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Forecasting with Option Implied Information

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Forecasting with Option Implied Information*

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Abstract

This chapter surveys the methods available for extracting forward-looking information from option prices. We consider volatility, skewness, kurtosis, and density forecasting. More generally, we discuss how any forecasting object which is a twice differentiable function of the future realization of the underlying risky asset price can utilize option implied information in a well-defined manner. Going beyond the univariate option-implied density, we also consider results on option-implied covariance, correlation and beta forecasting as well as the use of option-implied information in cross-sectional forecasting of equity returns.

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1 Introduction

We provide an overview of techniques used to extract information from derivatives, and document the applicability of this information in forecasting. The premise of this chapter is that derivative prices contain useful information on the conditional density of future underlying asset returns. This information is not easily extracted using econometric models of historical values of the underlying asset prices, even though historical information may also be useful for forecasting, and combining historical information with information extracted from derivatives prices may be especially effective.

A derivative contract is an asset whose future payoff depends on the uncertain realization of the price of an underlying asset. Many different types of derivative contracts exist: futures and forward contracts, interest rate swaps, currency and other plain-vanilla swaps, credit default swaps (CDS) and variance swaps, collateralized debt obligations (CDOs) and basket options, European style call and put options, and American style and exotic options. Several of these classes of derivatives, such as futures and options, exist for many different types of underlying assets, such as commodities, equities, and equity indexes.

Because of space constraints, we are not able to discuss available techniques and empirical evidence of forecastability for all these derivatives contracts. We therefore use three criteria to narrow our focus. First, we give priority to larger and more liquid markets, because they presumably are of greater interest to the reader, and the extracted information is more reliable. Second, we focus on methods that are useful across different types of securities. Some derivatives, such as basket options and CDOs, are multivariate in nature, and as a result techniques for information extraction are highly specific to these securities. While there is a growing literature on extracting information from these derivatives, the literature on forecasting using this information is as yet limited, and we therefore do not focus on these securities. Third, some derivative contracts such as forwards and futures are linear in the return on the underlying security, and therefore their payoffs are too simple to contain useful and reliable information. This makes these securities less interesting for our purpose. Other securities, such as exotic options, have path-dependent payoffs, which may make information extraction cumbersome.

Based on these criteria, we mainly focus on European-style options. European-style options hit the sweet spot between simplicity and complexity and will therefore be the main, but not the exclusive, focus of our survey.[1] Equity index options are of particular interest, because the underlying risky asset (a broad equity index) is a key risk factor in the economy. They are among the most liquid exchange-traded derivatives, so they have reliable and publicly available prices. The fact that the most often used equity index options are European-style also makes them tractable and computationally convenient.[2] For these reasons, the available empirical literature on equity

[1] Note that for American options the early exercise premium can usually be estimated (using binomial trees for example). By subtracting this estimate from the American option price, a synthetic European option is created which can be analyzed using the techniques we study in this Chapter.

[2] Most studies use options on the S&P500 index, which are European. Early studies used options on the S&P100,

index options is also the most extensive one.

Forecasting with option-implied information typically proceeds in two steps. First, derivative prices are used to extract a relevant aspect of the option-implied distribution of the underlying asset. Second, an econometric model is used to relate this option-implied information to the forecasting object of interest. For example, the Black-Scholes model can be used to compute implied volatility of an at-the-money European call option with 30 days to maturity. Then, a linear regression is specified with realized volatility for the next 30 days regressed on today's implied Black-Scholes volatility. We will focus on the first step in this analysis, namely extracting various information from observed derivatives prices. The econometric issues in the second step are typically fairly standard and so we will not cover them in any detail.

Finally, there are a great number of related research areas we do not focus on, even though we may mention and comment on some of them in passing. In particular, this chapter is not a survey of option valuation models (see Whaley (2003)), or of the econometrics of option valuation (see Garcia, Ghysels, and Renault (2010)), or of volatility forecasting in general (see Andersen, Bollerslev, Christoffersen, and Diebold (2006)). Our chapter exclusively focuses on the extraction of information from option prices, and only to the extent that such information has been used or might be useful in forecasting.

The remainder of the chapter proceeds as follows. Section 2 discusses methods for extracting volatility and correlation forecasts from option prices. Section 3 focuses on constructing option-implied skewness and kurtosis forecasts. Sections 4 covers techniques that enable the forecaster to construct the entire density, thus enabling event probability forecasts for example. Sections 2-4 cover model-based as well as model-free approaches. When discussing model-based techniques, we discuss in each section the case of two workhorse models, Black and Scholes (1973) and Heston (1993), as well as other models appropriate for extracting the object of interest. Sections 2-4 use the option-implied distribution directly in forecasting the physical distribution of returns. Section 5 discusses the theory and practice of converting option-implied forecasts to physical forecasts. Section 6 concludes.

2 Extracting Volatility and Correlation from Option Prices

Volatility forecasting is arguably the most widely used application of option implied information. When extracting volatility information from options, model-based methods were originally more popular, but recently model-free approaches have become much more important. We will discuss each in turn.

which was the most liquid market at the time. These options are American.

2.1 Model-Based Volatility Extraction

In this section we will review two of the most commonly used models for option valuation, namely the Black and Scholes (1973) and Heston (1993) models. The Black-Scholes model only contains one unknown parameter, namely volatility, and so extracting an option-implied volatility forecast from this model is straightforward. The Heston model builds on more realistic assumptions regarding volatility, but it also contains more parameters and so it is more cumbersome to implement.

2.1.1 Black-Scholes Implied Volatility

Black and Scholes (1973) assume a constant volatility geometric Brownian motion stock price process of the form

$$dS = rSdt + \sigma Sdz$$

where r is the risk-free rate, σ is the volatility of the stock price, and dz is a normally distributed innovation.[3] Given this assumption, the future log stock price is normally distributed and the option price for a European call option with maturity T and strike price X can be computed in closed form using

$$C^{BS}(T, X, S_0, r; \sigma) = S_0 N(d) - X \exp(-rT) N\left(d - \sigma\sqrt{T}\right) \tag{1}$$

where S_0 is the current stock price, $N(\cdot)$ denotes the standard normal CDF and where

$$d = \frac{\ln(S_0/X) + T\left(r + \frac{1}{2}\sigma^2\right)}{\sigma\sqrt{T}}. \tag{2}$$

European put options can be valued using the put-call parity

$$P_0 + S_0 = C_0 + X \exp(-rT)$$

which can be derived from a no-arbitrage argument alone and so is not model dependent.

The Black-Scholes option pricing formula has just one unobserved parameter, namely volatility, denoted by σ. For any given option with market price, C_0^{Mkt}, the formula therefore allows us to back out the value of σ which is implied by the market price of that option,

$$C_0^{Mkt} = C^{BS}(T, X, S_0, r; BSIV) \tag{3}$$

The resulting option-specific volatility, $BSIV$, is generically referred to as implied volatility (IV). To distinguish it from other volatility measures implied by options, we will refer to it as Black-Scholes IV, thus the $BSIV$ notation.

[3] Throughout this chapter we assume for simplicity that the risk-free rate is constant across time and maturity. In reality it is not and the time-zero, maturity-dependent risk-free rate, $r_{0,T}$ should be used instead of r in all formulas. Recently, the overnight indexed swap rate has become the most commonly used proxy for the risk-free rate. See Hull (2011) Chapter 7.

Although the Black-Scholes formula in (1) is clearly non-linear, for at-the-money options, the relationship between volatility and option price is virtually linear as illustrated in the top panel of Figure 1.

[Figure 1: Black-Scholes Price and Vega]

In general the relationship between volatility and option prices is positive and monotone. This in turn implies that solving for $BSIV$ is quick even if it must be done numerically. The so-called option Vega captures the sensitivity of the option price w.r.t. changes in volatility. In the Black-Scholes model it can be derived as

$$Vega_{BS} = \frac{\partial C_0^{BS}}{\partial \sigma} = S_0 \sqrt{T} N'(d)$$

where d is as defined in (2) and where $N'(d)$ is the standard normal probability density function.

The bottom panel of Figure 1 plots the Black-Scholes Vega as a function of moneyness. Note that the sensitivity of the options with respect to volatility changes is largest for at-the-money options. This in turn implies that changes in at-the-money option prices are the most informative about changes in expected volatility.

Table 1 reproduces results from Busch, Christensen, and Nielsen (2011), who regress total realized volatility (RV) for the current month on the lagged daily, weekly and monthly realized volatility, and subsequently use $BSIV$ as an extra regressor. Realized daily volatility is computed using intraday returns. Alternative specifications separate RV into its continuous (C) and jump components (not reported here). Panel A contains \$/DM FX data for 1987-1999, Panel B contains S&P 500 data for 1990-2002, and Panel C contains Treasury bond data for 1990-2002.

The results in Table 1 are striking. Option implied volatility has an adjusted R^2 of 40.7% for FX, 62.1% for S&P 500 and 35% for Treasury bond data. This compares with R^2 of 26.9%, 61.9% and 37% respectively for the best RV based model. The simple $BSIV$ forecast is thus able to compete with some of the most sophisticated historical return-based forecasts. The Treasury bond options contain wild-card features that increase the error in option implied volatility in this market. The fact that $BSIV$ performs worse in this case is therefore not surprising.

[Table 1: Forecasting Realized Volatility using Black-Scholes Implied Volatility]

In Figure 2 we plot $BSIV$s for S&P 500 call and put options quoted on October 22, 2009. In the top panel of Figure 2 the $BSIV$s on the vertical axis are plotted against moneyness (X/S_0) on the horizontal axis for three different maturities.

[Figure 2: Black-Scholes Implied Volatility as a Function of Moneyness and Maturity]

The index-option $BSIV$s in the top panel of Figure 2 display a distinct downward sloping pattern commonly known as the "smirk" or the "skew". The pattern is evidence that the Black-Scholes model–which relies on the normal distribution–is misspecified. Deep out-of-the-money

(OTM) put options ($X/S_0 << 1$) have much higher $BSIV$s than other options which from Figure 1 implies that they are more expensive than the normal-based Black-Scholes model would suggest. Only a distribution with a fatter left tail (that is negative skewness) would be able to generate these much higher prices for OTM puts. This finding will lead us to consider models that account for skewness and kurtosis in Section 3.

The bottom panel of Figure 2 shows that the $BSIV$ for at-the-money options ($X/S_0 \approx 1$) tends to be larger for long-maturity than short-maturity options. This is evidence that volatility changes over time although Black-Scholes assumes it is constant. We therefore consider models with stochastic volatility next.

2.1.2 Stochastic Volatility

For variances to change over time, we need a richer setup than the Black-Scholes models. The empirically most relevant model that provides this result is Heston (1993), who assumes that the price of an asset follows the so-called square-root process[4]

$$\begin{aligned} dS &= rSdt + \sqrt{V}Sdz_1 \\ dV &= \kappa\left(\theta - V\right)dt + \sigma_V\sqrt{V}dz_2 \end{aligned} \tag{4}$$

where the two innovations are correlated with parameter, ρ.

At time zero, the variance forecast for horizon T can be obtained as

$$VAR_0\left(T\right) \equiv E_0\left[\int_0^T V_t dt\right] = \theta T + \left(V_0 - \theta\right)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa} \tag{5}$$

The horizon-T variance $VAR_0(T)$ is linear in the spot variance V_0. Notice how the mean-reversion parameter κ determines the extent to which the difference between current spot volatility and long run volatility, $(V_0 - \theta)$, affects the horizon T forecast. The smaller the κ, the slower the mean reversion in volatility, and the higher the importance of current volatility for the horizon T forecast.

Figure 3 shows the volatility term structure in the Heston model, namely

$$\sqrt{VAR_0\left(T\right)/T} = \sqrt{\theta + \left(V_0 - \theta\right)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa T}} \tag{6}$$

when $\theta = 0.09$, $\kappa = 2$ and $V_0 = 0.36$ (dashed line) corresponding to a high current spot variance and $V_0 = 0.01$ (solid line) corresponding to a low current spot variance.

[Figure 3: Heston Volatility Term Structures]

A similar approach could be taken for the wide range of models falling in the affine class to which the Heston model belongs. Duffie, Pan, and Singleton (2000) provide an authoritative treatment

[4] Christoffersen, Jacobs, and Mimouni (2010) consider models with alternative drift and diffusion specifications.

of a general class of continuous time affine models. For examples of discrete time affine models, see for example Heston and Nandi (2000) and Christoffersen, Heston, and Jacobs (2006).

Note that whereas the Black-Scholes model only has one parameter, σ, the Heston model has four parameters, namely κ, θ, σ_V, and ρ, in addition to the spot variance, V_0. Estimation of the parameters and spot volatility in the model can be done using a data set of returns, but also using option prices. Bakshi, Cao, and Chen (1997) re-estimate the model daily treating V_0 as a fifth parameter to be estimated along with the structural parameters θ, κ, ρ, and σ_V. Bates (2000) and Christoffersen, Heston, and Jacobs (2009) keep the structural parameters fixed over time. They make use of an iterative two-step option valuation error minimization procedure where in the first step the structural parameters are estimated for a given path of $\{V_t\}_{t=1}^{N}$. In the second step V_t is estimated each period keeping the structural parameters fixed. Iterating between the first and second step provides the final estimates of structural parameters and spot volatilities. Alternatively, a more formal filtering technique can be used, which is econometrically more complex.

The complications involved in estimating the parameters and filtering the spot volatility in models such as Heston's–as well as the parametric assumptions required–have motivated the analysis of model-free volatility extraction to which we now turn.

2.2 Model-Free Volatility Extraction

2.2.1 Theory

Under the assumptions that investors can trade continuously, interest rates are constant, and the underlying futures price is a continuous semi-martingale, Carr and Madan (1998) show that the expected value of the future realized variance can be computed as,

$$E_0\left[\int_0^T V_t dt\right] = 2\int_0^\infty \frac{C_0^F\left(T,X\right) - \max\left(F_0 - X, 0\right)}{X^2} dX, \tag{7}$$

where F_0 is the forward price of the underlying asset and $C^F\left(T,X\right)$ is a European call option on the forward contract.

Britten-Jones and Neuberger (2000) show that the relationship also holds when V_t is replaced by the return, dS_t/S_t,

$$E_0\left[\int_0^T \left(dS_t/S_t\right)^2 dt\right] = 2\int_0^\infty \frac{C_0^F\left(T,X\right) - \max\left(F_0 - X, 0\right)}{X^2} dX. \tag{8}$$

Jiang and Tian (2005) generalize this result further and show that (8) holds even if the price process contains jumps.

When relying on options on the underlying spot asset rather than on the forward contract, the expected variance between now and horizon T is

$$VAR_0\left(T\right) = 2\int_0^\infty \frac{C_0\left(T, e^{-rT}X\right) - \max\left(S_0 - X, 0\right)}{X^2} dX.$$

Jiang and Tian (2005) and Jiang and Tian (2007) discuss the implementation of (8). In particular, they discuss potential biases that can arise from

1. Truncation errors: the integration is performed over a finite range of strike prices instead of from 0 to ∞.
2. Discretization errors: the integral over strikes is replaced by a sum.
3. Limited availability of strikes: the range of available strikes is narrow and/or has large gaps.

In practice, a finite range, $X_{\max} - X_{\min}$, of discrete strikes are available. Jiang and Tian (2005) consider using the trapezoidal integration rule

$$VAR_0(T) \approx \sum_{i=1}^{m} \left\{ \frac{\left[C_0^F(T, X_i) - \max(F_0 - X_i, 0)\right]}{X_i^2} + \frac{\left[C_0(T, X_{i-1}) - \max(F_0 - X_{i-1}, 0)\right]}{X_{i-1}^2} \right\} \Delta X \tag{9}$$

where $\Delta X = (X_{\max} - X_{\min})/m$, and the discrete (evenly spaced) strikes $X_i = X_{\min} + i\Delta X$.

In order to reduce the discretization error, ΔX needs to be reasonably small. Jiang and Tian (2005) fill in gaps in strikes by applying a cubic spline to the $BSIV$s of traded options, and demonstrate using a Monte Carlo experiment that this approaches works well. To overcome truncation problems, Jiang and Tian (2005) use a flat extrapolation outside of the strike price range, whereas Jiang and Tian (2007) use a linear extrapolation with smooth pasting. Figlewski (2010) proposes further modifications, including: (i) a fourth degree rather than a cubic spline, (ii) smoothing which does not require the interpolation function to fit the traded option prices exactly, and (iii) the application of extreme value functions for the tails of the distribution.

2.2.2 The VIX Volatility Index

The VIX volatility index is published by the Chicago Board of Options Exchange (CBOE). It is probably the best-known and most widely used example of option-implied information. It has become an important market indicator and it is sometimes referred to as "The Investor Fear Gauge" (Whaley (2000)).

The history of the VIX nicely illustrates the evolution in the academic literature, and the increasing prominence of model-free approaches rather than model-based approaches. Prior to 1993, the VIX was computed as the average of the $BSIV$ for four call and four put options just in- and out-of-the-money, with maturities just shorter and longer than thirty days. (See Whaley (2000) for a detailed discussion.) Since 2003, the new VIX relies on a model-free construction, and relies on the following general result.[5]

[5] The VIX calculation assumes a stock price process where the drift and diffusive volatility are arbitrary functions of time. These assumptions encompass for example implied tree models in which volatility is a function of stock price and time. See Dupire (1994) for a discussion of this type of models.

A variance swap is a contract that at time T pays integrated variance between time 0 and T less a strike price, X_{VS}. The strike is set so that the value of the variance swap is zero when written at time 0

$$e^{-rT} E_0 \left[\frac{1}{T} \int_0^T V_t dt - X_{VS} \right] = 0$$

Consider a stock price process with a generic dynamic volatility specification

$$dS = rSdt + \sqrt{V_t} S dz$$

From Ito's lemma we have

$$d\ln(S) = \left(r - \tfrac{1}{2} V_t\right) dt + \sqrt{V_t} dz$$

so that

$$\frac{dS}{S} - d\ln(S) = \tfrac{1}{2} V_t dt$$

This relationship shows that variance can be replicated by taking positions sensitive to the price, S, and the log price, $\ln(S)$, of the underlying asset. The idea of using log contracts to hedge volatility risk was first introduced by Neuberger (1994). Demeterfi, Derman, Kamal, and Zou (1999) use this result to derive the replicating cost of the variance swap as

$$VAR_0\left(T\right) = E_0 \left[\int_0^T V_t dt \right] = 2E_0 \left[\int_0^T \frac{dS}{S} - d\ln(S) \right] = 2E_0 \left[\int_0^T \frac{dS}{S} - \ln\left(\frac{S_T}{S_0} \right) \right] \tag{10}$$

CBOE (2009) implements the VIX as follows

$$VIX = 100 \sqrt{ \frac{2}{T} \sum_i \frac{\Delta X_i}{X_i^2} e^{rT} O\left(X_i\right) - \frac{1}{T} \left[\frac{F_0}{X_0} - 1 \right]^2 } \tag{11}$$

where X_0 is the first strike below F_0, $\Delta X_i = \left(X_{i+1} - X_{i-1}\right)/2$, and $O\left(X_i\right)$ is the midpoint of the bid-ask spread for an out of the money call or put option with strike X_i. The second term in (11) comes from the Taylor series expansion of the log function. Note that the VIX is reported in annual percentage volatility units.

The CBOE computes VIX using out-of-the-money and at-the-money call and put options. It calculates the volatility for the two available maturities that are the nearest and second-nearest to 30 days. Then they either interpolate, if one maturity is shorter and the other is longer than 30 days, or otherwise extrapolate, to get a 30 day index.

It is noteworthy that the implementation of this very popular index requires several ad hoc decisions which could conceivably affect the results. See for example Andersen and Bondarenko (2007), Andersen and Bondarenko (2009), and Andersen, Bondarenko, and Gonzalez-Perez (2011) for potential improvements to the VIX methodology. The latter paper shows that the time-varying range of strike prices available for the VIX calculation affects its precision and consequently suggests an alternative measure based on corridor variances that use a consistent range of strike prices over time.

Besides the underlying modeling approach, another important change was made to the computation of the VIX in 1993. Since 1993, the VIX is computed using S&P 500 option prices. Previously, it was based on S&P100 options. Note that the CBOE continues to calculate and disseminate the original-formula index, known as the CBOE S&P100 Volatility Index, with ticker VXO. This volatility series is sometimes useful because it has a price history going back to 1986.

The popularity of the VIX index has spawned the introduction of alternative volatility indexes in the U.S. and around the world. Table 2 provides an overview of VIX-like volatility products around the world. Table 2 also contains other option-implied products to be discussed below.

[Table 2: Volatility Indexes Around the World]

2.3 Volatility Forecasting Applications

A large number of studies test if option-implied volatility can forecast the future volatility of the underlying asset. The main market of interest has been the equity market, particularly stock market indices. Older studies typically used model-based estimates, mainly $BSIV$, whereas more recent studies focus more on model-free estimates.

Overall, the evidence indicates that option-implied volatility is a biased predictor of the future volatility of the underlying asset, but most studies find that it contains useful information over traditional predictors based on historical prices, and option-implied volatility by itself often outperforms historical volatility. A few studies investigate if option-implied volatility can predict variables other than volatility, such as stock returns and bond spreads. Table 3 contains a summary of existing empirical results. We now discuss these empirical results for different underlying assets.

[Table 3: Forecasting with Option-Implied Volatility]

2.3.1 Equity Index Applications

Almost all studies find that option-implied index volatility is useful in forecasting the volatility of the stock market index, a notable exception being Canina and Figlewski (1993). However, the evidence is mixed regarding the unbiasedness and efficiency of the option-implied estimates. Fleming, Ostdiek, and Whaley (1995), Fleming (1998), and Blair, Poon, and Taylor (2001) find that $BSIV$ is an efficient, but biased predictor, whereas Day and Lewis (1992) find that $BSIV$ is an unbiased, but inefficient predictor. Christensen and Prabhala (1998) find that $BSIV$ is unbiased and efficient. Busch et al. (2011) find that $BSIV$ is an efficient and unbiased predictor in equity index markets.

Jiang and Tian (2005) find that model-free option-implied volatility ($MFIV$) is biased, but efficient, subsuming all information in $BSIV$. Andersen and Bondarenko (2007) find a different result using a new measure of implied volatility, called Corridor IV (CIV). They compare the forecasting performance of the broad and narrow CIV, which are substitutes of the $MFIV$ and

$BSIV$ respectively, and find that the narrow CIV ($BSIV$) is biased, but subsumes the predictive content of the broad CIV ($MFIV$).

Harvey and Whaley (1992) test the predictability of $BSIV$ itself and find that $BSIV$ is predictable, but conclude that since arbitrage profits are not possible in the presence of transaction costs, this predictability is not inconsistent with market efficiency. Poon and Granger (2005) provide a comprehensive survey of volatility forecasting in general.

Many recent studies have started exploring other ways in which the implied volatility can be used in forecasting. Bollerslev, Tauchen, and Zhou (2009), Bekaert, Hoerova, and Lo Duca (2010), and Zhou (2010) find strong evidence that the variance risk premium, which is the difference between implied variance and realized variance, can predict the equity risk premium. Bakshi, Panayotov, and Skoulakis (2011) compute the forward variance, which is the implied variance between two future dates, and find that the forward variance is useful in forecasting stock market returns, T-bill returns, and changes in measures of real economic activity. A related paper by Feunou, Fontaine, Taamouti, and Tedongap (2011) find that the term structure of implied volatility can predict both the equity risk premium and variance risk premium.

2.3.2 Individual Equity Applications

Latané and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981), and Lamoureux and Lastrapes (1993) find that $BSIV$ is useful in forecasting the volatility of individual stocks. Swidler and Wilcox (2002) focus on bank stocks, and find that $BSIV$ outperforms historical predictors.

Implied volatility has also been used to predict future stock returns. Banerjee, Doran, and Peterson (2007) find that the VIX predicts the return on portfolios sorted on book-to-market equity, size, and beta. Diavatopoulos, Doran, and Peterson (2008) find that implied idiosyncratic volatility can forecast the cross-section of stock returns. Doran, Fodor, and Krieger (2010) find that the information in option markets leads analyst recommendation changes.

Ang, Hodrick, Xing, and Zhang (2006) have a somewhat different focus, investigating the performance of the VIX as a pricing factor: they find that the VIX is a priced risk factor with a negative price of risk, so that stocks with higher sensitivities to the innovation in VIX exhibit on average future lower returns. Delisle, Doran, and Peterson (2010) find that the result in Ang et al. (2006) holds when volatility is rising, but not when volatility is falling.

2.3.3 Other Markets

Fackler and King (1990) and Kroner, Kneafsey, and Claessens (1995) study the forecasting ability of implied volatility in commodity markets. For currencies, Jorion (1995) and Xu and Taylor (1995) find that $BSIV$ outperforms historical predictors. Pong, Shackleton, Taylor, and Xu (2004) compare $BSIV$ to predictors based on historical intraday data in currency markets, and find that historical predictors outperform $BSIV$ for one-day and one-week horizons, whereas $BSIV$ is at least as accurate as historical predictors for one-month and three-month horizons. Christoffersen and Mazzotta (2005) also find that the implied volatility yields unbiased and accurate forecast of exchange rate volatility.

Busch et al. (2011) investigate assets in three different markets: the S&P 500, the currency market, using the USD/DM exchange rate, and the fixed income market, using the 30-year US Treasury bond. They find that the $BSIV$ contains incremental information about future volatility in all three markets, relative to continuous and jump components of intraday prices. $BSIV$ is an efficient predictor in all three markets and is unbiased in foreign exchange and stock markets. Amin and Ng (1997) also find that implied volatility from Eurodollar futures options forecasts most of the realized interest rate volatility.

2.4 Extracting Correlations from Option Implied Volatilities

Certain derivatives contain very rich information on correlations between financial time series. This is especially the case for basket securities, written on a basket of underlying securities, such as collateralized debt obligations (CDOs). As mentioned in the introduction, because of space constraints we limit our survey to options.

We now discuss the extraction of information on correlations for two important security classes, currency and equity. In both cases, some additional assumptions need to be made. Despite the differences in assumptions, in both cases correlations are related to option implied volatilities. This is not entirely surprising, as correlation can be thought of as a second co-moment. Implied correlation information on equities is particularly relevant, because equity as an asset class is critically important for portfolio management. Table 4 contains a summary of existing empirical results on the use of option-implied correlations in forecasting.

[Table 4: Forecasting with Option-Implied Correlation]

2.4.1 Extracting Correlations From Triangular Arbitrage

Using the U.S. dollar, \$, the British Pound, £, and the Japanese Yen, ¥, as an example, from triangular arbitrage in FX markets we know that

$$S_{\$/£} = S_{\$/¥} S_{¥/£}.$$

From this it follows that for log returns

$$R_{\$/£} = R_{\$/¥} + R_{¥/£}.$$

From this we get that

$$VAR_{\$/£} = VAR_{\$/¥} + VAR_{¥/£} + 2COV(R_{\$/¥}, R_{¥/£})$$

so that the correlation must be

$$CORR(R_{\$/¥}, R_{¥/£}) = \frac{\left(VAR_{\$/£} - VAR_{\$/¥} - VAR_{¥/£}\right)}{2VAR_{\$/¥}^{1/2} VAR_{¥/£}^{1/2}}.$$

Provided we have option-implied variance forecasts for the three currencies, we can use this to get an option-implied covariance forecast. See Walter and Lopez (2000) and Campa and Chang (1998) for applications.

Siegel (1997) finds that option-implied exchange rate correlations for the DM/GBP pair and the DM/JPY pair predict significantly better than historical correlations between 1992 and 1993. Campa and Chang (1998) also find that the option-implied correlation for USD/DM/JPY predicts better than historical correlations between 1989 and 1995. The evidence in Walter and Lopez (2000), however, is mixed. They find that the option-implied correlation is useful for USD/DM/JPY (1990-1997), but much less useful for USD/DM/CHF (1993-1997), and conclude that the option-implied correlation may not be worth calculating in all instances.

Correlations have been extracted from options in fixed income markets. Longstaff, Santa-Clara, and Schwartz (2001) and de Jong et al. (2004) provide evidence that forward rate correlations implied by cap and swaption prices differ from realized correlations.

2.4.2 Extracting Average Correlations Using Index and Equity Options

Skintzi and Refenes (2005) and Driessen, Maenhout, and Vilkov (2009) propose the following measure of average option-implied correlation between the stocks in an index, I,

$$\rho_{ICI} = \frac{VAR_I - \sum_{j=1}^{n} w_j^2 VAR_j}{2\sum_{j=1}^{n-1}\sum_{i>j}^{n} w_i w_j VAR_i^{1/2} VAR_j^{1/2}} \tag{12}$$

where w_j denotes the weight of stock j in the index.

Note that the measure is based on the option-implied variance for the index, VAR_I, and the individual stock variances, VAR_j. Skintzi and Refenes (2005) use options on the DJIA index and its constituent stocks between 2001 and 2002, and find that the implied correlation index is biased upward, but is a better predictor of future correlation than historical correlation. Buss and Vilkov (2011) use the implied correlation approach to estimate option-implied betas and find that the option-implied betas predict realized betas well. DeMiguel, Plyakha, Uppal, and Vilkov (2011) use option-implied information in portfolio allocation. They find that option-implied volatility and correlation do not improve the Sharpe ratio or certainty-equivalent return of the optimal portfolio. However, expected returns estimated using information in the volatility risk premium and option-implied skewness increase both the Sharpe ratio and the certainty-equivalent return substantially. The CBOE has recently introduced an Implied Correlation Index (ICI) for S&P 500 firms based on (12).

3 Extracting Skewness and Kurtosis from Option Prices

The $BSIV$ smirk patterns in Figure 2 revealed that index options imply negative skewness not captured by the normal distribution. Prior to 1987, this pattern more closely resembled a symmetric "smile". Other underlying assets such as foreign exchange rates often display symmetric smile patterns in $BSIV$ implying evidence of excess kurtosis rather than negative skewness. In this section we consider methods capable of generating option-implied measures of skewness and kurtosis which can be used as forecasts for subsequent realized skewness and kurtosis.

3.1 Model-Free Skewness and Kurtosis Extraction

We will begin with model-free methods for higher moment forecasting because they are the most common. This section first develops the general option replication approach for which higher-moment extraction is a special case. We will then briefly consider other approaches.

3.1.1 The Option Replication Approach

Bakshi and Madan (2000) and Carr and Madan (2001) show that any twice continuously differentiable function, $H(S_T)$, of the terminal stock price S_T, can be replicated (or spanned) by a unique position of risk-free bonds, stocks and European options. Let $H(S_0) - H'(S_0) S_0$ denote units of the risk-free discount bond, which of course is independent of S_T, let $H'(S_0)$ denote units of the underlying risky stock, which is trivially linear in S_T, and let $H''(X)\,dX$ denote units of (nonlinear) out-of-the-money call and put options with strike price X. Then we have

$$\begin{aligned} H(S_T) &= \left[H(S_0) - H'(S_0) S_0\right] + H'(S_0) S_T \\ &\quad + \int_0^{S_0} H''(X) \max(X - S_T, 0)\, dX + \int_{S_0}^{\infty} H''(X) \max(S_T - X, 0)\, dX \end{aligned} \tag{13}$$

This result is clearly very general and we provide its derivation in Appendix A. From a forecasting perspective, for any desired function $H(\cdot)$ of the future realization S_T there is a portfolio of risk-free bonds, stocks, and options whose current aggregate market value provides an option-implied forecast of $H(S_T)$.

Let the current market value of the bond be e^{-rT}, and the current put and call prices be $P_0(T, X)$ and $C_0(T, X)$ respectively, then we have

$$\begin{aligned} E_0\left[e^{-rT} H(S_T)\right] &= e^{-rT}\left[H(S_0) - H'(S_0) S_0\right] + S_0 H'(S_0) \\ &\quad + \int_0^{S_0} H''(X) P_0(T, X)\, dX + \int_{S_0}^{\infty} H''(X) C_0(T, X)\, dX \end{aligned} \tag{14}$$

Bakshi, Kapadia, and Madan (2003) (BKM hereafter) apply this general result to the second, third, and fourth power of log returns. We provide their option implied moments in Appendix B. For simplicity we consider here higher moments of simple returns where $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^2$, $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^3$, and $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^4$.

We can use OTM European call and put prices to derive the quadratic contract as

$$M_{0,2}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T - S_0}{S_0}\right)^2\right] = \frac{2}{S_0^2}\left[\int_0^{S_0} P_0(T, X)\, dX + \int_{S_0}^{\infty} C_0(T, X)\, dX\right]. \tag{15}$$

The cubic contract is given by

$$M_{0,3}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T-S_0}{S_0}\right)^3\right] = \frac{6}{S_0^2}\left[\int_0^{S_0}\left(\frac{X-S_0}{S_0}\right)P_0(T,X)\,dX + \int_{S_0}^{\infty}\left(\frac{X-S_0}{S_0}\right)C_0(T,X)\,dX\right] \quad (16)$$

and the quartic contract is given by

$$M_{0,4}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T-S_0}{S_0}\right)^4\right] = \frac{12}{S_0^2}\left[\int_0^{S_0}\left(\frac{X-S_0}{S_0}\right)^2 P_0(T,X)\,dX + \int_{S_0}^{\infty}\left(\frac{X-S_0}{S_0}\right)^2 C_0(T,X)\,dX\right] \quad (17)$$

Notice how the quadratic contract–which is key for volatility–simply integrates over option prices. The cubic contract–which is key for skewness–integrates over option prices multiplied by moneyness, $\frac{X-S_0}{S_0} = \frac{X}{S_0} - 1$. The quartic contract–which is key for kurtosis–integrates over the option prices multiplied by moneyness squared. High option prices imply high volatility. High OTM put prices and low OTM call prices imply negative skewness (and vice versa). High OTM call and put prices at extreme moneyness imply high kurtosis.

We can now compute the option-implied volatility, skewness, and kurtosis (for convenience we suppress the dependence of M on T) as

$$VOL_0(T) \equiv [VAR_0(T)]^{1/2} = \left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{1/2} \quad (18)$$

$$SKEW_0(T) = \frac{e^{rT}M_{0,3} - 3M_{0,1}e^{rT}M_{0,2} + 2M_{0,1}^3}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{\frac{3}{2}}} \quad (19)$$

$$KURT_0(T) = \frac{e^{rT}M_{0,4} - 4M_{0,1}e^{rT}M_{0,3} + 6e^{rT}M_{0,1}^2M_{0,2} - 3M_{0,1}^4}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^2} \quad (20)$$

where

$$M_{0,1} \equiv E_0\left[\left(\frac{S_T-S_0}{S_0}\right)\right] = e^{rT} - 1 \quad (21)$$

BKM provide a model-free implied variance, like Britten-Jones and Neuberger (2000) in (8). BKM compute the variance of the holding period return, whereas Britten-Jones and Neuberger (2000) compute the expected value of realized variance. These concepts of volatility will coincide if the log returns are zero mean and uncorrelated.

Using S&P 500 index options from January 1996 through September 2009 Figure 4 plots the higher moments of log returns for the one-month horizon.

[Figure 4: Option-Implied Moments for One-Month S&P 500 Returns]

Not surprisingly, the volatility series is very highly correlated with the VIX index, with a correlation of 0.997 between January 1996 and September 2009. The annualized volatility varied between around 0.1 and 0.4 before the subprime crisis of 2008, but its level shot up to an unprecedented level of around 0.8 at the onset of the crisis, subsequently reverting back to its previous level by late 2009. The estimate of skewness is negative for every day in the sample, varying between minus three and zero. Interestingly, skewness did not dramatically change during the subprime crisis, despite the fact that option-implied skewness is often interpreted as the probability of a market crash or the fear thereof. The estimate of kurtosis is higher than three (i.e. excess kurtosis) for every day in the sample, indicating that the option-implied distribution has fatter tails than the normal distribution. Its level did not dramatically change during the sub-prime crisis, but the time series exhibits more day-to-day variation during this period.

The estimation of skewness and kurtosis using the BKM method is subject to the same concerns discussed by Jiang and Tian (2005) and Jiang and Tian (2007) in the context of volatility estimation. Chang, Christoffersen, Jacobs, and Vainberg (2011) present Monte Carlo evidence on the quality of skewness estimates when only discrete strike prices are available. Fitting a spline through the implied volatilities and integrating the spline, following the methods of Jiang and Tian (2005) and Jiang and Tian (2007), seems to work well for skewness too, and dominates simple integration using only observed contracts.

In February 2011, the CBOE began publishing the CBOE S&P 500 Skew Index. The skewness index is computed using the methodology in BKM described in this section combined with the interpolation/extrapolation method used in the VIX calculation described in Section 2.2.2. See CBOE (2011) for details.

3.1.2 Other Model-Free Measures of Option Implied Skewness

Many empirical studies on option implied skewness use the asymmetry observed in the implied volatility curve in Figure 2, often referred to as the smirk, to infer the skewness of the option-implied distribution. There are many variations in the choice of options used to measure the asymmetry of the implied volatility curve. The most popular method involves taking the difference of the out-of-the-money put $BSIV$ and out-of-the-money call $BSIV$. This measure, proposed by Bates (1991), reflects the different extent to which the left-hand tail and the right-hand tail of the option-implied distribution of the underlying asset price deviate from the lognormal distribution. Another approach is to take the difference between the out-of-the-money put $BSIV$ and at-the-money put (or call) $BSIV$ as in Xing, Zhang, and Zhao (2010). This measure only looks at the left-hand side of the distribution, and is often used in applications where the downside risk of the underlying asset is the variable of interest. Another variable that is also shown to be somewhat related to implied skewness is the spread of implied volatility of call and put options with the same maturity and same strike (Cremers and Weinbaum (2010) and Bali and Hovakimian (2009)).

Recently, Neuberger (2011) has proposed a model-free method that extends the variance swap methodology used to compute the VIX index. He shows that just as there is a model-free strategy to replicate a variance swap, a contract that pays the difference between option implied variance and realized variance, there is also a model-free strategy to replicate a skew swap, a contract that pays the difference between option implied skew and realized skew.

3.2 Model-Based Skewness and Kurtosis Extraction

In this section we first review two models that are based on expansions around the Black-Scholes model explicitly allowing for skewness and kurtosis. We then consider an alternative model-based approach specifying jumps in returns which imply skewness and kurtosis.

3.2.1 Expansions of the Black-Scholes Model

Jarrow and Rudd (1982) propose an option pricing method where the density of the security price at option maturity, T, is approximated by an alternative density using the Edgeworth series expansion. If we choose the lognormal as the approximating density, and use the shorthand notation for the Black-Scholes model

$$C_0^{BS}(T,X) \equiv C^{BS}(T,X,S_0,r;\sigma)$$

then the Jarrow-Rudd model is defined by

$$C_0^{JR}(T,X) \approx C_0^{BS}(T,X) - e^{-rT}\frac{(K_3 - K_3(\Psi))}{3!}\frac{d\psi(T,X)}{dX} + e^{-rT}\frac{(K_4 - K_4(\Psi))}{4!}\frac{d^2\psi(T,X)}{dX^2} \quad (22)$$

where K_j is the jth cumulant of the actual density, $K_j(\Psi)$ is the cumulant of the lognormal density, $\psi(T,X)$, so that

$$\begin{aligned}
\psi(T,X) &= \left(X\sigma\sqrt{T2\pi}\right)^{-1}\exp\left\{-\frac{1}{2}\left(d-\sigma\sqrt{T}\right)^2\right\} \\
\frac{d\psi(T,X)}{dX} &= \frac{\psi(T,X)\left(d-2\sigma\sqrt{T}\right)}{X\sigma\sqrt{T}} \\
\frac{d^2\psi(T,X)}{dX^2} &= \frac{\psi(T,X)}{X^2\sigma^2 T}\left[\left(d-2\sigma\sqrt{T}\right)^2 - \sigma\sqrt{T}\left(d-2\sigma\sqrt{T}\right) - 1\right]
\end{aligned}$$

and where d is as defined in (2).

In general we have the following relationships between cumulants and moments

$$K_2 = VAR, \qquad K_3 = K_2^{3/2} SKEW, \qquad K_4 = K_2^2(KURT - 3)$$

For the log normal density we have the following moments

$$\begin{aligned}
VAR(X) &= \exp\left(2\left(\ln(S_0) + \left(r - \frac{1}{2}\sigma^2\right)T\right) + \sigma^2 T\right)\left(\exp\left(\sigma^2 T\right) - 1\right) \\
SKEW(X) &= \left(\exp\left(\sigma^2 T\right) + 2\right)\sqrt{\exp\left(\sigma^2 T\right) - 1} \\
KURT(X) &= \exp\left(4\sigma^2 T\right) + 2\exp\left(3\sigma^2 T\right) + 3\exp\left(2\sigma^2 T\right) - 3
\end{aligned}$$

The cumulants corresponding to these moments provide the expressions for $K_3(X)$ and $K_4(X)$ in equation (22) above.

The Jarrow-Rudd model in (22) now has three parameters left to estimate, namely, σ, K_3, and K_4 or equivalently σ, $SKEW$ and $KURT$. In principle these three parameters could be solved for using three observed option prices. These parameters would then provide option-implied forecasts of volatility, skewness and kurtosis in the distribution of $\ln(S_T)$. Alternatively they could be estimated by minimizing the option valuation errors on a larger set of observed option prices. Christoffersen and Jacobs (2004) discuss the choice of objective function in this type of estimation problems.

As an alternative to the Edgeworth expansion, Corrado and Su (1996) consider a Gram-Charlier series expansion,[6] in which

$$C_0^{CS}(T,X) = C_0^{BS}(T,X) + Q_3 SKEW + Q_4 (KURT - 3) \tag{23}$$

where

$$\begin{aligned} Q_3 &= \frac{1}{3!} S_0 \sigma \sqrt{T} \left(\left(2\sigma\sqrt{T} - d \right) N'(d) + \sigma^2 T N(d) \right); \\ Q_4 &= \frac{1}{4!} S_0 \sigma \sqrt{T} \left(\left(d^2 - 1 - 3\sigma\sqrt{T} \left(d - \sigma\sqrt{T} \right) \right) N'(d) + \sigma^3 T^{3/2} N(d) \right) \end{aligned}$$

where $N'(d)$ is again the standard normal probability density function. Note that Q_4 and Q_3 represent the marginal effect of skewness and kurtosis respectively and note that d is as defined in (2). In the Corrado-Su model $SKEW$ and $KURT$ refer to the distribution of log return shocks defined by

$$Z_T = \left[\ln S_T - \ln(S_0) - \left(r - \frac{1}{2}\sigma^2 \right) T \right] / \left(\sigma\sqrt{T} \right)$$

Again, option-implied volatility, skewness and kurtosis can be estimated by minimizing the distance between $C_0^{CS}(T,X)$ and a sample of observed option prices or by directly solving for the three parameters using just three observed option prices.

3.2.2 Jumps and Stochastic Volatility

While the Black and Scholes (1973) and stochastic volatility option pricing models are often used to extract volatility, the study of higher moments calls for different models. The Black-Scholes model assumes normality, and therefore strictly speaking cannot be used to extract skewness and kurtosis from the data, although patterns in Black-Scholes implied volatility are sometimes used to learn about skewness.

Stochastic volatility models such as Heston (1993) can generate skewness and excess kurtosis, but fall short in reconciling the stylized facts on physical higher moments with the dynamics of higher option-implied moments (Bates (1996b) and Pan (2002)). Instead, generalizations of the Black and Scholes (1973) and Heston (1993) setup are often used, such as the jump-diffusion model of Bates (1991) and the stochastic volatility jump-diffusion (SVJ) model of Bates (1996b).

In Bates (2000), the futures price F is assumed to follow a jump-diffusion of the following form

$$\begin{aligned} dF/F &= -\lambda \bar{k} dt + \sqrt{V} dz_1 + k dq, \\ dV &= \kappa(\theta - V) dt + \sigma_V \sqrt{V} dz_2 \end{aligned} \tag{24}$$

[6] See also Backus, Foresi, Li, and Wu (1997).

where q is a Poisson counter with instantaneous intensity λ, and where k is a lognormally distributed return jump

$$\ln(1+k) \sim N\left[\ln\left(1+\bar{k}\right) - \delta^2/2, \delta^2\right]$$

As in Heston (1993) the return and variance diffusion terms are correlated with coefficient ρ.

Bates (2000) derives the n^{th} cumulant for horizon T to be

$$K_n(T) = \left[\frac{\partial^n A(T;\Phi)}{\partial \Phi^n} + \frac{\partial^n B(T;\Phi)}{\partial \Phi^n} V\right]_{\Phi=0} + \lambda T \left[\frac{\partial^n C(\Phi)}{\partial \Phi^n}\right]_{\Phi=0}$$

where

$$\begin{aligned}
A(T;\Phi) &= -\frac{\kappa\theta T}{\sigma_V^2}\left(\rho\sigma_V\Phi - \kappa - D(\Phi)\right) - \frac{2\kappa\theta}{\sigma_V^2}\ln\left[1 + \frac{1}{2}\left(\rho\sigma_V\Phi - \kappa - D(\Phi)\right)\frac{1-e^{D(\Phi)T}}{D(\Phi)}\right], \\
B(T;\Phi) &= \frac{-\left[\Phi^2 - \Phi\right]}{\rho\sigma_V\Phi - \kappa + D(\Phi)\left(\frac{1+e^{D(\Phi)T}}{1-e^{D(\Phi)T}}\right)}, \text{ and} \\
C(\Phi) &= \left[\left(1+\bar{k}\right)^{\Phi} e^{\frac{1}{2}\delta^2\left[\Phi^2-\Phi\right]} - 1\right] - \bar{k}\Phi, \text{ and where} \\
D(\Phi) &= \sqrt{\left(\rho\sigma_V\Phi - \kappa\right)^2 - 2\sigma_V^2\left\{\frac{1}{2}\left[\Phi^2 - \Phi\right]\right\}},
\end{aligned}$$

From the cumulants we have the following conditional moments for the log futures returns for holding period T

$$VAR_0(T) = K_2(T), \quad SKEW_0(T) = K_3(T)/K_2^{3/2}(T), \quad KURT_0(T) = K_4(T)/K_2^2(T) + 3$$

Besides the higher moments such as skewness and kurtosis, this model yields parameters describing the intensity and size of jumps, which can potentially be used to forecast jump-like events such as stock market crashes and defaults.

There is an expanding literature estimating models like (24) as well as more general models with jumps in volatility using returns and/or options. See for instance Bates (2000), Bates (2008), Andersen, Benzoni, and Lund (2002), Pan (2002), Huang and Wu (2004), Eraker, Johannes, and Polson (2003), Broadie, Chernov, and Johannes (2009), Li, Wells, and Yu (2008), and Chernov, Gallant, Ghysels, and Tauchen (2003).

3.3 Applications

As discussed in Section 2.3, many studies use option implied volatility to forecast the volatility of the underlying asset. A few studies have used option implied skewness and kurtosis to forecast the returns on the underlying, as well as cross-sectional differences in stock returns. Table 5 contains a summary of existing empirical results.

[Table 5: Forecasting with Option-Implied Skewness and Kurtosis]

3.3.1 Time Series Forecasting

Bates (1991) investigates the usefulness of jump parameters estimated using a jump diffusion model for forecasting the stock market crash of 1987. He also forecasts using a skewness premium constructed from prices of out-of-the-money puts and calls. Bates (1996a) examines option implied skewness and kurtosis of the USD/DM and USD/JPY exchange rates between 1984 and 1992, and finds that the option implied higher moments contain significant information for the future USD/DM exchange rate, but not for the USD/JPY rate. The option implied higher moments are again estimated both using a model-based approach, using a jump-diffusion dynamic, but also using a model-free measure of the skewness premium.

Navatte and Villa (2000) extract option implied moments for the CAC 40 index using the Gram-Charlier expansion. They find that the moments contain a substantial amount of information for future moments, with kurtosis contributing less forecasting power than skewness and volatility.

Carson, Doran, and Peterson (2006) find that the implied volatility skew has strong predictive power in forecasting short-term market declines. However, Doran, Peterson, and Tarrant (2007) find that the predictability is not economically significant.

For individual stocks, Diavatopoulos, Doran, Fodor, and Peterson (2008) look at changes in implied skewness and kurtosis prior to earnings announcements and find that both have strong predictive power for future stock and option returns. DeMiguel et al. (2011) propose using implied volatility, skewness, correlation and variance risk premium in portfolio selection, and find that the inclusion of skewness and the variance risk premium improves the performance of the portfolio significantly.

3.3.2 Option Implied Market Moments as Pricing Factors

Two recent studies investigate if option-implied higher moments of the S&P 500 index help explain the subsequent cross-section of returns. Chang, Christoffersen, and Jacobs (2009) test the cross-section of all stocks in the CRSP database, whereas Agarwal, Bakshi, and Huij (2009) investigate returns on the cross-section of hedge fund returns. Both studies use the model-free moments of BKM described in Section 3.1. Both studies find strong evidence that stocks with higher sensitivity to the innovation in option-implied skewness of the S&P 500 index exhibit lower returns in the future. Agarwal et al. (2009) also find a positive relationship between a stock's sensitivity to innovations in option-implied kurtosis of the S&P 500 index and future returns.

3.3.3 Equity Skews and the Cross-Section of Future Stock Returns

Several recent studies find a cross-sectional relationship between the option-implied skew of individual stocks and their subsequent returns. Xing et al. (2010) define skew as the difference in implied volatilities between out-of-the-money puts and at-the-money calls. They find that steeper smirks are associated with lower future stock returns. Doran and Krieger (2010) decompose the volatility skew into several components. They find that future stock returns are positively related to the difference in volatilities between at-the-money calls and puts, and negatively related to a measure of the left skew of the implied volatility curve. These results are consistent with those found in Cremers and Weinbaum (2010), Bali and Hovakimian (2009), and Xing et al. (2010). More importantly, the results in Doran and Krieger (2010) indicate that different measures of implied skewness can lead to different empirical results on the relationship between implied skewness and the cross-section of future stock returns.

Conrad, Dittmar, and Ghysels (2009) and Rehman and Vilkov (2010) both use the model-free skewness of Bakshi et al. (2003), but find the opposite relationship between implied skewness and the cross-section of future stock returns. Conrad et al. (2009) find a negative relationship while Rehman and Vilkov (2010) find a positive one. The difference between these two empirical studies is that Conrad et al. (2009) use average skewness over the last three months whereas Rehman and Vilkov (2010) use skewness measures computed only on the last available date of each month. Again, these conflicting results indicate that the relationship between equity skews and the cross-section of future stocks returns is sensitive to variations in empirical methodology.

3.3.4 Option Implied Betas

Section 2.4 above documents how option-implied correlation can be extracted from option data. Given the assumptions, correlations are a function of option-implied volatilities. Chang et al. (2011) provide an alternative approach, assuming that firm-specific risk has zero skewness. In this case it is possible to derive an option-implied beta based on the option-implied moments of firm j and the market index I as follows

$$\beta_j = \left(\frac{SKEW_j}{SKEW_I}\right)^{1/3} \left(\frac{VAR_j}{VAR_I}\right)^{1/2}, \tag{25}$$

where VAR and $SKEW$ can be computed from index options and from equity options for firm j using (18) and (19). Chang et al. (2011) find that, similar to the evidence for implied volatilities, historical betas and option-implied betas both contain useful information for forecasting future betas.

4 Extracting Densities from Option Prices

There are many surveys on density forecasting using option prices. See Söderlind and Svensson (1997), Galati (1999), Jackwerth (1999), Jondeau and Rockinger (2000), Bliss and Panigirtzoglou (2002), Rebonato (2004), Taylor (2005), Bu and Hadri (2007), Jondeau, Poon, and Rockinger (2007), Figlewski (2010), Fusai and Roncoroni (2008), and Markose and Alentorn (2011). We describe the details of only a few of the most popular methods in this section, and refer the readers interested in the details of other methods to these surveys. We start by discussing model-free estimation, and subsequently discuss imposing more structure on the problem using no-arbitrage restrictions or parametric models.

4.1 Model-Free Estimation

Breeden and Litzenberger (1978) and Banz and Miller (1978) show that the option-implied density of a security can be extracted from a set of European-style option prices with a continuum of strike prices. This result can be derived as a special case of the general replication result in (13).

The value of a European call option, C_0, is the discounted expected value of its payoff on the expiration date T. Under the option-implied measure, $f_0(S_T)$, the payoff is discounted at the risk-free rate

$$C_0(T,X) = e^{-rT} \int_0^{\infty} \max\{S_T - X, 0\} f_0(S_T)\, dS_T = e^{-rT} \int_X^{\infty} (S_T - X) f_0(S_T)\, dS_T \tag{26}$$

Take the partial derivative of C_0 with respect to the strike price X to get

$$\frac{\partial C_0(T,X)}{\partial X} = -e^{-rT}\left[1 - \tilde{F}_0(X)\right], \tag{27}$$

which yields the cumulative density function (CDF)

$$\tilde{F}_0(X) = 1 + e^{rT}\frac{\partial C_0(T,X)}{\partial X} \text{ so that } \tilde{F}_0(S_T) = 1 + e^{rT}\left.\frac{\partial C_0(T,X)}{\partial X}\right|_{X=S_T}. \tag{28}$$

The conditional probability density function (PDF) denoted by $f_0(X)$ can be obtained by taking the derivative of (28) with respect to X.

$$f_0(X) = e^{rT}\frac{\partial^2 C_0(T,X)}{\partial X^2} \text{ so that } f_0(S_T) = e^{rT}\left.\frac{\partial^2 C_0(T,X)}{\partial X^2}\right|_{X=S_T} \tag{29}$$

As noted above, the put-call parity states that $S_0 + P_0 = C_0 + Xe^{-rT}$, so that if we use put option prices instead, we get

$$\tilde{F}_0(S_T) = e^{rT}\left.\frac{\partial P_0(T,X)}{\partial X}\right|_{X=S_T} \text{ and } f_0(S_T) = e^{rT}\left.\frac{\partial^2 P_0(T,X)}{\partial X^2}\right|_{X=S_T}. \tag{30}$$

In practice, we can obtain an approximation to the CDF in (28) and (30) using finite differences of call or put option prices observed at discrete strike prices

$$\tilde{F}_0(X_n) \approx 1 + e^{rT}\left(\frac{C_0(T, X_{n+1}) - C_0(T, X_{n-1})}{X_{n+1} - X_{n-1}}\right) \tag{31}$$

or

$$\tilde{F}_0(X_n) \approx e^{rT}\left(\frac{P_0(T, X_{n+1}) - P_0(T, X_{n-1})}{X_{n+1} - X_{n-1}}\right). \tag{32}$$

Similarly, we can obtain an approximation to the PDF in (29) and (30) via

$$f_0(X_n) \approx e^{rT}\frac{C_0(T, X_{n+1}) - 2C_0(T, X_n) + C_0(T, X_{n-1})}{(\Delta X)^2} \tag{33}$$

$$f_0\left(X_n\right) \approx e^{rT} \frac{P_0\left(T, X_{n+1}\right) - 2P_0\left(T, X_n\right) + P_0\left(T, X_{n-1}\right)}{\left(\Delta X\right)^2}. \tag{34}$$

In terms of the log return, $R_T = \ln S_T - \ln S_0$, the CDF and PDF are

$$\tilde{F}_{0,R_T}\left(x\right) = F_0\left(e^{x+\ln S_0}\right) \qquad \text{and} \qquad f_{0,R_T}\left(x\right) = e^{x+\ln S_0} f_0\left(e^{x+\ln S_0}\right).$$

The most important constraint in implementing this method is that typically only a limited number of options are traded in the market, with a handful of strike prices. This approximation method can therefore only yield estimates of the CDF and the PDF at a few points in the domain. This constraint has motivated researchers to develop various ways of imposing more structure on the option-implied density. In some cases the additional structure exclusively derives from no-arbitrage restrictions, in other cases a parametric model is imposed. We now survey these methods, below in increasing order of structure imposed.

4.2 Imposing Shape Restrictions

Aït-Sahalia and Duarte (2003) propose a model-free method of option-implied density estimation based on locally polynomial regressions that incorporates shape restrictions on the first and the second derivatives of the call pricing function. Again, let $f_0\left(S_T\right)$ be the conditional density, then the call option prices are

$$C_0\left(T, X\right) = e^{-rT} \int_0^{+\infty} \max\left(S_T - X, 0\right) f_0\left(S_T\right) dS_T$$

By differentiating the call price C with respect to the strike X, we get

$$\frac{\partial C_0(T,X)}{\partial X} = -e^{-rT} \int_X^{+\infty} f_0\left(S_T\right) dS_T.$$

Since $f_0\left(S_T\right)$ is a probability density, it is positive and integrates to one across X. Therefore,

$$-e^{-rT} \leq \frac{\partial C_0(T,X)}{\partial X} \leq 0. \tag{35}$$

By differentiating the call price twice with respect to the strike price, we obtain as before

$$\frac{\partial^2 C_0(T,X)}{\partial X^2} = e^{-rT} f_0\left(X\right) \geq 0. \tag{36}$$

Two additional restrictions can be obtained using standard no arbitrage bounds of the call option prices,

$$\max\left(0, S_0 - Xe^{-rT}\right) \leq C_0\left(T, X\right) \leq S_0.$$

Li and Zhao (2009) develop a multivariate version of the constrained locally polynomial estimator in Aït-Sahalia and Duarte (2003) and apply it to interest rate options.

4.3 Using Black-Scholes Implied Volatility Functions

The simple but flexible Ad-Hoc Black-Scholes ($AHBS$) model in which the density forecast is constructed from Black-Scholes implied volatility curve fitting is arguably the most widely used method for option-implied density forecasting, and we now describe it in some more detail. The density construction proceeds in two steps.

First, we estimate a second-order polynomial or other well-fitting function for implied Black-Scholes volatility as a function of strike and maturity. This will provide the following fitted values for $BSIV$. We can write

$$BSIV\left(X,T\right)=a_0+a_1X+a_2X^2+a_3T+a_4T^2+a_5XT \tag{37}$$

Second, using this estimated polynomial, we generate a set of fixed-maturity implied volatilities across a grid of strikes. Call prices can then be obtained using the Black-Scholes functional form

$$C_0^{AHBS}\left(X,T\right)=C_0^{BS}\left(T,X,S_0,r;BSIV\left(X,T\right)\right). \tag{38}$$

Once the model call prices are obtained the option implied density can be obtained using the second derivative with respect to the strike price.

$$f_0\left(S_T\right)=e^{rT}\left.\frac{\partial^2 C_0^{AHBS}\left(T,X\right)}{\partial X^2}\right|_{X=S_T}.$$

Shimko (1993) was the first to propose this approach to constructing density forecasts from smoothed and interpolated $BSIV$s. Many variations on the Shimko approach have been proposed in the literature, and strictly speaking most of these are not entirely model-free, because some parametric assumptions are needed. The differences between these variations mainly concern three aspects of the implementation (See Figlewski (2010) for a comprehensive review):

1. Choice of independent variable: the implied volatility function can be expressed as a function of strike (X), or of moneyness (X/S), or of option delta (see Malz (1996)).

2. Choice of interpolation method: implied volatilities can be interpolated using polynomials (Shimko (1993) and Dumas, Fleming, and Whaley (1998)) or splines, which can be quadratic, cubic (Bliss and Panigirtzoglou (2002)) or quartic (Figlewski (2010)). Malz (1997), Rosenberg (1998), Weinberg (2001), and Monteiro, Tutuncu, and Vicente (2008) propose other methods.

3. Choice of extrapolation method: for strikes beyond the range of traded options, one can use extrapolation (Jiang and Tian (2005), Jiang and Tian (2007)), truncation (Andersen and Bondarenko (2007), Andersen and Bondarenko (2009) and Andersen et al. (2011)), or alternatively a parametric density function can be used. For instance Figlewski (2010) and Alentorn and Markose (2008) propose the Generalized Extreme Value distribution. Lee (2004), Benaim and Friz (2008), and Benaim and Friz (2009) derive restrictions on the slope of the implied volatility curve at tails based on the slope's relationship to the moments of the distribution.

Figure 5 shows the CDF and PDF obtained when applying a smoothing cubic spline using $BSIV$ data on 30-day OTM calls and puts on the S&P 500 index on October 22, 2009 together with the CDF and PDF of the lognormal distribution. The model-free estimate of the option-implied distribution is clearly more negatively skewed than the lognormal distribution. Note that we only draw the distribution for available strike prices and thus do not extrapolate beyond the lowest and highest strikes available.

[Figure 5: Option-Implied Distribution from BSIV Curve Fitting vs. Lognormal]

Related approaches are proposed by Madan and Milne (1994), who use Hermite polynomials, and Abadir and Rockinger (2003), who propose the use of hypergeometric functions. Empirical studies using these approaches include Abken, Madan, and Ramamurtie (1996), Jondeau and Rockinger (2001), Flamouris and Giamouridis (2002), Rompolis and Tzavalis (2008), and Giacomini, Härdle, and Krätschmer (2009).

Many alternative approaches have been proposed including 1) Implied binomial trees (Rubinstein (1994)) and its extensions (Jackwerth (1997), Jackwerth and Rubinstein (1996), Jackwerth (2000), and Dupont (2001)); 2) Entropy (Stutzer (1996), Buchen and Kelly (1996)); 3) Kernel regression (Aït-Sahalia and Lo (1998), Aït-Sahalia and Lo (2000)); 4) Convolution approximations (Bondarenko (2003)); and 5) Neural networks (Healy, Dixon, Read, and Cai (2007)). However, Black-Scholes implied volatility curve fitting remains the simplest and most widely used method.[7]

4.4 Static Distribution Models

As discussed above in Section 3.2.1, Jarrow and Rudd (1982) propose an Edgeworth expansion of the option-implied distribution around the lognormal density and Corrado and Su (1996) propose a related Gram-Charlier expansion which we also discussed above. These methods can be used to produce density forecasts as well as moment forecasts.

If we alternatively assume that S_T is distributed as a mixture of two lognormals, then we get

$$f_0(S_T) = w\psi(S_T, \mu_1, \sigma_1, T) + (1-w)\psi(S_T, \mu_2, \sigma_2, T) \tag{39}$$

The forward price for maturity T imposes the constraint

$$F_0 = w\mu_1 + (1-w)\mu_2$$

where μ_1 and μ_2 are parameters to be estimated, subject to the above constraint, along with the remaining parameters w, σ_1 and σ_2. The resulting option pricing formula is simply a weighted average of BS option prices.

$$C_0^{Mix}(T, X) = wC^{BS}(T, X, \mu_1, r; \sigma_1) + (1-w)C^{BS}(T, X, \mu_1, r; \sigma_2)$$

[7]See for example Christoffersen and Jacobs (2004) and Christoffersen, Heston, and Jacobs (2011).

Most applications assume a mixture of two or three lognormals. The resulting mixture is easy to interpret, especially when it comes to predicting events with a small number of outcomes. The moments can be obtained from

$$E_0\left[S_T^n\right] = w\mu_1^n \exp\left(\tfrac{1}{2}\left(n^2-n\right)\sigma_1^2 T\right) + (1-w)\,\mu_2^n \exp\left(\tfrac{1}{2}\left(n^2-n\right)\sigma_2^2 T\right)$$

The distribution is thus flexible enough to capture higher moments such as skewness and kurtosis. See for instance Ritchey (1990), Bahra (1997), and Melick and Thomas (1997) as well as Taylor (2005).

Alternative parametric distributions have been entertained by Bookstaber and McDonald (1991), who use a generalized beta distribution of the second kind, Sherrick, Garcia, and Tirupattur (1996), who use a Burr III distribution, Savickas (2002), who uses a Weibull distribution, and Markose and Alentorn (2011), who assume a Generalized Extreme Value (GEV) distribution. Other distributions used include generalized Beta functions (Aparicio and Hodges (1998) and Liu, Shackleton, Taylor, and Xu (2007)), generalized Lambda distribution (Corrado (2001)), generalized Gamma distribution (Tunaru and Albota (2005)), skewed Student-t (de Jong and Huisman (2000)), Variance Gamma (Madan, Carr, and Chang (1998)), and Lévy processes (Matache, Nitsche, and Schwab (2004)).

4.5 Dynamic Models with Stochastic Volatility and Jumps

There is overwhelming evidence that a diffusion with stochastic volatility (Heston (1993)), a jump-diffusion with stochastic volatility (Bates (1996b)), or an even more complex model for the underlying with jumps in returns and volatility is a more satisfactory description of the data than a simple Black-Scholes model. Nevertheless, these models are not the most popular choices in forecasting applications. This is presumably due to the significantly higher computational burden, which is especially relevant in a forecasting application which requires frequent re-calibration of the model.

The advantage of assuming a stochastic volatility model with jumps is that the primitives of the model include specification of the dynamic of the underlying at a frequency which can be chosen by the researcher. This not only adds richness to the model in the sense that it allows the multiperiod distribution to differ from the one-period distribution, it also allows consistent treatment of options of different maturities, and it ensures that the estimation results can be related in a straightforward way to estimation results for the underlying security.

In affine SVJ models closed form solutions are available for the conditional characteristic function for the log stock price at horizon T defined by

$$\Upsilon_0\left(i\phi, T\right) \equiv E_0[\exp\left(i\phi \ln\left(S_T\right)\right)].$$

The characteristic function can be used to provide call option prices as follows:

$$C_0(T,X) = S_0 P_1\left(X,T\right) - Xe^{-rT} P_2\left(X,T\right), \tag{40}$$

where P_1 and P_2 are obtained using numerical integration of the characteristic function

$$P_1(X,T) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi}\Upsilon_0(i\phi+1,T)}{i\phi S_0 e^{rT}}\right] d\phi \tag{41}$$

$$P_2(X,T) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi}\Upsilon_0(i\phi,T)}{i\phi}\right] d\phi \tag{42}$$

where $\Upsilon_0(\phi,T)$ denotes the characteristic function. The cumulative density forecast implied by the model is directly provided by $P_2(X,T)$ and the density forecast can be obtained from

$$f_0(S_T) = \left.\frac{\partial P_2(X,T)}{\partial X}\right|_{X=S_T},$$

which must be computed numerically.

4.6 Comparison of Methods

Many studies have compared the empirical performance of different density estimation methods. Jondeau and Rockinger (2000) compare semi-parametric methods based on Hermite and Edgeworth expansions, single and mixture lognormals, and methods based on jump diffusion and stochastic volatility models, and recommend using the mixture of lognormals model for the short-run options, and the jump-diffusion model for the long-run options. Coutant, Jondeau, and Rockinger (2001) compare Hermite expansion, maximum entropy, mixture of lognormals, and a single lognormal methods and conclude that all methods do better than a single lognormal method. They favor the Hermite expansion method due to its numerical speed, stability, and accuracy. Bliss and Panigirtzoglou (2002) compare double-lognormal and smoothed implied volatility function, focusing on the robustness of their parameter estimates, and conclude that the smoothed implied volatility function method dominates the double-lognormal method. Campa, Chang, and Reider (1998) and Jackwerth (1999) compare various parametric and nonparametric methods, and conclude that the estimated distributions from different methods are rather similar.

In summary, these and many other papers compare different estimation methods, and arrive at conclusions that are not always consistent with one another. Since the resulting densities are often not markedly different from each other using different estimation methods, it makes sense to use methods that are computationally easy and/or whose results are easy to interpret given the application at hand. Because of computational ease and the stability of the resulting parameter estimates, the smoothed implied volatility function method is a good choice for many purposes. The jump-diffusion model is useful if the event of interest is a rare event such as stock market crash. The lognormal-mixture is particularly useful when dealing with situations with a small number of possible outcomes, such as elections.

4.7 Density Forecasting Applications

Table 6 contains a summary of the existing empirical studies using option-implied densities (OID) in forecasting. Many early studies focus on the markets for commodities and currencies. Silva and

Kahl (1993) estimate soybean and corn futures price OIDs. Melick and Thomas (1997) estimate the distribution for crude oil during the Persian Gulf crisis. Høg and Tsiaras (2011) also examine options on crude oil futures. Leahy and Thomas (1996) estimate densities from Canadian dollar futures options around the referendum on Quebec sovereignty. Malz (1997) use s to explore issues related to the puzzle of excess returns in currency markets. Campa and Chang (1996), Malz (1996), and Haas, Mittnik, and Mizrach (2006) examine the information content of exchange rate OIDs around ERM crises. Bodurtha and Shen (1999) and Campa et al. (1998) both study the USD/DM and USD/Yen relationship using options. Campa, Chang, and Refalo (2002) apply the intensity of realignment and credibility measures developed in Campa and Chang (1996) to the "crawling peg" between the Brazilian Real and the USD between 1994 and 1999.

Using equity options, Gemmill and Saflekos (2000) study the predictive power of OID around four stock market crashes and three British elections between 1987 and 1997 using FTSE-100 index options. Mizrach (2006) examines whether the collapse of Enron was expected in option markets. Shackleton, Taylor, and Yu (2010) estimate S&P 500 index densities using various methods. Recently, Kostakis, Panigirtzoglou, and Skiadopoulos (2011) use estimated densities for portfolio selection.

[Table 6: Forecasting using Option-Implied Densities]

4.8 Event Forecasting Applications

There is a significant and expanding literature on prediction markets. The primary purpose of these markets is to forecast future events, and the contracts are designed to facilitate extracting information to use in forecasting. This literature is covered in detail in Snowberg, Wolfers, and Zitzewitz (2011) and is therefore not discussed here. We instead focus on the prediction of events using option data, where the primary function of the traded options is not the prediction itself. In this literature, which naturally overlaps with the density forecasting work discussed above, estimation methods vary greatly depending on the events to be forecast. We therefore do not describe the details of the estimation methods and focus our attention on the empirical results. Table 7 contains a summary of relevant empirical studies.

[Table 7: Forecasting with Option-Implied Event Probabilities]

Many stock market events are of great interest from a forecasting perspective, including stock market crashes and individual corporate events such as earnings announcements, stock splits, and acquisitions. Bates (1991) is the best known study of whether and how stock market index option prices reveal the market's expectation of future stock market crashes. He studies the behavior of S&P 500 futures options prices prior to the crash of October 1987, and finds unusually negative skewness in the option implied distribution of the S&P 500 futures between October 1986 and August 1987, leading to the conclusion that the crash was expected. He finds, however, that the same indicators do not exhibit any strong crash fears during the two months immediately preceding the crash. There are few other studies investigating if index option prices can predict stock market crashes. Doran et al. (2007) find that the option skew is useful in predicting stock market crashes and spikes, but conclude that the value of this predictive power is not always economically significant. Overall therefore, there is some evidence in favor of predictability, but the evidence is not conclusive.

Mizrach (2006) finds that option prices did not reflect the risk of Enron until just weeks before the firm's bankruptcy filing in 2001. Other studies examine corporate events other than crashes, and the results of these studies are more positive. Jayaraman, Mandelker, and Shastri (1991), Barone-Adesi, Brown, and Harlow (1994), Cao, Chen, and Griffin (2005), and Bester, Martinez, and Roşu (2011) all test the forecasting ability of variables in the option market (e.g. prices, trading volume, etc.) prior to corporate acquisitions. Jayaraman et al. (1991) find that implied volatilities increase prior to the announcement of the first bid for the target firm and decrease significantly at the announcement date, indicating that the market identifies potential targets firms prior to the first public announcement of the acquisition attempt. Cao et al. (2005) find that takeover targets with the largest preannouncement call-volume imbalance increases experience the highest announcement day returns. As for the probability of success and timing of announced acquisitions, Bester et al. (2011) show that their option pricing model yields better predictions compared to a "naive" method, although Barone-Adesi et al. (1994) find no evidence that option prices predict the timing of announced acquisitions.

5 Option-Implied Versus Physical Forecasts

So far in the chapter we have constructed various forecasting objects using the so-called risk-neutral or pricing measure implied from options. When forecasting properties of the underlying asset we ideally want to use the physical measure and not the risk-neutral measure which is directly embedded in option prices. Knowing the mapping between the two measures is therefore required. A fully specified option valuation model provides a physical measure for the underlying asset return as well as a risk-neutral measure for derivatives valuation, and therefore implicitly or explicitly defines the mapping. In this section we explore the mapping between measures. In this section use superscript Q to describe the option-implied density used above and we use superscript P to denote the physical density obeyed by the underlying asset.

5.1 Complete Markets

Black and Scholes (1973) assume a physical stock price process of the form

$$dS = (r + \mu) S dt + \sigma S dz \tag{43}$$

where μ is the equity risk premium. In the Black-Scholes model a continuously rebalanced dynamic portfolio consisting of one written derivative, C, and $\frac{\partial C}{\partial S}$ shares of the underlying asset has no risk and thus earns the risk-free rate. This portfolio leads to the Black-Scholes differential equation.

In the complete markets Black-Scholes world the option is a redundant asset which can be perfectly replicated by trading the stock and a risk-free bond. The option price is independent of the degree of risk-aversion of investors because they can replicate the option using a dynamic trading strategy in the underlying asset. This insight leads to the principle of risk-neutral valuation where all derivative assets can be valued using the risk-neutral expected pay-off discounted at the risk free rate. For example, for a European call option we can write

$$C_0(X,T) = \exp(-rT)\, E_0^Q\left[\max\{S_T - X, 0\}\right]$$

Using Ito's lemma on (43) we get

$$d\ln(S) = \left(r + \mu - \tfrac{\sigma^2}{2}\right)dt + \sigma dz$$

which implies that log returns are normally distributed

$$f_0^P(\ln(S_T)) = \frac{1}{\sqrt{2\pi\sigma^2 T}}\exp\left(\frac{1}{2\sigma^2 T}\left(\ln(S_T) - \ln(S_0) + \left(r + \mu - \tfrac{\sigma^2}{2}\right)T\right)^2\right)$$

Under the risk-neutral measure, $\mu = 0$, and we again have the lognormal density, but now with a different mean

$$f_0^Q(\ln(S_T)) = \frac{1}{\sqrt{2\pi\sigma^2 T}}\exp\left(\frac{1}{2\sigma^2 T}\left(\ln(S_T) - \ln(S_0) + \left(r - \tfrac{\sigma^2}{2}\right)T\right)^2\right)$$

In a Black-Scholes world, the option-implied density forecast will therefore have the correct volatility and functional form but a mean biased downward because of the equity premium, μ. Since the risk-neutral mean of the asset return is the risk-free rate, the option price has no predictive content for the mean return.

5.2 Incomplete Markets

The Black-Scholes model is derived in a complete market setting where risk-neutralization is straightforward. Market incompleteness arises under the much more realistic assumptions of market frictions arising for example from discrete trading, transactions costs, market illiquidity, price jumps and stochastic volatility or other non-traded risk factors.

5.2.1 Pricing Kernels and Investor Utility

In the incomplete markets case we can still assume a pricing relationship of the form

$$\begin{aligned} C_0(X,T) &= \exp(-rT)\, E_0^Q\left[\max\{S_T - X, 0\}\right] \\ &= \exp(-rT)\int_X^\infty \max\{\exp(\ln(S_T)) - X, 0\}\, f_0^Q(\ln(S_T))\, dS_T \end{aligned}$$

But the link between the option-implied and the physical distributions is not unique and a pricing kernel M_T must be assumed to link the two distributions. Define

$$M_T = \exp(-rT) \frac{f_0^Q(\ln(S_T))}{f_0^P(\ln(S_T))}$$

then we get

$$\begin{aligned} C_0(X,T) &= \exp(-rT) E_0^Q[\max\{S_T - X, 0\}] \\ &= E_0^P[M_T \max\{S_T - X, 0\}] \end{aligned}$$

The pricing kernel (or stochastic discount factor) describes how in equilibrium investors trade off the current (known) option price versus the future (stochastic) pay-off.

The functional form for the pricing kernel can be motivated by a representative investor with a particular utility function of terminal wealth. Generally, we can write

$$M_T \propto U'(S_T)$$

where $U'(S_T)$ is the first-derivative of the utility function.

For example, assuming exponential utility with risk-aversion parameter γ we have

$$U(S) = -\frac{1}{\gamma}\exp(-\gamma S)$$

so that $U'(S) = \exp(-\gamma S)$, and

$$f_0^Q(S_T) = \exp(rT) M_T f_0^P(\ln(S_T)) = \frac{\exp(-\gamma(S_T)) f_0^P(S_T)}{\int_0^\infty \exp(-\gamma(S)) f_0^P(S)\, dS}$$

where the denominator ensures that $f_0^Q(S_T)$ is a proper density.

Assuming instead power utility, we have

$$U(S) = \frac{S^{1-\gamma} - 1}{1-\gamma}, \tag{44}$$

so that $U'(S) = S^{-\gamma}$, and

$$f_0^Q(S_T) = \frac{S_T^{-\gamma} f_0^P(S_T)}{\int_0^\infty S^{-\gamma} f_0^P(S)\, dS}$$

Importantly, these results demonstrate that any two of the following three uniquely determine the third: 1) the physical density; 2) the risk-neutral density; 3) the pricing kernel. We refer to Hansen and Renault (2009) for a concise overview of various pricing kernels derived from economic theory.

The Black-Scholes model can be derived in a discrete representative investor setting where markets are incomplete. Brennan (1979) outlines the sufficient conditions on the utility function to obtain the Black-Scholes pricing result. Christoffersen, Elkamhi, Feunou, and Jacobs (2010) provide a general class of pricing kernels in a discrete time setting with dynamic volatility and non-normal return innovations.

5.2.2 Static Distribution Models

Liu et al. (2007) show that if we assume a mixture of lognormal option-implied distribution as in (39) and furthermore a power utility function with risk aversion parameter γ as in (44) then the physical distribution will also be a mixture of lognormals with the following parameter mapping

$$\begin{aligned} \mu_i^P &= \mu_i \exp\left(\gamma \sigma_i^2 T\right) \text{ for } i = 1, 2 \\ w^P &= \left[1 + \frac{1-w}{w}\left(\frac{\mu_2}{\mu_1}\right)^{\gamma} \exp\left(\frac{1}{2}\left(\gamma^2 - \gamma\right)\left(\sigma_2^2 - \sigma_1^2\right) T\right)\right]^{-1} \end{aligned}$$

where σ_1^2 and σ_1^2 do not change between the two measures.

The physical moments can now be obtained from

$$E_0^P\left[S_T^n\right] = w^P\left(\mu_1^P\right)^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_1^2 T\right) + \left(1 - w^P\right)\left(\mu_2^P\right)^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_2^2 T\right).$$

Keeping μ_1, μ_2, w, σ_1^2 and σ_1^2 fixed at their option-implied values, the risk aversion parameter γ can be estimated via maximum likelihood on returns using the physical mixture of lognormals defined by the parameter mapping above. Liu et al. (2007) also investigate other parametric distributions. See Fackler and King (1990) and Bliss and Panigirtzoglou (2004) for related approaches.

5.2.3 Dynamic Models with Stochastic Volatility

The Heston model allows for stochastic volatility implying that the option, which depends on volatility, cannot be perfectly replicated by the stock and bond. Markets are incomplete in this case and the model therefore implicitly makes an assumption on the pricing kernel or the utility function of the investor. Heston (1993) assumes that the price of an asset follows the following physical process

$$\begin{aligned} dS &= \left(r + \mu V\right) S dt + \sqrt{V} S dz_1 \\ dV &= \kappa^P\left(\theta^P - V\right) dt + \sigma_V \sqrt{V} dz_2 \end{aligned} \tag{45}$$

where the two diffusions are allowed to be correlated with parameter ρ. The mapping between the physical parameters in (45) and the option-implied parameters in (4) is given by

$$\kappa = \kappa^P + \lambda, \ \theta = \theta^P \frac{\kappa^P}{\kappa}$$

where λ is the price of variance risk.

Christoffersen et al. (2011) show that the physical and option-implied processes in (45) and (4) imply a pricing kernel of the form

$$M_T = M_0 \left(\frac{S_T}{S_0}\right)^{\gamma} \exp\left(\delta T + \eta \int_0^T V(s) ds + \xi(V_T - V_0)\right) \tag{46}$$

where ξ is a variance preference parameter. The risk premia μ and λ are related to the preference parameters γ and ξ via

$$\mu = -\gamma - \xi\sigma_V\rho$$
$$\lambda = -\rho\sigma_V\gamma - \sigma_V^2\xi$$

In order to assess the implication of the price of variance risk, λ, for forecasting we consider the physical expected integrated variance

$$\begin{aligned} VAR_0^P(T) &= \theta^P T + \left(V_0 - \theta^P\right)\frac{\left(1 - e^{-\kappa^P T}\right)}{\kappa^P} \\ &= \theta\frac{\kappa}{\kappa-\lambda}T + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1 - e^{-(\kappa-\lambda)T}\right)}{\kappa-\lambda} \end{aligned}$$

Under the physical measure the expected future variance in the Heston (1993) model of course differs from the risk-neutral forecast in (5) when $\lambda \neq 0$. If an estimate of λ can be obtained, then the transformation from option-implied to physical variance forecasts is trivial.

In Figure 6 we plot the physical volatility term structure per year defined by

$$\sqrt{VAR_0^P(T)/T} = \sqrt{\theta\frac{\kappa}{\kappa-\lambda} + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1 - e^{-(\kappa-\lambda)T}\right)}{(\kappa-\lambda)T}} \tag{47}$$

along with the option-implied term structure from (6). We use parameter values as in Figure 3, where $\theta = 0.09$ $\kappa = 2$, and we set $\lambda = -1.125$ which implies that $\theta/\theta^P = \frac{\kappa-\lambda}{\kappa} = (1.25)^2$ which corresponds to a fairly large variance risk premium. Figure 6 shows the effect of a large volatility risk premium on the volatility term structure. For short horizons and when the current volatility is low then the effect of the volatility risk premium is relatively small. However for long-horizons the effect is much larger.

5.2.4 Model-free Moments

Bakshi et al. (2003) also assume power utility with parameter γ, and show that option-implied and physical moments for the market index are approximately related as follows

$$VAR^Q \approx VAR^P\left(1 - \gamma SKEW^P\left(VAR^P\right)^2\right)$$
$$SKEW^Q \approx SKEW^P - \gamma(KURT^P - 3)\left(VAR^P\right)^2$$

Given a reasonable estimate for γ it is thus possible to convert option-implied estimates for VAR^Q and $SKEW^Q$ into physical moment forecasts, VAR^P and $SKEW^P$ without making explicit assumptions on the functional form of the distribution.

5.3 Pricing Kernels and Risk Premia

There is a related literature focusing on estimating pricing kernels from P and Q densities rather than on forecasting. Jackwerth (2000) estimates pricing kernels using Q-densities obtained from one day of option prices, and P-densities using returns for the previous month. Christoffersen et al. (2011) estimate pricing kernels using the entire return sample by standardizing returns by a dynamic volatility measure. Some authors assume that Q is time varying while P is constant over time. Aït-Sahalia and Lo (2000) use four years of data to estimate P when constructing pricing kernels. Rosenberg and Engle (2002) use returns data over their entire sample 1970-1995 to construct estimates of the pricing kernel.

Interestingly, recent evidence suggests that key features of the pricing kernel such as risk premia are useful for forecasting returns. This is not surprising, because as we saw above, the pricing kernel is related to preferences, and therefore changes in the pricing kernel may reflect changes in risk aversion, or, more loosely speaking, in sentiment.

Bollerslev et al. (2009), Zhou (2010), and Bekaert et al. (2010) find evidence that the variance risk premium, $VAR^Q - VAR^P$, can predict the equity risk premium.

Risk premia can be estimated in various ways. Parametric models can be used to jointly calibrate a stochastic model of stock and option prices with time-variation in the Q and P densities. For instance, Shackleton et al. (2010) and Pan (2002) calibrate stochastic volatility models to options and asset prices. Alternatively, (model-free) option-implied moments can be combined with separately estimated physical moments to compute risk premia. In this case, the question arises how to estimate the physical moments. The literature on the optimal type of physical information to combine with option-implied information is in its infancy. However, we have extensive knowledge about the use of different types of physical information from the literature on forecasting with historical information as chronicled in this and the previous Handbook of Economic Forecasting volume.

6 Summary and Discussion

The literature contains a large body of evidence supporting the use of option-implied information to predict physical objects of interest. In this chapter we have highlighted some of the key tools for extracting forecasting information using option-implied moments and distributions. We have also summarized the key theoretical relationships between option-implied and physical densities, enabling the forecaster to convert the option-implied forecasts to physical forecasts.

We hasten to add that it is certainly not mandatory that the option-implied information is mapped into the physical measure to generate forecasts. Some empirical studies have found that transforming option-implied to physical information improves forecasting performance in certain situations (see Shackleton et al. (2010) and Chernov (2007)) but these results do not necessarily generalize to other types of forecasting exercises.

We would expect the option-implied distribution or moments to be biased predictors of their physical counterpart, but this bias may be small, and attempting to remove it can create problems of its own, because the resulting predictor is no longer exclusively based on forward-looking information from option prices, but also on backward-looking information from historical prices as well as on assumptions on investor preferences.

More generally, the existence of a bias does not prevent the option-implied information from being a useful predictor of the future object of interest. Much recent evidence for example on volatility forecasting (See Table 1 from Busch et al. (2011)) strongly suggest that this is indeed the case empirically.

Appendix A: Option Spanning

Carr and Madan (2001) show that any twice continuously differentiable payoff function $H(S_T)$ can be replicated with bonds, the underlying stock and the cross section of out-of-the-money options. The fundamental theorem of calculus implies that for any fixed S_0

$$H(S_T) = H(S_0) + 1_{(S_T>S_0)}\int_{S_0}^{S_T} H'(u)du - 1_{(S_T<S_0)}\int_{S_T}^{S_0} H'(u)du$$

$$= H(S_0) + 1_{(S_T>S_0)}\int_{S_0}^{S_T}\left[H'(S_0) + \int_{S_0}^{u} H''(v)dv\right]du - 1_{(S_T<S_0)}\int_{S_T}^{S_0}\left[H'(S_0) + \int_{u}^{S_0} H''(v)dv\right]du$$

Because $H'(S_0)$ is not a function of u we are able to apply Fubini's theorem to get

$$H(S_T) = H(S_0) + H'(S_0)(S_T - S_0) + 1_{(S_T>S_0)}\int_{S_0}^{S_T}\int_{v}^{S_T} H''(v)dudv + 1_{(S_T<S_0)}\int_{S_T}^{S_0}\int_{S_T}^{v} H''(v)dudv$$

Now integrate over u

$$H(S_T) = H(S_0) + H'(S_0)(S_T - S_0) + 1_{(S_T>S_0)}\int_{S_0}^{S_T} H''(v)(S_T - v)dv + 1_{(S_T<S_0)}\int_{S_T}^{S_0} H''(v)(v - S_T)dv$$

$$= H(S_0) + H'(S_0)(S_T - S_0) + \int_{S_0}^{\infty} H''(v)(S_T - v)^+dv + \int_{0}^{S_0} H''(v)(v - S_T)^+dv$$

If we integrate over X instead of v, where X is interpreted as the strike, we are left with the spanning equation

$$H(S_T) = [H(S_0) - H'(S_0)S_0] + H'(S_0)S_T + \int_{S_0}^{\infty} H''(X)(S_T - X)^+dX + \int_{0}^{S_0} H''(X)(X - S_T)^+dX$$

From this equation we see that the payoff $H(S_T)$ is spanned by a $[H(S_0) - H'(S_0)S_0]$ position in bonds, $H'(S_0)$ position in shares of the stock and a $H''(X)dX$ position in out-of-the-money options.

Appendix B: Log Return Moments

Bakshi et al. (2003) apply the general result in (14) to the second, third, and fourth power of log returns, $H(S_T) = R_T^2 = \ln(S_T/S_0)^2$, $H(S_T) = R_T^3 = \ln(S_T/S_0)^3$, and $H(S_T) = R_T^4 = \ln(S_T/S_0)^4$. They get the quadratic contract to be

$$M_{0,2}(T) \equiv E_0\left[e^{-rT}R_T^2\right] = \int_{S_0}^{\infty} \frac{2(1-\ln[X/S_0])}{X^2} C_0(T,X)\,dX + \int_0^{S_0} \frac{2(1+\ln[S_0/X])}{X^2} P_0(T,X)\,dX, \tag{48}$$

The cubic and quartic contracts are given by

$$\begin{aligned} M_{0,3}(T) \equiv E_0\left[e^{-rT}R_T^3\right] &= \int_{S_0}^{\infty} \frac{6\ln[X/S_0] - 3(\ln[X/S_0])^2}{X^2} C_0(T,X)\,dX \\ &\quad - \int_0^{S_0} \frac{6\ln[S_0/X] + 3(\ln[S_0/X])^2}{X^2} P_0(T,X)\,dX, \end{aligned} \tag{49}$$

$$\begin{aligned} M_{0,4}(T) \equiv E_0\left[e^{-rT}R_T^4\right] &= \int_{S_0}^{\infty} \frac{12(\ln[X/S_0])^2 - 4(\ln[X/S_0])^3}{X^2} C_0(T,X)\,dX \\ &\quad + \int_0^{S_0} \frac{12(\ln[S_0/X])^2 + 4(\ln[S_0/X])^3}{X^2} P_0(T,X)\,dX. \end{aligned} \tag{50}$$

from this we can compute the option-implied volatility, skewness, and kurtosis (for convenience we suppress the dependence on T) as

$$VOL_0(T) \equiv [VAR_0(T)]^{1/2} = E_0\left[(R_T - E_0[R_T])^2\right]^{1/2} = \left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{1/2} \tag{51}$$

$$\begin{aligned} SKEW_0(T) &\equiv \frac{E_0\left[(R_T - E_0[R_T])^3\right]}{VOL_0(T)^3} \\ &= \frac{e^{rT}M_{0,3} - 3M_{0,1}e^{rT}M_{0,2} + 2M_{0,1}^3}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{\frac{3}{2}}} \end{aligned} \tag{52}$$

$$\begin{aligned} KURT_0(T) &\equiv \frac{E_0\left[(R_T - E_0[R_T])^4\right]}{VOL_0(T)^4} \\ &= \frac{e^{rT}M_{0,4} - 4M_{0,1}e^{rT}M_{0,3} + 6e^{rT}M_{0,1}^2M_{0,2} - 3M_{0,1}^4}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^2} \end{aligned} \tag{53}$$

where

$$M_{0,1}(T) \equiv E_0[R_T] \approx e^{rT} - 1 - \frac{e^{rT}}{2}M_{0,2}(T) - \frac{e^{rT}}{6}M_{0,3}(T) - \frac{e^{rT}}{24}M_{0,4}(T). \tag{54}$$

Bibliography

Abadir, K., & Rockinger, M. (2003). Density Functionals, with an Option-Pricing Application. *Economic Theory*, *19*, 778-811.

Abken, P. A., Madan, D. B., & Ramamurtie, S. (1996). Estimation of Risk-Neutral and Statistical Densities by Hermite Polynomial Approximation: With an Application to Eurodollar Futures Options. Working paper, Federal Reserve Bank of Atlanta.

Agarwal, V., Bakshi, G., & Huij, J. (2009). Do Higher-Moment Equity Risks Explain Hedge Fund Returns? Working paper, University of Maryland.

Aït-Sahalia, Y., & Duarte, J. (2003). Nonparametric Option Pricing Under Shape Restrictions. *Journal of Econometrics*, *116*, 9-47.

Aït-Sahalia, Y., & Lo, A. W. (1998). Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices. *The Journal of Finance*, *53*, 499-547.

Aït-Sahalia, Y., & Lo, A. W. (2000). Nonparametric Risk Management and Implied Risk Aversion. *Journal of Econometrics*, *94*, 9-51.

Alentorn, A., & Markose, S. (2008). Generalized Extreme Value Distribution and Extreme Economic Value at Risk (EE-VaR). In Kontoghiorghes, E. J., Rustem, B., & Winker, P. (Eds.). *Computational methods in Financial Engineering*, Springer-Verlag, 47-71.

Amin, K. I., & Ng, V. K. (1997). Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach. *The Review of Financial Studies*, *10*, 333-367.

Andersen, T. G., Benzoni, L., & Lund, J. (2002). An Empirical Investigation of Continuous-Time Equity Return Models. *The Journal of Finance*, *57*, 1239-1284.

Andersen, T. G., Bollerslev, T., Christoffersen, P. F., & Diebold, F. X. (2006). Volatility and Correlation Forecasting. In Elliott, G., Granger, C. W. J., & Timmermann, A. (Eds.). *Handbook of Economic Forecasting*, Elsevier Science, 778-878.

Andersen, T. G., & Bondarenko, O. (2007). Construction and Interpretation of Model-Free Implied Volatility. In Nelken, I. (Ed.). *Volatility as an Asset Class*, Risk Publications, 141-184.

Andersen, T. G., & Bondarenko, O. (2009). Dissecting the Pricing of Equity Index Volatility. Working paper, Northwestern University.

Andersen, T. G., Bondarenko, O., & Gonzalez-Perez, M. T. (2011). A Corridor Fix for VIX: Developing a Coherent Model-Free Option-Implied Volatility Measure. Working paper, Northwestern University.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The Cross-Section of Volatility and Expected Returns. *The Journal of Finance*, *61*, 259-299.

Aparicio, S. D., & Hodges, S. (1998). Implied Risk-Neutral Distribution: A Comparison of Estimation Methods. Working paper, University of Warwick.

Backus, D., Foresi, S., Li, K., & Wu, L. (1997). Accounting for Biases in Black-Shcholes. Working paper, New York University.

Bahra, B. (1997). Implied Risk-Neutral Probability Density Functions From Option Prices: Theory and Application. Working paper, Bank of England.

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical Performance of Alternative Option Pricing Models. *Journal of Finance*, *52*, 2003-2049.

Bakshi, G., Kapadia, N., & Madan, D. (2003). Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *Review of Financial Studies*, *16*, 101-143.

Bakshi, G., & Madan, D. (2000). Spanning and Derivative-Security Valuation. *Journal of Financial Economics*, *55*, 205-238.

Bakshi, G., Panayotov, G., & Skoulakis, G. (2011). Improving the Predictability of Real Economic Activity and Asset Returns with Forward Variances Inferred from Option Portfolios. *Journal of Financial Economics*, *100*, 475-495.

Bali, T. G., & Hovakimian, A. (2009). Volatility Spreads and Expected Stock Returns. *Management Science*, *55*, 1797-1812.

Banerjee, P. S., Doran, J. S., & Peterson, D. R. (2007). Implied Volatility and Future Portfolio Returns. *Journal of Banking & Finance*, *31*, 3183-3199.

Banz, R. W., & Miller, M. H. (1978). Prices for State-Contingent Claims: Some Estimates and Applications. *The Journal of Business*, *51*, 653-672.

Barone-Adesi, G., Brown, K. C., & Harlow, W. V. (1994). On the Use of Implied Volatilities in the Prediction of Successful Corporate Takeovers. In Chance, D. M., & Trippi, R. R. (Eds.). *Advances in Futures and Options Research*, Emerald Group Publishing, 147-165.

Bates, D. S. (1991). The Crash of '87: Was It Expected? The Evidence from Options Markets. *The Journal of Finance*, *46*, 1009-1044.

Bates, D. S. (1996a). Dollar Jump Fears, 1984-1992: Distributional Abnormalities Implicit in Currency Futures Options. *Journal of International Money and Finance*, *15*, 65-93.

Bates, D. S. (1996b). Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies*, *9*, 69-107.

Bates, D. S. (2000). Post-'87 Crash Fears in the S&P 500 Futures Option Market. *Journal of Econometrics*, *94*, 181-238.

Bates, D. S. (2008). The Market for Crash Risk. *Journal of Economic Dynamics and Control*, *32*, 2291-2321.

Beckers, S. (1981). Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability. *Journal of Banking & Finance*, *5*, 363-381.

Bekaert, G., Hoerova, M., & Lo Duca, M. (2010). Risk, Uncertainty and Monetary Policy. Working paper, National Bureau of Economic Research.

Benaim, S., & Friz, P. (2008). Smile Asymptotics II: Models with Known Moment Generating Function. *Journal of Applied Probability*, *45*, 16-32.

Benaim, S., & Friz, P. (2009). Regular Variation and Smile Asymptotics. *Mathematical Finance*, *19*, 1-12.

Bester, C. A., Martinez, V. H., & Roşu, I. (2011). Option Prices and the Probability of Success of Cash Mergers. Working paper.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, *81*, 637-654.

Blair, B. J., Poon, S.-H., & Taylor, S. J. (2001). Forecasting S&P 100 Volatility: The Incremental Information Content of Implied Volatilities and High-Frequency Index Returns. *Journal of Econometrics*, *105*, 5-26.

Bliss, R. R., & Panigirtzoglou, N. (2002). Testing the Stability of Implied Probability Density Functions. *Journal of Banking & Finance*, *26*, 381-422.

Bliss, R. R., & Panigirtzoglou, N. (2004). Option-Implied Risk Aversion Estimates. *The Journal of Finance*, *59*, 407-446.

Bodurtha, J. N. J., & Shen, Q. (1999). Historical and Implied Measures of "Value at Risk": The DM and Yen Case. Working paper, University of Michigan.

Bollerslev, T., Tauchen, G., & Zhou, H. (2009). Expected Stock Returns and Variance Risk Premia. *Review of Financial Studies*, *22*, 4463-4492.

Bondarenko, O. (2003). Estimation of Risk-Neutral Densities Using Positive Convolution Approximation. *Journal of Econometrics*, *116*, 85-112.

Bookstaber, R. M., & McDonald, J. B. (1991). Option Pricing for Generalized Distributions. *Communications in Statistics - Theory and Methods*, *20*, 4053-4068.

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of State-Contingent Claims Implicit in Option Prices. *The Journal of Business*, *51*, 621-651.

Brennan, M. J. (1979). The Pricing of Contingent Claims in Discrete Time Models. *The Journal of Finance*, *34*, 53-68.

Britten-Jones, M., & Neuberger, A. (2000). Option Prices, Implied Price Processes, and Stochastic Volatility. *Journal of Finance*, *55*, 839-866.

Broadie, M., Chernov, M., & Johannes, M. (2009). Understanding Index Option Returns. *Review of Financial Studies*, *22*, 4493-4529.

Bu, R., & Hadri, K. (2007). Estimating Option Implied Risk-Neutral Densities Using Spline and Hypergeometric Functions. *Econometrics Journal*, *10*, 216-244.

Buchen, P. W., & Kelly, M. (1996). The Maximum Entropy Distribution of an Asset Inferred from Option Prices. *The Journal of Financial and Quantitative Analysis*, *31*, 143-159.

Busch, T., Christensen, B. J., & Nielsen, M. O. (2011). The Role of Implied Volatility in Forecasting Future Realized Volatility and Jumps in Foreign Exchange, Stock, and Bond Markets. *Journal of Econometrics*, *160*, 48-57.

Buss, A., & Vilkov, G. (2011). Option-Implied Correlation and Factor Betas Revisited. Working paper, Goethe University.

Campa, J. M., Chang, P., & Reider, R. L. (1998). Implied Exchange Rate Distributions: Evidence from OTC Option Markets. *Journal of International Money and Finance*, *17*, 117-160.

Campa, J. M., & Chang, P. H. K. (1996). Arbitrage-Based Tests of Target-Zone Credibility: Evidence from ERM Cross-Rate Options. *The American Economic Review*, *86*, 726-740.

Campa, J. M., & Chang, P. H. K. (1998). ERM Realignment Risk and its Economic Determinants as Reflected in Cross-Rate Options. *The Economic Journal*, *108*, 1046-1066.

Campa, J. M., Chang, P. H. K., & Refalo, J. F. (2002). An Options-Based Analysis of Emerging Market Exchange Rate Expectations: Brazil's Real Plan, 1994-1999. *Journal of Development Economics*, *69*, 227-253.

Canina, L., & Figlewski, S. (1993). The Informational Content of Implied Volatility. *The Review of Financial Studies*, *6*, 659-681.

Cao, C., Chen, Z., & Griffin, J. M. (2005). Informational Content of Option Volume Prior to Takeovers. *Journal of Business*, *78*, 1073-1072.

Carr, P., & Madan, D. (1998). Towards a Theory of Volatility Trading. In Jarrow, R. A. (Ed.). *Volatility: New Estimation Techniques for Pricing Derivatives*, RISK Publications, 417-427.

Carr, P., & Madan, D. (2001). Optimal Positioning in Derivative Securities. *Quantitative Finance*, *1*, 19-37.

Carson, J. M., Doran, J. S., & Peterson, D. R. (2006). Market Crash Risk and Implied Volatility Skewness: Evidence and Implications for Insurer Investments. Working paper, Florida State University.

CBOE. (2009). The CBOE Volatility Index - VIX. Technical Report.

CBOE. (2011). The CBOE Skew Index - SKEW. Technical Report.

Chang, B.-Y., Christoffersen, P., & Jacobs, K. (2009). Market Skewness Risk and the Cross-Section of Stock Returns. Working paper, McGill University.

Chang, B.-Y., Christoffersen, P., Jacobs, K., & Vainberg, G. (2011). Option-Implied Measures of Equity Risk. *Review of Finance*, Forthcoming.

Chernov, M. (2007). On the Role of Risk Premia in Volatility Forecasting. *Journal of Business & Economic Statistics*, *25*, 411-426.

Chernov, M., Gallant, A. R., Ghysels, E., & Tauchen, G. (2003). Alternative Models for Stock Price Dynamics. *Journal of Econometrics*, *116*, 225-257.

Chiras, D. P., & Manaster, S. (1978). The Information Content of Option Prices and a Test of Market Efficiency. *Journal of Financial Economics*, *6*, 213-234.

Christensen, B. J., & Prabhala, N. R. (1998). The Relation between Implied and Realized Volatility. *Journal of Financial Economics*, *50*, 125-150.

Christoffersen, P., Elkamhi, R., Feunou, B., & Jacobs, K. (2010). Option Valuation with Conditional Heteroskedasticity and Nonnormality. *Review of Financial Studies*, *23*, 2139-2183.

Christoffersen, P., Heston, S. L., & Jacobs, K. (2006). Option Valuation with Conditional Skewness. *Journal of Econometrics*, *131*, 253-284.

Christoffersen, P., Heston, S. L., & Jacobs, K. (2009). The Shape and Term Structure of the Index Option Smirk: Why Multifactor Stochastic Volatility Models Work so Well. *Management Science*, *55*, 1914-1932.

Christoffersen, P., Heston, S. L., & Jacobs, K. (2011). A GARCH Option Model with Variance-Dependent Pricing Kernel. Working paper, University of Toronto.

Christoffersen, P., & Jacobs, K. (2004). The Importance of the Loss Function in Option Valuation. *Journal of Financial Economics*, *72*, 291-318.

Christoffersen, P., Jacobs, K., & Mimouni, K. (2010). Volatility Dynamics for the S&P500: Evidence from Realized Volatility, Daily Returns and Option Prices. *Review of Financial Studies*, *23*, 3141-3189.

Christoffersen, P., & Mazzotta, S. (2005). The Accuracy of Density Forecasts from Foreign Exchange Options. *Journal of Financial Econometrics*, *3*, 578-605.

Conrad, J., Dittmar, R. F., & Ghysels, E. (2009). Ex Ante Skewness and Expected Stock Returns. Working paper, University of North Carolina.

Corrado, C. J. (2001). Option Pricing Based on the Generalized Lambda Distribution. *Journal of Futures Markets*, *21*, 213-236.

Corrado, C. J., & Su, T. (1996). Skewness and Kurtosis in S&P 500 Index Returns Implied by Option Prices. *The Journal of Financial Research*, *19*, 175-192.

Coutant, S., Jondeau, E., & Rockinger, M. (2001). Reading PIBOR Futures Options Smiles: The 1997 Snap Election. *Journal of Banking & Finance*, *25*, 1957-1987.

Cremers, M., & Weinbaum, D. (2010). Deviations from Put-Call Parity and Stock Return Predictability. *Journal of Financial and Quantitative Analysis*, *45*, 335-367.

Day, T. E., & Lewis, C. M. (1992). Stock Market Volatility and the Information Content of Stock Index Options. *Journal of Econometrics*, *52*, 267-287.

De Jong, C., & Huisman, R. (2000). From Skews to a Skewed-t. Working paper, Erasmus Research Institute of Management.

De Jong, F., Driessen, J., & Pelsser, A. (2004). On the Information in the Interest Rate Term Structure and Option Prices. *Review of Derivatives Research*, *7*, 99-127.

Delisle, R. J., Doran, J. S., & Peterson, D. R. (2010). Implied Systematic Moments and the Cross-Section of Stock Returns. Working paper, Washington State University.

Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). More Than You Ever Wanted To Know About Volatility Swaps. Quantitative Strategies Research Notes, Goldman Sachs.

DeMiguel, V., Plyakha, Y., Uppal, R., & Vilkov, G. (2011). Improving Portfolio Selection Using Option-Implied Volatility and Skewness. Working paper, London Business School.

Diavatopoulos, D., Doran, J., & Peterson, D. (2008). The Information Content in Implied Idiosyncratic Volatility and the Cross-Section of Stock Returns: Evidence from the Option Markets. *The Journal of Futures Markets*, *28*, 1013-1039.

Diavatopoulos, D., Doran, J. S., Fodor, A., & Peterson, D. (2008). The Information Content of Implied Skewness and Kurtosis Changes Prior to Earnings Announcements for Stock and Option Returns. Working paper, Villanova University.

Doran, J. S., Fodor, A., & Krieger, K. (2010). Option Market Efficiency and Analyst Recommendations. *Journal of Business Finance & Accounting*, *37*, 560-590.

Doran, J. S., & Krieger, K. (2010). Implications for Asset Returns in the Implied Volatility Skew. *Financial Analysts Journal*, *66*, 65-76.

Doran, J. S., Peterson, D. R., & Tarrant, B. C. (2007). Is There Information in the Volatility Skew? *Journal of Futures Markets*, *27*, 921-959.

Driessen, J., Maenhout, P. J., & Vilkov, G. (2009). The Price of Correlation Risk: Evidence from Equity Options. *Journal of Finance*, *64*, 1377-1406.

Duffie, D., Pan, J., & Singleton, K. (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica*, *68*, 1343-1376.

Dumas, B., Fleming, J., & Whaley, R. E. (1998). Implied Volatility Functions: Empirical Tests. *The Journal of Finance*, *53*, 2059-2106.

Dupire, B. (1994). Pricing with a Smile. *Risk*, *7*, 18- 20.

Dupont, D. Y. (2001). Extracting Risk-Neutral Probability Distributions from Option Prices Using Trading Volume as a Filter. Economics Series, Institute for Advanced Studies.

Eraker, B., Johannes, M., & Polson, N. (2003). The Impact of Jumps in Volatility and Returns. *The Journal of Finance*, *58*, 1269-1300.

Fackler, P. L., & King, R. P. (1990). Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets. *American Journal of Agricultural Economics*, *72*, 73-83.

Feunou, B., Fontaine, J.-S., Taamouti, A., & Tedongap, R. (2011). The Equity Premium and the Maturity Structure of Uncertainty. Working paper, Bank of Canada.

Figlewski, S. (2010). Estimating the Implied Risk Neutral Density for the U.S. Market Portfolio. In Bollerslev, T., Russell, J. R., & Watson, M. (Eds.). *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Oxford University Press, 323-353.

Flamouris, D., & Giamouridis, D. (2002). Estimating Implied PDFs From American Options on Futures: A New Semiparametric Approach. *Journal of Futures Markets*, *22*, 1096-9934.

Fleming, J. (1998). The Quality of Market Volatility Forecasts Implied by S&P 100 Index Option Prices. *Journal of Empirical Finance*, *5*, 317-345.

Fleming, J., Ostdiek, B., & Whaley, R. E. (1995). Predicting Stock Market Volatility: A New Measure. *Journal of Futures Markets*, *15*, 265-302.

Fusai, G., & Roncoroni, A. (2008). *Implementing Models in Quantitative Finance: Methods and Cases*. Springer, Berlin.

Galati, G. (Ed.). (1999). *Proceedings of a Workshop on Estimating and Interpreting Probability Density Functions.* Bank for International Settlements.

Garcia, R., Ghysels, E., & Renault, E. (2010). Econometrics of Option Pricing Models. In Aït-Sahalia, Y., & Hansen, L. P. (Eds.). *Handbook of Financial Econometrics*, North Holland, 479-552.

Gemmill, G., & Saflekos, A. (2000). How Useful are Implied Distributions? Evidence from Stock Index Options. *Journal of Derivatives*, *7*, 83-98.

Giacomini, E., Härdle, W., & Krätschmer, V. (2009). Dynamic Semiparametric Factor Models in Risk Neutral Density Estimation. *Advances in Statistical Analysis*, *93*, 387-402.

Haas, M., Mittnik, S., & Mizrach, B. (2006). Assessing Central Bank Credibility During the ERM Crises: Comparing Option and Spot Market-Based Forecasts. *Journal of Financial Stability*, *2*, 28-54.

Hansen, L. P., & Renault, E. (2009). Pricing Kernels and Stochastic Discount Factors. Working paper, University of Chicago.

Harvey, C. R., & Whaley, R. E. (1992). Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market. *Journal of Financial Economics*, *31*, 43-73.

Healy, J. V., Dixon, M., Read, B. J., & Cai, F. F. (2007). Non-Parametric Extraction of Implied Asset Price Distributions. *Physica A: Statistical Mechanics and its Applications*, *382*, 121-128.

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies*, *6*, 327-343.

Heston, S. L., & Nandi, S. (2000). A Closed-Form GARCH Option Valuation Model. *Review of Financial Studies*, *13*(3), 585-625.

Høg, E., & Tsiaras, L. (2011). Density Forecasts of Crude Oil Prices Using Option-Implied and ARCH-Type Models. *Journal of Futures Markets*, *31*, 727-754.

Huang, J.-z., & Wu, L. (2004). Specification Analysis of Option Pricing Models Based on Time-Changed Lévy Processes. *The Journal of Finance*, *59*, 1405-1440.

Hull, J. C. (2011). *Options, Futures, and other Derivatives* (8th ed.). Pearson Prentice Hall.

Jackwerth, J. C. (1997). Generalized Binomial Trees. *Journal of Derivatives*, *5*, 7-17.

Jackwerth, J. C. (1999). Option-Implied Risk-Neutral Distributions and Implied Binomial Trees. *Journal of Derivatives*, *7*, 66-82.

Jackwerth, J. C. (2000). Recovering Risk Aversion from Option Prices and Realized Returns. *The Review of Financial Studies*, *13*, 433-451.

Jackwerth, J. C., & Rubinstein, M. (1996). Recovering Probability Distributions from Option Prices. *The Journal of Finance*, *51*, 1611-1631.

Jarrow, R., & Rudd, A. (1982). Approximate Option Valuation for Arbitrary Stochastic Processes. *Journal of Financial Economics*, *10*, 347-369.

Jayaraman, N., Mandelker, G., & Shastri, K. (1991). Market Anticipation of Merger Activities: An Empirical Test. *Managerial and Decision Economics*, *12*, 439-448.

Jiang, G. J., & Tian, Y. S. (2005). The Model-Free Implied Volatility and its Information Content. *The Review of Financial Studies*, *18*, 1305-1342.

Jiang, G. J., & Tian, Y. S. (2007). Extracting Model-Free Volatility from Option Prices: An Examination of the VIX Index. *Journal of Derivatives*, *14*, 35-60.

Jondeau, E., Poon, S.-H., & Rockinger, M. (2007). *Financial Modeling Under Non-Gaussian Distributions*. Springer-Verlag, London.

Jondeau, E., & Rockinger, M. (2000). Reading the Smile: The Message Conveyed by Methods which Infer Risk Neutral Densities. *Journal of International Money and Finance*, *19*, 885-915.

Jondeau, E., & Rockinger, M. (2001). Gram-Charlier Densities. *Journal of Economic Dynamics and Control*, *25*, 1457-1483.

Jorion, P. (1995). Predicting Volatility in the Foreign Exchange Market. *The Journal of Finance*, *50*, 507-528.

Kostakis, A., Panigirtzoglou, N., & Skiadopoulos, G. (2011). Market Timing with Option-Implied Distributions: A Forward-Looking Approach. *Management Science*, 1110-1346.

Kroner, K. F., Kneafsey, K. P., & Claessens, S. (1995). Forecasting Volatility in Commodity Markets. *Journal of Forecasting*, *14*, 77-95.

Lamoureux, C. G., & Lastrapes, W. D. (1993). Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities. *The Review of Financial Studies*, *6*, 293-326.

Latané, H. A., & Rendleman, R. J., Jr. (1976). Standard Deviations of Stock Price Ratios Implied in Option Prices. *The Journal of Finance*, *31*, 369-381.

Leahy, M. P., & Thomas, C. P. (1996). The Sovereignty Option: The Quebec Referendum and Market Views on the Canadian Dollar. International Finance Discussion Papers, Board of Governors of the Federal Reserve System.

Lee, R. W. (2004). The Moment Formula For Implied Volatility at Extreme Strikes. *Mathematical Finance*, *14*, 469-480.

Li, H., Wells, M. T., & Yu, C. L. (2008). A Bayesian Analysis of Return Dynamics with Lévy Jumps. *Review of Financial Studies*, *21*, 2345-2378.

Li, H., & Zhao, F. (2009). Nonparametric Estimation of State-Price Densities Implicit in Interest Rate Cap Prices. *Review of Financial Studies*, *22*, 4335-4376.

Liu, X., Shackleton, M. B., Taylor, S. J., & Xu, X. (2007). Closed-Form Transformations from Risk-Neutral to Real-World Distributions. *Journal of Banking & Finance*, *31*, 1501-1520.

Longstaff, F. A., Santa-Clara, P., & Schwartz, E. S. (2001). The Relative Valuation of Caps and Swaptions: Theory and Empirical Evidence. *The Journal of Finance*, *56*, 2067-2109.

Madan, D. B., Carr, P. P., & Chang, E. C. (1998). The Variance Gamma Process and Option Pricing. *European Finance Review*, *2*, 79-105.

Madan, D. B., & Milne, F. (1994). Contingent Claims Valued and Hedged by Pricing and Investing in a Basis. *Mathematical Finance*, *4*, 223-245.

Malz, A. M. (1996). Using Option Prices to Estimate Realignment Probabilities in the European Monetary System: The Case of Sterling-Mark. *Journal of International Money and Finance*, *15*, 717-748.

Malz, A. M. (1997). Option-Implied Probability Distribution and Currency Excess Returns. Staff Reports, Federal Reserve Bank of New York.

Markose, S., & Alentorn, A. (2011). The Generalized Extreme Value Distribution, Implied Tail Index, and Option Pricing. *Journal of Derivatives*, *18*, 35-60.

Matache, A., Nitsche, P., & Schwab, C. (2004). Wavelet Galerkin Pricing of American Options on Lévy Driven Asset. *Working paper*, Working paper, ETH Zürich.

Melick, W. R., & Thomas, C. P. (1997). Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil during the Gulf Crisis. *The Journal of Financial and Quantitative Analysis*, *32*, 91-115.

Mizrach, B. (2006). The Enron Bankruptcy: When did the Options Market in Enron Lose its Smirk? *Review of Quantitative Finance and Accounting*, *27*, 365-382.

Monteiro, A. M., Tutuncu, R. H., & Vicente, L. N. (2008). Recovering Risk-Neutral Probability Density Functions from Options Prices Using Cubic Splines and Ensuring Nonnegativity. *European Journal of Operational Research*, *187*, 525-542.

Navatte, P., & Villa, C. (2000). The Information Content of Implied Volatility, Skewness and Kurtosis: Empirical Evidence from Long-Term CAC 40 Options. *European Financial Management*, *6*, 41-56.

Neuberger, A. (1994). The Log Contract. *Journal of Portfolio Management*, *20*, 74-80.

Neuberger, A. (2011). Realized Skewness. Working paper, University of Warwick.

Pan, J. (2002). The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study. *Journal of Financial Economics*, *63*, 3-50.

Pong, S., Shackleton, M. B., Taylor, S. J., & Xu, X. (2004). Forecasting Currency Volatility: A Comparison of Implied Volatilities and AR(FI)MA Models. *Journal of Banking & Finance*, *28*, 2541-2563.

Poon, S.-H., & Granger, C. (2005). Practical Issues in Forecasting Volatility. *Financial Analysts Journal*, *61*, 45-56.

Rebonato, R. (2004). *Volatility and Correlation: The Perfect Hedger and the Fox*. John Wiley & Sons.

Rehman, Z., & Vilkov, G. (2010). Risk-Neutral Skewness: Return Predictability and its Sources. Working paper, Goethe University.

Ritchey, R. J. (1990). Call Option Valuation for Discrete Normal Mixtures. *The Journal of Financial Research*, *13*, 285-296.

Rompolis, L. S., & Tzavalis, E. (2008). Recovering Risk Neutral Densities from Option Prices: A New Approach. *Journal of Financial and Quantitative Analysis*, *43*, 1037-1053.

Rosenberg, J. V. (1998). Pricing Multivariate Contingent Claims Using Estimated Risk-Neutral Density Functions. *Journal of International Money and Finance*, *17*, 229-247.

Rosenberg, J. V., & Engle, R. F. (2002). Empirical Pricing Kernels. *Journal of Financial Economics*, *64*, 341-372.

Rubinstein, M. (1994). Implied Binomial Trees. *The Journal of Finance*, *49*, 771-818.

Savickas, R. (2002). A Simple Option-Pricing Formula. *The Financial Review*, *37*, 207-226.

Shackleton, M. B., Taylor, S. J., & Yu, P. (2010). A Multi-Horizon Comparison of Density Forecasts for the S&P 500 Using Index Returns and Option Prices. *Journal of Banking & Finance*, *34*, 2678-2693.

Sherrick, B. J., Garcia, P., & Tirupattur, V. (1996). Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions. *Journal of Futures Markets*, *16*, 545-560.

Shimko, D. C. (1993). Bounds of Probability. *Risk Magazine*, *6*, 33-37.

Siegel, A. F. (1997). International Currency Relationship Information Revealed by Cross-Option Prices. *Journal of Futures Markets*, *17*, 369-384.

Silva, E. M. d. S., & Kahl, K. H. (1993). Reliability of Soybean and Corn Option-Based Probability Assessments. *Journal of Futures Markets*, *13*, 765-779.

Skintzi, V. D., & Refenes, A.-P. N. (2005). Implied Correlation Index: A New Measure of Diversification. *Journal of Futures Markets*, *25*, 171-197.

Snowberg, E., Wolfers, J., & Zitzewitz, E. (2011). How Prediction Markets Can Save Event Studies. Working paper, CESifo Group.

Söderlind, P., & Svensson, L. (1997). New Techniques to Extract Market Expectations from Financial Instruments. *Journal of Monetary Economics*, *40*, 383-429.

Stutzer, M. (1996). A Simple Nonparametric Approach to Derivative Security Valuation. *Journal of Finance*, *51*, 1633-1652.

Swidler, S., & Wilcox, J. A. (2002). Information about Bank Risk in Options Prices. *Journal of Banking & Finance*, *26*, 1033-1057.

Taylor, S. J. (2005). *Asset Price Dynamics, Volatility, and Prediction.* Princeton University Press.

Tunaru, R., & Albota, G. (2005). Estimating Risk Neutral Density with a Generalized Gamma Distribution. Working paper, City University London.

Walter, C., & Lopez, J. A. (2000). Is Implied Correlation Worth Calculating? Evidence from Foreign Exchange Options and Historical Data. Working paper, Federal Reserve Bank of San Francisco.

Weinberg, S. A. (2001). Interpreting the Volatility Smile: An Examination of the Information Content of Option Prices. International Finance Discussion Papers, Board of Governors of the Federal Reserve System.

Whaley, R. E. (2000). The Investor Fear Gauge. *Journal of Portfolio Management*, *26*, 12-17.

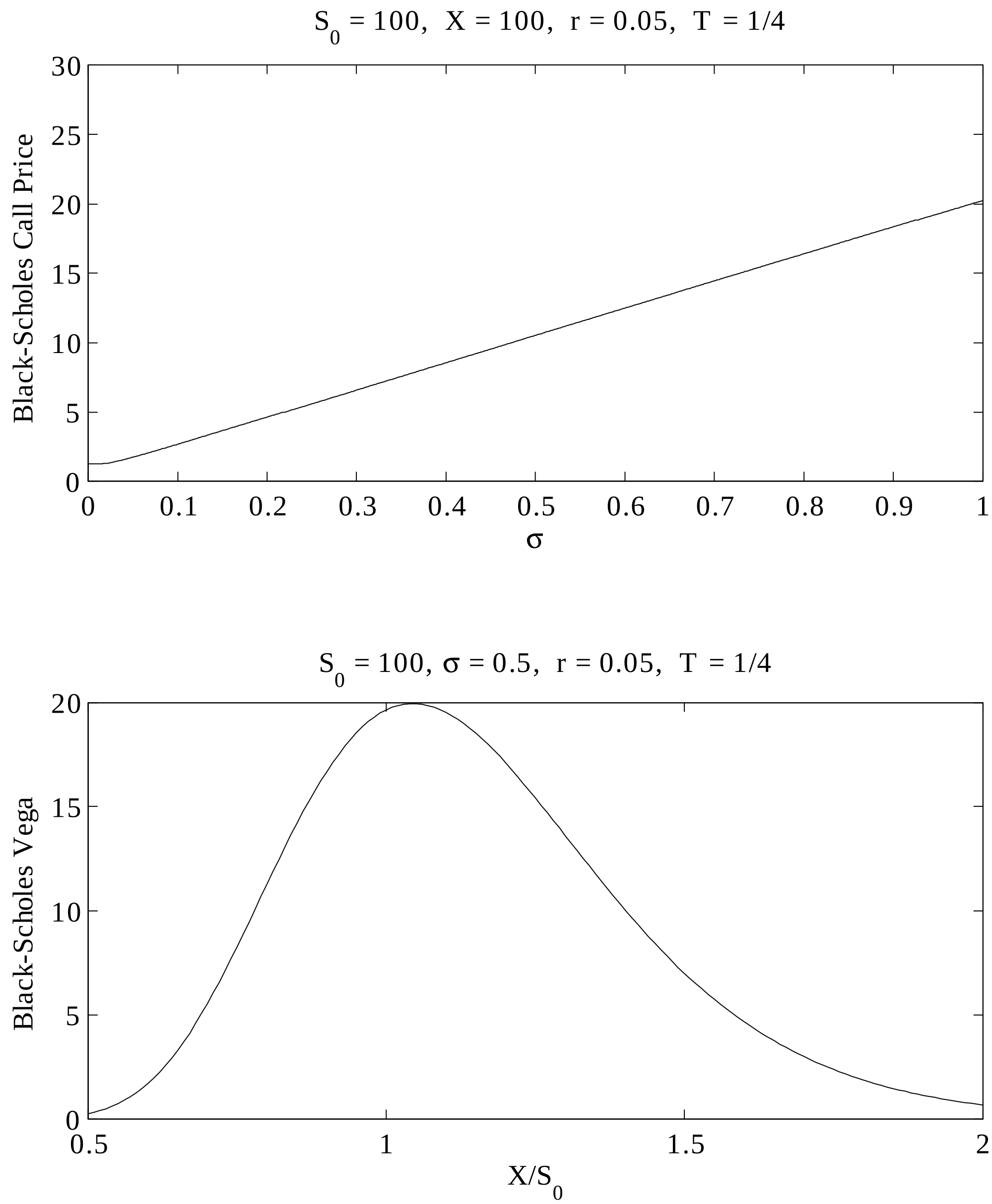
Whaley, R. E. (2003). Derivatives. In Constantinides, G., Harris, M., & Stulz, R. (Eds.). *Handbook of the Economics of Finance: Volume 1B Financial Markets and Asset Pricing*, Elsevier North-Holland Publishing, 1129-1206.

Xing, Y., Zhang, X., & Zhao, R. (2010). What Does the Individual Option Volatility Smirk Tell Us about Future Equity Returns? *Journal of Financial and Quantitative Analysis*, *45*, 641-662.

Xu, X., & Taylor, S. J. (1995). Conditional Volatility and the Informational Efficiency of the PHLX Currency Options Market. *Journal of Banking & Finance*, *19*, 803-821.

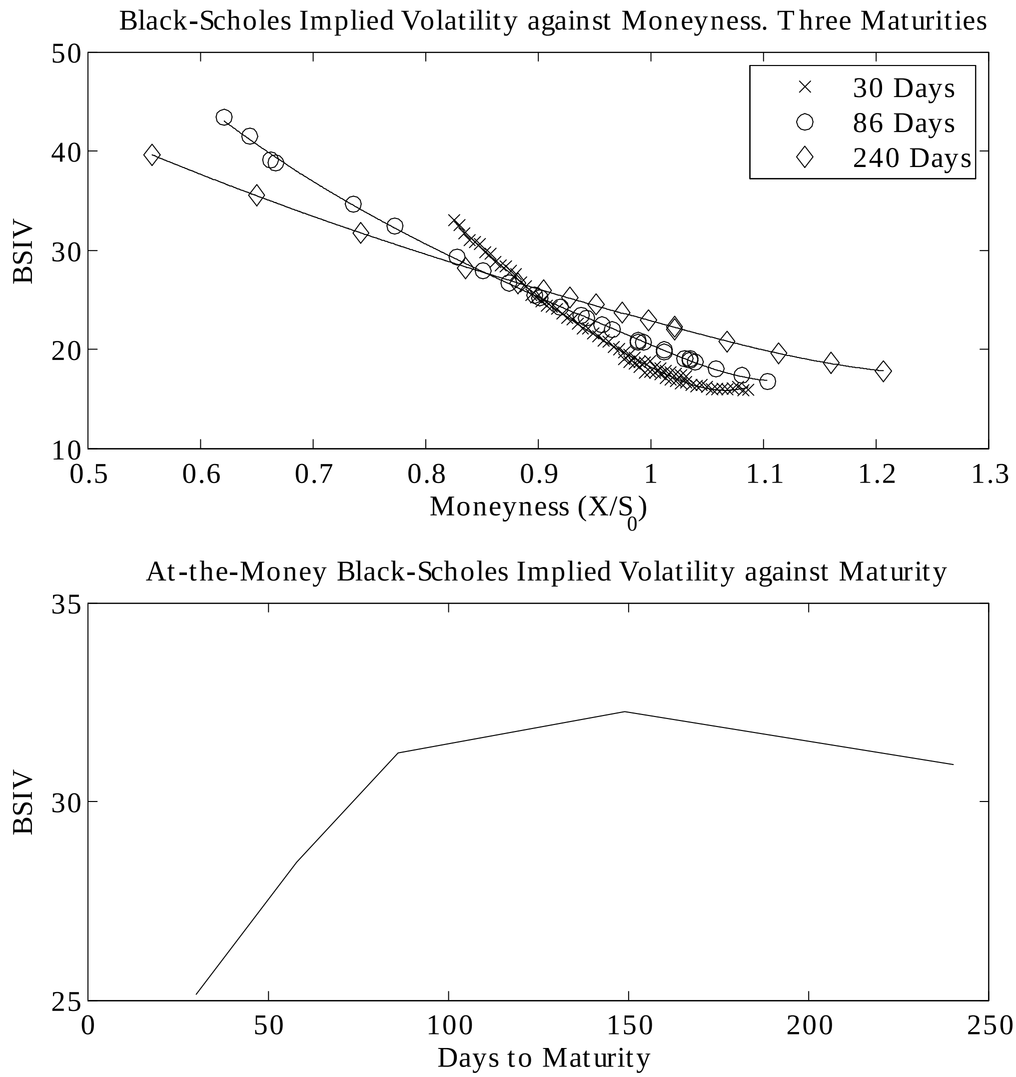
Zhou, H. (2010). Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty. Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System.

Figure 1: Black-Scholes Price and Vega



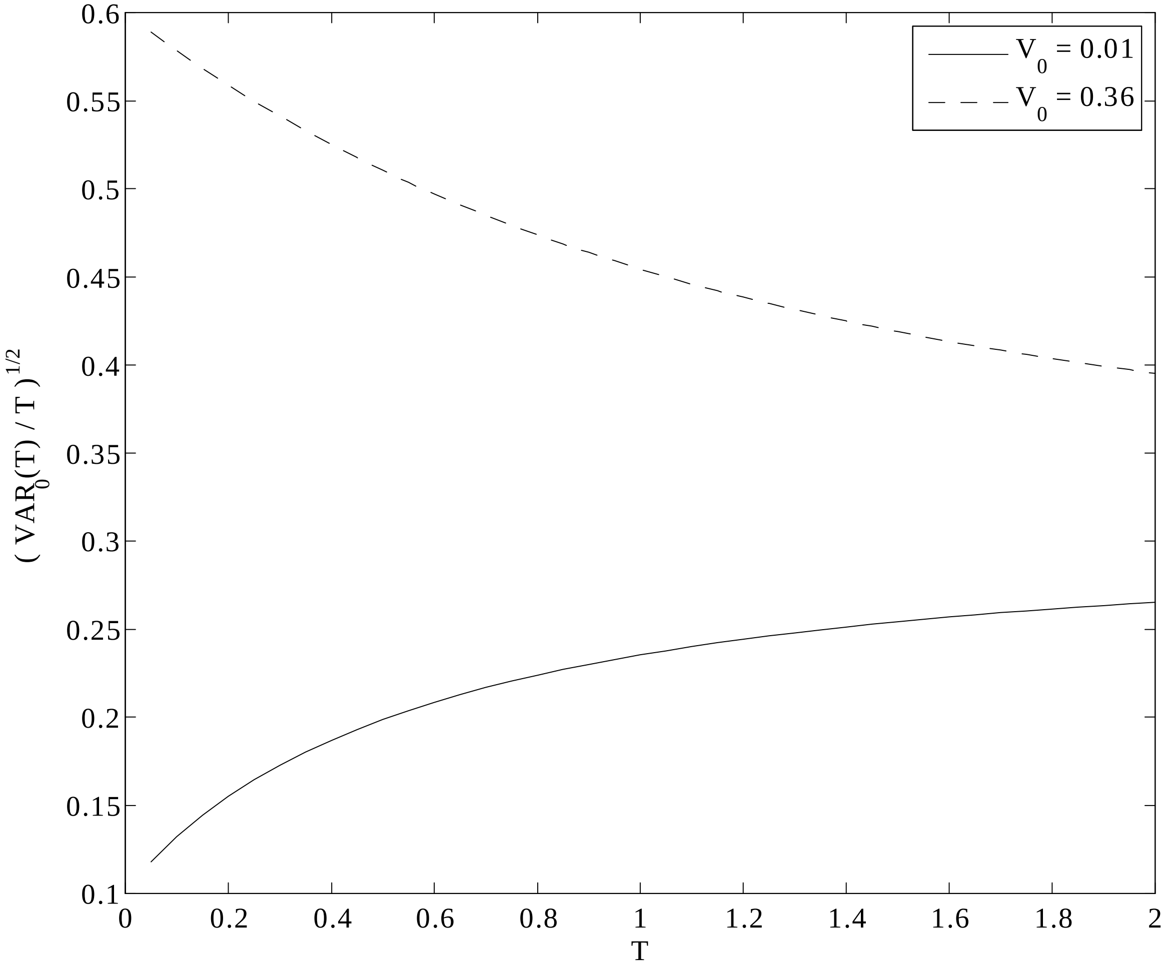
Notes to Figure: In the top panel, we plot the Black-Scholes call price as a function of volatility for an at-the-money option with a strike price of 100 and three months to maturity. The risk-free interest rate is 5% per year. In the bottom panel we plot the Black-Scholes Vega as a function of moneyness for a call option with a volatility of 50% per year.

Figure 2: Black-Scholes Implied Volatility as a Function of Moneyness and Maturity



Notes to Figure: In the top panel, we plot Black-Scholes implied volatility ($BSIV$) against moneyness, X/S_0, for various S&P 500 options quoted on October 22, 2009. In the bottom panel we plot at-the-money $BSIV$ against days to maturity (DTM).

Figure 3: Volatility Term Structures in the Heston Model

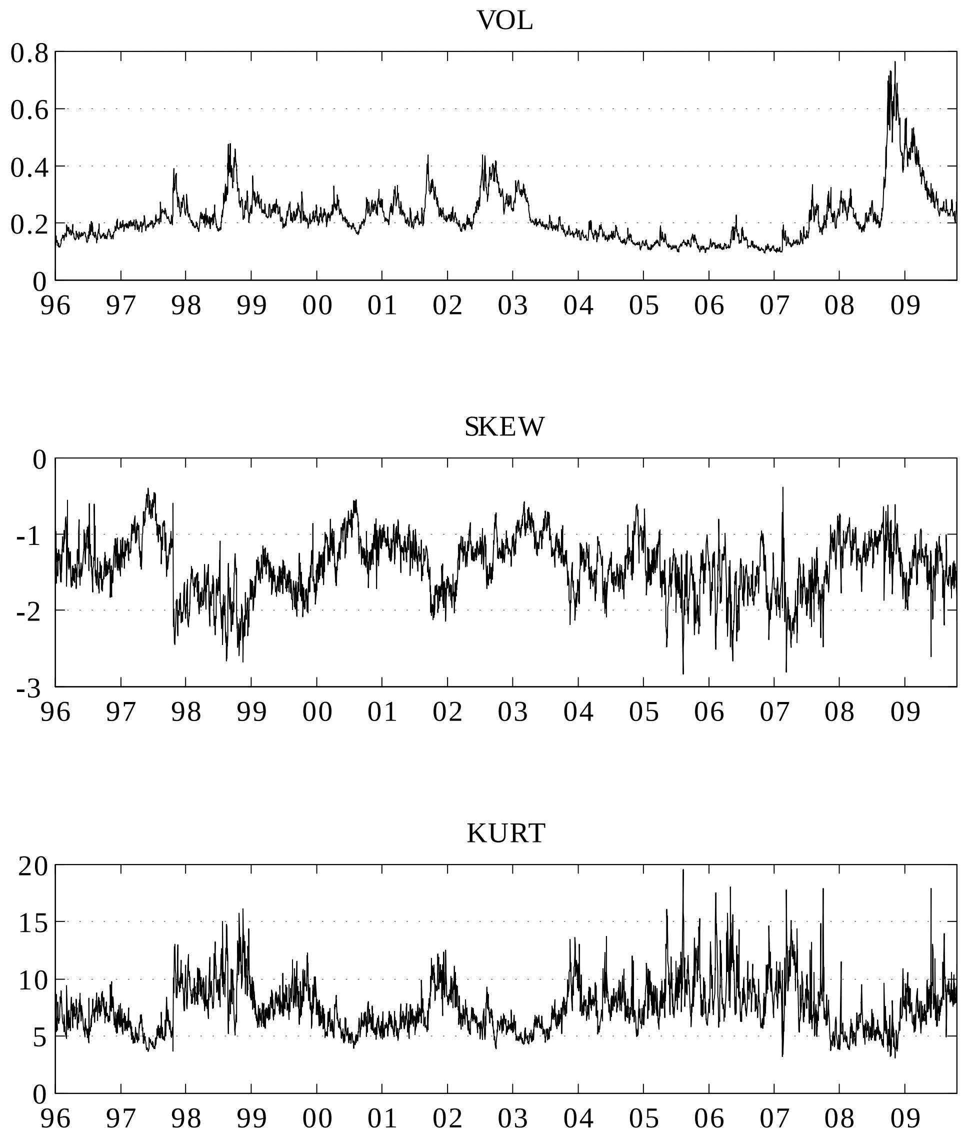


Notes to Figure: We plot the volatility term structure in the Heston model defined as

$$\sqrt{VAR_0\left(T\right)/T} = \sqrt{\theta + \left(V_0 - \theta\right)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa T}}$$

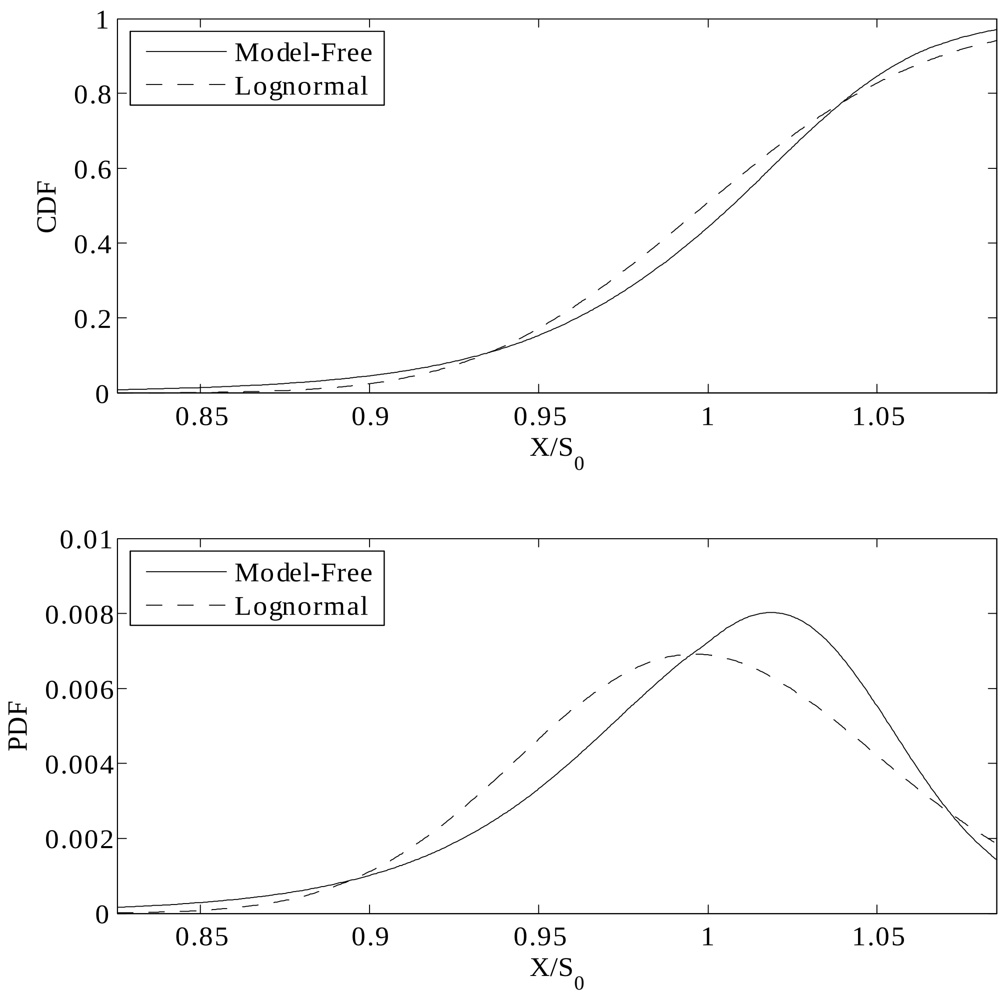
when $\theta = 0.09$, $\kappa = 2$ and $V_0 = 0.36$ (dashed line) corresponding to a high current spot variance and $V_0 = 0.01$ (solid line) corresponding to a low current spot variance.

Figure 4: Option-Implied Moments for One-Month S&P 500 Returns



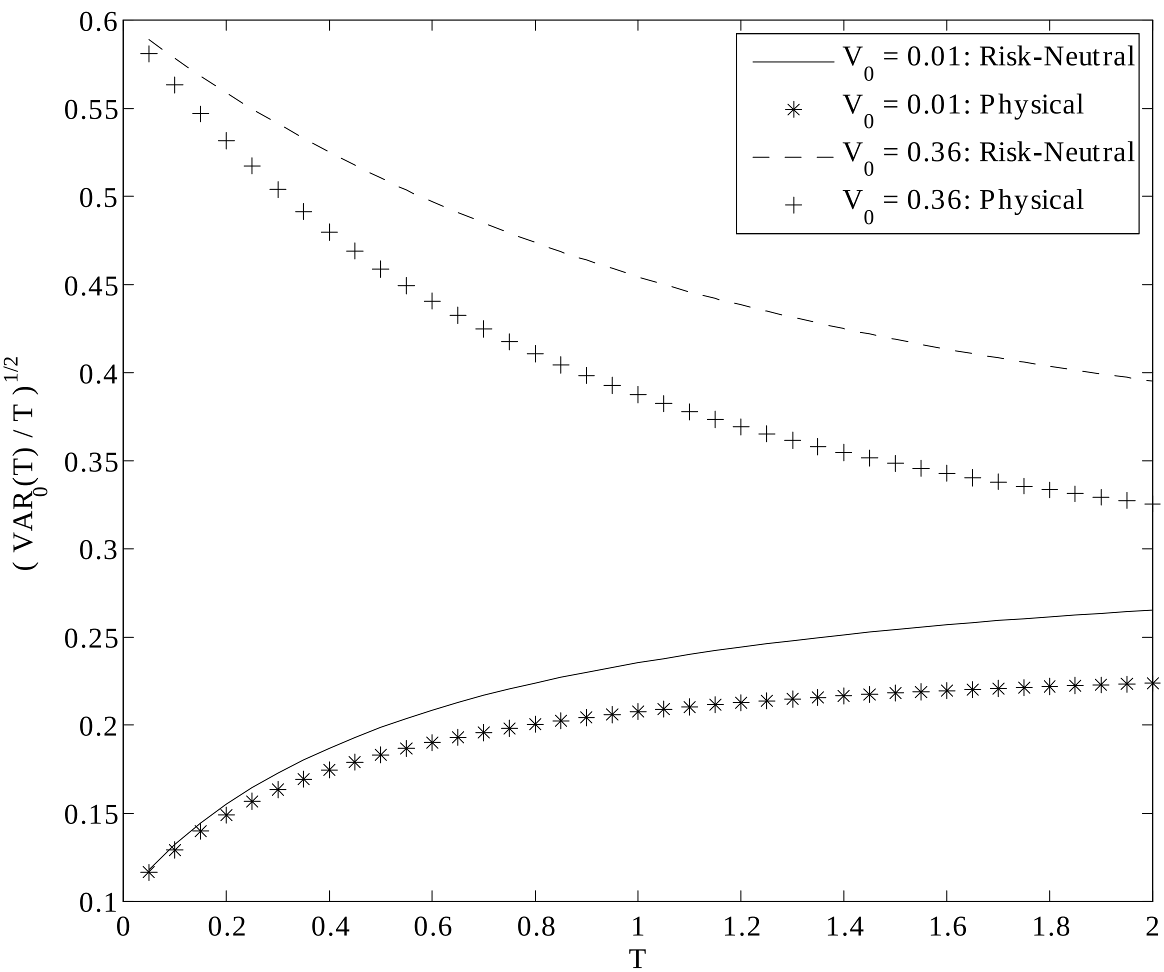
Notes to Figure: We plot the volatility, skewness, and kurtosis implied by S&P 500 index options using the methodology in Bakshi et al. (2003).

Figure 5: Option-Implied Distribution from BSIV Curve Fitting vs. Lognormal



Notes to Figure: We plot the CDF and PDF obtained from applying a smoothing cubic spline (solid lines) using data for S&P index options with thirty days to expiration on October 22, 2009, together with the CDF and PDF of the lognormal distribution (dashed lines).

Figure 6: Physical and Option-Implied Volatility Term Structures in the Heston Model



Notes to Figure: We plot the option-implied (from Figure 3) as well as the physical volatility term structure in the Heston model defined as

$$\sqrt{VAR_0^P\left(T\right)/T} = \sqrt{\theta\frac{\kappa}{\kappa-\lambda} + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1-e^{-(\kappa-\lambda)T}\right)}{\left(\kappa-\lambda\right)T}}$$

where $\theta = 0.09$, $\kappa = 2$, and $\lambda = -1.125$. Lines with '*' and '+' markers denote physical forecasts. The dashed line with $V_0 = 0.36$ shows the option-implied forecast from a a high current spot variance and the solid line with $V_0 = 0.01$ shows the option-implied forecasts from a low current spot variance.

Table 1: Forecasting Monthly Realized Volatility using Black-Scholes Implied Volatility

Constant	RV_M	RV_W	RV_D	C_M	C_W	C_D	$BSIV$	Adj. R^2
Panel A: Foreign exchange data								
0.0061	0.2186	0.0981	0.1706	–	–	–	–	26.0
(0.0011)	(0.1138)	(0.1438)	(0.0828)					
0.0061	–	–	–	0.2355	0.0871	0.2407	–	26.9
(0.0011)				(0.1597)	(0.1623)	(0.0930)		
0.0022	–	–	–	–	–	–	0.8917	40.7
(0.0011)							(0.0884)	
0.0021	−0.1483	0.0769	0.0765	–	–	–	0.8733	41.1
(0.0011)	(0.1178)	(0.1284)	(0.0754)				(0.1419)	
0.0022	–	–	–	−0.0517	0.0097	0.1114	0.8715	40.4
(0.0012)				(0.1526)	(0.1471)	(0.0869)	(0.1515)	
Panel B: S&P 500 data								
Constant	RV_M	RV_W	RV_D	C_M	C_W	C_D	$BSIV$	Adj. R^2
0.0053	0.6240	−0.3340	0.6765	–	–	–	–	53.0
(0.0025)	(0.1132)	(0.1039)	(0.1007)					
0.0037	–	–	–	0.1568	0.0407	0.9646	–	61.9
(0.0023)				(0.1327)	(0.1353)	(0.1088)		
−0.0050	–	–	–	–	–	–	1.0585	62.1
(0.0027)							(0.0667)	
−0.0052	0.0378	−0.1617	0.3177	–	–	–	0.9513	64.0
(0.0027)	(0.1311)	(0.0943)	(0.1026)				(0.1391)	
−0.0051	–	–	–	−0.1511	0.0633	0.6016	0.7952	68.2
(0.0027)				(0.1336)	(0.1237)	(0.1194)	(0.1447)	
Panel C: Treasury bond data								
Constant	RV_M	RV_W	RV_D	C_M	C_W	C_D	$BSIV$	Adj. R^2
0.0031	0.3600	0.1112	0.1389	–	–	–	–	32.5
(0.0005)	(0.1106)	(0.1143)	(0.0744)					
0.0037	–	–	–	0.4203	0.1436	0.0826	–	37.0
(0.0005)				(0.1347)	(0.1363)	(0.0776)		
0.0023	–	–	–	–	–	–	0.5686	35.0
(0.0006)							(0.0641)	
0.0018	0.0462	0.1835	0.0817	–	–	–	0.3933	40.4
(0.0006)	(0.1254)	(0.1086)	(0.0710)				(0.0882)	
0.0023	–	–	–	0.1736	0.1424	0.0318	0.4129	45.5
(0.0006)				(0.1355)	(0.1267)	(0.0729)	(0.0867)	

Note: We reproduce parts of Table 1 from Busch et al. (2011), who regress total realized volatility (RV) for the current month on the lagged monthly (subscript M), weekly (subscript W) and daily (subscript D) realized volatility. Alternative specifications separate RV into its continuous (C) and jump components (not reported here). Black-Scholes implied volatility ($BSIV$) is introduced in univariate regressions as well as an additional regression in the RV regressions. Panel A contains $/DM FX data for 1987-1999, Panel B contains S&P 500 data for 1990-2002, and Panel C contains Treasury bond data for 1990-2002.

<u>Table 2: Volatility Indexes Around the World</u>

Country	Exchange	Index	Underlying	Maturity	Launch Date	Method
US	Chicago Board Options Exchange (CBOE)	VIX	S&P 500	1 month	Sep 2003 (old index renamed VXO, 1993-)	Demeterfi, Derman, Kamal, and Zou (99) - Goldman Sachs (VIX methodology)
US	CBOE	VXV	S&P 500	3 months	Nov 2007	VIX
US	CBOE	VXO	S&P 100	1 month	1993	Whaley (1993)
US	CBOE	VXD	DJIA	1 month	Mar 2005	VIX
US	CBOE	VXN	Nasdaq 100	1 month		VIX
US	CBOE	VXAZN, VXAPL, VXGS, VXGOG, VXIBM	Stocks - Amazon, Apple, Goldman Sachs, Google, IBM	1 month	Jan 2011	VIX
US	CBOE	EVZ, GVZ, OVX, VXEEM, VXSLV, VXFXI, VXGDX, VXEWZ, VXXLE	ETFs - EuroCurrency, gold, crude oil, emerging markets, silver, China, gold miners, Brazil, energy sector	1 month	2008	VIX
US	CBOE	ICJ, JCJ, KCJ	S&P 500	As of May 2011, KCJ - Jan 2012, ICJ - Jan 2013, JCJ - Jan 2014, the tickers are to be recycled as they expire	Jul 2009	Skintzi and Refenes (2005)
Australia	Australian Securities Exchange	S&P/ASX 200 VIX (ASX code: XVI)	S&P/ASX 200 (XJO)	1 month	Sep 2010	VIX
Belgium	Euronext	VBEL	BEL 20	1 month	Sep 2007	VIX
Canada	TMX	S&P/TSX 60 VIX (VIXC)	S&P/TSX 60	1 month	Oct 2010	VIX
Europe	Eurex	VSTOXX	Euro STOXX 50	30, 60, 90,..., 360 days	Apr 20, 2005 (30 days); May 31, 2010 (60-360 days)	VIX
France	Euronext	VCAC	CAC 40	1 month	Sep 2007	VIX
Germany	Deutsche Borse	VDAX-NEW	DAX	1 month	Apr 2005 (previously VDAX, Dec 1994)	VIX
Hong Kong	Hong Kong Futures Exchange	VHSI	HSI	1 month	Feb 2011	VIX
India	National Stock Exchange of India	India VIX	NIFTY	1 month	Jul 2010	VIX
Japan	CSFI, Univ. of Osaka	CSFI - VXJ	Nikkei 225	1 month	Jul 2010	VIX
Mexico	Mexican Derivatives Exchange	VIMEX	Mexican Stock Exchange Price and Quotation Index (IPC)	3 months	Apr 2006	Fleming and Whaley (1995)
Netherlands	Euronext	VAEX	AEX	1 month	Sep 2007	VIX
South Africa	Johannesburg Stock Exchange	New SAVI	FTSE/JSE Top40	3 months	2010 (previously SAVI, 2007-)	VIX
South Korea	Korea Exchange	V-KOSPI	KOSPI200	1 month	Apr 2009	VIX
Switzerland	Six Swiss Exchange	VSMI	SMI	1 month	Apr 2005	VIX
UK	Euronext	VFTSE	FTSE 100	1 month	Jun 2008	VIX

Table 3: Forecasting with Option-Implied Volatility

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Fackler and King	1990	Commodity	Mean, vol	Mean, vol	Average of just OTM put and call IV	Corn and live cattle reliable, but overstate volatility of soybean and understate location of hog prices
Kroner, Kneafsey, and Claessens	1995	Commodity	Vol	Vol	Barone-Adesi and Whaley (87)	Combination of IV and historical outperform
Jorion	1995	Currency	Vol	Vol	Black (76) at the money	IV outperform historical, but biased
Taylor and Xu	1995	Currency	Vol	Vol	Barone-Adesi and Whaley (87)	IV outperform historical
Pong, Shackleton, Taylor, and Xu	2004	Currency	Vol	Vol	OTC quotes	IV as accurate as historical at 1 and 3 month horizons, but not better
Christoffersen and Mazzotta	2005	Currency	Vol, density, interval	Vol, density, interval	Malz (97)	Unbiased and accurate forecast
Day and Lewis	1992	Equity	Vol	Vol	Dividend adjusted BS + Whaley (82)	Add IV to GARCH and EGARCH. Both are unbiased, but inconclusive as for the relative performance.
Harvey and Whaley	1992	Equity	Vol	Vol	Cash-dividend adjusted binomial	IV predicts, but arbitrage profits are not possible, thus consistent with market efficiency
Canina and Figlewski	1993	Equity	Vol	Vol	Binomial tree adjusting for dividends and early exercise	IV does not predict
Fleming, Ostdiek, and Whaley	1995	Equity	Vol	Vol	Cash-dividend adjusted binomial, old VIX (Whaley (93))	Biased, but useful for forecasting
Christensen and Prabhala	1998	Equity	Vol	Vol	BS	Outperform historical
Fleming	1998	Equity	Vol	Vol	Modified binomial model of Fleming and Whaley (94)	IV is an upward biased forecast, but contains relevant information.
Blair, Poon, and Taylor	2001	Equity	Vol	Vol	VIX	VIX forecasts best and high-frequency intraday returns add no incremental information.
Poon and Granger	2003	Equity	Vol	Vol	N/A	Review of volatility forecasting, table with summary of literature
Jiang and Tian	2005	Equity	Vol	Vol	Britten-Jones et al. cubic spline	Model-free IV subsumes all info in BS IV and historical volatility.
Ang, Hodrick, Xing, and Zhang	2006	Equity	Vol	Cross-section of stock returns	VIX	Innovation in VIX is a priced risk factor with a negative price of risk.
Andersen and Bondarenko	2007	Equity	Vol	Vol	Corridor implied volatility (CIV)	Broad CIV related to model-free IV. narrow CIV related to BS IV. narrow IV is a better volatility predictor than model-free IV or BS IV.
Bollerslev, Tauchen, and Zhou	2009	Equity	Variance risk premium	Equity risk premium	VIX	VRP predicts stock market return
Bekaert, Hoerova, and Lo Duca	2010	Equity	Variance risk premium	Equity risk premium	VIX	A lax monetary policy decreases risk aversion after about five months. Monetary authorities react to periods of high uncertainty by easing monetary policy.
Zhou	2010	Equity	Variance risk premium	Equity risk premium	VIX	VRP predicts a significant positive risk premium across equity, bond, and credit markets in the short-run (1-4 months)

Table 3 (continued): Forecasting with Option-Implied Volatility

Bakshi, Panayotov, Skoulakis	2011	Equity	Forward variances	(i) Growth in measures of real economic activity, (ii) Treasury bill returns, (iii) stock market returns, and (iv) changes in variance swap rates	Forward variances extracted from the prices of exponential claims of different maturities (Carr and Lee (08))	The forward variances predict (i) growth in measures of real economic activity, (ii) Treasury bill returns, (iii) stock market returns, and (iv) changes in variance swap rates
Fenou, Fontaine, Taamouti, and Tedongap	2011	Equity	Term structure of implied voaltility	Equity risk premium, variance risk premium	VIX	Term structure of implied volatility predicts both equity risk premium and variance risk premium
DeLisle, Doran, and Peterson	2011	Equity	Vol	Cross-section of stock returns	VIX	Result in Ang et al. (2006) holds when volatility is rising, but not when volatility is falling.
Latane and Rendleman	1976	Equity (individual)	Vol	Vol	Vega-weighted average of individual stock option BS IVs	Outperform historical
Chiras and Manaster	1978	Equity (individual)	Vol	Vol	BS	Risk-free return using option trading strategies
Beckers	1981	Equity (individual)	Vol	Vol	Weighted average BS Ivs vs. at-the-money BS IV	At-the-money BS IV predicts better than weighted average of BS Ivs.
Sheikh	1989	Equity (individual)	Vol	Split announcement and ex-dates	Roll (1977), American option with dividends	No relative increase in IV of stocks announcing splits, but increase is detected at the ex-date.
Lamoureux and Lastrapes	1993	Equity (individual)	Vol	Vol	Hull and White (87), stochastic volatility option pricing model	IV contains incremental information to historical
Swidler and Wilcox	2002	Equity (individual)	Vol	Bank stock volatility	Old VIX	Outperform historical
Banerjee, Doran, and Peterson	2007	Equity (individual)	Vol	Return of characteristic-based portfolios	VIX	Strong predictive ability
Diavatopoulos, Doran, and Peterson	2008	Equity (individual)	Idiosyncratic volatility	Future cross-sectional stock returns		Strong positive link
Doran, Fodor, and Krieger	2010	Equity (individual)	Vol	Abnormal return after analyst recommendation change	Simulate Bates (1996) model of SVJ	Information in option markets leads analyst recommendation changes
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2004) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return
Amin and Ng	1997	FI	Vol	Volatility of interest rate	HJM (92)	Predicts well. show how to combine IV with historical.
Busch, Christensen, and Nielsen	2011	FI, equity, currency	Vol	Realized volatility, jump	Numerical inversion of Black (76)	Prediction in all three markets

Table 4: Forecasting with Option-Implied Correlation

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Siegel	1997	Currency	Correlation	Correlation: USD/DM/pound	Garman and Kohlhagen (83)	Outperform historical
Campa and Chang	1998	Currency	Correlation	Correlation between USD/DM and USD/YEN	From relationship between implied volatilities of three exchange rates	Outperforms forecast based on historical correlation
Walter and Lopez	2000	Currency	Correlation	Correlation: USD/DEM/JPY, USD/DEM/JPY	From relationship between implied volatilities of three exchange rates	Useful for USD/DEM/JPY, not for USD/DEM/JPY, so may not be useful in general
Skintzi and Refenes	2005	Equity (individual)	Correlation	Correlation	Implied correlation based on IV of index and individual stocks	Although the implied correlation index is a biased forecast of realized correlation, it has a high explanatory power, and it is orthogonal to the information set compared to a historical forecast.
Driessen, Maenhout and Vilkov	2009	Equity (individual)	Correlation	Correlation	Stock prices follow a geometric Brownian motion with constant drift and possibly stochastic diffusion. Assume that a single state variable drives all pairwise correlations	The entire index variance risk premium can be attributed to the high price of correlation risk.
Buss and Vilkov	2011	Equity (individual)	Correlation and factor betas	Factor betas	Stock prices follow a multifactor model. Assume that a single state variable drives all pairwise correlations	Most efficient and unbiased predictor of beta
Chang, Christoffersen, Jacobs, and Vainberg	2011	Equity (individual)	Beta, moments	Beta	Formula based on implied vol and skew of index and individual stocks	Forecast future beta
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2004) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return
Longstaff, Santa-Clara, and Schwartz	2003	FI	Correlation	Correlation	Caps and swaptions	Implied correlation is lower than historical correlation. Significant mispricings detected.
De Jong, Driessen, and Pelsser	2004	FI	Correlation	Correlation	Caps and swaptions	Implied correlation is different from historical correlation. Significant mispricings detected.

Table 5: Forecasting with Option-Implied Skewness and Kurtosis

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Bates	1996	Currency	Skew, kurt	Skew, kurt of USD/DM, USD/YEN, 1984-1992	Jump diffusion	The implicit abnormalities (e.g. moments) predict future abnormalities in log-differenced $/DM futures prices, but not S/yen.
Bates	1991	Equity	Skew premium, jump-diffusion parameters	Crash of 1987	Jump diffusion	RND negatively skewed one year before the crash, but no strong crash fears during 2 months immediately preceding the crash
Navatte and Villa	2000	Equity	Vol, skew, kurt	Moments of CAC 40 (Jan95-May97)	Gram-Charlier	Implied moments contain substantial amount of information, which decreases with the moment's order
Doran, Carson, and Peterson	2006	Equity	Skew	Crash	Barone-Adesi and Whaley (87)	Implied volatility skew has significant forecast power for assessing the degree of market crash risk
Agarwal, Bakshi, and Huij	2009	Equity	Vol, skew, kurt	Cross-section of hedge fund returns	Bakshi, Kapadia, Madan (03)	Innovations in implied market vol, skew, kurt are all priced risk factors for hedge fund returns.
Chang, Christoffersen, and Jacobs	2011	Equity	Vol, skew, kurt	Cross-section of stock returns	Bakshi, Kapadia, Madan (03)	RND negatively skewed one year before the crash, but no strong crash fears during 2 months immediately preceding the crash
Doran, Peterson and Tarrant	2007	Equity (individual)	Skew	Crash and spikes upward	Barone-Adesi and Whaley (87)	Reveal crashes and spikes with significant probability, but not economically significant
Diavatopoulos, Doran, Fodor, and Peterson	2008	Equity (individual)	Skew, kurt prior to earnings announcements	Stock and option returns	Bakshi, Kapadia, Madan (03)	Both have strong predictive power
Bali and Hovakimian	2009	Equity (individual)	Realized-implied volatility, call-putIV spread	Cross-section of stock returns		Negative relationship with realized-implied volatility. positive relationship with call-put IV spread.
Conrad, Dittmar and Ghysels	2009	Equity (individual)	Vol, skew, kurt	Cross-section of stock returns	Bakshi, Kapadia, Madan (03)	Negative relationship between volatility and skew with future return, and positive relationship between kurt and future return
Cremers and Weinbaum	2010	Equity (individual)	Call-putIV spread	Cross-section of stock returns	BS IV	Positive relationship
Doran and Krieger	2010	Equity (individual)	Volatility skew	Return	Five skew measures based on ATM, ITM, OTM IV and traded volume	Discourage the use of skew-based measures for forecasting equity returns without fully parsing the skew into its most basic portions.
Rehman and Vilkov	2010	Equity (individual)	Skew	Return	Bakshi, Kapadia, Madan (03)	Positive relationship
Xing, Zhao and Zhang	2010	Equity (individual)	Volatility skew	Cross-section of stock returns	OTM put IV - ATM call IV (highest volume, highest open-interest, or volume or open-interest weighted)	Stocks with steepest smirks underperform stocks with least pronounced smirks
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2004) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return

Table 6: Forecasting using Option-Implied Densities

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Silva and Kahl	1993	Commodity	PDF	PDF of soybean, corn	Log-normal VS. linear interpolation of cdf	Corroborate Fackler and King (90)
Melick, Thomas	1997	Commodity	PDF from American options	Crude oil price during gulf crisis	Mixture of two & three lognormals	Option prices were consistent with the market commentary. Use of lognormal model would have overestimated the prob. and price impact of major disruption.
Hog and Tsiaras	2010	Commodity	PDF	PDF and various intervals and regions of interest of crude oil	Generalized Beta of the second kind (GB2), Q to P using statistical recalibration (param. and non-param.)	Outperform historical
Leahy, Thomas	1996	Currency	PDF from American options	Canadian dollar around Quebec referendum	Mixture of three lognormals	Option prices were consistent with the market commentary.
Campa, Chang	1996	Currency	PDF, Intensity of realignment and credibility measures	ERM target-zone credibility and devaluation	Arbitrage bounds	Devaluation predicted with different time lags from a week to a year
Malz	1996	Currency	PDF from American options	PDF and realignment prb. of sterling/DM	Jump diffusion	Useful for defense of target zones against speculative attack
Malz	1997	Currency	PDF, moments	Excess return puzzle in currency markets	IV in function of ATM IV, risk reversal price, strangle price, etc.	Tests International CAPM using RN moments as explanatory variables and show that they have considerably greater explanatory power for excess returns in currency markets.
Campa, Chang, Reider	1998	Currency	PDF from American options	PDF of USD/DM and USD/YEN	Compare cubic splines, Implied binomial tree, mixture of normals	Use trimmed binomial tree
Bodurtha and Shen	1999	Currency	PDF of corr	Covariance VaR of mark and yen	Whaley (82)	Implied correlation provides incremental explanatory power over historical-based correlation estimates
Campa, Chang, Refalo	2002	Currency	PDF, Intensity and credibility measure of CC96	PDF and realignment prob. of Brazilian Real/USD, 1991-1994	Shimko (93) IV in quadratic function of strike	Anticipate realignments of exchange rate bands
Haas, Mittnik, and Mizrach	2006	Currency	PDF	PDF, central bank credibility during ERM crisis	Mixture of normals	Both historical and option based forecasts are useful
Gemmill and Saflekos	2000	Equity	PDF	PDF, four crashed, three British elections	Mixture of two lognormals	Little forecasting ability
Mizrach	2006	Equity	PDF	Enron's collapse	Mixture of lognormals	Market remained too optimistic until just weeks before the collapse
Shackleton, Taylor, Yu	2010	Equity	PDF	PDF, S&P 500 index return	Calibration of jump-diffusion model, statistical transformation from RN to physical	Compare historical, risk-neutral, and risk-transformed physical PDF. Performance depends on the forecast horizon.
Kostakis, Panigirtzoglou, Skiadopoulos	2010	Equity	PDF	Portfolio selection	Smoothed IV smile + Barone-Adesi and Whaley (87)	Improved portfolio

Table 7: Forecasting with Option-Implied Event Probabilities

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Melick, Thomas	1997	Commodity	PDF from American options	Crude oil price during gulf crisis	Mixture of two & three lognormals	Option prices were consistent with the market commentary. Use of lognormal model would have overestimated the prob. and price impact of major disruption.
Leahy, Thomas	1996	Currency	PDF from American options	CDN	Mixture of three lognormals	Option prices were consistent with the market commentary.
Campa, Chang	1996	Currency	PDF, Intensity of realignment and credibility measures	ERM target-zone credibility and devaluation	Arbitrage bounds	Devaluation predicted with different time lags from a week to a year
Malz	1996	Currency	PDF from American options	PDF and realignment prb. of sterling/DM	Jump diffusion	Useful for defense of target zones against speculative attack
Campa, Chang, Refalo	2002	Currency	PDF, Intensity of realignment and credibility measures	PDF and realignment prob. of Brazilian Real/USD, 1991-1994	Shimko (93) IV in quadratic function of strike	Anticipate realignments of exchange rate bands
Haas, Mittnik, and Mizrach	2006	Currency	PDF	PDF, central bank credibility during ERM	Mixture of normals	Both historical and option based forecasts are useful
Bates	1991	Equity	Jump-diffusion parameters	Crash of 1987	Jump diffusion	RND negatively skewed one year before the crash, but no strong crash fears during 2 months immediately preceding the crash
Gemmill and Saflekos	2000	Equity	PDF	PDF, four crashed, three British elections	Mixture of two lognormals	Little forecasting ability
Doran, Carson, and Peterson	2006	Equity	Skew	Crash	Baron-Adesi and Whaley (87)	Implied volatility skew has significant forecast power for assessing the degree of market crash risk
Mizrach	2006	Equity	PDF	Enron's collapse	Mixture of lognormals	Market remained too optimistic until just weeks before the collapse
Sheikh	1989	Equity (individual)	Vol	Split announcement and ex-dates	Roll (1977), American option with dividends	No relative increase in IV of stocks announcing splits, but increase is detected at the ex-date.
Jayaraman, Mandelker, and Shastri	1991	Equity (individual)	Premia on call options	Information leakage prior to merger announcement	BS	Implied volatility increases prior to the announcement of the first bid for the target firm and decrease significantly at the announcement date.
Barone-Adesi, Brown and Harlow	1994	Equity (individual)	Probability and timing of acquisitions	Probability and timing of acquisitions	Propose model of probability weighted IV	Cannot predict success or timing of acquisition
Cao, Chen, and Griffin	2005	Equity (individual)	Call volume imbalance	Takeover announcement day return	N/A	Takeover targets with the largest preannouncement call-volume imbalance increases experience the highest announcement day returns
Bester, Martinez, and Rosu	2008	Equity (individual)	Merger success probability	Merger success probability	New option pricing model with merger	Better prediction compared to naive method

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