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Discriminating between fractional integration and spurious long memory

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Abstract. Fractionally integrated processes have become a standard class of models to describe the long memory features of economic and financial time series data. However, it has been demonstrated in numerous studies that structural break processes and non-linear features can often be confused as being long memory. The question naturally arises whether it is possible empirically to determine the source of long memory as being genuinely long memory in the form of a fractionally integrated process or whether the long range dependence is of a different nature. In this paper we suggest a testing procedure that helps discriminating between such processes. The idea is based on the feature that nonlinear transformations of stationary fractionally integrated Gaussian processes decrease the order of memory in a specific way which is determined by the Hermite rank of the transformation. In principle, a non-linear transformation of the series can make the series short memory I(0). We suggest using the Wald test of Shimotsu (2007) to test the null hypothesis that a vector time series of properly transformed variables is I(0). Our testing procedure is designed such that even non-stationary fractionally integrated processes are permitted under the null hypothesis. The test is shown to have good size and to be robust against certain types of deviations from Gaussianity. The test is also shown to be consistent against a broad class of processes that are non-fractional but still exhibit (spurious) long memory. In particular, the test is shown to have excellent power against a class of stationary and non-stationary random level shift models as well as Markov switching GARCH processes where the break and transition probabilities are allowed to be time varying.

Keywords: Long memory, fractional integration, non-linear models, structural breaks, random level shifts, Hermite polynomials, realized volatility, inflation.
JEL Classification: C12, C2, C22

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1. Introduction

Recent empirical research indicates that many time series in economics and finance have long memory and belong to the class of fractionally integrated processes. This is especially the case for high frequency financial time series such as log squared returns, log implied and realized volatility, interest rate spreads etc., see e.g. Taylor (1986), Diebold and Rudebusch (1989), Ding *et al.* (1993), Baillie *et al.* (1996), Comte and Renault (1998) Andersen, *et al.* (2001, 2003), Christensen and Nielsen (2006) and Bollerslev *et al.* (2013). A univariate time series process x_t is said to have long memory if its autocorrelation function $\gamma_x(\tau)$ at long lags decays at a hyperbolic (rather than an exponential) rate, i.e. $\gamma_x(\tau) \simeq B\tau^{2d-1}$ as $\tau \to \infty$ where B is a finite constant and d is the memory index. Alternatively, the spectral density of the series is proportional to λ^{-2d} with $d \neq 0$ for low frequencies $\lambda \to 0$. This implies that distant observations tend to be highly correlated and hence the term long memory or long range dependence is used for such processes. Fractionally integrated processes have this property and are often used as a convenient model class for time series with seemingly long memory features. A vast literature exists in estimating the long memory parameter d. Many of these studies address the estimation problem in a semiparametric framework, e.g. Geweke and Porter-Hudak (1983), Künsch (1987), and Robinson (1995a, 1995b), but parametric models within the class of fractionally integrated processes have also attracted considerable attention, i.e. Fox and Taqqu (1986), Sowell (1992a,b), and Nielsen (2005). Recently Johansen and Nielsen (2010, 2012) and Lasak (2010) have developed estimation and testing procedures for co-fractional VAR models.

Despite its attractiveness for parsimonous model building, fractionally integrated processes are often found to be a too flexible model class in the sense that processes that are not truly fractional or genuinely long memory, can be fitted arbitrarily well to the data. For instance, the features of hyperbolic decay of autocorrelations and unboundedness of the spectral density at the origin can also be present when a short memory process is affected by a regime change, a smooth trend, or similar. Hence, in practice it can be hard to distinguish these processes from genuine long memory processes. Often this feature is referred to as "spurious" long memory, see e.g. Diebold and Inoue (2001), Gourieux and Jasiak (2001), Granger and Ding (1996), Granger and Hyung (2004), Shimotsu (2006), Ohanissian, Russell and Tsay (2008), Perron and Qu (2010) and Qu (2011). A commonly used example is a short memory process, (a summable and invertible ARMA process for instance), which is subject to a random level shift where the shifts are governed by a Bernoulli process with a shift probability p, see for instance Diebold and Inoue (2001), Lu and Perron (2010), Qu (2011), Perron and Qu (2010), and Xu and Perron (2014). Diebold and Inoue (2001) show that if the shift probability is allowed to depend on the sample size in a particular way, then the process mimics the long run behaviour of a long memory process. Even though fractionally integrated processes and the random level shift model for instance have common features in terms of long memory, fractionally integrated processes have the property of self-similarity which is a feature that the random level shift model and other apparent long memory processes do not share. The notion of a self-similar process is discussed in e.g. Mandelbrot and Van Ness (1968) and has the implication for instance that the long memory properties of the process are unaffected by the sampling frequency. To sum up, long memory features described only through the first two moments as in the autocorrelation and spectral density functions will generally be insufficient to discriminate fractional integration from spurious long memory processes.

A steadily growing literature has developed with emphasis on whether it is possible to empirically discriminate between genuine long memory processes and spurious long memory processes. This is an important question from both a statistical and an economic perspective. Estimation and inference under a stationary long memory model is quite different from structural break, non-linear, and non-stationary models and from an economic perspective the long lasting impact of shocks in presence of long memory is rather different from models where rare structural breaks occur for example. Also forecasts obtained from these models can be very different.

A number of tests have been developed which attempt to discriminate between true and spurious long memory. Ohanissian, Russell and Tsay (2008) exploit the self-similarity property of fractional processes to develop a test based on the invariance of the long memory parameter subject to different temporal aggregates of the process. Shimotsu (2006) develops two tests. One test is based on the estimation of the long memory parameter using subsamples and comparing these estimates with the estimate for the full sample. Under the null of true long memory the memory parameters are identical. The second test suggested by Shimotsu (2006) is based on the idea of estimating d for the full sample and test whether $\Delta^{\widehat{d}}x_t$ is short memory I(0). This is a property which does not generally apply for spurious long memory models. Qu (2011) proposes a test for the null of true stationary long memory. The test is a frequency domain test based on the derivative of the profiled local Whittle likelihood function in a shrinking neighborhood of the origin. For long memory processes the behaviour of the likelihood at the origin depends on the bandwidth parameter in a particular way and this is a feature that can be used as a basis for testing. Perron and Qu (2010) propose a test against the mean shift hypothesis based on the observation that under the alternative the estimate of d will depend on the number of frequencies included in the log-periodogram regression.

The present paper takes a different point of departure. More precisely, the null being tested is that a Gaussian univariate time series is fractionally integrated of some order d with $0 < d < 1/2$, i.e. the process is assumed to be a stationary long memory process. As for the second test of Shimotsu (2006) it is true that the dth difference is short memory I(0). However, there are other ways a Gaussian fractional long memory process can be transformed to be I(0). We exploit the feature that nonlinear transformations of a fractionally integrated process will have a lower order of memory compared with the original series, see e.g. Dittmann and Granger (2002) and Avarucci and Marinucci (2007). The actual reduction in memory is determined by the Hermite rank of the transformation. A Gaussian $FI(d)$ series x_t can thus be made I(0) by an appropriately chosen nonlinear transformation, possibly in combination with partial differencing of the series $\Delta^\delta x_t$ with $\delta < d$. In principle, a range of different transformations can be considered in constructing a vector series which is short memory I(0) under the null hypothesis. The combination of a non-linear transformation and partial pre-differencing also allows the case of *non*-stationary Gaussian $FI(d)$ processes with $d > 0.5$ and hence generalizes the applicability of the test to a broader class of fractionally integrated processes.

We suggest to use a multivariate Gaussian semiparametric estimator of long-range dependent processes proposed by Shimotsu (2007) to estimate the long memory parameters of a vector of transformed time series and to use a Wald-statistic to test the null that the vector time series is jointly I(0). The test has a limiting χ^2 distribution under the null and by appropriate choice of tuning parameters, i.e. the choice of Hermite rank and partial differencing parameter δ, the test is shown to have good empirical size properties. After appropriate pre-filtering the test is shown to be robust to the presence of serial correlation and conditional heteroscedasticity. The test is shown to have power against a variety of spurious long memory models that have been previously analyzed in the literature; this includes stationary and non-stationary random level shift models and Markov switching GARCH processes. Even though the power is comparable to the power of competing tests in the literature the test of Qu (2011) appears to be superior though. However, if the regime switching or transition probabilities are allowed to be time varying the test is shown to have excellent power and outperforms the Qu (2011) test in most cases.

The plan of the paper is as follows. In section 2, we define the class of models considered in the paper and in section 3 the theoretical results underlying our suggested testing procedure are presented and includes a review of how fractionally integrated Gaussian processes are affected by Hermite polynomial or other transformations. Section 4 presents the suggested testing procedure and subsequently the size and power properties are examined through simulations in sections 5-8. The paper presents an empirical application in section 9 that focuses on log realized volatility for stock market indices and US inflation time series. Finally, the paper is completed with a conclusion and suggestions for future extensions.

2. Fractional integration, structural breaks, and non-linear models

Consider a fractionally integrated time series process x_t generated according to

$$(1-L)^d x_t = u_t \tag{1}$$

where u_t is a covariance stationary process with spectral density $f_u(\lambda)$ being bounded (and bounded away from zero) at frequency zero. Initially we will assume that $0 \leq d < 1/2$ meaning that x_t is stationary with long memory, except for $d = 0$ where x_t is short memory. More generally, $-1/2 < d < 0$ means that the process is antipersistant whereas $1/2 < d < 1$ implies a non-stationary, mean-reverting, long memory process. Assume for instance that x_t follows a summable and invertible ARFIMA(p,d,q) process

$$\Phi(L)(1-L)^d x_t = \Theta(L)\varepsilon_t \tag{2}$$

where $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - .... - \phi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q$ and ε_t is white noise with variance $E(\varepsilon_t^2) = \sigma_\varepsilon^2$. Then the spectral density of x_t in a neighborhood of zero approximates

$$f_x(\lambda) \simeq \frac{\sigma_\varepsilon^2}{2\pi}\frac{|\Theta(1)|^2}{|\Phi(1)|^2}\lambda^{-2d} \quad \text{as } \lambda \to 0_+$$

where " $\simeq$ " signifies that the ratio on the right and left sides tends to unity. In general, a long memory process is defined to be a process with spectral density $f(\lambda) \simeq C\lambda^{-2d}$ as $\lambda \to 0_+$ for $-1/2 < d < 1/2$ and finite constant C. In other words, the spectral density of an ARFIMA process exhibit the shape of a long memory process around the origin and hence belongs to this class of models. Similarly, it can be shown that the auto correlation function of a long memory process will exhibit hyperbolic decay, i.e. $\gamma_x(\tau) \simeq H\tau^{2d-1}$ where H is a finite constant, and this is a property shared with ARFIMA process. We will denote a long memory process with memory parameter d an $LM(d)$ process. Note that an $FI(d)$ process is $LM(d)$, but the reverse is not generally the case. In the present paper the class of long memory processes we want to discriminate from spurious long memory processes are of the Gaussian ARFIMA type with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $0 \leq d < 1/2$.

3. Nonlinear transformations of Gaussian fractionally integrated processes

It is well known that stationary Gaussian fractionally integrated processes can be transformed to reduce their order of long memory, see e.g. Dittmann and Granger (2002) and Avarucci and Marinucci (2007). We will exploit this property to develop a test for fractional long memory. This section reviews some properties of non-linear transformations and Hermite polynomial expansions and we present the main result upon which our test is based.

Consider the case of a zero mean Gaussian random variable[1] x with variance σ^2 and some transformation given by the function $G(.)$ satisfying $EG^2(x_t) < \infty$. The function can be expanded in terms of a series of Hermite polynomials

$$G(x) = \sum_{k=0}^{\infty} \frac{c_k}{k!} H_k(x) \tag{3}$$

[1] It may be seen as a limitation that the following results build upon Gaussianity. Alternatively, Appell polynomial expansions could be considered which allow a more general class of distributions and encompasses the Gaussian distribution as a special case, see e.g. the review in Beran *et al.* (2013). However, because the specific moment generating function of the underlying distribution has to be known in this case and as its unknown parameters enter the Appell polynomials to be used, this approach also has its limitations. As we shall see, transforming data to approximate Gaussianity is sufficient for our testing procedure to have good properties in terms of size and power.

where the Hermite polynomials read

$$H_k(x) = (-1)^k \exp\left(\frac{x^2}{2\sigma^2}\right) \frac{\mathrm{d}^k}{\mathrm{d}^k x}\left(\exp\left(\frac{-x^2}{2\sigma^2}\right)\right). \tag{4}$$

The Hermite polynomials follow the recursion formulae $H_{k+1}(x) = xH_k(x) - kH_{k-1}(x) = xH_k(x) - H_k'(x)$ and apart from scaling the first 5 Hermite polynomials are given by:

$$\begin{aligned} H_0(x) &= 1 \\ H_1(x) &= x \\ H_2(x) &= x^2 - 1, \\ H_3(x) &= x^3 - 3x \\ H_4(x) &= x^4 - 6x^2 + 3 \\ H_5(x) &= x^5 - 10x^3 + 15x. \end{aligned} \tag{5}$$

The coefficients c_k in equation (3) are defined as

$$c_k = \mathrm{E}\left[G(x)H_k(x)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(x)H_k(x) \exp\left(\frac{-x^2}{2\sigma^2}\right) \mathrm{d}x \tag{6}$$

so for instance c_0 =E$[G(x)]$.

The Hermite rank of $G(.)$ is the index J of the lowest non-zero coefficient c_k, i.e.

$$\begin{aligned} &c_k = 0 \quad \text{for } k = 1, 2, ..., J-1 \\ &c_J = \mathrm{E}\left[G(x)H_J(x)\right] \neq 0 \end{aligned}$$

Interestingly, we have that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_l(x)H_k(x) \exp\left(\frac{-x^2}{2\sigma^2}\right) \mathrm{d}x = \begin{cases} 1 & \text{for } l = k \\ 0 & \text{for } l \neq k \end{cases}$$

such that the Hermite polynomials are orthogonal and hence, according to (6), a polynomial transformation $G(x)$ with Hermite rank J can always be found by selecting Hermite polynomials $G(x) = H_J(x)$, i.e. by construction transformations corresponding to Hermite polynomials of order J will have Hermite rank J.

Assume that we have two zero-mean Gaussian variables x and y scaled to have unit variance. Then

$$\mathrm{E}[H_k(x)H_l(y)] = \begin{cases} k!\,[\mathrm{E}(xy)]^k & \text{for } l = k \\ 0 & \text{for } l \neq k \end{cases} \tag{7}$$

Now, consider the Gaussian process $\Delta^d x_t$, $0 < d < 1/2$, i.e. x_t is a fractionally integrated Gaussian process whereby the autocorrelation function declines according to $\gamma_x(\tau) \simeq G\tau^{2d-1}$. It follows from (7) that

$$\mathrm{E}[H_k(x_t)H_k(x_{t-\tau})] = k!\gamma_x^k(\tau) \simeq C\tau^{k(2d-1)}, \text{ as } \tau \to \infty$$

In other words, the sequence $H_{k+1}(x_t)$ will exhibit less memory than the sequence $H_k(x_t)$. More precisely, if $x_t \sim FI(d)$, then the transformation $H_k(x_t)$ will be a long memory series $LM(d_k)$ with $d_k < d$ where

$$d_k = \max\left\{0, (d - \frac{1}{2})k + \frac{1}{2}\right\} \tag{8}$$

which follows from $2d_k - 1 = k(2d - 1)$.

Figure 1 displays the relationship (8) for transformations with Hermite rank J where we use Hermite polynomials as transformations, i.e. $G_{J=k}(x_t) = H_k(x_t)$. As seen, any tranformation with Hermite rank bigger than one will reduce the long memory index of the original fractionally integrated process. For instance, for Hermite rank $J = 2$ an $I(d)$ series can be made short memory $I(0)$ as long as $0 < d < 0.25$. For $J = 3$ the transformed series is $I(0)$ when the original series has d in the range $0 < d < .33$, and so forth. The memory range in which a series can be made $I(0)$ increases with the Hermite rank of the transformation. However, as d increases towards the boundary value of $d = 0.5$, it becomes difficult to make the transformed series $I(0)$ without choosing a transformation with a very high Hermite rank. There is a way of circumventing this problem. Assume that the original series is $I(d)$ with d being "close" to 0.5, say $d = 0.45$. For a Hermite rank of $J = 2$ it is not possible to make the series $I(0)$. In fact, in this case $d_2 = 0.4$ and only a minor reduction in memory results. However, it is possible to partially difference the series prior to non-linear transformation in order to amplify the reduction of memory order. In place of x_t, consider as an example the series $y_t = \Delta^\delta x_t$, where $\delta = 0.2$ such that $y_t \sim FI(0.25)$ as $d - \delta = 0.25$. Note that y_t is still a Gaussian process. A subsequent transformation $H_J(y_t)$ with Hermite rank $J = 2$ will thus make the original series I(0) after "partial" pre-differencing. We shall later see that such a combination of partial differencing and a non-linear transformation may be useful for testing purposes.

Figure 1 about here

As seen, an interesting feature of Gaussian $FI(d)$ processes is that the series can be made short memory $I(0)$ not only by taking a $d'th$ difference of the series, but also by non-linear transformation. This has the consequence, that a series may "cointegrate" non-linearly with itself even though common long memory rather than cointegration seems to be the appropriate notion in this case, see Engle and Granger (1987) for the definition of cointegration. Assume for instance the case where $x_t \sim FI(d)$ and consider the transformation $H_3(x_t)$. It follows that $H_3(x_t) = (x_t^3 - 3x_t)$ has memory $LM(d_3)$, with $d_3 = \max\{0, 3d - 1\} < d$. Note however, that the two terms x_t^3 and $3x_t$ both have Hermite rank $J = 1$ and hence have memory $LM(d)$. This shows that linear combinations of Gaussian $FI(d)$ processes may have common memory features with non-linear transformations of itself and for certain values of d the linear combination of the series may even be short memory. This property is somewhat related to the notion of summability and co-summability of linear and nonlinear stochastic processes that has recently been suggested by Berenguer-Rico and Gonzalo (2013, 2014). However, the two notions are distinct and only coincide in special cases.

After having presented the relevant results for nonlinear transformations of fractionally integrated processes, we turn to transformation of processes which generate spurious long memory. In particular, we focus on the widely studied random level shift process for an illustration. It is given by $\mu_t = \mu_{t-1} + \varepsilon_t$ with $\varepsilon_t = \pi_t \eta_t$, where $\pi_t \sim iidB(1,p)$ and $\eta_t \sim iidN(0,\sigma_\eta^2)$ being independent of each other. The process can be re-written as $\mu_t = \sum_{j=1}^{t} \varepsilon_j$ with $E(\varepsilon_j) = 0$ and $var(\varepsilon_j) = p\sigma_\eta^2$. This leads to $E(\mu_t) = 0$ and $var(\mu_t) = tp\sigma_\eta^2$. The autocovariance function is denoted as $C_{t,t-\tau} = E(\sum_{j=1}^{t} \varepsilon_j \sum_{j=1}^{t-\tau} \varepsilon_j)$ with $|\tau| \leq t-1$. Consider the nonlinear transformation of the standardized series $z_t = \mu_t / C_{t,t}^{1/2}$, i.e. $G(z_t) = \sum_{k=0}^{\infty} \alpha_k^{(t)} H_k(z_t)$, where $\alpha_k^{(t)} = \frac{1}{k!} E\left(\frac{d^k G(C_{t,t}^{1/2} z_t)}{dz_t^k}\right)$. In general, the autocovariance function of the nonlinearly transformed series is given by

$$\gamma_{G(z)}(\tau) = \sum_{k=1}^{\infty} (\alpha_k^{(t)} C_{t,t}^{-k/2})(\alpha_k^{(t-\tau)} C_{t-\tau,t-\tau}^{-k/2}) k! C_{t,t-\tau}^k$$

see Ermini and Granger (1993). For the case of a random level shift process and a transformation of $G(z_t) = H_2(z_t)$ with Hermite rank $J = 2$, we obtain after some algebra $\gamma_{G(z)}(\tau) \simeq C_z \tau^2 p^2$. According to Diebold and Inoue (2001), random level shift processes and fractionally integrated processes display the same long memory behaviour in levels if $p = O(T^{2d-2})$.[2] When applying this rate, we obtain that $G(z_t)$ is $LM(d^*)$ since

$$\gamma_{G(z)}(\tau) \simeq C_z \tau^{2d^*-1}$$

where $d^* = \left(\frac{1}{2}d + \frac{1}{4}\right) \in [1/4, 1/2)$ for $d \in [0, 1/2)$. Thus, the long memory behaviour of a squared random level shift process is very different to the one for a squared fractionally integrated process. While the two processes are hardly distinguishable when considering the *levels*, taking the *square* does reveal substantial differences in the decay of the autocovariance function. Such a distinct behaviour is the basis for our testing approach.

4. Testing the null of fractional integration

The properties of non-linear transformations of stationary $FI(d)$ processes discussed in the previous section will be used to develop a testing procedure to discriminate between stationary, fractionally integrated processes and spurious long memory processes.

The testing procedure consists of first estimating d for the original series, x_t, which is $FI(d)$ under the null hypothesis. We know that as long as the estimate of d is consistent, then in the limit $\Delta^{\widehat{d}} x_t$ is short memory $I(0)$, see Shimotsu (2006), and,

[2]Similar to the analysis in Diebold and Inoue (2001), the results derived below also extend to the richer "mean plus noise" model $x_t = \mu_t + u_t$ with $u_t \sim iidN(0, \sigma_u^2)$.

also in the limit, $y_t = \Delta^{\widehat{d}-\delta} x_t$ is $FI(\delta)$ for some target differencing δ. Consider next a Hermite polynomial transformation of y_t, i.e. $H_J(y_t)$ which is designed such that theoretically this will have short memory $LM(0)$[3]. Recall that the choice of δ and J are related to ensure this. In principle, a sequence of transformations with different Hermite ranks can be considered (and in different combinations) which leads to the vector time series $Y_t = (\Delta^{\widehat{d}} x_t, H_{J_1}(y_t), H_{J_2}(y_t), ..., H_{J_p}(y_t))'$ of dimension $q \times 1, (q = p + 1)$, being short memory $LM(\mathbf{0})$ under the null hypothesis. Observe that the first element in Y_t is x_t differenced by the estimate $\widehat{d}$ and hence does not involve a nonlinear transformation. If this element were the only one being tested in Y_t the test would correspond to the "differencing test" by Shimotsu (2006). Our test statistic is based on a multivariate estimate of the long memory parameters of the vector series Y_t and testing $Y_t \sim LM(\mathbf{0})$ using a Wald test due to Shimotsu (2007). Note that in the construction of the y_t series using different Hermite polynomial transformations, the possibility of using different target values δ for each transformation may be considered. In accordance with the discussion in section 3 a proper notation will include an index J of the partially differenced series to indicate that the target differencing may depend on the Hermite rank, i.e. $y_t^J = \Delta^{\widehat{d}-\delta_J} x_t$ is $FI(\delta_J)$. In section 5 we will use simulations to determine how the target differencing filter can be selected in practice to obtain an acceptable size-power trade-off of the test[4].

The design of the null hypothesis implies that a broader class of fractional Gaussian processes can be accomodated. The theory presented in section 3 only applies for stationary fractionally integrated Gaussian processes where $0 < d < 1/2$. However, because the series is pre-differenced to the stationary region before testing our procedure allows d to take on values in the non-stationary region $d \geq 1/2$ and hence can be used under more general conditions than initially thought.

Assume that the dimension of Y_t is $q \times 1, q = p+1$, with the memory index vector $\mathbf{d} = (d_0, d_1, d_2, ..., d_p)' = \mathbf{0}'$ under the null hypothesis, i.e.

$$H_0: \; Y_t = (\Delta^{\widehat{d}} x_t, H_{J_1}(y_t^{J_1}), H_{J_2}(y_t^{J_2}), ..., H_{J_p}(y_t^{J_p}))' \sim LM(\mathbf{0}). \tag{9}$$

In principle, a single element or a group of elements in Y_t can be tested under the null. The test we suggest is based on Shimotsu's (2007) multivariate Gaussian semiparametric estimator (GSE) of long memory processes and the associated Wald test of the null hypothesis (9). The estimator is defined as

$$\widehat{\mathbf{d}} = \arg\min_{\mathbf{d} \in \mathbf{\Theta}} R(\mathbf{d}) \tag{10}$$

[3] y_t is assumed to be measured in deviations from the mean and to have unit variance. For notational convenience, we assume that y_t is the demeaned series scaled by its standard deviation.

[4] The idea of differencing the data appropriately has also been used in Nielsen and Frederiksen (2011) in the context of weak fractional cointegration. As the authors note, the particular choice of the differencing parameter is user-specfic and different choices lead to different outcomes. In their case, there exists a choice which is best in a GLS sense. In our situation there is a best choice as well as the power of our tests are monotonically increasing in the target differencing parameter. Therefore, we are able to optimize the power of the tests with respect to δ, while controlling for the size.

where the objective function reads

$$R(\mathbf{d}) = \log\det\widehat{G}(\mathbf{d}) - 2\sum_{a=1}^{q} d_a \frac{1}{m}\sum_{j=1}^{m}\log\lambda_j \tag{11}$$

$$\widehat{G}(\mathbf{d}) = \frac{1}{m}\sum_{j=1}^{m}\mathrm{Re}\left[\Lambda_j(\mathbf{d})^{-1}I(\lambda_j)\Lambda_j^*(\mathbf{d})^{-1}\right] \tag{12}$$

The Fourier frequencies are given by $\lambda_j = 2\pi j/T$ with $j = 1, 2, ...m$ and $m = o(T)$ is the bandwidth parameter. We also have $\Lambda_j(\mathbf{d})$ =diag$(\Lambda_{ja}(\mathbf{d}))$, $\Lambda_{ja}(\mathbf{d}) = \lambda_j^{d_a}\exp(i(\pi - \lambda_j)d_a/2)$ and $\Lambda_j^*(\mathbf{d})$ is the conjugate transpose of $\Lambda_j(\mathbf{d})$. The admissable range of $\mathbf{d}$ in minimizing the objective function (11) is $\Theta = [\Delta_1, \Delta_2]^q$, with $-\frac{1}{2} < \Delta_1 < \Delta_2 < \frac{1}{2}$. Nielsen (2011) extends the Shimotsu (2007) estimator to an expanded range of d values given by $-\frac{1}{2} < d < \infty$, so in principle this estimator can be considered for an extended class of long memory processes that exhibit non-stationarity.

Shimotsu (2007) shows the consistency and asymptotic normality of the estimator in (10). He also shows that the estimator is more efficient that the two step GSE estimator of Lobato (1999). It is important to note that Gaussianity is not assumed in the asymptotic theory and a general class of multivariate long-range dependent processes is allowed for, including fractionally integrated processes. A test of the hypothesis $\mathrm{H}_0 : \mathbf{d} = \mathbf{0}$ is given by the Wald statistic

$$W = m\,\widehat{\mathbf{d}}'\widehat{\mathbf{\Omega}}\,\widehat{\mathbf{d}} \sim \chi^2(q) \text{ as } T \to \infty \tag{13}$$

where

$$\widehat{\mathbf{\Omega}} = 2\left[\widehat{G}(\widehat{\mathbf{d}}) \odot \widehat{G}(\widehat{\mathbf{d}})^{-1} + I_q + \frac{\pi^2}{4}(\widehat{G}(\widehat{\mathbf{d}}) \odot \widehat{G}(\widehat{\mathbf{d}})^{-1} - I_q)\right]$$

and $\odot$ denotes the Hadamard product.

With reference to work by Hurvich and Chen (2000) concerning properties of the GSE estimator in univariate settings, Shimotsu (2007) proposes a correction factor of the Wald test that appears to yield better size properties. The modified Wald test reads

$$W_c(\mathbf{J}, \delta_J) = c_m\widehat{\mathbf{d}}\widehat{\mathbf{\Omega}}\widehat{\mathbf{d}} \tag{14}$$

where $c_m = \sum_{j=1}^{m} v_j^2$, $v_j = \log\lambda_j - \frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$. Because $c_m/m \to 1$ as $m \to \infty$, the asymptotic distribution is unaffected by the correction factor. This is our preferred test to be used subsequently and in short we will use the notation $W(\mathbf{J}, \delta_j) = W((J_1, J_2, ..J_p), \delta_J)$.

Note that when the null hypothesis cannot be rejected it is appropriate to model the series as a fractional long memory process and if the null is rejected a search should be made for an adequate alternative model specification. The route outlined in Qu (2011) can be followed in this situation.

5. Test size and choice of δ for the Gaussian fractional noise case

In order to implement the testing procedure we will examine how appropriate target values δ_J can be determined to control the size of the test. The transformations we consider are themselves Hermite polynomials but in principle any transformation with Hermite rank larger than one can be considered. In case of no estimation uncertainty, the relationship (8) would hold exactly and finding the appropriate differencing $d - \delta_J$ that would make y_t^J exactly $LM(0)$ would be trivial. However, when estimating d the estimation error should be accounted for. We have empirically replicated the theoretical shapes displayed in Figure 1 by estimating d for a range of $FI(d)$ processes x_t with $d \in [0, 0.5[$. Next, we have estimated the memory parameter d_k of the transformed univariate series $H_J(x_t)$. Figures 2-3 display Monte Carlo averages of the combinations of $\widehat{d}$ and $\widehat{d}_k$ using local Whittle estimates of the long memory parameter estimated with the commonly used bandwidth parameter $m = [T^{0.65}]$, see Künsch (1987). The graphs are displayed for Hermite ranks 2-5 with $T = 500$ and $T = 3000$ observations. 2000 Monte Carlo replications were used to construct the graphs.

Figures 2-3 about here

As seen, the theoretical decline in long memory is only partially reflected in the empirical estimates. Generally, the reduction of memory is less steep than predicted by the theory and the point where the Monte Carlo average of $\widehat{d}_k$ is approximately zero is lower than the theoretical value. This suggests that the target differing δ_J should be chosen to be a smaller value than indicated by the theory. The simulations also confirm that δ_J can be chosen as an increasing function of J whereas the sample size seems to play a neglible role in the choice of δ_J.

To determine the desired target values δ_J we will follow a different route and consider the $W(\mathbf{J},\delta_J)$ test to see for which target values δ_J the test has acceptable size. It is clear that selecting a value of δ_J that is too large will size distort the test since the long memory is not completely removed after the appropriate transformation. On the other hand, by selecting a too low value of δ_J may be at the cost of power even though the overall size of the test is less affected.

We consider different variants of the test where $\delta_J \in [0, 0.4]$ and for the bivariate ($q = 2$) transformations choose $J = (2, 3, 4, 5)$ and for trivariate ($q = 3$) transformations $J = ((2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5))$. Recall that for all tests the first element in the vector Y_t given in (9) is the (empirically) differenced series $\Delta^{\widehat{d}} x_t$ using the local Whittle estimator to estimate d. Hence the dimension of Y_t being tested is respectively $q = 2$ and $q = 3$. The GSE (10) is constructed with the bandwidth parameter $m = [T^{0.65}]$ and asymptotic critical values are taken from the $\chi^2(q)$ distribution.

The benchmark model for the simulations is a Gaussian fractional noise process generated according to

$$\Delta^d x_t = u_t \qquad t = 1, 2, ..., T$$
$$u_t \sim N(0, 1)$$

Different values of $0 < d < 0.5$ are considered.

Figures 4-5 about here

Figure 4 reports the rejection frequencies for the two-dimensional tests ($q = 2$) for $d = 0.4$ with $J = 2, 3, 4, 5$ and $T = 500, 3000$. The graphs show that for small values of δ_J the tests are conservative with size less than the nominal 5% level. However, as δ_J increases the rejection frequencies increase as well. This is expected since too large values of δ_J results in remaining long memory and beyond a certain point the rejection frequencies increase as the sample size increases. This reflects that power and not size is measured when δ_J is large and hence demonstrates that the test is consistent. The reason why the tests are undersized for small values of δ_J is due to the prior differencing operation $\Delta^{\widehat{d}-\delta_J}$ which uses the estimate $\widehat{d}$ that is subject to estimation uncertainty. The optimal value δ_J^* is the largest possible value that can be achieved before the test size increases above its nominal level. This is because the power increases monotonically in δ_J. The simulations show that a choice of values robust to the sample size is in the neighborhood of $\delta_2^* = 0.225, \delta_3^* = 0.3$, and $\delta_4^* = \delta_5^* = 0.35$ and hence are slightly smaller than the theoretical values ($\delta_2 = 0.25, \delta_3 = 0.33, \delta_4 = 0.375, \delta_5 = 0.40$). These are the values we will use subsequently in simulations and applications.

Figure 5 displays graphs for the three dimensional case ($q = 3$) where pairs of Hermite transformations are compared. In theory, one would expect a test to have power increasing in q, but in practice the appropriate choice of target differencing may blur this intuition. Qualitatively, the conclusions regarding size for the $q = 3$ case are similar to the $q = 2$ case. The appropriate choice of δ_J^* should be in the range $0.2 - 0.3$ depending upon the test design. For practical purposes we suggest the target parameters $\delta_{2,3}^* = \delta_{2,4}^* = \delta_{2,5}^* = 0.2, \delta_{3,4}^* = 0.3, \delta_{3,5}^* = 0.275$, and $\delta_{4,5}^* = 0.3$. These tuning parameters will give a slightly conservative test, but larger target values will potentially result in excessive size distortions.

Figures 4 and 5 motivating our (practical) choice of target parameters δ_J^* are constructed for the situation with $d = 0.4$. This is a memory parameter in the range typically found in financial applications. When calculating rejection frequencies for a broader range of d values it can be shown to have only a minor effect on size. Table 1 documents this by showing the rejection frequencies for both the two-, and three- dimensional tests with d in the stationary range between 0.2 and 0.45 as well as the non-stationary range $0.55 - 0.65$. As seen the tests are slightly undersized for small sample sizes and close to the nominal 5% level for $T = 3000$. Due to the partial differencing as an integral part of the testing procedure, values of d in the non-stationary region $d > 0.5$ are permitted as well and as seen the sizes are similar to the stationary case. The fact that the test is performing well even for non-stationary fractional processes is a clear advantage of the present testing procedure since competing tests assume stationarity and can be heavily size distorted when the sample size is large. We have simulated the empirical size of the Qu (2011) test for $d = 0.55, 0.60$,and 0.65 and for $T = 3000$ the size of the test is respectively $0.077, 0.226$, and 0.455. For $T = 500$ the size distortion is moderate.

Table 1 about here

6. ROBUSTIFYING THE PROCEDURE TO ALLOW FOR GENERAL FRACTIONAL PROCESSES

The theoretical arguments motivating our test assume the underlying process is a fractional Gaussian noise under the null hypothesis. It is unknown how short-memory dynamics or conditional heteroscedasticity will affect the validity of the results, but it is likely that such dynamics may have an impact and hence there is a need for appropriately modifying the testing procedure to control the size of the test. Following the approach of Qu (2011), we will adopt a prewhitening procedure to remove a possible short memory component of the underlying series. This procedure is based on estimation of a low order ARFIMA model and filtering the series using the estimated autoregressive and moving average coefficients. A second prewhitening filter is considered for situations where the underlying process is suspected to be conditionally heteroscedastic. This variance prefiltering is based on a GARCH(1,1) specification and is thus comparable to the mean prewhitening procedure of the process. We do not assume that the underlying process under the null is an ARFIMA-GARCH process, but we suggest this model as a reasonable approximation to account for serial dependence and conditional heteroscedasticity.

6.1. ARFIMA prewhitening. The mean prefiltering procedure to be used follows Qu (2011). Initially ARFIMA(p, d, q) models are estimated for the time series x_t by maximizing the Gaussian likelihood for $p, q = 0, 1$ and using the Akaike information criterion (AIC) for model selection. The estimated AR and MA parameters are $\widehat{a}_1, \widehat{b}_1$ which are restricted to ensure stationarity and invertibility. In practice the restriction is implemented by requiring that $-1 + \kappa \leq \widehat{a}_1, \widehat{b}_1 \leq 1 - \kappa$ with $\kappa > 0$ being a small constant. Following Qu (2011) we set $\kappa = 0.01$ for practical applications. The filtered series reads

$$x_t^* = (1 - \widehat{a}_1 L)(1 + \widehat{b}_1 L)^{-1}(x_t - \overline{x}) \tag{15}$$

where $\overline{x} = \frac{1}{T}\sum_t x_t$. With no further filtering the series x_t^* in (15) is used in place of x_t in the testing procedure outlined in section 4.

6.2. GARCH prewhitening. In case the underlying series is suspected to be heteroscedastic we additionally suggest GARCH prefiltering of the x_t^* series, i.e. the series which has been mean prefiltered as in (15). The experience is that a GARCH(1,1) model approximates time series data with conditional heteroscedasticity rather well so for simplicity we preselect the orders of the GARCH model. The estimated conditional standard deviations obtained from maximum likelihood estimation is denoted $\widehat{\sigma}_t$ and accordingly, we construct the GARCH filtered series as $x_t^{**} = x_t^*/\widehat{\sigma}_t$ which is the series to be analyzed subsequently.

7. SIMULATION RESULTS FOR THE ROBUSTIFIED TESTING PROCEDURE

In order to evaluate the robustness of the testing procedure compared to the benchmark model, we examine a range of Monte Carlo designs. In all cases we set $d = 0.4$ in the following model specifications:

1. $\text{ARFIMA}(0, d, 0) : (1 - L)^d x_t = e_t,$
2. $\text{ARFIMA}(1, d, 0) : (1 - 0.8L)(1 - L)^d x_t = e_t,$
3. $\text{ARFIMA}(0, d, 1) : (1 - L)^d x_t = (1 + 0.5L)e_t,$
4. $\text{FI}(d) - GARCH(1, 1) : (1 - L)^d x_t = u_t$, where $u_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.1u_{t-1}^2 + 0.85\sigma_{t-1}^2$

For all models we let $e_t \sim iid$ $\text{N}(0, 1)$ in line with Qu (2011). Note that the $\text{ARFIMA}(0, d, 0)$ is the benchmark model which is also considered in the previous section. When implementing our testing procedure we used the ARFIMA prewhitening filter in all cases. For the case with conditional heteroscedasticity the GARCH prewhitening filter was used as well.

Figures 6 and 7 report the rejection frequencies as a function of δ_J for the two-dimensional tests ($q = 2$) with $J = 2, 3, 4, 5$ and $T = 500, 3000$, respectively. Figures 8 and 9 display the corresponding graphs for the three-dimensional tests ($q = 3$). The simulation results can be summarized very briefly: The shape of the curves is very similar compared with the benchmark model and hence the tuning parameters δ_J^* suggested in the previous section can be applied under more general situations when the testing procedure is combined with appropriate prewhitening of the series. As a consequence the size of the test is very reasonable even when the underlying process is governed by serially dependent and possibly conditionally heteroscedastic strongly persistent disturbances.

The case of GARCH(1,1) errors implies that the underlying distribution will have fat tails. In simulations that are not reported here we find that assuming errors to be governed by a t-distributions with six degrees of freedom will have very little impact on the size of the test for the target differencing parameters we are suggesting to use in practice. Our findings suggest that the $W(\mathbf{J},\delta_J)$ test generally is robust to fat-tailness of the underlying distribution under the null hypothesis.

Figures 6-9 about here.

8. POWER

8.1. Models with constant parameters. We will examine the power properties of the $W(\mathbf{J},\delta_J)$ test with respect to the hypothesis (9) for six different non-linear model specifications that are known to exhibit long memory. The models considered are all characterized by having constant parameters and are also included in the studies of Ohanissian, Russell and Tsay (2008) and Qu (2011) amongst others[5]. The models are:

1. Nonstationary random level shift model (RLS-NS): $y_t = \mu_t + \varepsilon_t, \mu_t = \mu_{t-1} + \pi_t \eta_t, \pi_t \sim iid$ B$(1,p), \varepsilon_t \sim iid$ N$(0,\sigma_\varepsilon^2), \eta_t \sim iid$ N$(0,\sigma_\eta^2)$ where $p = 6.1/T, \sigma_\varepsilon^2 = 5, \sigma_\eta^2 = 1$.
2. Stationary random level shift model (RLS-S): $y_t = \mu_t + \varepsilon_t, \mu_t = (1-\pi_t)\mu_{t-1} + \pi_t \eta_t, \pi_t \sim iid$ B$(1,p), \varepsilon_t$ and $\eta_t \sim iid$ N$(0,\sigma^2)$ where $p = 0.003, \sigma_\varepsilon^2 = \sigma_\eta^2 = 1$.
3. White noise with a monotonic deterministic trend (MONO): $y_t = at^{-\beta} + \varepsilon_t, \varepsilon_t \sim iid$ N$(0,\sigma_\varepsilon^2)$, where $a = 3, \beta = 0.1, \sigma_\varepsilon^2 = 1$.
4. White noise with a nonmonotonic deterministic trend (NON-MONO): $y_t = \sin(a\pi t/T) + \varepsilon_t, \varepsilon_t \sim iid$ N$(0,\sigma_\varepsilon^2)$, where $a = 4, \sigma_\varepsilon^2 = 3$.
5. Markov switching model with iid regimes (MS): $y_t \sim iid$ N(μ_0,σ_0^2) if $s_t = 0$ and $y_t \sim iid$ N(μ_1,σ_1^2) if $s_t = 1$, with state transition probabilities $p_{01} = p_{10}$. $\mu_0 = -\mu_1 = 1, \sigma_0^2 = \sigma_1^2 = 1, p_{01} = p_{10} = 0.001$.
6. Markov switching model with GARCH regimes (MS-GARCH): $y_t = \log r_t^2$ with $r_t = \sqrt{h_t}\varepsilon_t$, $h_t = \omega_0 + \omega_1 s_t + \alpha r_{t-1}^2 + \beta h_{t-1}$,and $\varepsilon_t \sim iid$ N$(0,\sigma_\varepsilon^2)$, $s_t = 0,1$. The transition probabilities are $p_{10} = p_{01}$. $\omega_0 = 1, \omega_1 = 2, \alpha = 0.4, \beta = 0.3$, $\sigma_\varepsilon^2 = 1$, and $p_{10} = p_{01} = .001$.

To make comparisons with previous tests analyzed in the literature the calibration of the model specifications above are as in Qu(2011). We denote this set of experiments as experiment A. The configurations chosen result in mean values of the estimated long memory parameters of around 0.25-0.30 for the RLS-NS model, the range [0.20-0.60] for the RLS-S model, around [0.12-0.17] for the MONO model, around 0.33 for the NON-MONO model, [0.15-0.60] for the MS model, and finally [0.12-0.20] for the MS-GARCH model. These are Monte Carlo averages of local Whittle estimates. Because the estimated memory parameter appears to be relatively low for several of the models in the experiments of Qu(2011) we conducted a second set of experiments, experiment B, which essentially corresponds to experiment A, but where each model was calibrated to have a larger mean value of $\widehat{d}$, typically in the

[5] See also Bos, Franses and Ooms (1998), Chen and Tiao (1990), Lu and Perron (2010), Li and Perron (2013), Qu and Perron (2013), and Xu and Perron (2014) for examples of these models generating apparent long memory in financial and inflation time series.

range [0.35-0.45], i.e. a value that is often found in empirical applications[6]. In a final set of experiments, experiment C, we modified the calibration of experiment A by introducing an AR component in ε_t to check the influence of a slightly persistent element governing the series. The AR coefficient selected was 0.8 and the variance was chosen to make the unconditional variance of the ε_t term identical across experiments A and C, i.e. the ε_t was scaled by its theoretical standard deviation $(1-0.8^2)^{-1/2}$.

Tables 2, 3, and 4 report the results for experiments A, B, and C, respectively. For each model the $W(\mathbf{J},\delta_j)$ test was calculated with bandwidth parameter $m = [T^{0.65}]$ and test rejection frequencies were estimated using 2000 replications. The sample size studied are as reported in Qu(2011) and varies between 500 and 9000. Different combinations of Hermite rank J were considered in combination with the target differencing parameter δ_J selected in accordance with the previous suggestions. For experiment A we compare with other tests and additionally report the power results for the Qu (2011) test (QU), the Ohanissian, Russell, and Tsay (2008) test, (ORT), the split-sample (S-SPLIT) and the difference (S-DIFF) tests of Shimotsu (2006). Finally, the mean-t_d test (PQ) of Perron and Qu (2010) is compared with. The rejection frequencies reported for these competing tests are taken from Qu (2011). With respect to experiments B and C the powers of the QU test are included for comparison.

In experiment A the Qu (2011) test is generally seen to perform best amongst most competitors, especially for the RLS-NS, RLS-S, and MS models. The different variants of the $W(\mathbf{J},\delta_j)$ test are shown to do at least as well as several of the other competing tests, but the performance varies depending upon the Hermite rank combinations $\mathbf{J}$ being considered. A pattern for the models mentioned above is that the $W(\mathbf{J},\delta_j)$ test with $J=2$ generally comes out as the second best in terms of power when compared to the QU test. For the MONO model the $W(\mathbf{J},\delta_j)$ test with $J=2$ has better power than the QU test but is slightly dominated by the S-DIFF test. When testing against the MS-GARCH model the $W(\mathbf{J},\delta_j)$ test is doing especially well when a Hermite rank of $J=4$ is included in the transformations and dominates the QU test even for small sample sizes.

When designing the models to exhibit a different memory structure (experiment B, Table 3), the $W(\mathbf{J},\delta_j)$ test generally has larger power compared to experiment A. Qualitatively, the conclusions from experiment A carry over to experiment B for the RLS-NS, RLS-S, and MS models, and again the model design with $J=2$ performs best. For the MONO model the $W(\mathbf{J},\delta_j)$ test outperforms the QU test for a wide range of J values. With respect to the MS-GARCH model the $W(\mathbf{J},\delta_j)$ tests involving $J=4$ have better power compared with the QU test.

The overall picture from these experiments is unaffected when the error term of the models is modified to have AR error dynamics as in experiment C, Table 4. One

[6]The calibrations for experiment B are as follows. RLS-NS: $p=6/T, \sigma_\varepsilon^2=2.5, \sigma_\eta^2=1$. RLS-S: $p=5.5/T, \sigma_\varepsilon^2=\sigma_\eta^2=1$. MONO: $a=10, \beta=0.1, \sigma_\varepsilon^2=1$. NON-MONO: $a=25$, $\sigma_\varepsilon^2=3$. MS: $\mu_0=-\mu_1=0.6$, $\sigma_0^2=\sigma_1^2=1$, $p_{01}=p_{10}=10/T$. MS-GARCH: $\omega_0=1$, $\omega_1=5$, $\alpha=0.4$, $\beta=0.5$, $\sigma_\varepsilon^2=1$, and $p_{10}=p_{01}=10/T$.

modification of the conclusions is that for the MS-GARCH model the $W(\mathbf{J},\delta_j)$ test with $J=2$ dominates the other tests including the QU test. In fact, the preferred test design across all model structures is the one with $J=2$.

To conclude, the $W(\mathbf{J},\delta_j)$ test is shown to be consistent and has power comparable with competing tests for several designs of the alternative hypothesis. The QU test generally seems to perform best, especially against the class of stationary and non-stationary random level shift models as well as the Markov switching model with iid regimes. However, for the specific design where $J=2$ the $W(\mathbf{J},\delta_j)$ test still has excellent power and for practical applications the advice points towards using the $W(2, 0.225)$ test together with the QU test.

Tables 2-4 about here

8.2. Time varying parameter models. The (spurious) long memory models considered in section 8.1 are a reference class of models in the literature. However, many other models will have spurious (non-fractional) long memory features. Here we will consider an extended class of models that exhibit stochastic shifts but as opposed to the previous models we let the switching or transition probabilities be time varying. Basically we consider time varying parameter versions of the RLS-NS, RLS-S, and MS-GARCH models. Variants of these models have also been considered in Xu and Perron (2014), building on Lu and Perron (2010), and have been shown to be empirically relevant. See also Diebold *et al.* (1994) and Kim *et al.* (2008) for further references. For these models the jump or switching probabilities are made dependent on some covariate variable and allows a more comprehensive and realistic structure for the level shift model. The design is in the spirit of the "news impact curve" motivated by Engle and Ng (1993).

We specify the jump probability to be $p_t = f(p, r_{t-1})$ where p is a constant probability and r_{t-1} is a covariate variable (e.g. the lagged returns series). $f(\cdot)$ is a function that ensures $p_t \in [0,1]$ and here we choose the standard normal cdf $\Phi(\cdot)$. More specifically we let

$$f(p, r_{t-1}) = \begin{cases} \Phi(\Phi^{-1}(p) + \gamma_1 \mathbf{I}(r_{t-1}<0) + \gamma_2 \mathbf{I}(r_{t-1}<0)|r_{t-1}|) \text{ for } |r_{t-1}| > c_{0.01} \\ \Phi(p) \text{ otherwise} \end{cases} \tag{16}$$

where $\mathbf{I}(r_{t-1}<0)$ is the indicator variable and $c_{0.01}$ is a preselected threshold which we choose to be the bottom say 1% of the sample distribution of returns. The main idea is that the break probability is largely increased when we have an extremely negative outcome of r_t in the previous period and hence capturing the leverage effect that is well documented in many empirical studies.

We define the models RLS-NS-TVP and RLS-S-TVP which are both variants of the RLS models presented in section 8.1 but now with the break probability modelled as in (16). We also expand the models to allow (weak) dependence and conditional heteroscedasticity. More specifically we let $\varepsilon_t = \phi\varepsilon_{t-1} + u_t$ with $\phi = 0.2$ and $u_t \sim iid$ $\mathrm{N}(0,1)$. The innovations are scaled by the inverse of the standard deviation to ensure a unit variance. The returns series r_t is generated according to a standard GARCH(1,1) model $r_t = h_t^{\frac{1}{2}} \varepsilon_t, h_t = \omega_0 + \alpha r_{t-1}^2 + \beta h_{t-1}$ where we set $\omega_0 = 1, \alpha = 0.1$ and $\beta = 0.85$. Given the sequence of returns r_t the 1%-quantile denoted $c_{0.01}$ can be found. This provides the input for calculating the probabilities (16) where we set $p = 6.1/T$ as in experiment A, while $\gamma_1 = 2$ and $\gamma_2 = 0.3$. π_t is then simulated from a Bernouilli$(1, p_t)$ distribution. Finally, the random level shift processes are generated with π_t and ε_t as for the previous RLS models where we set $\sigma_\varepsilon^2 = 0.75^2$ and $\sigma_\eta^2 = 0.4^2$.

In a similar fashion we can define a MS-GARCH-TVP model with time varying transition probabilities. The important feature of this model is that it allows for a higher probability to switch from a low ($s_t = 0$) to a high volatility regime ($s_t = 1$) when past returns are extremely negative and thereby stressing the leverage effect. Importantly, after such a break we wish to switch back to a low volatility regime with a similarly high probability. Hence volatility shifts and jumps are possible. The MS-GARCH-TVP model has time varying switching probabilities $p_{01,t}$ generated as in (16) with $p = p_{01}$. Moreover, we let $p_{10,t} = 1 - p_{01,t}$ to ensure that the process is likely to switch back to the low volatility regime ($s_t = 0$). The parameters are set as follows: $\omega_0 = 1, \omega_1 = 25, \alpha = 0.4, \beta = 0.3, p_{01} = p_{10} = 0.001$.

Table 5 about here

As can be seen from Table 5 the three processes have varying pseudo memory estimates with the RLS-NS-TVP model being relatively more persistent for the present experimental design. For this model the $W(\mathbf{J}, \delta_j)$ test is doing very well with excellent power, especially for $J = (2, (2,3), (2,4), (2,5))$. The QU test has also high power. Turning to the RLS-S-TVP model the same pattern is revealed for the $W(\mathbf{J}, \delta_j)$ test. However, in this case the test is clearly outperforming the QU test almost regardless the choice of J, i.e. for 9 out of 10 tests. Finally, for the MS-GARCH-TVP model 6 out of 10 $W(\mathbf{J}, \delta_j)$ tests have superior power compared with to the QU test. For all three TVP models especially the test with $J = 2$ has excellent power.

To conclude the power analysis it is seen that no test is uniformly most superior compared to other tests and the actual test performance depends upon the actual model being considered under the alternative. In the choice of an appropriate test design there is a general pattern however that the $W(\mathbf{J}, \delta_j)$ test with $J = 2$ is performing well for a broad range of models under the alternative. Hence we suggest to use this test in practical applications together with other tests with good power, the Qu (2011) test in particular.

9. Empirical illustration

To illustrate, we analyze three log realized volatility series and three inflation series displayed in Figure 10. The log realized volatility series are for the DJIA, FTSE-100 and SP stock index series using daily observations from January 3, 2000 to June 25, 2013, which yields a total of $T = 3413$ observations. The three US inflation series are CPI (Consumer Price Index for all Urban Consumers: all items), PPI (Producer

Price Index: finished goods), and PCE (Personal Consumption Expenditures). The inflation series are recorded monthly and are seasonally adjusted. CPI and PPI cover the period 1947-2014 yielding 807 and 804 observations respectively, whilst the PCE series covers the period 1959-2014, (662 observations).

For the realized volatility series the unfiltered local Whittle estimates of d are in the range [0.64-0.68] whilst the inflation series have estimates in the range [0.40-0.55]. When implementing the $W(\mathbf{J}, \delta_j)$ test the realized volatility series needed ARFIMA and GARCH pre-filtering. The inflation series were only GARCH filtered as the selected lag lengths for the ARMA component were (0,0). The decision to GARCH pre-filter the data was based on the Lagrange Multiplier tests for ARCH effects up to 12 lags and these tests strongly rejected the null of no ARCH effects. The filtered volatility series have estimates of d in the range $[0.39 - 0.48]$ whilst the GARCH-filtered inflation series have d estimates in the interval $[0.27 - 0.47]$. The filtered series appear to be approximately symmetrically distributed with some degree of excess kurtosis (in comparison to the normal distribution) in case of realized volatilities.

In Table 6 the test values for the Qu(0.02) and the $W(2, 0.225)$ tests are reported. The two tests provide rather contradicting evidence for the realized volatility series. Whilst the Qu test accepts the null for all the volatility series, the $W(2, 0.225)$ test rejects the null at a 1% level for the DJIA and SP indices and at a 5% level for the FTSE index. The evidence for the inflation series is more in line for the two tests: Both tests reject the null for the CPI series (at 1% and 5% levels, respectively), and cannot reject the null for the PPI series. The two tests are in conflict regarding the PCE series.

Figure 10 about here
Table 6 about here

10. Conclusion and extensions

A new procedure to discriminate between Gaussian fractional long memory and spurious long memory processes has been suggested. The testing procedure is based on the feature that non-linear transformations of (stationary) Gaussian fractional processes will reduce the memory when the Hermite rank of the transformation exceeds one. By appropriate difference filtering combined with a nonlinear transformation, the series can thus be made short memory. This property is used as a framework to test that an appropriately designed vector time series is short memory under the null hypothesis and we suggest using the Wald test of Shimotsu (2007). The test has a limiting χ^2 distribution under the null and by appropriate choice of tuning parameters the test is shown to have good empirical size properties. After appropriate pre-filtering the test is shown to be robust to the presence of serial correlation and conditional heteroscedasticity. The test is formulated such that, as opposed to competing tests in the literature, even non-stationary fractional processes are permitted under the null. The suggested test has good power against a variety of spurious long memory models that have been previously analyzed in the literature including stationary and non-stationary random level shift and Markov switching GARCH processes. Even though the power is comparable and outperforms competing tests in the literature, the test of Qu (2011) appears to be superior in many cases. However, if the regime switching or transition probabilities are allowed to be time varying the test is shown to have really good power and outperforms the Qu (2011) test in most cases. For practical applications our suggestion to use our new test in combination with the Qu (2011) test.

In principle, our testing approach can be extended to cover the case of multiple series in a straightforward manner. For instance, if one wants to test the null hypothesis of true (non-)stationary fractional integration for more than one single time series, then a multivariate testing framework is desirable. Our approach allows to consider multiple series in a simple way: If we would consider two series $x_t^{(1)}$ and $x_t^{(2)}$ and the Hermite rank $J = 2$, the resulting Wald statistic $W(2, \delta)$ would be build from the time series vector $Y_t = (\Delta^{\widehat{d}_1} x_t^{(1)}, \Delta^{\widehat{d}_2} x_t^{(2)}, H_2(y_t^{(1),2}, \delta), H_2(y_t^{(2),2}, \delta))$. Importantly, the same target values δ can still be used. A possible situation where such a setup is of interest is for instance the relation between international stock market volatilities. The bivariate test can be extended to a K-variate time series vector along the same lines. The number of zero-restrictions to be tested is then $2K$ and the limiting distribution of the corresponding Wald test is $\chi^2(2K)$. Thus, critical values from a standard distribution also applies in the multivariate setting. The originally proposed test by Qu (2011) is univariate. Obviously, a multivariate extension of Qu's test can be achieved by considering a multivariate local Whittle likelihood function and by deriving a similar statistic. The exact from of the test statistic is, however, more complicated to obtain and the limiting distribution will be non-standard (as in the univariate case). Estimation of the memory parameter can carried out via the Shimotsu (2007) estimator.

Recently there has been some focus on making inference on the long memory index whilst simultaneously allowing for zero frequency contamination of the process in the form of structural breaks or shifts, see e.g. McCloskey and Perron (2013) and Delle Monache *et al.* (2013) for nonparametric and parametric approaches, respectively. It should be noted that the class of models considered in the present paper does not allow for the joint presence of fractional long memory and a break or structural change component. It is not clear how one should proceed in this situation but a possible approach would be to first model possible break or level shift components if the test for fractional long memory initially rejects the null, and then test for remaining (fractional) long memory accounting for breaks, level shifts, etc. It remains for future research to further analyze such extensions.

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11. Tables and Figures

Table 1. Test rejection frequencies (empirical sizes) of $W(J,\delta_J)$ test at nominal 5% level

T	d	$W(2,0.225)$	$W(3,0.3)$	$W(4,0.35)$	$W(5,0.35)$	$W(23,0.2)$	$W(24,0.2)$	$W(25,0.2)$	$W(34,0.3)$	$W(35,0.275)$	$W(45,0.3)$
500	0,2	0,031	0,022	0,031	0,030	0,044	0,043	0,045	0,060	0,140	0,058
	0,25	0,032	0,032	0,037	0,030	0,040	0,042	0,042	0,050	0,136	0,050
	0,3	0,027	0,033	0,032	0,028	0,030	0,034	0,040	0,048	0,133	0,038
	0,35	0,026	0,032	0,029	0,035	0,035	0,034	0,036	0,062	0,136	0,052
	0,4	0,034	0,033	0,033	0,031	0,034	0,032	0,044	0,054	0,124	0,054
	0,45	0,036	0,036	0,032	0,040	0,038	0,047	0,044	0,046	0,127	0,051
	0,55	0,031	0,034	0,037	0,030	0,037	0,037	0,033	0,047	0,113	0,049
	0,6	0,024	0,024	0,035	0,037	0,042	0,048	0,043	0,043	0,117	0,054
	0,65	0,032	0,028	0,038	0,022	0,040	0,049	0,036	0,066	0,109	0,038
3000	0,2	0,054	0,047	0,048	0,054	0,049	0,041	0,051	0,060	0,069	0,066
	0,25	0,057	0,052	0,062	0,048	0,046	0,048	0,044	0,070	0,064	0,064
	0,3	0,055	0,053	0,060	0,050	0,042	0,048	0,052	0,062	0,057	0,062
	0,35	0,054	0,046	0,048	0,051	0,042	0,042	0,046	0,058	0,057	0,069
	0,4	0,050	0,043	0,054	0,049	0,048	0,045	0,044	0,060	0,062	0,056
	0,45	0,052	0,055	0,050	0,056	0,044	0,040	0,046	0,055	0,059	0,051
	0,55	0,056	0,047	0,047	0,050	0,049	0,046	0,054	0,064	0,064	0,060
	0,6	0,054	0,042	0,048	0,047	0,033	0,039	0,045	0,062	0,057	0,062
	0,65	0,056	0,044	0,059	0,062	0,053	0,057	0,053	0,069	0,062	0,062

Note: The table reports test rejection frequencies for the various tests based on 1000 Monte Carlo replications. The $W(J, \delta_J)$ test is the modified Shimotsu Wald test for design parameters J and δ_J .

Table 2. Finite sample power of tests at a 5% nominal level for experiment A, the parameter setting of Qu (2011).

											Other existing tests					
T	*W*(2,0.225)	*W*(3,0.3)	*W*(4,0.35)	*W*(5,0.35)	*W*(23,0.2)	*W*(24,0.2)	*W*(25,0.2)	*W*(34,0.3)	*W*(35,0.275)	*W*(45,0.3)	QU(0.02)	ORT	S-SPLIT	S-DIFF	PQ	*mean(d^)*
RLS-NS: Nonstationary Random Level Shift Model																
500	0,03	0,04	0,03	0,04	0,04	0,06	0,06	0,06	0,11	0,06	0,19	0,09	0,17	**0,37**	0,19	0,27
1000	0,07	0,06	0,05	0,06	0,05	0,05	0,06	0,09	0,12	0,08	0,27	0,11	0,26	**0,48**	0,25	0,28
3000	0,26	0,17	0,16	0,09	0,14	0,16	0,16	0,19	0,14	0,15	**0,76**	0,26	0,51	0,74	0,51	0,30
5000	0,42	0,26	0,31	0,14	0,20	0,22	0,20	0,28	0,18	0,20	**0,92**	0,34	0,76	0,84	0,64	0,29
7000	0,54	0,47	0,49	0,21	0,30	0,30	0,30	0,39	0,24	0,27	**0,97**	0,44	0,75	0,89	0,74	0,31
9000	0,57	0,53	0,61	0,25	0,36	0,38	0,37	0,53	0,30	0,42	**0,98**	0,53	0,81	0,93	0,77	0,31
RLS-S: Stationary Random Level Shift Model																
500	0,07	0,04	0,03	0,05	0,05	0,07	0,08	0,09	0,12	0,07	0,21	0,08	**0,33**	0,32	0,14	0,21
1000	0,19	0,13	0,08	0,07	0,12	0,17	0,16	0,14	0,15	0,12	0,44	0,10	**0,60**	0,42	0,19	0,33
3000	0,58	0,34	0,24	0,15	0,41	0,43	0,42	0,33	0,30	0,22	**0,92**	0,11	0,82	0,33	0,15	0,48
5000	0,67	0,40	0,29	0,18	0,52	0,53	0,54	0,39	0,34	0,28	**0,98**	0,10	0,76	0,21	0,07	0,55
7000	0,74	0,43	0,36	0,20	0,60	0,63	0,62	0,46	0,44	0,33	**1,00**	0,14	0,71	0,13	0,04	0,57
9000	0,72	0,42	0,36	0,21	0,64	0,64	0,67	0,45	0,42	0,36	**1,00**	0,16	0,66	0,08	0,02	0,58
MONO: White Noise with a Monotonic Trend																
500	0,05	0,04	0,03	0,04	0,04	0,06	0,06	0,07	0,12	0,06	0,02	0,06	0,10	**0,37**	0,08	0,12
1000	0,11	0,07	0,07	0,05	0,09	0,10	0,10	0,11	0,13	0,09	0,03	0,07	0,12	**0,43**	0,14	0,14
3000	0,52	0,32	0,18	0,10	0,32	0,32	0,33	0,31	0,24	0,17	0,13	0,13	0,22	**0,91**	0,32	0,15
5000	0,81	0,63	0,34	0,19	0,49	0,50	0,50	0,53	0,38	0,27	0,44	0,19	0,35	**0,99**	0,44	0,16
7000	0,93	0,71	0,40	0,24	0,70	0,71	0,69	0,73	0,59	0,38	0,77	0,24	0,51	**1,00**	0,55	0,16
9000	0,97	0,82	0,55	0,25	0,83	0,83	0,82	0,85	0,68	0,43	0,93	0,30	0,62	**1,00**	0,62	0,17
NONMONO: White Noise with a Nonmonotonic Trend																
500	0,02	0,02	0,02	0,04	0,03	0,03	0,04	0,04	0,13	0,03	**0,83**	0,09	0,05	0,01	0,11	0,33
1000	0,01	0,02	0,02	0,02	0,03	0,02	0,03	0,04	0,10	0,02	**0,95**	0,09	0,03	0,00	0,17	0,33
3000	0,03	0,07	0,09	0,03	0,01	0,02	0,02	0,07	0,08	0,06	**1,00**	0,12	0,01	0,06	0,27	0,34
5000	0,03	0,19	0,27	0,07	0,02	0,03	0,03	0,20	0,12	0,12	**1,00**	0,15	0,00	0,48	0,34	0,34
7000	0,04	0,39	0,59	0,13	0,03	0,02	0,02	0,39	0,16	0,20	**1,00**	0,16	0,00	0,95	0,39	0,34
9000	0,05	0,58	0,78	0,24	0,02	0,03	0,02	0,61	0,22	0,35	**1,00**	0,17	0,00	**1,00**	0,43	0,34
MS: Markov Switching with iid regimes																
500	0,10	0,07	0,07	0,06	0,07	0,07	0,08	0,10	0,14	0,09	0,17	0,07	**0,32**	0,21	0,07	0,17
1000	0,20	0,12	0,12	0,09	0,12	0,14	0,16	0,12	0,16	0,12	0,46	0,09	**0,59**	0,36	0,16	0,30
3000	0,59	0,33	0,18	0,14	0,45	0,43	0,45	0,33	0,28	0,22	**0,91**	0,12	0,89	0,47	0,19	0,48
5000	0,76	0,35	0,17	0,13	0,51	0,53	0,55	0,32	0,29	0,18	**0,98**	0,10	0,79	0,41	0,15	0,52
7000	0,81	0,31	0,13	0,09	0,61	0,65	0,63	0,32	0,28	0,15	**1,00**	0,09	0,66	0,32	0,08	0,55
9000	0,83	0,30	0,15	0,11	0,65	0,66	0,64	0,28	0,22	0,13	**1,00**	0,08	0,51	0,22	0,05	0,57
MS-GARCH: Markov Switching with GARCH regimes																
500	0,04	0,02	0,15	0,03	0,09	0,09	0,06	0,45	0,08	**0,33**	0,02	0,07	0,12	0,08	0,06	0,12
1000	0,07	0,03	0,17	0,05	0,11	0,09	0,05	0,51	0,06	**0,54**	0,06	0,07	0,11	0,20	0,13	0,13
3000	0,14	0,04	0,30	0,05	0,16	0,14	0,08	0,69	0,10	**0,86**	0,47	0,15	0,20	0,42	0,34	0,17
5000	0,15	0,04	0,37	0,05	0,18	0,17	0,09	0,86	0,12	**0,93**	0,78	0,21	0,27	0,47	0,45	0,19
7000	0,19	0,05	0,51	0,08	0,17	0,22	0,12	0,90	0,17	**0,96**	0,92	0,23	0,29	0,48	0,47	0,20
9000	0,18	0,06	0,51	0,06	0,14	0,21	0,10	0,94	0,18	**0,99**	0,98	0,26	0,30	0,47	0,48	0,21

Note: The table reports test rejection frequencies for the various tests based on 1000 Monte Carlo replications. The W(ℓ, δ_J) test is the modified Shimotsu Wald test for design parameters δJ and J. QU(0.02) is the Qu (2011) test with trimming porpotion ε=0.02, ORT is the test of Ohanissian, Russell, and Tsay (2008), S-SPLIT and S-DIFF are the split and difference tests of Shimotsu (2006), and finally PQ is the Perron and Qu (2010) test. Bold face values indicate the tests with the largest power for a given configuration.

Table 3. Finite sample power of tests at a 5% nominal level for experiment B, parameter setting with high persistency.

T	$W(2,0.225)$	$W(3,0.3)$	$W(4,0.35)$	$W(5,0.35)$	$W(23,0.2)$	$W(24,0.2)$	$W(25,0.2)$	$W(34,0.3)$	$W(35,0.275)$	$W(45,0.3)$	QU(0.02)	mean(d^)
RLS-NS: Nonstationary Random Level Shift Model												
500	0,05	0,03	0,03	0,03	0,05	0,07	0,09	0,09	0,13	0,06	**0,30**	0,33
1000	0,11	0,08	0,07	0,04	0,07	0,09	0,09	0,08	0,12	0,07	**0,46**	0,36
3000	0,37	0,20	0,19	0,09	0,20	0,23	0,22	0,22	0,19	0,16	**0,85**	0,37
5000	0,51	0,34	0,35	0,16	0,34	0,33	0,34	0,37	0,28	0,24	**0,94**	0,36
7000	0,62	0,49	0,47	0,22	0,42	0,43	0,42	0,49	0,33	0,33	**0,98**	0,36
9000	0,69	0,56	0,55	0,29	0,49	0,46	0,47	0,57	0,39	0,39	**0,99**	0,38
RLS-S: Stationary Random Level Shift Model												
500	0,10	0,09	0,07	0,04	0,14	0,12	0,16	0,10	0,15	0,09	**0,24**	0,41
1000	0,25	0,11	0,10	0,10	0,19	0,22	0,20	0,16	0,18	0,11	**0,50**	0,42
3000	0,51	0,34	0,22	0,16	0,46	0,46	0,48	0,36	0,33	0,26	**0,90**	0,44
5000	0,68	0,52	0,36	0,17	0,55	0,54	0,55	0,49	0,40	0,32	**0,97**	0,44
7000	0,77	0,63	0,46	0,27	0,64	0,64	0,66	0,61	0,51	0,41	**0,98**	0,45
9000	0,82	0,72	0,61	0,32	0,74	0,74	0,74	0,74	0,63	0,51	**0,99**	0,45
MONO: White Noise with a Monotonic Trend												
500	0,51	0,32	0,28	0,22	0,38	0,53	0,54	**0,55**	0,53	0,48	0,01	0,40
1000	**0,91**	0,73	0,73	0,46	0,72	0,86	0,86	0,89	0,85	0,82	0,01	0,40
3000	**1,00**	**1,00**	0,99	0,88	0,99	**1,00**	**1,00**	**1,00**	**1,00**	0,99	0,14	0,40
5000	**1,00**	**1,00**	**1,00**	0,97	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	0,57	0,40
7000	**1,00**	**1,00**	**1,00**	0,98	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	0,88	0,40
9000	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	0,97	0,40
NONMONO: White Noise with a Nonmonotonic Trend												
500	0,02	0,03	0,03	0,02	0,03	0,03	0,03	0,05	0,14	0,03	**0,89**	0,25
1000	0,02	0,02	0,02	0,01	0,03	0,03	0,03	0,03	0,07	0,02	**1,00**	0,36
3000	0,03	0,02	0,01	0,02	0,03	0,03	0,03	0,02	0,03	0,02	**1,00**	0,42
5000	0,03	0,02	0,02	0,02	0,03	0,03	0,03	0,02	0,02	0,02	**1,00**	0,44
7000	0,02	0,03	0,02	0,01	0,03	0,03	0,04	0,03	0,03	0,02	**1,00**	0,44
9000	0,02	0,03	0,02	0,02	0,03	0,03	0,03	0,03	0,02	0,02	**1,00**	0,44
MS: Markov Switching with iid regimes												
500	0,02	0,03	0,03	0,02	0,04	0,04	0,03	0,06	**0,12**	0,04	0,07	0,37
1000	0,03	0,04	0,03	0,03	0,04	0,03	0,03	0,06	0,09	0,03	**0,15**	0,41
3000	0,13	0,08	0,04	0,03	0,11	0,11	0,11	0,10	0,08	0,05	**0,78**	0,44
5000	0,30	0,15	0,07	0,05	0,19	0,19	0,18	0,15	0,09	0,05	**0,97**	0,45
7000	0,43	0,22	0,09	0,04	0,28	0,29	0,30	0,20	0,12	0,08	**1,00**	0,45
9000	0,59	0,27	0,10	0,05	0,41	0,39	0,38	0,27	0,13	0,08	**1,00**	0,45
MS-GARCH: Markov Switching with GARCH regimes												
500	0,18	0,03	0,13	0,04	0,29	0,23	0,16	**0,49**	0,07	0,42	0,01	0,34
1000	0,21	0,03	0,14	0,03	0,23	0,15	0,10	0,56	0,05	**0,67**	0,01	0,35
3000	0,17	0,02	0,25	0,03	0,24	0,19	0,12	0,75	0,09	**0,90**	0,04	0,34
5000	0,17	0,05	0,39	0,05	0,25	0,23	0,11	0,84	0,15	**0,93**	0,14	0,33
7000	0,20	0,06	0,53	0,07	0,25	0,26	0,11	0,87	0,18	**0,95**	0,30	0,32
9000	0,17	0,09	0,47	0,05	0,25	0,25	0,13	0,96	0,21	**0,96**	0,47	0,31

Note: The table reports test rejection frequencies for the various tests based on 1000 Monte Carlo replications. The W(J, δ_J) test is the modified Shimotsu Wald test for design parameters J and δ_J. QU(0.02) is the Qu (2011) test with trimming proportion ε=0.02. Bold face values indicate the tests with maximal

Table 4. Finite sample power of tests at a 5% nominal level for experiment C, the parameter setting of Qu (2011) modified to have AR errors.

T	*W* (2,0.225)	*W* (3,0.3)	*W* (4,0.35)	*W* (5,0.35)	*W* (23,0.2)	*W* (24,0.2)	*W* (25,0.2)	*W* (34,0.3)	*W* (35,0.275)	*W* (45,0.3)	QU(0.02)	mean(d^)
RLS-NS: Nonstationary Random Level Shift Model												
500	0,02	0,02	0,05	0,03	0,03	0,03	0,05	0,06	**0,13**	0,05	0,00	0,47
1000	0,03	0,02	0,05	0,04	0,04	0,06	0,05	0,05	**0,10**	0,07	0,01	0,38
3000	0,10	0,07	0,09	0,07	0,05	0,05	0,04	0,08	0,07	0,06	**0,14**	0,28
5000	0,19	0,13	0,16	0,10	0,06	0,08	0,07	0,15	0,10	0,12	**0,35**	0,25
7000	0,26	0,20	0,24	0,13	0,10	0,10	0,10	0,21	0,11	0,13	**0,49**	0,25
9000	0,38	0,26	0,35	0,19	0,14	0,14	0,13	0,26	0,13	0,18	**0,61**	0,23
RLS-S: Stationary Random Level Shift Model												
500	0,03	0,04	0,02	0,04	0,03	0,05	0,04	0,05	**0,13**	0,06	0,01	0,46
1000	0,06	0,04	0,03	0,04	0,05	0,08	**0,09**	0,07	**0,09**	0,06	0,03	0,42
3000	0,28	0,12	0,09	0,06	0,20	0,23	0,22	0,15	0,14	0,09	**0,32**	0,42
5000	0,53	0,19	0,11	0,08	0,34	0,38	0,39	0,25	0,24	0,15	**0,61**	0,41
7000	0,59	0,27	0,20	0,10	0,45	0,51	0,51	0,31	0,28	0,18	**0,79**	0,40
9000	0,70	0,29	0,21	0,12	0,54	0,60	0,58	0,31	0,31	0,20	**0,89**	0,41
MONO: White Noise with a Monotonic Trend												
500	0,03	0,03	0,03	0,02	0,05	0,05	0,05	0,05	**0,07**	0,06	0,00	0,41
1000	0,04	0,04	0,04	0,03	0,03	0,04	0,04	0,04	**0,08**	0,06	0,01	0,34
3000	**0,13**	0,05	0,07	0,05	0,05	0,06	0,07	0,07	0,08	0,06	0,01	0,22
5000	**0,17**	0,07	0,07	0,06	0,11	0,13	0,13	0,12	0,10	0,09	0,02	0,18
7000	**0,24**	0,11	0,11	0,06	0,10	0,12	0,13	0,13	0,08	0,08	0,05	0,16
9000	**0,31**	0,16	0,11	0,07	0,14	0,13	0,11	0,15	0,09	0,09	0,06	0,15
NONMONO: White Noise with a Nonmonotonic Trend												
500	0,02	0,03	0,02	0,02	0,03	0,03	0,05	0,07	**0,11**	0,06	0,01	0,48
1000	0,02	0,03	0,03	0,04	0,04	0,04	0,04	0,04	**0,09**	0,05	0,03	0,39
3000	0,06	0,03	0,05	0,04	0,02	0,02	0,03	0,06	0,09	0,06	**0,23**	0,30
5000	0,09	0,06	0,07	0,05	0,04	0,04	0,03	0,11	0,08	0,07	**0,55**	0,27
7000	0,11	0,15	0,15	0,07	0,07	0,05	0,05	0,11	0,07	0,09	**0,79**	0,25
9000	0,14	0,17	0,24	0,08	0,05	0,06	0,05	0,19	0,10	0,14	**0,91**	0,24
MS: Markov Switching with iid regimes												
500	0,05	0,03	0,04	0,02	0,06	0,07	0,07	0,10	**0,11**	0,07	0,01	0,45
1000	0,09	0,04	0,05	0,05	0,07	0,10	0,08	0,08	**0,11**	0,07	0,03	0,42
3000	0,22	0,14	0,10	0,07	0,12	0,14	0,12	0,13	0,10	0,08	**0,41**	0,40
5000	0,25	0,18	0,10	0,06	0,19	0,21	0,18	0,16	0,12	0,08	**0,75**	0,40
7000	0,21	0,19	0,13	0,06	0,20	0,24	0,23	0,18	0,13	0,07	**0,91**	0,40
9000	0,24	0,23	0,10	0,03	0,17	0,19	0,20	0,22	0,15	0,06	**0,98**	0,40
MS-GARCH: Markov Switching with GARCH regimes												
500	0,13	0,04	0,07	0,02	0,22	0,18	0,11	0,24	0,07	**0,51**	0,00	0,43
1000	0,16	0,09	0,06	0,02	0,28	0,24	0,12	0,32	0,15	**0,50**	0,00	0,40
3000	0,34	0,32	0,15	0,07	0,57	0,45	0,27	**0,58**	0,38	0,46	0,01	0,32
5000	0,51	0,55	0,29	0,17	0,69	0,59	0,38	**0,73**	0,54	0,53	0,02	0,30
7000	0,63	0,65	0,38	0,23	0,85	0,74	0,51	**0,86**	0,70	0,58	0,06	0,26
9000	0,72	0,73	0,43	0,28	**0,93**	0,84	0,64	**0,93**	0,80	0,57	0,12	0,25

Note: The table reports test rejection frequencies for the various tests based on 1000 Monte Carlo replications. The $W(J, \delta_J)$ test is the modified Shimotsu Wald test for design parameters J and δ_J. QU(0.02) is the Qu (2011) test with trimming proportion ε=0.02. Bold face values indicate the tests with maximal power for a given configuration.

Table 5. Finite sample power of tests at a 5% nominal level for time varying parameter models.

T	*W* (2,0.225)	*W* (3,0.3)	*W* (4,0.35)	*W* (5,0.35)	*W* (23,0.2)	*W* (24,0.2)	*W* (25,0.2)	*W* (34,0.3)	*W* (35,0.275)	*W* (45,0.3)	QU(0.02)	*mean(d^)*
RLS-NS-TVP: Nonstationary Time Varying Parameter Random Level Shift Model												
500	**0,75**	0,24	0,17	0,12	0,74	0,72	0,72	0,33	0,26	0,25	0,44	0,37
1000	**0,98**	0,36	0,28	0,16	0,95	0,95	0,94	0,47	0,40	0,34	0,69	0,41
3000	**1,00**	0,53	0,56	0,31	**1,00**	**1,00**	**1,00**	0,72	0,58	0,58	0,92	0,52
5000	**1,00**	0,50	0,64	0,32	**1,00**	**1,00**	**1,00**	0,80	0,64	0,70	0,99	0,57
7000	**1,00**	0,52	0,73	0,41	**1,00**	**1,00**	**1,00**	0,84	0,68	0,76	**1,00**	0,58
9000	**1,00**	0,56	0,76	0,42	**1,00**	**1,00**	**1,00**	0,86	0,68	0,81	**1,00**	0,61
RLS-S-TVP: Stationary Time Varying Parameter Random Level Shift Model												
500	**0,79**	0,27	0,18	0,13	0,74	0,73	0,71	0,32	0,30	0,25	0,28	0,23
1000	**0,97**	0,40	0,31	0,19	0,95	0,94	0,94	0,49	0,40	0,34	0,43	0,27
3000	**1,00**	0,52	0,54	0,32	**1,00**	**1,00**	**1,00**	0,79	0,62	0,58	0,52	0,35
5000	**1,00**	0,55	0,62	0,31	**1,00**	**1,00**	**1,00**	0,85	0,68	0,70	0,53	0,37
7000	**1,00**	0,59	0,71	0,36	**1,00**	**1,00**	**1,00**	0,91	0,72	0,76	0,50	0,39
9000	**1,00**	0,55	0,77	0,37	**1,00**	**1,00**	**1,00**	0,93	0,74	0,81	0,52	0,40
MS-GARCH-TVP : Time Varying Parameter Markov Switching with GARCH regimes												
500	0,12	0,02	0,10	0,04	0,17	0,15	0,10	**0,37**	0,08	0,41	0,05	0,23
1000	0,23	0,04	0,13	0,02	0,26	0,24	0,14	0,37	0,08	**0,57**	0,13	0,23
3000	0,49	0,12	0,20	0,04	0,63	0,57	0,36	0,55	0,18	**0,75**	0,34	0,30
5000	0,62	0,15	0,27	0,04	0,69	0,71	0,45	0,57	0,17	**0,85**	0,44	0,32
7000	0,66	0,16	0,33	0,04	0,74	0,78	0,49	0,58	0,17	**0,89**	0,53	0,34
9000	0,70	0,17	0,36	0,02	0,80	0,78	0,56	0,57	0,14	**0,93**	0,55	0,33

Note: The table reports test rejection frequencies for the various tests based on 1000 Monte Carlo replications. The $W(J, \delta_J)$ test is the modified Shimotsu Wald test for design parameters δJ and J. QU(0.02) is the Qu (2011) test with trimming porpotion ε=0.02. Bold face values indicate the tests with the largest power for a given configuration.

Table 6. Testing Fractional Integration against spurious (non-fractional) long memory for log realized volatility and inflation time series

	QU(0.02)	W(2,0.225)
Log Realized Volatility		
DJIA	0,642	15,657 ***
FTSE	0,827	8,725 **
SP	0,656	15,864 ***
Inflation		
CPI	1,539 ***	16,514 **
PCE	1,577 ***	1,564
PPI	0,849	2,054

Note: The log realized volatility series are sampled 2000-01-03 to 2013-06-25, T=3414.
Source: Oxford-Man realized library.
Inflation series: CPI, monthly, percentage change, 1947-01 to 2014-04, T=807.
PPI, monthly, percentage change, 1947-04 to 2014-04, T=804
Source: US Department of Labor: Bureau of Labor Statistics.
PCE, monthly, percentage change, 1959-01 to 2014-03, T=662
Source: US Department of Commerce: Bureau of Economic Analysis.
***, **, * indicates significance at 1%,5%, and 10%.

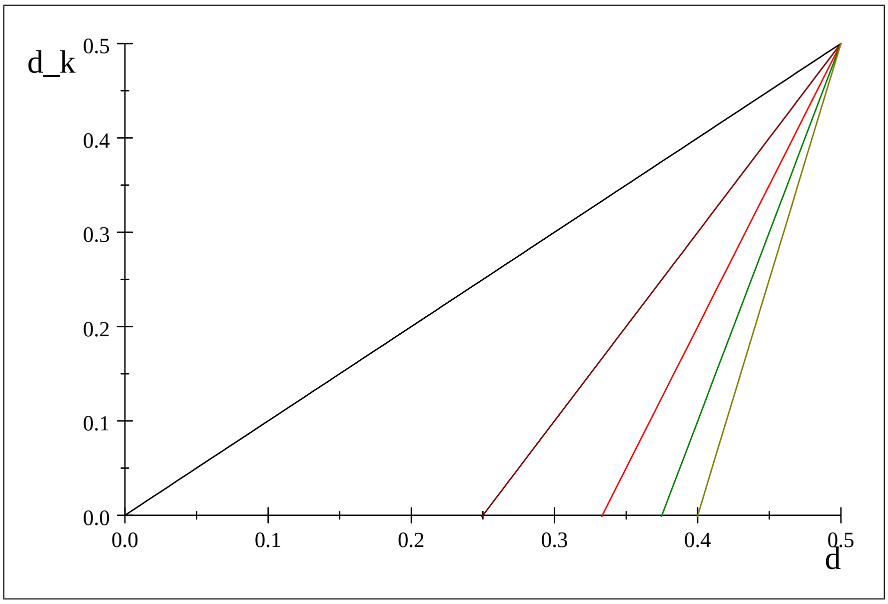

Figure 1: Plot of the function $d_J = \max\{0, (d-0.5)J + 0.5\}$ for transformations with Hermite ranks $J = 1, 2, 3, 4, 5$.

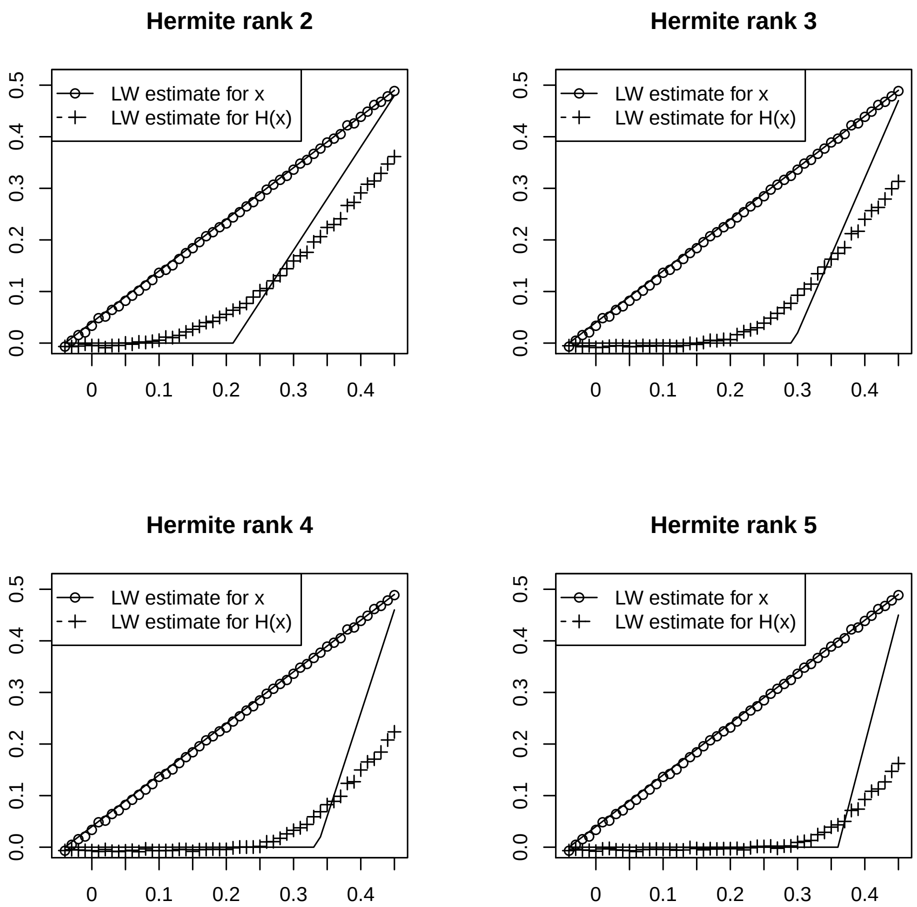

Figure 2: Mean values of local Whittle estimates of FI(d) processes, x-axis, and the estimate of d(J), y-axis, for Hermite ranks 2-5 and T=500. The curves are based on 2000 MC replications.

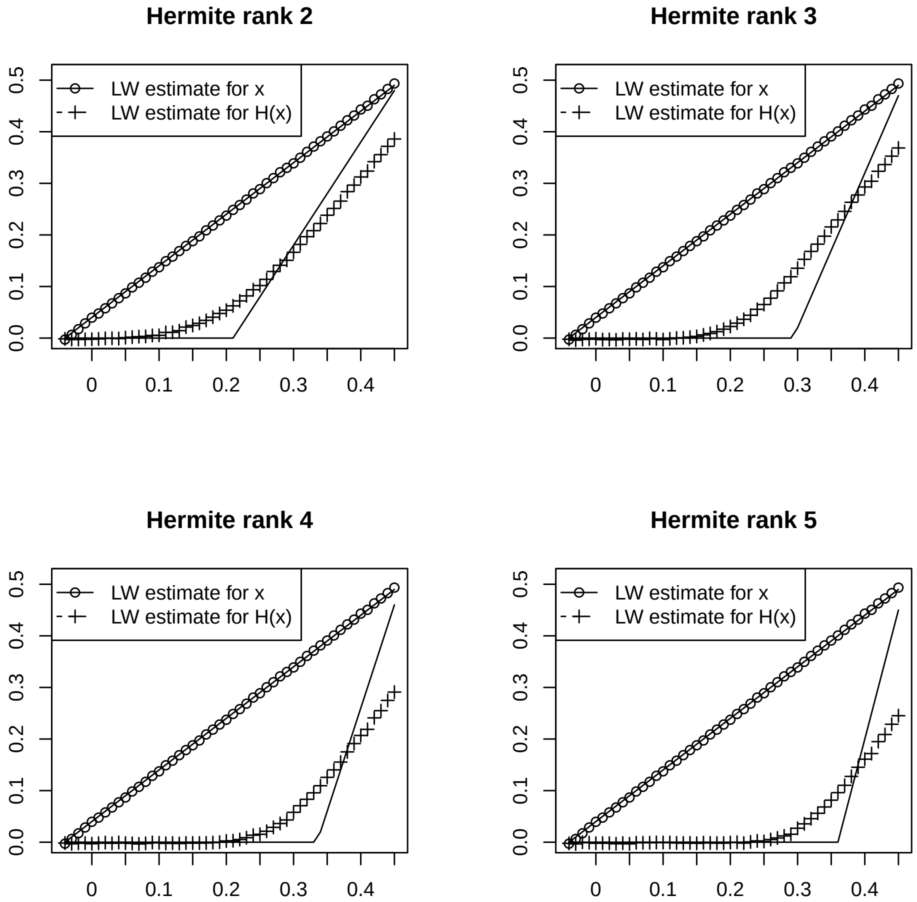

Figure 3: Mean values of local Whittle estimates of FI(d) processes, x-axis, and the estimate of d(J), y-axis, for Hermite ranks 2-5 and T=3000. The curves are based on 2000 MC replications.

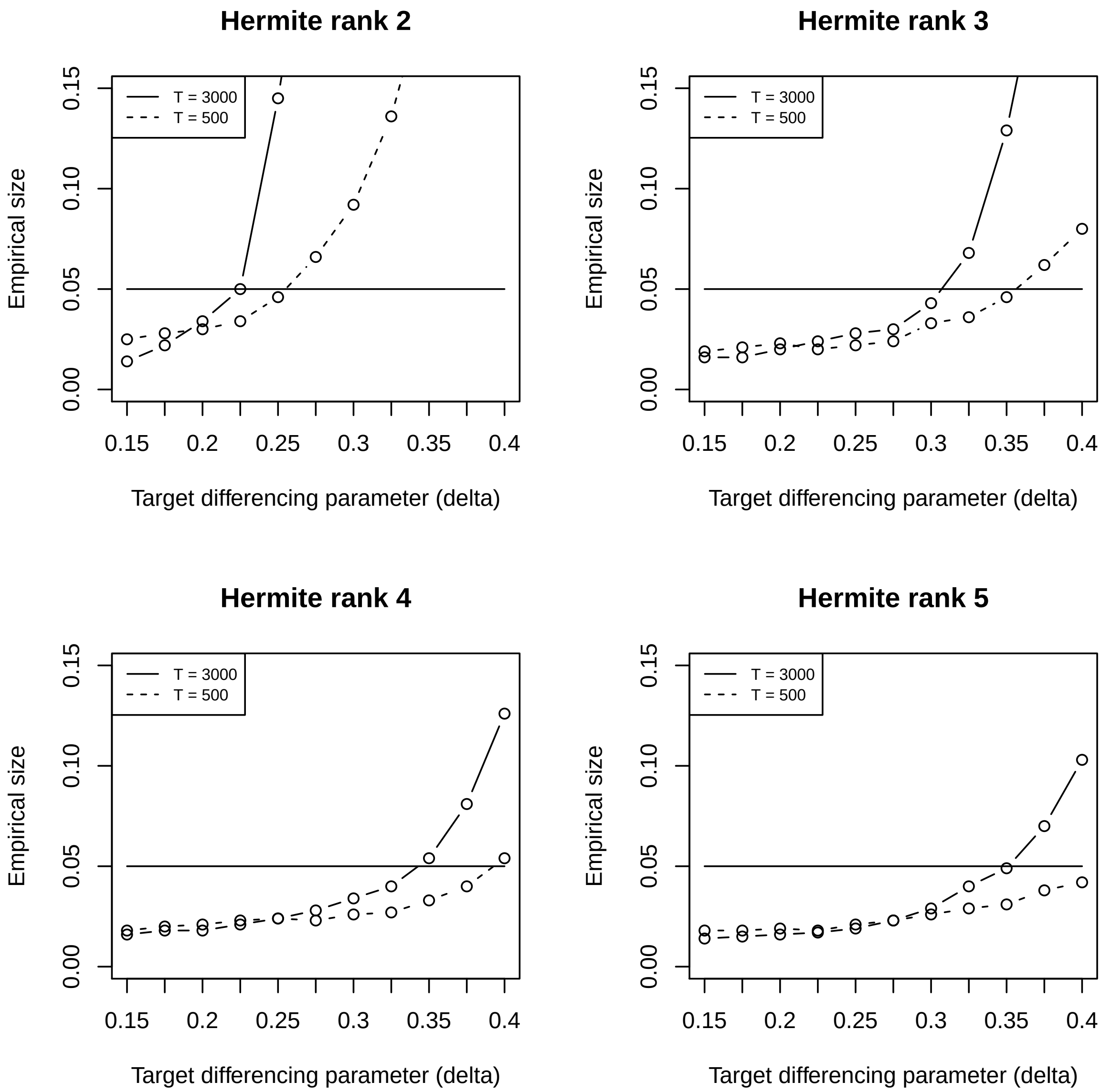

Figure 4: Size of $W(\delta_J, J)$ test as a function of δ for Hermite ranks J=2,3,4,5. The sample sizes are T=500 and T=3000. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID(0,1) are displayed.

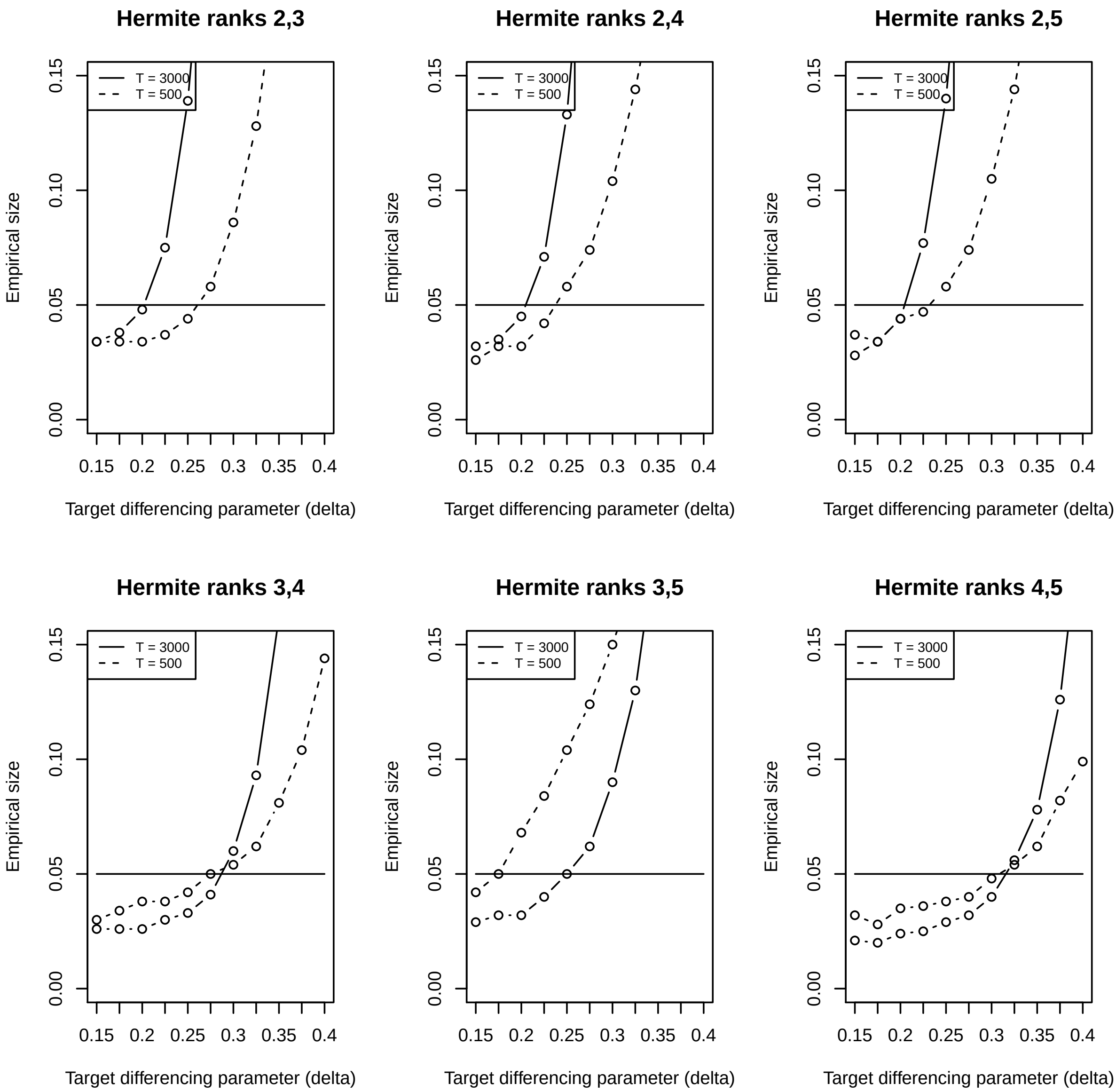

Figure 5: Size of $W(\delta_J, J)$ test as a function of δ for pairwise Hermite rank combinations of J=2,3,4,5. The sample sizes are T=500 and T=3000. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID(0,1) are displayed.

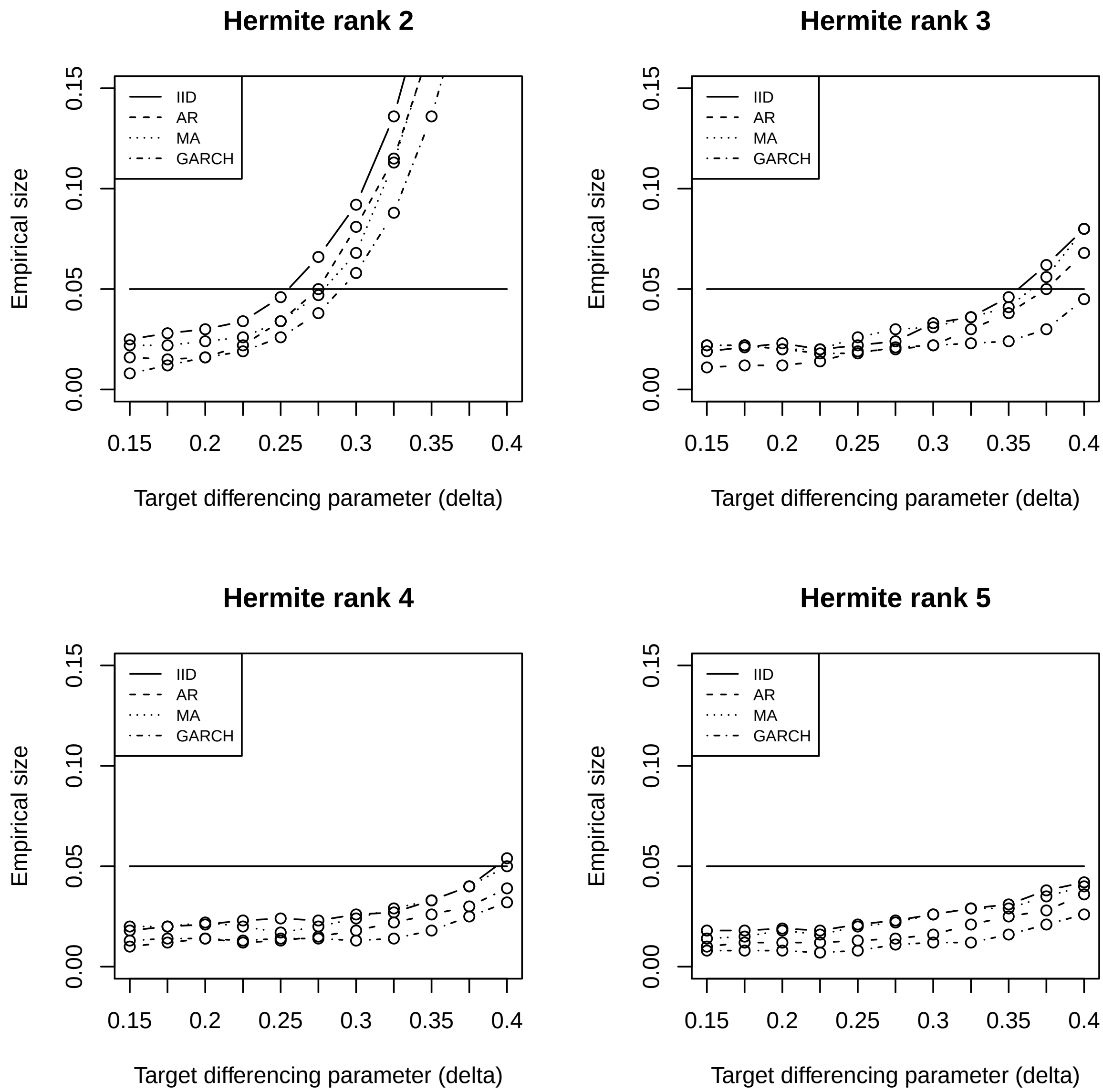

Figure 6: Size of $W(\delta_J, J)$ test as a function of δ for Hermite ranks J=2,3,4,5. The sample size is T=500. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID, AR(1), MA(1) and GARCH(1,1) are displayed.

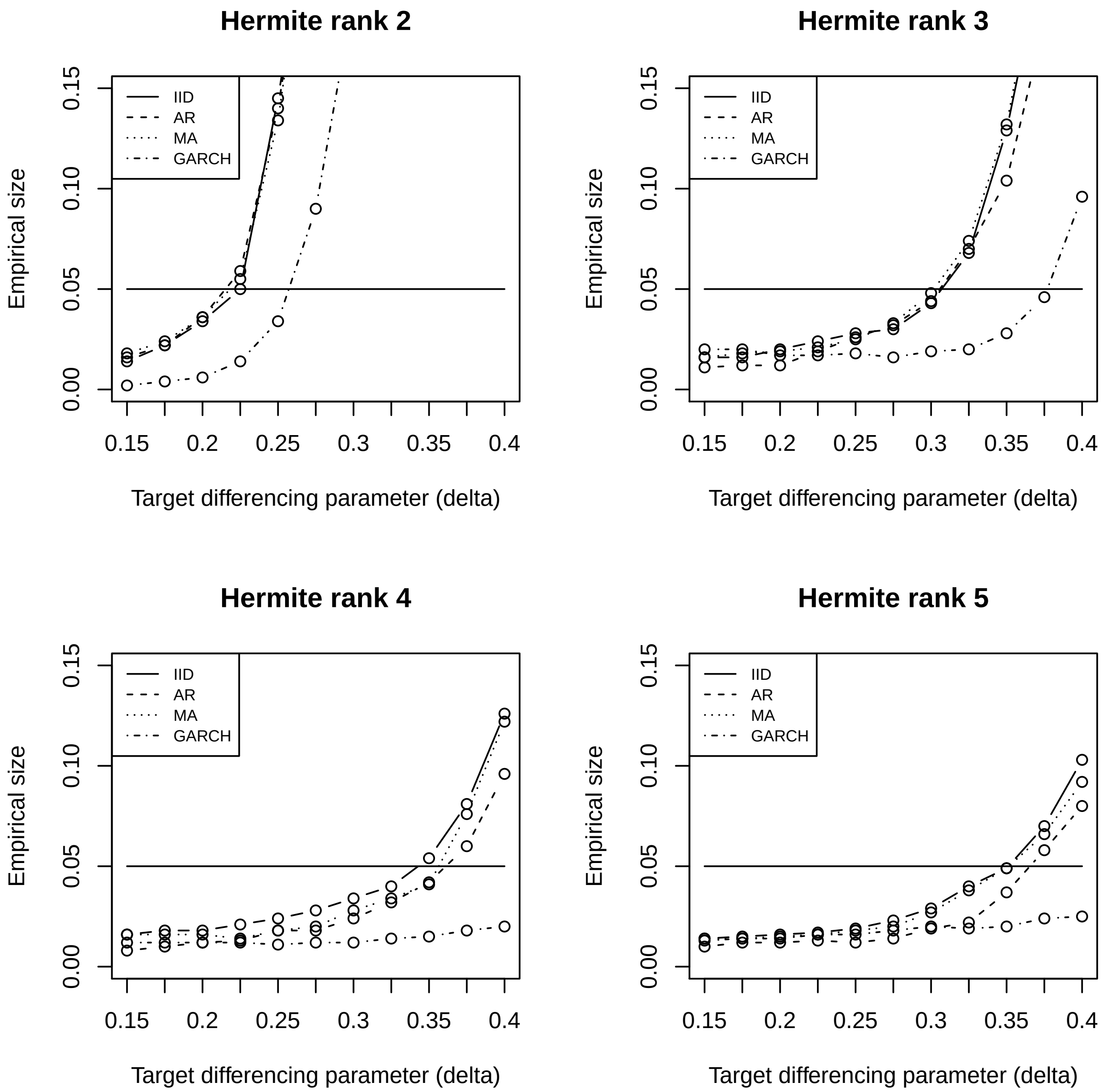

Figure 7: Size of $W(\delta_J, J)$ test as a function of δ for Hermite ranks J=2,3,4,5. The sample size is T=3000. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID, AR(1), MA(1) and GARCH(1,1) are displayed.

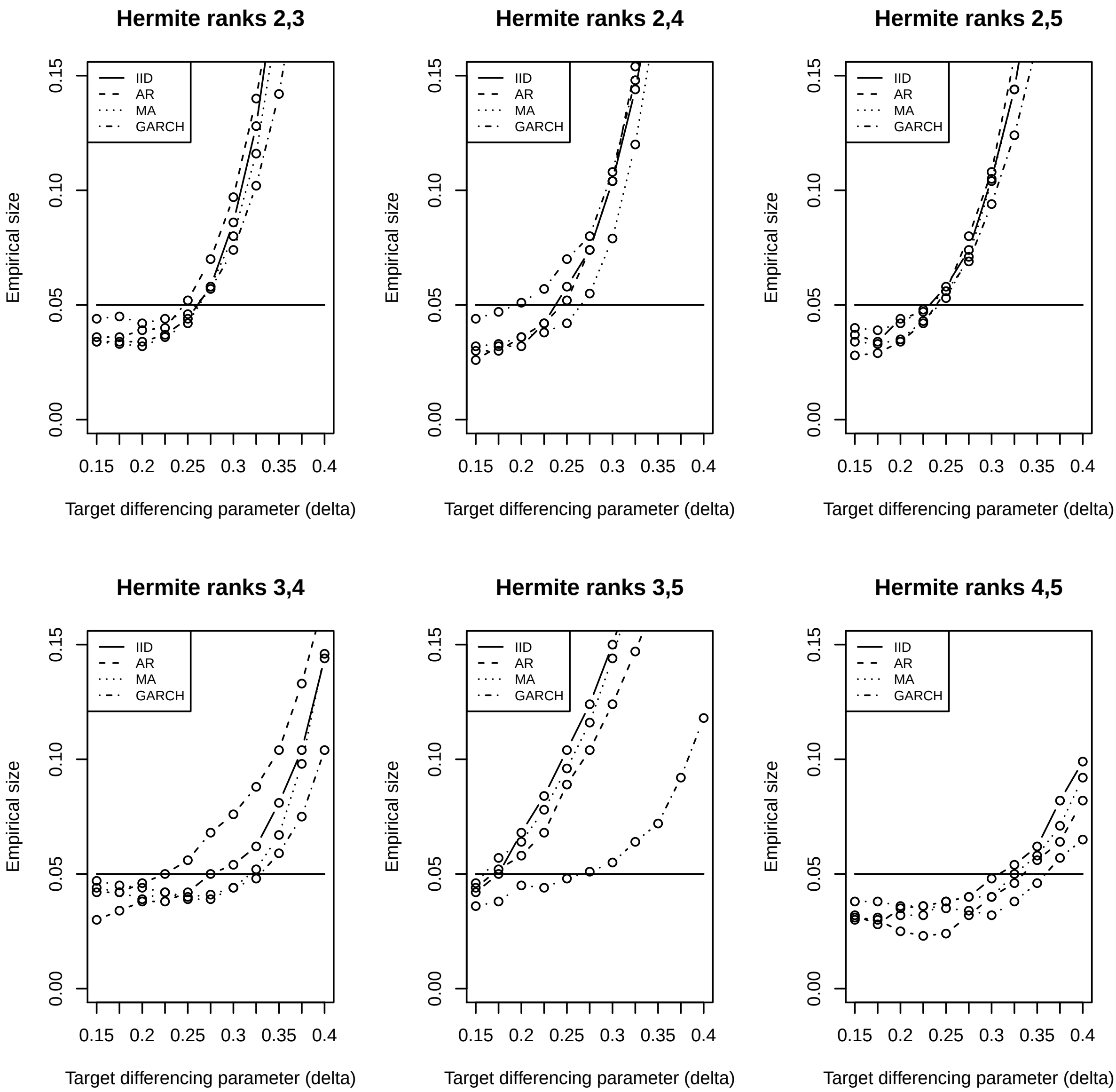

Figure 8: Size of $W(\delta_J, J)$ test as a function of δ for pairwise Hermite rank combinations of J=2,3,4,5. The sample size is T=500. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID, AR(1), MA(1) and GARCH(1,1) are displayed.

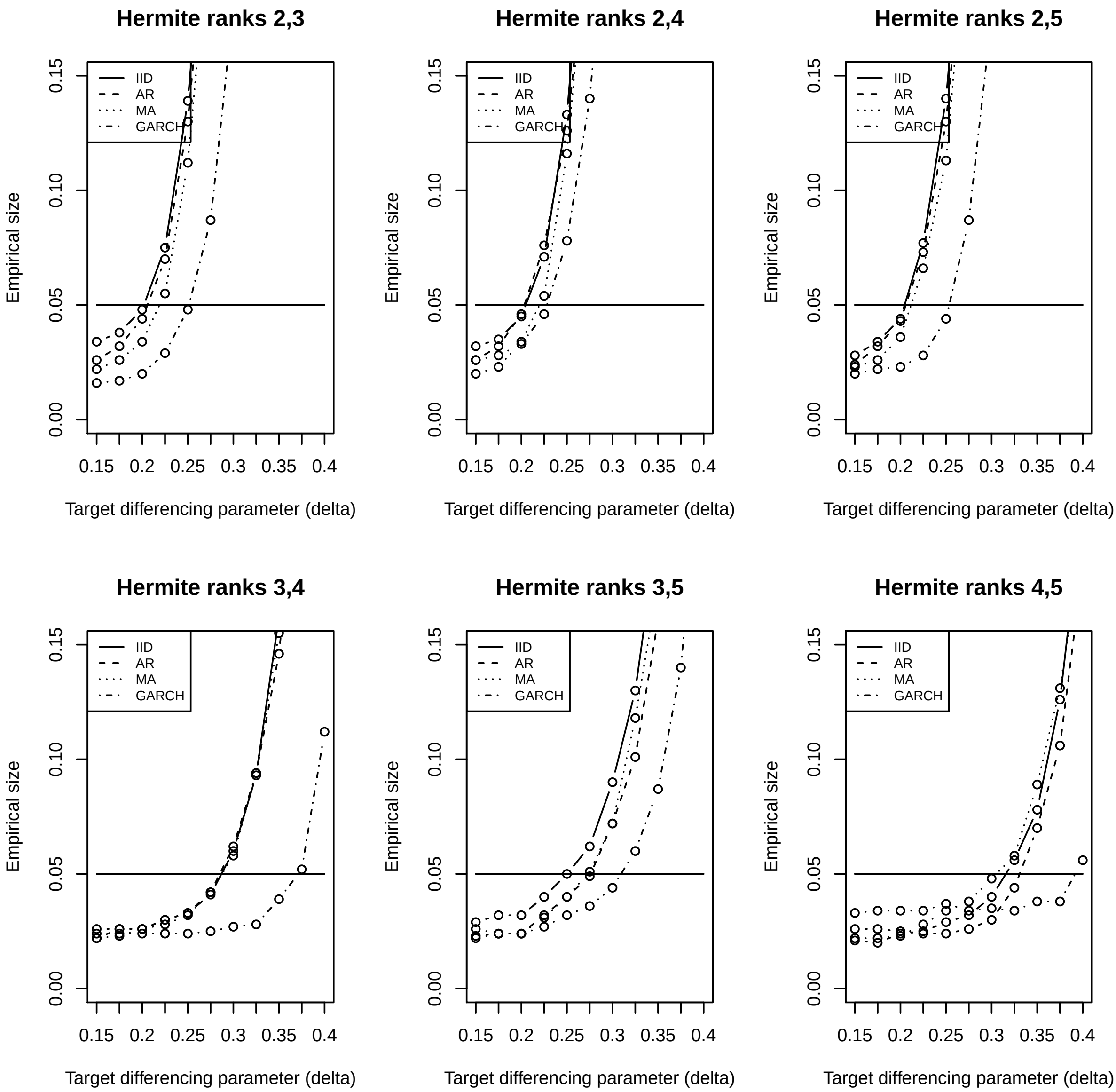

Figure 9: Size of $W(\delta_J, J)$ test as a function of δ for pairwise Hermite rank combinations of J=2,3,4,5. The sample size is T=3000. x_t is $FI(d)$ with d=0.40. Sizes for errors being NID, AR(1), MA(1) and GARCH(1,1) are displayed.

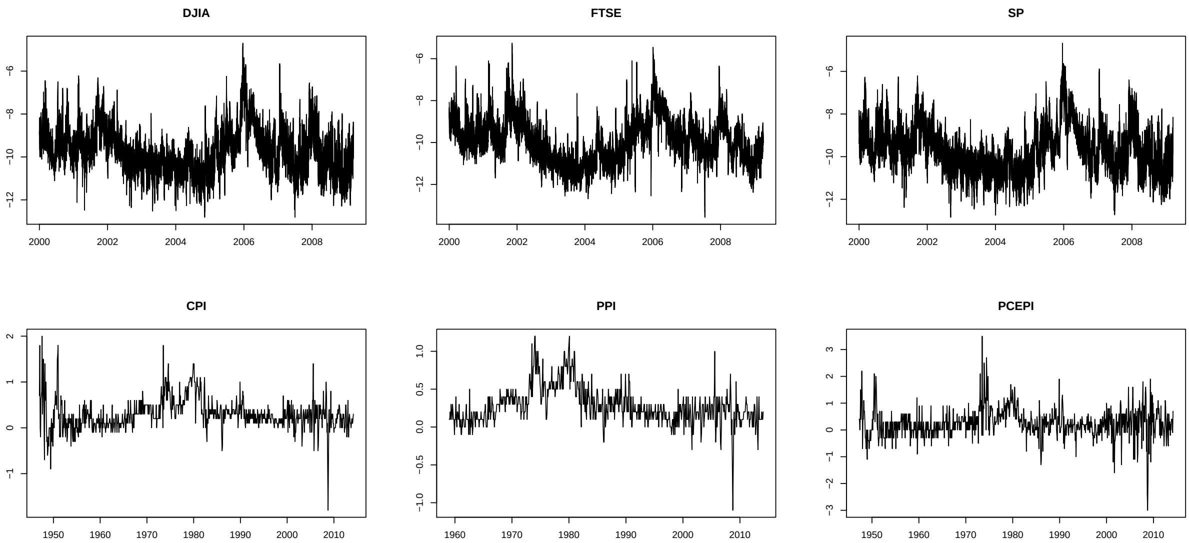

Figure 10: Log realized volatility series for DJIA, FTSE-100 and SP stock index series using daily observations from January 3, 2000 to June 25, 2013. The three US inflation series are CPI (Consumer Price Index for all Urban Consumers: all items), PPI (Producer Price Index: finished goods), and PCE (Personal Consumption Expenditures). The inflation series are recorded monthly and are seasonally adjusted. CPI and PPI cover the period 1947-2014. The PCE series covers the period 1959-2014.

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