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Abstract

In this paper we propose a new multivariate GARCH model with time-varying conditional correlation structure. The time-varying conditional correlations change smoothly between two extreme states of constant correlations according to a predetermined or exogenous transition variable. An LM–test is derived to test the constancy of correlations and LM– and Wald tests to test the hypothesis of partially constant correlations. Analytical expressions for the test statistics and the required derivatives are provided to make computations feasible. An empirical example based on daily return series of five frequently traded stocks in the S&P 500 stock index completes the paper.

JEL classification: C12; C32; C51; C52; G1

Key words: Multivariate GARCH; Constant conditional correlation; Dynamic conditional correlation; Return comovement; Variable correlation GARCH model; Volatility model evaluation

Running head: Modelling conditional correlations

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1 Introduction

"...During major market events, correlations change dramatically ..." Bookstaber (1997)

Financial decision makers usually deal with many financial assets simultaneously. Modelling individual time series separately is thus an insufficient method as it leaves out information about comovements and interactions between the instruments of interest. Investors are facing risks that affect the assets in their portfolio in various ways, which encourages them to find a position that allows hedging against losses. In practice, this is often done by trying to diversify across many stock markets. When optimizing a portfolio, correlations among, say, international stock returns are needed to determine gains from international portfolio diversification, and also the calculation of minimum variance hedge ratio needs updated correlations between assets in the hedge. Evidence that the correlations between national stock markets increase during financial crises but remain more or less unaffected during other times can be found for instance in King and Wadhwani (1990), Lin, Engle, and Ito (1994), de Santis and Gerard (1997), and Longin and Solnik (2001). As further examples, options depending on multiple underlying assets are very sensitive to correlations among those assets, and asset pricing models as well as some risk measures need measures of covariance between the assets in a portfolio. It is clear that there is a need for a flexible and accurate model that can incorporate the information of possible comovements between the assets.

Volatility in multivariate financial data has been typically modelled by applying the concept of conditional heteroskedasticity originally introduced by Engle (1982); see Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009) for recent reviews on multivariate GARCH models. In the multivariate context, one also has to model the conditional covariances, not only the conditional variances. One possibility is to model the former directly and another is to do that through conditional correlations. One of the most frequently used multivariate GARCH models is the Constant Conditional Correlation (CCC) GARCH model of Bollerslev

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(1990). In this model comovements between heteroskedastic time series are modelled by allowing each series to follow a separate GARCH process while restricting the conditional correlations between the GARCH processes to be constant. The estimation of parameters of the CCC–GARCH model is relatively simple and the model has thus become popular among practitioners.

In practice, the assumption of constant conditional correlations has often been found too restrictive. In order to mitigate this problem, Tse and Tsui (2002) and Engle (2002) defined dynamic conditional correlation GARCH models (VC–GARCH and DCC–GARCH, respectively) that impose GARCH-type dynamics on the conditional correlations as well as on the conditional variances. These models are flexible enough to capture many kinds of heteroskedastic behaviour in multivariate series. However, due to their structure, these models have limited capability to explain what drives correlations.

Pelletier (2006) proposed a model with a regime-switching correlation structure driven by an unobserved state variable that follows a K-dimensional first-order Markov chain. The regime-switching model asserts that the correlations remain constant in each regime and the change between the states is abrupt and governed by transition probabilities. There exists a parsimonious version of this model that contains two (extreme) correlation matrices, one of which has all correlations equal to zero. In order to compensate for that restriction, the number of states can be made large so that the correlations are described by as many linear combinations of the two extreme correlation matrices as there are states. This model is motivated by the empirical finding that the correlations among asset returns tend to increase during periods of distress whereas the series behave in a more independent manner in tranquil periods.

In this paper we introduce another way of modelling comovements in the returns. The Smooth Transition Conditional Correlation (STCC) GARCH model allows the conditional correlations to change smoothly from one state to another as a function of a transition variable. This continuous variable may be a combination of observable stochastic variables, or a function

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of lagged error terms.

The model has the appealing feature that it provides a framework in which constancy of the correlations, and thus the adequacy of the model, can be tested in a straightforward fashion. Implications of the STCC–GARCH correlation structure for the effects of news on the covariances can be considered through news impact surfaces, introduced by Kroner and Ng (1998). This concept can be easily adapted to the STCC–GARCH context.

A special case of the STCC–GARCH model was independently introduced by Berben and Jansen (2005). Their model is bivariate, and the variable controlling the transition between the extreme regimes is simply the time.

The paper is organized as follows. In Section 2 the model is introduced and the estimation of its parameters by maximum likelihood considered. Section 3 is devoted to tests of constant correlations. An application to illustrate the capabilities of the model can be found in Section 4. Section 5 concludes. Technical derivation of the test statistics in the paper and other relevant tests is available in 'Additional Material' (AM) at `http://econ.au.dk/research/research-centres/creates/research/research-papers/supplementary-downloads/`.

2 The Smooth Transition Conditional Correlation GARCH model

2.1 The general multivariate GARCH model

Consider the following stochastic N–dimensional vector process with the standard representation

$$\boldsymbol{y}_t = E\left[\boldsymbol{y}_t \mid \mathcal{F}_{t-1}\right] + \boldsymbol{\varepsilon}_t \qquad t = 1, 2, \dots, T \tag{1}$$

where $\mathcal{F}_{t-1}$ is the sigma-field generated by all the information until time $t-1$. Each of the univariate error processes has the specification

$$\varepsilon_{it} = h_{it}^{1/2} z_{it},$$

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where the errors z_{it} form a sequence of independent random variables with mean zero and variance one, for each $i = 1, \ldots, N$. The conditional variance h_{it} follows a univariate GJR–GARCH process of Glosten, Jagannathan, and Runkle (1993):

$$h_{it} = \alpha_{i0} + \sum_{j=1}^{q} \alpha_{ij}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{ij}h_{i,t-j} + \sum_{j=1}^{q} \delta_{ij}(\varepsilon_{i,t-j}^-)^2 \tag{2}$$

where $\varepsilon_{it}^- = \min(\varepsilon_{it}, 0)$, with the non-negativity and stationarity restrictions imposed on the parameters. Other GARCH models may also be considered as the GJR–GARCH model offers just one way of introducing asymmetry in the conditional variance.

The second conditional moment of the vector $\boldsymbol{z}_t = (z_{1t}, \ldots, z_{Nt})'$ is given by

$$E\left[\boldsymbol{z}_t\boldsymbol{z}_t' \mid \mathcal{F}_{t-1}\right] = \boldsymbol{P}_t. \tag{3}$$

Since z_{it} has unit variance for all i, $\boldsymbol{P}_t = [\rho_{ij,t}]_{i,j=1,\ldots,N}$ is the conditional correlation matrix for the ε_t. The correlations $\rho_{ij,t}$ are allowed to be time-varying in a manner that will be defined later on. In this paper it will, however, be assumed that $\boldsymbol{P}_t \in \mathcal{F}_{t-1}$, but extensions are possible.

To establish the connection to the approach often used in context of conditional correlation models, let us denote the conditional covariance matrix of ε_t as

$$E\left[\varepsilon_t\varepsilon_t' \mid \mathcal{F}_{t-1}\right] = \boldsymbol{H}_t = \boldsymbol{S}_t\boldsymbol{P}_t\boldsymbol{S}_t$$

where $\boldsymbol{P}_t$ is the conditional correlation matrix as in equation (3) and $\boldsymbol{S}_t = diag(h_{1t}^{1/2}, .., h_{Nt}^{1/2})$ with elements defined in (2). For the positive definiteness of $\boldsymbol{H}_t$ it is sufficient to require the correlation matrix $\boldsymbol{P}_t$ to be positive definite. The total number of parameters in (2) and (3) equals $N(p + 2q + 1) + N(N - 1)/2$.

The individual GARCH processes (2) contain a component that allows asymmetric volatility, which enables us to account for potential leverage effects. This is important in modelling stock returns. Another asymmetry that has recently attracted attention is the asymmetry of correlations. This may mean that the correlation between a pair of individual returns increases more

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after a negative shock to the system than it does when the shock is positive and of the same size; see the discussion in Cappiello, Engle, and Sheppard (2006). (In the latter case, the correlation need not change at all.) Thorp and Milunovich (2007) recently provided empirical evidence suggesting that accounting both for asymmetric volatility and asymmetric correlations in a multivariate GARCH model can improve the accuracy of volatility forecasts. It will be seen that our GARCH model is eminently suitable for modelling conditional correlations with an asymmetric response to shocks. In fact, our model allows much more flexible cases of asymmetry than the simple example given here.

2.2 Smooth transitions in conditional correlations

In order to complete the definition of the model we have to specify the time-varying structure of the conditional correlations. We propose the Smooth Transition Conditional Correlation GARCH (STCC–GARCH) model, in which the conditional correlations are assumed to change smoothly over time depending on a transition variable. In the simplest case there are two extreme states of nature with state-specific constant correlations among the variables. The correlation structure changes smoothly between the two extreme states of constant correlations as a function of the transition variable. More specifically, the conditional correlation matrix $\boldsymbol{P}_t$ is defined as follows:

$$\boldsymbol{P}_t = (1 - G_t)\, \boldsymbol{P}_{(1)} + G_t \boldsymbol{P}_{(2)} \tag{4}$$

where $\boldsymbol{P}_{(1)}$ and $\boldsymbol{P}_{(2)}$ are positive definite correlation matrices. Furthermore, G_t is a transition function whose values are bounded between 0 and 1. This structure ensures $\boldsymbol{P}_t$ to be positive definite with probability one, because it is a convex combination of two positive definite matrices.

The transition function is chosen to be the logistic function

$$G_t = \left(1 + e^{-\gamma(s_t - c)}\right)^{-1}, \quad \gamma > 0 \tag{5}$$

where s_t is the transition variable, c determines the location of the transition and $\gamma > 0$ the slope

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of the function, that is, the speed of transition. Increasing γ increases the speed of transition from 0 to 1 as a function of s_t, and the transition between the two extreme correlation states becomes abrupt as $\gamma \to \infty$. For simplicity, the parameters c and γ are assumed to be the same for all correlations. This assumption may sometimes turn out to be restrictive, but letting different parameters control the location and the speed of transition in correlations between different series may cause conceptual difficulties. This is because then $\boldsymbol{P}_{(1)}$ and $\boldsymbol{P}_{(2)}$ being positive definite does not imply the positive definiteness of every $\boldsymbol{P}_t$. The difference between this model and that of Pelletier (2006) is that in this one, the variable controlling the transitions is continuous and observable. In Pelletier's, the corresponding variable is latent and discrete.

The choice of transition variable s_t in (5) depends on the process to be modelled. An important feature of the STCC–GARCH model is that the investigator can choose s_t to fit the research problem. In some cases, economic theory proposals may determine the transition variable, in others the available empirical information may be used for this purpose. Possible choices include time as in Berben and Jansen (2005), or functions of past values of one or more of the return series. Yet another option would be to use an exogenous variable, which is a natural idea, for example when co-movements of individual stock returns are linked to the behaviour of the stock market itself. In that case, s_t could be a function of lagged values of the whole index. The Chicago Board Options Exchange index VIX constitutes an example. One could use the past conditional variance of index returns, which Lanne and Saikkonen (2005) suggested when they constructed a univariate smooth transition GARCH model.

Another point worth considering in this context is the number of parameters. It increases rapidly with the number of series in the model, although the current parameterization is still quite parsimonious. However, if one wishes to model the dynamic behaviour between the series, a very small number of parameters may not be enough. Simplifications that are too radical are likely to lead to models that do not capture the behaviour that is the objective of the

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modelling. It is possible to simplify the STCC–GARCH model to some extent such that it may still be useful in certain applications. As an example, one may restrict one of the two extreme correlation states to be that of complete independence ($\boldsymbol{P}_k = \boldsymbol{I}_N$, $k = 1$ or 2). This is a special case of a model where $\boldsymbol{P}_k = [\rho_{(k),ij}]$ such that $\rho_{(k),ij} = \rho$, $i \neq j$. Another possibility is to allow some of the conditional correlations to be time-varying, while the others remain constant over time. Examples of this will be discussed both in connection with testing and in the empirical application.

2.3   Estimation of the STCC–GARCH model

For the maximum likelihood estimation of parameters we assume joint conditional normality of the errors:

$$\boldsymbol{z}_t \mid \mathcal{F}_{t-1} \sim N\left(\boldsymbol{0}, \boldsymbol{P}_t\right).$$

Denoting by $\boldsymbol{\theta}$ the vector of all the parameters in the model, the log-likelihood for observation t is

$$l_t\left(\boldsymbol{\theta}\right) = -\frac{N}{2}\log\left(2\pi\right) - \frac{1}{2}\sum_{i=1}^{N}\log h_{it} - \frac{1}{2}\log|\boldsymbol{P}_t| - \frac{1}{2}\boldsymbol{z}_t'\boldsymbol{P}_t^{-1}\boldsymbol{z}_t, \quad t = 1,\ldots,T \qquad (6)$$

and maximizing $\sum_{t=1}^{T} l_t(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ yields the maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$.

Asymptotic properties of the maximum likelihood estimators in the present case remain to be established. Bollerslev and Wooldridge (1992) provided a proof of consistency and asymptotic normality of the quasi maximum likelihood estimators in the context of general dynamic multivariate models. The required conditions for their results to hold, however, have not yet been verified. Recently, Ling and McAleer (2003) considered a class of vector ARMA–GARCH models and established strict stationarity and ergodicity as well as consistency and asymptotic normality of the QMLE under reasonable moment conditions. Extending their results to the present situation would be interesting. The STCC–GARCH model is an inherently nonlinear model. At the moment, however, the asymptotic theory for nonlinear GARCH models only covers a class

of univariate GARCH models. Meitz and Saikkonen (2008) have recently proven consistency and asymptotic normality of maximum likelihood estimator for this class of models that includes the smooth transition GARCH model. Their results cannot, however, be immediately generalized to the STCC–GARCH model. Proving asymptotic normality for the maximum likelihood estimator for the parameters of the multivariate STCC–GARCH model would therefore be a tedious task and is beyond the scope of this paper.[1]

Nevertheless, since we find our model useful and the simulation results do not provide any evidence that asymptotic normality does not hold, we simply proceed by assuming that asymptotic normality holds, that is

$$\sqrt{T}\left(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0\right) \xrightarrow{d} N\left(\mathbf{0}, \mathfrak{I}^{-1}(\boldsymbol{\theta}_0)\right)$$

where $\boldsymbol{\theta}_0$ is the true parameter and $\mathfrak{I}(\boldsymbol{\theta}_0)$ the population information matrix evaluated at $\boldsymbol{\theta} = \boldsymbol{\theta}_0$.

Before estimating the STCC–GARCH model, however, it is necessary to first test the hypothesis that the conditional correlations are constant. The reason for this is that some of the parameters of the STCC–GARCH model are not identified if the true model has constant conditional correlations. Estimating an STCC–GARCH model without first testing the constancy hypothesis could thus lead to inconsistent parameter estimates. The same is true if one wishes to increase the number of transitions in an already estimated model. Testing constancy of conditional correlations will be discussed in the next section.

Maximization of the log-likelihood (6) with respect to all the parameters at once can be difficult due to numerical problems. For the DCC–GARCH model, Engle (2002) proposed a two-step estimation procedure based on the decomposition of the likelihood into a volatility and a correlation component. The univariate GARCH models are estimated first, independently of each other, and the correlations thereafter, conditionally on the GARCH parameter estimates.

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[1] It may be mentioned that at the moment asymptotic normality of maximum likelihood estimators of correlation parameters remains unproven for other CC–GARCH models such as the ones by Engle (2002), Tse and Tsui (2002), and Pelletier (2006) as well.

This implies that the dynamic behaviour of each return series, characterized by an individual GARCH process, is not linked to the time-varying correlation structure. Under this assumption, the parameter estimates of the DCC–GARCH model are consistent under reasonable regularity conditions; see Engle (2002) and Engle and Sheppard (2001) for discussion. For comparison, in the STCC–GARCH model the dynamic conditional correlations form a channel of interaction between the volatility processes. Parameter estimation accommodates this fact: the parameters are estimated simultaneously by conditional maximum likelihood.

Due to the large number of parameters in the model, estimation of the STCC–GARCH model is carried out iteratively by concentrating the likelihood. The parameters are divided into three sets: parameters in the GARCH equations, correlations, and parameters of the transition function, and the log-likelihood is maximized by sequential iteration over these sets. After the first completed iteration, the parameter estimates correspond to the estimates obtained by a two-step estimation procedure. Even if the parameter estimates do not change much during the sequence of iterations, the iterative method increases efficiency by yielding smaller standard errors than the two-step method. Furthermore, convergence is generally achieved after a reasonably small number of iterations.

It should be pointed out, however, that estimation requires care. The log-likelihood may have several local maxima, so estimation should be initiated from a set of different starting-values of the nonlinear parameters γ and c, and the obtained maxima compared before settling for final estimates.

3   Testing constancy of correlations

The modelling of time-varying conditional correlations must begin by testing the hypothesis of constant correlations, as previously discussed. Tse (2000), Bera and Kim (2002), and Engle and Sheppard (2001) already proposed tests for this purpose. We shall present an LM–type

test of constant conditional correlations against the STCC–GARCH alternative. A rejection of the null hypothesis supports the hypothesis of time-varying correlations or other types of misspecification but does not imply that the data have been generated from an STCC–GARCH model. For this reason our LM–type test can also be seen as a general misspecification test of the CCC–GARCH model. As we shall see, it is also useful in selecting an appropriate transition variable if it has not been chosen in advance.

In order to derive the test, consider an N–variate case where we wish to test the assumption of constant conditional correlations against conditional correlations that are time-varying with a simple transition of type (4) with a transition function defined by (5). For simplicity, assume that the conditional variance of each of the individual series follows a GJR–GARCH$(1, 1)$ process and let $\boldsymbol{\omega}_i = (\alpha_{i0}, \alpha_i, \beta_i, \delta_i)'$ be the vector of parameters for conditional variance h_{it}. Generalizing the test to other types of GARCH models for the individual series is straightforward. The STCC–GARCH model collapses into a constant correlation model under the null hypothesis of $\gamma = 0$ in (5). When this restriction holds, however, some of the parameters of the model are not identified. To circumvent this problem, we follow Luukkonen, Saikkonen, and Teräsvirta (1988) and consider an approximation of the alternative hypothesis. It is obtained by a first-order Taylor approximation around $\gamma = 0$ to the transition function G_t:

$$G_t = \left(1 + e^{-\gamma(s_t - c)}\right)^{-1} \cong 1/2 + (1/4)\left(s_t - c\right)\gamma. \tag{7}$$

Applying (7) to (4) linearizes the time-varying correlation matrix $\boldsymbol{P}_t$ as follows:

$$\boldsymbol{P}_t^* = \boldsymbol{P}_{(1)}^* + s_t \boldsymbol{P}_{(2)}^*$$

where

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\begin{aligned}
\boldsymbol{P}_{(1)}^* &= \frac{1}{2}\left(\boldsymbol{P}_{(1)} + \boldsymbol{P}_{(2)}\right) + \frac{1}{4}c\left(\boldsymbol{P}_{(1)} - \boldsymbol{P}_{(2)}\right)\gamma, \\
\boldsymbol{P}_{(2)}^* &= \frac{1}{4}\left(\boldsymbol{P}_{(2)} - \boldsymbol{P}_{(1)}\right)\gamma.
\end{aligned}
\tag{8}
$$

If $\gamma = 0$, then $\boldsymbol{P}^*_{(2)} = 0$ and the correlations are constant. Thus we construct an auxiliary null hypothesis $H_0^{\text{aux}} : \boldsymbol{\rho}^*_{(2)} = 0$ where $\boldsymbol{\rho}^*_{(2)} = vecl\boldsymbol{P}^*_{(2)}$.[2]

This null hypothesis can be tested by an LM–test. Note that when H_0 holds, there is no approximation error because then $G_t \equiv 1/2$, and the standard asymptotic theory remains valid. Let $\boldsymbol{\theta} = (\boldsymbol{\omega}'_1, \ldots, \boldsymbol{\omega}'_N, \boldsymbol{\rho}^{*\prime}_{(1)}, \boldsymbol{\rho}^{*\prime}_{(2)})'$, where $\boldsymbol{\rho}^*_{(j)} = vecl\boldsymbol{P}^*_{(j)}$, $j = 1, 2$, be the vector of all parameters of the model. Assuming asymptotic normality of the score, the LM–statistic

$$LM_{CCC} = T^{-1} \left(\sum_{t=1}^{T} \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\rho}^{*\prime}_{(2)}} \right) \left[\hat{\mathfrak{I}}_T(\hat{\boldsymbol{\theta}}) \right]^{-1}_{(\boldsymbol{\rho}^*_{(2)}, \boldsymbol{\rho}^*_{(2)})} \left(\sum_{t=1}^{T} \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\rho}^*_{(2)}} \right), \tag{9}$$

evaluated at the maximum likelihood estimators under the restriction $\boldsymbol{\rho}^*_{(2)} = 0$, has an asymptotic χ^2 distribution with $N(N-1)/2$ degrees of freedom. In expression (9), $[\hat{\mathfrak{I}}_T(\hat{\boldsymbol{\theta}})]^{-1}_{(\boldsymbol{\rho}^*_{(2)}, \boldsymbol{\rho}^*_{(2)})}$ is the south-east $\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ block of the inverse of $\hat{\mathfrak{I}}_T$, where $\hat{\mathfrak{I}}_T$ is a consistent estimator for the asymptotic information matrix. For derivation and details of the statistic, as well as the suggested consistent estimator for the asymptotic information matrix, see AM.

A straightforward extension is to test the constancy of conditional correlations against partially constant correlations:

$$H_0 : \quad \gamma = 0 \qquad H_1 : \quad \rho_{(1),ij} = \rho_{(2),ij} \quad \text{for } (i, j) \in N_1$$

where $N_1 \subset \{1, \ldots, N\} \times \{1, \ldots, N\}$. Under the null hypothesis we again face the identification problem which is solved by linearizing the transition function. For details, see AM.

These tests involve a particular transition variable. Thus a failure to reject the null of constant correlations is just an indication that there is no evidence of time-varying correlations, given this transition variable. Evidence of time-varying correlations may still be found in case of another indicator. In practice it may be useful to consider several alternatives unless restrictions implied by economic theory or other considerations make the choice unique. If there is uncertainty about which transition variable to use, the strongest rejection rule may be applied.

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[2]The *vecl* operator stacks the columns of the strict lower diagonal (obtained by excluding the diagonal elements) of the square argument matrix.

It should be mentioned that Berben and Jansen (2005) have in a bivariate context coincidentally proposed a test of the correlations being invariant with respect to calendar time. Their test is derived using an approach similar to ours, but they choose a different estimator for the information matrix in (9). Based on our simulation experiments, the estimator they use is substantially less efficient than ours in finite samples, especially when the number of series in the model is large.

Finite-sample properties of the test of constancy of correlations have been studied by simulation in a bivariate case and found satisfactory, see AM for details. Selected results of power simulations can be found in AM as well.

4   Application to daily stock returns

The data set of our application consists of daily returns of five S&P 500 composite stocks traded at the New York Stock Exchange and the S&P 500 index itself. The main criterion for choosing the stocks is that they are frequently traded and that the trades are often large. The stocks are Ford, General Motors, Hewlett-Packard, IBM, and Lockheed Martin, and the observation period begins May 25, 1984 and ends November 23, 2009. As usual, closing prices are transformed into returns by taking natural logarithms, differencing, and multiplying by 100, which gives a total of 6432 observations for each of the series. To avoid problems in the estimation of the univariate GARCH equations, the observations in the series are truncated such that extremely large positive (negative) returns are set to $+ (-)$ $4\times$ standard deviation of the series. This is preferred to removing them altogether, because we do not want to remove the information in comovements related to very large negative returns.[3] Descriptive statistics of the original and truncated return series, including skewness and kurtosis, can be found in AM.

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[3]The estimation of the correlation parameters is not affected by the truncation.

4.1 Choosing the transition variable

We consider the possibility that common shocks affect conditional correlations between daily returns. The transition variable in the transition function is a function of lagged returns of the S&P 500 index. As discussed in Section 2.2, several choices are available. A question frequently investigated, see for instance Andersen, Bollerslev, Diebold, and Labys (2001) and Chesnay and Jondeau (2001), is whether comovements in the returns are stronger during general market turbulence than they are during more tranquil times. In that case, a lagged squared or absolute daily return, or a sum of lags of either ones, would be an obvious choice. Following Lanne and Saikkonen (2005), one could also consider the conditional variance of the S&P 500 returns. A model-based estimate of this quantity may be obtained by specifying and estimating an adequate GARCH model for the S&P 500 return series. Yet another possibility would be VIX, the volatility index based on implied volatilities.

We restrict our attention to different functions of lagged squared and absolute returns of the index. Specifically, we consider up to five-day lags of both the lagged squared and lagged absolute returns, as well as equally weighted averages of both the lagged squared and lagged absolute returns over periods ranging from two days, and one to four weeks, and finally weighted averages of the same quantities with exponentially decaying weights with the discount ratios 0.9, 0.7, and 0.3. The constant conditional correlations hypothesis is then tested using each of the 26 transition variables in the complete five-dimensional model as well as in every one of its submodels. The strongest overall rejection occurs (these results are not reported here) when the transition variable is the equally weighted ten-day average of lagged absolute returns. The graph of this transition variable is presented in the mid-panel of Figure 1.

Table 1 contains the p–values of the constancy test based on this transition variable for all bivariate models and the full five-variable CCC–GARCH model. The test rejects the null hypothesis of constant correlations at significance level 0.01 for all models except the Ford-

General Motors one. The high p–value for this model (0.86) indicates that the correlation dynamics between these two automotive companies is not directly related to the level of volatility in the markets. The rejections grow stronger as the dimension of the model increases, the only exception being when moving from four-variate models to the full five-variate one.

If the interest lies in finding out whether the direction of the price movement as well as its strength affect conditional correlations, a function of lagged returns that preserves the sign of the returns is an appropriate transition variable. In order to accommodate this possibility, we consider the following three sets of lagged returns: $\{r_{t-j} : j = 1, \ldots, 5\}$ and $\{1/j \sum_{i=1}^{j} r_{t-i} : j = 2, 5, 10, 15, 20\}$; note that $\sum_{i=1}^{j} r_{t-i} = 100(p_{t-1} - p_{t-(j+1)})$ where p_t is the log-price of the stock, and finally weighted averages of the lagged returns with exponentially decaying weights with the discount ratios 0.9, 0.7, and 0.3. The constant conditional correlations hypothesis is tested using these thirteen choices of transition variables. The strongest rejection most frequently occurs (results not reported here) when the transition variable is the lagged average four-week return $100 \times 1/20(p_{t-1} - p_{t-20})$. In this case, all CCC–GARCH models are clearly rejected at the 0.01 level and the strength of rejection again grows with the dimension of N (see Table 1). The transition variable is depicted in the bottom panel of Figure 1.

4.2 Effects of market turbulence on conditional correlations

We shall first investigate the case in which the conditional correlations are assumed to fluctuate as a result of time-varying market distress which is measured by the lagged ten-day averages of absolute S&P 500 returns. Three remarks are in order. First, we only consider first-order STCC–GARCH models. In order to account for the leverage effect present in stock returns, the univariate volatilities are modelled using the GJR–GARCH model. Second, the STCC–GARCH model is only fitted to data for which the constant correlations hypothesis is rejected at the 5% level. Third, estimation of parameters is carried out either by the iterative full maximum

likelihood (STCC–GARCH) or the two-step method (DCC–GARCH). The standard errors of the parameter estimates of the STCC–GARCH model are calculated using numerical second derivatives for all estimates with an occasional exception for the estimate of the velocity of transition parameter γ, for details see AM.

When the transition variable is a function of lagged absolute S&P 500 returns, positive and negative returns of the same size have the same effect on the correlations, and the absolute magnitude of the returns carries all the information of possible comovements in the returns. Small ten-day averages of absolute returns are associated with the conditional correlation matrix $\boldsymbol{P}_{(1)}$, whereas the large ones are related to $\boldsymbol{P}_{(2)}$. We begin by considering the bivariate models. Results from the five-dimensional model are discussed thereafter. The estimation results are presented in Table 2. The estimated correlations are plotted in Figure 3 for the recent high volatility period 2008–09. The close-up graph of the transition variable during this period appear in Figure 2.

In all bivariate models, with the exception of the F–GM one which failed to reject the constancy of correlations hypothesis at 1% level, the correlations increase with an increased level of volatility, and most of them quite dramatically. The strength of the correlation between Ford and GM shows a slight decrease when the market volatility peaks. The transition between the two extreme states of correlations is a step-function in the model for F and GM, and in the models that are a combination HPQ, IBM, or LMT whereas all other models exhibit genuine smooth transition between the states. However, for most of the models, the transitions are quite rapid. For the models for IBM and either of the two automotive companies F and GM, the correlations spend most of the time between the states, and the slower the velocity of the transition, the less likely the correlations are to reach the extreme states. The correlation between IBM and LMT is fluctuating around 0.2 about 83% of the time, and the correlations only decrease when the markets are very calm. Quite the opposite happens in all other models

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(with the exception of the F–GM one). In those cases, reasonably turbulent market conditions are required to shift the correlation levels from low to high (the estimated location for GM–IBM is at 70% quantile and for the rest the locations range between 86% and 96% quantiles of the observed two-week returns).

Before combining the assets into a five-dimensional model, we note that in the STCC–GARCH model the location of the transition is governed by one parameter common to all assets regardless of the dimension of the model. Naturally, as the dimension increases, the point estimate of the location parameter will have to accommodate the different needs of each of the dynamics between any two asset returns. Hence the resulting estimate can be seen as an average of the locations from the bivariate relations, weighted by the relative strength of their dynamics. One could argue that having a single location parameter for the correlation dynamics in a high-dimensional model is too coarse a simplification. However, the estimated location reveals the point around which the data provides strongest evidence of changing correlation dynamics, given the specific transition variable. Finer details of the dynamics can be obtained by studying the submodels.

The estimation results from the five-variate model are presented in Table 3. In the estimated bivariate models, the estimated locations are scattered between the 17% and 96% quantiles of the observed two-week returns, with most of them at the high end. With the above note in mind, it is not surprising that the estimated location in the five-variate model is at the 72% quantile. This has further effects on the speed of the transition whose estimate from the five-variate model is slightly higher than the slowest transitions in the bivariate models, but dramatically lower than the estimates from the majority of the bivariate models. Now that the transition has changed location and speed, the estimates of the extreme levels of correlations adjust accordingly. An interesting finding is that the single location seems to replicate the bivariate dynamics in the five-dimensional model for all but one combination: the F–IBM model finds extreme levels of

correlations that are closer together than the corresponding correlation levels from the bivariate model. Furthermore, the direction of change in the correlations in the model for F and GM is opposite to all the other ones in the five-dimensional, and also in every bivariate model, i.e. when the fluctuations are small, the correlation between F and GM is higher than during turbulent times.

4.3 Effects of shock asymmetry on conditional correlations

As already mentioned, asymmetric correlations have recently attracted attention. We use a market indicator to represent price changes and study time-variation in correlations by again assuming the transition variable to be a function of the S&P 500 index. Since the interest lies in the direction and size of the price movements, we select the lagged average four-week return to be the transition variable as discussed in Section 4.1. The results of the constancy tests appear in Table 1. The tests of constant correlations reject constancy for each model. An STCC–GARCH$(1, 1)$ model is thus estimated for all combinations. The S&P 500 twenty-day average returns below the estimated location imply a correlation state approaching that of $\boldsymbol{P}_{(1)}$, whereas the returns greater than the estimated location result in correlations closer to the other extreme state, $\boldsymbol{P}_{(2)}$. The estimation results for the bivariate STCC–GARCH models are presented in Table 4. The estimated correlations are depicted in Figure 4 for the period 2008–09. The transition variable during this period is shown in Figure 2.

The estimation results support the theory, see e.g. Hong and Stein (2003), that pessimistic market conditions lead to higher correlations than optimistic views do. However, the magnitude and sign of the four-week return required to alter the correlations varies across the models. For the models F–IBM, GM–IBM, HPQ–IBM, and GM–HPQ, large negative four-week return on the S&P 500 index implies high correlations, whereas for the remaining models (F–HPQ being an exception) correlations decrease only after large positive index returns. The model for F

and HPQ has a weak positive correlation that increases slightly after relatively large positive four-week index return (the estimated location is at the 70% quantile of the observed four-week returns).

The transitions in the correlations are quite smooth for most of the models, whereas the correlations seem to show genuine or close to regime switching behaviour for four of the models. Comparison of the estimated correlations with the ones from the previous subsection shows that they behave differently when the transition variable differentiates the direction of price movements from general market turbulence. This is to be expected as times of distress are characterized by high volatility which results from large, positive or negative, shocks. For instance, the correlation in the F–IBM model was close to zero most of the time and peaked at 0.7 with very high volatility in the previous subsection. However, when the transition variable allows the correlations to depend on the direction of change, they increase from a level of 0.3 to a somewhat higher one when the price change is sufficiently negative and large in absolute value.

When combining the assets into a five-variate model, the estimation results provide somewhat surprising outcomes. These results are presented in Table 5. While the estimates for the location of the transition in the bivariate models ranged from the 1% to 74% quantile of the twenty-day return distribution, in the five-variate model the estimate for the location parameter is close to the 75% quantile. While most of the models have no significant differences between the bivariate and five-variate correlation estimates, some differences emerge. In fact, the correlation between GM and IBM is now deemed constant. The direction of change in correlations, however, remains the same as in the bivariate models, except for the model for F and HPQ.

In theory, as a solution to the 'multilocation problem' one could generalize the STCC–GARCH model such that it would allow different slope and location parameters for each pair of correlations. However, as already mentioned, such an extension entails the statistical problem

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of ensuring positive definiteness of the correlation matrix at each point of time.

4.4 Comparison

We conclude this section with a brief informal comparison of the time-varying correlations implied by the STCC– and DCC–GARCH models. The DCC–GARCH model is chosen because it is the most frequently applied conditional correlation GARCH model. To keep the comparison transparent, we only consider bivariate models, and focus on the similarities and differences in the correlation dynamics implied by the two modelling approaches. These aspects can clearly be seen by looking at the specific time periods.

A fundamental difference between the two models is that in the DCC–GARCH model, the correlation dynamics only uses the past returns of the series to be modelled. On the other hand, the STCC–GARCH models make use of the two transition variables discussed in the previous subsections. One can therefore expect the dynamics implied by the two models to be somewhat different. The bivariate DCC–GARCH$(1, 1)$ models are estimated using the two-step estimation method of Engle (2002). The estimated GARCH equations in the DCC–GARCH model differ slightly from the ones in the STCC–GARCH models due to the two-step procedure, and the correlation dynamics are very persistent (for conciseness we do not present the estimation results). The estimated correlations from the bivariate DCC–GARCH models are shown in Figure 5 for the period 2008–09.

By comparing Figures 3, 4, and 5, it is seen that the correlations from the two families of models are quite different. The STCC–GARCH model finds evidence for the correlations to increase quite rapidly. The DCC–GARCH model suggests that the correlations respond very slowly to the turbulence: the correlations merely show an upward sloping trend. Interestingly, the correlations from a few of the DCC–GARCH models show a sudden decrease right before the crisis hit. The STCC–GARCH counterparts tend to indicate an increase in the correlations

at the same time frame.

The bivariate GM–IBM model deserves a closer look as it offers an example of differences that have to do with the fundamental structure of the models. When the STCC–GARCH model uses the lagged absolute S&P 500 returns averaged over a two-week period, the estimated correlations are similar to the ones obtained from the DCC–GARCH model. The major difference is the rate at which the correlations revert back towards the pre-crisis levels. This points at the fact that the transition variable in question responds to general market turbulence, as does the DCC–GARCH model. The situation is quite different when one considers the estimated correlations from the STCC–GARCH model that uses the lagged S&P 500 return over twenty days. They show a clear increase, albeit short-lived, in the correlations to levels that the other two models could not produce. This is due to sufficiently large negative shocks during the crisis.

These two approaches thus lead to rather different conclusions about the conditional correlations between the return series. Since the correlations cannot be observed, it is not possible to decide whether the results from the STCC–GARCH models are closer to the 'truth' than the ones from the DCC–GARCH model or vice versa. In theory, testing the models against each other may be possible but would be computationally demanding. These models may also be compared by investigating their out-of-sample forecasting performance, which is left for the future. In practice, financial decisions that benefit from analysing correlations are linked to market conditions. For this reason, the STCC–GARCH model can be found useful as it enables one to investigate the correlation dynamics with respect to their response to different variables.

5 Conclusions

We propose a new multivariate conditional correlation model with time-varying correlations, the STCC–GARCH model. The conditional correlations are changing smoothly between two extreme states according to a transition variable that can be exogenous to the system. The tran-

sition variable controlling the time-varying correlations can be chosen quite freely, depending on the modelling problem at hand. The STCC–GARCH model may thus be used for investigating the effects of numerous potential factors, lagged predetermined as well as exogenous, on conditional correlations. In this respect the model differs from most other dynamic conditional correlation models such as the ones proposed by Tse and Tsui (2002), Engle (2002), and Pelletier (2006).

The STCC–GARCH model is applied to up to a five-variable set of daily returns of frequently traded stocks included in the S&P 500 index. When using the two-week lagged average of the daily absolute return of the index as the transition variable we find that the conditional correlations are substantially higher during periods of high volatility than otherwise. Asymmetric response of correlations to shocks is examined using the one-day lag of the four-week average index returns. In that case market optimism weakens the conditional correlations between the asset returns.

In its present form the model allows for a single transition with location and smoothness parameters common to all series. In theory this restriction can be relaxed, but finding a useful way of doing it is left for future work. The model may be further refined by allowing specifications of the univariate GARCH equations beyond the standard GJR–GARCH$(1, 1)$ model. For example, the transition between the regimes could be made smooth or nonstationarities could be introduced. A point worth considering is incorporating higher frequency data into the model. Recent research has emphasized the importance of information present in the high-frequency data but lost in aggregation. One could use the realized volatility or bipower variation of stock index returns over a day or a number of days as the transition variable in a model for stock returns. This possibility is left for future research.

Figures
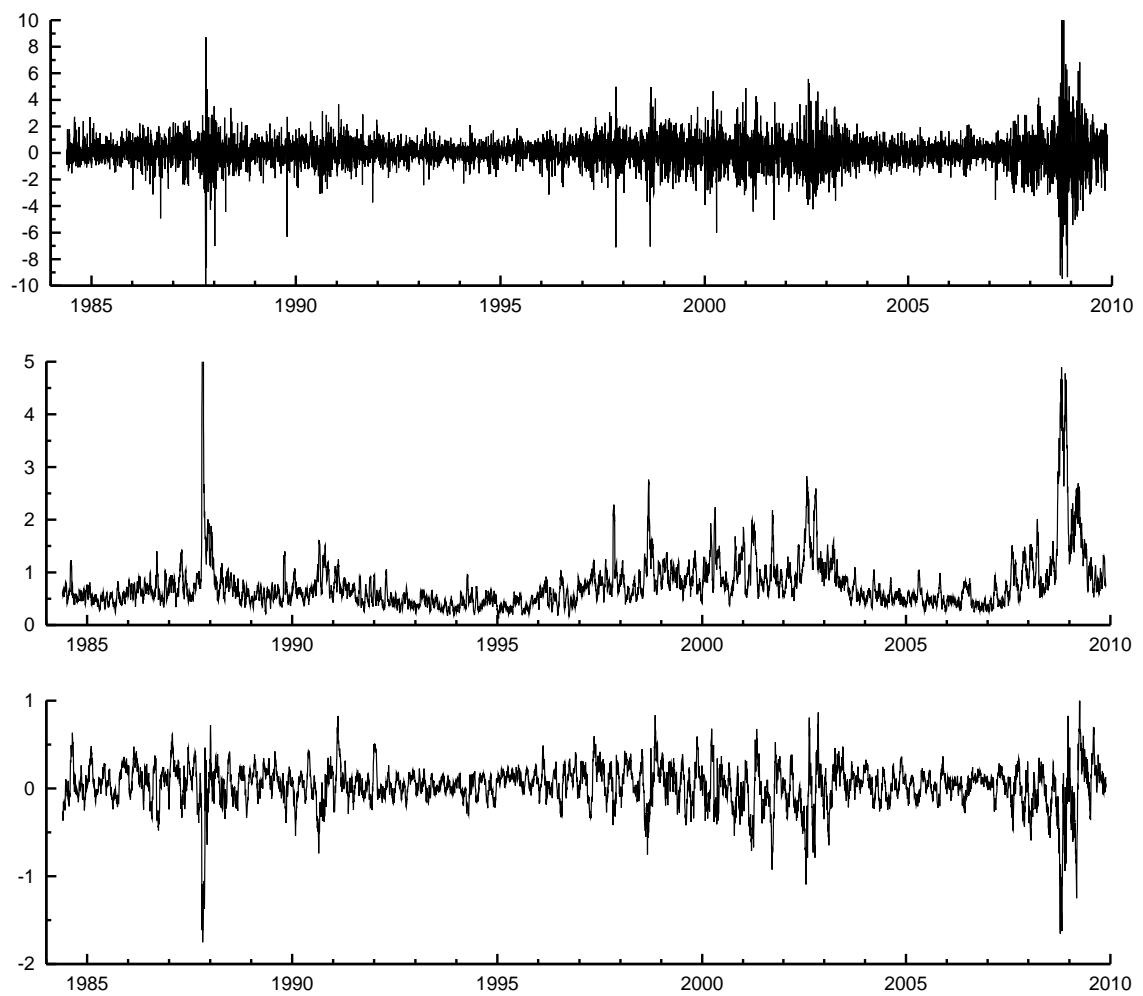
Figure 1: The S&P 500 returns from May 25, 1984 to November 23, 2009. The top panel shows the returns (three observations falls outside the presented range), the mid-panel shows the average of the absolute value returns over ten days (eight observations fall outside the presented range), and the bottom panel shows the log of the price difference averaged over twenty days, or average return over four weeks (three observations fall outside the presented range).
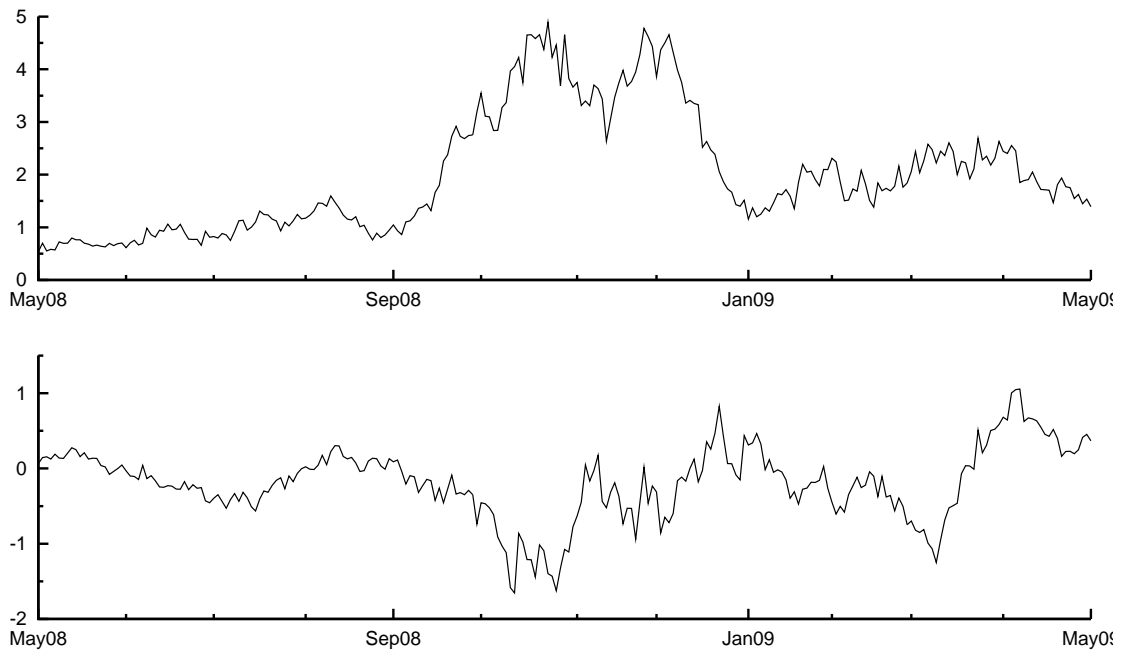
Figure 2: The close-up graphs of the two transition variables from May 2008 to May 2009. The upper panel shows the average of the absolute value returns over ten days, and the lower panel shows the log of the price difference averaged over twenty days, or average return over four weeks.

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Figure 3: The estimated time-varying conditional correlations from the bivariate STCC–GARCH models when the transition variable is the lagged absolute S&P 500 index returns averaged over ten days, see Table 2. The time period covers the year from May 2008 to May 2009.

Figure 4: The estimated time-varying conditional correlations from the bivariate STCC–GARCH models when the transition variable is the lagged average S&P 500 index return over twenty days, see Table 4. The time period covers the year from May 2008 to May 2009.
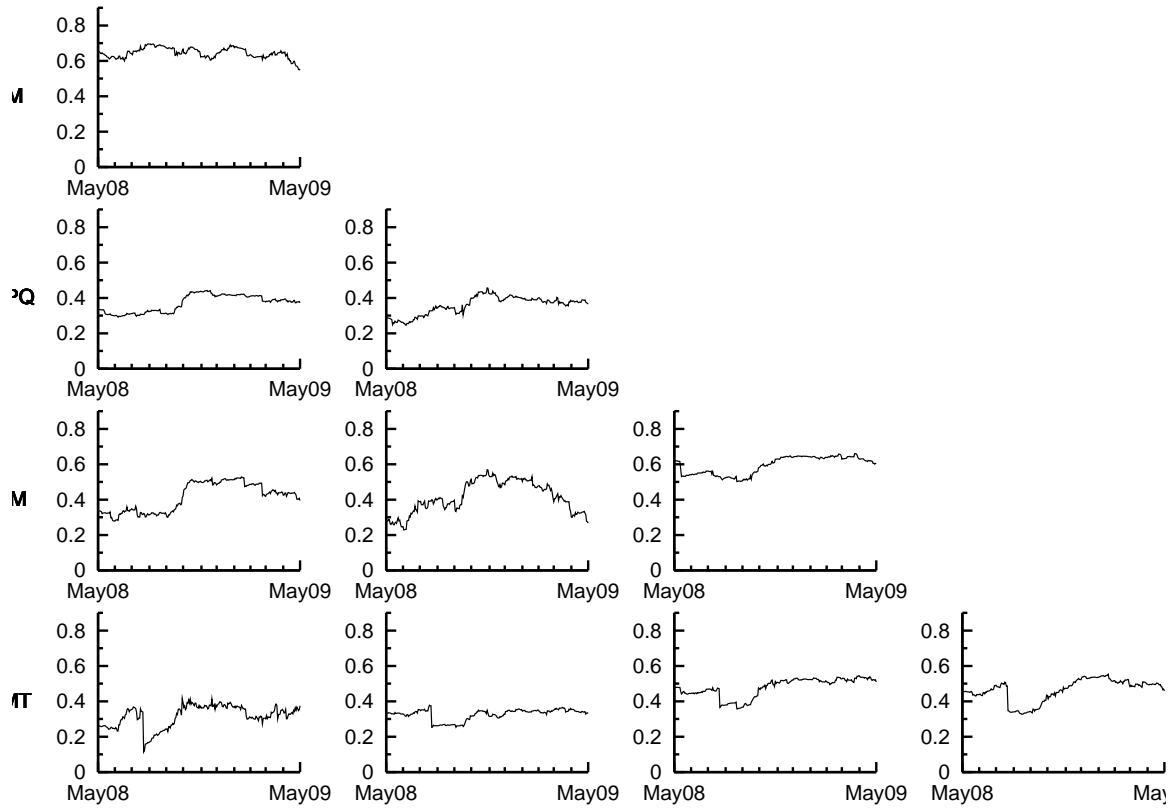
Figure 5: The estimated conditional correlations from the bivariate DCC–GARCH model. The time period covers the year from May 2008 to May 2009.

Tables

Table 1: Test of constant conditional correlation against STCC–GARCH model for all combinations of assets. The transition variables are $s_t^{(1)}$,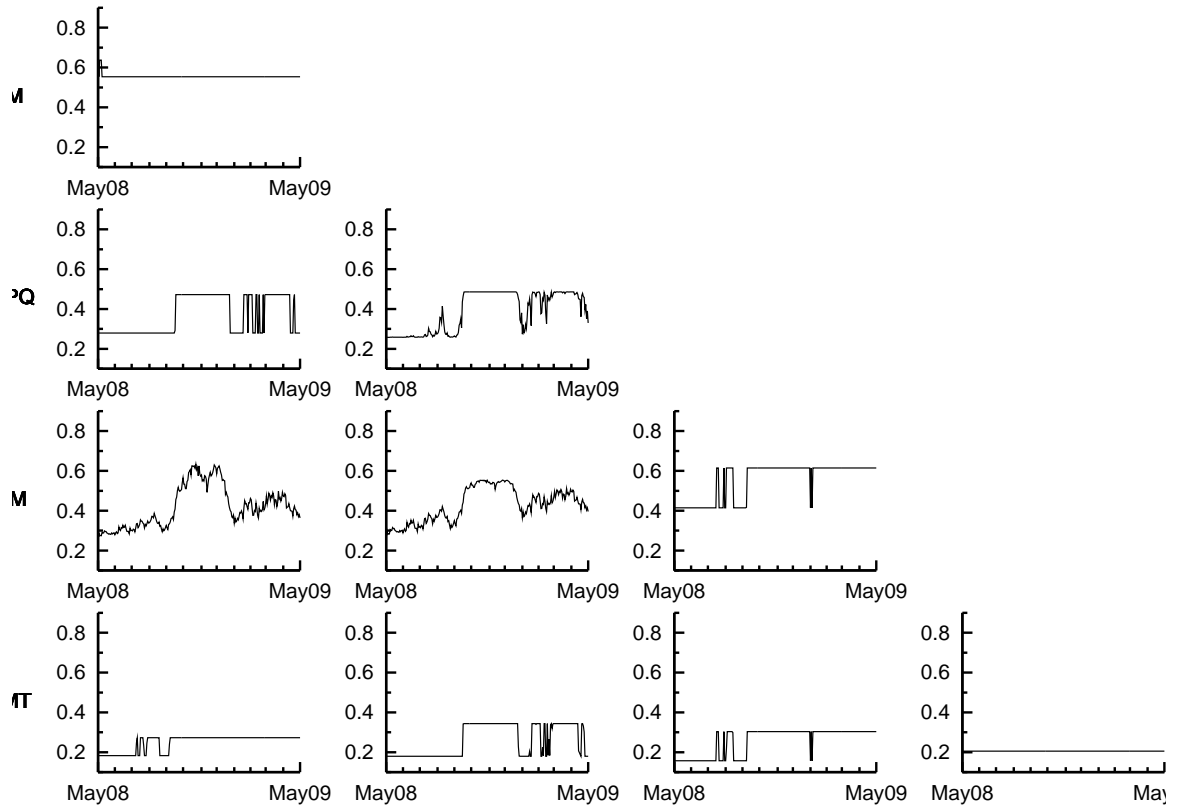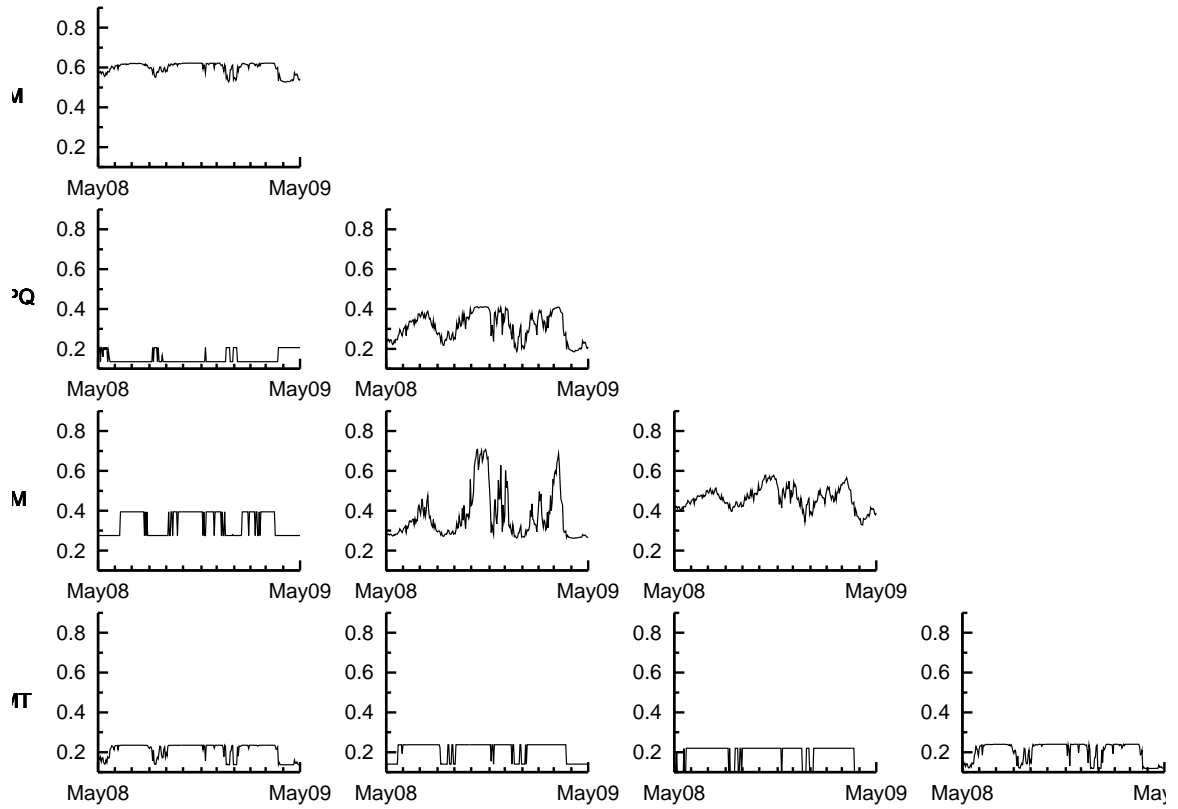 the lagged absolute S&P 500 index returns averaged over ten days, and $s_t^{(2)}$, the lagged S&P 500 index average return over 20 days.

	$s_t^{(1)}$		$s_t^{(2)}$	
	LM_{CCC}	p–value	LM_{CCC}	p–value
F – GM	6.54	0.0106	11.05	0.0009
F – HPQ	17.77	2×10^{-5}	33.64	7×10^{-9}
F – IBM	31.18	2×10^{-8}	21.73	3×10^{-6}
F – LMT	13.32	0.0003	11.43	0.0007
GM – HPQ	19.40	1×10^{-5}	26.90	2×10^{-7}
GM – IBM	35.02	3×10^{-10}	25.51	4×10^{-7}
GM – LMT	8.73	0.0031	11.82	0.0006
HPQ – IBM	43.19	5×10^{-11}	15.68	8×10^{-5}
HPQ – LMT	25.44	5×10^{-7}	37.96	7×10^{-10}
IBM – LMT	22.77	2×10^{-6}	24.91	6×10^{-7}
F – GM – HPQ – IBM – LMT	102.04	2×10^{-17}	71.72	2×10^{-11}

Table 2: Estimation results for bivariate models (standard errors in parentheses) when the transition variable is the lagged absolute S&P 500 index returns averaged over ten days.

model	α_0	α	δ	β	$\rho_{(1)}$	$\rho_{(2)}$	c	γ/s_{s_t}
F	0.0196 (0.0049)	0.0258 (0.0038)	0.0089 (0.0042)	0.9663 (0.0035)	0.6369 (0.0105)	0.5535 (0.0109)	0.6046 (0.0022)	1000 (−−)
GM	0.0350 (0.0057)	0.0144 (0.0035)	0.0363 (0.0049)	0.9608 (0.0040)				
F	0.0259 (0.0064)	0.0258 (0.0045)	0.0148 (0.0049)	0.9619 (0.0044)	0.2794 (0.0119)	0.4722 (0.0387)	1.8036 (0.0142)	302.68 (2.2452)
HPQ	0.0441 (0.0099)	0.0222 (0.0044)	0.0136 (0.0051)	0.9630 (0.0051)				
F	0.0270 (0.0070)	0.0300 (0.0050)	0.0145 (0.0052)	0.9580 (0.0049)	0.0201 (0.2399)	0.6932 (0.2005)	1.3274 (0.6966)	0.6557 (0.4717)
IBM	0.0214 (0.0047)	0.0291 (0.0049)	0.0411 (0.0070)	0.9443 (0.0067)				
F	0.0273 (0.0070)	0.0271 (0.0048)	0.0182 (0.0054)	0.9588 (0.0049)	0.1829 (0.0133)	0.2725 (0.0342)	1.1204 (0.0139)	363.29 (2.6798)
LMT	0.0429 (0.0113)	0.0516 (0.0093)	0.0236 (0.0093)	0.9242 (0.0122)				
GM	0.0442 (0.0043)	0.0179 (0.0075)	0.0452 (0.0060)	0.9513 (0.0052)	0.2577 (0.0151)	0.4862 (0.0477)	1.4941 (0.1451)	7.5969 (8.0501)
HPQ	0.0498 (0.0047)	0.0229 (0.0113)	0.0148 (0.0052)	0.9606 (0.0058)				
GM	0.0426 (0.0073)	0.0202 (0.0044)	0.0455 (0.0062)	0.9498 (0.0050)	0.0839 (0.1355)	0.5575 (0.1052)	0.8377 (0.5134)	1.1921 (0.6568)
IBM	0.0222 (0.0048)	0.0314 (0.0051)	0.0390 (0.0071)	0.9429 (0.0067)				
GM	0.0498 (0.0083)	0.0185 (0.0045)	0.0507 (0.0067)	0.9471 (0.0056)	0.1795 (0.0149)	0.3439 (0.0453)	1.7394 (0.0717)	76.96 (114.68)
LMT	0.0470 (0.0132)	0.0553 (0.0105)	0.0237 (0.0093)	0.9193 (0.0148)				
HPQ	0.0621 (0.0143)	0.0276 (0.0056)	0.0129 (0.0057)	0.9547 (0.0070)	0.4140 (0.0123)	0.6143 (0.0251)	1.2169 (0.0035)	1000 (−−)
IBM	0.0241 (0.0050)	0.0268 (0.0048)	0.0373 (0.0064)	0.9465 (0.0067)				
HPQ	0.0425 (0.0100)	0.0208 (0.0044)	0.0169 (0.0052)	0.9632 (0.0052)	0.1570 (0.0141)	0.3033 (0.0395)	1.2280 (0.0056)	1000 (−−)
LMT	0.0369 (0.0097)	0.0472 (0.0086)	0.0219 (0.0077)	0.9312 (0.0111)				
IBM	0.0236 (0.0051)	0.0299 (0.0053)	0.0446 (0.0078)	0.9413 (0.0071)	0.0854 (0.0381)	0.2050 (0.0138)	0.4217 (0.0027)	1000 (−−)
LMT	0.0438 (0.0116)	0.0533 (0.0093)	0.0224 (0.0087)	0.9229 (0.0125)				

Table 3: Estimation results for the five-variate STCC–GARCH model (standard errors in parentheses) when the transition variable is the lagged absolute S&P 500 index returns averaged over ten days.

model	α_0	α	δ	β		c	γ/s_{s_t}
F	0.0191 (0.0048)	0.0267 (0.0038)	0.0059 (0.0041)	0.9669 (0.0034)		0.8579 (0.0781)	1.6147 (0.4788)
GM	0.0333 (0.0055)	0.0164 (0.0036)	0.0319 (0.0047)	0.9614 (0.0039)			
HPQ	0.0452 (0.0106)	0.0244 (0.0047)	0.0095 (0.0047)	0.9626 (0.0055)			
IBM	0.0169 (0.0038)	0.0248 (0.0043)	0.0283 (0.0053)	0.9554 (0.0056)			
LMT	0.0404 (0.0106)	0.0504 (0.0090)	0.0195 (0.0078)	0.9279 (0.0118)			

	$\boldsymbol{P}_{(1)}$					$\boldsymbol{P}_{(2)}$			
	F	GM	HPQ	IBM		F	GM	HPQ	IBM
GM	0.6895 (0.0314)				GM	0.4745 (0.0337)			
HPQ	0.2092 (0.0412)	0.1629 (0.0447)			HPQ	0.3830 (0.0418)	0.4135 (0.0447)		
IBM	0.1910 (0.0447)	0.1741 (0.0475)	0.2694 (0.0458)		IBM	0.4249 (0.0461)	0.4712 (0.0478)	0.6347 (0.0425)	
LMT	0.1433 (0.0432)	0.1325 (0.0416)	0.0862 (0.0515)	0.1013 (0.0492)	LMT	0.2592 (0.0444)	0.2505 (0.0428)	0.2768 (0.0523)	0.2778 (0.0510)

Table 4: Estimation results for bivariate STCC–GARCH models (standard errors in parentheses) when the transition variable is the lagged average S&P 500 index return over twenty days.

model	α_0	α	δ	β	$\rho_{(1)}$	$\rho_{(2)}$	c	γ/s_{s_t}
F	0.0177 (0.0048)	0.0258 (0.0039)	0.0112 (0.0043)	0.9657 (0.0036)	0.6219 (0.0196)	0.5267 (0.0415)	0.1558 (0.1606)	7.1487 (3.7453)
GM	0.0332 (0.0054)	0.0141 (0.0034)	0.0384 (0.0050)	0.9607 (0.0039)				
F	0.0281 (0.0068)	0.0253 (0.0045)	0.0165 (0.0049)	0.9610 (0.0046)	0.1354 (0.0130)	0.2062 (0.0222)	0.1354 (0.0038)	821.58 (6.0923)
HPQ	0.0456 (0.0103)	0.0220 (0.0044)	0.0141 (0.0051)	0.9627 (0.0053)				
F	0.0290 (0.0072)	0.0288 (0.0049)	0.0156 (0.0052)	0.9579 (0.0050)	0.3944 (0.0303)	0.2753 (0.0129)	−0.1534 (0.0010)	1000 (−−)
IBM	0.0226 (0.0051)	0.0293 (0.0053)	0.0423 (0.0074)	0.9432 (0.0074)				
F	0.0280 (0.0069)	0.0263 (0.0047)	0.0193 (0.0052)	0.9590 (0.0048)	0.2351 (0.0185)	0.1367 (0.0325)	0.1054 (0.0770)	19.90 (3.2218)
LMT	0.0462 (0.0117)	0.0527 (0.0093)	0.0250 (0.0086)	0.9213 (0.0125)				
GM	0.0450 (0.0076)	0.0174 (0.0043)	0.0455 (0.0060)	0.9514 (0.0052)	0.4113 (0.0583)	0.1842 (0.0649)	−0.0817 (0.1948)	4.5927 (2.5045)
HPQ	0.0505 (0.0114)	0.0222 (0.0047)	0.0155 (0.0052)	0.9608 (0.0057)				
GM	0.0440 (0.0075)	0.0192 (0.0043)	0.0457 (0.0061)	0.9500 (0.0051)	0.7127 (0.1870)	0.2609 (0.0368)	−0.5874 (0.2772)	4.1809 (2.9079)
IBM	0.0240 (0.0051)	0.0316 (0.0051)	0.0410 (0.0073)	0.9410 (0.0069)				
GM	0.0505 (0.0082)	0.0182 (0.0044)	0.0515 (0.0065)	0.9469 (0.0054)	0.2372 (0.0165)	0.1406 (0.0174)	0.0457 (0.0070)	950.86 (7.0509)
LMT	0.0496 (0.0127)	0.0555 (0.0098)	0.0255 (0.0089)	0.9174 (0.0133)				
HPQ	0.0651 (0.0151)	0.0249 (0.0053)	0.0152 (0.0054)	0.9554 (0.0072)	0.5898 (0.0879)	0.2984 (0.1299)	−0.0340 (0.3932)	1.9828 (1.4297)
IBM	0.0268 (0.0055)	0.0268 (0.0048)	0.0401 (0.0069)	0.9443 (0.0071)				
HPQ	0.0443 (0.0107)	0.0205 (0.0046)	0.0176 (0.0052)	0.9627 (0.0056)	0.2195 (0.0160)	0.0808 (0.0227)	0.1267 (0.0014)	1000 (−−)
LMT	0.0428 (0.0128)	0.0510 (0.0109)	0.0236 (0.0085)	0.9248 (0.0148)				
IBM	0.0233 (0.0051)	0.0291 (0.0051)	0.0457 (0.0076)	0.9416 (0.0070)	0.2396 (0.0274)	0.1188 (0.0282)	0.0522 (0.0716)	17.39 (15.53)
LMT	0.0458 (0.0117)	0.0538 (0.0095)	0.0231 (0.0085)	0.9213 (0.0125)				

Table 5: Estimation results for the five-variate STCC–GARCH model (standard errors in parentheses) when the transition variable is the lagged average S&P 500 index return over twenty days.

model	α_0	α	δ	β	c	γ/s_{s_t}
F	0.0183 (0.0047)	0.0255 (0.0037)	0.0084 (0.0041)	0.9671 (0.0034)	0.1610 (0.0239)	50.81 (21.26)
GM	0.0323 (0.0053)	0.0154 (0.0034)	0.0333 (0.0047)	0.9619 (0.0038)		
HPQ	0.0496 (0.0112)	0.0221 (0.0043)	0.0098 (0.0046)	0.9637 (0.0055)		
IBM	0.0204 (0.0044)	0.0254 (0.0044)	0.0304 (0.0058)	0.9527 (0.0062)		
LMT	0.0472 (0.0124)	0.0537 (0.0097)	0.0211 (0.0083)	0.9216 (0.0131)		

	$\boldsymbol{P}_{(1)}$					$\boldsymbol{P}_{(2)}$			
	F	GM	HPQ	IBM		F	GM	HPQ	IBM
GM	0.6027 (0.0091)				GM	0.5483 (0.0200)			
HPQ	0.3223 (0.0132)	0.3032 (0.0133)			HPQ	0.2013 (0.0300)	0.2158 (0.0287)		
IBM	0.3119 (0.0131)	0.3172^{R} (0.0257)	0.4542 (0.0116)		IBM	0.2685 (0.0293)	0.3172^{R} (0.0257)	0.3996 (0.0236)	
LMT	0.2183 (0.0146)	0.2102 (0.0147)	0.2106 (0.0165)	0.2073 (0.0152)	LMT	0.1382 (0.0275)	0.1195 (0.0290)	0.0711 (0.0288)	0.1125 (0.0287)

The superscript R indicates that the correlation is restricted to be constant.

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