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Nonlinear dynamic interrelationships between real activity and stock returns*

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Abstract

We explore the differences between the causal and noncausal vector autoregressive (VAR) models in capturing the real activity-stock return-relationship. Unlike the conventional linear VAR model, the noncausal VAR model is capable of accommodating various nonlinear characteristics of the data. In quarterly U.S. data, we find strong evidence in favor of noncausality, and the best causal and noncausal VAR models imply quite different dynamics. In particular, the linear VAR model appears to underestimate the importance of the stock return shock for the real activity, and the real activity shock for the stock return.

Keywords: Noncausal VAR model, non-Gaussianity, generalized forecast error variance decomposition, business cycles, fundamentals.
JEL Classification: C32, C58, E17, E44.

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1 Introduction

To what extent can stock price movements be attributed to market fundamentals, and vice versa? Standard asset pricing models imply a close relation between stock returns and (expected future) real activity. In the same vein, real activity movements should be related to stock prices inasmuch as they contain information about future economic developments. However, the presence of certain stylized features of observed asset prices, such as excess volatility and the predictability of stock returns, suggest that a substantial fraction of stock price variation arises from (non-fundamental) factors other than the real activity.

Much of the previous research on the interlinkages between stock markets and the real economy is confined to linear vector autoregressive (VAR) models, on which long-run restrictions are imposed to identify shocks arising from the stock market and the real economy, often called 'non-fundamental' and 'fundamental' shocks, respectively. Binswanger (2004a,b), and Groenewold (2004), among others, employed this structural VAR (SVAR) approach, which, in our view, suffers from three shortcomings. First, at least implicitly Gaussian errors have been assumed in this line of research although there is mounting evidence in favor of nonlinear and non-Gaussian dependencies, such as volatility clustering and regime switches, in macroeconomic and financial time series. Previously reached conclusions may thus be misleading due to overlooking nonlinearities and deviations from normality. Second, the commonly employed bivariate VAR model is likely to lack relevant variables, but it may be difficult to find an adequate linear specification, and, in addition, it is not clear how the shocks should be identified in a higher-dimensional model. Finally, as pointed out by Faust and Leeper (1997) and discussed in more detail in Section 3, identification of structural shocks by long-run restrictions is prone to a number of econometric problems.

In order to alleviate potential misspecification, we propose the noncausal VAR model of Lanne and Saikkonen (2013) as a non-Gaussian nonlinear alternative to the causal linear SVAR model. Our main goal is to compare the conclusions concerning the dynamic relationship between stock return and real activity implied by the conventional SVAR and noncausal VAR models. Noncausal models have not yet been much employed in empirical economic and financial research, but they are steadily gaining ground (see, for instance, the financial applications in Gourieroux and Zakoïan (2013), Lof (2013), Karapanagiotidis (2014), Hencic and Gourieroux (2015), and Lof and Nyberg (2015)).

The noncausal VAR model has at least two advantages over the traditional linear causal VAR model. First, it may provide a parsimonious representation of the data. The distinguishing property of the noncausal VAR model is the explicit dependence of the current value on future values of the time series. This, in turn, implies the predictability of future values of the error term by past values of the time series. Intuitively, these predictable errors may be thought of as containing effects of variables that are excluded from the model and have predictive power for the included variables. Hence, the bivariate noncausal VAR model containing only the variables of interest may well be adequate, whereas a causal VAR model would have to be augmented with potentially a large number of variables to capture all relevant effects.

Second, the noncausal VAR model is capable of accommodating nonlinearities. As discussed in Lanne and Saikkonen (2013), it has a causal nonlinear representation albeit, in general, it cannot be given in closed form. Thus, in addition to robustness with respect to omitted variables, the noncausal VAR model has the benefit of capturing various nonlinear features. These features include, for example, financial market bubbles, mild conditional heteroskedasticity, and regime switches as brought up by Gourieroux and Zakoïan (2013), and Lanne and Saikkonen (2013), Lof (2013), respectively. Results based on a noncausal VAR model are thus expected to be robust with respect to various kinds of nonlinearities. Although the conventional causal VAR model could be augmented with a generalized autoregressive conditional heteroskedasticity (GARCH) component, this would increase the number of parameters considerably and be difficult to take into account in structural analysis.

Nonlinearity and the predictability of errors complicate structural analysis in the noncausal VAR model. In particular, identification of structural shocks is not possible by long-run restrictions that have typically been entertained in the previous related literature. Instead, we employ the generalized methods put forth by Gallant, Rossi and Tauchen (1993) and Koop, Pesaran and Potter (1996) to compute generalized impulse response functions and forecast error variance decompositions. On the positive side, we hence avoid the problems associated with identification by long-run restrictions alluded to above. However, impulse responses implied by nonlinear models, including the noncausal VAR model, depend on the history and the size and sign of the shock, which brings about certain complications. On the other hand, the model facilitates studying the relative importance of shocks separately, for example, in business cycle expansions and recessions as well as bear and bull stock markets. Differences across such regimes have often been examined in the previous financial literature.

We consider bivariate causal and noncausal VAR models for the quarterly growth rates of the real U.S. Gross Domestic Product (GDP) and the real stock return from 1953:Q1 to 2012:Q4. Strong evidence in favor of noncausality is found, which indicates inadequacy of the bivariate linear VAR model for studying the interlinkages of stock market and real activity. Specifically, both the stock return and GDP growth appear strongly dependent on the future GDP growth rate. Compared to the causal VAR model, the generalized forecast error decompositions emphasize the relative importance of stock market shocks for the real activity. On the other hand, the real activity seems to have a greater impact on the stock returns than the linear SVAR model suggests. In other words, controlling for omitted factors (like time-varying discount factors, interest rates and other potentially informative variables) and various nonlinearities via allowing for noncausality appears to emphasize the role of economic fundamentals in determining stock prices, in line with standard asset pricing models.

The structure of the paper is as follows. In Section 2, we briefly review how the bidirectional linkages between real activity and stock returns have been empirically examined with linear Gaussian VAR models in the previous literature. The noncausal VAR model and the related econometric methods are discussed in Section 3. The empirical results are reported in Section 4. Finally, Section 5 concludes.

2 Review of Previous Research

The literature on the linkages between real economic activity and stock market returns is quite voluminous. The bulk of the previous studies are confined to linear regression models for studying the predictability of the real activity by lagged stock returns, and vice versa (i.e., Granger causal relationships between stock returns and real activity).[1]

Fama (1990) and Schwert (1990) also regressed stock returns on leads of real activity variables and found them to have predictive power. Their idea was to use those leads as proxies of expected business conditions to capture the extra information that agents may have about future macroeconomic developments when predicting stock returns. In a similar setup, Campbell and Diebold (2009) replaced the leads of real activity variables by survey expectations of future business conditions. They found the expected business conditions to predict the excess stock return, undermining the predictive power of the conventional financial predictors (dividend yield, default premium and term premium).

Linear single-equation models are appealing because of their simplicity, but they have little to say about more structural issues. In particular, they do not facilitate identifying the economic shocks that are driving stock prices or real activity. Structural analysis can be conducted in an identified structural VAR

[1] Nonlinear regression models have been entertained only in a few previous papers. Domian and Louton (1999) found evidence of a threshold-type asymmetry in the stock return-real activity linkage in a (single-equation) threshold autoregressive model, while Henry et al. (2004) found allowing for regime switches related to business cycles important in a panel regression model.

model containing the stock return and a real activity variable (typically the real GDP growth). Much of the previous literature is based on the bivariate VAR(p) model,

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{u}_t \tag{1}$$

or

$$\mathbf{A}\left(L\right)\mathbf{y}_t = \boldsymbol{u}_t,$$

where $\mathbf{y}_t = [x_t \quad r_t]'$ with x_t a real activity variable and r_t the real stock return, and $\boldsymbol{u}_t$ is an independently and identically distributed error term with mean zero and finite positive definite covariance matrix, invariably assumed Gaussian in this literature. $\mathbf{A}\left(L\right) = I_2 - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p$ is a pth order polynomial in the lag operator L (i.e. $L^k\mathbf{y}_t = \mathbf{y}_{t-k}$), and $\mathbf{A}_1 \ldots, \mathbf{A}_p$ are 2×2 coefficient matrices. Because $\mathbf{y}_t$ depends only on its past values, model (1) is termed causal. Note that if $\boldsymbol{u}_t$ is assumed normally distributed, the bivariate VAR model (1) reduces to the two separate predictive linear regression models discussed above.

In structural analysis, the real activity and stock market shocks have typically been identified by means of long-run restrictions. Specifically, it has been assumed that the erros $\boldsymbol{u}_t$ are linear combinations of the economic shocks $\boldsymbol{\varepsilon}_t$, i.e., $\boldsymbol{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$, where $\mathbf{B}$ is a full-rank 2×2 matrix, and one of them (the stock market shock) is restricted to have no impact on the level of real activity in the long run. In the literature, the stock market shock is also referred to as the non-fundamental shock, while the unrestricted shock is typically called fundamental. Notice that this identification requires the real activity variable x_t to be a stationary ($I(0)$) variable obtained by differencing a variable integrated of order one ($I(1)$).

The long-run restriction is explicitly imposed on and the impulse response functions are computed from the infinite-order vector moving average representation of model (1),

$$\mathbf{y}_t = \mathbf{A}(L)^{-1}\boldsymbol{u}_t = \sum_{k=0}^{\infty} \Psi_k \boldsymbol{u}_{t-k}, \tag{2}$$

which exists if the lag polynomial $\mathbf{A}(L)$ is invertible. Substituting $\boldsymbol{u}_t = \mathbf{B}\boldsymbol{\varepsilon_t}$, we obtain

$$\mathbf{y}_t = \sum_{k=0}^{\infty} \Psi_k \mathbf{B} \boldsymbol{\varepsilon}_t = \sum_{k=0}^{\infty} \mathbf{C}_k \boldsymbol{\varepsilon}_t, \tag{3}$$

and the long-run identification restriction discussed above just sets the (1,2) element of the long-run impact matrix $\sum_{k=0}^{\infty} \mathbf{C}_k$ equal to zero, i.e.,

$$\sum_{k=0}^{\infty} c_{k,12} = 0. \tag{4}$$

This type of long-run restriction in a bivariate VAR model is often attributed to Blanchard and Quah (1989), who employed it in studying U.S. output growth and unemployment, and showed that it is sufficient for exact identification (see also Shapiro and Watson, 1988).

The impulse response function (IRF) of shock i ($i = 1, 2$), tracing out its impact on variable j ($j = 1, 2$) in periods $h = 0, 1, ...,$ is obtained by collecting the (i, j) elements $c_{h,ij}$ of the matrices $\mathbf{C}_h$, $h = 0, 1, ...$ A more convenient measure of the relative importance of each shock at forecast horizon h is provided by the forecast error variance decomposition (FEVD)

$$\frac{\sum_{l=0}^{h} c_{l,ij}^2}{\sum_{l=0}^{h} c_{l,1j}^2 + \sum_{l=0}^{h} c_{l,2j}^2}, \tag{5}$$

which can be interpreted as the portion of the h-period forecast error variance of variable j accounted for by shock i.

The setup above has been applied in several studies. Binswanger (2004a,b) and Groenewold (2004) included the growth rate of the gross domestic product (or, alternatively, industrial production) and a stock return in the bivariate VAR model. Groenewold (2004) considered quarterly Australian data from 1959:Q4 to 1999:Q1, and found that while both shocks mattered for both stock price and real activity, the stock market shock only had a short-lived impact on the latter. In quarterly U.S. data from 1953:Q1 to 2002:Q4, Binswanger (2004a) found a break in the relative importance of the shocks for the stock price, with the stock market shock dominating in the past few decades. He also considered a VAR model augmented with a third variable, measuring either (real) dividends, earnings or an interest rate, and found only a slight increase in the relative importance of the real activity shock. Binswanger (2004b) reconfirmed this finding in quarterly U.S., Japanese, and aggregate European data (comprising France, Germany, Italy, and the U.K.) data from 1960:Q1 to 1999:Q4.

Velinov and Chen's (2014) recent study differs from the basic setup in that they allowed for Markov switching in the error term of model (2), which facilitates testing the long-run identification restriction. In an international dataset comprising six of the G7 countries (France, Germany, Italy, Japan, the U.K., and the U.S.) from 1960:Q1 to 2013:Q3, they found that in only one (Italy) could the identification restriction be rejected at the 5% level of significance. With the exception of the U.S., the stock market shock was found to dominate the determination of the real stock price. In the same vein, Lütkepohl and Velinov (2015) found support for similar long-run identification restrictions in a trivariate SVAR model (containing also an interest rate) for quarterly U.S. data from 1947:Q1 to 2012:Q3.

As pointed out in the Introduction, identification of structural VAR models by long-run restrictions has been criticized quite forcefully by Faust and Leeper (1997) (see also Taylor (2004), and Gospodinov (2010)). Faust and Leeper's (1997) main point is that it is difficult to estimate the long-run behavior of time series from a short span data, and, hence, there is great uncertainty concerning the estimate of $\sum_{k=0}^{\infty} \mathbf{C}_k$ in (3), upon which the long-run restriction is imposed. This uncertainty, in turn shows up as uncertainty in the impulse response function and forecast error variance decompositions of the identified shocks, which can be quite imprecise. In addition, they point out that because of aggregation effects, the structural shocks actually are likely to be combinations of multiple economic shocks.

3 Structural Methods for Noncausal VAR Model

As discussed in the Introduction, while relatively straightforward to apply, the conventional causal structural VAR approach described in Section 2 has a number of problems in the context of real activity and stock returns. In particular, the bivariate VAR model typically considered is likely to miss relevant variables and nonlinearities.[2] In this section, we show how the noncausal VAR framework can solve these problems.

Below in Section 3.1, we discuss the noncausal model and review in more detail its potential benefits in modeling the dynamics of real activity and stock return. In order to avoid the problems related to long-run identification restrictions, we propose to use the generalized forecast error variance decomposition, whose computation in the noncausal VAR model is discussed in Section 3.2. Generalized methods are a natural choice since the noncausal VAR model is nonlinear.

3.1 Model

The particular noncausal VAR specification employed in this study is that of Lanne and Saikkonen (2013)[3], where the stochastic process $\mathbf{y}_t = [x_t \quad r_t]'$ is assumed to be generated by

$$\Pi(L)\,\Phi\left(L^{-1}\right)\mathbf{y}_t = \boldsymbol{u}_t. \tag{6}$$

The error term $\boldsymbol{u}_t$ (2×1) is an independently and identically distributed non-Gaussian error term with zero mean and finite positive definite covariance matrix, and $\Pi(L) = I_n - \Pi_1 L - \cdots - \Pi_r L^r$ and $\Phi\left(L^{-1}\right) = I_n - \Phi_1 L^{-1} - \cdots - \Phi_s L^{-s}$ are 2×2 matrix polynomials in the lag operator L ($L^k \mathbf{y}_t = \mathbf{y}_{t-k}$, $k = 0, \pm 1, \ldots$). Moreover, the determinants of the matrix polynomials $\Pi(z)$ and $\Phi(z)$ are assumed

[2]Higher-dimensional VAR models have been entertained in a few previous studies. Lee (1998) and Binswanger (2004a) included variables such as dividends, earnings and real interest rate as the third variable in a trivariate SVAR model identified with long-run restrictions analogous to (4). In contrast, Lee (1992) estimated a recursively identified four-variable SVAR model for the stock return, real interest rate, growth rate of industrial production and inflation. James, Koreisha and Partch (1985) investigated the relations among the stock return, real activity, money supply, and inflation in a vector autoregressive moving average (VARMA) model, but they did not impose any explicit identification restrictions.

[3]Gourieroux and Jasiak (2014) also consider this specification, while Davis and Song (2012) propose a different formulation that does not facilitate separating the lead and lag (or "causal" and "noncausal") polynomials.

to have their zeros outside the unit disc, so that

$$\det \Pi (z) \neq 0, \quad |z| \leq 1, \quad \text{and} \quad \det \Phi (z) \neq 0, \quad |z| \leq 1,$$

which guarantees the stationarity of $\mathbf{y}_t$.

We refer to the noncausal VAR model (6) as the VAR(r, s) model, where r and s denote the orders of the lag-polynomials. If $\Pi_1 = \cdots = \Pi_r = 0$ (i.e., $r = 0$), the model is called purely noncausal because then the time series $\mathbf{y}_t$ depends only on its future values. If $\Phi_1 = \cdots = \Phi_s = 0$ (i.e., $s = 0$), the model reduces to the conventional causal VAR(r) model with dependence only on the past values of $\mathbf{y}_t$.

The reason for assuming the distribution of the error term $\boldsymbol{u}_t$ non-Gaussian is identification. It is only under this assumption that the different pth-order VAR(r, s) models (with $r + s = p$) are discernible, which follows from the well-known fact that noncausality is not identified by second-order properties (see, e.g., Breidt et al. (1991)). In this paper, we specifically assume that $\boldsymbol{u}_t$ follows a multivariate t-distribution with λ degrees of freedom.[4] This assumption is well in line with the heavy-tailed distributions of the residuals often encountered in macroeconomic and financial time series applications, and our empirical results in Section 4 lend support to its adequacy in the bivariate model for the U.S. real activity and stock return. Following Lanne and Saikkonen (2013), in the empirical analysis, we take as a starting point the causal Gaussian VAR(p) model that is deemed adequate in that it produces serially uncorrelated residuals. Then, the VAR(r, s) model among the models with $r + s = p$ is selected that maximizes the log-likelihood function, and its adequacy is checked by diagnostic tests of remaining autocorrelation and conditional heteroskedasticity in the residuals, as well as quantile-quantile plots of the residuals.

As shown by Lanne and Saikkonen (2013), under stationarity, the VAR(r, s) process $\mathbf{y_t}$ has a two-sided infinite-order vector moving average (VMA) represen-

[4]The log-likelihood function for model (6) in the case of the t-distribution is given in Lanne and Saikkonen (2013). They also show that under regularity conditions, the maximum likelihood estimator is consistent and asymptotically normally distributed.

tation (cf. the corresponding moving average representation of the causal model (2) with dependence on past errors only). To gain further insight, the VAR(r, s) model (6) can be rewritten as

$$\mathbf{y}_t = a_1 \mathbf{y}_{t-1} + \cdots + a_{nr} \mathbf{y}_{t-2r} + \sum_{k=-r}^{\infty} N_k \boldsymbol{u}_{t+k}, \tag{7}$$

where the parameters $a_1, \ldots, a_{nr}$ and the sequence N_k are functions of the coefficient matrices of the lag polynomials in (6) (for details, see Nyberg and Saikkonen (2014)). Expression (7) shows how the VAR(r, s) model implies dependence of $\mathbf{y}_t$ on its past values and future error terms. This shows that unlike the causal VAR model, in the noncausal VAR model $(s \geq 1)$, future errors are predictable by past values of $\mathbf{y}_t$. In the purely noncausal case $(r = 0, s \geq 1)$, the past values of $\mathbf{y}_t$ vanish completely from the right hand side of (7), and $\mathbf{y}_t$ only depends on future errors.

As pointed out by Lanne and Saikkonen (2013), the predictability of the error term suggests that the errors incorporate factors excluded from the model and predictable by the real activity and stock return included in the model. In other words, the noncausal VAR model is capable of capturing effects of omitted variables, and the empirical results should thus to some extent be robust with respect to such omissions. Lof's (2013) recent simulation results provide confirming evidence in favor of this conjecture. This is a highly relevant feature because the bivariate model is likely to be inadequate to address our question of interest (see, e.g., Lee (1998) and Binswanger (2004a,b) for a discussion on this point), yet it may be difficult to find the correct causal specification. In particular, the mere number of variables required to that end might render conventional SVAR analysis infeasible in practice. The availability of all relevant data may also be an issue; for instance, among potential relevant variables, dividends and earnings are not directly observable at the quarterly frequency, and are thus prone to interpolation errors that may affect the results.[5]

[5] A closely related issue is the heterogeneity of the beliefs held by investors that can give rise to dependence on future errors if overlooked by the econometrician modeling the data (see Kasa

In addition to robustness with respect to omitted variables, the noncausal VAR model has the benefit of capturing nonlinear effects and thus yielding results robust with respect to nonlinearities. As discussed by Lanne and Saikkonen (2013), the linear noncausal VAR process (6) has a causal nonlinear representation (whose error term is not predictable) although, in general, it cannot be given in closed form.[6] Presumably many different kinds of nonlinearities relevant to macroeconomic and financial data are covered. Lof's (2013) simulation results suggest that noncausality gets easily mixed up with logistic smooth transition (LSTAR) type nonlinearity,[7] while Gourieroux and Zakoïan (2013) suggest noncausal autoregressions for modeling financial market bubbles.

Particularly relevant to our empirical application is the ability of the noncausal VAR model to capture mild conditional heteroskedasticity prevalent in quarterly financial and macroeconomic data (see Lanne and Saikkonen (2013)). Although the conventional causal VAR model could be augmented with a generalized autoregressive conditional heteroskedasticity (GARCH) component, this would increase the number of parameters considerably and be difficult to take into account in structural analysis. Conversely, taking GARCH effects into account requires no additional effort in the noncausal VAR framework. The presence of conditional heteroskedasticity can also be made use of in selecting the correct noncausal specification because we can expect to find autocorrelation in the squared residuals of an estimated VAR(r, s) model with incorrect orders r and s.

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et al. (2014) for a formal presentation, where often documented properties of stock returns, such as excess volatility and predictability are attributed to unmodeled heterogenous beliefs). Yet another phenomenon giving rise to noncausality are news shocks that affect the agents' behavior, but cannot be observed by the econometrician, and, hence, are omitted from the empirical model (for a recent survey, see Beaudry and Portier (2014)).

[6] For a special univariate case, where a a closed-form nonlinear representation exists, see Gourieroux and Zakoïan (2013).

[7] This suggests that the noncausal VAR model should also be robust against other types of regime-switching models, such as Markov-switching models commonly employed in economics and finance although, to the best of our knowledge, this issue has not been explicitly studied.

3.2 Generalized impulse response analysis

For the linear causal SVAR model, computing the impulse response function is straightforward as discussed in Section 2. For nonlinear models, including the noncausal vector autoregression, simulation methods are called for, and the generalized impulse response function (GIRF) put forth by Gallant et al. (1993), and Koop et al. (1996) offers a viable alternative way of conducting structural analysis. The basic idea in both papers is the same: The GIRF is defined as the difference between two conditional expectations (forecast paths), one of which contains the effect of the examined shock, while the other one (the benchmark path) does not involve the shock.

The major difference between the two approaches is related to the treatment of the shock. While Gallant et al. (1993) impose the shock directly on $\mathbf{y}_t$, the GIRF of Koop et al. (1996) traces out the effect of a disturbance to the error term of the model. We employ the former approach, i.e., we add a shock to each component of $\mathbf{y}_t$ in turn. The reason for this is that we are interested in the effects of unanticipated shocks, and, therefore, tracing out the effects of a disturbance to the predictable error term of the noncausal VAR model would not yield the desired impulse response function. An alternative would be to trace out the effects of a shock on the error term of the nonlinear causal representation of the model that is unpredictable, but the causal representation is not known, and even if it were known, its error term would not necessarily enter it in an additive manner, as required by Koop et al. (cf. their equation (1)).

In the bivariate noncausal VAR model, we define the GIRF of shock δ_{it}, $i = 1, 2$, to the ith element of $\mathbf{y}_t$ at horizon l as

$$GI(l, \delta_{it}, \boldsymbol{\omega}_{t-1}) = E(\mathbf{y}_{t+l}|\delta_{it}, \boldsymbol{\omega}_{t-1}) - E(\mathbf{y}_{t+l}|\boldsymbol{\omega}_{t-1}), \quad l = 0, 1, 2, \ldots, \tag{8}$$

where $\boldsymbol{\omega}_{t-1}$ and δ_{it} are the history and the shock to the ith equation that we condition on when computing the conditional expectations, respectively. Notice that the GIRF depends on the sign and size of the shock as well as the history (the state) of the process at time t. The conditional expectation $E(\mathbf{y}_{t+l}|\delta_{it}, \boldsymbol{\omega}_{t-1})$

is taken conditional on a fixed history $\boldsymbol{\omega}_{t-1}$ and a fixed value of the ith shock at time t, while integrating out all other contemporaneous and future shocks. The GIRF, interpreted as the time profile of the effect of the shock δ_{it} hitting at time t, is hence obtained as the difference between two conditional expectations: one conditioned on the shock and history $\boldsymbol{\omega}_{t-1}$ and another conditioned only on the same history $\boldsymbol{\omega}_{t-1}$. For the noncausal VAR$(r, s)$ model with $s \geq 1$, the computation of the conditional expectations in (8) calls for a simulation-based method, such as that recently proposed by Nyberg and Saikkonen (2014) that we employ in this paper.

As suggested by Koop et al. (1996), the shocks δ_{it} $(i = 1, 2)$ can be sampled (bootstrapped) from among the innovations (prediction errors) of the estimated model because they are unpredictable and can, thus, be interpreted as unanticipated shocks. In most models, the residuals are the innovations, but the residuals of the noncausal VAR model explicitly depend on future values of $\mathbf{y}_t$ (reflecting the fact that the error term of the noncausal VAR model is predictable), and hence cannot be considered innovations. Therefore, one-step prediction errors computed by the methods of Nyberg and Saikkonen (2014) are used instead.

A general overview of the dynamic effects of the shocks is given by the GIRF conditioned on all histories and shocks. This GIRF is characteristic of the data at hand, and solves the problem of selecting the size and sign of the shocks to each equation. In addition, GIRFs can be computed over different subsets of histories and shocks to answer specific research questions (see Koop et al. (1996), and Lanne (2015) specifically in the case of univariate noncausal AR models). In Section 4 below, we are, in particular, interested in finding the business cycle-specific GIRFs computed separately over the histories and shocks related to business cycle expansions and recessions. Similarly, we also separately report results related to histories of bear and bull stock markets. It is worth pointing out that such computations are particularly straightforward in the noncausal VAR model compared to regime-switching models, as the problem of increasing complexity due to taking the possibility of future regime changes into account is sidestepped (see Karamé (2012) for a discussion on this in the context of a Markov-switching structural VAR model). This is a manifestation of the ability of the noncausal model to accommodate nonlinearities discussed in Section 3.1.

The GIRF can be presented graphically, as shown in Gallant et al. (1993), Koop et al. (1996) and Teräsvirta et al. (2010, Section 15.2). However, instead of the GIRF, in line with the previous literature, we report a measure of the relative importance of the real activity and stock market shocks for the future evolution of GDP growth and stock return. To that end, we employ the generalized forecast error variance decomposition (GFEVD) recently proposed for nonlinear models by Lanne and Nyberg (2014) that is analogous to the FEVD of the linear causal VAR model in (5). For shock i ($i = 1, 2$), variable j ($j = 1, 2$), horizon h ($h = 0, 1, 2, \ldots$) and history $\boldsymbol{\omega}_{t-1}$, the GFEVD is obtained as a function of the GIRFs in (8) as

$$\lambda_{ij,\boldsymbol{\omega}_{t-1}}(h) = \frac{\sum_{l=0}^{h} GI(l, \delta_{it}, \boldsymbol{\omega}_{t-1})_j^2}{\sum_{l=0}^{h} GI(l, \delta_{1t}, \boldsymbol{\omega}_{t-1})_j^2 + \sum_{l=0}^{h} GI(l, \delta_{2t}, \boldsymbol{\omega}_{t-1})_j^2}. \tag{9}$$

The denominator measures the aggregate cumulative effect of both shocks, while the numerator is the cumulative effect of the ith shock. By construction, (9) lies between 0 and 1, measuring the relative contribution of the ith shock in relation to the total impact of the two shocks after h periods on the jth variable in $\mathbf{y}_t$, and these contributions sum to unity. In the same way as the GIRF, the GFEVD can be conditioned on any subset of histories and shocks of interest, and in Section 4, we separately report results for business cycle expansions and recessions, and bull and bear market periods, in addition to the GFEVD based on all data. See details on computing the GFEVD (9) in the Appendix.

The GIRFs and GFEVDs can also be computed for a linear VAR model, and in order to assess the importance of long-run identification restrictions in the causal VAR model, we also report such GFEVDs in Section 4.3. In that case, the computations are actually greatly simplified by the fact that the GIRFs do not depend on the history $\boldsymbol{\omega}_{t-1}$, and they can be obtained with explicit formulae (see, e.g., Pesaran and Shin, 1998) and plugged into (9) to compute the GFEVDs.

4 Empirical Results

4.1 Dataset

Our empirical analysis concentrates on quarterly U.S. real GDP growth and stock return series from 1953:Q1 to 2012:Q4. The quarterly returns are obtained by summing up the monthly continuously compounded value-weighted returns in each quarter. The return series is compiled by the Center of Research in Security Prices (CRSP), and adjusted for monthly U.S. consumer price inflation. The quarterly real GDP growth rate is obtained as the logarithmic difference of the real GDP (vintage November 2013). The CRSP returns are downloaded from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html), while the source of the rest of the data is the FRED databank of the Federal Reserve Bank St. Louis.

In line with the previous literature, we concentrate on the (generalized) forecast error variance decomposition in gauging the relative importance of GDP growth and stock return. As pointed out in Section 3.2, the GFEVD is history- and shock-dependent in the noncausal VAR model. We thus also report the GFEVDs specific to business cycle recession and expansion as well as bear and bull stock market histories. In bull market periods, stock prices tend to rise, whereas in bear market periods they tend to fall, and it is interesting to see whether this shows up in the relative importance of the two shocks.

To identify recessions and expansions, in accordance with much of past research, we use the official U.S. business cycle recession and expansion periods determined by the National Bureau of Economic Research (NBER). The U.S. stock market turning points are taken from Pagan and Sossounov (2003) and Nyberg (2013). In both cases, the turning points have been determined on a monthly basis. To aggregate them to the quarterly level, we follow the rule that a recession (bear market) starts in a given quarter if the monthly peak turning point occurs in the first or second month of that quarter. The endpoints of the quarterly recession (bear market) periods are determined in an analogous way: a quarter is classified as a recession (bear market) quarter if a trough occurs in the second or third month of the quarter. The quarters not classified as recession (bear market) quarters, are expansion (bull market) quarters. The business cycle and stock market periods overlap, but they are not identical: 77% of the expansion quarters are also bull market quarters, and 59% of the recession quarters are also bear market quarters.

4.2 Model selection

We start the empirical analysis by selecting the appropriate VAR(r, s) specification for the bivariate vector $\mathbf{y}_t$ consisting of the U.S. real GDP growth and stock return. An issue of particular interest is the presence of noncausality, in which case the preferred model is nonlinear and the methods discussed in Section 3 are needed in gauging the relative importance of the two GDP growth and stock return shocks and in selecting the model, we follow the procedure suggested by Lanne and Saikkonen (2013) discussed in Section 3.1.

Among the Gaussian linear VAR models, the Akaike and Bayesian information criteria recommend the second-order model. The p-values of the Ljung–Box and McLeod–Li tests for remaining residual autocorrelation and conditional heteroskedasticity, respectively, are reported in the second column of Table 1. The residuals of the VAR(2) model seem serially uncorrelated, and there is only slight evidence in favor of remaining conditional heteroskedasticity at the 10% level. However, the quantile-quantile (Q-Q) plots (not shown) as well as normality tests of the residuals indicate that the assumption of Gaussian errors is not reasonable. In particular, the p-values of the Jarque-Bera and Lütkepohl's (2006, Section 4.5) normality test are virtually zero. The residuals appear to be leptokurtic, suggesting a non-Gaussian error distribution with fat tails, such as a multivariate t-distribution. Thus, we proceed by estimating all second-order VAR(r, s)-t models with t-distributed errors.

The results of the VAR(r, s)-t models are also summarized in Table 1, and they lend support to non-Gaussianity and noncausality, questioning the adequacy of the conventional linear Gaussian VAR model. Of the second-order models, the purely noncausal VAR(0,2)-t model maximizes the likelihood function. According to the residual diagnostics, there is neither autocorrelation nor conditional heteroskedasticity remaining in the residuals of this model, and it is also the only specification passing the diagnostic tests at the 10% level.[8] Moreover, the Q-Q plots of the residuals in Figure 1 attest to the adequacy of the multivariate t-distribution for the errors. While the fit of the t-distribution for the residuals of the equation for GDP growth is excellent, the distribution of the residuals of the equation for the stock return appears to be slightly skewed to the right. All in all, the VAR(0,2)-t model thus seems the preferred second-order VAR specification. As a final diagnostic check, we estimated VAR(1,2)-t and VAR(0,3)-t models (i.e., we augmented the preferred VAR(0,2)-t specification by one lag or lead) and tested for the significance of the additional parameters by the likelihood ratio test. The null hypothesis of the additional coefficient matrix equalling zero could not be rejected at conventional significance levels, which lends additional support to the selected model.

Details on the estimated VAR(0,2)-t model are presented in Table 2. The estimate of the degree-of-freedom parameter λ of the t-distribution is quite small, indicating indeed a fat-tailed error distribution. Furthermore, the fact that a purely noncausal model is selected, means that GDP growth and stock return depend on their future expected values, but not on their past realized values. This can be seen by taking the conditional expectation of both sides of (6) (with $r = 0$ and $s = 2$) with respect to the information set $\{\mathbf{y}_t, \mathbf{y}_{t-1}, \ldots\}$:

$$\mathbf{y}_t = \Phi_1 E_t(\mathbf{y}_{t+1}) + \Phi_2 E_t(\mathbf{y}_{t+2}) + E_t(\boldsymbol{u}_t).$$

Interestingly the future leads of the GDP growth are found statistically sig-

[8]These tests are not exactly valid in the case of the noncausal VAR model as they do not take estimation error correctly into account, and, hence, the p-values should only be taken as convenient summary measures of autocorrelation remaining in the residuals and their squares (see Lanne and Saikkonen (2011)).

nificant predictors of the stock return (at the 1% significance level). The GDP growth rate is also highly dependent on its own future values, whereas the leads of the stock return are not statistically significant predictors of either variable.[9] This is in accordance with the results of Fama (1990) and Schwert (1990), who found leads of future real activity to have useful predictive power for U.S. stock returns. Binswanger (2000) also provided similar evidence in the subsample period preceding the early 1980s, but a breakdown in this relationship thereafter. However, his latest observations were from the year 1997.

The strong evidence in favor of non-Gaussianity and noncausality advises against basing structural analysis on the causal SVAR model identified by long-run restrictions. This conclusion is reinforced by the good fit of the noncausal VAR model, and we thus proceed the structural analysis based on the VAR(0,2)-t model.

4.3 Forecast error variance decomposition analysis

We computed generalized forecast error variance decompositions in order to study the relative importance of the shocks to the GDP growth and the real stock return. For simplicity, we call these two shocks the real activity (RA) and stock market (SM) shocks. For comparison, we also report the forecast error decomposition based on the Gaussian VAR(2) model and the long-run identification restrictions entertained in the previous literature. The differences in the results highlight the facts that the latter model is misspecified, and, in fact, different shocks are being considered. Indeed, recall that the long-run restrictions are supposed to be identifying the fundamental (F) and non-fundamental (NF) shocks that are not likely to exactly match the RA and SM shocks of our generalized setup.

As discussed in Section 3.2, in the noncausal VAR model the relevant unan-

[9] We also compared the out-of-sample forecasting performance of the VAR(0,2)-t model with that of the causal VAR(2)-N and VAR(2,0)-t models, which can be seen as a robustness check against overfitting. The differences are minor and statistically insignificant although the noncausal model turned out slightly superior, especially at one and two-quarter forecast horizons (details of the results are available upon request).

ticipated shock is the one-step forecast error that is unpredictable by past values of GDP growth and stock return, and these shocks are computed first from the VAR(0,2)-t model presented in Table 2.

The FEVDs of the SVAR(2) model and the GFEVDs of the VAR(0,2)-t model in Table 3 are based on all the histories. It is immediately noticed that the relative importance of the shocks hardly depends on the the horizon in the case of either model, which is quite a common finding also in the previous literature (see, e.g., Binswanger (2004a,b), and Velinov and Chen (2014)), and results from the fact that the (generalized) impulse responses decay to zero very quickly. The non-fundamental (NF) and the stock market (SM) shocks seem to dominate the FEVD of the stock return, accounting for about 65% of its variation in both the causal Gaussian and noncausal models. Thus, in both cases, the general conclusion is more or less the same. The rather minor role played by the real activity challenges the view that fundamentals determine stock prices. This result is in line with Binswangers's (2004a,b) results for the post mid-1980s period, while Chen and Velinov (2014) found the fundamental and non-fundamental shocks approximately equally important in the U.S. data from 1960 to 2013.

When looking at the figures for the GDP growth, large differences between the two models are seen. According to the causal model, the fundamental shock clearly dominates, accounting for more than three quarters of the forecast error variance, while the corresponding proportion for the real activity shock in the noncausal model is only 40–44%. One possible explanation to this finding lies in the news view literature of business cycle fluctuations, according to which news about future events affects the real activity (see, e.g., Beaudry and Portier (2014)). A shock to the stock return can be thought of as news concerning the future that has immediate real economic effects. Taken at face value, this empirical finding thus emphasizes the importance of news for the real activity and business cycle fluctuations.[10]

[10]The FEVD of the causal VAR(2,0)-t model identified by the long-run restriction yields essentially the same conclusions as the Gaussian SVAR(2) model. In fact, the importance of the

Due to the nonlinearity of the noncausal VAR model, the generalized forecast error variance decompositions need not be constant in time. In Figures 2 and 3, we depict the relative importance of the real activity shock to GDP growth and stock returns four quarters ahead ($h = 4$) with 68% level confidence bands (see the details on computing the confidence bands in the Appendix). The time-varying GFEVDs, and their eight-quarter moving averages exhibit some fluctuations, but no clear development patterns can be detected. In particular, there is no clear change around the beginning of the 1980s, in contrast to Binswanger's (2004a,b) findings based on linear SVAR models.

In Table 4, we report the business cycle-specific GFEVDs implied by the noncausal VAR model. The relative contribution of the real activity shock to the stock return is clearly greater in both expansion and recession periods than implied by the noncausal VAR model based on all the histories in Table 3. Bearing in mind the capability of the noncausal model to accommodate missing variables, this outcome is in accordance with Lee's (1998) finding that the relative importance of fundamentals for the stock return increases with the number of (relevant) variables in the VAR model.

As far as the GDP growth is concerned, the relative importance of the real activity and stock market shocks does not seem to depend on the business cycle. However, the real activity shock seems to have a markedly greater relative importance in expansion and recession periods than implied by the GFEVDs based on all histories (a rise of up to ten percentage points). Nevertheless, the relative importance of the real activity shock still remains much smaller than that of the fundamental shock in the linear SVAR model.

The differences between Tables 3 and 4 are likely to be a consequence of the fact that the different GFEVDs are now based on different shocks: in computing the GFEVDs in Table 3, the shocks are bootstrapped from among all the shocks, while those in the left and right panels of Table 4 are based on shocks related to

non-fundamental shock to the stock return was found even slightly smaller than in the Gaussian model reported in Table 3.

only expansion and recession periods, respectively. In a way, the results in Table 4 thus respect the structure of the data more accurately, as each GFEVD is based on shocks typical to one phase of the business cycle only. Both shocks tend to be negative in recession periods, while the shocks related to expansions are more often positive. In recessions, both shocks also have higher variance. Because there really are differences in the shocks across the business cycle phases, we should probably rely on the business cycle specific results.

In Table 5, we report the GFEVDs in bull and bear stock market periods. The results are, in general, quite similar in both periods. It thus seems that the relative importance of the two shocks does not seem to depend strongly on whether stock prices are going up or down, despite the fact that shocks have different properties in bull and bear markets. In particular, in the bear market periods, both shocks tend to be negative, and also have greater variance than when stock prices are going up (bull market). Nevertheless, compared to the GFEVDs for all histories in Table 3, the relative importance of the real activity shock for both variables seems somewhat greater when the phase of the stock market is taken into account. This is in line with the findings concerning the business cycle phase specific results in Table 4, and presumably follows similarly from the fact that these figures are based on a better match of histories and shocks.

Finally, we also computed the GFEVDs in the linear VAR model in order to assess the relative importance of noncausality and long-run identification restrictions for the results.[11] Compared to the FEVDs of fundamental and nonfundamental shocks in Table 3, the GFEVDs of the RA and SM shocks turned out to be quite different. Quite remarkably, the stock market shock was virtually irrelevant for the GDP growth, which also differs from the conclusions based on the noncausal VAR model. This suggests that for the real activity, both the long-run identification restrictions and noncausality matter. In contrast, for the stock return, both the SM and RA shocks seemed almost equally important. Hence, as far as the stock

[11] To save space, the detailed results are not reported, but they are available upon request.

return is concerned, these results are quite well in line with those based on the noncausal VAR model, suggesting that noncausality only plays a minor role, while the differences mostly stem from the long-run identification restrictions.

In sum, our main findings from the noncausal models are twofold. First, the (correctly specified) noncausal VAR model strongly emphasizes the relative importance of the stock market shock for the real activity. In particular, it is far more important than the non-fundamental shock in the linear structural VAR model identified by long-run restrictions. This effect seems to arise from noncausality rather than the long-run identification restrictions since according to the generalized forecast error variance decomposition of the causal model, the stock market shock plays hardly any role in real activity dynamics. Second, the noncausal VAR model suggests a greater role for the real activity in stock return dynamics than the linear SVAR model. This conclusion is shared with the generalized forecast error variance decomposition of the causal model, which suggests that the discrepancy can be attributed to the long-run identification restrictions rather than noncausality.

5 Conclusions

There is a large literature on the relationship between stock returns and real activity. Much of the previous research is confined to structural analysis in the bivariate linear VAR framework, concentrating on the relative importance of 'fundamental' and 'non-fundamental' shocks identified via long-run restrictions. The relative importance of the shocks is typically measured by means of the forecast error variance decomposition. In this study, we have examined the robustness of this methodology vis-à-vis an alternative identification scheme and nonlinearity. Because of the well-known problems related to long-run identification in SVAR models, we have considered the generalized forecast error variance decomposition, replacing the 'fundamental' and 'non-fundamental' shocks by shocks to stock return and real activity. As a versatile nonlinear model, we have considered the noncausal VAR model of Lanne and Saikkonen (2013), which has been shown to be capable of accommodating many kinds of nonlinearities. In addition, it has the benefit of being able to capture effects of missing variables, which may be an issue in the small model typically considered.

Our empirical results concern the postwar quarterly U.S. real activity and stock return data. A noncausal specification was clearly selected among the adequate bivariate VAR models, suggesting omitted nonlinearities or missing variables in the commonly employed linear specification. The generalized forecast error variance decompositions based on the noncausal VAR model indicated greater relative importance of shocks to the real activity on the stock return than implied by the linear SVAR model. In the same vein, the linear model appeared to underestimate the importance of the shock to the stock return on the real activity. When interpreting these results, it should be kept in mind that there is not necessarily a one-to-one correspondence between the shocks to real activity and stock return, and the fundamental and non-fundamental shocks, respectively. The generalized forecast error variance decompositions in the causal and noncausal VAR models turned out quite similar for the stock return, indicating the importance of the long-run identification restriction as the driver of the results. For the GDP growth, in contrast, these decompositions were quite different, pointing at the dominating role of noncausality.

All in all, it seems that the conclusions on the dynamics of stock return and real activity do depend on identification and taking nonlinearities into account. Besides real activity and stock returns, the same SVAR methodology has been applied to dividends, earnings and interest rates (see, e.g., Lee (1992, 1995, 1998)) and stock returns and inflation (see Hess and Lee (1999)). In future work, it would be interesting to study the robustness of these conclusions with respect to identification and nonlinearities using the methods put forth in this paper.

Appendix: Details on Computing GFEVD (9) in Noncausal VAR Model

In this Appendix, we give practical details on how to compute the GFEVD (9) in the case of the noncausal VAR model. The step-by-step procedure given below closely follows Lanne and Nyberg (2014) with the important modifications implied by the noncausal VAR model. As discussed in Section 3.2, we recommend computing the GFEVD as the average of $\lambda_{ij,\boldsymbol{\omega}_{t-1}}(h)$ over all the histories, and shocks obtained by bootstrapping from the one-step-ahead forecast errors of the selected noncausal model. This yields the GFEVD characteristic of the data at hand, solving the problem of selecting the size of shocks to each equation in $\mathbf{y}_t$.

The GFEVDs (9) for the bivariate ($K = 2$) noncausal model are computed by the following steps:

1. Select an appropriate noncausal VAR model (i.e. select r and s in the VAR(r, s) model (6)).
2. Construct (one-step-ahead) forecasts using the selected noncausal model for different histories $\boldsymbol{\omega}_{t-1}$ and compute forecast errors in the usual way. Notice that the history $\boldsymbol{\omega}_{t-1}$ consists of the information (i.e. the lags of $\mathbf{y}_t$) used to compute the forecasts (i.e. the conditional expectation given in (8) without the effect of the shock). Details on computing forecasts in the noncausal VAR model can be obtained from Nyberg and Saikkonen (2014).
3. Draw N vectors of shocks $(\delta_{1t}, \delta_{2t}, \ldots, \delta_{Kt})'$ from the set of forecast errors obtained in Step 2.
4. Pick a history $\boldsymbol{\omega}_{t-1}$ from among the set of histories which are of interest in the analysis (for example, all histories or business cycle-specific histories, as analyzed in Section 4.3).
5. Pick a shock vector, and compute GI($\cdot$) (see (8)) for each $\delta_{it}, i = 1, \ldots, K$. In (8), the benchmark conditional expectations (without the shock) are already computed in Step 2. For the first sequence of conditional expectations,

following Gallant et al. (1993) and as discussed in Section 3.2, we insert the shock δ_{it} directly to the value of $\mathbf{y}_{t-1}$ (included in the history $\boldsymbol{\omega}_{t-1}$).

6. Plug the GIs computed in Step 5 into (9) to obtain $\lambda_{ij,\boldsymbol{\omega}_{t-1}}(h)(h = 0, 1, 2, \ldots)$ for the particular history and shock.

7. Repeat Steps 5 and 6 for all N vectors of shocks.

8. Repeat Steps 4–7 for all the histories of interest.

9. Finally, compute the average of $\lambda_{ij,\boldsymbol{\omega}_{t-1}}(h)$, $(h = 0, 1, 2, \ldots)$ over all the histories and shocks.

Based on the procedure above, we can also compute the confidence bands of the GFEVD components. This can be done in various ways. We recommend computing all the history-specific averages of $\lambda_{ij,\boldsymbol{\omega}_{t-1}}(h)$ over the shocks (i.e. Steps 5–8 above). This yields an empirical distribution of GFEVDs, where we can easily compute the pointwise confidence bands for the selected confidence levels. For example, in Figures 2 and 3 we depict the 68% confidence bands of the GFEVD components.

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Tables and Figures

Table 1: Diagnostic checks of the second-order causal and noncausal VAR models.

	Model			
	VAR(2)-N	VAR(2,0)-t	VAR(1,1)-t	VAR(0,2)-t
Log-likelihood	-593.055	-586.619	-584.745	-584.601
Ljung-Box(4), GDP growth	0.369	0.486	0.888	0.921
Ljung-Box(4), Stock return	0.874	0.814	0.105	0.189
McLeod-Li(4), GDP growth	0.298	0.229	0.004	0.179
McLeod-Li(4), Stock return	0.081	0.090	0.718	0.306

Notes: VAR(p)-N and VAR(r, s)-t denote the Gaussian pth order VAR model, and the noncausal VAR model with r lags and s leads and a t-distributed error term, respectively. The p-values of the Ljung-Box and McLeod-Li tests for error autocorrelation and conditional heteroskedasticity with four lags are reported for both equations.

Table 2: Estimation results of the VAR(0,2) model.

	0.311	-0.001
Φ_1	(0.063)	(0.006)
	2.260	0.048
	(0.606)	(0.061)
	0.117	-0.007
Φ_2	(0.067)	(0.006)
	1.973	-0.006
	(0.663)	(0.058)
	0.051	-0.029
Σ	(0.007)	(0.037)
	-0.029	5.056
	(0.037)	(0.724)
λ	5.527	
	(1.642)	

Notes: Parameter estimates of the bivariate VAR(0,2) model for the real GDP growth (x_t) and the real stock return (r_t) (i.e., $\mathbf{y}_t = [x_t \quad r_t]'$). The figures in the parentheses are standard errors.

Table 3: Forecast error variance decompositions of the selected SVAR and noncausal VAR models for different forecast horizons h (quarters).

	SVAR(2)-N				VAR(0,2)-t			
	GDP growth		Stock return		GDP growth		Stock return	
h	F	NF	F	NF	RA	SM	RA	SM
0	0.76	0.24	0.34	0.66	0.44	0.56	0.30	0.70
1	0.78	0.22	0.34	0.66	0.41	0.59	0.36	0.64
2	0.77	0.23	0.35	0.65	0.40	0.60	0.36	0.64
3	0.77	0.23	0.35	0.65	0.40	0.60	0.36	0.64
4	0.77	0.23	0.35	0.65	0.40	0.60	0.36	0.64
8	0.77	0.23	0.35	0.65	0.40	0.60	0.36	0.64
12	0.77	0.23	0.35	0.65	0.40	0.60	0.36	0.64

Notes: For the VAR(0,2)-t model, the GFEVD (9) is reported. F and NF denote the fundamental and non-fundamental shocks, respectively, in the linear VAR model identified by the long-run identification restrictions discussed in Section 2, whereas RA and SM denote the real activity and stock market shocks, respectively. The GFEVD is based on 100 shocks bootstrapped from among the one-step-ahead forecast errors of the VAR(0,2)-t model presented in Table 2. For each pair of shocks, the GIRFs (8) used in the construction of the GFEVD are computed for each of the 238 histories (i.e. all the histories), yielding, in total, 23 800 GIRFs, over which the GFEVD (9) is averaged. The conditional expectations in the GIRFs (8) are based on 100 000 simulated realizations of the model. See details on computing the GFEVD (9) in the Appendix.

Table 4: GFEVDs of the noncausal model for business cycle-specific histories.

	Expansion				Recession			
	GDP growth		Stock return		GDP growth		Stock return	
h	RA	SM	RA	SM	RA	SM	RA	SM
0	0.50	0.50	0.47	0.53	0.50	0.50	0.44	0.56
1	0.49	0.51	0.48	0.52	0.49	0.51	0.47	0.53
2	0.48	0.52	0.48	0.52	0.49	0.51	0.47	0.53
3	0.48	0.52	0.48	0.52	0.49	0.51	0.46	0.54
4	0.48	0.52	0.48	0.52	0.49	0.51	0.46	0.54
8	0.48	0.52	0.48	0.52	0.49	0.51	0.46	0.54
12	0.48	0.52	0.48	0.52	0.49	0.51	0.46	0.54

Notes: Different histories ($\boldsymbol{\omega}_{t-1}$) are classified as expansions and recessions based on the official U.S. business cycle turning points determined by the NBER (see Section 4.1). Similarly as in Table 3, the real activity (RA) and stock market (SM) shocks are denoted by RA and SM, respectively. See also other notes to Table 3.

Table 5: GFEVDs of the noncausal model for bear and bull stock market-specific histories.

	Bull				Bear			
	GDP growth		Stock return		GDP growth		Stock return	
h	RA	SM	RA	SM	RA	SM	RA	SM
0	0.50	0.50	0.44	0.56	0.48	0.52	0.44	0.56
1	0.48	0.52	0.47	0.53	0.46	0.54	0.47	0.53
2	0.48	0.52	0.47	0.53	0.46	0.54	0.47	0.53
3	0.48	0.52	0.47	0.53	0.46	0.54	0.47	0.53
4	0.48	0.52	0.47	0.53	0.46	0.54	0.46	0.54
8	0.48	0.52	0.46	0.54	0.46	0.54	0.46	0.54
12	0.47	0.53	0.46	0.54	0.46	0.54	0.46	0.54

Notes: Different histories ($\boldsymbol{\omega}_{t-1}$) are classified as bull and bear stock market periods based on the turning points determined by Pagan and Sossounov (2003) and Nyberg (2013) (see Section 4.1). See also the notes to Table 3.

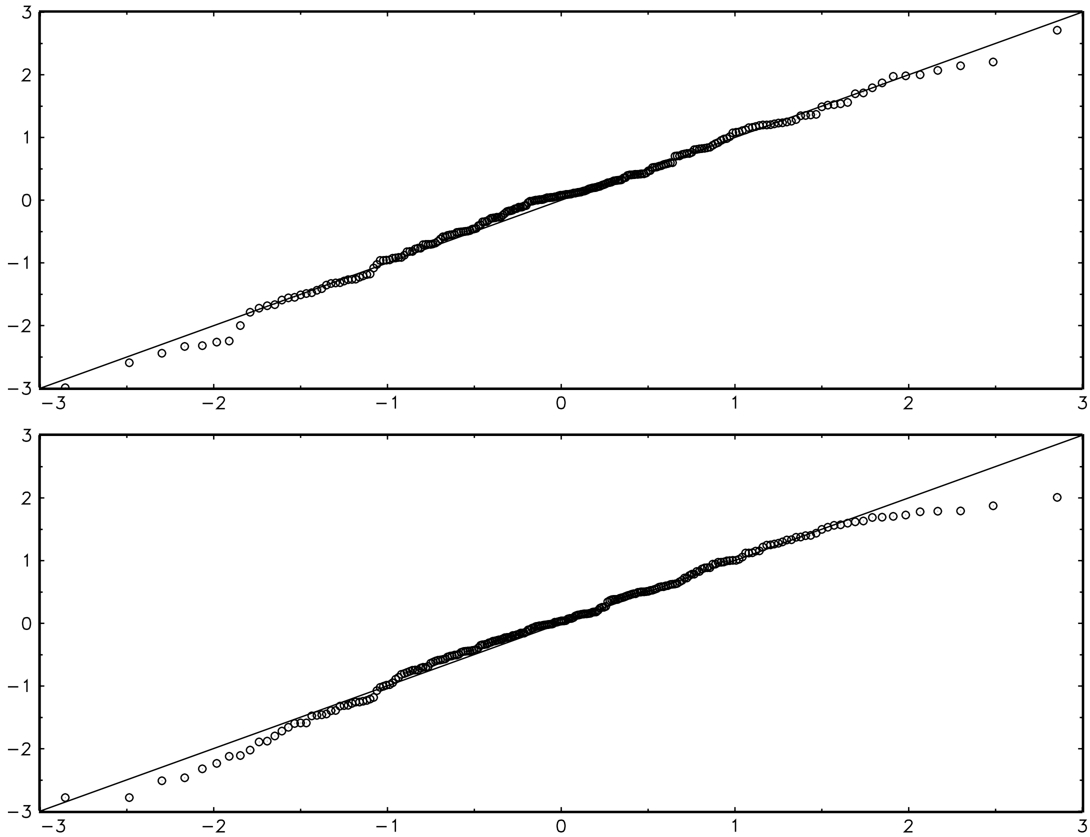

Figure 1: Quantile-quantile plots of the residuals of the VAR(0,2)-t model. The residuals of the GDP growth (stock return) in the upper (lower) panel.

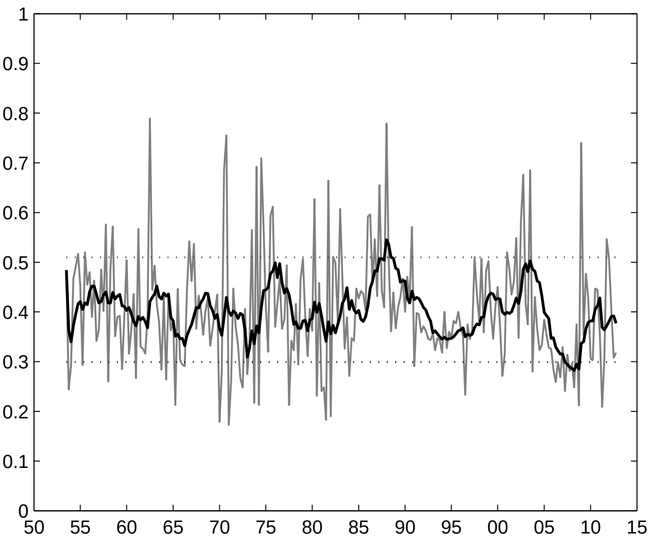

Figure 2: Relative importance of the real activity shock to GDP growth (1953-2012). The time-varying GFEVDs (the black line is their 8-quarter moving average) are averages over different shocks for each history with 68% confidence intervals depicted with dashed lines.

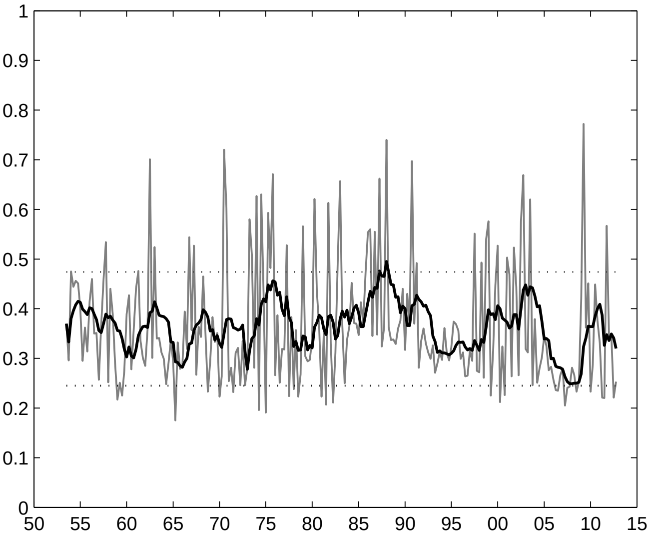

Figure 3: Relative importance of the real activity shock to stock return (1953-2012). The time-varying GFEVDs (the black line is their 8-quarter moving average) are averages over different shocks for each history with 68% confidence intervals depicted with dashed lines.

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