



# Volatility jumps and their economic determinants

Massimiliano Caporin, Eduardo Rossi and Paolo Santucci de Magistris

**CREATES Research Paper 2014-27** 

Department of Economics and Business Aarhus University Fuglesangs Allé 4 DK-8210 Aarhus V Denmark Email: oekonomi@au.dk Tel: +45 8716 5515

# Volatility jumps and their economic determinants \*

Massimiliano Caporin<sup>†</sup>

Edward Rossi<sup>‡</sup> Paolo Santucci de Magistris<sup>§</sup>

August 20, 2014

#### Abstract

The volatility of financial returns is characterized by rapid and large increments. We propose an extension of the Heterogeneous Autoregressive model to incorporate jumps into the dynamics of the ex-post volatility measures. Using the realized-range measures of 36 NYSE stocks, we show that there is a positive probability of jumps in volatility. A common factor in the volatility jumps is shown to be related to a set of financial covariates (such as variance risk premium, S&P500 volume, credit-default swap, and federal fund rates). The credit-default swap on US banks and variance risk premium have predictive power on expected jump moves, thus confirming the common interpretation that sudden and large increases in equity volatility can be anticipated by credit deterioration of the US bank sector as well as changes in the market expectations of future risks. Finally, the model is extended to incorporate the credit-default swap and the variance risk premium in the dynamics of the jump size and intensity.

Keywords: Volatility jumps, Realized range, HAR-V-J, CDS. J.E.L. codes: C22, C58, G01

<sup>\*</sup>We are grateful to Asger Lunde for providing us with the data used in this study. We would like to thank Torben Andersen, Giampiero M. Gallo, Alfonso Novales, Juan-Angel Jimenez Martin, Teodosio Amaral, Miguel Dominguez, and the participants at the 4th CSDA International Conference on Computational and Financial Econometrics 2010 (London), the High Frequency Research Training Workshop 2011 (Berlin), the Time series and Financial econometrics Workshop (Bocconi University, Milan), the 2011 European meeting of the Econometric Society (ESEM, Oslo), the SCO11 Conference (Padova University, Padova), the NBER-NSF 2011 Time Series Conference (Michigan State University, East Lansing), the SIE 2011 Conference (University Rome III, Rome), and the seminars held at the Complutense University of Madrid (Spain), at CREATES (Aarhus, Denmark), and at CASS Business School (London) for useful comments on previous versions of this paper. We would also like to thank the Associate Editor and two anonymous referees for their precious suggestions that have led to significant improvements of the paper. Massimiliano Caporin acknowledges financial support from the MIUR PRIN project MISURA - Multivariate Statistical Models for Risk Assessment. Paolo Santucci de Magistris acknowledges financial support from the Danish National Research Foundation (DNRF78).

<sup>&</sup>lt;sup>†</sup>Dipartimento di scienze economiche e aziendali "Marco Fanno", Via del Santo 33, University of Padova, 35123 Padova, Italy

<sup>&</sup>lt;sup>‡</sup>Dipartimento di scienze economiche e aziendali, Via San Felice 5, University of Pavia, Pavia, 27100 Italy. Corresponding author. Tel.: +39 0382/986207 Fax: +39 0382/304226. *E-mail address*: erossi@eco.unipv.it

<sup>&</sup>lt;sup>§</sup>CREATES, University of Aarhus, Fuglesangs Alle 4, 8210, Aarhus V, Denmark.

#### 1 Introduction

Recent empirical studies indicate that diffusive stochastic volatility and jumps in returns are incapable of capturing the empirical features of equity index returns. Instead, it has been stressed that jumps in volatility can improve the overall fitting of stochastic volatility models. Eraker et al. (2003), for instance, report convincing evidence that volatility of financial returns is affected by rapid and large increments. The results of Eraker et al. (2003) point toward the inclusion of both jumps in volatility and in returns, and analogous conclusions are reached by Broadie et al. (2007). Duffie et al. (2000) provide some evidence, based on calibration, that the introduction of a volatility jump may attenuate the option overpricing observed in stochastic volatility models with jumps only in the returns. Todorov and Tauchen (2011), by means of a non-parametric analysis on the VIX volatility index, find that market volatility involves many small changes as well as occasional big moves, where the latter justifies the use of jumps in volatility modeling. They also find strong correlation between return and volatility jumps measured as realized jumps in both series. In general, both the return-based and option-based evidence support the presence of jumps in returns as well as in volatility.

As noted by Giot et al. (2010), market participants usually care as much about the nature of volatility as about its level. For example, all traders make the distinction between *good* and *bad* volatilities. *Good* volatility is directional, persistent and relatively easy to anticipate. *Bad* volatility is jumpy and relatively difficult to foresee. As such, *good* and *bad* volatilities can be associated with the continuous, persistent part and with the discontinuous, jump component of volatility, respectively.

From an econometric perspective, and focusing on continuous time stochastic volatility models, the introduction of jumps requires an additional set of latent state variables. Therefore, the estimation of stochastic volatility models with jumps in returns and/or volatilities can be possible only if the unobserved state variables are filtered out. For instance, Chernov et al. (2003), Eraker et al. (2003), and Eraker (2004), estimate their models by means of simulation-based methods, while Pan (2002) adopts the implied-state technique to fit her models to returns and option prices, and Li et al. (2008) employ MCMC techniques for models with infinite-activity Lévy jumps.

We contribute to this strand of the financial econometrics literature by focusing on the modeling and on the estimation of the volatility jump component in a discrete time setting. As a distinctive feature of our contribution, we use the realized range as non parametric ex-post measure of the daily integrated variance. Such a choice allows us to simplify the computational burden of estimating the jumps in volatility. In fact it circumvents the need to integrate out unobservable quantities. As suggested by Todorov (2009, 2011) we can make inference on the volatility jumps regardless of how complicated the model for the stochastic volatility is. Furthermore, recent theoretical findings by Christensen and Podolskij (2007, 2012) prove that realized range is a very efficient estimator of the quadratic variation of the returns. In our framework, efficiency of the integrated variance estimation is a crucial element, since the potential reduction in the measurement error obtained with realized-range measures can lead to more precise evaluations of the volatility jump component.

In order to evaluate the contribution of jumps to the daily volatility dynamics we specify and estimate a parametric model in discrete time. In particular, given the well documented long-range dependence of the realized variance estimators, see Andersen et al. (2003) among others, and the persistent effects of jumps in volatility, we propose a conditional model that generalizes the HAR model, introduced by Corsi (2009). The asymmetric Heterogeneous Autoregressive-Volatility-Jump (HAR-V-J) model includes an additive volatility jump term, which is modeled as a compound Poisson process allowing for multiple jumps per day, as in Chan and Maheu (2002) and Maheu and McCurdy (2004), whose intensity and magnitude parameters are varying over time according to an autoregressive specification. In this way, we are able to model and identify periods with higher volatility jump activity, that are also periods of high market stress.

The empirical analysis focuses on 36 stocks quoted at the New York Stock Exchange, representing nine sectors of the U.S. economy: banks, insurance and financial services, oil gas and basic materials, food beverage and leisure, health care, industrial goods, retail and telecommunications, services, and technology. The estimation results point out that the jump activity is characterized by two different periods. The first one, from 2004 to 2007, of low jump activity, the second, from mid-2008 to mid-2009, of high jump activity. In particular, during the second period the jump component represents a relevant part of the estimated conditional volatility. Such a finding is perfectly in line with the known feature of equity data during the sample period we consider. Furthermore, we find an ex-post positive correlation between volatility and price jumps, in line with Bandi and Renò (2014).

The second part of the analysis is devoted to investigate the relationship between the estimated volatility jump sequences and some economic covariates. A similar analysis is conducted by Rangel (2011) who evaluates the impact that scheduled announcements have on the conditional jump intensity of daily market returns. Firstly, we regress the estimated volatility jump sequences on a set of daily financial variables, such as the variance risk premium (VRP), trading volume of S&P 500, Federal Funds rate, and credit default swaps (CDS) on the US banks in order to investigate

the potential determinants of occasional big moves in asset's volatility. The results indicate that the CDS, the lagged market return and, the VRP are significant across the stocks considered. Secondly, we look at the presence of a common factor in the estimated volatility jump components. We find that the first principal component, that we call jump volatility factor, is highly correlated with the same explanatory variables. The main result is that VRP and CDS capture large part of the expected jumps moves, verifying the common interpretation that large and sudden increases in the volatility of stock markets over some days in the recent financial crisis have been anticipated by credit deterioration of US bank sector, and by an underestimate of volatility by market operators. We also argue that this result is not driven by the inclusion of the financial sector in the analysis. In fact, a robustness check made excluding the volatility jumps associated with financial companies, lead to almost identical results. The introduction of CDS and VRP as exogenous variables in the dynamics of volatility jumps provides an improvement in the ex-ante and ex-post jump probabilities. The evidence we provide thus suggests that our modeling strategy could be used in the forecast of the integrated variance, employed within risk management, or used for policy and market intervention purposes. The empirical validation of those additional elements is not included in this present contribution and is left for future researches.

This paper is organized as follows. In Section 2 the econometric model is set out and the estimation procedure is outlined. Section 3 illustrates the estimation results of the HAR-V-J model with data on 36 NYSE stocks. Section 4 investigates the determinants of the common component of estimated expected jumps and presents the estimates of an extended version of the HAR-V-J. Section 6 concludes. The Appendix summarizes some results associated with the realized range and presents the bias-corrected realized range-based bipower variation that is employed in the empirical analysis.

# 2 A model for realized range with jumps

A wide empirical literature focuses on modeling daily realized volatility measures, see e.g. Andersen and Bollerslev (1998), Andersen et al. (2001), Andersen et al. (2003), Andersen et al. (2007). More recently, the estimates of the components of the total daily price variation (the integrated variance and the price jump) have been separately modeled, see e.g. Bollerslev et al. (2009a), Busch et al. (2011), and Andersen et al. (2011). In the same spirit, we investigate the volatility jumps contribution to the integrated variance, using an ex-post realized range estimator. The range estimator can produce considerable efficiency gains relative to a standard return-based estimator, even when the latter employs subsampling to exhaust the entire database (e.g. Zhang et al., 2005). The range partially distills some of the information contained in intermediate data not used by a sparsely sampled return-based estimator, and this turns out to be a more effective way of doing it compared to subsampling of low-frequency returns (see Christensen and Podolskij, 2012).

In our analysis, we employ the bias-corrected realized range-based bipower variation  $(RBV_{m,BC}^{\Delta})$ of Christensen et al. (2009) as an ex-post measure of the integrated variance. The Appendix discusses the notation, the assumptions and the properties of the range-based estimator. The  $RBV_{m,BC}^{\Delta}$  has been shown to be a consistent estimator of the integrated variance in presence of jumps in prices and microstructure noise, as well as periodicities and long memory in the instantaneous volatility process. As long as jumps in prices cause discontinuities in the price trajectories, jumps in volatility temporarily amplify the diffusive component of the price dynamics. The  $RBV_{m,BC}^{\Delta}$  disentangles the contribution to the total price variation of the jumps in prices, but it retains the contribution of the jumps in volatility. The  $RBV_{m,BC}^{\Delta}$  is evaluated at the daily level from stock prices sampled at 1 minute intervals.

We propose an extension of the HAR model by Corsi (2009), which incorporates a jump term into the conditional mean of the realized range. The HAR model mimics the asymmetric propagation of volatility, due to the presence of heterogeneous market participants. It is an additive cascade model of different volatility components each of which is generated by the actions of different types of market players. This additive volatility cascade generates a simple long AR-type model, as it considers averages of realized measures over different time horizons. In the most common HAR specification, the actual log-realized variance is regressed on its past daily, weekly and monthly averages, together with a leverage term capturing the asymmetric relation between returns and volatility, see Martens et al. (2009), among others. The main advantage of the HAR-type specifications is represented by its estimation simplicity, given that the model can be estimated by ordinary last squares.

Let  $X_t = \log RBV_{m,BC,t}^{\Delta}$  and  $I^{t-1}$  be the time t-1 information set, the asymmetric HAR-Volatility Jump (HAR-V-J) model for  $X_t$  is given by,

$$X_{t} = \mu + \phi_{D} X_{t-1} + \phi_{W} X_{t-1}^{W} + \phi_{M} X_{t-1}^{M} + \gamma r_{t-1} \mathbf{I}(r_{t-1} < 0) + Z_{t} + \epsilon_{t}$$
(1)

where  $\epsilon_t | I^{t-1} \sim N(0, \sigma_t^2)$  and

$$X_t^W = \frac{1}{5} \sum_{j=0}^4 X_{t-j}, \text{ and } X_t^M = \frac{1}{22} \sum_{j=0}^{21} X_{t-j}$$

represent the weekly and monthly volatility components, respectively, see also Corsi (2009); the asymmetric component is driven by  $I(\cdot)$ , i.e. the indicator function which takes unit value when asset daily return is negative. The conditional variance of  $\epsilon_t$  is allowed to be time-varying and it can be modeled as a GARCH process. The additional jump term allows us to estimate the probability and the impact of volatility jumps on the dynamic and the conditional moments of the log  $RBV_{m,BC}^{\Delta}$ . This model implies that the  $RBV_{m,BC}^{\Delta}$  is given by a multiplicative structure such as

$$RBV_{m,BC,t}^{\Delta} = \exp\left\{\bar{X}_{t-1}\right\} \exp\left\{Z_t\right\} \exp\left\{\epsilon_t\right\}$$

where  $\bar{X}_{t-1} = \mu + \phi_D X_{t-1} + \phi_W X_{t-1}^W + \phi_M X_{t-1}^M + \gamma r_{t-1} I(r_{t-1} < 0)$ . Hence, the jump term  $J_t = \exp\{Z_t\}$  acts as a multiplicative term in the volatility process, such that it can be considered as a burst factor of the volatility dynamics. In case of no jumps, i.e. when  $J_t = 1$ , the volatility follows a HAR process. In period t, the jump term,  $Z_t$ , is given by

$$Z_t = \sum_{k=1}^{N_t} Y_{t,k}$$

where the jump size, conditional on the information set at time t - 1, is independent and normally distributed

$$Y_{t,k}|I^{t-1} \sim N\left(\Theta_t, \Delta_t\right),$$

and  $\epsilon_t$  and  $Y_{t,k}$  are assumed to be independent of each other. Chan and Maheu (2002) specify the dynamics of  $\Theta_t$  and  $\Delta_t$  as affine functions of the lagged dependent variable. In our case,  $\Theta_t$  and  $\Delta_t$  are assumed to be

$$\Theta_t = \zeta_0 + \zeta' W_{t-1} \tag{2}$$

and

$$\Delta_t = \eta_0 + \eta' W_{t-1},\tag{3}$$

where  $W_{t-1}$  are vectors containing variables included in the information set at time t-1, and  $\zeta$ and  $\eta$  are vectors of parameters conformable with  $W_{t-1}$ . For example,

The jump component has a compound Poisson structure where the number of jumps arrivals at time t,  $N_t$ , is a Poisson counting process with intensity parameter  $\Lambda_t > 0$  and density

$$P(N_t = j | I^{t-1}) = \frac{e^{-\Lambda_t} \Lambda_t^j}{j!}, \quad j = 0, 1, 2, \dots$$

This implies that

$$\mathbf{E}\left[N_t|I^{t-1}\right] = \mathbf{Var}\left[N_t|I^{t-1}\right] = \Lambda_t$$

so that the conditional density of  $Z_t$  given  $N_t$  and  $I^{t-1}$  is

$$Z_t | N_t = j, I^{t-1} \sim N\left(j\Theta_t, j\Delta_t\right). \tag{4}$$

Since  $E\left[Z_t|N_t=j,I^{t-1}\right]=j\Theta_t$ , the conditional expected value of the jump component is

$$\mathbf{E}\left[Z_t|I^{t-1}\right] = \Theta_t \Lambda_t \tag{5}$$

where  $\Theta_t$  is assumed to be measurable with respect to  $I^{t-1}$ , as in (2). Given the conditional density of  $Z_t$  in (4), the conditional variance of the jump component is

$$\operatorname{Var}\left[Z_t|I^{t-1}\right] = \left(\Delta_t + \Theta_t^2\right)\Lambda_t,\tag{6}$$

where  $\Delta_t$  is assumed to be measurable with respect to  $I^{t-1}$ , see (3). Whereas, as in Chan and Maheu (2002), the unobserved log-volatility jump intensity is assumed to follow an autoregressive specification

$$\Lambda_t = \lambda_0 + \lambda_1 \Lambda_{t-1} + \psi \xi_{t-1}. \tag{7}$$

As a result, the conditional jump intensity in period t depends on its own lag and on the lag of the innovation term  $\xi_t$ , which represents the measurable shock constructed ex-post. This shock, or jump intensity residual, is defined as

$$\xi_t = \mathbf{E}\left[N_t | I^t\right] - \Lambda_t.$$

Therefore,  $\xi_t$  depends on the expected number of jumps measured with respect to the information set including the contemporaneous information, i.e. at time t. It follows that the jump intensity equation can be rewritten as

$$\Lambda_t = \lambda_0 + (\lambda_1 - \psi) \Lambda_{t-1} + \psi \operatorname{E} \left[ N_{t-1} | I^{t-1} \right]$$

with

$$\mathbf{E}\left[N_t|I^t\right] = \sum_{j=0}^{\infty} jP\left(N_t = j|I^t\right).$$
(8)

As noted by Chan and Maheu (2002), other functional forms that include nonlinearity also may be very useful. For example, in Bandi and Renò (2014), the intensities of the jumps are nonlinear functions of the underlying variance level. An alternative observation-driven specification for the dynamic parameter  $\Lambda_t$  could be obtained in the GAS framework, discussed in Harvey (2013) and Creal et al. (2013) among others.<sup>1</sup>

The filtered probabilities  $P(N_t = j | I^t)$  are obtained by means of the Bayes' law

$$P\left(N_{t}=j|I^{t}\right) = \frac{P\left(X_{t}|N_{t}=j,I^{t-1}\right)P\left(N_{t}=j|I^{t-1}\right)}{P\left(X_{t}|I^{t-1}\right)}, \quad j=0,1,2,\dots$$
(9)

where

$$P(X_t|I^{t-1}) = \sum_{j=0}^{\infty} P(X_t|N_t = j, I^{t-1}) P(N_t = j|I^{t-1})$$

and  $P(X_t|N_t = j, I^{t-1})$  is given by the density of  $\epsilon_t$ . Analogously, we can compute the conditional probability of tail events, such as  $P(X_t > u|I^{t-1})$ . This allows us to compare the probability of extreme events implied by the HAR-V-J model, with those implied by the Gaussian HAR.

#### 2.1 Log-likelihood

Given equation (4), the first two conditional moments of  $X_t$  are given by

$$\mathbf{E}\left[X_t|N_t=j,I^{t-1}\right] = \bar{X}_{t-1} + j\Theta_t$$

and

$$\operatorname{Var}\left[X_t|N_t=j,I^{t-1}\right] = \sigma_t^2 + j\Delta_t,$$

with  $\sigma_t^2$  modeled as a GARCH(1,1), i.e.

$$\sigma_t^2 = \omega + \alpha \tilde{\epsilon}_{t-1}^2 + \beta \sigma_{t-1}^2$$

$$\tilde{\epsilon}_{t-1} = X_t - \bar{X}_{t-1} - \Lambda_t \Theta_t$$

<sup>&</sup>lt;sup>1</sup>Creal et al. (2013) derives the GAS representation for both the time-varying intensity Poisson process and the dynamic mixtures of models. However, the derivation of the GAS specification for  $\Lambda_t$  in the HAR-V-J model in (1) is complicated by the fact that  $Z_t$  is an infinite countably mixture of Gaussian terms whose weights depend on the realization of a random variable,  $N_t$ , which is Poisson distributed. We believe that an extension of the HAR-V-J model within the GAS framework is a natural advancement but this is left to future investigation.

The likelihood function of the model, conditional on the number of arrivals,  $N_t = j$ , and on  $I^{t-1}$ , is therefore given by

$$f(X_t|N_t = j, I^{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_t^2 + j\Delta_t)}} \exp\left(-\frac{(X_t - \bar{X}_{t-1} - j\Theta_t)^2}{2(\sigma_t^2 + j\Delta_t)}\right)$$

so that the log likelihood function conditional on  $I^{t-1}$  is

$$\ell(X_t|I^{t-1}) = \log\left(\sum_{j=0}^{\infty} P(N_t = j|I^{t-1}) \cdot f(X_t|N_t = j, I^{t-1})\right).$$
(10)

The likelihood function is then maximized with respect to the parameter vector,

$$\theta = \{\mu, \phi_D, \phi_W, \phi_M, \zeta_0, \zeta, \eta_0, \eta, \lambda_0, \lambda_1, \psi, \omega, \alpha, \beta\}.$$

In the empirical applications presented below the expression in (10) is approximated by a finite sum, where we employ a truncation value of  $20.^2$ 

#### 2.2 Conditional moments

Adopting the expression in Maheu and McCurdy (2004, p.766), the first four conditional moments of  $X_t$  are:

$$\mathbf{E}[X_t|I^{t-1}] = \bar{X}_{t-1} + \Lambda_t \Theta_t,\tag{11}$$

$$\operatorname{Var}[X_t|I^{t-1}] = \sigma_t^2 + \left(\Theta_t^2 + \Delta_t\right)\Lambda_t,\tag{12}$$

$$\operatorname{Sk}[X_t|I^{t-1}] = \frac{\Lambda_t \left(\Theta_t^3 + 3\Theta_t \Delta_t\right)}{\left[\sigma_t^2 + \left(\Theta_t^2 + \Delta_t\right)\Lambda_t\right]^{3/2}},\tag{13}$$

$$\operatorname{Kur}[X_t|I^{t-1}] = 3 + \frac{\Lambda_t \left(\Theta_t^4 + 6\Theta_t^2 \Delta_t + 3\Delta_t^2\right)}{\left[\sigma_t^2 + \left(\Theta_t^2 + \Delta_t\right)\Lambda_t\right]^2}.$$
(14)

Interestingly, the introduction of a jump component in the mean evolution of the log-RBV provides a number of relevant features. First of all, as compared to the simple HAR model, the conditional higher moments, i.e. the conditional skewness and kurtosis, are time-varying when  $\Lambda_t > 0$ . This makes the HAR-V-J model more flexible than the HAR without jumps, as it allows to relax the assumption of conditional normality. The jump component also contributes to the volatilityof-volatility evolution through the time-varying specification of the intensity and of the size and

 $<sup>^{2}</sup>$ Larger truncation values did not provide sensible improvements in the likelihood or relevant changes in the results.

variance of  $Y_{k,t}$ . In fact, the HAR-V-J can be seen as a model designed to parameterize the timevarying conditional heteroskedasticity observed in the log-realized measures, in the same spirit of Corsi et al. (2008). In the HAR-V-J model the conditional heteroskedasticity of  $X_t$  is decomposed into two sources of variation: one is the volatility-of-volatility the other is the jump component.

The expected value of  $RBV_{m,BC,t}^{\Delta} = \exp\{X_t\}$  conditional on  $I^{t-1}$  is

$$E[RBV_{m,BC,t}^{\Delta}|I^{t-1}] = \sum_{j=0}^{\infty} \left[ P\left(N_t = j|I^{t-1}\right) \cdot \exp\left\{\bar{X}_{t-1} + j\Theta_t + \frac{1}{2}(\sigma_t^2 + j\Delta_t)\right\} \right]$$
(15)

Finally, the conditional expectation of  $J_t \equiv \exp\{Z_t\}$  is given by

$$J_{t|t-1} \equiv E[J_t|I^{t-1}] = \sum_{j=0}^{\infty} P(N_t = j|I^{t-1}) \cdot \exp(j\Theta_t + \frac{1}{2}j\Delta_t).$$
(16)

In the next section, we discuss the results of the ML estimates of the HAR-V-J model, providing a detailed investigation of the conditional volatility jump component,  $J_{t|t-1}$ .

# 3 Volatility jumps in the US stock market

#### 3.1 Data description

Our empirical analysis is based on the intradaily returns of 36 equities of the S&P 500 index. The companies considered are shown in Table 1. Prices are sampled at one minute frequency, from January 2, 2004 to December 31, 2009, for a total of 1510 trading days. The start of the sample period is motivated by the availability of the *CDS* data, which are employed in Section 4. We compute the  $RBV_{m,BC,t}^{\Delta}$ , for each stock, according to (26), using one-minute returns. The parameter m is set equal to the average number of returns in one minute interval in each trading day for each stock considered, while the number of intradaily ranges is n = 390. Figure 1 plots the dynamic behavior of the annualized volatility of BA, IBM, JPM, and UPS.<sup>3</sup> The volatility is characterized by two dominant regimes. A long period of low volatility, approximately from 2004 to 2007, which is followed by a period of high volatility in correspondence of the financial crisis. It is interesting to note that the first part of the sample is not characterized by large jumps, while the period in correspondence of the recent financial crisis has many large spikes. As expected, this suggests that during financial crises, the probability and the magnitude of the jumps could be higher. Instead, the volatility of UPS is lower but with a large spike on October 10, 2008.

<sup>&</sup>lt;sup>3</sup>To save space the plots are displayed only for these four stocks.

Sector	Ticker	Company
BANK	BAC C JPM WFC	Bank of America Citygroup JP Morgan Wells Fargo
INSURANCE AND FIN. SERVICES	AXP GS MET MS	American Express Goldman & Sachs Met Life Morgan Stanley
OIL, GAS AND BASIC MATERIALS	XOM CVX FCX NEM	Exxon Chevron Freeport-McMoRan Copper Newmont Mining Corporation
FOOD, BEVERAGE AND LEISURE	TWX PEP KFT MCD	Time Warner Pepsi Cola Kraft Mc Donalds
HEALTH CARE AND CHEMICAL	JNJ PFE PG DD	Johnson & Jonhson Pfizer Procter & Gamble Du Pont
INDUSTRIAL GOODS	CAT BA HON F	Caterpillar Boeing Honeywell Ford
RETAIL AND TELECOMMUNICATIONS	WMT T HD VZ	Wall-Mart AT&T Home Depot Verizon
SERVICES	FDX UPS GE EMR	Fed-Ex UPS General Electric Emerson Electric
TECHNOLOGY	AAPL IBM HPQ TXN	Apple International Business Machines Hewlett Packard Texas Instruments

Table 1: Sector, Companies and Ticker



Figure 1: Annualized volatilities of Boeing, IBM, JP Morgan, and UPS computed as  $100 \times \sqrt{252} \times \sqrt{RBV_{BC,t}}$ .

As it is apparent from the second and third columns of Table 2, the series are right-skewed and leptokurtic. The sample skewness is around 1, on average, while the kurtosis is generally higher than three but smaller than six. This positive skewness could be related to the presence of a few large values in the log  $RBV^{\Delta}_{m,BC,t}$  series, such as those observed during the financial crisis (2008-2009). The unconditional non-normality could stem from the presence of jumps in the volatilities, as well as to changes in the conditional behavior of the series. However, explicitly accounting for the presence of jumps and GARCH effects reduces the tail fatness of the conditional distribution of the disturbances.

#### 3.2 Estimation results

We estimate the model in (1) according to the maximum likelihood procedure outlined in previous section. Preliminary estimates obtained by including  $X_{t-1}$  and  $X_{t-1}^2$  in equation (5) and (6) as in Chan and Maheu (2002), suggest that past volatility does not significantly affect the dynamics of  $\Theta_t$  and  $\Delta_t$ .<sup>4</sup> Consequently, the dynamics in the jump size mean and variance are initially restricted

<sup>&</sup>lt;sup>4</sup>These estimates are not reported here to save space, but they are available upon request from the authors. All computations have been performed with MATLAB.

	$\bar{\sigma}$	Skewness	Kurtosis	$\bar{\Lambda}_t$	$\overline{\mathrm{E}(N_t I^t)}$	$\overline{\operatorname{Var}(X_t I^{t-1})}$	$\overline{\mathrm{Sk}(X_t I^{t-1})}$	$\overline{\mathrm{Kur}(X_t I^{t-1})}$	$ \rho(J_{t t-1}, \Xi_t)_{04:07} $	$ \rho(J_{t t-1}, \Xi_t)_{08:09} $	$ \rho(J_{t t-1}, \Xi_t)_{04:09} $
AAPL	24.8368	0.7437	4.5830	0.1480	0.1400	0.2217	0.9208	5.1104	0.2494	0.4999	0.4256
AXP	20.5084	0.7587	2.4834	0.6060	0.0666	0.1798	0.4528	3.4386	0.5013	0.5036	0.5080
BA	15.9610	1.1382	4.7531	0.1554	0.0478	0.1757	0.4285	3.7419	0.2424	0.5739	0.5646
BAC	23.1555	0.9767	2.9254	0.1486	0.0751	0.2090	0.4766	3.8997	0.5659	0.3181	0.4260
С	23.9415	0.9290	3.0168	0.3935	0.0659	0.2345	0.4384	3.5649	0.6778	0.3841	0.4694
CAT	18.4294	1.1565	4.1235	0.2610	0.1497	0.1747	0.5149	3.7756	0.2937	0.4249	0.4193
CVX	15.9767	0.9214	4.7157	0.3389	0.1030	0.1655	0.1745	3.2217	0.2896	0.4359	0.3910
DD	16.5392	1.1019	4.1269	0.1213	0.0493	0.1648	0.3263	3.6093	0.4055	0.5818	0.6312
EMR	16.3422	1.3718	5.3417	0.1836	0.0666	0.1692	0.4619	3.7992	0.5721	0.5933	0.6162
F	30.0859	1.2691	5.7290	0.1802	0.0679	0.2676	0.4130	4.4640	0.2101	0.3250	0.3451
FCX	29.1566	1.0974	4.6910	0.1982	0.1488	0.1518	0.3964	3.6045	0.3604	0.6971	0.6603
FDX	16.0122	0.7384	3.4815	0.0325	0.0328	0.1706	0.6708	4.8614	0.2918	0.3988	0.4608
GE	15.6763	1.1663	3.7924	0.7652	0.2988	0.2050	0.5304	3.5059	0.6066	0.5950	0.6084
GS	21.5953	1.0772	3.7746	0.4591	0.1381	0.1609	0.2821	3.2560	0.6884	0.5114	0.5230
HD	17.8131	0.9366	3.4931	0.1000	0.0750	0.1844	0.5439	4.1896	0.2495	0.4384	0.3489
HON	16.7893	1.3807	6.2263	0.1838	0.1212	0.1793	0.3969	3.6082	0.1443	0.5774	0.4987
HPQ	16.6704	1.1246	4.8770	0.4664	0.3733	0.2022	0.5125	3.5950	-0.0581	0.1372	-0.0101
IBM	13.3862	1.3166	5.2915	0.2066	0.0870	0.1645	0.3525	3.4803	0.4385	0.6549	0.6343
JNJ	10.1048	1.3872	5.7007	0.2531	0.1674	0.1855	0.5560	3.8558	0.2228	0.3743	0.3568
JPM	22.0841	0.9047	2.7714	0.2547	0.5170	0.1536	0.4135	3.5565	0.6403	0.4611	0.5254
KFT	16.1119	0.6633	5.0948	0.0608	0.1022	0.2428	0.7285	5.0754	0.3174	0.5157	0.4491
MCD	14.1048	1.3491	6.3214	0.0432	0.0338	0.2014	0.7700	6.0775	0.1775	0.4657	0.4478
MET	23.0263	1.1781	4.1188	0.1401	0.0512	0.1977	0.4269	3.8061	0.2048	0.3711	0.2632
MS	28.5541	1.1187	3.8033	0.1938	0.0223	0.1888	0.4847	3.9082	0.5801	0.5995	0.5596
NEM	21.5462	0.8663	3.8108	0.0690	0.0306	0.1261	0.3079	3.6153	0.3457	0.7390	0.6432
PEP	11.7077	1.4604	6.2378	0.1232	0.1789	0.1947	0.6286	4.3648	0.3239	0.4726	0.4590
PFE	13.8933	1.2437	4.9546	0.1554	0.1328	0.2659	1.1211	6.2036	0.0750	0.4942	0.3931
$\mathbf{PG}$	11.5246	1.3308	5.6124	0.1631	0.1299	0.1936	0.4304	3.9387	0.1674	0.4948	0.4707
Т	17.0354	0.8744	4.9006	0.4070	0.1671	0.1864	0.1584	3.3035	0.2213	0.3552	0.3703
TWX	17.6097	1.1112	4.6496	0.2773	0.1168	0.1822	0.5774	3.9257	0.2555	0.4777	0.5038
TXN	20.3096	0.9976	4.2807	0.0618	0.0290	0.1668	0.2262	3.5960	0.0446	0.3653	0.4378
UPS	13.1429	1.0870	4.4766	0.0316	0.0227	0.1732	0.6783	5.5076	0.3128	0.4657	0.4562
VZ	14.5341	1.2889	5.2242	0.3394	0.0931	0.2119	0.6441	3.9441	0.2738	0.4439	0.4265
WFC	22.6472	0.8660	2.6464	0.3683	0.0941	0.1799	0.4766	3.6784	0.5368	0.5066	0.5270
WMT	13.1333	1.1507	4.4018	0.2287	0.0968	0.1928	0.5371	3.9605	0.4041	0.4520	0.4961
XOM	15.1828	0.9625	4.5853	0.6287	0.2017	0.1597	0.2592	3.1982	0.3859	0.5668	0.4894

Table 2: Summary statistics of  $\log(RBV_{BC})$ , and conditional jumps and conditional moments of log-volatility. First column reports the average percentage volatility on annual basis, that is  $\bar{\sigma} = \frac{1}{T} \sum_{t=1}^{T} \sqrt{RBV_{BC,t}} \times 100 \times \sqrt{252}$ . Second and third columns report sample skewness and kurtosis of  $\log(RBV_{BC})$ , see (13) and (14). Column 4 reports the sample averages of  $\Lambda_t$ , which is the average expected number of jumps.  $\overline{E(N_t|I^t)}$ ,  $\overline{Var(X_t|I^{t-1})}$ ,  $\overline{Sk(X_t|I^{t-1})}$ ,  $\overline{Kur(X_t|I^{t-1})}$  are the sample average of the number of expected jumps, and the sample average of the conditional variance, conditional skewness, and conditional kurtosis of log-volatility, respectively, implied by the HAR-V-J model. The last three columns report the jump correlations for the periods 2004-2007, 2008-2009 and 2004-2009.  $J_{t|t-1} = E[J_t|I^{t-1}]$  is the expected jump component in the  $BPV_{BC,t}$ , that is obtained from the estimates of the HAR-V-J model, see (16). Squared jump price is estimated by  $\Xi_t = \lambda_{2,m} (RRG_{BC} - RBV_{BC})$ .  $\rho(J_{t|t-1}, \Xi_t)$  is the estimated linear correlation coefficient between  $J_{t|t-1}$  and  $\Xi_t$ .

to be constant. Therefore, the specification we adopt here is the following

$$X_{t} = \bar{X}_{t-1} + \sum_{k=1}^{N_{t}} Y_{t,k} + \epsilon_{t}, \quad \epsilon_{t} | I^{t-1} \sim N(0, \sigma_{t}^{2}), \quad Y_{t,k} | I^{t-1} \sim N(\zeta_{0}, \eta_{0}),$$

$$P\left(N_{t} = j | I^{t-1}\right) = \frac{e^{-\Lambda_{t}} \Lambda_{t}^{j}}{j!} \quad j = 0, 1, 2, \dots$$

$$\Lambda_{t} = \lambda_{0} + (\lambda_{1} - \psi) \Lambda_{t-1} + \psi \operatorname{E} \left[ N_{t-1} | I^{t-1} \right]$$

$$\sigma_{t}^{2} = \omega + \alpha \tilde{\epsilon}_{t-1}^{2} + \beta \sigma_{t-1}^{2}$$
(17)

In Section 5, the dynamic equations for  $\Lambda_t$  and  $\Theta_t$  are extended by the inclusion of economic/financial covariates.

Table 3 reports the estimated parameters of the HAR-V-J for the 36 stocks under analysis, while Table 4 contains some diagnostic checks. Looking at the Ljung-Box test on the model residuals, see the last columns of Table 4, for some series the HAR-V-J model is not able to completely capture the dynamic of the log-volatility series. This is due to the peculiar autoregressive lag structure of the HAR-V model which appears to be too restrictive for many series under exam, thus leaving some autocorrelation in the residuals. It is important to stress that the autocorrelation in the residuals is not due to the inclusion of the jump term in the HAR-V-J model. The coefficient of the HAR part of the model are in line with values observed in previous studies, positive and highly significant. In addition, the leverage term is characterized by negative coefficients, an expected result. In fact, when the past returns are negative, we observe an increase in the log-realized measure, that is an increase in volatility. For what concerns the jump size mean, we note that the estimates of  $\zeta_0$  in (2) are positive and statistically significant, in most cases at the 5% confidence level. Only in five cases we have a jump mean size coefficient which is not statistically significant. In addition, the jump size variance ( $\eta_0$ ) is statistically significant in most cases at the 10% confidence level; the few exceptions are AXP, JPM, and WFC.

The estimates of the GARCH(1,1) parameters for the conditional variance of  $\epsilon_t$  are significant in almost all cases. It should also be noted that the estimated GARCH shows a moderate persistence. Notably, Table 4 reports in the first column the LR test for the presence of GARCH effects (volatilityof-volatility effects). The null cannot be rejected in just 8 cases, many of them associated with a very limited persistence. To understand how the jump component contributes to the overall conditional variance of  $X_t$ , we plot in Figure 2 the ratio of the conditional variance of the HAR-V-J model to the one obtained from a HAR-V model, both with a GARCH(1,1) specification for the error conditional variance. From the plots it is evident how the inclusion of the jumps not only changes

		HA	AR		Leverage	Mean	Variance		Intensity			GARCH	
	$\mu$	$\phi_D$	$\phi_W$	$\phi_M$	$\gamma$	$\zeta_0$	$\eta_0$	$\lambda_0$	$\lambda_1$	$\psi$	ω	α	eta
AAPL	$-0.6157^{a}$	$0.3576^{a}$	$0.3376^{a}$	$0.1843^{a}$	$-0.0780^{a}$	$0.5597^{a}$	$0.2714^{a}$	$0.0017^{a}$	$0.9881^{a}$	$0.0080^{a}$	$0.0081^{a}$	$0.0225^{a}$	$0.9028^{a}$
AXP	$-0.4493^{a}$	$0.3290^{a}$	$0.4238^{a}$	$0.1821^{a}$	$-0.0589^{a}$	0.1568	0.1129	0.0002	$0.9990^{a}$	$0.0504^{a}$	$0.0127^{a}$	$0.0772^{a}$	$0.7120^{a}$
BA	$-0.6180^{a}$	$0.2699^{a}$	$0.4097^{a}$	$0.2117^{a}$	$-0.0603^{a}$	$0.3290^{a}$	$0.2358^{a}$	$0.0001^{a}$	$0.9983^{a}$	$0.0491^{a}$	$0.0196^{a}$	$0.0299^{a}$	$0.7949^{a}$
BAC	$-0.4515^{a}$	$0.3951^{a}$	$0.4179^{a}$	$0.1180^{a}$	$-0.0571^{a}$	$0.3980^{a}$	$0.2344^{a}$	0.0001	$0.9991^{a}$	$0.0296^{a}$	$0.0078^{a}$	$0.0432^{a}$	$0.8854^{a}$
С	$-0.5890^{a}$	$0.3463^{a}$	$0.4349^{a}$	$0.1244^{a}$	$-0.0451^{a}$	$0.2358^{a}$	$0.1885^{a}$	0.0007	$0.9972^{a}$	$0.0838^{a}$	$0.0399^{a}$	$0.1162^{a}$	$0.5035^{a}$
CAT	$-0.4242^{a}$	$0.3180^{a}$	$0.3634^{a}$	$0.2467^{a}$	$-0.0480^{a}$	0.2299	$0.2121^{a}$	$0.0210^{a}$	$0.9182^{a}$	$0.3081^{a}$	$0.0980^{a}$	$0.0429^{a}$	0.0000
CVX	$-0.4272^{a}$	$0.3442^{a}$	$0.5105^{a}$	$0.0725^{a}$	$-0.0796^{a}$	0.0804	$0.1708^{a}$	$0.0234^{a}$	$0.9318^{a}$	$0.5688^{a}$	$0.0909^{a}$	$0.0213^{a}$	0.1041
DD	$-0.5028^{a}$	$0.2756^{a}$	$0.4565^{a}$	$0.1812^{a}$	$-0.0878^{a}$	0.2494	$0.2757^{a}$	0.0001	$0.9982^{a}$	$0.0477^{b}$	0.0058	$0.0341^{a}$	$0.9056^{a}$
EMR	$-0.7156^{a}$	$0.2431^{a}$	$0.4026^{a}$	$0.2244^{a}$	$-0.0341^{a}$	$0.3935^{a}$	$0.1584^{a}$	0.0003	$0.9976^{a}$	$0.1013^{a}$	$0.0578^{a}$	$0.0856^{a}$	$0.3502^{a}$
F	$-0.3243^{a}$	$0.2513^{a}$	$0.3106^{a}$	$0.3676^{a}$	$-0.0244^{a}$	$0.2056^{a}$	$0.4833^{a}$	0.0002	$0.9983^{a}$	$0.0498^{a}$	$0.0091^{a}$	$0.0981^{a}$	$0.7894^{a}$
FCX	$-0.3613^{a}$	$0.3242^{a}$	$0.4288^{a}$	$0.1726^{a}$	$-0.0469^{a}$	$0.1872^{a}$	$0.2082^{c}$	$0.0024^{a}$	$0.9874^{a}$	$0.0893^{a}$	$0.0522^{a}$	$0.0901^{a}$	$0.3644^{a}$
FDX	$-0.3175^{a}$	$0.2554^{a}$	$0.3492^{a}$	$0.3423^{a}$	$-0.0498^{a}$	$1.1303^{a}$	0.0000	0.0002	$0.9944^{a}$	0.0057	$0.0233^{a}$	$0.0421^{a}$	$0.7627^{a}$
GE	$-0.7683^{a}$	$0.2992^{a}$	$0.4357^{a}$	$0.1585^{a}$	$-0.0991^{a}$	$0.1810^{b}$	$0.1087^{a}$	0.0004	$0.9990^{a}$	$0.0593^{a}$	$0.0672^{a}$	$0.1362^{a}$	0.0000
GS	$-0.4776^{a}$	$0.3943^{a}$	$0.3698^{a}$	$0.1542^{a}$	$-0.0674^{a}$	$0.1339^{a}$	$0.1181^{a}$	$0.0119^{a}$	$0.9743^{a}$	$0.4588^{a}$	$0.0750^{a}$	$0.0575^{a}$	$0.1416^{a}$
HD	$-0.3299^{a}$	$0.3241^{a}$	$0.3739^{a}$	$0.2500^{a}$	$-0.0619^{a}$	$0.4886^{a}$	$0.2226^{a}$	$0.0120^{a}$	$0.8785^{a}$	$0.1405^{a}$	$0.0338^{a}$	$0.0543^{a}$	$0.6814^{a}$
HON	$-0.6342^{a}$	$0.2562^{a}$	$0.4339^{a}$	$0.1929^{a}$	$-0.0635^{a}$	$0.2510^{b}$	$0.1971^{a}$	0.0002	$0.9985^{a}$	$0.0292^{a}$	$0.0067^{b}$	$0.0468^{a}$	$0.8831^{a}$
HPQ	$-0.5188^{a}$	$0.2967^{a}$	$0.4565^{a}$	$0.1623^{a}$	$-0.0876^{a}$	$0.1639^{a}$	$0.1735^{a}$	$0.2858^{a}$	$0.3872^{a}$	0.0000	$0.0183^{a}$	$0.1058^{a}$	$0.6296^{a}$
IBM	$-0.7510^{a}$	$0.3173^{a}$	$0.4513^{a}$	$0.1092^{a}$	$-0.1344^{a}$	$0.2682^{a}$	$0.1529^{a}$	0.0004	$0.9972^{a}$	$0.0897^{a}$	$0.0679^{a}$	$0.0443^{a}$	$0.3615^{a}$
JNJ	$-0.7980^{a}$	$0.2455^{a}$	$0.4093^{a}$	$0.2297^{a}$	$-0.1659^{a}$	$0.3005^{a}$	$0.1775^{a}$	0.0003	$0.9981^{a}$	$0.0415^{a}$	$0.0544^{a}$	$0.1115^{a}$	$0.3523^{a}$
JPM	$-0.4327^{a}$	$0.3964^{a}$	$0.4297^{a}$	$0.1080^{a}$	$-0.0700^{a}$	$0.2616^{a}$	0.1223	0.0001	$0.9990^{a}$	$0.0452^{a}$	0.0056	0.0240	$0.9097^{a}$
KFT	$-0.6929^{a}$	$0.1845^{a}$	$0.4481^{a}$	$0.2356^{a}$	$-0.0476^{a}$	$0.9152^{a}$	$0.2294^{b}$	0.0003	$0.9953^{a}$	$0.0113^{a}$	$0.0087^{a}$	$0.0317^{a}$	$0.9069^{a}$
MCD	$-0.5563^{a}$	$0.2327^{a}$	$0.4274^{a}$	$0.2432^{a}$	$-0.0879^{a}$	$0.6798^{a}$	$0.6780^{a}$	0.0002	$0.9957^{a}$	$0.0175^{a}$	$0.0272^{a}$	$0.0402^{a}$	$0.7668^{a}$
MET	$-0.3332^{a}$	$0.2465^{a}$	$0.5128^{a}$	$0.1856^{a}$	$-0.0538^{a}$	$0.3027^{a}$	$0.2899^{a}$	$0.0258^{a}$	$0.8134^{a}$	$0.3922^{a}$	0.0003	$0.0051^{a}$	$0.9907^{a}$
MS	$-0.3154^{a}$	$0.4011^{a}$	$0.3492^{a}$	$0.1951^{a}$	$-0.0427^{a}$	$0.2565^{a}$	$0.2865^{a}$	$0.0136^{a}$	$0.9297^{a}$	$0.3328^{a}$	$0.0519^{a}$	$0.0534^{a}$	$0.4797^{a}$
NEM	$-0.2662^{a}$	$0.3116^{a}$	$0.4565^{a}$	$0.1819^{a}$	$-0.0364^{a}$	$0.3405^{a}$	$0.2282^{a}$	0.0002	$0.9959^{a}$	$0.0814^{a}$	$0.0989^{a}$	$0.0268^{a}$	0.0000
PEP	$-0.6877^{a}$	$0.1852^{a}$	$0.4874^{a}$	$0.2190^{a}$	$-0.0838^{a}$	$0.5384^{a}$	$0.1779^{a}$	0.0003	$0.9974^{a}$	$0.0206^{a}$	$0.0072^{a}$	$0.0318^{a}$	$0.9002^{a}$
PFE	$-0.4111^{a}$	$0.3066^{a}$	$0.3225^{a}$	$0.3108^{a}$	$-0.0560^{a}$	$0.5111^{a}$	$0.5400^{a}$	$0.0046^{a}$	$0.9704^{a}$	$0.0307^{a}$	$0.1189^{a}$	$0.0821^{a}$	0.0000
$\mathbf{PG}$	$-0.5535^{a}$	$0.2967^{a}$	$0.4362^{a}$	$0.1776^{a}$	$-0.0778^{a}$	$0.2121^{a}$	$0.3340^{a}$	$0.0008^{c}$	$0.9945^{a}$	$0.0380^{a}$	$0.0084^{a}$	$0.0387^{a}$	$0.8774^{a}$
Т	$-0.3665^{a}$	$0.2570^{a}$	$0.4834^{a}$	$0.1940^{a}$	$-0.0604^{a}$	0.0598	$0.2043^{a}$	0.0007	$0.9971^{a}$	$0.1168^{a}$	$0.0675^{a}$	$0.0939^{a}$	$0.1528^{a}$
TWX	$-0.5057^{a}$	$0.2326^{a}$	$0.3945^{a}$	$0.2806^{a}$	$-0.0144^{a}$	$0.2467^{a}$	$0.2240^{a}$	$0.0014^{a}$	$0.9940^{a}$	$0.0904^{a}$	$0.0266^{a}$	$0.0604^{a}$	$0.6320^{a}$
TXN	$-0.3507^{a}$	$0.2517^{a}$	$0.4364^{a}$	$0.2397^{a}$	$-0.0406^{a}$	$0.2404^{a}$	$0.3873^{a}$	$0.0008^{a}$	$0.9884^{a}$	$0.1020^{a}$	0.0028	$0.0261^{a}$	$0.9491^{a}$
UPS	$-0.4609^{a}$	$0.2371^{a}$	$0.5352^{a}$	$0.1529^{a}$	$-0.0840^{a}$	$0.8774^{a}$	$0.4138^{a}$	0.0001	$0.9974^{a}$	$0.0155^{a}$	0.0037	$0.0193^{a}$	$0.9478^{a}$
VZ	$-0.6315^{a}$	$0.2652^{a}$	$0.3471^{a}$	$0.2905^{a}$	$-0.0925^{a}$	$0.2791^{a}$	$0.2043^{a}$	$0.0371^{a}$	$0.8895^{a}$	$0.2846^{a}$	$0.0850^{a}$	$0.0526^{a}$	$0.1673^{a}$
WFC	$-0.5353^{a}$	$0.2536^{a}$	$0.3634^{a}$	$0.3051^{a}$	$-0.0517^{a}$	$0.3611^{b}$	0.0564	$0.0424^{a}$	$0.8858^{a}$	$0.4381^{a}$	0.0021	$0.0182^{b}$	$0.9515^{a}$
WMT	$-0.5045^{a}$	$0.2363^{a}$	$0.4587^{a}$	$0.2264^{a}$	$-0.0873^{a}$	$0.2397^{a}$	$0.2812^{a}$	$0.0019^{a}$	$0.9914^{a}$	$0.0934^{a}$	$0.0971^{a}$	$0.0938^{a}$	0.0000
XOM	$-0.4863^{a}$	$0.3548^{a}$	$0.4629^{a}$	$0.1057^{a}$	$-0.0978^{a}$	$0.0870^{b}$	$0.1081^{a}$	$0.0327^{a}$	$0.9471^{a}$	$0.4356^{a}$	$0.0849^{a}$	$0.0129^{a}$	0.0002

Table 3: Estimated parameters of the HAR-V-J model using price data from January 2, 2004 to December 31, 2009. The first column reports the ticker of the stocks, see Table 1. a, b, and c denote significance at the 1%, 5% and 10% confidence levels, respectively.



Figure 2: The ratio between the conditional variances of HAR-V-J and HAR-V both with GARCH(1,1) errors.

the evolution of the conditional variance of  $X_t$  but also, and more relevantly, contributes in the second period of the sample to the increase of the level of the conditional variance, inducing large variations on some days.

Now, if we turn our attention to the estimates of the parameters of  $\Lambda_t$ , the jump intensity, we have that the persistence parameter,  $\lambda_1$ , is strongly significant and greater than 0.9 in most cases. This result confirms the evidence in Eraker et al. (2003) and Duffie et al. (2000), where the jump arrivals in volatility are highly persistent, producing jump clusters. The close-to-unit-root behavior of the jumps intensity could stem from a change of regime in the number of jumps arrivals during the financial crisis. Interestingly, the plots of the expected number of jumps in Figure 3 suggest the presence of three regimes in the jumps intensity, with the exception of UPS. The first period, from 2004 to the beginning of 2007, is characterized by an absence of jumps in volatility (the number of jumps is on average one in twenty days). In the second period, the estimated jump arrivals sharply increase to a daily average of 0.3-0.4, while between mid-2008 and mid-2009, the average number of jump arrivals dramatically increases, implying approximately more than one jump every second day. This result is in line with the findings in Dotsis et al. (2007), who estimate a constant probability of jumps equal to 0.4 for a set of implied volatility series, during the period 1997-2004.

	$LR_{\alpha=\beta=0}$	$LR_{\lambda_1=\psi=0}$	$Q_{\epsilon}^5$	$Q_{\epsilon}^{20}$	$Q^5_{\tilde{\epsilon}^2}$
AAPL	0.0018	0.8297	0.0187	0.2522	0.8322
AXP	0.0000	0.0038	0.0001	0.0061	0.9653
BA	0.0929	0.0001	0.0242	0.1165	0.4044
BAC	0.0001	0.0010	0.0391	0.1196	0.9475
С	0.0000	0.0001	0.0019	0.0060	0.8166
CAT	0.1608	0.0001	0.0095	0.0368	0.5304
CVX	0.7221	0.1013	0.1487	0.0000	0.7789
DD	0.0002	0.0015	0.0026	0.0018	0.9581
EMR	0.0003	0.0000	0.0506	0.0134	0.2179
F	0.0000	0.0001	0.4440	0.0242	0.5470
FCX	0.0036	0.0305	0.0004	0.0011	0.8963
FDX	0.0000	0.3897	0.0430	0.1471	0.0509
GE	0.0001	0.0000	0.0087	0.0334	0.2245
GS	0.1431	0.0010	0.0001	0.0004	0.8328
HD	0.0011	0.1057	0.0002	0.0018	0.8850
HON	0.0007	0.0527	0.0372	0.1377	0.7482
HPQ	0.0000	0.9049	0.0000	0.0045	0.8425
IBM	0.5876	0.0710	0.0000	0.0003	0.9988
JNJ	0.0000	0.0003	0.0070	0.0005	0.9901
JPM	0.0036	0.0073	0.1217	0.0850	0.9661
KFT	0.0001	0.4424	0.0502	0.5993	0.9349
MCD	0.0005	0.9836	0.0774	0.0544	0.7593
MET	0.0005	0.0594	0.0598	0.1334	0.9744
MS	0.0079	0.0596	0.0451	0.1429	0.9917
NEM	0.3122	0.0005	0.3007	0.2246	0.3791
PEP	0.0000	0.0172	0.0064	0.1446	0.4376
PFE	0.0000	0.0001	0.0768	0.1315	0.1536
$\mathbf{PG}$	0.0083	0.0592	0.0189	0.2739	0.8504
Т	0.0036	0.0525	0.0003	0.0036	0.9287
TWX	0.0037	0.0000	0.0129	0.0411	0.0898
TXN	0.0079	0.1295	0.1781	0.0023	0.5562
UPS	0.0000	0.0154	0.1071	0.0562	0.9669
VZ	0.0189	0.0183	0.0004	0.0177	0.9091
WFC	0.0000	0.0000	0.2706	0.2265	0.6970
WMT	0.0003	0.0002	0.0024	0.0031	0.6492
XOM	0.9041	0.7370	0.0948	0.0006	0.8634

Table 4: Diagnostic test. The first column reports the company ticker. Column  $LR_{\alpha=\beta=0}$  reports the *p*-value of the likelihood-ratio test for the null hypothesis  $\alpha = \beta = 0$ , i.e. absence of GARCH effects. Column  $LR_{\lambda_1=\psi=0}$  reports the *p*-value of the likelihood-ratio test for the null hypothesis  $\lambda_1 = \psi = 0$ , i.e. the jump intensity is constant. Columns  $Q_{\epsilon}^5$  and  $Q_{\epsilon}^{20}$  contain the *p*-values of the Ljung-Box test on the residuals of the estimated HAR-V-J model in (17), with 5 and 20 lags, respectively. The last column,  $Q_{\epsilon^2}^5$  reports the *p*-values of the Ljung-Box test on the squared standardized residuals,  $\frac{\hat{\epsilon}_t}{\sqrt{\sigma_t^2 + (\Theta_t^2 + \Delta_t)\Lambda_t}}$ .

.



Figure 3: The expected number of jump arrivals,  $\Lambda_t$ .

This is a common characteristic of the estimated jump intensities of the financial stocks in the sample and it could be the outcome of the financial crisis, which hits the bank sector more than others. This result is in line with the findings of Todorov and Tauchen (2011), where the high number of jumps in volatility is attributed to the pure jump nature of the volatility process. The changing-regime feature in the intensity process could be modeled with a dummy variable or with a smooth transition function. We don't pursue this possibility here because we don't want to further complicate the estimation of the model. We obtain a different picture for UPS where the jump activity is very low for the entire sample period. This result is consistent with the evolution of the corrected-realized range shown in Figure 1.

The estimated ex-post probability of at least one jump, shown for the four stocks in Figure 4, reflects the high persistence in the  $\Lambda_t$  estimates, consistently with the jump clustering which characterizes all the series in the sample. Moreover, in the crisis period (2008-2009), the ex-post probability of observing at least one log-volatility jump approaches one. The higher estimated jump activity in the second part of the sample explains the increase in the unconditional expected value of the  $\Lambda_t$  process, see Table 2.

Our results suggest that time variation in the jumps intensity is not negligible, so that the



Figure 4: Ex-post probability of at least one jump,  $P(N_t \ge 1|I^t)$ .

assumption of constant jump arrival probability turns out to be unrealistic, especially during periods of financial turnoil. This is also confirmed by looking at the parameter  $\psi$  in the intensity equation which is always positive, and significant in most of the cases. As a consequence the unobserved past innovation has always a positive and significant impact on the jump intensity. Some of the  $\psi$  coefficients are, however, not significant or very small, thus raising some doubts on the dynamic evolution of the jump intensity in these cases. This hypothesis is verified by means of a LR test, reported in Table 4. Only in six cases the LR test cannot reject the null of constant jump intensity at any significance level.<sup>5</sup> In the remaining cases, three have p-values around 10%, another ten assets show p-values between 1% and 10%, and the remaining 17 assets have p-values below 1%. Those results further support the dynamic evolution of jump intensity which captures both volatility bursts as well as the heteroskedasticity of realized range volatilities.

Figure 5 reports the estimated expected exponential jumps,  $\hat{J}_{t|t-1}$ . In all cases, the expected exponential jumps increase during the period 2008-2009, that is, jumps in volatility constitute

<sup>&</sup>lt;sup>5</sup>Since when jumps are absent, i.e.  $\Lambda_t = 0$ , the parameters of  $\Theta_t$  and  $\Delta_t$  are not identified, it is not possible to evaluate the significance of the jump term by a standard LR test, see the discussion in Hansen (1996). In this case simulation based approaches can be used to recover likelihood ratio test critical values following Hansen (1996). We don't pursue this strategy due to the computational burden implied in the estimation of the HAR-V-J, but rather we evaluate, in the next section, the consequences of the inclusion of the jump term in characterizing the moments and the tails of the distribution of  $X_t$ .



Figure 5: Expected exponential jump,  $J_{t|t-1}$ .

an important source of price variability. By looking at the volatility of JPM, the expected jump component increases already after 2007. The role of the jumps for IBM and BA is relevant in the period between mid-2008 and mid-2009, namely during the recent financial crisis. A completely different picture emerges from the plot of the expected exponential jumps of UPS, which remain rather low and stable for the entire period. Overall, the results we obtain are somewhat expected, and show the ability of the model in capturing the occurrences of jumps in volatility during the recent financial crisis. Furthermore, such a result suggests the relevance of volatility jumps in addition to the price jumps.

#### 3.3 Volatility jump features

With the HAR-V-J model we can study the difference between the ex-ante and ex-post probabilities of jumps during a given day. This can be done in our setup, simply comparing  $P(N_t = j|I^{t-1})$  with  $P(N_t = j|I^t)$ , where the latter is obtained by the Bayes law in (9). In particular, the recent financial crisis peaked on October 10, 2008, when annualized volatility of the S&P 500 index reached a peak of 120%. As it is clear from Figure 6, the estimated ex-ante and ex-post probabilities on October 10, 2008 have different patterns. In particular, the ex-ante probabilities of more than one jump,



Figure 6: Ex-ante (black solid line) and ex-post (red dashed line) probability of jumps on October 10, 2008, i.e.  $P(N_t = j | I^{t-1})$  and  $P(N_t = j | I^t)$ , respectively.

calculated using  $\Lambda_t$ , are already high and centered on 1-2 jumps. On the other hand, after the arrival of the information on the volatility for October 10, 2008, the ex-post jump probability distribution is shifted to the right, such that we have a higher probability of observing 3-4 jumps on that day.

In Table 2 we report the sample averages of conditional moments for the expected number of jumps, as computed in (8), and for  $X_t$ , as in (12)-(14). The average ex-post number of jumps is very close to the expected number of jumps, meaning that the specification of  $\Lambda_t$  correctly estimates  $E[N_t|I^t]$ . Since, the HAR-V-J model is a non-Gaussian conditional model for  $X_t$  due to the presence of jumps it generates positive skewness and mild leptokurtosis. The sample averages for the 36 stocks of  $Sk(X_t|I^{t-1})$  are always positive and smaller than one, with the exception of PFE. This means that including the jumps in the model allows us to partially account for the positive skewness observed in the log-volatility. Analogous considerations hold for the sample averages of the conditional kurtosis.

Table 2 also reports the sample correlations between the expected exponential jump component, i.e.  $E[J_t|I^{t-1}]$ , and the squared price jumps, defined as  $\sum_{i=1}^{N_p(t)} \varsigma_i^2$ , as in (24) (see Appendix). The jumps are estimated by  $\Xi_t = \lambda_{2,m} (RRG_{BC} - RBV_{BC})$ , see Appendix for details. The interesting finding is a positive correlation with the squared jumps-in-price component (see for an analogous result Todorov and Tauchen, 2011). This result is in line with the evidence in Eraker et al. (2003) which suggest a positive association in the price and volatility jumps. This is also confirmed in a recent paper by Jacod et al. (2013), which rejects the null hypothesis of no correlation between price and volatility jumps. Since we look at the correlation between the cumulated squared price jumps and the volatility jumps, we cannot provide an evidence on the sign of the correlation between the jump sizes. Correlation is strengthened during the financial crisis, since the arrival of bad news induces not only jumps in prices, but also a sharp increase in the volatility. A possible explanation of this is that as traders receive new information, they revise their expectations, causing an increase in the disagreement on the fair price that leads to higher volatility. This result provides further support to the model specification in Bandi and Renò (2014) where the price and volatility co-jumps intensity is function of the volatility level.

In order to highlight the ability of the HAR-V-J model in predicting the log-volatility one-day ahead, we compute  $P(RBV_{m,BC,t}^{\Delta} > u|I^{t-1})$ , where *u* corresponds to a certain level of the annualized volatility. We choose October 10, 2005, as a day of low volatility, and October 10, 2008, as a day of extremely high volatility. For both days we calibrate *u* on the basis of the annualized realized variance: if ARBV is the annualized realized bipower on a given day, we set  $u = 0.9 \times ARBV$ . In such a way, we evaluate the probability of tail events setting the threshold in accordance with both the features of each series and the scale of the volatility observed in a specific day. The tail probability is given by

$$\Pr\left[RBV_{m,BC,t}^{\Delta} > u|I^{t-1}\right] = \sum_{j=0}^{\infty} \left[P\left(N_t = j|I^{t-1}\right) \cdot \left(1 - \Phi\left(\frac{\log u - E_t}{V_t}\right)\right)\right]$$
(18)

where

$$E_t = \mathbb{E}[X_t | N_t = j, I^{t-1}] = \bar{X}_{t-1} + j\Theta_t$$
$$V_t^2 = \operatorname{Var}[X_t | N_t = j, I^{t-1}] = \sigma_t^2 + j\Delta_t$$

and  $\Phi(\cdot)$  is the standard Normal CDF. This simple exercise allows us to evaluate how much the introduction of the jump component in the HAR increases the conditional probability of observing abnormal levels of volatility. The results in Table 5 clearly illustrates that the HAR-V-J behaves better than the Gaussian HAR in days of high volatility. This is not surprising because the positive conditional probability of observing more than one jump, as already seen above, dramatically increases the conditional probability that the log-volatility is above a given threshold. In general, the performances of the two models for October 10, 2005, are very similar, and this is not surprising as the probability of jumps is low in that day. There is one exception, represented by *Ford*, for

	Octo	ober 10, 2	2005	Octo	ber 10, 20	)08	Kupie	ec (1995) Test	Christof	fersen (1998) Test	Bate	s (2000) Test
	ARBV	1	2	ARBV	3	4	HAR	HAR - V - J	HAR	HAR - V - J	HAR	HAR - V - J
AAPL	20.0188	0.0144	0.0474	140.4111	0.0000	0.1430	0.0000	0.9813	0.0000	0.8496	0.0000	0.0001
AXP	7.0985	0.0000	0.0000	195.9950	0.5728	0.4604	0.0000	0.9813	0.0000	0.8496	0.0000	0.0674
BA	11.2431	0.0000	0.0000	123.5293	0.0000	0.0832	0.0000	0.9813	0.0000	0.8496	0.0000	0.0001
BAC	7.0298	0.0000	0.0011	199.4987	0.9966	0.6161	0.0000	0.9813	0.0000	0.8496	0.0000	0.0102
С	9.5267	0.0000	0.0503	255.5481	0.9590	0.5605	0.0006	0.4360	0.0000	0.5853	0.0000	0.0642
CAT	11.9894	0.0000	0.0000	152.8040	0.0000	0.2902	0.0047	0.6113	0.0000	0.7770	0.0000	0.1261
CVX	15.9539	0.0000	0.0029	192.1001	0.1177	0.3846	0.0047	0.5947	0.0000	0.7052	0.0000	0.5761
DD	9.8219	0.0000	0.0000	141.2881	0.0000	0.4489	0.0167	0.5947	0.0000	0.7052	0.0000	0.0037
EMR	11.0407	0.0000	0.0000	131.8486	0.0000	0.3037	0.0047	0.4360	0.0000	0.5853	0.0000	0.0020
$\mathbf{F}$	37.9658	0.0041	0.3158	411.7023	0.9971	0.4972	0.0000	0.2080	0.0000	0.3401	0.0000	0.0034
FCX	26.1148	0.0000	0.0002	211.2908	0.9995	0.7842	0.0167	0.2080	0.0000	0.3401	0.0000	0.2485
FDX	12.2305	0.0000	0.0004	93.8766	0.0000	0.0004	0.0001	0.5947	0.0000	0.7052	0.0000	0.0001
GE	6.8006	0.0000	0.0138	153.0170	0.0080	0.4054	0.0000	0.9813	0.0000	0.8496	0.0000	0.3794
GS	7.7381	0.0000	0.0000	272.7749	0.9902	0.5476	0.0006	0.4360	0.0000	0.5853	0.0000	0.1449
HD	10.2281	0.0000	0.0036	171.6845	0.0000	0.1765	0.0024	0.7792	0.0000	0.7995	0.0000	0.0034
HON	13.4460	0.0000	0.0000	150.3984	0.0000	0.2942	0.0024	0.3072	0.0000	0.4586	0.0000	0.5446
HPQ	14.7037	0.0000	0.0399	110.7953	0.0000	0.3583	0.0012	0.9834	0.0000	0.8498	0.0000	0.2329
IBM	11.7421	0.0000	0.0000	137.9708	0.0000	0.2377	0.0024	0.5947	0.0000	0.7052	0.0000	0.0000
JNJ	9.6341	0.0000	0.0000	40.6092	0.0000	0.1435	0.0000	0.3072	0.0000	0.2936	0.0000	0.6413
JPM	10.8244	0.0000	0.0017	206.8594	0.9941	0.6079	0.0024	0.6113	0.0000	0.7770	0.0000	0.1238
KFT	14.9079	0.0000	0.0006	100.5462	0.0000	0.0003	0.0000	0.7812	0.0000	0.8002	0.0000	0.1006
MCD	6.8673	0.0000	0.0000	146.5338	0.0000	0.0088	0.0297	0.4360	0.0000	0.5853	0.0000	0.1434
MET	12.8613	0.0000	0.0003	147.9437	0.6029	0.3764	0.0091	0.5965	0.0000	0.7063	0.0000	0.5361
MS	10.5996	0.0000	0.0000	554.0050	1.0000	0.7507	0.0167	0.9813	0.0000	0.8496	0.0000	0.0001
NEM	19.6853	0.0000	0.0000	126.4511	0.0000	0.0199	0.0167	0.5947	0.0000	0.7052	0.0000	0.0020
PEP	9.6105	0.0000	0.0046	139.3136	0.0000	0.0297	0.0003	0.5965	0.0000	0.7063	0.0000	0.8380
PFE	12.0183	0.0000	0.0001	31.1800	0.0000	0.0060	0.0000	0.8109	0.0000	0.8429	0.0000	0.0298
$\mathbf{PG}$	8.5149	0.0000	0.0004	125.7594	0.0000	0.2665	0.0024	0.9813	0.0000	0.8496	0.0000	0.0696
Т	17.2465	0.0000	0.0000	152.2023	0.0000	0.3278	0.0002	0.0813	0.0000	0.0268	0.0000	0.0030
TWX	16.4170	0.0000	0.0191	140.8583	0.0000	0.3516	0.0000	0.2080	0.0000	0.3401	0.0000	0.6530
TXN	22.9959	0.0000	0.0035	115.7267	0.0000	0.3441	0.1354	0.3072	0.0000	0.4586	0.0000	0.0012
UPS	11.6728	0.0000	0.0000	164.6850	0.0000	0.1117	0.0047	0.1354	0.0000	0.2395	0.0000	0.2345
VZ	12.1461	0.0000	0.0263	134.7510	0.0000	0.2273	0.0003	0.6113	0.0000	0.7770	0.0000	0.6056
WFC	8.0200	0.0000	0.0020	204.3211	0.0000	0.5856	0.0006	0.4360	0.0000	0.5853	0.0000	0.7212
WMT	9.5847	0.0000	0.0000	129.5273	0.0000	0.1325	0.0090	0.7792	0.0000	0.7995	0.0000	0.7262
XOM	13.0649	0.0000	0.0512	188.5670	0.0000	0.4604	0.0297	0.4360	0.0000	0.5853	0.0000	0.5977

Table 5: Probability of tail events. ARBV is the annualized percentage RBV,  $1 = P(ARBV_t^{HAR} > u_1|I^{t-1})$ ,  $2 = P(ARBV_t^{HAR-V-J} > u_1|I^{t-1})$ ,  $3 = P(ARBV_t^{HAR} > u_2|I^{t-1})$ ,  $4 = P(ARBV_t^{HAR-V-J} > u_2|I^{t-1})$ .  $P(ARBV_t^{HAR} > u_1|I^{t-1})$  and  $P(ARBV_t^{HAR-V-J} > u_1|I^{t-1})$  are the estimated conditional probabilities obtained from the HAR and HAR-V-J model, respectively.  $P(ARBV_t^{HAR-V-J} > u_2|I^{t-1})$  is calculated as in (18) and  $P(ARBV_t^{HAR-V-J} > u_2|I^{t-1}) = \left(1 - \Phi\left(\frac{\log u - \mu_t}{\sigma_t}\right)\right)$  with  $\mu_t = \bar{X}_{t-1}$  and  $\sigma_{\epsilon}^2$ .  $u_1 = 0.9 \times ARBV$  and  $u_2 = 0.9 \times ARBV$  represent the threshold values and they are set proportional to the ex-post realization of volatility. Columns 8 and 9 report the *p*-values of the Kupiec (1995) test, while the *p*-values of the Christoffersen (1998) test are included in columns 10 and 11. Columns 12 and 13 report the *p*-values of the Bates (2000) test for goodness-of-fit of the 99-th quantile.

which the conditional probability that the annualized RBV is larger than u in October 10, 2005 is correctly anticipated by the HAR-V-J while the one obtained with the Gaussian HAR is close to zero. A completely different picture is obtained on October 10, 2008, when the average level of volatility is much higher and the ex-ante probability of observing a volatility level higher than u is much higher. In this case, the HAR-V-J is able, in the majority of the stocks considered, to give a conditional probability of an extreme realization much higher than the probability implied by the Gaussian HAR. For the HAR-V-J model, in 3 cases the expected tail probability is below 1% and in 29 cases we have a probability larger than 10%. Differently, the HAR model results include only 10 cases with a probability of tail events higher than 5%, suggesting that the HAR-V-J model is a more appropriate specification.

Finally, to assess the model's capability of predicting tail events, like the ones so far discussed, we report the results of the Kupiec (1995), the Christoffersen (1998) and the Bates (2000) tests. The unconditional coverage test of Kupiec (1995) verifies the statistical significance of the difference between the expected (1%) and the actual number of exceedances. The null hypothesis of the Christoffersen (1998) test is the independence of the observed exceedances. The test of Bates (2000) takes a different point of view and verifies the correct model specification by testing the Uniform distribution of tail probabilities after a monotonic transformation. The tests are computed for both Gaussian HAR (i.e. without jumps) and HAR-V-J models. While for the former the right tail quantile (at 1%) of the conditional distribution of  $X_t$  is known, for the HAR-V-J it has to be calculated by simulation for each t. The results, displayed in Table 5, shows that for the Gaussian HAR, the unconditional coverage test rejects in 30 out of 36, at 1% significance level the null of an accurate interval forecast, since the actual fraction of violations is statistically different from the expected fraction, in our case 1%. Moreover, both the Bates (2000) and Christoffersen (1998) tests reject the null in all cases; this latter finding might be associated with the clustering of exceedances during the financial crisis (after 2007). A completely different evidence emerges from the results for the HAR-V-J. In this case, the Kupiec and Christoffersen tests cannot reject the null for all stocks (at the 1% confidence level), while the Bates test show evidences of some rejections (11 out of 36 cases at the 1% confidence level). These findings can be interpreted as an evidence in favour of the inclusion of the jump component in the HAR model, as it improves the prediction accuracy of extreme events.

As shown by Maheu and McCurdy (2011) the full characterization of the distribution of the returns helps in constructing the forecast of return densities. From this point of view the inclusion of a jump term in the log-volatility specification provides better fit to the ex-post volatility dynamics

and, through this, to the forecast density of the daily returns. A detailed study of this is left for future research.

# 4 Volatility jumps and financial covariates

Understanding the origins of jumps in returns and volatility is a topic of considerable interest to both theorists and market practitioners. In this section we focus on the economic determinants of volatility jumps. To this end, we investigate to what extent the estimated jump components in  $\log RBV_{m,BC,t}^{\Delta}$  are driven by common factors. We aim at identify variables that have predictive power for the future occurrence of jumps for the stocks considered. We consider in our analysis financial and policy variables that are informative on the market expectations of the future economic activity and on the perceived risks of the financial system.

We thus regress the change in the estimated exponential jump components,  $\Delta \hat{J}_{t|t-1}$ , of each stock on a set of lagged financial variables<sup>6</sup>: a proxy of the *VRP* computed as the difference between the VIX and the realized volatility of the S&P 500 index.<sup>7</sup> The *VRP* measures the mismatch between the market expectation of risk and the statistical measure of market risk; the first difference of the logarithm of S&P 500 volume,  $\Delta V$ ; the daily log-return of S&P,  $\Delta S\&P$ ; the log-return of the DJ-UBS Commodity Index,  $\Delta CM$ ; the first difference of the logarithm of the Federal Reserve trade-weighted US dollar index,  $\Delta Ex$ ; the first difference of the excess yield on Moody's seasoned Baa corporate bond over the Moody's seasoned Aaa corporate bond, the credit spread or  $\Delta CS$ ; the change in the difference between the 10-Year and 3-months Treasury constant maturity rates, the term spread or  $\Delta TS$ ; the difference between the effective and the target Federal Funds rates, FF; and the US Banks sector credit default swap index<sup>8</sup>, CDS.<sup>9</sup>

The estimated parameters are shown in Table 6. From the results it is evident that only two variables, namely VRP and CDS, are significant for almost all stocks. Moreover, it should be noted that the signs of the estimated coefficients are the same across the individual stocks. The coefficient of the lagged CDS is significant, at the 1% level, in 20 cases out of 36, and not significant at the

 $<sup>^{6}</sup>$ We follow here the setup in Fernandes et al. (2014).

<sup>&</sup>lt;sup>7</sup>As in Bollerslev et al. (2009b), we define the VRP as the difference between a forward looking market a forward looking market measure of return variation, based on VIX, and the realized return variation, based on RV. As noted by Bollerslev et al. (2009b, p.4475), this definition of VRP has the advantage, over other specifications, to be *model-free*, i.e. both VIX and realized volatility are directly observable at time t. This makes this definition of VRP suitable for the purpose of the present application.

 $<sup>^{8}</sup>$ We do not consider a more general CDS index but prefer focusing on a sectoral index as this is closely related to the financial crisis.

<sup>&</sup>lt;sup>9</sup>The financial variables have been recovered from different sources including Datastream, the Federal Reserve Economic Data website, and the Oxford-Mann Realized Library. The list of regressors include variables in level or in change, depending on the results of an Augmented Dickey-Fuller test.

	const	$VRP_{t-1}$	$\Delta V_{t-1}$	$\Delta \mathrm{CM}_{t-1}$	$\Delta \mathbf{E} \mathbf{x}_{t-1}$	$\Delta S\&P_{t-1}$	$\Delta CS_{t-1}$	$\Delta T S_{t-1}$	$FF_{t-1}$	$CDS_{t-1}$	$R^2$
AAPL	$0.0007^{a}$	$-0.0001^{a}$	0.0005	-0.0072	$0.0282^{c}$	$-0.0166^{a}$	-0.0010	-0.0003	-0.0001	0.0001	0.0137
AXP	$0.0017^{a}$	$-0.0003^{a}$	-0.0001	0.0122	-0.0213	$-0.0236^{c}$	-0.0028	-0.0006	0.0001	$0.0005^{a}$	0.0809
BA	$0.0028^{a}$	$-0.0005^{a}$	0.0008	-0.0086	0.0302	-0.0191	0.0102	-0.0034	-0.0007	$0.0008^{a}$	0.1381
BAC	$0.0014^{a}$	$-0.0002^{a}$	$0.0020^{b}$	0.0028	0.0305	$-0.0456^{a}$	-0.0020	0.0017	0.0005	$0.0005^{b}$	0.0911
С	$0.0043^{a}$	$-0.0007^{a}$	$0.0051^{c}$	0.0168	0.0264	$-0.1159^{a}$	0.0074	0.0036	-0.0005	$0.0015^{b}$	0.1113
CAT	$0.0099^{a}$	$-0.0018^{a}$	$0.0145^{b}$	-0.1287	0.1875	-0.1931	-0.0161	-0.0200	0.0132	$0.0044^{a}$	0.0980
CVX	$0.0049^{a}$	$-0.0009^{a}$	0.0021	-0.0540	0.1560	-0.1278	0.0146	-0.0073	$0.0099^{c}$	$0.0023^{a}$	0.0480
DD	$0.0015^{a}$	$-0.0003^{a}$	0.0010	-0.0083	-0.0094	$-0.0210^{c}$	-0.0070	-0.0022	0.0000	$0.0005^{a}$	0.1225
EMR	$0.0100^{a}$	$-0.0018^{a}$	0.0015	-0.0276	0.1943	$-0.1980^{b}$	0.0029	-0.0137	0.0062	$0.0044^{a}$	0.2456
F	$0.0010^{a}$	$-0.0002^{a}$	-0.0003	-0.0232	0.0202	$-0.0457^{b}$	-0.0006	-0.0037	0.0007	$0.0006^{b}$	0.0354
FCX	$0.0019^{a}$	$-0.0003^{a}$	0.0014	$-0.0414^{a}$	0.0022	-0.0137	-0.0109	-0.0004	0.0014	$0.0008^{a}$	0.0923
FDX	$0.0003^{a}$	$-0.0000^{a}$	-0.0002	-0.0003	0.0042	-0.0034	0.0012	-0.0004	0.0001	0.0001	0.0149
GE	$0.0033^{a}$	$-0.0005^{a}$	$0.0035^{b}$	0.0172	-0.0096	$-0.0719^{a}$	-0.0001	0.0007	0.0002	$0.0011^{a}$	0.1156
GS	$0.0100^{a}$	$-0.0019^{a}$	$0.0150^{b}$	-0.0160	$0.4593^{b}$	$-0.3937^{b}$	-0.0055	0.0217	$0.0273^{b}$	$0.0057^{a}$	0.1628
HD	$0.0066^{a}$	$-0.0012^{a}$	0.0022	-0.0572	0.1017	-0.0758	0.0014	-0.0174	0.0073	$0.0027^{a}$	0.0854
HON	$0.0010^{a}$	$-0.0002^{a}$	-0.0005	$-0.0120^{b}$	$0.0350^{b}$	$-0.0180^{a}$	-0.0045	-0.0006	-0.0005	$0.0003^{a}$	0.0857
HPQ	0.0001	0.0000	0.0000	0.0013	0.0112	-0.0027	0.0005	0.0001	0.0002	-0.0001	0.0007
IBM	$0.0040^{a}$	$-0.0007^{a}$	$0.0037^{b}$	0.0039	0.0547	$-0.0925^{a}$	0.0084	-0.0071	-0.0002	$0.0014^{a}$	0.1858
JNJ	$0.0025^{a}$	$-0.0004^{a}$	0.0008	-0.0144	0.0423	$-0.0429^{b}$	0.0091	-0.0017	-0.0005	$0.0007^{a}$	0.1340
JPM	$0.0019^{a}$	$-0.0003^{a}$	$0.0026^{b}$	-0.0054	$0.0437^{c}$	$-0.0475^{a}$	0.0042	0.0025	0.0008	$0.0005^{b}$	0.1618
KFT	$0.0011^{a}$	$-0.0002^{a}$	-0.0008	-0.0090	-0.0290	-0.0021	0.0038	-0.0002	0.0005	$0.0003^{c}$	0.0385
MCD	$0.0012^{a}$	$-0.0002^{a}$	-0.0002	-0.0131	0.0169	$-0.0244^{b}$	0.0094	0.0002	-0.0003	0.0002	0.0551
MET	$0.0054^{b}$	$-0.0010^{b}$	$0.0233^{a}$	-0.0828	0.1210	-0.0643	-0.0092	-0.0039	0.0072	$0.0030^{c}$	0.0281
MS	$0.0066^{c}$	$-0.0014^{b}$	0.0116	0.0238	$0.7203^{a}$	$-0.4620^{b}$	0.0012	0.0084	$0.0542^{a}$	$0.0066^{b}$	0.0990
NEM	$0.0022^{a}$	$-0.0004^{a}$	$0.0019^{b}$	$-0.0486^{b}$	0.0050	-0.0289	0.0043	0.0042	0.0017	$0.0009^{a}$	0.1329
PEP	$0.0020^{a}$	$-0.0003^{a}$	0.0006	$-0.0174^{c}$	0.0278	$-0.0227^{c}$	0.0071	-0.0006	-0.0001	$0.0006^{a}$	0.1464
PFE	$0.0024^{a}$	$-0.0004^{a}$	0.0003	0.0127	0.0797	-0.0291	0.0095	-0.0030	$0.0044^{a}$	$0.0012^{a}$	0.0452
$\mathbf{PG}$	$0.0012^{a}$	$-0.0002^{a}$	-0.0001	-0.0123	0.0307	$-0.0206^{b}$	0.0038	-0.0024	0.0003	$0.0003^{b}$	0.0816
Т	$0.0018^{a}$	$-0.0003^{a}$	0.0009	0.0140	0.0028	$-0.0477^{a}$	0.0039	$-0.0044^{c}$	-0.0011	$0.0004^{b}$	0.0699
TWX	$0.0014^{a}$	$-0.0002^{a}$	-0.0014	0.0079	0.0345	$-0.0373^{a}$	0.0119	-0.0034	0.0001	$0.0004^{c}$	0.0585
TXN	$0.0014^{b}$	$-0.0002^{a}$	$-0.0033^{c}$	-0.0129	-0.0173	0.0080	0.0184	-0.0013	0.0000	$0.0006^{b}$	0.0666
UPS	$0.0014^{a}$	$-0.0002^{a}$	$0.0013^{a}$	-0.0079	0.0146	-0.0098	0.0035	-0.0011	-0.0004	$0.0004^{b}$	0.0992
VZ	$0.0143^{a}$	$-0.0025^{a}$	0.0096	-0.1724	0.1006	$-0.3224^{b}$	0.0784	-0.0172	0.0156	$0.0061^{a}$	0.0912
WFC	$0.0246^{a}$	$-0.0045^{a}$	$0.0853^{b}$	$-0.5725^{c}$	0.0698	-0.3699	0.0597	-0.0105	$0.0733^{c}$	$0.0133^{a}$	0.0935
WMT	$0.0038^{a}$	-0.0006 <sup>a</sup>	-0.0006	$-0.0482^{c}$	0.0917	$-0.0588^{b}$	0.0049	-0.0020	0.0016	$0.0012^{a}$	0.0878
XOM	$0.0051^{a}$	$-0.0009^{a}$	0.0047	$-0.0966^{c}$	$0.2121^{c}$	$-0.1902^{b}$	-0.0242	-0.0095	0.0084	$0.0023^{a}$	0.0808

Table 6: Regression results. The first difference of the estimated conditional jump for each stock is regressed on the lagged values of the volatility risk premium,  $VRP_{t-1} = VIX_{t-1} - RV_{t-1}$ , the S&P volume change,  $\Delta V_{t-1}$ ; the daily log return of S&P,  $\Delta$ S&P; the log return of the DJ-UBS Commodity Index,  $\Delta CM_{t-1}$ ; the first difference of the logarithm of the foreign exchange value of the US dollar,  $\Delta Ex_{t-1}$ ; the first difference of the credit spread,  $\Delta CS_{t-1}$ ; the first difference of the term spread,  $\Delta TS_{t-1}$ ; the difference between the effective and the target Federal Funds rates,  $FF_{t-1}$ ; the US Banks sector credit default swap index,  $CDS_{t-1}$ . The Table reports the OLS estimates of the regression coefficients. *a*, *b*, and *c* stand for significance at 1%, 5% and 10% respectively. Standard errors are computed with the Newey-West method.

10% level in just four cases. The sign is, as expected, positive, since an increase in the default risk associated with the bank sector provides an increase in the estimated jump component. In turn, this is associated with the increase in unexpected volatility burst that might occur when even single financial institutions news report a deterioration of their credit worthiness. The coefficient of the VRP, is significant in 33 cases at the 1% level, all coefficients have a negative sign. This somewhat surprising result can be interpreted by looking at the evolution of the risk premium in the analysed sample. In fact, during the crises, VRP becomes negative and has a positive impact on the expected jump. When the market (the VIX index) underestimates the volatility level (as measured by the realized variance of the S&P index) this positively impacts on the volatility jump component on the next day. On the contrary, a positive and increasing VRP might negatively impact on the expected jump as it can be associated with a market already pricing (at least in part) the risk of a volatility burst.<sup>10</sup> This evidence suggests that the increase in volatility bursts (driven by the jump intensity) can be due to an underestimation of the risk by the market participants. In fact, this occurs when the forward looking measure of volatility, the VIX, is lower than the actual measure of volatility (i.e. the realized volatility of S&P500 index).

Among the remaining variables, two show some significance across the stocks we consider. The first is the difference between the effective and target FED fund rates, FF, which is significant, up to the 10% level, in 5 cases, with three of them being financial companies. The coefficients are in all instances positive suggesting that a decrease in the effective rate leads to a decrease in the expected jump, as the market view is concordant with the view of the FED. Discordant views increase the probability of volatility bursts. The second relevant variable is the market return, S&P, that is significant, again up to the 10% level, in 21 cases out of 36. For that variable, the coefficients are always negative, suggesting a kind of leverage effect also for the expected jumps. When negative returns are observed, we face an increase in the probability of sudden changes in the volatility.

#### 4.1 Volatility jump factor

Overall, the previous outcomes suggest that there could be a common factor in the estimated jump components which can be predicted on the basis of lagged economic and financial variables. To get further insights into the presence of a common factor in the individual jump components we extract the first principal component,  $PC_1$ , computed from the correlation matrix of the 36 estimated conditional jump series,  $\hat{J}_{t|t-1}$ .  $PC_1$ , explains approximately 60% of the overall variation

 $<sup>^{10}</sup>$ We stress the increase in VRP might be due both to a decrease of the market realized volatility or to an increase in the VIX index. Nevertheless, our interpretation remains valid in both cases.



Figure 7: First principal component of estimated conditional jumps in daily volatilities.

(considering the first three components one arrives at 73%). The weights<sup>11</sup> in  $PC_1$  are all positive.  $PC_1$ , plotted in Figure 7(a), is therefore a good proxy of the latent volatility jump factor. The dynamic pattern of  $PC_1$  closely follows the behavior of the volatility series, and, as illustrated above, a sharp increase occurs in the levels of the expected volatility jumps during the financial crisis period. In order to investigate the determinants of the volatility jump series, we regress the first difference of  $PC_1$ ,  $\Delta PC_1$ , on the same set of variables previously used.

Table 7 reports the result of the regressions. The  $R^2$  is higher than 20%, suggesting that VRP, CDS,  $\Delta$  S& P and FF have predictive power for jumps in volatility. Further, the partial  $r^2$  reported in the last column, are consistent with the findings observed at the single asset level: the most relevant variable is the VRP, that negatively impacts on the first principal component; then, the second relevant variable is the CDS of the banking sector, with a positive impact; finally, the FED fund rates and the market returns have minor, but significant, impact, positive the first, negative the second. Jointly removing the cited variables from the regression drastically reduces the  $R^2$ . However, as the significant covariates are, in most cases, finance-related, the result might be driven by the inclusion in our sample of the financial sector.

To address this issue, we extract the first principal component of the volatility jumps of 28 equities, excluding banks and insurance companies. The first PC still explains about 60% of the total variation and its pattern, shown in Figure 7(b), is similar to that of the entire set of assets, with peaks located during the crisis period. Furthermore, the second panel of Table 7 shows that the coefficients' sign and magnitude are only slightly affected. The coefficient of CDS maintains its significance, even if its magnitude is somewhat smaller. This might be explained by the direct effect that CDS have on the banks volatility. In addition, the coefficient of FF is now not significant, and

<sup>&</sup>lt;sup>11</sup>Not reported to save space but available upon request from the authors.

	$\beta$	s.e.	<i>t</i> -stat	p-value	$r^2$
const	0.0431	0.0065	6.5907	0.0000	0
$VRP_{t-1}$	-0.0074	0.0011	-7.0295	0.0000	0.1446
$\Delta V_{t-1}$	0.0292	0.0187	1.5590	0.1192	0.0038
$\Delta CM_{t-1}$	-0.3459	0.1972	-1.7544	0.0796	0.0020
$\Delta E x_{t-1}$	0.7661	0.4615	1.6600	0.0971	0.0019
$\Delta S \& P_{t-1}$	-0.8476	0.3185	-2.6609	0.0079	0.0164
$\Delta CS_{t-1}$	0.0643	0.1229	0.5230	0.6010	0.0004
$\Delta T S_{t-1}$	-0.0343	0.0492	-0.6957	0.4867	0.0011
$\Delta FF_{t-1}$	0.0410	0.0201	2.0432	0.0412	0.0051
$CDS_{t-1}$	0.0169	0.0037	4.5407	0.0000	0.0256
	(b) Evel	luding the	Financial a		
	(D) Exc.	luang the	r manciai se	ector	
	$\beta$	s.e.	<i>t</i> -stat	<i>p</i> -value	$r^2$
const	$\beta$ 0.0435	s.e. 0.0068	<i>t</i> -stat 6.4244	<i>p</i> -value 0.0000	$r^2$ 0
$const  VRP_{t-1}$	β 0.0435 -0.0074	s.e. 0.0068 0.0011	<i>t</i> -stat 6.4244 -6.8315	<i>p</i> -value 0.0000 0.0000	$r^2$ 0 0.1430
$const  VRP_{t-1}  \Delta V_{t-1}$	$\beta$ 0.0435 -0.0074 0.0133	s.e. 0.0068 0.0011 0.0167	<i>t</i> -stat 6.4244 -6.8315 0.7916	p-value           0.0000           0.0000           0.4287	$r^2$ 0 0.1430 0.0008
$const  VRP_{t-1}  \Delta V_{t-1}  \Delta CM_{t-1}$	$\frac{\beta}{0.0435}$ -0.0074 0.0133 -0.3681	0.0068           0.0011           0.0167           0.1916	<i>t</i> -stat 6.4244 -6.8315 0.7916 -1.9207	p-value           0.0000           0.0000           0.4287           0.0550	
$const VRP_{t-1} \Delta V_{t-1} \Delta CM_{t-1} \Delta Ex_{t-1}$	$\begin{array}{c} \beta\\ \hline 0.0435\\ -0.0074\\ 0.0133\\ -0.3681\\ 0.6464 \end{array}$	0.0068           0.011           0.0167           0.1916           0.4796	$\begin{array}{c} t - \text{stat} \\ \hline \\ 6.4244 \\ -6.8315 \\ 0.7916 \\ -1.9207 \\ 1.3476 \end{array}$	$\begin{array}{c} p \text{-value} \\ \hline p \text{-value} \\ \hline 0.0000 \\ 0.0000 \\ 0.4287 \\ 0.0550 \\ 0.1780 \end{array}$	$r^2$ 0 0.1430 0.0008 0.0023 0.0014
$const VRP_{t-1} \Delta V_{t-1} \Delta CM_{t-1} \Delta Ex_{t-1} \Delta S \& P_{t-1}$	$\begin{array}{c} \beta\\ \hline 0.0435\\ -0.0074\\ 0.0133\\ -0.3681\\ 0.6464\\ -0.7419 \end{array}$	s.e. 0.0068 0.0011 0.0167 0.1916 0.4796 0.3116	<i>t</i> -stat 6.4244 -6.8315 0.7916 -1.9207 1.3476 -2.3811	p-value           0.0000           0.0000           0.4287           0.0550           0.1780           0.0174	$\begin{array}{c} r^2 \\ 0 \\ 0.1430 \\ 0.0008 \\ 0.0023 \\ 0.0014 \\ 0.0126 \end{array}$
$const VRP_{t-1} \Delta V_{t-1} \Delta CM_{t-1} \Delta Ex_{t-1} \Delta S \& P_{t-1} \Delta CS_{t-1}$	$\begin{array}{c} \beta\\ \hline \\ 0.0435\\ -0.0074\\ 0.0133\\ -0.3681\\ 0.6464\\ -0.7419\\ 0.0764 \end{array}$	0.0068           0.0167           0.1916           0.4796           0.3116           0.1350	$\begin{array}{c} t - \text{stat} \\ \hline \\ 6.4244 \\ -6.8315 \\ 0.7916 \\ -1.9207 \\ 1.3476 \\ -2.3811 \\ 0.5657 \end{array}$	$\begin{array}{c} \text{p-value} \\ \hline p \text{-value} \\ \hline 0.0000 \\ 0.0000 \\ 0.4287 \\ 0.0550 \\ 0.1780 \\ 0.0174 \\ 0.5717 \end{array}$	$\begin{array}{c} r^2 \\ 0 \\ 0.1430 \\ 0.0008 \\ 0.0023 \\ 0.0014 \\ 0.0126 \\ 0.0006 \end{array}$
$const VRP_{t-1} \Delta V_{t-1} \Delta CM_{t-1} \Delta Ex_{t-1} \Delta S \& P_{t-1} \Delta CS_{t-1} \Delta TS_{t-1}$	$\begin{array}{c} \beta\\ \hline \\ 0.0435\\ -0.0074\\ 0.0133\\ -0.3681\\ 0.6464\\ -0.7419\\ 0.0764\\ -0.0512 \end{array}$	0.0068           0.011           0.0167           0.1916           0.4796           0.3116           0.1350           0.0501	$\begin{array}{c} t - \text{stat} \\ \hline t - \text{stat} \\ \hline 6.4244 \\ -6.8315 \\ 0.7916 \\ -1.9207 \\ 1.3476 \\ -2.3811 \\ 0.5657 \\ -1.0216 \end{array}$	$\begin{array}{c} p\text{-value} \\ \hline p\text{-value} \\ \hline 0.0000 \\ 0.0000 \\ 0.4287 \\ 0.0550 \\ 0.1780 \\ 0.0174 \\ 0.5717 \\ 0.3072 \end{array}$	$\begin{array}{c} r^2 \\ 0 \\ 0.1430 \\ 0.0008 \\ 0.0023 \\ 0.0014 \\ 0.0126 \\ 0.0006 \\ 0.0024 \end{array}$
$const VRP_{t-1} \Delta V_{t-1} \Delta CM_{t-1} \Delta Ex_{t-1} \Delta S \& P_{t-1} \Delta CS_{t-1} \Delta TS_{t-1} \Delta FF_{t-1}$	$\begin{array}{c} \beta\\ \hline 0.0435\\ -0.0074\\ 0.0133\\ -0.3681\\ 0.6464\\ -0.7419\\ 0.0764\\ -0.0512\\ 0.0244 \end{array}$	0.0068           0.011           0.0167           0.1916           0.4796           0.3116           0.1350           0.0501           0.0188	$\begin{array}{c} t - \text{stat} \\ \hline t - \text{stat} \\ \hline 6.4244 \\ -6.8315 \\ 0.7916 \\ -1.9207 \\ 1.3476 \\ -2.3811 \\ 0.5657 \\ -1.0216 \\ 1.2960 \end{array}$	p-value           0.0000           0.0000           0.4287           0.0550           0.1780           0.0174           0.5717           0.3072           0.1952	$\begin{array}{c} r^2 \\ 0 \\ 0.1430 \\ 0.0008 \\ 0.0023 \\ 0.0014 \\ 0.0126 \\ 0.0006 \\ 0.0024 \\ 0.0018 \end{array}$

(a) All stocks

Table 7: OLS regression: The difference of the first principal component of the excess jump,  $\Delta PC1_t$ , is regressed on the lagged values of volatility risk premium,  $VRP_{t-1} = VIX_{t-1} - RV_{t-1}$ , on the S&P volume change,  $\Delta V_{t-1}$ ; the daily log return of S&P,  $\Delta S\&P_{t-1}$ ; the log return of the DJ-UBS Commodity Index,  $\Delta CM_{t-1}$ ; the first difference of the logarithm of the foreign exchange value of the US dollar,  $\Delta Ex_{t-1}$ ; the first difference of credit spread,  $\Delta CS_{t-1}$ ; the first difference of term spread,  $\Delta TS_{t-1}$ ; the difference between the effective and the target Federal Funds rates,  $FF_{t-1}$ ; the US Banks sector credit default swap index,  $CDS_{t-1}$ . The Newey-West standard errors are reported. The last column reports the partial  $r^2$ . The Financial sector is given by the union of the Bank and Insurance sectors.



Figure 8: Time-varying parameter estimates. The figure displays the estimated time-varying coefficients (black-solid line) of VRP and CDS with the 95% confidence interval (red-dotted line), obtained in a time-varying parameter linear regression where the variables are those in Table 7. The estimation is based on the on-line method by Raftery et al. (2010) and Koop and Korobilis (2012).

this depend on the relevance of that variable mostly for the financial institutions. The partial  $r^2$ , are substantially unchanged with the total value stills around 20%.

Finally, to get some insights into the effect that the financial crisis have on the estimates of the regression of  $\Delta PC_1$  for the entire set of assets, we perform a time-varying parameters estimation following the on-line method proposed by Raftery et al. (2010) and Koop and Korobilis (2012). The results confirm the relevance of the VRP, whose coefficient is statistically significant for the entire sample, see Figure 8, peaking during the subprime crisis. More importantly, the *CDS* turns out to be significant only after 2007. We can thus conclude that the financial crisis amplifies the effect that financial covariates have on the volatility jump factor.<sup>12</sup> These further results confirm our previous intuitions that the change in the volatility jump intensity, and thus the increase in the occurrence of volatility bursts, can be associated with a worsening of the bank sector risk, and with a negative value of the *VRP*.

### 5 Extended HAR-V-J model

Given the predictive ability of VRP and CDS previously documented, we extend the HAR-V-J model including both variables in the jump intensity and in the expected jump size dynamic equations, as follows:

$$\Theta_t = \zeta_0 + \zeta_{CDS} CDS_{t-1} + \zeta_{VRP} VRP_{t-1}, \tag{19}$$

<sup>&</sup>lt;sup>12</sup>The other covariates rolling estimates confirm their irrelevance, confidence intervals always include the zero. Plots are available upon requests.

$$\Lambda_t = \lambda_0 + \lambda_1 \Lambda_{t-1} + \beta_{CDS} CDS_{t-1} + \beta_{VRP} VRP_{t-1} + \psi \xi_{t-1}.$$
(20)

In order to allow for a possible negative impact of VRP on the jump intensity, and given the presence of negative values in the time series of VRP, we remove the positivity constraints on the coefficient  $\beta_{VRP}$ , required to guarantee the positivity of the jump intensity. We estimate the model by imposing a minimum bound at zero to the jump intensity  $\Lambda_t$ . The bound has never been reached thus supporting our choice.<sup>13</sup>

In addition, we can't a priori exclude that CDS and VRP have a linear effect on the log-volatility. Thus we also include both in the conditional mean. In this way the two covariates can have linear and nonlinear effects on the log-volatility. The latter is simply induced by the fact the conditional mean of  $X_t$  depends on  $\Lambda_t \Theta_t$ , as shown in (11). The lagged CDS and VRP in the conditional mean anticipates the evolution of the continuous component of the log-volatility, while the presence in the expected size and intensity contributes to account for unpredictable variations induced by the credit worsening of US banks or by the change in the mismatch between market perception of risk and realized risk. The estimation results for selected coefficients are presented in Table 8.<sup>14</sup> Since the effect of the two covariates on the jump intensity is minimum, with the coefficients equal to zero for almost all stocks, Table 8 reports the estimates for the extended HAR-V-J model under the assumption of  $\beta_{CDS} = \beta_{VRP} = 0$ .

The CDS has a larger impact on the conditional mean of log-volatility compared to VRP. The former impacts on the mean in 30 cases out of 36 (at the 1% level) compared to the 12 cases for the risk premium. Moreover, while the impact of the CDS is in almost all cases positive (only one case shows evidence of a negative impact), the sign turns to be negative for VRP (it is negative in 10 cases). When looking at the jump mean size, results are similar with a somewhat higher relevance for CDS compared to VRP: the CDS impacts positively in 27 cases, negatively in just 4, and it is not significant in 5 cases; the VRP now plays a more relevant role, but its impact is not so clear, being positive in 7 cases, negative in 13, and not significant in 16. The results suggest a potential impact of covariates on the size of jumps, but not on the process driving the jump intensity.

Table 9 reports further tests on the extended model with the purpose of evaluating the benefits coming from the inclusion of covariates. We first consider a likelihood ratio test verifying the null that all coefficients associated with the covariates are jointly equal to zero (second column of Table

<sup>&</sup>lt;sup>13</sup>Imposing the positivity of jump intensity could have been achieved in other ways. However, in facing the trade-off between model flexibility and the appropriateness of the model estimation approach, we gave a preference to the former. We thus made a sub-optimal choice bearing the risk that predicted jump intensity might be negative when the model is used for forecasting purposes.

<sup>&</sup>lt;sup>14</sup>The full table has been omitted for space reasons but can be made available upon request.

	Conditio	nal Mean	Jum	p Size Mea	an and Vari	ance	Ju	mp Intens	ity
	$\phi_{CDS}$	$\phi_{VRP}$	$\zeta_0$	$\zeta_{CDS}$	$\zeta_{VRP}$	$\eta_0$	$\lambda_0$	$\lambda_1$	$\psi$
AAPL	$0.0179^{a}$	-0.0049	$0.3422^{a}$	$0.0332^{a}$	$0.0205^{b}$	$0.2865^{a}$	$0.0021^{b}$	$0.9866^{a}$	$0.0203^{a}$
AXP	$0.1851^{a}$	0.0012	$0.2452^{a}$	-0.0341	-0.0070	$0.0974^{a}$	0.0002	$0.9990^{a}$	$0.0531^{b}$
BA	$0.0627^{a}$	-0.0048	$0.3052^{a}$	$0.0569^{a}$	-0.0083	$0.2477^{a}$	0.0002	$0.9981^{a}$	$0.0474^{a}$
BAC	$0.1807^{a}$	-0.0056	0.0019	$0.1075^{a}$	$0.0072^{b}$	$0.1039^{a}$	$0.0487^{a}$	$0.9406^{a}$	$0.2636^{a}$
С	$0.1253^{a}$	$-0.0113^{a}$	$0.0423^{a}$	$0.0806^{a}$	$0.0052^{b}$	$0.0378^{a}$	$0.3050^{a}$	$0.8493^{a}$	$0.4926^{a}$
CAT	$0.0250^{a}$	0.0020	$0.1460^{a}$	$0.1269^{a}$	$-0.0139^{a}$	$0.1393^{a}$	$0.0431^{a}$	$0.9018^{a}$	$0.2366^{a}$
CVX	$0.0197^{a}$	$0.0027^{c}$	$0.1829^{a}$	$0.0930^{a}$	$-0.0217^{a}$	$0.1732^{a}$	$0.0401^{a}$	$0.8312^{a}$	$0.4244^{a}$
DD	$0.0082^{a}$	$0.0075^{a}$	$0.0928^{a}$	$0.1055^{a}$	$-0.0121^{a}$	$0.0759^{a}$	0.0035	$0.9952^{a}$	$0.2045^{a}$
EMR	$0.0245^{a}$	0.0002	$0.0247^{a}$	$0.1573^{a}$	-0.0014	$0.0613^{a}$	$0.0801^{a}$	$0.9217^{a}$	$0.4150^{a}$
F	0.0088	$-0.0091^{b}$	$0.1656^{a}$	$0.0840^{a}$	-0.0072	$0.4797^{a}$	0.0001	$0.9989^{a}$	$0.0437^{a}$
FCX	$0.0783^{a}$	$-0.0082^{a}$	$0.1045^{a}$	$-0.0373^{a}$	$0.0132^{b}$	$0.2041^{a}$	0.0019	$0.9902^{a}$	$0.0495^{a}$
FDX	$0.0790^{a}$	$-0.0047^{b}$	$1.1797^{a}$	-0.0122	-0.0077	0.0000	0.0002	$0.9951^{a}$	0.0042
GE	$0.0724^{a}$	0.0001	$0.0603^{a}$	$0.0441^{a}$	-0.0005	$0.0560^{a}$	$0.1936^{a}$	$0.9260^{a}$	$0.2894^{a}$
GS	$0.1709^{a}$	0.0018	$0.0845^{c}$	$0.1373^{a}$	0.0019	0.0210	0.0089	$0.9920^{a}$	$0.2745^{a}$
HD	$0.1262^{a}$	$-0.0093^{a}$	$0.1253^{a}$	$0.0657^{a}$	$0.0046^{c}$	$0.0860^{a}$	$0.0624^{a}$	$0.8926^{a}$	$0.3473^{a}$
HON	$0.0421^{a}$	-0.0033	$0.2654^{a}$	$0.0960^{a}$	$-0.0160^{a}$	$0.1963^{a}$	0.0005	$0.9973^{a}$	$0.0158^{c}$
HPQ	0.0752	-0.0074	$0.1035^{a}$	-0.0150	0.0072	$0.1532^{a}$	$0.0766^{a}$	$0.8665^{a}$	0.0000
IBM	$-0.0206^{b}$	$0.0063^{b}$	$0.0271^{b}$	$0.0786^{a}$	$-0.0043^{a}$	$0.0568^{a}$	$0.0134^{a}$	$0.9931^{a}$	$0.1680^{a}$
JNJ	$0.0385^{a}$	-0.0004	$0.2947^{a}$	$0.0884^{a}$	$-0.0148^{a}$	$0.1514^{a}$	0.0009	$0.9963^{a}$	$0.0549^{a}$
JPM	$0.1863^{a}$	0.0018	0.0602	0.0820	-0.0005	0.0451	0.0350	$0.9761^{a}$	$0.2944^{a}$
KFT	0.0234	-0.0102	$0.5803^{a}$	$0.1373^{a}$	-0.0105	$0.1733^{a}$	$0.0174^{a}$	$0.8700^{a}$	0.0000
MCD	0.0607	-0.0099	$0.7348^{a}$	$0.2339^{a}$	-0.0451	$0.6073^{a}$	0.0003	$0.9942^{a}$	$0.0182^{b}$
MET	$0.0864^{a}$	-0.0056	$0.5658^{a}$	$0.0453^{c}$	-0.0014	$0.2993^{a}$	0.0076	$0.8595^{a}$	0.0000
MS	$0.1323^{a}$	0.0023	$0.2861^{a}$	$0.0830^{a}$	-0.0079	$0.1945^{a}$	0.0054	$0.9749^{a}$	$0.1800^{a}$
NEM	$0.0723^{a}$	$-0.0111^{a}$	$1.0517^{a}$	$-0.5423^{a}$	$0.0196^{a}$	$0.0523^{a}$	0.0000	$0.9975^{a}$	$0.0226^{a}$
PEP	$0.0569^{a}$	$-0.0053^{a}$	$0.3408^{a}$	$0.1291^{a}$	$-0.0072^{a}$	$0.1090^{a}$	$0.1414^{a}$	$0.4681^{a}$	$0.2079^{a}$
PFE	$0.0993^{a}$	-0.0025	$0.8874^{a}$	$-0.0564^{a}$	$-0.0382^{a}$	$0.4737^{a}$	$0.0035^{a}$	$0.9748^{a}$	0.0071
$\mathbf{PG}$	$0.0339^{a}$	0.0038	$0.0605^{a}$	$0.1608^{a}$	$-0.0090^{a}$	$0.1408^{a}$	0.0101	$0.9811^{a}$	$0.1669^{a}$
Т	-0.0376	-0.0032	0.0216	$0.1249^{a}$	-0.0115	$0.1259^{a}$	0.0014	$0.9970^{a}$	$0.1602^{a}$
TWX	0.0148	-0.0014	$0.2563^{a}$	$0.0985^{a}$	$-0.0183^{a}$	$0.2183^{a}$	0.0018	$0.9922^{a}$	$0.0961^{a}$
TXN	$0.0929^{a}$	$-0.0082^{a}$	$0.1607^{a}$	$-0.0812^{a}$	$0.0103^{a}$	$0.4226^{a}$	$0.0011^{c}$	$0.9856^{a}$	$0.1492^{a}$
UPS	$0.0719^{a}$	-0.0021	$0.7245^{a}$	$0.1689^{a}$	$-0.0074^{a}$	$0.3757^{a}$	0.0001	$0.9970^{a}$	$0.0132^{a}$
VZ	$0.0362^{a}$	$-0.0053^{a}$	$0.1924^{a}$	$0.1038^{a}$	$-0.0122^{a}$	$0.1284^{a}$	$0.0603^{a}$	$0.9102^{a}$	$0.2372^{a}$
WFC	$0.1567^{a}$	-0.0025	$0.0517^{b}$	$0.0932^{a}$	0.0011	$0.0345^{a}$	$0.1130^{a}$	$0.9403^{a}$	$0.4322^{a}$
WMT	$0.0683^{a}$	0.0034	$0.3331^{a}$	$0.0226^{a}$	$-0.0194^{a}$	$0.2277^{a}$	$0.0029^{a}$	$0.9898^{a}$	$0.1214^{a}$
XOM	$0.0523^{a}$	$-0.0066^{a}$	$0.0569^{a}$	$0.0469^{a}$	$0.0068^{a}$	$0.0877^{a}$	$0.0570^{a}$	$0.8995^{a}$	$0.4302^{a}$

Table 8: Estimated parameters of the model with jump intensity and expected size specified as in (19) and (20).  $\phi_{CDS}$  denotes the coefficient of  $CDS_{t-1}$  in the conditional mean. Similarly,  $\phi_{VRP}$  denotes the coefficient of  $VRP_{t-1}$  in the conditional mean. a, b, and c stand for significance at 1%, 5% and 10% respectively.

9). We reject the null in 31 cases at the 1% confidence level. However, such a rejection can be due either to coefficients in the mean or in the jump dynamics (size and intensity). Therefore, we also consider as models under the alternative hypothesis restricted versions of the most general one that includes CDS and VRP in conditional mean and jump parameters. We first test the null hypothesis that the covariates have no impact on the jump intensity ( $\beta_{CDS} = \beta_{VRP} = 0$ ), column 3. Secondly, we contrast a model with covariates in the jump size and intensity against a model without covariates (a standard HAR-V-J), column 4. Finally, we evaluate the inclusion of covariate only in the conditional mean dynamic ( $\phi_{CDS} = \phi_{VRP} = 0$ ). The overall picture does not change: the covariates are not relevant for the jump intensity, while their role is much more evident for both the mean volatility and the jump size. The last claim is further confirmed if we compare the HAR-V-J model with covariates in the mean with the HAR-V-J model with covariates in the jump dynamics. Being the two models non-nested, we resort to a Diebold-Mariano testing framework. We thus compare the in-sample performances of the two models in fitting the log-RBV dynamic adopting a mean squared error (MSE) loss function. Negative values represent over-performance of the HAR-V-J with explanatory variables in conditional mean. The tests suggest that the two specifications are substantially equivalent, without thus giving a predominance to the inclusion of covariates in the mean volatility dynamic or in the jump dynamic.

#### 5.1 Structural break analysis

The analysis presented so far does not consider the possibility of a structural break associated with the beginning of the financial crisis. However, this cannot be excluded a priori. In fact, in subsection 4.1, the analysis suggests the presence of a structural break in the coefficients of *CDS*. We reformulate the HAR-V-J model, allowing for a structural break in all the covariates coefficients, as follows

$$X_{t} = \bar{X}_{t-1} + Z_{t} + \phi_{CDS,1}(1 - D_{t})CDS_{t-1} + \phi_{CDS,2}D_{t}CDS_{t-1} + \phi_{VRP,1}(1 - D_{t})VRP_{t-1} + \phi_{VRP,2}D_{t}VRP_{t-1} + \epsilon_{t}$$
(21)  

$$\Theta_{t} = \zeta_{0} + \zeta_{CDS,1}(1 - D)CDS_{t-1} + \zeta_{CDS,2}D_{t}CDS_{t-1} + \zeta_{VRP,1}(1 - D_{t})VRP_{t-1} + \zeta_{VRP,2}D_{t}VRP_{t-1}$$
(22)

$$\Lambda_t = \lambda_0 + \lambda_1 \Lambda_{t-1} + \psi \xi_{t-1} \tag{23}$$

where  $D_t = 1$  when t > 750, corresponding to January 3, 2007, and 0 elsewhere. Note that we exclude the covariates from the jump intensity equation as the previous estimates suggest their

		Constant	Parameter	r Model			Structu	ıral Break N	Iodel	
	$LR(6)_0^E$	$LR(2)_{\beta=0}^E$	$LR(4)_0^J$	$LR(2)_0^M$	$DM_J^M$	$LR(8)_0^{bE}$	$LR(4)^{bE}_{\beta=0}$	$LR(2)^{bE}_{bJ}$	$LR(2)^{bE}_{bM}$	$DM_{bM}^{bJ}$
AAPL	0.7767	0.3680	0.9257	0.8508	0.4493	0.0420	0.0052	0.0022	0.9820	6.1108
AXP	0.0000	0.0673	0.0080	0.0000	-0.5979	0.0000	0.0606	0.0125	0.0181	3.1441
BA	0.0004	0.0808	0.1587	0.0001	-0.8459	0.0000	0.0001	0.0692	0.0001	-0.8902
BAC	0.0000	0.1543	0.0011	0.0000	-0.0473	0.0000	0.0012	0.8387	0.0008	0.3635
С	0.0000	0.2540	0.0000	0.0000	1.2703	0.0000	0.0061	0.4693	0.0098	-0.6121
CAT	0.0004	0.2896	0.0002	0.0015	-0.1600	0.0002	0.1063	0.1793	0.0197	1.4988
CVX	0.0063	0.9049	0.0071	0.0023	-0.7208	0.0002	0.0135	0.9442	0.0153	0.8673
DD	0.0002	0.1095	0.0021	0.0003	-0.6262	0.0000	0.0081	0.0159	0.0011	3.7768
EMR	0.0000	0.0147	0.0000	0.0035	1.1358	0.0000	0.0024	0.0028	0.0308	3.1248
F	0.0041	0.0194	0.0020	0.0023	0.6431	0.0002	0.0008	0.0000	0.0002	2.1806
FCX	0.0071	0.6750	0.6776	0.0138	-1.2573	0.0037	0.2068	0.8692	0.3058	0.4970
FDX	0.0004	0.5471	0.0484	0.0000	-1.7543	0.0010	0.6043	0.8690	0.2675	3.4024
GE	0.0000	0.0544	0.0000	0.0000	0.6006	0.0000	0.0000	0.0000	0.0000	-7.4576
GS	0.0000	0.9048	0.0018	0.0000	-0.1418	0.0000	0.0024	0.2604	0.4674	-3.1338
HD	0.0001	0.3679	0.0017	0.0000	0.0099	0.0000	0.0248	0.5117	0.0185	0.9235
HON	0.0101	0.4348	0.0015	0.0036	-0.2057	0.0000	0.0006	0.0031	0.0003	5.9157
HPQ	0.0053	0.1772	0.0665	0.0006	-0.5072	0.0062	0.1693	0.3608	0.8918	0.5600
IBM	0.0000	0.9477	0.0000	0.0179	0.5613	0.0000	0.0007	0.0063	0.0002	-4.8731
JNJ	0.0006	0.5924	0.0004	0.0258	-0.2347	0.0000	0.0026	0.0180	0.0137	-7.0145
JPM	0.0000	0.7783	0.0000	0.0000	0.1221	0.0000	0.0013	0.0500	0.0005	-4.6967
KFT	0.0019	1.0000	0.0310	0.0003	-0.6924	0.0000	0.0000	0.0002	0.0014	4.8857
MCD	0.0001	0.9783	0.0082	0.0000	-0.3524	0.0000	0.0002	0.0018	0.0117	4.0945
MET	0.0005	0.1079	0.0087	0.0001	-0.7784	0.0001	0.0109	0.0076	0.0135	4.4704
MS	0.0000	0.2212	0.0088	0.0000	-0.1682	0.0000	0.1017	0.0361	0.6952	1.9340
NEM	0.0009	0.7823	0.1493	0.0008	-1.1983	0.0000	0.0015	0.3502	0.0002	0.9102
PEP	0.0001	0.5515	0.0006	0.0001	0.4998	0.0000	0.0021	0.0300	0.0006	1.5866
PFE	0.0000	0.3272	0.0002	0.0000	-1.3600	0.0000	0.3638	0.4604	0.2545	-7.7316
$\mathbf{PG}$	0.0000	0.9150	0.0006	0.0012	-0.4952	0.0000	0.1039	0.5346	0.0623	-4.8856
Т	0.0000	0.3806	0.0000	0.0000	-0.6432	0.0000	0.0502	0.0000	0.1540	4.8602
TWX	0.0001	0.0133	0.0000	0.0060	-0.5696	0.0001	0.0229	0.0090	0.0229	2.5300
TXN	0.0006	0.3071	0.0002	0.0001	-0.8320	0.0008	0.2504	0.8976	0.0451	-0.2880
UPS	0.0030	0.7918	0.2277	0.0002	-1.9748	0.0008	0.1195	0.8993	0.0454	-0.8010
VZ	0.0000	0.5043	0.0000	0.0000	-0.6122	0.0000	0.0012	0.0408	0.0132	1.7701
WFC	0.0000	0.3062	0.0000	0.0000	1.7818	0.0000	0.0143	0.5869	0.5123	2.6738
WMT	0.0001	0.6673	0.0165	0.0001	-0.6298	0.0000	0.0055	0.0024	0.9092	4.4961
XOM	0.0011	0.9990	0.0105	0.0006	-1.0185	0.0000	0.0115	0.1243	0.4205	4.3562

Table 9: Model Evaluations. The table reports the *p*-values of several LR statistics for the HAR-V-J model with explanatory variables and breaks. In column 2,  $(LR(6)_{E}^{O})$  is relative to the null  $\phi_{CDS} = \phi_{VRP} = \zeta_{CDS} = \zeta_{VRP} = \beta_{CDS} = \beta_{VRP} = 0$  in the HAR-V-J with *CDS* and *VRP* in conditional mean, jump size and jump intensity (E) (see (21)-(23)), 6 represents the number of tested restrictions/degrees of freedom. In column 3, the null is  $\beta_{CDS} = \beta_{VRP} = 0$ ,  $LR(2)_{\beta=0}^{E}$ . In column 4, the null is  $\zeta_{CDS} = \zeta_{VRP} = \beta_{CDS} = \beta_{VRP} = 0$  in the HAR-V-J with *CDS* and *VRP* only in jump size and jump intensity (J). In column 5, the null is  $\phi_{CDS} = \phi_{VRP} = 0$  in the HAR-V-J with *CDS* and *VRP* in conditional mean only (M). In column 7 the null is  $\phi_{CDS,1} = \phi_{CDS,2} = \phi_{VRP,1} = \phi_{VRP,2} = \zeta_{CDS,1} = \zeta_{CDS,2} = \zeta_{VRP,1} = \zeta_{VRP,2} = 0$  in the HAR-V-J model in (21)-(23) (*bE*). In column 8 the model under the null is the HAR-V-J with jump intensity specified as in (23) (i.e.  $\beta = 0$ ) and with no breaks in the covariates coefficients versus the model with breaks (*bE*). In column 9 the model under the null is the HAR-V-J with breaks in  $\Theta_t$  only (*bJ*) versus the model *bE*. In column 10 the model under the null has breaks in conditional mean only (*bM*) versus *bE*. Column 6 reports the in-sample Diebold-Mariano test of the HAR-V-J with explanatory variables in conditional mean against the HAR-V-J with breaks only in jumps (*bJ*) versus breaks only in the conditional mean (*bM*). Positive values represent under-performance of the latter.

irrelevance.<sup>15</sup>

Table 10 reports the estimated coefficients. The results substantially confirm our previous findings. Looking at the estimated coefficients in the conditional mean of  $X_t$ , the impact of VRP is more evident and pronounced in the second subsample with 17 significant coefficients (at the 1% level), 15 of them being negative, compared to 10 significant coefficients in the first subsample, with mixed sign (4 negative, 6 positive). This is supporting our preliminary claim, associated with the sign of the VRP. Similarly to VRP, the impact of the CDS is more heterogeneous in the first subsample the coefficients are, as expected, always positive (in all the 33 cases where we have a significant estimate at the 1% level).

When considering the covariates impact on the jump size, we have a larger heterogeneity when looking at VRP, in particular in the first subsample, where 13 coefficients out of the 23 statistically significant are positive. Differently, in the crisis period, VRP is less relevant, with just 18 significant estimates, but 12 of them are negative, in accordance to the sign of the covariate (negative), and thus leading to an increase in the jump intensity. The CDS seems more relevant in both periods. In the crisis period, the CDS has a positive impact in 22 cases and a negative impact in 9 (all significant at the 1% level), consistently with what one might expect. On the contrary, in the first period, the CDS has a negative impact in 28 cases and a positive one in just 7 cases. The empirical evidences thus suggest that in the pre-crisis period an increase in the CDS reduces the size of jumps but increases the volatility level. It should be noted that, in the first sample, the CDS has a positive impact on both the jump size as well as the mean volatility.

To get further insight on these results we perform additional diagnostic checks. In Table 9 (column 7) we report the likelihood ratio test statistics for the model with structural breaks in the covariate coefficients. A general test for the null of no impact of covariates is always rejected (with the exception of AAPL), as expected. Moreover, when testing the equality of covariate coefficients across the two subsamples (i.e. no break), null is rejected in 25 cases out of 36 at the 5% confidence level (see column 8 of Table 9). We thus test restrictions in the conditional mean dynamic only, the null hypothesis is that  $\phi_{CDS,1} = \phi_{CDS,2}$  and  $\phi_{VRP,1} = \phi_{VRP,2}$ . The null is rejected in 18 out of 36 cases at 5%, confirming the presence of a structural break in the conditional mean for half of the stocks (see column 9 of Table 9). Similarly, we test for the absence of a break in the mean

<sup>&</sup>lt;sup>15</sup>We have also tried to include them, allowing for a break in their coefficients. However, they were not-significant in both subsamples.

		Condition	nal Mean			Jum	p Size Mea	n and Var	iance		Ju	Jump Intensity		
	$\phi_{CDS,1}$	$\phi_{VRP,1}$	$\phi_{CDS,2}$	$\phi_{VRP,2}$	$\zeta_0$	$\zeta_{CDS,1}$	$\zeta_{VRP,1}$	$\zeta_{CDS,2}$	$\zeta_{VRP,2}$	$\eta_0$	$\lambda_0$	$\lambda_1$	$\psi$	
AAPL	$0.1533^{a}$	$0.0087^{a}$	$0.0624^{a}$	$-0.0059^{a}$	$0.3740^{a}$	$-0.1416^{a}$	$0.0202^{a}$	$0.0148^{a}$	$0.0212^{a}$	$0.2818^{a}$	0.0021	$0.9866^{a}$	$0.0349^{a}$	
AXP	$-0.0839^{a}$	$0.0041^{c}$	$0.1746^{a}$	-0.0002	$0.2333^{a}$	$0.0708^{a}$	$-0.0088^{a}$	$-0.0274^{a}$	-0.0068	$0.1120^{a}$	0.0002	$0.9990^{a}$	$0.0496^{a}$	
BA	$-0.0710^{a}$	0.0076	$0.0925^{a}$	$-0.0065^{b}$	$0.9388^{a}$	$-1.3666^{a}$	$-0.0561^{a}$	$0.0776^{a}$	0.0046	0.0000	0.0001	$0.9984^{a}$	0.0000	
BAC	$0.2430^{a}$	0.0011	$0.1932^{a}$	-0.0004	$0.9777^{a}$	$-6.9558^{a}$	$0.1358^{a}$	$-0.0858^{a}$	$0.0168^{a}$	$0.0059^{b}$	$0.0316^{a}$	$0.6333^{a}$	0.0000	
С	$0.3504^{a}$	-0.0036	$0.1381^{a}$	$-0.0108^{a}$	$0.1362^{a}$	$-0.3603^{a}$	$0.0017^{b}$	$0.0468^{a}$	$0.0060^{a}$	$0.0354^{b}$	$0.6726^{a}$	$0.6011^{a}$	$0.5373^{a}$	
CAT	$0.2147^{a}$	0.0078	$0.0438^{a}$	0.0014	$0.2491^{a}$	$-0.9547^{a}$	0.0004	$0.0808^{a}$	$-0.0146^{a}$	$0.1196^{a}$	$0.0603^{a}$	$0.8841^{a}$	$0.2003^{a}$	
CVX	$-0.0399^{a}$	0.0009	$0.0306^{a}$	0.0002	$0.5497^{a}$	$-2.5750^{a}$	0.0057	$0.0173^{b}$	$-0.0342^{a}$	$0.1646^{a}$	$0.1136^{a}$	$0.2969^{a}$	$0.2771^{a}$	
DD	$0.3674^{a}$	$-0.0041^{c}$	$0.0130^{a}$	$0.0088^{a}$	$0.1003^{a}$	$-0.5667^{a}$	$0.0148^{a}$	$0.1086^{a}$	$-0.0129^{a}$	$0.0749^{a}$	$0.0035^{a}$	$0.9950^{a}$	$0.2081^{a}$	
EMR	$1.0869^{a}$	$-0.0127^{b}$	$0.0665^{a}$	0.0000	$0.1596^{a}$	$-1.2083^{a}$	$0.0210^{a}$	$0.1064^{a}$	-0.0017	$0.0515^{a}$	$0.1050^{a}$	$0.8844^{a}$	$0.3516^{a}$	
F	$-0.4568^{a}$	$-0.0044^{b}$	0.0046	-0.0024	0.0000	$3.8208^{a}$	$-0.0733^{a}$	$0.0964^{a}$	$-0.0096^{c}$	$0.3145^{a}$	0.0001	$0.9990^{a}$	$0.0662^{a}$	
FCX	$-0.0556^{a}$	$-0.0080^{c}$	$0.0745^{a}$	$-0.0081^{a}$	$0.1109^{a}$	$-1.6505^{a}$	$0.0659^{a}$	$-0.0215^{a}$	$0.0100^{b}$	$0.1873^{a}$	0.0030	$0.9860^{a}$	$0.0695^{a}$	
FDX	-0.0025	$-0.0038^{a}$	$0.0787^{a}$	$-0.0048^{a}$	$1.3575^{a}$	$0.0306^{a}$	$-0.0693^{a}$	$-0.0727^{a}$	$-0.0097^{c}$	0.0000	0.0002	$0.9936^{a}$	0.0000	
GE	$0.0981^{a}$	0.0107	$0.1053^{a}$	-0.0028	0.0940	-0.2333	0.0044	$0.0190^{b}$	0.0013	$0.0570^{a}$	$0.5830^{a}$	$0.7712^{a}$	$0.3967^{a}$	
$\operatorname{GS}$	$-0.2537^{a}$	$0.0159^{a}$	$0.1563^{a}$	-0.0019	$0.1921^{a}$	$-0.8042^{a}$	$0.0171^{c}$	$0.1071^{a}$	0.0030	$0.0232^{a}$	$0.0106^{a}$	$0.9818^{a}$	$0.1921^{a}$	
HD	$0.0850^{a}$	$-0.0103^{b}$	$0.1100^{a}$	$-0.0055^{a}$	$0.7280^{a}$	$-3.1663^{a}$	$0.0956^{a}$	$0.0085^{a}$	$-0.0109^{a}$	$0.0337^{a}$	$0.0139^{c}$	$0.8912^{a}$	$0.0794^{a}$	
HON	$0.4953^{a}$	-0.0010	$0.0888^{a}$	$-0.0050^{a}$	$0.7432^{a}$	$-2.4900^{a}$	$0.0101^{c}$	$-0.0200^{b}$	-0.0098	$0.0514^{a}$	0.0007	$0.9964^{a}$	0.0000	
HPQ	$-0.0847^{a}$	$0.0052^{b}$	$0.1110^{a}$	$-0.0114^{a}$	$0.1119^{a}$	$-0.1885^{a}$	0.0111	$-0.0440^{a}$	$0.0107^{a}$	$0.1503^{a}$	$0.2993^{a}$	$0.4934^{a}$	0.0000	
IBM	$1.4756^{a}$	$-0.0383^{a}$	$-0.0048^{c}$	$0.0077^{a}$	$0.0618^{a}$	$-0.9265^{a}$	$0.0276^{a}$	$0.0646^{a}$	$-0.0051^{a}$	$0.0539^{a}$	$0.0162^{a}$	$0.9925^{a}$	$0.1264^{a}$	
JNJ	$1.0683^{a}$	$-0.0061^{c}$	$0.0535^{b}$	0.0073	$0.2177^{a}$	$-1.0880^{a}$	0.0301	$0.0318^{a}$	$-0.0091^{a}$	$0.0530^{a}$	0.0005	$0.9990^{a}$	$0.0752^{a}$	
JPM	$0.9191^{a}$	$-0.0190^{a}$	$0.1883^{a}$	0.0022	$0.0730^{a}$	$-1.2697^{a}$	$0.0453^{a}$	$0.0976^{a}$	-0.0024	$0.0458^{a}$	$0.0191^{a}$	$0.9841^{a}$	$0.2568^{a}$	
KFT	$0.5509^{a}$	-0.0015	$0.0841^{a}$	$-0.0107^{a}$	$1.1552^{a}$	$-3.2988^{a}$	-0.0030	$-0.0358^{a}$	$-0.0166^{a}$	$0.0599^{a}$	$0.0846^{a}$	$0.4239^{a}$	0.0000	
MCD	$-0.1877^{a}$	$0.0123^{a}$	$0.0926^{a}$	$-0.0117^{a}$	$1.5885^{a}$	$-1.4982^{a}$	$-0.1446^{a}$	$0.0779^{a}$	$-0.0493^{a}$	$0.1100^{a}$	$0.0434^{a}$	$0.0214^{a}$	0.0182	
MET	$0.2279^{a}$	$0.0059^{a}$	$0.1135^{a}$	$-0.0064^{a}$	$0.6484^{a}$	$-1.9484^{a}$	-0.0054	$-0.0721^{a}$	$-0.0035^{a}$	$0.2076^{a}$	$0.0111^{a}$	$0.9193^{a}$	0.0000	
MS	$-0.4524^{a}$	$0.0147^{a}$	$0.1274^{a}$	0.0001	$0.2896^{a}$	$1.8178^{a}$	$-0.0661^{a}$	$0.0843^{a}$	-0.0060	$0.2504^{a}$	0.0025	$0.9832^{a}$	$0.1278^{a}$	
NEM	$0.2985^{a}$	$-0.0116^{b}$	$0.0946^{a}$	$-0.0115^{a}$	$0.8508^{a}$	$-3.1712^{a}$	$-0.0192^{a}$	$-0.4547^{a}$	$0.0203^{a}$	0.0000	0.0002	$0.9982^{a}$	0.0000	
PEP	$0.8871^{a}$	0.0018	$0.1039^{a}$	-0.0021	$0.4463^{a}$	$-1.1102^{a}$	-0.0084	-0.0065	-0.0080	$0.0430^{a}$	$0.4307^{a}$	$0.4709^{a}$	$0.2051^{a}$	
PFE	$-0.2005^{a}$	$0.0057^{c}$	$0.0949^{a}$	-0.0031	$0.6814^{a}$	$2.9508^{a}$	$-0.0810^{a}$	$0.0113^{a}$	$-0.0334^{a}$	$0.4633^{a}$	0.0020	$0.9836^{a}$	$0.0157^{a}$	
$\mathbf{PG}$	$0.3096^{a}$	0.0008	$0.0466^{a}$	0.0036	$0.1104^{a}$	$-1.0082^{a}$	$0.0207^{a}$	$0.1445^{a}$	-0.0100	$0.1261^{a}$	$0.0130^{a}$	$0.9787^{a}$	$0.1731^{a}$	
Т	$0.9859^{a}$	-0.0026	$0.0607^{a}$	$-0.0037^{a}$	$0.1769^{a}$	$-0.5343^{a}$	$-0.0114^{c}$	$0.1049^{a}$	$-0.0120^{a}$	$0.1310^{a}$	0.0022	$0.9959^{a}$	$0.1314^{a}$	
TWX	$0.4440^{a}$	$-0.0098^{a}$	$0.0188^{a}$	$0.0023^{b}$	$0.1845^{a}$	$-0.8068^{a}$	$0.0213^{a}$	$0.1053^{a}$	$-0.0163^{a}$	$0.1577^{a}$	0.0087	$0.9816^{a}$	$0.1913^{a}$	
TXN	$0.0150^{a}$	-0.0071	$0.0922^{a}$	$-0.0087^{a}$	0.0000	$8.8607^{a}$	$-0.2940^{a}$	$-0.0298^{a}$	$0.0105^{b}$	$0.3129^{a}$	0.0011	$0.9909^{a}$	$0.1904^{a}$	
UPS	$-0.0202^{a}$	0.0000	$0.0736^{a}$	-0.0025	$0.9636^{a}$	$-5.7507^{a}$	$0.0981^{a}$	$0.1635^{a}$	-0.0057	$0.1125^{a}$	0.0002	$0.9957^{a}$	0.0000	
VZ	$0.5172^{a}$	-0.0044	$0.0563^{c}$	$-0.0052^{a}$	$0.3923^{a}$	$-1.3138^{a}$	0.0052	$0.0486^{a}$	$-0.0164^{c}$	$0.1081^{a}$	$0.2380^{a}$	$0.6074^{a}$	$0.3075^{a}$	
WFC	$0.1662^{a}$	-0.0140	$0.1498^{a}$	$-0.0034^{b}$	$0.0702^{a}$	$-0.3057^{a}$	$0.0113^{a}$	$0.0721^{a}$	0.0010	$0.0306^{a}$	$0.2452^{a}$	$0.8791^{a}$	$0.4750^{a}$	
WMT	$-0.5291^{a}$	$0.0282^{a}$	$0.0889^{a}$	-0.0005	$0.3301^{a}$	$0.3074^{a}$	$-0.0317^{a}$	0.0070	$-0.0145^{b}$	$0.2334^{a}$	0.0032	$0.9885^{a}$	$0.1224^{a}$	
XOM	$-0.4568^{a}$	$0.0062^{c}$	$0.0548^{a}$	$-0.0087^{a}$	$0.0767^{a}$	$-0.3253^{a}$	$0.0190^{a}$	$0.0427^{a}$	$0.0083^{a}$	$0.0927^{a}$	$0.0541^{a}$	$0.8830^{a}$	$0.4102^{a}$	

Table 10: Estimated parameters of the model with jump intensity and expected size specified as in (21) and (22). The subscripts identity the two subsamples for the CDS and VRP coefficients. a, b, and c stand for significance at 1%, 5% and 10% respectively.



Figure 9: Ex-ante (black solid line) and ex-post (red dashed line) probability of jumps on October 10 2008, implied by the HAR-V-J model with CDS and VRP, i.e.  $P(N_t = j|I^{t-1})$  and  $P(N_t = j|I^t)$ , respectively.

of the jump size only,  $\zeta_{CDS,1} = \zeta_{CDS,2}$  and  $\zeta_{VRP,1} = \zeta_{VRP,2}$ , (see column 10). The null is rejected in 24 cases at 5%, thus confirming the presence of a break in the mean of the jump size for many assets. The analysis is complemented with a Diebold-Mariano test to compare the in-sample fit of the model with breaks only in jumps and model with breaks only in the conditional mean. In 15 cases the Diebold-Mariano test has a positive and significant value, meaning that the model with breaks only in jumps provides better predictions than the model with breaks only in the conditional mean. In 7 cases the Diebold-Mariano test has a negative and significant, while in 14 cases is not significant. Summarizing, despite the Diebold-Mariano test does not discriminate between the two models in all cases, there is some evidence that it is important to allow for a breaks in the impact of the covariates in the jump size equation more than in the conditional mean.

Concluding, a worsening in the credit risk of financial intermediaries, as represented by an increase in CDS, or a change in the VRP, increases the expected size of the volatility jumps, and thus the market risk, especially during the financial crises. These results confirm and extend those in Zhang et al. (2009) on the relation between credit risk, volatility risk and jumps. In particular, we stress that the risk of default directly impacts on the expected size of the volatility jumps, causing

a rapid variation in the price. The importance of the inclusion of the CDS in the jump equations is also shown by the improvement of the in-sample fit over the classic HAR model.

Finally, we calculate and plot in Figure 9 the ex-ante and ex-post probability of jumps on October 10, 2008 with the extended HAR-V-J. Those probabilities are different from those obtained with the standard HAR-V-J (see Figure 6). In particular, the densities are assign larger probabilities to a higher number of jumps. The differences between the probabilities in Figure 6 and those in Figure 9 can be explained by the introduction of the covariates in the model. As expected, once the *CDS* and the *VRP* are included in the model, the probability of having a jump during the crisis changes and is higher than that obtained without covariates. Overall, the improvement made by the introduction of additional variables in the model information set is evident. This outcome might open the door to the development of risk management strategies which are trying to anticipate (or to take into account) the arrival of volatility jumps. At this stage we must also make clear that the choice of the exogenous variables to be introduced in the HAR-V-J model might depend on several elements: market evolution, macroeconomic framework, economic sector structure, without excluding qualitative and fundamentals subjective choices. Different explanatory variables could become relevant in other sample periods or with different dependent variables.

# 6 Concluding remarks

This paper studies the contribution of volatility jumps to the evolution of volatility. Differently from some earlier contribution we propose a modified version of the HAR, the HAR-V-J, for modeling the realized range instead of relying on continuous-time stochastic volatility specification.

We model the corrected bipower realized range, a consistent estimator of the integrated variance in presence of jumps in prices and microstructure noise, with a HAR-V-J model that allows for the presence of jumps in volatility. The inference on the parameters of the model is carried out utilizing maximum likelihood estimation, after having specified the dynamics of the jumps sizes and intensities. The estimation results of the HAR-V-J model based on high-frequency data from 36 NYSE stocks suggest that jumps in volatility are more likely to happen during the financial crises, i.e., when the level of volatility is high, and they are positively correlated with jumps in prices. The second part of the analysis focuses on the common determinants of the jump component to the volatilities of individual stocks. It turns out that the variability of a common factor of the estimated jumps, obtained by principal components, can be predicted by using a set of leading financial variables. In particular, *CDS* on US banks appears particularly significant in explaining the observed common jump component.

This result reinforces the idea that the increase in volatility observed during the 2008-2009 US stock market turmoil has been provoked by the worsening of the credit risk of financial institutions and by an underestimate of the risk by market participants. From this point of view the paper contributes to the understanding of the volatility evolution and in particular to the nature and the sources of volatility jumps. Finally, the HAR-V-J model is modified to incorporate the information content of CDS and VRP in the dynamics of the jump size and intensity. The estimation results of the extended HAR-V-J model confirm the significant contribution of the CDS to the jump size dynamics, and some impact coming from the VRP. The proposed modeling approach provides a better understanding of realized measures behavior which could be relevant for risk management strategies, policy interventions or trading activities.

# References

- Andersen, T. and Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 39:885–905.
- Andersen, T. G., Bollerslev, T., and Diebold, F. X. (2007). Roughing it up: Including jump components in the measurement, modeling and forecasting of return volatility. *The Review of Economics and Statistics*, 2007:701–721.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Ebens, H. (2001). The distribution of stock return volatility. *Journal of Financial Economics*, 61:43–76.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, 71:579–625.
- Andersen, T. G., Bollerslev, T., and Huang, X. (2011). A reduced form framework for modeling volatility of speculative prices based on realized variation measures. *Journal of Econometrics*, 160(1):176 – 189.
- Bandi, F. and Renò, R. (2014). Price and volatility co-jumps. Technical report, University of Siena.
- Bates, D. (2000). Post-87 crash fears in the sp 500 futures option market. *Journal of Econometrics*, 1-2(94):181–238.
- Bollerslev, T., Kretschmer, U., Pigorsch, C., and Tauchen, G. E. (2009a). A discrete-time model for daily S&P 500 returns and realized variations: Jumps and leverage effects. *Journal of Econometrics*, 150:151–166.
- Bollerslev, T., Tauchen, G., and Zhou, H. (2009b). Expected stock returns and variance risk premia. *Review of Financial Studies*, 22(11):4463–4492.
- Broadie, M., Chernov, M., and Johannes, M. (2007). Model specification and risk premia: Evidence from futures options. *The Journal of Finance*, 62:1453–1490.
- Busch, T., Christensen, B. J., and Nielsen, M. Ø. (2011). The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets. *Journal of Econometrics*, 160(1):48 – 57.
- Chan, W. H. and Maheu, J. M. (2002). Conditional jump dynamics in stock market returns. *Journal* of Business and Economic Statistics, 20(3):377–389.

- Chernov, M., Gallant, R., Ghysels, E., and Tauchen, G. (2003). Alternative models for stock price dynamics. *Journal of Econometrics*, 116:225–257.
- Christensen, K. and Podolskij, M. (2007). Realized range-based estimation of integrated variance. Journal of Econometrics, 141:323–349.
- Christensen, K. and Podolskij, M. (2012). Asymptotic theory of range-based multipower variation. Journal of Financial Econometrics, 10:417–456.
- Christensen, K., Podolskij, M., and Vetter, M. (2009). Bias-correcting the realized range-based variance in the presence of market microstructure noise. *Finance and Stochastics*, 13(2):239–268.
- Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review*, 39(4):841–862.
- Corsi, F. (2009). A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics, 7:174–196.
- Corsi, F., Mittnik, S., Pigorsch, C., and Pigorsch, U. (2008). The volatility of realized volatility. *Econometric Reviews*, 27:46–78.
- Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.
- Dotsis, G., Psychosios, D., and Skiadopoulos, G. (2007). An empirical comparison of continuoustime models of implied volatility indices. *Journal of Banking & Finance*, 31:3584–3603.
- Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jumpdiffusions. *Econometrica*, 68:1343–1376.
- Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. Journal of Finance, 59(2):1367–1403.
- Eraker, B., Johannes, M. S., and Polson, N. G. (2003). The impact of jumps in returns and volatility. Journal of Finance, 53(3):1269–1300.
- Fernandes, M., Medeiros, M. C., and Scharth, M. (2014). Modeling and predicting the CBOE market volatility index. Journal of Banking & Finance, 40(0):1 – 10.
- Giot, P., Laurent, S., and Petitjean, M. (2010). Trading activity, realized volatility and jumps. Journal of Empirical Finance, 17(1):168–175.

- Hansen, B. (1996). Inference when a nuisance parameter is not identified under the null hypothesis. *Econometrica*, 64:413–430.
- Harvey, A. (2013). Dynamic models for volatility and heavy tails. Econometric Society Monograph. Cambridge University Press, Cambridge.
- Jacod, J., Klüppelberg, C., and Müller, G. (2013). Testing for non-correlation between price and volatility jumps. Technical report, TUM Münich.
- Koop, G. and Korobilis, D. (2012). Forecasting inflation using dynamic model averaging. International Economic Review, 53:867–886.
- Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. Journal of Derivatives, 3(2):73–84.
- Li, H., Wells, M. T., and Yu, C. L. (2008). A Bayesian analysis of return dynamics with Lévy jumps. *Review of Financial Studies*, 21:2345–2378.
- Maheu, J. M. and McCurdy, T. H. (2004). News arrival, jump dynamics, and volatility components for individual stock returns. *Journal of Finance*, LIX:755–793.
- Maheu, J. M. and McCurdy, T. H. (2011). Do high-frequency measures of volatility improve forecasts of return distributions? *Journal of Econometrics*, 160(1):69 76.
- Martens, M., Van Dijk, D., and De Pooter, M. (2009). Forecasting S&P 500 volatility: Long memory, level shifts, leverage effects, day-of-the-week seasonality, and macroeconomic announcements. *International Journal of Forecasting*, 25:282–303.
- Pan, J. (2002). The jump-risk premia implicit in options: evidence from an integrated time-series study. *Journal of Financial Economics*, 63(1):3 50.
- Raftery, A., Karny, M., and Ettler, P. (2010). Online prediction under model uncertainty via dynamic model averaging: Application to a cold rolling mill. *Technometrics*, 52:52–66.
- Rangel, J. G. (2011). Macroeconomic news, announcements, and stock market jump intensity dynamics. *Journal of Banking & Finance*, 35:1263–1276.
- Rossi, E. and Spazzini, F. (2009). Finite sample results of range-based integrated volatility estimation. Technical Report WP-CEA-4-2009, Center for Econometric Analysis, Cass Business School.

- Todorov, V. (2009). Estimation of continuous-time stochastic volatility models with jumps using high-frequency data. *Journal of Econometrics*, 148(2):131 148.
- Todorov, V. (2011). Econometric analysis of jump-driven stochastic volatility models. Journal of Econometrics, 160(1):12–21.
- Todorov, V. and Tauchen, G. (2011). Volatility jumps. *Journal of Business and Economic Statistics*, 29(3):356–371.
- Zhang, B. Y., Zhou, H., and Zhu, H. (2009). Explaining credit default swap spreads with the equity volatility and jump risks of individual firms. *Review of Financial Studies*, 22:5099–5131.
- Zhang, L., Mykland, P. A., and Aït-Sahalia, Y. (2005). A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Association*, 100:1364–1411.

# Appendix: Realized range estimators

In this Appendix, we describe the integrated variance estimator that we adopt in our empirical study. Our objective is to estimate the return variation over the trading day, that is disentangled from the jump in prices component and robust to microstructure noise. Christensen and Podolskij (2007) show that the realized range is a consistent estimator of the integrated variance and it is, in ideal situation of a fully observed price sequence, five times more efficient than the realized variance. Recent Monte Carlo experiments support this finding in more complex setups, see e.g. Rossi and Spazzini (2009) and Christensen and Podolskij (2012).

Suppose first that the log-price of an asset, p(t), follows a stochastic volatility model (SV), that is the price follows a jump-diffusion process

$$p(t) = p(0) + \int_0^t \mu(u) du + \int_0^t \sigma(u) dW(u) + \sum_{i=1}^{N_p(t)} \varsigma_i$$
(24)

where  $N_p(t)$  counts the jumps arrivals at time t, and  $\varsigma_i$  is the jump size. Consider the equidistant partition  $0 = t_0 < t_1 < \ldots < t_n = 1$  where  $t_i = i/n$  and  $\Delta = 1/n$  for  $i = 1, \ldots, n$ . We assume as in Christensen and Podolskij (2007) that mn + 1 equally spaced observations of the price are available. The log-price for each time in the interval (0,1) is denoted as  $p_{\frac{i-1}{n} + \frac{t}{mn}}$ , where  $i = 1, \ldots, n$  and  $t = 0, \ldots, m$ . The observed range over the *i*-th interval is given by  $s_{p_{i\Delta,\Delta},m} =$  $\max_{0 \le s,t \le m} \left\{ p_{\frac{i-1}{n} + \frac{t}{mn}} - p_{\frac{i-1}{n} + \frac{s}{mn}} \right\}$ .

When there are no jumps in prices but the price is contaminated by the microstructure noise,  $\eta_t$ , which is modeled as an i.i.d. sequence of random variables with mean zero and finite variance  $\omega^2$ , Christensen et al. (2009) show that the estimator of the integrated variance is

$$RRG^{\Delta}_{m,BC} = \frac{1}{\widetilde{\lambda}_{2,m}} \sum_{i=1}^{n} (s_{\widetilde{p}_{i\Delta,\Delta},m} - 2\widehat{\omega}_N)^2$$
(25)

where

$$\widetilde{\lambda}_{r,m} = E\left[\left|\max_{t:\eta\frac{t}{m}=\omega,s:\eta\frac{s}{m}=-\omega} \left(W_{\frac{t}{m}} - W_{\frac{s}{m}}\right)\right|^{r}\right].$$

where  $\lambda_{r,m} = E[s_{W,m}^r] = E\left[\max_{0 \le s,t \le m} \left\{W_{t/m} - W_{s/m}\right\}^r\right]$  is the *r*-th moment of the range of a standard Brownian motion over a unit interval when we observe only *m* increments of the underlying continuous time process. The value of  $\lambda_{r,m}$  is obtained through numerical simulation of a standard Brownian motion observed *m* times over the unit interval and  $\lambda_{2,m} \to \lambda_2 = 4 \log (2)$  as  $m \to \infty$ . The variance of the noise process  $\omega^2$  can be consistently estimated with  $\hat{\omega}_N^2 = \frac{RV^N}{2N}$ , where  $RV^N$  is

the realized variance computed using N intraday returns, with N = nm, i.e. the total number of log-returns, and  $N^{1/2} \left( \widehat{\omega}_N^2 - \omega^2 \right) \stackrel{d}{\to} \mathcal{N}(0, \omega^4)$ .

Christensen et al. (2009) show that the bias-corrected realized range-based bipower variation, defined as:

$$RBV_{m,BC}^{\Delta} = \frac{1}{\widetilde{\lambda}_{1,m}^2} \sum_{i=1}^{n-1} |s_{p_{i\Delta},\Delta,m} - 2\widehat{\omega}_N| |s_{p_{(i+1)\Delta},\Delta,m} - 2\widehat{\omega}_N|$$
(26)

is a consistent and robust estimator of the integrated variance in the presence of stochastic volatility, jumps and noise. Furthermore, the price jumps can be determined as

$$\lambda_{2,m} \left( RRG_{m,BC}^{\Delta} - RBV_{m,BC}^{\Delta} \right) \xrightarrow{p} \sum_{i=1}^{N_p(t)} \varsigma_i^2$$

where  $N_p(t)$  cumulates the number of jumps in a given discrete interval. In this paper, we have used (26) to estimate the daily integrated variance. Given the properties outlined above, the bias-corrected realized range-based bipower variation represents a suitable expost-measure of the integrated variance.

# Research Papers 2013



- 2014-11: Yukai Yang: Testing Constancy of the Error Covariance Matrix in Vector Models against Parametric Alternatives using a Spectral Decomposition
- 2014-12: Stefano Grassi, Nima Nonejad and Paolo Santucci de Magistris: Forecasting with the Standardized Self-Perturbed Kalman Filter
- 2014-13: Hossein Asgharian, Charlotte Christiansen and Ai Jun Hou: Macro-Finance Determinants of the Long-Run Stock-Bond Correlation: The DCC-MIDAS Specification
- 2014-14: Mikko S. Pakkanen and Anthony Réveillac: Functional limit theorems for generalized variations of the fractional Brownian sheet
- 2014-15: Federico Carlini and Katarzyna Łasak: On an Estimation Method for an Alternative Fractionally Cointegrated Model
- 2014-16: Mogens Bladt, Samuel Finch and Michael Sørensen: Simulation of multivariate diffusion bridges
- 2014-17: Markku Lanne and Henri Nyberg: Generalized Forecast Error Variance Decomposition for Linear and Nonlinear Multivariate Models
- 2014-18: Dragan Tevdovski: Extreme negative coexceedances in South Eastern European stock markets
- 2014-19: Niels Haldrup and Robinson Kruse: Discriminating between fractional integration and spurious long memory
- 2014-20: Martyna Marczak and Tommaso Proietti: Outlier Detection in Structural Time Series Models: the Indicator Saturation Approach
- 2014-21: Mikkel Bennedsen, Asger Lunde and Mikko S. Pakkanen: Discretization of Lévy semistationary processes with application to estimation
- 2014-22: Giuseppe Cavaliere, Morten Ørregaard Nielsen and A.M. Robert Taylor: Bootstrap Score Tests for Fractional Integration in Heteroskedastic ARFIMA Models, with an Application to Price Dynamics in Commodity Spot and Futures Markets
- 2014-23: Maggie E. C. Jones, Morten Ørregaard Nielsen and Michael Ksawery Popiel: A fractionally cointegrated VAR analysis of economic voting and political support
- 2014-24: Sepideh Dolatabadim, Morten Ørregaard Nielsen and Ke Xu: A fractionally cointegrated VAR analysis of price discovery in commodity futures markets
- 2014-25: Matias D. Cattaneo and Michael Jansson: Bootstrapping Kernel-Based Semiparametric Estimators
- 2014-26: Markku Lanne, Jani Luoto and Henri Nyberg: Is the Quantity Theory of Money Useful in Forecasting U.S. Inflation?
- 2014-27: Massimiliano Caporin, Eduardo Rossi and Paolo Santucci de Magistris: Volatility jumps and their economic determinants