31 December 2012 (6939 observations)

	Testing constancy in GARCH framework				Specification test	
	$\widehat{\alpha}_1$	$\widehat{\beta}_1$	$\widehat{\alpha}_1 + \widehat{\beta}_1$	TR^2	TR^2	p
Indonesia	0.052	0.948	1.000	16.839	245.997	0.000
Korea	0.073	0.927	1.000	26.648	43.695	0.000
Taiwan	0.087	0.905	0.992	11.655	171.839	0.000

2 January 1994 – 15 June 2000 (2357 observations)

	Testing constancy in GARCH framework				Specification test	
	$\widehat{\alpha}_1$	$\widehat{\beta}_1$	$\widehat{\alpha}_1 + \widehat{\beta}_1$	TR^2	TR^2	p
Indonesia	0.067	0.933	1.000	9.914	133.129	0.000
Korea	0.084	0.916	1.000	12.566	85.632	0.000
Taiwan	0.120	0.880	1.000	5.396	39.108	0.000

Table 11: Estimation and misspecification test results as well as the specification test results for the exchange rates for Indonesia, Korea, and Taiwan. The top panel shows the results for the period 1994–2012 and the bottom panel for 1994–2000, the time period considered in Davidson (2004). The p-value is calculated using χ^2_3 distribution.

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