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Daily House Price Indices: Construction, Modeling, and Longer-Run Predictions^{*}

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Abstract

We construct *daily* house price indices for ten major U.S. metropolitan areas. Our calculations are based on a comprehensive database of several million residential property transactions and a standard repeat-sales method that closely mimics the methodology of the popular monthly Case-Shiller house price indices. Our new daily house price indices exhibit dynamic features similar to those of other daily asset prices, with mild autocorrelation and strong conditional heteroskedasticity of the corresponding daily returns. A relatively simple multivariate time series model for the daily house price index returns, explicitly allowing for commonalities across cities and GARCH effects, produces forecasts of monthly house price changes that are superior to various alternative forecast procedures based on lower frequency data.

JEL classification: C22, C32, C53, G17, R21

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"There are many ways to measure changes in house prices, but the Standard & Poor's/Case-Shiller index has become many economists' favored benchmark in recent years."

Wall Street Journal, September 25, 2012

1 Introduction

For many U.S. households their primary residence represents their single largest financial asset holding: the Federal Reserve estimated the total value of the U.S. residential real estate market at \$16 trillion at the end of 2011, compared with \$18 trillion for the U.S. stock market (as estimated by the Center for Research in Security Prices). Consequently, changes in housing valuations importantly affect households' saving and spending decisions, and in turn the overall growth of the economy (see also the discussion in Holly, Pesaran and Yamagata, 2010). A number of studies (e.g., Case, Quigley and Shiller, 2011) have also argued that the wealth effect of the housing market for aggregate consumption is significantly larger than that of the stock market. The recent economic crisis, which arguably originated with the precipitous drop in housing prices beginning in 2006, directly underscores this point.

Set against this background, we: i) construct new *daily* house price indices for ten major U.S. metropolitan areas based on a comprehensive database of publicly recorded residential property transactions;¹ ii) show that the dynamic dependencies in the new daily housing price series closely mimic those of other asset prices, and that these dynamic dependencies along with the cross-city correlations are well described by a standard multivariate GARCH type model; and iii) demonstrate that this relatively simple daily model allows for the construction of improved longer-run monthly and quarterly housing price forecasts compared with forecasts based on existing monthly and/or quarterly indices.

Our new daily house price indices are based on the same "repeat-sales" methodology underlying the popular S&P/Case-Shiller monthly indices (Shiller, 1991), and the Federal

¹To the best of our knowledge, this represents the first set of house price indices at the daily frequency analyzed in the academic literature. Daily residential house price indices constructed on the basis of a patent-pending proprietary algorithm are available commercially from Radar Logic Inc.

Housing Finance Agency's quarterly indices. Measuring the prices at a daily frequency help alleviate potential "aggregation biases" that may plague the traditional coarser monthly and quarterly indices if the true prices change at a higher frequency. More timely house prices are also of obvious interest to policy makers, central bankers, developers and lenders alike, by affording more accurate and timely information about the housing market and the diffusion of housing prices across space and time (see, e.g., the analysis in Brady, 2011).² Even though actual housing decisions are made relatively infrequently, potential buyers and sellers may also still benefit from more timely price indicators.

The need for higher frequency daily indexing is perhaps most acute in periods when prices change rapidly, with high volatility, as observed during the recent financial crisis and its aftermath. To illustrate, Figure 1 shows our new daily house price index along with the oft-cited monthly S&P/Case-Shiller index for Los Angeles from September 2008 through September 2010. The precipitous drop in the daily index over the first six months clearly leads the monthly index. Importantly, the daily index also shows the uptick in housing valuations that occurred around April 2009 some time in advance of the monthly index. Similarly, the more modest rebound that occurred in early 2010 is also first clearly manifest in the daily index.

Systematically analyzing the features of the dynamics of the new daily house price indices for all of the ten metropolitan areas in our sample, we find that, in parallel to the daily returns on most other broadly defined asset classes, they exhibit only mild predictability in the mean, but strong evidence of volatility clustering. We show that the volatility clustering within and across the different house price indices can be satisfactorily described by a multivariate GARCH model. The correlation between the daily returns on the city indices is much lower than the correlation observed for the existing monthly return indices. However, as we temporally aggregate the daily returns to monthly and quarterly frequen-

 $^{^{2}}$ Along these lines, the analysis in Anundsen (2014) also suggests that real time econometric modeling could have helped in earlier detection of the fundamental imbalances underlying the recent housing market collapse.

cies, we find that the correlations increase to levels consistent with the ones observed for existing lower frequency indices. Furthermore, we document that the new daily indices do indeed result in improved house price index forecasts, not solely in that they more quickly identify turning points as suggested by Figure 1 for Los Angeles, but also more generally for longer forecast horizons and other sample periods. This holds true for the city-specific housing returns and a composite index, thus directly underscoring the informational advantages of the new daily index developed here vis-a-vis the existing monthly published indices.

The rest of the paper is organized as follows. The next section provides a review of house price index construction and formally describes the S&P/Case-Shiller methodology. Section 3 describes the data and the construction of our new daily prices series. Section 4 briefly summarizes the dynamic and cross-sectional dependencies in the daily series, and presents our simple multivariate GARCH model designed to account for these dependencies. Section 5 demonstrates how the new daily series and our modeling thereof may be used in more accurately forecasting the corresponding longer-run returns. Section 6 concludes. Additional analysis and empirical results are provided in the online Supplementary Appendix.

2 House price index methodologies

The construction of house price indices is plagued by two major difficulties. Firstly, houses are heterogeneous assets; each house is a unique asset, in terms of its location, characteristics, maintenance status, etc., all of which affect its price. House price indices aim to measure the price movements of a hypothetical house of average quality, with the assumption that average quality remains the same across time. In reality, average quality has been increasing over time, because newly-built houses tend to be of higher quality and more in line with current households' requirements than older houses. Detailed house qualities are not always available or not directly observable, so when measuring house prices at an aggregate level, it is difficult to take the changing average qualities of houses into consideration. The second major difficulty is sale infrequency. For example, the average time interval between two successive transactions of the same property is about six years in Los Angeles, based on our data set described in Section 3 below. Related to that, the houses sold at each point in time may not be a representative sample of the overall housing stock.

Three main methodologies have been used to overcome the above-mentioned difficulties in the construction of reliable house price indices (see, e.g., the surveys by Cho, 1996; Rappaport, 2007; Ghysels, Plazzi, Torous and Valkanov, 2013). The simplest approach relies on the median value of all transaction prices in a given period. The National Association of Realtors employ this methodology and publishes median prices of existing home sales monthly for both the national and four Census regions. The median price index has the obvious advantage of calculation simplicity, but it does not control for heterogeneity of the houses actually sold.

A second, more complicated, approach uses a hedonic technique, to price the "average quality" house by explicitly pricing its specific attributes. The U.S. Census Bureau constructs its Constant Quality (Laspeyres) Price Index of New One-Family Houses Sold using a hedonic method. Although this method does control for the heterogeneity of houses sold, it also requires more advanced estimation procedures and much richer data than are typically available (see, e.g., the recent study by Baltagi, Bresson and Etienne, 2014, who rely on a sophisticated unbalanced spatial panel model).

A third approach relies on repeat sales. This is the method used by both Standard & Poor's and the Office of Federal Housing Finance Agency (FHFA). The repeat sales model was originally introduced by Bailey, Muth and Nourse (1963), and subsequently modified by Case and Shiller (1989). The specific model currently used to construct the S&P/Case-Shiller indices was proposed by Shiller (1991) (see Clapp and Giaccotto, 1992; Meese and Wallace, 1997, for a comparison of the repeat-sales method with other approaches).³

As the name suggests, the repeat sales method estimates price changes by looking at repeated transactions of the same house. This provides some control for the heterogeneity in the characteristics of houses, while only requiring data on transaction prices and dates. The basic models, however, are subject to some strong assumptions (see, e.g., the discussion in Cho, 1996; Rappaport, 2007). Firstly, it is assumed that the quality of a given house remains unchanged over time. In practice, of course, the quality of most houses changes through aging, maintenance or reconstruction. This in turn causes a so-called "renovation bias." Secondly, repeat sales indices exploit information only from houses that have been sold at least twice during the sampling period. This subset of all houses may not be representative of the entire housing stock, possibly resulting in a "sample-selection bias." Finally, as noted above, all of the index construction methods are susceptible to "aggregation bias" if the true average house price fluctuates within the estimation window.⁴

Our new daily home price indices are designed to mimic the popular S&P/Case-Shiller house price indices for the "typical" prices of single-family residential real estate. They are based on a repeat sales method and the transaction dates and prices for all houses that sold at least twice during the sample period. If a given house sold more than twice, then only the non-overlapping sale pairs are used. For example, a house that sold three times generates sale pairs from the first and second transaction, and the second and third transaction; the pair formed by the first and third transaction is not included.

More precisely, for a house j that sold at times s and t at prices $H_{j,s}$ and $H_{j,t}$, the

³Meese and Wallace (1997), in particular, point out that repeat-sales models can be viewed as special cases of hedonic models, assuming that the attributes, and the shadow prices of these attributes, do not change between sales. Thus, if the additional house characteristic data were widely available, it would clearly be preferable to use a hedonic pricing model.

⁴Calhoun, Chinloy and Megbolugbe (1995) compare repeat sales indices over annual, semiannual, quarterly as well as monthly intervals, and conclude that aggregation bias arises for all intervals greater than one month. By analogy, if the true housing values fluctuate *within* months, the standard monthly indices are likely to be biased. We formally test this conjecture below.

standard repeat sales model postulates that,

$$\beta_t H_{j,t} = \beta_s H_{j,s} + \sqrt{2}\sigma_w w_{j,t} + \sqrt{(t-s)}\sigma_v v_{j,t}, \qquad 0 \le s < t \le T, \tag{1}$$

where the house price index at any given time τ , computed across all houses j that sold between time 0 and T, is defined by the inverse of β_{τ} . The last two terms on the righthand side account for "errors" relative to the prices predicted by the aggregate index, in the sale pairs, with $\sqrt{2}\sigma_w w_{j,t}$ representing the "mispricing error," and $\sqrt{(t-s)}\sigma_v v_{j,t}$ representing the "interval error." Mispricing errors are included to allow for imperfect information between buyers and sellers, potentially causing the actual sale price of a house to differ from its "true" value. The interval error represents a possible drift over time in the value of a given house away from the overall market trend, and is therefore scaled by the (square root of the) length of the time interval between the two transactions. The error terms $w_{j,t}$ and $v_{j,t}$ are assumed independent and identically standard normal distributed.

The model in (1) lends itself to estimation by a multi-stage generalized least square type procedure (for additional details, see Case and Shiller, 1987), and each pair of sales of a given house $(H_{j,s}, H_{j,t})$ represents a data point to be used in estimation. We adopt a modified version of this method to construct our daily indices, described in detail in Section 3.1 below. In the standard estimation procedure, a "base" period must be chosen, to initialize the index, and the S&P/ Case-Shiller indices use January 2000. All index values prior to the base period are estimated simultaneously. After the base period, the index values are estimated using a chain-weighting procedure that conditions on previous values. This chain-weighting procedure is used to prevent revisions of previously published index values. Finally, the S&P/Case-Shiller indices are smoothed by repeating a given transaction in three successive months, so that the index for a given month is based on sale pairs for that month and the preceding two months (see the Index Construction Section of S&P/Case-Shiller Home Price Index Methodology for additional details).

3 Daily house price indices

The transaction data used in our daily index estimation is obtained from DataQuick, a property information company. The database contains information about more than one hundred million property transactions in the United States from the late 1990s to 2012. We focus our analysis on the ten largest Metropolitan Statistical Areas (MSAs), as measured in the year 2000. Further details pertaining to the data and the data cleaning procedures are provided in the Supplementary Appendix.

3.1 Estimation

The repeat-sales index estimation based on equation (1) is not computationally feasible at the daily frequency, as it involves the simultaneous estimation of several thousand parameters: the daily time spans for the ten MSAs range from 2837 for Washington D.C. to 4470 days for New York. To overcome this difficulty, we use an expanding-window estimation procedure: we begin by estimating daily index values for the final month in an initial startup period, imposing the constraint that all of the earlier months in the period have only a single *monthly* index value. Restricting the daily values to be the same within each month for all but the last month drastically reduces the dimensionality of the estimation problem. We then expand the estimation period by one month, and obtain daily index values for the new "last" month. We continue this expanding estimation proof," in that earlier values of the index do not change when later data becomes available. Finally, similar to the S&P/Case-Shiller methodology, we normalize all of the individual indices to 100 based on their average values in the year 2000.

One benefit of the estimation procedure we adopt is that it is possible to formally test whether the "raw" daily price series actually exhibit significant intra-monthly variation. In particular, following the approach used by Calhoun, Chinloy and Megbolugbe (1995) to test for "aggregation biases," we test the null hypothesis that the estimates of $\beta_{i,\tau}$ for MSA i are the same for all days τ within a given calendar month against the alternative that these estimates differ within the month. These tests strongly reject the null for all months and all ten metropolitan areas; further details concerning the actual F-tests are available upon request. We show below that this statistically significant intra-monthly variation also translates into economically meaningful variation and corresponding gains in forecast accuracy compared to the forecasts based on coarser monthly index values only.

3.2 Noise filtering

Due to the relatively few transactions that are available on a given day, the raw daily house price indices are naturally subject to measurement errors, an issue that does not arise so prominently for monthly indices.⁵ To help alleviate this problem, it is useful to further clean the data and extract more accurate estimates of the true latent daily price series. Motivated by the use of similar techniques for extracting the "true" latent price process from high-frequency data contaminated by market microstructure noise (e.g., Owens and Steigerwald, 2006; Corsi et al., 2014), we rely on a standard Kalman filter-based approach to do so.

Specifically, let $P_{i,t}$ denote the true latent index for MSA *i* at time *t*. We assume that the "raw" price indices constructed in the previous section, $P_{i,t}^* = 1/\beta_{i,t}$, are related to the true latent price indices by,

$$\log P_{i,t}^* = \log P_{i,t} + \eta_{i,t},\tag{2}$$

where the $\eta_{i,t}$ measurement errors are assumed to be serially uncorrelated. For simplicity of the filter, we further assume that the true index follows a random walk with drift,

$$r_{i,t} \equiv \Delta \log P_{i,t} = \mu_i + u_{i,t},\tag{3}$$

⁵The average number of transactions per day ranges from a low of 49 for Las Vegas to a high of 180 for Los Angeles. Measurement errors are much less of an issue for monthly indices, as they are based on approximately 20 times as many observations; i.e., around 1000 to 3500 observations per month.

where $\eta_{i,t}$ and $u_{i,t}$ are mutually uncorrelated. It follows readily by substitution that,

$$r_{i,t}^* \equiv \Delta \log P_{i,t}^* = r_{i,t} + \eta_{i,t} - \eta_{i,t-1}.$$
(4)

Combining (3) and (4), this in turn implies an MA(1) error structure for the "raw" returns, with the value of the MA coefficient determined by the variances of $\eta_{i,t}$ and $u_{i,t}$, σ_{η}^2 and σ_u^2 . This simple MA(1) structure is consistent with the sample autocorrelations for the raw return series reported in Figure A.1 in the Supplementary Appendix.

Interpreting equations (3) and (4) as a simple state-space system, μ , σ_{η}^2 and σ_u^2 may easily be estimated by standard (quasi-)maximum likelihood methods. This also allows for the easy filtration of of the "true" daily returns, $r_{i,t}$, by a standard Kalman filter; see, e.g., Hamilton (1994). The Kalman filter implicitly assumes that $\eta_{i,t}$ and $u_{i,t}$ are *iid* normal. If the assumption of normality is violated, the filtered estimates are interpretable as best linear approximations. The Kalman filter parameter estimates reported in the Supplementary Appendix imply that the noise-to-signal (σ_{η}/σ_u) ratios for the daily index returns range from a low of 6.48 (Los Angeles) to a high of 15.18 (Boston), underscoring the importance of filtering out the noise.

The filtered estimates of the latent "true" daily price series for Los Angeles are depicted in Figure 2 (similar plots for all ten cities are available in Figure A.2 in the Supplementary Appendix). For comparison, we also include the raw daily prices and the monthly S&P/Case-Shiller index. Looking first at the top panel for the year 2000, the figure clearly illustrates how the filtered daily index mitigates the noise in the raw price series. At the same time, the filtered prices also point to discernable within month variation compared to the step-wise constant monthly S&P/Case-Shiller index.

The bottom panel of Figure 2 reveals a similar story for the full 1995-2012 sample period. The visual differences between the daily series and the monthly S&P/Case-Shiller index are obviously less glaring on this scale. Nonetheless, the considerable (excessive) variation in the raw daily prices coming from the noise is still evident. We will consequently refer to and treat the filtered series as *the* daily house price indices in the analysis below.⁶

The online Supplementary Appendix provides further frequency-based comparisons of the daily indices with the traditional monthly S&P/Case-Shiller indices, and the potential loss of information in going from a daily to a monthly observation frequency. In sum, we find that the monthly S&P/Case-Shiller indices essentially "kill" all of the within quarter variation inherent in the new daily indices, while delaying all of the longer-run information by more than a month. We turn next to a more detailed analysis of the time series properties of the new daily indices.

4 Time series modeling of daily housing returns

To facilitate the formulation of a multivariate model for all of the ten city indices, we restrict our attention to the common sample period from June 2001 to September 2012. Excluding weekends and federal holidays, this yields 2,843 daily observations.

4.1 Summary statistics

Summary statistics for each of the ten daily series are reported in Table 1. Panel A gives the sample means and standard deviations for each of the index levels. Standard unit root tests clearly suggest that the price series are non-stationary, and as such the sample moments in Panel A need to be interpreted with care; further details concerning the unit root tests are available upon request. In the following, we therefore focus on the easierto-interpret daily return series.

The daily sample mean returns reported in Panel B are generally positive, ranging from a low of -0.006 (Las Vegas) to a high of 0.015 (Los Angeles and Washington D.C.).

 $^{^{6}}$ The "smoothed" daily prices constructed from the full sample look almost indistinguishable from the filtered series shown in the figures. We purposely rely on filtered rather than smoothed estimates to facilitate the construction of meaningful forecasts.

The standard deviation of the most volatile daily returns 0.599 (Chicago) is double that of the least volatile returns 0.291 (New York). The first-order autocorrelations are fairly close to zero for all of the cities, but the Ljung-Box χ^2_{10} tests for up to tenth order serial correlation indicate significant longer-run dynamic dependencies in many of the series.

The corresponding results for the squared daily returns reported in Panel C indicate very strong dynamic dependencies. This is also evident from the plot of the ten daily return series in Figure 3, which show a clear tendency for large (small) returns in an absolute sense to be followed by other large (small) returns. This directly mirrors the ubiquitous volatility clustering widely documented in the literature for other daily speculative returns. Further, consistent with the evidence for other financial asset classes, there is also a commonality in the volatility patterns across most of the series. In particular, the magnitude of the daily price changes for each of the ten cities were generally fairly low from 2004 to 2007 compared to their long-run average values. Correspondingly, and directly in line with the dynamic dependencies observed for other asset prices, there was a sizeable increase in the magnitude of the typical daily house price change for the majority of the cities concurrent with the onset of the 2008-2010 financial crisis, most noticeably so for Mi-ami, Las Vegas and San Francisco.

4.2 Modeling conditional mean dependencies

The summary statistics discussed above point to the existence of some, albeit relatively mild, dynamic dependencies in the daily conditional means for most of the cities. Some of these dependencies may naturally arise from a common underlying dynamic factor that influences housing valuations nationally. In order to accommodate both city specific and national effects within a relatively simple linear structure, we postulate the following model for the conditional means of the daily returns,

$$E_{t-1}(r_{i,t}) = c_i + \rho_{i1}r_{i,t-1} + \rho_{i5}r_{i,t-5} + \rho_{im}r_{i,t-1}^m + b_{ic}r_{c,t-1}^m,$$
(5)

where $r_{i,t}^m$ refers to the (overlapping) "monthly" returns defined by the summation of the corresponding daily returns,

$$r_{i,t}^m = \sum_{j=0}^{19} r_{i,t-j},\tag{6}$$

and the composite (national) return $r_{c,t}$ is defined as a weighted average of the individual city returns,

$$r_{c,t} = \sum_{i=1}^{10} w_i r_{i,t},\tag{7}$$

with the weights identical to the ones used in the construction of the composite ten city monthly S&P/Case Shiller index, which are 0.212, 0.074, 0.089, 0.037, 0.050, 0.015, 0.055, 0.118, 0.272, and 0.078. The own fifth lag of the returns is included to account for any weekly calendar effects. The inclusion of the own monthly returns and the composite monthly returns provides a parsimonious way of accounting for longer-run city-specific and common national dynamic dependencies. This particular formulation is partly motivated by the Heterogeneous Autoregressive (HAR) model proposed by Corsi (2009) for modeling socalled realized volatilities, and we will refer to it as an HAR-X model for short. This is not the absolutely *best* time series model for each of the ten individual daily MSA indices. The model does, however, provide a relatively simple and easy-to-implement common parametric specification that fits all of the ten cities reasonably well.⁷

We estimate this model for the conditional mean simultaneously with the model for the conditional variance described in the next section via quasi-maximum likelihood. The estimation results in Table 2 reveal that the ρ_1 and ρ_5 coefficients associated with the own lagged returns are mostly, though not uniformly, insignificant when judged by the robust standard errors reported in parentheses. Meanwhile, the b_c coefficients associated with the composite monthly return are significant for nine out of the ten cities. Still, the one-day re-

⁷Importantly, for the proper modeling of longer-run dynamic dependencies and forecast horizons beyond the ones analyzed here, the model does not incorporate any cointegrating relationships among the MSA indices. More sophisticated structural panel data models involving longer time spans of data explicitly allowing for cointegration between housing prices and real income have been estimated by Holly, Pesaran and Yamagata (2010) among others.

turn predictability implied by the model is fairly modest, with the average daily R^2 across the ten cities equal to 0.024, ranging from a low of 0.007 (Denver) to a high of 0.049 (San Francisco). This mirrors the low R^2 s generally obtained from time series modeling of other daily financial returns (e.g., Tsay, 2010).

The adequacy of the common specification for the conditional mean in equation (5) is broadly supported by the tests for up to tenth-order serial correlation in the residuals $\varepsilon_{i,t} \equiv r_{i,t} - E_{t-1}(r_{i,t})$ from the model reported in Panel C of Table 2. Only two of the tests are significant at the 5% level (San Francisco and Washington, D.C.) when judged by the standard χ^2_{10} distribution. At the same time, the tests for serial correlation in the squared residuals $\varepsilon^2_{i,t}$ from the model, given in the bottom two rows of Panel C, clearly indicate strong non-linear dependencies in the form of volatility clustering.

4.3 Modeling conditional variance and covariance dependencies

Numerous parametric specifications have been proposed in the literature to describe volatility clustering in asset returns. Again, in an effort to keep our modeling procedures simple and easy to implement, we rely on the popular GARCH(1,1) model (Bollerslev, 1986) for describing the dynamic dependencies in the conditional variances for all of the ten cities,

$$Var_{t-1}(r_{i,t}) \equiv h_{i,t} = \omega_i + \kappa_i \varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}.$$
(8)

The results from estimating this model jointly with the the conditional mean model described in the previous section are reported in Panel B of Table 2 together with robust standard errors following Bollerslev and Wooldridge (1992) in parentheses.

The estimated GARCH parameters are all highly statistically significant and fairly similar across cities. Consistent with the results obtained for other daily financial return series, the estimates for the sum $\kappa + \lambda$ are all very close to unity (and just above for Chicago, at 1.002) indicative of a highly persistent, but eventually mean-reverting, time-varying volatility process. The high persistence might also in part reflect breaks in the overall levels of the volatilities, most notably around 2007 for several of the cities. As such, it is possible that even better fitting in-sample models could be obtained by explicitly allowing for structural breaks. At the same time, with the time of the breaks unknown a priori, these models will not necessarily result in better out-of-sample forecasts (see, e.g., the discussion in Pesaran and Timmermann, 2007; Anderson and Tian, 2014).

Wald tests for up to tenth-order serial correlation in the resulting standardized residuals, $\varepsilon_{i,t}/h_{i,t}^{1/2}$, reported in Panel C, suggest that little predictability remains, with only two of the cities (Las Vegas and San Francisco) rejecting the null of no autocorrelation at the 5% level, and none at the 1% level. The tests for serial correlation in the squared standardized residuals, $\varepsilon_{i,t}^2/h_{i,t}$, reject the null for four cities, perhaps indicative of some remaining predictability in volatility not captured by this relatively simple model. However for the majority of cities the specification in equation (8) appears to provide a satisfactory fit. The dramatic reduction in the values of the test statistics for the squared residuals compared to the values reported in the second row of Panel C is particularly noteworthy.

The univariate HAR-X-GARCH models defined by equations (5) and (8) indirectly incorporate commonalities in the cross-city returns through the composite monthly returns $r_{c,t}$ included in the conditional means. The univariate models do not, however, explain the aforementioned commonalities in the volatilities observed across cities and the corresponding dynamic dependencies in the conditional covariances of the returns.

The Constant Conditional Correlation (CCC) model proposed by Bollerslev (1990) provides a particularly convenient framework for jointly modeling the ten daily return series by postulating that the temporal variation in the conditional covariances are proportional to the products of the conditional standard deviations. Specifically, let $\mathbf{r}_t \equiv$ $[r_{1,t}, ..., r_{10,t}]'$ and $D_t \equiv diag \{h_{1t}^{1/2}, ..., h_{10,t}^{1/2}\}$ denote the 10 × 1 vector of daily returns and 10 × 10 diagonal matrix with the GARCH conditional standard deviations along the diagonal, respectively. The GARCH-CCC model for the conditional covariance matrix of the

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returns may then be succinctly expressed as,

$$Var_{t-1}(\mathbf{r_t}) = D_t R D_t,\tag{9}$$

where R is a 10 × 10 matrix with ones along the diagonal and the conditional correlations in the off-diagonal elements. Importantly, the R matrix may be efficiently estimated by the sample correlations for the 10 × 1 vector of standardized HAR-X-GARCH residuals; i.e., the estimates of $D_t^{-1} [\mathbf{r_t} - E_{t-1}(\mathbf{r_t})]$. The resulting estimates are reported in Table A.5 in the Supplementary Appendix.

We also experimented with the estimation of the Dynamic Conditional Correlation (DCC) model of Engle (2002), resulting in only a very slight increase in the maximized value of the (quasi-) log-likelihood function. Hence, we conclude that the relatively simple multivariate HAR-X-GARCH-CCC model defined by equations (5), (8), and (9) provides a satisfactory fit to the joint dynamic dependencies in the conditional first and second order moments of the ten daily housing return series.

4.4 Temporal aggregation and housing return correlations

The estimated conditional correlations from the HAR-X-GARCH-CCC model for the daily index returns reported in the Supplementary Appendix average only 0.022. By contrast the unconditional correlations for the monthly S&P/Case Shiller index returns calculated over the same time period average 0.708, and range from 0.382 (Denver–Las Vegas) to 0.926 (Los Angeles–San Diego). The discrepancy between the two sets of numbers may appear to call into question the integrity of our new daily indices and/or the time-series models for describing the dynamic dependencies therein, however conditional daily correlations and the unconditional monthly correlations are not directly comparable. In an effort to more directly compare the longer-run dependencies inherent in our new daily indices with the traditional monthly S&P/Case Shiller indices, we aggregate our daily return indices to a monthly level by summing the daily returns within a month (20 days). The unconditional sample correlations for these new monthly returns are reported in the lower triangle of Panel B in Table 3. These numbers are obviously much closer, but generally still below the 0.708 average unconditional correlation for the published monthly S&P/Case Shiller indices.

However, as previously noted, the monthly S&P/Case Shiller indices are artificially "smoothed," by repeating each sale pair in the two months following the actual sale. As such, a more meaningful comparison of the longer-run correlations inherent in our new daily indices with the correlations in the S&P/Case Shiller indices is afforded by the unconditional quarterly (60 days) correlations reported in the upper triangle of Panel B in Table 3. There, we find an average correlation of 0.668, and a range of 0.317 (Denver–Las Vegas) to 0.906 (Los Angeles–San Diego), which are quite close to the corresponding numbers for the published S&P/Case Shiller index returns.

These comparisons, of course, say nothing about the validity of the HAR-X-GARCH-CCC model for the daily returns, and the low daily *conditional* correlations estimated by that model. As a further model specification check, we therefore also consider the modelimplied longer-run correlations, and study how these compare with the sample correlations for the actual longer-run aggregate returns.

The top number in each element of Panels A and B of Table 3 gives the median modelimplied unconditional correlations for the daily, weekly, monthly, and quarterly return horizons, based on 500 simulated sample paths. The bottom number in each element is the corresponding sample correlations for the actual longer-run aggregated returns. Although the daily unconditional correlations in Panel A are all close to zero, the unconditional correlations implied by the model gradually increase with the return horizon, and almost all of the quarterly correlations are in excess of one-half. Importantly, the longer-run modelimplied correlations are all in line with their unconditional sample analogues.

To further illuminate this feature, Figure 4 presents the median model-implied and

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sample correlations for return horizons ranging from one-day to a quarter, along with the corresponding simulated 95% confidence intervals implied by the model for the Los Angeles–New York city pair. The model provides a very good fit across all horizons, with the actual correlations well within the confidence bands. The corresponding plots for all of the 45 city pairs, presented in Figure A.3 in the Supplementary Appendix, tell a similar story.

Taken as whole these results clearly support the idea that the longer-run cross-city dependencies inherent in our new finer sample daily house price series are consistent with those in the published coarser monthly S&P/Case Shiller indices. The results also confirm that the joint dynamic dependencies in the daily returns are well described by the relatively simple HAR-X-GARCH-CCC model, in turn suggesting that this model could possibly be used in the construction of improved house price index forecasts over longer horizons.

5 Forecasting housing index returns

One of the major potential benefits from higher frequency data is the possibility of constructing more accurate forecasts by using models that more quickly incorporate new information. The plot for Los Angeles discussed in the introduction alludes to this idea. In order to more rigorously ascertain the potential improvements afforded by the daily house price series and our modeling thereof, we consider a comparison of the forecasts from the daily HAR-X-GARCH-CCC model with different benchmark alternatives.

Specifically, consider the problem of forecasting the 20-day ("monthly") return on the house price index for MSA i,

$$r_{i,t}^{(m)} \equiv \sum_{j=0}^{19} r_{i,t-j} \tag{10}$$

for forecast horizons ranging from h = 20 days ahead to h = 1 day ahead.⁸ When h = 20this corresponds to a simple one-step ahead forecast for one-month returns, but for h < 20

⁸In the forecast literature, this is commonly referred to as a "fixed event" forecast design; see Nordhaus (1987) for an early analysis of such problems.

an optimal forecast will contain a mixture of observed data and a forecast for the return over the remaining part of the month. We will use the period June 2001 to June 2009 as our in-sample period, and the period July 2009 to September 2012 as our out-of-sample period, with all of the model parameters estimated once over the fixed in-sample period. ⁹

Our simplest benchmark forecast is based purely on end-of-month data, and is therefore *not* updated as the horizon shrinks. We will consider a simple AR(1) for these monthly returns,

$$r_{i,t}^{(m)} = \phi_0 + \phi_1 r_{i,t-20}^{(m)} + e_{i,t}.$$
(11)

As the forecast is not updated through the month, the forecast made at time t-h is simply the AR(1) forecast made at time t - 20,

$$\hat{r}_{i,t-h}^{Mthly} = \hat{\phi}_0 + \hat{\phi}_1 r_{i,t-20}^{(m)}.$$
(12)

Our second benchmark forecast is again purely based on monthly data, but now we allow the forecaster to update the forecast at time t - h, which may be in the middle of a month. We model the incorporation of observed data by allowing the forecaster to take a linear combination of the monthly return observed on day t - h and the one-month-ahead forecast made on that day,

$$\hat{r}_{i,t-h}^{Interp} = \left(1 - \frac{h}{20}\right) r_{i,t-h}^{(m)} + \frac{h}{20} \left(\hat{\phi}_0 + \hat{\phi}_1 r_{i,t-h}^{(m)}\right).$$
(13)

Our third forecast fully exploits the daily return information, by using the actual returns from time t - 19 to t - h as the first component of the forecast, as these are part of the information set at time t - h, and then using a "direct projection" method to obtain a forecast for the remaining *h*-day return based on the one-month return available at time

⁹In a preliminary version of the paper we used an earlier vintage of the DataQuick database that ended in June 2009, which is how we chose this particular sample-split point. That preliminary version of the paper did not consider any out-of-sample comparisons, and so the results presented here are close to "true," rather than "pseudo," out-of-sample.

t - h. Specifically,

$$\hat{r}_{i,t-h}^{Direct} = \sum_{j=h}^{19} r_{i,t-j} + \hat{\beta}_0^{(h)} + \hat{\beta}_1^{(h)} r_{i,t-h}^{(m)}, \tag{14}$$

where $\beta_0^{(h)}$ and $\beta_1^{(h)}$ are estimated from the projection:

$$\sum_{j=0}^{h-1} r_{i,t-j} = \beta_0^{(h)} + \beta_1^{(h)} r_{i,t-h}^{(m)} + u_{i,t}.$$
(15)

Finally, we consider a forecast based on the HAR-X-GARCH-CCC model presented in the previous section. Like the third forecast, this forecast uses the actual returns from time t - 19 to t - h as the first component, and then iterates the expression for the conditional daily mean in equation (5) forward to get forecasts for the remaining h days,

$$\hat{r}_{i,t-h}^{HAR} = \sum_{j=h}^{19} r_{i,t-j} + \sum_{j=0}^{h-1} \hat{E}_{t-h} \left[r_{i,t-j} \right].$$
(16)

Given the construction of the target variable, we expect the latter three forecasts ("Interp", "Direct", "HAR") to all beat the "Mthly" forecast for all horizons less than 20 days. If intra-monthly returns have dynamics that differ from those of monthly returns, then we expect the latter two forecasts to beat the "Interp" forecast. Finally, if the HAR-X-GARCH-CCC model presented in the previous section provides a better description of the true dynamics than a simple direct projection, then we would expect the fourth forecast to beat the third.

Figure 5 shows the resulting Root Mean Squared Errors (RMSEs) for the four forecasts as a function of the forecast horizon, when evaluated over the July 2009 to September 2012 out-of-sample period. The first striking, though not surprising, feature is that exploiting higher frequency (intra-monthly) data leads to smaller forecast errors than a forecast based purely on monthly data. All three of the forecasts that use intra-monthly information out-perform the model based solely on end-of-month data. The only exception to this is for Las Vegas at the h = 20 horizon, where the HAR model slightly under-performs the monthly model.

Another striking feature of Figure 5 is that the more accurate modeling of the daily dynamic dependencies afforded by the HAR-X-GARCH-CCC model results in lower RM-SEs across *all* forecast horizons for eight of the ten cities. For San Francisco and Las Vegas the direct projection forecasts perform essentially as well as the HAR forecasts, and for Denver and Los Angeles the improvement of the HAR forecast is small (but positive for all horizons). For some of the cities (Boston, Miami and Washington D.C., in particular) the improvements are especially dramatic over longer horizons.

The visual impression from Figure 5 is formally underscored by Diebold-Mariano tests, reported in Table 4. Not surprisingly, the HAR forecasts significantly outperform the monthly forecasts for horizons of 1, 5 and 10 days, for all ten cities and the composite index. At the one-month horizon, a tougher comparison for the model, the HAR forecasts are significantly better than the monthly model forecasts for four out of ten cities, as well as the composite index, and are never significantly beaten by the monthly model forecasts. Almost identical conclusions are drawn when comparing the HAR forecasts to the "interpolation" forecasts, supporting the conclusion that the availability of daily data clearly holds the promise of more accurate forecasts, particularly over shorter horizons, but also even at the monthly level.

The bottom row of each panel in Table 4 compares the HAR forecasts with those from a simple direct projection model. Such forecasts have often been found to perform well in comparison with "iterated" forecasts from more complicated dynamic models. By contrast, the Diebold-Mariano tests reported here suggest that the more complicated HAR forecasts generally perform better than the direct projection forecasts. For no city-horizon pair does the direct projection forecast lead to significantly lower out-of-sample forecast RMSE than the HAR forecasts, while for many city-horizon pairs the reverse is true. In particular, for Boston, Miami and Washington D.C., the HAR forecasts significantly beat the direct projection forecasts across all four horizons, and for the composite index this is true for all but the shortest horizon.

6 Conclusion

We present a set of new *daily* house price indices for ten major U.S. Metropolitan Statistical Areas spanning the period from June 2001 to September 2012. The indices are based on the repeat sales method of Shiller (1991), and use a comprehensive database of several million publicly recorded residential property transactions. We demonstrate that the dynamic dependencies in the new daily housing price series closely mimic those of other financial asset prices, and that the dynamics, along with the cross-city correlations, are well described by a standard multivariate GARCH-type model. We find that this simple daily model allows for the construction of improved daily, weekly, and monthly housing price index forecasts compared to the forecasts based solely on monthly price indices.

The new "high frequency" house price indices developed here open the possibility for many other applications. Most directly, by providing more timely estimates of movements in the housing market, the daily series should be of immediate interest to policy makers and central banks. In a related context, the series may also prove useful in further studying the microstructure of the housing market. At a broader level, combining the daily house price series with other daily estimates of economic activity should afford better and more up-to-date insights into changes in the macro economy. Along these lines, the series also hold the promise for the construction of more accurate forecasts for other macro economic and financial time series. We leave all of these issues for future research.

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	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
					Pane	Panel A: Level				
Mean	177.764	145.561	128.901	118.049	162.896	136.511	164.473	137.702	159.450	170.039
Std. dev.	41.121	13.381	21.631	4.605	48.351	48.568	34.058	27.169	25.877	34.830
					Panel	Panel B: Returns				
Mean	0.015	0.008	-0.002	0.003	0.006	-0.006	0.010	0.005	0.011	0.015
Std.dev.	0.347	0.351	0.599	0.303	0.428	0.370	0.387	0.509	0.291	0.502
$\operatorname{AR}(1)$	-0.059	0.047	0.008	-0.018	-0.034	0.061	-0.005	-0.113	0.049	-0.018
LB(10)	67.877	21.935	24.362	16.838	17.742	59.549	15.065	269.509	13.335	24.977
					Panel C: 5	Panel C: Squared returns	IS			
Mean	0.121	0.123	0.358	0.092	0.183	0.137	- 0.150	0.259	0.085	0.252
Std. dev.	0.200	0.260	1.269	0.242	0.336	0.369	0.270	0.616	0.170	0.607
AR(1)	0.113	0.102	0.075	0.021	0.107	0.071	0.037	0.042	0.042	0.132
LB(10)	182.307	109.914	102.316	33.414	445.189	85.348	50.715	179.632	53.109	106.434

 Table 1: Daily summary statistics

Table 2: Daily HAR-X-GARCH models $r_{i,t} = c_i + \rho_{i,1}r_{i,t-1} + \rho_{i,5}r_{i,t-5} + \rho_{i,m}r_{i,t-1}^m + b_{i,c}r_{c,t-1}^m + \varepsilon_{i,t}$ $\varepsilon_{i,t}|\Omega_{t-1} \sim N(0, h_{i,t})$ $h_{i,t} = \omega_i + \kappa_i \varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}$ Washington, D.C. (0.024)(0.884)(0.007)(0.010)(0.020)-0.0351.2450.0120.0320.0840.027New York (0.538)(0.020)(0.017)(0.006)(0.006)-0.0290.9080.0400.0040.0540.033San Francisco (0.018)-0.222(0.223)-0.094(0.022)-0.014(0.006)(0.00)0.1600.0560.049San Diego (3.221)(0.020)(0.006)(0.010)(0.022)-0.013-0.037-0.0360.2400.0600.030Las Vegas (0.004)-0.111(0.368)(0.016)(0.039)0.017(0.007)Panel A: Mean 0.0040.0060.0350.044Miami (0.942)(0.006)(0.020)(0.032)-0.008 (0.008)1.152-0.034-0.0060.0600.027Denver -0.074(5.338)-0.015(0.052)(0.101)(0.008)(0.018)-0.0110.0100.0200.007Chicago (0.011)(0.163)(0.010)(0.007)(0.008)-0.006-0.0230.0940.0050.0490.009Boston -0.302(0.769)(0.022)(0.017)-0.014(0.005)(0.007)0.0300.0090.0390.018Los Angeles (0.678)(0.020)(0.020)(0.007)-0.080-0.014(0.009)1.7100.0540.0590.039c (×10^{-2}) ρ_5 ρ_m R^2 ρ_1 b_c

					Table 7	Table 2: Continued				
	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
					Panel]	Panel B: Variance				
$\omega~(\times 10^{-2})$	0.013	0.230	0.075	0.215	0.016	0.014	0.024	0.023	0.041	0.067
	(0.015)	(0.074)	(0.058)	(0.103)	(0.014)	(0.013)	(0.028)	(0.026)	(0.023)	(0.043)
Ľ	0.020	0.056	0.056	0.034	0.013	0.017	0.014	0.016	0.026	0.032
	(0.008)	(0.010)	(0.009)	(0.012)	(0.003)	(0.006)	(0.007)	(0.006)	(0.005)	(0.006)
Y	0.979	0.926	0.946	0.943	0.986	0.982	0.985	0.983	0.969	0.965
	(0.00)	(0.012)	(0.009)	(0.017)	(0.002)	(0.006)	(0.008)	(0.007)	(0.006)	(0.007)
$\kappa + \lambda$	0.999	0.982	1.002	0.977	0.999	0.999	0.999	0.999	0.995	0.998
				<u>[</u>]	anel C: Seri	Panel C: Serial correlation tests	tests			
$\varepsilon_{i,t}$	16.325	10.934	15.178	11.144	8.952	18.086	8.953	25.641	7.133	18.906
	(0.091)	(0.363)	(0.126)	(0.346)	(0.537)	(0.054)	(0.537)	(0.004)	(0.713)	(0.042)
$\varepsilon^{2}_{i,t}$	92.430	62.011	56.910	22.875	150.471	46.849	41.513	72.156	36.577	36.247
	(0.000)	(0.000)	(0.000)	(0.011)	(0.00)	(0.00)	(0.00)	(0.000)	(0.00)	(0.000)
$\varepsilon_{i,t} h_{i,t}^{-1/2}$	11.003	11.878	15.071	14.344	6.576	20.148	7.677	18.762	6.386	12.855
	(0.357)	(0.293)	(0.130)	(0.158)	(0.765)	(0.028)	(0.660)	(0.043)	(0.782)	(0.232)
$\varepsilon_{i,t}^2 h_{i,t}^{-1}$	12.511	24.289	24.616	25.424	9.426	4.946	16.156	40.312	8.650	11.998
	(0.252)	(0.007)	(0.006)	(0.005)	(0.492)	(0.895)	(0.095)	(0.00)	(0.566)	(0.285)

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)		o Quantina	DOILVOI	TITIOTTAT	Las vegas	San Diego	San Francisco	New York	Washington, D.C.
			Panel A: D	Daily (lowe	er triangle)) and Weekly	aily (lower triangle) and Weekly (upper triangle)	<u>e)</u>		
Los Angeles	Ι	$\underset{0.065}{0.117}$	$\underset{0.124}{0.061}$	$\underset{0.073}{0.066}$	$\underset{0.219}{0.197}$	$\underset{0.250}{0.172}$	$\underset{0.240}{0.198}$	$\underset{0.309}{0.309}$	$\underset{0.145}{0.145}$	$\underset{0.204}{0.156}$
Boston	$\underset{0.026}{0.026}$	I	$\begin{array}{c} 0.033 \\ 0.068 \end{array}$	$\underset{0.128}{0.068}$	$\underset{0.130}{0.130}$	$\underset{0.121}{0.121}$	$\underset{0.063}{0.143}$	$\underset{0.054}{0.118}$	$\underset{0.128}{0.128}$	$\underset{0.129}{0.129}$
Chicago	$\underset{0.019}{0.019}$	-0.007 -0.001	I	$\underset{0.108}{0.108}$	$\underset{0.149}{0.077}$	$\begin{array}{c} 0.058 \\ 0.064 \end{array}$	$\underset{0.042}{0.042}$	$\underset{0.148}{0.148}$	$\underset{0.115}{0.102}$	$\begin{array}{c} 0.068 \\ 0.089 \end{array}$
Denver	$\begin{array}{c} 0.001 \\ -0.003 \end{array}$	$\underset{0.031}{0.031}$	-0.002 -0.003	I	$\underset{0.100}{0.100}$	$\underset{0.110}{0.092}$	$\underset{0.090}{0.090}$	$\underset{0.106}{0.060}$	$\begin{array}{c} 0.053 \\ 0.006 \end{array}$	$\underset{0.090}{0.090}$
Miami	$\underset{0.069}{0.069}$	$\underset{0.043}{0.043}$	$\underset{0.046}{0.046}$	$\underset{0.047}{0.047}$	I	$\underset{0.239}{0.173}$	$\underset{0.214}{0.178}$	$\underset{0.176}{0.165}$	$\substack{0.187\\0.169}$	$\underset{0.183}{0.150}$
Las Vegas	$0.060\\0.077$	$\underset{0.049}{0.049}$	$\begin{array}{c} 0.015 \\ 0.032 \end{array}$	$\begin{array}{c} 0.038 \\ 0.027 \end{array}$	$\underset{0.054}{0.053}$	I	$\underset{0.209}{0.165}$	$\underset{0.162}{0.147}$	$\underset{0.060}{0.123}$	$\underset{0.173}{0.173}$
San Diego	$\begin{array}{c} 0.077 \\ 0.072 \end{array}$	$\begin{array}{c} 0.059 \\ 0.053 \end{array}$	-0.006 0.022	$\begin{array}{c} 0.045 \\ 0.042 \end{array}$	$\underset{0.060}{0.056}$	$\begin{array}{c} 0.058 \\ 0.065 \end{array}$	I	$\underset{0.263}{0.171}$	$\begin{array}{c} 0.148 \\ 0.169 \end{array}$	$\underset{0.127}{0.127}$
San Francisco	$\underset{0.235}{0.183}$	$\underset{0.038}{0.038}$	$\begin{array}{c} 0.037 \\ 0.065 \end{array}$	$\underset{-0.003}{0.006}$	$\underset{0.060}{0.057}$	$\begin{array}{c} 0.052 \\ 0.068 \end{array}$	$\underset{0.066}{0.066}$	I	$\begin{array}{c} 0.138 \\ 0.137 \end{array}$	$\underset{0.151}{0.151}$
New York	$\underset{0.041}{0.041}$	$\begin{array}{c} 0.011 \\ 0.000 \end{array}$	$\begin{array}{c} 0.047 \\ 0.061 \end{array}$	-0.009 -0.002	$\begin{array}{c} 0.065 \\ 0.063 \end{array}$	$\begin{array}{c} 0.010 \\ -0.002 \end{array}$	$\underset{0.029}{0.029}$	$\underset{0.031}{0.031}$	I	$\underset{0.088}{0.149}$
Washington, D.C.	$\underset{0.045}{0.045}$	$\begin{array}{c} 0.038 \\ 0.034 \end{array}$	$\underset{0.024}{0.017}$	$\begin{array}{c} 0.032 \\ 0.041 \end{array}$	$\underset{0.038}{0.038}$	$\underset{0.034}{0.049}$	$\underset{0.027}{0.033}$	$\begin{array}{c} 0.038 \\ 0.038 \end{array}$	$\underset{0.043}{0.043}$	1

	Los Angeles	Boston	Chicago	Denver	Miami	Miami Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
			Panel B: Mo	onthly (low	er triangle)	Panel B: Monthly (lower triangle) and Quarterly (upper triangle)	ly (upper triar	ıgle)		
Los Angeles	I	$\underset{0.621}{0.621}$	$0.530 \\ 0.602$	$\underset{0.506}{0.463}$	$\underset{0.852}{0.730}$	$\underset{0.837}{0.600}$	$\underset{0.906}{0.731}$	$\underset{0.834}{0.724}$	$\underset{0.747}{0.747}$	$\underset{0.733}{0.733}$
Boston	$\underset{0.348}{0.348}$	l	$\begin{array}{c} 0.451 \\ 0.655 \end{array}$	$\underset{0.559}{0.400}$	$\underset{0.507}{0.616}$	$\underset{0.522}{0.522}$	$\underset{0.673}{0.673}$	$\underset{0.623}{0.594}$	$\underset{0.735}{0.643}$	$\begin{array}{c} 0.627 \\ 0.688 \end{array}$
Chicago	$\underset{0.344}{0.266}$	$\underset{0.320}{0.320}$	I	$\underset{0.502}{0.323}$	$\begin{array}{c} 0.519 \\ 0.612 \end{array}$	$\substack{0.417\\0.510}$	$\underset{0.567}{0.513}$	$0.500 \\ 0.667$	$\begin{array}{c} 0.572 \\ 0.767 \end{array}$	$\begin{array}{c} 0.532 \\ 0.675 \end{array}$
Denver	$\begin{array}{c} 0.251 \\ 0.355 \end{array}$	$\underset{0.254}{0.254}$	$\underset{0.293}{0.138}$	I	$\underset{0.370}{0.457}$	$\underset{0.317}{0.391}$	$\underset{0.557}{0.454}$	$\underset{0.625}{0.416}$	$\underset{0.411}{0.411}$	$\begin{array}{c} 0.456 \\ 0.513 \end{array}$
Miami	$\underset{0.619}{0.493}$	$\underset{0.277}{0.384}$	$\underset{0.355}{0.355}$	$\underset{0.239}{0.239}$	I	$\underset{0.797}{0.591}$	$\underset{0.769}{0.769}$	$\underset{0.754}{0.669}$	$\underset{0.761}{0.734}$	0.697
Las Vegas	$\begin{array}{c} 0.395 \\ 0.633 \end{array}$	$\underset{0.322}{0.322}$	$\underset{0.233}{0.233}$	$\begin{array}{c} 0.229 \\ 0.201 \end{array}$	$\underset{0.547}{0.404}$	I	$\begin{array}{c} 0.589 \\ 0.782 \end{array}$	$0.558 \\ 0.657$	$0.599 \\ 0.659$	$\begin{array}{c} 0.582 \\ 0.708 \end{array}$
$\operatorname{San}\operatorname{Diego}$	$\begin{array}{c} 0.497 \\ 0.626 \end{array}$	$\begin{array}{c} 0.388 \\ 0.307 \end{array}$	$\underset{0.276}{0.276}$	$\underset{0.351}{0.266}$	$\underset{0.570}{0.468}$	$\underset{0.497}{0.497}$	I	$\underset{0.822}{0.678}$	$\underset{0.711}{0.731}$	$\underset{0.824}{0.694}$
San Francisco	$\underset{0.623}{0.511}$	$\underset{0.288}{0.288}$	$\underset{0.404}{0.253}$	$\underset{0.427}{0.216}$	$\underset{0.527}{0.424}$	$\underset{0.417}{0.343}$	$\begin{array}{c} 0.435 \\ 0.600 \end{array}$	I	$\begin{array}{c} 0.700 \\ 0.663 \end{array}$	$\underset{0.791}{0.677}$

Table 3: Continued

Note: Model-implied correlations are upper numbers and data-based correlations are in smaller font just below. Daily, weekly, monthly and quarterly horizons correspond to 1, 5, 20, $\underset{0.437}{0.437}$ $\underset{0.486}{0.486}$ $\underset{0.551}{0.433}$ $\underset{0.444}{0.368}$ $\begin{array}{c} 0.444 \\ 0.515 \end{array}$ $\begin{array}{c} 0.253 \\ 0.309 \end{array}$ $\underset{0.385}{0.385}$ $\begin{array}{c} 0.384 \\ 0.415 \\ 0.366 \\ 0.375 \end{array}$ $\underset{0.603}{0.469}$ 60 days respectively.

 $\underset{0.761}{0.738}$ T

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 $\underset{0.394}{0.431}$

 $\underset{0.430}{0.430}$

 $\underset{0.354}{0.354}$

 $\underset{0.496}{0.496}$

 $\underset{0.149}{0.247}$

 $\underset{0.427}{0.318}$

 $\underset{0.478}{0.478}$

New York

Washington, D.C.

30

-		Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
				Paı	nel A: One	→day-ahe	Panel A: One-day-ahead $(h = 1)$				
Mthly v.s. HAR	9.240	8.337	7.378	10.060	9.845	8.680	9.981	9.929	8.067	8.981	9.142
Interp v.s. HAR	8.707	10.171	7.623	6.242	9.249	11.415	8.569	10.786	7.865	8.609	10.293
Direct v.s. HAR	1.599	1.381	2.943	-0.176	1.224	2.785	0.126	-0.276	3.139	-0.012	2.173
				Pan	el B: One	-week-ah€	Panel B: One-week-ahead $(h = 5)$				
Mthly v.s. HAR	4.956	4.458	3.876	5.412	5.126	5.087	6.682	6.581	4.258	5.268	4.981
Interp v.s. HAR	4.071	2.964	4.856	5.466	6.724	5.882	4.501	4.761	5.349	4.588	5.304
Direct v.s. HAR	4.495	1.200	3.580	1.514	1.141	2.669	-0.298	0.768	-0.373	0.562	3.212
				Pane	1 C: Two-	weeks-ah€	Panel C: Two-weeks-ahead $(h = 10)$				
Mthly v.s. HAR	4.544	2.751	3.799	6.647	4.343	4.078	5.204	5.847	3.453	5.261	4.392
Interp v.s. HAR	4.372	1.478	3.617	4.586	4.042	3.333	2.489	3.598	2.954	2.973	3.798
Direct v.s. HAR	5.668	0.828	3.567	2.640	0.763	2.585	-0.214	1.342	-0.381	0.964	3.563
				Pane	l D: One-r	nonth-ah	Panel D: One-month-ahead $(h = 20)$				
Mthly v.s. HAR	I	I	Ι		I	I			I	I	I
Interp v.s. HAR	6.762	0.623	3.553	4.117	0.830	2.211	-0.511	1.777	0.941	1.909	4.268
Direct v.s. HAR	I	I	I	Ι	I	I	I	I	I	I	Ι

Table 4: Diebold-Mariano forecast comparison tests



Figure 1: Daily and monthly house price indices for Los Angeles



Figure 2: Raw and filtered daily house price indices for Los Angeles



Figure 3: Daily housing returns



Figure 4: Unconditional return correlations for Los Angeles and New York


Figure 5: Forecast RMSEs as a function of forecast horizon (1 to 20 days)

Supplementary Appendix

Daily House Price Indices: Construction, Modeling, and Longer-Run Predictions

Tim Bollerslev, Andrew J. Patton and Wenjing Wang

January 12, 2015

A.1 Data and data cleaning

The historical transaction records in DataQuick extends from the late 1990s to 2012 (exact dates are given in Talbe A.2 below) with some large metropolitan areas, such as Boston and New York, having transactions recorded as far back as 1987. Properties are uniquely identified by property IDs, which enable us to identify sale pairs. We rely U.S. Standard Use Codes contained in the DataQuick database to identify transactions of single-family residential homes. The specific counties included in each of the ten MSAs are listed in Table A.1.

Our data cleaning rules are based on the same filters used by S&P/Case-Shiller in the construction of their monthly indices. In brief, we remove all transactions that are not "arms length," using a flag for such transactions available in the database. We also remove transactions with "unreasonably" low or high sale prices (below \$5000 or above \$100 million, and those generating an average annual return of below -50% or above 100%), as well as any sales pair with an interval of less than six months. Sale pairs are also excluded if there are indications that major improvements have been made between the two transactions, although such indications are not always present in the database.

Once these filters are imposed, we use all remaining sale pairs to estimate the repeat-sales model presented in equation (1) using the estimation procedure described in Section 3.1. For the Los Angeles MSA, for example, we have a total of 877,885 "clean" sale pairs, representing an average of 180 *daily* sale pairs over the estimation period. Details for all ten MSAs are provided in Table A.2 below.

1

MSA	Represented counties	Counties in our indices
Los Angeles-Long Beach-Santa Ana, CA	Los Angeles CA, Orange CA	Los Angeles CA, Orange CA
Metropolitan Statistical Area (Los Angeles)		
Boston-Cambridge-Quincy, MA-NH	Essex MA, Middlesex MA, Norfolk MA, Plymouth	Essex MA, Middlesex MA, Norfolk MA, Plymouth
Metropolitan Statistical Area (Boston)	MA, Suffolk MA, Rockingham NH, Strafford NH	MA, Suffolk MA, Rockingham NH, Strafford NH
Chicago-Naperville-Joliet, IL Metropolitan	Cook IL, DeKalb IL, Du Page II, Kane IL, Kendal	Cook IL, DeKalb IL, Du Page II, Kane IL, Kendal
Division (Chicago)	IL, McHenry IL, Will IL, Grundy IL	IL, McHenry IL, Will IL, Grundy IL
Denver-Aurora, CO Metropolitan Statistical	Adams CO, Arapahoe CO, Broomfield CO, Clear	Adams CO, Arapahoe CO, Broomfield CO, Clear
Area (Denver)	Creek CO, Denver CO, Douglas CO, Elbert CO,	Creek CO, Denver CO, Douglas CO, Elbert CO,
	Gilpin CO, Jefferson CO, Park CO	Gilpin CO, Jefferson CO, Park CO
Miami-Fort Lauderdale-Pompanp Beach, FL	Broward FL, Miami-Dade FL, Palm Beach FL	Broward FL, Miami-Dade FL, Palm Beach FL
Metropolitan Statistical Area (Miami)		
Las Vegas-Paradise, NV Metropolitan Statis-	Clark NV	Clark NV
tical Area (Las Vegas)		
San Diego-Carlsbad-San Marcos, CA	San Diego CA	San Diego CA
Metropolitan Statistical Area (San Diego)		
San Francisco-Oakland-Fremont, CA	Alameda CA, Contra Costa CA, Marin CA, San	Alameda CA, Contra Costa CA, Marin CA, San
Metropolitan Statistical Area (San Fran-	Francisco CA, San Mateo CA	Francisco CA, San Mateo CA
cisco)		

	Lable A.I: Continued	
MSA	Represented counties	Counties in our indices
New York City Area (New York)	Fairfield CT, New Haven CT, Bergen NJ, Essex	Fairfield CT, New Haven CT, Bergen NJ, Essex
	NJ, Hudson NJ, Hunterdon NJ, Mercer NJ, Mid-	NJ, Hudson NJ, Hunterdon NJ, Mercer NJ, Mid-
	dlesex NJ, Monmouth NJ, Morris NJ, Ocean NJ,	dlesex NJ, Monmouth NJ, Morris NJ, Ocean NJ,
	Passaic NJ, Somerset NJ, Sussex NJ, Union NJ,	Passaic NJ, Somerset NJ, Sussex NJ, Union NJ,
	Warren NJ, Bronx NY, Dutchess NY, Kings NY,	Warren NJ, Bronx NY, Dutchess NY, Kings NY,
	Nassau NY, New York NY, Orange NY, Putnam	Nassau NY, New York NY, Orange NY, Putnam
	NY, Queens NY, Richmond NY, Rockland NY,	NY, Queens NY, Richmond NY, Rockland NY,
	Suffolk NY, Westchester NY, Pike PA	Suffolk NY, Westchester NY
Washington-Arlington-Alexandria, DC-	District of Columbia DC, Calvert MD, Charles MD,	District of Columbia DC, Calvert MD, Charles MD,
VA-MD-WV Metropolitan Statistical Area	Frederick MD, Montgomery MD, Prince Georges	Frederick MD, Montgomery MD, Prince Georges
(Washington, D.C.)	MD, Alexandria City VA, Arlington VA, Clarke	MD, Alexandria City VA, Arlington VA, Fairfax
	VA, Fairfax VA, Fairfax City VA, Falls Church	VA, Falls Church City VA, Fauquier VA, Freder-
	City VA, Fauquier VA, Fredericksburg City VA,	icksburg City VA, Loudoun VA, Manassas City
	Loudoun VA, Manassas City VA, Manassas Park	VA, Manassas Park City VA, Prince William VA,
	City VA, Prince William VA, Spotsylvania VA,	Spotsylvania VA, Stafford VA
	Stafford VA, Warren VA, Jefferson WV	
17 - 14 - 15 - 16 - 17 - 17 - 17 - 17 - 17 - 11 - 11		

Table A.1: Continued

Note: The table reports the counties included in the ten Metropolitan Statistical Areas (MSAs) underlying the S&P/Case-Shiller indices. The name of each MSA is abbrevi-

ated by that of its major city or county, as indicated in parenthesis.

				Lable A	Ladie A.z.: Data summary	mmary			
Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
				Panel.	Panel A: Data availability	bility			
Full sample start date	art date								
04/01/88	02/01/87	01/08/96	02/01/98	02/01/97	04/01/88	04/01/88	04/01/88	02/01/87	01/10/96
Daily index start date	tart date								
03/01/95	03/01/95	09/01/99	05/03/99	04/01/98	01/03/95	01/02/96	01/03/95	01/03/95	06/01/01
Full sample end date	nd date								
10/23/12	10/11/12	10/12/12	10/17/12	10/15/12	10/17/12	10/23/12	10/18/12	10/23/12	10/23/12
				$\overline{\mathrm{Pan}_{\epsilon}}$	Panel B: Transactions	<u>ons</u>			
Total transactions	tions								
10,285,770	2,121,471	3,948,706	1,672,669	3,689,159	2,236,138	2,845,804	3,778,446	5,943,114	2,168,018
Single family	Single family residential housing transactions	using transact	tions						
5,970,536	1,141,930	1,886,433	1,000,785	1,366,745	1,479,872	1,584,732	2,331,860	2,951,031	1,055,537
Arms-length transaction	transaction								
2,562,884	975,964	1,157,215	672,512	935,985	915,408	755,440	1,031,261	2,307,079	759, 752
After excludir	After excluding transaction value <= 5000 or >= 100,000,000	value $\leq = 500$	00 or >= 100	,000,000					
2,555,165	917,039	1,156,042	671,605	935,178	913,682	754,106	1,030,384	2,271,467	757,675
After excludir	After excluding houses sold only once	only once							
1,980,740	638,577	659, 732	475,481	668, 552	729,365	579, 152	757, 379	1,234,074	459,842

Table A.2: Data summary

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				Table	Table A.2: Continued	nued			
Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
				Panel B: T	Panel B: Transactions (continued)	$\overline{\text{ontinued}}$			
After excluding transactions happen within 6 months	g transactions	s happen with	uin 6 months						
1,627,149	532,761	561,945	374,045	576,810	645,869	510, 450	688,869	1,026,836	325, 251
After excluding $\ge 2 \times$ standard deviations and	$g >= 2 \times stan$	ndard deviatic		\times median tra	$>= 6 \times$ median transaction values	Ñ			
1,578,869	514, 356	543,038	360,944	561,805	628, 790	494,894	665,537	999,284	313,777
				Par	Panel C: Sale pairs	IS			
Total pairs									
939,476	294,101	292,737	198,608	321, 358	378,093	296,985	397, 229	544, 326	162,693
After excluding renovation/reconstruction between two sales	g renovation/	reconstruction	a between two) sales					
899,573	286,760	292,737	187, 977	287,790	226,701	244,059	350,500	540, 235	151,203
After excluding abnormal annual returns (less than -50% or more than 100%)	g abnormal aı	nnual returns	(less than -50)% or more t	han $100\%)$				
878,017	272,858	277,160	181,633	281, 393	221,877	239, 232	341,878	512, 251	143,481
After excluding sale pairs with second transaction on weekends	g sale pairs w	ith second tr_{ε}	ansaction on v	veekends					
878,002	272, 727	277,095	180,504	277,442	221,876	239, 215	341,858	508,860	143, 433
After excluding sale pairs with second transaction on federal holidays	g sale pairs w	ith second tr_{ε}	ansaction on f	ederal holida	ys				
877,885	272,414	277,079	180,003	276,676	221,554	239,041	341,469	508,548	143, 431
Average daily sale pairs for the daily index estimation period	sale pairs for	the daily ind ϵ	x estimation	period					
180	55	84	53	77	49	51	70	109	49

 Table A.2:
 Continued

A.2 Supplementary tables and figures

Tables A.3-A.5 and Figures A.1-A.3 contain additional empirical results for each of the ten MSAs pertaining to: the noise filter estimates; the daily HAR-X-GARCH-CCC correlation estimates; the unconditional correlations of the monthly Case-Shiller index returns; the sample autocorrelations for the raw daily index returns; time series plots of the raw and filtered daily house price indices; and the unconditional return correlations as a function of the return horizon.

	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
- 1	0.018	0.019	0.002	0.009	0.013	0.001	0.020	0.018	0.016	0.017
	(0.006)	(0.007)	(0.001)	(0.008)	(0.010)	(0.00)	(0.006)	(0.001)	(0.005)	(0.00)
σ_{η}	2.457	5.888	4.668	3.779	4.113	5.362	3.746	4.925	4.349	4.612
	(0.039)	(0.140)	(0.155)	(0.139)	(0.076)	(0.212)	(0.058)	(0.105)	(0.132)	(0.108)
σ_{u}	0.379	0.388	0.593	0.327	0.497	0.568	0.407	0.525	0.376	0.501
	(0.022)	(0.034)	(0.057)	(0.040)	(0.035)	(0.056)	(0.022)	(0.029)	(0.034)	(0.037)
σ_n/σ_u	6.478	15.180	7.866	11.544	8.273	9.448	9.204	9.376	11.576	9.200

 Table A.3: Noise filter estimates

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	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	$\operatorname{San}\operatorname{Diego}$	San Francisco	New York	Washington, D.C.
Los Angeles	1.000	-0.001	-0.004	-0.009	0.049	0.033	0.056	0.177	0.011	0.031
Boston		1.000	-0.013	0.014	0.023	0.025	0.041	0.023	-0.010	0.022
Chicago			1.000	-0.004	0.018	0.006	-0.014	0.036	0.048	0.015
Denver				1.000	0.030	0.024	0.031	-0.004	-0.022	0.023
Miami					1.000	0.017	0.025	0.038	0.038	0.018
Las Vegas						1.000	0.028	0.030	-0.023	0.026
San Diego							1.000	0.054	0.002	0.011
San Francisco								1.000	0.005	0.023
New York									1.000	0.025
Washington, D.C.										1.000

	Los Angeles Boston Chicago	Boston	Chicago	Denver	Miami	Las Vegas	$\operatorname{San}\operatorname{Diego}$	San Francisco	New York	Washington, D.C.
Los Angeles	1.000	0.651	0.658	0.543	0.870	0.875	0.926	0.835	0.778	0.881
Boston		1.000	0.767	0.749	0.527	0.495	0.672	0.693	0.725	0.773
Chicago			1.000	0.679	0.637	0.544	0.567	0.688	0.818	0.762
Denver				1.000	0.398	0.382	0.545	0.693	0.496	0.666
Miami					1.000	0.799	0.782	0.743	0.795	0.802
Las Vegas						1.000	0.819	0.663	0.684	0.748
San Diego							1.000	0.833	0.712	0.839
San Francisco								1.000	0.659	0.855
New York									1.000	0.816
Washington, D.C.										1.000



Figure A.1: Sample autocorrelations for raw daily index returns, with 95% confidence intervals



Figure A.2: Raw and filtered daily house price indices for ten MSAs



Figure A.3: Unconditional return correlations as a function of return horizon

A.3 Frequency-based comparisons with monthly S&P/Case-Shiller index In parallel to the monthly S&P/Case-Shiller indices, our daily house price indices are based on all publicly available property transactions. However, the complicated non-linear transformations of the data used in the construction of the indices prevent us from expressing the monthly indices as explicit functions of the corresponding daily indices. Instead, as a simple way to help gauge the relationship between the indices, and the potential loss of information in going from the daily to the monthly frequency, we consider the linear projection of the monthly S&P/Case-Shiller returns for MSA *i*, denoted $r_{i,t}^{S\&P}$, on 60 lagged values of the corresponding daily index returns,

$$r_{i,t}^{S\&P} = \delta(L)r_{i,t} + \varepsilon_{i,t} \equiv \sum_{j=0}^{59} \delta_j L^j r_{i,t} + \varepsilon_{i,t}, \qquad (6.1)$$

where $L^{j}r_{i,t}$ refers to the daily return on the j^{th} day before the last day of month t. Since all of the price series appear to be non-stationary, we formulate the projection in terms of returns as opposed to the price levels. The inclusion of 60 daily lags match the three-month smoothing window used in the construction of the monthly S&P/Case-Shiller indices, discussed in Section 2. The true population coefficients in the linear $\delta(L)$ filter are, of course, unknown, however they are readily estimated by ordinary least squares (OLS).

The OLS estimates for $\delta_{j=0,...,59}$ obtained from the single regression that pools the returns for all ten MSAs are reported in the top panel of Figure A.4. Each of the individual coefficients are obviously subject to a fair amount of estimation error. At the same time, there is a clear pattern in the estimates for δ_j across lags, naturally suggesting the use of a polynomial approximation in j to help smooth out the estimation error. The solid line in the figure shows the resulting nonlinear least squares (NLS) estimates obtained from a simple quadratic approximation. The corresponding R^2 s for the unrestricted OLS and the NLS fit ($\hat{\delta}_j = 0.1807 + 0.0101j - 0.0002j^2$) are 0.860 and 0.851, indicating only a slight deterioration in the accuracy of the fit by imposing a quadratic approximation to the lag coefficients. Moreover, even though the monthly

S&P/Case-Shiller returns are not an exact linear function of the daily returns, the simple relationship dictated by $\delta(L)$ accounts for the majority of the monthly variation.

To further illuminate the features of the approximate linear filter linking the monthly returns to the daily returns, consider the gain,

$$G(\omega) = \left[\sum_{j=0}^{59} \sum_{k=0}^{59} \delta_j \delta_k \cos(|j-k|\omega)\right]^{1/2}, \quad \omega \in (0,\pi),$$
(6.2)

and the phase

$$\theta(\omega) = \tan^{-1}\left(\frac{\sum_{j=0}^{59} \delta_j \sin(j\omega)}{\sum_{j=0}^{59} \delta_j \cos(j\omega)}\right), \quad \omega \in (0,\pi), \tag{6.3}$$

of $\delta(L)$. Looking first at the gains in Figures A.4b and A.4c, the unrestricted OLS estimates and the polynomial NLS estimates give rise to similar conclusions. The filter effectively down-weights all of the high-frequency variation (corresponding to periods less than around 70 days), while keeping all of the low-frequency information (corresponding to periods in excess of 100 days). As such, potentially valuable information for forecasting changes in house prices is obviously lost in the monthly aggregate. Further along these lines, Figures A.4d and A.4e show the estimates of $\frac{\theta(\omega)}{\omega}$, or the number of days that the filter shifts the daily returns back in time across frequencies. Although the OLS and NLS estimates differ somewhat for the very highest frequencies, for the lower frequencies (periods in excess of 60 days) the filter systematically shifts the daily returns back in time by about 30 days. This corresponds roughly to one-half of the three month (60 business days) smoothing window used in the construction of the monthly S&P/Case-Shiller index.



Figure A.4: Characteristics of the $\delta(L)$ filter

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