

Cross listing: price discovery dynamics and exchange rate effects

Cristina M. Scherrer

CREATES Research Paper 2014-53

Cross listing: price discovery dynamics and exchange rate effects*

Cristina Mabel Scherrer[†]

Aarhus University, Department of Economics and Business, and CREATES

This version: December 10, 2014

Abstract

The paper investigates the dynamics of price discovery for cross-listed firms and the impact of exchange rate shocks on firm value. A simple price discovery model is proposed in which prices in the home and foreign markets react to shocks on two latent prices, namely, the efficient firm value and the efficient exchange rate. I disentangle the effects on firm value from the exchange rate from the other determinants of a firm's cash flow. I use high-frequency data and find that a depreciation/appreciation of the home currency decreases/increases firm value. This finding is consistent with currency fluctuation affecting discount rates.

JEL classification numbers: G15, G12, G14, G32, C32, F31

Keywords: price discovery, exchange rate, market microstructure, structural VECM, high frequency data

Acknowledgments: I am indebted to Marcelo Fernandes, Angelo Ranaldo, Carsten Tanggaard, Neil Pearson, Tim Bollerslev, Erik Hjalmarsson, George Tauchen, Fotis Papailias and Gustavo Fruct Dias for valuable comments as well as the seminar participants at Sandbjerg Conference (Denmark), The Arne Ryde Workshop (Lund), Oxmetrics Conference, First International Workshop in Financial Econometrics (Natal, Brazil), Brazilian Econometric Society Meeting, Duke Financial Econometrics Lunch Group, Queen Mary Econometrics Reading Group, CREATES, Bank of England, Stockholm Business School and University of St. Andrews. I acknowledge support from CREATES - Center for Research in Econometric Analysis of Time Series (DNRF78), funded by the Danish National Research Foundation.

*A modified version of this paper has previously circulated with the title "Price discovery and instantaneous effects among cross-listed stocks".

[†]Department of Economics and Business, Aarhus University, Fuglesangs All 4, Building 2628, DK-8210 Aarhus V, Denmark. E-mail: cscherrer@creates.au.dk. Tel:+45 8716 6146.

1 Introduction

Cross-listed foreign firms have become more popular among investors in recent years. U.S.-listed American Depositary Receipts (ADR) traded 130 billion shares in 2011, up from 38 billion shares in 2005, an astonishing increase of 240%. Raising capital through depositary receipts has also increased (178% from 2013 to 2014), primarily through initial public offerings¹. This highly positive trend shows that investors are increasingly willing to take foreign firm risks (in exchange for perhaps a higher expected return), even if they come with the cost of greater exchange rate risk exposure. Non-arbitrage among markets implies that ADR prices should not deviate from their analogous shares traded on the home market, entailing that at least one of the markets should also incorporate shocks to the exchange rate. For an investor holding ADR shares, it is of interest not only to understand the impact of exchange rates on ADR prices but also to separate and quantify the impact from non-arbitrage adjustments from the potential impact on the present value of the firm cash flows. If the latter is significant, the net effect on the ADR prices may exceed the magnitude of the exchange rate shock. Measuring the impact of exchange rates on firm value has been the target of several studies in the international finance literature. The consensus is that exchange rates may affect firm value in two ways: future cash flows and the discount rate. The results using low-frequency data, however, are mixed and data-dependent. [Jorion \(1991\)](#) and [Bartov and Bodnar \(1994\)](#) find no strong evidence using U.S. data, while [He and Ng \(1998\)](#), [Dominguez and Tesar \(2001\)](#) and [Muller and Verschoor \(2006a\)](#) encounter significant exposure for other countries (see [Muller and Verschoor \(2006b\)](#) for a survey).

This paper investigates the dynamics of price discovery for cross-listed firms and the effect on firm value from shocks to the exchange rate. The use of high-frequency data allows the price discovery methodology to be applied to disentangle the effect of exchange rate shocks on firm value from the non-arbitrage adjustment of cross-listed stocks. To the best of my knowledge, this is the first study using high-frequency data to relate firm value and exchange

¹Market Fragmentation: Does it Really Matter?, transaction services, Citi

rate shocks. I formalize this question using a simple price discovery model for cross-listed firms. The model has transaction prices as a function of two common factors: the efficient price (firm value) and the efficient exchange rate. The effect that the exchange rate has on firm value depends on the parameters driving the correlation of common factor innovations. I write the short- and long-run solutions for this model as functions of uncorrelated innovations in common factors and, hence, explicitly show the effect of exchange rates on the efficient price of the firm.

I work with a two-year high-frequency data set from BM&FBovespa (the Brazilian stock exchange), NYSE and ARCA. The use of Brazilian firms is very interesting, as these firms trade on the U.S. markets as ADR and have significant activity in the cross-listing equity market. Some Brazilian firms are among the top 10 most liquid ADR programs in terms of volume and value movers, showing an increase of 20% in investors' positions from 2008 to 2010. A number of companies may even have more intense trading activity in the U.S. market than in the Brazilian market (see [Fernandes and Scherrer \(2014\)](#)), presenting liquidity in all markets that provides sufficient data for the study. Brazilian and U.S. data present the additional advantage of offering more overlapping trading hours, which allows much more information to be gathered (usually, studies of price discovery work only with overlapping hours) compared to the overlap of European and U.S. markets. Additionally, I work with companies that possess distinctive characteristics in their core business, ownership structure, global insertion, and strategic and political relevance to allow for robust conclusions.

This paper uses the price discovery framework to address the exchange rate effect on firm value in cross-listed firms. The two most prominent measures of price discovery are the information share (IS) of [Hasbrouck \(1995\)](#) and the component share (CS) as based on the work of [Gonzalo and Granger \(1995\)](#). These methodologies and their numerous variations were broadly applied to different markets, assets and financial instruments. These studies have primarily focused on identifying either the market or the financial instrument that is the fastest at impounding new information. The above methodologies are somehow

static measures that do not allow for an analysis of the speed and dynamics of market adjustment. Specifically, to measure price discovery, one would be interested in examining the instantaneous and total effects of the uncorrelated shocks that drive the common factors, the efficient firm value and the exchange rate. Therefore, to appropriately address price discovery dynamics, it is paramount to adopt a structural methodology that allows the uncorrelated innovations to be correctly assigned to markets. [Yan and Zivot \(2007\)](#) moves in this direction by introducing a dynamic measure of price discovery. They use a modification of [Gonzalo and Ng \(2001\)](#) and introduce a structural measure of price discovery in the context of one common factor (the efficient firm value). [Kim \(2010\)](#) expands the work of [Yan and Zivot \(2007\)](#) to two common factors but imposes restrictions on the correlation of their innovations and rules out part of the feedback between the exchange rate and firm value. Hence, the price discovery literature has unfortunately been unable to provide either an identification strategy for the parameters driving the correlation among the changes in common factors or answers regarding the relationship between the exchange rate and firm value.

I propose a strategy that allows for the identification of the orthogonal shocks (structural innovations) that drive the common factors. I compute impulse response functions for the structural innovations, quantifying the effect of exchange rate shock on firm value. Therefore, methodologically, my work differs from [Yan and Zivot \(2007\)](#) and [Kim \(2010\)](#) in two ways. First, I introduce a theoretical model in which innovations to common factors are contemporaneously correlated; hence, short-run and total effect solutions are identified as functions of the parameters driving the correlation between the efficient firm value and the exchange rate innovations. Second, I impose direct identification restrictions only on the variance of the structural innovations driving the common factors rather than zero restrictions on the off-diagonal elements of the covariance matrix of changes in common factors. This strategy is sufficient to provide the identification of the short-run and the total effect parameters of the structural innovations. With this identification strategy on hand, I am

now in a position to formulate a different research question than the standard one in the price discovery literature: what is the impact of exchange rate shocks on firm value?

By using high-frequency data, I am able to identify the intrinsic value of the exchange rate and measure its impact on the fundamental value of the firm using only past price information and without any other information/assumptions regarding the firm. I find a positive relation between the value of the domestic currency and the value of the firm, meaning that a depreciation/appreciation in the domestic currency negatively/positively affects the fundamental value of the firm. This finding is consistent across all companies (regardless of degrees of internationalization or ownership), leading to the conclusion that the discount rate (through market risk) is the one being affected. This result links the exchange rate to market risk along the lines of [Minton and Schrand \(1999\)](#) and [Bartov, Bodnar, and Kaul \(1996\)](#). Their findings indicate that exchange rate volatility increases market risk (and the cost of capital) and hence lowers firm value. Furthermore, the empirical results corroborate the price discovery model, implying that innovations associated with the latent processes are indeed contemporaneously correlated, leading to a feedback effect between the efficient exchange rate and firm value. This finding shows that measuring price discovery independently of the exchange rate as well as using a methodology that imposes zero restrictions on the off-diagonal elements of the covariance matrix may deliver misleading results. I also find that in general, ARCA is faster than the NYSE and Bovespa. The results are consistent across all companies as well as at different sampling frequencies.

The remainder of the paper proceeds as follows. Section [2](#) introduces the simple theoretical model. Section [3](#) describes the estimation procedure and shows the identification strategy. Section [4](#) presents the primary data features and documents the empirical results for Brazilian firms, whereas Section [5](#) offers some concluding remarks. The appendix presents additional results regarding the identification strategy and a simple Monte Carlo study addressing the performance of the estimation methodology.

2 A simple model for price discovery

I present a model in which a firm cross-lists its shares in a foreign market. Prices in the home and foreign markets cannot drift apart because they reflect the value of the same security. Because shares are traded in different currencies, there are two fundamental values linking these prices: the firm value and the currency value. Hence, transaction prices in these markets share two common factors, seen as the fundamental value of the firm (the efficient price) and the fundamental link between the two currencies (the efficient exchange rate).

Assume that the firm efficient price (m_t) and the efficient exchange rate (e_t) are expressed in logarithmic terms and modeled as random walk processes. They are latent prices driven by two uncorrelated innovations: one associated with the firm efficient price (η_t^m) and another with the efficient exchange rate (η_t^e), respectively. In this context, η_t^m summarizes all of the information affecting the present value of the firm's future cash flows but the one contained in η_t^e , as below:

$$e_t = e_{t-1} + \eta_t^e + \lambda \eta_t^m \quad (1)$$

$$m_t = m_{t-1} + \eta_t^m + \rho \eta_t^e, \quad (2)$$

where $\eta_t^P = (\eta_t^e, \eta_t^m)'$, $\mathbb{E}(\eta_t^P) = 0$, $\text{Var}(\eta_t^P)$ is a diagonal matrix with $\text{Var}(\eta_t^e) = \varsigma_e^2$ and $\text{Var}(\eta_t^m) = \varsigma_m^2$, and e_t is defined in terms of the home currency (home over foreign currency).

The model in (1) and (2) accommodates non-zero contemporaneous correlation between the returns of common factors. This correlation is driven by the parameters λ and ρ as in (3):

$$\text{Var}([\Delta e_t, \Delta m_t]') = \begin{pmatrix} \varsigma_e^2 + \lambda^2 \varsigma_m^2 & \rho \varsigma_e^2 + \lambda \varsigma_m^2 \\ \rho \varsigma_e^2 + \lambda \varsigma_m^2 & \varsigma_m^2 + \rho^2 \varsigma_e^2 \end{pmatrix} \quad (3)$$

where $\Delta e_t = e_t - e_{t-1}$ and $\Delta m_t = m_t - m_{t-1}$.

The next step consists of modeling the observed transaction prices. Transaction prices may not be equal to the efficient prices in (1) and (2) at every point in time because markets may process information differently. Liquidity issues and asymmetric information may cause transaction prices to adjust to the efficient prices at various speeds. Assume that partial adjustments to transaction prices from m_t and e_t are given by γ_i and $\dot{\gamma}_i$, respectively (i accounts for the venue). This model setup is consistent with other partial adjustment models used in the literature (see [Amihud and Mendelson \(1987\)](#), [Hasbrouck and Ho \(1987\)](#) and [Yan and Zivot \(2010\)](#)). If all trading venues incorporated changes at the same time, transaction prices would be equal to the efficient price plus microstructure noise. Transitory innovations (η_t^T) reflect the presence of market microstructure noise (bid-ask bounces, inventory effects, price discreteness, etc). It is assumed that these innovations do not affect latent prices and therefore have zero long-term impact, as opposed to innovations to the common factors (η_t^e and η_t^m in (1) and (2)), which are seen as permanent. Permanent innovations (η_t^P) are related to the stochastic process that drives the fundamental values of assets². Denote P_t as a 4×1 vector containing the logarithm of the observed exchange rate (w_t) and the logarithm of transaction prices on different venues ($p_{i,t}$, where $i = 2, 3, 4$ and $i = 2$ account for the home market, whereas $i = 3, 4$ accounts for venues in the foreign market). The price process for each element of P_t is as follows:

$$w_t = w_{t-1} + \gamma_1 (m_t - m_{t-1}) + \dot{\gamma}_1 (e_t - w_{t-1}) + b_1 \eta_t^T \quad (4)$$

$$p_{2,t} = p_{2,t-1} + \gamma_2 (m_t - p_{2,t-1}) + \dot{\gamma}_2 (e_t - w_{t-1}) + b_2 \eta_t^T \quad (5)$$

$$p_{3,t}^* = p_{3,t-1}^* + \gamma_3 (m_t - p_{3,t-1}) + \dot{\gamma}_3 (e_t - w_{t-1}) + b_3 \eta_t^T \quad (6)$$

$$p_{4,t}^* = p_{4,t-1}^* + \gamma_4 (m_t - p_{4,t-1}) + \dot{\gamma}_4 (e_t - w_{t-1}) + b_4 \eta_t^T, \quad (7)$$

where b_1, b_2, b_3 and b_4 are 1×2 vectors and η_t^T is a 2×1 vector³.

²A formal (econometric) definition of permanent and transitory innovations can be found in [Gonzalo and Granger \(1995\)](#), and it is briefly presented in Section 3.

³For identification purposes, it is necessary to have two transitory innovations such that the total number of innovations is equal to the number of markets in line with [Gonzalo and Ng \(2001\)](#) and [Yan and Zivot](#)

Equation (4) has w_t as the observed exchange rate, (5) displays the transaction price on the home market, while (6) and (7) express prices in foreign markets (two different trading venues). To keep the model as flexible as possible, all transaction prices adjust to innovations to the two latent prices (the efficient price and exchange rate). Prices traded in different currencies are distinguished such that $p_{3,t}^*$ and $p_{4,t}^*$ entail prices in foreign currencies (the currencies they are actually traded in), whereas $p_{3,t}$ and $p_{4,t}$ are expressed in the home currency. The relation between them is given as

$$p_{3,t}^* = p_{3,t} - w_t$$

$$p_{4,t}^* = p_{4,t} - w_t,$$

where w_t is denominated as the home currency over the foreign currency.

Given this setup, one is interested in examining the changes in transaction prices as a function of uncorrelated permanent innovations (η_t^P). Because the model allows for partial adjustments, one can see this impact instantaneously and in total terms. To assess these quantities, I make use of the impulse response function obtained from the infinite vector moving average (VMA(∞)) in (8).

$$\Delta P_t = d_0 \eta_t + d_1 \eta_{t-1} + d_2 \eta_{t-2} + \dots = \sum_{i=0}^{\infty} d_i \eta_{t-i}. \quad (8)$$

where $\Delta P_t = (\Delta w_t, \Delta p_{2,t}, \Delta p_{3,t}^*, \Delta p_{4,t}^*)'$, $\eta_t = (\eta_t^P, \eta_t^T)'$, $\eta_t^P = (\eta_t^e, \eta_t^m)$, η_t^T is a 2×1 vector and d_0, d_1, d_2, \dots are 4×4 matrices.

Using (4) to (7), one can obtain d_0 (the instantaneous impact) and the total impact, D (2007). Hence, in a model with n markets, it is necessary to have $n - x$ transitory innovations, where x is the number of permanent innovations. Therefore, the four-variable model discussed in this section carries two permanent and two transitory innovations.

(where $D = d_0 + d_1 + d_2 + \dots = \sum_{i=0}^{\infty} d_i$ from (8)). The solution for d_0 is as follows:

$$d_0 = \begin{pmatrix} \dot{\gamma}_1 + \gamma_1 \rho & \dot{\gamma}_1 \lambda + \gamma_1 & b_1 \\ \dot{\gamma}_2 + \gamma_2 \rho & \dot{\gamma}_2 \lambda + \gamma_2 & b_2 \\ \dot{\gamma}_3 + \gamma_3 \rho & \dot{\gamma}_3 \lambda + \gamma_3 & b_3 \\ \dot{\gamma}_4 + \gamma_4 \rho & \dot{\gamma}_4 \lambda + \gamma_4 & b_4 \end{pmatrix}, \quad (9)$$

where b_1, b_2, b_3 and b_4 are 1×2 vectors.

The solution for D is as follows:

$$D = \begin{pmatrix} 1 & \lambda & 0 \\ \rho & 1 & 0 \\ \rho - 1 & 1 - \lambda & 0 \\ \rho - 1 & 1 - \lambda & 0 \end{pmatrix}, \quad (10)$$

where 0 are 1×2 vectors. The detailed steps of these solutions are in Appendix 6.1.

Each market's importance to the price discovery process is given by (9), which shows a combination of parameters from (1), (2) and (4) to (7). Matrix (10) reflects the total impact on observed prices given innovations to the latent common factors. Note that (10) is solely a combination of parameters from (1) and (2). This follows because γ_i and $\dot{\gamma}_i$ are equal to either 1 or 0 in the long run because prices ultimately incorporate all information. Therefore, one can gauge price discovery by how fast markets incorporate new information, (9), and analyze the total impact, including feedback effects, using the magnitude of the elements in (10). Interest lies only in the parameters accompanying η_t^e and η_t^m (permanent innovations) because transitory shocks are not cash flow-related innovations and have zero effect in the long run. Therefore, the analysis focuses on the sub-matrices of (9) and (10), which are related to η_t^e and η_t^m (first and second column, respectively).

The solution in (10) shows that innovations to the efficient exchange rate have a long-term effect on the home price and foreign prices equal to ρ and $(\rho - 1)$, respectively. Hence, ρ gives the net effect on the efficient price of the firm. This impact may come through

expected future cash flows or discount rate effects. Exchange rate fluctuation may affect firms' cash flows in a variety of ways: transactions (imports and exports), competitors' and suppliers' currency, access to international capital markets, assets' physical location, etc⁴. Exchange rate fluctuations can also affect the discount rate that investors use with expected future cash flows to calculate present value. [Minton and Schrand \(1999\)](#) show that higher cash flow volatility increases the cost to access capital markets, impacting discount rates and reducing firm value. If investors perceive changes in exchange rates that lead to more volatile expected future cash flows, it may drive higher costs of capital and discount rates. [Bartov, Bodnar, and Kaul \(1996\)](#) suggest a relation between an increase in exchange rate variability, greater asset return volatility and higher market risk. The hypothesis drawn here is that a depreciation/appreciation of the home currency provokes the investment in domestic firms to become riskier/less risky, increasing/decreasing market risk and the cost of capital and thereby reducing/augmenting firm value. Therefore, there would be a positive relation between the value of the home currency and the value of the domestic asset, which implies a negative relation between exchange rates (home/foreign currency) and firm value. This relation translates into $\rho < 0$.

Innovations to the efficient price are not expected to lead to prominent effects on the efficient exchange rate, given by λ in (10). Small effects, however, could occur if a firm is large enough and its import or export activities are of a significant size, thereby impacting the exchange rate. A more likely situation arises when overall movements of the stock market in the home country (coming from overall good or bad news in the economy) causes a significant inflow or outflow of foreign money in the domestic market, impacting the exchange rate⁵. If this is the situation, one would expect to find $\lambda < 0$, meaning that good/bad news for firms on average would attract more/less foreign capital, leading to an appreciation/depreciation

⁴A useful literature review on the effects of exchange rates on firm value can be found in [Muller and Verschoor \(2006b\)](#).

⁵For instance, the correlation between returns on the exchange rate (Brazilian currency over U.S. dollars) and the main index of the Brazilian stock exchange (Ibovespa) was equal to -0.60 during December 2007 and November 2009. This correlation is computed using daily transaction prices; however, it suggests that the same could be true for the latent process.

of the home currency.

In summary, the model presented in this section illustrates the importance of allowing for non-zero correlations between returns on the firm efficient price and the efficient exchange rate because this feature allows exchange rate shocks to affect the efficient firm value. In other words, a non-zero correlation allows a split between exchange rate shocks and all other shocks that affect the present value of the firm's future cash flows. Moreover, the theoretical model motivates the methodology discussed in Section 3 and the empirical results in Section 4.

3 A structural measure of price discovery

Keeping in mind the model presented in the previous section, I construct a methodology to estimate the parameters in (9) and (10) and, hence, λ and ρ in (1) and (2), respectively.

In summary, I use a slightly modified version of the [Gonzalo and Granger \(1995\)](#) and [Gonzalo and Ng \(2001\)](#) two-step methodology. The primary difference between the two methods is that the identification strategy implemented in this section does not impose ex ante zero restrictions on the parameters driving the correlation among common factor innovations and therefore does not force the parameters λ or ρ to be zero⁶.

Cross-listed stocks have prices based on the same fundamental value: the intrinsic value of the firm. They share a stochastic trend (common factor) and therefore should not drift apart. In econometric terms, they are cointegrated. Assume that these prices can be approximated by a vector error correction model (VECM) as follows:

$$\Delta P_t = \xi_1 \Delta P_{t-1} + \xi_2 \Delta P_{t-2} + \dots + \xi_l \Delta P_{t-l} + \zeta + \xi_0 P_{t-1} + \epsilon_t, \quad (11)$$

where P_t is a $k \times 1$ vector containing the logarithm of both the exchange rate and prices in different markets; $\xi_0 = \alpha\beta'$; α is the error correction matrix; β is the cointegrating vector;

⁶See the covariance matrix of common factor innovation in (3).

and ϵ_t is a zero mean white noise process with a non-diagonal covariance matrix Ω . I impose restrictions on the constant term for the absence of a deterministic time trend.

VECM parameters can be used to back out the VMA coefficients as in (12) through dynamic simulation (see [Hamilton \(1994\)](#)).

$$\Delta P_t = \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \dots = \Psi(L)\epsilon_t, \quad (12)$$

where L is the lag operator. Note that the infinite VMA process in (12) is driven by ϵ_t , which is likely to present contemporaneous correlation. Hence, (12) is expressed in its reduced form, whereas the instantaneous impact and total impact derived from (8) are driven by contemporaneously uncorrelated innovations. Taking these factors into consideration, the target is now to have a VMA expression as a function of contemporaneously uncorrelated (structural) innovations.

Moving from the reduced-form VMA to its structural counterpart requires a series of identification restrictions. These restrictions usually require prior knowledge of the importance of each market, which might be difficult or perhaps questionable⁷. A way to partly overcome this issue is to consider assumptions regarding permanent and transitory innovations as in [Gonzalo and Granger \(1995\)](#) and [Gonzalo and Ng \(2001\)](#). The first step to obtain the structural counterpart of (12) does not require any prior judgement about the market's importance and can be reasonably justified. [Gonzalo and Granger \(1995\)](#) define η_t^P and η_t^T as permanent and transitory innovations, respectively, such that the following conditions hold: $\lim_{h \rightarrow \infty} \partial E_t(P_{t+h}/\partial \eta_t^{P'}) \neq 0$ and $\lim_{h \rightarrow \infty} \partial E_t(P_{t+h}/\partial \eta_t^{T'}) = 0$, where E_t denotes the conditional expectation in relation to past information up to time t . The underlying assumption in this identification strategy is that only η_t^P has a permanent impact on the level of P_t . Assuming that the number of permanent and transitory innovations is the same as

⁷Restrictions that require prior knowledge of the importance of markets are usually related to orthogonalization procedures that rely on decompositions that use lower (upper) triangular matrices, as is the case for the Cholesky decomposition.

the number of variables in the system, the structural counterpart of (12) is given by

$$\Delta P_t = d_0 \eta_t + d_1 \eta_{t-1} + d_2 \eta_{t-2} + \dots = \sum_{i=0}^{\infty} d_i \eta_{t-i} = D(L) \eta_t, \quad (13)$$

where $\eta_t = (\eta_t^P, \eta_t^T)'$ is a white noise process with a diagonal covariance matrix. Therefore, the infinite VMA process in (13) is solely driven by uncorrelated permanent and transitory shocks (as opposed to market innovations, as in (12)).

The main procedure consists of building a bridge between (12) and (13) such that the identification of d_0 and D is possible. The first step of the procedure in Gonzalo and Ng (2001) consists of rotating the reduced-form market innovations, ϵ_t in (11), and decomposing them into permanent and transitory innovations in their reduced form using matrix G as defined in (14):

$$G = [\alpha'_{\perp}, \beta']', \quad (14)$$

where α_{\perp} is a $(k-r) \times k$ matrix, the orthogonal projection of α , that summarizes the entire permanent portion of prices, whereas β is an $r \times k$ matrix providing the transitory portion of prices. The caveat associated with using matrix G to rotate the reduced-form innovations is that this may lead to estimates of d_0 and D that are not order-invariant⁸. To overcome this issue, I define G^* as in Warne (1993) because it delivers an order-invariant measure of d_0 and D . Matrix G^* is constructed with $\alpha' \Omega^{-1}$ instead of β' , viz.

$$G^* = [\alpha'_{\perp}, \alpha' \Omega^{-1}]' \quad (15)$$

Although α' is not unique, changing the order of variables in the system does not affect the estimates of d_0 and D . The construction of G^* is straightforward from the results of

⁸When the number of cointegrated vectors is larger than one, Gonzalo and Ng (2001) suggest that β should be expressed in its triangular representation. This solution has a drawback in that matrix G becomes order-variant and, consequently, the estimates of d_0 and D become order-variant, following the triangular representation of β .

the VEC model, and the intuition is the same as when using β' , i.e., to identify everything that is transitory and therefore vanishes away. The Monte Carlo exercises in Appendix 6.3 compare these two approaches and show the advantages of using G^* instead of G .

The rotation of the reduced-form market innovations is implemented by multiplying G^* by ϵ_t , providing permanent and transitory innovations in their reduced form, defined as ε_t ⁹ as follows:

$$\varepsilon_t = G^* \epsilon_t, \quad (16)$$

where $\varepsilon_t = (\varepsilon_t^P, \varepsilon_t^T)'$ are the unorthogonalized shocks, and $\varepsilon_t^P = \alpha'_\perp \epsilon_t$ and $\varepsilon_t^T = \alpha' \Omega^{-1} \epsilon_t$.

Define Ξ as the variance of ε_t such that

$$\Xi = G^* \Omega G^{*'}, \quad (17)$$

where $\Omega = E(\epsilon_t \epsilon_t')$ is the covariance matrix from the disturbances of the VEC model. Matrix Ξ is still not diagonal, giving way for the second step: decompose Ξ in such a way as to obtain a relation between ε_t and η_t as well as one between $D(L)$ in (12) and $\Psi(L)$ in (13). There are two methods for implementing this step in the literature. [Gonzalo and Ng \(2001\)](#) decompose Ξ using the well-known Cholesky decomposition such that $\Xi = FF' = FIF'$, where F is a lower triangular matrix. Setting $\text{Var}(\eta_t)$ equal to an identity matrix gives the easy relation between the covariance matrices of ε_t and η_t : $\Xi = FIF' = F\text{Var}(\eta_t)F'$. Given this relation, one can then find η_t as a function of ε_t and ϵ_t , as $\eta_t = F^{-1}\varepsilon_t = F^{-1}G\epsilon_t$. [Yan and Zivot \(2007\)](#) propose a different approach. They apply the so-called *HCH* decomposition, where $\Xi = HCH'$, H is a unique lower triangular matrix with ones in its main diagonal and C is a diagonal matrix with positive entries. Matrices F and H deliver the relation between correlated (ε_t) and uncorrelated (η_t) permanent and transitory innovations. Both Cholesky

⁹Note that permanent and transitory innovations in their reduced form may still be contemporaneously correlated, and therefore, they are not the ones contained in η_t , which are assumed to be orthogonal (structural innovations).

and HCH decompositions impose zero restrictions on the off-diagonal elements of F and H . Using these decompositions would impose restrictions to the parameters in (1) and (2) in Section 2. For instance, when the exchange rate is ordered first, λ is zero, whereas when it is ordered second, ρ is zero. This strategy would nullify the research question relating exchange rates and firm value because both the Cholesky and the HCH decompositions assume ex ante that some of the parameters driving the correlation between changes in the two common factors (efficient exchange rate and efficient price) are zero.

What I address differently in this paper is the importance of implementing an identification strategy that does not impose zero restrictions when decomposing Ξ , allowing ρ in (1) and λ in (2) to be different than zero. By taking this step, I allow innovations to the efficient exchange rate to affect the firm efficient price (firm value) contemporaneously. To achieve this strategy, I use spectral decomposition on a normalized covariance matrix (no longer symmetric) such that no zero restrictions on the off-diagonal elements of the decomposed matrix are imposed. To this end, I normalize Ξ such that $\tilde{\Xi} = \Xi\Theta^{-1}$, with Θ being a diagonal matrix constructed with the diagonal elements of Ξ . I then implement the spectral decomposition (which does not deliver a lower triangular matrix) on $\tilde{\Xi}$:

$$\tilde{\Xi} = \tilde{S}\tilde{S} \tag{18}$$

where \tilde{S} is the square root of $\tilde{\Xi}$ obtained from an eigenvalue decomposition, as below.

$$\tilde{\Xi} = V\Lambda V^{-1} \Rightarrow \tilde{\Xi}^{1/2} = V\Lambda^{1/2}V^{-1}, \tag{19}$$

where the columns of V are the eigenvectors of $\tilde{\Xi}$, and Λ is a diagonal matrix with the corresponding eigenvalues. Note that I can recover Ξ just by multiplying back Θ as in

$$\Xi = \tilde{\Xi}\Theta = \tilde{S}\tilde{S}\Theta. \tag{20}$$

This strategy renders $(k^2 - k)/2$ new equations in my system of equations, which are now used to avoid zero restrictions on the off-diagonal elements of \tilde{S} (details of the identification strategy are in Appendix 6.2.2). The primary motivation behind decomposing $\tilde{\Xi}$ instead of Ξ is that it allows one to impose restrictions on the variance of the structural innovations rather than on the off-diagonal elements of \tilde{S} . Hence, by obtaining $(k^2 - k)/2$ new equations, I only need to impose that the variances of the structural innovations are the same as the variances of their reduced-form counterparts, i.e., $\text{Var}(\eta_t) = \Theta$. This assumption becomes very mild because, unlike with the Cholesky and HCH decompositions, no zero restriction on the off-diagonal elements of \tilde{S} is imposed. Provided that $\tilde{S}\tilde{S}\Theta = \tilde{S}\Theta\tilde{S}'$ holds (Appendix 6.2.3 presents the proof of this equality) and setting $\text{Var}(\eta_t) = \Theta$, then

$$\begin{aligned}\tilde{S}\Theta\tilde{S}' &= \Xi \\ \text{Var}(\eta_t) = \Theta &= \tilde{S}^{-1}\tilde{\Xi}\tilde{S}^{-1'}.\end{aligned}\tag{21}$$

Given (21), the relation between η_t and ε_t is retrieved by

$$\eta_t = \tilde{S}^{-1}\varepsilon_t.\tag{22}$$

Note that the permanent and transitory innovations in their reduced form can be expressed as $\varepsilon_t = \tilde{S}\eta_t$. Because \tilde{S} is allowed to be a full matrix, no zero restrictions are imposed on the parameters in (1) and (2). Hence, the identification strategy adopted in this section allows permanent innovations from a given common factor to have a permanent impact on any other common factor, as opposed to the identification strategy used in Yan and Zivot (2007) and Gonzalo and Ng (2001), which rules out this possibility.

Finally, the relation between (12) and (13) is given as

$$\Delta P_t = \Psi(L)G^{*-1}\tilde{S}\tilde{S}^{-1}G^*\varepsilon_t = D(L)\eta_t,\tag{23}$$

which delivers $D(L) = \Psi(L)G^{*-1}\tilde{S} = \Psi(L)d_0$, with $d_0 = G^{*-1}\tilde{S}$ and $D = \Psi(1)G^{*-1}\tilde{S}$.

In summary, this section describes an identification strategy that allows changes to the efficient exchange rate and the efficient price to be correlated, i.e., $\text{Var}([\Delta e_t, \Delta m_t]')$ to be a non-diagonal matrix. As an outcome of this identification strategy, instantaneous and total effects (d_0 and D , respectively) are obtained without imposing zero restrictions to their off-diagonal elements. This outcome is crucial for disentangling the effect of innovations to exchange rates on firm value. Furthermore, one can use the off-diagonal elements of D to draw statistical inferences regarding the parameters ρ in (1) and λ in (2), which ultimately drive the correlation between common factor changes. The empirical results show that these parameters are statistically different from zero, indicating that innovations to common factors are indeed contemporaneously correlated, and therefore, the effect of exchange rate innovations can be disentangled from all other effects on firm value. I show using a simple Monte Carlo simulation (Appendix 6.3) that the methodology discussed in this section achieves the best finite sample results (in terms of mean squared error) for models with one and two common factors. Extensions for a case with more common factors can be easily implemented.

4 Price discovery for Brazilian cross-listed stocks

4.1 Institutional background

The BM&FBovespa is the only stock exchange in Brazil and the leading exchange in Latin America in terms of the number of contracts traded. It is a very active market, with a 2013 average daily trading volume of BRL 7.4 billion (equivalent to USD 3.1 billion) in the Bovespa segment, making it one of the top 10 stock exchanges in the world (in terms of market capitalization). BM&FBovespa is a fully electronic exchange (the end of open outcry transactions and of derivatives transactions at Bovespa took place in 2005 and 2009, respectively) and operates under the supervision of the CVM (Brazilian Securities Commission).

BM&FBovespa markets include equity, commodities and futures, foreign exchange, securities and ETFs (exchange-traded funds). Brazil achieved an investment grade rating from Standard & Poor's in April 2008. Fitch and Moody's increased Brazilian ratings in May 2008 and September 2009, respectively¹⁰.

Cross-listed Brazilian companies are traded and listed in the U.S. market through American Depositary Receipts (ADR). An ADR is a physical certificate evidencing ownership of a U.S. dollar denominated form of equity in a foreign company. It represents the shares of the company held on deposit by a custodian bank in the company's home country and carries the corporate and economic rights of the foreign shares, subject to the terms specified on the ADR certificate.

I start with a tick-by-tick data set of Brazilian blue-chip companies traded at three venues: Bovespa, the NYSE and ARCA. The sample period is beneficial (December 2007 to November 2009) because it is large enough to encompass a variety of movements in the stock markets, including the 2008/2009 financial crisis. I perform a rolling window exercise to explore this feature as a robustness exercise. The use of a high-frequency data set is crucial to this analysis because it provides the timely incorporation of new information in each market. A daily data set would not provide the information necessary to measure price discovery. On a daily basis, all markets would have fully incorporated new information, and one would not be able to capture which market achieved it first and/or faster.

There are two benefits of using this data set. Firstly, there is a large time intersection between Brazil and the U.S. The U.S. and Brazilian markets have an overlap period of six and a half hours during the majority of the year, from mid-February to mid-November, with the Brazilian stock exchange open for only thirty minutes while the NYSE is closed. The Brazilian stock exchange actually changes its trading hours from mid-October to mid-February to keep the largest time overlap possible when both countries adjust their clocks for daylight savings time. The lowest intersection period during the year occurs from mid-

¹⁰These three dates are part of the sample period.

November to mid-February, when they remain jointly open for five and a half hours. Hence, a very reduced amount of information is left out of the analysis, providing more accuracy to isolate the different aspects that drive the price dynamics in all markets. Secondly, Brazilian companies are very liquid in the U.S. market, sometimes showing more trading activity in the U.S. than they do in Brazil. In fact, some Brazilian firms are among the top 10 most liquid ADR programs as well as top volume and value movers. Furthermore, there was an increase of 20% on investors' positions in ADR from Brazilian companies from 2008 to 2010. Among the Brazilian firms cross-listed in the U.S., I choose those that are very liquid in the three markets considered in this study. This strategy delivers a large amount of information to analyze the effect of exchange rate innovations on transaction prices in all markets and hence to distinguish the exchange rate effect from the other effects on future cash flows of the firm. The idea is not to lose information during the aggregation process between a very illiquid venue and a liquid one. I work with firms from a variety of industries so that the results are not sector- or industry-specific. The firms are Ambev (beverage), BR Telecom (telecommunication), Bradesco (finance), Gerdau (steel), Vale (mining) and Petrobras (oil). Apart from BR Telecom, they are all part of IBOVESPA. Preferred shares of Vale and Petrobras are the two most heavily traded shares on the Brazilian market, with Gerdau and Bradesco coming in in the top 15. The number of trades for Petrobras is approximately 9 million on Bovespa for the two-year data set. For Vale, it is 6.4 million, Gerdau, 3.2 million, Bradesco, 3 million, Ambev, 0.7 million and BR Telecom, 0.6 million.

For a tick-by-tick database, two important steps must be implemented before one can actually estimate measures of price discovery. The first step relies on cleaning the data because they may present entries that are implausible considering normal market activity. I use the algorithm proposed by [Brownlees and Gallo \(2006\)](#) to clean the data. Secondly, there is the data set non-synchronization issue. Some stocks are more intensively traded than others; hence, they are aggregated based on a time interval ranging from 30s to 300s. I use the 'replace all' method that [Harris, McInish, L.Shoesmith, and Wood \(1995\)](#) applied

for fixed time intervals. Aggregating at a higher frequency would result in a large number of missing observations derived from non-synchronous trading, which could lead to artificial serial autocorrelation. Figure 1 shows the price evolution of the shares used in this study. Additional information and more details on the data handling can be found in [Fernandes and Scherrer \(2014\)](#).

4.2 Markets' importance and exchange rate effects

Using the methodology presented in Section 3, I aim to answer two research questions. The first question asks how investors update their beliefs regarding firm value given exchange rate fluctuations in a high-frequency data world. What is the effect of a home currency depreciation/appreciation in the intrinsic value of the firm? The methodology presented in the previous section does not impose zero restrictions on the off-diagonal elements of matrix \tilde{S} in (22). This matrix relates ε_t to η_t (see (22)), as does the matrix formed by the parameters ρ and λ in the theoretical model in (1) and (2)¹¹. Therefore, a conclusion can be drawn about the net feedback effect of exchange rate innovations on firm value. The second research question addresses the standard questions in price discovery. Where are prices formed? How quickly do prices incorporate innovations from different sources? When a company decides to cross-list its shares, is the domestic market more important than any foreign market? I also present insights about whether the model presented in Section 2 suits the data set well.

I assume that prices can be approximated by a VEC model and estimate (11) using the full-information maximum likelihood (FIML) approach proposed by [Johansen \(1988\)](#) and [Johansen \(1991\)](#) and discussed in [Hamilton \(1994\)](#). I test for cointegrating vectors using the same methodology and choose the lag length based on the Schwarz criteria. For model diagnostic purposes, Table 1 shows the results of the Breusch Godfrey Lagrange multiplier

¹¹In the theoretical model presented in Section 2, the matrix relating correlated and uncorrelated permanent innovations is $\varepsilon_t = \begin{pmatrix} 1 & \lambda \\ \rho & 1 \end{pmatrix} \eta_t$.

test for the autocorrelation of the residuals of the VEC model and the structural residuals for lags 5, 10, 15 and 50. I cannot reject the null hypothesis of no serial correlation for all lags. The closest to rejection of the null hypothesis is Bradesco for lag 50¹². These results indicate that the model is well-specified.

Tables 2, 3, 4, 5, 6 and 7 report the results. There are four markets for the first five companies (Gerdau, Petrobras, Bradesco, Ambev and BR Telecom): the exchange rate (Brazilian Reais/U.S. dollars [USD]), shares traded on the Brazilian market, those traded on the NYSE and those traded on ARCA. For Vale, there are three markets: the exchange rate and shares traded on the Brazilian market and those traded on the NYSE. Shares traded on Brazil are quoted in Brazilian Reais (R\$) and shares traded on the U.S. market are expressed in USD.

The Johansen test determines that there are two cointegrating vectors (β , reported in the last two columns of each table) for all firms (apart from Vale, where there is only one, given the smaller price system). The vectors show the behavior that one would expect: $(0, 0, 1, -1)$ and $(-1, 1, 0, -1)$ with the exchange rate as the first variable followed by prices in the home markets, the NYSE and ARCA. The two cointegrating vectors yield two common factors for all firms, seen as the efficient exchange rate and the efficient price of the firm. Hence, the analysis focuses on the first two columns of d_0 and D (as defined and computed in Section 3), which are related to the two permanent innovations. I denote them as η_t^e and η_t^m , which are associated with the efficient exchange rate and efficient price, respectively (just as in the model in Section 2). The matrix d_0 has the instantaneous effect of a permanent innovation, whereas D has the total effect.

The first research question addresses a common debate in the international finance literature: the impact of the exchange rate on firm value. The literature has documented mixed results regarding firm exposure to exchange rate fluctuations. Jorion (1991) suggests that there is no evidence that the exchange rate is priced in the U.S. stock market, and Bar-

¹²However, when the lag length is determined by the Akaike criteria, I strongly do not reject the null at lag 50, and all other results remain the same.

Barboza and Bodnar (1994) find that abnormal returns for a sample of American firms have no correlation with the variation in the U.S. dollar. The weak U.S. evidence of exchange rate exposure led to international studies using more open economy countries. He and Ng (1998) find significant exposure for Japanese firms, and Dominguez and Tesar (2001) identify a high degree of exposure for a variety of countries. Muller and Verschoor (2006a) find evidence of exposure for European countries where the depreciation/appreciation of the Euro leads to a negative/positive effect on European stock returns.

The parameters in the total effect columns for innovations to the exchange rate are all negative for the Brazilian market and higher than one in absolute value for the U.S. market (same amount as the Brazilian plus one unit, all negative). For Gerdau, for instance, a unit innovation in the exchange rate has an effect of -0.62 on the home asset price and an effect of -1.58 on the foreign market (-1 as a result of the non-arbitrage adjustment). This result implies a net negative effect on the efficient price of the firm, or a negative ρ . This finding is strong because it shows how exchange rates affect firm value for different companies without using any firm-related variables and therefore without assumptions regarding the degree of internationalization, ownership, etc.

The results show that a depreciation of the Brazilian currency reduces the intrinsic value of Brazilian assets in real terms, that is, net of arbitrage adjustments. This conclusion is in line with the work of Minton and Schrand (1999) and Bartov, Bodnar, and Kaul (1996), indicating that exchange rate volatility increases market risk and the cost of capital, hence lowering firm value. The results show a clear and significant negative/positive impact on firm value, which could come from a higher/lower cost of capital (caused by greater/smaller market risk) once investors perceive a weaker/stronger currency following a depreciation/appreciation. I find that a one percent depreciation in the home currency leads to a net significant decrease in firm value. This result holds for all companies, although the magnitude of the impact varies (ranging from 0.26 to 0.62 percent)¹³. This finding implies

¹³This difference could reflect the different sensitivity of each individual stock to the market portfolio.

$\rho < 0$ in the theoretical model.

Considering the price discovery analysis, the results suggest, in general, that the U.S. market is the most important for the price discovery process of cross-listed Brazilian firms. In particular, ARCA instantaneously impounds more information than the NYSE. This finding might be explained by ARCA's smarter order-router system. Its router is able to check other exchanges to determine if there is a better quote than that currently provided at ARCA; it then executes the order at the venue where the best quote is available. This special system appears to be more efficient than those in other trading venues, giving a more important role to ARCA in the price discovery process when compared to the NYSE. This finding goes in line with others in the literature, where the NYSE is less important to the price discovery process (compared to NASDAQ), as its specialist quotes are less closely related to the efficient price (see [Hansen and Lunde \(2006\)](#)). In addition, it appears to be important to pay some attention to how actively traded these stocks are in each exchange (before any aggregation step). ARCA has a similar or higher number of transactions than the NYSE for the majority of stocks, apart from BR Telecom and Ambev. For these two companies, ARCA presents half the number of trades seen on the NYSE, which affects the importance of ARCA, leading to the conclusion that liquidity matters. In terms of total effects, ARCA and the NYSE are equally important, as one would expect. If this were not the case, it would mean that arbitrage transactions could occur in these markets. The larger importance attributed to the U.S. market may depend on a range of reasons. The type of platform, variety in the group of investors, supply of other assets (which could attract more- or less-informed investors) and transaction fees are among the characteristics that may impact this behavior.

Because the U.S. market is the fastest in incorporating news on the efficient price, one would expect a similar pattern for the exchange rate. The results indeed show that U.S. prices adjust instantaneously to a change in the exchange rate.

I document an instantaneous overshooting of the observed exchange rate once there is

an innovation to the efficient exchange rate. Given a unit shock on the efficient exchange rate (R\$/USD), i.e., a depreciation of the Brazilian currency, there is a higher depreciation instantaneously than in the long run. This behavior occurs for all stocks¹⁴. This overshooting is related to the parameter γ_1 in (4) being higher than that in the theoretical model. Intuitively, the overshooting could be a signal of herd behavior during turbulent periods. Indeed, the Brazilian currency depreciated 49% over 90 days during mid-July 2008 and in the beginning of October 2008 and partially recovered a few months later.

So far, I have focused on the instantaneous and total impacts. It might also be interesting to look at how long it takes for new information to be completely impounded by all markets. Impulse response functions for Gerdau are shown in Figure 2¹⁵. The first part displays the effect of an innovation on the efficient price in the three trading venues in 10 and 30 minutes. The majority of information processing takes place in the first 10 minutes, although we can see minor adjustments afterwards. This result is similar across companies, with very small variations. The second and third columns of Figure 2 depict the impact of an exchange rate innovation. Similar to the efficient price case, markets take approximately 10 minutes to incorporate all of the information. This result proves the importance of using high-frequency data in these studies; otherwise, no conclusions could be drawn. Moreover, it shows that high-frequency data have additional information.

How well do the data fit the model described in Section 2? There is a significant difference between the instantaneous and the total effects, averring that γ_i (in (4) to (7)) is different than the unit. This result shows that there is a partial adjustment process that demonstrates that markets do not impound all information instantaneously. I find the estimate of the elements of D in (10) to be statistically different from zero in all companies. Hence, the model stated in Section 2 appears to fit well in terms of partial adjustments. As expected, I find both λ and ρ to be negative and statistically different from zero. Table 8 presents the

¹⁴For the BR Telecom case, I find that exchange rate overshooting is significantly smaller than that reported for the other companies. This result can be well-explained by the fact that I use a 300s interval, leading to an artificially longer period assigned as the short run.

¹⁵Results for other firms are available upon request.

results for the over-identified parameters ρ and λ , which are statistically equal to each other, showing robustness in the results.

4.3 Robustness

Two data-related questions may arise when computing market leadership. The first question regards data handling. The choice of interval frequency must account for market liquidity and the presence of microstructure noise. The second issue is whether there is a difference in terms of market leadership across different periods of time. I try to tackle both questions in this subsection.

4.3.1 Interval Frequency

Section 4.1 briefly explains how non-synchronous trading is treated. Considering the number of observations for each price series, I adjust the interval length to aggregate the series¹⁶. I also estimate the covariance matrix using the Newey-West estimator to avoid the serial correlation issue. Additionally, I sample the data at different frequencies, checking whether the primary conclusions regarding market leadership change. This check is especially important for stocks for which the aggregation occurred at a much lower frequency, for instance, 240 seconds. The results for total impacts do not alter with this change, implying that they are robust to different sampling frequencies¹⁷.

4.3.2 Rolling Window

The structural approach implemented in this article ignores two important issues (as do many other methods in the literature, such as [Hasbrouck \(1995\)](#), [Grammig, Melvin, and](#)

¹⁶For instance, suppose share A has ten trades for each thirty-second interval, whereas share B has ten for each three hundred-second interval. If I aggregate them in thirty-second intervals, I would incur a high risk of serial correlation for share B, given the many missing observation intervals. At the same time, if I aggregate at three hundred-second intervals, I would lose some important information from share A. An in-between solution is needed.

¹⁷Results are available upon request.

Schlag (2005), Grammig and Peter (2013), and Yan and Zivot (2007), to name a few): time-varying variance and parameters. I address these issues in a very simplistic way by implementing a rolling window exercise. Others implement a similar procedure by estimating daily VEC models (see Hasbrouck (2003), Mizrach and Neely (2008), Chakravarty, Gulen, and Mayhew (2004) and Hansen and Lunde (2006)). A rolling window is used to capture a smooth transition of the parameter estimates and the covariance matrix. I estimate the model considering a smaller sample size such that each regression accounts for approximately two months. The shift in the window size is set to have the number of observations closely resemble two weeks.

Figure 3 displays the price dynamics over time for Gerdau. A few conclusions arise: although the majority of measures are considerably stable over time (especially if one considers the bootstrap intervals also graphed), there are changes in the market's importance, primarily during the second half of 2008 and the first half of 2009. I claim that this change in behavior comes from uncertainties derived from the 2008/2009 crises because stability measures stabilize in the second half of 2009. The primary conclusions derived in Section 4.2 appear to be stable over time, although the magnitude of the parameters may vary. The relation between exchange rate fluctuations and firm value is the same across time: a depreciation of the home currency leads to a decrease in firm value. ARCA appears to be faster than the NYSE during the majority of periods in incorporating innovations to the efficient price and in determining the efficient exchange rate (with a negative sign). Most of this behavior is seen during the crisis period (approximately windows 15 to 30).

5 Conclusion

I investigate price discovery for cross-listed Brazilian companies. The two primary targets of my investigation are to infer the relation between exchange rate fluctuations and firm value and to measure how quickly permanent innovations are impounded by the different

platforms (as well as to determine which markets are the most important in incorporating new information).

I present a simple price discovery model that guides the understanding of the empirical results. The model allows the observed prices to depend on two common factors: the efficient exchange rate and the efficient asset price. Moreover, I allow changes to the common factors to be contemporaneously correlated, yielding the necessary conditions to answer the research questions. The proposed theoretical model elucidates the stocks' price-formation process once it allows shocks to the efficient exchange rate to lead to a real (after arbitrage adjustments) effect on firm value. I provide short- and long-run solutions as a function of the structural parameters as well as dynamic price discovery measures.

I propose an alternative methodology to measure the instantaneous effects of permanent shocks on prices. By using the structural framework, one is able to isolate the impact of the exchange rate on firm value from the other impacts affecting the present value of the firm's cash flow. This type of question has to date not been analyzed using the price discovery framework or high-frequency data. The methodology is order-invariant and works properly even for a large number of variables and cointegrating vectors.

In the empirical results, I find that a depreciation/appreciation of the home currency leads to a decrease/increase in firm value. I then link the empirical results to my theoretical model, identifying the parameters that give this result. I also conclude that the trading platform ARCA is the most efficient market for instantaneously incorporating shocks, and the U.S. market is the one that adjusts for exchange rate shocks.

References

- AMIHUD, Y., AND H. MENDELSON (1987): “Trading Mechanisms and Stock Returns: An Empirical Investigation,” *The Journal of Finance*, 42, 533–553.
- BARTOV, E., AND G. M. BODNAR (1994): “Firm valuation, earnings expectations, and the exchange rate exposure effect,” *The Journal of Finance*, 49, 1755–1785.
- BARTOV, E., G. M. BODNAR, AND A. KAUL (1996): “Exchange rate variability and the riskiness of U.S. multinational firms: Evidence from the breakdown of the Bretton Woods system,” *Journal of Financial Economics*, 42, 105–132.
- BROWNLEES, C., AND G. GALLO (2006): “Financial econometric analysis at ultra-high frequency: Data handling concerns,” *Computational Statistics & Data Analysis*, 51, 2232–2245.
- CHAKRAVARTY, S., H. GULEN, AND S. MAYHEW (2004): “Informed Trading in Stock and Option Markets,” *The Journal of Finance*, LIX, NO. 3, 1235–1257.
- DOMINGUEZ, K. M. E., AND L. L. TESAR (2001): “A Reexaminations of Exchange-Rate Exposure,” *The American Economic Review*, 91, 396–399.
- FERNANDES, M., AND C. SCHERRER (2014): “Price discovery on common and preferred shares across multiple markets,” Discussion paper, CREATES Research Paper 2014-10, Aarhus University.
- GONZALO, J., AND C. GRANGER (1995): “Estimation of Common Long-Memory Components in Cointegrated Systems,” *Journal of Business & Statistics*, 13, 27–35.
- GONZALO, J., AND S. NG (2001): “A systematic framework for analyzing the dynamic effects of permanent and transitory shocks,” *Journal of Economics Dynamics & Control*, 25, 1527–1546.

- GRAMMIG, J., M. MELVIN, AND C. SCHLAG (2005): “Internationally cross-listed stock prices during overlapping trading hours: price discovery and exchange rate effects,” *Journal of Empirical Finance*, 12, 139–164.
- GRAMMIG, J., AND F. J. PETER (2013): “Telltale Tails: A New Approach to Estimating Unique Market Information Shares,” *Journal of Financial and Quantitative Analysis*, 48, Issue 02, 459–488.
- HAMILTON, J. D. (1994): *Time Series Analysis*. Princeton University Press.
- HANSEN, P. R., AND A. LUNDE (2006): “Realized Variance and Market Microstructure Noise,” *Journal of Business & Economic Statistics*, 24, NO. 2, 127–161.
- HARRIS, F. H., T. H. MCINISH, G. L. SHOESMITH, AND R. A. WOOD (1995): “Cointegration, Error Correction, and Price Discovery on Informationally Linked Security Markets,” *Journal of Financial and Quantitative Analysis*, 4, 563–578.
- HASBROUCK, J. (1995): “One Security, Many Markets: Determining the Contributions to Price Discovery,” *The Journal of Finance*, 50, 1175–1198.
- (2003): “Intraday Price Formation in U.S. Equity Index Markets,” *The Journal of Finance*, 58, 2375–2399.
- HASBROUCK, J., AND T. HO (1987): “Order Arrival, Quote Behavior, and the Return-Generating Process,” *The Journal of Finance*, 42, 1035–1048.
- HE, J., AND L. K. NG (1998): “The Foreign Exchange Exposure of Japanese Multinational Corporations,” *The Journal of Finance*, 53, 733–753.
- JOHANSEN, S. (1988): “Statistical Analysis of Cointegration Vectors,” *Journal of Economic Dynamics and Control*, 12, 231–254.
- (1991): “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models,” *Econometrica*, 59, 1551–1580.

- JORION, P. (1991): “The pricing of exchange rate risk in the stock market,” *Journal of Financial and Quantitative Analysis*, 26, 363-376.
- KIM, C. (2010): “Cross-Border Listing and Price Discovery: Canadian Blue Chips Traded in TSX and NYSE,” Discussion paper, University of Washington.
- LUTKEPOHL, H. (2007): *New Introduction to Multiple Time Series Analysis*. Springer.
- MINTON, B. A., AND C. SCHRAND (1999): “The impact of cash flow volatility on discretionary investment and the costs of debt and equity financing,” *Journal of Financial Economics*, 54, 423–460.
- MIZRACH, B., AND C. J. NEELY (2008): “Information shares in the US Treasury market,” *Journal of Banking & Finance*, 32, 1221-1233.
- MULLER, A., AND W. VERSCHOOR (2006a): “European Foreign Exchange Risk Exposure,” *European Financial Management*, 12, 195–220.
- (2006b): “Foreign exchange rate exposure: Survey and suggestions,” *Journal of Multinational Financial Management*, 16, 385–410.
- WARNE, A. (1993): “A common trends model: identification, estimation and inference,” Discussion paper, Stockholm School of Economics.
- YAN, B., AND E. ZIVOT (2007): “The Dynamics of Price Discovery,” Discussion paper, Department of Economics, University of Washington, Seattle, WA.
- YAN, B., AND E. ZIVOT (2010): “A structural analysis of price discovery measures,” *Journal of Financial Markets*, 13, 1–19.

6 Appendix

6.1 Model derivation

This section discusses the steps adopted to obtain the instantaneous and total effect measures given in (9) and (10), respectively. The target is to express the changes in the observed prices solely as function of the structural innovations given by $\eta_t = (\eta_t^P, \eta_t^T)$, where $\eta_t^P = (\eta_t^e, \eta_t^m)$. Similarly to (8), consider the infinite VMA process such that

$$\Delta P_t = d_0 \eta_t + d_1 L \eta_t + d_2 L^2 \eta_{t-2} + \dots = \sum_{i=0}^{\infty} d_i L^i \eta_{t-i}. \quad (24)$$

The instantaneous effect is simply achieved by making the lag operator equal to 0 ($L = 0$), whereas the total effect is obtained by setting $L = 1$, which gives the infinite summation of the parameter matrices in (24). To see how each price reacts to uncorrelated innovations, it is important to write d_0 and D as defined in (9) and (10) as functions of the structural parameters, λ , ρ in (2) and (1). For this purpose, I first express the observed prices (from (4) to (7)) as first differences. Secondly, the structural innovations that drive the efficient prices in (1) and (2) are substituted accordingly, and the lag operator (L) is set to 0 and 1 to find the instantaneous and total impacts, respectively.

Starting by solving the model to obtain the the first row of (9) and (10) (relative to the exchange rate), subtract w_{t-1} from both sides of (4) and collect the terms such that

$$\begin{aligned} w_t - w_{t-1} &= w_{t-1} - w_{t-2} + \dot{\gamma}_1 (\Delta e_t - \Delta w_{t-1}) + \gamma_1 (\Delta m_t - \Delta m_{t-1}) + b_1 (\eta_t^T - \eta_{t-1}^T) \\ (1 - L + L\dot{\gamma}_1) \Delta w_t &= \dot{\gamma}_1 (\Delta e_t) + \gamma_1 (\Delta m_t - L\Delta m_t) + b_1 (\eta_t^T - L\eta_t^T) \end{aligned} \quad (25)$$

Instantaneous impacts on the observed exchange rate given innovations in η_t^e and η_t^m are

obtained by setting $L = 0$ as

$$\begin{aligned}
\Delta w_t &= \dot{\gamma}_1 (\Delta e_t) + \gamma_1 (\Delta m_t) + b_1 \eta_t^T \\
\Delta w_t &= \dot{\gamma}_1 (\eta_t^e + \lambda \eta_t^m) + \gamma_1 (\eta_t^m + \rho \eta_t^e) + b_1 \eta_t^T \\
\Delta w_t &= (\dot{\gamma}_1 + \gamma_1 \rho) \eta_t^e + (\dot{\gamma}_1 \lambda + \gamma_1) \eta_t^m + b_1 \eta_t^T.
\end{aligned} \tag{26}$$

Equation (26) gives the instantaneous changes in the observed exchange rate solely as a function of permanent and transitory errors, η_t^P and η_t^T , respectively. The parameters in (26) account for the first row of d_0 as in (9).

The total impact is obtained by setting $L = 1$ in (25) such that

$$\begin{aligned}
\dot{\gamma}_1 \Delta w_t &= \dot{\gamma}_1 \Delta e_t \\
\Delta w_t &= \Delta e_t \\
\Delta w_t &= \eta_t^e + \lambda \eta_t^m.
\end{aligned} \tag{27}$$

Equation (27) delivers the the first row of the D measure depicted in (10). Note that the total effect solution only depends on the permanent innovations and the structural parameter that drives the efficient exchange rate.

Similar steps are implemented to obtain the remaining rows of d_0 and the D measures. By taking the first difference of the observed prices in (5), (6) and (7) and collecting the terms, it is possible to express the observed prices as functions of the permanent and transitory shocks.

$$(1 - L + L\gamma_2) \Delta p_{2,t} = \gamma_2 (\Delta m_t) + \dot{\gamma}_2 (\Delta e_t - L\Delta w_t) + b_2 (\eta_t^T - L\eta_t^T) \tag{28}$$

$$(1 - L + L\gamma_3) \Delta p_{3,t} = \Delta w_t - L\Delta w_t + \gamma_3 \Delta m_t + \dot{\gamma}_3 (\Delta e_t - L\Delta w_t) + b_3 (\eta_t^T - L\eta_t^T) \tag{29}$$

$$(1 - L + L\gamma_4) \Delta p_{4,t} = \Delta w_t - L\Delta w_t + \gamma_4 \Delta m_t + \dot{\gamma}_4 (\Delta e_t - L\Delta w_t) + b_4 (\eta_t^T - L\eta_t^T) \tag{30}$$

The instantaneous dynamics are obtained by setting $L = 0$ such that the parameters attached to η_t^e , η_t^m and η_t^T on the right-hand side of (31), (32) and (33) are the second, third and fourth rows of matrix d_0 , respectively. It is important to highlight that the instantaneous impact depends on γ_i and $\dot{\gamma}_i$ with $i = 2, 3, 4$, which are not the structural parameters that drive the efficient exchange rate and firm value but instead the parameters that drive the price discovery dynamics.

$$\Delta p_{2,t} = (\gamma_2 \rho + \dot{\gamma}_2) \eta_t^e + (\gamma_2 + \dot{\gamma}_2 \lambda) \eta_t^m + b_2 \eta_t^T \quad (31)$$

$$\Delta p_{3,t}^* = (\gamma_3 \rho + \dot{\gamma}_3) \eta_t^e + (\gamma_3 + \dot{\gamma}_3 \lambda) \eta_t^m + b_3 \eta_t^T \quad (32)$$

$$\Delta p_{4,t}^* = (\gamma_4 \rho + \dot{\gamma}_4) \eta_t^e + (\gamma_4 + \dot{\gamma}_4 \lambda) \eta_t^m + b_4 \eta_t^T \quad (33)$$

where $\Delta p_{3,t}^* = \Delta p_{3,t} - \Delta w_t$ and $\Delta p_{4,t}^* = \Delta p_{4,t} - \Delta w_t$.

The total impact dynamics are obtained as they are in the exchange rate case by setting $L = 1$ in (28), (29) and (30). As expected, the total impact only depends on the structural parameters. This conclusion follows because in the long run, permanent innovations must be fully assimilated by all markets, and transitory innovations must vanish away.

$$\Delta p_{2,t} = \rho \eta_t^e + \eta_t^m \quad (34)$$

$$\Delta p_{3,t}^* = (\rho - 1) \eta_t^e + (1 - \lambda) \eta_t^m \quad (35)$$

$$\Delta p_{4,t}^* = (\rho - 1) \eta_t^e + (1 - \lambda) \eta_t^m \quad (36)$$

Note that $\Delta w_t = \Delta e_t$ holds if $L = 1$ as in (27). This equality is used to obtain the total impact of permanent innovations in shares traded at the home and foreign markets as in (34), (35) and (36). The parameters attached to η_t^e and η_t^m in (34), (35) and (36) are the elements of the second, third and fourth rows of matrix D in (10), respectively.

6.2 Identification issues

There are three issues regarding the identification procedure in Section 3 that need to be addressed in more detail. The first issue arises with the computation of matrix G^* , which requires the identification of α_\perp ; the second refers to the implementation of the spectral decomposition instead of those previously adopted in the literature, HCH and Cholesky; and the third point elucidates the normalization computed in (22).

6.2.1 Identifying α'_\perp

The first identification issue is addressed, as in [Yan and Zivot \(2007\)](#), by showing that α'_\perp in (15) is identified by a linear combination of the rows of matrix $\Psi(1)$, which is obtained by setting $L = 1$ in (12). Using the Granger Representation Theorem as in [Lutkepohl, 2007](#), pg. 252, denote $\Gamma = \beta_\perp \left[\alpha'_\perp \left(I_k - \sum_{i=1}^l \xi_i \right) \beta_\perp \right]^{-1}$ such that the matrix $\Psi(1)$ can be decomposed as

$$\Psi(1) = \beta_\perp \left[\alpha'_\perp \left(I_k - \sum_{i=1}^l \xi_i \right) \beta_\perp \right]^{-1} \alpha'_\perp = \Gamma \alpha'_\perp, \quad (37)$$

where ξ_i with $i = 1, 2, \dots, l$ are the parameter matrices from the VECM model in (11); β_\perp and α'_\perp are the orthogonal projections of β and α , respectively. Multiplying both sides by the error term obtained from the reduced-form VEC model delivers

$$\Psi(1)\epsilon_t = \Gamma \alpha'_\perp \epsilon_t. \quad (38)$$

Recall that permanent and transitory innovations in their reduced form in (16) are obtained by multiplying matrix G^* and ϵ_t . From (15), the upper part of G^* contains $\alpha'_\perp \epsilon_t$, which is the portion of ε_t related to permanent innovations in their reduced form, $\varepsilon_t^P = (\varepsilon_t^e, \varepsilon_t^m)'$. Note that the right-hand side of (38) contains exactly the portion of ε_t related to permanent

innovations. Hence, I write $\Psi(1)\epsilon_t$ as

$$\Psi(1)\epsilon_t = \Gamma \varepsilon_t^P \quad (39)$$

The term on the left-hand side of (39) has a long-run impact on prices given the market innovations obtained from the VECM model in (11), whereas the right-hand side shows its counterpart as a function of the permanent innovations in their reduced form. Hence, the matrix Γ gives the long-run impact on market prices following innovations to ε_t^P . Considering a model that accounts for the exchange rate and shares traded on both domestic and foreign markets similar to the model discussed in Section 2, it is necessary to add assumptions that allow for the identification of α'_\perp solely using the rows of $\Psi(1)$. Assuming a simpler model, [Yan and Zivot \(2010\)](#) and [Kim \(2010\)](#) identify α'_\perp by imposing long-run restrictions on ε_t^P to justify the use of common rows in $\Psi(1)$. The identification strategy I implement follows along the same lines. I impose restrictions such that an innovation to ε_t^e is, in the long run, fully incorporated by the the shares traded on the foreign market. No restrictions are imposed on the short-run dynamics. Similarly, an innovation to ε_t^m should be fully incorporated by the shares traded on all markets in the long run. These sets of restrictions are translated in the following matrix Γ :

$$\Gamma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ -1 & 1 \end{pmatrix} \quad (40)$$

Combining (39) with (40), the first two rows of $\Psi(1)$ can be used in place of the first two rows of α'_\perp , and the third row minus the second row of $\Psi(1)$ can be used as the third row of α'_\perp . These restrictions also imply that the reduced-form innovations associated with the efficient price do not have a permanent impact on the exchange rate in the long run. This implication

is not harmful in this analysis for two reasons: first, there is no strong intuition regarding an innovation to the efficient price affecting the exchange rate, apart from contemporaneous correlation aspects governing the structural innovations associated with the common factors. Second, such a correlation among the structural errors can still be captured by the model because the restriction is to ε_t^m and not to η_t^m . Moreover, Γ imposes that changes in the reduced-form innovations associated with the exchange rate affect only the foreign market. Again, this requirement is not harmful because both restrictions are constructed in terms of ε_t^e .

6.2.2 Number of restrictions

This section elucidates the orthogonalization procedure that rotates the permanent and transitory innovations in their reduced form to their structural counterparts, as discussed in Section 3. Hence, I am interested in finding a full matrix \tilde{S} such that $\eta_t = \tilde{S}^{-1}\varepsilon_t$ holds and $Var(\eta_t)$ is a diagonal covariance matrix. The usual decompositions, HCH and Cholesky, impose zero restrictions on the decomposed matrix¹⁸, making it either a lower or an upper triangular matrix. When decomposing the $k \times k$ covariance matrix $Var(\varepsilon) = \Xi$, both decompositions input $(k^2 - k)/2$ zeros into the decomposed matrix. The need for such restrictions comes from having k^2 unknown variables to be identified in the decomposed matrix and only $((k^2 - k)/2) + k$ unique equations in Ξ , following the fact that Ξ is a symmetric matrix. To overcome the problem that both HCH and the Cholesky decompositions produce triangular matrices, I propose decomposing a normalized version of the covariance matrix of ε_t using the spectral decomposition such that no zero restrictions are imposed on \tilde{S} .

To this end, I perform a normalization and decompose a non-symmetric matrix $\tilde{\Xi} = \Xi\Theta^{-1}$, allowing the set of restrictions to be on the variance of η_t rather than imposing zero restrictions to the decomposed matrix. Decomposing $\tilde{\Xi}$ gives $(k^2 - k)/2$ further equations because $\tilde{\Xi}$ is no longer a symmetric matrix, implying that there are now k^2 equations to

¹⁸The decomposed matrix is F for Cholesky such that $\Xi = FF'$ and H for HCH such that $\Xi = HCH$.

identify $k^2 + k^2$ unknowns in $\tilde{\Xi} = \tilde{S} \text{Var}(\eta_t) \tilde{S}'$. Hence, k^2 further restrictions are necessary to identify the system. Matrix $\text{Var}(\eta_t)$ must be a diagonal matrix given that it is the covariance matrix of innovations in their structural form. This conclusion eliminates $(k^2 - k)$ unknown variables. There are, however, k unknowns left in the system. Imposing that the permanent and transitory innovations in their reduced form have the same variance as their structural counterpart adds another k restrictions to the system, making it just identified.

6.2.3 Normalization proof

Section 3 shows that structural innovations can be recovered through $\eta_t = \tilde{S}^{-1} \varepsilon_t$ provided that $\tilde{S} \tilde{S} \Theta = \tilde{S} \Theta \tilde{S}'$ holds. This section discusses the proof of this equality, considering the special case where Ξ is a 2×2 matrix. For this purpose, define Ξ as a symmetric matrix (covariance matrix) as below:

$$\Xi = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (41)$$

Define Θ as a diagonal matrix containing the vector $\theta = (a, c)'$ on its diagonal, and compute $\tilde{\Xi}$ as

$$\tilde{\Xi} = \Xi \Theta^{-1} = \begin{pmatrix} 1 & \frac{b}{c} \\ \frac{b}{a} & 1 \end{pmatrix}. \quad (42)$$

Define V as the matrix containing the eigenvectors associated with $\tilde{\Xi}$ and Λ as the diagonal matrix with the eigenvalues of $\tilde{\Xi}$ on its diagonal, such that

$$V = \begin{pmatrix} -\frac{\sqrt{a}}{\sqrt{c}} & \frac{\sqrt{a}}{\sqrt{c}} \\ 1 & 1 \end{pmatrix} \quad (43)$$

$$\Lambda = \begin{pmatrix} \frac{\sqrt{ac}-b}{\sqrt{ac}} & 0 \\ 0 & \frac{b+\sqrt{ac}}{\sqrt{ac}} \end{pmatrix} \quad (44)$$

By applying the spectral decomposition, $\tilde{\Xi} = \tilde{S}\tilde{S}$ and $\Xi = \tilde{S}\tilde{S}\Theta$, with \tilde{S} given by

$$\tilde{S} = V\Lambda^{1/2}V^{-1} = \begin{pmatrix} \frac{1}{2} \left[\left(\frac{\sqrt{ac}-b}{\sqrt{ac}} \right)^{1/2} + \left(\frac{b+\sqrt{ac}}{\sqrt{ac}} \right)^{1/2} \right] & \frac{\sqrt{a}}{2\sqrt{c}} \left[\left(\frac{b+\sqrt{ac}}{\sqrt{ac}} \right)^{1/2} - \left(\frac{\sqrt{ac}-b}{\sqrt{ac}} \right)^{1/2} \right] \\ \frac{\sqrt{c}}{2\sqrt{a}} \left[\left(\frac{b+\sqrt{ac}}{\sqrt{ac}} \right)^{1/2} - \left(\frac{\sqrt{ac}-b}{\sqrt{ac}} \right)^{1/2} \right] & \frac{1}{2} \left[\left(\frac{\sqrt{ac}-b}{\sqrt{ac}} \right)^{1/2} + \left(\frac{b+\sqrt{ac}}{\sqrt{ac}} \right)^{1/2} \right] \end{pmatrix} \quad (45)$$

By computing $\tilde{S}\tilde{S}\Theta$ and $\tilde{S}\Theta\tilde{S}'$ as in (46), I show that these two quantities are equal to each other, proving that the normalization holds ¹⁹.

$$\tilde{S}\tilde{S}\Theta = \tilde{S}\Theta\tilde{S}' = \begin{pmatrix} \frac{a}{2} \left[\left(1 - \frac{b}{\sqrt{ac}} \right) + \left(\frac{b}{\sqrt{ac}} + 1 \right) \right] & \frac{\sqrt{ac}}{2} \left[\left(\frac{b}{\sqrt{ac}} + 1 \right) - \left(1 - \frac{b}{\sqrt{ac}} \right) \right] \\ \frac{\sqrt{ac}}{2} \left[\left(\frac{b}{\sqrt{ac}} + 1 \right) - \left(1 - \frac{b}{\sqrt{ac}} \right) \right] & \frac{c}{2} \left[\left(1 - \frac{b}{\sqrt{ac}} \right) + \left(\frac{b}{\sqrt{ac}} + 1 \right) \right] \end{pmatrix} \quad (46)$$

6.3 Simulations

This section illustrates the proposed estimation methodology by comparing it with existing methodologies in the literature. Using the identification strategy discussed in Section 3, it is possible to isolate the relative performance of the two methodological changes implemented in this article, namely, the computation of matrix G^* using $\alpha'\Omega^{-1}$ instead of β' and the use of the spectral decomposition rather than the usual HCH or Cholesky decompositions.

The model used in this set of simulations is a simplified version of the one presented in Section 2. I work with two common factors, but the extension to the case with more

¹⁹A numerical exercise showing that (46) holds for matrices with dimensions greater than two is available upon request.

common factors is straightforward. I also assume that the parameters λ and ρ are zero to avoid imposing any prior benefits to any of the methodologies investigated. Given these restrictions, the elements of d_0 are the parameters giving the partial adjustment between efficient and observed prices as below.

$$d_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma_2 & b_{1i} & b_{2i} \\ \dot{\gamma}_3 & \gamma_3 & b_{1i} & b_{3i} \\ \dot{\gamma}_4 & \gamma_4 & b_{1i} & b_{4i} \end{pmatrix} \quad (47)$$

I additionally assume that the efficient exchange rate is an observed process. Therefore, the data-generation process is given by

$$e_t = e_{t-1} + \eta_t^e \quad (48)$$

$$m_t = m_{t-1} + \eta_t^m$$

where e_t is the efficient exchange rate and m_t is the asset efficient price. The structural innovations η_t^e and η_t^m are random normal processes generated with a diagonal covariance matrix. The transitory innovations η_t^T are also normally distributed. The observed prices are given by

$$\Delta p_{2,t} = \gamma_2 (m_t - p_{2,t-1}) + b_2 \eta_t^T \quad (49)$$

$$\Delta p_{3,t}^* = \gamma_3 (m_t - p_{3,t-1}) - \dot{\gamma}_3 (e_t - e_{t-1}) + b_3 \eta_t^T$$

$$\Delta p_{4,t}^* = \gamma_4 (m_t - p_{4,t-1}) - \dot{\gamma}_4 (e_t - e_{t-1}) + b_4 \eta_t^T$$

where $p_{2,t}$ are the transaction prices observed in the domestic market; $p_{3,t}^*$ and $p_{4,t}^*$ are prices observed in the foreign market and expressed in foreign currency; and the 1×2 vector b_i has the parameters that accompany the transitory innovations.

Table 9 reports the results based on the four comparisons. Firstly, I seek to measure the benefit of computing d_0 using the matrix G constructed with $\alpha'\Omega^{-1}$. Therefore, I compare \tilde{d}_0 versus \dot{d}_0 , where \dot{d}_0 represents d_0 computed using $\alpha'\Omega^{-1}$ and decomposed with HCH , whereas \tilde{d}_0 represents d_0 calculated with the matrix G computed using β' and the HCH decomposition. The second comparison assesses the benefit of using only the spectral decomposition. Hence, I compute two estimates of d_0 : the first one uses the $\alpha'\Omega^{-1}$ expression in G and the spectral decomposition (denoted as \hat{d}_0), whereas the second measure uses $\alpha'\Omega^{-1}$ and the HCH decomposition (denoted as \dot{d}_0). The third comparison addresses the benefits of combining the two methodological changes discussed in this work. I compute d_0 using both the $\alpha'\Omega^{-1}$ expression and the spectral decomposition (denoted as \hat{d}_0), and I denote \tilde{d}_0 as the estimates computed using β' and the HCH decomposition. Finally, I also aim to compare \hat{d}_0 with the methodology suggested by Gonzalo and Ng (2001) (computing with β' and using the Cholesky decomposition). I denote it as \bar{d}_0 .

I report the results in terms of the mean, relative mean squared errors (RelMSE) and relative root mean squared error (RelRMSE). I also display a ratio that offers information about how the RelMSE and RelRMSE measures are computed. For instance, the ratio \hat{d}_0/\bar{d}_0 implies that the relative measures of this column in the table are computed with \bar{d}_0 in the denominator and \hat{d}_0 in the numerator. Thus, relative measures smaller than one in this column indicate that the \hat{d}_0 outperforms \bar{d}_0 .

The results show that \tilde{d}_0 is biased for systems with more than one cointegrating vector (computations of \tilde{d}_0 for a smaller system with only one cointegrating vector to eliminate the bias). Using $\alpha'\Omega^{-1}$ to construct matrix G eliminates the finite sample bias even when the HCH decomposition is adopted (see the results of \dot{d}_0). Hence, \dot{d}_0 , \bar{d}_0 and \hat{d}_0 are not biased. By analyzing the relative measures, I show that \hat{d}_0 presents massive gains when compared to the \tilde{d}_0 measures. Similar results are obtained when \dot{d}_0 is compared to \tilde{d}_0 , showing that the use of $\alpha'\Omega^{-1}$ instead of β' considerably improves estimates of the d_0 matrix. In summary, the proposed measure outperforms all competitors.

Table 1: Breusch Godfrey LM test: p values

lags	ex rate	Bovespa	NYSE	ARCA	η_t^e	η_t^m
Gerdau						
5	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00
50	0.37	0.74	1.00	0.99	0.37	0.97
Petro						
5	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00
15	0.66	1.00	1.00	1.00	1.00	1.00
50	0.27	0.99	0.88	0.79	0.95	0.41
Bradesco						
5	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00
15	0.91	0.98	1.00	1.00	0.99	1.00
50	0.05	0.83	0.96	0.90	0.08	0.75
Ambev						
5	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00
50	0.98	0.97	0.99	1.00	0.97	1.00
BR Telecom						
5	1.00	1.00	1.00	1.00	1.00	1.00
10	0.99	0.85	1.00	1.00	1.00	1.00
15	0.98	0.96	0.91	0.99	0.99	1.00
50	0.30	0.53	0.44	0.19	0.38	0.87
Vale						
5	1.00	1.00	1.00		1.00	1.00
10	1.00	1.00	1.00		1.00	1.00
15	1.00	0.76	1.00		1.00	1.00
50	0.41	0.94	0.56		0.87	0.72

BreuschGodfrey Lagrange multiplier test for serial correlation at lags 5, 10, 15 and 50. P values are shown for the residuals of the reduced form model (exchange rate, Bovespa, NYSE and ARCA) and for the structural residuals, efficient exchange rate (η_t^e) and efficient price (η_t^m). Vale does not trade at ARCA.

Table 2: Price discovery Gerdau

		Inst. effect		Total effect			
		η_t^e	η_t^m	η_t^e	η_t^m	β	
	<i>ExRate</i>	1.34 (0.018)	0.00 (0.007)	0.97 (0.003)	-0.09 (0.005)	0	-1
BR	<i>GGBR4</i>	0.19 (0.047)	0.84 (0.029)	-0.62 (0.027)	0.97 (0.003)	0	1
NYSE	<i>GBR_n</i>	-0.82 (0.094)	0.74 (0.033)	-1.58 (0.025)	1.07 (0.003)	1	0
ARCA	<i>GBR_p</i>	-1.34 (0.087)	0.90 (0.029)	-1.58 (0.025)	1.07 (0.003)	-1	-1

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE and ARCA are foreign markets. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. Gerdau prices are sampled at 30 seconds frequency ($T = 352, 159$). The bootstrap standard errors are in parenthesis.

Table 3: Price discovery Petrobras

		Inst. effect		Total effect			
		η_t^e	η_t^m	η_t^e	η_t^m	β	
	<i>ExRate</i>	1.21 (0.026)	0.05 (0.014)	0.97 (0.004)	-0.11 (0.007)	0	-1
BR	<i>Petr4</i>	-0.18 (0.05)	0.88 (0.021)	-0.52 (0.036)	0.97 (0.004)	0	1
NYSE	<i>PBRa_N</i>	-1.20 (0.07)	0.89 (0.022)	-1.49 (0.033)	1.08 (0.004)	1	0
ARCA	<i>PBRa_P</i>	-1.52 (0.034)	1.00 (0.011)	-1.49 (0.033)	1.08 (0.004)	-1	-1

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE and ARCA are foreign markets. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. Petrobras prices are sampled at 30 seconds frequency ($T = 352, 676$). The bootstrap standard errors are in parenthesis.

Table 4: Price discovery Bradesco

		Inst. effect		Total effect			
		η_t^e	η_t^m	η_t^e	η_t^m	β	
	<i>ExRate</i>	1.30 (0.019)	0.01 (0.009)	0.98 (0.003)	-0.10 (0.007)	0	-1
BR	<i>BBDC4</i>	0.31 (0.032)	0.78 (0.028)	-0.47 (0.024)	0.98 (0.003)	0	1
NYSE	<i>BBD_n</i>	-1.10 (0.061)	0.81 (0.025)	-1.45 (0.022)	1.08 (0.004)	1	0
ARCA	<i>BBD_p</i>	-1.29 (0.051)	0.97 (0.02)	-1.45 (0.022)	1.08 (0.004)	-1	-1

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE and ARCA are foreign markets. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. Bradesco prices are sampled at 30 seconds frequency ($T = 352, 183$). The bootstrap standard errors are in parenthesis.

Table 5: Price discovery Ambev

		Inst. effect		Total effect				
		η_t^e	η_t^m	η_t^e	η_t^m	β		
	<i>ExRate</i>	1.14 (0.017)	-0.03 (0.011)	0.99 (0.002)	-0.09 (0.008)	0	-1	
BR	<i>AMBEV4</i>	0.17 (0.034)	0.95 (0.032)	-0.26 (0.022)	0.99 (0.002)	0	1	
NYSE	<i>ABV_n</i>	-0.76 (0.07)	0.85 (0.036)	-1.25 (0.02)	1.09 (0.007)	1	0	
ARCA	<i>ABV_p</i>	-0.91 (0.072)	0.96 (0.038)	-1.25 (0.02)	1.09 (0.007)	-1	-1	

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE and ARCA are foreign markets. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. Ambev prices are sampled at 90 seconds frequency ($T = 117,087$). The bootstrap standard errors are in parenthesis.

Table 6: Price discovery BR Telecom

		Inst. effect		Total effect				
		η_t^e	η_t^m	η_t^e	η_t^m	β		
	<i>ExRate</i>	1.07 (0.018)	-0.06 (0.008)	0.98 (0.003)	-0.08 (0.007)	0	-1	
BR	<i>BRT04</i>	0.21 (0.074)	0.89 (0.031)	-0.41 (0.039)	0.98 (0.003)	0	1	
NYSE	<i>BTM_n</i>	-0.88 (0.087)	1.00 (0.035)	-1.40 (0.036)	1.07 (0.005)	1	0	
ARCA	<i>BTM_p</i>	-0.83 (0.082)	0.96 (0.034)	-1.39 (0.036)	1.07 (0.005)	-1	-1	

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE and ARCA are foreign markets. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. BR Telecom prices are sampled at 300 seconds frequency ($T = 35,229$). The bootstrap standard errors are in parenthesis.

Table 7: Price discovery Vale

		Inst. effect		Total effect				
		η_t^e	η_t^m	η_t^e	η_t^m	β		
	<i>ExRate</i>	1.14 (0.019)	-0.05 (0.012)	0.96 (0.005)	-0.12 (0.008)	-1		
BR	<i>Vale5</i>	-0.46 (0.057)	0.97 (0.018)	-0.58 (0.044)	0.96 (0.005)	1		
NYSE	<i>Riop_N</i>	-1.58 (0.044)	1.11 (0.01)	-1.55 (0.04)	1.09 (0.004)	-1		

Exchange rate is in R\$ per US dollars. BR is the home market and NYSE is the foreign market. η_t^e and η_t^m are permanent shocks related to the efficient exchange rate and the efficient price of the underlying security, respectively. Lag length in the VEC model is determined through Schwarz criteria. Vale prices are sampled at 30 seconds frequency ($T = 352,344$). The bootstrap standard errors are in parenthesis.

Table 8: Inference on theoretical model parameters

	Gerdau		Petro		Bradesco	
	λ	ρ	λ	ρ	λ	ρ
ExRate	-0.09 (0.005)	—	-0.11 (0.007)	—	-0.10 (0.007)	—
BR	—	-0.62 (0.027)	—	-0.52 (0.036)	—	-0.47 (0.024)
NYSE	-0.07 (0.003)	-0.58 (0.025)	-0.08 (0.004)	-0.49 (0.033)	-0.08 (-0.004)	-0.45 (0.022)
ARCA	-0.07 (0.003)	-0.58 (0.025)	-0.08 (0.004)	-0.49 (0.033)	-0.08 (-0.004)	-0.45 (0.022)

	Ambev		BRTelecom		Vale	
	λ	ρ	λ	ρ	λ	ρ
ExRate	-0.09 (0.008)	—	-0.08 (0.007)	—	-0.12 (0.008)	—
BR	—	-0.26 (0.022)	—	-0.41 (0.039)	—	-0.58 (0.044)
NYSE	-0.09 (0.007)	-0.25 (0.02)	-0.07 (0.005)	-0.40 (0.036)	-0.09 (0.004)	-0.55 (0.04)
ARCA	-0.09 (0.007)	-0.25 (0.02)	-0.07 (0.005)	-0.39 (0.036)	—	—

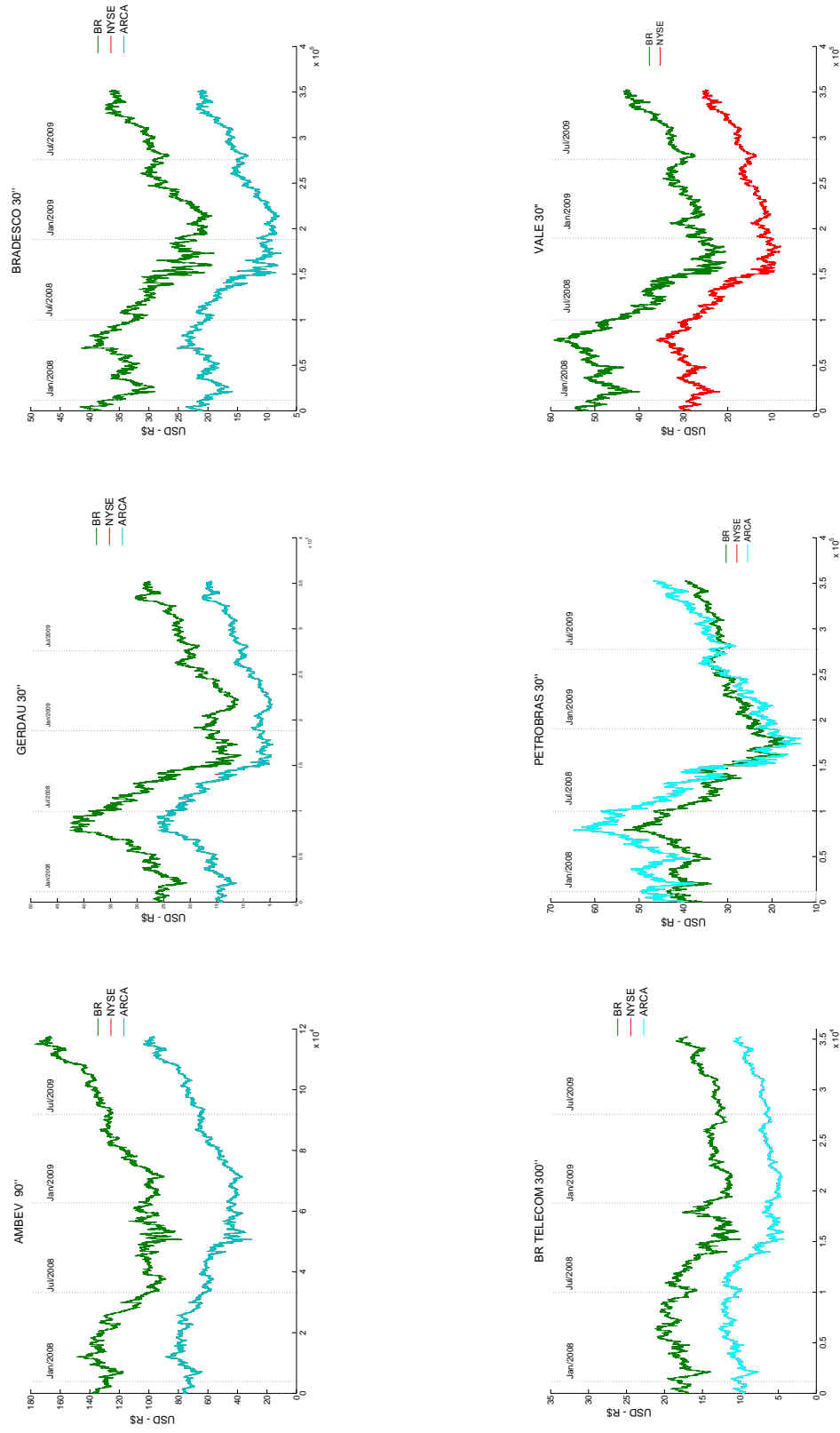
Inference on parameters from (10) are computed from the columns of total effect in each of the companies. Parameter λ is over identified and can be found on position 12 of total effect (first row, second column) as well as 1 - value in position 32 and 1 - value in position 42. Parameter ρ is also over identified and can be found on position 21 of total effect (second row, first column) as well as 1 + value in position 31 and 1 + value in position 41. '—' means that the respective parameter cannot be inferred from the equation, see (10) for details on this.

Table 9: Monte Carlo simulations

True value	Mean				RMSE				RRMSE			
	β_{HCH}	α_{HCH}	β_C	α_S	$\frac{\alpha_{HCH}}{\beta_{HCH}}$	$\frac{\alpha_S}{\beta_{HCH}}$	$\frac{\alpha_S}{\beta_{HCH}}$	$\frac{\alpha_S}{\beta_C}$	$\frac{\alpha_{HCH}}{\beta_{HCH}}$	$\frac{\alpha_S}{\alpha_{HCH}}$	$\frac{\alpha_S}{\beta_{HCH}}$	$\frac{\alpha_S}{\beta_C}$
d_0	\tilde{d}_0	\hat{d}_0	\bar{d}_0	\hat{d}_0	\hat{d}_0/\tilde{d}_0	\hat{d}_0/\hat{d}_0	\hat{d}_0/\tilde{d}_0	\hat{d}_0/\bar{d}_0	\hat{d}_0/\tilde{d}_0	\hat{d}_0/\hat{d}_0	\hat{d}_0/\tilde{d}_0	\hat{d}_0/\bar{d}_0
$d_0^{11} = 1.0$	1.03	0.99	0.99	0.99	0.05	1.00	0.05	0.92	0.22	1.00	0.22	0.96
$d_0^{21} = 0.0$	0.46	0.00	0.00	0.00	0.00	0.80	0.00	0.65	0.03	0.64	0.02	0.81
$d_0^{31} = 0.2$	1.15	0.20	0.20	0.20	0.00	0.99	0.00	0.97	0.01	0.99	0.01	0.99
$d_0^{41} = 0.5$	0.95	0.50	0.50	0.50	0.00	0.95	0.00	0.87	0.05	0.91	0.05	0.93
$d_0^{12} = 0.0$	0.01	0.00	0.00	0.00	0.20	0.70	0.10	0.51	0.45	0.50	0.32	0.71
$d_0^{22} = 0.8$	0.87	0.79	0.79	0.79	0.01	1.00	0.01	0.99	0.09	1.00	0.09	0.99
$d_0^{32} = 0.2$	0.45	0.20	0.20	0.20	0.00	0.96	0.00	0.93	0.04	0.93	0.04	0.96
$d_0^{42} = 0.5$	0.54	0.50	0.49	0.50	0.01	0.93	0.01	0.86	0.10	0.87	0.10	0.93

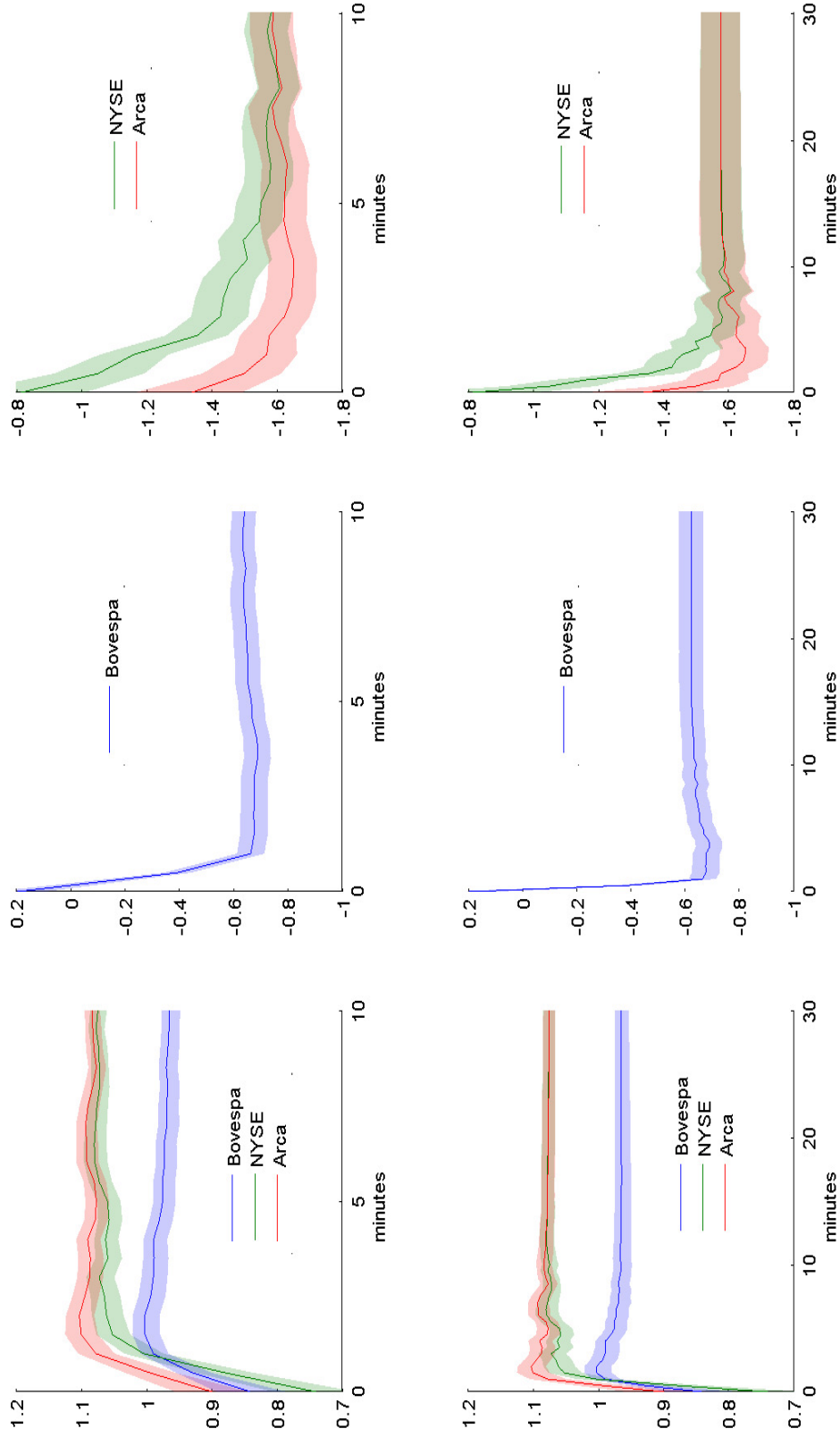
Results are expressed in terms of Relative Mean Squared Error (RMSE) and Relative Root Mean Squared Error (RRMSE). Sample size and replication number are fixed at 10,000 and 1,000 respectively. The variable $d_0^{i,j}$ denotes the ij^{th} element of the d_0 matrix.

Figure 1: Price Evolution



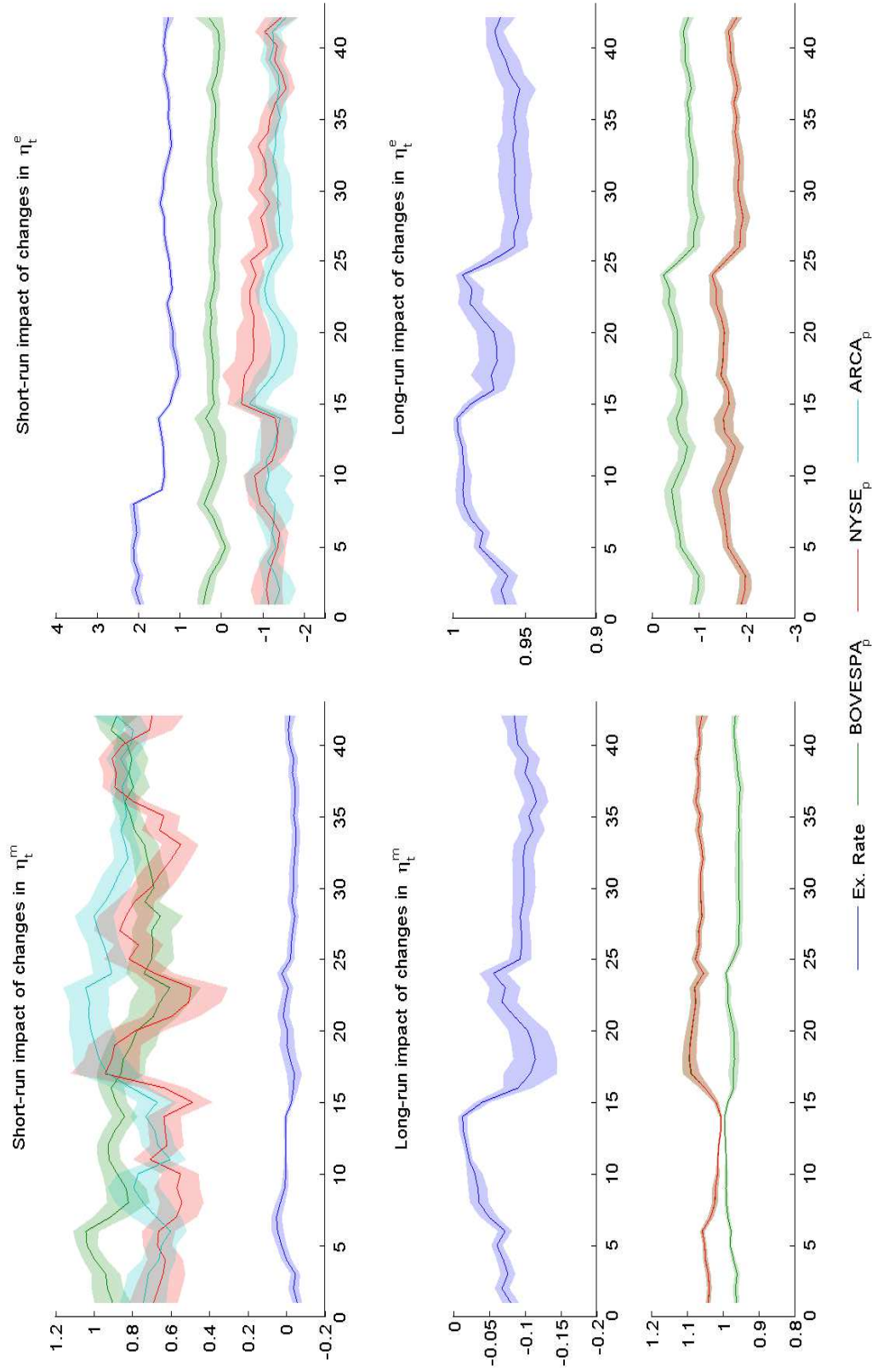
Displays price evolution of Ambev, Gerdau, Bradesco, BR Telecom, Petrobras and Vale stocks traded at BOVESPA, NYSE and ARCA. Sampling frequency are fixed as follows: Ambev, 90"; Gerdau, 30"; Bradesco, 30"; BR Telecom, 300"; Petrobras 30"; and Vale 30". Prices are aggregated following [Harris, McInish, L. Shoenish, and Wood \(1995\)](#) and free of non plausible values.

Figure 2: Gerdau: IRF - Innovation on efficient price and on efficient exchange rate



First column displays the cumulative impulse response functions showing the effect at Bovespa, ARCA and NYSE of an innovation on the firm efficient price over 10 (upper graph) and 30 (lower graph) minutes. Second and third column displays cumulative impulse response functions showing the effect of an innovation on the efficient exchange rate over 10 (upper graph) and 30 (lower graph) minutes at Bovespa (left) and at ARCA and NYSE (right), respectively. The empirical 95% confidence intervals are obtained using the bootstrap standard errors.

Figure 3: Gerdau



Displays rolling window estimates of the short- and long-run structural polynomials, $D(0)$ and $D(1)$ respectively. Sample size and window size are fixed equal to 30,547 and 7,637 observations respectively, resulting in 42 windows. Prices are aggregated at 30 seconds. *BOVESPA* accounts for shares traded at BOVESPA (Brazil), *NYSE* for the ADR's on shares traded at NYSE and *ARCA* for shares traded at ARCA. Exchange rate (ExRate) is expressed in R\$ per US dollars. 95% confidence intervals are obtained using the bootstrap standard errors.

Research Papers 2013



- d2014-36: Mehmet Caner and Anders Bredahl Kock: Asymptotically Honest Confidence Regions for High Dimensional
- 2014-37: Gustavo Fruet Dias and George Kapetanios: Forecasting Medium and Large Datasets with Vector Autoregressive Moving Average (VARMA) Models
- 2014-38: Søren Johansen: Times Series: Cointegration
- 2014-39: Søren Johansen and Bent Nielsen: Outlier detection algorithms for least squares time series regression
- 2014-40: Søren Johansen and Lukasz Gatarek: Optimal hedging with the cointegrated vector autoregressive model
- 2014-41: Laurent Callot and Johannes Tang Kristensen: Vector Autoregressions with Parsimoniously Time Varying Parameters and an Application to Monetary Policy
- 2014-42: Laurent A. F. Callot, Anders B. Kock and Marcelo C. Medeiros: Estimation and Forecasting of Large Realized Covariance Matrices and Portfolio Choice
- 2014-43: Paolo Santucci de Magistris and Federico Carlini: On the identification of fractionally cointegrated VAR models with the $F(d)$ condition
- 2014-44: Laurent Callot, Niels Haldrup and Malene Kallestrup Lamb: Deterministic and stochastic trends in the Lee-Carter mortality model
- 2014-45: Nektarios Aslanidis, Charlotte Christiansen, Neophytos Lambertides and Christos S. Savva: Idiosyncratic Volatility Puzzle: Influence of Macro-Finance Factors
- 2014-46: Alessandro Giovannelli and Tommaso Proietti: On the Selection of Common Factors for Macroeconomic Forecasting
- 2014-47: Martin M. Andreasen and Andrew Meldrum: Dynamic term structure models: The best way to enforce the zero lower bound
- 2014-48: Tim Bollerslev, Sophia Zhengzi Li and Viktor Todorov: Roughing up Beta: Continuous vs. Discontinuous Betas, and the Cross-Section of Expected Stock Returns
- 2014-49: Tim Bollerslev, Viktor Todorov and Lai Xu: Tail Risk Premia and Return Predictability
- 2014-50: Kerstin Gärtner and Mark Podolskij: On non-standard limits of Brownian semi-stationary
- 2014-51: Mark Podolskij : Ambit fields: survey and new challenges
- 2014-52: Tobias Fissler and Mark Podolskij: Testing the maximal rank of the volatility process for continuous diffusions observed with noise
- 2014-53: Cristina M. Scherrer: Cross listing: price discovery dynamics and exchange rate effects