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### **Coherent Model-Free Implied Volatility: A Corridor Fix for High-Frequency VIX**

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# Coherent Model-Free Implied Volatility: A Corridor Fix for High-Frequency VIX\*

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## Abstract

The VIX index is computed as a weighted average of SPX option prices over a range of strikes according to specific rules regarding market liquidity. It is explicitly designed to provide a model-free option-implied volatility measure. Using tick-by-tick observations on the underlying options, we document a substantial time variation in the coverage which the stipulated strike range affords for the distribution of future S&P 500 index prices. This produces idiosyncratic biases in the measure, distorting the time series properties of VIX. We introduce a novel “Corridor Implied Volatility” index (CX) computed from a strike range covering an “economically invariant” proportion of the future S&P 500 index values. We find the CX measure superior in filtering out noise and eliminating artificial jumps, thus providing a markedly different characterization of the high-frequency volatility dynamics. Moreover, the VIX measure is particularly unreliable during periods of market stress, exactly when a “fear gauge” is most valuable.

*JEL Classification:* G13, C58

*Keywords:* VIX, Model-Free Implied Volatility, Corridor Implied Volatility, Time Series Coherence

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# 1 Introduction

The VIX volatility index disseminated by the Chicago Board Options Exchange (CBOE) has attracted much attention in recent years. It is constructed to serve as a Model-Free option-Implied return Volatility (MFIV) measure for the S&P 500 index over the coming 30 days, expressed in annualized percentage terms. The index is computed from concurrent price quotes on a wide range of out-of-the-money European put and call options written on the S&P 500 equity index, and is released in real-time by the CBOE throughout the trading day, with index values being updated about every fifteen seconds. The CBOE launched futures contracts on the VIX index in February 2004 and options on VIX futures in February 2006, enabling investors to obtain direct exposure to market volatility. The trading activity in VIX related derivatives has grown dramatically in recent years and the CBOE considers the VIX options the most successful new product launch in their history. Moreover, both the VIX futures and options have received “most innovative index product” awards at industry conferences.

The success of the VIX index and the VIX derivatives is attributable to several factors. First, there is a strong negative correlation between VIX index changes and returns on the underlying S&P 500 index. This negative relationship with market returns implies that long VIX positions can serve as a hedge against market risk and may be useful in providing diversification for portfolios that are exposed to the equity-index. On the other hand, the existence of a large volatility risk premium means that the average return on long VIX futures positions are significantly negative, implying that speculators may desire short VIX positions. Various trading strategies exploiting the latter feature have been launched under the label “volatility arbitrage” and associated performance measures are widely followed. Hence, there are natural industry constituents for the trading of VIX derivatives.

Second, the pronounced negative correlation with the overall market returns has also captured the attention of the popular press and the more specialized financial media. In fact, the VIX is often called a “fear gauge” due to the rapid spikes it displays during periods of market stress. Naturally, this moniker became even more prominent as the index soared to dramatic heights during the financial crisis of 2008-2009, reaching an intraday high of 89.5% on October 24, 2008, compared with an average value of about 19% in the period from 1990 until October 2008. In short, the VIX is now routinely referenced by the main media when characterizing the current market conditions or “investor sentiment”.

Finally, the availability of a model-free market-wide measure of return volatility has proven convenient for empirical work in financial economics. It circumvents the need to specify and estimate a particular volatility model. Moreover, since it is based on option prices from a liquid market, it reflects all current relevant public information and should provide a rational signal about future return variation. One set of studies directly explores the forecast power of variation in the VIX for future realized return volatility. For example, Jiang & Tian (2005) deem the information content of the VIX volatility forecast superior to alternative implied volatility measures as well as forecasts based on historical volatility. However, a second group of studies emphasize that the VIX measure incorporates the pricing of variance risk that is embedded within option prices under the so-called risk-neutral or  $Q$ -measure. Hence, the wedge between regular time series forecasts for volatility, developed under the objective or  $P$ -measure, and the VIX, representing a risk-neutral forecast, constitutes a volatility risk premium, see, e.g., Bondarenko (2010), Bollerslev, Tauchen & Zhou (2009) and Carr & Wu (2009). The evidence favors a large, negative, and strongly time-varying variance risk premium that is negatively correlated with market returns. Interestingly, this variance risk premium has been found to carry predictability for future equity returns and to be priced in the cross-section of equity returns. Further studies document that jumps in prices and volatilities are critical for the dynamics of the variance risk premium and suggest that the premium is tied to time-varying compensation for tail risks, see Todorov (2010) and Bollerslev & Todorov (2011b). A third type of studies seeks to draw inference regarding

the stochastic properties of market return volatility from the high-frequency behavior of the VIX. A jump in the volatility of the S&P 500 index may be hard to identify, even from high-frequency equity-index observations, while it should manifest itself directly in the VIX index. This insight motivates Todorov & Tauchen (2010) to explore the relative importance and finer probabilistic structure of jumps in high-frequency VIX series.

In a parallel development, the use of VIX style indices is moving far beyond the S&P 500 index. In recent years, the same basic methodology has been applied by leading exchanges across North America, Europe, Asia, and Australia to a broad range of asset classes and international indices. As a result, VIX measures are currently released in real-time for a host of U.S. and international equity indices as well as individual stocks, energy, metal, foreign exchange, and agricultural products. Likewise, most of these exchanges have initiated, or soon plan to initiate, trading in derivative contracts written on these volatility measures, and they are actively seeking to harmonize and standardize their procedures for measuring and trading such indices to facilitate market acceptance and depth. In short, exchange-traded products representing pure volatility bets are rapidly expanding worldwide, and the CBOE VIX methodology is emerging as the blueprint for the industry.

Given these trends, it is important to be cognizant of potential problems with the VIX measure. How robust is the measure? Does it display particular biases? Under what conditions is it likely to fail? These questions are intrinsically linked to features of the underlying market for equity-index options. This article explores the high-frequency properties of the VIX index and compares it to a number of alternative model-free volatility measures constructed from the identical SPX options. At first blush, the VIX may appear robust to market microstructure issues because it typically is computed from more than 200 distinct option mid-quotes. As such, any idiosyncratic errors will tend to cancel. Unfortunately, a range of other practical features produce systematic biases that render precise real-time measurement difficult. Many of these issues have previously been noted by Jiang & Tian (2007) who discuss how to alleviate some specific concerns. However, they do not address the lack of intertemporal consistency of the VIX index. We document that this feature induces dramatic distortions in the time series characteristics of the high-frequency VIX series and we set out to remedy them.

Initially, to establish a proper benchmark, we seek to replicate the VIX using the exact CBOE computational procedure and using the high-frequency option quotes provided by the CBOE vendor. Overall, we find a reasonable coherence between our constructed index and the official VIX. Nonetheless, there are occasional extreme deviations that have a pronounced impact on the high-frequency volatility series. The main reason is a substantial, and sometimes abrupt, time variation in the range of options exploited in the VIX computations. While we often can pinpoint the source of those shifts, they are not always consistent with our option quote record, thus driving a wedge between our replication series and the VIX. Even more importantly, however, is the fact that such sudden changes in the range of covered strikes induce artificial breaks or “jumps” in the associated volatility indices. Given these problems, we conclude that the high-frequency VIX data are not suitable for analyzing critical aspects of the volatility dynamics due to inconsistencies and noise in the time series. We also find that any alternative computation of the ideal MFIV series must confront serious obstacles in order to construct coherent index measures over time. Inevitably, the pronounced variability in the available strike coverage necessitates *ad hoc* corrections that will infuse the series with non-trivial measurement errors.

Consequently, we turn to a different methodology for volatility index construction, designed to alleviate the more critical problem with the VIX computations. Rather than using all available strikes at a given point in time, we deliberately focus on a limited strike range that, measured by an option pricing metric, covers an invariant portion of the underlying risk-neutral density. Hence, we introduce a

new cut-off criterion, determined endogenously by option prices, that allows the measure to reflect the pricing of volatility across an economically equivalent fraction of the strike range, ensuring intertemporal coherence of the volatility measure. Any such measure is termed a model-free corridor implied volatility index (CX) and is closely related to the corresponding theoretical concept discussed by Carr & Madan (1998). Obviously, different corridor measures may be obtained depending on the width and positioning of the strike range. We vary the corridors moderately to explore the advantages and drawbacks of a given choice.

Our empirical results indicate that the CX indices dominate VIX in terms of providing accurate and coherent volatility measures. Specifically, the CX series contain fewer “erroneous” jumps, display stronger time series persistence, and has a higher degree of high-frequency correlation with the market returns than VIX, reflecting the improved consistency and the lower level of idiosyncratic noise of the CX measures. In addition, the measurement problems for VIX worsen during tumultuous market conditions, when the behavior of the “fear gauge” is of particular interest for traders, exchanges, regulators, and academics alike. This seriously undermines the function of the index as a real-time thermometer of market stress. Finally, we document that the quality of the end-of-day VIX value is akin to that of the intraday measure, so it is equally deficient. Hence, the distortions in the day-to-day changes in the VIX measure are not trivial.<sup>1</sup> However, since the (absolute) errors of the daily and the fifteen second measures are similar, but the standard deviation of daily volatility exceeds that of the high-frequency series by an order of magnitude, the relative error is much smaller when assessing the VIX at the daily level.

We conclude that our CX measures facilitate model-free identification of important underlying features of the volatility series that are useful in discriminating between alternative models and explanations for observed high-frequency asset price dynamics. Moreover, they are critical for monitoring the real-time evolution of volatility during turbulent conditions where the traditional VIX measure is especially prone to erratic behavior. The latter feature is dramatically exemplified by the event known as the “flash crash.”

## 2 High-Frequency Volatility During the Flash Crash

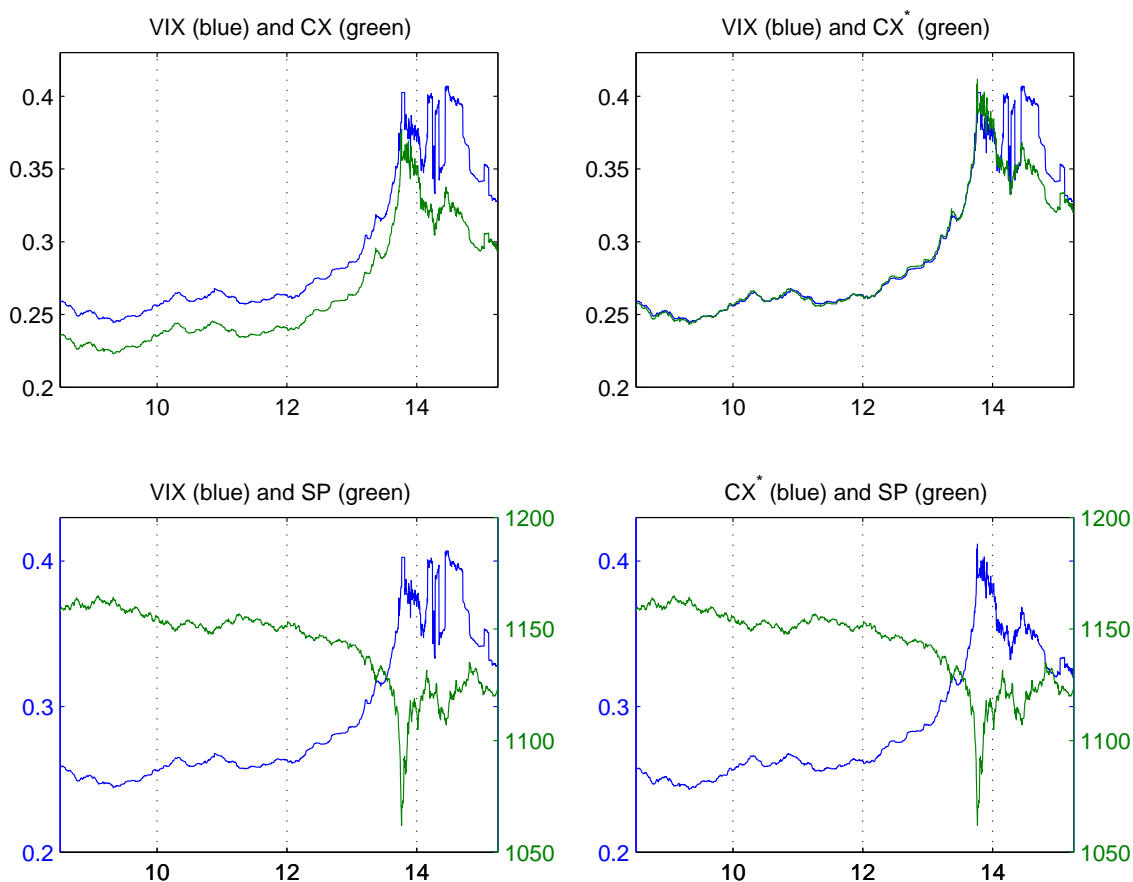
On May 6, 2010, just after 13:30 CT, the S&P 500 index dived by more than five percent in a few minutes, only to rapidly recover by a similar amount immediately thereafter. Most other U.S. securities markets underwent corresponding whipsawing trading patterns, pointing towards a widespread temporary breakdown in cross-market liquidity. This decline and rebound in the markets was unprecedented in speed and scope, and the event is now referred to as the “flash crash.” In the CFTC-SEC Report, exploring the market dynamics of this episode, the intraday behavior of the VIX measure feature prominently in the account of the escalating uncertainty gripping the market. Given the prior discussion of the potential problems with VIX during turbulent market conditions, we constructed our alternative corridor volatility, or CX, series for this trading day. The upper left panel of Figure 1 portrays the official intraday VIX along with this CX index for May 6, 2010, while the upper right panel displays the VIX and a scaled version of the CX index, labeled CX\*, that more closely matches the general level of the VIX index, in order to more easily compare the two series.<sup>2</sup> The bottom panels depict the E-mini

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<sup>1</sup>Those closing values are identical to the daily SPX option quotes available through the Wharton Research Data Services (WDRS) that form the basis for a large number of empirical option pricing studies.

<sup>2</sup>The low value of the unscaled CX series simply reflects the narrow corridor of strikes used in the computation of this index relative to the strikes exploited by the VIX.

Figure 1: VIX and Corridor Implied Volatility Indices on May 6, 2010



S&P 500 futures price and either the VIX or the  $CX^*$  indices.<sup>3</sup>

A few features of Figure 1 are striking. First, as evident from the upper right panel, during the early part of the day the volatility indices are near indistinguishable and both hit a local peak at the trough of the equity-index around 13:45. Second, both indices decline in erratic fashion over the following 20-25 minutes. Third, the indices then diverge sharply over the next hour, until after 15:00. The VIX spikes up again and, in fact, attains its maximum around 14:30, while  $CX^*$  is more than ten percent lower and well below its high of the day. Moreover, the path of the VIX is now characterized by abrupt jumps, up and down, while  $CX^*$  evolves more smoothly, almost as a perfect reverse, or mirror, image of the S&P 500 index. Hence, the two series portray vastly different trading environments following the flash crash. The real-time VIX index conveys a picture of sustained, or even elevated, uncertainty for a prolonged period. The corridor index instead settles down reasonably well around 14:00, and subsequently never reaches a level comparable with the peak observed at 13:45.

For a market observer eyeing the VIX index in real-time following the crash, the successive spikes of more than 10% around 14:12 and 14:30 would be rather unsettling. However, if  $CX^*$  provides an accurate picture of the volatility dynamics, these eruptions in volatility were fictitious, arising solely

<sup>3</sup>The futures price series stems from the CME Group. The E-mini futures contract is extraordinarily liquid and these futures prices are widely viewed as perhaps the most timely indicators of the high-frequency developments in the S&P 500 index available.

from inherent instabilities in the computational procedure for the VIX. We postpone a more detailed explanation for the observed discrepancies until the end of the paper, after we have more formally defined these volatility indices and analyzed their basic properties more carefully.

### 3 The Methodology behind the VIX Computation

#### 3.1 The Origins of Model-Free Implied-Volatility

Over-the-counter trading of volatility contracts emerged in the 1990s with liquidity building primarily in the so-called variance swaps. Despite the label, these swaps are forward contracts involving no initial or intermediary cash flows. At expiry, the long party pays a fixed premium, determined at contract initiation, and in exchange receives a payment reflecting the (random) realized variance of the underlying index. Typically, the realized variance would be measured by the cumulative sum of daily squared index returns from initiation to expiry. A critical backdrop for these developments was breakthroughs in theory demonstrating the feasibility of replicating the realized return variation of an asset over a given future horizon. This path-dependent payoff will, under general conditions, equal the payoff from a strategy involving only a static options portfolio combined with delta hedging in the underlying asset.<sup>4</sup> These results provide practical pricing and hedging tools necessary for sustaining market liquidity and depth. In particular, they imply that the “fair” value of a claim promising to pay the future index return variation is given by the market price of the replicating options portfolio. As such, the premium on a variance swap directly values the future return variation. Moreover, since this reasoning is model-independent, the corresponding notion is referred to as the model-free implied variance, and its square-root as the *model-free implied volatility*, or simply MFIV.

In order to facilitate the development of an exchange traded market for volatility derivatives, the CBOE, on September 22, 2003, aligned their methodology for computing the volatility index, VIX, with that of the trading community. Hence, while the “old” VIX, now denoted VXO, was based on at-the-money Black-Scholes implied volatility, the “new” VIX is designed to closely approximate the MFIV. In this manner, market participants can exploit established procedures for pricing and hedging of over-the-counter volatility contracts in dealing with exchange traded VIX products.

#### 3.2 Theory and Implementation

In theory, the computation of the model-free implied variance for a given maturity requires the availability of market prices for a continuum of European-style options with strike prices spanning the support of the possible future index prices, from zero to infinity, for that specific maturity. A convenient representation of the model-free implied variance takes the form,

$$\sigma_T^2 = \frac{2e^{rT}}{T} \left[ \int_0^F \frac{P(T,K)}{K^2} dK + \int_F^\infty \frac{C(T,K)}{K^2} dK \right] = \frac{2e^{rT}}{T} \left[ \int_0^\infty \frac{Q(T,K)}{K^2} dK \right], \quad (1)$$

where  $r$  is the (annualized) risk-free interest rate for the period  $[0, T]$ , as represented by the corresponding U.S. Treasury bill rate,  $T$  is time-to-maturity measured in units of a year,  $F$  denotes the forward price for maturity  $T$ ,  $P(T, K)$  and  $C(T, K)$  are the prices for European put and call options with strike  $K$  and time-to-maturity  $T$ , and  $Q(T, K) = \min\{C(T, K), P(T, K)\}$  denotes the price of the OTM option at

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<sup>4</sup>Initial work leading to these insights is represented by Neuberger (1994), Dupire (1993) and Dupire (1996), with the latter published in Dupire (2004). Building on these insights, Carr & Madan (1998) constructed an explicit and robust replication strategy for the pay-off on the variance swap, while Britten-Jones & Neuberger (2000) provide additional results.

strike  $K$ . Hence, for  $K < F$  ( $K > F$ ),  $Q(T, K)$  is the price of an OTM put (call). Correspondingly, we label an option with strike  $K = F$ , and thus  $C(T, K) = P(T, K)$ , an “at-the-money” (ATM) option.<sup>5</sup>

The terminology “model-free implied variance” is motivated by a direct theoretical link between  $\sigma_T^2$  and the market price of a contract with a terminal payoff equal to the future return variation. Let the logarithmic equity-index value at time  $t$  be denoted  $\log(S_t)$ . In a frictionless market, if the price path is continuous and there are no arbitrage opportunities, then the price must evolve according to a Brownian semi-martingale, see, e.g., Back (1991),

$$d\log(S_t) = \mu_t dt + v_t dW_t,$$

where  $W_t$  denotes a standard Brownian motion. Hence, the log price dynamics is characterized by a (stochastic volatility) diffusion. Under weak regularity conditions, and absent specific parametric assumptions regarding the drift and diffusion coefficients,  $\mu_t$  and  $v_t$ , the quadratic return variation, or *integrated variance*, for this process over the time interval  $[0, T]$ , denoted  $IVar$ , is given by,

$$IVar = \int_0^T v_u^2 du.$$

The integrated variance is the natural notion of return variation in this setting, see, e.g., Andersen, Bollerslev & Diebold (2010). The key result is that the model-free implied variance equals the expected future quadratic return variation under the risk-neutral measure. Hence, the linear combination of option prices in equation (1) represents the fair value of a payoff that equals the (annualized) future return variation. Formally, we have,

$$\sigma_T^2 = \frac{1}{T} E^* \left[ \int_0^T v_u^2 du \right] = \frac{1}{T} E^* [IVar], \quad (2)$$

where  $E^*[\cdot]$  denotes the expected value operator under the risk-neutral measure.<sup>6</sup>

Of course, in practice the number of available strikes is limited so it is not feasible to compute the expression in equation (1). Instead, the CBOE approximates  $\sigma_T^2$  by the following quantity,

$$\hat{\sigma}_T^2 = \underbrace{\frac{2e^{rT}}{T} \sum_{i=1}^n \frac{\Delta K_i}{K_i^2} Q(T, K_i)}_{\text{Discrete Approximation}} - \underbrace{\frac{1}{T} \left[ \frac{F}{K_f} - 1 \right]^2}_{\text{Correction Term}}, \quad (3)$$

where  $0 < K_1 < \dots < K_f \leq F < K_{f+1} < \dots < K_n$  refer to the strikes included in the computation,  $K_f$  denotes the first strike price available below the forward rate,  $F$ , where the index  $f$  is a positive integer, and we assume a reasonable cross-section of options are available so that,  $2 < f < n - 1$ . Moreover, the increments in the strike ranges are calculated as  $\Delta K_1 = K_2 - K_1$ ,  $\Delta K_n = K_n - K_{n-1}$ , and for  $1 < i < n$ ,  $\Delta K_i = (K_{i+1} - K_{i-1})/2$ , while the term  $Q(T, K_i)$  is defined as the midpoint of the bid-ask spread for the OTM option with strike  $K_i$ . More specifically,  $Q(T, K_i)$  is equal to the put price when  $K_i < K_f$ , the call price when  $K_i > K_f$ , and the average of the put and call prices when  $K_i = K_f$ . Finally, the second term in (3) reflects a correction for the discrepancy between  $K_f$  and the forward price.<sup>7</sup>

<sup>5</sup>Henceforth, we follow this standard terminology in spite of this option formally being “at-the-forward.”

<sup>6</sup>Bondarenko (2010) demonstrates that this formula for pricing the future expected quadratic return variation generally provides a very good approximation even in the presence of jumps in the asset price process, as long as the integrated return variation is appropriately defined to include realized squared jumps along with the integrated variance.

<sup>7</sup>Since  $K_f$  is below  $F$ , the first term in (3) relies in part on higher priced in-the-money call options over the region  $(K_f, F)$ ,



On organized exchanges, only a few option maturity dates are quoted at any given time. The *VIX* is defined for a fixed calendar maturity of  $T_M = \frac{30}{365}$ , or thirty days. The CBOE obtains the *VIX* measure by linearly combining  $\hat{\sigma}_1^2$  and  $\hat{\sigma}_2^2$  for the two expiration dates closest to thirty calendar days with time-to-maturity of  $T_1$  and  $T_2$ , but excluding options with less than seven calendar days to expiry. The interpolated quantity is annualized and quoted in volatility units,<sup>8</sup>

$$VIX = 100 \times \sqrt{\left[ w_1 (\hat{\sigma}_1^2 T_1) + w_2 (\hat{\sigma}_2^2 T_2) \right]} \times \frac{365}{30}, \quad (4)$$

where  $w_1 = \frac{T_2 - T_M}{T_2 - T_1}$  and  $w_2 = \frac{T_M - T_1}{T_2 - T_1}$ , so that, obviously,  $w_1 + w_2 = 1$ .

There are various sources of measurement error in the *VIX*. Jiang & Tian (2005) classify them as follows: (i) truncation errors – the minimum and maximum strikes are far from zero and infinity in equation (3); (ii) discretization errors – piecewise linear functions approximate the integrals in equation (3); (iii) “expansion” errors – a Taylor series expansion is used to approximate the log function in deriving the correction term in equation (3); and (iv) interpolation errors – linear interpolation of the maturities in equation (4).

In the remainder of this paper, our main concern is an additional, and critical, source of *idiosyncratic variation* in *VIX*, namely the procedure to determine the minimum and maximum strikes in computing  $\hat{\sigma}^2$ . The cut-off rule for the OTM options induces a *time-varying* effective strike range that produces spurious breaks in the *VIX* series, entirely unrelated with contemporaneous developments in the underlying return series (*artificial jumps*). In order to quantify the implications of this bias, we formalize the notion of measuring the expected return variation via option prices covering only a limited range of strikes. Specifically, let the indicator function  $I_t(B_1, B_2)$ , for a given pair of barriers  $B_1$  and  $B_2$ ,  $0 < B_1 < B_2 < \infty$ , be defined as,

$$I_t(B_1, B_2) = 1[B_1 \leq S_t \leq B_2],$$

which takes the value of unity if the asset price at time  $t$  is within  $[B_1, B_2]$  and zero otherwise.

Carr & Madan (1998) demonstrate that equation (2) naturally generalizes to the case involving a limited strike range, as the measure then captures the so-called *Corridor Integrated Variance*, or *CIVar*,

$$\frac{1}{T} E^* [CIVar(K_1, K_n)] \equiv \frac{1}{T} E^* \left[ \int_0^T v_u^2 I_u(K_1, K_n) du \right] = \frac{2e^{rT}}{T} \int_{K_1}^{K_n} \frac{Q(T, K)}{K^2} dK. \quad (5)$$

In other words, the (annualized) corridor integrated variance reflects the return variation that is realized only while the asset price is within prespecified barriers. Hence, the use of a limited strike range implies that the return variation over the tail areas,  $[0, K_1]$  and  $[K_n, \infty]$  is not accounted for. Thus, the *VIX* is based on a corridor integrated variance measure rather than the full-blown integrated variance. Moreover, if the relative significance of the return variation in the tails fluctuates substantially, the *VIX* measure becomes *incoherent*, as the index does not refer to a comparable underlying quantity (corridor integrated volatility) at different points in time.

We argue this problem can be greatly alleviated by implementing the Corridor Implied Volatility index (*CX*) as described in Andersen & Bondarenko (2007). This corridor construction explicitly controls the range  $(K_1, K_n)$  in a time-consistent manner to ensure a coherent basis for computing model-free volatility.

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resulting in an upward bias. The second negative term provides an approximate correction for this bias.

<sup>8</sup>Detailed information is available at the URL: <http://www.cboe.com/micro/vix/vixwhite.pdf>.

## 4 The Globalization of VIX

On March 16, 2011, the CBOE and Standard & Poor's announced the formation of a global "VIX Network" of exchanges with agreements regarding the use of CBOE's VIX methodology. The objective is to provide an information-sharing venue for current and potential users of the VIX methodology and to promote VIX as the global standard for measuring market volatility. At the time, agreements related to the use of VIX style measures were in place with the Australian Securities Exchange, the CME Group, Deutsche Börse, the Hong Kong Stock Exchange, the National Stock Exchange of India, Euronext LIFFE, the Taiwan Futures Exchange, and the TMX Group in Canada.

Table 1 summarizes the key features behind some existing and, as of March 2011, impending volatility indices offered by exchanges within the VIX Network. The membership list includes most of the world's premier venues for equity and derivatives trading. It is evident that VIX related measures for equity indices are now a truly global phenomenon. Concurrently, the main U.S. exchanges have initiated VIX measures for individual stocks as well as securities linked to commodity, energy, foreign exchange and global equity exchange traded funds and futures contracts. Moreover, a number of these exchanges have introduced futures and options that trade with these volatility indices as the underlying, thus greatly expanding the set of exchange-traded "pure volatility" products.

From Table 1, it is evident that the methodology behind the index computation already is highly standardized, with exchanges largely being adhering to the CBOE VIX procedure. Some deviations arise from institutional features or liquidity concerns, while others reflect historical conventions. We briefly review the main differences here, while deferring detailed discussions until we have identified the critical aspects of the index construction.

As indicated in column two of Table 1, the CBOE determines the forward rate via put-call parity for the ATM strike. This approach is accurate and robust only if the measurement errors for the quote midpoint of the ATM options are small and the quotes are current. In turn, this hinges on features like the size of the bid-ask spread and market liquidity. As an alternative, a wider set of put-call option pairs may be used to determine the forward rate in a more robust, albeit often also less precise, manner, as noted in the Eurex regulations. Another alternative is to use observed futures prices for the underlying asset in lieu of the forward price, as done by the National Stock Exchange of India.

A second feature is the "risk-free" interest rate used for discounting the option payoffs. The CBOE and CME Group interpolate these rates from U.S. Treasury bill rates, while the other exchanges rely on interbank rates, thus reflecting the costs of unsecured borrowing for major financial institutions. Since the VIX methodology is mostly applied for shorter maturities, this difference has a negligible impact, but for volatility indices covering longer maturities and during periods of financial stress with an elevated gap between interbank and treasury rates, the difference can become meaningful.

The rules governing the range of options included in the index computation are most critical. They are determined by two complementary criteria, namely the explicit conditions on the range *and* the filters invoked to eliminate faulty or excessively noisy quotes. The restrictions on the range, listed in column 4 of Table 1, display interesting variation. The CBOE applies a stopping rule centered on the ATM strike: moving into the OTM region, all options with positive bid quotes are included until two consecutive zero bid quotes are encountered, after which all further OTM options are excluded. This alleviates the noise stemming from low-priced and illiquid options, but it also induces randomness in the effective strike range. In contrast, Eurex eliminates options with a mid-quote below 0.50 index points, while other exchanges, in principle, allow all quoted options to contribute. Finally, the Hong Kong Stock Exchange uses only OTM options with exercise prices within 20% of the ATM strike.<sup>9</sup>

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<sup>9</sup>This is an example of a (very inflexible) corridor implied volatility index – a notion we discuss extensively below.

Table 1: **Volatility Indices Exploiting the VIX Methodology**

Points (1)-(4) refer to the procedures used in computing the VIX, as reported in the CBOE’s White Paper and also detailed in this paper. In summary, (1)  $F_i = K_i^* + e^{r_i T_i} [C(K_i^*, T_i) - P(K_i^*, T_i)]$ , where  $F_i$  is the forward price for the  $i^{\text{th}}$  maturity,  $K_i^*$  is the ATM strike price,  $r_i$  is the annualized risk-free interest rate,  $T_i$  is the time-to-maturity, in years, and  $C(K_i^*, T_i)$  ( $P(K_i^*, T_i)$ ) is the price for the ATM call (put) with maturity  $T_i$ . CBOE defines the ATM strike as the one which minimizes the distance between the call and put price. Moreover, the price of the OTM option is denoted  $Q(K, T) = \min(C(K, T), P(K, T))$ , and “min(Q)” refers to a minimum option price for inclusion in the computation. (1.1) is equivalent to (1), but with the proviso that “if a clear minimum does not exist, the average value of the relevant forward prices will be used instead” to determine  $F_i$ . (2) The annualized yield of the U.S. Treasury bill maturing closest to the expiration dates of the relevant options. (3) Let the OTM put (call) options be sorted in descending (ascending) order according to strike price. The minimum (maximum) strike  $K_{\min}$  ( $K_{\max}$ ) is the OTM put (call) strike closest to  $K^*$  for which the next two consecutive strikes, representing further OTM put (call) options, have zero bid quotes. (4) Finally, delete any remaining OTM options with zero bid prices. The acronym “ETF” denotes an exchange traded fund, while “n.a.” indicates that information is “not available” from material provided by the exchange. “RS” denotes the “Relative Spread,” computed as  $RS = (A - B) / [(A + B) / 2]$ , where  $A$  and  $B$  denote ask and bid option quotes, while “MS” signifies that a “maximum spread” rule must be satisfied for an option quote to be included in the index computation.

<b>Volatility Index Ticker (Underlying Asset)</b>	<b>Forward</b>	<b>Int. Rate</b>	<b>(<math>K_{\min}, K_{\max}</math>)</b>	<b>Filters</b>
<b>CBOE; United States</b>				
<b>Equity Index:</b> VIX (S&P 500), VXV (S&P 500, 3 month) VXN (Nasdaq 100), VXD (DJIA 30), RVX (Russell 2000)	(1)	(2)	(3)	(4)
<b>Individual Stock:</b> VXAPL (Apple), VXAZN (Amazon) VXGS (Goldman Sach), VXGOG (Google), VXIBM (IBM)	(1)	(2)	(3)	(4)
<b>ETF:</b> OVX (Oil), GVZ (Gold), VXSLV (Silver), VXXLE (Energy) VXEEM (Emerging Markets), VXFXI (China), VXEWZ (Brazil) EVZ (Euro-currency), VXGDY (Gold Miners)	(1)	(2)	(3)	(4)
<b>CME Group; United States</b>				
<b>Commodity Futures:</b> OIV (NYMEX Light Sweet Crude Oil), GIV (COMEX Gold), SIV (CBOT Soybean), CIX (CBOT Corn)	(1)	(2)	(3)	(4)
<b>TMX Group; Canada</b>				
<b>Equity Index:</b> S&P/TSX 60 VIX (S&P/TSX 60)	(1)	CORRA/ CDOR	(3)	(4)
<b>Eurex; Eurozone, Switzerland</b>				
<b>Equity Index:</b> VDAX-NEW (DAX, Germany), VSTOXX (Euro Stoxx 50)	(1)	EONIA/ EURIBOR	min(Q)	(4), A=0, MS
<b>Equity Index:</b> VSMI (SMI, Switzerland)	(1)	LIBOR	min(Q)	(4), A=0, MS
<b>NYSE Euronext; Eurozone, United Kingdom</b>				
<b>Equity Index:</b> VAEX (AEX, Netherlands), VBEL (BEL 20, Belgium), VFTSE (FTSE 100, U.K.), VCAC (CAC 40, France)	(1.1)	n.a.	All	(4), RS>50%
<b>ASX; Australia</b>				
<b>Equity Index:</b> XVI (S&P/ASX 200)	(1)	RBA BBSW	All	(4)
<b>HKEX; Hong Kong</b>				
<b>Equity Index:</b> VHSI (HSI)	(1)	HIBOR	(.8K*, 1.2K*)	(4), B≥A
<b>NSE; India</b>				
<b>Equity Index:</b> India VIX (S&P CNX Nifty)	NIFTY Futures	NSE MIBOR	All	(4), RS>30%

The seemingly disparate rules for the option ranges are mitigated by the differences in the filters, summarized in the last column of Table 1. Exchanges that explicitly restrict the range typically only apply mild additional filtering. For example, the CBOE only adds the constraint that any remaining options with a zero bid quote are excluded, while Eurex imposes a maximum spread rule that simply enforces the existing regulations for posting valid quotes in their electronic system. On the other hand, a number of exchanges that, in principle, allow all options to enter the index computation, indirectly eliminate illiquid or low-priced options by imposing a maximum percentage spread rule. This again induces random variation in the range of options exploited for the index calculation across time. The remaining discrepancies mostly reflect institutional differences.<sup>10</sup>

In summary, the CBOE VIX methodology has proven extraordinarily successful. It has become the blueprint for a rapid expansion in volatility indices and products across the globe. It has captured the imagination of the public press and established itself as a gauge for market conditions. The establishment of the VIX Network solidifies the methodology’s position as the worldwide standard for measuring and trading volatility across all asset classes. As such, it is imperative to study the design of the measure and understand its potential limitations for practical applications as well as academic research. We dedicate the remainder of the paper to this task.

## 5 Data

We compute model-free volatility indices from SPX option quotes obtained from the official vendor of CBOE data, Market Data Express, or MDX. The data stem from an electronic feed supplied by two of the lead market makers in the SPX options. The series are based on MDR (Market Data Retrieval) quotes captured by CBOE’s internal system. The underlying tick-by-tick data cover the period June 2, 2008 – June 30, 2010, encompassing 525 individual trading days, and contain a huge number of quotes.<sup>11</sup> For the two maturities involved in the construction of the implied volatility measures, we have an average of about 213,000 (147,000) OTM put (call) option quotes per day, or around 52 ( $33\frac{1}{2}$ ) million OTM put (call) option quotes in total across the full sample.

We construct fifteen second series for each individual option using the “previous tick” method, implying that we retain the last available quotes prior to the end of each 15 second interval throughout the active trading day, from 8:30:15 to 15:15:00 CT.<sup>12</sup> If no new quote arrives in a 15 second interval, the last available quote prior to the interval is retained. Hence, in the case of inactivity, the quotes may become stale. Consequently, for a variety of analysis, we impose a limit on the duration of widespread inactivity as described below. In addition, as a benchmark, we will on occasion exploit the corresponding intraday quotes for the S&P 500 futures from the CME Group.

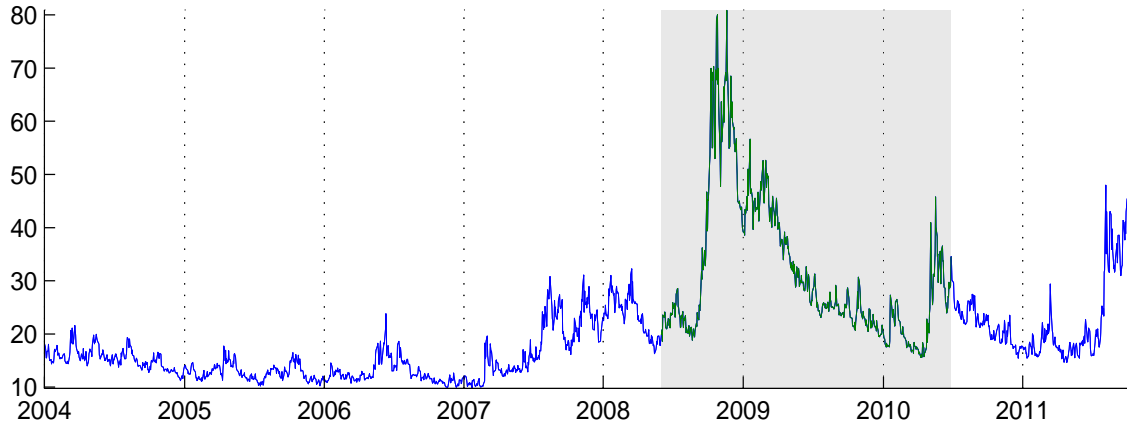
A few filters are applied to guard against data errors. First, we avoid excessive staleness by deeming an option missing if the quote time-stamp precedes the computation of the volatility index by more than five minutes. If quotes for nearby strikes are available, the VIX computation interpolates the option value for this strike during the calculation via the formula in equation (3), so this does not create a gap in the volatility series. Second, if an entire block of adjacent and relevant OTM option prices have not

<sup>10</sup>For example, some exchanges stipulate that the ask quote cannot be lower than the bid quote. Other exchanges enforce this constraint within their electronic quoting system and do not add this requirement to their explicit filtering procedure.

<sup>11</sup>The size of the full data set is around 263 GB, and it is constructed from 525 daily CSV files of an average size of about 0.5 GB each. The files include tick-by-tick quotes and trades for options on the S&P 500 index across all active maturities and strikes. Apart from five Holidays with only partial trading, we include all these days in our empirical analysis. To the best of our knowledge, this is the first time a tick-by-tick option data set of this magnitude has been used in this capacity.

<sup>12</sup>For each 15 second cross-section, we exploit an average of 66 (39) strikes for OTM put (call) options for the first maturity, and an average of 69 (43) strikes for OTM put (call) options at the second maturity.

Figure 2: **Daily Closing Value for VIX, January 2004 – August 2010.** Our sample period, June 2, 2008 – June 30, 2010, is indicated in green and is shaded.



been updated for five minutes, we deem the index itself “not available” (n.a.) and eliminate this time period to avoid errors due to “systemic staleness.” This condition is essential for avoiding artificial jumps in the volatility index when an entire set of stale option quotes are updated simultaneously after a temporary malfunction of the quote dissemination system. Third, we impose a maximum threshold for the degree of no-arbitrage violations implied by the option mid-quotes, and we declare the volatility index n.a. whenever this “non-convexity” threshold is exceeded. We lose close to 1.2% of the 15 second observations due to our filters, with about half of the missing values stemming from the non-systemic staleness requirement and half from the non-convexity condition.<sup>13</sup>

The corresponding fifteen second series for VIX is obtained from different sources although all quotes originate with the CBOE. The first data set, covering June 2008 to September 2008, provides the VIX values at fifteen second intervals and was obtained directly from the exchange. The second source is MDR-VIX which contains tick-by-tick quotes of VIX options for October 2008 to April 2009. This source indicates the value of the underlying VIX index whenever a VIX option quote is recorded, so a complete fifteen second series may be extracted. Finally, the May 2009 – June 2010 period is again covered by the fifteen second VIX series, but now secured via MDX.

We apply only a very mild filter to the intraday VIX series: we remove observations that fall outside the daily high-low range for VIX. Since this range is reported at the end of trading, it may reflect the correction of errors in the real-time VIX values disseminated during the day. We denote this very lightly filtered series  $VIX^*$ . Figure 2 shows the evolution of the daily VIX closing value over recent years, with the green line denoting the sample period exploited in this article.

## 6 Practical Issues with the Computation of the VIX

### 6.1 VIX Replication Indices, RX1 and RX2

We first seek to replicate the official VIX index from the underlying SPX options using the exact CBOE methodology. The initial step involves the computation of  $\hat{\sigma}^2$  for each relevant maturity. However, the forward price,  $F$ , and the strike price just below the forward price,  $K_f$ , in equation (3) are not directly observable. The CBOE determines these basic variables from the available option quotes in three steps.

<sup>13</sup>The procedures for implementing these filters are detailed in the Appendix.

First, for each maturity separately, they identify the strike price  $K^*$  for which the distance between the quoted midpoints of the call and put prices is minimal. This strike is then used to compute the “implied” forward according to put-call parity,  $F^* = K^* + e^{rT} \times [C(K^*, T) - P(K^*, T)]$ . Finally,  $K_f$  is determined as the first available strike price below  $F^*$ . We refer to the VIX index constructed from (3) and (4), using  $F^*$  and the associated  $K_f$ , as our first “Replication index”, or RX1. This index constitutes our direct equivalent of the CBOE VIX, computed according to official rules.

Our analysis has revealed that this feature of the CBOE methodology entails a certain lack of robustness, arising from the use of only a single ( $K^*$ ) put-call option price pair to infer  $F^*$  and  $K_f$ . Occasional problems, like a temporary gap in the quote updating for a subset of options or a basic recording error, can cause the minimum distance between the call and put prices to, erroneously, appear at a point far from the true ATM strike. In turn, this can generate a large bias in the VIX, and may even render its computation infeasible.

Motivated by this finding, we develop a more robust procedure for determining the implied forward. In a first step, for the given maturity, we identify the set of strikes for which the discrepancy between the price of the put and call is below \$25. Secondly, for each of these put-call option pairs, we compute the implied forward price. Third, we designate the “robust implied forward” to be the median of this set of implied forward prices. Using a robust statistic defined over multiple option pairs prunes the implied forward series of major erroneous outliers. On the other hand, the wider set may include less liquid options with large relative pricing errors. Consequently, as our final step, we retain the original  $F^*$  value, rather than the robust forward, unless the two values differ substantially. Following diagnostic checks, we chose a relative threshold of 0.5%.<sup>14</sup> Hence, if the two implied forwards are close, we stick with  $F^*$ , but if they differ by more than 0.5%, we instead adopt the robust implied forward. This procedure generates an implied forward price,  $F$ , that typically coincides with  $F^*$ , but deviates whenever  $F^*$  may contain a sizeable error. The VIX replication index, computed from  $F$ , is denoted RX2. Importantly, it retains the feature that, apart from the risk-free interest rate, it is computed exclusively from CBOE option prices, as is the case for the official VIX. We note that this approach is consistent with the spirit of the forward price computation employed by NYSE Euronext, as indicated in Table 1. Throughout our analysis we include both RX1 and RX2 as candidate VIX measures although the conclusions generally are identical.

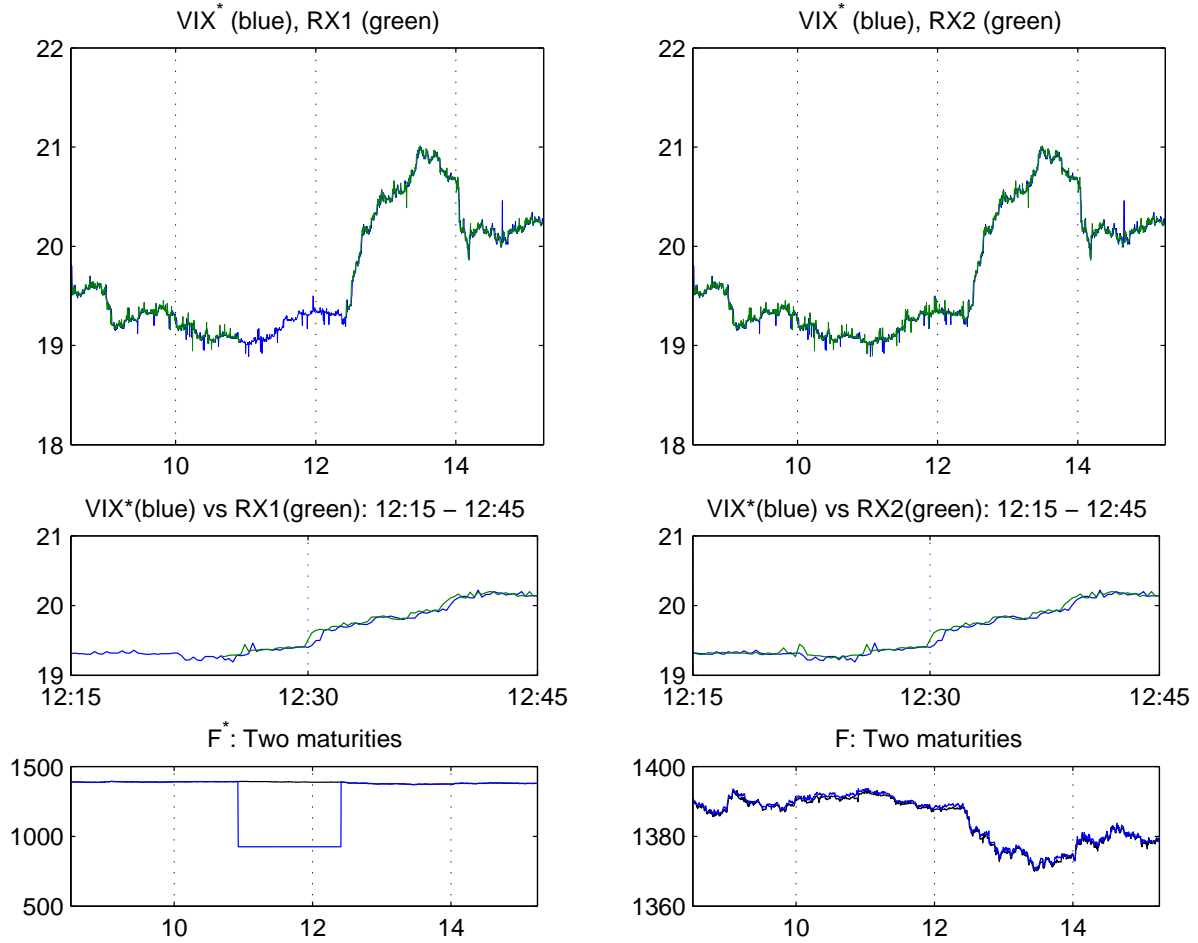
The top panels of Figure 3 depict the evolution of VIX\*, RX1 and RX2 on June 3, 2008. It provides an example of near perfect replication of the VIX index by RX2 and, whenever it may be computed, RX1. The problem with RX1 between 11:00 and 12:30 is that the implied forward price, erroneously, is determined to be below 900 for the second (less liquid) maturity, while it should be close to 1390. This is clearly due to a faulty option quote record and exemplifies, in an admittedly extreme manner, the type of error that can arise from relying on a single option quote pair for determining the implied forward. In this case, it renders RX1 impossible to compute, while there is no problem with RX2. These incidents occur, but are rare, so RX1 and RX2 are almost identical throughout the sample.

Unfortunately, the near ideal replication of VIX\* in Figure 3 is somewhat deceptive. The panels in the middle row of Figure 3 zoom in on a shorter period within the day. Close inspection reveals that the real-time VIX lags our RX series by about 15-30 seconds. The delay is not unique to this period or trading day, but is observed consistently throughout the sample. It is likely caused by capacity constraints of the CBOE server which processes data for a large set of derivatives contracts simultaneously.<sup>15</sup> Since the reporting delay varies over time in ways that, given the nature of the CBOE system, are not observ-

<sup>14</sup>For a level of the S&P 500 around 1000, the threshold is about 5 which is the same order of magnitude as the gap between the strike prices.

<sup>15</sup>This explanation arose from conversations with John Hiatt, Director of Research at CBOE.

Figure 3: **CBOE Forward vs Robust Forward in Replicating VIX.** The top panels depict the RX1, RX2 and VIX\* indices on June 3, 2008. The middle panels depict the same series as the top panels, but they zoom in on a period in the middle of the trading day. The bottom panels plot the implied forward price for the first and second maturity, according to the CBOE methodology ( $F^*$ ) and the more robust procedure ( $F$ ), respectively.



able and cannot be reconstructed ex post, it is infeasible to construct a record of the actual quotes used for the real-time VIX computation. Consequently, it is impossible to match the high-frequency VIX values perfectly, even using the official methodology. Moreover, the discrepancies arising from this delay can be non-trivial, even within a fairly calm trading environment like this. For example, just after 12:30, the RX2 measure rises quickly from 19.4% to above 19.6%. This will mechanically, given the delay, create a proportional gap relative to VIX\* of about 0.75%, even if the two indices are constructed from the identical option prices. Moreover, this type of deviation, due purely to a dissemination lag, becomes more prominent in times of rapidly shifting volatility, or high volatility of volatility, when the VIX index is of particular interest as a gauge of evolving market conditions.

While the above issues raise some concerns regarding the integrity of the real-time VIX series, they are probably not – in and of themselves – sufficient to cause major reservations. First, large discrepancies due to the implied forward price are rare and readily handled. Second, some real-time delays in the dissemination are to be expected, and if market movements are reflected correctly with only a small delay the associated distortions of the statistical properties of the index will be minor.

Third, such problems constitute much less of a problem for assessing the volatility dynamics at lower frequencies. In particular, the typical shift in volatility over 15-30 seconds is minuscule compared to those observed over daily horizons, so the relative errors due to the delay are trivial for daily data. This does not imply that such discrepancies are immaterial, but rather that their importance hinges on the intended use of the volatility measure.

Our main concerns about the VIX index reside elsewhere and run deeper as we explain below.

## 6.2 A Broader VIX Replication Index, RX3

In principle, all (available) options should be exploited in computing a model-free volatility index and hence also the VIX. However, this strategy faces some practical problems. First, far OTM options are near worthless and trade infrequently, if at all. Second, partially as a result, the relative bid-ask spread is very high for far OTM options and even the midpoint quotes may be poor indicators of underlying value. As discussed previously, the CBOE adapts to this feature by invoking a specific cut-off rule: moving away from the forward price, once two consecutive strikes with zero bid quotes are encountered, all further OTM options are discarded. Hence, market liquidity and pricing jointly determines the cut-off level. The rule has intuitive appeal. For example, as volatility rises and option (bid) quotes increase, then additional, and newly valuable, options are included in the VIX calculation. As a result, the computation should generally include all options with non-trivial (bid) valuations and it should avoid the noise and distortions associated with far OTM options which, in theory, have only a limited impact on the overall measure. On the other hand, this introduces the potential for random variation in the strike range that may complicate the comparison of VIX measures across time and distort the time series properties of the index.

We rely on the concept of an *effective range*, or ER, to gauge the coverage of options in the VIX computation. Formally, the effective range is defined as,

$$ER = \left[ \frac{\ln(K_1/F)}{\hat{\sigma}_{BS} \sqrt{T}}, \frac{\ln(K_n/F)}{\hat{\sigma}_{BS} \sqrt{T}} \right], \quad (6)$$

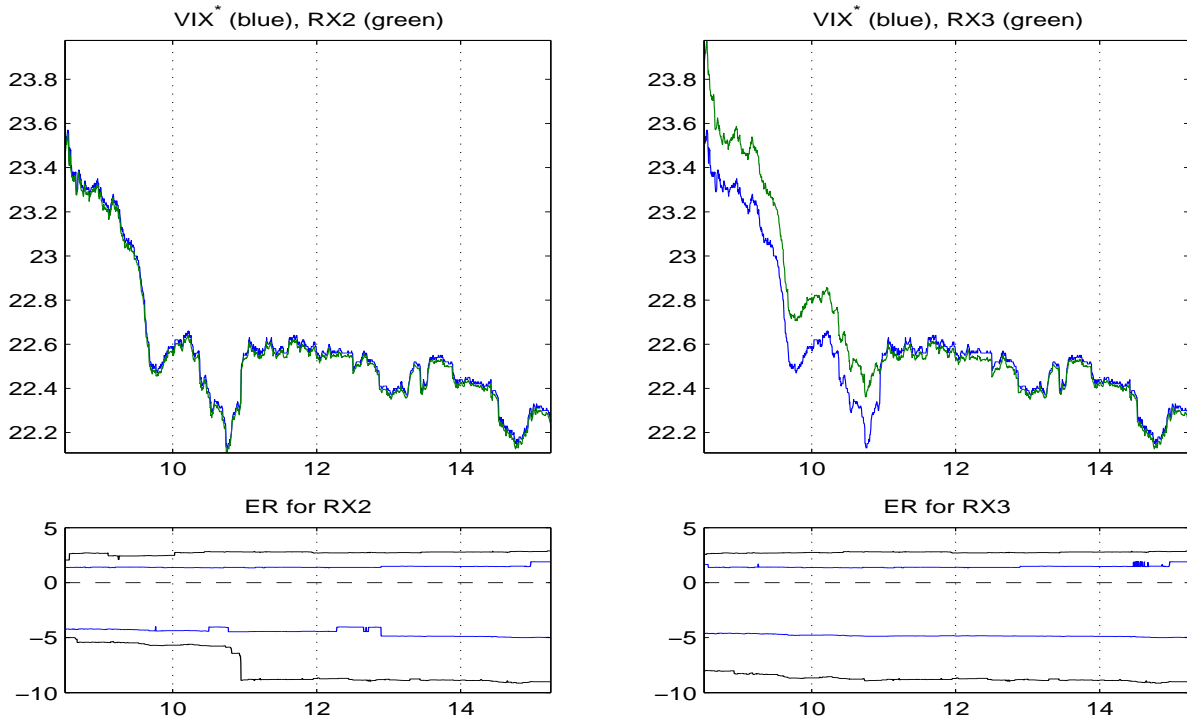
where  $\hat{\sigma}_{BS}$  is an ATM implied Black-Scholes volatility measure, obtained from a linear interpolation of four Black-Scholes implied volatilities, extracted from options with strike prices just above and below the forward price for the two maturities closest to 30 calendar days.

The effective range reflects the degree of coverage afforded by the given set of options, controlling for the concurrent level of volatility. It allows us to assess whether the CBOE stopping rule produces a coherent strike range across time for the computation of VIX. If we instead ignore the stopping rule, and exploit all options with positive bid quotes, we obtain a broader range, reflecting the *maximum* coverage offered by the prevailing market liquidity. We denote this broader index RX3. As for RX2, we rely on the robust forward price,  $F$ , in computing RX3. It should provide an upper bound on the VIX values reported by the CBOE. Moreover, whenever there are no positive bid quotes for OTM options beyond the CBOE truncation point, RX2 and RX3 coincide. We also note that, as indicated in Table 1, the use of all (valid) option quotes is consistent with the conventions adopted by NYSE Euronext, and the main exchanges of Australia and India.

Figure 4 illustrates why the strike range underlying the VIX computation may be of concern. On February 16, 2010, VIX starts out, at 8:30, around 23.5 which is consistent with RX2, but more than two percent below the RX3 value of 24. As the trading day progresses the gap between VIX and RX3 shrinks to less than half the original size by 10:30, but the discrepancy is not closed until just prior to 11:00, when the VIX and RX2 indices “jump” up to RX3. After this point, both RX2 and RX3 replicate



Figure 4: **RX2, RX3 and VIX\***. The top panels depict the RX2, RX3 and VIX\* volatility indices for February 16, 2010. The bottom panels display the effective strike ranges used by RX2 and RX3 throughout the day, separately for the first (black) and second (blue) maturities.

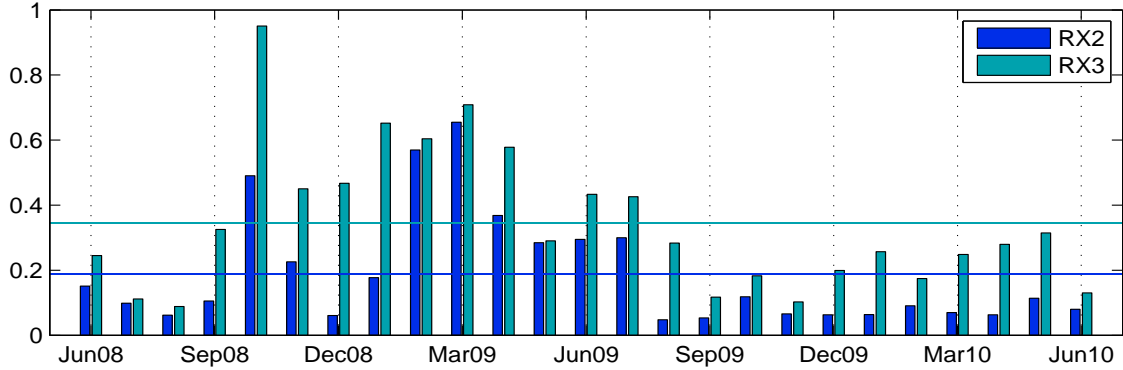


the VIX series closely for the remainder of the trading day.

At some level, the example represents a successful high-frequency replication of VIX. Following the official rules, RX2 closely mimics the real-time evolution of the index. On the other hand, the most notable high-frequency feature of VIX throughout this trading day is the “jump” just prior to 11:00. In a matter of 15 seconds, the volatility index increases by about one percent. However, had we used all available options to compute the VIX index, i.e., relied on RX3, the series would barely have moved at this point – there is simply no indication of a discontinuity in RX3. The explanation is apparent from the bottom panel of Figure 4, where we notice a dramatic expansion in the lower part of the strike range (OTM put options at the nearby maturity) used for computing RX2, exactly at the point of the jump. Moreover, there is a drift towards a larger strike range between 8:35 and 10:50 which coincides with the slowly narrowing gap between VIX and RX3 over this period. In contrast, the RX3 measure is computed from an near invariant strike range throughout the trading day. As a result, the evolution of RX3, almost exclusively, reflect shifts in the underlying option prices and thus provide a direct gauge on the change in the (implied) volatility expectations. In other words, variation in the effective strike range was an important driver behind the observed high-frequency movements of the VIX (and RX2). Based on the alternative RX3 measure, we can only conclude that the observed jump in VIX is “dubious.”

In summary, there are examples for which the cut-off rule for the strike range determined by the CBOE appears not to produce the intended result. Using the full range of available strikes would likely have been preferable for the particular day explored above. The issue is whether this is an atypical case. The next section explores more thoroughly how well VIX may be replicated by RX2 and RX3.

Figure 5: **Mean Absolute Percentage Approximation Error.** Mean Absolute Percentage Error (MAPE) for the RX2 and RX3 indices relative to VIX\*. The bars represent the monthly MAPE, while the straight lines indicate full sample values.



### 6.3 The Coherence between the VIX and RX Indices

We have argued that perfect replication of the high-frequency VIX series is infeasible due to a time-varying dissemination delay at the CBOE. However, as long as the VIX and RX2 indices are based on the same methodology and a similar set of underlying option quotes, the average discrepancies should be minor, even if they can be non-trivial during high volatility periods. Figure 5 depicts the Mean Average Percentage Error (MAPE) for the high-frequency RX2 and RX3 series relative to VIX\*, where

$$MAPE = 100 \times \frac{1}{T} \sum_{t=1}^T \left| \frac{VIX_t^* - RX_t}{VIX_t^*} \right|, \quad RX = RX2, RX3.$$

As expected, the deviations are larger in the depth of the financial crisis. Nonetheless, the overall coherence is good, with an average discrepancy of about 0.35% for RX3 and less than 0.20% for RX2. Moreover, it is 0.1% or less for RX2 over the last eleven months of the sample, when market conditions were less turbulent.

We also compute the fraction of time the indices coincide versus differ, using a 0.50% (0.25%) relative deviation as the criterion for satisfactory replication. The numbers are reported in the first (second) two columns of Table 2. The first column reflects the fraction of the 15-second VIX observations that are replicated within the given threshold for accuracy, while the second column provides the corresponding number for the VIX based on end-of-day observations. The latter measure constitutes the daily (closing) value of the VIX and is available from the CBOE web-site.

Within the trading day, the VIX\* is matched by RX3 about 81% (64%) of the time, while the corresponding number for the end-of-day indices is 79% (62%).<sup>16</sup> Consistent with the fact that RX2 is designed to explicitly mimic the CBOE cut-off rule, this index achieves an even higher degree of replication of 90% (80%). Hence, most of the time the VIX\* values are compatible with having been computed from all quoted options across both maturities and coincides with both RX indices. However, in 20% (around 35%) of the cases we cannot provide a close replication of the VIX index by RX3 and in about half of these instances the discrepancy can be directly attributed to the CBOE cut-off rule, as RX2 provides acceptable replication. This still leaves close to 10% (20%) of the observed VIX

<sup>16</sup>These values are obtained by adding to entries in the first two rows of Table 2, whereas the replication frequency of VIX\* by RX2 is given by the sum of the entries in row one and three of the table.

Table 2: **The Coherence between the VIX and RX Indices.** The table reports the fraction of cases for which the VIX\* is matched by RX2 and/or RX3 indices at a given threshold for relative accuracy. Statistics are reported separately for the 15-second (intraday) and end-of-day (EOD) cases.

	Accuracy of 0.50%		Accuracy of 0.25%	
	15-sec	EOD	15-sec	EOD
$VIX \approx RX2, VIX \approx RX3$	0.80	0.77	0.63	0.60
$VIX > RX2, VIX \approx RX3$	0.01	0.02	0.01	0.02
$VIX \approx RX2, VIX < RX3$	0.10	0.13	0.17	0.18
Unmatched	0.09	0.08	0.19	0.20

values unaccounted for.<sup>17</sup> It is also striking that the percentages are very similar for the end-of-day VIX values. Hence, our findings for the 15-second frequency carry over to the daily VIX measure!

The descriptive statistics in Table 2 suggest that the VIX series oscillates, sometimes abruptly, between the RX3 benchmark and lower values, which may or may not be consistent with RX2. This is exactly what we observed in Figure 4 and it is, indeed, a pattern observed across many other days. Figure 6 further illustrates the phenomenon. Here, the pattern is reversed relative to Figure 4, as VIX is matched by both indices during most of the early trading, but later drops abruptly along with RX2. Thus, at the close of trading, VIX is significantly below RX3 due to the smaller effective strike range dictated by the CBOE stopping rule. For both February 16 and March 2, 2010, it is evident that the “jumps” in the VIX, up to or away from RX3, are driven by sudden shifts in the effective range and not by any coincident change in volatility. Moreover, RX3 is computed from a largely invariant strike range across these days and it evolves quite smoothly throughout.

In summary, considering the inevitable discrepancies arising from the CBOE dissemination delay, we are for the most part able to replicate the VIX index with reasonable precision. Nonetheless, we do identify troublesome features. Most importantly, the VIX, on occasion, displays large discrete changes solely due to abrupt shifts in the range of underlying option strikes. The RX3 measure is less susceptible to such errant movements and may be a more reliable volatility gauge than the VIX series. However, since RX3 uses all available strikes it will be impacted by the relatively large bid-ask spread, or noise, associated with far OTM options. Moreover, the effective strike range for RX3 may also, occasionally, vary significantly with overall option market liquidity, thus generating the same type of problem as observed for VIX\* and RX2. We can only assess the extent of this problem by explicitly controlling for the effective range used in the computation of the index.

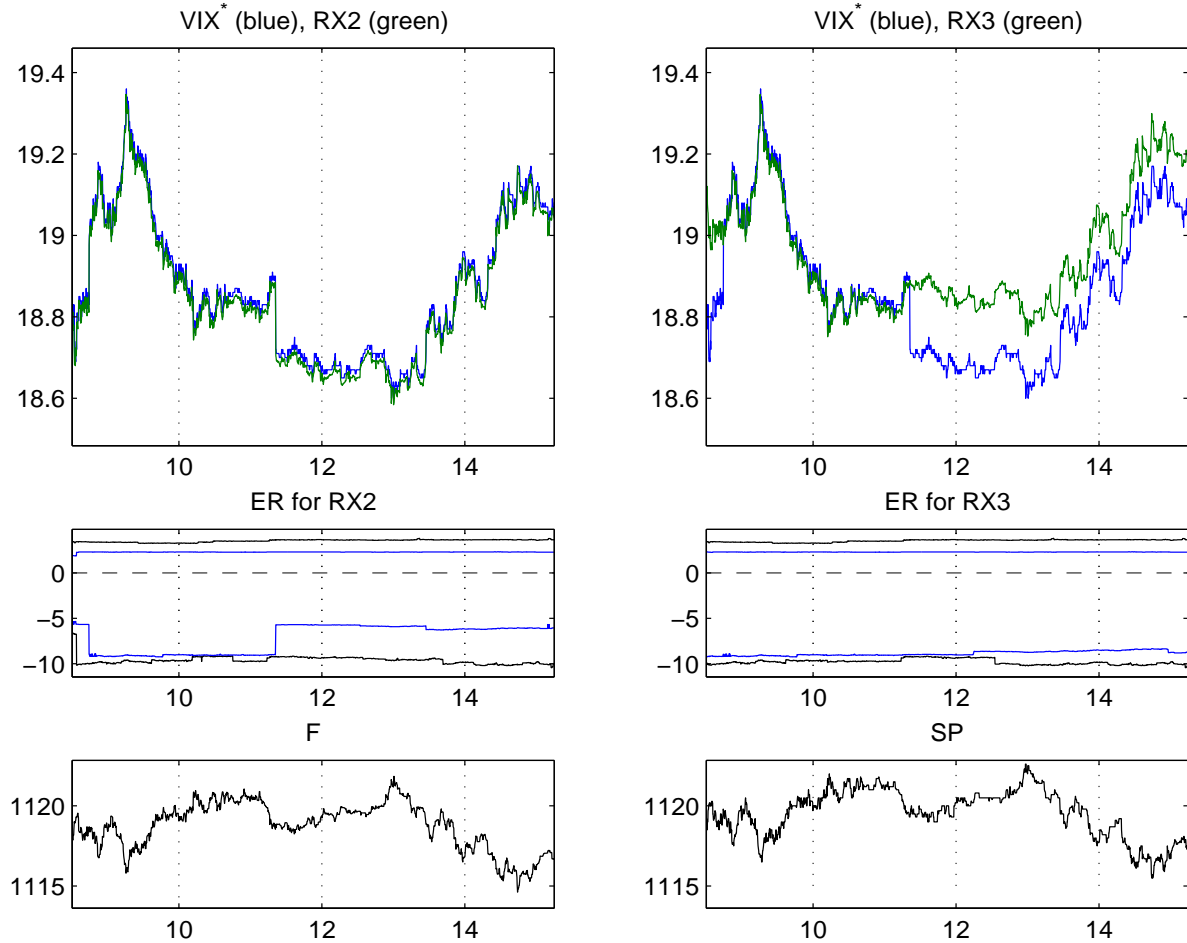
## 7 Corridor Implied Volatility, or CX, Indices

### 7.1 Defining Corridor Volatility

We turn to the concept of corridor implied volatility, introduced by Carr & Madan (1998) and explored empirically in Andersen & Bondarenko (2007), to assess how variation in the strike range impacts the

<sup>17</sup>We performed a few robustness checks to further gauge these relationships. Most importantly, as discussed in connection with Figure 3, there is a time-varying delay, averaging between 15 and 30 seconds, in the reporting of the official VIX. Consequently, any abrupt change in volatility will induce an artificial, short-lived deviation between the VIX and RX series. Hence, we explored the match between the VIX\* and 15 or 30 second lagged RX series. While the matched observations increased, the effect was marginal. Thus, our count of unexplained deviations may be mildly inflated relative to the “true” discrepancies but it is a minor bias.

Figure 6: **RX2, RX3 and VIX\***. The top panels depict the RX2, RX3 and VIX\* volatility indices for March 2, 2010. The middle panels display the effective strike ranges used by RX2 and RX3 throughout the day, separately for the first (black) and second (blue) maturities. Finally, the bottom panels indicate the implied forward (left), computed using a linear interpolation of the implied forward for the two maturities, and the CME Group S&P 500 futures (right).



volatility measures. The point is, a priori, to fix the range of strikes at a level that provides broad coverage but avoids excessive extrapolation of noisy or non-existing quotes for far OTM options. Since it operates with a fairly narrow range, the Corridor Index, or CX, will usually be lower than RX3. However, the invariant strike coverage ensures that the measure is coherent in the time series dimension and alleviates the variation induced by idiosyncratic shifts in the effective range.

The main issue is how to define an economically invariant portion of the strike range which ensures that the associated implied volatility measures are compatible over time. We emphasize a few desirable features. First, the corridor should be linked to observable option prices to ensure transparency and facilitate real-time computation. Second, the forward price constitutes a focal point by determining the ATM strike and thus identifying the set of OTM options to use in the calculation. Thus, it is natural to construct a metric which associates the forward rate with the 50<sup>th</sup> percentile of the range and allows us to define the corridor relative to this reference point. Third, the range should endogenously adjust over time to accommodate variation in the relative importance of strikes in the tail of the distribution.

One common approach is to measure moneyness relative to the forward price via the distance

measured in Black-Scholes (BS) ATM implied volatilities. However, this relies on a model-dependent, and clearly misspecified, assumption of constant volatility and log-normality. In particular, the Black-Scholes volatility fails to adapt suitably to variation in the shape of the risk-neutral distribution (RND) caused by shifts in the skewness or by asymmetric variation in the tail thickness.<sup>18</sup> Another common approach is to rely on the percentiles of the RND. Unfortunately, percentile corridors have significant drawbacks. First, they are not centered on the forward price. This is particularly troublesome for equities due to the pronounced skew in the RND. In fact, across our sample, the forward price is located anywhere between the 38<sup>th</sup> and 47<sup>th</sup> percentiles of the distribution with a mean around the 43<sup>th</sup>. Second, RND percentiles are not reliable indicators of the importance of the associated strikes, as time-varying tail-thickness has a first order impact on option prices. Finally, estimation of the RND requires auxiliary assumptions that render the computations less transparent.<sup>19</sup>

In response to these shortcomings, we adopt an alternative approach developed in Andersen & Bondarenko (2010). It builds on the fact that option prices reflect **tail moments**. Hence, by defining the truncation criterion directly via tail moments, we ensure that the option prices enter explicitly into the procedure. The left and right tail moments of a positive random variable,  $x$ , with strictly positive density  $f(x)$  for all  $x > 0$ , are given by,

$$LT(K) = \int_0^K (K-x)f(x)dx \quad \text{and} \quad RT(K) = \int_K^\infty (x-K)f(x)dx. \quad (7)$$

We define the **ratio statistic**,  $R(K)$ , as an indicator of how far in the tail a given point,  $K$ , is located within the support of  $x$ ,

$$R(K) = \frac{LT(K)}{LT(K) + RT(K)}. \quad (8)$$

We note that  $R(K)$  constitutes a valid cumulative density function, or *CDF*, as it is strictly increasing on  $(0, \infty)$  with  $R(0) = 0$  and  $R(\infty) = 1$ . Moreover, it is centered on the mean of  $x$  as  $R(\bar{K}) = \frac{1}{2}$ , where  $\bar{K} = E[x] = \int_0^\infty x f(x) dx$ . Second, for a given point, or percentile,  $q$ , in the range of the  $R(K)$  function, we define the quotient,  $K_q$ , as

$$K_q = R^{-1}(q) \quad \text{for any } q \in [0, 1]. \quad (9)$$

Now, letting  $f(x)$  denote the risk-neutral density for the S&P 500 forward price for the maturity date  $T = \frac{1}{12}$ , or one month ahead, and  $K$  be the strike price of European style put and call options, we obtain the following expression,

$$R(K) = \frac{P(K)}{P(K) + C(K)}. \quad (10)$$

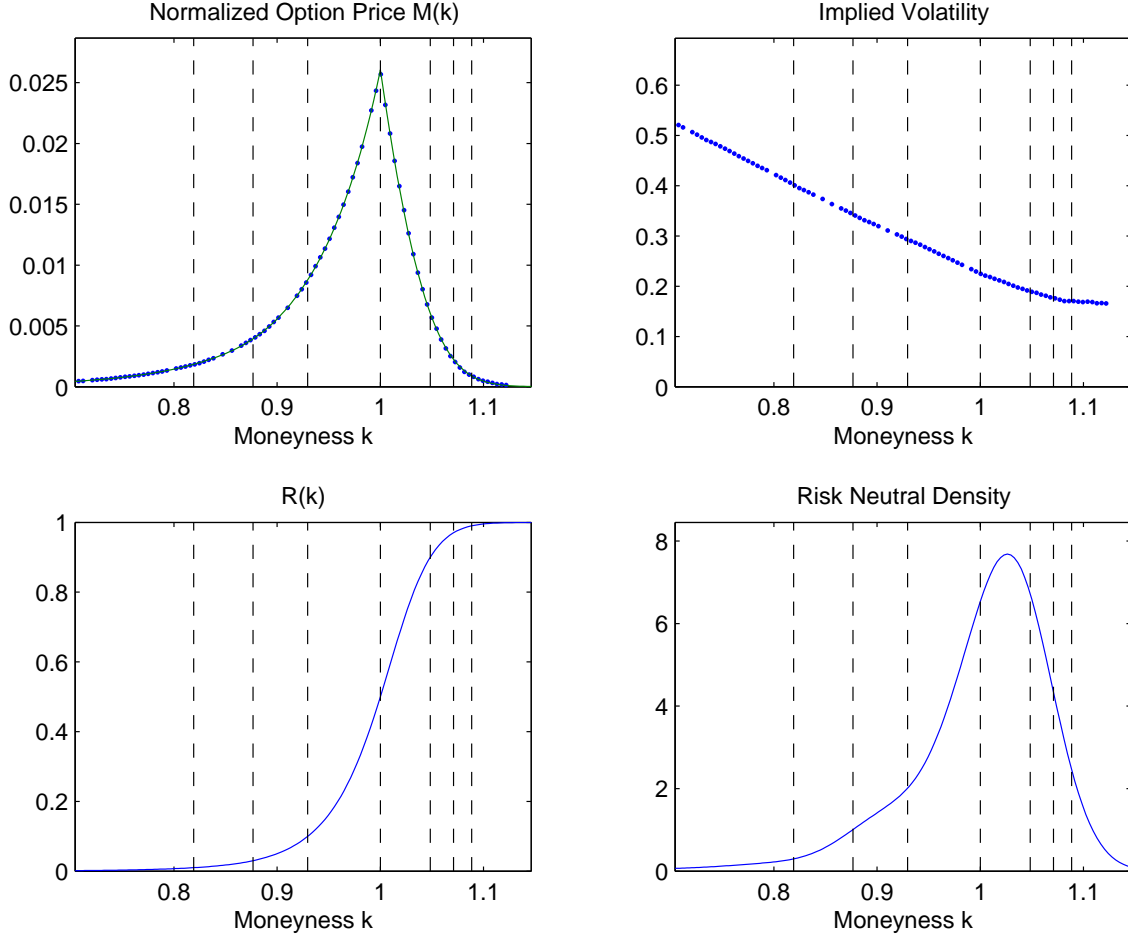
Hence, the ratio statistic may be computed directly from the prices of put and call options. Thus, as long as we stay within the range where option prices may be extracted reliably from the available quotes,  $R(K)$  is trivial to compute, and it is independent of any estimates for the underlying RND.

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<sup>18</sup>As noted in Table 1, the Hong Kong Stock Exchange uses a corridor defined in terms of the moneyness of the strikes. This corridor is unresponsive to shifts in volatility and does not adapt to variation in the shape of the RND at all. As a result, it will cover a highly varying proportion of the underlying RND over time, and produce an intertemporally incoherent implied volatility measure.

<sup>19</sup>The convention of defining corridors via the percentiles of the RND was adopted by Andersen & Bondarenko (2007), so the approach developed below is novel.

Figure 7: **Determination of the CX Truncation.** The top left panel displays the normalized out-of-the-money option prices at the end of trading on June 16, 2010 for the thirty day maturity. Moneyness is defined as  $k = K/F$ , while the normalized option prices are given as  $M(k) = Q(T, K)/F$  and  $Q(T, K) = \min(C(T, K), P(T, K))$ . The top right panel depicts the corresponding Black-Scholes implied volatilities. The bottom right panel provides an estimate of the corresponding risk-neutral density, whereas the left bottom panel shows the extracted  $R(k)$  function. The vertical dashed lines indicate the 1, 3, 10, 50, 90, 97 and 99 percentile quotients of  $R(k)$ .



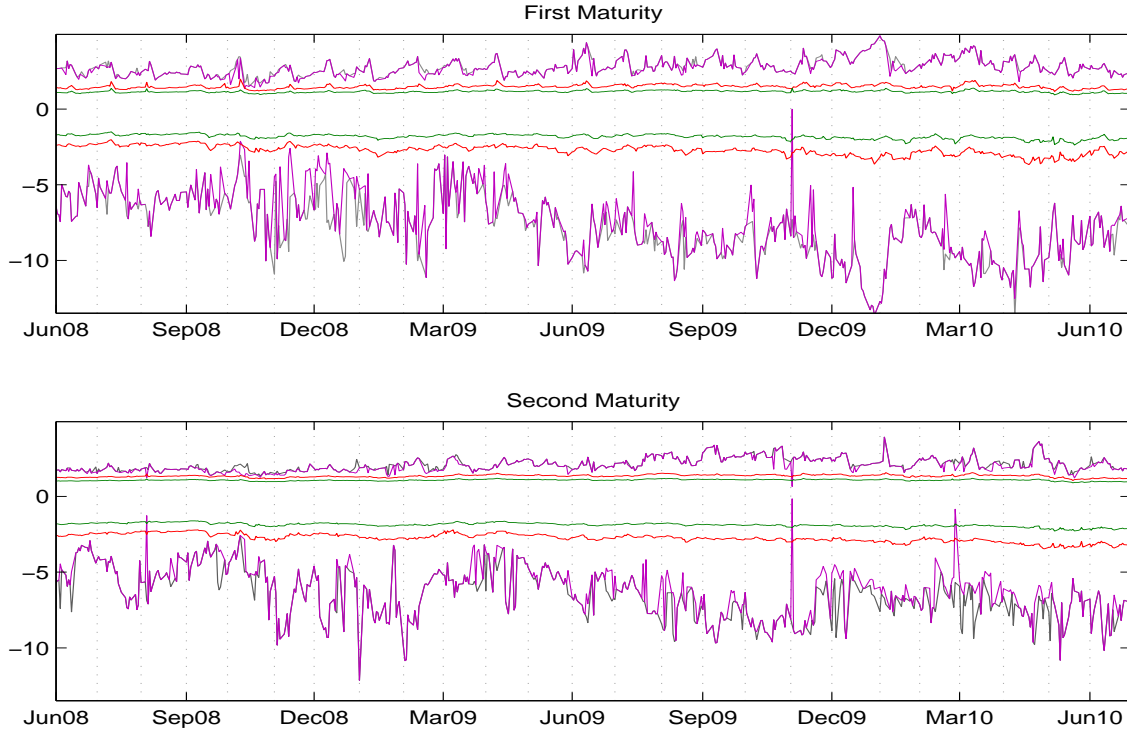
Moreover, the median, or 50<sup>th</sup> percentile, of  $R(K)$  corresponds to the expected value of the S&P 500 index at  $T$ , as the forward rate equals the mean of the RND,

$$\bar{K} = F \quad \text{and} \quad K_{0.50} = R^{-1}(0.50) = F. \quad (11)$$

This implies that the forward price,  $F$ , separating the strike range into OTM put and call regions, also represents the median of the  $R(K)$  function. Consequently, the percentiles of the ratio function conveniently center the strike range on the focal point for the computation of model-free implied volatility. This is particularly useful when considering a set of nested CX measures as we do below.

It is natural to define the truncation points for the implied volatility computation via symmetric percentiles of the  $R$  function, so that the truncation points reflect the relative importance of the right and left tails for option pricing in a consistent manner across time. For example, we may use the left

Figure 8: **Effective Strike Range for End-of-Day Implied Volatility Measures.** The effective strike range, expressed in Black-Scholes at-the-money volatilities, is shown for various indices at the end-of-the-day, separately for the first and second maturities used in the calculation. Moving away from zero, indicating the at-the-money strike, the lines correspond to CX2, then CX1, RX2, and finally RX3.



and right truncation strikes obtained as,

$$K_{0.01} = R^{-1}(0.01) \quad \text{and} \quad K_{0.99} = R^{-1}(0.99). \quad (12)$$

As illustrated in Figure 7, option prices across a wide range of strikes are readily deduced from the available quotes. Only towards the tails of the RND are option prices, and thus the distribution function itself, hard to ascertain with precision. This issue is circumvented by the  $R$  function which only depends on the option prices inside the truncation points. Specifically, we construct the CX1 and CX2 series by exploiting the  $[0.01, 0.99]$  and  $[0.03, 0.97]$  quotients for the  $R$  function, respectively.

## 7.2 Assessing the Stability of the Corridor Strike Range

We have established that shifts in the effective strike range can be an important source of idiosyncratic jumps in implied volatility measures constructed along the lines of the VIX. This motivates our introduction of the CX indices which compute volatility measures from a consistent span of option strikes over time. This section compares the stability of the effective strike ranges associated with alternative MFIV indices. For conciseness, we focus on the strike coverage at the end of the day. We verified, in Table 2, that this period is representative of the behavior across the entire trading day. Hence, Figure 8 depicts the end-of-day coverage for various indices measured in terms of implied ATM Black-Scholes volatilities.

First, we observe the pronounced asymmetry in the coverage of OTM puts and calls, with the put

range typically being more than double the call range. This reflects the relative importance of the OTM put versus call options for pricing volatility. Second, we note that the variation in the strike coverage of the RX measures is dramatic, especially in contrast to the stability attained by the CX measures. This suggests that corridor indices will provide superior intertemporal coherence in the volatility measures. Third, comparing the individual RX measures, we note that RX2 and RX3 have largely matching outliers, but RX2 sports more frequent inlier observations which arise from an intermediate truncation of the RX2 strike coverage. Since such “premature” truncation has a significant impact on implied volatility measures because closer-to-the-money options carry a higher price tag, the RX3 series may be more reliable than RX2 in terms of capturing fluctuations in market volatility. Nonetheless, the strike coverage of RX3 remains extraordinarily variable, with the downside truncation point ranging across  $[-3, -13]$  at the nearby maturity and  $[-2\frac{1}{2}, -12]$  at the longer maturity. In comparison, the variation in the downside truncation level for CX1 and CX2 is minuscule, falling within the intervals  $[-1\frac{1}{2}, -2\frac{1}{2}]$  and  $[-2, -3\frac{1}{2}]$ , respectively.

We next explore whether the variation in the effective range for RX3 has a systematic and economically meaningful effect via a simple regression based test. The dependent variable,  $Z_t$ , captures the relative value of RX3 and CX2 at the end of trading day  $t$ ,  $Z_t = RX3_t/CX2_t - 1$ , and the explanatory variable,  $DR_t$ , denotes the difference in the strike range of RX3 and CX2.<sup>20</sup>

$$Z_t = 0.0340 + 0.0089 DR_t, \quad \bar{R}^2 = 67.2. \quad (6.76) \quad (11.36)$$

The robust t-statistics below the regression coefficients reflect Newey-West HAC standard errors based on twenty lags. They indicate that the discrepancy in the effective strike range is overwhelmingly significant and the adjusted regression  $R^2$  signifies that this single variable has very strong explanatory power for the observed variation in the relative size of RX3 and CX2. The standard deviation of  $DR_t$  is 1.88, so the regression implies that an exogenous increase in the effective range for RX3 by one standard deviation will inflate the relative value of RX3 compared to CX2 by an additional 1.7% ( $1.88 \cdot 0.89\% = 1.7\%$ ). This suggests that purely idiosyncratic variation in the strike range, unrelated to shifts in the underlying option prices, induces economically significant distortions in RX3.

We can estimate similar regression for the index  $VIX^*$  with  $Z_t = VIX_t^*/CX2_t - 1$ . However, in this case we do not observe the range of  $VIX^*$  directly, so we proceed as follows. Define

$$D2_t := \frac{RX2_t}{VIX_t^*} - 1, \quad D3_t := \frac{RX3_t}{VIX_t^*} - 1.$$

We only estimate the regression on those days for which either (1)  $DR2 \leq 0.5\%$ , or (2)  $DR3 \leq 0.5\%$ , or (3) both (1) and (2) applies. The “implicit” range  $DR_t$  is then defined as  $DR2_t$  for cases (1) and (3), and as  $DR3_t$  for case (2). We then obtain,

$$Z_t = 0.0331 + 0.0091 DR_t, \quad \bar{R}^2 = 66.8. \quad (6.12) \quad (11.01)$$

The standard deviation of  $DR_t$  is 1.86, so the regression implies that an exogenous increase in the implicit effective range by one standard deviation will inflate the relative value of  $VIX^*$  compared to CX2 by an additional 1.7% ( $1.86 \cdot 0.91\% = 1.7\%$ ).

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<sup>20</sup>The effective strike ranges are first constructed for each of the two nearby maturities and each index. These ranges are then linearly interpolated to provide the effective 30-day effective range for each measure. Finally,  $DR_t$  is the difference between these two effective strike ranges for the 30-day maturity.



In summary, the variation in the strike range of the various implied volatility measures, relative to the range of the CX2 index, has overwhelming explanatory power for the relative size of the index values. In other words, idiosyncratic variation in the strike range induces a highly significant distortion in associated volatility measure.

### 7.3 A Pair of Illustrative Trading Days

Figure 9 depicts the evolution of the indices during a volatile day, October 14, 2008, where the intraday range of the VIX spans values below 50 and above 60 percent. There is an apparent problem with the real-time computation of the official index, as VIX\* is frozen at an inexplicably high level of about 55% from 8:30-8:40, only to later drop to a similarly incomprehensibly low set of values around 46% and 47%. These observations cannot be reconciled with the contemporaneous values for RX2 and RX3. After additional wild gyrations, the VIX\* values start roughly mimicking the RX2 index in the period 9:40-12:55 and then, after an apparent jump at 12:55, coinciding reasonably well with both RX2 and RX3 until about 14:30. Finally, from this point onwards, RX2 drops below RX3 while VIX\* remains fairly close to RX3 until the market close. The reason for the “jumps” in RX2 at 12:55 and after 14:30 is readily identified from the second panel as the effective strike range for the nearby maturity widens and then narrows later in concert with the shifts in the volatility index level.

Even disregarding the problems during the early parts of the trading day, the VIX series appears “schizophrenic” as it first is aligned with RX2 over 9:40-12:55 and then roughly follows the RX3 index for the remainder of the day. Such random oscillation between two distinct volatility levels induces artificial VIX breaks unrelated to any features of the underlying equity index. Of course, the identical reasoning applies to the RX2 series, as it is subject to the same jump-like behavior due to shifts in its strike range. Finally, the RX3 index indicates a strongly elevated volatility level over 10:05-10:25 which again may be attributed to an expansion of the associated strike range for the nearby maturity. This “jump” increases RX3 by close to 4%, while none of the other indices displays any major discontinuity. In short, the RX3 measure is also vulnerable to idiosyncratic shifts in the strike range. In contrast, the CX2 measure evolves continuously, albeit somewhat erratically, throughout the trading day. Moreover, the qualitative pattern mirrors the movements in the RX indices apart from the absence of glaring, and suspicious, jumps.<sup>21</sup>

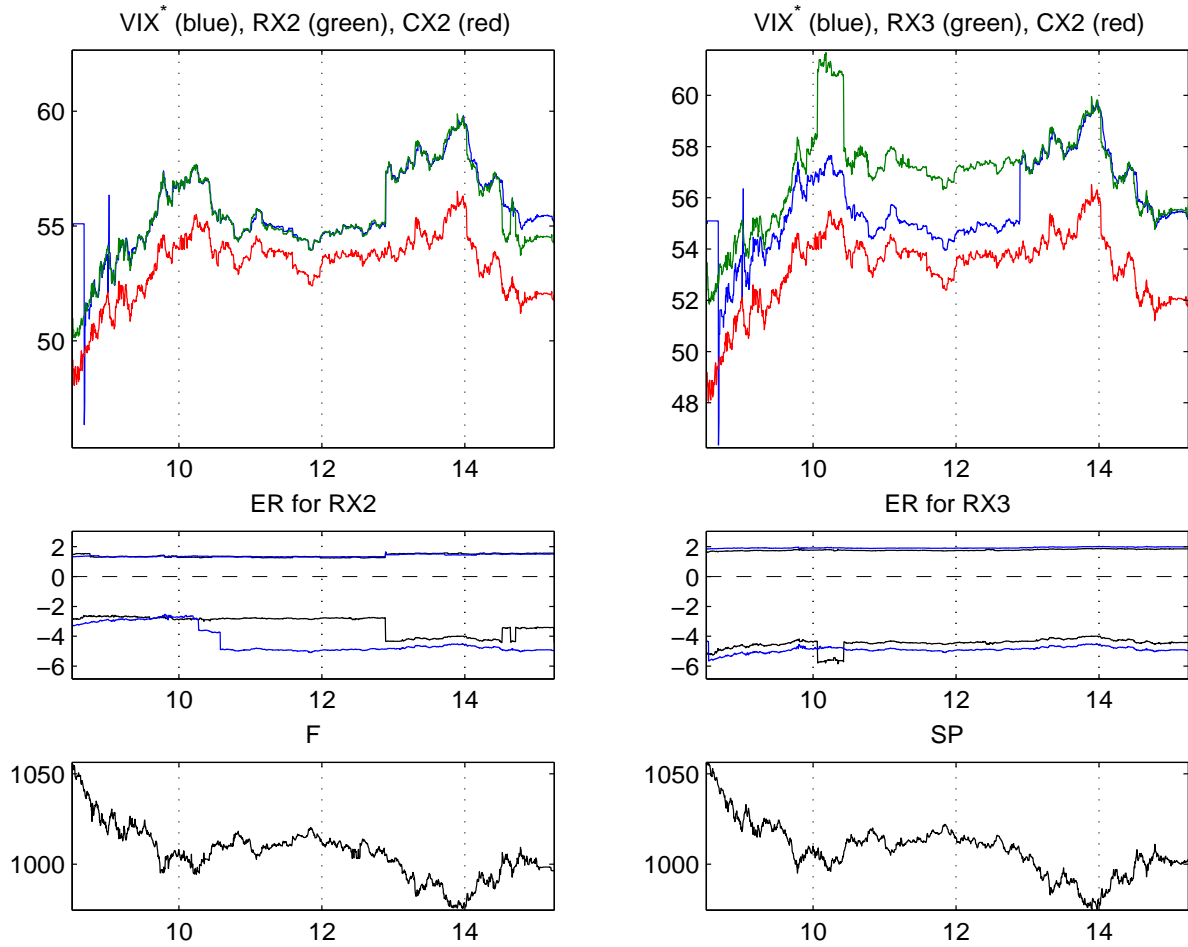
Figure 10 depicts a more typical trading day with a distinctly lower volatility level, namely February 9, 2010. In this case, RX2 generally replicates the official VIX series very well, although there are a number of striking outliers in both the RX2 and VIX\* series between 11:00 and 11:30, and a fairly abrupt drop in RX2 around 3:00. In contrast, the RX3 index is more erratic between 10:50 and 11:45 and then again after 15:00. Not surprisingly, over these exact time spans, there is a widening in the strike range for RX3. As before, CX2 appears to capture the qualitative features in the evolution of volatility satisfactorily while avoiding any abnormal jumps. The only sharp movement in the CX2 series, just after 11:40, is associated with a contemporaneous drop in the S&P 500 forward price, as indicated in the bottom panel, and thus likely represents a genuine change in implied volatility.

These illustrations suggest that the CX measure may provide a superior measure of the intertemporal variation in volatility relative to the officially released VIX series or our replication indices, RX2 and RX3. We now turn towards a more comprehensive investigation of the alternative volatility indices.

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<sup>21</sup>Since the qualitative behavior of the two CX measures is near indistinguishable, we include only the more narrow corridor measure, CX2, in these illustrations.

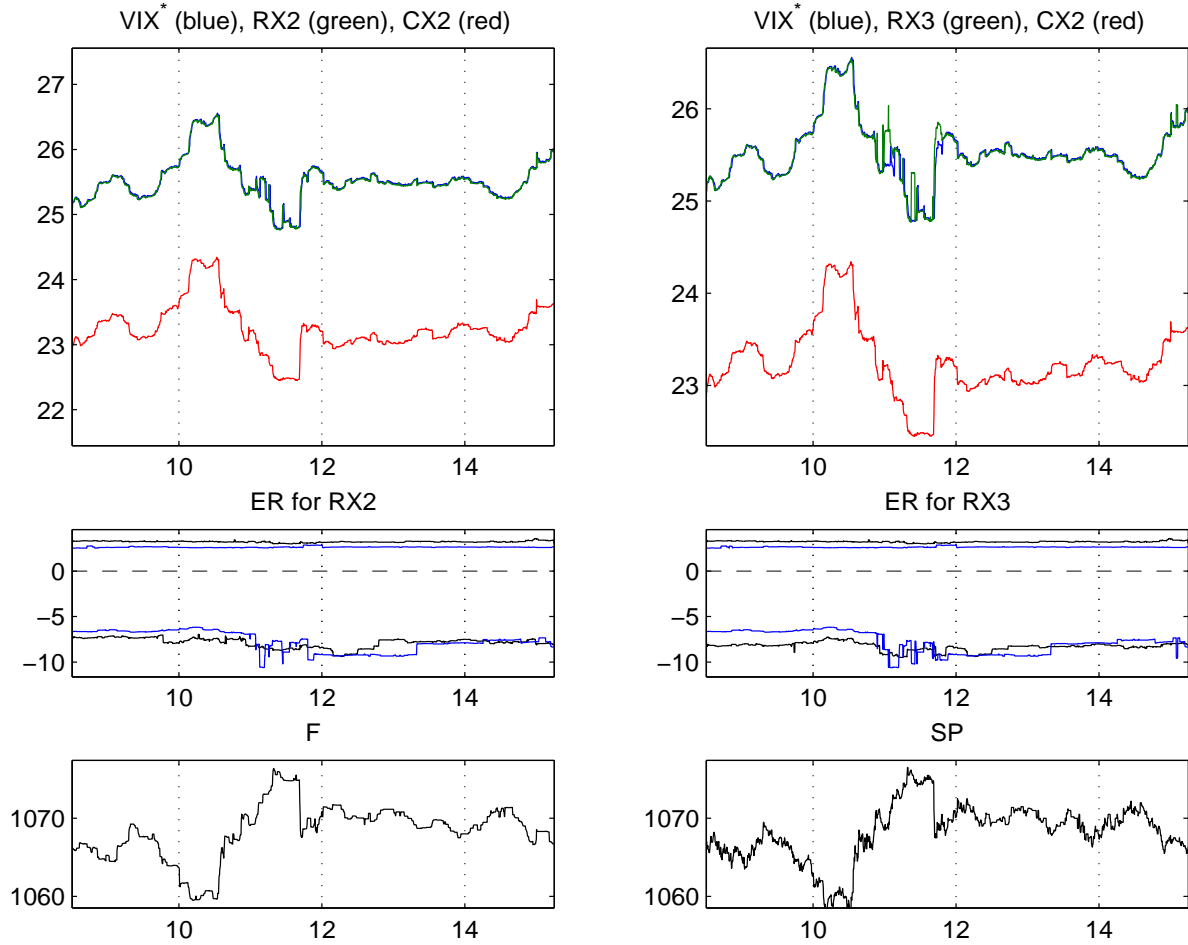
Figure 9: **VIX\***, **RX2**, **RX3** and **CX2**. The top panels depict the VIX\*, RX2, RX3 and CX2 volatility indices for October 14, 2008. The middle panels display the effective strike ranges used by RX2 and RX3 throughout the day, separately for the first (black) and second (blue) maturities. Finally, the bottom panels indicate the implied forward (left), computed using a linear interpolation of the implied forward for the two maturities, and the CME Group S&P 500 futures (right).



## 8 High-Frequency Properties of the Volatility Indices

We now explore whether the observed differences in strike coverage between the VIX, RX and CX indices impact the statistical properties of the series. We focus on critical features of return volatility such as the prevalence of abrupt moves, or jumps, the serial correlation, or persistence, and the extent of (negative) correlation between changes in volatility and the underlying equity returns, often labeled the leverage effect. To alleviate potential distortions arising from noise in the 15-second series, we focus on one-minute changes, or returns, for the volatility indices. In addition, since the market opening for options is characterized by an unusual degree of volatility and occasional failures in establishing adequate liquidity during the initial minutes of trading, we compute all statistics in this section from data covering the period 8:35-15:15, thus excluding the first five minutes of trading each day.

Figure 10: **VIX\***, **RX2**, **RX3** and **CX2**. The top panels depict the VIX\*, RX2, RX3 and CX2 volatility indices for February 9, 2010. The middle panels display the effective strike ranges used by RX2 and RX3 throughout the day, separately for the first (black) and second (blue) maturities. Finally, the bottom panels indicate the implied forward (left), computed using a linear interpolation of the implied forward for the two maturities, and the CME Group S&P 500 futures (right).



## 8.1 Volatility Jumps

We proceed nonparametrically by defining large moves relative to a robust measure of concurrent volatility. We first obtain a robust estimate of the volatility for each index series across every trading day, accounting both for shifts in volatility across days and the pronounced intraday volatility pattern.<sup>22</sup> Exploiting this robust standard deviation measure, we sort the one-minute returns for each volatility measure into a set of mutually exclusive size categories. Table 3 tabulates the findings.

The table reveals a startling discrepancy in the number of large moves across the alternative indices, irrespective of the threshold adopted for identifying “jumps.” This is also visually evident in Figure 11. For example, including moves beyond six standard deviations on either the upside or downside, we find around around 900 outliers in the VIX indices and close to 775 for RX1 and RX2. In contrast, the numbers are approximately 570 for RX3, and below 350 and 320, respectively, for CX1 and CX2.

<sup>22</sup>A description of this procedure is given in the appendix. Moreover, we have confirmed that the qualitative results are robust to a number of alternative ways of estimating intraday volatility.

Table 3: **Distribution of Extreme Returns (“Jumps”), 1-min Frequency.** Range is measured in multiples of the (robust) standard deviation.

	RX1	RX2	RX3	CX1	CX2	VIX*	VIX
$(-\infty, -30)$	9	8	2	0	0	9	15
$(-30, -15)$	37	37	18	7	4	49	50
$(-15, -9)$	88	88	61	18	23	111	114
$(-9, -6)$	242	241	195	141	128	272	272
$(-6, -4)$	713	716	682	637	613	742	739
$(4, 6)$	729	728	655	603	592	730	731
$(6, 9)$	251	251	210	164	140	255	254
$(9, 15)$	98	97	65	17	11	130	131
$(15, 30)$	43	44	18	5	4	55	56
$(30, \infty)$	6	5	2	0	0	5	11

Given that the sample covers 25 months, all indices display, on average, more than one large move per two trading days, while the VIX measures top the count with well in excess of one per day. Even more telling is the discrepancy in the number of extremely large moves of more than 15 standard deviations. The CX2 measure have 8 such moves, while the RX3 index has five times more, and the other indices exceed CX2 more than tenfold. This is obviously anomalous: any genuinely large shift in volatility should manifest itself in a significant elevation of option prices across a broad range of strikes and should be reflected in all broadly based model-free volatility indices. This suggests that various sources of measurement error, including the documented idiosyncratic variation in the strike range, are sufficiently commonplace that they severely inflate the outlier count for some indices.<sup>23</sup>

A second intriguing finding is the near symmetry of positive and negative “jumps” within each size category. Part of the explanation may be that misclassified jumps naturally revert, as an unusual widening of the strike range, say, is followed by a sudden return to the usual range. However, even for measures less prone to this type of error, we observe a near symmetric distribution of positive and negative jumps. This suggests that volatility frequently moves significantly in either direction. The evidence of numerous *negative* volatility jumps is inconsistent with many popular parametric specifications of asset price dynamics allowing only for positive jumps in volatility.

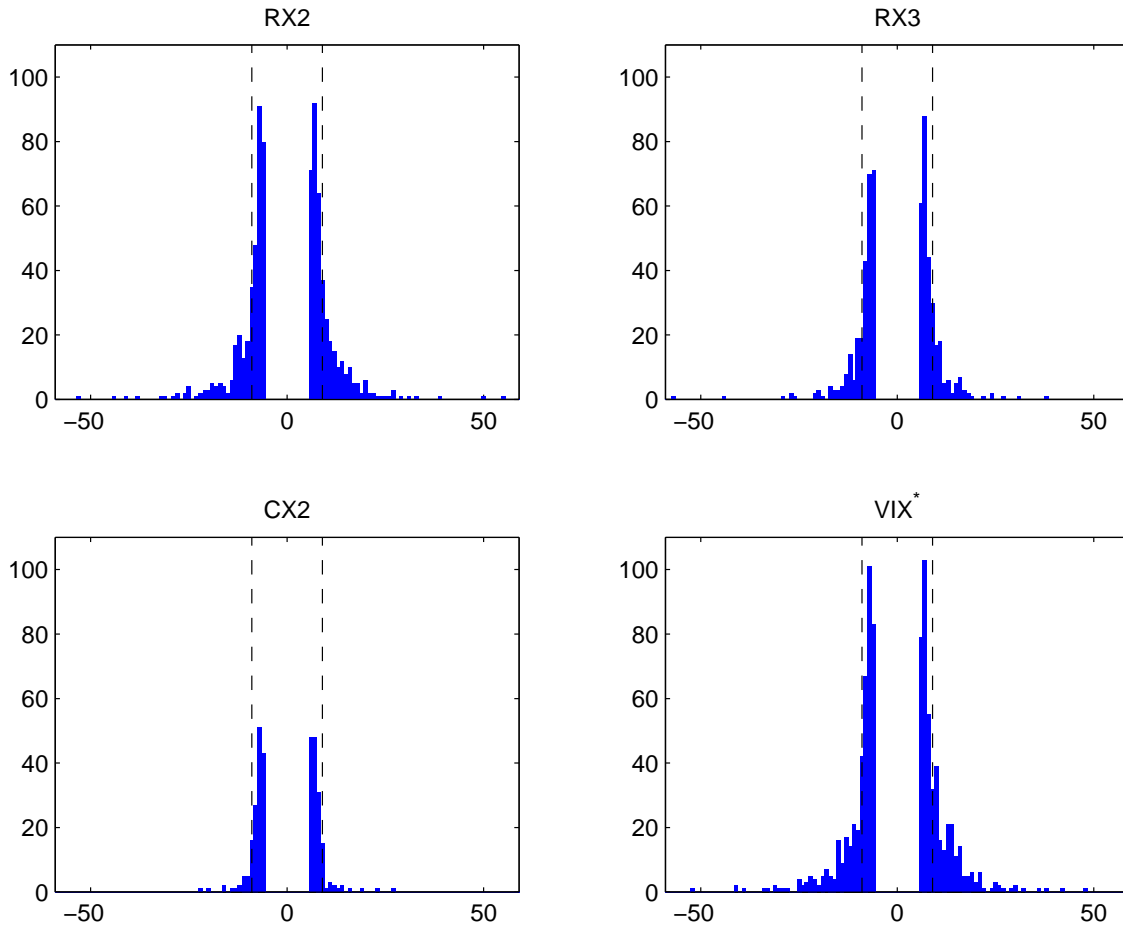
## 8.2 Distributional Features

The pronounced variation in the jump intensity and jump sizes across indices is likely to leave a mark on their relative distributional characteristics. Table 4 provides an overview of the summary statistics for the alternative volatility measures.

The most noteworthy aspect of Table 4 is the huge variation in the kurtosis statistic for the volatility changes, or returns, in Panel B. While the sample kurtosis for the CX series are sizeable, falling in the range of 25 to 27, the values for RX3 and RX2 are rather imposing at 91 and 213, while they attain truly outsized values of 751, 2386 and 3758 for the VIX\*, RX1 and VIX indices. Notice also that mild filtering reduces the kurtosis of RX2 and VIX\* dramatically relative to RX1 and VIX, suggesting that erroneous data are an important source of the inflated statistics. Overall, these findings are consistent

<sup>23</sup>The indices in Table 3 contain a slightly different number of one-minute observations due to the difference in the filters applied to the series. The qualitative results are unaffected by restricting the sample to the smaller set of one-minute returns that are common to the series. These findings are available upon request.

Figure 11: **Return Histograms.** This figure depicts distributions of one-minute returns which fall into one of the two groups: SJ (*Small Jumps*) and LJ (*Large jumps*). Dashed lines separate SJ and LJ.



with the view that all series, bar perhaps the CX indices, suffer from sizeable artificial outliers. Another point of note is the relatively small skewness statistics of the volatility returns. There is again little evidence of any pronounced asymmetry in volatility changes, contradicting the idea that (high-frequency) volatility jumps only, or predominantly, are positive. Finally, we observe that there is positive serial correlation in the volatility returns, but the effect is largely dissipated after five minutes and entirely gone after twenty minutes. If the VIX measures were error-free and direct trading on the VIX index was feasible, one would expect the VIX returns to display minimal short-term autocorrelation as competing agents eliminate simple predictable return patterns. Hence, the short term return persistence is likely indicative of some staleness in the underlying option quotes.<sup>24</sup>

In spite of pronounced discrepancies across the return series, the indices are near identical in their portrayal of the general volatility level, as is apparent from Panel A of Table 4. Except from the fact that the CX indices, as a consequence of deliberate truncation, represent slightly down-scaled versions of model-free implied volatility, the statistics are strikingly similar across the board, e.g., the kurtosis measures all take values close to 4.2. Hence, apart from a slight reduction in the mean level and standard deviation, the CX measures capture the identical features of overall volatility as the VIX and

<sup>24</sup>Trading on the VIX is feasible, e.g., via the VIX futures contracts offered by the CBOE Futures Exchange. However, the VIX “cash index” explored here is not directly traded.

Table 4: **Summary Statistics, 1-min Frequency.** For Panel A, the percent of missing is 1.20% for *RX1*, 1.16% for *RX2-CX2*, 0.03% for *VIX\**, and 0% for *VIX*. For Panel B, the percent of missing is 1.46% for *RX1*, 1.42% for *RX2-CX2*, 0.04% for *VIX\**, and 0% for *VIX*.  $\rho_i$  indicates the  $i^{\text{th}}$  autocorrelation coefficient, while  $\rho_{i,j}$  denotes the average of the  $i^{\text{th}}$  through the  $j^{\text{th}}$  autocorrelation coefficients.

	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
Mean	31.58	31.58	31.64	30.27	29.03	31.72	31.72
Std Dev	13.28	13.28	13.33	12.78	12.31	13.42	13.42
Skewness	1.33	1.33	1.33	1.32	1.32	1.32	1.32
Kurtosis	4.21	4.21	4.19	4.20	4.20	4.17	4.17

	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
Mean $\times 10^4$	-0.11	-0.11	-0.16	-0.12	-0.14	-0.14	-0.15
Std Dev $\times 10^4$	24.43	22.59	20.69	21.21	21.82	25.42	31.75
Skewness	0.10	-0.27	-0.52	0.34	0.33	-0.46	-0.71
Kurtosis	2386	213.0	91.47	26.30	24.80	751.0	3758
$\rho_1 \times 10^2$	4.44	4.41	5.66	5.24	4.60	4.07	3.74
$\rho_{2,5} \times 10^2$	2.46	2.48	2.88	2.67	2.28	2.80	2.68
$\rho_{6,21} \times 10^2$	0.95	0.96	1.05	0.98	0.88	0.92	0.91
$\rho_{21,45} \times 10^2$	0.03	0.03	0.04	0.03	0.00	-0.02	-0.02

RX indices. This translates into an extraordinarily high degree of correlation between the index levels, as documented in Panel A of Table 5.

Of course, Tables 3 and 4 imply that the correlations are much lower for the index changes which is verified by Panel B of Table 5. In fact, the indices fall into three distinct categories with similar high-frequency characteristics, namely the *RX1* and *RX2* measures in one group, the *VIX* and *VIX\** in another, and the *CX1* and *CX2* in a third, with *RX3* being loosely associated with the latter. Most noteworthy is the poor coherence between the *VIX* and *RX1* (*RX2*) returns as the latter is designed to mimic the former. Moreover, the *VIX* returns display the lowest degree of correlation with the “cleaner” *CX* returns. This is likely due to the tendency of *VIX* to randomly oscillate between *RX2* and *RX3*, rendering it an excessively noisy indicator of short term volatility fluctuations.

It is worth reflecting on the striking discrepancies between Panels A and B of Table 5. It highlights the point that even exceptionally high levels of correlation (0.999!) do not ensure that the high-frequency dynamic behavior of the series is similar. Instead, the question of whether a given series provides a suitable representation of volatility hinges critically on the application. For example, it is evident that the degree of low frequency correlation of *MFIV* with other economic variables may be addressed equally well using any of the volatility series in the table, even if they offer quite different depictions of volatility changes. Finally, as we have seen, the indices have radically different implications for the nature of the volatility generating process at the high-frequency level.<sup>25</sup>

<sup>25</sup>Informally, it may be instructive to think of the indices as fractionally co-integrated so that, even if they are highly persistent, they move in unison over time. While pronounced deviations from the typical relation between the series may be observed, for example due to idiosyncratic variation in the strike ranges, such “errors” tend to be relatively short-lived. This

Table 5: Correlations, 1-min Frequency

**Panel A: Volatility Index Levels**

	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
<i>RX1</i>	1.0000						
<i>RX2</i>	1.0000	1.0000					
<i>RX3</i>	0.9998	0.9998	1.0000				
<i>CX1</i>	0.9995	0.9995	0.9996	1.0000			
<i>CX2</i>	0.9991	0.9991	0.9992	0.9999	1.0000		
<i>VIX*</i>	0.9999	0.9999	0.9997	0.9995	0.9991	1.0000	
<i>VIX</i>	0.9999	0.9999	0.9997	0.9995	0.9991	1.0000	1.0000

**Panel B: Volatility Index Returns**

	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
<i>RX1</i>	1.000						
<i>RX2</i>	1.000	1.000					
<i>RX3</i>	0.821	0.820	1.000				
<i>CX1</i>	0.831	0.829	0.904	1.000			
<i>CX2</i>	0.828	0.826	0.887	0.964	1.000		
<i>VIX*</i>	0.600	0.599	0.543	0.564	0.557	1.000	
<i>VIX</i>	0.600	0.599	0.543	0.564	0.557	1.000	1.000

### 8.3 Volatility-Return Correlations

We now explore the dynamic relation between the returns (changes) of the volatility indices and the underlying stock index returns. The pronounced negative correlation between daily VIX changes and daily S&P 500 returns is well established and serves as an argument for diversifying long equity positions with an exposure to the volatility index. The CBOE web-site provides year-by-year estimates for the sample correlation of the two series in the range of -0.75 to -0.85 at the daily level for 2004–2009, with the -0.75 estimate referring to the year 2009 which constitutes about half of our sample. The origin of such large negative correlations has been much debated. For example, it is unclear whether it arises from a corresponding correlation at the very highest frequencies, often labeled a “leverage effect,” or whether it is generated through a dynamic feedback mechanism where an elevation of volatility, say, increases the required rate of return on the equity index, inducing further drops in stock prices. Such dynamic effects may be hard to ascertain from daily data, as the aggregation of high-frequency returns and volatility into daily measures make them appear contemporaneous.

In order to gauge whether the presence of artificial jumps may distort the results, we split the observations into three sets, reflecting the size of the volatility returns relative to the robust estimate of the standard deviation, namely (0,6) (NJ, or “No Jump”), (6,9) (SJ, or “Small Jump”) or (9,∞) (LJ, or “Large Jump”), thus exploiting the classifications from Table 3. The one-minute correlations between VIX and S&P 500 returns are provided in Panel A of Table 6. Since the S&P 500 cash index, inevitably, will suffer from non-synchronous updating of prices and quotes across the constituent stocks, resulting in spurious serial correlation in the returns, we instead exploit the forward price,  $F$ , extracted from the S&P 500 index options, to construct the high-frequency equity return. However, this forward price is

“error-correcting” feature restores “equilibrium” to the system. Although formal modeling along these lines may provide interesting insights it will lead us too far astray in the current context.

Table 6: Correlations of Volatility Indexes with S&P 500 Return, 1-min Frequency

<b>Panel A: S&amp;P 500 Implied Forward</b>							
	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
(0,6)	-0.68	-0.68	-0.70	-0.72	-0.73	-0.48	-0.48
(6,9)	-0.56	-0.56	-0.57	-0.73	-0.74	-0.49	-0.49
(9,∞)	-0.16	-0.23	-0.41	-0.58	-0.56	-0.21	-0.11
(0,∞)	-0.56	-0.61	-0.66	-0.71	-0.73	-0.41	-0.33

<b>Panel B: S&amp;P 500 Futures</b>							
	<i>RX1</i>	<i>RX2</i>	<i>RX3</i>	<i>CX1</i>	<i>CX2</i>	<i>VIX*</i>	<i>VIX</i>
(0,6)	-0.64	-0.64	-0.65	-0.68	-0.68	-0.50	-0.50
(6,9)	-0.55	-0.55	-0.57	-0.74	-0.75	-0.40	-0.40
(9,∞)	-0.22	-0.31	-0.34	-0.59	-0.62	-0.18	-0.14
(0,∞)	-0.53	-0.58	-0.62	-0.67	-0.68	-0.40	-0.32

also a direct input into the computation of the volatility indices, so any noise or error in the forward price will simultaneously impact the implied volatility and equity returns, possibly inducing artificial correlation. Hence, we also report, in Panel B, the corresponding correlations using the S&P 500 futures returns obtained from the CME Group. In this case there might be a slight mismatch in the timing of the volatility and equity index returns across the exchanges which could induce a downward bias in the (absolute) value of the estimated correlations.

Table 6 conveys a clear message. For the smallest volatility returns, the correlations with the equity returns are uniformly strongly negative, coming in at around -0.70 for the RX and CX series and around -0.50 for the VIX indices. For the “small” jumps, in the (6,9) category, the correlations for the CX indices remain equally negative, but they drop off noticeably for the RX indices. Finally, for the “larger” volatility jumps, the correlations drop even more dramatically, but less so for the CX indices, and in particular less so for the correlations with the futures returns which likely are more reliable if there are large contemporaneous changes in the option-implied forward rates. Hence, the CX indices are the only ones to display reasonable consistency across the size categories. This is also manifest in the overall, or unconditional, correlations, reported in the last row, which are much higher for the CX measures than for the rest.

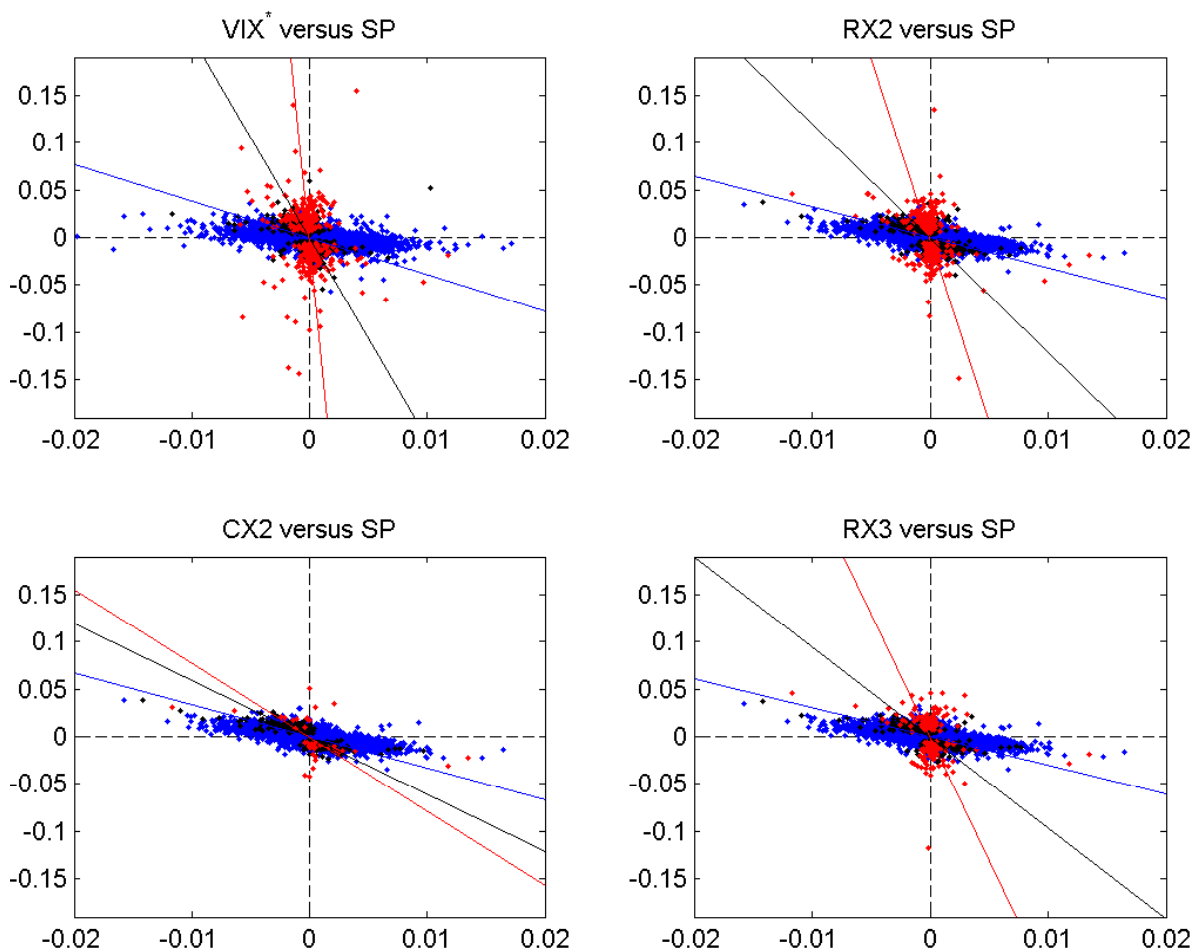
Figure 12 affords a visual representation of the volatility-equity return correlations. It depicts the outcome from an orthogonal regression of the S&P 500 returns on the volatility index returns.<sup>26</sup> The graphs corroborate our prior conclusions. For example, it is evident that the near vertical regression lines for the largest volatility returns of RX2 and VIX\* stem from the numerous large volatility moves that have no counterpart in unusual equity index returns. In contrast, the regressions involving the RX3 “jump” returns feature a dramatic reduction in the scattering of observations along the y-axis and the regression slopes tilt correspondingly, indicating a stronger negative association with the equity returns. Finally, for the regressions involving CX2, almost all large volatility returns are accompanied by large equity-index returns in the opposite direction. Consequently, all the associated scatter plots are clustered quite tightly around similar well-defined and negatively sloped regression lines.

In summary, the CX series contain many fewer abrupt movements than the other measures, and when the CX indices “jump” the associated volatility returns, are strongly, and negatively, related to

<sup>26</sup>The concept of orthogonal regression is reviewed in the Appendix C.



Figure 12: **Volatility Index Return vs. S&P 500 Return Regressions.** This figure depicts the orthogonal regressions between 1-minute returns on the volatility indices and the underlying equity futures where we decompose the (absolute) volatility returns into: *Small Moves or No Jump, NJ* (returns fall within the (0,6) group; Blue), *Small Jump Returns, SJ* (fall within the (6,9) group; Black) and *Large Jump Returns, LJ* (the remainder; Red).

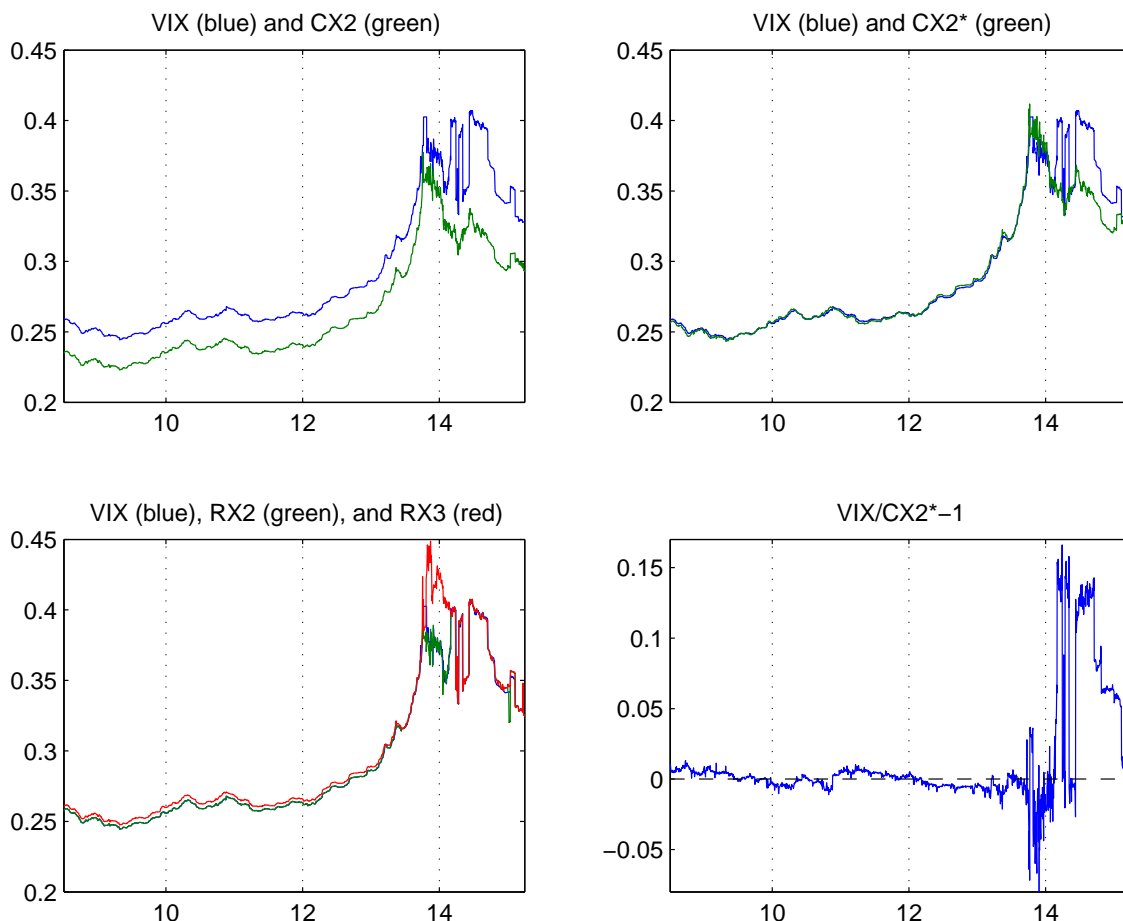


the equity returns. On the other hand, jumps in RX2 and VIX\*, that do not also manifest themselves in the CX indices, are generally unrelated to the concurrent S&P 500 returns. This suggests that they arise from deficiencies in the real-time volatility index construction rather than from true volatility jumps. In particular, these events are not related to any broad-based increase in option prices across the strike range – as this also would also induce significant shifts in the CX and RX3 values.

These findings corroborate the hypothesis that large high-frequency moves are only credible in the CX indices, while many outliers in the alternative volatility measures originate from idiosyncratic errors or unwarranted variation in their effective strike ranges. Since artificial outliers, by definition, have no systematic relation to the contemporaneous equity returns, the dramatic reduction in the correlation measures is a logical corollary. Moreover, the intraday correlations between implied volatility and equity index returns obtained via the CX series mirror the values observed at the daily level and the relation appears stable across the size categories. This has interesting implications for practical risk management and derivatives pricing as well as the more general specification of asset pricing models.<sup>27</sup>

<sup>27</sup>Moreover, our results, based on the CX implied volatility measures, nicely complement the high-frequency S&P 500

Figure 13: VIX and Corridor Implied Volatility Indices on May 6, 2010



## 9 The Flash Crash Revisited

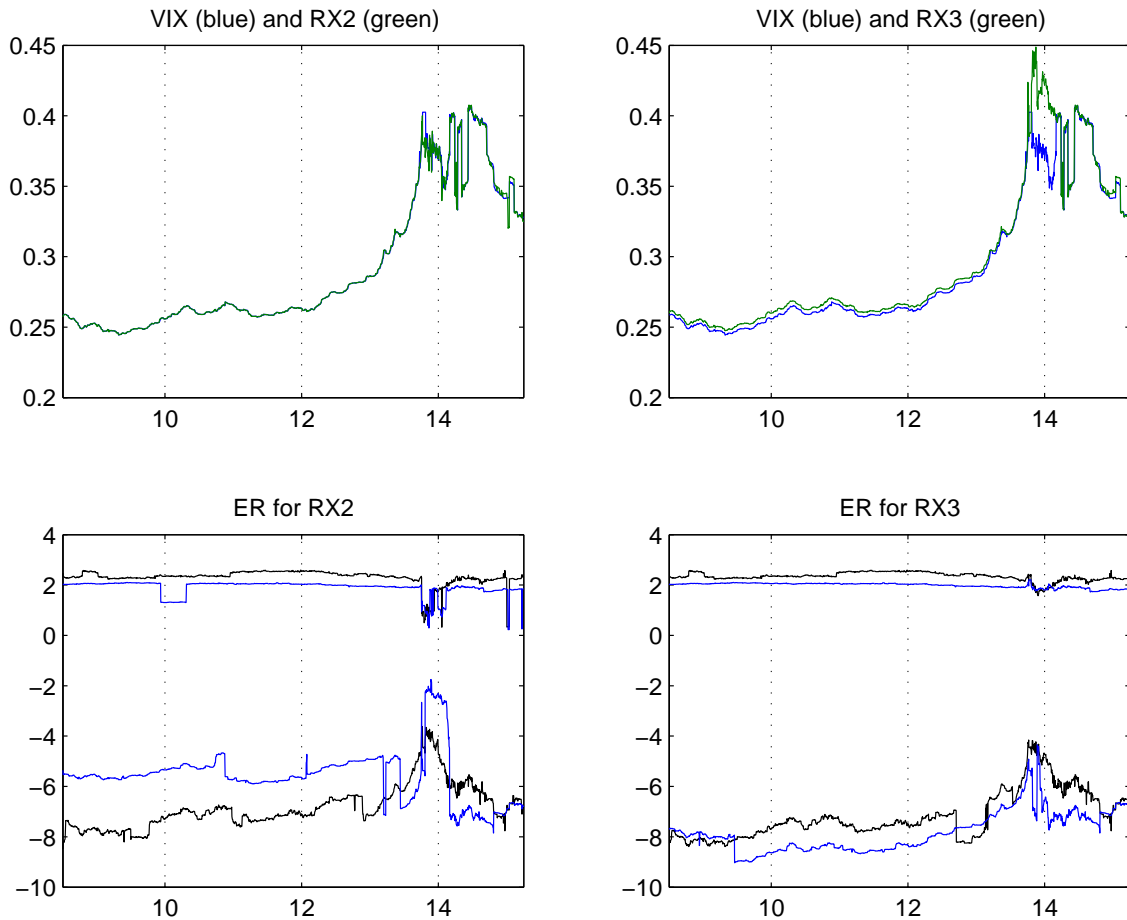
We are now in a position to better rationalize the behavior of the VIX measure during the Flash Crash on May 6, 2010, which was briefly reviewed in Section 2. Given the findings from the preceding sections, it is evident that it is absolutely essential to retain a coherent strike range for computing the MFIV measure across such a turbulent day. The equity index level, the market volatility, the market depth and liquidity, and the individual option prices all experienced dramatic swings across the trading period, potentially inducing severe irregularities in the factors critical for the VIX calculation. One may circumvent many of these problems by constructing the implied volatility measure from a relatively narrow corridor index like CX2 which relies only on relatively liquid options around the ATM strike.

The top panels of Figure 13 depict the evolution of the VIX\* measure along with our corridor measure, CX2, in the left panel and the scaled version, CX2\*, in the right panel.<sup>28</sup> The bottom panels display the RX series along with the VIX\* series on the left and the relative discrepancy between VIX\* and CX2\* series on the right. There are a couple of striking features in these displays. First, from the

future study by Bollerslev, Litvinova & Tauchen (2006) who also conclude that the evidence favors the leverage type effect. Their study relies on absolute five-minute equity returns as their volatility proxy.

<sup>28</sup>These indices were the ones displayed in the original Figure 1. At that stage we were not in a position to explain how they were constructed.

Figure 14: **RX2 and RX3 Indices on May 6, 2010** The top panels depict the RX2, RX3, and VIX indices on the day of the Flash Crash. The bottom panels display the effective strike ranges used by RX2 and RX3 throughout the day, separately for the first (black) and second (blue) maturities.



bottom left panel, we notice that RX2 almost perfectly replicates the VIX index throughout the entire day. Hence, the features shaping the evolution of the RX2 index serve as direct indicators of the factors governing the observed VIX dynamics as well. Second, the CX2\* index replicates VIX very well until about 13:00. After that point, the relative value of the VIX starts deviating increasingly erratically from our CX2\* benchmark. Initially, after 13:00, there is a small downward drift in VIX relative to CX2\*, followed by a period of more dramatic instability, culminating in an extreme VIX undervaluation of up to 5% during the beginning phase of the flash crash, succeeded by rapid wild oscillatory upward swings, resulting in periodic 10-15% overvaluation of VIX relative to CX2\* thereafter.

Since CX2 is obtained from a largely invariant strike range over the entire day, a natural explanation for the startling variation in the relative value of MFIV, on display in the lower right panel of Figure 13, is that there were corresponding shifts in the effective strike range exploited for computing VIX over this period. Using the strike range for RX2 as an indicator for the (unobserved) range employed by the CBOE in computing high-frequency VIX, Figure 14 provides a striking confirmation of this hypothesis.

In particular, the bottom panels of Figure 14 reveal that the strike range, especially for the OTM put options, at the nearby maturity starts shrinking after 13:00 and through the trough, and then only slowly expands back out to about the original range. This implies that both RX2 and RX3 underestimate the

spike in implied volatility at the peak of the crash. However, the short maturity carried relatively low weight in the computation of the VIX on this day. The second, longer, maturity was closer to the target thirty day horizon and the corresponding option prices had a greater impact on the dynamics of the measure through the crash episode. The blue curve associated with RX2 in the bottom left panel reveals a fairly constant strike range past 13:00. Thereafter, it becomes markedly unstable, with the range first expanding significantly, just as the crash gets underway, and then monotonically declining until just prior to the height of the crash, when it implodes to a level of about one fourth of that experienced earlier in the day. This narrow range persists until about 14:10, when it suddenly expands dramatically and now exceeds the levels observed earlier in the trading day. The results in a large jump in the VIX measure, induced solely by this erratic shift in the underlying strike range.

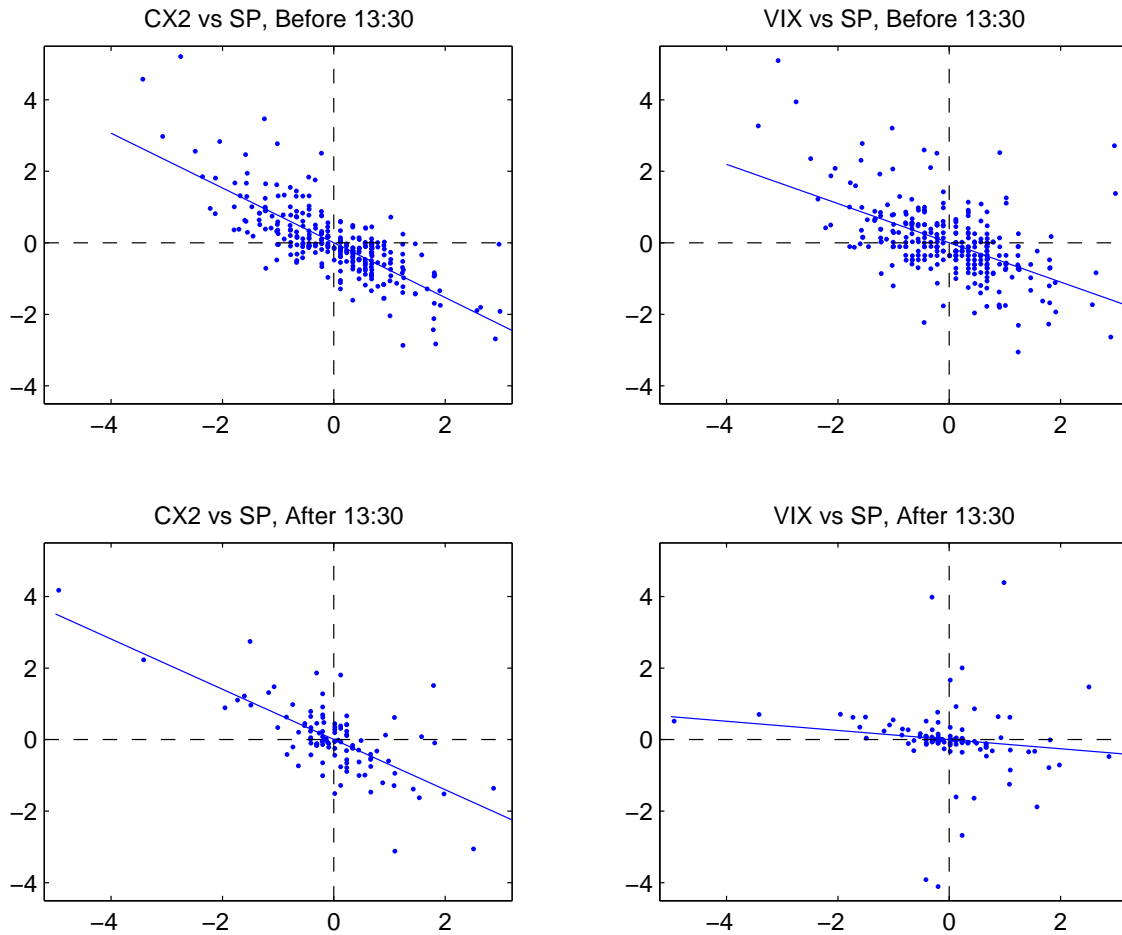
We conclude that the official high-frequency VIX series was severely downward biased during the height of the flash crash due to a collapse of the liquidity in the option market. Equally remarkably, the VIX became strongly upward biased shortly thereafter as the effective strike range expanded well beyond the levels observed prior to the crash. The latter phenomenon likely reflects a partial restoration of confidence among market makers along with residual customer interest in purchasing downside insurance during the still volatile aftermath of the crash. This results in the VIX measure, erroneously, reaching a maximum for the day at around 14:30, about 45 minutes after the market trough.

The relative value of VIX\* versus CX2\* in Figure 13 reflects these dramatic fluctuations in the effective strike range underlying the VIX calculations. In effect, the picture portrays the large biases that arose in the high-frequency VIX measure as the crisis unfolded. Just as it was most critical to have a gauge of the fear gripping the market, the measure was at its worse, thrown off track by irregularities in the underlying equity-index options market. By focusing on the subset of most liquid and reliable option valuations, the CX2\* index largely avoids such confounding effects, and it is thus useful as an anchor for assessing the market pricing of volatility during the critical phase of the crash.

Figure 15 provides an alternative view of these developments. By standardizing both the equity-index and volatility-index returns separately, prior to and following the onset of the crash, we obtain comparable depictions of the interactions between the equity-index and volatility returns throughout the day. This relation is remarkably stable for the CX2 index *vis-a-vis* the equity index, as captured in the two panels on the left. In contrast, the relationship between VIX and the equity-index degenerates into a flat line at the onset of the crash, reflecting the excessive noise of the high-frequency VIX measures. This suggests, once again, that a deliberate and coherent choice for the effective strike range of the MFIV index is necessary to provide a degree of robustness that is crucial for extracting sensible information about concurrent market developments within a turbulent environment.

We conclude by quantifying these effects through the correlations between the volatility indices and the S&P 500 returns. Table 7 shows that all the volatility index returns were strongly negatively correlated with the equity-index returns before 13:30, as expected, even if the correlations were somewhat smaller for the VIX than the others. After 13:30, these correlations shrink dramatically for all indices except CX2. It is worth keeping in mind that the market disruptions also might have impaired the quality of the implied forward price computation, implying that the correlation numbers provided for the futures returns in Panel B may be more reliable. In either case, only CX2 retains a near invariant relation to the underlying S&P 500 index returns throughout the day. As before, the officially disseminated high-frequency VIX series provides the poorest possible coherence with the equity-index throughout the crisis period. In short, the VIX index fails when it is most needed – it is almost entirely ineffective and displays large idiosyncratic biases, just as turbulence engulfs the markets.

Figure 15: **Volatility Index Return vs. S&P 500 Futures Return on May 6, 2010** This figure depicts 1-minute returns of CX2 and VIX versus returns of S&P 500 futures, before and after 13:30 on the Flash Crash day. For each panel, returns are first standardized to have the mean of zero and standard deviation of one.



## 10 Conclusions

We pursue two primary objectives. First, we seek to replicate the official real-time fifteen second VIX series from tick-by-tick quotes on S&P 500 option prices obtained from the official CBOE vendor. The purpose is to gauge the reliability of high-frequency movements in the VIX index and understand which features of the real-time series are robust to noise and measurement errors. This first step provides valuable insights into potential problems plaguing the VIX series. We identify a particularly important deficiency: the rule for determining the effective strike range used to compute the volatility index is not robust to common data errors or random shifts in market liquidity. This induces spurious jumps in VIX as well as any alternative index constructed on the basis of similar rules for the determination of the effective strike range. These “artificial” jumps have a profound impact on the high-frequency characteristics of the VIX returns. Importantly, the identical issue affects the daily closing values of the VIX which are used in numerous academic studies and are widely cited in the financial press. Day-to-day changes in the index are not reliable indicators of the corresponding *shifts* in implied volatility. On the other hand, all the implied volatility measures we explored convey near

Table 7: Volatility and S&P 500 Return Correlations, May 6, 2010, 1-min Frequency

**Panel A: S&P 500 Implied Forward**

	<i>RX2</i>	<i>RX3</i>	<i>CX2</i>	<i>VIX</i>
Before 13:30	-0.79	-0.82	-0.84	-0.65
After 13:30	-0.19	-0.06	-0.56	-0.21
All	-0.21	-0.09	-0.58	-0.23

**Panel B: S&P 500 Futures**

	<i>RX2</i>	<i>RX3</i>	<i>CX2</i>	<i>VIX</i>
Before 13:30	-0.70	-0.72	-0.77	-0.55
After 13:30	-0.24	-0.25	-0.70	-0.13
All	-0.26	-0.26	-0.71	-0.15

identical messages concerning the evolution in the overall *level* of volatility. Critical discrepancies arise only when focusing on the *changes* in the indices at an intraday or daily frequency.

Our second goal is to construct corridor implied volatility indices, *CX*, that retain the theoretical advantages of the model-free volatility concept while avoiding the pitfalls associated with the official *VIX* series. These *CX* indices may be computed directly from real-time option quotes via a simple and transparent computational rule. The main change from the *VIX* is the determination of a different truncation point for when to exclude far OTM options from the calculation. By ensuring intertemporal consistency in this dimension, we obtain *CX* indices that cover economically equivalent parts of the strike range. Consequently, they are internally consistent and the index values may be meaningfully compared across time, independently of the liquidity of the options market. As a result, we find that the number of spurious jumps in the volatility index are dramatically reduced, if not entirely eliminated, and the jump returns now consistently display a pronounced negative relation to the underlying equity returns. In short, the high-frequency *CX* and *VIX* return series display strikingly different time series characteristics along critical dimensions relevant for real-time financial decision-making and risk management. The same issues afflict the widely used daily *VIX* series, available from the CBOE web-site. The daily volatility returns implied by this *VIX* series are inherently noisy and caution should be exercised if the analysis is focused on the daily shifts in volatility. In such instances, our *CX* measures represent a superior alternative and they are readily computed from the underlying SPX option quotes.

Our findings point towards a number of issues for future research related to the use and construction of implied volatility measures. One important application is the identification and modeling of the equity variance risk premium. The premium is given by the wedge between the (expected) realized variance of the equity index returns and the MFIV measure over the corresponding horizon. In the literature, this is most often approximated by the difference between an (expected) realized variance measure, constructed from the cumulative sum of squared high-frequency equity-index returns, and the (squared) implied volatility index, represented by the CBOE *VIX* measure. Thus, to the extent *VIX* is a noisy, downward-biased, and excessively volatile measure of implied volatility, the quality of the risk premium measures are similarly degraded. It is evident that a precise and coherent identification of the variance risk premium requires the development of techniques to estimate the MFIV from options with strikes that span the full range of potential future values of the equity index. There are two obvious approaches to this issue. First, one may directly seek to extrapolate prices for deep OTM options so that the effective strike range is expanded to ensure all non-trivial contributions to the measure are included.

Since observed quotes for far OTM options are unreliable, or even unobserved, such procedures hinge on specific assumptions about the shape and tail-fatness of the RND. One popular approach is to apply extrapolation in the Black-Scholes implied volatility space, either assuming implied volatility remains constant beyond the lowest and highest strike price for which reliable option values are available, as in Jiang & Tian (2005) and Carr & Wu (2009), or extending the implied volatility curves linearly from these extreme endpoints, as in Jiang & Tian (2007). A more recent sophisticated approach towards capturing the time-variation in the shape of the tails is provided by Bollerslev & Todorov (2011a), and it is also the subject of ongoing work in Andersen & Bondarenko (2011). Second, one may resort to exploring the variance risk premium only within ranges, or corridors, of the strike range for which we can reliably measure both the model-free implied variance and the corresponding realized return variation of the underlying equity-index returns. This approach provides a decomposition of the variance risk premium into segments of the return space, thus enabling direct identification of the size of the variance risk premium across different equity return states. For example, one may explore how much of the premium stems from high valuations of return variation in states where the equity prices are declining. This approach is explored in detail in Andersen & Bondarenko (2010).

A second important avenue for future research is to enhance our understanding of the nature of the random variation in the quality and strike coverage of the SPX option quotes that constitute the most critical input to the computation of the implied volatility measures. The SPX options market is primarily a floor-based open outcry system. However, the electronic data feed providing the official SPX quotes does not originate from this market but are instead constructed from real-time quotes issued by two lead market maker firms at the CBOE as well as customer and broker orders listed on the same electronic platform. These electronically disseminated quotes generally have significantly wider spreads than the effective bid-ask spreads prevailing in the SPX trading pit. By studying the interaction of the quoting patterns from the lead market makers, customers and brokers, we may obtain a deeper understanding of the features in the data that induce noise as well as spurious jumps into the real-time VIX measures. As a consequence, improved procedures for extracting the relevant information from the real-time option quotes may be within reach. Gonzalez-Perez (2011) is currently exploring the relevant microstructure features of the SPX option market and the associated electronic quoting mechanism in order facilitate such direct inquiry into the origins of the distortions in the VIX index.

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# Appendix

## A Data Filtering

### A.1 SPX Option Classes

We acquired the MDR (Market Data Retrieval) data for S&P 500 options from the CBOE subsidiary, Market Data Express (<http://www.marketdataexpress.com/>). The MDR data include tick-by-tick quotes and transactions throughout the trading day for all option classes issued by the CBOE on the S&P 500. Each option class is characterized by a letter code. We only consider options that the CBOE actually used in their computation of the VIX over our sample period, namely those in the SPB, SPQ, SPT, SPV, SPX, SPZ, SVP, SXB, SXM, SXY, SXZ, SYG, SYU, SYV and SZP categories. The latter are generally known as SPX equity options and they mature on the Saturday immediately following the third Friday of the expiration month (see <http://www.cboe.com/Products/EquityOptionSpecs.aspx>).

### A.2 Systemic Staleness in Option Quotes

By far, the most influential options for the computation of model-free implied volatility measures are the out-of-the-money (OTM) put and call options with strikes close to the at-the-money (ATM) strike which is given by the forward price. Hence, we actively monitor the liquidity of the ATM option as well as the set of twenty OTM put and twenty OTM call options closest to ATM for the two maturities exploited for the VIX computation. We label this group of options the “pivotal” ones.

Our systemic staleness filter flags episodes where there are no quote update among the pivotal options in the bid *or* the ask price at *either* of the two maturities for five minutes. Hence, this dummy variable equals one if all options within one of these four pivotal option groups have not been updated for five minutes and it is zero otherwise. When the flag is activated (equals unity), we classify the *entire* inactive period of five minutes or more as unreliable, and the volatility indices are not available (n.a.).

### A.3 Non-Convexity Filter

To preclude arbitrage opportunities, theoretical call and put prices must be monotonic and convex functions of the strike. In particular, the call prices must satisfy the following convexity restriction:

$$D_i = \frac{C(K_{i+1}) - C(K_i)}{K_{i+1} - K_i} - \frac{C(K_i) - C(K_{i-1}))}{K_i - K_{i-1}} \geq 0,$$

and a similar restriction for the put prices. We obtain the option prices as the average of the bid and ask quotes and use the above restriction to identify “suspect” cross-section with apparent arbitrage violations, which could arise from recording errors, staleness, and other issues. Specifically, for each strike  $K_i$ , we compute the following measure of local non-convexity:

$$NC_i = -\min\{D_i, 0\}.$$

For low strikes ( $K_i \leq F$ ), we compute  $NC_i$  from OTM puts and, for high strikes ( $K_i > F$ ), we use OTM calls. We then average  $NC_i$  across all strikes to obtain the aggregate measure of non-convexity  $NC$ .

When  $NC > 0.1$ , we deem a cross-section unreliable and do not use it in our econometric analysis. For those cross-sections, the option prices indicate sizeable apparent arbitrage opportunities.

However, for some illustrations, we need to compute volatility indices even when the quality of data is poor ( $NC > 0.1$ ). In those cases, prior to computing the volatility indices, we adjust option prices by running the so-called *Constrained Convex Regression* (CCR). This procedure has been implemented in Bondarenko (2000). Intuitively, CCR searches for the smallest (in the sense of least squares) perturbation of option prices that restores the no-arbitrage restrictions.

## B Robust Volatility Estimation

For a given volatility index, let the associated one-minute log return at time  $t$  on trading day  $d$  by  $r_{dt}$ . We assume this return may be represented as

$$r_{dt} = \sigma_d f_t z_{dt},$$

where  $\sigma_d$  is average volatility for trading day  $d$ ,  $f_t$  is a scaling factor which adjusts for the intraday volatility pattern, and  $z_{dt}$  are i.i.d. random variables with zero mean and unit standard deviation, but not necessary normally distributed.

We estimate the daily volatilities  $\{\sigma_d\}_1^{N_d}$  and intraday adjustment factors  $\{f_t\}_1^{N_t}$  as follows. First, for fixed  $t$ , we compute the average squared return across all trading days,

$$\frac{1}{N_d} \sum_d^{N_d} r_{dt}^2 = f_t^2 \cdot \frac{1}{N_d} \sum_d^{N_d} \sigma_d^2 z_{dt}^2.$$

Since  $z_{dt}$  are i.i.d. with  $E[z_{dt}] = 0$  and  $\text{Var}(z_{dt}) = 1$ , we obtain,

$$E \left[ \frac{1}{N_d} \sum_d^{N_d} r_{dt}^2 \right] = f_t^2 \cdot \left( \frac{1}{N_d} \sum_d^{N_d} \sigma_d^2 \right) = V_{mean} \cdot f_t^2,$$

where  $V_{mean}$  is a constant independent of  $t$ . Therefore, the adjustment factor  $f_t$  may be estimated as

$$f_t^2 = \frac{\frac{1}{N_d} \sum_d^{N_d} r_{dt}^2}{V_{mean}},$$

where  $V_{mean}$  now is determined from the condition that,

$$\sigma_d^2 = \frac{1}{N_t} E \left[ \sum_t^{N_t} r_{dt}^2 \right] = \frac{1}{N_t} \sum_t^{N_t} \sigma_d^2 \cdot f_t^2,$$

or

$$\frac{1}{N_t} \sum_t^{N_t} f_t^2 = 1. \quad (13)$$

The above approach provides an unbiased estimator of  $f_t^2$ . However, it might not be robust in the presence of extreme returns in the  $r_{dt}$  series. Therefore, we implement a robustified version of the same approach, based on the sample median rather than the mean. Specifically,

$$E [\text{Median}_d \{r_{dt}^2\}] = f_t^2 \text{Median}_d \{\sigma_d^2\} = V_{med} \cdot f_t^2,$$

where  $V_{med}$  again denotes a constant independent of  $t$ . The adjustment factor  $f_t$  is then estimated as

$$f_t^2 = \frac{\text{Median}_d \{r_{dt}^2\}}{V_{med}},$$

subject to the constraint in (13). For additional robustness, the estimated intraday factors are smoothed by averaging them over 10-minute windows.<sup>29</sup> Armed with  $\{f_t\}_1^{N_t}$ , the daily volatility  $\sigma_d$  can now be estimated as the volatility of the re-scaled returns,  $u_{dt} = \frac{r_{dt}}{f_t}$ . This could be done in many ways, but we focus on the robust estimator based on 5-to-95 percentile range. Specifically, we sort the re-scaled returns  $u_{dt}$  for a given day  $d$  and determine their 5- and 95-percentiles,  $P(0.05)$  and  $P(0.95)$ . Then,

$$\sigma_d = \frac{P(0.95) - P(0.05)}{3.2898},$$

<sup>29</sup>For all volatility indices, the intraday factors range from about 0.6 to 2.0.

where the denominator equals the 5-to-95 percentile range of the standard normal random variable.<sup>30</sup>

Finally, when defining large index returns, we take into account both the daily measure of volatility  $\sigma_d$  and the intraday adjustment factor  $f_t$ . That is, the move is deemed large if the absolute value of the ratio,

$$\left| \frac{r_{dt}}{\sigma_d f_t} \right| = \left| \frac{u_{dt}}{\sigma_d} \right|$$

exceeds a pre-specified threshold.

## C Orthogonal Regression

The Orthogonal regression (OR), also known as Total Least Squares (TLS) regression, has a long history in statistics and economics, see, e.g. the discussion in Anderson (1984). It has been viewed as more appropriate than the Ordinary Least Squares (OLS) regression in some circumstances, including when both the predictor and response variables are measured with error. In the bivariate case, OR minimizes the sum of squared perpendicular (or total) distances from the data to the fitted regression line, in contrast to OLS which minimizes the sum of squared vertical distances. The slope of OR line is related to the first Principal Component.

In Figure 12 we regress volatility index returns onto the underlying futures return, using OR. When a volatility index contains large “artificial” jumps, those observations tend to be located along the vertical axis (a small move in the underlying futures but a very large return in the volatility index). The slope of OLS regression has a very low sensitivity to such outliers and is less informative for our purposes. In contrast, the slope of OR is affected by the presence of extreme volatility returns along the vertical axis.

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<sup>30</sup>We also experimented with other robust estimators of  $\sigma_d$ , and the results are quantitatively similar.

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