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Correlation Dynamics and International Diversification Benefits*

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Abstract

Forecasting the evolution of security co-movements is critical for asset pricing and portfolio allocation. Hence, we investigate patterns and trends in correlations over time using weekly returns for developed markets (DMs) and emerging markets (EMs) during the period 1973-2012. We show that it is possible to model co-movements for many countries simultaneously using BEKK, DCC, and DECO models. Empirically, we find that correlations have significantly trended upward for both DMs and EMs. Based on a time-varying measure of diversification benefit, we find that it is not possible in a long-only portfolio to circumvent the increasing correlations by adjusting the portfolio weights over time. However, we do find some evidence that adding EMs to a DM-only portfolio increases diversification benefits.

JEL Classification: G12

Keywords: asset pricing, asset allocation, dynamic conditional correlation (DCC), dynamic equicorrelation (DECO).

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1 Introduction

Forecasting the dynamics of co-movements in international equity returns is of paramount importance for international finance. The traditional case for international diversification benefits has relied largely on the existence of low and stable cross-country correlations. Initially, the literature studied developed markets (DMs), but over the last few decades much of the focus has shifted to the diversification benefits offered by emerging markets (EMs).

Have cross-country correlations remained low and stable through time? It is far from straightforward to address this ostensibly simple question without making additional assumptions. Computing rolling correlations is subject to well-known drawbacks. Multivariate GARCH models, as for example in Longin and Solnik (1995), seem to provide a solution. However, as discussed by Solnik and Roulet (2000), the implementation of these models using large numbers of countries is subject to well known dimensionality problems. As a result, most of the available evidence on the time-variation in cross-country correlations is based on factor models. For example, Bekaert, Hodrick, and Zhang (2009) investigate international stock return co-movements for 23 DMs during 1980-2005, and find an upward trend in return correlations only among the subsample of European stock markets, but not for North American and East Asian markets.

We argue that recent advances make it feasible to overcome dimensionality and convergence problems. We characterize time-varying correlations using weekly returns during the 1973-2012 period for a large number of EMs AND DMs without relying on a factor model. We implement models that overcome the dimensionality problems, and that are easy to estimate. To do so, we rely on the variance targeting idea in Engle and Mezrich (1996) and the numerically efficient composite likelihood procedure proposed by Engle, Shephard and Sheppard (2008). We use the flexible dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002). As a robustness exercise, we report on the dynamic equicorrelation (DECO) model of Engle and Kelly (2012), which can be estimated on large sets of assets using conventional maximum likelihood estimation. We also contrast our findings with those obtained using the more traditional scalar BEKK model from Engle and Kroner (1995). We thus demonstrate that it is possible to estimate correlation patterns in international markets using large number of countries and extensive time series, without relying on a factor model that may bias inference. Our implementation is relatively straightforward and computationally fast, which allows us to report results using several estimation approaches, while assessing the robustness of our findings.

Our results based on BEKK, DCC, and DECO models are extremely robust and suggest that correlations have been trending upward for both DMs and EMs. We find that the correlation between DMs is higher than the correlation between EMs at all times in the sample. For emerging markets, the correlation with developed markets is generally somewhat higher than the correlation with the other emerging markets, however, the differences are small. While the range of correlations for DMs has narrowed around the increasing trend in correlation levels, this is not the case for EMs. Although these results are robust across methodologies, the BEKK model yields substantially more outliers compared to the DCC approach, suggesting that the latter allows for more realistic modeling of correlation patterns, due to a richer parameterization.

Our robust finding of an upward trend in correlations is all the more remarkable because the parametric models we use enforce mean-reversion in volatilities and correlation, and we estimate the models using long samples of weekly returns. The data clearly pull the models away from the average correlation, and any reversion to the mean is temporary in the samples we investigate. Christoffersen, Errunza, Jacobs, and Langlois (2012) build on this and allow for a parametric trend in their model of dynamic copula correlations. The implementation of their model is more complex and relies heavily upon Monte Carlo simulation.

We develop a time-varying measure of diversification benefits that is based on time-varying optimal portfolio weights and the dynamic correlations. We find that it is not possible in a long-only portfolio to circumvent the increasing correlations by adjusting the portfolio weights over time. Consistent with the patterns in correlations, diversification benefits have decreased for emerging markets as well as developed markets. However the level of diversification benefits is still higher in emerging markets, and emerging markets thus still offer correlation-based diversification benefits to investors.

The paper proceeds as follows. Section 2 provides a brief outline of BEKK, DCC, and DECO correlation models, with special emphasis on the estimation of large systems. Section 3 presents the data, as well as empirical results on time variation in international equity market correlations. Section 4 conducts a real-time forecasting exercise and Section 5 concludes.

2 Correlation Dynamics and Diversification Measures

This section outlines the various models we use to capture the dynamic dependence across equity markets. We first describe how the conventional scalar BEKK model of Engle and Kroner (1995) can be implemented simultaneously on many assets. We then introduce the dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002), which allows for added flexibility in that it separates the modeling of volatility dynamics from correlation dynamics the latter of which is our main focus. Finally we describe the DECO model which can be viewed as a special case of DCC and so is included mainly to assess the robustness of our empirical findings.

2.1 The Scalar BEKK Approach

In the existing literature, the estimation of dynamic dependence models for large-scale systems of countries using extended time periods has been judged impractical and/or impossible because of dimensionality problems.[1] Existing implementations of multivariate GARCH models have therefore traditionally used a limited number of countries. We argue that it is feasible to estimate such large systems using a number of recent advances. Because it is not always obvious to relate some of the models we use, such as DCC, to models previously used in the literature, we start by explaining how these innovations help to estimate more traditional multivariate GARCH models. We illustrate this using the scalar BEKK model, which is arguably the most often-used empirical model for capturing dynamic dependence in large systems.[2] In this model the return on asset i at time t is assumed to follow the dynamic

$$R_{i,t} = \mu_{i,t} + \varepsilon_{i,t} = \mu_{i,t} + \sigma_{i,t} z_{i,t} \tag{2.1}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha \varepsilon_{i,t-1}^2 + \beta \sigma_{i,t-1}^2 \tag{2.2}$$

where $\sigma_{i,t}^2$ denotes the conditional variance, and where the conditional mean dynamic, $\mu_{i,t}$, can be specified using an asset pricing model that captures the equity premium or using a simple univariate autoregressive model as we do below.

The covariances between assets i and j follow the dynamic

$$\sigma_{ij,t} = \omega_{ij} + \alpha \varepsilon_{i,t-1} \varepsilon_{j,t-1} + \beta \sigma_{ij,t-1} \tag{2.3}$$

The defining characteristic of the scalar BEKK model is that the persistence parameters α and β are identical across all conditional variances in (2.2) and across all conditional covariances in (2.3). This requirement ensures that the conditional covariance matrix for all assets is positive semi-definite at all time and therefore ensures that the portfolio variance will be positive for any given set of portfolio weights.

The common persistence across all variances and covariances is clearly restrictive. Equally important is the restriction that the functional form of the variance dynamic in (2.2) is required to be identical to the form of the covariance dynamic in (2.3). This rules out for example the so-called

[1] See for instance Solnik and Roulet (2000) for an excellent discussion. See Longin and Solnik (1995) and Karolyi (1995) for early examples of bivariate models.

[2] The BEKK model is most often used to estimate factor models with a GARCH structure. See for instance DeSantis and Gerard (1997, 1998), and Carrieri, Errunza, and Hogan (2007) for examples. See Ramchand and Susmel (1998), Baele (2005), and Baele and Inghlebrecht (2009) for more general multivariate GARCH models with regime switching.

leverage effect in volatility, which has been found to be an important stylized fact in equity index returns (see for example Black, 1976, and Engle and Ng, 1993).[3] The leverage effect is really an asymmetric volatility response that captures the fact that a large negative shock to an equity market increases the equity market volatility by much more than a positive shock of the same magnitude. We discuss this more explicitly in equation (2.8) below.

If N denotes the number of equity markets under study then the scalar BEKK model has $N(N+1)/2+2$ parameters to be estimated. Below we will study up to 16 emerging markets and 16 developed markets, thus $N = 32$ and so the BEKK model will have 530 parameters. It is well recognized in the literature that it is impossible to estimate these parameters reliably due to the need to use numerical optimization techniques, see for instance Solnik and Roulet (2000) for a discussion. To operationalize estimation, some existing implementations (see for example DeSantis and Gerard (1997)) have relied on the targeting idea in Engle and Mezrich (1996).

Written in matrix form, the BEKK model is

$$\Sigma_t = \Omega + \alpha \varepsilon_{t-1} \varepsilon_{t-1}' + \beta \Sigma_{t-1}$$

Taking expectations on both sides and solving for the unconditional variance-covariance matrix, Σ, yields

$$\Sigma = \Omega / (1 - \alpha - \beta) \tag{2.4}$$

If we use the sample variance-covariance matrix, $\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \varepsilon_t'$, as an estimate of the unconditional variance-covariance matrix, then the Ω matrix can be estimated via the relationship

$$\hat{\Omega} = (1 - \alpha - \beta) \hat{\Sigma} \tag{2.5}$$

Using the pre-estimated $\hat{\Sigma}$, the numerical optimizer now only has to search in two dimensions, namely over α and β, rather than in the original 530 dimensions. Note that this implementation also ensures that the estimated BEKK model yields a positive semi-definite covariance matrix, because $\hat{\Sigma}$ and $\varepsilon_{t-1} \varepsilon_{t-1}'$ are positive semi-definite by construction. Note also that (2.4) provides us with a more intuitive interpretation of the BEKK model via

$$\Sigma_t = (1 - \alpha - \beta) \Sigma + \alpha \varepsilon_{t-1} \varepsilon_{t-1}' + \beta \Sigma_{t-1} \tag{2.6}$$

which shows that the conditional covariance in BEKK is a weighted average of the long-run covariance, yesterday's innovation cross-product, and yesterday's conditional covariance. The dynamic

[3] See Bekaert and Wu (2000) for a model that introduces the leverage effect in a multivariate GARCH setup.

correlations in the BEKK model are given by the usual definition

$$\Gamma_{ij,t} = \Sigma_{ij,t}/\sqrt{\Sigma_{ii,t}\Sigma_{jj,t}}.$$

Even when using covariance targeting, estimation is cumbersome in large-dimensional problems due to the need to invert the N by N covariance matrix, Σ_t, on every day in the sample for every likelihood evaluation. The likelihood in turn must be evaluated many times in the numerical optimization routine. More importantly, Engle, Shephard and Sheppard (2008) find that in large-scale estimation problems, the parameters α and β which drive the covariance dynamics are estimated with bias when using conventional estimation techniques. They propose an ingenious solution based on the composite likelihood defined as

$$CL(\alpha, \beta) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln f(\alpha, \beta; R_{it}, R_{jt}) \tag{2.7}$$

where $f(\alpha, \beta; R_{it}, R_{jt})$ denotes the bivariate normal distribution of asset pair i and j and where covariance targeting is imposed.

The composite log-likelihood is thus based on summing the log-likelihoods of pairs of assets. Each pair yields a valid (but inefficient) likelihood for α and β, but summing over all pairs produces an estimator which is relatively efficient, numerically fast, and free of bias even in large-scale problems. We use the composite log-likelihood in all our estimations below. We have found it to be very reliable and robust, effectively turning a numerically impossible task into a manageable one. To the best of our knowledge, we are the first to apply the composite likelihood estimation procedure to the estimation of large systems of international equity data using long time series of weekly returns, which are needed for the identification of variance and covariance patterns, and therefore the first to be able to estimate dynamic correlation models for such large systems.

Fully efficient MLE is attractive in theory but infeasible for our application due to the challenges of numerical optimization in large systems. Indeed, we are not aware of feasible alternatives to composite likelihood for estimation of large systems such as ours. We refer to Engle, Shephard and Sheppard (2008) for Monte Carlo evidence on its finite sample performance.

2.2 The Dynamic Conditional Correlation Approach

While it is thus possible to operationalize the scalar BEKK model for large systems, the restrictions imposed on the variance and covariance dynamics are a cause for concern. Cappiello, Engle and Sheppard (2006), for example, have found that the persistence in correlation differs from that in variance when looking at stock and bond markets.[4] We therefore implement the dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002). Allowing for the leverage effect, we assume that the conditional variance of asset i at time t follows the dynamic

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \left(\varepsilon_{i,t-1} - \theta_i \sigma_{i,t-1}\right)^2 + \beta_i \sigma_{i,t-1}^2 \tag{2.8}$$

Note that the conditional variance parameters are now allowed to vary across firms.

Because the covariance is just the product of correlations and standard deviations, we can write

$$\Sigma_t = D_t \Gamma_t D_t$$

where D_t has the standard deviations $\sigma_{i,t}$ on the diagonal and zeros elsewhere, and where Γ_t has ones on the diagonal and conditional correlations off the diagonal. The correlation dynamics are driven by the cross-products of the return shocks

$$\tilde{\Gamma}_t = (1 - \alpha_\Gamma - \beta_\Gamma)\tilde{\Gamma} + \alpha_\Gamma (z_{t-1} z_{t-1}') + \beta_\Gamma \tilde{\Gamma}_{t-1} \tag{2.9}$$

which are used to define the conditional correlations via the normalization

$$\Gamma_{ij,t} = \tilde{\Gamma}_{ij,t} / \sqrt{\tilde{\Gamma}_{ii,t} \tilde{\Gamma}_{jj,t}}.$$

The normalization ensures that all correlations remain in the -1 to 1 interval.[5]

Note that the DCC model allows for a leverage effect in conditional variance, and it does not require the correlation persistence to match the variance persistence as is the case in the BEKK model. Note also that the DCC model has $N(N-1)/2+2$ correlation parameters and $4N$ variance parameters. But we estimate the 4 variance parameters per country one market at a time, and the correlation parameters are estimated using $\frac{1}{T}\sum_{t=1}^{T} z_t z_t'$ as a pre-specified estimate of $\tilde{\Gamma}$. Thus only two correlation parameters are estimated simultaneously using numerical optimization.

More generally, note that the DCC model in (2.9) is designed directly to model correlation dynamics via the standardized shock cross products, whereas in the BEKK model in (2.6) the correlation dynamics are merely implied by the specification of the covariance and variance dynamics. The direct modeling of the correlation dynamics in the DCC model is potentially an advantage in our application which is focused exactly on correlation dynamics.

[4] See Kroner and Ng (1998) and Solnik and Roulet (2000) for a more elaborate discussion of the restrictions imposed in the first generation of multivariate GARCH models.

[5] We have verified that implementing the adjustment to DCC suggested in Aielli (2011) does not affect our results.

We again rely on composite likelihood estimation for the DCC parameters. We now have

$$CL(\alpha_\Gamma, \beta_\Gamma) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln f(\alpha_\Gamma, \beta_\Gamma; z_{it}, z_{jt}) \tag{2.10}$$

2.3 The Dynamic EquiCorrelation Approach

The dynamic equicorrelation (DECO) model in Engle and Kelly (2012) can be viewed as a special case of the DCC model in which the correlations are equal across all pairs of countries but where this common so-called equicorrelation is changing over time. The resulting dynamic correlation can be thought of as an average dynamic correlation between the countries included in the analysis. Following Engle and Kelly (2012), we parameterize the dynamic rank equicorrelation matrix as

$$\Gamma_t = (1 - \rho_t) I_N + \rho_t J_{N \times N}$$

where I_N denotes the n-dimensional identity matrix and $J_{N \times N}$ is an $N \times N$ matrix of ones. The inverse and determinants of the rank equicorrelation matrix, Γ_t, are given by

$$\Gamma_t^{-1} = \frac{1}{(1 - \rho_t)} [I_N - \frac{\rho_t}{1 + (N-1)\rho_t} J_{N \times N}]$$

The ease with which the correlation matrix is inverted ensures that the model can be estimated on large sets of assets using conventional maximum likelihood estimation.

The dynamic equicorrelation parameter, ρ_t follows the simple linear form

$$\rho_{t+1} = \omega_\rho + \alpha_\rho u_t + \beta_\rho \rho_t$$

where u_t represents the equicorrelation update. In our empirical application, we apply the following correlation updating rule

$$u_t = \frac{\sum_{i \neq j} z_{i,t} z_{j,t}}{(N-1) \sum_i z_{i,t}^2}$$

Note that u_t by construction lies within the range $\left(\frac{-1}{N-1}, 1\right)$.[6]

While the DECO model may appear to be restrictive, it may lead to superior correlation estimates when the true correlations are close to each other across assets. In that case the DCC and BEKK model may well both provide noisy estimates of the correlation paths. The relative performance of the DECO to the DCC and BEKK models is thus very much sample dependent.

[6] Note that we rely on the original version of the DECO model here. Our results are very similar when we use other versions of the DECO model including that in the published version of Engle and Kelly (2012).

2.4 A Measure of Diversification Benefits from Dynamic Correlation

If correlations are changing over time, then the benefits of diversification will be changing as well. We therefore need to develop a dynamic measure of diversification benefits.

To this end we consider the conditional portfolio variance which is given by

$$\begin{aligned}\sigma_{PF,t}^2 &= \sum_i \sum_j w_{i,t} w_{j,t} \sigma_{i,jt} \\ &= \sum_i \sum_j w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t} \rho_{i,j,t}\end{aligned}$$

Assume that the conditional volatility changes over time but that it is the same across assets. This is clearly not realistic but it allows us to focus on the implications of changing correlations.

$$\sigma_{i,t} = \sigma_{j,t}, \text{ for all } i, j$$

and call this $\sigma_{A,t}$ for "asset" volatility. Then we have

$$\sigma_{PF,t}^2 = \sigma_{A,t}^2 \sum_i \sum_j w_{i,t} w_{j,t} \rho_{i,j,t}$$

so that

$$\sigma_{PF,t}^2 / \sigma_{A,t}^2 = \sum_i \sum_j w_{i,t} w_{j,t} \rho_{i,j,t}.$$

We compute this variance ratio using the dynamic weights $w_{i,t}^*$ that minimize $\sigma_{PF,t}^2 / \sigma_{A,t}^2$ subject to these weights summing to one and subject to short-sale constraints. Taking one less the variance ratio and using the optimal portfolio weights defines our measure of correlation-based diversification benefits

$$CDB_t = 1 - \sum_i \sum_j w_{i,t}^* w_{j,t}^* \rho_{i,j,t}. \tag{2.11}$$

Recall further that in the DECO model all pairwise correlations are identical so that

$$CDB_t = 1 - \rho_t \sum_i \sum_j w_{i,t}^* w_{j,t}^* = 1 - \rho_t \tag{2.12}$$

In this case the optimal weight in each asset is simply $1/N$, and the dynamic measure of diversification benefits is therefore equal to one minus the DECO correlation path. In the empirical section, we can thus view the DECO correlation path directly as one minus the measure of conditional diversification benefits for this special case. Equation (2.11) also shows that, in the DCC and BEKK models, when all correlations are zero, the CDB measure is one, and when all correlations are one, the CDB measure is zero.

While we are using CDB as a measure of diversification benefit it can also be viewed as a portfolio weighting scheme. Amenc, Goltz and Martellini (2013) compare our CDB approach with a number of standard portfolio allocation approaches in the literature. Consider first a dynamic version of the Markowitz (1952) allocation which maximizes the Sharpe ratio

$$w_t^* = \frac{\Sigma_t^{-1}\mu_t}{\mathbf{1}'\Sigma_t^{-1}\mu_t}.$$

One can view many of the alternatives suggested in the literature as special cases of the Markowitz allocation. At the extreme, the equal-weighted allocation

$$w_t^* = \frac{1}{N}\mathbf{1}$$

is optimal only when the means, variances and correlations each are identical across assets. The global minimum variance portfolio

$$w_t^* = \frac{\Sigma_t^{-1}\mathbf{1}}{\mathbf{1}'\Sigma_t^{-1}\mathbf{1}}$$

is optimal only when the means are identical across assets. When short-selling is allowed, our CDB measure implies an allocation of

$$w_t^* = \frac{\Gamma_t^{-1}\mathbf{1}}{\mathbf{1}'{\Gamma_t}^{-1}\mathbf{1}},$$

and it is only optimal when means and variances are identical across assets.

In reality, means, variances, and correlations are potentially dynamic and unknown quantities that must be estimated and so a portfolio allocation rule that is suboptimal in theory may in fact be optimal in practice. Regardless, our use of the CDB measure is simply driven by our focus on correlations: We want a portfolio allocation–and a portfolio risk measure–that emphasizes the implications of correlation dynamics and CDB does exactly that.

3 Empirical Correlation Analysis

This section contains our empirical correlation results. We first describe the different data sets that we use and briefly discuss the univariate models. We then analyze the time-variation in linear correlations. Subsequently we measure the dispersion in correlations across pairs of assets at each point in time and check if this dispersion has changed over time.

3.1 Data

As in Christoffersen, Errunza, Jacobs and Langlois (2012), we employ three data sets:

- Weekly closing U.S. dollar returns for 16 developed markets from DataStream over the period January 12, 1973 through June December 28, 2012.[7]

- Weekly closing U.S. dollar returns for 13 emerging markets from Standard and Poor's/ International Finance Corporation Global (IFCG) indices over the period January 6, 1989 through July 25, 2008. The IFCG data set spans a longer time period, and represents a broad measure of emerging market returns, but is not available after July 25, 2008.[8]

- Weekly closing U.S. dollar returns for 16 emerging markets from Standard and Poor's/ International Finance Corporation Investable (IFCI) indices over the period July 7, 1995 through December 28, 2012. The IFCI data set tracks returns that are legally and practically available to foreign investors. The index construction takes into account portfolio flow restrictions, liquidity, size, and float. It continues to be updated but the shorter sample period is a disadvantage in model estimation and in assessing long-term trends in correlation.[9]

Table 1 contains descriptive statistics using IFCI and developed market data for the 1995-2012 period. The results for the IFCG sample are available upon request. While the cross-country variations are large, Table 1 shows that the average annualized return in the developed markets was 9.89%, versus 13.95% in the emerging markets. This emerging market premium is reflective of an annual standard deviation of 33.88% in emerging markets versus only 22.22% in developed markets. Excess kurtosis is on average slightly higher in emerging markets indicating more tail risk. But skewness is close to zero in emerging markets and considerably negative in mature markets, suggesting that emerging markets are not more risky from this perspective.

3.2 Univariate Models

Table 1 indicates that the first-order autocorrelations are fairly small for most countries. The Ljung-Box (LB) test that the first 20 weekly autocorrelations are zero is rejected in most markets. We use

[7] The 16 developed markets are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, Switzerland, U.K., and U.S.

[8] The 13 IFCG countries are Argentina, Brazil, Chile, Colombia, India, Jordan, Korea, Malaysia, Mexico, Philippines, Taiwan, Thailand, and Turkey.

[9] The 16 IFCI countries are Brazil, Chile, China, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, South Africa, Taiwan, Thailand, and Turkey. We omit Argentina due to missing observations starting in October 2009.

an autoregressive model of order two, AR(2), for each market to pick up this return dependence. The Ljung-Box test that the first 20 autocorrelations in absolute returns are zero is strongly rejected for all 29 markets. Remember that in the DECO and DCC models, we employ the NGARCH(1,1) model of Engle and Ng (1993) in (2.8) for each market to pick up this second-moment dependence and to account for asymmetries.

Table 2 reports the results from the estimation of the AR(2)-NGARCH(1,1) models on each market for the 1995-2012 data set. The results are fairly standard. The volatility updating parameter, α, is around 0.1, and the autoregressive variance parameter, β, is around 0.8. The parameter θ governs the volatility asymmetry also known as the leverage effect. It is commonly found to be large and positive in developed markets and we find that here as well. The literature contains less evidence on emerging markets but we find that the leverage effect is positive for all countries and on average similar to the developed markets. The model-implied variance persistence is high for all countries, as is commonly found in the literature.

The Ljung-Box (LB) test on the model residuals show that the AR(2) models are able to pick the weak evidence of return predictability found in Table 1. Impressively, the GARCH models are also able to pick up the strong persistence in absolute returns found in Table 1. Note also that the GARCH model has picked up much of the excess kurtosis found in Table 1.

We conclude from Tables 1 and 2 that the AR(2)-NGARCH(1,1) models are successful in delivering the white-noise residuals that are required to obtain unbiased estimates of the dynamic correlations. We will therefore use the AR(2)-NGARCH(1,1) model in the DECO and DCC applications. As discussed in Section 2.1, in the scalar BEKK model all variance and covariances have the same dynamics, and so in that case we will use the AR(2) model for the conditional mean.

3.3 Correlation Patterns Over Time

Table 3 reports the parameter estimates and log likelihood values for the DECO, DCC and BEKK correlation models. We report results for the three data sets introduced above. For each set of the countries, we estimate two versions of each model: one version allowing for correlation dynamics and another where the correlation dynamics are shut down, and thus $\alpha = \beta = 0$. A conventional likelihood ratio test would suggest that the restricted model is rejected for all sets of countries, but unfortunately the standard chi-squared asymptotics are not available for composite likelihoods. Note that the improvement of the unrestricted over the restricted model is greatest for the BEKK model, but that is because the model restrictions imply neither variance nor correlation dynamics in this case.

Note that the likelihood values differ across models in the case of No Dynamics in Table 3. This is because in the DECO model, the unconditional correlations are the same across all assets, and because in the BEKK model, neither variances nor correlations are dynamic. In the DCC model, the unconditional correlations differ across assets and the variance dynamics have been removed from the return residuals.

In order to ensure that the likelihoods are comparable across models, we report composite likelihoods for all models, including DECO where regular likelihood optimization is feasible. The likelihoods include the contributions from the univariate AR-GARCH models for each asset in DECO and DCC. This is also to ensure comparability with the BEKK models where the GARCH variance and covariance dynamics are estimated jointly. Notice that the DCC likelihoods are higher than the DECO and BEKK likelihoods for all sets of countries.

The correlation persistence $(\alpha + \beta)$ is close to one in all models implying very slow mean-reversion in correlations. In the DECO model persistence is estimated to be essentially one, reflecting the upward trend in correlation which we now discuss.

We present time series of dynamic equicorrelations (DECOs) for several samples. The left panels in Figure 1 present results for twenty-nine developed and emerging markets for the sample period January 20, 1989 to July 25, 2008. As explained in Section 3.1, sixteen of these markets are developed and thirteen are emerging markets. We also present DECOs for each group of countries separately. We refer to this sample as the 1989-2008 sample.

The right panels in Figure 1 present results for thirty-two developed and emerging markets for the sample period July 21, 1995 to December 28, 2012. This sample contains the same sixteen developed markets, and sixteen emerging markets. There is considerable overlap between this sample of emerging markets and the one used in the left panels of Figure 1. Section 3.1 discusses the differences. We refer to this sample as the 1995-2012 sample.

Figure 2 contains the same data as Figure 1 but reports the average (across all pairs of countries) model-free rolling correlations using a relatively short 6-month estimation window (denoted by grey lines) and using a relatively long 2-year estimation window (denoted by black lines). Figure 2 clearly illustrates the drawbacks of rolling correlations: the measurement of the conditional correlation critically depends on the estimation window.

The left-side panels in Figure 3 contain time series of DECOs for the group of sixteen developed markets between January 26, 1973 and December 28, 2012. We refer to this sample as the 1973-2012 sample. Figure 3 also shows results for the 1989-2008 and the 1995-2012 data for comparison.

These figures contain some of the main messages of our paper. The DECOs in Figures 1 and 3, which can usefully be thought of as the average of the pairwise correlations between all pairs of countries in the sample, fluctuate considerably from year to year, but have been on an upward trend since the early 1970s. Figure 3 shows that for the sixteen developed markets, the DECO increased from approximately 0.3 in the mid-1970s to between 0.7 and 0.8 in 2012. Figure 1 indicates that over the 1989-2012 period, the DECO correlations between emerging markets are lower than those between developed markets, but that they have also been trending upward, from approximately $0.1 - 0.2$ in the early nineties to around 0.5 in 2012.

Figure 2 shows that it is not the DECO model structure that is driving the upward-sloping trend result. The model-free estimates of dynamic correlation in Figure 2 show the same upward trend in correlation evident in Figure 1. The model-free estimates of dynamic correlation have the disadvantage that they depend greatly on the width of the data window chosen: A long window will result in stable but potentially biased estimates of the true dynamic correlation whereas a very short window will result in very noisy estimates. The dynamic models we apply have the advantage of letting the data choose–via maximum likelihood estimation–the optimal weights on past data points.

Because the DECO approach models correlation as time-varying with a model-implied long-run mean, one may wonder whether the sample selection strongly affects inference on correlation estimates at a particular point in time. Figure 3 addresses this issue by reporting DECO estimates for the sixteen developed markets for three different sample periods. Whereas there are some differences, the correlation estimate at a particular point in time is remarkably robust to the sample period used, and the conclusion that correlations have been trending upward clearly does not depend on the sample period used. Comparing the left and right panels of Figure 1, it can be seen that a similar conclusion obtains for the emerging markets, even though this comparison is more tenuous as the sample composition and the return data used for the emerging markets are somewhat different across panels.

3.4 Cross-Sectional Differences in Correlations

The DECO gives us a good idea of the evolution of correlations over time in a given sample of markets. They can usefully be thought of as an average of all possible permutations of pairwise correlations in the sample. The next question is how much cross-sectional heterogeneity there is in the correlations. The question whether correlations between emerging markets and developed markets have evolved differently is of special interest because emerging markets are sometimes viewed as being inherently more risky. The DCC framework discussed in Section 2.2 is designed to address this question. It yields a time-varying correlation series for each possible permutation of markets in the sample.

Reporting on all these time-varying pairwise correlation paths is not feasible, and we have to aggregate the correlation information in some way. Figures 3-5 provide an overview of the results.

The right-side panels in Figure 3 provide the average across all sixteen developed markets of the DCC paths, and compare them with the DECO paths. The top-right panel provides the average DCC from 1973 through 2012, the middle-right panel for the 1989-2008 sample period, and the bottom-right panel for the 1995-2012 period. The left-side panels provide the DECO correlations. Figure 3 demonstrates that the DECO can indeed be thought of as an average of the DCCs. Moreover, Figure 3 demonstrates that the average DCC correlation at each point in time is robust to the sample period used in estimation, as is the case for the DECO.[10]

Figure 4 uses the 1995-2012 sample to report, for each of the thirty-two countries in the sample, the average of its DCC correlations with all other countries using light grey lines. Figure 4.A contains the 16 developed markets and Figure 4.B contains the 16 emerging markets. While these paths are averages, they give a good indication of the differences between individual countries, and they are also informative of the differences between developed and emerging markets. In order to further study these differences, each figure also gives the average of the market's DCC correlations with all (other) developed markets using black lines and all (other) emerging markets using dark grey lines. Figure 4 yields some very interesting conclusions. First, the DCC correlation paths display an upward trend for all 32 countries. Second, for developed markets the average correlation with other developed markets is higher than the average correlation with emerging markets at virtually each point in time for virtually all markets. Third, for emerging markets the correlation with developed markets is generally higher than the correlation with other emerging markets. However, the difference between the two correlation paths is much smaller than in the case of developed markets. In several cases the average correlation paths are very similar. Note that in Figure 4.A the trend patterns for European countries are also not very different from those for other DMs. Notice in particular that the correlations for Japan and the US have increased just as for the European countries during the last decade even if the level of correlation is still somewhat lower in Japan. Inspection of the pairwise DCC paths, which are not reported because of space constraints, reveals that the trend patterns are remarkably consistent for almost all pairs of countries, and there is no meaningful difference between European countries and other DMs.

Figure 4 reports the average correlation between the DCC of each market and that of other markets. It could be argued that instead the correlation between each market and the average return of the other markets ought to be considered. We have computed these correlations as well. While the correlation with the average return is nearly always higher than the average correlation from Figure 4, the conclusion that the correlations are trending upwards is not affected. In order

[10] In Figure 3, and throughout the paper, we report equal-weighted averages of the pairwise correlations from the DCC and BEKK models. Value-weighted correlations (not reported here) also display an increasing pattern during the last 10-15 years. Note that in the benchmark DECO model all pairwise correlations are identical and so the weighting is irrelevant.

to save space we do not show the plots of the correlation with average returns on other markets.

Figure 4 does not tell the entire story, because we have to resort to reporting correlation averages due to space constraints. Figure 5 provides additional perspective by providing correlation dispersions for the developed markets, emerging markets, and all markets respectively. In particular, at each point in time, the top left of panel in Figure 5 considers all DCC correlations for the sixteen developed markets, and reports the 10th and 90th percentile of these pairwise correlations. The shaded area shows the range between the 10th and 90th percentile. The middle left panel in Figure 5 reports the same statistics for the emerging markets for the 1995-2012 sample and the bottom left panel shows all 32 markets together. While the increasing level of correlations is evident, Figure 5 also shows that the entire range of correlations has increased rendering it difficult to avoid the rising correlations via active portfolio allocation.

The evidence in Figures 3 through 5 discussed so far is obtained using the DCC techniques outlined in Section 2.2. In studies of international markets, the BEKK technique has been used more often. We do not engage in a detailed comparison of the different models here. Rather, we note that it is currently feasible to implement multivariate correlation techniques in a way that allows us to study large cross-sections of returns and capture important stylized facts. For instance, our implementation of the DCC technique in Section 2.2 allows for leverage effects, which is critical for capturing the negative skewness in index returns.

We therefore present a comparison of the DCC results with BEKK results implemented using variance targeting, as discussed in Section 2.1. This exercise is meant to provide some insight into how our results compare with existing studies that use the BEKK approach.

Figure 5 provides evidence on the differences between BEKK and DCC, by giving correlation dispersions obtained using both techniques. The correlation dispersions for the BEKK model suggest that this model yields substantially more outliers and noise.

The top row of panels in Figure 6 presents the time series of the average DCC correlation for sixteen developed countries for the 1973-2012 sample, and compares it with the average BEKK correlation. The middle and bottom rows do the same for sixteen emerging markets and for all markets together for the 1995-2012 sample. The main conclusions are consistent across the two samples. First, the BEKK correlation path confirms the conclusion obtained using DCC and DECO techniques that correlations have trended upward over time. Second, while the DCC and BEKK paths are quite similar, the DCC paths contain far fewer outliers and less noise compared to BEKK.

Figure 7 plots the conditional diversification benefit measure developed in (2.11) for developed, emerging, and all markets using the dynamic correlations from the DCC model. The black lines in Figure 7 uses CBD-optimal portfolio weights, the dark grey lines use equal weights across countries and the light grey lines use market capitalization weights.

Notice that the solid lines look very similar to (one minus) the equal-weighted correlations in the left panels of Figure 6. Figure 7 thus shows that it is not possible in a long-only portfolio to avoid the increasing correlations by adjusting the portfolio weights over time. The optimally weighted portfolio in Figure 7 shows a decreasing trend in diversification benefits, matching the increasing trend found in the equal-weighted correlations in Figure 6: Correlations have been rising rapidly and the benefits of diversification have been decreasing during the last ten years in developed markets. Diversification benefits have also decreased in emerging markets but the level of benefits is still higher than in developed markets. When combining the developed and emerging markets, the diversification benefits are declining as well but the level is again higher than when considering developed markets alone. Emerging markets thus still offer some correlation-based diversification benefits to investors.

The black lines in Figure 7 represent the highest possible level of CDB attainable to investors. The dark grey and light grey lines report CDB for equal and market capitalization weighted portfolios, respectively. It is interesting to note that while the equal-weighted portfolio CDB is often fairly close to the optimal CDB the market cap weighted CDB is typically much lower. During most of the sample smaller countries thus offer better diversification benefits.

Table 4 reports various statistics from the distribution of portfolio weights for each country used to construct the optimal CDB represented by the black lines in Figure 7. The CDB portfolio weights are allowed to change each week, thus creating a distribution of weights over time for each country. The top half of Table 4 reports the minimum, 25th, 50th and 75th percentile as well as the maximum weight over time for each country for the developed market CDB in the top panel of Figure 7. The last column in Table 4 reports the fraction of weeks (in percent) each country has an allocation of exactly zero. The bottom half of Table 4 reports the corresponding distributions for the emerging market CDB in the middle panel of Figure 7. Recall that we have imposed no-shortsale constraints so that the weights cannot be negative. As the weights have to sum to one the maximum possible weight is one.

The top half of Table 4 shows that all countries except for Japan indeed have a zero allocation in at least one week in the sample. The Netherlands is never included in the CDB portfolio and France, Germany and the UK have zero allocations virtually in the entire sample. They are poor investments from a CDB diversification perspective. Japan has the largest maximum allocation at 41%. Japan, the United States, and Hong Kong have the highest median allocations. No country ever has a maximum allocation of one.

The bottom half of Table 4 shows that all emerging markets have zero allocations in some part of the sample. Brazil and Korea have the largest fraction of weeks with zero portfolio weights. Turkey has the highest median portfolio weight and Thailand has the highest max portfolio weight at 33%.

In general the CDB optimal portfolio for emerging markets seems to be more evenly spread across countries than is the case for developed markets: The last column in Table 4 shows that the average ratio of zero-weights across countries is close to 48% for DMs and only 28% for EMs. This suggests that more EMs provide diversification benefits than do DMs in their respective portfolios.

4 Real-Time CDB Evaluation

In this section we conduct a real time CDB optimization exercise, which allows us to conduct an out-of-sample forecasting exercise.

Using weekly IFCI returns, we reestimate the univariate AR-NGARCH models, and the multivariate DECO, DCC, and BEKK models on an expanding sample. Our first estimation sample is July 1995 through December 2000. We reestimate each model at the end of each calendar year, keeping the estimation starting date fixed at July 1995. We then use each model to deliver a real-time one-week ahead correlation matrix $\Gamma_{t+1|t}$ which is used to construct portfolio weights in real time.

Starting in January 2001, we use the previous year's parameter estimates to compute one-week ahead forecasts for the correlation matrix $\Gamma_{t+1|t}$ which incorporates this weeks returns. Using these forecasts, we compute the weights, $w^{*\prime}_{t+1|t}$, that maximize expected CDB one-week ahead portfolio variance for week $t+1$, again subject to short-sale constraints.

Recall that in matrix form we have

$$CDB_t = 1 - w_t^{*\prime}\Gamma_t w_t^*. \tag{4.1}$$

where the CDB-optimal portfolio weights w_t^* depend on the correlation forecasting model. We can compute real-time CDB weights and forecasts using the recursive estimation methodology to get

$$CDB^F_{t+1|t} = 1 - w_t^{*\prime}\Gamma_{t+1|t} w_t^*. \tag{4.2}$$

CDB realizations can be computed via

$$CDB^R_{t+1} = 1 - w_t^{*\prime}\Gamma^R_{t+1} w_t^*. \tag{4.3}$$

where Γ^R_{t+1} is the realized correlation matrix. To compute realized CDB, we use four different realized correlation matrices: First, correlations from each individual forecasting model based on the corresponding full-sample estimator; second, DCC correlations based on full-sample estimator;

third, rolling regressions on a +/- 6 month window around the forecast week of interest; and fourth, a +/- 2 year window around the forecast week.

For each forecasting model and for each ex-post correlation method, we can now run the regression

$$CDB_{t+1}^{R} = b_0 + b_1 CDB_{t+1|t}^{F} + e_{t+1}$$

and check the regression fit measured by R^2.

The CDB forecasting exercise is carried out for three sets of portfolio weights: First, optimal weights computed by maximizing the CDB measure each week; second, market capitalization weights; and third, equal weights across countries which are constant through time. We construct CDB forecasts using correlation models estimated in real time on our three different sets of data: 16 developed markets, 16 emerging markets using IFCI data, and all 32 markets. The results are reported in Table 5.

Panel A of Table 5 reports regression R^2 when using the own-model, in-sample realized CDB in the forecasting regressions. Panel A shows that all models do well from this perspective. The DECO model is the only model for which the R^2 is not above 0.90.

Panel B of Table 5 reports regression R^2 when using the DCC-model in-sample realized CDB in the forecasting regressions. The DCC numbers are thus the same in Panel A and Panel B. The exercise in Panel B is useful to demonstrate how close the alternative forecasting models are to the DCC model. The BEKK model is generally closest to the DCC.

Panel C computes realized CDB from rolling correlation on a two-sided 6-month window of weekly returns. From this perspective, the BEKK correlations provide good CDB forecasts. Interestingly, the 6-month (one-sided) rolling correlations do not perform particularly well.

Panel D computes realized CDB from rolling correlation on a two-sided 2-year window of weekly returns. From this perspective, the DCC provides good CDB forecasts. Interestingly, the 6-month (one-sided) rolling correlations do not perform particularly well here either.

It is clear that the forecasting results depend on the measure used for realized correlations. The optimal choice of realized correlation measure is thus an important but unexplored issue. It would be interesting to employ intraday data to compute model-free realized weekly correlations. We leave this task for future work.

5 Summary and Conclusion

We characterize time-varying correlations using long samples of weekly returns for systems consisting of large number of countries. We implement models that overcome econometric complications arising from the dimensionality problem, and that are easier to estimate, using variance targeting and the composite likelihood procedure. Results based on BEKK, DCC and DECO models are extremely robust and suggest that correlations have been significantly trending upward for both the DMs and EMs. Correlations between DMs have exceeded correlations between EMs throughout the 1989-2012 period. Moreover, for developed markets, the average correlation with other developed markets is higher than the average correlation with emerging markets. For emerging markets, the correlation with developed markets is generally somewhat higher than the correlation with the other emerging markets, however, the differences are small.

These results have important implications for portfolio management. We compute measures of diversification benefits with dynamic weights. We find that it is not possible in a long-only portfolio to circumvent the increasing correlations by adjusting the portfolio weights over time. Consistent with the patterns in correlations, diversification benefits have decreased for emerging markets as well as developed markets, but the level of diversification benefits is still higher in emerging markets.

It may prove interesting to further explore the implications for portfolio management in future work. It may also prove useful to investigate the robustness of our findings to allowing for multiple regimes, or to the inclusion of multiple components, as for example in the model of Colacito, Engle, and Ghysels (2011).

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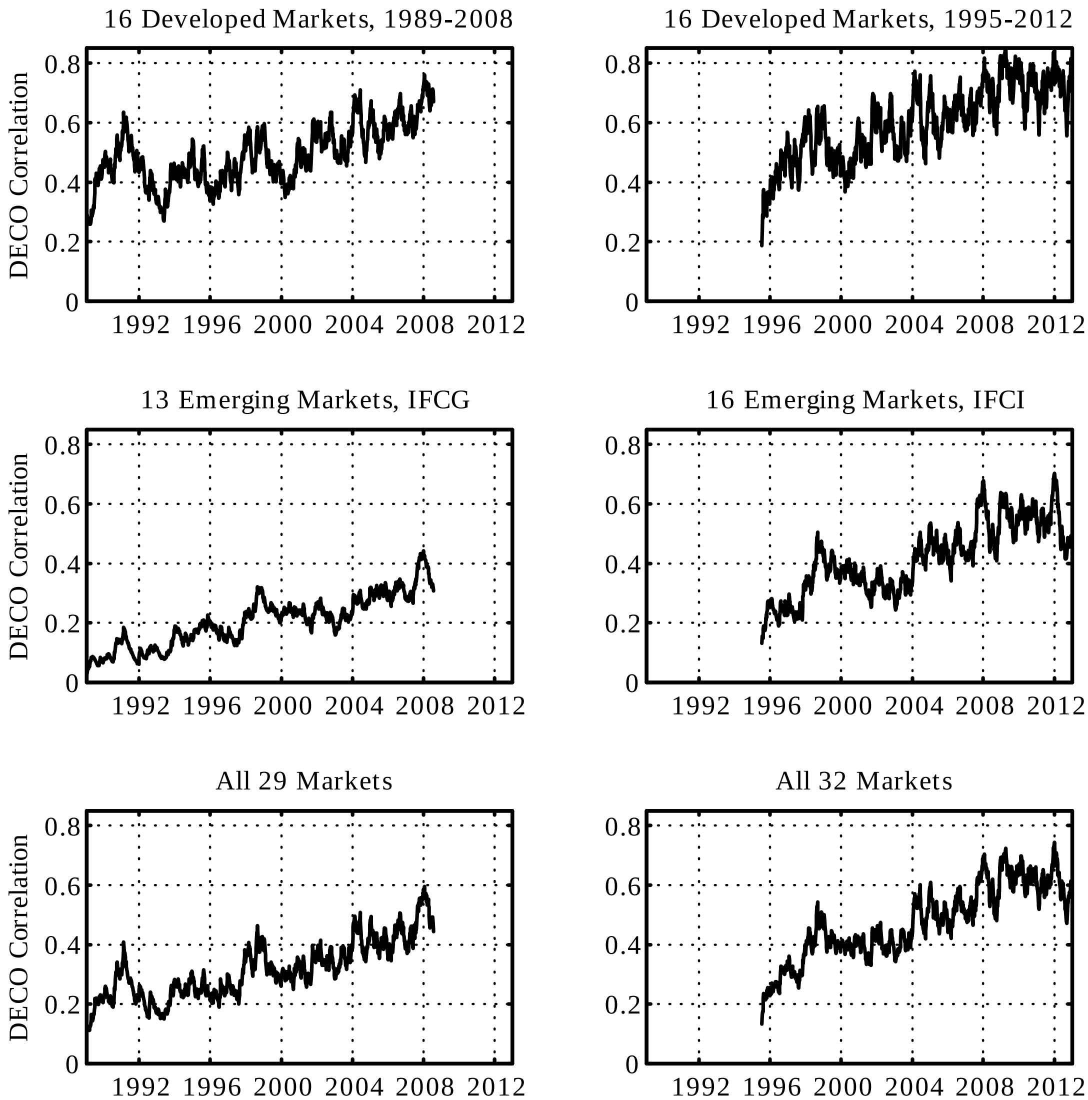
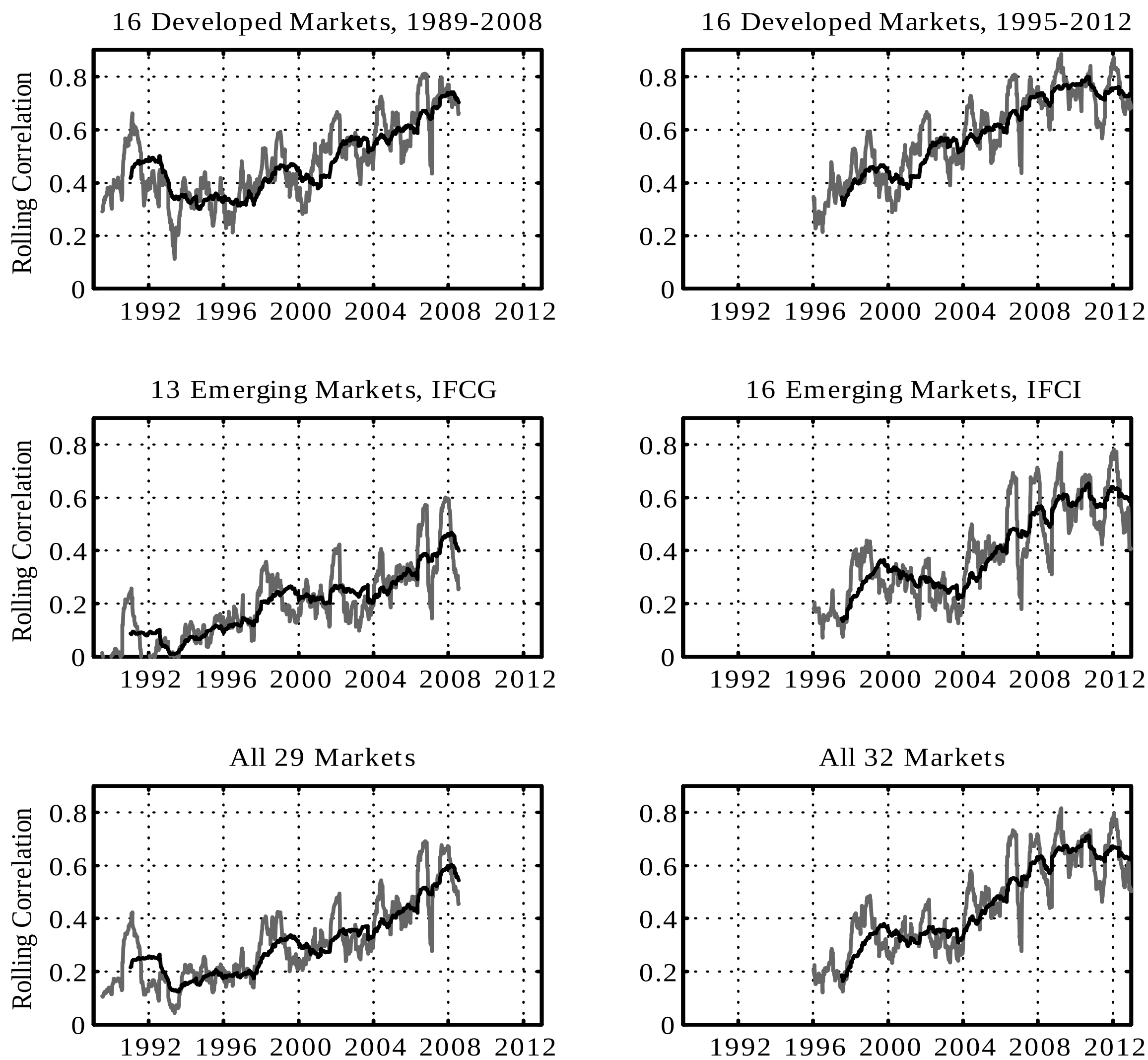
Figure 1: Dynamic (DECO) Correlations for Developed, Emerging, and All Markets



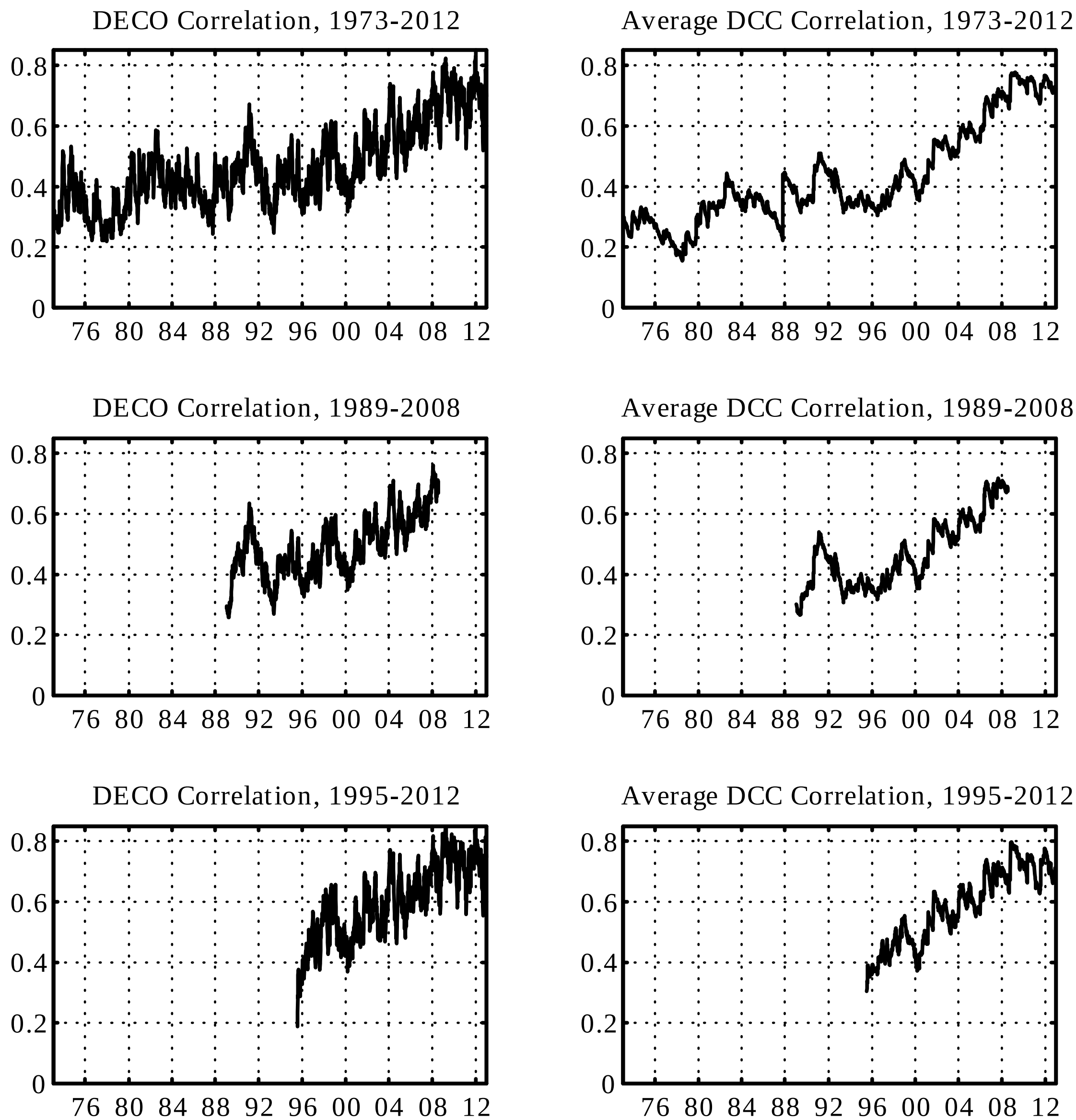
Notes to Figure: We report dynamic equicorrelations (DECOs) for two sample periods. The left-side panels report on the period January 20, 1989 to July 25, 2008. The right-side panels report on the period July 21, 1995 to December 28, 2012. The top panels report on developed markets, the middle panels report on emerging markets, and the bottom panels report on samples consisting of developed and emerging markets.

Figure 2: Rolling Correlations for Developed, Emerging, and All Markets. Two Estimates



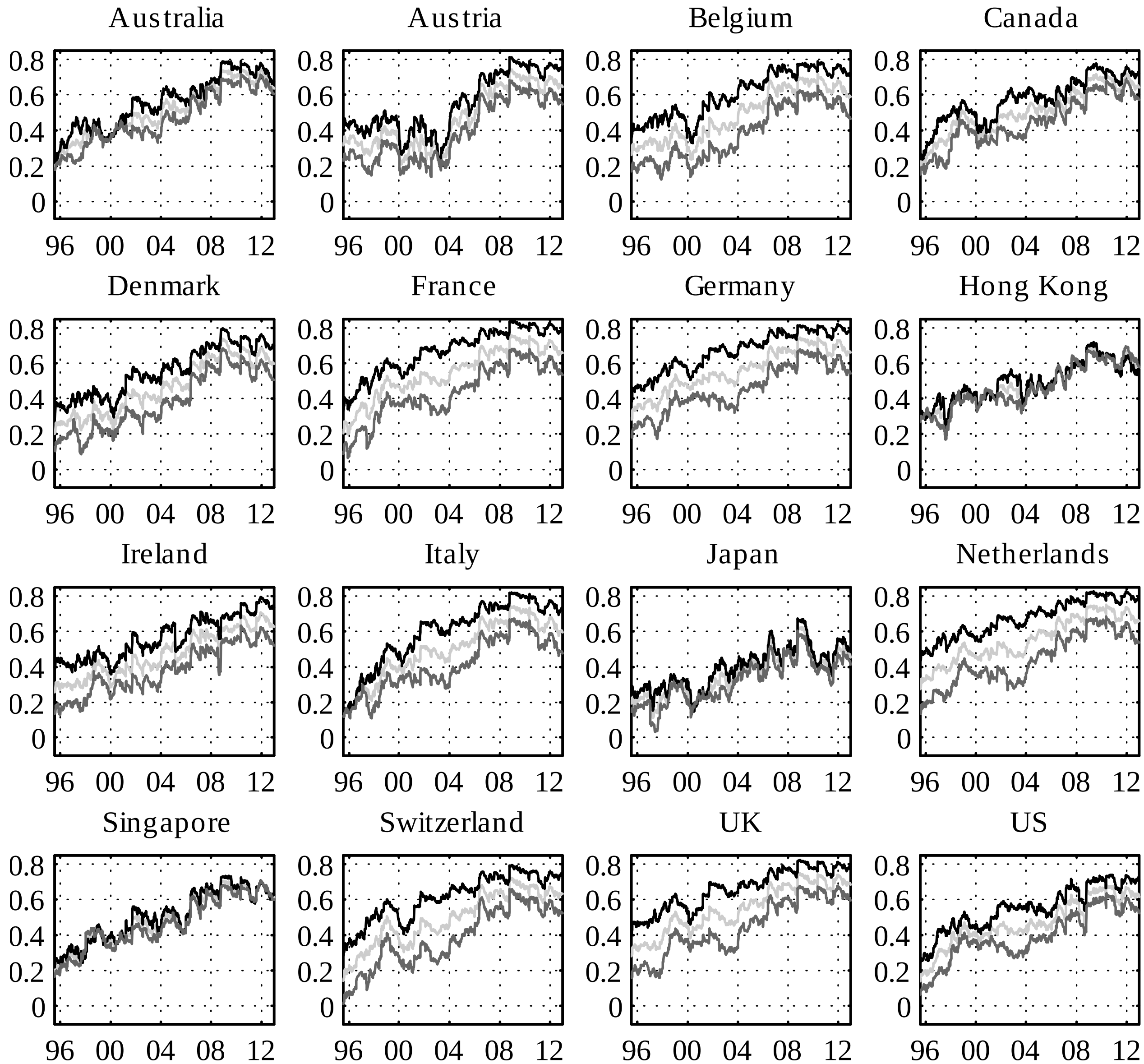
Notes to Figure: We report rolling correlations for two sample periods. The left-side panels report on the period January 20, 1989 to July 25, 2008. The right-side panels report on the period July 21, 1995 to December 28, 2012. The top panels report on developed markets, the middle panels report on emerging markets, and the bottom panels report on samples consisting of developed and emerging markets. We use 6-month (grey lines) and 2-year (black lines) windows to estimate rolling correlations for each pair of markets which are then averaged across pairs to produce the plot.

Figure 3: Comparing DECO and DCC Correlations. Developed Markets. Various Sample Periods



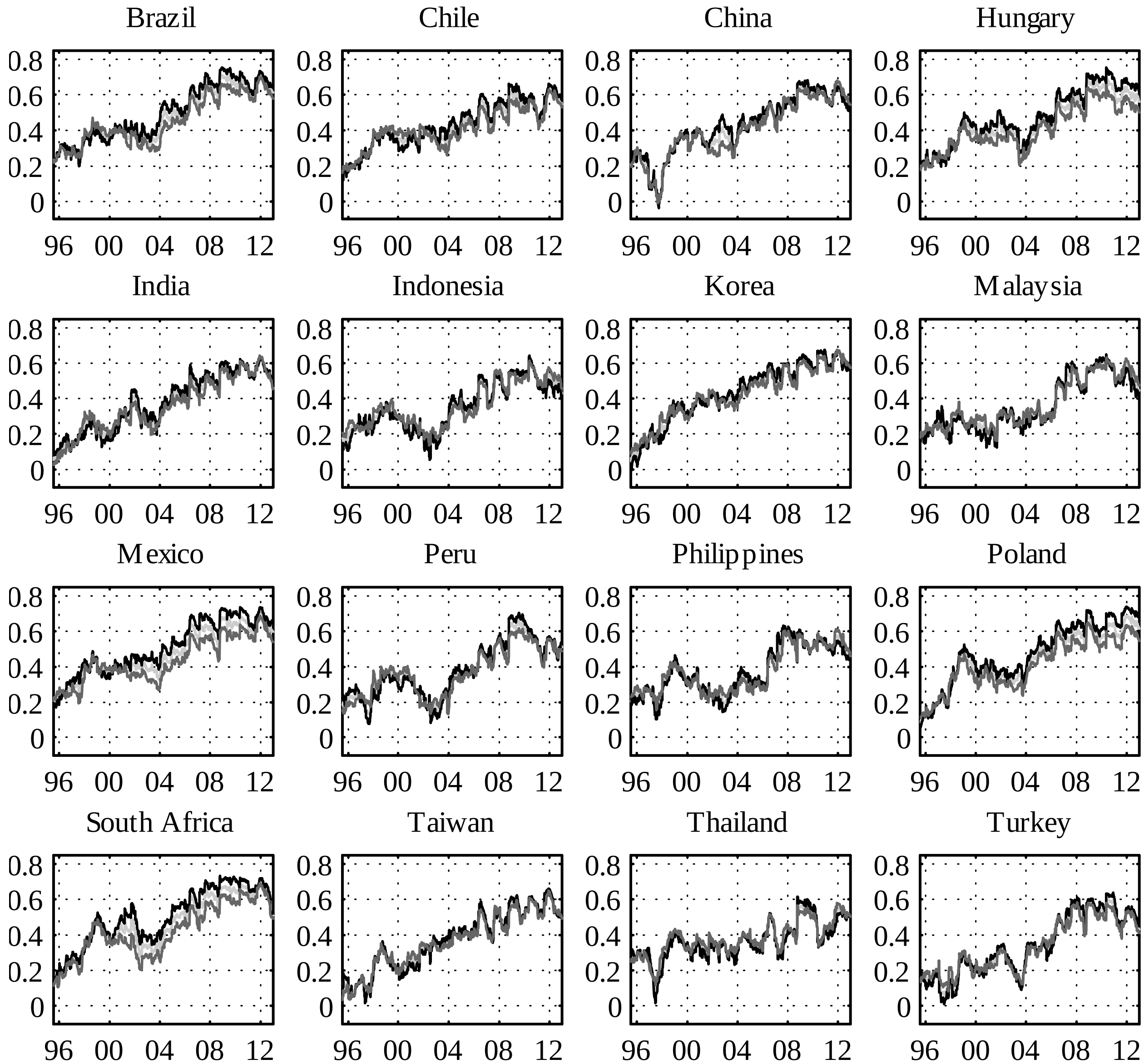
Notes to Figure: We report dynamic equicorrelations (DECOs) and dynamic conditional correlations (DCCs) for sixteen developed markets for three sample periods. The top panels report on the period January 26, 1973 to December 28, 2012. The middle panels report on the period January 20, 1989 to July 25, 2008. The bottom panels report on the period July 21, 1995 to December 28, 2012.

Figure 4.A: Correlations for Each Developed Market



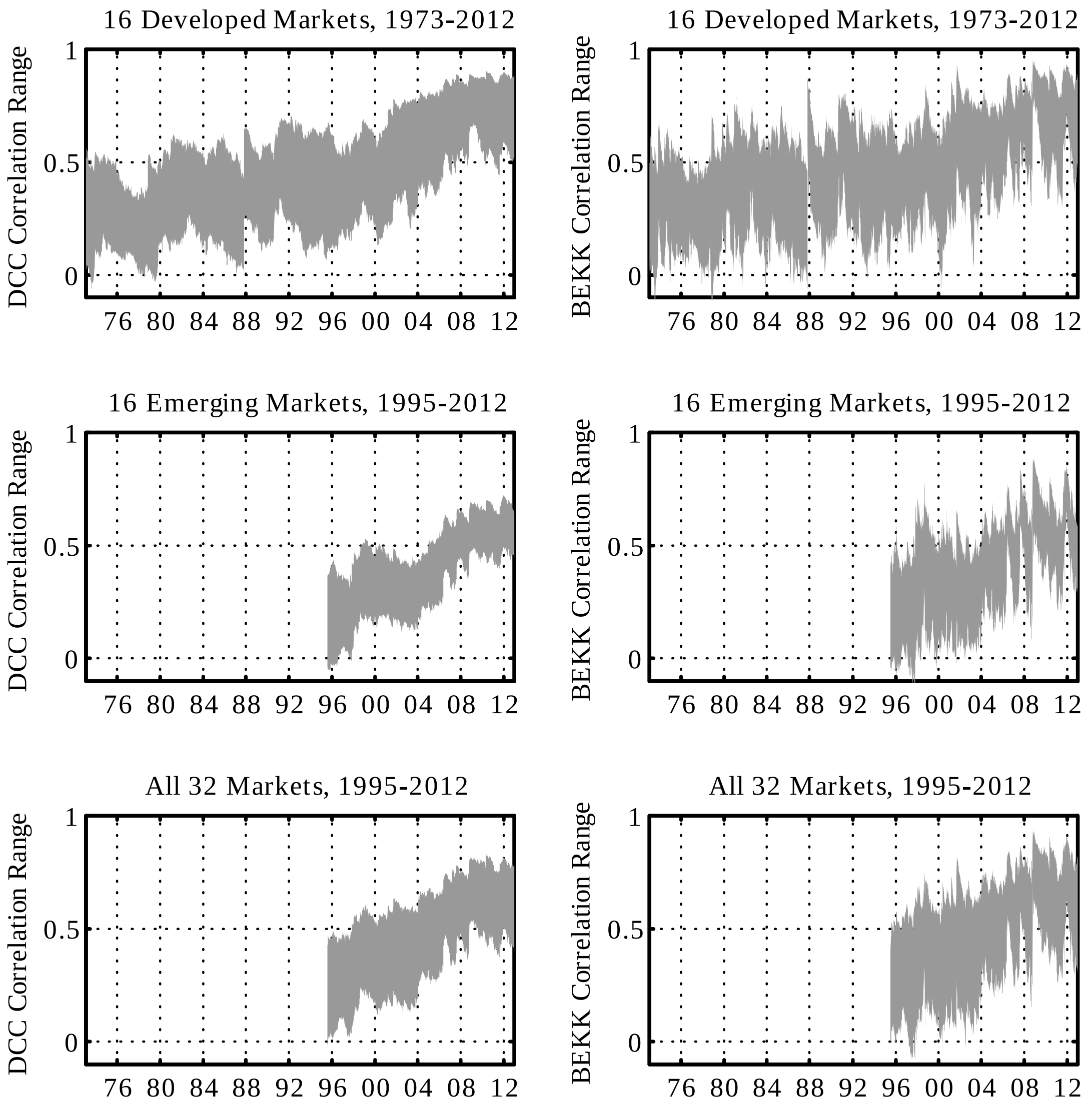
Notes to Figure: We report dynamic conditional correlations for sixteen developed markets for the period July 21, 1995 to December 28, 2012. For each country, at each point in time we report three averages of conditional correlations with other countries: the average of correlations with the fifteen other developed markets (black line), with the sixteen emerging markets (dark grey line), and with the fifteen developed and sixteen emerging markets (light grey line).

Figure 4.B: Correlations for each Emerging Market



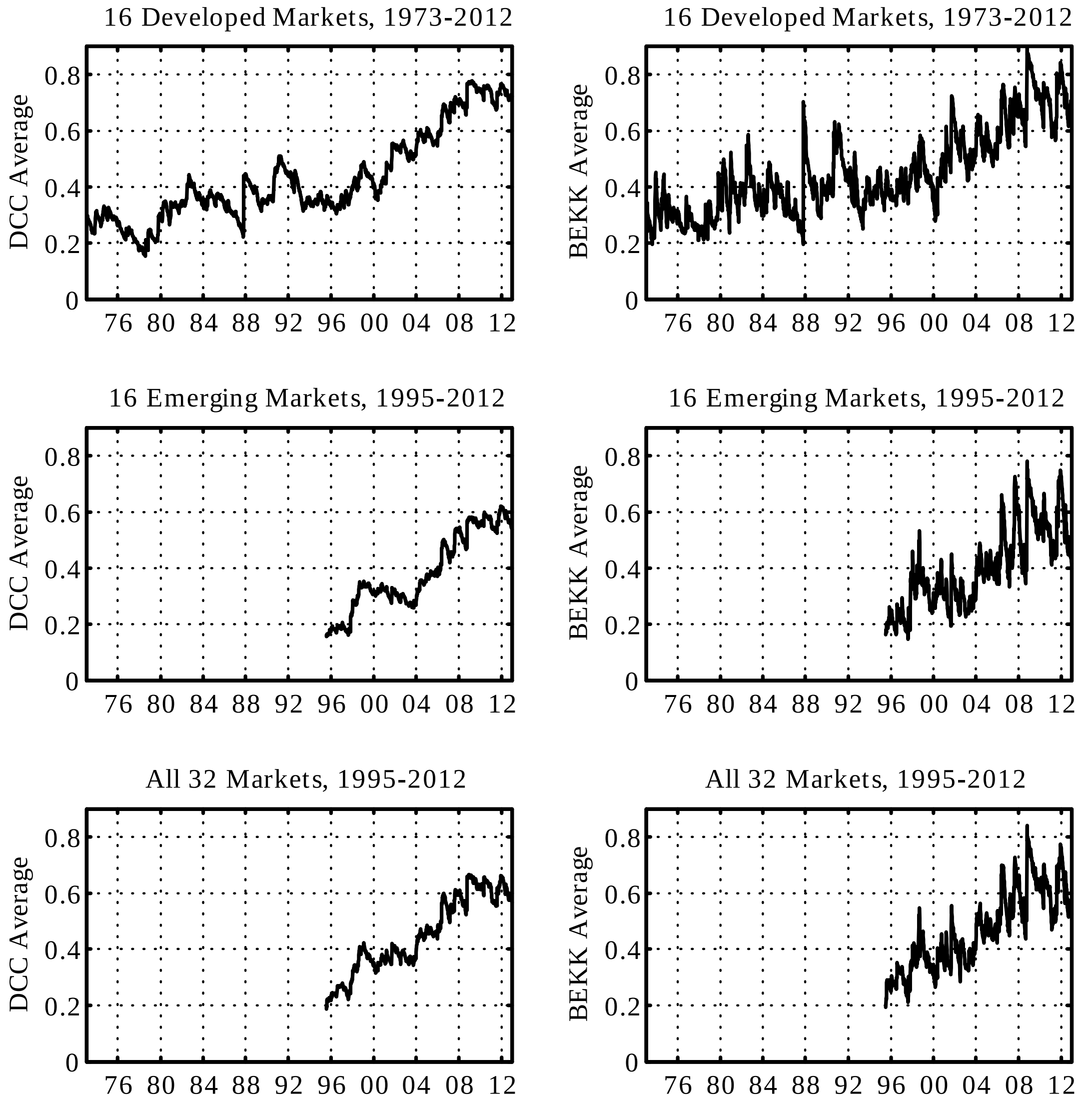
Notes to Figure: We report dynamic conditional correlations for sixteen emerging markets for the period July 21, 1995 to December 28, 2012. For each country, at each point in time we report three averages of conditional correlations with other countries: the average of correlations with sixteen developed markets (black line), with the fifteen other emerging markets (dark grey line), and with the sixteen developed and fifteen other emerging markets (light grey line).

Figure 5: Correlation Range (90th and 10th Percentile). Developed, Emerging and All Markets



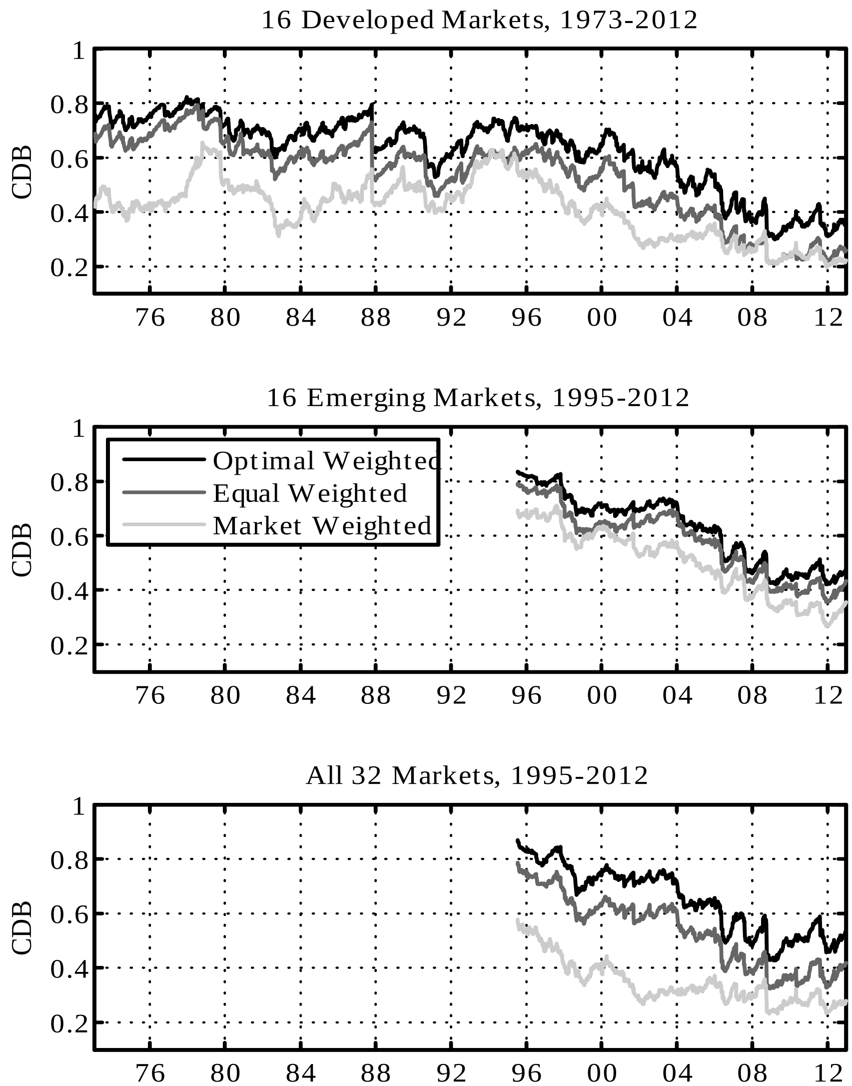
Notes to Figure: The shaded areas show the correlation range between the 90th and 10th percentiles for DCCs (left panels) and BEKKs (right panels). The top panels report on sixteen developed markets for the period January 26, 1973 to December 28, 2012. The middle panels report on sixteen emerging markets for the period July 21, 1995 to December 28, 2012. The bottom panels report on sixteen developed and sixteen emerging markets for the period July 21, 1995 to December 28, 2012.

Figure 6: Comparing Average Correlations from DCC and BEKK



Notes to Figure: We report averages of DCCs (left panels) and BEKKs (right panels). The top panels report on sixteen developed markets for the period January 26, 1973 to December 28, 2012. The middle panels report on sixteen emerging markets for the period July 21, 1995 to December 28, 2012. The bottom panels report on sixteen developed and sixteen emerging markets for the period July 21, 1995 to December 28, 2012.

Figure 7: Conditional Diversification Benefits (CDB) using the DCC Model.
Developed, Emerging and All Markets



Notes to Figure: For each set of countries, we use the dynamic conditional correlation (DCC) model to compute the correlation-based conditional diversification benefits (CDB) as defined in (2.11). the black lines are using CDB-optimal portfolio weights, the dark grey lines are using equal weights and the light grey lines are using market capitalization weights.

Table 1: Descriptive Statistics for Weekly Returns on 16 DM and 16 EM (IFCI)
July 21, 1995 to December 28, 2012

	Annual Mean (%)	Annual Standard Deviation	Skewness	Excess Kurtosis	1st Order Auto-Correlation	LB(20) P-Value on Returns	LB(20) P-Value on Absolute Returns
Developed Markets							
Australia	14.03	23.26	-1.209	10.64	-0.044	0.0395	0.0000
Austria	8.63	22.96	-1.273	9.49	0.029	0.0013	0.0000
Belgium	10.38	21.51	-0.902	6.12	0.020	0.0022	0.0000
Canada	13.88	22.50	-0.787	6.89	-0.086	0.0021	0.0000
Denmark	12.87	21.90	-1.092	6.87	-0.055	0.0089	0.0000
France	10.65	22.51	-0.677	4.63	-0.025	0.0021	0.0000
Germany	9.38	22.98	-0.550	4.85	-0.020	0.1725	0.0000
Hong Kong	11.31	24.51	-0.237	3.02	0.021	0.1820	0.0000
Ireland	8.53	23.73	-1.192	8.04	-0.003	0.0098	0.0000
Italy	8.91	24.85	-0.501	5.03	0.013	0.0109	0.0000
Japan	2.10	20.85	0.081	1.63	-0.087	0.1558	0.0000
Netherlands	8.92	22.87	-0.809	6.94	0.025	0.0023	0.0000
Singapore	9.47	23.00	-0.262	5.19	0.031	0.0090	0.0000
Switzerland	10.32	18.97	-0.640	6.39	-0.062	0.0129	0.0000
United Kingdom	9.42	20.30	-0.759	9.16	-0.085	0.0000	0.0000
United States	9.42	18.77	-0.512	5.07	-0.080	0.0045	0.0000
Average	9.89	22.22	-0.708	6.25	-0.026	0.0385	0.0000
Median	9.45	22.69	-0.718	6.25	-0.023	0.0090	0.0000
Emerging Markets							
Brazil	21.39	38.56	-0.256	3.67	-0.087	0.0017	0.0000
Chile	10.91	23.56	-0.841	8.43	0.019	0.0491	0.0000
China	12.22	33.41	0.066	2.96	-0.009	0.6726	0.0000
Hungary	17.22	37.74	-0.479	4.99	0.005	0.0010	0.0000
India	12.35	28.19	-0.158	2.10	0.086	0.0043	0.0000
Indonesia	16.54	48.26	0.692	16.18	-0.093	0.0000	0.0000
Korea	14.45	40.46	0.071	7.35	-0.091	0.0025	0.0000
Malaysia	7.67	28.94	1.433	26.99	0.005	0.0000	0.0000
Mexico	17.91	30.88	-0.213	6.25	-0.058	0.0021	0.0000
Peru	19.53	26.37	-0.258	3.93	0.034	0.2499	0.0000
Philippines	6.27	28.82	-0.388	4.62	0.005	0.0443	0.0000
Poland	13.46	33.52	-0.290	2.68	0.013	0.0314	0.0000
South Africa	14.06	28.63	0.102	5.26	-0.026	0.3596	0.0000
Taiwan	7.19	28.23	0.156	2.42	-0.012	0.5137	0.0000
Thailand	7.37	35.37	0.194	3.82	0.020	0.0000	0.0000
Turkey	24.65	51.17	0.232	7.70	-0.057	0.1464	0.0000
Average	13.95	33.88	0.004	6.84	-0.015	0.1299	0.0000
Median	13.76	32.15	-0.046	4.81	-0.002	0.0179	0.0000

Notes to Table: We report the first four sample moments and the first order autocorrelation of the 16 DM and 16 EM (IFCI) returns. We also report the p-value from a Ljung-Box test that the first 20 autocorrelations are zero for returns and absolute returns. The sample period is from July 21, 1995 to December 28, 2012.

Table 2: Parameter Estimates from NGARCH(1,1) on 16 DM and 16 EM (IFCI)
July 21, 1995 to December 28, 2012

	α	β	θ	Variance Persistence	LB(20) P-Value on Residuals	LB(20) P-Value on Absolute Residuals	Residual Skewness	Residual Excess Kurtosis
Developed Markets								
Australia	0.122	0.808	0.501	0.960	0.616	0.145	-0.684	1.96
Austria	0.085	0.888	0.364	0.985	0.192	0.247	-0.554	1.90
Belgium	0.149	0.678	0.800	0.922	0.232	0.302	-0.434	1.27
Canada	0.119	0.778	0.641	0.947	0.348	0.736	-0.483	0.99
Denmark	0.057	0.931	0.313	0.994	0.575	0.579	-0.696	2.44
France	0.110	0.733	0.908	0.933	0.187	0.100	-0.398	1.09
Germany	0.151	0.702	0.731	0.933	0.899	0.173	-0.411	0.92
Hong Kong	0.089	0.854	0.612	0.977	0.526	0.781	-0.237	0.69
Ireland	0.042	0.894	1.048	0.981	0.244	0.763	-0.726	2.12
Italy	0.162	0.730	0.548	0.940	0.567	0.056	-0.271	0.73
Japan	0.046	0.919	0.542	0.978	0.823	0.813	-0.101	1.21
Netherlands	0.118	0.705	1.051	0.953	0.731	0.493	-0.465	0.90
Singapore	0.072	0.858	0.849	0.982	0.138	0.702	-0.367	2.02
Switzerland	0.110	0.576	1.275	0.866	0.461	0.818	-0.475	1.56
United Kingdom	0.105	0.750	0.906	0.941	0.626	0.407	-0.614	1.70
United States	0.148	0.675	0.950	0.957	0.421	0.338	-0.541	1.32
Average	0.105	0.780	0.752	0.953	0.474	0.466	-0.466	1.427
Median	0.110	0.764	0.765	0.955	0.494	0.450	-0.470	1.294
Emerging Markets								
Brazil	0.086	0.805	0.783	0.944	0.684	0.272	-0.502	1.02
Chile	0.100	0.833	0.500	0.958	0.673	0.895	-0.277	1.62
China	0.146	0.812	0.360	0.977	0.696	0.610	-0.064	0.77
Hungary	0.140	0.637	0.944	0.902	0.073	0.577	-0.543	2.43
India	0.072	0.868	0.584	0.964	0.193	0.194	-0.110	1.06
Indonesia	0.102	0.865	0.534	0.995	0.261	0.030	-0.368	2.14
Korea	0.157	0.783	0.482	0.976	0.427	0.417	-0.317	0.58
Malaysia	0.109	0.874	0.275	0.991	0.526	0.734	-0.328	2.81
Mexico	0.134	0.684	0.930	0.934	0.760	0.821	-0.365	0.85
Peru	0.092	0.870	0.181	0.965	0.786	0.630	-0.243	1.03
Philippines	0.045	0.885	0.953	0.971	0.930	0.783	-0.424	1.81
Poland	0.082	0.812	0.684	0.932	0.203	0.825	-0.208	0.89
South Africa	0.086	0.778	0.858	0.927	0.884	0.640	-0.458	1.11
Taiwan	0.067	0.874	0.709	0.974	0.862	0.887	-0.145	1.46
Thailand	0.048	0.910	0.757	0.985	0.598	0.155	-0.259	1.51
Turkey	0.058	0.918	0.526	0.992	0.099	0.711	0.023	2.94
Average	0.095	0.826	0.629	0.962	0.541	0.574	-0.287	1.503
Median	0.089	0.849	0.634	0.968	0.635	0.635	-0.297	1.283

Notes to Table: We report parameter estimates and residual diagnostics for the NGARCH(1,1) models. The sample period for 16 DM and 16 EM (IFCI) weekly returns is from July 21, 1995 to December 28, 2012. The conditional mean is modeled by an AR(2) model. The coefficients from the AR models are not shown. The constant term in the GARCH model is fixed by variance targeting.

Table 3: Parameter Estimates for DECO, DCC, and BEKK Models. Emerging Markets (EM) and Developed Markets (DM)

	DECO					DCC				BEKK			
	ω	α	β	Persistence	Composite Likelihood	α	β	Persistence	Composite Likelihood	α	β	Persistence	Composite Likelihood
Weekly Returns, January 26, 1973 to December 28, 2012													
16 Developed Markets	1.68E-02	0.0979	0.9021	1.000	9530.17	0.0208	0.9777	0.998	9585.46	0.0685	0.9170	0.985	9532.95
No Dynamics	0.4450	0	0	0	9471.67	0	0	0	9499.93	0	0	0	9097.73
Weekly IFCG Returns, January 20, 1989 to July 25, 2008													
16 Developed Markets	1.17E-02	0.0683	0.9317	1.000	4880.29	0.0249	0.9711	0.996	4912.09	0.0501	0.9370	0.987	4890.48
No Dynamics	0.4846	0	0	0	4866.39	0	0	0	4887.70	0	0	0	4770.28
13 Emerging Markets	3.22E-03	0.0419	0.9581	1.000	3760.82	0.0135	0.9822	0.996	3770.90	0.0652	0.9214	0.987	3733.96
No Dynamics	0.2032	0	0	0	3754.69	0	0	0	3761.08	0	0	0	3496.26
All 29 Markets	7.41E-03	0.0695	0.9305	1.000	4344.19	0.0186	0.9776	0.996	4370.70	0.0572	0.9299	0.987	4340.05
No Dynamics	0.3106	0	0	0	4334.93	0	0	0	4354.87	0	0	0	4168.80
Weekly IFCI Returns, July 21, 1995 to December 28, 2012													
16 Developed Markets	2.33E-02	0.0980	0.8967	0.995	4246.79	0.0355	0.9534	0.989	4286.57	0.0702	0.9145	0.985	4253.16
No Dynamics	0.6012	0	0	0	4224.91	0	0	0	4252.49	0	0	0	4059.28
16 Emerging Markets	1.07E-02	0.0693	0.9307	1.000	3385.97	0.0159	0.9814	0.997	3393.97	0.0753	0.9112	0.986	3354.45
No Dynamics	0.4096	0	0	0	3372.15	0	0	0	3376.98	0	0	0	3129.06
All 32 Markets	1.18E-02	0.0703	0.9297	1.000	3786.69	0.0222	0.9716	0.994	3809.35	0.0731	0.9125	0.986	3770.69
No Dynamics	0.4711	0	0	0	3768.82	0	0	0	3786.29	0	0	0	3557.21

Notes to Table: We report parameter estimates for the DCC, DECO and BEKK models for the 13 emerging markets (IFCG), 16 emerging markets (IFCI), 16 developed markets, and all markets. The composite likelihood is the average of the quasi-likelihoods (correlation log likelihood + all marginal log likelihoods) of all pairs of assets. We also report the special case of no dynamics.

Table 4: Statistics for CDB Portfolio Weights (%). DM and EM Portfolios
July 21, 1995 to December 28, 2012

	Min	Precentiles 25%	50%	75%	Max	Fraction of Zeros (%)
Developed Markets Portfolio						
Australia	0.00	0.00	0.00	0.42	17.77	73.11
Austria	0.00	0.00	7.54	18.63	34.37	28.65
Belgium	0.00	0.00	2.84	7.11	17.62	34.69
Canada	0.00	0.00	2.45	9.20	19.09	37.54
Denmark	0.00	4.68	8.47	12.14	21.93	11.86
France	0.00	0.00	0.00	0.00	5.19	98.24
Germany	0.00	0.00	0.00	0.00	5.74	98.68
Hong Kong	0.00	6.17	12.36	18.43	26.58	8.12
Ireland	0.00	0.00	6.43	10.80	28.96	26.02
Italy	0.00	0.00	2.17	12.59	29.61	42.59
Japan	7.11	18.24	22.97	27.42	40.86	0.00
Netherlands	0.00	0.00	0.00	0.00	0.00	100.00
Singapore	0.00	0.00	4.90	11.68	20.52	29.09
Switzerland	0.00	0.00	0.00	0.00	22.68	76.18
United Kingdom	0.00	0.00	0.00	0.00	3.01	99.23
United States	0.00	7.82	13.36	17.99	31.88	0.55
Average	0.44	2.31	5.22	9.15	20.36	47.78
Median	0.00	0.00	2.64	10.00	21.23	36.11
Emerging Markets Portfolio						
Brazil	0.00	0.00	0.00	0.74	10.37	72.45
Chile	0.00	0.59	3.76	8.87	22.24	21.73
China	0.00	0.00	0.87	7.55	23.67	47.75
Hungary	0.00	0.00	2.29	6.36	15.99	36.99
India	0.00	7.22	9.97	12.44	21.10	4.28
Indonesia	0.00	6.11	9.60	12.36	20.49	1.54
Korea	0.00	0.00	0.00	5.26	14.79	61.03
Malaysia	0.00	4.54	7.86	11.07	17.84	11.31
Mexico	0.00	0.00	0.00	4.25	10.41	51.92
Peru	0.00	5.34	9.29	13.25	21.79	10.76
Philippines	0.00	1.20	6.16	9.55	22.43	20.53
Poland	0.00	0.00	1.83	8.34	17.09	41.49
South Africa	0.00	0.00	1.72	5.72	17.92	38.53
Taiwan	0.00	4.96	9.66	14.06	23.39	1.54
Thailand	0.00	0.00	4.42	11.95	33.07	30.19
Turkey	0.00	10.29	14.67	18.98	29.07	0.55
Average	0.00	2.52	5.13	9.42	20.10	28.29
Median	0.00	0.29	4.09	9.21	20.79	25.96

Notes to Table: We report statistics from the distribution of portfolio weights (in percent) for each country in the developed market CDB measure and the emerging market CDB measure. The final column shows the percent of weeks in which the portfolio weight is exactly zero for a given country. We impose no-short sale constraints so that the minimum possible weight is zero.

Table 5: R^2 from CDB Forecasting Regressions

	Developed Markets			Emerging Markets			All Markets		
	Portfolio Weights			Portfolio Weights			Portfolio Weights		
	Optimal	Market	Equal	Optimal	Market	Equal	Optimal	Market	Equal
Forecast Model	Panel A: Realized CDB from Own-Model Correlations								
DCC	0.957	0.822	0.962	0.971	0.973	0.975	0.968	0.774	0.976
BEKK	0.969	0.960	0.974	0.981	0.977	0.982	0.980	0.955	0.981
DECO	0.873	0.814	0.873	0.873	0.873	0.873	0.873	0.789	0.873
6-Month Rolling	0.956	0.930	0.960	0.972	0.975	0.977	0.971	0.929	0.976
2-Year Rolling	0.995	0.989	0.996	0.997	0.997	0.997	0.997	0.988	0.997
Forecast Model	Panel B: Realized CDB from DCC Model Correlations								
DCC	0.957	0.822	0.962	0.971	0.973	0.975	0.968	0.774	0.976
BEKK	0.913	0.831	0.899	0.916	0.913	0.923	0.920	0.844	0.932
DECO	0.848	0.485	0.848	0.914	0.901	0.914	0.910	0.388	0.910
6-Month Rolling	0.768	0.709	0.818	0.843	0.877	0.892	0.822	0.736	0.891
2-Year Rolling	0.858	0.741	0.888	0.953	0.951	0.959	0.934	0.672	0.950
Forecast Model	Panel C: Realized CDB from Two-Sided 6-Month Rolling Correlations								
DCC	0.491	0.231	0.594	0.635	0.657	0.681	0.613	0.212	0.704
BEKK	0.551	0.352	0.606	0.662	0.649	0.703	0.640	0.327	0.714
DECO	0.495	0.141	0.495	0.635	0.595	0.635	0.628	0.111	0.628
6-Month Rolling	0.418	0.208	0.487	0.553	0.545	0.630	0.567	0.184	0.644
2-Year Rolling	0.377	0.113	0.431	0.642	0.620	0.663	0.594	0.101	0.655
Forecast Model	Panel D: Realized CDB from Two-Sided 2-Year Rolling Correlations								
DCC	0.782	0.583	0.865	0.792	0.868	0.842	0.776	0.518	0.865
BEKK	0.556	0.279	0.638	0.666	0.632	0.685	0.661	0.203	0.708
DECO	0.848	0.633	0.848	0.876	0.901	0.876	0.860	0.606	0.860
6-Month Rolling	0.426	0.168	0.540	0.611	0.599	0.676	0.575	0.122	0.686
2-Year Rolling	0.757	0.410	0.785	0.876	0.886	0.894	0.852	0.373	0.890

Notes to Figure: We report the R^2 from the predictive regressions of realized CDB on forecasted CDB using different correlation forecasts. The regressions are run on developed, emerging and all markets. Each panel corresponds to different versions of realized CDB. The sample period is January 2, 2001 to December 28, 2012. For the emerging markets we use IFCI data.

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