

CREATES Research Paper 2011-44

Do Realized Skewness and Kurtosis Predict the Cross-Section of Equity Returns?

Diego Amaya, Peter Christoffersen, Kris Jacobs
and Aurelio Vasquez

Do Realized Skewness and Kurtosis Predict the Cross-Section of Equity Returns?*

Diego Amaya
HEC Montreal and UQAM

Peter Christoffersen
Rotman, CBS and CREATES

Kris Jacobs
University of Houston

Aurelio Vasquez
ITAM School of Business

July 29, 2011

Abstract

Yes. We use intraday data to compute weekly realized variance, skewness and kurtosis for individual equities and assess whether this week's realized moments predict next week's stock returns in the cross-section. We sort stocks each week according to their past realized moments, form decile portfolios and analyze subsequent weekly returns. We find a very strong negative relationship between realized skewness and next week's stock returns, and a positive relationship between realized kurtosis and next week's stock returns. We do not find a strong relationship between realized volatility and stock returns. A trading strategy that buys stocks in the lowest realized skewness decile and sells stocks in the highest realized skewness decile generates an average weekly return of 43 basis points with a t-statistic of 8.91. A similar strategy that buys stocks with high realized kurtosis and sells stocks with low realized kurtosis produces a weekly return of 16 basis points with a t-statistic of 2.98. Our results are robust across sample periods, portfolio weightings, and proxies for firm characteristics, and they are not captured by the Fama-French and Carhart factors.

JEL Codes: G11, G12, G17

Keywords: Realized volatility; skewness; kurtosis; equity markets; return prediction.

*We would like to thank seminar participants at McGill University, Rutgers University, Ryerson University, University of British Columbia, the 2010 FMA and EFMA meetings, the IFM² Mathematical Finance Conference, Stanford SITE, and the Market Microstructure Conference in Paris for helpful comments. We thank IFM², SSHRC, and Asociación Mexicana de Cultura A.C. for financial support. Any remaining inadequacies are ours alone. Correspondence to: Peter Christoffersen, Phone: 416-946-5511, Email: peter.christoffersen@rotman.utoronto.ca.

1 Introduction

We examine the relationship between higher moments computed from intraday returns and future stock returns. Extending the well-known concept of realized volatility (Hsieh (1991), Andersen, Bollerslev, Diebold, and Ebens (2001)), computed from intraday squared returns, we compute realized skewness and kurtosis from intraday cubed and quartic returns. The realized moments are model-free and do not require assumptions regarding the underlying moment dynamics. We show that the realized moments have well-defined convergence limits under realistic assumptions and that they are measured reliably in finite samples.

We find a strong relation between realized skewness and future stock returns, and also between realized kurtosis and future stock returns. Our findings complement the results obtained using alternative skewness and kurtosis estimates in Boyer, Mitton, and Vorkink (2010), Zhang (2006), Xing, Zhang and Zhao (2010), Dittmar (2002) and Conrad, Dittmar, and Ghysels (2008), as well as the tail-risk estimates in Kelly (2011).

Our empirical strategy uses a very extensive sample of weekly data. We aggregate daily realized moments to obtain weekly realized volatility, skewness, and kurtosis measures for over two million firm-week observations. We sort stocks into deciles based on the current-week realized moment and compute the subsequent one-week return of the trading strategy that buys the portfolio of stocks with a high realized moment (volatility, skewness or kurtosis) and sells the portfolio of stocks with a low realized moment.

When sorting on realized volatility, the resulting portfolio return differences are not statistically significant. However, when sorting by realized skewness, the long-short equal-weighted portfolio produces an average weekly return of -43 basis points with a t -statistic of -8.91 . This value is of a similar order of magnitude than the premium of -67 basis points per month reported in Boyer, Mitton, and Vorkink (2010) and -36 basis points per month reported in Zhang (2006). The resulting four factor Carhart risk adjusted alpha for the long-short skewness portfolio is -46 basis points per week. Consistent with the findings of Conrad, Dittmar, and Ghysels (2008) on risk-neutral kurtosis, we find a positive relation between realized kurtosis and subsequent stock returns. For realized kurtosis, the long-short equal-weighted portfolio generates a weekly return of 16 basis points with a t -statistic of 2.98 . The four factor Carhart alpha of 13 basis points per week further supports the value of realized kurtosis as a predictor of stock returns.

We confirm the negative relation between realized skewness and future returns, and the positive relation between realized kurtosis and future returns using Fama-MacBeth regressions. We also investigate the robustness of these findings to controlling for a number of well-documented determinants of returns: lagged return (Jegadeesh (1990), Lehmann (1990) and Gutierrez and Kelley (2008)), realized volatility, market size (Fama and French (1993)), the book-to-market ratio (Fama and French (1993)), market beta, historical skewness, idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), coskewness (Harvey and Siddique (2000)), maximum return (Bali, Cakici, and

Whitelaw (2009)), the number of analysts that follow the firm (Arbel and Strebel (1982)), illiquidity (Amihud (2002)), and the number of intraday transactions. Two-way sorts on realized skewness and the firm characteristics also confirm that the relationship between realized skewness and returns is significant. However, two-way sorts show that the relationship between realized kurtosis and returns is not always significant when controlling for other firm characteristics. Finally, results for realized skewness and realized kurtosis are robust to the January effect and are significant when considering only NYSE stocks.

Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility earn low returns, invalidating the mean-variance model. Motivated by their findings, we further explore the relationship between realized skewness, idiosyncratic volatility and subsequent stock returns. We find that when idiosyncratic volatility increases, low skewness stocks are compensated with higher returns while high skewness stocks are compensated with lower returns. Therefore, skewness provides a partial explanation of the idiosyncratic volatility puzzle. We also show that similar findings obtain when using realized volatility instead of idiosyncratic volatility.

To verify that our measures of higher moments are not contaminated by microstructure noise, and to make sure that we are effectively measuring asymmetry and fat tails, we also investigate two additional measures of skewness and kurtosis using high frequency data. The first measure is an enhanced version of the realized moment that uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005) to compute realized volatility. This subsampling methodology ensures that useful data is not ignored and provides a more robust estimator of the realized moment. The second approach uses percentiles of the high-frequency return distribution as alternative measures to capture skewness and kurtosis. We confirm that both the negative relation between realized skewness and future stock returns and the positive relation between realized kurtosis and future stock returns are robust to using different measures of skewness and kurtosis. We also show that our cross-sectional results are robust to using monthly holding periods for returns instead of weekly.

Finally, we verify the limiting properties of realized higher moments, based on a continuous-time specification of equity price dynamics that includes stochastic volatility and jumps. We show how the limits of the higher realized moments are determined by the jump parameters of the continuous-time price process. Using Monte Carlo techniques, we verify that the measurement of the realized higher moments is robust to the presence of market microstructure noise as well as to quote discontinuities in existence prior to decimalization. We also show that our cross-sectional results hold up when using jump-robust measures of realized volatility.

The relationship between skewness and asset prices has been a topic of study since Kraus and Litzenberger (1976). Recent papers find that higher moments of the underlying stock return distribution are related to future returns. Ang, Hodrick, Xing, and Zhang (2006) find that stocks with higher idiosyncratic volatility have lower subsequent returns. Boyer, Mitton, and Vorkink (2010) find that stocks with higher expected idiosyncratic skewness yield lower future returns.

Zhang (2006) and Xing, Zhang and Zhao (2010) document a negative relation between skewness and future returns in the cross-section. Using the methodology in Bakshi, Kapadia, and Madan (2003), Conrad, Dittmar, and Ghysels (2008) extract higher individual risk neutral moments from equity options and find that subsequent stock returns are negatively related to volatility, negatively related to skewness, and positively related to kurtosis. Theoretical models developed by Barberis and Huang (2008), Mitton and Vorkink (2007), and Brunnermeier, Gollier, and Parker (2007) provide support for a negative relationship between skewness and future returns.

The remainder of the paper is organized as follows. Section 2 estimates the weekly realized higher moments from intraday returns and constructs portfolios based on these moments. Section 3 computes raw and risk-adjusted returns on portfolios sorted on realized volatility, skewness and kurtosis, and estimates Fama-MacBeth regressions including various control variables. Section 3 also investigates the interaction of volatility, skewness and returns. Section 4 contains a series of robustness checks. Section 5 investigates the limiting properties of the realized higher moments as well as the significance of our results when using jump-robust realized volatility estimators. Section 6 concludes.

2 Constructing Moment-Based Portfolios

We first describe the data. We then show how the realized higher moments are computed. Finally, we form portfolios by sorting stocks into deciles based on the weekly realized moments, and then report on the characteristics of these portfolios.

2.1 Data

We analyze every listed stock in the Trade and Quote (TAQ) database from January 4, 1993 to September 30, 2008. TAQ provides historical tick by tick data for all stocks listed on the New York Stock Exchange, American Stock Exchange, Nasdaq National Market System, and SmallCap issues. We record prices every five minutes starting at 9:30 EST and construct five-minute log-returns for the period 9:30 EST to 16:00 EST for a total of 78 daily returns. We construct the five minute grid by using the last recorded price within the preceding five-minute period. If there is no price in a period, the return for that period is set to zero.

To ensure sufficient liquidity, we require that a stock has at least 80 daily transactions to construct a daily measure of realized moments.¹ The average number of intraday transactions per day for a stock is over one thousand. The weekly realized moment estimator is the average of the available daily estimators (Wednesday through Tuesday). Only one valid day of the realized moment is required to have a weekly estimator and the maximum number of daily estimators is five. Stocks with prices below \$5 are excluded from the analysis.

¹We repeated the analysis using a minimum of 100, 250 and 500 transactions instead. The results are similar.

We use data from three additional databases. From the Center for Research and Security Prices (CRSP) database, we use daily returns of each firm to calculate weekly returns (from Tuesday close to Tuesday close), historical equity skewness, market beta, lagged return, idiosyncratic volatility, maximum return over the previous month, and illiquidity; we use monthly returns to compute coskewness as in Harvey and Siddique (2000);² we use daily volume to compute illiquidity; and we use outstanding shares and stock price to get the market capitalization. COMPUSTAT is used to extract the Standard and Poor’s issuer credit ratings and book values to calculate book-to-market ratios of individual firms. From Thomson Returns Institutional Brokers Estimate System (I/B/E/S), we obtain the number of analysts that follow each individual firm. These variables are discussed in more detail in Appendix A.

2.2 Computing Realized Higher Moments

We first define the intraday log returns for each firm. On day t , the i th intraday return is given by

$$r_{t,i} = p_{t-1+\frac{i}{N}} - p_{t-1+\frac{i-1}{N}}, \quad (1)$$

where p_l is the natural logarithm of the price observed at time l and N is the number of return observations in a trading day. We use five-minute returns so that in 6.5 trading hours we have $N = 78$.

The well-known daily realized variance (Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold and Labys (2003)) is obtained by summing squares of intraday high-frequency returns

$$RDVar_t = \sum_{i=1}^N r_{t,i}^2. \quad (2)$$

As is standard, we do not estimate the mean of the high-frequency return because it is dominated by the variance at this frequency.

An appealing characteristic of this volatility measure compared to other estimation methods is its model-free nature (see Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) for details). Moreover, as we will discuss below, realized variance converges to a well-defined quadratic variation limit as the sampling frequency N increases.

Given that we are interested in measuring the asymmetry of the daily return’s distribution, we construct a measure of ex-post realized daily skewness based on intraday returns standardized by the realized variance as follows

$$RDSkew_t = \frac{\sqrt{N} \sum_{i=1}^N r_{t,i}^3}{RDVar_t^{3/2}}. \quad (3)$$

The interpretation of this measure is straightforward: negative values indicate that the stock’s return distribution has a left tail that is fatter than the right tail, and positive values indicate the

²To compute co-moments with high-frequency data is not straightforward due to synchronicity problems between stock and index returns. We leave this problem for future research.

opposite.

We are interested in extremes of the return distribution more generally, and so we also construct a measure of realized daily kurtosis defined by

$$RDKurt_t = \frac{N \sum_{i=1}^N r_{t,i}^4}{RDVar_t^2}. \quad (4)$$

The limits of the third and fourth moment when the sampling frequency N increases will be analyzed below as well.

Our cross-sectional asset pricing analysis below is conducted at the weekly frequency. We therefore construct weekly realized moments from their daily counterparts as follows. If t is a Tuesday then we compute

$$RVol_t = \left(\frac{252}{5} \sum_{i=0}^4 RDVar_{t-i} \right)^{1/2}, \quad (5)$$

$$RSkew_t = \frac{1}{5} \sum_{i=0}^4 RDSkew_{t-i}, \quad (6)$$

$$RKurt_t = \frac{1}{5} \sum_{i=0}^4 RDKurt_{t-i}. \quad (7)$$

Our cross-sectional analysis below is conducted at the weekly frequency and t will therefore denote a week from this point on. Note that, as is standard, we have annualized the realized volatility measure to facilitate the interpretation of results.

We compute the $RVol_t$, $RSkew_t$, and $RKurt_t$ for more than two million firm-week observations during our January 1993 to September 2008 sample period. Figure 1 summarizes the realized moments. The top-left panel of Figure 1 displays a histogram of the realized volatility measure pooled across firms and weeks. As often found in the realized volatility literature, the unconditional distribution of realized equity volatility appears to be roughly log-normally distributed. The top-right panel in Figure 1 shows the time-variation in the cross-sectional percentiles using three-month moving averages. The cross-sectional dispersion in realized equity volatility is clearly not constant over time and seems to have decreased through our sample period.

The middle-left panel of Figure 1 shows the histogram of realized equity skewness. The skewness distribution is very fat-tailed and strongly peaked around zero. The middle-right panel of Figure 1 shows the time-variation in the cross-sectional skewness percentiles. The cross-sectional dispersion in realized equity skewness has increased through our sample.

The bottom-left panel of Figure 1 shows the histogram of realized equity kurtosis. Similar to realized volatility, realized kurtosis appears to be approximately log-normally distributed. The vast majority of our sample has a kurtosis above 3, strongly suggesting fat-tailed returns. The bottom-right panel of Figure 1 shows that the cross-sectional distribution of realized equity kurtosis has become more disperse over time, matching the result found for realized skewness.

2.3 Portfolio Sort Characteristics

Each Tuesday, we form portfolios by sorting stocks into deciles based on the weekly realized moments. Table 1 reports the time-series sample averages for the moments and different firm characteristics, by decile. Panel A reports the time-series averages for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Column 1 represents the portfolio of stocks with the smallest average realized moment, and column 10 is for the portfolio of stocks with the highest realized moment. The characteristics include firm size, book-to-market ratio, realized volatility over the previous week, historical skewness using daily returns from the previous month, market beta from the market model regression, lagged return, illiquidity as in Amihud (2002), coskewness as in Harvey and Siddique (2000), idiosyncratic volatility as in Ang, Hodrick, Xing, and Zhang (2006), the number of analysts from IBES, credit rating, stock price, the number of intraday transactions, and the number of stocks per decile. On average there are 257 companies per decile each week.

Table 1, Panel A displays results for the ten decile portfolios based on realized volatility. Realized volatility increases from 18.8% for the first decile to 145.0% for the highest decile. Interestingly, realized skewness has a negative relation with realized volatility and realized kurtosis shows an increasing pattern through the volatility deciles. Furthermore, companies with high realized volatility tend to be small, followed by fewer analysts, less co-skewed with the market and have a lower stock price. A positive relation exists between realized volatility and historical skewness, market beta, lagged return, idiosyncratic volatility and maximum return. Finally, no pattern is observed between realized volatility and book-to-market, number of intraday transactions, and credit rating.

Panel B of Table 1 shows that realized skewness equals -1.04 for the first decile portfolio and 1.02 for the tenth decile. Firms with a high degree of asymmetry, either positive or negative, are small, highly illiquid, followed by fewer analysts and with lower number of intraday transactions.

Panel C of Table 1 reports on the decile portfolios based on realized kurtosis. The average kurtosis ranges from 3.9 to 16.6 across the deciles. Firm characteristics that are positively related to realized kurtosis include realized volatility, historical skewness, lagged return, idiosyncratic skewness, illiquidity and maximum return. Variables that have a negative relation with realized kurtosis include size, market beta, co-skewness, number of IBES analysts, stock price and number of intraday transactions.

In summary, Table 1 strongly suggest that firm-specific realized volatility, skewness, and kurtosis all contain unique information about the cross-sectional and temporal distribution of equity returns. We now attempt to exploit this moment-based information for predicting the cross-section of equity returns.

3 Realized Moments and the Cross-Section of Stock Returns

In this section, we first analyze the relationship between the current week's returns and the previous week's realized volatility, realized skewness, and realized kurtosis. Second, we use the Fama and

MacBeth (1973) methodology to conduct cross-sectional regressions and to determine the significance of each higher realized moment individually and simultaneously, as well as when controlling for firm-specific factors. Third, we investigate the interaction of returns, realized volatility and skewness.

3.1 Sorting Stock Returns on Realized Volatility

Every Tuesday, stocks are ranked into deciles according to their realized volatility. Then, using returns over the following week, we construct equal- and value-weighted portfolios. Table 2, Panel A reports the time series average of weekly returns for decile portfolios based on the level of realized volatility.

The equal-weighted returns show an increasing pattern, from 22 basis points for decile 1 to 27 basis points for decile 10. On the other hand, value-weighted returns decrease from 20 basis points to 8 basis points. Thus, the returns of the long-short portfolio, namely one that buys stocks in decile 10 and sells stocks in decile 1, are positive for equal-weighted portfolios and negative for value weighted ones. The negative relation between individual volatility and stock returns for value-weighted portfolios is consistent with Ang, Hodrick, Xing, and Zhang (2006). However, neither the equal-weighted nor the value-weighted long-short portfolios are statistically significant, and this is the case for raw returns as well as for alphas from the Carhart four factor model. The four factor model employs the three Fama and French (1993) factors (excess market-return, size and book-to-market factors) and the Carhart (1997) momentum factor.

We conclude that realized volatility and future stock returns are not robustly related when using our measure of realized volatility.

3.2 Sorting Stock Returns on Realized Skewness

Table 2, Panel B reports the time-series average of weekly returns for decile portfolios grouped by realized skewness.

The equal-weighted and value-weighted returns both show a monotonically decreasing pattern between realized skewness and the average stock returns over the subsequent week. The return for the portfolio of stocks with the lowest level of skewness is 55 basis points for equal-weighted portfolios and 40 basis points for value-weighted portfolios, while the returns for stocks with the highest level of realized skewness is 12 basis points for equal-weighted and 17 for value-weighted portfolios. The weekly return difference between portfolio 10 and 1 is -43 basis points for equal-weighted returns and -24 for value-weighted returns. Both differences are statistically significant at the one percent level. This result is consistent with recent theories stating that stocks with lower skewness command a risk premium. For prominent examples, see Barberis and Huang (2008), Brunnermeier, Gollier, and Parker (2007), and Mitton and Vorkink (2007).

We also assess the empirical relationship between realized skewness and stock returns by adjusting for standard measures of risk. Panel B of Table 2 presents, for each decile, alphas relative to

the Carhart four factor model. Note that alphas are large and statistically significant for equal- and value-weighted portfolios across deciles. In addition, the difference between the alphas of the tenth and first deciles is -46 and -23 basis points for equal- and value-weighted portfolios, respectively. Note also that the magnitude of the alphas is very similar to that of raw returns, which shows that standard measures of risk do not account for the return provided by the realized skewness exposure.

The magnitude and sign of the cross-sectional relationship between skewness and subsequent stock returns is consistent with the findings of other studies that use different measures of skewness. Boyer, Mitton, and Vorkink (2010) use a model that incorporates firm characteristics in order to measure the expected skewness over a given horizon. They report that a strategy that buys stocks with the highest one-month expected skewness and sells stocks with the smallest one-month expected skewness generates an average return of -67 basis points per month. Zhang (2006) measures expected skewness for a stock by allocating it into a peer group (e.g. industry) and uses recent returns from this group to compute its skewness measure. In this case the long-short strategy produces risk-adjusted returns of -36 basis points per month.

In conclusion, we find strong evidence that realized skewness predicts the cross section of stock returns. Realized skewness is an important determinant of the cross-sectional variation in subsequent one-week returns, and its effect is not captured by standard measures of risk.

3.3 Sorting Stock Returns on Realized Kurtosis

Panel C of Table 2 documents the average next-week stock returns for decile portfolios based on realized kurtosis. Equal-weighted and value-weighted portfolio returns both increase with the level of realized kurtosis. For equal weighted portfolios, decile 1 has an average weekly return of 23 basis points, compared to 38 basis points for decile 10. Thus, the long-short portfolio generates a return of 16 basis points with a t-statistic of 2.98. A similar result is found for the value-weighted portfolio, where the long-short realized kurtosis premium equals 14 basis points with a t-statistic of 2.12.

The results for the Carhart four-factor alpha are of the same magnitude as those of the raw returns. The equal-weighted alpha for the long-short portfolio is 13 basis points and the value-weighted alpha is 11 basis points. The latter alpha is only significant at the 5% level.

Comparing Panels A, B and C in Table 2, we conclude that, while the results for kurtosis are fairly strong, realized skewness appears to be the most reliable moment-based predictor of subsequent one-week equity returns in the cross section.

3.4 Fama-MacBeth Regressions

To further assess the relationship between future returns and realized volatility, realized skewness, and realized kurtosis, we carry out various cross-sectional regressions using the method proposed in Fama and MacBeth (1973). Each week t , we compute the realized moments for firm i and estimate

the following cross-sectional regression on the week $t + 1$ returns

$$r_{i,t+1} = \gamma_{0,t} + \gamma_{1,t}RVol_{i,t} + \gamma_{2,t}RSkew_{i,t} + \gamma_{3,t}RKurt_{i,t} + \phi_t^l Z_{i,t} + \varepsilon_{i,t+1}, \quad (8)$$

where $r_{i,t+1}$ is the weekly return (in bps) of the i th stock for week $t + 1$, and where $Z_{i,t}$ represent a vector of characteristics and controls for the i th firm observed at the end of week t . The characteristics and controls included are the week t return (in bps), firm size, book-to-market, market beta, historical skewness, idiosyncratic volatility, co-skewness, maximum monthly return (in bps), number of analysts, illiquidity and number of intraday transactions.

Table 3 reports the time-series average of the γ and ϕ coefficients for six cross-sectional regressions. The first column presents the results of the regression of the stock return on lagged realized volatility. The coefficient associated with realized volatility is 3.5 with a Newey-West t-statistic of 0.26. This confirms that there does not seem to be a significant relationship between realized volatility and stock returns. The second and third columns confirm the relation between the stock return and lagged realized skewness and realized kurtosis respectively. In column 2, the coefficient associated with realized skewness is -22.4 with a Newey-West t-statistic of -7.90 . Similarly, in column 3, the coefficient on realized kurtosis is 1.3 with a t-statistic of 3.49. In the fourth column, we report regression results using all higher moments simultaneously. The coefficients on lagged realized skewness and realized kurtosis remain statistically significant, and are again negative and positive respectively. The third and fourth realized moments appear to explain different aspects of stock returns.

In the fifth column, we include lagged returns in the regression, given the strong evidence of the return reversal effect in short run returns (Jegadeesh (1990), Gutierrez and Kelley (2008)). Even though the coefficient of realized skewness decreases to -3.9 , it remains significant and negative. The coefficient of realized kurtosis increases to 1.6 with a Newey-West t-statistic of 3.83. The coefficient on lagged return is negative and statistically significant, as expected.

In the last column, we add all control variables to ensure that realized skewness and realized kurtosis are not a manifestation of previously documented relationships between firm characteristics and stock returns. We find that the coefficients of realized skewness and realized kurtosis are still significant, with Newey-West t-statistics of -2.73 and 2.15, and preserve their signs with coefficients of -4.7 and 0.7, respectively. The negative sign on the coefficient related to size and the positive sign of the coefficient related to book-to-market confirm existing results in the literature. We include control variables related to the illiquidity and visibility of individual stocks. This includes the number of intraday transactions, the measure of illiquidity proposed in Amihud (2002), and the number of analysts following a stock (see Arbel and Strebel (1982)). We also control for the previously documented relationships between stock returns and firm characteristics, such as idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), the maximum daily return over the previous month (Bali, Cakici, and Whitelaw (2009)), and the stock's coskewness, as measured

by the variability of the stock’s return with respect to changes in the level of volatility following Harvey and Siddique (2000). Finally, we control for the market beta computed with a regression using daily returns on the market over the previous 12 months.

The results in the last column of Table 3 show that the economic and statistical significance of realized skewness and realized kurtosis for the cross-section of weekly returns is robust to the inclusion of various control variables. Variables such as realized volatility, idiosyncratic volatility, and coskewness do not play a significant role in the cross-section of returns at a weekly level, while variables such as lagged return, maximum daily return, and size are relevant.

3.5 Realized Skewness and Realized Volatility

We now further examine the interaction between the effects of realized skewness and realized volatility on returns. We construct portfolios using a double sort on realized skewness and realized volatility and then examine subsequent stock returns. First, we form five quintile portfolios with different levels of realized skewness. Within each of these portfolios, we form five portfolios that have different levels of realized volatility.³ Panel A of Table 4 reports the equal-weighted returns for the 25 portfolios as well as the difference between the high realized volatility quintiles and low realized volatility quintiles. We observe that in low skewness portfolios, higher realized volatility translates into higher returns. The portfolio that buys quintile 5 (stocks with high realized volatility and low realized skewness) and sells quintile 1 (stocks with low realized volatility and low realized skewness) has a weekly return of 47 basis points with a t-statistic of 3.31. Hence, in the case of low skewness stocks, investors are compensated with higher returns when holding high volatility stocks. However, for stocks with high skewness, we find that portfolios containing stocks with low volatility have higher subsequent returns than portfolios containing stocks with high volatility. In this case, the long-short portfolio return premium is -28 basis points with a t-statistic of -2.22 . Overall, the portfolio with the lowest return is the one with stocks that have high skewness and high volatility. Panel B of Table 4 demonstrates that the same conclusion obtains for value-weighted portfolios.

Panel A of Figure 2 shows the equal-weighted returns for the 25 portfolios double sorted on realized skewness and realized volatility. Realized skewness increases from -0.772 to $+0.762$ and realized volatility increases from 20% to about 120% for the 25 portfolios. Portfolios with low realized volatility of 20% have very similar returns for all five levels of realized skewness: between 20 and 30 basis points. However, as realized volatility increases, the return of low and high realized skewness portfolios strongly diverges. Portfolios with high realized volatility of 120% report the highest and the lowest return of all 25 portfolios. Stocks with the lowest realized skewness earn the highest average return of 77 basis points; and stocks with the highest realized skewness earn the lowest average return of -9 basis points. Hence, it is important to account for skewness when analyzing the return/volatility relationship. Highly volatile stocks may earn low returns, which

³Sorting on realized volatility first, and subsequently on realized skewness, does not change the results.

seems counterintuitive, but the reason for this is that their skewness is high.⁴

3.6 Realized Skewness and Idiosyncratic Volatility

Building on our findings regarding volatility and skewness, we now investigate whether realized skewness can explain the idiosyncratic volatility puzzle uncovered by Ang, Hodrick, Xing, and Zhang (2006). They find that stocks with high idiosyncratic volatility earn lower returns than stocks with low idiosyncratic volatility, contradicting the implications of mean-variance models. Table 5 replicates the idiosyncratic volatility puzzle in Ang, Hodrick, Xing, and Zhang (2006) for our sample. For value-weighted portfolios (Panel B of Table 5), we find a weekly premium of -0.24% with a t-statistic of -1.73 for the period 1993-2008. This result is comparable to that of Ang, Hodrick, Xing, and Zhang (2006) who find a monthly premium of -1.06% with a t-statistic of -3.10 for the period 1963-2000. Interestingly, Panel A of Table 5 indicates that the puzzle does not obtain for equal-weighted portfolios, where we do not observe significant differences across quintiles.

To study how realized skewness and idiosyncratic volatility interact in impacting stock returns, we double sort using both variables. We first sort stocks by realized skewness and form quintile portfolios. Quintile 1 has stocks with the lowest level of realized skewness and quintile 5 has stocks with the highest level of realized skewness. Then, within each quintile portfolio, we sort stocks by idiosyncratic volatility. Table 6 reports the results for equal-weighted and value-weighted portfolios. The equal-weighted portfolio results (Panel A) are very similar to those for realized volatility reported in Panel A of Table 4. In particular, we observe that the premium of the high idiosyncratic volatility portfolio (quintile 5) minus the low idiosyncratic volatility portfolio (quintile 1) decreases as the level of skewness increases. The premium for low realized skewness is 35 basis points and decreases to -43 basis points for high realized skewness. The highest return of 64 basis points is for the portfolio with high idiosyncratic volatility and low skewness and the lowest return of -20 basis points is for the portfolio with high idiosyncratic volatility and high skewness. Panel B of Figure 2 shows the returns of the 25 equal-weighted portfolios for different levels of idiosyncratic volatility. Just as with realized volatility in Panel A of Figure 2, high idiosyncratic volatility is compensated with high returns only if skewness is low. Investors are willing to accept low returns and high idiosyncratic volatility in exchange for high positive skewness.

Panel B of Table 6 indicates that value-weighted portfolios display a pattern similar to equal-weighted-portfolios, but with two main differences. First, the high-low premium is negative for all levels of skewness, even though it decreases as the skewness increases. Related to this, the portfolio of stocks with low skewness and high idiosyncratic volatility does not earn the highest returns among all portfolios. Apart from these two differences, we still observe that returns of stocks with high volatility and high skewness yield the lowest average returns. Investors trade high idiosyncratic volatility and low returns for high skewness, because they prefer skewness. Preference

⁴This finding is supported by Golec and Tamarkin (1998), who show that gamblers at horse races accept bets with low returns and high volatility only because they enjoy the high positive skewness offered by these bets.

for skewness seems to partly explain the idiosyncratic volatility puzzle.

4 Robustness Analysis

In this section, we further explore the relation between realized moments and stock returns. First, we investigate if the relation between current-week realized moments and next-week returns is present in different subsamples. Second, we check if our findings are robust to alternative measures of skewness and kurtosis. Third, we use monthly rather than weekly holding periods for returns.

4.1 Subsamples

Table 7, Panel A, reports value- and equal-weighted returns of portfolios sorted on realized skewness across different subsamples. Keim (1983) documents calendar-related anomalies for the month of January, in which stocks have higher returns than in the rest of the year. Panel A of Table 7 presents the average weekly returns for the month of January and for the rest of the year for both equal- and value-weighted portfolios. As expected, returns for the month of January are consistently higher than returns for the rest of the year.

The difference between the returns of portfolios with high-skewness stocks and portfolios with low-skewness stocks is still negative and significant for both January and non-January groups. This is true for equal-weighted as well as value-weighted portfolios.

We previously documented that stocks with high and low levels of skewness tend to be small. Hence, we examine if the effect of skewness is exclusively driven by small NASDAQ stocks. By only including stocks from the New York Stock Exchange (NYSE), row 3 of Table 7 shows that the effect of realized skewness is present among NYSE stocks. Hence, small NASDAQ stocks are not driving our results.

In Table 7, Panel B, we analyze the equal-weighted and value-weighted returns of portfolios sorted on realized kurtosis for different subsamples. The long-short portfolio returns are positive for all subsamples. As expected, in January the long-short portfolio earns higher returns compared to the rest of the year. We also confirm that the effect of realized kurtosis is not driven by small NASDAQ stocks, and find that the returns remain positive and significant for equal-weighted portfolios. For value-weighted portfolios the realized kurtosis premium is positive, but often not statistically significant.

4.2 Alternative Measures of Skewness and Kurtosis

We investigate the robustness of realized skewness by using two additional measures of skewness. The first estimator uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005), which provides measures robust to microstructure noise. This method consists of constructing subsamples that are spaced every minute. This means that instead of working with one realized measure based on a five-minute return grid, we end up working with five estimators

of realized skewness using subsamples of 5-minute returns for the period 9:30 EST to 16:00 EST. Subsamples start every minute (at 9:00, 9:01, 9:02, 9:03 and 9:04), but we use 5-minute returns. Subsequently, the realized skewness estimator is computed as the average of the five (overlapping) estimators obtained from the subsamples.

The second alternative to intraday skewness depends solely on quartiles from the intraday return distribution.⁵ As proposed in Bowley (1920), a measure of skewness that is based on quartiles can be defined as

$$SK2_t = (Q_3 + Q_1 - 2Q_2)/(Q_3 - Q_2),$$

where Q_i is the i^{th} quartile of the five-minute return distribution F , that is $Q_1 = F^{-1}(0.25)$, $Q_2 = F^{-1}(0.5)$, and $Q_3 = F^{-1}(0.75)$.

We find that the relationship between skewness and next-week returns is negative and statistically significant for both measures of intraday skewness. We do not report these results to save space, but they are available on request.

To gauge the robustness of realized kurtosis, we construct two alternative measures of kurtosis similar to the ones constructed for intraday skewness. The first measure uses the subsampling methodology proposed by Zhang, Mykland, and Ait-Sahalia (2005). Five measures of realized kurtosis are computed and we obtain the estimator by averaging those five estimators. The second measure of intraday kurtosis uses the octiles of the intraday return distribution as proposed by Moors (1988). In particular, the centered kurtosis measure is defined as

$$KR2_t = ((E_7 - E_5) + (E_3 - E_1))/(E_6 - E_2) - 1.23,$$

where E_i is the i^{th} octile of the five-minute return distribution F , that is $E_i = F^{-1}(i/8)$. Confirming our other results, we find that the long-short equal-weighted portfolio generates a significantly positive average return. The average returns on the long-short value-weighted portfolio are positive but not statistically significant.

4.3 Realized Moments and other Firm Characteristics

This section further analyzes the interaction between realized moments and other firm characteristics. As pointed out by Fama and French (2008), to ensure the validity of an anomaly, small (microcaps), medium and large firms ought to all exhibit the anomaly. We use a double sorting methodology to analyze the realized skewness premium and the realized kurtosis premium for five different size portfolios. We first sort stocks into quintiles by size and then, within each quintile, we sort stocks again by realized skewness (or realized kurtosis) into quintiles. Then we compute the difference of return between the highest and lowest realized skewness (or kurtosis) quintiles. This difference represents a realized skewness (or kurtosis) premium conditional on size. This double

⁵Neuberger (2011) has recently suggested another interesting alternative measure of realized skewness which we have not pursued here.

sorting methodology analyzes the long-short portfolio, quintile 5 minus quintile 1, for each size quintile. With this methodology, we can assess if the realized moment premium is economically significant for all size levels. We also provide a similar analysis for the following firm characteristics: lagged return, market beta, BE/ME, realized volatility, historical skewness, illiquidity, number of intraday transactions, maximum return over the previous month, number of I/B/E/S analysts, idiosyncratic volatility and coskewness. We perform all analyses for realized skewness and realized kurtosis. Realized volatility is not included, because its relation with future stock returns is not statistically significant.

Panel A of Table 8 reports the results for skewness. In row 1, we double sort stock returns on realized skewness across different levels of firm size and, for each firm size quintile, we compute the return of the portfolio that buys the highest realized skewness stocks (within a given size quintile) and sells the lowest realized skewness stocks (within that same size quintile). The realized skewness premium of -77 basis points for quintile 1 can be earned by buying small stocks (microcaps) with high realized skewness and selling small stocks with low realized skewness. For big firms in quintile 5, the corresponding premium is -17 basis points. All five size groups exhibit the realized skewness anomaly. The stronger negative effect of skewness for small firms is consistent with Chan, Chen, and Hsieh (1985), who show that there are risk differences between small and large firms. This finding also explains why the effect of realized skewness is weaker for value-weighted portfolios when compared to equal-weighted portfolios.

Table 8, Panel A indicates that the realized skewness premia are negative and statistically significant for all firm characteristics. The relationship between realized skewness and subsequent returns is robust to all firm characteristics and is not a proxy for any of them. The premium is larger (more negative) for small companies with low BE/ME, low lagged returns, high realized variance, high idiosyncratic volatility, high illiquidity, high maximum return over the previous month, low number of analysts that follow the stock and high market beta. In addition, we find that the premium does not vary across different levels of historical skewness, coskewness, or the number of intraday transactions.

In the previous section, Fama-MacBeth regressions showed that lagged returns have explanatory power for next-week returns. The third row in Panel A of Table 8 shows that realized skewness is negatively associated with next week returns for any level of lagged return, and that the effect is stronger for past losers (stocks with the smallest lagged return). Furthermore, as the market beta increases, the long-short skewness premium becomes more negative. This pattern is also observed for idiosyncratic volatility or realized volatility, confirming the results in Tables 4 and 6.

Hou and Moskowitz (2005) find that small, highly illiquid stocks with low visibility respond slowly to new information. As information diffusion decreases, investors' disagreement on how to interpret new signals increases. According to Xu (2007), the more investors disagree on the information, the higher the skewness of the stock is. Therefore, such firms will be more prone to the effects of realized skewness, which is confirmed by the skewness return patterns observed as a

function of size, illiquidity and the number of analysts.

Table 8, Panel B reports the results from a double sort on realized kurtosis and firm characteristics. The results are not as strong as those for realized skewness since not all realized kurtosis premia are statistically significant. The realized kurtosis premium is not statistically significant for different illiquidity and size quintiles, and for quintile 4 and 5 the premium is negative. The kurtosis premium is significant for one quintile only when double sorting by the number of analysts.

The results versus certain other firm characteristics are stronger and confirm the predictive power of realized kurtosis. Conditioning on the lagged return, the realized kurtosis premium is always positive and statistically significant. Some characteristics provide mixed evidence. The realized kurtosis premium is positive and significant in at least 3 quintiles for the following firm characteristics: book-to-market, realized volatility, intraday transactions, maximum previous month return, market beta, idiosyncratic volatility and co-skewness. It is clear from Table 8 that realized kurtosis is a weaker predictor of future stock returns than realized skewness.

4.4 Monthly Returns

Thus far our empirics have been based on weekly returns and weekly realized moments. In this section we keep the weekly frequency when computing realized moments but we increase the return holding period from one week to one month.

Table 9 contains the results for overlapping monthly returns. As for the weekly returns in Table 2, we report the equal-weighted and value-weighted returns of decile portfolios formed from realized moments, and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment). Each panel reports on both equal-weighted and value-weighted portfolios and include t-statistics computed from robust standard errors. Alpha is again computed using the Carhart four factor model.

Panel A reports the results for realized volatility. Perhaps not surprisingly, we see that the insignificant relationship between volatility and weekly returns found in Panel A of Table 2 is evident for monthly returns in Table 9 as well.

Panel B in Table 9 reports the relationship between monthly returns and realized skewness. The strong negative relationship between realized skewness and weekly returns found in Table 2 is very clear when using monthly returns as well. This relationship is significant for raw returns as well as alphas, and for equal-weighted and value-weighted portfolios.

Finally, Panel C in Table 9 reports the cross-sectional relationship between weekly realized kurtosis and monthly returns. The positive relationship found for weekly returns in Table 2 is also clearly evident for monthly returns in Table 9. The relationship appears to be stronger for equal-weighted returns than for value-weighted returns when using a one-month holding period. This was also the case for weekly returns in Table 2.

5 Properties of Realized Moments

The limiting properties of realized variance have been studied in detail in the econometrics literature, however, much less is known about realized skewness and kurtosis. In this section we therefore investigate the realized moments when assuming that the underlying continuous time price process follows a jump-diffusion with stochastic volatility. First, we derive closed-form solutions for the limits of the realized moments. Second, we allow for market microstructure noise and discontinuities in quoted prices, and provide Monte Carlo evidence on the realized higher moments. Third, we assess the significance of our cross-sectional return results when using jump-robust measures of realized volatility.

5.1 The Equity Price Process

To illustrate the properties of the realized moments defined in (2), (3), and (4), we assume that the log-price p_t of a security evolves according to the stochastic differential equation

$$dp_t = (\mu - V_t - \mu_J \lambda) dt + \sqrt{V_t} dW_t^{(1)} + J dN_t, \quad (9)$$

$$dV_t = \kappa(\theta - V_t) dt + \sigma \sqrt{V_t} dW_t^{(2)}, \quad (10)$$

where μ is the drift parameter, κ is the mean reversion speed to the long-term volatility mean θ , and σ is the diffusion coefficient of the volatility process V_t . $W_t^{(1)}$ and $W_t^{(2)}$ denote two standard Brownian motions with correlation ρ , and N_t is an independent Poisson process with arrival rate λ . The jump size J is distributed $N(\mu_J, \sigma_J^2)$.

5.2 The Limits of the Realized Moments

Suppose that in the time interval $[0, T]$, for example a day, $N + 1$ observations are available on p , and the distance between these observations is $\tau = T/N$, that is, the observation times are $t_i = i\tau$, for $i = 0, \dots, N$. Then we define the realized moments by

$$RM(j) = \sum_{i=0}^N (p_{t_{i+1}} - p_{t_i})^j \quad (11)$$

for $j = 1, 2, 3, 4$. The limits of these realized moments are given in the following proposition:

Proposition 1 *The realized moments defined in (11) for $j = 1, 2, 3$, and 4 converge in mean*

square to the integrated moments

$$IM(1) = \left(\mu - \frac{\theta}{2}\right) T + (\theta - V_0) \frac{(1 - e^{-\kappa T})}{\kappa}, \quad (12)$$

$$IM(2) = (\theta + \lambda(\mu_J^2 + \sigma_J^2)) T - (\theta - V_0) \frac{(1 - e^{-\kappa T})}{\kappa}, \quad (13)$$

$$IM(3) = \lambda(\mu_J^3 + 3\mu_J\sigma_J^2) T, \quad (14)$$

$$IM(4) = \lambda(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4) T. \quad (15)$$

Proof. See Appendix B. ■

This result is quite revealing. Note that while the limit for $j = 2$ contains both jump and diffusion parameters, the limits for $j = 3$ and $j = 4$ depend exclusively on jump parameters. This means that realized skewness and realized kurtosis, as defined in equations (3) and (4), complement the information captured by realized volatility. $IM(3)$, which is the limit of the numerator in $RDSkew$, is the only realized moment that accounts for the jump direction, since its sign depends on that of the average jump size, μ_J . $IM(4)$, which is the limit of the numerator in $RDKurt$, captures the magnitude of the jump.

5.3 Allowing for Market Microstructure Noise

So far, we have studied the realized moments in the idealized case where observed prices correspond to their theoretical counterparts. In practice, microstructure noise is present in high frequency prices. To simulate market microstructure noise, we define the observed log price p_t^* as

$$p_t^* = p_t + u_t, \quad (16)$$

where u_t is i.i.d. Gaussian noise with mean zero and variance σ_u^2 . Hence, the observed log price p_t^* is a noisy observation of the non-observable true price p_t .

Several studies use Monte Carlo simulations to investigate the properties of realized variance estimators when allowing for market microstructure noise, see for instance Andersen, Bollerslev, and Meddahi (2011), Gonçalves and Meddahi (2009), and Ait-Sahalia and Yu (2009). Following their work, we conduct the following Monte Carlo study. We simulate 100,000 paths of the log price process p_t using the Euler scheme at a time interval $\tau = 1$ second. The parameters for the continuous part of the process are set to $\mu = 0.05$, $\kappa = 5$, $\theta = 0.04$, $\sigma = 0.5$, $V_0 = 0.09$, and $\rho = -0.5$. The parameters for the jump component are set at $\lambda = 100$, $\mu_J = 0.01$ and $\sigma_J = 0.05$. The microstructure noise, u_t , is modeled with a normal distribution of mean zero and standard deviation of 0.05%. These parameter values are similar to those employed by Ait-Sahalia and Yu (2009).

To assess the impact of the microstructure noise at different sampling frequencies, we use signature plots as proposed in Andersen, Bollerslev, Diebold, and Labys (2000). The signature

plots provide the sample mean of a daily realized moment based on returns sampled at different intraday frequencies. We take as an observation period $T = 1$ day, that is $T = 1/252$, and we assume a day has 6.5 trading hours.

Panel A of Figure 3 shows the signature plots of $RM(j)$ (as defined in (11)) for $j = 2, 3$, and 4. The signature plot for the second moment $RM(2)$ depicts the well-known effect that microstructure noise has on realized volatility: as the sampling frequency increases (moving from right to left in the figure), the variance of the noise dominates that of the price process; but for lower frequencies, this effect attenuates. In contrast, the microstructure noise does not affect the signature plots of $RM(3)$ and $RM(4)$ in the same way. There is a small and insignificant bias in $RM(3)$ and $RM(4)$ relative to $IM(3)$ and $IM(4)$ but the bias does not increase with the intraday frequency.

5.4 Allowing for Quoted Price Discontinuity

Chakravarty, Wood, and Van Ness (2004) document that bid-ask spreads declined significantly following the decimalization of NYSE-listed companies in 2001. This indicates that pre-decimalization prices exhibit an additional bid-ask spread generated by fractional minimum increments. To gauge the effect of this discontinuity on the realized moment measures, we conduct a Monte Carlo study similar to the one above, with the exception that observed prices are now measured in sixteenths of a dollar. To isolate the effect of fractional minimum increments, we assume here that observed prices are not affected by microstructure noise.

Panel B of Figure 3 shows the signature plots for the realized moments. The plots reveal that realized volatility is the only moment affected by fractional changes in observed prices. As the frequency increases, the discontinuity of observed prices creates noise that is picked up by the volatility measure. However, the noise does not affect the third and fourth moments.

Figure 3 includes 99% confidence bands around the Monte Carlo estimates. For the second moments in the first row of panels the confidence intervals are very tight around the Monte Carlo estimate making them barely noticeable in the plot. For the third and fourth moments (the second and third row of panels), the 99% confidence intervals contain the Monte Carlo estimate as well as the theoretical limit which shows that the Monte Carlo average is not significantly different from the limit value.

In summary, we find that the third and fourth moments used in this paper have well-defined limits. Moreover, when estimated with an adequate sampling frequency, realized moments are not contaminated by market microstructure noise or discontinuous quotes.

5.5 Alternative Realized Volatility Estimators

For the affine jump-diffusion model that we assumed in (9)-(10) the limit of the sum of intraday squared returns in (13) can be written as the sum of jump variation and integrated variance

$$IM(2) = JV + IV$$

where

$$\begin{aligned} JV &\equiv \lambda (\mu_J^2 + \sigma_J^2) T \\ IV &\equiv \theta T + (V_0 - \theta) \frac{(1 - e^{-\kappa T})}{\kappa} \end{aligned}$$

The $RDVar_t$ estimator in (2) that we have used in the empirics so far will capture both jumps and diffusive volatility in the limit. This does not invalidate it as an ex-post measure for the total daily quadratic variation, but it does suggest the use of more refined procedures for separately estimating IV and JV .

Several volatility estimators that are robust to jumps have been developed in the literature. They are designed to only capture IV and not JV in the limit. The so-called bipower variation estimator of Barndorff-Nielsen and Shephard (2004) is defined by

$$BPV_t = \frac{\pi}{2} \frac{N}{N-1} \sum_{i=1}^{N-1} |r_{t,i+1}| |r_{t,i}|$$

which converges in the limit to integrated variance, IV_t , when N approaches infinity, even in the presence of jumps.

Motivated by the presence of large jumps that may bias upward the bipower variation measure in realistic settings when N is finite, Andersen, Dobrev and Schaumburg (2010) have recently developed two alternative jump-robust estimators, defined by

$$\begin{aligned} MinRV_t &= \frac{\pi}{\pi - 2} \left(\frac{N}{N-1} \right) \sum_{i=1}^{N-1} \min \{ |r_{t,i}|, |r_{t,i+1}| \}^2, \text{ and} \\ MedRV_t &= \frac{\pi}{6 - 4\sqrt{3} + \pi} \left(\frac{N}{N-2} \right) \sum_{i=2}^{N-1} \text{median} \{ |r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}| \}^2 \end{aligned}$$

These estimators will also both converge to IV_t when N goes to infinity and in the presence of large jumps they typically have better finite sample properties than BPV_t .

To assess the robustness of our cross-sectional return results, we report in Table 10 the equal-weighted and value-weighted weekly returns of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) when using the three alternative realized volatility estimators, BPV_t , $MinRV_t$ and $MedRV_t$. To facilitate comparisons, the first column of Table 10 uses the standard $RVol_t$ from (5) and thus reproduces the last column of Table 2. Each panel in Table 10 reports the equal-weighted and the value-weighted long-short returns. Alpha is again computed using the Carhart four factor model.

Panel A in Table 10 reports the long-short results for the three alternative realized volatility estimators. We find that the insignificant relationship between return and volatility remains when alternative estimators of realized volatility are used.

Panel B in Table 10 shows the long-short results for realized skewness when scaling by the three alternative realized volatility estimators. We see that the strong negative relationship between realized skewness and return found in Table 2 is robust to changing the denominator in

$$RDSkew_t = \frac{\sqrt{N} \sum_{i=1}^N r_{t,i}^3}{RDVar_t^{3/2}}. \quad (17)$$

to be any of the three jump-robust volatility estimators defined in this section.

Panel C in Table 10 presents the realized kurtosis results when scaling by the three alternative realized volatility estimators. Panel C shows that the positive relationship between return and realized kurtosis is also robust to changing the denominator in $RDKurt_t$ to be any of the jump-robust volatility estimators.

We conclude that the strong negative relationship between skewness and return in the cross-section, as well as the positive relationship between return and kurtosis, is not an artifact of the particular measure of realized volatility that we used above. The results hold up when we use estimators of realized volatility that are jump-robust.

6 Conclusions

We document the relationship between realized higher moments of individual stocks and future stock returns. We first introduce model-free estimates of higher moments based on the methodology used by Andersen, Bollerslev, Diebold, and Ebens (2001) to estimate realized volatility. We use five-minute returns to obtain a daily measure of realized volatility, realized skewness and realized kurtosis, and subsequently aggregate this measure up to the weekly frequency. We find that realized skewness and realized kurtosis predict next week’s stock returns in the cross-section, but realized volatility does not.

Realized skewness is negatively related to future stock returns. Portfolios with low skewness outperform portfolios with high skewness by 43 basis points per week. Realized kurtosis is positively related to future stock returns. A portfolio that buys stocks with high realized kurtosis and sells stocks with low realized kurtosis generates an average weekly return of 16 basis points.

Fama-MacBeth regressions and double sorting confirm that realized skewness is not a proxy for firm characteristics such as lagged return, size, book-to-market, realized volatility, market beta, historical skewness, idiosyncratic volatility, coskewness, maximum return over the previous month, analysts coverage, illiquidity or number of intraday transactions. The forecasting ability of realized kurtosis is also found to be robust to these firm characteristics when employing Fama-MacBeth regressions. However, when double sorting using firm characteristics, the predictive power of realized kurtosis weakens.

We analyze the relationship between realized skewness and realized volatility in more detail. When double sorting on realized skewness and volatility, we find that stocks with negative skewness

are compensated with high future returns. However, as skewness increases and becomes positive, the positive relation between volatility and returns turns into a negative relation. We conclude that investors may accept low returns and high volatility because they are attracted to high positive skewness.

We perform a similar analysis for realized skewness and idiosyncratic volatility. We find that portfolios with high idiosyncratic volatility compensate investors with higher returns only for low levels of skewness. For high levels of skewness, high idiosyncratic volatility leads to lower returns. This finding may help explain the idiosyncratic volatility puzzle in Ang, Hodrick, Xing, and Zhang (2006), who document that stocks with high idiosyncratic volatility earn low returns.

References

- Ait-Sahalia, Y., and J. Yu, 2009, High Frequency Market Microstructure Noise Estimates and Liquidity Measures, *Annals of Applied Statistics* 3, 422-457.
- Amihud, Y., 2002, Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, *Journal of Financial Markets* 5, 31-56.
- Andersen, T.G., and T. Bollerslev, 1998, Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts, *International Economic Review* 39, 885-905.
- Andersen, T.G., T. Bollerslev, F. Diebold, and H. Ebens, 2001, The Distribution of Realized Stock Return Volatility, *Journal of Financial Economics* 61, 43-76.
- Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys, 2000, Great Realizations, *Risk*, 105-108.
- Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys, 2001, The Distribution of Realized Exchange Rate Volatility, *Journal of the American Statistical Association* 96, 42-55.
- Andersen, T., Bollerslev, T., Diebold, F.X. and P. Labys, 2003, Modeling and Forecasting Realized Volatility, *Econometrica*, 71, 529-626.
- Andersen, T.G., T. Bollerslev, and N. Meddahi, 2011, Realized Volatility Forecasting and Market Microstructure Noise, *Journal of Econometrics* 160, 220-234.
- Andersen, T.G., Dobrev, D. and E. Schaumburg, 2010, Jump-Robust Volatility Estimation Using Nearest Neighbor Truncation, *Federal Reserve Bank of New York Staff Report No. 465*.
- Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang, 2006, The Cross-Section of Volatility and Expected Returns, *Journal of Finance* 61, 259-299.
- Arditti, F., 1967, Risk and the Required Return on Equity, *Journal of Finance* 22, 19-36.
- Arbel, A., and P. Strebler, 1982, The Neglected and Small Firm Effects, *Financial Review* 17, 201-218.
- Bakshi, G., N. Kapadia, and D. Madan, 2003, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, *Review of Financial Studies* 16, 101-143.
- Bali, T., N. Cakici, and R. Whitelaw, 2009, Maxing Out: Stocks as Lotteries and the Cross-Section of Expected Returns, *Journal of Financial Economics* 99, 427-446.
- Barndorff-Nielsen, O.E., and N. Shephard, 2002, Econometric Analysis of Realised Volatility and its Use in Estimating Stochastic Volatility Models, *Journal of the Royal Statistical Society* 64, 253-280.

- Barndorff-Nielsen, O.E., and N. Shephard, 2004, Power and Bipower Variation with Stochastic Volatility and Jumps, *Journal of Financial Econometrics* 2, 1-37.
- Barberis, N., and M. Huang, 2008, Stocks as Lotteries: The Implications of Probability Weighting for Security Prices, *American Economic Review* 98, 2066–2100.
- Bowley, A.L., 1920, *Elements of Statistics*, Scribner's, New York.
- Boyer, B., T. Mitton, and K. Vorkink, 2010, Expected Idiosyncratic Skewness, *Review of Financial Studies* 23, 169-202.
- Brunnermeier, M., C. Gollier, and J. Parker, 2007, Optimal Beliefs, Asset Prices and the Reference for Skewed Returns, *American Economic Review* 97, 159-165.
- Campbell, R. H., and A. Siddique, 2000, Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263–1295.
- Carhart, M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57-82.
- Chan, K.C., N. Chen, and D.A. Hsieh, 1985, An Exploratory Investigation of the Firm Size Effect, *Journal of Financial Economics* 14, 451-471.
- Chakravarty, S., R. A. Wood, and R. A. Van Ness, 2004, Decimals And Liquidity: A Study Of The NYSE, *Journal of Financial Research* 27, 75–94.
- Conrad, J., R.F. Dittmar, and E. Ghysels, 2008, Ex Ante Skewness and Expected Stock Returns, University of North Carolina, Working Paper.
- Dittmar, R.F., 2002, Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity, *Journal of Finance* 57, 369-403.
- Duffie, D., J. Pan, and K. Singleton, 2000, Transform Analysis and Asset Pricing for Affine Jump-Diffusions, *Econometrica* 68, 1343–1376.
- Fama, E., and K. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56.
- Fama, E., and K. French, 2008, Dissecting Anomalies, *Journal of Finance* 63, 1653-1678.
- Fama, E., and M. J. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607-636.
- Golec, J., and M. Tamarkin, 1998, Bettors Love Skewness, Not Risk, at the Horse Track, *Journal of Political Economy* 106, 205-225.
- Gonçalves, S., and N. Meddahi, 2009, Bootstrapping Realized Volatility, *Econometrica* 77, 283-306.

- Gutierrez Jr, R., and E. Kelley, 2008, The Long-Lasting Momentum in Weekly Returns, *Journal of Finance* 63, 415-447.
- Harvey, C., and A. Siddique, 2000, Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263-1295.
- Hou, K., and T.J. Moskowitz, 2005, Market Frictions, Price Delay, and the Cross-Section of Expected Returns, *Review of Financial Studies* 18, 981-1020.
- Hsieh, D., 1991, Chaos and Nonlinear Dynamics: Application to Financial Markets, *Journal of Finance*, 46, 1839-1877.
- Jegadeesh, N., 1990, Evidence of Predictable Behavior of Security Returns, *Journal of Finance* 45, 881-898.
- Keim, D.B., 1983, Size-Related Anomalies and Stock Return Seasonality, *Journal of Financial Economics* 12, 13-32.
- Kelly, B., 2011, Tail Risk and Asset Prices, Chicago Booth Research Paper No. 11-17.
- Kraus, A., and R. Litzenberger, 1976, Skewness Preference and the Valuation of Risk Assets, *Journal of Finance* 31, 1085-1100.
- Lehmann, B.N., 1990, Fads, Martingales, and Market Efficiency, *Quarterly Journal of Economics* 105, 1-28.
- Mitton, T., and K. Vorkink, 2007, Equilibrium Underdiversification and the Preference for Skewness, *Review of Financial Studies* 20, 1255-1288.
- Moors, J.J.A., 1988, A Quantile Alternative for Kurtosis, *The Statistician* 37, 25-32.
- Neuberger, A., 2011, Realized Skewness, Working Paper, Warwick Business School.
- Scott, R., and P. Horvath, 1980, In the Direction of Preference for Higher Order Moments, *Journal of Finance* 35, 915-919.
- Xing, Y., X. Zhang, and R. Zhao, 2010, What Does Individual Option Volatility Smirks Tell Us about Future Equity Returns?, *Journal of Financial and Quantitative Analysis* 45, 641-662.
- Xu, J., 2007, Price Convexity and Skewness, *Journal of Finance* 62, 2521-2552.
- Zhang, L., P. Mykland, and Y. Ait-Sahalia, 2005, A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-Frequency Data, *Journal of the American Statistical Association* 100, 1394-1411.
- Zhang, Y., 2006, Individual Skewness and the Cross-Section of Average Stock Returns, Working Paper.

Appendix A: Data

- Following Fama and French (1993), size (in billions of dollars) is computed each June as the stock price times the number of outstanding shares. The market equity value is held constant for a year.
- Following Fama and French (1993), book-to-market is computed as the ratio of book common equity over market capitalization (size). Book common equity is defined using COMPUSTAT's book value of stockholders' equity plus balance-sheet deferred taxes and investment tax credit minus the book value of preferred stock. The ratio is then computed as the book common equity at the end of the fiscal year over size at the end of December.
- Historical skewness for stock i on day t is defined as

$$HSkew_{i,t} = \frac{1}{N} \sum_{s=0}^N \left(\frac{r_{i,t-s} - \mu_i}{\sigma_i} \right)^3, \quad (18)$$

where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, μ_i is the mean over the last month for stock i and σ_i is the standard deviation of stock i for that month. We use 20 trading days to estimate historical skewness.

- Market beta is computed at the end of each month using a regression of daily returns over the past 12 months.
- Following Harvey and Siddique (2000), coskewness is defined as

$$CoSkew_{i,t} = \frac{E[\varepsilon_{i,t} \varepsilon_{m,t}^2]}{\sqrt{E[\varepsilon_{i,t}^2] E[\varepsilon_{m,t}^2]}}, \quad (19)$$

where $\varepsilon_{i,t}$ is obtained from $\varepsilon_{i,t} = r_{i,t} - \alpha_i - \beta_i r_{m,t}$, where $r_{i,t}$ is the monthly return of stock i on month t , $r_{m,t}$ is the market monthly return on month t . This regression is estimated at the end of each month using monthly returns for the past 24 months.⁶

- Following Ang, Hodrick, Xing, and Zhang (2006), idiosyncratic volatility is defined as

$$idvol_{i,t} = \sqrt{\text{var}(\varepsilon_{i,t})}, \quad (20)$$

where $\varepsilon_{i,t}$ is the error term of the three-factor Fama and French (1993) regression. The regression is estimated with daily returns over the previous 20 trading days.

- Maximum return is defined as the maximum daily return over the previous month.

⁶Harvey and Siddique (2000) use data for the past 60 months to estimate coskewness. This approach considerably reduces our sample of firms. We therefore estimate coskewness using only 24 months, which is also suggested in Harvey and Siddique (2000).

- Following Amihud (2002), stock illiquidity on day t is measured as the average of the ratio of the absolute value of the return over the dollar value of the trading volume over the previous year

$$illiquidity_{i,t} = \frac{1}{N} \sum_{s=0}^N \left(\frac{|r_{i,t-s}|}{|volume_{i,t-s} * price_{i,t-s}|} \right), \quad (21)$$

where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, $volume_{i,t-s}$ is the daily volume of stock i on day $t-s$ and $price_{i,t-s}$ is the price of stock i on day $t-s$. We use 252 trading days to estimate illiquidity.

- The credit rating is retrieved from COMPUSTAT and is then assigned a numerical value as follows: AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21 and D=22. When no rating is available, the default credit rating value is 8.

Appendix B: Limits of Realized Moments

The process for the log-price p_t in equation (9) belongs to the class of models commonly known as affine jump diffusion models. The affine structure of this process yields closed-form solutions for the moment generating function, MGF. In this appendix, we find an explicit representation of the MGF of $p_{t_i} - p_{t_{i-1}}$, which is then used to prove the limits of the realized measures.

Corollary 1 *The MGF of p_t is given by*

$$\begin{aligned} \psi_t(u) &= E[\exp(\tilde{u} \cdot [p_T, V_T]^\top) | \mathcal{F}_t] \\ &= e^{\alpha(u, T-t) + u p_t + \beta_2(u, T-t) V_0}, \end{aligned}$$

where

$$\begin{aligned} \alpha(u, t) &= (\mu - \lambda \mu_J) u t + \frac{\kappa \theta}{\sigma^2} \left((\gamma + b) t + 2 \log \left(1 - \frac{\gamma + b}{2\gamma} (1 - e^{-\gamma t}) \right) \right) + \lambda t (\phi(u) - 1), \\ \beta_2(u, t) &= -\frac{a(1 - e^{-\gamma t})}{2\gamma - (\gamma + b)(1 - e^{-\gamma t})}, \end{aligned}$$

where $a = u - u^2$, $b = \sigma \rho u - \kappa$, $\gamma = \sqrt{b^2 + a\sigma^2}$, and $\tilde{u} = [u, 0]^\top$. \mathcal{F}_t denotes the information set generated by the process $[p_t, V_t]^\top$.

Proof. Using the transform analysis in Duffie, Pan, and Singleton (2000), we have that

$$E[\exp(\tilde{u} \cdot [p_T, V_T]^\top) | \mathcal{F}_t] = \exp(\mathcal{A}(\tilde{u}, t) + \mathcal{B}(\tilde{u}, t) \cdot [p_t, V_t]^\top),$$

where $\mathcal{B}(t) = [\beta_1(t), \beta_2(t)]^\top$ and $\tilde{u} = [u, 0]^\top$. Solving the system of ODEs

$$\begin{aligned}\dot{\mathcal{B}}(t) &= \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & -\kappa \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix}^\top \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & -\kappa \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix}, \\ \dot{\mathcal{A}}(t) &= \begin{bmatrix} \mu - \lambda\mu_J \\ \kappa\theta \end{bmatrix} \cdot \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix} + \lambda(\phi(u) - 1),\end{aligned}$$

with $\mathcal{A}(T) = u$ and $\mathcal{B}(T) = [u, 0]^\top$ concludes the proof. ■

Armed with the MGF of p_t , we can now derive the MGF for $p_{t_{i+1}} - p_{t_i}$.

Corollary 2 *The MGF of $p_{t_{i+1}} - p_{t_i}$ is given by*

$$\begin{aligned}\varphi_t(u, \tau) &= E[\exp(u(p_{t+\tau} - p_t)) | \mathcal{F}_0] \\ &= \exp(\alpha(u, \tau)) \exp\left(\frac{\beta(u, \tau) e^{-\kappa t}}{1 - \beta(u, \tau) \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa t})}\right) \left(1 - \beta(u, \tau) \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa t})\right)^{-\frac{2\kappa\theta}{\sigma^2}}\end{aligned}$$

Proof. From the definition of the MGF and using the law of iterated expectations, we have:

$$\begin{aligned}E[\exp(u(p_{t+\tau} - p_t)) | \mathcal{F}_0] &= E[E[\exp(u(p_{t+\tau} - p_t)) | \mathcal{F}_t] | \mathcal{F}_0] \\ &= E[\exp(\alpha(u, \tau) + \beta(u, \tau) V_t) | \mathcal{F}_0] \\ &= \exp(\alpha(u, \tau)) E[\exp(\beta(u, \tau) V_t) | \mathcal{F}_0] \\ &= \exp(\alpha(u, \tau)) \exp(-A(-\beta(u, \tau), t) - B(-\beta(u, \tau), t) V_0),\end{aligned}$$

where

$$A(\varsigma, t) = \frac{\varsigma e^{-\kappa t}}{1 + \varsigma \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa t})}, \quad B(\varsigma, t) = \frac{2\kappa\theta}{\sigma^2} \log\left(1 + \varsigma \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa t})\right),$$

are the coefficients of the affine representation of the MGF for V_t . ■

To prove Proposition 1, we need to find the expected value of the limits of the realized moments defined in (11). The following proposition establishes these limits.

Corollary 3 *The limit*

$$\lim_{N \rightarrow \infty} E \left[\sum_{i=1}^N (p_{t_i} - p_{t_{i-1}})^j \right] \quad (22)$$

converges to (12), (13), (14) and (15) for $j = 1, 2, 3$ and 4, respectively.

Proof. We start by rewriting equation (22) as

$$\begin{aligned}\lim_{N \rightarrow \infty} E \left[\sum_{i=1}^N (p_{t_i} - p_{t_{i-1}})^j \right] &= \lim_{N \rightarrow \infty} \sum_{i=1}^N E \left[(p_{t_i} - p_{t_{i-1}})^j \right] \\ &= \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{\partial^j \varphi_t}{\partial u^j}(u, \tau) \Big|_{u=0}.\end{aligned}$$

The Taylor series expansion of $\frac{\partial^j \varphi_t}{\partial u^j}(u, \tau) \Big|_{u=0}$ around $\tau = 0$ yields

$$\frac{\partial \varphi_t}{\partial u}(u, \tau) \Big|_{u=0} = \left(\mu - \frac{\theta}{2} + \frac{1}{2} e^{-\kappa t} (\theta - V_0) \right) \tau + O(\tau^2), \quad (23)$$

$$\frac{\partial^2 \varphi_t}{\partial u^2}(u, \tau) \Big|_{u=0} = \left(\theta + \lambda (\mu_J^2 + \sigma_J^2) - \frac{1}{2} e^{-\kappa t} (\theta - V_0) \right) \tau + O(\tau^2), \quad (24)$$

$$\frac{\partial^3 \varphi_t}{\partial u^3}(u, \tau) \Big|_{u=0} = \lambda (\mu_J^3 + 3\mu_J \sigma_J^2) \tau + O(\tau^2), \quad (25)$$

$$\frac{\partial^4 \varphi_t}{\partial u^4}(u, \tau) \Big|_{u=0} = \lambda (\mu_J^4 + 6\mu_J^2 \sigma_J^2 + 3\sigma_J^4) \tau + O(\tau^2), \quad (26)$$

where $O(\tau)^2$ denotes all terms of order 2 and above. As N tends to infinity, τ converges to zero, so that the only remaining terms in equations (23), (24), (25), and (26) are those of order 1. The limit of the sum of these terms coincides with the definition of the Riemann-Stieltjes integral, so that integrating these terms with respect to t over the sampling interval $[0, T]$ gives (12), (13), (14), and (15). ■

Corollary 4 *The limit of the variance of the realized moments is*

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \text{Var} \left[(p_{t_i} - p_{t_{i-1}})^j \right] = 0,$$

for $j = 1, 2, 3$ and 4.

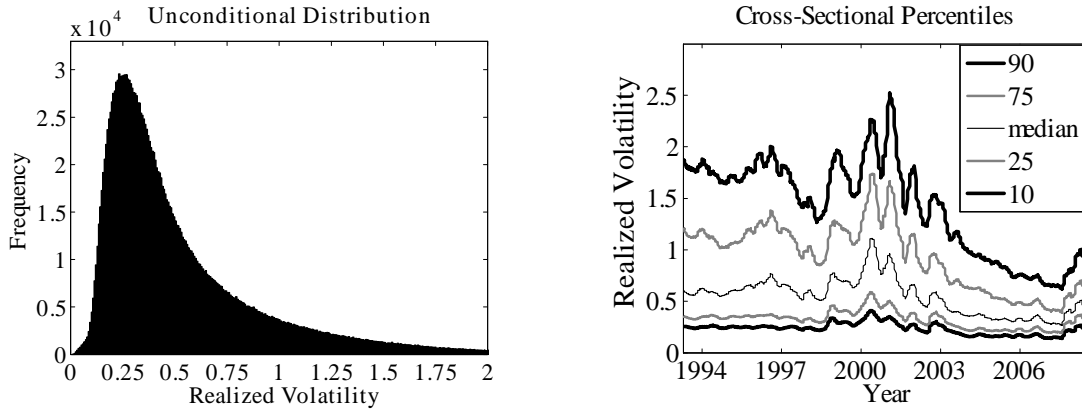
Proof. Follows by arguments similar to those made to prove (22). ■

The results in this appendix ensure mean-squared or L^2 convergence of the realized moments.

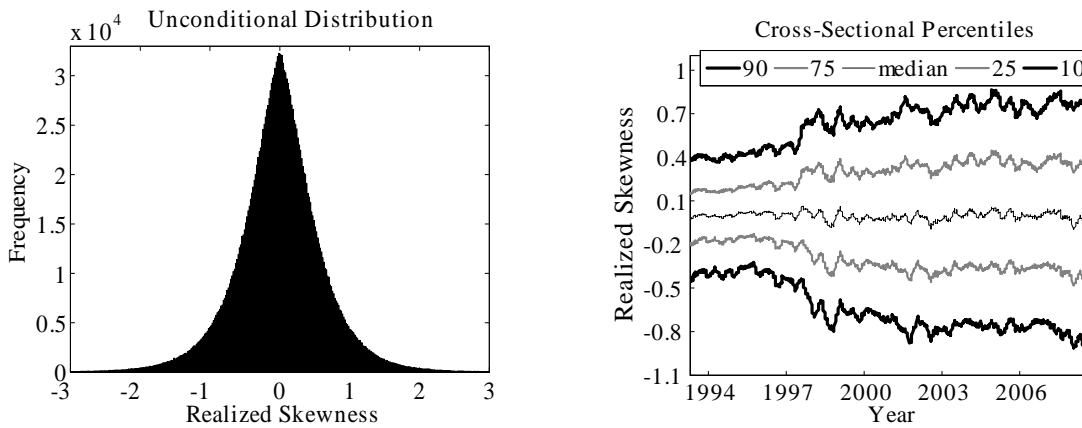
Figure 1
Histogram and Percentiles of Realized Moments

We display the histograms and various percentiles of realized moments for the cross-section of stocks over the period January 1993 to September 2008. Figures for realized volatility, realized skewness and realized kurtosis are reported in Panel A, Panel B and Panel C respectively. The sample contains 2,052,752 firm-week observations.

Panel A: Realized Volatility



Panel B: Realized Skewness



Panel C: Realized Kurtosis

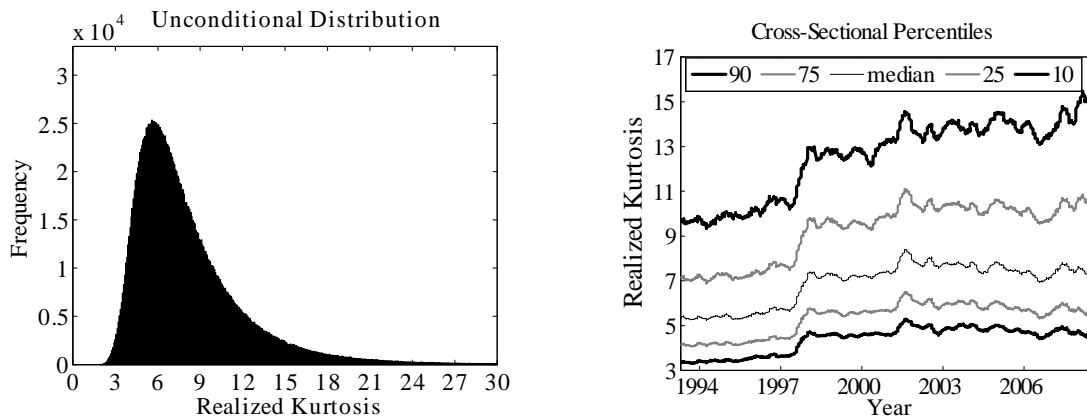
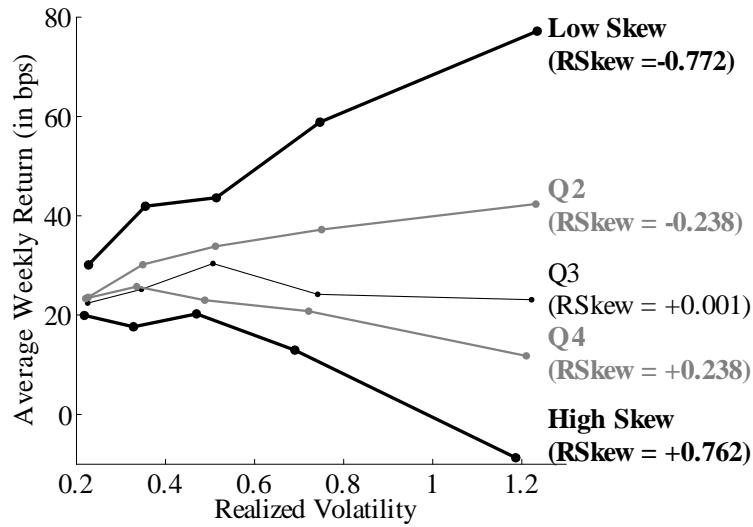


Figure 2

Interaction between Volatility, Realized Skewness, and Stock Returns

Each week, stocks are first ranked by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles by realized volatility (Panel A) and idiosyncratic volatility (Panel B). We report equal-weighted returns (in bps) for different levels of volatility and realized skewness. Each line represents different quintiles of realized skewness. The average realized skewness in each quintile is reported in parentheses.

Panel A: Interaction between Realized Volatility, Realized Skewness and Stock Returns



Panel B: Interaction between Idiosyncratic Volatility, Realized Skewness and Stock Returns

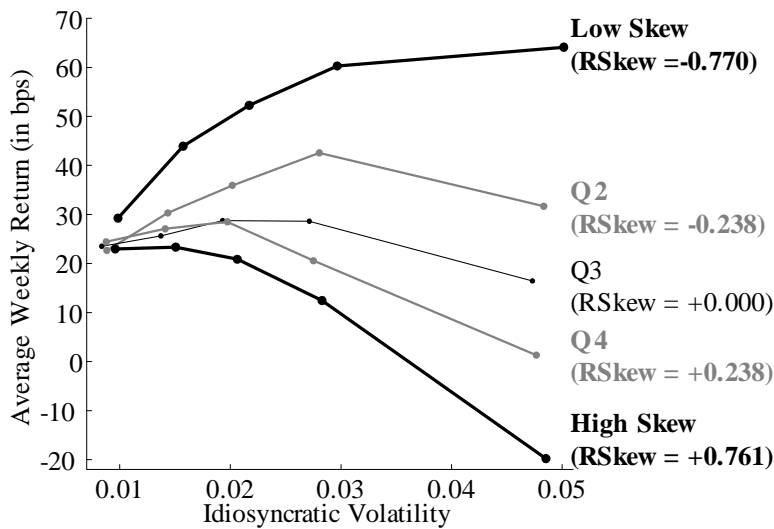


Figure 3
Signature Plots for Realized Moments

We show signature plots for the three daily realized moments, $RM(2)$, $RM(3)$, and $RM(4)$. The intraday sampling frequency on the horizontal axis is in seconds. The dotted lines represent the theoretical limit of realized moments as given by equations (13), (14), and (15). Monte Carlo estimates are plotted in continuous dark lines. Confidence intervals at 99% are shown in grey lines. In Panel A, observed prices have a microstructure noise component, which is simulated from a mean-zero normal distribution with a standard deviation of 0.05%. In Panel B, prices are only observed in sixteenths of a dollar.

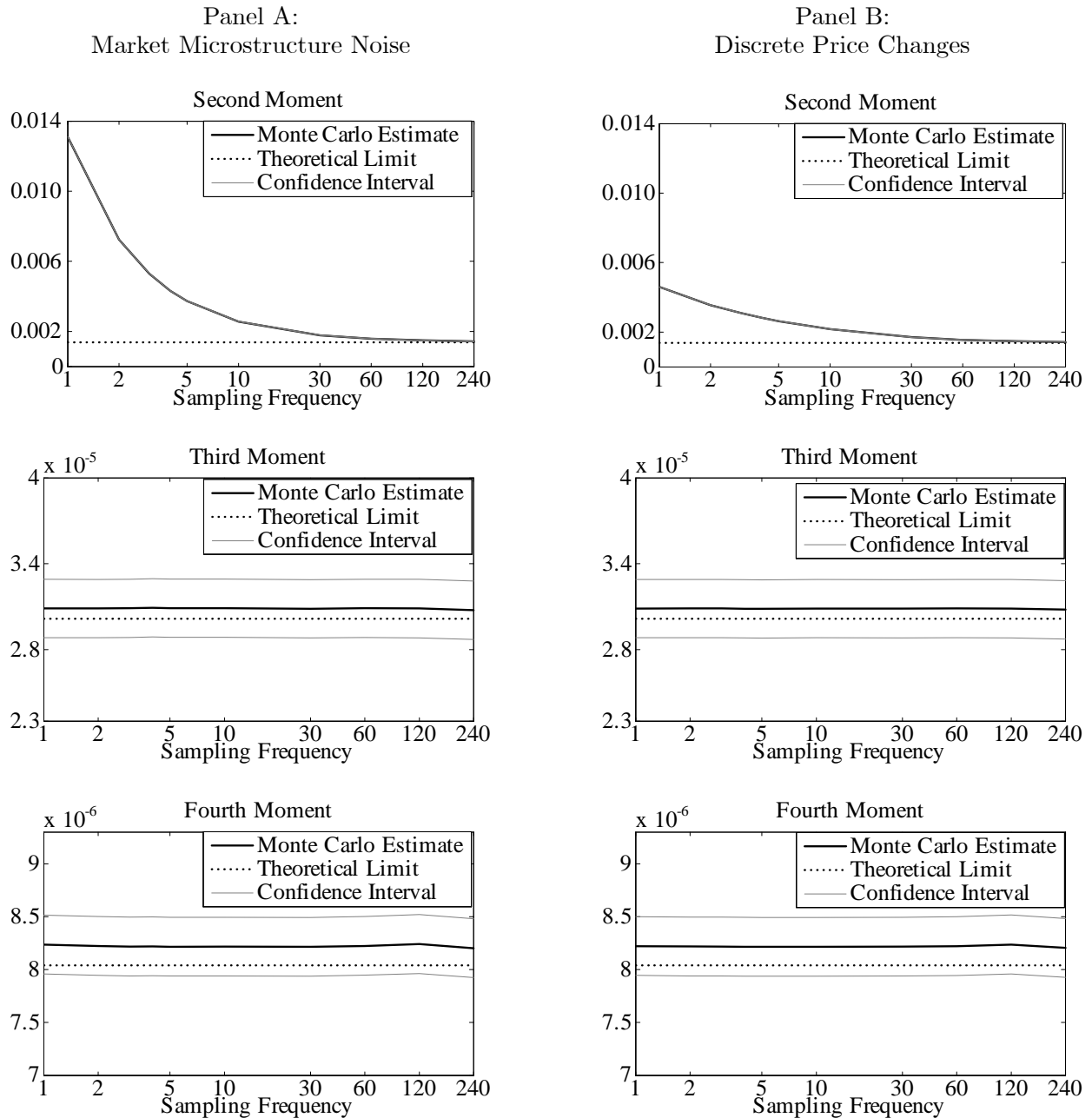


Table 1
Characteristics of Portfolios Sorted by Realized Moments

Each week, stocks are ranked by their realized moment and sorted into deciles. The equal-weighted characteristics of those deciles are computed over the same week, and repeated for every week from January 1993 through September 2008. Panel A displays the results for realized volatility, Panel B for realized skewness and Panel C for realized kurtosis. Average characteristics of the portfolios are reported for the realized moment, Size (\$ market capitalization in \$billions), BE/ME (book-to-market equity ratio), Realized volatility (weekly realized volatility computed with high-frequency data), HSkew (one month historical skewness from daily returns), Market Beta, Previous Week Return, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000)), Maximum Return (of the previous month), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6 , as in Amihud (2002)), Number of Analysts (from I/B/E/S), Credit Rating (1= AAA, 8= BBB+, 17= CCC+, 22=D), Price (stock price), Intraday Transactions (intraday transactions per day) and Number of Stocks.

Panel A: Characteristics of Portfolios Sorted by Realized Volatility

| Deciles | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Realized Volatility | 0.188 | 0.254 | 0.308 | 0.369 | 0.447 | 0.543 | 0.660 | 0.803 | 1.001 | 1.450 |
| Realized Skewness | 0.024 | 0.020 | 0.012 | 0.004 | 0.000 | -0.004 | -0.009 | -0.014 | -0.024 | -0.026 |
| Realized Kurtosis | 7.0 | 6.9 | 7.0 | 7.2 | 7.5 | 7.8 | 8.1 | 8.4 | 8.8 | 9.4 |
| Size | 13.84 | 12.14 | 9.04 | 6.19 | 4.26 | 2.89 | 1.91 | 1.21 | 0.79 | 0.46 |
| BE/ME | 0.654 | 0.573 | 0.509 | 0.507 | 0.488 | 0.459 | 0.428 | 0.421 | 0.417 | 0.440 |
| Historical Skewness | 0.109 | 0.102 | 0.109 | 0.126 | 0.144 | 0.166 | 0.191 | 0.212 | 0.241 | 0.311 |
| Market Beta | 0.72 | 0.82 | 0.89 | 0.97 | 1.07 | 1.18 | 1.28 | 1.36 | 1.36 | 1.32 |
| Lagged Return | 0.003 | 0.004 | 0.004 | 0.005 | 0.005 | 0.006 | 0.007 | 0.008 | 0.010 | 0.028 |
| Idiosyncratic Volatility | 0.011 | 0.013 | 0.015 | 0.017 | 0.020 | 0.023 | 0.027 | 0.031 | 0.036 | 0.048 |
| Co-Skewness | 0.003 | -0.007 | -0.018 | -0.028 | -0.032 | -0.035 | -0.041 | -0.045 | -0.049 | -0.050 |
| Maximum Return | 0.032 | 0.036 | 0.041 | 0.047 | 0.055 | 0.064 | 0.074 | 0.085 | 0.099 | 0.137 |
| Illiquidity | 0.005 | 0.005 | 0.006 | 0.009 | 0.013 | 0.020 | 0.035 | 0.080 | 0.126 | 0.562 |
| Number of Analysts | 10.0 | 10.6 | 10.2 | 9.5 | 8.8 | 7.9 | 6.9 | 5.8 | 4.7 | 3.2 |
| Credit Rating | 7.0 | 7.4 | 7.9 | 8.4 | 8.7 | 8.9 | 8.9 | 8.8 | 8.7 | 8.4 |
| Price | 62.2 | 53.6 | 38.9 | 33.5 | 29.6 | 27.7 | 24.7 | 21.4 | 17.8 | 15.0 |
| Intraday Transactions | 1,216 | 1,499 | 1,576 | 1,568 | 1,628 | 1,617 | 1,558 | 1,438 | 1,268 | 1,189 |
| Number of Stocks | 256 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 |

Table 1 (Continued)

Panel B: Characteristics of Portfolios Sorted by Realized Skewness

| Deciles | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Realized Skewness | -1.04 | -0.50 | -0.31 | -0.17 | -0.05 | 0.05 | 0.17 | 0.31 | 0.50 | 1.02 |
| Realized Volatility | 0.618 | 0.613 | 0.614 | 0.614 | 0.610 | 0.606 | 0.600 | 0.590 | 0.577 | 0.579 |
| Realized Kurtosis | 11.4 | 7.9 | 7.0 | 6.6 | 6.4 | 6.4 | 6.6 | 7.0 | 7.8 | 11.1 |
| Size | 2.96 | 4.41 | 5.22 | 5.75 | 6.22 | 6.22 | 6.09 | 5.69 | 4.91 | 3.49 |
| BE/ME | 0.476 | 0.471 | 0.480 | 0.483 | 0.486 | 0.488 | 0.500 | 0.495 | 0.488 | 0.490 |
| Historical Skewness | 0.140 | 0.160 | 0.163 | 0.167 | 0.162 | 0.167 | 0.171 | 0.173 | 0.184 | 0.209 |
| Market Beta | 1.05 | 1.14 | 1.12 | 1.11 | 1.09 | 1.10 | 1.11 | 1.11 | 1.10 | 1.02 |
| Lagged Return | -0.041 | -0.025 | -0.014 | -0.006 | 0.003 | 0.011 | 0.020 | 0.030 | 0.042 | 0.061 |
| Idiosyncratic Volatility | 0.026 | 0.025 | 0.024 | 0.024 | 0.023 | 0.023 | 0.023 | 0.024 | 0.024 | 0.025 |
| Co-Skewness | -0.038 | -0.031 | -0.028 | -0.029 | -0.027 | -0.026 | -0.025 | -0.024 | -0.025 | -0.027 |
| Maximum Return | 0.067 | 0.067 | 0.066 | 0.065 | 0.065 | 0.065 | 0.066 | 0.068 | 0.069 | 0.072 |
| Illiquidity | 0.116 | 0.071 | 0.082 | 0.066 | 0.076 | 0.059 | 0.058 | 0.067 | 0.102 | 0.158 |
| Number of Analysts | 6.5 | 7.6 | 7.9 | 8.1 | 8.1 | 8.1 | 8.2 | 8.2 | 8.0 | 6.9 |
| Credit Rating | 8.4 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.4 |
| Price | 29.8 | 32.6 | 33.2 | 32.2 | 31.5 | 31.4 | 33.0 | 33.7 | 33.9 | 33.1 |
| Intraday Transactions | 998 | 1,324 | 1,516 | 1,613 | 1,676 | 1,683 | 1,633 | 1,595 | 1,426 | 1,095 |
| Number of Stocks | 256 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 |

Panel C: Characteristics of Portfolios Sorted by Realized Kurtosis

| Deciles | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Realized Kurtosis | 3.9 | 4.7 | 5.3 | 5.9 | 6.5 | 7.2 | 8.0 | 9.2 | 10.9 | 16.6 |
| Realized Volatility | 0.485 | 0.537 | 0.570 | 0.589 | 0.603 | 0.614 | 0.623 | 0.638 | 0.654 | 0.711 |
| Realized Skewness | 0.005 | 0.007 | 0.006 | 0.006 | 0.007 | 0.003 | 0.000 | -0.006 | -0.012 | -0.034 |
| Size | 15.59 | 8.80 | 6.22 | 4.89 | 3.88 | 3.27 | 2.80 | 2.38 | 1.98 | 1.36 |
| BE/ME | 0.481 | 0.492 | 0.476 | 0.502 | 0.477 | 0.478 | 0.476 | 0.480 | 0.480 | 0.514 |
| Hskew | 0.129 | 0.149 | 0.156 | 0.165 | 0.170 | 0.166 | 0.180 | 0.177 | 0.186 | 0.220 |
| Beta | 1.09 | 1.15 | 1.16 | 1.15 | 1.14 | 1.13 | 1.12 | 1.08 | 1.03 | 0.90 |
| Lagged Return | 0.004 | 0.006 | 0.007 | 0.007 | 0.008 | 0.008 | 0.008 | 0.008 | 0.009 | 0.015 |
| Idiosyncratic Volatility | 0.019 | 0.022 | 0.023 | 0.024 | 0.024 | 0.025 | 0.025 | 0.026 | 0.026 | 0.027 |
| Co-Skewness | -0.012 | -0.017 | -0.020 | -0.023 | -0.027 | -0.030 | -0.031 | -0.036 | -0.040 | -0.046 |
| Maximum Return | 0.053 | 0.061 | 0.065 | 0.067 | 0.068 | 0.069 | 0.070 | 0.070 | 0.071 | 0.076 |
| Illiquidity | 0.014 | 0.029 | 0.035 | 0.046 | 0.061 | 0.070 | 0.074 | 0.109 | 0.112 | 0.306 |
| Number of Analysts | 11.4 | 10.1 | 9.1 | 8.4 | 7.8 | 7.4 | 6.9 | 6.4 | 5.8 | 4.4 |
| Credit Rating | 7.8 | 8.1 | 8.3 | 8.3 | 8.4 | 8.4 | 8.4 | 8.5 | 8.5 | 8.5 |
| Price | 36.4 | 37.9 | 36.2 | 34.0 | 33.6 | 32.4 | 30.2 | 29.5 | 28.6 | 25.6 |
| Intraday Transactions | 3,250 | 2,195 | 1,795 | 1,553 | 1,349 | 1,217 | 1,067 | 917 | 736 | 483 |
| Number of Stocks | 256 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 | 257 |

Table 2
Realized Moments and the Cross-Section of Stock Returns

We report the equal-weighted and value-weighted weekly returns (in bps) of decile portfolios formed from realized moments, their t-statistics (in parentheses), and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to September 2008. Panel A displays the results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Each panel reports the equal-weighted portfolios and the value-weighted portfolios. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio using the Carhart four factor model.

Panel A: Realized Volatility and the Cross-Section of Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 21.96 (3.93) | 24.10 (3.62) | 25.82 (3.55) | 30.19 (3.75) | 28.56 (3.13) | 31.92 (2.97) | 29.15 (2.35) | 32.04 (2.29) | 34.04 (2.16) | 27.04 (1.55) | 5.08 (0.33) |
| Alpha, C4 | 21.17 (3.73) | 23.83 (3.53) | 26.29 (3.57) | 30.33 (3.71) | 29.51 (3.19) | 33.36 (3.07) | 30.61 (2.44) | 34.57 (2.44) | 37.08 (2.32) | 29.83 (1.70) | 7.00 (0.45) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 19.98 (3.24) | 21.66 (3.06) | 26.28 (3.27) | 20.59 (2.19) | 18.41 (1.69) | 18.10 (1.39) | 19.51 (1.32) | 14.09 (0.81) | 17.01 (0.93) | 8.07 (0.41) | -11.92 (-0.65) |
| Alpha, C4 | 20.34 (3.25) | 22.40 (3.13) | 28.11 (3.48) | 22.93 (2.42) | 20.87 (1.90) | 20.48 (1.56) | 21.56 (1.44) | 17.61 (1.01) | 21.40 (1.16) | 12.58 (0.63) | -9.42 (-0.51) |

Panel B: Realized Skewness and the Cross-Section of Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 54.99 (5.06) | 45.73 (4.12) | 36.08 (3.31) | 30.78 (2.91) | 29.43 (2.87) | 21.13 (2.11) | 23.78 (2.41) | 18.22 (1.87) | 13.29 (1.41) | 11.54 (1.29) | -43.44 (-8.91) |
| Alpha, C4 | 55.92 (5.07) | 47.21 (4.20) | 37.47 (3.39) | 32.50 (3.03) | 30.40 (2.93) | 22.08 (2.18) | 24.96 (2.50) | 19.81 (2.01) | 14.34 (1.50) | 12.06 (1.33) | -45.52 (-9.26) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 40.49 (4.12) | 33.84 (3.49) | 27.24 (2.77) | 19.93 (1.96) | 26.64 (2.83) | 15.61 (1.76) | 20.58 (2.29) | 9.00 (1.00) | 5.25 (0.61) | 16.69 (1.98) | -23.80 (-3.65) |
| Alpha, C4 | 39.56 (3.97) | 35.70 (3.63) | 28.49 (2.87) | 21.19 (2.06) | 29.81 (3.14) | 16.80 (1.88) | 22.34 (2.46) | 10.45 (1.15) | 8.06 (0.93) | 18.67 (2.19) | -22.55 (-3.42) |

Panel C: Realized Kurtosis and the Cross-Section of Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 22.68 (2.28) | 25.66 (2.52) | 25.81 (2.48) | 25.25 (2.37) | 27.30 (2.58) | 27.00 (2.59) | 27.86 (2.70) | 30.47 (2.96) | 34.39 (3.45) | 38.45 (4.21) | 15.77 (2.98) |
| Alpha, C4 | 23.70 (2.35) | 27.90 (2.71) | 27.32 (2.60) | 26.94 (2.50) | 28.85 (2.69) | 28.39 (2.69) | 29.17 (2.79) | 31.22 (2.99) | 35.03 (3.47) | 38.12 (4.12) | 12.75 (2.41) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 17.17 (1.80) | 20.83 (2.32) | 18.30 (2.06) | 27.24 (3.06) | 17.55 (1.93) | 15.04 (1.61) | 21.81 (2.47) | 26.27 (2.81) | 18.91 (2.16) | 30.74 (3.58) | 13.57 (2.12) |
| Alpha, C4 | 18.67 (1.94) | 23.12 (2.55) | 20.76 (2.33) | 29.29 (3.26) | 18.68 (2.04) | 15.23 (1.62) | 22.61 (2.53) | 27.25 (2.88) | 20.98 (2.37) | 31.18 (3.57) | 10.85 (1.67) |

Table 3
Fama-MacBeth Cross-Sectional Regressions

We report results from Fama-MacBeth cross-sectional regressions of weekly stock returns (in bps) on firm characteristics for the period January 1993 to September 2008. Firm characteristics are Realized Volatility, Realized Skewness, Realized Kurtosis, Lagged Return (in bps), Size (market capitalization in \$billions), BE/ME (book-to-market equity ratio), Market Beta, HSkew (one month historical skewness from daily returns), Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data), Maximum Return (of previous month in bps), Number of Analysts (from I/B/E/S), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6 , as in Amihud (2002)), and Number of Intraday Transactions. We report the average of the coefficient estimates for the weekly regressions along with the Newey-West t-statistic (in parentheses).

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---------------------------------------|----------------|------------------|----------------|------------------|--------------------|--------------------|
| Intercept | 27.7 (3.80) | 31.5 (3.23) | 21.0 (1.99) | 20.7 (2.54) | 16.8 (2.03) | 187.6 (6.76) |
| Realized Volatility | 3.5 (0.26) | | | 0.1 (0.01) | 6.7 (0.46) | -3.1 (-0.40) |
| Realized Skewness | | -22.4 (-7.90) | | -24.3 (-9.21) | -3.9 (-2.07) | -4.7 (-2.73) |
| Realized Kurtosis | | | 1.3 (3.49) | 1.3 (2.99) | 1.6 (3.83) | 0.7 (2.15) |
| Lagged Return | | | | | -0.032 (-10.55) | -0.041 (-14.27) |
| log (Size) | | | | | | -13.2 (-5.48) |
| log (BE/ME) | | | | | | 3.2 (1.80) |
| Market Beta | | | | | | -8.1 (-1.20) |
| Historical Skewness | | | | | | 17.4 (9.98) |
| Idiosyncratic Volatility | | | | | | -284.0 (-1.17) |
| Co-skewness | | | | | | 15.4 (0.99) |
| Maximum Monthly Return | | | | | | -0.014 (-2.09) |
| log (Number of Analysts+1) | | | | | | 0.28 (0.12) |
| Illiquidity | | | | | | -8.2 (-0.48) |
| log (Number of Intraday transactions) | | | | | | 13.6 (5.31) |
| Rsquare | 0.025 | 0.003 | 0.003 | 0.031 | 0.039 | 0.096 |

Table 4
Double Sorting on Realized Skewness and Realized Volatility

Each week, stocks are first sorted by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles by realized volatility. Panel A reports equal-weighted returns and Panel B reports value-weighted returns. The equal-weighted and value-weighted average weekly returns (in bps) are reported for all double sorted portfolios as well as for the difference between portfolio five (High volatility) and one (Low volatility) for each level of realized skewness. T-statistics are in parentheses.

| | | Panel A: Equal-Weighted | | | | | |
|----------------------------|---|-------------------------|----------------|----------------|----------------|-----------------|------------------|
| | | Low | 2 | 3 | 4 | High | High-Low RVol |
| 1 (Low Realized Skewness) | | 30.0 (4.50) | 42.0 (4.92) | 43.2 (3.70) | 59.0 (4.16) | 77.0 (4.47) | 47.2 (3.31) |
| | 2 | 23.8 (3.73) | 30.2 (3.74) | 33.5 (3.09) | 37.2 (2.61) | 43.0 (2.37) | 18.9 (1.23) |
| | 3 | 22.5 (3.70) | 25.3 (3.28) | 30.0 (3.02) | 24.2 (1.77) | 23.0 (1.36) | 0.6 (0.04) |
| | 4 | 23.4 (3.88) | 25.3 (3.33) | 23.1 (2.40) | 20.8 (1.60) | 12.0 (0.70) | -11.7 (-0.80) |
| 5 (High Realized Skewness) | | 19.9 (3.32) | 17.6 (2.33) | 20.2 (2.20) | 13.0 (1.09) | -9.0 (-0.56) | -28.4 (-2.22) |
| | | Panel B: Value-Weighted | | | | | |
| | | Low | 2 | 3 | 4 | High | High-Low RVol |
| 1 (Low Realized Skewness) | | 32.7 (4.32) | 43.1 (4.47) | 32.7 (2.45) | 42.0 (2.47) | 40.8 (2.07) | 8.1 (0.46) |
| | 2 | 25.0 (3.40) | 25.6 (2.70) | 24.1 (1.82) | 25.6 (1.50) | 14.8 (0.70) | -10.2 (-0.55) |
| | 3 | 24.9 (3.59) | 19.1 (2.11) | 20.8 (1.68) | 21.5 (1.35) | 1.1 (0.06) | -23.7 (-1.35) |
| | 4 | 15.4 (2.27) | 25.5 (2.96) | 10.6 (0.88) | 0.7 (0.04) | -6.9 (-0.35) | -22.2 (-1.25) |
| 5 (High Realized Skewness) | | 12.6 (1.89) | 8.7 (0.95) | 14.6 (1.34) | 6.1 (0.44) | -9.6 (-0.53) | -22.2 (-1.34) |

Table 5
Idiosyncratic Volatility and the Cross-Section of Stock Returns

This table reports the equal-weighted and value-weighted weekly returns (in bps) of quintile portfolios ranked by idiosyncratic volatility, their t-statistics (in parentheses), and the difference between portfolio 5 (highest idiosyncratic volatility) and portfolio 1 (lowest idiosyncratic volatility) over the period January 1993 to September 2008. Panel A reports the equal-weighted portfolios and Panel B reports the value-weighted portfolios. Raw returns (in bps) are obtained for quintile portfolios sorted solely on the idiosyncratic volatility measure. Alpha is the intercept from the time-series regressions for the return on the portfolio that buys portfolio 5 and sells portfolio 1, using the Carhart four-factor model.

| Panel A: Equal-Weighted | | | | | | |
|-------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns | 23.61 (3.96) | 30.38 (3.96) | 32.69 (3.30) | 32.23 (2.50) | 20.96 (1.30) | -2.64 (-0.20) |
| Alpha, C4 | 23.59 (3.91) | 30.91 (3.98) | 33.50 (3.34) | 34.64 (2.65) | 23.11 (1.42) | -2.14 (-0.16) |
| Panel B: Value-Weighted | | | | | | |
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns | 24.61 (3.46) | 22.76 (2.79) | 15.05 (1.46) | 18.40 (1.39) | 0.48 (0.03) | -24.14 (-1.73) |
| Alpha, C4 | 25.95 (3.63) | 24.54 (2.99) | 16.36 (1.57) | 21.71 (1.62) | 5.09 (0.30) | -22.53 (-1.59) |

Table 6
Double Sorting on Realized Skewness and Idiosyncratic Volatility

Each week, stocks are first ranked by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles based on idiosyncratic volatility. Panel A reports equal-weighted returns and Panel B reports value-weighted returns. The equal-weighted and value-weighted average weekly returns (in bps) are reported for all double sorted portfolios as well as for the difference between portfolio five (high idiosyncratic volatility) and one (low idiosyncratic volatility) for each level of realized skewness. T-statistics are in parentheses.

| | | Panel A: Equal-Weighted | | | | | |
|----------------------------|---|-------------------------|----------------|----------------|----------------|------------------|------------------|
| | | Low | 2 | 3 | 4 | High | High-Low |
| 1 (Low Realized Skewness) | | 29.3 (4.44) | 44.0 (5.05) | 51.9 (4.53) | 60.5 (4.27) | 64.0 (3.76) | 34.8 (2.48) |
| | 2 | 22.8 (3.64) | 30.2 (3.74) | 36.0 (3.26) | 42.4 (2.97) | 32.0 (1.83) | 9.0 (0.61) |
| | 3 | 23.5 (3.99) | 26.1 (3.37) | 28.4 (2.79) | 28.3 (2.14) | 17.0 (0.98) | -6.8 (-0.47) |
| | 4 | 24.5 (4.07) | 26.4 (3.47) | 28.6 (2.93) | 20.3 (1.56) | 2.0 (0.09) | -22.9 (-1.64) |
| 5 (High Realized Skewness) | | 23.0 (3.79) | 23.3 (3.10) | 21.0 (2.31) | 12.4 (1.06) | -20.0 (-1.32) | -42.6 (-3.40) |

| | | Panel B: Value-Weighted | | | | | |
|----------------------------|---|-------------------------|----------------|-----------------|----------------|------------------|------------------|
| | | Low | 2 | 3 | 4 | High | High-Low |
| 1 (Low Realized Skewness) | | 32.1 (3.98) | 36.1 (3.81) | 48.9 (3.89) | 46.6 (2.99) | 25.1 (1.35) | -6.9 (-0.43) |
| | 2 | 23.2 (2.91) | 26.4 (2.95) | 26.5 (2.19) | 28.1 (1.83) | 2.6 (0.13) | -20.6 (-1.22) |
| | 3 | 30.2 (3.98) | 22.3 (2.54) | 12.1 (1.09) | 21.1 (1.46) | 0.9 (0.05) | -29.4 (-1.90) |
| | 4 | 16.8 (2.25) | 22.8 (2.67) | 17.5 (1.56) | 0.1 (0.01) | -9.5 (-0.53) | -26.3 (-1.67) |
| 5 (High Realized Skewness) | | 17.0 (2.29) | 9.0 (1.01) | -1.1 (-0.11) | 8.3 (0.65) | -16.4 (-1.03) | -33.4 (-2.44) |

Table 7
Realized Moments and Returns for Different Subsamples

We report weekly returns (in bps) and t-statistics of quintile portfolios formed from ranking stocks by their realized moments. We also report the difference between portfolio 5 (highest realized moment) and portfolio 1 (lowest realized moment). Panel A displays the results for realized skewness and Panel B for realized kurtosis. Each panel reports the equal-weighted and value-weighted quintile portfolio returns for the month of January, for all months excluding January, and only for NYSE stocks.

Panel A: Realized Skewness Effects for Different Subsamples

| | Equal weighted | | | | | |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|--------------------|
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns, January | 93.27 (2.72) | 55.86 (1.74) | 59.54 (1.98) | 54.28 (1.76) | 37.21 (1.31) | -56.05 (-4.13) |
| Raw Returns, Non-January | 46.36 (4.05) | 31.37 (2.78) | 22.15 (2.08) | 17.90 (1.75) | 10.11 (1.06) | -36.25 (-8.36) |
| Raw Returns, NYSE | 66.84 (4.81) | 44.06 (3.08) | 28.57 (2.06) | 18.80 (1.40) | 2.48 (0.21) | -64.36 (-12.27) |

| | Value weighted | | | | | |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns, January | 54.76 (2.11) | 43.49 (1.45) | 36.75 (1.43) | 18.08 (0.68) | 24.20 (0.90) | -30.57 (-1.72) |
| Raw Returns, Non-January | 34.31 (3.41) | 20.93 (2.03) | 19.47 (2.08) | 13.99 (1.52) | 8.09 (0.93) | -26.22 (-4.80) |
| Raw Returns, NYSE | 44.61 (3.08) | 31.23 (2.04) | 26.48 (1.82) | 13.90 (1.01) | 13.06 (1.02) | -31.55 (-3.51) |

Panel B: Realized Kurtosis Effects for Different Subsamples

| | Equal weighted | | | | | |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns, January | 47.14 (1.54) | 58.03 (1.82) | 63.79 (1.97) | 60.40 (1.93) | 70.76 (2.34) | 23.62 (1.59) |
| Raw Returns, Non-January | 22.05 (2.10) | 22.56 (2.05) | 23.74 (2.16) | 26.26 (2.44) | 33.23 (3.35) | 11.18 (2.66) |
| Raw Returns, NYSE | 25.85 (1.63) | 31.95 (2.16) | 28.70 (2.11) | 33.81 (2.67) | 40.34 (3.72) | 14.49 (1.88) |

| | Value weighted | | | | | |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | Low | 2 | 3 | 4 | High | High-Low |
| Raw Returns, January | 31.25 (1.20) | 38.43 (1.44) | 37.81 (1.38) | 30.27 (1.26) | 58.70 (2.10) | 27.44 (1.46) |
| Raw Returns, Non-January | 17.49 (1.82) | 21.28 (2.35) | 13.61 (1.45) | 23.45 (2.51) | 19.44 (2.24) | 1.95 (0.39) |
| Raw Returns, NYSE | 25.41 (1.75) | 22.47 (1.59) | 16.71 (1.24) | 33.42 (2.71) | 32.15 (2.89) | 6.75 (0.75) |

Table 8
Double Sorting on Firm Characteristics, then on Realized Moments

Based on a given firm characteristic, stocks are sorted into five quintiles each week. Then, within each quintile, stocks are sorted once again into five quintiles based on the realized moment measure. We then compute equal-weighted average weekly returns for the realized moment quintile difference between portfolio five and one, along with the t-statistic (in parentheses). Panel A displays the results for realized skewness and Panel B for realized kurtosis. Firm characteristics are Size (\$ market capitalization in \$billions), BE/ME (book-to-market equity ratio), Realized Volatility (weekly realized volatility computed with high-frequency data), Lagged return, HSkew (one month historical skewness from daily returns), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6 , as in Amihud (2002)), Number of Intraday Transactions, Maximum Return (of previous month), Number of Analysts (from I/B/E/S), Market Beta, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)) and Coskewness (computed as in Harvey and Siddique (2000)).

Panel A: Double Sorting on Firm Characteristics, then on Realized Skewness

| | Characteristics | | | | |
|---|-----------------|---------|---------|---------|----------|
| | Low | 2 | 3 | 4 | High |
| Across Size quintiles | -77.0 | -36.9 | -33.7 | -9.5 | -16.9 |
| Realized Skewness 5-1 | (-10.23) | (-6.17) | (-6.49) | (-1.91) | (-3.66) |
| Across BE/ME quintiles | -58.7 | -39.7 | -36.0 | -29.3 | -24.6 |
| Realized Skewness 5-1 | (-7.91) | (-6.97) | (-6.70) | (-5.59) | (-4.33) |
| Across Lagged Return quintiles | -30.5 | -9.9 | -9.9 | -3.3 | -14.3 |
| Realized Skewness 5-1 | (-5.47) | (-2.28) | (-2.50) | (-0.68) | (-2.32) |
| Across Realized Volatility quintiles | -10.1 | -26.3 | -31.7 | -46.6 | -88.9 |
| Realized Skewness 5-1 | (-3.70) | (-7.45) | (-6.45) | (-8.17) | (-11.50) |
| Across Realized Kurtosis quintiles | -24.9 | -29.5 | -38.5 | -46.0 | -48.1 |
| Realized Skewness 5-1 | (-3.84) | (-4.77) | (-6.98) | (-8.33) | (-8.68) |
| Across Historical Skewness quintiles | -31.9 | -37.8 | -37.4 | -38.1 | -43.6 |
| Realized Skewness 5-1 | (-5.88) | (-7.14) | (-7.20) | (-6.87) | (-7.22) |
| Across Illiquidity quintiles | -15.3 | -18.7 | -27.6 | -40.3 | -77.6 |
| Realized Skewness 5-1 | (-3.01) | (-3.64) | (-5.52) | (-7.41) | (-11.55) |
| Across Intraday Transactions quintiles | -37.0 | -37.6 | -37.8 | -39.4 | -37.7 |
| Realized Skewness 5-1 | (-8.02) | (-7.53) | (-6.99) | (-6.13) | (-5.01) |
| Across Maximum Return quintiles | -17.5 | -25.9 | -5.3 | -39.7 | -57.6 |
| Realized Skewness 5-1 | (-2.20) | (-1.56) | (-0.39) | (-2.40) | (-5.76) |
| Across Number of Analysts quintiles | -55.8 | -50.1 | -39.2 | -18.6 | -19.4 |
| Realized Skewness 5-1 | (-8.45) | (-8.72) | (-7.38) | (-3.67) | (-3.72) |
| Across Market Beta quintiles | -29.6 | -31.9 | -32.8 | -44.2 | -62.0 |
| Realized Skewness 5-1 | (-6.34) | (-7.55) | (-6.93) | (-8.06) | (-9.01) |
| Across Idiosyncratic Volatility quintiles | -6.3 | -20.8 | -32.3 | -54.0 | -82.4 |
| Realized Skewness 5-1 | (-2.45) | (-6.02) | (-6.62) | (-8.68) | (-11.04) |
| Across Co-Skewness quintiles | -37.9 | -16.9 | -27.7 | -32.9 | -24.3 |
| Realized Skewness 5-1 | (-6.75) | (-3.03) | (-5.42) | (-6.42) | (-5.09) |

Table 8 (Continued)

Panel B: Double Sorting on Firm Characteristics, then on Realized Kurtosis

| | Characteristics | | | | |
|---|-----------------|---------|--------|---------|---------|
| | Low | 2 | 3 | 4 | High |
| Across Size quintiles | 4.3 | 12.1 | 1.8 | -1.0 | -0.6 |
| Realized Kurtosis 5-1 | (0.42) | (1.53) | (0.31) | (-0.19) | (-0.13) |
| Across BE/ME quintiles | 14.3 | -3.6 | 9.5 | 11.6 | 21.4 |
| Realized Kurtosis 5-1 | (2.05) | (-0.61) | (1.83) | (2.21) | (3.54) |
| Across Lagged Return quintiles | 16.7 | 8.9 | 9.6 | 21.2 | 15.8 |
| Realized Kurtosis 5-1 | (2.10) | (1.76) | (2.28) | (4.16) | (2.24) |
| Across Realized Volatility quintiles | 3.8 | 1.7 | 15.3 | 16.3 | 20.3 |
| Realized Kurtosis 5-1 | (1.22) | (0.45) | (2.82) | (1.91) | (1.88) |
| Across Realized Skewness quintiles | 14.1 | 7.2 | 13.1 | 9.5 | 8.5 |
| Realized Kurtosis 5-1 | (2.28) | (1.24) | (2.27) | (1.72) | (1.65) |
| Across Historical Skewness quintiles | 12.3 | 17.5 | 11.7 | 13.0 | 2.7 |
| Realized Kurtosis 5-1 | (2.34) | (3.45) | (2.20) | (2.19) | (0.41) |
| Across Illiquidity quintiles | 3.7 | 0.6 | 2.5 | 2.6 | 12.6 |
| Realized Kurtosis 5-1 | (0.77) | (0.12) | (0.51) | (0.40) | (1.37) |
| Across Intraday Transactions quintiles | 25.1 | 17.7 | 13.1 | 11.4 | -2.4 |
| Realized Kurtosis 5-1 | (5.23) | (3.48) | (2.41) | (1.84) | (-0.35) |
| Across Maximum Return quintiles | 14.3 | 11.5 | 30.0 | 28.1 | -0.8 |
| Realized Kurtosis 5-1 | (1.87) | (0.45) | (2.17) | (1.61) | (-0.08) |
| Across Number of Analysts quintiles | 17.9 | 5.9 | 3.6 | -1.9 | 4.3 |
| Realized Kurtosis 5-1 | (2.08) | (0.92) | (0.69) | (-0.41) | (0.82) |
| Across Market Beta quintiles | 27.7 | 14.1 | 11.2 | -0.8 | -4.9 |
| Realized Kurtosis 5-1 | (5.32) | (3.29) | (2.34) | (-0.15) | (-0.68) |
| Across Idiosyncratic Volatility quintiles | 3.5 | 9.0 | 10.9 | 19.4 | 19.7 |
| Realized Kurtosis 5-1 | (1.17) | (2.33) | (2.08) | (2.68) | (2.04) |
| Across Co-Skewness quintiles | 13.5 | 17.1 | 4.1 | 10.8 | 5.2 |
| Realized Kurtosis 5-1 | (2.17) | (2.81) | (0.80) | (2.12) | (1.08) |

Table 9
Weekly Realized Moments and the Cross-Section of Monthly Stock Returns

We report the equal-weighted and value-weighted monthly returns (in bps) of decile portfolios formed from realized moments, their Newey-West t-statistics (in parentheses) and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to September 2008. Panel A displays the results for realized volatility, Panel B for realized skewness and Panel C for realized kurtosis. Each panel reports the equal-weighted portfolios and the value-weighted portfolios. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model.

Panel A: Realized Volatility and Monthly Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|-----------------|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------------|-------------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 89.54 (4.58) | 94.83 (4.29) | 98.43 (4.14) | 107.19 (4.01) | 102.54 (3.32) | 111.95 (3.00) | 103.15 (2.32) | 102.50 (1.99) | 93.36 (1.63) | 81.74 (1.25) | -7.80 (-0.13) |
| Alpha, C4 | 90.74 (7.68) | 96.73 (6.50) | 101.45 (5.99) | 110.41 (5.10) | 107.55 (4.23) | 119.58 (3.85) | 111.45 (2.84) | 113.46 (2.48) | 105.05 (2.02) | 95.69 (1.62) | 3.29 (0.06) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 78.20 (4.04) | 78.93 (3.66) | 88.84 (3.66) | 91.56 (3.12) | 79.02 (2.30) | 65.72 (1.64) | 71.95 (1.52) | 49.58 (0.92) | 10.37 (0.18) | -10.54 (-0.15) | -88.74 (-1.42) |
| Alpha, C4 | 80.87 (6.70) | 82.55 (5.54) | 94.28 (5.49) | 96.90 (3.90) | 88.86 (3.17) | 73.98 (2.12) | 81.11 (1.89) | 65.02 (1.31) | 20.66 (0.38) | 3.35 (0.06) | -79.18 (-1.39) |

Panel B: Realized Skewness and Monthly Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|------------------|------------------|------------------|------------------|------------------|-----------------|------------------|-----------------|-----------------|-----------------|-------------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 128.61 (3.37) | 114.74 (2.97) | 102.22 (2.72) | 96.81 (2.66) | 102.05 (2.82) | 86.29 (2.49) | 97.79 (2.86) | 84.10 (2.47) | 84.18 (2.54) | 88.65 (2.80) | -39.96 (-3.29) |
| Alpha, C4 | 135.21 (5.80) | 122.44 (4.71) | 109.63 (4.39) | 103.28 (3.65) | 108.85 (3.73) | 92.72 (3.21) | 104.18 (3.34) | 91.94 (2.98) | 90.10 (2.90) | 94.06 (3.18) | -42.81 (-3.73) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 112.31 (3.77) | 84.25 (2.80) | 81.82 (2.78) | 83.82 (2.86) | 89.20 (3.09) | 66.91 (2.47) | 71.81 (2.52) | 64.06 (2.33) | 64.49 (2.54) | 77.01 (3.10) | -35.30 (-2.63) |
| Alpha, C4 | 113.21 (5.83) | 90.22 (4.21) | 87.41 (4.16) | 88.15 (3.59) | 95.03 (3.93) | 72.04 (3.06) | 76.23 (2.90) | 68.93 (2.70) | 71.94 (2.95) | 83.01 (3.56) | -31.86 (-2.43) |

Panel C: Realized Kurtosis and Monthly Stock Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | High-Low |
|----------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-----------------|
| Equal weighted | | | | | | | | | | | |
| Raw Returns | 78.77 (2.48) | 87.09 (2.53) | 89.19 (2.54) | 95.85 (2.63) | 95.81 (2.62) | 97.60 (2.62) | 93.39 (2.56) | 109.74 (2.96) | 116.13 (3.22) | 121.75 (3.44) | 42.98 (2.54) |
| Alpha, C4 | 84.93 (4.24) | 95.24 (4.06) | 96.24 (4.12) | 103.37 (3.61) | 102.92 (3.47) | 104.74 (3.36) | 100.63 (3.11) | 116.01 (3.50) | 122.31 (3.65) | 125.89 (3.93) | 39.30 (2.75) |
| Value weighted | | | | | | | | | | | |
| Raw Returns | 62.01 (2.04) | 78.22 (2.81) | 72.57 (2.64) | 99.75 (3.69) | 76.19 (2.66) | 79.11 (2.84) | 80.72 (2.89) | 93.86 (3.39) | 94.12 (3.52) | 92.40 (3.32) | 30.39 (1.78) |
| Alpha, C4 | 67.54 (3.54) | 84.49 (4.37) | 79.56 (4.12) | 104.51 (4.64) | 80.99 (3.35) | 83.10 (3.45) | 85.44 (3.35) | 97.57 (3.80) | 98.17 (3.88) | 94.97 (3.64) | 25.77 (1.64) |

Table 10
Alternative Realized Volatility Estimators and the Cross-Section of Stock Returns

We report the equal-weighted and value-weighted weekly returns (in bps) of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) during January 1993 to September 2008. Stocks are ranked based on their realized moments using different realized volatility estimators. Panel A displays the results for alternative realized volatility estimators, Panel B for realized skewness scaled by alternative realized volatility estimators, and Panel C for realized kurtosis scaled by alternative realized volatility estimators. Each panel reports the equal-weighted and the value-weighted long-short returns. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model.

Panel A: Long-Short Returns for Alternative Realized Volatility Estimators

| | RV | BPV | minRV | medRV |
|----------------|---------|---------|---------|---------|
| Equal weighted | | | | |
| Raw Returns | 5.08 | 0.39 | -0.26 | 0.14 |
| | (0.33) | (0.02) | (-0.02) | (0.01) |
| Alpha, C4 | 7.00 | 2.85 | 1.54 | 2.01 |
| | (0.45) | (0.17) | (0.10) | (0.13) |
| Value weighted | | | | |
| Raw Returns | -11.92 | -21.75 | -20.21 | -21.66 |
| | (-0.65) | (-1.15) | (-1.10) | (-1.18) |
| Alpha, C4 | -9.42 | -17.72 | -17.33 | -18.77 |
| | (-0.51) | (-0.92) | (-0.93) | (-1.01) |

Panel B: Long-Short Returns for Realized Skewness Scaled by
Alternative Realized Volatility Estimators

| Realized Skewness scaled by | RV | BPV | minRV | medRV |
|-----------------------------|---------|---------|---------|---------|
| Equal weighted | | | | |
| Raw Returns | -43.44 | -24.31 | -27.29 | -31.22 |
| | (-8.91) | (-7.14) | (-7.95) | (-8.58) |
| Alpha, C4 | -45.52 | -26.58 | -29.38 | -33.45 |
| | (-9.26) | (-7.74) | (-8.46) | (-9.09) |
| Value weighted | | | | |
| Raw Returns | -23.80 | -16.64 | -19.96 | -21.75 |
| | (-3.65) | (-3.23) | (-3.86) | (-3.72) |
| Alpha, C4 | -22.55 | -18.35 | -22.64 | -24.50 |
| | (-3.42) | (-3.51) | (-4.31) | (-4.14) |

Panel C: Long-Short Returns for Realized Kurtosis Scaled by
Alternative Realized Volatility Estimators

| Realized Kurtosis scaled by | RV | BPV | minRV | medRV |
|-----------------------------|--------|--------|--------|--------|
| Equal weighted | | | | |
| Raw Returns | 15.77 | 14.69 | 18.13 | 18.54 |
| | (2.98) | (2.45) | (3.32) | (3.59) |
| Alpha, C4 | 12.75 | 10.68 | 14.77 | 15.54 |
| | (2.41) | (1.81) | (2.72) | (3.02) |
| Value weighted | | | | |
| Raw Returns | 13.57 | 11.69 | 13.14 | 12.44 |
| | (2.12) | (1.88) | (2.21) | (2.06) |
| Alpha, C4 | 10.85 | 8.32 | 9.79 | 8.98 |
| | (1.67) | (1.33) | (1.64) | (1.48) |

Research Papers 2011



- 2011-28: Anders Bredahl Kock and Timo Teräsvirta: Forecasting performance of three automated modelling techniques during the economic crisis 2007-2009
- 2011-29: Yushu Li: Wavelet Based Outlier Correction for Power Controlled Turning Point Detection in Surveillance Systems
- 2011-30: Stefano Grassi and Tommaso Proietti: Stochastic trends and seasonality in economic time
- 2011-31: Rasmus Tangsgaard Varneskov: Generalized Flat-Top Realized Kernel Estimation of Ex-Post Variation of Asset
- 2011-32: Christian Bach: Conservatism in Corporate Valuation
- 2011-33: Adrian Pagan and Don Harding: Econometric Analysis and Prediction of Recurrent Events
- 2011-34: Lars Stentoft: American Option Pricing with Discrete and Continuous Time Models: An Empirical Comparison
- 2011-35: Rasmus Tangsgaard Varneskov: Flat-Top Realized Kernel Estimation of Quadratic Covariation with Non-Synchronous and Noisy Asset Prices
- 2011-36: David F. Hendry and Søren Johansen: The Properties of Model Selection when Retaining Theory Variables
- 2011-37: Torben G. Andersen, Tim Bollerslev, Peter F. Christoffersen and Francis X. Diebold: Financial Risk Measurement for Financial Risk Management
- 2011-38: Malene Kallestrup-Lamb: The Role of the Spouse in Early Retirement Decisions for Older Workers
- 2011-39: Torben Schmith, Søren Johansen and Peter Thejll: Statistical analysis of global surface air temperature and sea level using cointegration methods
- 2011-40: Søren Johansen and Bent Nielsen: Asymptotic theory for iterated one-step Huber-skip estimators
- 2011-41: Luc Bauwens, Arnaud Dufays and Jeroen V.K. Rombouts: Marginal Likelihood for Markov-switching and Change-point Garch Models
- 2011-42: Manuel Lukas: Utility-based Forecast Evaluation with Multiple Decision Rules and a New Maxmin Rule
- 2011-43: Peter Christoffersen, Ruslan Goyenko, Kris Jacobs, Mehdi Karoui: Illiquidity Premia in the Equity Options Market
- 2011-44: Diego Amaya, Peter Christoffersen, Kris Jacobs and Aurelio Vasquez: Do Realized Skewness and Kurtosis Predict the Cross-Section of Equity Returns?