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Generalizing smooth transition autoregressions

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Abstract

We introduce a variant of the smooth transition autoregression - the GSTAR model - capable to parametrize the asymmetry in the tails of the transition equation by using a particular generalization of the logistic function. A General-to-Specific modelling strategy is discussed in detail, with particular emphasis on two different LM-type tests for the null of symmetric adjustment towards a new regime and three diagnostic tests, whose power properties are explored via Monte Carlo experiments. Four classical real datasets illustrate the empirical properties of the GSTAR, jointly to a rolling forecasting experiment to evaluate its point and density forecasting performances. In all the cases, the dynamic asymmetry in the cycle is efficiently captured by the new model. The GSTAR beats AR and STAR competitors in point forecasting, while this superiority becomes less evident in density forecasting, specially if robust measures are considered.

Keywords: Dynamic Asymmetry, Smooth Transition, Testing, Estimation, (Density) Forecasting Performance.
JEL: C22, C51, C52

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1 Introduction

Many of the economic and natural sciences time series show asymmetric fluctuations, see Tong (1990); Teräsvirta, Tjøstheim, and Granger (2010) *inter alia*. Sichel (1993) gives a double definition of asymmetry in Business Cycle: the first - the *steepness* - happens when contractions in the levels are steeper than expansions (symmetry in the level axis); the second - the *deepness* - when the series undergoes at an accelerating time until a minimum after which it starts to recover with high, decreasing acceleration, until to smoothly recover the peak (symmetry in time axis). When these two definitions are combined, we call this *dynamic asymmetry*.

Smooth transition autoregressions (STAR), originated by the pioneering contribution by Bacon and Watts (1971) in Biostatistics, then developed in time series by Haggan and Ozaki (1981); Chan and Tong (1986) and Teräsvirta (1994), are currently one of the most simple and successful tools to model the nonlinear dynamics in the conditional mean and/or variance. In particular, a logistic transition is commonly postulated when the series under consideration is assumed having asymmetric oscillations from its conditional mean. We argue that, being the logistic function reflectively symmetric by construction, the resulting logistic STAR does not match the theoretical definition of dynamic asymmetry. In other words, the available models allow the econometrician, at the best, to answer to the question: *Does the series return to its original regime and when?* Here, our objective is to answer to another, more challenging question: *Is the rate of change (if any) in the left tail of the logistic transition different with respect of the right tail and how much?* As we will show, an appropriate solution to this methodological question, per se interesting for descriptive aims, improves the forecasting ability of STAR family.

The econometric literature provides two strategies: the first, proposed by Sollis, Leybourne, and Newbold (1999) (SLN1), is to raise the STAR's transition function to an exponent using an idea by Nelder (1961); the second, suggested by Sollis, Leybourne, and Newbold (2002) (SLN2) is to add a parameter inside the transition function in such a way to control for the asymmetry of both the tails of the transition function by simply using a Heaviside indicator. Both of these solutions have been successfully applied to some classical macroeconomic series. Lundbergh and Teräsvirta (2006) (LT) provides three diagnostic tests for the assessment of the estimated asymmetric model for exchange rate with GARCH errors.

Unfortunately, both of these solutions present some criticality: Figure 1, panel (a) clearly shows that in the SLN2 case, the transition function could be non-smooth; on the other hand, the SLN1 and LT parametrization, plotted in panel (b) conveys a smooth transition, but the effect of increasing of the asymmetry parameter could translate just in a shift effect, if not properly restricted as stated in the same article; moreover, this parametrization does not provide an immediate description of the behavior of each tail of the transition function (which is instead the beauty of SLN2). Thus, the detection and assessment of the dynamic asymmetry in a statistically well-specified time series model seem still an open issue. This work nests this strand of literature and represents a step ahead for what concerns the basic parametrization of the STAR family.

The literature on point forecast combination and on evaluation of individual density forecasts is nowadays established, see Timmermann (2006) and Corradi and Swanson (2006). The literature on aggregation of more density forecasts is instead in a development phase, and focuses on the so called *scoring rules* (or *opinion pools*), peculiar functions enabling the forecaster to properly aggregate the set of conditional predictive density as well as more common measures as Mean Square Forecast Error *et similia* do for point forecasts. Despite their dated origins in statistics, as documented by Gneiting and Raftery (2007), scoring rules are becoming increasingly applied by contemporaneous econometric literature only recently; see, *inter alia*, Mitchell and Hall (2005); Kascha and Ravazzolo (2010); Geweke and Amisano (2011); Ravazzolo and Vahey (2013) and therein mentioned literature. We contribute to this strand of literature by investigating if dynamic asymmetry accounts for density combination.

The next Section 2 applies to the classical STAR model a generalized version of the logistic transition function with two parameters governing the the two tails of the logistic sigmoid and a logarithmic/exponential rescaling able to preserve the smoothness of the transition without requiring any restriction in the parameters. The resulting Generalized STAR (GSTAR) model encloses the symmetric STAR, so we modify the general-to-specific modeling procedure following Granger and Teräsvirta (1993) (GT); this is done in Section 3. Two different LM-type tests for the null hypothesis that the two tails of the transition function are reflexively symmetric - a situation which is called *dynamic symmetry* for what follows - are built-up in Section 4: the first is a classical Score test on the two slope parameters, while the second is modified version of the Taylor-expansion-based test by Luukkonen, Saikkonen, and Teräsvirta (1988) (LST). Section 5 modifies three diagnostic tests originally introduced by Eitrheim and Teräsvirta (1996) (ET). Section 6 provides a simulation study according to which the SLT-type test seems less restrictive than the Score test. Four different case studies on U.S. industrial production and unemployment rate, International Sunspot Number and Canadian Lynx data are illustrated in Section 7, jointly with a rolling forecasting exercise where both point and density forecasting evaluation are investigated: in all these examples, the dynamic asymmetry is found to be a non negligible feature to deal with.

2 The Model

Definition 1. Let be y_t a realization of a time series observed at $t = 1-p, 1-(p-1), \ldots, -1, 0, 1, \ldots, T-1, T$. Then the univariate process $\{y_t\}_t^T$ follows a GSTAR(p) model if

$$y_t = \boldsymbol{\phi}'\mathbf{z}_t + \boldsymbol{\theta}'\mathbf{z}_t G(\boldsymbol{\gamma}, h(c_k, s_t)) + \epsilon_t, \quad \epsilon_t \sim I.I.D.(0, \sigma^2), \tag{1}$$

$$G(\boldsymbol{\gamma}, h(c_k, s_t)) = \left(1 + \exp\left\{ -\prod_{k=1}^{K} h\big(c_k, s_t\big)\right\}\right)^{-1}, \tag{2}$$

$$h(c_k, s_t) = \begin{cases} \gamma_1^{-1}\exp(\gamma_1|s_t - c_k| - 1) & \text{if } \gamma_1 > 0, \\ s_t - c_k & \text{if } \gamma_1 = 0, \\ -\gamma_1^{-1}\log(1 - \gamma_1|s_t - c_k|) & \text{if } \gamma_1 < 0, \end{cases} \tag{3}$$

for $(s_t - c_k) > 0$ (or, equivalently, $h(c, s_t) > 1/2$) and

$$h(c_k, s_t) = \begin{cases} -\gamma_2^{-1}\exp(\gamma_2|s_t - c_k| - 1) & \text{if } \gamma_2 > 0, \\ s_t - c_k & \text{if } \gamma_2 = 0, \\ \gamma_2^{-1}\log(1 - \gamma_2|s_t - c_k|) & \text{if } \gamma_2 < 0, \end{cases} \tag{4}$$

for $(s_t - c_k) \leq 0$ (or, equivalently, $h(c_k, s_t) < 1/2$), where y_t is a dependent variable, $\mathbf{z}_t = (1, y_{t-1}, \ldots, y_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)'$ are parameter vectors, the transition function $G(\cdot, \cdot, \cdot)$ is a continuous function in the vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)$ and in the function $h(c_k, s_t)$, which is strictly increasing in the transition variable $s_t = y_{t-d}$, $d > 0$ is a delay parameter, and the $K = \{1, 2\}$ location parameter(s) c_k.

In what follows we simplify the notation by denoting the kernel of the model corresponding to the k-esim location with $\eta_{k,t} \equiv s_t - c_k$ and by $h(\eta_{k,t})$ the associated function, so that the general form of the transition function $G(\cdot)$ can be written as:

$$G(\boldsymbol{\gamma}, h(\eta_{k,t})) = \left(1 + \exp\left\{ -\prod_{k=1}^{K} \Big[h\big(\eta_{k,t}\big) I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h\big(\eta_{k,t}\big) I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + \right.\right.$$
$$\left.\left. + h\big(\eta_{k,t}\big) I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h\big(\eta_{k,t}\big) I_{(\gamma_1 > 0, \gamma_2 > 0)}\Big] \right\}\right)^{-1}. \tag{5}$$

Equation (3) (equation (4)) models the higher (lower) tail of the probability function, so allowing for the asymmetric behavior introduced by the slope parameter γ_1 (γ_2) which controls the velocity of the transition. The case in which $h(\eta_{k,t}) = \eta_{k,t}$ implies that the function nests a one-parameter symmetric logistic STAR model with slope $\gamma_1 = \gamma_2 = \gamma$. When $\gamma_1, \gamma_2 > 0$ ($\gamma_1, \gamma_2 < 0$), $h(\eta_{k,t})$ is an exponential (logarithmic) rescaling which increases more quickly (more slowly) than a standard logistic function. Model (1) can be generalized to other distributions of exponential family. The Indicator functions in (5) stress that slope parameters are not constrained, as in the classical STAR model (whereas the positiveness of the slope parameter was an identifying condition). When $\boldsymbol{\gamma} \to +\infty$, both the models nests an indicator function $I_{(s_t > c)}$, in which case the model become a (Self Exciting) Threshold Autoregression (SETAR), see Tong (1983); on the other side, they nest a straight line around 1/2 for each s_t when $\boldsymbol{\gamma} \to -\infty$.

The Generalized Logistic is plotted in Figure 2: the resulting sigmoid is clearly consistent with the Sichel (1993) definition of dynamic asymmetry (see, e.g., the case in which $\gamma_1 = -2$ and $\gamma_2 = 4$) and maintains the global slope of the transition function unchanged with respect to the traditional LSTAR one, so that no additional identification restriction is needed with respect to the traditional STAR model.

Remark 1. The model described in this section is the time series variant of the original generalized logistic model proposed by Stukel (1988), which differs for the definition of $\mathbf{z} = (x_1, \dots, x_N)'$, for $\{x_i\}_{i=1}^N$ being N exogenous regressors, and consequently, $\eta_t = \boldsymbol{\phi}'\mathbf{z}$.

Remark 2. The GLSTAR model described by equation (1)-(4) nests a linear AR model for $\boldsymbol{\gamma} = \mathbf{0}$ if $h(\eta_t)$ is modified as follows:

$$h(\eta_t)^{EZC} = \begin{cases} \gamma_1^{-1} \exp(\gamma_1 |\eta_t| - 1) & \text{if } \gamma_1 > 0, \\ 0 & \text{if } \gamma_1 = 0, \\ -\gamma_1^{-1} \log(1 - \gamma_1 |\eta_t|) & \text{if } \gamma_1 < 0, \end{cases} \tag{6}$$

for $\eta_t \geq 0$ ($\mu > 1/2$) and

$$h(\eta_t)^{EZC} = \begin{cases} -\gamma_2^{-1} \exp(\gamma_2 |\eta_t| - 1) & \text{if } \gamma_2 > 0, \\ 0 & \text{if } \gamma_2 = 0, \\ \gamma_2^{-1} \log(1 - \gamma_2 |\eta_t|) & \text{if } \gamma_2 < 0, \end{cases} \tag{7}$$

for $\eta_t < 0$ ($\mu < 1/2$). The label "EZC" distinguishes this version from the the original Stukel' s generalized logistic function for exposition matter. This special case is necessary in order to build a test for the null of linearity against of dynamic asymmetry, see next Section 4.

Remark 3. As in the traditional STAR, the process $\{\epsilon_t\}_t^T$ is assumed to be a martingale difference sequence with respect to the history of the time series up to time t - 1, denoted as $\Omega_{t-1} = [y_{t-1}, \ldots, y_{t-p}]$, i.e., $E[\epsilon_t|\Omega_{t-1}] = 0$. This is sufficient to built up tests based on artificial regressions as demonstrated in Davidson and McKinnon (1990) and has important consequence for applied aims, in what the "All-in-One" test discussed in Section 4 and the three diagnostic tests discussed in Section 5 can still be meaningful if the normality test reject this hypothesis. For expositional purposes, we restrict the conditional variance of the process $\{\epsilon_t\}_t^T$ to be constant, $E[\epsilon_t^2|\Omega_{t-1}] = \sigma^2$. Moreover the parameter vectors $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ are assumed to not change in time and the number of regimes is assumed to not exceed $K = 2$. However, these restriction could be relaxed and tested, see Section 5.

Remark 4. As in the traditional STAR, if process is characterized by $G(\mathbf{0}, h(\eta_t)^{EZC})$, we assume $Q(z) = z^p - \phi_1 z^{p-1} - \cdots - \phi_p = 0$ has its roots inside the unit circle, since this implies that the model is stationary and ergodic under the null hypothesis of linearity.

We now discuss three relevant cases of GSTAR model.

Example 1. If $K = 1$, the parameters $\boldsymbol{\phi} + \boldsymbol{\theta} G(\boldsymbol{\gamma}, \mathbf{c}, s_t)$ change monotonically as a function of s_t from $\boldsymbol{\phi}$ to $\boldsymbol{\phi} + \boldsymbol{\theta}$. The corresponding transition function is:

$$G(\boldsymbol{\gamma}, h(\eta_{1t})) = \Big(1 + \exp\Big\{ - \Big[h(\eta_{1,t}) I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_{1,t}) I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h(\eta_{1,t}) I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + + h(\eta_{1,t}) I_{(\gamma_1 > 0, \gamma_2 > 0)}\Big]\Big\}\Big)^{-1}, \tag{8}$$

with $h(\eta_{1,t})$ corresponding to (3) and (4).

Example 2. When $K = 2$ and $c_1 \neq c_2 = \mathbf{c}$, the model (1) nests the following STAR model with second order Generalized Logistic (GLSTAR2) function:

$$G(\boldsymbol{\gamma}, h(\eta_t)) = 1 - \exp\Big\{ - h(\eta_{2,t})\Big\}, \tag{9}$$

where:

$$h(\eta_{2,t}) = \begin{cases} \gamma_1^{-1}\exp(\gamma_1|(s_t - c_1)(s_t - c_2)| - 1) & \text{if } \gamma_1 > 0, \\ (s_t - c_1)(s_t - c_2) & \text{if } \gamma_1 = 0, \\ -\gamma_1^{-1}\log(1 - \gamma_1|(s_t - c)(s_t - c_2)|) & \text{if } \gamma_1 < 0, \end{cases} \tag{10}$$

for $(s_t - c)^2 > 0$ (or, equivalently, $h(\eta_t) > 1/2$) and

$$h(\eta_{2,t}) = \begin{cases} -\gamma_2^{-1}\exp(\gamma_2|(s_t - c)(s_t - c_2)| - 1) & \text{if } \gamma_2 > 0, \\ (s_t - c)(s_t - c_2) & \text{if } \gamma_2 = 0, \\ \gamma_2^{-1}\log(1 - \gamma_2|(s_t - c_1)(s_t - c_2)|^2) & \text{if } \gamma_2 < 0, \end{cases} \tag{11}$$

for $(s_t - c_1)(s_t - c_2) < 0$ (or, equivalently, $h(\eta_{2,t}) < 1/2$), whith $\eta_t \equiv \eta_t = (s_t - c_1)(s_t - c_2)$. Figure 3 shows the transition function for a set of different combinations of γ_1 for fixed γ_2 (upper panel) and viceversa (lower panel).

Example 3. A particular case of GLSTAR2 holds when $K = 2$ and $c_1 = c_2 = \mathbf{c}$, in which case the model (1) nests an exponential generalized exponential autoregressive (GESTAR) model, which is defined as in (9) - (11), apart the fact that $h(\eta_{2,t}) = (s_t - c)^2$ if $\gamma_1 = 0$ for $(s_t - c)^2 > 0$ and $\gamma_2 = 0$ for $(s_t - c)^2 \leq 0$ In this case, the parameters $\boldsymbol{\phi} + \boldsymbol{\theta}G(\cdot)$ change asymmetrically at some (undefined) point where the function reaches its own minimum.

A simulated example of GLSTAR model (in both Stukel' s and EZC' s versions), jointly with its symmetric Teräsvirta (1994) counterpart, is shown in Figure 4. For each of these three models, we used two different specifications, which differ for the location parameter c. As easy seen in panel (a), the Stukel and EZC model coincides; the associated transition functions versus time plotted in panel (b) and versus ordered η_t in panel (c) confirm this finding; on the other hand, the plot of $h(\eta_t)$ versus ordered η_t in panel (d) is more informative with respect to the effect of the different kind of asymmetry in the process: $h(\eta_t)^{EZC}$ is a 45° angle straight line, while the rescaling effect is visible in the Stukel' s $h(\eta_t)$ parametrization.

3 Modelling Strategy and Estimation

According to GT, the investigator should always be interested in testing whether a linear AR(p) representation is adequate when building a GSTAR model. If the answer is negative, then the second step will be the selection of a nonlinear symmetric model. Then, the issue of testing for dynamic symmetry hypothesis arises as further step, when finding good specifications of STAR models becomes too difficult or whenever suggested by the economic theory. The resulting General-to-Specific modelling strategy consists in the following 7 steps:

1. Specify a linear autoregressive model.
2. Test linearity for different values of d, and if rejected, determining d in (2) or (9).
3. Choose between LSTAR, LSTAR2 or ESTAR by the Teräsvirta's rule.
4. Test the symmetry of the tails transition function according to the result in Step 3.
5. If the hypothesis of symmetry is rejected, estimate the GSTAR model with the most appropriate transition function given by step 3.
6. Evaluate the new parametrization by some diagnostic tests.
7. Use the estimated GSTAR model for forecasting aims.

The autoregressive order p is selected according to Bayesian Information Criterion (Schwarz, 1978), which is combined with the result with a portmanteau test for serial correlation in order to avoid a wrong rejection of symmetry hypothesis. This is due to the fact that the GSTAR model requires a lower autoregressive order with respect to its symmetric counterpart.

For what concerns the Step 4, the dynamic symmetry hypothesis is tested by two different LM-type tests. In the first test the series is assumed to follow a STAR model, so that testing for symmetry is a second step with respect to testing for linearity. Hence, we will refer to this test as "Two-Step test". In the second test we do not assume any prior of the nonlinearity of the series, so that it enclose all steps from 2) to 5) of the General-to-Specific modelling strategy above mentioned; hence, the use of the label "All-in-one" to distinguish it from the different null hypothesis of "Two-Step" test. The choice of what test to use depends on the needs of the investigator[1]. Our experience suggests to perform the "All-in-One" test should be used if the investigator wants to be conservative against evidence of asymmetric dynamics, while the "Two-Step" tends to not reject the null unless extreme situations (see Section 7 for details). Both the tests will be discussed in the next Section 4.

The choice of the delay parameter d and the choice of the transition function can be done with the same procedure adopted in Tsay (1989) and Teräsvirta (1994).

Following Leybourne, Newbold, and Vougas (1998), estimation is done by concentrating the Sum of Square Residuals function with respect to $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, that is minimizing:

$$SSR = \sum_{t=1}^{T} \left(y_t - \hat{\boldsymbol{\psi}}' \mathbf{x}_\mathbf{t}' \right)^2, \tag{12}$$

where:

$$\hat{\boldsymbol{\psi}} = [\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\theta}}] = \left(\sum_{t=1}^{T} \mathbf{x}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) \mathbf{x}_\mathbf{t}(\boldsymbol{\gamma}, \mathbf{c}) \right)^{-1} \left(\sum_{t=1}^{T} \mathbf{x}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) y_t \right), \tag{13}$$

and

$$\mathbf{x}_\mathbf{t}(\hat{\gamma}, \hat{\mathbf{c}}) = \left[\mathbf{z}, \mathbf{z}_\mathbf{t}' G(\hat{\gamma}, h(\hat{\mathbf{c}}, s_t) \right]. \tag{14}$$

[1]Our simulation study shows that the two tests behave differently in terms of empirical power. See Section 6 for details.

This is possible because if $\boldsymbol{\gamma}$ and $\mathbf{c}$ are known and fixed, the GSTAR model is linear in $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, which can be easy computed via conditional OLS. In a such a way, the nonlinear least square minimization problem, otherwise necessary, more demanding in terms of parameters to estimate and not available in closed-form, reduces to a minimization on three (four) parameters, and is solved via a grid search over γ_1, γ_2, c (c_1, c_2 in case of GLSTAR2).

In our applications, both γ_1 and γ_2 are chosen between a minimum value of -10 and a maximum of 10 with rate 0.5 in the first three examples (-150 and 150 with rate 15 in the fourth one); the grid for parameter c_1 (c_2) is the set of values computed between the 10^{th} and 90^{th} percentile of s_t with rate computed as the difference of the two and divided for an arbitrarily high number (here, 200).

The one-step forecast is immediately available if knowing the nonlinear function in what, by least-square criterion, $E(\epsilon_{t+1}|I_t) = 0$, $I_t = y_{t-i}, i \geq 1$ in (1). The multi-step ahead forecast is not available in closed form and requires numerical integration. Hence at t+1, we generate, $1, \ldots, m, \ldots, M$ draws conditionally on the estimated parameters and obtain the forecast $y_{t+1} \sim f(y_{t+1}|I_t)$; in turn, this is collected to draw, at $t+2$, the forecast $y_{t+2} \sim f(y_t + 2|I_t, y_{t+1}^{(m)})$, and so on until, at $t+h$,the forecast $y_{t+h|t} = f(t+h|I_t, y_{t+1}^{(m)}, \ldots, y_{t+h-1}^{(m)})$ is obtained and then evaluated as:

$$\hat{y}_{t+h} = \frac{1}{M}\sum_{m=1}^{M} \hat{y}_{t+h|t}^{(m)} \tag{15}$$

4 Testing for Dynamic Symmetry

In this section we discuss two LM-type test for the null of dynamic symmetry according to the General-to-Specific strategy stated in the previous section 3. The "Two-Step" test, illustrated in Subsection 4.1, is an adaptation for time series of the original Stukel' s parametrization. On the other side, the "All-in-One" test, derived in Subsection 4.2, takes the idea by LST to linearize the $G(\boldsymbol{\gamma}, h(\eta_t^{EZC}))$ by third-order Taylor expansion of $G(\cdot)$, which leads to an augmented artificial model which in turn can be investigated by a classical χ^2 or F-test. This is due to the fact that the Information matrix is the same as in GT[2].

4.1 "Two-Step" Test

Consider the general formulation (1)-(2). Then, the null hypothesis of no logarithmic (exponential) deviations from the logistic transition in systems (1)-(8) or (1)-(11) can be tested by setting the following hypotheses testing system:

$$H_{0i} : (\gamma_1, \gamma_2) = (0, 0) \;\text{ vs }\; H_{1i} : (\gamma_1, \gamma_2) \neq (0, 0), \;\; i = 1, 2, 3 \;, \tag{16}$$

with subscript i indicating the type of underlining transition function, namely $i = 1$ for generalized logistic (eq. (5)), $i = 2$ for generalized second order logistic (eq. (9)) and $i = 3$ for the generalized exponential one.

This hypothesis system requires a simple score test. Let denote by $\Xi = [\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\gamma}, c]$ the hyper-parameter vector of the model, so that the log-likelihood function of the T observations can be denoted by $\mathcal{L}_t(\mathbf{z_t}, \Xi)$ and the score vector by $\mathbf{q_t}(\mathbf{z_t}, \Xi) = \sum_t \mathbf{q}(\mathbf{z_t}, \Xi) = \partial\mathcal{L}_t(\mathbf{z_t}, \Xi)/\partial\Xi$ evaluated at $(\boldsymbol{\theta_0}, \boldsymbol{\phi_0}, \mathbf{0}, c_0)$. Then, standard results lead to the following log-likelihood function:

$$\begin{aligned} \mathcal{L}_t(\mathbf{z_t}, \Xi) &= const + \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\sigma^2 \sum_t (y_t - \boldsymbol{\phi}'\mathbf{z_t} - \boldsymbol{\theta}'\mathbf{z_t}G)^2 \\ &= const + \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\sigma^2 \sum_t u_t^2(\Xi), \end{aligned} \tag{17}$$

with *const* denoting a constant and u_t the model's residual, and to the score:

$$\mathbf{q_t}(\mathbf{z_t}, \Xi) = \sum_t \mathbf{q}(\mathbf{z_t}, \Xi) = \frac{\partial\mathcal{L}_t(\mathbf{z_t}, \Xi)}{\partial\Xi} = \frac{1}{\sigma^2}\sum_t u_t(\Xi)\mathbf{k_t} \;, \tag{18}$$

[2] See GT, pp. 64-5, adjust the notation for an autoregressive framework and notice that we only modify the definition of nonlinear part $f_t = f(\boldsymbol{w_t}; \boldsymbol{\psi})$, which does not vary the general result.

where

$$\mathbf{k_t} = \frac{\partial u_t(\Xi)}{\boldsymbol{\partial \Xi}} = (\mathbf{z_t}, \mathbf{z_t}G, \boldsymbol{\theta}'\mathbf{z_t}G_{\gamma_1}, \boldsymbol{\theta}'\mathbf{z_t}G_{\gamma_2}, \boldsymbol{\theta}'\mathbf{z_t}G_{t,c}), \tag{19}$$

with $G_{t,\gamma_1} = \partial G/\partial\gamma_1$, $G_{t,\gamma_2} = \partial G/\partial\gamma_2$, and $G_c = \partial G/\partial\gamma_c$ being defined in Appendix A.1.

Under H_0, the test statistic is:

$$S_1(\Xi)^{LM} = \frac{1}{\hat{\sigma}^2}\hat{\mathbf{u}}'\mathbf{H}(\mathbf{H}'\mathbf{H} - \mathbf{H}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{H})^{-1}\mathbf{H}'\hat{\mathbf{u}}, \tag{20}$$

where $\hat{\mathbf{u}} = [\hat{u}_1, \dots \hat{u}_T]'$, $\mathbf{Z} = (\mathbf{z_1'}, \dots, \mathbf{z_T'})'$, $\mathbf{H} = [(\mathbf{h_1^0})', \dots, (\mathbf{h_T^0})']'$, with $(\mathbf{h_t^0})' = \mathbf{k_t^G}$, $\mathbf{k_t^G}$ denoting the sub-vector $[\boldsymbol{\theta'}\mathbf{z_t}G_{\gamma 1}, \boldsymbol{\theta'}\mathbf{z_t}G_{\gamma 2}, \boldsymbol{\theta'}\mathbf{z_t}G_c]'$ and $n = dim(\mathbf{k_t^G})$. Under H_0, statistic S1 is asymptotically distributed as a χ^2_n. Just minor modifications are needed in notation of $\mathbf{k_t}$ and $\mathbf{q_t}$ in case of GLSTAR2 model due to an additional c parameter with respect to the GLSTAR.

4.2 "All-in-One" Test

Consider (2) with $G(\boldsymbol{\gamma}, h(\eta_t^{EZC}))|_{\boldsymbol{\gamma}=\mathbf{0}}$ and define $\boldsymbol{\tau} = (\boldsymbol{\tau_1}, \boldsymbol{\tau_2})'$, where $\boldsymbol{\tau_1} = (\phi_0, \boldsymbol{\phi}')'$, $\boldsymbol{\tau_2} = \boldsymbol{\gamma}$. Let $\hat{\boldsymbol{\tau}}_\mathbf{1}$ the LS estimator of $\boldsymbol{\tau_1}$ under $H_0 : \boldsymbol{\gamma} = \mathbf{0}$, $\hat{\boldsymbol{\tau}} = (\boldsymbol{\tau}_\mathbf{1}', \mathbf{0}')'$. Moreover, let $\mathbf{z_t}(\boldsymbol{\tau}) = \frac{\partial \epsilon_t}{\partial \boldsymbol{\tau}}$ and $\hat{\mathbf{z}}_\mathbf{t} = \mathbf{z_t}(\hat{\boldsymbol{\tau}}) = (\hat{\mathbf{z}}_{\mathbf{1,t}}, \hat{\mathbf{z}}_{\mathbf{2,t}})$, where the partition conforms to that of $\boldsymbol{\tau}$. Then the general form of LM statistic is:

$$S_2(\Xi)^{LM} = \frac{1}{\hat{\sigma}^2}\hat{\mathbf{u}}'\hat{\mathbf{Z}}_\mathbf{2}(\hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{Z}}_\mathbf{2} - \hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{Z}}_\mathbf{1}(\hat{\mathbf{Z}}_\mathbf{1}'\hat{\mathbf{Z}}_\mathbf{1})^{-1}\hat{\mathbf{Z}}_\mathbf{1}'\hat{\mathbf{Z}}_\mathbf{2})^{-1}\hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{u}}, \tag{21}$$

where $\hat{\mathbf{u}}$ is previously defined, $\hat{\sigma}^2 = \frac{1}{T}\sum_1^T \hat{u}_t^2$ and $\hat{u}_t = y_t - \hat{\boldsymbol{\tau}}_\mathbf{1}'\mathbf{z_t}$, $\hat{\mathbf{Z}}_\mathbf{i} = (\hat{\mathbf{z}}_{\mathbf{i1}}, \dots, \hat{\mathbf{z}}_{\mathbf{it}}, \dots, \hat{\mathbf{z}}_{\mathbf{iT}})'$, $i = \{1, 2\}$, $t = 1, \dots, T$. When the model is an GLSTAR, $\hat{\mathbf{z}}_{\mathbf{1,t}} = -\mathbf{z_t} = -(1, y_{t-1}, \dots, y_{t-p})'$ while $\hat{\mathbf{z}}_{\mathbf{2t}} \equiv \frac{\partial^2 u_t}{\partial\boldsymbol{\gamma}\partial\boldsymbol{\gamma}'}\big|_{\boldsymbol{\gamma}=\mathbf{0}} = -\frac{1}{2}\big\{\theta_{20}[y_t(y_{t-d})] - cy_t\boldsymbol{\theta}'\mathbf{z_t} + \boldsymbol{\theta}_\mathbf{2}'\mathbf{z_t}y_t y_{t-d}\big\}$, where d is the delay parameter. The change in the definition of $\mathbf{z_{2t}}$ is not significant in terms of LM stastistic build-up. This implies that no change of treatment with respect to the original parametrization is needed. In particular, in order to circumvent the Davies

(1977)' s problem of unidentification of nuisance parameters θ_0 and $\bar{\boldsymbol{\theta}} = [\theta_1, \ldots, \theta_p]'$ under the null hypothesis, the same LST approach can be used. The linearized GLSTAR model

$$y_t = \phi'\mathbf{z_t} + \theta'\mathbf{z_t}T_3\Big[h\big(\eta_{k,t}\big)I_{(\gamma_1\le 0,\gamma_2\le 0)} + h\big(\eta_{k,t}\big)I_{(\gamma_1\le 0,\gamma_2>0)} + h\big(\eta_{k,t}\big)I_{(\gamma_1>0,\gamma_2\le 0)} + h\big(\eta_{k,t}\big)I_{(\gamma_1>0,\gamma_2>0)}\Big] + \epsilon'_t \ , \tag{22}$$

leads to the following auxiliary regression for testing linearity and symmetry:

$$\hat{u}_t = \hat{\mathbf{z}}'_{\mathbf{1t}}\tilde{\boldsymbol{\beta}}_{\mathbf{1}} + \sum_{j=1}^{p}\beta_{2j}y_{t-j}y_{t-d} + \sum_{j=1}^{p}\beta_{3j}y_{t-j}y_{t-d}^2 + \sum_{j=1}^{p}\beta_{4j}y_{t-j}y_{t-d}^3 + \upsilon_t \ , \tag{23}$$

where υ_t is a $N.I.D.(0,\sigma^2)$ process, $\tilde{\boldsymbol{\beta}}_{\mathbf{1}} = (\beta_{10}, \boldsymbol{\beta}'_{\mathbf{1}})'$, $\beta_{10} = \phi_0 - (c/4)\theta_0$, $\boldsymbol{\beta}_{\mathbf{1}} = \boldsymbol{\phi} - (c/4)\boldsymbol{\theta} + (1/4)\theta_0\mathbf{e}_d$, $\mathbf{e}_d = (0,0,\ldots,0,1,0,\ldots,0)'$ with the d-th element equal to unit and $T_3(G) = f_1G + f_3G^3$ is the third-order Taylor expansion of $G(\Xi)$ at $\boldsymbol{\gamma} = \mathbf{0}$, $f_1 = \partial G(\Xi)/\partial\Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$ and $f_3 = (1/6)\partial^3 G(\Xi)/\partial\Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$, $G(\Xi)$ being defined in previous section. The null hypothesis is

$$H_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \ \ j = 1, \ldots, p, \tag{24}$$

The test statistic:

$$LM_1 = (SSR_0 - SSR)/\hat{\sigma_v}^2 \ , \tag{25}$$

with SSR_0 and SSR denoting the sum of squared estimated residuals from the estimated auxiliary regression (23) and under the null and alternative, respectively and $\sigma_v^2 = (1/T)SSR$, has an asymptotic χ^2_{3p} distribution under H_0.

If the model is an GESTAR(p), then $\hat{\mathbf{z}}_{\mathbf{1}} = -\mathbf{z_t}$ as in the generalized logistic case, while $\hat{\mathbf{z}}_{\mathbf{2,t}} = -2\boldsymbol{\theta}'_{\mathbf{2}}\mathbf{z_t}y_{t-2}^2 - 2\theta_{20}y_{t-d}^2 + 4c\boldsymbol{\theta}'_{\mathbf{2}}\mathbf{w_t}y_{t-d} - 2c^2\boldsymbol{\theta}'_{\mathbf{2}}\mathbf{z_t}y_t + 4c\theta_{20}y_{t-d} - 2c^2\theta_{20} = 2\hat{\mathbf{z}}^{ESTAR}_{\mathbf{2,t}}$, where $ESTAR$ denotes the vector $\hat{\mathbf{z}}_{\mathbf{2,t}}$ for the ESTAR model. That is, the vector $\hat{\mathbf{z}}_{\mathbf{2,t}}$ of the generalized ESTAR model is found to be two times the symmetric one. The corresponding auxiliary regression is

$$\hat{\epsilon}_t = \tilde{\boldsymbol{\beta}}_{\mathbf{1}}'\hat{\mathbf{z}}_{\mathbf{1}} + \boldsymbol{\beta}_{\mathbf{2}}'\mathbf{z_t}y_{t-d} + \boldsymbol{\beta}_{\mathbf{3}}'\mathbf{z_t}y_{t-d}^2 + v_t' \,, \tag{26}$$

where v_t' is a $N.I.D.(0, \sigma^2)$ error term and $\tilde{\boldsymbol{\beta}}_{\mathbf{1}} = (\beta_{10}, \boldsymbol{\beta_1'})'$, with $\beta_{10} = \phi_0 - c^2\theta_0$ and $\boldsymbol{\beta_1} = \boldsymbol{\phi} - c^2\boldsymbol{\theta} + 2c\theta_0\boldsymbol{e}_d$; moreover $\boldsymbol{\beta_2} = 2c\boldsymbol{\theta} - \theta_0\mathbf{e}_d$ and $\boldsymbol{\beta_3} = -\boldsymbol{\theta}$. Thus the null hypothesis of linearity is

$$H_0' : \boldsymbol{\beta_2} = \boldsymbol{\beta_3} = 0, \tag{27}$$

which can be tested by the test statistic:

$$LM_2 = (SSR_0 - SSR)/\hat{\sigma}_{v1}^2, \tag{28}$$

where SSR_0 and SSR are the sum of squared residuals from (26) under the null and the alternative respectively, $\hat{\sigma}_{v1}^2 = (1/T)SSR$. When the null is true, the statistic (28) is asymptotically χ_p^2 distributed.

5 Evaluation

For what concerns the diagnostics, the new parametrization can be applied directly to the three tests developed by ET, which will be discussed in detail.

5.1 Serial independence

Consider the general additive model (1), where:

$$\epsilon_t = a'v_t + u_t = \sum_{j=1}^{q} a_j L^j \epsilon_t + u_t, \quad u_t \sim I.I.D.(0, \sigma^2), \tag{29}$$

with L^j denoting the lag operator, $v_t = (u_{t-1}, \ldots, u_{t-q})'$, $a = (a_1, \ldots, a_q)'$, $a_q \neq 0$. Under the assumption of stationarity and ergodicity (see Section 2), the null hypothesis of serial independence is $H_0 : a = 0$. By pre-multiplying eq. (2) by

$1 - \sum_{j=1}^{q} a_j L^j$ we get:

$$y_t = \sum_j a_j L^j y_t + \boldsymbol{\phi}'\mathbf{z_t} - \sum_j a_j L^j \boldsymbol{\phi}'\mathbf{z_t} + \boldsymbol{\theta}'\mathbf{z_t} G(\cdot) - \sum_j a_j \boldsymbol{\theta}' G(\cdot) + \epsilon_t \ , \tag{30}$$

hence, assuming the necessary initial values $y_0, y_{-1}, \ldots, y_{-(p+q)+1}$ fixed, the pseudo normal loglikelihood for $t = 1, \ldots, T$ is:

$$\begin{aligned} \mathcal{L}_t &= constant + \frac{1}{2}\ln\sigma^2 - \frac{\epsilon_t^2}{2\sigma^2}, \\ \epsilon_t &= y_t - \sum_j a_j L^j y_t - \boldsymbol{\phi}'\mathbf{z_t} + \sum_j a_j L^j \boldsymbol{\phi}'\mathbf{z_t} - \boldsymbol{\theta}' G(\mathbf{z_{t-j}}, \Xi) + \sum_j a_j \boldsymbol{\theta}' G(\mathbf{z_{t-j}}, \Xi) \ . \end{aligned} \tag{31}$$

Consistently with the model initial assumptions, the information matrix is block diagonal, hence we can consider σ^2 fixed for the rest of the derivations. So we have:

$$\frac{\partial \mathcal{L}_t}{\partial a_j} = \frac{\epsilon_t}{\sigma^2}[y_{t-j} - \boldsymbol{\phi}'\mathbf{z_{t-j}} - \boldsymbol{\theta}' G(\mathbf{z_{t-j}}, \Xi)] \tag{32}$$

$$\frac{\partial \mathcal{L}_t}{\partial \Xi} = \frac{\epsilon_t}{\sigma^2}\left[\boldsymbol{\theta}'\mathbf{z_t}\frac{\partial G(\mathbf{z_{t-j}}, \Xi)}{\partial \Xi} - \sum_j a_j \boldsymbol{\theta}' \frac{\partial G(\mathbf{z_{t-j}}, \Xi)}{\partial \Xi}\right] \ . \tag{33}$$

Under H_0, consistent estimators of (32) - (33) are:

$$\left.\frac{\partial \hat{\mathcal{L}}_t}{\partial a_t}\right|_{H_0} = \frac{1}{\sigma^2}\hat{\mathbf{u}}_\mathbf{t}\hat{\mathbf{v}}_\mathbf{t} \quad \left.\frac{\partial \hat{\mathcal{L}}_t}{\partial \boldsymbol{\Xi}_t}\right|_{H_0} = -\frac{1}{\sigma^2}\hat{\mathbf{u}}_\mathbf{t}\hat{\mathbf{z}}_\mathbf{t} \ , \tag{34}$$

where $\hat{\mathbf{u}}_\mathbf{t} = (\hat{\mathbf{v}}_\mathbf{t-1}, \ldots, \hat{\mathbf{v}}_\mathbf{t-q})'$, $\hat{\mathbf{v}}_\mathbf{t-j} = y_{t-j} - \boldsymbol{\phi}'\mathbf{z_{t-j}} - \boldsymbol{\theta}' G(\mathbf{z_{t-j}}, \hat{\Xi})$, $j = 1, \ldots, q$, $\hat{\boldsymbol{\Xi}}$ is the QMLE of $\boldsymbol{\Xi}$ and $\hat{\mathbf{z}}_\mathbf{t} = \frac{\partial G(\mathbf{z_t}, \hat{\boldsymbol{\Xi}})}{\partial \hat{\boldsymbol{\Xi}}} = \boldsymbol{k}_t^{\boldsymbol{G}} = [\boldsymbol{\theta}'\mathbf{z_t} G_{\gamma 1}, \boldsymbol{\theta}'\mathbf{z_t} G_{\gamma 2}, \boldsymbol{\theta}'\mathbf{z_t} G_{\gamma c}]$. The resulting LM statistic is:

$$LM = \frac{1}{\hat{\sigma}}\left(\hat{\mathbf{u}}_\mathbf{t}'\hat{\mathbf{v}}_\mathbf{t}\right)\left\{\hat{\mathbf{v}}_\mathbf{t}'\hat{\mathbf{v}}_\mathbf{t} - \hat{\mathbf{v}}_\mathbf{t}'\hat{\mathbf{z}}_\mathbf{t}\left(\hat{\mathbf{z}}_\mathbf{t}\hat{\mathbf{z}}_\mathbf{t}'\right)^{-1}\hat{\mathbf{z}}_\mathbf{t}'\hat{\mathbf{v}}_\mathbf{t}\right\}^{-1}\left(\hat{\mathbf{v}}'\hat{\mathbf{u}}_\mathbf{t}\right), \tag{35}$$

with $\hat{\sigma}^2 = \frac{1}{T}\sum_t u_t^2$. Under the null hypothesis, statistics (35) is asymptotically χ_q^2 distributed. The partial derivatives of $G(\cdot)$ are shown in Appendix A.1. Another possibility is to use the same three-step procedure for carrying an F-test:

1. Estimate the GSTR model under the assumption of uncorrelated errors and compute the residual sum of squares $SSR_0 = \sum_{t=1}^{T} \hat{u}_t^2$.

2. Regress $\hat{u}_t$ on $\hat{v}_t$, $\mathbf{z_t}$, $\mathbf{z_t}\hat{G}(\mathbf{z_{t-d}})$, $\hat{G}_{\gamma 1}$, $\hat{G}_{\gamma 2}$, $\hat{G}_c$ (eventually $\hat{G}_{c2}$ in case of GLSTR2) and compute SSR;

3. Compute the test statistic $F_{LM} = \frac{SSR_0 - SSR}{q} / \frac{SSR}{T-n-q}$, where $n = \dim(\hat{\mathbf{z}}_\mathbf{t})$

The $F-$statistics is preferable to the χ^2 statistics which may suffer from size problems when the number of lags is high and time series is short, so that the estimated residuals can be non-orthogonal to the gradient vector $\hat{\mathbf{z}}_\mathbf{t}$. In this case ET suggests to add an extra-step to the step (i), consisting in regressing the estimated errors to $\mathbf{z_t}$, $\mathbf{z_t}$, $\mathbf{z_t}\hat{G}(\mathbf{z_{t-j}})$, $\hat{G}_{\gamma 1}$, $\hat{G}_{\gamma 2}$, $\hat{G}_c$; the resulting errors $\tilde{u}_t$ is used to derive the $SSR_1 = \sum_{t=1}^{T} \tilde{u}_t^2$.

5.2 No remaining asymmetry

As in the symmetric STAR model, we are interested to detect possible misspecification. In this case there are two plausible issues to investigate: neglected (additive) nonlinearity and, in our case, neglected asymmetry. Consider the additive GSTAR model:

$$y_t = \boldsymbol{\phi}'\mathbf{z_t} + \boldsymbol{\theta}'\mathbf{z_t} G_1(\boldsymbol{\gamma}, h(\eta_t)^{(1)}) + \boldsymbol{\pi}'\mathbf{z_t} G_2(\boldsymbol{\gamma}, h(\eta_t)^{(2),EZC}) + u_t, \tag{36}$$

with $u_t \sim I.I.D.\ (0, \sigma^2)$. The null of neglected asymmetry is:

$$H_0 : h(\eta_t)^{(2),EZC} = 0 \quad \text{vs} \quad H_0 : h(\eta_t)^{(2),EZC} \neq 0. \tag{37}$$

If $\boldsymbol{\gamma}$ is found being not null, the investigator can easily check if the additive nonlinear part is significant. The EZC version of $h(\eta_t)$ is necessary in order to nest the discussion to ET framework. We assume that, under H_0, $\boldsymbol{\Xi}$ can be consistently estimated by QML. Similarly to the symmetric case, it should be noticed that the model is not identified under H_0, so that the Taylor expansion of the $G(\cdot)$ suggested by LST can be used in order to circumvent this problem. In this case, we assume $G_2(\cdot)$ as generalized logistic and replace it with its third-order Taylor expansion about $h(\boldsymbol{\gamma})^{(2)} = 0$. This implies:

$$T_2 = g_{20} + g_{21} y_{t-l} + g_{22} y_{t-l}^2 + g_{23} y_{t-l}^3 \ , \tag{38}$$

where g_{2j}, $j = 0, 1, 2, 3, 4$ are functions of $\boldsymbol{\gamma}^{(2)}$ such that $g_{20} = g_{21} = g_{22} = g_{23} = 0$ for $\boldsymbol{\gamma}^{(2)} = \mathbf{0}$, consistently with the definition of $h_\gamma(s_t)$. By re-parametrizing, the model (36) became:

$$y_t = \beta_0' \mathbf{z_t} + \boldsymbol{\theta}' \mathbf{z_t} G_1(\cdot) + \beta_1' \tilde{\mathbf{z}}_\mathbf{t} y_{t-l} + \beta_2' \tilde{\mathbf{z}}_\mathbf{t} y_{t-l}^2 + \beta_3' \tilde{\mathbf{z}}_\mathbf{t} y_{t-l}^3 + r_t \ , \tag{39}$$

where $\tilde{\mathbf{z}}_\mathbf{t} = (y_{t-1}, \ldots, y_{t-p})'$. The null hypothesis of no additive nonlinearity is $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$, and, as in the symmetric case, under H_0, $r_t = u_t$. The LM statistics distributes as a $\chi^2(3p)$. As in the symmetric case, the test preserves power also against generalized exponential transition. Since there are no modifications in the statistical assumptions concerning the errors distribution, the asymptotic theory is the same of the symmetric STAR case. The test statistic is (21) with $\hat{\mathbf{z}}_\mathbf{t} = (\mathbf{z_t}, \mathbf{z_t} \hat{G}(\cdot), \hat{G}_{\gamma 1}, \hat{G}_{\gamma 2}, \hat{G}_c)'$ (or $\hat{G}_{c1}$, $\hat{G}_{c2}$ in case of GLSTR2), whereas $\mathbf{v_t} = (\tilde{\mathbf{z}}_\mathbf{t}' y_{t-l}, \tilde{\mathbf{z}}_\mathbf{t}' y_{t-l}^2, \tilde{\mathbf{z}}_\mathbf{t}' y_{t-l}^3)'$. As in the symmetric STAR model, the test is implemented with the same procedure for serial correlation, the F-test has $(3p)$ and $(T - n - 3p)$ degrees of freedom and the Teräsvirta rule can be applied to (39) in order to select the form of the transition. If this selection is not desirable, a polynomial expansion of (36) can be performed to build up an omnibus test, but in this case, a rejection of the null of no additive nonlinearity will not give any qualitative information, that is why we do not take in consideration this scenario.

5.3 Parameter constancy

Consider the model:

$$y_t = \boldsymbol{\phi}(\boldsymbol{t})'\bar{\mathbf{z}}_{\mathbf{t}} + \boldsymbol{\theta}(\boldsymbol{t})'\tilde{\mathbf{z}}_{\mathbf{t}} G(\boldsymbol{\gamma}, h(\eta_t)) + u_t, \quad u_t \sim I.I.D.\ (0, \sigma^2)\ , \tag{40}$$

with $\bar{\mathbf{z}}_{\mathbf{t}}$ denoting the $k \leq p+1$ element of $\mathbf{z_t}$ for which the corresponding element of ϕ is not assumed zero a priori, $\tilde{\mathbf{z}}_{\mathbf{t}}$ is the same $(l \times 1)'$ for the element of $\boldsymbol{\theta}$. Let $\tilde{\boldsymbol{\phi}}$ and $\tilde{\boldsymbol{\theta}}$ denote the equivalent $(k+1)$ and $(l+1)$ parameter vectors, $\boldsymbol{\phi}(\boldsymbol{t}) = \tilde{\boldsymbol{\phi}} + \lambda_1 G_j(t; \boldsymbol{\gamma}, h(\eta_t)^{(1)})$, and $\boldsymbol{\theta}(\boldsymbol{t}) = \tilde{\boldsymbol{\theta}} + \lambda_2 G_j(t; \boldsymbol{\gamma}, h(\eta_t)^{(2)})$ with λ_1 and λ_2 being a $(k \times 1)$ and $(l \times 1)$ vectors respectively. Then the null of parameter constancy in (40) is

$$H_0 : G_j(t; \boldsymbol{\gamma}, h(\eta_t)) \equiv 0 \ \ (\text{or} \equiv \text{const}). \tag{41}$$

Three forms for G_j can be considered:

$$\begin{aligned} G_1(t; \boldsymbol{\gamma}, h(\mathbf{c}, s_t)) &= (1 + \exp\{-h(\eta_t^{GL})\})^{-1} \ \text{with} \\ \eta_t^{GL} &\equiv t - c\,, \\ G_2(t; \boldsymbol{\gamma}, h(\mathbf{c}, s_t)) &= 1 + \exp\{-h(\eta_t^{GE})\} \ \text{with} \\ \eta_t^{GE} &\equiv (t-c)^2\,, \\ G_3(t; \boldsymbol{\gamma}, h(\mathbf{c}, s_t)) &= (1 + \exp\{-h(\eta_t^{C})\})^{-1} \text{ with} \\ \eta_t^{C} &\equiv (t^3 + c_{12}t^2 + c_{11}t + c_{10}) \end{aligned} \tag{42}$$

The null of parameter constancy is $H_0 : \boldsymbol{\gamma} = \mathbf{0}$. Notice that in this case the model is identified also in case of $\boldsymbol{\gamma} < \mathbf{0}$, so that the only identifying restriction is that $\boldsymbol{\gamma} \neq \mathbf{0}$. G_1 and G_2 are the Generalized Logistic and Exponential smooth transition of the change in parameters, while G_3 is a cubic function which allows for both monotonically and non-monotonically changing parameters and can be seen as a general case of G_1 and G_2 when building up a test. As suggested by the literature, we use a third-order Taylor expansion of G_3 about $\boldsymbol{\gamma} = \mathbf{0}$:

$$T_3(t; \boldsymbol{\gamma}, h(\eta_t)) = \frac{1}{4} h(\boldsymbol{\gamma})(t^3 + c_{12}t^2 + c_{11}t + c_{10}) + R(t, \boldsymbol{\gamma}, h(\eta_t)). \tag{43}$$

in order to approximate $\boldsymbol{\phi(t)}$ and $\boldsymbol{\theta(t)}$ in (40) using (43). This yields to:

$$\begin{aligned} y_t &= \boldsymbol{\beta_0'}(\bar{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_1'}(t\bar{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_2'}(t^2\bar{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_3'}(t^3\bar{\mathbf{z}}_{\mathbf{t}}) + \\ &+ \{\boldsymbol{\beta_4'}(\tilde{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_5'}(t\tilde{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_6'}(t^2\tilde{\mathbf{z}}_{\mathbf{t}}) + \boldsymbol{\beta_7'}(t^3\tilde{\mathbf{z}}_{\mathbf{t}})\} G(t; \boldsymbol{\gamma}, h(\eta_t)) + r_t^*, \end{aligned} \tag{44}$$

where $r_t^* = u_t + R(t; \boldsymbol{\gamma}, h(\eta_t))$. Under H_0, $r_t^* = u_t$. In (44), $\boldsymbol{\beta_j} = h(\eta_t)\bar{\boldsymbol{\beta}}$, $j = 1, \ldots, 7$, hence the null hypothesis in terms of (44) becomes $H_0 : \boldsymbol{\beta_j} = \mathbf{0}$, $j = 1, \ldots, 7$. Consequently, the locally approximated pseudo normal log-likelihood under H_0 (ignoring R) is

$$\begin{aligned} \mathcal{L}_t = const - \frac{1}{2} \ln \sigma^2 - \frac{1}{2}\sigma^2 [y_t - \boldsymbol{\beta_0'}\mathbf{w_t} - \boldsymbol{\beta_1'}(t\bar{\mathbf{w}}_{\mathbf{t}}) - \boldsymbol{\beta_2'}(t^2\bar{\mathbf{w}}_{\mathbf{t}}) - \boldsymbol{\beta_3'}(t^3\bar{\mathbf{w}}_{\mathbf{t}}) - \\ -\{\boldsymbol{\beta_4'}(\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_5'}(t\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_6'}(t^2\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_7'}(t^3\tilde{\mathbf{w}}_{\mathbf{t}})\} G(y_{t-d}; \boldsymbol{\gamma}, h(\eta_t)]^2. \end{aligned} \tag{45}$$

The partial derivatives are:

$$\frac{\partial \mathcal{L}_t}{\partial \boldsymbol{\beta_j}} = \frac{1}{\sigma^2} u_t (t^j \bar{\mathbf{w}}_{\mathbf{t}}), \qquad \text{j=0, \ldots, 3,} \tag{46}$$

$$\frac{\partial \mathcal{L}_t}{\partial \boldsymbol{\beta_j}} = \frac{1}{\sigma^2} u_t (t^j \tilde{\mathbf{w}}_{\mathbf{t}}) G(y_{t-d}; \boldsymbol{\gamma}, h(\eta_t)), \qquad \text{j=4, \ldots, 7,} \tag{47}$$

$$\frac{\partial \mathcal{L}_t}{\partial \gamma_1} = \frac{1}{\sigma^2} u_t \{\boldsymbol{\beta_4'}(\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_5'}(t\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_6'}(t^2\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_7'}(t^3\tilde{\mathbf{w}}_{\mathbf{t}})\} G_{\gamma_1}, \tag{48}$$

$$\frac{\partial \mathcal{L}_t}{\partial \gamma_2} = \frac{1}{\sigma^2} u_t \{\boldsymbol{\beta_4'}(\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_5'}(t\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_6'}(t^2\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_7'}(t^3\tilde{\mathbf{w}}_{\mathbf{t}})\} G_{\gamma_2}, \tag{49}$$

$$\frac{\partial \mathcal{L}_t}{\partial c} = \frac{1}{\sigma^2} u_t \{\boldsymbol{\beta_4'}(\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_5'}(t\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_6'}(t^2\tilde{\mathbf{w}}_{\mathbf{t}}) + \boldsymbol{\beta_7'}(t^3\tilde{\mathbf{w}}_{\mathbf{t}})\} G_c, \tag{50}$$

where G_{γ_1}, G_{γ_2}, G_c are the derivatives of $G(y_{t-d}, \boldsymbol{\gamma}, h(\eta_t))$ with respect to γ_1, γ_2 and c. With this notation, the estimators of $\frac{\partial \mathcal{L}_t}{\partial \gamma_1}$, $\frac{\partial \mathcal{L}_t}{\partial \gamma_2}$ and $\frac{\partial \mathcal{L}_t}{\partial c}$ are $\frac{\partial \hat{\mathcal{L}}_t}{\partial \gamma_1} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_{\gamma_1}$, $\frac{\partial \hat{\mathcal{L}}_t}{\partial \gamma_2} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_{\gamma_2}$, $\frac{\partial \hat{\mathcal{L}}_t}{\partial c} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_c$ respectively, so that: $\hat{\mathbf{z}}_{\mathbf{t}} = (1, \bar{\mathbf{z}}_{\mathbf{t}}', \tilde{\mathbf{z}}_{\mathbf{t}}' \hat{G}(y_{t-d}; \cdot), \hat{G}_{\gamma_1}, \hat{G}_{\gamma_2}, \hat{G}_{\gamma_c})'$ and $\hat{u}_t = (t\bar{\mathbf{z}}_{\mathbf{t}}', t^2\bar{\mathbf{z}}_{\mathbf{t}}', t^3\bar{\mathbf{z}}_{\mathbf{t}}', t\tilde{\mathbf{z}}_{\mathbf{t}}'\hat{G}(y_{t-d}, \cdot), t^2\tilde{\mathbf{z}}_{\mathbf{t}}'\hat{G}(y_{t-d}, \cdot), t^3\tilde{\mathbf{z}}_{\mathbf{t}}'\hat{G}(y_{t-d}, \cdot))$. Like in the symmetric scenario, under H_0, the statistic (35) has a χ^2 distribution with $3(k+l)$ degrees of freedom and the equivalent F-distribution has $3(k+l)$ and $T-4(k+l)-2$ degrees of freedom (the stastistic is denoted LM_3). The following rule is used: if H_1 is (40) with transition function G_3, then (35) is based on (44) assuming $\boldsymbol{\beta_3}=\mathbf{0}$ and $\boldsymbol{\beta_7}=\mathbf{0}$ (statistic LM_2) and, if the same alternative hypothesis has the transition function G_2, the test is based on (44), assuming $\boldsymbol{\beta_2}=\boldsymbol{\beta_3}=\mathbf{0}$ and $\boldsymbol{\beta_6}=\boldsymbol{\beta_7}=\mathbf{0}$ (statistic LM_2).

6 Simulation Study

6.1 Simulation design

A Monte Carlo simulation experiment is settled in order to investigate the empirical properties of the proposed asymmetry tests. We consider two different data generating processes (DGP):

$$y_{1,t}^{(i)} = 0.4y_{1,t-1}^{(i)} - 0.25y_{1,t-2}^{(i)} + (0.02 - 0.9y_{1,t-1}^{(i)} + 0.795y_{1,t-2})^{(s)}G^{(i)}(\Xi) + \epsilon_{1,t}^{(s)}\,, \quad (51)$$

and

$$y_{2,t}^{(i)} = 0.8y_{2,t-1}^{(i)} - 0.7y_{2,t-2}^{(i)} + (0.01 - 0.9y_{2,t-1}^{(i)} + 0.795y_{2,t-2}^{(i)})G^{(i)}(\Xi) + \epsilon_{2,t}^{(s)}\,, \quad (52)$$

where

$$G^{(i)}(\Xi) = \Big(1 + \exp\Big\{ - h(\eta_t)^{(i)} I_{(\gamma_1<0,\gamma_2<0)} + h(\eta_t)^{(i)} I_{(\gamma_1>0,\gamma_2<0)} + h(\eta_t)^{(i)} I_{(\gamma_1<0,\gamma_2>0)} + \\ + h(\eta_t)^{(i)} I_{(\gamma_1>0,\gamma_2>0)}\Big]\Big\}\Big)^{-1}, \quad (53)$$

with $\epsilon_t^{(i)} \sim N(0,1)$, $i=\{1,\ldots,I\}$ denoting the i-esim simulation of the process $\{y_t\}_{t=1}^{T}$ with $s=y_{t-1}$, $c=\frac{1}{T}y_t^{(i)}$, $I=1,000$.

$y_{2,t}^{(i)}$ (henceforth "DGP 1") is an additive nonlinear model with accentuated nonlinear behavior, due to the high autoregressive parameters driving $G(\Xi)$ and the low ones driving the linear part; this can be the case of a macroeconomic indicator affected by an unexpected shocks affecting the whole dynamics. On the other hand, $y_{2,t}^{(i)}$ (henceforth "DGP 2") describes a more balanced scenario.

In order to simulate the function $h(\eta_t)$ we use a set of values of vector $\boldsymbol{\gamma}$. The same different combinations of (γ_1, γ_2) of the two symmetry tests has been used to investigate the empirical size and the empirical power of the three diagnostic test described in Section 5. These combinations allow us to investigate: i) the different cases of null, medium and extreme asymmetry respectively; ii) the effect of having different kinds of asymmetry, due to the different signs in the two γ-s. Moreover, we consider three different hypotheses for T and the size α, namely $T = \{100, 300, 1000\}$ and $\alpha = \{1\%, 5\%, 10\%\}$. For each DGP we explore the possibility of both types of different functional form of asymmetry in $G(\cdot)$ and compute the corresponding statistics (25) - (28), jointly to the "Two-Step" test hypothesis corresponding to statistics (20). In this experiment, the first 100 simulations have been discarded in order to avoid the initialization effect.

For what concerns the three diagnostic tests, in the error autocorrelation test we assumed the errors of the generating process followed an AR(1) process $u_t = \rho u_{t-1} + \epsilon$, $\epsilon \sim NID(0,1)$ and $\rho = \{0.2, 0.4\}$. In the test for no additive asymmetry we added to the previously described DGP a generalized logistic function $G_2(\boldsymbol{\gamma}^{(2)}, h(\boldsymbol{c}, y_{t-1}))$ with coefficients $\pi_0 = 0.01$, $\pi_2 = -1.8$, $\pi_3 = 1.6$, $\boldsymbol{\gamma}^{(2)} = (\gamma_3, \gamma_4) = \{(5,2), (50,20), (500,200)\}$ and $\boldsymbol{\gamma}^{(1)}$ fixed at (120,70); this ensures that the behavior of the additive component remains isolated from the second; our experience shows that if higher parameter are set, the inversion become problematic. For the test for parameter constancy, the coefficients has been simulated according to a generalized logistic smooth change with $\lambda_1 = (0, 0.4, -0.25)'$ and $\lambda_2 = (0.2, -0.9, -0.795)'$. All these devices should make our simulation exercise comparable to the ET results.

6.2 Results

The results of the "All-in-one" and "Two-step" tests discussed in Section 4 for single DGP 1 and DGP 2 are reported in Table 1 and Table 2, respectively. Several findings can be easily noticed: first, the two tests have good and similar size properties. Only in large samples the two models behave in a slightly different way because of the statistics LM_2 , being its size in for DGP 1 (0.0735 at a nominal size 5%) slightly oversized with respect to DGP 2 (0.0391), while statistics LM_1 and the "Two-Step" test are more consistent.

Second, both the tests react similarly to different DGPs: the statistics LM_1 is more powerful of LM_2, regardless to the DGP 2 as sample size grows, although the empirical power is similar for moderate asymmetry and T=100. The "Two-Step" test makes an exception: under $DGP1$, the power of S_1 is very similar to LM_1 and LM_2, while, under DGP 2, the S_1 power is full when one of the slopes is 0 and the other is positive (see rows 4, 6 or 10 and 12 in Table 2). An important difference between the two scenarios is the change is scale of the empirical power under DGP 2; for example, when T=300, LM_1 statistic passes from 0.99 to 0.13 for $\alpha = 5\%$. This implies that the detection of a dynamic asymmetric movement of the series when the underlining process is not unambiguously nonlinear remains critical.

Third, both the tests are quite sensitive to different couples of (γ_1, γ_2) with respect to signs and scale: the empirical power of both tests tends to decline while $\boldsymbol{\gamma}$ has opposite signs. In particular, for $\gamma_1 < 0$, the power decays up to one third (see the case of $\boldsymbol{\gamma} = (50, 10)$ for $\alpha = \%5$ in statistic LM_2 at DGP1). In any case, all the statistics requires high slopes (500, 100 and similar) to get power in low sample. Heuristically, this is justified with the fact that the Stukel' s function approximates a near-to-linear function for extreme negative slopes, implying the possibility of a missed dynamic asymmetry problem. This argument will be actively invoked in our empirical applications in Section 7.

With reference to diagnostic tests, the results reported in Table 3 and Table 4 deliver a similar picture: on one hand, all the tests have good empirical size properties for both the DGP used. Some *caveat* are required to interpret the empirical power properties: under DGP 1, all the tests have good power, in particular for serial correlation test; the test of no additive asymmetry and parameter constancy are characterized by a duality: when the two slopes are high, that is $\boldsymbol{\gamma} = (500, 200)$, the power is extreme, while it decays for low-medium asymmetry (0.21 vs 1.00 at $\alpha = 5\%$, T=100 in no additive asymmetry test, 0.44 vs 0.87 for LM_2 statistic at the same nominal and sample size for parameter constancy). On the other hand, under DGP 2, the change in scale of the power is evident only for the parameter constancy test; interestingly, the test for no serial correlation is more powerful. Thus, we are substantially confident in the use of diagnostic tests in empirical analysis, conditionally on high dynamic asymmetry.

7 Illustrations

7.1 Data and Methods

In this section the GSTAR model is applied to four time series, namely: the U.S. index of industrial production and unemployment (IP and UN, respectively); the yearly average of daily International Sunspot Number (YSSN), and the Canadian Lynx data (LYNX). We consider also the monthly average of Sunspot Number from January 1850 to December 2013 (1962 observations) for which three different kinds of data transformations are compared to link our model to the existing literature: the logarithmic (logMSSN), square root (sqrtMSSN) and the growth rate (DLMSSN); in this case, the Kalman-smoothed version of the series is available and necessary to avoid inversion problems due to the high noise. Further informations on the dataset can be found in Table 5. The data and the resulting (multiple) transition function(s), plotted versus time are reported in Figure 5, while the same transitions plotted versus the selected transition variable are shown in Figure 6.

The out-of-sample predictive properties of the estimated models are investigated via rolling forecast experiment, according to which the series y_t is divided in a "pre-forecast" period (going from time $\{1 \dots t\}$) from which the model is estimated and the h-step-ahead forecast are computed and compared with the "test" period, going from time $\{T^s \dots T\}$ where $T^s = t + h$; this allows to measure $T - T^s - h + 1$ out-of-sample forecasts. Let denote the corresponding realization of the series as y_t, y_T^s and y_T, as well as the corresponding density forecasts as f_t, f_T^s and f_T. Since our interest lies in short-run forecasting we consider $h = \{1, 3, 6, 12\}$. The point predictive performances of the model j are investigated by four different measures: the mean forecast error (MFE), the symmetric mean absolute percentage error (sMAPE), the median relative absolute error (mRAE) and the root mean square forecast error (RMSFE)[3]:

$$MFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right) \tag{54}$$

$$sMAPE_{j,h} = \frac{100|y_{t+h} - \hat{y}^j_{t+h}|}{0.5(y_{t+h} - \hat{y}^j_{t+h|t})} \tag{55}$$

$$mRAE_{j,h} = \frac{|y_{t+h} - \hat{y}^j_{t+h}|}{|y_{t+h} - \hat{y}^{(1)}_{t+h}|}, \quad \text{with (1) indexing the benchmark model;} \tag{56}$$

$$RMSFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right)^2 \tag{57}$$

In a similar fashion, four different scoring rules are used for aggregate the $T - T^s - h + 1$ density forecasts produced by the same forecasting exercise[4]:

[3]In particular, sMAPE and mRAE are recommended when the series is known to present volatility effects or skewness, two features often associated to nonlinearity; see the discussion in Tashman (2000).

[4]The scoring rules here considered are just a fraction of the many nowadays available. The choice of the scores has been done for easy of treatment and does not imply any preference.

- the logarithmic score (LogS) (Good, 1952):

$$LogS_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} log \hat{f}^j_{t+h|t} \tag{58}$$

corresponding to a Kullback-Liebler distance from the true density; models with higher LogS are preferred.

- The quadratic score (Brier, 1950), somehow the equivalent of the MSFE in point forecasting, is defined as:

$$QSR_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \sum_{k=i}^{K} (f^j_{t+h|t} - d_{kt})^2 \tag{59}$$

where $d_{kt} = 1$ if $k = t$ and 0 otherwise; models with lower QSR are preferred.

- The (aggregate) continuous-ranked probability (CRPS) score (Epstein, 1969), equivalent to the sMAPE, is defined as:

$$CRPS_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(|f_{t+h} - \hat{f}^j_{t+h|t}| - 0.5|f_{t+h} - f'_{t+h}| \right), \tag{60}$$

where f and f' are independent random draws from the predictive density and $f_{t+h|t}$ the observed value; models with lower CRPS are preferred.

- Finally, the quantile score (qS) (Cervera and Munoz, 1996) can be obtained if $f^j_{t+h|t}$ is replaced in (58) by a predictive α-level quantile $q^{\alpha}_{t+h|t}$ (and the logarithmic function removed); this score is used in risk analysis because provide information about deviations from the true tail of the distribution.

Finally the p-values of the Giacomini and White (2006) test for equal predictive ability are reported for completeness of analysis.

7.2 Results

Tables 6 and 7 report the results of the General-to-Specific modelling strategy discussed in Section 3, and specifically: the descriptive analysis of the series, using basic statistics and a battery of test for normality via Jarque-Brera test (JB), ARCH-effect (via Engle's test), serial uncorrelation via Durbin-Watson test (DW), identical distribution via Kolmogorov-Smirnov test (KS) and the t-statistics of the Dikey-Fuller test augmented with two lags and constant as deterministic kernel (ADF) in the first panel; the result of the LST linearity test, the selected model according to the Teräsvirta rule and two symmetry tests introduced in Section 4 (second panel); parameter estimates and HAC standard errors of the selected GSTAR model with its equivalent STAR specification, for which the possibility of multiple regimes has been taken in consideration (third panel); the diagnostic tests (fourth panel). The rolling forecasting exercise is shown in Table 8.

Several facts arises: (i) first, the dynamic asymmetry here introduced is never rejected if a GSTAR model is assumed, as the "All-in-One" test suggests; however, the "Two-Step" approach, which strictly follows the original Stukel's methodology changes this result; this seems reasonable at least for case of UN. This is not the case of monthly sunspots series, for which the "Two-Step" test, although still not able to reject the null for the selected model, starts to reject if different data and models are used[5]. (ii) The GSTAR transition function differs from its symmetric equivalent and this holds also when the two estimated slopes are very similar. In particular, Figure 5, panels (a), (c) and (d), shows that the estimated asymmetric $G(\boldsymbol{\gamma}, s_t, c)$ functions tends to concentrate in the upper part of the of the space of continuum of states (between 0.5 and 1); on the other side, in panel (b) of the same figure, the estimated GLSTAR transition for UN (where $\gamma_1 = \gamma_2 = 0.001$) reduces of 30% from the full [0, 1] range (from 1950 to 1980) to a [0.2, 0.9] range (after first 1980s), allowing to reproduce the cyclical movements of the data and their change

[5]The results, not shown here, can be provided upon request.

in scale better than the traditional parametrization. The differences between transition functions are still more evident in Figure 6.

(iii) The GSTAR(p) specification allows the modeller to gain in terms of parsimony, since in all the examples the dynamics of the series is found being well replicated by using just one asymmetric transition, while the same does not hold for its symmetric counterpart. This is immediately evident in the LYNX example, where an autoregressive order 7 is sufficient to pass all the diagnostic tests, whereas more lags was required by previous literature[6] and monthly sunspot data enforces this finding.

(iv) Coherently with the logarithmic (exponential) rescaling imposed by h-functions (3) - (4) and with the evidence from application of SETAR and STAR models obtained by previous studies, the GSTAR model is sensitive to changes in scale, so that a further transformation tends to over-smoothing (exacerbate) the nonlinear dynamics of the process if further transformations are applied. In this sense, Figure 7 is almost self-explanatory.

(v) The dynamic asymmetry is an important feature to take in account for forecasting aims: in particular, according to mRAE, the GSTAR model beats almost always its symmetric counterpart, while, in terms of RMSFE, the GSTAR wins in many forecast horizons of YSSN and LYNX and at longest horizons of UN; similar evidence is provided by MFE criterion: the new model prevails in two cases (UN and YSSN) whereas at very short term, the AR is still a good model for IP and LYNX. The superiority is less evident if considering sMAE: the two nonlinear specification almost equivalent for IP, while, for other three cases the MR-STAR prevails with a factor of less than 0.1%.

(vi) The results of density forecasting are quite more challenging. In particular, according to LogS, the GSTAR wins for UN, while AR does for IP and shot horizons of YSSN and MR-STAR for LYNX. The GSTAR returns to outperform if QSR is used, beating its competitors for YSSN and LYNX and at long horizons of IP and

[6] Tong (1977) and Teräsvirta (1994) suggested p=11.

UN. Differently, the CRPS conveys a clear superiority of MR-STAR, which win in almost all cases, with the exception of YSSN and short-run horizons of IP (where AR better). The qS^{α} enforces this result by providing evidence in favor linear specifications with the only exception of IP, and STAR being still the second best for IP and UN.

(vii) Finally, according to the results for Giacomini and White (2006) test, there is evidence of significant improvement in prediction until h=6 if a GSTAR(p) is considered with respect to a linear AR specification.

8 Conclusions

The Generalized Logistic function is applied to STAR family as simple, statistically feasible way to capture the dynamic asymmetry in the conditional mean of a time series. The resulting GSTAR model ensures the smoothness of the transition function by construction without requiring further efforts for what concerns identification and estimation, is able to characterize some of the most prominent examples of nonlinear time series also when the estimated asymmetry coefficients are very similar and presents good point forecasting properties.

The results of density forecasting exercise confirm - and possibly enforce - the Kascha and Ravazzolo (2010) evidence that the relation between highest LogS and lower RMSFE is not one-to-one. In addiction to this, we find that such a relation breaks under CRPS and reverts under qS^{α}. This means that dynamic asymmetric models are superior to traditional STARs if classical measures are used and not dissimilar if robust measures are. In any case, nonlinear specifications remains preferable to linear ones.

The GSTAR model is feasible of several developments, first of all for what concerns modelling the conditional variance in long samples (Gonzáles-Rivera, 1998; Amado and Teräsvirta, 2013; Silvennoinen and Teräsvirta, 2013) and multiple time series analysis, see Rothman, van Dijk, and Franses (2001); Milas and Rothman (2008) and Camacho (2004) for an economic examples and Hubrich and Teräsvirta (2013) for a survey. The properties of the GSTAR in non-stationary time series is still unknown and constitutes an important issue for financial applications; Kapetanios, Shin, and Snell (2003), Vougas (2006) and Addo, Billio, and Guégan (2014) discuss this problem in a traditional STAR and MT-STAR framework and provide a basis to start with. Finally, the dynamic asymmetry here introduced has been modelled by implicitly assuming autoregressive structure. Wecker (1981), and Brännäs and De Gooijer (1994, 2004) provide interesting asymmetric moving-average models which can be compared and possibly merged with GSTAR features.

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A Appendix

A.1 Mathematical derivations

A.1.1 Preliminar notation

Let denote $G_t = G(\Xi)$, $\Xi = [\gamma_1, \gamma_2, c]$ or $[\gamma_1, \gamma_2, c_1, c_2]$ in case of GLSTR1 and GESTAR (GESTR). Then we can re-define $G(\Xi)$ as:

$$
\begin{aligned}
G^{\{i\}}(\Xi) &= [1 + g(f^{\{i\}}(\Xi))]^j, \\
f^{\{GLSTR\}}(\Xi) &= -[h(\eta_t^L)I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_t^L)I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h(\eta_t^L)I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h(\eta_t^L)I_{(\gamma_1 > 0, \gamma_2 > 0)}], \\
f^{\{GLSTR2\}}(\Xi) &= -[h(\eta_t^{2L})I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_t^{2L})I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h(\eta_t^{2L})I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h(\eta_t^{2L})I_{(\gamma_1 > 0, \gamma_2 > 0)}], \\
f^{\{GESTR\}}(\Xi) &= -[h(\eta_t^E)I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_t^E)I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h(\eta_t^E)I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h(\eta_t^E)I_{(\gamma_1 > 0, \gamma_2 > 0)}],
\end{aligned}
$$

with $i = \{L, 2L, E\}$, denoting the Logistic, Double Logistic and Exponential parametrization, $j = \{1; -1\}$, with $j = 1$ only if $f(\Xi) = f^{GESTR}(\Xi)$, $\eta_t^L = s_t - c$, $\eta_t^{2L} = (s_t - c_1)(s_t - c_2)$, $\eta_t^L = (s_t - c)^2$. Moreover, let $f'(\Xi) = -[h'(\eta_t)I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h'(\eta_t)I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h'(\eta_t)I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h'(\eta_t)I_{(\gamma_1 > 0, \gamma_2 > 0)}]$ define the first derivative of $f(\Xi)$ and $D = 1 + g(\Xi)$ denote the denominator of the fraction which is the result of the computation of the second derivatives so that:

$$
\begin{aligned}
D^2 &= 1 + g(\Xi)^2 + 2g(\Xi), \\
g^{\{i\}}(\Xi)^2 &= 1 + \exp\{-2(h(\eta_t^{\{i\}})I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_t^{\{i\}})I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h(\eta_t^{\{i\}})I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + \\
&\quad + h(\eta_t^{\{i\}})I_{(\gamma_1 > 0, \gamma_2 > 0)})\} + 2\exp\{h(\eta_t^{\{i\}})I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h(\eta_t^{\{i\}})I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + \\
&\quad + h(\eta_t^{\{i\}})I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h(\eta_t^{\{i\}})I_{(\gamma_1 > 0, \gamma_2 > 0)}\},
\end{aligned}
$$

A.1.2 LSTR1 case

When the transition equation is a Generalized Logistic, we have the following derivatives:

(i)

$$G_{\gamma_1}(\Xi) = -\frac{g'(f(\Xi)) \cdot f'(\Xi)}{D^2} \tag{61}$$

where:

$$h'(\eta_t)I_{(\eta_t>0)} = \frac{\partial}{\partial \gamma_1} h(\boldsymbol{\gamma}) = \begin{cases} -\frac{1}{\gamma_1^2} \cdot \exp(|\eta_t|-1)(|\eta_t|-1) & \text{if } \gamma_1 > 0 \\ 0 & \text{if } \gamma_1 = 0 \\ -\frac{1}{\gamma_1^2} \cdot \ln(1-\gamma_1|\eta_t|) + \frac{|\eta_t|}{1-\gamma_1|\eta_t|}) & \text{if } \gamma_1 < 0 \end{cases} \tag{62}$$

and

$$h'(\eta_t)I_{(\eta_t \leq 0)} = \begin{cases} 0 \ \text{if } \ \gamma_2 > 0 \\ 0 \ \text{if } \ \gamma_2 = 0 \\ 0 \ \text{if } \ \gamma_2 < 0 \end{cases} \tag{63}$$

(ii) $G_{\gamma_2}(\cdot)$: equal to (61) but with

$$h'(\eta_t)I_{(\eta_t>0)} = \frac{\partial}{\partial \gamma_2} h(\boldsymbol{\gamma}) = \begin{cases} 0 \ \text{if } \ \gamma_1 > 0 \\ 0 \ \text{if } \ \gamma_1 = 0 \\ 0 \ \text{if } \ \gamma_1 < 0 \end{cases} \tag{64}$$

and

$$h'(\eta_t)I_{(\eta_t \leq 0)} = \begin{cases} \frac{1}{\gamma_2} \exp(1-\gamma_2|\eta_t|) \cdot (\frac{1}{\gamma_2} + |\eta_t|) & \text{if } \gamma_2 > 0 \\ 0 & \text{if } \gamma_2 = 0 \\ -\frac{1}{\gamma_2}\left[\frac{1}{\gamma_2} \ln(\gamma_2|\eta_t|-1) + \frac{|\eta_t|}{\gamma_2|\eta_t|-1}\right] & \text{if } \gamma_2 < 0 \end{cases} \tag{65}$$

(iii) $G_c(\cdot)$: equal to (61) but with

$$f'(\Xi) = h(\eta_t)I_{(\eta_t \leq 0)} + h(\eta_t)I_{(\eta_t>0)} \tag{66}$$

A.1.3 LSTR2 case

When the transition equation is a (Generalized) Double Logistic as in model (**??**), we have the following derivatives:

(i) $G_{\gamma_1}(\cdot)$ equal to equation (61) with: $f^{\{i\}}(\Xi) = f^{\{GLSTR2\}}(\Xi) = -[h(\eta_t^{2L})I_{(\gamma_1\leq 0,\gamma_2\leq 0)} + h(\eta_t^{2L})I_{(\gamma_1\leq 0,\gamma_2>0)} + h(\eta_t^{2L})I_{(\gamma_1>0,\gamma_2\leq 0)} + h(\eta_t^{2L})I_{(\gamma_1>0,\gamma_2>0)}]$, $g^{\{GLSTR2\}} = \exp\{f^{\{GLSTR2\}}(\Xi)\}$, $h'(\eta_t^{2L})I_{(\eta_t^{2L}>0)}$ and $h'(\eta_t^{2L})I_{(\eta_t^{2L}\leq 0)}$ equal to systems (62) and (63).

(ii) $G_{\gamma_2}(\cdot)$: equal to equation (61) with: $f^{\{GLSTR2\}}(\Xi)$ and $g^{\{GLSTR2\}}(\Xi)$ above defined as in case (i) and $h'(\eta_t^{2L})I_{(\eta_t^{2L}\geq 0)}$ and $h'(\eta_t^{2L})I_{(\eta_t^{2L}<0)}$ equal to systems (64) and (65) respectively.

(iii) $G_{c_1}(\cdot)$: equal to equation (61) with: $f^{\{GLSTR2\}}(\Xi)$ and $g^{\{GLSTR2\}}(\Xi)$ defined as in case (i) and

$$f'(\Xi) = h'(\eta_t^{2L})I_{(\eta_t^{2L}\leq 0)}(s_t - c_2) + h'(\eta_t^{2L})I_{(\eta_t^{2L}>0)}(s_t - c_2) \tag{67}$$

(iv) $G_{c_2}(\cdot)$: equal to equation (61) with: $f^{\{GLSTR2\}}(\Xi)$ and $g^{\{GLSTR2\}}(\Xi)$ defined as in case i) and

$$f'(\Xi) = h'(\eta_t^{2L})I_{(\eta_t^{2L}\leq 0)}(s_t - c_1) + h'(\eta_t^{2L})I_{(\eta_t^{2L}>0)}(s_t - c_1) \tag{68}$$

A.1.4 ESTR case

When the transition equation is an exponential as in model (**??**), we have: $f^{\{ESTR\}}(\Xi) = -[h(\eta_t^E)I_{(\gamma_1\leq 0,\gamma_2\leq 0)} + h(\eta_t^E)I_{(\gamma_1\leq 0,\gamma_2>0)} + h(\eta_t^E)I_{(\gamma_1>0,\gamma_2\leq 0)} + h(\eta_t^E)I_{(\gamma_1>0,\gamma_2>0)}]$, $g^{\{ESTR\}}(\Xi) = -\exp\{f^{\{E\}}\}$, hence the following derivatives:

(i) $G_{\gamma_1}(\cdot) = f^{\{ESTR\}*'(\Xi)}$ with: $f'(\Xi) = -[h(\eta_t^E)I_{(\eta_t^E\leq 0)}(s_t - c)^2 + h'(\eta_t^E)I_{(\eta_t^E\leq 0)}(s_t - c)^2)$, $h'(\eta_t^E)I_{(\eta_t^E>0]}$ and $h'(\eta_t^E)I_{(\eta_t^E\leq 0)}$ being the same of systems (62) and (63).

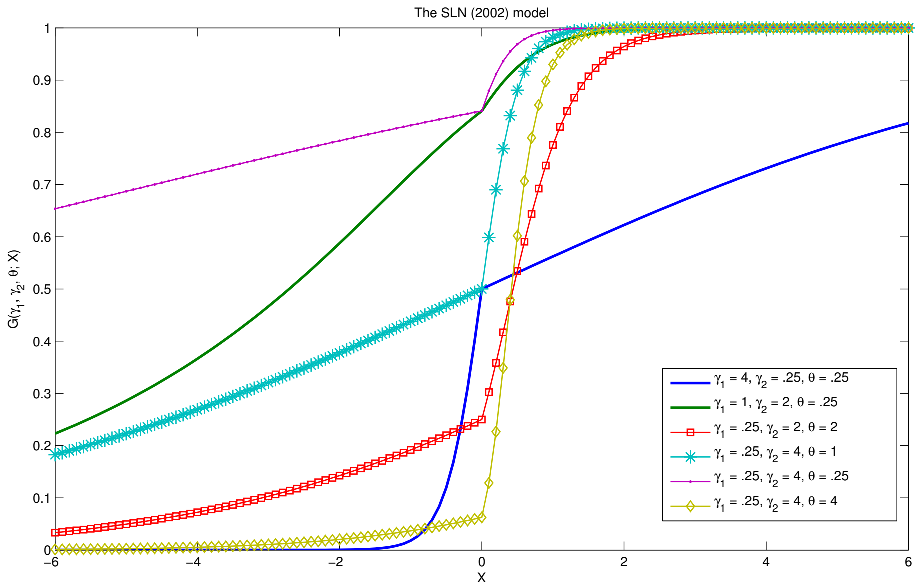
(ii) $G_{\gamma_2}(\cdot)$: same as $G_{\gamma_1}(\cdot)$ with $h'(\eta_t^E)I_{(\eta_t^E>0]}$ and $h'(\eta_t^E)I_{(\eta_t^E\leq 0)}$ being the same of systems (64) and (65).

(iii) $G_c(\cdot) = f^{\{ESTR\}*'}(\Xi)$ with $f'(\Xi) = h'(\eta_t^E)I_{(\eta_t^E\leq 0)}(2c) + h'(\eta_t^E)I_{(\eta_t^E>0)}(2c)$, with $h'(\eta_t^E)I_{(\eta_t^E>0]}$ and $h'(\eta_t^E)I_{(\eta_t^E\leq 0)}$ being the same of systems (64) and (65).

A.2 Tables and Graphs

Figure 1: Transition function for different parametrizations of Asymmetric STAR.

(a) Sollis et al. (2002)



(b) Sollis et. al (1999) - Lundbergh and Terasvirta (2006)

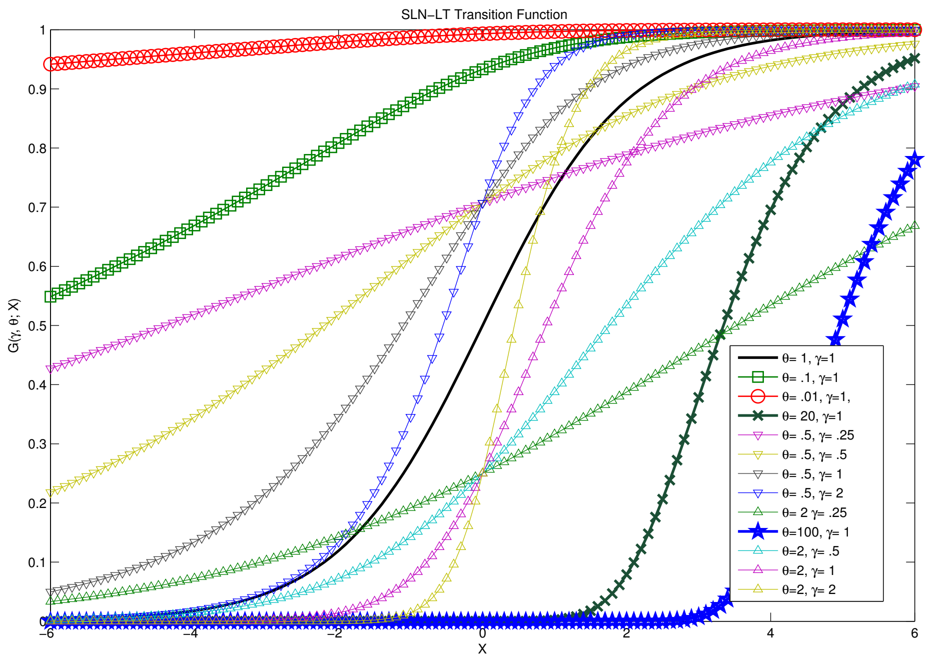


Figure 2: The Generalized Logistic function.

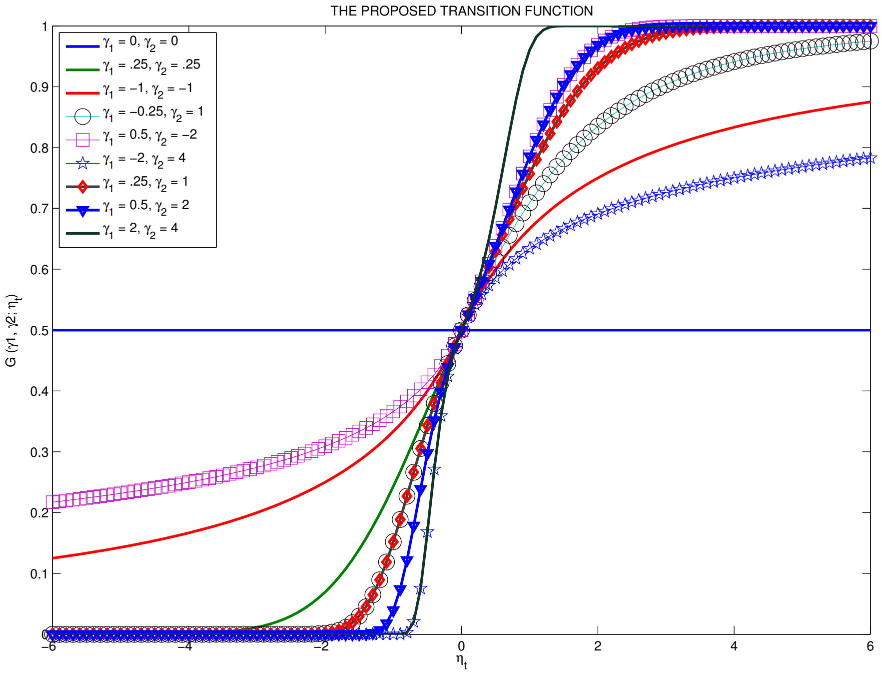


Figure 3: The Generalized Second-Order Logistic function.

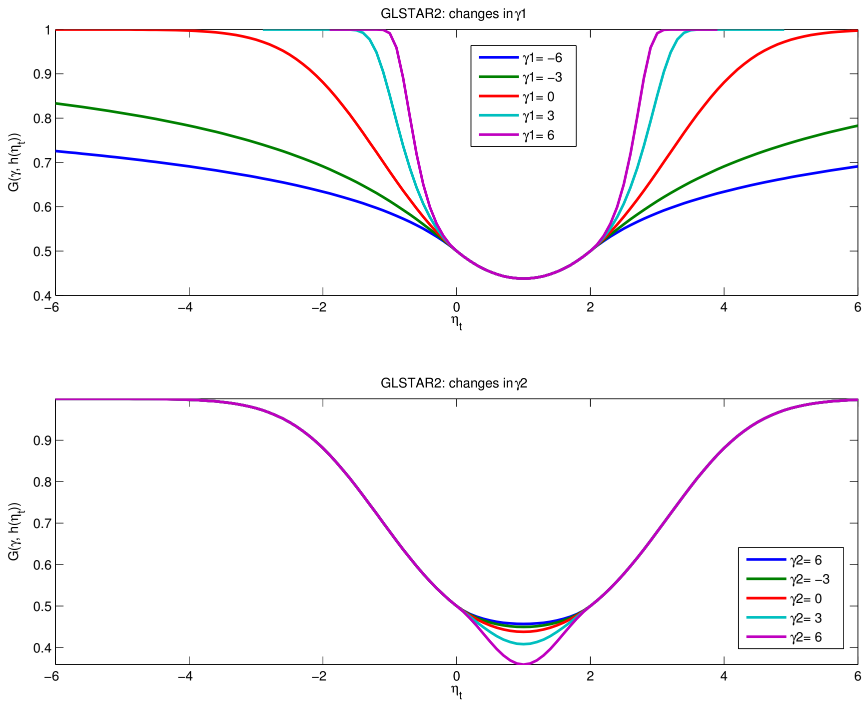
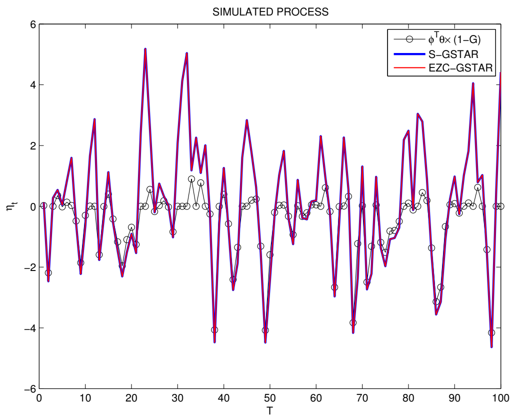
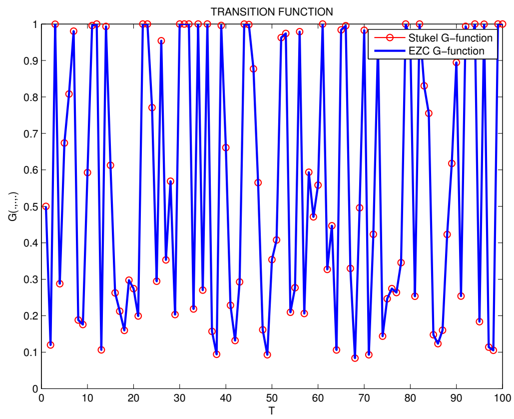


Figure 4: An example of GLSTAR model.

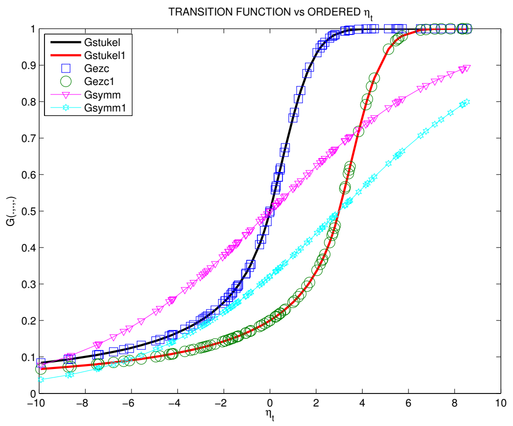
(a)



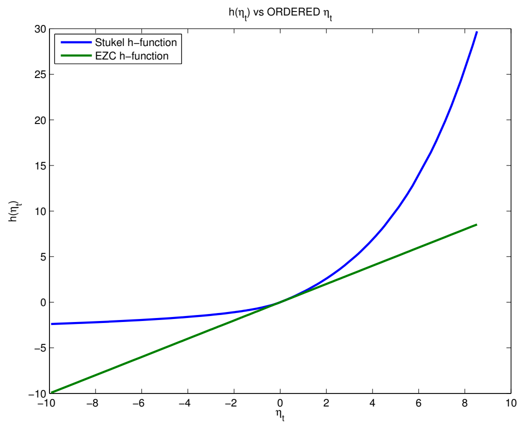
(b) Transition function



(c) Transition functions vs η_t



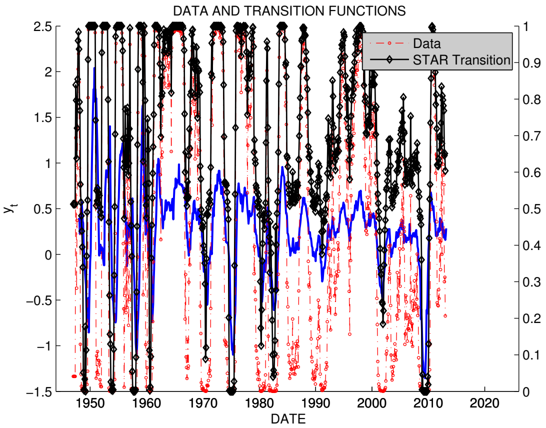
(d) The rescaling effect of h(η_t)



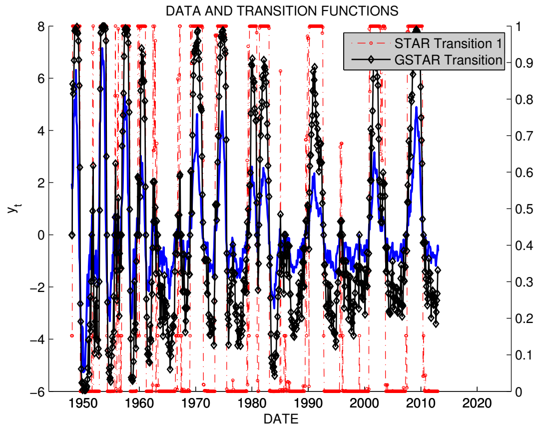
Simulation performed with following parameters: $\phi_0 = 0.05$; $\phi_1 = 0.4$; $\phi_2 = 0.25$; $\theta_0 = 0.2$; $\theta_1 = 0.4$; $\theta_2 = 0.25$; $\gamma_1 = 0.25$; $\gamma_2 = -1.0$; $c = 0$; $c_1 = 3$; $c_2 = 5$; $T = 100$

Figure 5: Estimated transition function for (MR)STAR and GSTAR specifications.

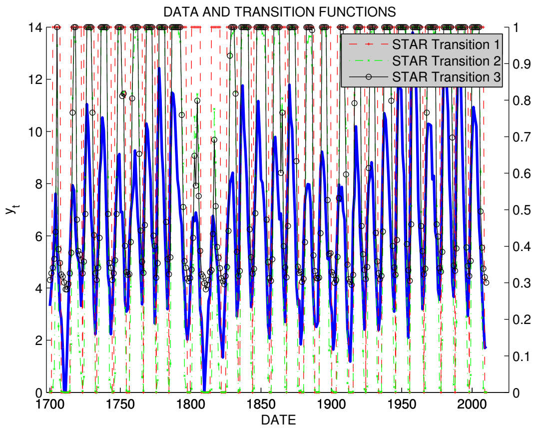
(a) U.S. Industrial Production



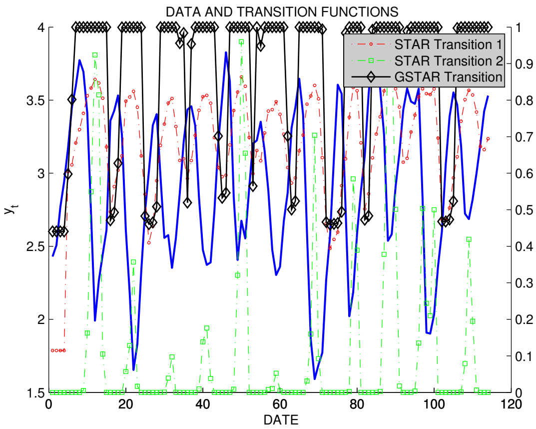
(b) U.S. Unemployment



(c) Yearly Sunspot Number



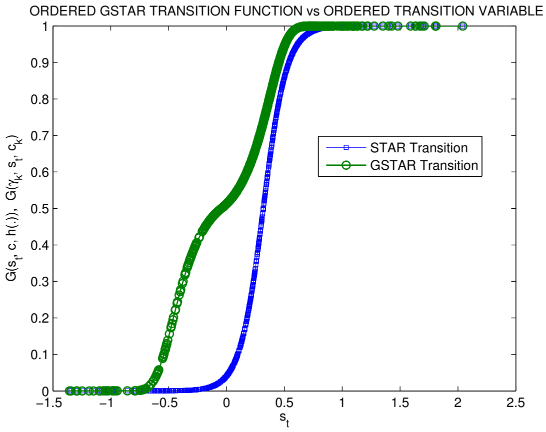
(d) Canadian Lynx



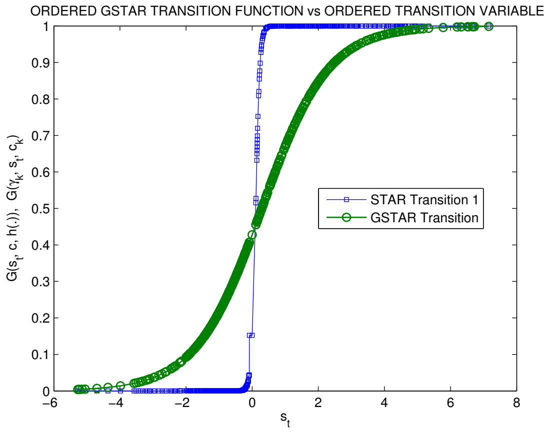
NOTE: The data are plotted in blue line.

Figure 6: Estimated transition functions vs transition variable.

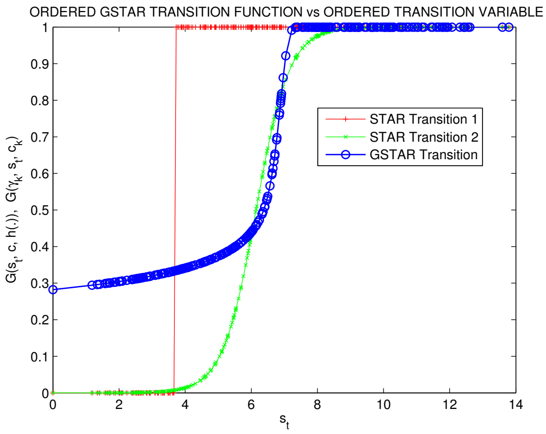
(a) U.S. Industrial Production



(b) U.S. Unemployment



(c) Yearly Sunspot Number



(d) Canadian Lynx

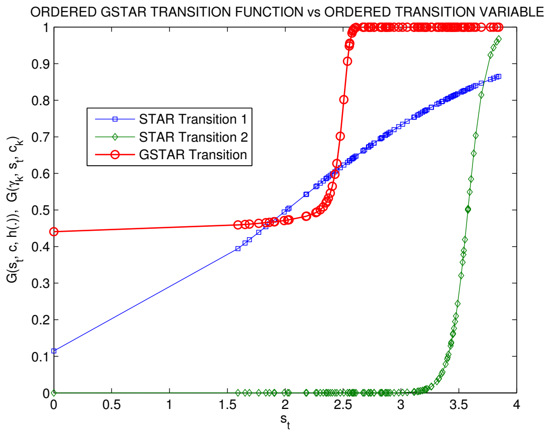
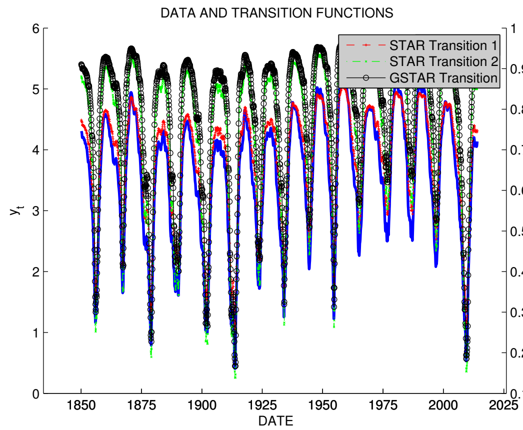
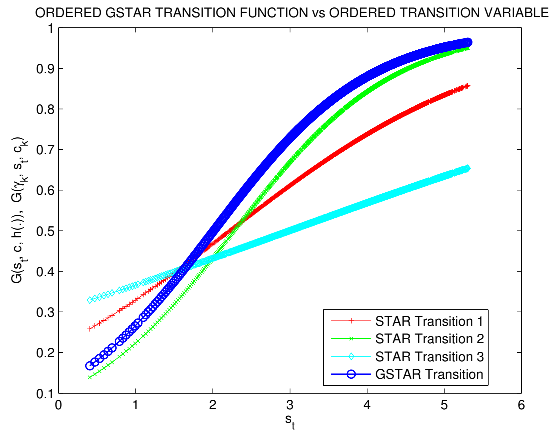


Figure 7: Monthly SSN: estimated transition functions for different data transformations.

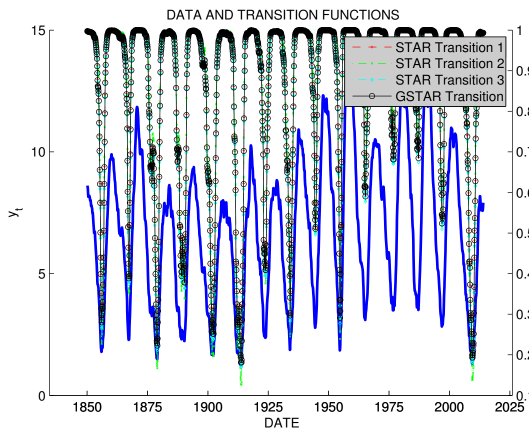
(a) logMSSN: G(Ξ) vs t



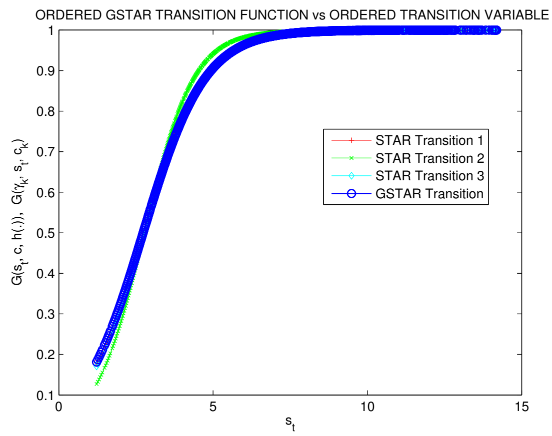
(b) logMSSN: G(Ξ) vs s_t



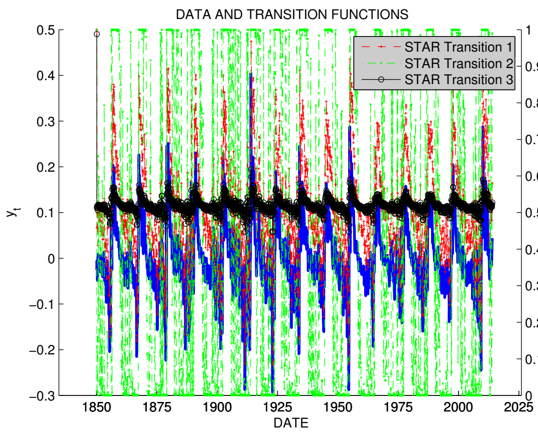
(c) sqrtMSSN: G(Ξ) vs t



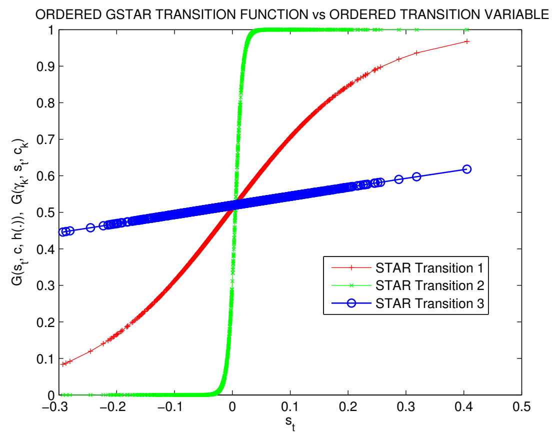
(d) sqrtMSSN: G(Ξ) vs s



(e) DLMSSN: G(Ξ) vs t



(f) DLMSSN: G(Ξ) vs s



NOTE: The data are plotted in blue line of left-had subfigures.

Table 1: Empirical Size and Power of "All-in-One" and "Two-Step" test for dynamic asymmetry under DGP 1.

			Empirical Size								
T	γ_1	γ_2	LM_1			LM_2			S_1		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
100	0	0	0.0003	0.0221	0.0458	0.0000	0.0230	0.0467	0.0025	0.0135	0.0430
300	0	0	0.0058	0.0255	0.0565	0.0070	0.0288	0.0653	0.0039	0.0168	0.0433
1000	0	0	0.0073	0.0384	0.0798	0.0208	0.0735	0.1297	0.0088	0.0527	0.0882

			Empirical Power								
T	γ_1	γ_2	LM_1			LM_2			S_1		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
	50	10	0.0315	0.1382	0.2366	0.0142	0.0951	0.1561	0.0271	0.0646	0.1006
	-50	10	0.0197	0.0737	0.1502	0.0054	0.0668	0.1246	0.0231	0.0694	0.1093
	50	-10	0.0180	0.0939	0.1904	0.0100	0.0747	0.1405	0.0175	0.0554	0.0790
	50	0	0.0170	0.0836	0.1601	0.0169	0.0731	0.1448	0.0006	0.0252	0.0443
	-50	0	0.0094	0.0801	0.1288	0.0164	0.0767	0.1364	0.0072	0.0416	0.0765
	0	50	0.0128	0.0732	0.1486	0.0128	0.0732	0.1486	0.0034	0.0226	0.0491
100	500	100	0.7930	0.8165	0.8335	0.7927	0.7987	0.8191	0.7986	0.8049	0.8130
	-500	100	0.0173	0.0890	0.1645	0.0079	0.0760	0.1238	0.0215	0.0640	0.1173
	500	-100	0.7931	0.8073	0.8241	0.7927	0.8031	0.8155	0.7965	0.8004	0.8259
	500	0	0.7954	0.8059	0.8215	0.7947	0.8067	0.8189	0.7928	0.7959	0.8004
	-500	0	0.0094	0.0782	0.1325	0.0164	0.0776	0.1394	0.0114	0.0466	0.0795
	0	500	0.8037	0.8083	0.8303	0.8037	0.8083	0.8303	0.8012	0.8083	0.8165
	50	10	0.1098	0.2570	0.3719	0.0281	0.1295	0.2018	0.0681	0.1501	0.2173
	-50	10	0.0207	0.1266	0.1941	0.0102	0.0640	0.1392	0.0390	0.0946	0.1496
	50	-10	0.0420	0.1458	0.2253	0.0132	0.0729	0.1561	0.0287	0.0890	0.1509
	50	0	0.0353	0.1339	0.2059	0.0212	0.0823	0.1496	0.0103	0.0363	0.0634
	-50	0	0.0190	0.1033	0.1476	0.0323	0.1179	0.2085	0.0117	0.0402	0.0697
	0	50	0.0176	0.0964	0.1898	0.0176	0.0964	0.1898	0.0092	0.0359	0.0729
300	500	100	0.9958	0.9961	0.9980	0.9958	0.9958	0.9958	0.9959	0.9999	1.0000
	-500	100	0.0360	0.1485	0.2147	0.0096	0.0765	0.1389	0.0468	0.1197	0.1856
	500	-100	0.9958	0.9961	0.9961	0.9958	0.9961	0.9961	1.0000	1.0000	1.0000
	500	0	1.0000	1.0000	1.0000	0.9958	0.9961	0.9961	1.0000	1.0000	1.0000
	-500	0	0.0190	0.1039	0.1434	0.0326	0.1275	0.2074	0.0122	0.0428	0.0702
	0	500	0.9917	0.9966	0.9972	0.9917	0.9966	0.9972	1.0000	1.0000	1.0000
	50	10	0.4727	0.7274	0.8419	0.1173	0.2498	0.3634	0.2139	0.3545	0.4518
	-50	10	0.0830	0.2032	0.2996	0.0274	0.0822	0.1506	0.0580	0.1589	0.2205
	50	-10	0.1411	0.2863	0.4373	0.0312	0.0942	0.1778	0.0866	0.2054	0.2599
	50	0	0.1344	0.3275	0.4455	0.0476	0.1435	0.2856	0.0107	0.0635	0.1281
	-50	0	0.0446	0.1193	0.2482	0.0846	0.2535	0.3520	0.0110	0.0471	0.0893
	0	50	0.0654	0.1799	0.2643	0.0654	0.1799	0.2643	0.0116	0.0586	0.1325
1000	500	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-500	100	0.1281	0.2695	0.4265	0.0242	0.0719	0.1607	1.0000	1.0000	1.0000
	500	-100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.1181	0.2180	0.2752
	500	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-500	0	0.0462	0.1223	0.2390	0.0890	0.2565	0.3556	1.0000	1.0000	1.0000
	0	500	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2: Empirical Size and Power of "All-in-One" and "Two-Step" test for dynamic asymmetry under DGP 2.

			Empirical Size								
T	γ_1	γ_2	LM_1			LM_2			S_1		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
100	0	0	0.0024	0.0164	0.0185	0.0014	0.0131	0.0237	0.0006	0.0264	0.0423
300	0	0	0.0000	0.0081	0.0272	0.0002	0.0123	0.0242	0.0112	0.0292	0.0469
1000	0	0	0.0036	0.0460	0.1018	0.0015	0.0391	0.0787	0.0126	0.0482	0.0863

			Empirical Power								
T	γ_1	γ_2	LM_1			LM_2			S_1		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
	50	10	0.0308	0.1249	0.2454	0.0111	0.0328	0.1117	0.0031	0.0261	0.0585
	-50	10	0.0185	0.1021	0.1847	0.0117	0.0262	0.0520	0.0100	0.0393	0.0698
	50	-10	0.0308	0.1249	0.2454	0.0111	0.0328	0.1117	0.0031	0.0411	0.0652
	50	0	0.0308	0.1249	0.2454	0.0111	0.0328	0.1117	1.0000	1.0000	1.0000
	-50	0	0.0185	0.1021	0.1847	0.0111	0.0262	0.0520	0.0098	0.0354	0.0697
	0	50	0.0185	0.0930	0.1705	0.0108	0.0262	0.0560	1.0000	1.0000	1.0000
100	500	100	0.0192	0.0950	0.1893	0.0108	0.0247	0.0544	0.0006	0.0159	0.0416
	-500	100	0.0189	0.1028	0.1786	0.0117	0.0262	0.0520	0.0111	0.0485	0.0800
	500	-100	0.0192	0.0950	0.1893	0.0108	0.0247	0.0544	0.0044	0.0349	0.0574
	500	0	0.0192	0.0950	0.1893	0.0108	0.0247	0.0544	1.0000	1.0000	1.0000
	-500	0	0.0189	0.1028	0.1786	0.0117	0.0262	0.0520	1.0000	1.0000	1.0000
	0	500	0.0185	0.0930	0.1705	0.0108	0.0262	0.0560	1.0000	1.0000	1.0000
	50	10	0.1044	0.2473	0.3516	0.0510	0.1327	0.2083	0.0099	0.0230	0.0409
	-50	10	0.0221	0.0957	0.1983	0.0103	0.0398	0.0681	0.0141	0.0315	0.0757
	50	-10	0.1044	0.2473	0.3516	0.0510	0.1327	0.2083	0.0138	0.0258	0.0511
	50	0	0.1044	0.2473	0.3516	0.0510	0.1327	0.2083	0.0099	0.0230	0.0387
	-50	0	0.0221	0.0957	0.1983	0.0103	0.0398	0.0681	0.0141	0.0305	0.0757
	0	50	0.0190	0.0929	0.1907	0.0084	0.0379	0.0666	0.0112	0.0292	0.0478
300	500	100	0.0329	0.1381	0.2106	0.0165	0.0575	0.0906	0.0131	0.0189	0.0369
	-500	100	0.0221	0.0957	0.1953	0.0103	0.0394	0.0623	0.0167	0.0443	0.0820
	500	-100	0.0329	0.1381	0.2106	0.0165	0.0575	0.0906	0.0131	0.0252	0.0533
	500	0	0.0329	0.1381	0.2106	0.0165	0.0575	0.0906	1.0000	1.0000	1.0000
	-500	0	0.0221	0.0957	0.1953	0.0103	0.0394	0.0623	1.0000	1.0000	1.0000
	0	500	0.0190	0.0929	0.1907	0.0084	0.0379	0.0666	1.0000	1.0000	1.0000
	50	10	0.5292	0.7540	0.8404	0.3287	0.5776	0.6915	0.0082	0.0251	0.0541
	-50	10	0.0964	0.2863	0.4173	0.0240	0.1301	0.2150	0.0073	0.0370	0.0527
	50	-10	0.5292	0.7540	0.8404	0.3287	0.5776	0.6915	0.0087	0.0322	0.0686
	50	0	0.5292	0.7540	0.8404	0.3287	0.5776	0.6915	1.0000	1.0000	1.0000
	-50	0	0.0964	0.2863	0.4173	0.0240	0.1301	0.2150	1.0000	1.0000	1.0000
	0	50	0.0888	0.2829	0.4009	0.0208	0.1199	0.2118	1.0000	1.0000	1.0000
1000	500	100	0.1872	0.4034	0.5708	0.0693	0.2030	0.3046	0.0020	0.0181	0.0379
	-500	100	0.0926	0.2848	0.4135	0.0233	0.1267	0.2122	0.0087	0.0385	0.0656
	500	-100	0.1872	0.4034	0.5708	0.0693	0.2030	0.3046	0.0057	0.0290	0.0533
	500	0	0.1872	0.4034	0.5708	0.0693	0.2030	0.3046	1.0000	1.0000	1.0000
	-500	0	0.0926	0.2848	0.4135	0.0233	0.1267	0.2122	1.0000	1.0000	1.0000
	0	500	0.0888	0.2829	0.4009	0.0208	0.1199	0.2118	1.0000	1.0000	1.0000

Table 3: Empirical Size and Empirical Power of test for serial correlation, no additive asymmetry and parameter constancy under DGP 1.

Empirical Size

T	γ_1	γ_2	γ_3	γ_4	Nominal size	No error autocorrelation $\rho = 0$				No additional asymmetry	Parameter constancy		
						q=1	q=2	q=4	q=10	H_0	LM_1	LM_2	LM_3
					$\alpha = 0.01$	0.0060	0.0041	0.0087	0.0053	0.0020	0.0000	0.0000	0.0005
	2	1			$\alpha = 0.05$	0.0628	0.0482	0.0354	0.0435	0.0034	0.0000	0.0000	0.0005
					$\alpha = 0.10$	0.0991	0.1008	0.0931	0.0995	0.0063	0.0000	0.0000	0.0029
					$\alpha = 0.01$	0.0045	0.0134	0.0070	0.0040	0.0000	0.0000	0.0000	0.0000
100	20	10			$\alpha = 0.05$	0.0526	0.0392	0.0186	0.0167	0.0007	0.0000	0.0000	0.0023
					$\alpha = 0.10$	0.0970	0.0953	0.0515	0.0393	0.0020	0.0000	0.0000	0.0092
					$\alpha = 0.01$	0.0102	0.0149	0.0098	0.0047	0.0000	0.0000	0.0000	0.0000
	200	100			$\alpha = 0.05$	0.0459	0.0478	0.0389	0.0318	0.0000	0.0000	0.0000	0.0012
					$\alpha = 0.10$	0.0921	0.1024	0.1008	0.0491	0.0080	0.0000	0.0012	0.0040
					$\alpha = 0.01$	0.0136	0.0114	0.0112	0.0057	0.0024	0.0000	0.0000	0.0000
	2	1			$\alpha = 0.05$	0.0462	0.0527	0.0616	0.0436	0.0065	0.0000	0.0000	0.0023
					$\alpha = 0.10$	0.1078	0.0969	0.1075	0.0983	0.0122	0.0000	0.0000	0.0081
					$\alpha = 0.01$	0.0087	0.0084	0.0043	0.0014	0.0018	0.0000	0.0000	0.0041
300	20	10			$\alpha = 0.05$	0.0622	0.0563	0.0287	0.0112	0.0018	0.0000	0.0000	0.0297
					$\alpha = 0.10$	0.1245	0.1210	0.0708	0.0241	0.0099	0.0000	0.0023	0.0607
					$\alpha = 0.01$	0.0250	0.0137	0.0149	0.0000	0.0142	0.0000	0.0000	0.0045
	200	100			$\alpha = 0.05$	0.0621	0.0043	0.0618	0.0208	0.0657	0.0000	0.0023	0.0344
					$\alpha = 0.10$	0.1221	0.0124	0.1195	0.0356	0.1083	0.0000	0.0057	0.0836

Empirical Power

T	γ_1	γ_2	γ_3	γ_4	Nominal size	No error autocorrelation $\rho = 0.2$				No error autocorrelation $\rho = 0.4$				No additional asymmetry	Parameter constancy		
						q=1	q=2	q=4	q=10	q=1	q=2	q=4	q=10	H_1	LM_1	LM_2	LM_3
					$\alpha = 0.01$	0.0075	0.0074	0.1261	0.6185	0.0276	0.0295	0.8493	0.9916	0.1744	0.2313	0.3280	0.5316
	2	1	5	2	$\alpha = 0.05$	0.0520	0.0594	0.3388	0.7702	0.0840	0.1011	0.9535	1.0000	0.2157	0.2957	0.4577	0.6858
					$\alpha = 0.10$	0.1111	0.0952	0.4723	0.8719	0.1530	0.1897	0.9706	1.0000	0.2516	0.3333	0.5354	0.7378
					$\alpha = 0.01$	0.0112	0.0138	0.0863	0.5425	0.0227	0.0228	0.7557	0.9930	0.1747	0.2270	0.3257	0.5356
100	20	10	50	20	$\alpha = 0.05$	0.0589	0.0621	0.2024	0.7090	0.0777	0.1014	0.9013	0.9959	0.2161	0.2768	0.4403	0.6958
					$\alpha = 0.10$	0.1033	0.1007	0.3262	0.7988	0.1371	0.1834	0.9465	1.0000	0.2496	0.3228	0.5361	0.7517
					$\alpha = 0.01$	0.0075	0.0203	0.0934	0.5540	0.0158	0.0290	0.7552	0.9931	1.0000	0.7871	0.8489	0.8924
	200	100	500	200	$\alpha = 0.05$	0.0611	0.1011	0.2538	0.7341	0.0759	1.0000	0.9035	1.0000	1.0000	0.8239	0.8770	0.9230
					$\alpha = 0.10$	0.1027	0.1146	0.3696	0.8176	0.1376	0.1767	0.9490	1.0000	1.0000	0.8296	0.8934	0.9331
					$\alpha = 0.01$	0.0290	0.0312	0.8014	1.0000	0.1047	0.1235	1.0000	1.0000	0.0118	0.0215	0.3353	0.7728
	2	1	5	2	$\alpha = 0.05$	0.0940	0.0953	0.9070	1.0000	0.2974	0.2724	1.0000	1.0000	0.0535	0.0835	0.6957	0.9442
					$\alpha = 0.10$	0.1859	0.1914	0.9467	1.0000	0.4006	0.4229	1.0000	1.0000	0.1149	0.1804	0.8281	0.9917
					$\alpha = 0.01$	0.0987	0.0810	0.7414	0.9968	0.0890	0.1331	1.0000	1.0000	0.1747	0.0215	0.3353	0.7728
300	20	10	50	20	$\alpha = 0.05$	0.1866	0.1652	0.8827	0.9980	0.2430	0.2848	1.0000	1.0000	0.2161	0.0835	0.6957	0.9442
					$\alpha = 0.10$	0.2372	0.2323	0.9069	1.0000	0.3436	0.4272	1.0000	1.0000	0.2496	0.1804	0.8281	0.9940
					$\alpha = 0.01$	0.0216	0.0419	0.7046	0.9968	0.0860	0.1422	1.0000	1.0000	1.0000	0.9961	0.9961	0.9961
	200	100	500	200	$\alpha = 0.05$	0.1106	0.1110	0.8476	1.0000	0.2156	0.2980	1.0000	1.0000	1.0000	0.9961	0.9970	1.0000
					$\alpha = 0.10$	0.1755	0.1986	0.8963	1.0000	0.3349	0.4364	1.0000	1.0000	1.0000	0.9968	0.9991	1.0000

Table 4: Empirical Size and Empirical Power of test for serial correlation, no additive asymmetry and parameter constancy under DGP 2.

Empirical Size

T	γ_1	γ_2	γ_3	γ_4	Nominal size	No error autocorrelation $\rho = 0$				No additional asymmetry	Parameter constancy		
						q=1	q=2	q=4	q=10	H_0	LM_1	LM_2	LM_3
					$\alpha = 0.01$	0.0082	0.0064	0.0107	0.0039	0.0055	0.0000	0.0000	0.0000
	2	1			$\alpha = 0.05$	0.0534	0.0453	0.0503	0.0443	0.0464	0.0000	0.0000	0.0000
					$\alpha = 0.10$	0.1120	0.1124	0.1119	0.0781	0.0945	0.0000	0.0000	0.0000
					$\alpha = 0.01$	0.0059	0.0069	0.0074	0.0029	0.0011	0.0000	0.0000	0.0000
100	20	10			$\alpha = 0.05$	0.0488	0.0421	0.0400	0.0333	0.0425	0.0000	0.0000	0.0041
					$\alpha = 0.10$	0.0978	0.0948	0.1062	0.0819	0.0812	0.0000	0.0000	0.0053
					$\alpha = 0.01$	0.0084	0.0059	0.0046	0.0018	0.0345	0.0000	0.0000	0.0000
	200	100			$\alpha = 0.05$	0.0433	0.0470	0.0446	0.0163	0.0700	0.0000	0.0000	0.0000
					$\alpha = 0.10$	0.0983	0.1089	0.0977	0.0455	0.1089	0.0000	0.0000	0.0005
					$\alpha = 0.01$	0.0096	0.0098	0.0159	0.0058	0.0118	0.0000	0.0000	0.0000
	2	1			$\alpha = 0.05$	0.0448	0.0479	0.0506	0.0532	0.0512	0.0000	0.0000	0.0000
					$\alpha = 0.10$	0.0899	0.0890	0.1006	0.0926	0.0925	0.0000	0.0000	0.0000
					$\alpha = 0.01$	0.0099	0.0083	0.0164	0.0051	0.0127	0.0000	0.0000	0.0000
300	20	10			$\alpha = 0.05$	0.0463	0.0446	0.0484	0.0461	0.0505	0.0000	0.0000	0.0048
					$\alpha = 0.10$	0.0884	0.0920	0.0882	0.0905	0.0916	0.0000	0.0000	0.0052
					$\alpha = 0.01$	0.0128	0.0134	0.0111	0.0100	0.0381	0.0000	0.0000	0.0000
	200	100			$\alpha = 0.05$	0.0584	0.0413	0.0611	0.0481	0.0640	0.0000	0.0000	0.0000
	(120)	(70)			$\alpha = 0.10$	0.0891	0.0853	0.1013	0.0968	0.1217	0.0000	0.0000	0.0000

Empirical Power

T	γ_1	γ_2	γ_3	γ_4	Nominal size	No error autocorrelation $\rho = 0.2$				No error autocorrelation $\rho = 0.4$				No additional asymmetry	Parameter constancy		
						q=1	q=2	q=4	q=10	q=1	q=2	q=4	q=10	H_1	LM_1	LM_2	LM_3
					$\alpha = 0.01$	0.0538	0.0819	0.2704	0.6581	0.3615	0.4917	0.9581	0.9892	0.1299	0.0019	0.0042	0.0277
	2	1	5	2	$\alpha = 0.05$	0.1772	0.2302	0.5272	0.8088	0.5759	0.7035	0.9842	0.9919	0.1675	0.0075	0.0456	0.1205
					$\alpha = 0.10$	0.2873	0.3440	0.6532	0.8812	0.7180	0.7885	0.9881	0.9959	0.2029	0.0197	0.0880	0.1992
					$\alpha = 0.01$	0.0540	0.0633	0.2863	0.6560	0.3746	0.4811	0.9618	0.9892	0.1299	0.0016	0.0186	0.1731
100	20	10	50	20	$\alpha = 0.05$	0.1817	0.2241	0.5427	0.8127	0.6207	0.7083	0.9858	0.9960	0.1675	0.0121	0.0807	0.4039
					$\alpha = 0.10$	0.2749	0.3312	0.6689	0.8792	0.7587	0.8168	0.9881	0.9960	0.2029	0.0223	0.1597	0.5689
					$\alpha = 0.01$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0059	0.0566	0.1340
	200	100	500	200	$\alpha = 0.05$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0207	0.1591	0.3303
					$\alpha = 0.10$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0503	0.2475	0.4782
					$\alpha = 0.01$	0.3156	0.3919	0.9576	1.0000	0.9431	0.9979	1.0000	1.0000	0.3567	0.0000	0.0015	0.0649
	2	1	5	2	$\alpha = 0.05$	0.5697	0.6956	0.9806	1.0000	0.9873	0.9997	1.0000	1.0000	0.3777	0.0000	0.0225	0.2348
					$\alpha = 0.10$	0.7064	0.7885	0.9890	1.0000	0.9948	1.0000	1.0000	1.0000	0.4125	0.0001	0.0509	0.3599
					$\alpha = 0.01$	0.3455	0.4193	0.9622	1.0000	0.9565	0.9983	1.0000	1.0000	0.3567	0.0000	0.0015	0.0609
300	20	10	50	20	$\alpha = 0.05$	0.6141	0.7093	0.9819	1.0000	0.9921	1.0000	1.0000	1.0000	0.3777	0.0000	0.0225	0.2349
					$\alpha = 0.10$	0.7394	0.8015	0.9922	1.0000	0.9964	1.0000	1.0000	1.0000	0.4125	0.0001	0.0509	0.3599
					$\alpha = 0.01$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0015	0.0649
	200	100	500	200	$\alpha = 0.05$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0225	0.2348
					$\alpha = 0.10$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0001	0.0509	0.3599

Table 5: Datasets

Series	Sample	T^s	Testing Period	Source	Previous Studies
IP	1947 M1 — 2013 M3	500	1982 M9+h — 2013M3-12+h	OECD Main Economic Indicators	Anderson and Teräsvirta (1992) Proietti (1998) Teräsvirta, van Dijk, and Medeiros (2005)
UN	1948 M1 — 2013 M3	500	1983 M9+h — 2013M3-12+h	OECD Main Economic Indicators	Rothman (1991) Rothman (1998) Montgomery, Zarnowitz, Tsay, and Tiao (1998) Skalin and Teräsvirta (2002) Proietti (2003)
YSSN	1700 — 2013	150	1851+h — 2013-12+h	Solar Influences Data Analysis Center	Tong and Lim (1980) Tsay (1989) Hansen (1999) Strikholm and Teräsvirta (2005)
LYNX	1820 — 1934	50	1871+h — 1934-12+h	The Encyclopedia of Mathematics wiki	Elton and Nicholson (1942) Moran (1953) Tong (1977) Tsay (1989) Teräsvirta (1994)

Table 6: Empirical application of the (G)STAR model to four real data samples

Descriptive statistics

Series	Mean	Median	MAD	Std.Dev.	Skewness	Kurtosis	JB	ARCH-effects	DW	KS	ADF(1)(3)
IP	0.2490	0.2790	0.3474	0.4780	-0.2815	4.5233	0.0010	0.0000	0.0293	0.0000	-11.1962**
UN	0.0974	-0.3704	1.3182	1.8054	0.9802	4.9645	0.0010	0.0000	0.9341	0.0001	-10.7775**
YSSN	6.3981	6.3246	2.4452	2.9486	0.1790	2.2904	0.0226	0.0000	0.2978	0.0000	-5.2865**
LYNX	2.9037	2.8870	0.4721	0.5584	-0.3620	2.2664	0.0582	0.0024	0.0176	0.0000	-5.8245**

Linearity and Asymmetry tests (p-values)

Series	Linearity Test(2)				Model(2)	Symmetry Test	
	F_L	F_3	F_2	F_1		All-in-One	Two-Step
IP	0.053	0.016	0.223	0.560	LSTAR1	0.0150	1.0000
UN	$2.1e^{-5}$	0.016	0.569	$1.6\ e^{-5}$	LSTAR1	0.0005	1.0000
YSSN	$3.03e^{-5}$	0.140	0.940	$2.99e^{-7}$	LSTAR1	0.0209	1.0000
LYNX	0.002	$1.6\ e^{-4}$	0.614	0.166	LSTAR1	0.0001	1.0000

Estimates

Parameter	(MR)STAR								GSTAR							
	IP		UN		YSSN		LYNX		IP		UN		YSSN		LYNX	
	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE
ϕ_0	0.0007	0.0105	-0.0217	0.0319	0.8386	0.4665	3.6129	8.9173	-0.0317	0.0304	-0.1318	0.1212	0.0016	0.8946	3.0571	3.5488
ϕ_1	1.2982	0.0534	0.8624	0.0511	1.3673	0.1859	-0.2229	1.2204	1.2509	0.0809	0.8283	0.0856	0.0017	0.5732	-0.8190	3.7252
ϕ_2	-0.2449	0.0846	0.2670	0.0659	-0.6531	0.2167	0.2898	0.7223	-0.1757	0.1228	0.2150	0.0903	-0.0013	1.0925	0.9297	3.0689
ϕ_3	0.0303	0.0837	-0.2039	0.0490	-0.0246	0.2418	0.0388	0.1648	0.0389	0.1263	-0.1499	0.0751	0.0001	0.5034	-0.1618	1.0542
ϕ_4	-0.0289	0.0821	-	-	-0.0596	0.2221	2.5471	6.5730	0.0422	0.1323	-	-	-0.0001	0.3340	0.6786	1.9507
ϕ_5	-0.0671	0.0818	-	-	0.2429	0.1534	0.1100	0.9837	-0.1608	0.1318	-	-	0.0002	0.3513	0.3750	0.7873
ϕ_6	-0.1055	0.0555	-	-	-	-	-0.9993	1.4859	-0.1499	0.0835	-	-	-	-	-0.9116	1.8615
ϕ_7	-	-	-	-	-	-	-0.0281	0.5210		-	-	-	-	-	0.2160	0.4393
θ_{10}	0.0626	0.0320	0.0805	0.0582	1.8438	1.1950	-10.7093	41.3355	0.0787	0.0433	0.3241	0.2318	0.7902	1.2649	-2.0815	3.5317
θ_{11}	-0.1975	0.0893	0.3500	0.0725	0.2799	0.2938	2.2348	2.7972	-0.0582	0.1109	0.4011	0.1103	-0.5954	0.5798	2.1757	3.7350
θ_{12}	0.2327	0.1359	-0.2121	0.1058	-0.5836	0.3459	-1.3248	1.6504	0.0566	0.1668	-0.1428	0.1525	1.1665	1.0964	-1.7943	3.1006
θ_{13}	-0.1057	0.1362	-0.1326	0.0708	-0.0460	0.3298	0.1796	0.4185	-0.0944	0.1733	-0.2521	0.1081	-0.5191	0.5365	0.5972	1.1143
θ_{14}	0.0044	0.1390	-	-	0.2602	0.3379	-1.7566	2.7348	-0.1125	0.1839	-	-	0.0003	0.3830	-1.1562	2.0178
θ_{15}	0.1746	0.1411	-	-	-0.3025	0.2201	-0.1107	1.6046	0.0269	0.1806	-	-	-0.0003	0.3680	-0.2925	0.8333
θ_{16}	0.2427	0.0891	-	-	-	-	1.5962	2.8549	0.2337	0.1092	-	-	-	-	1.0047	1.8314
θ_{17}	-	-	-	-	-	-	0.1031	0.7856		-	-	-	-	-	-0.1760	0.4510
θ_{20}	-	-	-	-	-0.3877	1.0656	0.5321	3.5232	-	-	-	-	-	-	-	-
θ_{21}	-	-	-	-	-0.5768	0.2432	-0.7867	0.7169	-	-	-	-	-	-	-	-
θ_{22}	-	-	-	-	1.1635	0.3362	-1.3103	0.7344	-	-	-	-	-	-	-	-
θ_{23}	-	-	-	-	-0.3871	0.2829	3.0855	1.2053	-	-	-	-	-	-	-	-
θ_{24}	-	-	-	-	-0.0173	0.3069	-2.8533	1.4792	-	-	-	-	-	-	-	-
θ_{25}	-	-	-	-	-0.0099	0.1914	3.9498	1.7185	-	-	-	-	-	-	-	-
θ_{26}	-	-	-	-	-	-	-2.8365	1.6553	-	-	-	-	-	-	-	-
θ_{27}	-	-	-	-	-	-	0.1947	0.5976								
γ_1	4.7500	0.0320	29.0890	0.0582	129.56	1.1950	0.7807	41.3355	5.0000	0.0433	0.0001	0.2318	3.3171	1.2649	-15.000	0.202
γ_2	-	-	-		5.9	0.0010	9.8119	46.587	3.0000	0.1109	0.0001	0.1103	-3.3171	0.5798	17.000	0.334
c_1	0.6644	0.0893	0.0590	0.0725	1.2482	0.2938	2.6201	2.7972	-0.0446	0.1668	0.2900	0.1525	6.3746	1.0964	2.3006	1.475
c_2	-	-	-	-	2.0760	0.0001	4.6587	-	-	-	-	-	-	-	-	-

Diagnostics (p-values)

Diagnostic Test	STAR				GSTAR			
No error autocorrelation								
q=1	0.1785	0.1014	0.3341	0.2398	0.3336	0.1434	0.2411	0.6612
q=2	0.4028	0.2616	0.6278	0.5018	0.5507	0.3430	0.5036	0.9089
q=4	0.0001	0.6130	0.9206	0.8486	0.0086	0.7106	0.8499	0.9958
q=10	0.0000	0.9882	0.9999	0.9993	0.0000	0.9953	0.9993	1.0000
No rem. nonlinearity / No rem. asymmetry	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Parameter constancy								
H1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
H2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
H3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

NOTE: (1)Values expressed in test-statistics, significance denoted by '*' (5%), '**' (1%); (2) Results acquired by JMulTi; (3)Results acquired by RATS

Table 7: Empirical application of the (G)STAR model to Monthly Smoothed Sunspot Number from 1850 to 2013.

Descriptive statistics

Series	Mean	Median	MAD	Std.Dev.	Skewness	Kurtosis	JB	ARCH-effects	DW	KS	ADF[(1)(3)]
logSSN	3.5843	3.8597	0.8641	1.0484	-0.7149	2.7842	0.0010	0.0000	0.0000	0.0000	-4.3619**
sqrtSSN	6.7691	6.8884	2.5196	3.0051	0.1714	2.1951	0.0010	0.0000	0.0000	0.0000	-5.37187**
DLSSN	0.0021	-0.0076	0.0563	0.1224	22.6304	798.4572	0.0010	0.9901	0.0000	0.0000	-36.5383**

Linearity and Asymmetry tests (p-values)

Series	Linearity Test[(2)]					Symmetry Test	
	F_L	F_3	F_2	F_1	Model[(2)]	All-in-One	Two-Step
logSSN	$1.05e^{-35}$	0.0027	$1.20e^{-14}$	$5.92e^{-23}$	LSTAR1	0.0000	1.0000
sqrtSSN	0.0002	0.2410	$5.57e^{-5}$	0.1210	LSTAR2	0.0000	1.0000
DLSSN	0.0012	0.9780	$4.57e^{-4}$	0.0123	LSTAR2	0.0000	1.0000

Estimates

Parameter	(MR)STAR						GSTAR					
	logSSN		sqrtSSN		DLSSN		logSSN		sqrtSSN		DLSSN	
	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE
ϕ_0	-2,495.1000	0.3420	-21.8270	1.4187	-0.3034	0.2572	0.5143	0.0555	1.0461	0.1757	-9.2453	48.1169
ϕ_1	-464.4000	2.1995	-32.574	0.9746	0.1760	0.5590	1.9919	0.1757	1.0461	0.3804	-5.4026	30.5430
ϕ_2	-67.1761	0.7843	-1.412	0.2425	-1.6014	0.8251	-2.0721	0.3724	-2.8801	0.8284	2.5303	10.4673
ϕ_3	31.4922	0.5348	3.1192	0.2704	1.5567	0.9119	1.1495	0.3527	1.6989	0.7672	-2.1641	8.9374
ϕ_4	-16.3030	0.4879	-3.2854	0.2380	-1.0027	0.6947	-0.3047	0.3030	-0.3700	0.6026	1.2448	5.2104
ϕ_5	18.6892	0.9051	0.9386	0.1156	0.7084	0.5401	-0.3126	0.1556	-0.3191	0.2778	-0.7517	3.2121
ϕ_6	-	-	-	-	-0.3175	0.3528	-	-	-	-	-0.3563	1.9530
ϕ_7	-	-	-	-	-	-	-	-	-	-	-	
θ_{10}	-328.351	0.2302	7,618.1000	0.3182	0.6107	0.3457	0.3626	0.1929	-0.8152	0.1908	17.8136	29.8383
θ_{11}	-160.294	0.9647	-1,196.8000	0.5750	-0.5527	0.7197	-1.1847	0.1659	-1.2356	0.3808	3.5103	34.6796
θ_{12}	-336.763	0.6516	-17.7000	1.5695	3.5177	0.8990	2.2391	0.3769	2.8006	0.8309	-4.9874	3.6295
θ_{13}	107.133	1.5158	-57.3000	1.8193	-3.6285	1.1869	-1.2748	0.3821	-1.7257	0.7710	4.2078	3.8702
θ_{14}	-53.4889	2.4075	90.6000	2.1883	2.3600	1.0980	0.3345	0.3389	0.3700	0.6074	-2.3867	3.5481
θ_{15}	130.2654	1.5391	31.1000	2.2599	-1.5697	0.9668	0.2752	0.1738	0.2100	0.2815	1.4728	3.1431
θ_{16}	-	-	-	-	0.7868	0.7366	-	-	-	-	0.7243	2.1704
θ_{17}	-	-	-	-	-	-	-	-	-	-	-	
θ_{20}	-98.2139	2.6406	-147.2963	0.8512	-0.0045	0.0128	-	-	-	-	-	-
θ_{21}	26.5350	0.7790	96.2139	1.6993	0.3015	0.3592	-	-	-	-	-	-
θ_{22}	79.8829	1.1839	-6.6805	3.0229	-1.8244	0.4337	-	-	-	-	-	-
θ_{23}	-15.5313	1.2734	20.4282	3.1806	1.8168	0.5167	-	-	-	-	-	-
θ_{24}	6.9089	1.5679	-17.8232	2.7499	-1.1288	0.4262	-	-	-	-	-	-
θ_{25}	-37.3372	1.3244	3.8179	1.2654	0.6836	0.3218	-	-	-	-	-	-
θ_{26}	-	-	-	-	-0.2950	0.2336	-	-	-	-	-	-
θ_{27}	-	-	-	-	-	-	-	-	-	-	-	-
γ_1	0.6114	0.2302	3.0383	0.0331	1.0173	0.3457	0.0001	0.1929	0.0001	0.1908	0.0001	29.8383
γ_2	1.0247	0.8703	3.7500	0.0492	19.5033	10.9862	-0.0001	0.1659	-0.0001	0.3808	0.0001	34.6796
γ_3	0.2893	0.8703	3.0452	0.0333	-	-	-	-	-	-	-	-
c_1	2.1121	-	0.9085	0.0180	-0.0410	0.7197	2.0109	3769	2.7331	0.8309	-0.0761	3.6295
c_2	2.1673	-	0.9214	0.0146	0.0406	0.0450	-	-	-	-	-	-
c_3	2.8470	-	0.9267	0.0185	-	-	-	-	-	-	-	-

Diagnostics (p-values)

Diagnostic Test	STAR			GSTAR		
No error autocorrelation						
q=1	0.8947	0.7664	0.7563	0.0031	0.0001	0.3836
q=2	0.9913	0.9569	0.9530	0.0125	0.0004	0.6842
q=4	1.0000	0.9991	0.9989	0.0673	0.0034	0.9439
q=10	1.0000	1.0000	1.0000	0.5558	0.1089	1.0000
No rem. nonlinearity / No rem. asymmetry	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Parameter constancy						
H1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994
H2	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991
H3	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991

NOTE: (1)Values expressed in test-statistics, significance denoted by '*' (5%), '**' (1%); (2) Results acquired by JMulTi; (3)Results acquired by RATS

Table 8: Predictive performances of GSTAR model for different forecast horizons and comparison with (MR)-STAR and AR models

		Point predictive performances											
Forecast horizon	Forecast Error Measure	AR				(MR)STAR				GSTAR			
		IP	UN	YSSN	LYNX	IP	UN	YSSN	LYNX	IP	UN	YSSN	LYNX
	MFE												
1		-0.0011	0.0009	-0.0300	0.0029	-0.0065	0.0156	0.0807	-0.0013	-0.0093	0.0095	0.0744	-0.0003
3		-0.1108	0.0340	-0.8837	-3.2369	-0.0071	0.0168	0.0975	-0.0058	-0.0098	0.0108	0.0881	-0.0031
6		-0.3024	0.0607	-0.0221	-8.6402	-0.0072	0.0150	0.1007	-0.0101	-0.0099	0.0093	0.0986	-0.0066
12		-1.4908	0.1117	-7.8745	-25.2732	-0.0068	0.0163	0.0984	-0.0176	-0.0094	0.0107	0.0957	-0.0115
	sMAE												
1		0.0011	0.0058	0.0009	0.0008	0.0011	0.0061	0.0008	0.0008	0.0011	0.0060	0.0008	0.0010
3		0.0037	0.0087	0.0016	0.0116	0.0011	0.0059	0.0008	0.0008	0.0011	0.0058	0.0008	0.0010
6		0.0070	0.0128	0.8572	0.0203	0.0012	0.0056	0.0008	0.0009	0.0012	0.0055	0.0008	0.0011
12		0.0124	0.0190	0.0050	0.0308	0.0012	0.0050	0.0009	0.0011	0.0012	0.0049	0.0009	0.0013
	mRAE												
1		1.0000	1.0000	1.0000	1.0000	1.0324	1.0224	1.7950	1.8697	1.0290	1.0107	1.6913	1.6567
3		1.0000	1.0000	1.0000	1.0000	1.1560	1.0552	2.0764	9.6346	1.1241	1.0369	2.1851	8.2831
6		1.0000	1.0000	1.0000	1.0000	1.4886	1.2189	2.5357	15.3676	1.4546	1.1693	2.7649	13.2310
12		1.0000	1.0000	1.0000	1.0000	2.0115	1.4965	9.4989	0.0260	2.1142	1.5085	9.4308	13.0395
	RMSFE												
1		0.0698	0.2856	1.0050	0.1878	0.0041	0.0178	0.0860	0.0243	0.0042	0.0176	0.0868	0.0283
3		0.2894	0.4234	2.0443	3.2842	0.0042	0.0179	0.0863	0.0249	0.0043	0.0177	0.0874	0.0291
6		0.7552	0.5915	1.8299	8.7639	0.0042	0.0181	0.0876	16.9274	0.0043	0.0179	0.0885	0.0305
12		0.7638	0.8545	9.0253	25.6618	0.0042	0.0184	0.0907	0.0281	0.0043	0.0182	0.0917	0.0336
		Density predictive performances											
Forecast horizon	Giacomini-White Test												
1	(p-values)	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.2030	0.1112	0.0000	0.0919	0.0002
3		0.0000	0.0000	0.0250	0.3923	0.0001	0.0000	0.0040	0.0000	0.0000	0.0003	0.0078	0.0836
6		0.0843	0.0241	0.0921	0.0000	0.1558	0.1292	0.0000	0.0000	0.1112	0.5175	0.0000	0.0038
12		0.1023	0.1004	0.0921	0.9042	0.0106	0.0033	0.0000	0.9217	0.0170	0.0002	0.0000	0.8646
Forecast horizon	Scoring Rule												
	LogS												
1		0.0020	0.0031	0.0073	0.0064	0.0017	0.0033	0.0072	0.0065	0.0018	0.0036	0.0072	0.0048
3		0.1108	0.0031	0.0072	0.0062	0.0017	0.0033	0.0072	0.0061	0.0018	0.0035	0.0072	0.0047
6		0.0020	0.0031	0.0071	0.0067	0.0017	0.0033	0.0072	0.0064	0.0018	0.0035	0.0071	0.0050
12		0.0020	0.0031	0.0071	0.0067	0.0017	0.0033	0.0072	0.0063	0.0018	0.0035	0.0072	0.0051
	QS												
1		1.1568	0.7986	0.2619	1.4829	2.1303	0.8511	0.2340	1.3515	2.1689	0.8683	0.2269	1.1717
3		1.1644	0.7998	0.2624	1.4698	2.1386	0.8537	0.2333	1.3384	2.1821	0.8698	0.2276	1.1385
6		1.1665	0.8125	0.2620	1.5058	2.1391	0.8550	0.2357	1.3762	2.1895	0.8698	0.2268	1.1677
12		1.2024	0.6741	0.2613	1.6211	2.1479	0.6412	0.2340	1.3106	2.1921	0.6354	0.2271	1.1668
	CRPS												
1		0.2142	1.6750	4.2628	1.5977	0.2170	1.6090	4.6027	1.4743	0.2201	1.6094	4.2893	1.4906
3		0.2161	1.6850	4.2932	1.6689	0.2189	1.6193	4.2935	1.5435	0.2224	1.6197	4.3243	1.5545
6		0.2188	1.6983	4.3551	1.7700	0.2212	1.6317	4.3787	1.6279	0.2252	1.6344	4.3942	1.6375
12		0.2255	1.7295	4.5782	2.0151	0.2279	1.6594	4.6027	1.8482	0.2318	1.6617	4.6064	1.8558
	qS												
1		0.0084	-0.0713	0.6198	0.2557	0.0089	-0.0727	0.6060	0.2571	0.0087	-0.0723	0.6067	0.2468
3		0.0083	-0.0706	0.6267	0.2527	0.0085	-0.0721	0.6119	0.2544	0.0086	-0.0717	0.6122	0.2439
6		0.0082	-0.0695	0.6329	0.2511	0.0083	-0.0708	0.6176	0.2537	0.0085	-0.0705	0.6173	0.2434
12		0.0077	-0.0667	0.6218	0.2455	0.0071	-0.0682	0.6068	0.2496	0.0080	-0.0679	0.6065	0.2387

NOTE: Forecast windows size is set to 500 for IP and UN, 50 for LYNX and 150 for YSSN. For the Giacomini-Wight test, an AR(1) model is used as benchmark.

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