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Abstract

A parametric portfolio policy function is considered that incorporates common stock volatility dynamics to optimally determine portfolio weights. Reducing dimension of the traditional portfolio selection problem significantly, only a number of policy parameters corresponding to first- and second-order characteristics are estimated based on a standard method-of-moments technique. The method, allowing for the calculation of portfolio weight and return statistics, is illustrated with an empirical application to 30 U.S. industries to study the economic activity before and after the recent financial crisis.

Keywords: Parametric portfolio policy, stock characteristics, volatility common factors.

JEL classification: C13, C21, C23, C58, G11, G15.

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1 Introduction

Portfolio selection problems have been traditionally studied based on the portfolio theory by Markowitz (1952), which requires modeling the joint distribution of returns. Portfolios selected based on Markowitz approach, however, do not completely take into account the risk borne by the investor because only the mean and variance are known but not the entire distribution.

Brand et al. (2009) (BSCV (2009) hereafter) proposes a parametric portfolio policy in that weights of stocks depend on stock characteristics. Their approach removes the necessity of modeling the joint distribution of returns and only a small number of parameters are estimated to determine optimal portfolio weights. While this approach is much easier to use in practice compared to the traditional Markowitz approach, it also lacks the ability to explicitly account for the risk borne by the investor in the weights function.

This paper considers a parametric portfolio policy with common volatility dynamics to explicitly incorporate the impact of risk borne by the investor in portfolio selection decisions. Our portfolio policy function is based on stock characteristics as proposed by BSCV (2009), but unlike theirs, ours is augmented by the estimates of volatility common factors. This way, the portfolio policy not only accounts for the first-order (stock) characteristics but also the second-order (volatility) characteristics thus providing the investor with the ability to base his decision also on risk.

Our portfolio policy contains only a number of stock characteristics and nests long-short portfolios of Fama and French (1993), Carhart (1997) and Fama and French (2015), but it additionally accounts for common volatility dynamics of the stocks. Since only a number of common stock characteristics are considered instead of historical stock returns and their joint distribution, dimensionality is significantly reduced. Therefore our approach is easy to implement in practice and it avoids possible imprecision due to overfitting.

In the analysis, volatility common factors are estimated first. Stock realized volatilities (RV's hereafter), which we calculate based on the jump-robust realized bipower variation measure due to Barndorff-Nielsen and Shephard (2004), exhibit fractional long-range dependence as shown by Bollerslev et al. (2013). This requires that stock RV's be appropriately differenced with their corresponding integration orders so that a principal components (PC) estimation can be employed to obtain the estimates of volatility common factors. These estimates are then plugged in to the parametric portfolio policy function of BSCV (2009) to determine optimal portfolio weights.

In the estimation of portfolio policy parameters, a generalized method-of-moments estimation is employed that is shown to produce consistent, asymptotically normal and efficient estimates as shown by Hansen (1982) within the class of estimators that employ the same set of moment conditions as ours. Based on these estimates, portfolio weight and return statistics can be calculated.

To illustrate the effectiveness of our approach, we use montly return data on 30 U.S. industries spanning the time period January 1966 - December 2014, which we split to January 1966 - August 2008 in-sample and September 2008 - December 2014 out-of-sample periods with the purpose of studying the impact of the recent crisis. We compare the performance of the portfolio policy that incorporates the common volatility dynamics to that which only considers first-order (stock) characteristics. The findings indicate that accounting for common volatility dynamics leads the investor to select an optimal portfolio with higher returns, reduced risk, higher Sharpe ratios and positive skewness in sample and out of sample.

The remainder of the paper is organized as follows. Next section explains the estimation of volatility common factors. Section 3 gives details on the parametric policy function incorporating common volatility dynamics. Section 4 provides an empirical illustration with data, and finally Section 5 concludes the paper.

2 Common Dynamics in Realized Volatilities

It is intuitive and clear that risk associated with the volatility of a stock affects the investment decision taken by the investor. That said, volatility associated with each stock can be treated separately to make allocation decisions but when large number of assets are analyzed instead, volatility-return assessment becomes cumbersome from an empirical point of view. With this in mind, we suggest using a common-factor model to capture the information about realized volatilities to reduce the dimension of the problem significantly. Common factors in the treatment of high-dimensional data has been used in several different setups; see e.g. Pesaran (2006) and Bai and Ng (2013).

We first construct the realized volatility measures based on bipower variation that is robust to jumps, following Barndorff-Nielsen and Shephard (2004). Let us denote an excess return at time t corresponding to industry i, $r_{i,t}$. Then the monthly realized bipower variation (RBV) is given by

$$RBV_{i,t} = \sum_{j=1}^{M-1} |r_{i,j}| \, |r_{i,j+1}| \, , \tag{1}$$

where M is the number of trading days in a month. Barndorff-Nielsen and Shephard (2004) argue that RBV converges to realized variance in the limit assuming asset prices follow a stochastic-volatility process and the limiting RBV measure is robust to rare jumps. Therefore, a jump-robust realized volatility measure can be envisaged as the square-root of RBV in (1).

To investigate the common dynamics of RV's, a common factor model can be employed as follows:

$$RV_{i,t} = \lambda_i' f_t + \epsilon_{i,t} \tag{2}$$

where λ_i are unobserved factor loadings indicating how much each cross-section unit is affected by the unobserved common factors f_t, and $\epsilon_{i,t}$ are assumed to be identically and independently distributed volatility shocks with mean zero and variance σ_i^2. In the estimation of common factor models, the use of principal components (PC) analysis, see e.g. Bai and Ng (2002, 2004, 2013), is standard to get the estimates of factor loadings and common factors, $\hat{\lambda}_i$ and $\hat{f}_t$. Restricting the attention to (2), the estimates $\hat{f}_t$ constitute the common dynamics of RV's and are much easier to use in portfolio choice problems than individual RV's due to reduced dimensionality providing a portfolio policy rather than requiring a stock-specific treatment. Asymptotic theory for $\hat{\lambda}_i$ and $\hat{f}_t$ is derived by Bai and Ng (2002, 2004) in case of stationary $I(0)$ and nonstationary $I(1)$ dependent variables, respectively.

Among others, Bollerslev et al. (2013) show that RV's exhibit long memory properties. This requires that RV's be appropriately differenced to stationarity before attempting to estimate (2). Bai and Ng (2004) use a similar approach in that they first-difference $I(1)$ data to obtain stationary variables to get factor structure estimates. Let us denote the fractional integration order of $RV_{i,t}$ by δ_i so that $RV_{i,t}$ is $I(\delta_i)$, where δ_i is positive. Then, using that $\Delta = 1 - L$ with the lag operator L, the common-factor structure estimates are obtained from the equation,

$$\Delta_t^{\delta_i} RV_{i,t} = \lambda_i' f_t + \epsilon_{i,t}. \tag{3}$$

For some $\delta > 0$,

$$\Delta_t^{\delta} = \Delta^{\delta} 1(t > 0) = \sum_{j=0}^{t-1} \pi_j(\delta) L^j, \tag{4}$$
$$\pi_j(\delta) = \frac{\Gamma(j-\delta)}{\Gamma(j+1)\Gamma(-\delta)},$$

where $1(\cdot)$ is the indicator function, and $\Gamma(\cdot)$ denotes the gamma function such that $\Gamma(d) = \infty$ for $d = 0, -1, -2, \ldots$, but $\Gamma(0)/\Gamma(0) = 1$. The expression in (4) bestows long-memory dynamics, in which autocorrelations show an algebraic rather than exponential decay because $\pi_j(\mu) \sim Cj^{-\mu-1}$ as $j \to \infty$ for $\mu > 0$. So, these weights are appropriate to control for inherent long memory in RV's as shown by Bollerslev et al. (2013) and $\Delta_t^{\delta_i} RV_{i,t}$ becomes $I(0)$.

When δ_i are known, this differencing can be directly carried out. However, in practice δ_i are unknown and must be estimated. For the estimation, a parametric approach or a semiparametric approach such as a local Whittle estimation, e.g. by Robinson (1995), can be used to obtain consistent estimates for δ_i. Then, we are simply interested in obtaining factor-structure estimates using a standard PC approach on the equation,

$$\Delta_t^{\hat{\delta}_i} RV_{i,t} = \lambda_i' f_t + \epsilon_{i,t}, \tag{5}$$

for which limiting theory is readily established in the literature, e.g. by Bai and Ng (2013). The number of common factors to be retained in the analysis can be determined based on the number of eigenvalues exceeding the mean eigenvalue. Denote $\hat{f}_t^*$ the vector of retained common factor estimates that is a subset of the factor estimates obtained from (5). Then, $\hat{f}_t^*$ can be used in different regression settings as plug-in estimates to serve, for example, as volatility common factor augmentation. The estimates $\hat{f}_t^*$ can also be used solely to capture the common volatility information, measuring whose impact on invesment decisions is generally of interest.

3 Optimal portfolio policy with common dynamics of volatility

In the setup, we consider that at time t, there are N_t number of stocks that are investable. Each stock i has a return of $r_{i,t+1}$ from time t to $t+1$ and is associated with a vector of firm characteristics $x_{i,t}$ and retained estimates of common volatility factors $\hat{f}_t^*$ observed at time t. The stock characteristics can contain, among others, the market capitalization of the stock and the book-to-market ratio of the stock. The investor's problem is then to maximize the conditional expected utility of the portfolio return $r_{p,t+1}$ by choosing the weights $w_{i,t}$ optimally, i.e.,

$$\max_{\{w_{i,t}\}_{i=1}^{N_t}} E_t[u(r_{p,t+1})] = E_t\left[u\left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t+1}\right)\right]. \tag{6}$$

Adopting BSCV (2009), we parameterize the portfolio weights as a function of stock characteristics as well as the common dynamics of stock volatilities,

$$w_{i,t} = g(x_{it}, \hat{f}_t^*; \theta, \gamma). \tag{7}$$

In particular, we focus on a linear specification of the portfolio weight function:

$$w_{i,t} = \bar{w}_{i,t} + \frac{1}{N_t}\left(\theta' \tilde{x}_{i,t} + \gamma' \hat{f}_t^*\right), \tag{8}$$

where $\bar{w}_{i,t}$ is the weight of the stock i at time t in a benchmark portfolio, e.g. the value-weighted market portfolio, θ and γ are coefficients to be estimated, $\hat{f}_t^*$ is the vector of common factors of volatilities, and $\tilde{x}_{i,t}$ are the characteristics of stock i, standardized cross-sectionally to have zero mean and unit standard deviation across all stocks at time t. The interest is in estimating weights as a single function of characteristics, as in BSCV (2009), and also common volatility drivers that applies to all stocks over time.

The parameterization in (8) brings in the possibility to deviate from the benchmark portfolio, whose weights are given by $\bar{w}_{i,t}$, based on $\tilde{x}_{i,t}$ and $\hat{f}_t^*$. In practice, standardization of characteristics and the normalization factor $1/N_t$ are necessary to ensure that weights are not mischosen; see BSCV (2009) for a discussion.

The coefficient vectors to be estimated, θ and γ, do not vary over time, which implies that portfolio weights depend only on firm and common volatility characteristics and not on historical returns. Time-invariant coefficients also imply that the coefficients that maximize the conditional expected utility of the investor also maximize his unconditional expected utility. Therefore, the maximization problem can be formulated using (7) as

$$\max_{\theta,\gamma} E\left[u\left(r_{p,t+1}\right)\right] = E\left[u\left(\sum_{i=1}^{N_t} g(x_{it}, \hat{f}_t^*; \theta, \gamma) r_{i,t+1}\right)\right]. \tag{9}$$

Since, under some regularity conditions, the empirical moment of the expected utility function converges to the theoretical one, in practice θ and γ will be estimated by maximizing the sample analogue of the unconditional expected utility,

$$\max_{\theta,\gamma}\left\{\frac{1}{T}\sum_{t=0}^{T-1} u(r_{p,t+1})\right\} = \max_{\theta,\gamma}\left\{\frac{1}{T}\sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{n} g(x_{it}, \hat{f}_t^*; \theta, \gamma) r_{i,t+1}\right)\right]\right\}, \tag{10}$$

for some prespecified choice of $u(\cdot)$, e.g. log, quadratic or a general constant relative risk aversion (CRRA) function. While the specification of $u(\cdot)$ is a matter of choice, the power-utility function of the form

$$u(c) = \frac{(1+c)^{1-\zeta}}{1-\zeta} \tag{11}$$

helps realize the implicit assumption made by time-invariant coefficients in (7) that the stock characteristics fully capture all aspects of the joint distribution of returns that are relevant for forming optimal portfolios because (11) not only takes into account the mean and variance, but also higher-order moments such as skewness and kurtosis. Moreover, CRRA is directly imposed by this functional form which shows sensitivity to different risk aversion levels through the parameter ζ.

Using (8), (10) can be expressed as

$$\max_{\theta,\gamma}\left\{\frac{1}{T}\sum_{t=0}^{T-1} u(r_{p,t+1})\right\} = \max_{\theta,\gamma}\left\{\frac{1}{T}\sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{n}\left(\bar{w}_{i,t} + \frac{1}{N_t}\left(\theta'\tilde{x}_{i,t} + \gamma'\hat{f}_t^*\right)\right) r_{i,t+1}\right)\right]\right\}. \tag{12}$$

It is important to note that (12) contains parameter vectors θ and γ that are of small dimensions because there are only a limited number of stock characteristics and very few (just one or two) common drivers of stock volatility, which makes their estimations computationally easy. Using this parametric portfolio policy also reduces the risk of imprecise estimation due to overfitting.[1]

A portfolio policy generated by (8) nests the long-short portfolios. Let us write the return of the portfolio policy in (8),

$$
\begin{aligned}
r_{p,t+1} &= \sum_{i=1}^{N_t} \bar{w}_{i,t+1} r_{i,t+1} + \sum_{i=1}^{N_t} \left(\frac{1}{N_t} \left(\theta' \tilde{x}_{i,t} + \gamma' \hat{f}_t^* \right) \right) r_{i,t+1} \\
&= r_{m,t+1} + r_{h,t+1},
\end{aligned} \tag{13}
$$

where m denotes the benchmark value-weighted market, and h denotes a long-short hedge fund with weights $\frac{1}{N_t}\left(\theta' \tilde{x}_{i,t} + \gamma' \hat{f}_t^*\right)$ summing up to zero. The linear portfolio policy weights in (8) therefore also nests the popular portfolios of Fama and French (1993, 2015) and Carhart (1997). For example, the return of the three-factor portfolio by Fama and French (1993) additionally incorporating volatility common factors can be expressed as

$$
r_{p,t+1} = r_{m,t+1} + \theta_{smb} r_{smb,t+1} + \theta_{hml} r_{hml,t+1} + \gamma' \hat{f}_t{}^* \frac{1}{N_t} \sum_{i=1}^{N_t} r_{i,t+1} \tag{14}
$$

where $r_{smb,t+1}$ and $r_{hml,t+1}$ are the returns to small-minus-big and high-minus-low portfolios, respectively.

Having formulated the optimal portfolio weights selection problem as an expected utility maximization problem, we can obtain the estimates $\hat{\theta}$ and $\hat{\gamma}$ resorting to methods of moments estimation. The estimates $\hat{\theta}$ and $\hat{\gamma}$, defined by the optimization problem in (12) satisfy the first-order conditions

$$
\frac{1}{T} \sum_{t=0}^{T-1} \left\{ u_\theta(r_{p,t+1}) \left(\frac{1}{N_t} \hat{x}_t' r_{t+1} \right) + u_\gamma(r_{p,t+1}) \left(\hat{f}_t^* \frac{1}{N_t} \sum_{i=1}^{N_t} r_{i,t+1} \right) \right\} = 0
$$

where $u_\varsigma = (\partial/\partial\varsigma)\, u$. The asymptotic variance-covariance matrix and its estimate can be envisaged following Hansen (1982) who shows that GMM estimates such as the ones we have are consistent, asymptotically normal and efficient within the class of

[1]For an extensive discussion see BSCV (2009).

estimators employing the same set of moment conditions. In practice, estimation may be performed based on multi-step or continuous-updating GMM procedures to acquire a desired level of parameter convergence.

4 Empirical illustration with data

4.1 Data description and empirical strategy

To illustrate the impact of incorporating common volatility dynamics into the parametric portfolio policy function by BSCV (2009), we explore the performance of industry portfolios because they are more informative about economic activity rather than being of specific investment interest.

We use daily return data on 30 U.S. industries and the composite average index of NYSE, NASDAQ and AMEX for the time period January 1966 - December 2014 downloaded from Ken French's Data Library along with the risk-free rates to calculate monthly industry and market RV's employing (1). We otherwise use the monthly data readily available for the three Fama-French factors in French's Data library. In the application, the investor is restricted to invest only in stocks. As also discussed by BSCV (2009), the reason for not including the risk-free asset as an investment opportunity is that the varying leverage induced by the risk-free asset only corresponds to a change in the scale of the stock portfolio weights.

The raw data requires standardization so that the results become comparable. The stock characteristics, x_{it}, show varying cross-sectional means and standard deviations that we take into account. The risk aversion is taken to be five. The CRRA utility function in (11) is used in a two-step GMM setting to determine the optimal portfolio weights.

With the goal of studying the predictive ability of the portfolio using common volatility factors, we divide the study sample into two groups: the in-sample analysis uses equity return data from January 1966 to August 2008 (512 data points), and the out-of-sample analysis focuses on the period September 2008 - December 2014 (76 data points), including the recent financial crisis. There is no specific reason as to why we split the sample to these two periods apart from the interest in investigating whether there are huge differences in terms of portfolio performance between pre- and post-crisis periods. Clearly different out-of-sample periods can also be considered.

We first estimate the common factors of industry RV's to be able to use them as further characteristics in the portfolio weight function. We then estimate the parameters of the portfolio whose returns are given by (14). Based on these estimates, we calculate portfolio weight statistics alongside with the unconditional mean, standard deviation, skewness and Sharpe ratio of the optimal portfolio.

4.2 Estimation of the common factors in industry RV's

First, we estimate the fractional integration orders of industry and market RV's based on Robinson (1995)'s local Whittle method that requires specifying the number of Fourier frequencies to be used. It is well known that long memory should be investigated in lower frequencies since higher frequencies are susceptible to short-memory contamination. This is why, we focus on $m = 45, 71$ Fourier frequencies corresponding to $T^{.6}$ and $T^{.67}$ with $T = 588$ the time-series length in our dataset.

The nonstationarity bound for long-memory processes is $\delta_i = 0.5$, so an indicator exhibits nonstationary long memory for $\delta_i \geq 0.5$ and stationary long memory for $\delta_i < 0.5$ and $\delta_i \neq 0$. Based on the results in Table 1, industry RV's show some heterogeneity in terms of stationarity while the market RV is stationary. This stresses the importance of appropriately differencing the RV's before carrying out PC estimation to obtain factor structure estimates.

After differencing the industry RV's by their corresponding integration orders[2], we carry out a PC estimation on (5) to get the common factor estimates. The PC estimation indicates that there is only one common factor driving the industry RV's, as can also be seen from the screeplot in Figure 1. This common factor explains 69.64%

[2] $m = 45$ Fourier frequencies were used.

of the total variation in the industry RV's.

It is also important to show that a common-factor model fits the industry RV's well. This can be checked by the uniqueness of variances that are not captured by the common factor: if uniqueness ratios are small, or equivalently if communality=1-uniqueness is large, then there is evidence that a common-factor model is well suited to the analysis of industry RV's. Table 2 below shows that the factor loadings estimates are positive and large while the uniqueness ratios are small. So, a common-factor model indeed fits industry RV's well.

4.3 Portfolio performance incorporating the common factor of industry RV's

In Section 3, we have shown that the linear portfolio policy in (8) nests many widely analyzed portfolios, such as those of Fama and French (1993), Carhart (1997) and Fama and French (2015). To simply illustrate the impact of incorporating common volatility dynamics into the parametric policy function of BSCV (2009), we restrict our attention to the portfolio of Fama and French (1993) that we discussed in (14). That said, obviously other portfolios can also be analyzed but the impact of common volatility dynamics on portfolio selection can be determined more easily in this less complicated setting.

We first consider the optimization problem in (12) as is, and then restrict $\gamma = 0$ to be able to determine the impact of $\hat{f}_t^*$ on optimal portfolio selection. A generalized method of moments estimation for the portfolio policy incorporating volatility common factor in (8) based on (11) leads to the results in Table 3.

The first six rows of Table 3 present the estimated coefficients of parametric portfolio policy function with volatility common factor along with their standard errors. These coefficients indicate that the optimal portfolio is determined by choosing small firms, value stocks and less volatile stocks since the coefficients are positive and statistically significant for *smb* and *hml* while it is negative for $\hat{f}_t^*$. The finding that the deviation of the optimal weights from the benchmark weights increases with *smb* and *hml* and decreases with $\hat{f}_t^*$ is quite intuitive and mirrors the findings in the literature.

Rows seven to eleven of Table 3 describe the weights of the optimized portfolio. The average absolute weight of the optimal portfolio is equal to 0.3871% in sample and 1.6822% out of sample. The average (over time) maximum and minimum weights of the optimal portfolio are 1.0639% and -4.4701% for the in-sample period and 4.0439% and -3.6111% for the out-of-sample period, respectively. The average sum of negative weights in the optimal portfolio is -0.4930 in sample and -0.1308 out of sample. The average fraction of negative weights (shorted equities) in the optimal portfolio is 0.2047 for in sample and 0.0933 for out of sample. Therefore, the optimal portfolio using common RV factor does not reflect unreasonably extreme bets on individual equities and could well be implemented by a combination of an index fund that reflects the market and a long-short equity hedge fund as in (13).

The remaining rows of Table 3 characterize the performance of the optimal portfolio. The optimal portfolio has an average monthly return of 0.51% in sample and 1.87% out of sample. The standard deviation of the optimal portfolio returns is 0.0161 and 0.0359, respectively, for in sample and out of sample that translates into Sharpe ratios of 0.3158 and 0.5211, respectively. Skewness is positive and large for both split-sample periods indicating that there is a decreased likelihood of encountering a large negative return.

In order to show that accounting for common volatility dynamics leads to better portfolio performance, we consider the parametric portfolio policy restricting the attention to *smb* and *hml* only, i.e. $\gamma = 0$. The estimation results along with portfolio weight and return statistics are reported in Table 4.

The estimated coefficients are positive for both *smb* and *hml* in sample and out of sample. That is, small firms and value stocks are positively weighed in for the selection of the optimal portfolio, which is in line with the findings in the literature. In the out-of-sample period, *smb* does not have a significant role in the determination of optimal portfolio weights but the coefficient of *hml* remains significant, indicating that in the post-crisis period the investment decision is based on high value stocks regardless of firm size.

Rows seven to eleven of Table 4 describe the weights of the optimized portfolio that does not account for common volatility dynamics. The average absolute weight of this portfolio is equal to 0.1949% in sample and 1.3333% out of sample. The average (over time) maximum and minimum weights of this portfolio are 0.2113% and 0.1807% for the in-sample period and 1.3984% and 1.2521% for the out-of-sample period, respectively. The average fraction of negative weights (shorted equities) in the optimal portfolio is 0 for in sample and out of sample, indicating that this portfolio policy recommends not shorting any of the equities. These findings contrast with the portfolio weight statistics in Table 3 in that accounting for common volatility dynamics leads to the recommendation to short equities whose risk is high.

The remaining rows of Table 4 summarizes the optimal portfolio return statistics. The optimal portfolio has an average monthly return of 0.19% in sample and 1.63% out of sample. The standard deviation of the optimal portfolio returns is 0.0182 and 0.0762, respectively, for in sample and out of sample that translates into Sharpe ratios of 0.1044 and 0.2138, respectively. Skewness is negative for both split-sample periods indicating that there is a likelihood of encountering a large negative return. These results contrast poorly to the optimal portfolio return statistics in Table 3 in that the portfolio policy accounting for common volatility dynamics has higher average monthly returns, reduced portfolio risk, higher Sharpe ratios and positive skewness both in sample and out of sample.

4.4 The relationship between common factor of industry RV's and variance risk premium

When an analysis is carried out at the macroeconomic level based on industry portfolios, it may also be interesting to establish the ties between the factor-structure estimates obtained from (5) and a general measure such as variance risk premium (VRP) since an economic discussion can then be pursued.

Common volatility dynamics can be linked to variance risk premium that is defined as the difference between the ex-ante risk neutral expectation of the future stock market return variance and the expectation of the stock market return variance between time t and $t+1$:

$$VRP_t \equiv E_t^{\mathbb{Q}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right) - E_t^{\mathbb{P}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right),$$

where "$E_t^{\mathbb{P}}$" denotes the conditional expectation with respect to physical probability. VRP_t is unobservable and can be estimated by replacing $E_t^{\mathbb{Q}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right)$ and $E_t^{\mathbb{P}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right)$ by their estimates $\hat{E}_t^{\mathbb{Q}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right)$ and $\hat{E}_t^{\mathbb{P}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right)$, respectively,

$$\widehat{VRP}_t \equiv \hat{E}_t^{\mathbb{Q}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right) - \hat{E}_t^{\mathbb{P}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right),$$

where in practice $\hat{E}_t^{\mathbb{Q}}\left(Var_{t,t+1}\left(r_{t+1}\right)\right)$ and the true variance $Var_{t,t+1}\left(r_{t+1}\right)$ are replaced by the squared VIX and realized variance, respectively.

We then consider the regression for the time period January 1990 - December 2012 whose data we borrow from Zhou (2010):

$$\widehat{VRP}_t = \xi_0 + \xi_1' \hat{f}_t^* + \varepsilon_{i,t}. \tag{15}$$

The estimation results are summarized in Table 5. These results indicate that the common factor of industry RV's are positively linked to the estimate of variance risk premium. The common factor of industry RV's is a systematic risk measure while VRP is a measure of the degree of risk aversion in an economy rather than a market risk measure as argued by Bollerslev et al. (2009). The positive relationship between VRP and common factor of industry RV's can then be explained as follows: an increase (decrease) in systematic risk leads risk-averse agents to cut (increase) their consumption and investment expenditures and shift their portfolios from more (less) risky assets to less (more) risky ones, which is also a consequence of an increase (decrease) in the degree of risk aversion, as reflected by VRP.

5 Conclusion

We have proposed incorporating common volatility dynamics as a determinant of the optimal portfolio weights that contrasts well with both the traditional Markowitz approach and the approach by BSCV (2009) who did not account for volatility effects in their portfolio selection methods. We have empirically illustrated the positive impact of accounting for common volatility dynamics on portfolio performance in a parametric portfolio setting, and linked the common volatility factor to VRP, which is widely used in empirical analyses.

While we restricted our attention to industry portfolios in the empirical analysis to be able to understand general economic activity, further research can be undertaken considering other investment-purpose portfolios. It could be also interesting to develop forecasting methods using the parametric portfolio policy that incorporates common volatility dynamics. Finally, further work is warranted for additional portfolio statistics, such as turnover ratios and truncated weights, which we purposefully neglect in this paper to focus on the main ideas.

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Table 1: Estimated Integration Orders of Industry Realized Volatilities

$\underline{m = 45:}$

Food	Bvrgs	Tobac	Games	Books	Hshld	Clths	Hlth	Chems	Txtls	**Market**
0.36	0.47	0.61	0.45	0.50	0.37	0.48	0.35	0.49	0.53	0.41
Cnstr	Steel	FabPr	ElcEq	Autos	Carry	Mines	Coal	Oil	Util	
0.48	0.49	0.44	0.38	0.45	0.41	0.52	0.61	0.49	0.45	
Telcm	Servs	BusEq	Paper	Trans	Whlsl	Rtail	Meals	Finan	Other	
0.46	0.45	0.56	0.42	0.39	0.34	0.49	0.48	0.52	0.49	

$\underline{m = 71:}$

Food	Bvrgs	Tobac	Games	Books	Hshld	Clths	Hlth	Chems	Txtls	**Market**
0.35	0.45	0.49	0.41	0.51	0.33	0.47	0.33	0.45	0.57	0.40
Cnstr	Steel	FabPr	ElcEq	Autos	Carry	Mines	Coal	Oil	Util	
0.44	0.51	0.45	0.42	0.50	0.40	0.45	0.53	0.43	0.44	
Telcm	Servs	BusEq	Paper	Trans	Whlsl	Rtail	Meals	Finan	Other	
0.48	0.42	0.54	0.42	0.40	0.34	0.42	0.45	0.65	0.47	

Note: *This table reports the local Whittle estimation results of the individual integration orders of industry and market realized volatilities with* $m = 45, 71$ *Fourier frequencies. Estimates are rounded to two digits after zero. Standard errors of the estimates are 0.0745 and 0.0593 respectively for* $m = 45, 71$.

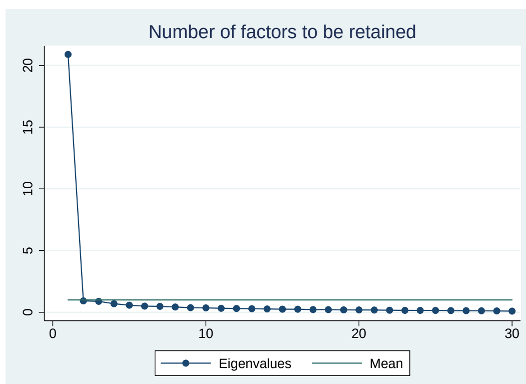

Figure 1: This screeplot draws the eigenvalues associated with factors and the mean eigenvalue which is equal to 1. Only eigenvalues greater than 1 are retained.

Table 2: Estimated Factor Loadings and Uniqueness of Variances

RV_i	Factor loadings	Ratio of variance unique to RV_i
food	0.8743	0.2357
beer	0.7593	0.4235
smoke	0.5088	0.7411
games	0.8544	0.2699
books	0.8530	0.2724
hshld	0.8622	0.2566
clths	0.8600	0.2605
hlth	0.8230	0.3227
chems	0.8934	0.2018
txtls	0.8017	0.3572
cnstr	0.9080	0.1755
steel	0.8537	0.2712
fabpr	0.9286	0.1377
elceq	0.8920	0.2044
autos	0.8528	0.2727
carry	0.8516	0.2748
mines	0.7192	0.4828
coal	0.6890	0.5252
oil	0.8161	0.3340
util	0.7699	0.4073
telcm	0.8178	0.3312
servs	0.8708	0.2418
buseq	0.8055	0.3512
paper	0.8852	0.2165
trans	0.8692	0.2444
whlsl	0.9059	0.1793
rtail	0.8707	0.2418
meals	0.8156	0.3348
fin	0.8397	0.2948
other	0.8696	0.2439

Note: *This table reports the PC estimation results for industry RV's. The uniqueness ratios are quite small indicating that the common factor explains much of the variance of each industry RV.*

Table 3: Portfolio performance with common volatility factor

Parameters	In-Sample	Out-of-Sample
$\hat{\theta}_{smb}$	0.0217***	0.0067***
	(0.0042)	(0.0015)
$\hat{\theta}_{hml}$	0.0084***	0.0033***
	(0.0022)	(0.0012)
$\hat{\gamma}$	-0.0756***	-0.0254***
	(0.0107)	(0.0058)
$\lvert w_i \rvert \times 100$	0.3871	1.6822
$\max w_i \times 100$	1.0639	4.0439
$\min w_i \times 100$	-4.4701	-3.6111
$\sum w_i I(w_i < 0)$	-0.4930	-0.1308
$\sum I(w_i \leq 0)/n$	0.2047	0.0933
$\bar{r}$	0.51%	1.87%
$\sigma(r)$	0.0161	0.0359
Skewness	5.4814	3.1426
Sharpe Ratio	0.3158	0.5211

Note: This table reports the estimation results of portfolio policy in (8). In-sample study covers the period from January 1966 to August 2008, and the out-of-sample study, carried out based on a rolling window of 12 months, covers the period from September 2008 to December 2014. Rows 7 to 11 show statistics of the portfolio weights averaged across time. These statistics include average absolute portfolio weight ($\lvert w_i \rvert \times 100$), the average maximum ($\max w_i \times 100$) and minimum ($\min w_i \times 100$) portfolio weights, the average sum of negative portfolio weights ($\sum w_i I(w_i < 0)$) and the fraction of the negative portfolio weights ($\sum I(w_i \leq 0)/n$), respectively. Rows 12 to 15 display the monthly portfolio statistics: average monthly return ($\bar{r}$), standard deviation ($\sigma(r)$), skewness and Sharpe ratio. Risk aversion is assumed to be equal to five. "***" indicates statistical significance at the 1% level.

Table 4: Portfolio performance without common volatility factor

Parameters	In-Sample	Out-of-Sample
$\hat{\theta}_{smb}$	0.00018**	0.00011
	(0.00008)	(0.00015)
$\hat{\theta}_{hml}$	0.00058***	0.00061***
	(0.00008)	(0.00011)
$\lvert w_i \rvert \times 100$	0.1949	1.3333
$\max w_i \times 100$	0.2113	1.3984
$\min w_i \times 100$	0.1807	1.2521
$\sum w_i I(w_i < 0)$	0	0
$\sum I(w_i \leq 0)/n$	0	0
$\bar{r}$	0.19%	1.63%
$\sigma(r)$	0.0182	0.0762
Skewness	-0.4519	-0.5559
Sharpe Ratio	0.1044	0.2138

Note: This table reports the estimation results of portfolio policy in (8) without the common factor of industry RV's, i.e. $\gamma = 0$. In-sample study covers the period from January 1966 to August 2008, and the out-of-sample study, carried out based on a rolling window of 12 months, covers the period from September 2008 to December 2014. Rows 7 to 11 show statistics of the portfolio weights averaged across time. These statistics include average absolute portfolio weight ($\lvert w_i \rvert \times 100$), the average maximum ($\max w_i \times 100$) and minimum ($\min w_i \times 100$) portfolio weights, the average sum of negative portfolio weights ($\sum w_i I(w_i < 0)$) and the fraction of the negative portfolio weights ($\sum I(w_i \leq 0)/n$), respectively. Rows 12 to 15 display the monthly portfolio statistics: average monthly return ($\bar{r}$), standard deviation ($\sigma(r)$), skewness and Sharpe ratio. Risk aversion is assumed to be equal to five. "***" and "**" indicate statistical significance at the 1% and 5% level, respectively.

Table 5: VRP and Common Factor of Industry RV's

Estimates	$\hat{\xi}_0$	$\hat{\xi}_1$
	0.0088	0.5459***
	(0.0479)	(0.0433)
	[0.8550]	[0.0000]

Note: *This table reports the regression results of the variance risk premium estimate on the common factor of industry RV's based on (15). Heteroskedasticity and autocorrelation robust standard errors are reported in parantheses and the corresponding p-values in square brackets. *** indicates significance at the 1% level.*

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