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Abstract

Risk premia between spot and forward prices play a key role in energy markets. This paper derives analytic expressions for such risk premia when spot prices are modelled by Lévy semistationary processes. While the relation between spot and forward prices can be derived using classical no-arbitrage arguments as long as the underlying commodities are storable, the situation changes in the case of electricity. Hence, in an empirical study based on electricity spot prices and futures from the European Energy Exchange market, we investigate the empirical behaviour of electricity risk premia from a statistical perspective. We find that a model-based prediction of the spot price has some explanatory power for the corresponding forward price, but there is a significant additional amount of variability, the risk premium, which needs to be accounted for. We demonstrate how a suitable model for electricity forward prices can be formulated and we obtain promising empirical results.

Keywords: Lévy semistationary process, energy market, spot price, forward price, futures, risk premia, stochastic volatility, European Energy Exchange market.

JEL classification codes: C10, C51, G00, G130.

1 Introduction

Energy forward and future contracts are one of the most important derivatives traded on modern energy markets. Several models have been proposed to describe forward price dynamics directly. The relationship between spot and forward prices, however, is still not well understood in energy markets.

In contrast to classical financial markets, such as the stock markets, classical arguments from the no-arbitrage pricing theory cannot directly be applied to all assets traded on energy markets. The classical example is (physical) spot electricity. Since electricity is not storable one cannot construct replicating portfolios and hence the forward price of electricity will not just be a suitably discounted version of the current spot price. Other assets traded on energy markets, such as gas, are storable and hence some classical arguments can be applied to these types of assets.

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Recently, several authors advocated the use of *Lévy-semistationary* ($\mathcal{LSS}$) processes to model spot prices in energy markets, see e.g. Barndorff-Nielsen et al. (2013+). Veraart & Veraart (2013+) extended the univariate model by Barndorff-Nielsen et al. (2013+) to a multivariate framework and showed that this multivariate version can be successfully fitted to hourly electricity spot prices data from the EEX. The model proposed by Benth (2011) also falls within this model class, and was shown to provide a good empirical fit to data from the UK gas spot market.

The purpose of this paper is to draw conclusions about forward prices and corresponding risk premia resulting from the class of Lévy-semistationary ($\mathcal{LSS}$) spot price models. For this we study both theoretical risk premia which result from an appropriate change of measure and empirical risk premia observed on the European Energy Exchange (EEX).

In general, one distinguishes between ex-ante and ex-post risk premia. Let $S(t)$ be the spot price of an asset at time t and let $F(t,T)$ denote the forward price observed at time t for delivery at time $T \geq t$. Then the ex-ante risk premium at time t is $F(t,T) - \mathbb{E}_t^P(S(T))$, where $\mathbb{E}_t^P(\cdot)$ denotes the expectation under the physical probability measure P conditional on the σ-algebra containing the information available at time t. The ex-post risk premium at time t is then given by $F(t,T) - S(T)$. The main difference between these two risk premia is that the ex-ante risk premium is not directly observable from market data, but is always dependent on the spot price model chosen, whereas the ex-post risk premium is completely observable from market data.

In general, one can observe changes in the sign of the risk premium in energy market. On the one hand, risk premia tend to be positive at the short-end of the forward curve, i.e. close to the start of the delivery period. It is usually argued that this is due to consumers trying to hedge risk that is due to possible price spikes, i.e. they are prepared to pay a premium for fixing prices in advance, because they do not want to accept the risk that prices might jump up considerably by the time they need to buy the energy on the spot market.

On the other hand, risk premia tend to be negative at the long-end, i.e. far from the begin of the delivery period. This is usually explained by producers wishing to hedge their revenues risk by selling forward contracts. Since having this hedge is considered to be an advantage, they are prepared to accept prices below the expected spot price.

Several approaches have been proposed to explain the behaviour, sign and magnitude of ex-ante and ex-post risk premia. An interesting approach considering the ex-ante risk premium can be found in Bessembinder & Lemmon (2002), where an equilibrium model is suggested that provides a link between spot and forward prices.

Furthermore, as soon as the forward price is computed as an expectation of the future spot price under a suitable pricing measure Q, i.e. $F(t,T) = \mathbb{E}_t^Q(S(T))$, the ex-ante risk premium is just given by $\mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T))$ and can be studied analytically. E.g. Benth & Sgarra (2012) provide a theoretical justification via a change of measure argument for the sign change observed in empirical data on risk premia. Our new theoretical results are in the spirit of the work by Benth & Sgarra (2012), which we extend to allow for a stochastic volatility component. We show that the stochastic volatility has an important impact on the (stochastic) dynamics of the risk premium. The empirical literature has mainly focused on the ex-post risk premia, see e.g. Lucia & Torró (2011) and the references therein.

Benth, Cartea & Kiesel (2008) provide an explanation for the sign and the magnitude of the market risk premium by modelling market players and their risk preferences directly by applying the certainty equivalence principle. Benth & Meyer-Brandis (2009) model forward prices as conditional expectations where forward-looking information are incorporated into the conditional expectation. This leads to an information based approach to explaining the risk-premium.

This paper is organized as follows. In Section 2 we briefly recall the class of Lévy-semistationary ($\mathcal{LSS}$) spot price models. Section 3 contains all definitions for different types of risk premia that are considered in energy markets. In particular, we briefly recall the risk premium approach to pricing forward and future contracts. We then derive analytic expressions for the conditional expectation of future spot prices for $\mathcal{LSS}$ models under the physical probability measure. In Section 5 these expressions will be used to investigate the explanatory power of this expectation for the forward price. Section 4 contains analytic expressions for theoretical risk premia which are obtained by considering forward prices as expectations of the future spot price under a probability measure which is equivalent to the physical probability measure. In particular, we discuss the influence of stochastic volatility in the spot model on the risk premium. Section 5 contains an empirical analysis of risk premia observed on the European Energy Exchange (EEX). For this we study empirical data of electricity spot prices and futures contracts which have delivery period over one month. We only consider the corresponding futures prices during the months prior to the begin of the delivery period. Hence, our empirical study focus on the short-end of the forward curve. Finally, Section 6 concludes.

2 Modelling spot prices by Lévy semistationary processes

In this section we will briefly recall the central properties of the class of *Lévy-semistationary* ($\mathcal{LSS}$) processes.

Throughout the paper, we denote by $(\Omega, \mathcal{F}, P)$ a probability space with a filtration $\mathfrak{F} = \{\mathcal{F}_t\}_{t\in\mathbb{R}}$ satisfying the 'usual conditions', see (Karatzas & Shreve 2005, p. 10).

2.1 The driving two-sided Lévy process

Let $L = (L(t))_{t\geq 0}$ denote a càdlàg modification of a Lévy process with characteristic triplet given by (γ, c, ν), where $\gamma \in \mathbb{R}$, $c \geq 0$ and ν denotes the Lévy measure on $\mathbb{R}$ satisfying $\nu(\{0\}) = 0$, and $\int_{\mathbb{R}}(|x|^2 \wedge 1)\nu(dx) < \infty$. The corresponding Lévy–Khintchine representation of L is hence given by

$$\mathbb{E}^P(\exp(i\theta L(t))) = \exp(t\Psi_L^P(\theta)), \qquad \text{for } \theta \in \mathbb{R},$$

where

$$\Psi_L^P(\theta) = i\gamma\theta - \frac{1}{2}c\theta^2 + \int_{\mathbb{R}} \left(e^{i\theta x} - 1 - i\theta x \mathbb{I}_{\{|x|\leq 1\}}\right)\nu(dx)$$

denotes the characteristic exponent of L under P. Also, let $\psi_L^P(\theta) := \Psi_L^P(-i\theta)$ for $\theta \in \mathbb{R}$ be the logarithm of the moment generating function of L, provided it exists.

In addition, we have the following Lévy–Itô representation

$$L(t) = \gamma t + \sqrt{c}B(t) + \int_0^t \int_{\{|x|\leq 1\}} x(N(dx, ds) - \nu(dx)ds) + \int_0^t \int_{|x|\geq 1\}} xN(dx, ds),$$

where B is a standard Brownian motion and N denotes the Poisson random measure associated with L.

In a next step, we extend the definition of a Lévy process defined on $\mathbb{R}_+$ to a two-sided Lévy process defined on $\mathbb{R}$. In order to do that, let L' denote another Lévy process (and as before we consider its càdlàg modification) that is independent of L and has the same characteristic triplet as L. We define a new process L^* by setting

$$L^*(t) := \begin{cases} L(t), & \text{for } t \geq 0, \\ -L'(-(t-)), & \text{for } t < 0. \end{cases} \tag{1}$$

The process $L^* = (L^*(t))_{t\in\mathbb{R}}$ is called a *two-sided* Lévy process, see e.g. Brockwell (2009), which is càdlàg. Note, however, that in the following, we will always write L rather than L^* to simplify notation.

2.2 Definition of $\mathcal{LSS}$ processes

An $\mathcal{LSS}$ process on $\mathbb{R}$ is defined as

$$Y(t) := \int_{-\infty}^{t} g(t-s)\sigma(s-)dL(s), \tag{2}$$

where L denotes the two–sided Lévy process defined in (1). Further, $g : \mathbb{R} \to \mathbb{R}$ denotes a deterministic, measurable function satisfying $g(t-s) = 0$ whenever $t < s$, and $\sigma = (\sigma(t))_{t\in\mathbb{R}}$ denotes a càdlàg, adapted and positive stochastic process, which we typically refer to as *stochastic volatility*. Throughout the paper we will assume that the stochastic processes L and σ are independent.

We define the stochastic process $(\phi^{(t)}(s))_{s\in\mathbb{R}}$ with $\phi^{(t)}(s) := g(t-s)\sigma(s-)$, which is predictable and $\phi^{(t)}(s) = 0$ for $s > t$. According to (Basse-O'Connor et al. 2013+, Corollary 4.1), the stochastic integral in (2) is well–defined if and only if the following three conditions hold almost surely:

$$\begin{gathered}\int_{-\infty}^{t} c\left(\phi_i^{(t)}(s)\right)^2 ds < \infty, \qquad \int_{-\infty}^{t}\int_{\mathbb{R}} \left(1 \wedge \left|\phi_i^{(t)}(s)z\right|^2\right)\nu(dz)ds < \infty, \\ \int_{-\infty}^{t} \left|\gamma\phi_i^{(t)}(s) + \int_{\mathbb{R}} \left(h\left(z\phi_i^{(t)}(s)\right) - \phi_i^{(t)}(s)h(z)\right)\nu(dz)\right| ds < \infty,\end{gathered} \tag{3}$$

where $h(x) = \mathbb{I}_{\{x\in\mathbb{R}:|x|\le 1\}}$.

Remark 1. *It should be noted that the integrability conditions above do not ensure the* square-integrability *of the $\mathcal{LSS}$ process. This can only be achieved under slightly stronger conditions, see Barndorff-Nielsen et al. (2013+, Section 2.3.1) for the details.*

Remark 2. *$\mathcal{LSS}$ processes are in general not semimartingales. If however this is a concern, then one can impose an additional smoothness condition on the kernel function g, which will lead to a subclass of $\mathcal{LSS}$ processes which are indeed semimartingales, see Barndorff-Nielsen et al. (2013+) and Barndorff-Nielsen et al. (2012, Proposition 5).*

In order to simplify the exposition, we will sometimes work with a different representation of an $\mathcal{LSS}$ process, which is obtained by splitting the stochastic integral from $-\infty$ to t as follows

$$Y(t) = X_0(t) + X(t), \qquad \text{for } t \ge 0,$$

where $X_0(t) = \int_{-\infty}^{0} g(t-s)\sigma(s-)dL(s)$ is $\mathcal{F}_0$-measurable for all $t \ge 0$, and $X(t) = \int_0^t g(t-s)\sigma(s-)dL(s)$. Two choices for X_0 are of particular interest to us: Either, we have the trivial choice that $X_0(t) = x_0$, for a constant $x_0 \in \mathbb{R}$ and for all $t \ge 0$, or $X_0(t) = \int_{-\infty}^{0} g(t-s)\sigma(s-)dL(s)$. To shorten the notation, we often write $dM(s) = \sigma(s-)dL(s)$.

Example 1. *Consider the Ornstein-Uhlenbeck process with parameter $\lambda > 0$: $Y(t) = \int_{-\infty}^{t} e^{-\lambda(t-s)}dM(s)$. Clearly, this process can be decomposed as above, where $X_0(t) = e^{-\lambda t}Y(0)$ and $X(t) = \int_0^t e^{-\lambda(t-s)}dM(s)$.*

In the following, we will always present the theoretical results for general $\mathcal{LSS}$ processes. In addition, we will show how the results simplify for two particularly relevant sub-classes of $\mathcal{LSS}$ processes: Let $i \in \{1, 2\}$. Then

$$Y^{(i)}(t) = X_0^{(i)}(t) + X^{(i)}(t), \quad t \ge 0, \qquad \text{where } X^{(i)}(t) = \int_0^t g(t-s)dM^{(i)}(s),$$

with

$$\begin{aligned} dM^{(1)}(s) &= \sigma(s-)\left(dB(s) + \gamma ds\right), \\ dM^{(2)}(s) &= dL(s). \end{aligned} \tag{4}$$

The first case, where it is implicitly assumed that $c = 1$, focuses on the case of a Brownian motion with drift modulated by a stochastic volatility process. The second case focuses on a Lévy process without stochastic volatility.

2.3 Geometric and arithmetic model

Energy spot prices typically exhibit seasonalities. We therefore incorporate seasonal behaviour and possibly a trend in our model in terms of a deterministic function $\Lambda : \mathbb{R}_+ \mapsto \mathbb{R}$. Throughout the paper, we denote by $S = (S(t))_{t\geq 0}$ the energy spot/day-ahead price. In the literature, we find both geometric and arithmetic models and hence we will study both cases in the following. We define the *geometric spot price model* by

$$S(t) = \Lambda(t)\exp(Y(t)), \qquad t \geq 0, \tag{5}$$

and the *arithmetic spot price model* by

$$S(t) = \Lambda(t) + Y(t), \qquad t \geq 0. \tag{6}$$

It should be noted that in a concrete application the seasonality and trend function Λ is typically not the same. However, we do not distinguish this in our notation to keep the exposition as simple as possible.

Also, if we work with the special cases of $M^{(1)}$ and $M^{(2)}$ as defined in (4) above, we denote the corresponding (geometric/ arithmetic) spot price by $S^{(1)}$ and $S^{(2)}$, respectively.

3 The risk premium and forward prices

3.1 Definition and basic properties of the risk premium

In the following we always consider a market with finite time horizon denoted by T^*, where $0 < T^* < \infty$. There are two types of (forward) risk premia that are usually considered: the ex-ante and the ex-post (or realised) risk premium. They are defined as follows.

Definition 1. *Let $S = (S(t))_{t\geq 0}$ denote the spot price and let $(F(t,T))_{0\leq t\leq T}$ denote the forward price with time of delivery $0 < T < T^*$. In the following, let $0 \leq t \leq T$.*

1. *The* ex-ante risk premium *at time t is defined as*

$$R(t,T) = F(t,T) - \mathbb{E}^P(S(T)|\mathcal{F}_t).$$

2. *The* ex-post risk premium *at time t is defined as*

$$r(t,T) = F(t,T) - S(T). \tag{7}$$

3. *The* basis *at time t is defined as*

$$B(t,T) = F(t,T) - S(t).$$

Note, that the ex-ante risk premium is the difference between the forward price and the (conditional) expected spot price. Since the latter is not observable, the ex-ante risk premium itself is not directly observable. In order to compute it, one needs to propose a model for the spot price and compute the corresponding conditional expectation. Therefore, the ex-ante risk premium will always be model-dependent.

The ex-post risk premium, is the difference between the forward price and the spot price at expiry of the forward contract. In contrast to the ex-ante risk premium it is directly observable from market data but only after the expiry of the forward contract.

The basis is the difference between the forward price and the current spot price and is therefore directly observable already prior to expiry of the forward contract.

These three quantities are related. The ex-post risk premium is equal to the ex-ante risk premium plus a prediction error of the spot as the following Lemma shows:

Lemma 1. *The ex-ante, ex-post risk premia and the basis are related as follows:*

$$r(t,T) = R(t,T) + \left(\mathbb{E}^P(S(T)|\mathcal{F}_t) - S(T)\right).$$

Also,

$$B(t,T) = R(t,T) + \mathbb{E}^P\left[(S(T) - S(t))\,|\mathcal{F}_t\right].$$

The proof is straightforward and hence omitted.

In the context of energy markets, we usually do not have delivery at a point in time T, but rather over a delivery period $[T_1, T_2]$, say, for $0 \leq T_1 \leq T_2 < T^*$. We call such contracts swap contracts and denote them by $F(t, T_1, T_2)$. Then we get the following expressions for the corresponding risk premia.

Definition 2. *In the following, let $0 \leq t \leq T_1$.*

1. *The* ex-ante swap risk premium *at time t is defined as*

$$R(t,T_1,T_2) = F(t,T_1,T_2) - \mathbb{E}^P\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(T)dT \,\middle|\, \mathcal{F}_t\right).$$

2. *The* ex-post swap risk premium *at time t is defined as*

$$r(t,T_1,T_2) = F(t,T_1,T_2) - \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(T)dT. \tag{8}$$

3. *The* swap basis *at time t is defined as*

$$B(t,T_1,T_2) = F(t,T_1,T_2) - S(t).$$

Remark 3. *Note that from a stochastic Fubini theorem, we obtain*

$$\mathbb{E}^P\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(T)dT \,\middle|\, \mathcal{F}_t\right) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \mathbb{E}^P\left(S(T)|\,\mathcal{F}_t\right)dT.$$

3.2 Pricing with a risk premium

When deriving forward or future prices in classical financial markets, such as the stock market, arguments from classical no-arbitrage theory and hedging arguments lead to forward prices. In particular, forward prices are directly linked to underlying spot prices since they are (conditional) expectations of the spot price under a risk-neutral pricing measure, i.e.

$$F(t,T) = \mathbb{E}^Q(S(T)|\mathcal{F}_t), \tag{9}$$

where Q is a risk-neutral probability measure, see e.g. Bjork (2004). A risk-neutral probability measure Q is a probability measure that is equivalent to the physical measure P, such that discounted prices of all tradable assets are martingales with respect to this measure. In the following we will always assume that there is a riskless asset available in the market, such that one unit (e.g. 1 Euro) invested in this riskless asset is worth e^{rt} units at time t. Here, $r > 0$ denotes the constant riskless (continuously compounded) interest rate. In such a situation prices of forward and futures coincide and hence we will not distinguish between the two.

In energy markets not all asset are directly tradable and hence the relationship between their forward and spot prices is far less clear, see Vehviläinen (2002) and (Benth, Šaltytė Benth & Koekebakker 2008, Chapter 4) for some further discussion. Consider the asset electricity, for example, we cannot hold a portfolio of electricity and hence one needs to be very cautious when applying hedging arguments to assets such as electricity spot prices.

Hence, other approaches for deriving forward prices need to be considered. One natural approach is to assume that forward prices are partially determined by the expected value of the underlying spot under the physical measure P and some risk premium that can be either positive or negative. Then one can define the time-t forward price for delivery at time T by

$$F(t,T) := \mathbb{E}_t^P(S(T)) + R(t,T),$$

for some stochastic process $(R(t,T))_{0\leq t\leq T}$. Since forwards are tradable assets their prices need to be consistent with no arbitrage assumptions. We will not model the risk-premium directly, but will later draw conclusion about its dynamics from empirical data. Note that for this approach it is crucial that we can evaluate the expression $\mathbb{E}_t^P(S(T))$ and we will do this in the following for the class of models that were introduced in Section 2. In our empirical study in Section 5 we will investigate, the explanatory power of expectations under the physical measure for the forward price.

3.3 The conditional expectation $\mathbb{E}_t^P(S(T))$

3.3.1 The geometric model

In the following, we provide analytical results for the conditional expectation of the spot price and the integrated spot price under the physical probability measure.

Proposition 1. *Let S be defined as in (5).*

1. *Let $0 \leq t \leq T$, then*

$$\mathbb{E}^P\left[S(T)|\mathcal{F}_t\right] = \Lambda(T)\exp\left(X_0(T) + \int_0^t g(T-s)\sigma(s-)dL(s)\right) \mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P(g(T-s)\sigma(s-))ds\right)\right).$$

2. *Let $0 \leq t \leq T_1 < T_2$, then*

$$\begin{aligned}&\mathbb{E}^P\left(\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S(T)dT\middle|\mathcal{F}_t\right)\\ &= \frac{1}{T_2-T_1}\int_{T_1}^{T_2}\Lambda(T)\exp\left(X_0(T) + \int_0^t g(T-s)\sigma(s-)dL(s)\right)\\ &\qquad \mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P(g(T-s)\sigma(s-))ds\right)\right)dT.\end{aligned} \tag{10}$$

From this, one can immediately derive the following results for two special cases:

Corollary 1. *Let S be defined as in (5). For the two special cases defined in (4) it holds that for $0 \leq t \leq T$*

$$\begin{aligned}\mathbb{E}^P\left[S^{(1)}(T)|\mathcal{F}_t\right] &= \Lambda(T)\exp\left(X_0(T) + \int_0^t g(T-s)\sigma(s-)dB(s)\right)\\ &\quad \mathbb{E}_t^P\left(\exp\left(\int_t^T \left\{\frac{1}{2}g^2(T-s)\sigma^2(s-) + \gamma g(T-s)\sigma(s-)\right\}ds\right)\right),\end{aligned} \tag{11}$$

$$\mathbb{E}^P\left[S^{(2)}(T)|\mathcal{F}_t\right] = \Lambda(T)\exp\left(X_0(T) + \int_0^t g(T-s)dL(s)\right)\exp\left(\int_t^T \psi_L^P(g(T-s))ds\right). \tag{12}$$

Let us focus on the Brownian case with stochastic volatility and $\gamma = 0$. Let

$$\sigma^2(t) = \int_{-\infty}^{t} e^{-\alpha(t-u)} dJ(u), \tag{13}$$

where $\alpha > 0$ and J is a subordinator. A straightforward computation leads to

$$\sigma^2(s) = e^{-\alpha(s-t)}\sigma^2(t) + \int_t^s e^{-\alpha(t-u)} dJ(u), \quad s \geq t. \tag{14}$$

With this special volatility structure, the expression in (11) can be expressed even more explicitly based on the following result.

Proposition 2. *Suppose σ^2 is given by* (13). *Then*

$$\begin{aligned}
&\mathbb{E}_t^P \left(\exp\left(\frac{1}{2} \int_t^T g^2(T-s)\sigma^2(s) ds \right) \right) \\
&= \exp\left(\frac{1}{2}\sigma^2(t) \int_t^T g^2(T-s) e^{-\alpha(s-t)} ds \right) \exp\left(\int_t^T \psi_J^P \left(\frac{1}{2} \int_u^T g^2(T-s) e^{-\alpha(t-u)} ds \right) du \right).
\end{aligned}$$

3.3.2 The arithmetic model

Proposition 3. *Let S be defined as in (6).*

1. *Let $0 \leq t \leq T$, then*

$$\begin{aligned}
\mathbb{E}^P [S(T)|\mathcal{F}_t] = \Lambda(T) + X_0(T) &+ \int_0^t g(T-s)\sigma(s-)dL(s) \\
&+ \mathbb{E}^P(L(1)) \int_t^T g(T-s)\mathbb{E}_t^P (\sigma(s-))\, ds.
\end{aligned} \tag{15}$$

2. *Let $0 \leq t \leq T_1 < T_2$, then*

$$\begin{aligned}
&\mathbb{E}^P \left(\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(T) dT \,\middle|\, \mathcal{F}_t \right) \\
&= \frac{1}{T_2 - T_1} \left[\left(\int_{T_1}^{T_2} (\Lambda(T) + X_0(T))\, dT + \int_0^t \left(\int_{T_1}^{T_2} g(T-s) dT \right) \sigma(s-) dL(s) \right.\right. \\
&\qquad \left.\left. + \mathbb{E}^P(L(1)) \int_t^T \left(\int_{T_1}^{T_2} g(T-s) dT \right) \mathbb{E}_t^P (\sigma(s-))\, ds \right) \right].
\end{aligned} \tag{16}$$

As soon as we specify a suitable model for the stochastic volatility process σ, we obtain an explicit formula for the conditional expectation of the stochastic volatility process and hence expression (15) can be simplified further. In the following we consider the two cases where the volatility is modelled as a non-Gaussian Ornstein-Uhlenbeck process or a Cox-Ingersoll-Ross process.

Assumption 1. *Under the measure P, suppose the stochastic volatility process σ satisfies either*

$$d\sigma(u) = -\alpha\sigma(u)du + dJ(\alpha u), \tag{17}$$

for $\alpha > 0$ and a (two-sided) Lévy subordinator J, which is independent of L, and $\beta := \mathbb{E}(J(1))$, or

$$d\sigma(u) = -\alpha(\sigma(u) - \beta)du + \upsilon\sqrt{\sigma(u)}dW(u), \tag{18}$$

for $\alpha, \beta, \upsilon > 0$ and where W is a (two-sided) standard Brownian motion independent of L.

Proposition 4. *Let S be defined as in (6) and suppose that Assumption 1 holds. Then,*

$$\mathbb{E}^P(L(1))\int_t^T g(T-s)\mathbb{E}_t^P\left(\sigma(s-)\right)ds = \sigma(t)h_1(t,T)+h_2(t,T),$$

where h_1 and h_2 are deterministic functions given by

$$h_1(t,T) = \mathbb{E}^P(L(1))\int_t^T g(T-s)\exp(-\alpha(s-t))ds,$$
$$h_2(t,T) = \mathbb{E}^P(L(1))\beta\int_t^T g(T-s)(1-\exp(-\alpha(s-t)))ds.$$

Remark 4. *Note that the stochastic volatility model given in* (13) *is the so-called Barndorff-Nielsen & Shephard (BNS) model, see Barndorff-Nielsen & Shephard (2001, 2002), whereas the model defined in* (17) *is a different stochastic volatility model motivated by the BNS model. The reason for considering these two different models is analytical tractability. In the geometric model, the squared volatility $\sigma^2(t)$ appears in the conditional expectation which motivates modelling this term directly, whereas, in the arithmetic model, we get the term $\sigma(t)$ in the conditional expectation and, hence, it is natural to model this term directly. The latter variant of the BNS model has been used in the literature before, see e.g. Veraart (2011). Moreover, Konaris (2002) has shown that while these two models are clearly not the same, in many empirical applications, either one can potentially provide a good fit when calibrated to financial data.*

4 The risk premium based on classical no-arbitrage theory

In the following we will use the classical no-arbitrage theory to study risk premia. For this we assume that the forward price is given by

$$F(t,T) = \mathbb{E}^Q(S(T)|\mathcal{F}_t), \tag{19}$$

where Q is a risk-neutral probability measure. Then, the (ex-ante) risk premium is given by

$$R(t,T) = \mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T)).$$

Note that in incomplete markets, the risk-neutral probability measure Q is not unique. This is usually the case in energy markets. Furthermore, note that if a asset is not directly tradable it does not have to be a martingale (after discounting) under the risk-neutral probability measure, see (Benth, Šaltytė Benth & Koekebakker 2008, Chapter 4) for further details.

Recall that we denote by P the physical probability measure. We will now introduce a new equivalent probability measure. For this, let Z denote a positive martingale with mean 1. Then we define for $0 \le t \le T^*$: $Q_t(A) := \mathbb{E}(\mathbb{I}_A Z_t) = \int_A Z_t d\mathbb{P}$, for all $A \in \mathcal{F}_t$. Then Q_t is a probability measure on $\mathcal{F}_t$ with $\frac{dQ_t}{dP} = Z_t$. In addition, we have $Q_{T^*}(A) = Q_t(A)$ for all $A \in \mathcal{F}_t$, since Z is a martingale.

To simplify the notation, we often drop the subscript and only write Q for an equivalent measure.

We briefly review the Bayes rule, which will be central in deriving our first results on the risk premium.

Proposition 5. [Bayes rule (Karatzas & Shreve (1991, p.193))] *Suppose $0 \le T^* < \infty$ is fixed and let Z denote a martingale. For any $\mathcal{F}_t$ measurable random variable ξ with $\mathbb{E}^{Q_{T^*}}|\xi| < \infty$ and for $0 \le s \le t \le T^*$, we have $\mathbb{E}_s^{Q_{T^*}}(\xi) = \frac{1}{Z(s)}\mathbb{E}_s(\xi Z(t))$, and, hence $\mathbb{E}_s(\xi) - \mathbb{E}_s^{Q_{T^*}}(\xi) = \mathbb{E}_s\left(\xi\left(1-\frac{Z(t)}{Z(s)}\right)\right)$.*

4.1 Risk premium in the geometric model

4.1.1 The general result

The spot price in the geometric model is given by

$$S(t) = \Lambda(t) \exp(Y(t)).$$

Proposition 6. *Assume that P and Q are equivalent probability measures. Let Z denote the Radon-Nikoým derivative of Q relative to P. Then the risk premium is given by*

$$R(t,T) = \mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T)) = \Lambda(T) \exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right) \xi(t;T), \quad (20)$$

where

$$\xi(t;T) = \mathbb{E}_t^P\left[\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\left(\frac{Z(T)}{Z(t)} - 1\right)\right]. \quad (21)$$

We find that the risk premium in the geometric model always has at least one stochastic component, no matter what kind of equivalent probability measure one uses for the change of measure, namely the factor $\exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right)$. This result is in line with the results reported by Benth & Sgarra (2012). Note also that the term consisting of the conditional expectation, i.e. $\xi(t;T)$, can also be stochastic. We will study such cases in the next Subsection.

Corollary 2. *Assume that P and Q are equivalent probability measures. Let Z denote the Radon-Nikoým derivative of Q relative to P. Assume that the kernel function is given by $g(T-s) = \exp(-\lambda(T-s))$, for $\lambda > 0$, corresponding to an Ornstein-Uhlenbeck process. Clearly, we can write $g(T-s) = g_1(T)g_2(s)$ for $g_1(T) = \exp(-\lambda T)$ and $g_2(s) = \exp(\lambda s)$. Then the risk premium is given by*

$$R(t,T) = \Lambda(T) \exp\left(\frac{g_1(T)}{g_1(t)} Y(t)\right) \xi(t;T) = \Lambda(T) \exp\left(e^{-\lambda(T-t)} Y(t)\right) \xi(t;T),$$

where $\xi(t;T)$ is defined as in (21).

4.1.2 Structure preserving change of measure

Proposition 7. *Assume that P and Q are related through a structure preserving change of measure. Then the risk premium is given by*

$$R(t,T) = \Lambda(T) \exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right) \xi(t;T), \quad (22)$$

where

$$\xi(t;T) = \left[\mathbb{E}_t^Q\left(\exp\left(\int_t^T \psi_L^Q\left(g(T-s)\sigma(s-)\right) ds\right)\right) - \mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P\left(g(T-s)\sigma(s-)\right) ds\right)\right)\right], \quad (23)$$

where ψ_L^P and ψ_L^Q denote the logarithm of the moment generating function of L, provided it exists, under P and Q, respectively.

4.1.3 The Gaussian case

The result given in Proposition 7 is already quite specific. Next we will show that when we work with a concrete model specification for the stochastic volatility process σ as well as with a distributional assumption for the driving Lévy process, we get even more explicit results.

Let us impose an additional assumption.

Assumption 2. *Suppose that σ^2 is given by*

$$\sigma^2(s) = \int_{-\infty}^{s} i(s,u)dJ(u), \qquad \textit{for } s \in \mathbb{R},$$

where J is a (two-sided) Lévy subordinator, which is independent of L, and i is a positive, measurable deterministic function which satisfies the integrability conditions for Volterra processes, see Barndorff-Nielsen et al. (2013+). Also, let ψ_J^P and ψ_J^Q denote the logarithm of the moment generating function (provided it exists) of J under P and Q, respectively.

Proposition 8. *Assume that P and Q are related through a structure preserving change of measure. Also, suppose that Assumption 2 holds and that $L = B$ is a two-sided standard Brownian motion. Then the risk premium is given by*

$$R(t,T) = \Lambda(T)\exp\left(\int_{-\infty}^{t} g(T-s)\sigma(s-)dL(s)\right)\xi(t;T), \tag{24}$$

where

$$\begin{aligned}\xi(t;T) = \exp&\left(\frac{1}{2}\int_{-\infty}^{t}\int_{t}^{T} g^2(T-s)i(s,u)dsdJ(u)\right)\\ &\left[\exp\left(\int_t^T \psi_J^Q\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right)\right.\\ &\quad\left.-\exp\left(\int_t^T \psi_J^P\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right)\right].\end{aligned} \tag{25}$$

Interestingly, we find that although the price process is driven by a Brownian motion, which in the case of a sufficiently smooth kernel function g, results in a continuous process, the risk premium could potentially exhibit jumps which are due to the driving subordinator of the stochastic volatility component.

4.2 Risk premium in the arithmetic model

4.2.1 The general result

The spot price in the arithmetic model is given by

$$S(t) = \Lambda(t) + Y(t).$$

Proposition 9. *Assume that P and Q are equivalent probability measures. Let Z denote the Radon-Nikoým derivative of Q relative to P. Then the risk premium is given by*

$$R(t,T) = \mathbb{E}_t^P\left[\int_t^T g(T-s)\sigma(s-)dL(s)\left(\frac{Z(T)}{Z(t)}-1\right)\right]. \tag{26}$$

The result differs significantly from the result we obtained in the case of a geometric model. While the risk premium always has a stochastic component in the geometric model, this is not necessarily the case in the arithmetic case. However, we will show in the following that the risk premium in the arithmetic model is for instance stochastic in the presence of a stochastic volatility process and if the change of measure is structure preserving.

4.2.2 Structure preserving change of measure

We obtain a more explicit result for the risk premium if we assume a certain structure of the change of measure. As in the geometric case, we assume in the following that the risk neutral probability measure Q and the physical measure P are linked by a structure preserving change of measure (preserving independence of σ and L and Lévy properties under Q).

Proposition 10. *Assume that P and Q are related through a structure preserving change of measure. Then the risk premium is given by*

$$R(t,T) = \mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T)) = \int_t^T g(T-s)\left(\mathbb{E}^Q(L_1)\mathbb{E}_t^Q(\sigma(s-)) - \mathbb{E}^P(L_1)\mathbb{E}_t^P(\sigma(s-))\right)ds.$$

4.2.3 Volatility modelled as a non-Gaussian OU process or CIR process

As soon as we specify a suitable model for the stochastic volatility process, we obtain an explicit formula for the risk premium.

Remark 5. *Note that under a structure preserving change of measure the two stochastic differential equations in Assumption 1 change to*

$$d\sigma(u) = -\alpha\sigma(u)du + dJ^Q(\alpha u),$$

where J^Q a (two-sided) Lévy subordinator under Q, see Nicolato & Venardos (2003, Theorem 3.2); also

$$d\sigma(u) = -\alpha^Q(\sigma(u) - \beta^Q)du + v\sqrt{\sigma(u)}dW^Q(u),$$

where W^Q is a (two-sided) standard Brownian motion under Q, and $a^Q, \beta^Q > 0$ denote parameters under Q, see Wong & Heyde (2006) for details.

Proposition 11. *Assume that P and Q are related through a structure preserving change of measure. In addition, suppose that Assumption 1 holds. Then the risk premium is given by*

$$R(t,T) = \sigma(t)h_1(t,T) + h_2(t,T),$$

where h_1 and h_2 are deterministic functions, which in the case of model (17) *are given by*

$$h_1(t,T) = \left(\mathbb{E}^Q(L(1)) - \mathbb{E}^P(L(1))\right)\int_t^T g(T-s)\exp(-\alpha(s-t))ds,$$
$$h_2(t,T) = \left[\mathbb{E}^Q(L(1))\mathbb{E}^Q(J(1)) - \mathbb{E}^P(L(1))\mathbb{E}^P(J(1))\right]\int_t^T g(T-s)(1-\exp(-\alpha(s-t)))ds,$$

and in the case of model (18) *are given by*

$$h_1(t,T) = \int_t^T g(T-s)\left\{\mathbb{E}^Q(L(1))\exp(-\alpha^Q(s-t)) - \mathbb{E}^P(L(1))\exp(-\alpha^P(s-t))\right\}ds,$$
$$\begin{aligned} h_2(t,T) = \int_t^T g(T-s)\{&\mathbb{E}^Q(L(1))\beta^Q(1-\exp(-\alpha^Q(s-t))) \\ &-\mathbb{E}^P(L(1))\beta^P(1-\exp(-\alpha^P(s-t)))\}ds. \end{aligned}$$

Note that the mean of L changes under P and Q. Hence, the difference of the two means, which enters as a multiplicative factor in h_1, will not be zero.

5 Empirical study

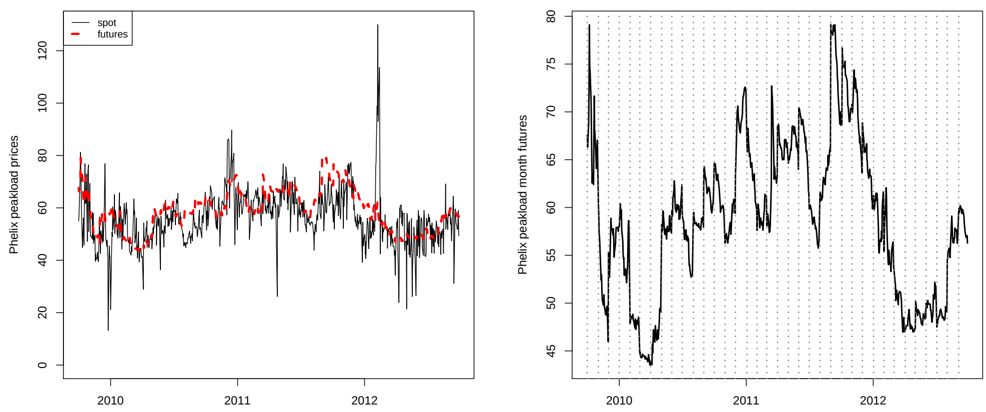
In general, forward prices and the corresponding risk premia are well understood if the underlying asset is storable. For electricity, the relationship between spot and forward contracts is far less clear. The purpose of the empirical study is to shed some light on this issue. For this, we fit a spot price model from the class of $\mathcal{LSS}$ processes to electricity spot price data and compute the corresponding conditional expectation of the future electricity price (under the physical measure) from this model. We investigate whether this expectation has any explanatory power for the forward price.

5.1 The data

In the following we consider data from the European Energy Exchange (EEX). For the electricity spot prices we consider the Phelix Day Peakload. Phelix stands for "Physical Electricity Index" and is a reference price for the European power wholesale market. The Phelix Day Peakload consists of the arithmetic mean of the 12 hourly prices between 08:00 am (CET) to 08:00 pm (CET) derived on the weekdays of the daily auction on the spot market of EPEX Spot for the German/Austrian market area, see European Energy Exchange AG (2012). All prices are specified in Euro per MWh.

In addition to the spot data we also consider Phelix Peakload Futures, i.e. these are future contracts for which the Phelix Day Peakload is the underlying. The delivery rate of these future contracts is 1MW electricity per hour. Usually, these future contracts are settled financially and not physically. There are several delivery periods available for Phelix Peakload Futures. We will concentrate on the Month Futures Peakload contracts. In general these are traded until the last day of the delivery period, but we will only look at the front-months data, i.e. we consider the prices of the future contracts during the months before the start of the delivery period. That means the prices for a futures contract observed in January correspond to a future contract with delivery in February etc..

The data were obtained from Datastream, code T43117 (Phelix Spot Peakload) and code EPMCS00 (Phelix Month Futures Peakload) and comprise the days 01/10/2009 until 28/9/2012 excluding weekends. Overall, we have 782 observations over three years which corresponds to approximately 261 observations per year.



(a) Spot and futures

(b) Futures

Figure 1: Daily prices of Phelix Day Peakload and Phelix Peakload Month Futures (front-month data) from 01/10/2010-30/09/2012 (excluding weekends).

Figure 1(a) shows the observed daily prices of the Phelix Day Peakload and Phelix Peakload

Month Futures from 01/10/2010-30/09/2012 (excluding weekends). We only consider the so-called front-month data for the prices of the month futures. Hence, even though we have plotted the future prices as one time series over 36 months, we are effectively observing the prices of 36 different futures contracts always during the last month before the beginning of their delivery period. To illustrate this in more detail we included Figure 1(b) which contains these front month future data with additional vertical line indicating the start and end date of a new futures contract. Hence we see that the majority of jumps observed in the data of the futures are due to different contracts that are considered and not due to *real* price jumps.

The spot data show several well-known stylized effects. We observe some mean-reversion in the data and occasional price spikes. Compared to earlier data from the EEX, however, the magnitude of the spikes is far less sever. We also see that in this data set there were not any negative prices in the peakload spot price. Hence, we can fit either a geometric or an arithmetic model to this data set.

5.2 The ex-post risk premium

Before we start with the model dependent analysis we look at the model independent risk premium - the ex-post risk premium. In (8) the ex-post risk premium for contracts with delivery period $[T_1, T_2]$ was defined. The corresponding empirical risk-premium for delivery during one month M for the peakload futures contract is then

$$r^{\text{empirical}}(t; M) = F(t; M) - \frac{1}{\#\{\text{weekdays in month } M\}} \sum_{T \in \{\text{weekday in month } M\}} S(T).$$

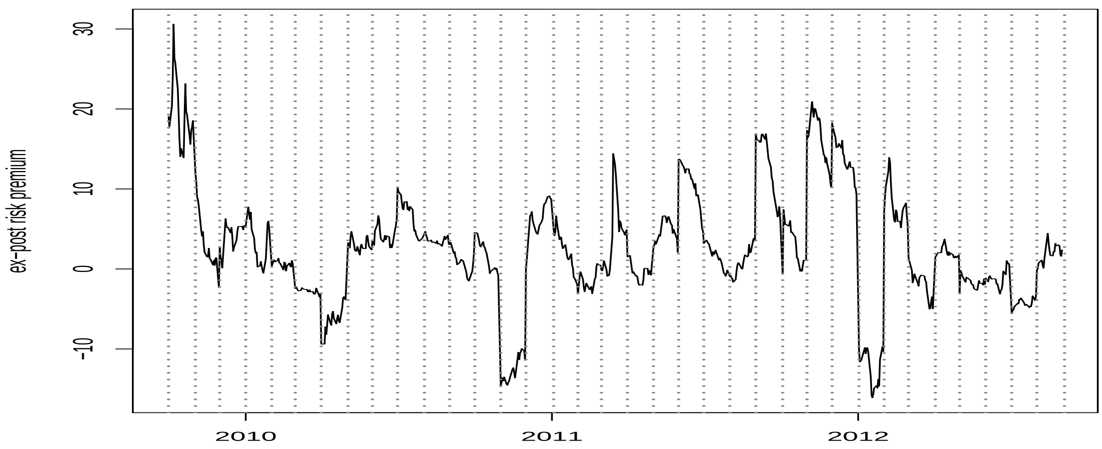
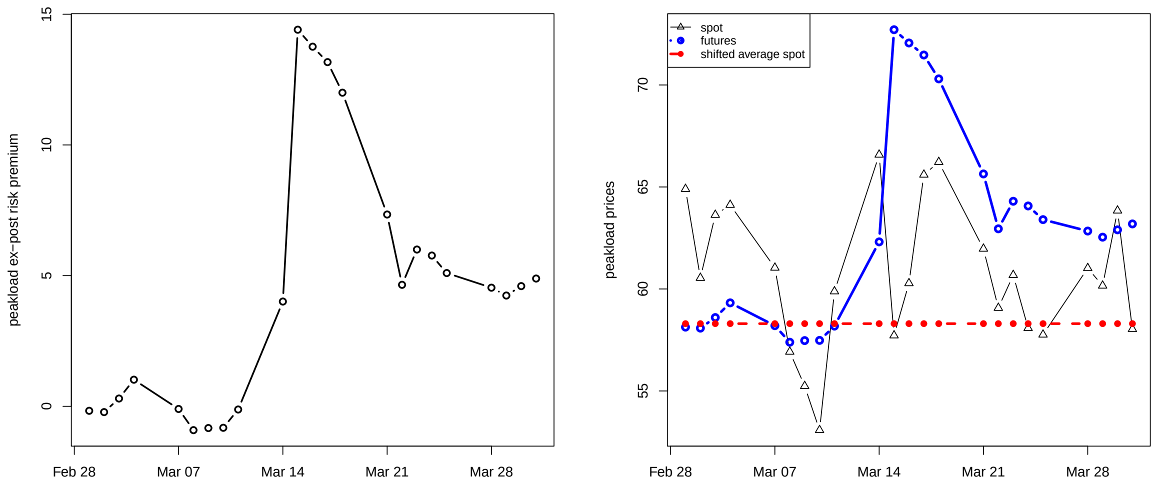


Figure 2: Empirical ex-post risk premium corresponding to the Phelix Peakload Month Futures.

Figure 2 shows this empirical ex-post risk premium for our data set. It seems to be stochastic in nature and no clear pattern seems to occur. Even the sign of the risk premium seems to vary. We will look at a particular month in more detail.

We consider data from March 2011 since we can observe some interesting effects here. In March 2011 the German government announced major changes to their energy policy due to the nuclear catastrophe that occurred in Japan as a consequence of a devastating earthquake on 11th March 2011 and a tsunami that hit the Fukushima Daiichi Nuclear Power Plant. On 14th March the German government announced a (temporary) moratorium on the lifespan extension of nuclear power plants. On 15th March, it was announced that several nuclear power plants in Germany would be temporarily shut, see Market Surveillance of European Energy Exchange AG (2011) for further discussion. These developments clearly affected the evolution of the future prices during this time. We observe a large upward jump of the future prices on 14th and 15th March. Not surprisingly, however, the spot price was unaffected by this, see Figure 3(b). This then lead to a large ex-post risk premium, see Figure 3(a). Since the shut-down of several power plants, decreased future supply of electricity for the German market, market participants were trying to hedge their risk by buying future contracts and they were prepared to pay high prices for this hedge. The futures prices increased significantly over this short period. This price jump can be observed in all futures traded on the Phelix on that day. In addition to this upward price jump, also the trading volume increased significantly, see Market Surveillance of European Energy Exchange AG (2011). This example clearly shows that it will always be difficult to link futures prices on electricity to current spot prices. Including an information premium in the modelling of forward prices along the lines of Benth & Meyer-Brandis (2009) could be promising in such a situation.



(a) Empirical ex-post risk premium

(b) Futures and spot

Figure 3: Fukushima effect in March 2011. Here shifted average spot in Figure 3(b) refers to the average spot price observed in April 2011.

5.3 Fitting the spot model

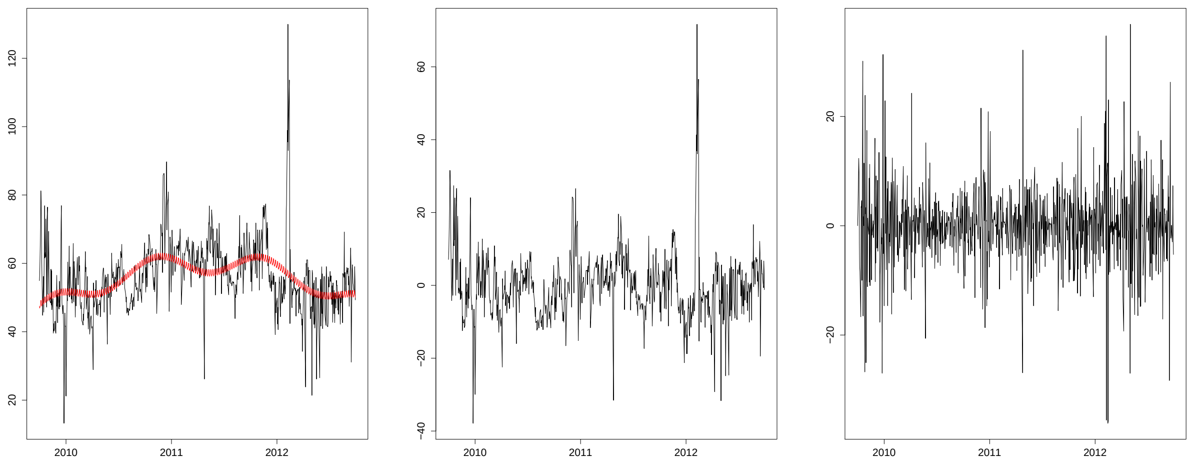
Next, we draw our attention to the ex-ante risk premium. In the following, we estimate a specific spot price model, which we will later use to derive the ex-ante risk premium for our data set. Here we will work with an arithmetic model based on an $\mathcal{LSS}$ process.

In a first step, we need to deseasonalise the data. Since, we want to make predictions from our model later on, we choose a parametric model for the seasonal function. We found that a linear combination of two sinusoidal functions, which take weekly and yearly seasonalities into account, as well as a polynomial of second order capture the overall trend and seasonality well. More precisely, we estimated the function

$$\Lambda(t) = c_0 + c_1 t + c_2 t^2 + c_3 \cos((\tau_1 + 2\pi t)/5) + c_4 \cos((\tau_2 + 2\pi t)/261),$$

using a robust estimation method, where the coefficients were estimated to be $c_0 = 45.45, c_1 = 0.07, c_2 = -8.57e^{-5}, c_3 = 1.08, c_4 = 3.02, \tau_1 = 35.45, \tau_2 = -5103.71$. We subtracted the estimated seasonal function from the data, and, in addition, de-meaned the resulting time series to obtain a times series which has zero mean.

The spot prices with the fitted seasonality function as well as the deseasonalised prices and price increments are depicted in Figure 4.

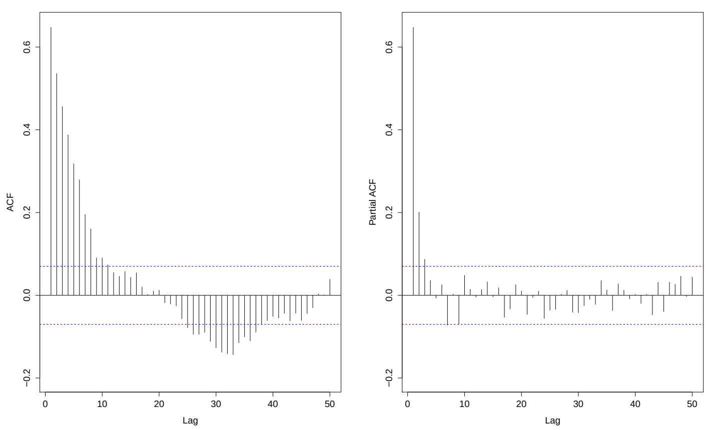


(a) (b) (c)

Figure 4: (a) Fitted parametric trend, (b) Detrended and deseasonalised spot prices, (c) Detrended and deseasonalised spot increments.

After having deseasonalised and detrended the data we run the augmented Dickey-Fuller Test and the Phillips-Perron Unit Root Test. Both tests rejected the null-hypothesis of a unit root against the alternative of stationarity at a significance level below 1%.

While the class of $\mathcal{LSS}$ processes is very general, in our empirical work, we are particularly interested in rather simple models which can be easily estimated. The exponential decay of the autocorrelation function (ACF) together with the clear cut-off of the partial autocorrelation function (PACF) of the prices after lag three, see Figure 5, suggests that we can work with linear models. In our case, an autoregressive process of order three seems to be a good starting point.



(a) (b)

Figure 5: (a) ACF and (b) PACF of detrended and deseasonalised spot prices.

The class of AR processes has a continuous-time analogue, called the continuous-time autoregressive processes of order $p \in \mathbb{N}$ (CAR(p)), see e.g. Benth, Šaltytė Benth & Koekebakker (2008, p. 281-285) for details. Such processes have also been studied in the context of electricity markets by e.g. Benth et al. (2012) and Garcia et al. (2011).

Such a model corresponds to choosing

$$g(t-s) = \mathbf{b}_1' \exp(\mathbf{A}(t-s))\mathbf{b}_p,$$

where $\mathbf{b}_1$ and $\mathbf{b}_p$ denote the first and pth unit vector in $\mathbb{R}^p$, respectively, and $\mathbf{A}$ is a $p \times p$ matrix given by

$$\mathbf{A} = \left[\begin{array}{cc} \mathbf{0} & \mathbf{I}_{p-1} \\ -\alpha_p & -\alpha_{p-1} \cdots - \alpha_1 \end{array} \right],$$

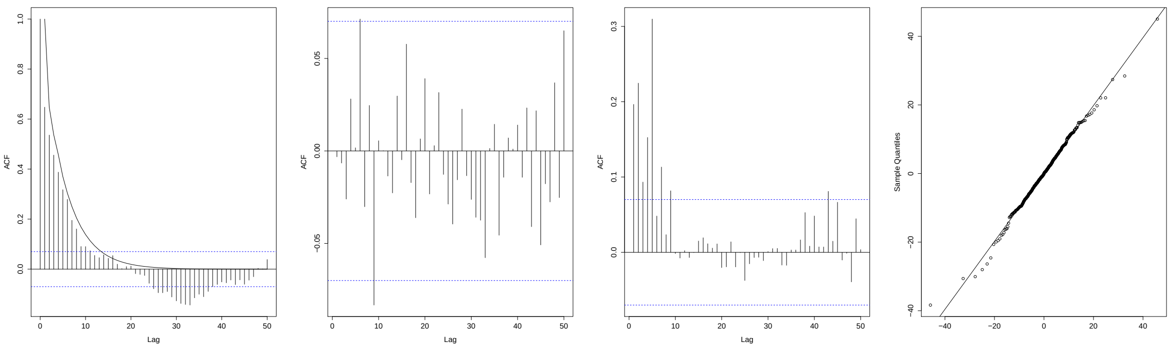
where $\mathbf{I}_{p-1}$ denotes the $(p-1) \times (p-1)$-identity matrix, $\mathbf{0}$ is a $p-1$-dimensional vector consisting of zeros, and $\alpha_1, \ldots, \alpha_p$ are constants which are typically assumed to be positive.

The Euler-discretisation of a CAR(3) process leads to an AR(3) process $(\Xi_n)_{n \in \mathbb{N}}$ of the form

$$\Xi(n) = b_1 \Xi(n-1) + b_2 \Xi(n-2) + b_3 \Xi(n-3) + \epsilon(n),$$

where $\epsilon(n)$ corresponds to the Euler-discretisation of the process M, i.e. for a step size of one we have $\epsilon(n) = \sigma_{n-1}(L(n) - L(n-1))$. Note that in the case when $L = B$ is a Brownian motion, then the ϵ are mixed-normal random variables.

Also, the parameters (b_1, b_2, b_3) are linked to the parameters of the continuous-time model by the relation $b_1 = 3 - \alpha_1$, $b_2 = 2\alpha_1 - \alpha_2 - 3$, $b_3 = \alpha_2 + 1 - \alpha_1 - \alpha_3$, see Benth, Šaltytė Benth & Koekebakker (2008, p. 285). In our empirical example, we get the following estimates $\alpha_1 = 2.50, \alpha_2 = 1.84, \alpha_3 = 0.25$. The diagnostic plots provided in Figure 6 show, that the autocorrelation structure of the AR(3) model is suitable for describing the empirical ACF and lead to uncorrelated residuals. Interestingly, we find signficant short-term autocorrelation in the squared residuals, indicating that we are not dealing with i.i.d. residuals. Fortunately, our general model structure allows for stochastic volatility, and hence we suggest to work with a modelling framework which accounts for clusters in the volatility.



(a) (b) (c) (d)

Figure 6: (a) Empirical and fitted ACF of the prices, (b) ACF of the AR(3) residuals, (c) ACF of the squared AR(3) residuals, (d) Quantile-Quantile plot comparing the empirical distribution of the AR(3) residuals with the symmetric Student-t distribution.

Motivated by previous related work by Barndorff-Nielsen et al. (2013+) and Veraart & Veraart (2013+), we fitted the class of generalised hyperbolic (GH) distributions to the residuals. Using the Akaike Information Criterion to select the model amongst eleven different cases of the GH distribution, including the symmetric and asymmetric versions of the general GH, the normal inverse Gaussian, variance gamma, generalised hyperbolic, Student-t and Gaussian distribution, we find that the symmetric Student-t distribution with parameters $\nu = 3.35, \mu = -0.02, \sigma = 7.17$ (see Breymann & Lüthi (2010) for details regarding the parametrisation) is the preferred choice. The symmetric Student-t distribution can be obtained as a mean-variance mixture of the Gaussian distribution with the inverse Gamma (IG) distribution. As such, we could think of the driving process $dM(t) = \sigma(t)dB(t)$ as a volatility modulation of a Brownian motion B, where σ^2 is an Ornstein-Uhlenbeck process with IG stationary distribution.

5.4 The ex-ante risk premium

Based on the CAR(3) model where we have a Brownian motion as the driving process, which is modulated by a stationary stochastic volatility process σ such that σ^2 has inverse Gamma stationary distribution, we compute the conditional expectation of the future averaged spot price over each month and compare it to the corresponding forward price, which gives us the ex-ante swap premium, see Definition 2.

Remark 6. *It should be noted here that we carry out the empirical study of the ex-ante risk premium as an* in-sample *study, meaning that we have used the entire data set to estimate the model parameters of the CAR(3) process as well as the seasonality and trend function and use this information to forecast the spot price (in-sample) using the conditional expectation operator as the optimal predictor in the mean-square sense.*

In our idealised situation of an in-sample study of the ex-ante risk premium, we try to answer the question whether the predicted spot price under the measure P, i.e. the market expectation, has any predictive power for the corresponding forward prices. Figure 7 depicts the predicted average spot prices (solid line) for each of the 36 months in the sample and the corresponding futures (dashed line). While in some months there seems to be a good overlap between the market expectation based on the spot model and the futures, there are quite a few months, where the two prices seem to behave almost independently from each other. In particular, when looking at Figure 7(d), we note that our predicted average spot prices mainly follow the seasonality and trend function and the additional information coming from our stochastic model when doing long-term forecasting is almost negligible.

This finding becomes also very apparent, when we look at the time series plot of ex-ante risk premium, see Figure 8. On most of the days, the ex-ante risk premium is positive, meaning that the futures prices were higher than our predicted average spot price. As already discussed before, there are events which impact only the futures prices, such as the temporary moratorium announced in March 2011, or only the spot prices, which result in significant ex-ante and ex-post risk premia. Note here that we observe the Fukushima-effect also in the ex-ante premium, since the spot price did not rise following the press conference and hence did not anticipate the sudden rise in the forward prices.

Let us briefly mention that a linear regression analysis where one analyses the relation of the futures to the (shifted) average spot prices leads to an R^2 of 32%, which increases to an R^2 of 47% when regressing the futures on our predicted average spot prices. So, we find that the predicted average spot prices do have some explanatory power for the futures, but clearly, there are other factors which need to be accounted for. It should be noted here, that if the ex-ante premium at time t was computed based on parameter estimates from the data up to time t only, rather than using the estimates from the full data set, we expect to obtain an even worse explanatory power of the expected average spot prices.

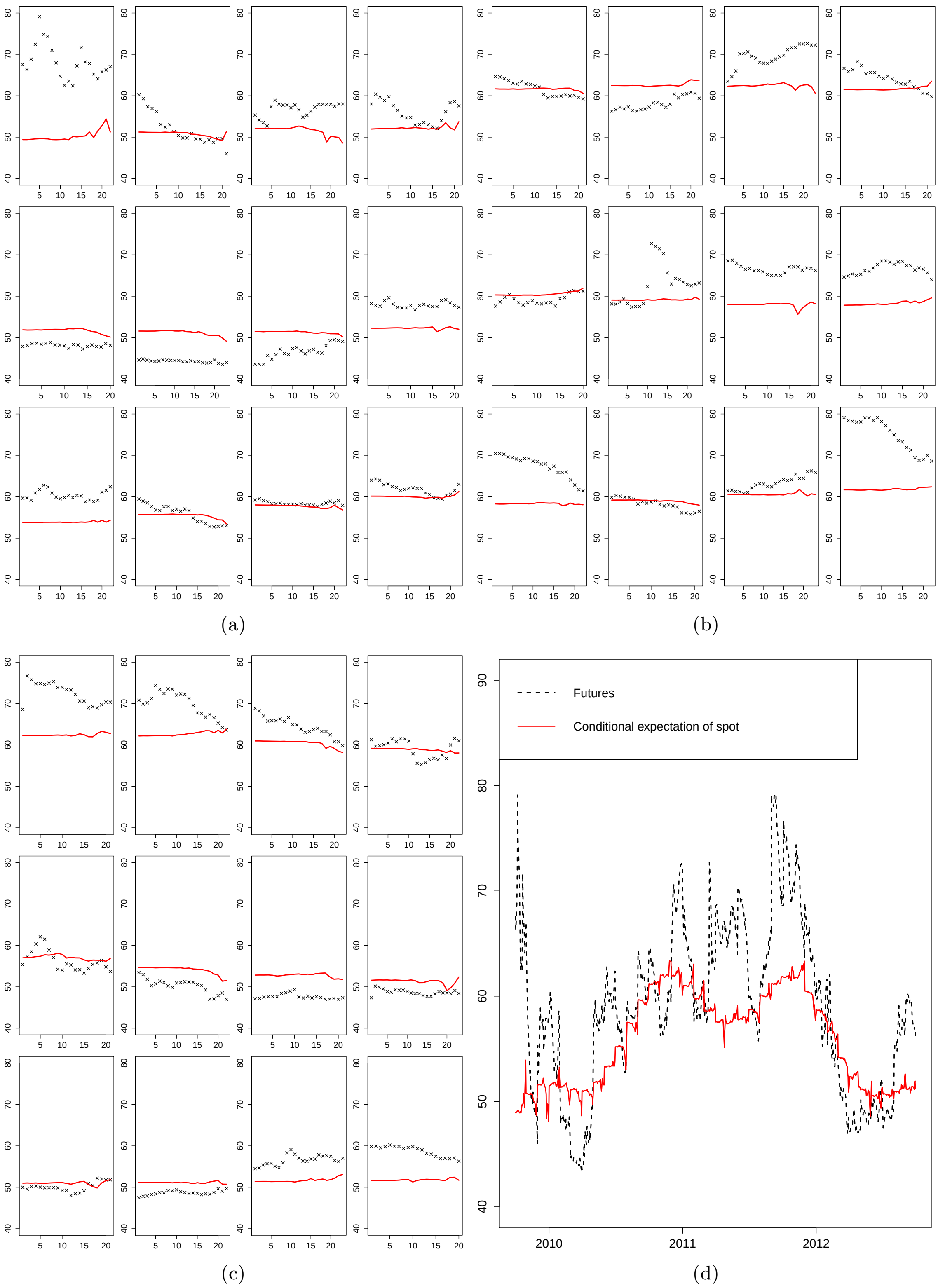


Figure 7: Predicted average spot prices (solid line) for each of the 36 months in the sample and corresponding futures (dashed line). (a) October 2009 - September 2010, (b) October 2010 - September 2011, (c) October 2011 - September 2012, (d) Entire sample from October 2009 - September 2012.

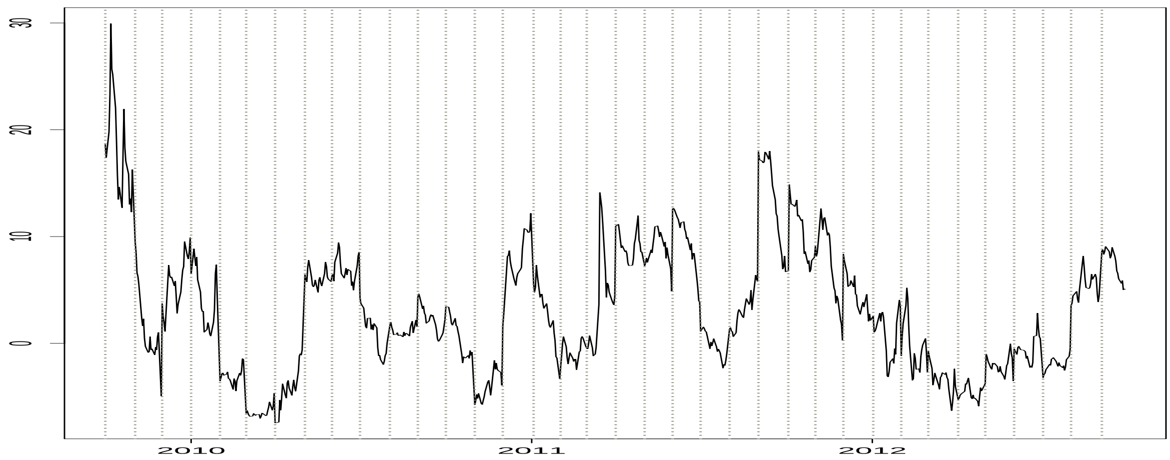


Figure 8: Ex-ante risk premium.

5.5 Fitting the forward model

Motivated by the findings in the previous Subsection, we follow a different strategy next to find a good model for the futures. Rather than calibrating the model parameters to the spot price data, one could attempt to model the forward price directly either by the geometric model, given by the right hand side (RHS) in formula (10), or by the arithmetic model, given on the RHS of equation (16).

We demonstrate this method using the arithmetic model. As a starting point, we focus on the special case when $X_0(T) \equiv 0$, $\mathbb{E}^P(L(1)) = 0$ and there is no stochastic volatility. Then formula (16) simplifies significantly, and it suggests that we can model a futures price at time t with delivery period $[T_1, T_2]$ directly by

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(T) dT + \int_0^t h(T_1, T_2, s) dL(s), \tag{27}$$

for $h(T_1, T_2, s) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} g(T - s) dT$. Figure 9 depicts the corresponding empirical findings. For the seasonality and trend function, we chose a step function mimicking the mean level of the futures each month. After we de-meaned the data, we did not find additional seasonal effects in our time series of the front-month futures. An Euler-discretisation of the stochastic integral in (27) leads to $h(T_1, T_2, n-1)(L(n-1) - L(n))$ (for a stepsize of one). Figure 9(d) shows that there is almost no significant autocorrelation in the increments, and no significant autocorrelation in the squared increments, see Figure 9(e). This finding together with the fact that the increments look quite stationary supports a model based on Lévy processes (or additive processes, if one does not impose the stationarity assumption on the increments) and $g(T - s) \equiv 1$. In that case, we have

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(T) dT + L(t).$$

Finally, we looked at the distribution of the increments and we found that the asymmetric generalised hyperbolic distribution seems to fit the increments of the futures rather well, see Figure 9(f).

Our empirical findings suggests that the formulae for conditional expectations of the average spot price, see (10) in the geometric case and (16) in the arithmetic case, can indeed describe the empirical behaviour of futures rather well when we calibrate them directly to the futures prices.

In future research one can explore how our models for the spot and future prices, which provide a good empirical fit, could be linked together through a suitable change of measure, which could

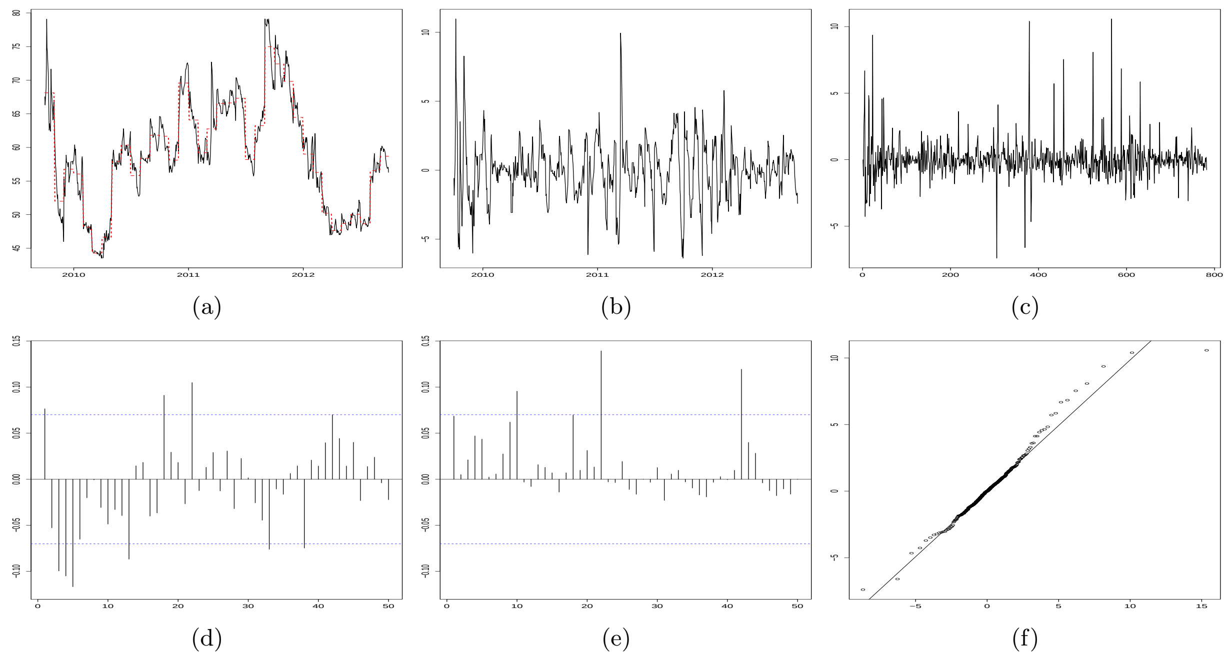


Figure 9: (a) Futures (solid) and a step function (dotted) indicating the mean level each month, (b) de-meaned futures, (c) increments of the de-meaned futures, (d) ACF of the increments, (e) ACF of the squared increments, (f) quantile-quantile plot comparing the empirical distribution of the increments with the asymmetric generalised hyperbolic distribution.

then be estimated from empirical data to provide some information on the so-called market price of risk.

6 Conclusion

This paper has investigated the role of risk premia in energy markets when energy spot prices are modelled by Lévy semistationary ($\mathcal{LSS}$) processes. For storable energy commodities, we have derived explicit theoretical results for the corresponding risk premia based on no-arbitrage arguments. We find that there is a structural difference between geometric and arithmetic models based on $\mathcal{LSS}$ processes. While there is always a stochastic component in the risk premium in a geometric model, see Proposition 6, this is not necessarily true in an arithmetic model, see Proposition 9. Moreover, we have shown that when working with a structure-preserving change of measure between the physical and the risk-neutral probability measure, the stochastic volatility is a key component, in particular in the arithmetic model, where it can solely introduce stochastic behaviour in the risk premium. For particular choices of the stochastic volatility process, e.g. for a square-root diffusion or a non-Gaussian Ornstein-Uhlenbeck process, we showed how the dynamics of the stochastic volatility process lead to stochastic dynamics of the risk premium.

Electricity is a particular case within the class of energy commodities since it is essentially non-storable. Hence it is not clear whether the theoretical results derived in this paper should also hold in the case of electricity. Therefore we have focused on this rather difficult special case in our empirical work and have investigated to what extent the conditional expectation of the average spot price under the physical probability measure has any explanatory power for the corresponding electricity futures. In our empirical study, we focused on the short-end of the forward curve only and used a daily time series of the front months of Phelix Peakload Month Futures from the EEX market from Oct 2009 - Sep 2012. Despite the fact that our $\mathcal{LSS}$-based model fits the spot prices very well, we find that the corresponding conditional expectation has only some explanatory power for the futures. There is still a significant amount of variability ($> 50\%$) in the futures which cannot be explained by our predicted average spot prices. Hence, either one models the risk premium directly to find a suitable model for electricity futures or one could model the futures directly. Here we followed the latter approach, where we postulated a model for electricity futures based on the theoretical results obtained for the conditional expectation of the average spot (based on an $\mathcal{LSS}$ process), see (10) and (16). We calibrated such a model directly to the futures rather than using parameter estimates based on the spot prices, which resulted in a good model fit.

While we have studied risk premia from a purely probabilistic and statistical perspective in this paper, future research could try to identify the key economic factors which drive the risk premia in energy markets.

A Proofs

Proof of Proposition 1.

1. For $0 \leq t \leq T$, we have

$$S(T) = \Lambda(T) \exp\left(X_0(T) + \int_0^t g(T-s)dM(s)\right) \exp\left(\int_t^T g(T-s)dM(s)\right).$$

Hence

$$\mathbb{E}_t^P(S(T)) = \Lambda(T) \exp\left(X_0(T) + \int_0^t g(T-s)dM(s)\right) \mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)dM(s)\right)\right).$$

Then

$$\begin{aligned}
&\mathbb{E}^P\left[S(T)|\mathcal{F}_t\right] \\
&= \Lambda(T) \exp\left(X_0(T) + \int_0^t g(T-s)\sigma(s-)dL(s)\right) \mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right) \\
&= \Lambda(T) \exp\left(X_0(T) + \int_0^t g(T-s)\sigma(s-)dL(s)\right) \mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P(g(T-s)\sigma(s-))ds\right)\right),
\end{aligned}$$

where we used the well-known result on the moment generating function of an integral of a deterministic function (after conditioning on the stochastic volatility) with respect to a Lévy process, see e.g. Cont & Tankov (2004, Lemma 15.1).

2. This follows directly from the stochastic Fubini Theorem.

□

Proof of Proposition 2. Using the identity (14), we obtain

$$\begin{aligned}
&\mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s)ds\right)\right) \\
&= \mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\left\{e^{-\alpha(s-t)}\sigma^2(t) + \int_t^s e^{-\alpha(t-u)}dJ(u)\right\}ds\right)\right) \\
&= \exp\left(\frac{1}{2}\sigma^2(t)\int_t^T g^2(T-s)e^{-\alpha(s-t)}ds\right) \mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\int_t^s e^{-\alpha(t-u)}dJ(u)ds\right)\right).
\end{aligned}$$

From the stochastic Fubini theorem, we get

$$\begin{aligned}
&\mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\int_t^s e^{-\alpha(t-u)}dJ(u)ds\right)\right) \\
&= \mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T \int_u^T g^2(T-s)e^{-\alpha(t-u)}dsdJ(u)\right)\right)
\end{aligned}$$

$$= \exp\left(\int_t^T \psi_J^P\left(\frac{1}{2}\int_u^T g^2(T-s)e^{-\alpha(t-u)}ds\right)du\right),$$

where ψ_J^P denotes the logarithm of the moment generating function of J. $\square$

Proof of Proposition 3. 1. For $0 \le t \le T$, we have

$$S(T) = \Lambda(T) + X_0(T) + \int_0^t g(T-s)dM(s) + \int_t^T g(T-s)dM(s).$$

Hence

$$\begin{aligned}\mathbb{E}_t^P(S(T)) &= \Lambda(T) + X_0(T) + \int_0^t g(T-s)dM(s) + \mathbb{E}_t^P\left(\int_t^T g(T-s)dM(s)\right)\\ &= \Lambda(T) + X_0(T) + \int_0^t g(T-s)dM(s) + \mathbb{E}^P(L_1)\int_t^T g(T-s)\mathbb{E}_t^P\left(\sigma(s-)\right)ds.\end{aligned}$$

2. Using the Fubini Theorem, we get

$$\begin{aligned}&\mathbb{E}^P\left(\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S(T)dT\middle|\mathcal{F}_t\right)\\ &= \frac{1}{T_2-T_1}\left[\left(\int_{T_1}^{T_2}\left(\Lambda(T)+X_0(T)\right)dT + \int_{T_1}^{T_2}\int_0^t g(T-s)\sigma(s-)dL(s)dT\right.\right.\\ &\quad\left.\left.+\mathbb{E}^P(L(1))\int_{T_1}^{T_2}\int_t^T g(T-s)\mathbb{E}_t^P\left(\sigma(s-)\right)ds\right)dT\right]\\ &= \frac{1}{T_2-T_1}\left[\left(\int_{T_1}^{T_2}\left(\Lambda(T)+X_0(T)\right)dT + \int_0^t\left(\int_{T_1}^{T_2} g(T-s)dT\right)\sigma(s-)dL(s)\right.\right.\\ &\quad\left.\left.+\mathbb{E}^P(L(1))\int_t^T\left(\int_{T_1}^{T_2} g(T-s)dT\right)\mathbb{E}_t^P\left(\sigma(s-)\right)ds\right)\right].\end{aligned}$$

$\square$

Proof of Proposition 4. For $s \ge t$, $\mathbb{E}_t^P(\sigma(s)) = \mathbb{E}_t^P(\sigma(s)-\sigma(t)) + \sigma(t)$. Then $\mathbb{E}_t^P(\sigma(s)-\sigma(t)) = \mathbb{E}_t^P\left(\int_t^s d\sigma(u)\right)$. If σ is given as in (17), then

$$\mathbb{E}_t^P(\sigma(s)-\sigma(t)) = \mathbb{E}_t^P\left(\int_t^s d\sigma(u)\right) = -\alpha\int_t^s \mathbb{E}_t^P\left(\sigma(u)\right)du + \alpha\mathbb{E}^P(J(1))(s-t).$$

Hence for $s \ge t$ and $Z(s) = E_t^P(\sigma(s))$ we obtain the SDE

$$dZ(s) = -\alpha Z(s)ds + \alpha\mathbb{E}^P(J(1))ds = \alpha(\mathbb{E}^P(J(1)) - Z(s))ds = \alpha(\beta - Z(s))ds,$$

with $Z(t) = \sigma(t)$. We obtain the same SDE also for model (18). Solving this SDE gives

$$\begin{aligned}Z(s) &= Z(t)\exp(-\alpha(s-t)) + \beta(1-\exp(-\alpha(s-t)))\\ &= \sigma(t)\exp(-\alpha(s-t)) + \beta(1-\exp(-\alpha(s-t))).\end{aligned}$$

Hence, for $s \ge t$,

$$E_t^P(\sigma(s)) = \sigma(t)\exp(-\alpha(s-t)) + \beta(1-\exp(-\alpha(s-t))).$$

Altogether we have

$$\mathbb{E}^P(L(1))\int_t^T g(T-s)\mathbb{E}_t^P\left(\sigma(s-)\right)ds = \sigma(t)h_1(t,T) + h_2(t,T),$$

where h_1 and h_2 are deterministic functions given by

$$h_1(t,T) = \mathbb{E}^P(L(1)) \int_t^T g(T-s)\exp(-\alpha(s-t))ds,$$
$$h_2(t,T) = \mathbb{E}^P(L(1))\beta \int_t^T g(T-s)(1-\exp(-\alpha(s-t)))ds.$$

□

Proof of Proposition 6. For $T \geq t$, we have

$$S(T) = \Lambda(T)\exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right)\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right).$$

Hence

$$\mathbb{E}_t^P(S(T)) = \Lambda(T)\exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right)\mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right),$$
$$\mathbb{E}_t^Q(S(T)) = \Lambda(T)\exp\left(\int_{-\infty}^t g(T-s)\sigma(s-)dL(s)\right)\mathbb{E}_t^Q\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right).$$

Then

$$\begin{aligned}\xi(t;T) &= \mathbb{E}_t^Q\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right) - \mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right)\\ &= \mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\left(\frac{Z(T)}{Z(t)}-1\right)\right),\end{aligned}$$

where we applied the Bayes Rule, see Proposition 5. □

Proof of Corollary 2. From Theorem 6, we have

$$\begin{aligned}R(t,T) &= \Lambda(T)\exp\left(\int_{-\infty}^t g_1(T)g_2(s)\sigma(s-)dL(s)\right)\xi(t;T)\\ &= \Lambda(T)\exp\left(\frac{g_1(T)}{g_1(t)}\int_{-\infty}^t g_1(t)g_2(s)\sigma(s-)dL(s)\right)\xi(t;T)\\ &= \Lambda(T)\exp\left(\frac{g_1(T)}{g_1(t)}Y(t)\right)\xi(t;T).\end{aligned}$$

□

Proof of Proposition 7. For $T \geq t$, we have

$$\mathbb{E}_t^P\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right) = \mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P\left(g(T-s)\sigma(s-)\right)ds\right)\right).$$

Similarly, under a structure preserving change of measure, we get

$$\mathbb{E}_t^Q\left(\exp\left(\int_t^T g(T-s)\sigma(s-)dL(s)\right)\right) = \mathbb{E}_t^Q\left(\exp\left(\int_t^T \psi_L^P\left(g(T-s)\sigma(s-)\right)ds\right)\right).$$

Then the result follows. □

Proof of Proposition 8. In the Gaussian case, when $L = W$ is a standard Brownian motion, we obtain

$$\mathbb{E}_t^P\left(\exp\left(\int_t^T \psi_L^P\left(g(T-s)\sigma(s-)\right)ds\right)\right) = \mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s-)ds\right)\right).$$

Now note that, for $s \geq t$,

$$\sigma^2(s) = \int_{-\infty}^t i(s,u)dJ(u) + \int_t^s i(s,u)dJ(u).$$

An application of the stochastic Fubini Theorem leads to

$$\begin{aligned}\int_t^T g^2(T-s)\sigma^2(s)ds &= \int_t^T g^2(T-s)\int_{-\infty}^t i(s,u)dJ(u)ds + \int_t^T g^2(T-s)\int_t^s i(s,u)dJ(u)ds \\ &= \int_{-\infty}^t \int_t^T g^2(T-s)i(s,u)dsdJ(u) + \int_t^T \int_u^T g^2(T-s)i(s,u)dsdJ(u).\end{aligned}$$

Hence

$$\begin{aligned}&\mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s-)ds\right)\right) \\ &= \exp\left(\frac{1}{2}\int_{-\infty}^t\int_t^T g^2(T-s)i(s,u)dsdJ(u)\right)\mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T\int_u^T g^2(T-s)i(s,u)dsdJ(u)\right)\right) \\ &= \exp\left(\frac{1}{2}\int_{-\infty}^t\int_t^T g^2(T-s)i(s,u)dsdJ(u)\right)\exp\left(\int_t^T \psi_J^P\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right).\end{aligned}$$

Similarly,

$$\begin{aligned}&\mathbb{E}_t^Q\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s-)ds\right)\right) \\ &= \exp\left(\frac{1}{2}\int_{-\infty}^t\int_t^T g^2(T-s)i(s,u)dsdJ(u)\right)\exp\left(\int_t^T \psi_J^Q\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right).\end{aligned}$$

Hence

$$\begin{aligned}\xi(t;T) &= \mathbb{E}_t^Q\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s-)ds\right)\right) - \mathbb{E}_t^P\left(\exp\left(\frac{1}{2}\int_t^T g^2(T-s)\sigma^2(s-)ds\right)\right) \\ &= \exp\left(\frac{1}{2}\int_{-\infty}^t\int_t^T g^2(T-s)i(s,u)dsdJ(u)\right) \\ &\quad \left[\exp\left(\int_t^T \psi_J^Q\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right)\right. \\ &\quad\quad \left. - \exp\left(\int_t^T \psi_J^P\left(\frac{1}{2}\int_u^T g^2(T-s)i(s,u)ds\right)du\right)\right].\end{aligned}$$

□

Proof of Proposition 9. A straightforward computation leads to

$$\mathbb{E}_t^P(S(T)) = \Lambda(T) + \int_{-\infty}^t g(T-s)\sigma(s-)dL(s) + \mathbb{E}_t^P\left[\int_t^T g(T-s)\sigma(s-)dL(s)\right],$$

and, likewise,

$$\mathbb{E}_t^Q(S(T)) = \Lambda(T) + \int_{-\infty}^{t} g(T-s)\sigma(s-)dL(s) + \mathbb{E}_t^Q\left[\int_t^T g(T-s)\sigma(s-)dL(s)\right].$$

Hence,

$$\begin{aligned}
R(t;T) &= \mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T)) \\
&= \mathbb{E}_t^Q\left[\int_t^T g(T-s)\sigma(s-)dL(s)\right] - \mathbb{E}_t^P\left[\int_t^T g(T-s)\sigma(s-)dL(s)\right] \\
&= \mathbb{E}_t^P\left[\int_t^T g(T-s)\sigma(s-)dL(s)\left(\frac{Z(T)}{Z(t)} - 1\right)\right].
\end{aligned}$$

□

Proof of Proposition 10. The result is an immediate consequence from Theorem 9. □

Proof of Proposition 11. For $s \geq t$, $\mathbb{E}_t^Q(\sigma(s)) = \mathbb{E}_t^Q(\sigma(s)-\sigma(t)) + \sigma(t)$. Then $\mathbb{E}_t^Q(\sigma(s)-\sigma(t)) = \mathbb{E}_t^Q\left(\int_t^s d\sigma(u)\right)$. If σ is given as in (17), then

$$\mathbb{E}_t^Q(\sigma(s)-\sigma(t)) = \mathbb{E}_t^Q\left(\int_t^s d\sigma(u)\right) = -\alpha\int_t^s \mathbb{E}_t^Q\left(\sigma(u)\right)du + \alpha\mathbb{E}^Q(J(1))(s-t).$$

Hence for $s \geq t$ and $Z(s) = E_t^Q(\sigma(s))$ we obtain the SDE

$$dZ(s) = -\alpha Z(s)ds + \alpha\mathbb{E}^Q(J(1))ds = \alpha(\mathbb{E}^Q(J(1)) - Z(s))ds,$$

with $Z(t) = \sigma(t)$. Solving this SDE gives

$$\begin{aligned}
Z(s) &= Z(t)\exp(-\alpha(s-t)) + \mathbb{E}^Q(J(1))(1-\exp(-\alpha(s-t))) \\
&= \sigma(t)\exp(-\alpha(s-t)) + \mathbb{E}^Q(J(1))(1-\exp(-\alpha(s-t))).
\end{aligned}$$

Therefore we obtain

$$\begin{aligned}
R(t,T) &= \mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T)) \\
&= \int_t^T g(T-s)\left(\mathbb{E}^Q(L(1))\mathbb{E}_t^Q(\sigma(s-)) - \mathbb{E}^P(L(1))\mathbb{E}_t^P(\sigma(s-))\right)ds \\
&= \sigma(t)h_1(t,T) + h_2(t,T),
\end{aligned}$$

where h_1 and h_2 are deterministic functions given by

$$\begin{aligned}
h_1(t,T) &= \left(\mathbb{E}^Q(L(1)) - \mathbb{E}^P(L(1))\right)\int_t^T g(T-s)\exp(-\alpha(s-t))ds, \\
h_2(t,T) &= \left[\mathbb{E}^Q(L(1))\mathbb{E}^Q(J(1)) - \mathbb{E}^P(L(1))\mathbb{E}^P(J(1))\right]\int_t^T g(T-s)(1-\exp(-\alpha(s-t)))ds.
\end{aligned}$$

Similarly, if σ is given as in (18), then

$$\mathbb{E}_t^P(\sigma(s)-\sigma(t)) = \mathbb{E}_t^P\left(\int_t^s d\sigma(u)\right) = -\alpha\int_t^s \mathbb{E}_t^P\left(\sigma(u)\right)du + \alpha\beta(s-t).$$

Hence, for $s \geq t$, we obtain as before

$$E_t^P(\sigma(s) = \sigma(t)\exp(-\alpha(s-t)) + \beta(1-\exp(-\alpha(s-t))).$$

Also,

$$E_t^Q(\sigma(s)) = \sigma(t)\exp(-\alpha^Q(s-t)) + \beta^Q(1-\exp(-\alpha^Q(s-t))).$$

Altogether we have

$$R(t,T) = \sigma(t)h_1(t,T) + h_2(t,T),$$

where h_1 and h_2 are deterministic functions given by

$$\begin{aligned}
h_1(t,T) &= \int_t^T g(T-s)\left\{\mathbb{E}^Q(L(1))\exp(-\alpha^Q(s-t)) - \mathbb{E}^P(L(1))\exp(-\alpha^P(s-t))\right\} ds,\\
h_2(t,T) &= \int_t^T g(T-s)\left\{\mathbb{E}^Q(L(1))\beta^Q(1-\exp(-\alpha^Q(s-t)))\right.\\
&\qquad\qquad\left. -\mathbb{E}^P(L(1))\beta^P(1-\exp(-\alpha^P(s-t)))\right\} ds.
\end{aligned}$$

□

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