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Return Predictability, Model Uncertainty, and Robust Investment

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Abstract

Stock return predictability is subject to great uncertainty. In this paper we use the model confidence set approach to quantify uncertainty about expected utility from investment, accounting for potential return predictability. For monthly US data and six representative return prediction models, we find that confidence sets are very wide, change significantly with the predictor variables, and frequently include expected utilities for which the investor prefers not to invest. The latter motivates a robust investment strategy maximizing the minimal element of the confidence set. The robust investor allocates a much lower share of wealth to stocks compared to a standard investor.

Keywords: Return predictability, Model uncertainty, Model confidence set, Portfolio choice, Loss function.

1 Introduction

There is substantial disagreement regarding existence, stability, specification, and economic significance of stock return predictability. The large literature on return predictability documents that certain variables, for example valuation ratios, help predicting stock market excess returns (see, e.g., Fama and French, 1988; Barberis, 2000; Lettau and Ludvigson, 2001; Lewellen, 2004; Ang and Bekaert, 2007, among many other studies). Strongly supportive evidence of predictive power mostly stems from in-sample analysis. Robustness, stability, and economic significance of predictability is still disputed, as out-of-sample result are much less conclusive (Timmermann, 2008). For example, Welch and Goyal (2008) find that forecasts based on the historical average (HA) are not consistently outperformed by a wide range of predictor variables in univariate predictive regression, and that the performance of predictive regressions changes over time. In some periods certain variables seems to predict excess returns, while in other periods return prediction models perform poorly.

The evidence on return predictability is not only sensitive to whether we look at in-sample or out-of-sample performance, but also to the measure by which return forecasts are evaluated (see, e.g., Pesaran and Timmermann, 1995). Kandel and Stambaugh (1996) use an economic measure based on the real-time performance of an investor, which provides a more relevant performance measure than statistical criteria. Cenesizoglu and Timmermann (2012) document that statistical measures are not very informative about the performance with economic measures.

Several empirical studies accounted for model uncertainty, rather than investigating return predictability for single model specifications. Cremers (2002) documents that even when taking model uncertainty into account by Bayesian model averaging, return prediction models are superior to unconditional forecasts. Using Bayesian model

averaging followed by optimal investment within the average model, Avramov (2002) finds that the Bayesian investor successfully uses return prediction models for portfolio choice. Wachter and Warusawitharana (2009) consider a Bayesian investor who puts low prior probability on return predictability. Even though this investor is skeptical about return predictability, the data are strong enough that investment decisions are influenced by the predictor variables. Aiolfi and Favero (2005) document that asset allocation based on multiple models, rather than a single model, can increase investors' utility. Using forecast combination, Rapach et al. (2010) find that the historical average can be significantly outperformed, even when the individual forecasts perform poorly.

Overall, there is evidence that return prediction models can benefit investors, even when the investment decision takes into account model uncertainty. However, previous approaches do not exploit the implications of the resulting investment decision for the individual return prediction models. In particular, it is ignored how the resulting investment performs within the individual return prediction models, and thus what the range of possible expected utilities under different models is. The range of expected utilities reflects the relevant uncertainty from potential return predictability.

In this paper we use the model confidence set approach of Hansen et al. (2011) to quantify the uncertainty stemming from potential return predictability. In particular, we construct confidence sets for the expected utility from investment. For this, we consider a small investor with CRRA utility, who allocates wealth to stocks and the risk-free asset. The confidence sets contain expected utility under return models which are not rejected by the data for a given confidence level. Return predictability implies that expected utility, and thus the confidence sets, depend on the conditioning variables of the return prediction models. First, we construct such confidence sets for a standard investor who does not use a return prediction model, but relies on the historical average (HA) of

returns to estimate expected returns. Second, we consider an investment strategy that is designed to reduce uncertainty about expected performance for a given confidence set of models. This robust strategy chooses investment such that the minimal element of the confidence set is maximized. This investment corresponds to an robust investor who maximizes to minimal expected utility over all models in the confidence set. The robust investment gives insight into how the uncertainty regarding expected utility depends on the investment.

The methodology described above is applied to monthly returns on the US stock market for 1945:12-2011:12. The potential predictors are 14 variables in the data set of Welch and Goyal (2008), which we group into valuation, financial market, corporate finance, and bond market variables. The candidate models are based on either or all of these four groups of variables and are in addition compared to the unconditional HA model. Forecasts of expected returns from each model are constructed using the Rapach et al. (2010) forecast combination approach. For the valuation variables the sum-of-part forecasting approach of Ferreira and Santa-Clara (2011) is also considered as an alternative. Conditional variances are modeled as GARCH(1,1).

Even for this small set of six models, the resulting model uncertainty is overwhelming: No model can be rejected in real-time at common confidence levels in any month in our out-of-sample period 1966:1-2011:12. Thus, the model confidence set contains all models considered at all times, and their implied expected utilities enter the confidence set of expected utilities. The large model uncertainty translates into large economic uncertainty regarding expected utility. The magnitude of uncertainty, measured by width of confidence sets, changes significantly with the predictor variables. Thus, it is important for investors to look at conditional uncertainty. In particular, we find that during recessions, when return predictability is strongest, uncertainty is very high.

Frequently, the confidence sets for the standard investor contain expected utilities lower than utility from holding the risk-free asset only. The robust investment strategy leads to investments that are much lower than for the standard investor, and frequently to holding only the risk-free asset. A range of modifications is considered, and we conclude that the above findings are not sensitive to specification of investors, estimation strategy or model confidence set construction.

Our findings add to the literature on model uncertainty in stock return prediction. The magnitude of uncertainty from potential return predictability in economic terms and the strong dependence on the predictor variables have not been documented before. It is statistically impossible to discriminate between different models for stock returns from the available data, as the model confidence set approach reveals. This model uncertainty translates into uncertainty about expected utility from investing in stocks. Our findings show that expected utility from investment is subject to large model uncertainty, even if the investment decision is based on multiple models.

The remainder of the paper is structured as follows. In Section 2 we present the investment problem, the econometric approach for constructing confidence sets, and the robust investment strategy. Section 3 discusses data and models used in the empirical analysis. Section 4 presents the empirical results. The sensitivity analysis in Section 5 shows that our findings are robust to modifications of investor and models. Concluding remarks are given in Section 6.

2   Investment and Confidence Sets

This section sets up the investment problem, presents the econometric methodology for confidence set construction, and introduces robust investment.

2.1 The Investment Problem

We study the real-time investment decisions of an small investor in the spirit of Kandel and Stambaugh (1996). The investor faces a one-period portfolio selection problem. The return on the risk-free asset is r_{t+1}^f and the excess return on stocks, the risky asset, is r_{t+1}. Returns are continuously compounded. At time t the risk free rate r_{t+1}^f is known, while the excess stock return r_{t+1} is uncertain. The investor has initial wealth of 1 to invest at every time t. At time t the investor has to decide what share of wealth θ_t to invest in stocks. The remaining wealth $1 - \theta_t$ is held in the risk free asset. The investor's final wealth at time $t+1$ is

$$W_{t+1} = \theta_t \exp(r_{t+1}^f + r_{t+1}) + (1 - \theta_t) \exp(r_{t+1}^f). \tag{1}$$

The investor's utility for wealth level W is given by constant relative risk aversion (CRRA) utility,

$$U(W) = \frac{W^{1-\gamma}}{1 - \gamma}, \tag{2}$$

with constant relative risk aversion coefficient $\gamma > 1$. As a function of investment and excess return, the utility is $U(\theta_t, r_{t+1}) = \frac{1}{1-\gamma}(\theta_t \exp(r_{t+1}^f + r_{t+1}) + (1 - \theta_t) \exp(r_{t+1}^f))^{1-\gamma}$. In order to calculate expected utilities, the investor needs a model for r_{t+1}. Given a model of conditional returns the investor can maximize expected utility. The *expected utility* from investing θ_t in stocks, is

$$\mathbb{E}_t \left[U(\theta_t, r_{t+1}) \right] = \frac{1}{1 - \gamma} \mathbb{E}_t \left[(\theta_t \exp(r_{t+1}^f + r_{t+1}) + (1 - \theta_t) \exp(r_{t+1}^f))^{1-\gamma} \right]. \tag{3}$$

The investor maximizes his expected utility in period t by investing

$$\theta_t = \arg\max_{\theta \in [0,1]} \mathbb{E}_t \left[U(\theta_t, r_{t+1}) \right]. \tag{4}$$

We impose the standard restriction $\theta \in [0,1]$, such that the investor can neither short-sell nor borrow.

The optimal portfolio requires a model for the conditional expectation as input. Given the high uncertainty regarding existence and form of return predictability, the investor might be unable to specify a unique conditional model for returns. Therefore, we next consider confidence sets for return models.

2.2 Confidence Sets of Expected Utility

Assume there is a set $\mathcal{M}_t = \{1, \ldots, m\}$ of potential return prediction models, including the unconditional HA model. Every model specifies a conditional density for return r_{t+1}, and thus a conditional expectation. Let $\mathbb{E}_{t,i}$ be the conditional expectation under model $i \in \mathcal{M}_t$. For such a set of models $\mathcal{M}_t$, we construct the model confidence set (MCS) at every time t. We denote the MCS by $\mathcal{M}_t^*$.

Loosely speaking, the MCS of Hansen et al. (2011) is a subset of the models, $\mathcal{M}_t^* \subseteq \mathcal{M}_t$, which contains the best model with $1-\alpha$ confidence. The best model is the one with highest expected utility in our setting. The MCS is constructed using past observation on outcomes (returns) and past predictions (in our case, optimal investments) from all models in $\mathcal{M}_t$. The confidence level $1 - \alpha$ controls how strong the statistical evidence against a model needs to be in order to exclude it from the MCS. The MCS approach captures statistical model uncertainty. The harder it is to identify the best model, the more models are included in the MCS. If one model performs significantly better than all competitors, then it becomes the only element of $\mathcal{M}_t^*$. Details on the implementation

of the MCS approach are given in Section 3.3.

Based on model confidence set $\mathcal{M}_t^*$ we can construct a confidence set for expected utility from investment θ_t as

$$\mathcal{C}_t(\theta_t) = \{\mathbb{E}_{t,i}[U(\theta_t, r_{t+1})] : i \in \mathcal{M}_t^*\}. \tag{5}$$

The confidence set $\mathcal{C}_t(\theta_t)$ is a measure of uncertainty about expected utility for investment θ_t. It contains the expected utility for all models that cannot be excluded from the model confidence set. As the expected utility depends on investment θ_t, so does the confidence set.

For easier interpretation, we transform expected utilities to the corresponding certainty equivalent returns. The certainty equivalent return (CER) under model i at time t for investment θ_t is

$$CER_{i,t}(\theta_t) = ((1-\gamma)\mathbb{E}_{t,i}[U(\theta_t, r_{t+1})])^{1/(1-\gamma)}.$$

Calculating the CER for all elements of $\mathcal{C}_t(\theta_t)$ we get a time t confidence set for the CER of investment θ_t.

The confidence set presented above is a tool to quantify uncertainty regarding expected utility associated with a certain investment strategy. It allows us to quantify uncertainty for a standard investor who uses the historical mean to guide his investment decision. Beyond this, we are interested in characterizing investment for which the uncertainty is lower, in a way that we shall discuss in the next section.

2.3 Robust Investment

The confidence sets for expected utility are a functions of investment θ_t. We use this to explore how the investor needs to set investment in order to reduce uncertainty. Specifically, we construct a robust investment strategy that maximizes the lowest expected utility in the confidence set.

At every time t we obtain a confidence set of expected utility $\mathcal{C}_t(\theta)$, for each possible investment θ. We want to find the investment $\theta_t^R \in [0,1]$ for which the minimal element of the resulting confidence set, $\min \mathcal{C}_t(\theta_t^R)$, is as high as possible:

$$\theta_t^R = \arg\max_{\theta \in [0,1]} \left(\min \mathcal{C}_t(\theta) \right). \tag{6}$$

The investment θ_t^R is the robust investment. By construction, θ_t^R has (weakly) higher expected utility than holding the risk free asset under all models in the MCS, because we can always set $\theta_t = 0$ to get the same expected utility under all models. The robust investment only allocates wealth to the risky asset if expected utility increases under all models compared to setting $\theta_t = 0$.

The robust investment in equation (6) is a special version of maxmin investment. Maxmin investment rules have drawn some attention in the portfolio choice literature (see, e.g., Epstein and Wang, 1994; Maenhout, 2004; Garlappi et al., 2007). Maxmin rules reflect an extreme attitude toward model uncertainty, i.e., they reflect model uncertainty *aversion* (see, e.g., Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001). Our robust investment strategy applies the maxmin rule over the model confidence set, such that it can be interpreted as an investor who is averse to uncertainty over the set of models, that are not rejected by the data.

3 Models and Data

This section discusses models, data, estimation, and model confidence set construction.

3.1 Variables and Data

A large portion of the uncertainty regarding return predictability stems from uncertainty regarding which variables should be used as predictors. The benchmark model, over which the investor wishes to improve expected utility, is the unconditional model:

- Using no predictor variables yields the historical average (HA) model, which specifies expected excess returns as a constant.

We consider predictors from the popular data set[1] of Welch and Goyal (2008). Stock returns are calculated from Center for Research in Security Prices (CRSP) data on the S&P 500 index. We follow Welch and Goyal (2008) in the construction of the variables from this data set. The variables are grouped in four categories for the further analysis:

- Financial Market (*fin*): Long-Term Rate of Return (*ltr*) and the variance of stock returns computed from daily returns (*vars*).

- Corporate Finance (*cor*): Dividend-earnings ratio (*d/e*), ratio of 12-month net equity issues over end-of-year market capitalization (*ntis*).

- Bond market (*bond*): Default yield spreads (*dfy*) measured by yield difference between AAA and BAA -rated corporate bonds, term spread between long-term bond and Treasury bill yields (*tms*), default return spread (*dfr*) between long-term corporate bonds and long term government bonds, long-term yields (*ltr*), and inflation (*inf*).

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[1] An updated data set until end of 2011 is available from Amit Goyal's homepage `http://www.hec.unil.ch/agoyal/`.

- Valuation ratios (*val*): Two methods are considered for forecasting with valuation ratios, for which different variables are used. For the sum-of-parts forecasting method of Ferreira and Santa-Clara (2011) we use earnings per share (*e/s*) and dividend-price ratio (*d/p*). When the sum-of-parts forecasting method is not used, then the valuation ratios are the ones considered by Welch and Goyal (2008): dividend-price ratio (*d/p*), dividend yield (*d/y*), 10-year moving average of earnings-price ratio (*e10/p*), and the book-to-market ratio (*b/m*).

Additionally, we consider a model with all variables:

- *All* of the above 13 variables as predictors.

In total this gives six candidate model that can be used to predict returns by the investor, including the unconditional HA model.

3.2 Return Prediction Models and Forecasts

In this next section we discuss the estimation and forecasting approach taken to turn the groups of variables into return prediction models. We use normal densities to model the conditional distribution of monthly excess returns,

$$r_{t+1|t} \sim N(\mu_{t+1}, \sigma_{t+1}^2),$$

where conditional mean and conditional variance are functions of the predictor variables.

For the conditional mean, linear models are considered. In the context of predictive regression, multivariate least-squares regression is known to produce noisy estimates, and very poor out-of-sample performance (see, e.g., the kitchen sink model in Welch and Goyal, 2008). To deal with this problem, we follow Rapach et al. (2010) by using combinations of univariate forecasts for the conditional mean. For every variable x_v we

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estimate a univariate linear model,

$$r_{t+1} = c_v + \beta_v' x_{v,t} + \epsilon_{v,t+1}, \tag{7}$$

where c_v is the intercept, β_v is the slope parameter, and $\epsilon_{v,t+1}$ are zero mean error terms. For the HA model, equation (7) only features a constant and no predictors. Using data up to time t, we get the least-squares estimates $\hat{c}_{v,t}$ and $\hat{\beta}_{v,t}$ using an expanding estimation window. From this estimated model, we get the conditional mean forecast,

$$\hat{\mu}_{v,t+1} = \hat{c}_{v,t} + \hat{\beta}_{v,t} x_{v,t},$$

for the univariate predictive regression with variable v. The conditional mean forecast for a group of variables $v = 1, \ldots, j$, for example the five *bond* variables, is the simple average of the univariate conditional mean forecasts from each variable in the group:

$$\hat{\mu}_{t+1,c} = \frac{1}{j} \sum_{k=1}^{j} \hat{\mu}_{t+1,k}.$$

Other weights than uniform weights could be applied, in particular weights could be made data-dependent. Using equal weighting avoids additional estimation error and has been found to perform well for predictive regression (see Rapach et al., 2010).

For valuation ratios (*val*) we do not use the forecast combination approach described above. Instead we use the sum-of-part (SOP) forecasting approach of Ferreira and Santa-Clara (2011), which does not require parameter estimation. This method exploits the time series characteristics of three return components (dividend-price ratio, earnings growth rate, and price-earnings ratio growth rate) to obtain return forecasts. The SOP

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conditional mean forecast is given by

$$\hat{\mu}_{t+1,SOP} = \hat{\mu}_{ge} + \hat{\mu}_{dp} - r_{t+1}^f.$$

The first component $\hat{\mu}_{ge}$ is the average growth in earnings per share over the past 20 years, estimating the expected earnings growth. The second component $\hat{\mu}_{dp}$ is the time t dividend-price ratio, which is the forecast of the dividend price-ratio under a random walk model. For the price-earnings ratio this forecast implies no growth.

The conditional variance, σ_{t+1}^2, is modeled as GARCH(1,1), such that

$$
\begin{aligned}
\epsilon_{t+1} &\sim N(0,\sigma_{t+1}^2), \\
\sigma_{t+1}^2 &= \omega + \gamma_1 \epsilon_t^2 + \gamma_2 \sigma_t^2,
\end{aligned}
$$

where ω, γ_1, and γ_2 are parameters. An estimated GARCH(1,1) model based on the residuals of the univariate regressions with variable v gives a variance forecast $\hat{\sigma}_{v,t+1}^2$. For the density forecast based on a group of variables, we construct a variance forecast as the simple average of variance forecasts based on residuals from the univariate regressions:

$$\hat{\sigma}_{t+1,c}^2 = \frac{1}{j} \sum_{v=1}^{j} \hat{\sigma}_{v,t+1}^2.$$

The volatility forecasts based on different residuals are very similar, such that the results are not sensitive to the way we construct the volatility forecast.

3.3 Model Confidence Set Construction

Next we discuss the exact implementation of the model confidence set (MCS) procedure of Hansen et al. (2011) used in this paper. The MCS is a subset of $\mathcal{M}_t$ that contains the

best model with $1 - \alpha$ confidence level. The best model is the one with lowest expected loss for a given loss function. The MCS at time t is denoted by $\mathcal{M}_t^*$, suppressing the dependence on the confidence level $1 - \alpha$. To construct $\mathcal{M}_t^*$, a sample of E losses up to time t for each model in $\mathcal{M}_t$ is needed. The MCS algorithm uses sequential testing of equal predictive ability. At every step of the sequential testing, critical values are obtained using a moving-block bootstrap. Based on this sequential testing a p-value for each model is obtained. These p-values tell us whether a certain model is member of the MCS for a given confidence level.

The investor's relevant loss function, here taken as the negative of his CRRA utility, is used to obtain the sample of E losses for each model. Results from forecast comparison for models of financial returns and volatility depend on the loss function and can, e.g., differ between utility-based and statistical loss functions (see, e.g., West et al., 1993; Gonzàlez-Rivera et al., 2004; Skouras, 2007; Cenesizoglu and Timmermann, 2012). The investor's realized losses are based on forecasts, and thus cannot be computed from the beginning of the available sample. We therefore reserve the first M observations for initial parameter estimation, such that when we have a sample of N observations at time t, the MCS is based on $E = N - M$ losses:

$$\underbrace{\overbrace{t - N + 1, \ldots, t - E}^{M}, \overbrace{t - E + 1, \ldots, t - 1, t}^{E}}_{N}.$$

Later in the sample more data are available to construct the MCS. We consider both expanding sample and rolling windows approaches to construct the MCS. A larger sample will give the MCS more power to exclude models. If, however, the performance of models varies over time, having a longer history of past losses is not necessarily more informative regarding expected performance.

4 Empirical Results

Our sample spans the period 1946:1 to 2011:12 ($N = 792$). All variables are at monthly frequency. The first 120 observations are reserved for initial estimation ($M = 120$). Another 120 observations are used for construction of the first model confidence set. The model confidence sets are constructed from a rolling window of fixed length 120 in this section. Thus, the out-of-sample period, for which we observe investments and confidence sets, is 1966:01-2011:12 (552 observations). All results in this section are for a risk aversion parameter $\gamma = 5$.

Return predictability appears to interact strongly which the business cycle. There is evidence that expected excess returns are higher during recessions (see, e.g., Fama and French, 1989; Henkel et al., 2011). We therefore identify NBER recessions in the results.

Before looking at the confidence sets, we consider the performance of the return prediction models in our sample. For this purpose the certainty equivalent return relative to the historical average investment, ΔCER, is computed for the return prediction models. For model i, ΔCER is given by

$$\Delta CER_i = \left((1-\gamma) \frac{1}{S} \sum_{t=1}^{S} U(\theta_{i,t}, r_{t+1}) \right)^{1/(1-\gamma)} - \left((1-\gamma) \frac{1}{S} \sum_{t=1}^{S} U(\theta_t^A r_{t+1}) \right)^{1/(1-\gamma)}, \quad (8)$$

where θ_t^A is the time t investment based on the HA model, $\theta_{i,t}$ is the investment for model i, and $S = 552$ is the number of observations.

Table 1 shows that the considered return prediction models do produce higher CER than for the HA investment, with the exception of the model that only uses the corporate finance (*cor*) variables. The improvements are of economically significant magnitude. Consistent with previous studies, we find that this predictability is concentrated on re-

cession times. Even though the return prediction models seem to improve the investor's performance, the improvement might not be statistically significant, such that we cannot conclude from Table 1 that there is return predictability. For statistical inference regarding performance of the models, we apply the model confidence set approach in the following.

Table 1: Annualized ΔCER in percentage

	1966:01	1976:01	2000:01	Recession	No Recession
All	1.86	2.07	4.09	9.06	0.54
Val	1.37	1.82	4.71	6.91	0.36
Fin	1.65	1.21	1.15	9.46	0.23
Cor	-0.29	-0.25	-1.34	-1.39	-0.08
Bon	1.36	0.47	-1.02	6.54	0.41

Note: Annualized certainty equivalent return difference to historical average investment (ΔCER) in percentages. First column identifies the group of predictor variables used. The dates indicate the starting date for the (sub-)sample. End date is 2011:12 for all (sub-)samples.

4.1 Confidence Sets and Investment

First we look at the evidence of real-time model uncertainty by computing series of model confidence sets for different confidence levels $1 - \alpha$. For every month in the out-of-sample period, Figure 1 shows which models are included in the MCS. For $\alpha = 0.25$ in Panel (a), and thus a confidence level of 0.75, the HA model and all five return prediction models are included in the MCS every single month. This suggests that real-time model uncertainty is very high, and that investors could not reject any of the models based on past performance. Also, it is not possible to infer from the data whether returns are predictable or not. For lower confidence level, with $\alpha = 0.4$ and $\alpha = 0.5$ in Panel (b) and (c), resp., models start getting excluded from the MCS. However, this does not correspond to common confidence levels. Therefore we will in the following focus on

$\alpha = 0.25$. Any other $\alpha < 0.25$ will give identical results by construction, as the MCS will also contain all models every month.
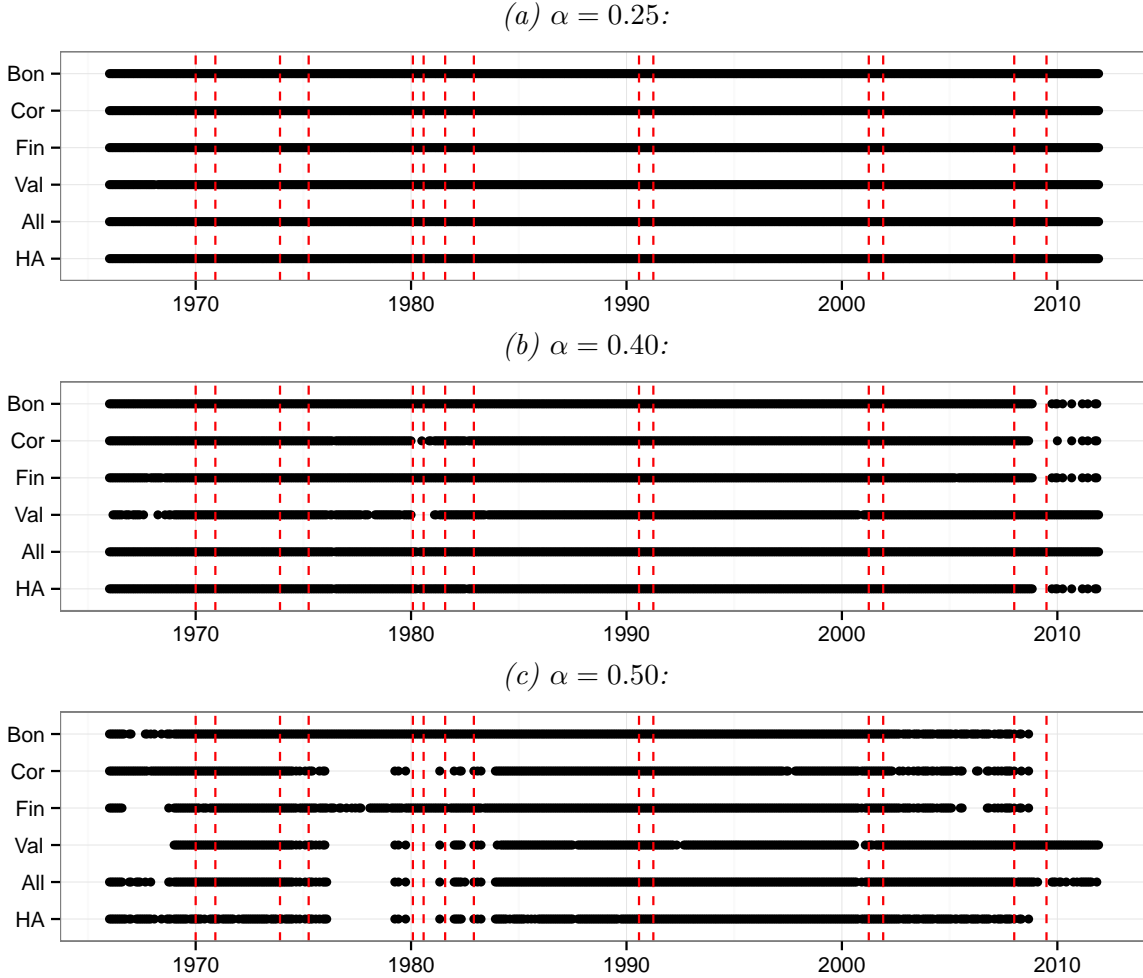

(a) $\alpha = 0.25$:

(b) $\alpha = 0.40$:

(c) $\alpha = 0.50$:

Figure 1: Inclusion in Model Confidence Set for different α. A dot indicates that the model is included in the real-time MCS for this month. Model confidence set are constructed with rolling window of 120 observations. Sample period is 1966:1-2011:12. Loss is based on risk aversion of $\gamma = 5$. Dashed red lines indicate start- and end-dates of NBER recessions.

Figure 2 shows the series of investments in stocks for the HA model and the model using all variables, as well as the robust investments. The HA investment series is more stable than the other two series. The upper bound of 1 is binding during some periods, while investment is always above the lower bound of zero. Investment based on all

variables is more volatile, and both lower and upper bound on investment are binding during some periods. This can be explained by the fact that return prediction model can produce negative forecasts for expected excess returns (see, e.g., Campbell and Thompson, 2008). The robust investment is much lower than the other two investment series, as we can see from Panel (d), where all three investment series are shown. Frequently the robust investment is zero.

In Table 2 the certainty equivalent return gains, as defined in equation (8), are shown for the robust investment. Like the return prediction models, the robust investment strategy achieves a higher CER than the HA model. During recessions, however, the CER gain is much smaller than for the return prediction models.

Table 2: Robust Investment annualized ΔCER in percentage

	1966:01	1976:01	2000:01	Recession	No Recession
$\alpha = 0.25$	1.30	1.53	0.81	0.57	1.42
$\alpha = 0.4$	1.27	1.72	1.182	2.12	1.12
$\alpha = 0.5$	0.92	0.94	-0.72	0.58	0.98

Note: Annualized certainty equivalent return difference to historical average investment (ΔCER) in percentages. The dates indicate the starting date for the (sub-)sample. End date is 2011:12 for all (sub-)samples.

Figure 3 shows confidence sets for certainty equivalent returns that are constructed from model confidence sets. For HA investment, the size of the confidence sets varies considerably over time, and exhibits persistence. Potential predictor variables matter tremendously for the size of the confidence sets and thus for the uncertainty that investors face. During some periods, confidence sets are narrow and all elements imply a CER higher than the risk free rate. During other periods, the lowest CER in the confidence set is below the risk free rate. NBER recessions fall into latter category. This is interesting, because return predictability is strongest during recession, when also uncertainty is very high. Before 1990, both lowest and highest element of the confidence

set are more stable than after 1990. For robust investment, the confidence set do not go below the risk free rate by construction.

The empirical findings can be summarized as follows. Model uncertainty is very high. At no point in our sample can any return prediction model be excluded from the MCS for $\alpha = 0.25$. This model uncertainty translates into very wide confidence sets for expected utility for the standard investors who uses the HA model. The uncertainty, measured by width of the confidence sets, shows large variations over time. During recessions, when return prediction models perform best, uncertainty about expected utility is high. A robust investment strategy maximizing the minimal element of the expected utility confidence set leads to much lower investment in stocks and frequently to holding only the risk free asset.

5 Sensitivity Analysis

We investigate the sensitivity of the empirical findings in three directions. First, the the risk aversion γ of the investor is varied. Second, changes to parameter estimation of the prediction models are considered. Third, model confidence sets are constructed with an expanding rather than rolling windows. Finally, we discuss likely consequences of parameter uncertainty, which we have ignored so far.

5.1 Risk Aversion

To assess the sensitivity to changes in the specification of the investor, we repeat the analysis for different values of the risk aversion parameter γ. Changing γ affects the utility and loss function. Thus, the series of optimal investments changes. Therefore, the MCS and CER confidence set are potentially affected as well, as they are based on a different loss function. In Figure 4 results for a less risk averse investor with $\gamma = 2$ are

shown. For this loss function the model based on the *val* variables is excluded from the MCS during the early to mid-90s. The bounds of the CER confidence sets still show the same characteristics. For investment based on HA investment, the no-borrowing restriction is binding most of the time for the investors. The robust investment strategy fluctuates strongly and still sets investment to zero frequently. For a higher risk aversion of $\gamma = 10$, shown in Figure 5, it is also the model based on *val* that is excluded from the MCS for certain periods, but now earlier in the sample than for $\gamma = 2$. The dynamics of the CER confidence sets for HA investment do not change. The HA investment series is much smoother, and the short-selling and borrowing constraints are never binding. The robust investment is also smoother than for lower risk aversion, but still sets investment to zero frequently.

5.2 Estimation and MCS construction

We repeat the analysis with all models estimated by multiple least-squares regression instead of the forecast combination from univariate regressions. This approach is applied for all groups of variables, including valuation ratios. Figure 6 shows that with the multiple regression approach, all models except the *cor* model get excluded during certain periods. The CER confidence sets have changed significantly. Both the lower and upper bound fluctuate much more over time. Also, the lower bound is below the risk-free rate more frequently. As a consequence, the robust investment is very low on average.

As an additional robustness check we replace the sum-of-parts (SOP) forecast method for valuation variables with forecast combination of univariate regressions, just as for the other groups of variables. The *val* variables used for this are dividend-price ratio (d/p), dividend yield (d/y), 10-year moving average of earnings-price ratio ($e10/p$), and

the book-to-market ratio (b/m). Figure 7 shows that the performance of the *val* model deteriorates significantly, compared the the SOP method. The *val* model is excluded from the MCS during 1996:10 to 2005:10. This does have a strong effect on the CER confidence sets. The bounds fluctuate much less than when SOP forecasts are used for the *val* model. Thus, the *val* forecast must have been behind these large fluctuations, because all other models remain unchanged. The impact on robust investment is that investment is now positive most of the time, and zero investments are very infrequent. However, the robust investment is still much lower than HA investment. Overall, it appears that the valuation variable group, and the way it is used for prediction, is very important for uncertainty about expected utility. With the SOP approach, the model is quite influential, but the SOP model performs better, as the model used in this modifications leads to frequent exclusion from the MCS.

In the previous analysis, model confidence sets are based on a fixed window of 10 year of past data (120 observation). This approach is motivated by time-varying performance of the models documented in the literature. The costs of limiting the sample size with a fixed window is that the MCS will have low power to reject models. To investigate whether the high model uncertainty we found above comes from low power of the MCS, we consider estimating the MCS based on all available past data at every time t, i.e., using an expanding window to construct the MCS. The finding is that the MCS for $\alpha = 0.25$ still contains all six models in all months in our sample, like in the case of rolling windows. Thus, the confidence sets and investment series remain unchanged and we therefore do not report the results for this case.

The focus of our analysis is on model uncertainty, and parameter uncertainty is ignored completely. Additionally accounting for parameter uncertainty by, for example, considering confidence intervals for model parameters, would increase the uncertainty

regarding return predictions for the individual models. Confidence sets for expected utility would be even wider. For the robust investment for the case allowing also for parameter uncertainty would even be higher.

Overall, we conclude that the empirical findings presented in Section 4 are not sensitive to the specification of the investor, model confidence sets, and forecast construction.

6 Conclusion

We have used the model confidence set approach to describe the uncertainty from potential return predictability. We found that this uncertainty is substantial both in statistical and economic terms. Uncertainty varies with potential predictor variables. Reducing the uncertainty requires lowering investment substantially compared to investment based on expected return forecasts from historical averages.

Our findings add to the literature by providing a new view on model uncertainty in return predictability. Rather that considering investment decisions based on multiple models, which is how previous studies dealt with the model uncertainty, we quantify the uncertainty for a given investment decision. This shows that potential return predictability entails large uncertainty regarding expected performance, even if the investment decision is based on multiple models.

Our result are obtained for a given set of models using common predictor variables, which is motivated be previous findings in the literature. Indeed, the models considered produce economic gains from return prediction in our sample. Modifications to the investor, model confidence sets, and model estimation have been shown not to change the empirical findings. We have ignored parameter uncertainty, but taking this into account would lead to more uncertainty, such that the findings could not change qualitatively.

One possibility how a different set of models could change the results qualitatively

is if a model is included which significantly outperforms other models, such that they are excluded from the model confidence set. This would reduce model uncertainty and thus potentially the uncertainty regarding expected utility. At this point, however, no such model seems to be available for the prediction of stock returns.

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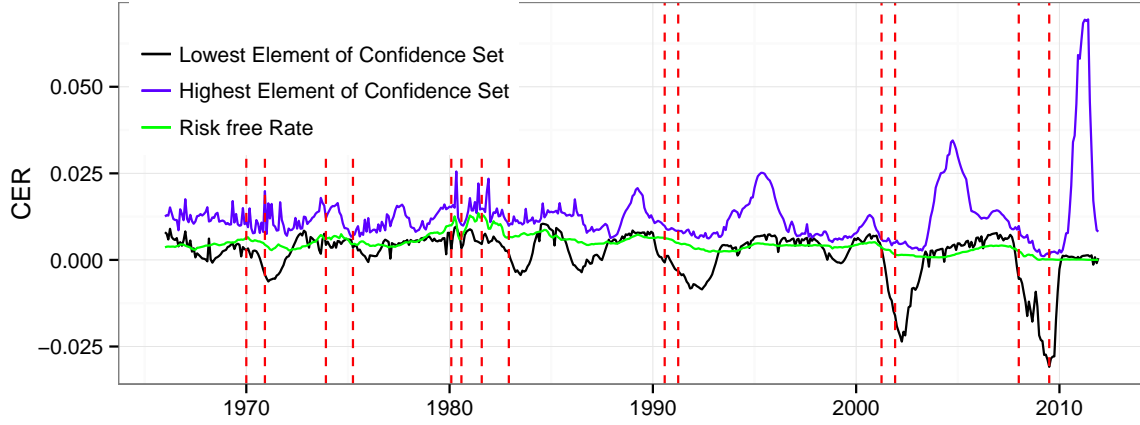
(a) HA investment:

(b) All investment:

(c) Robust investment:

(d) All three investment series:
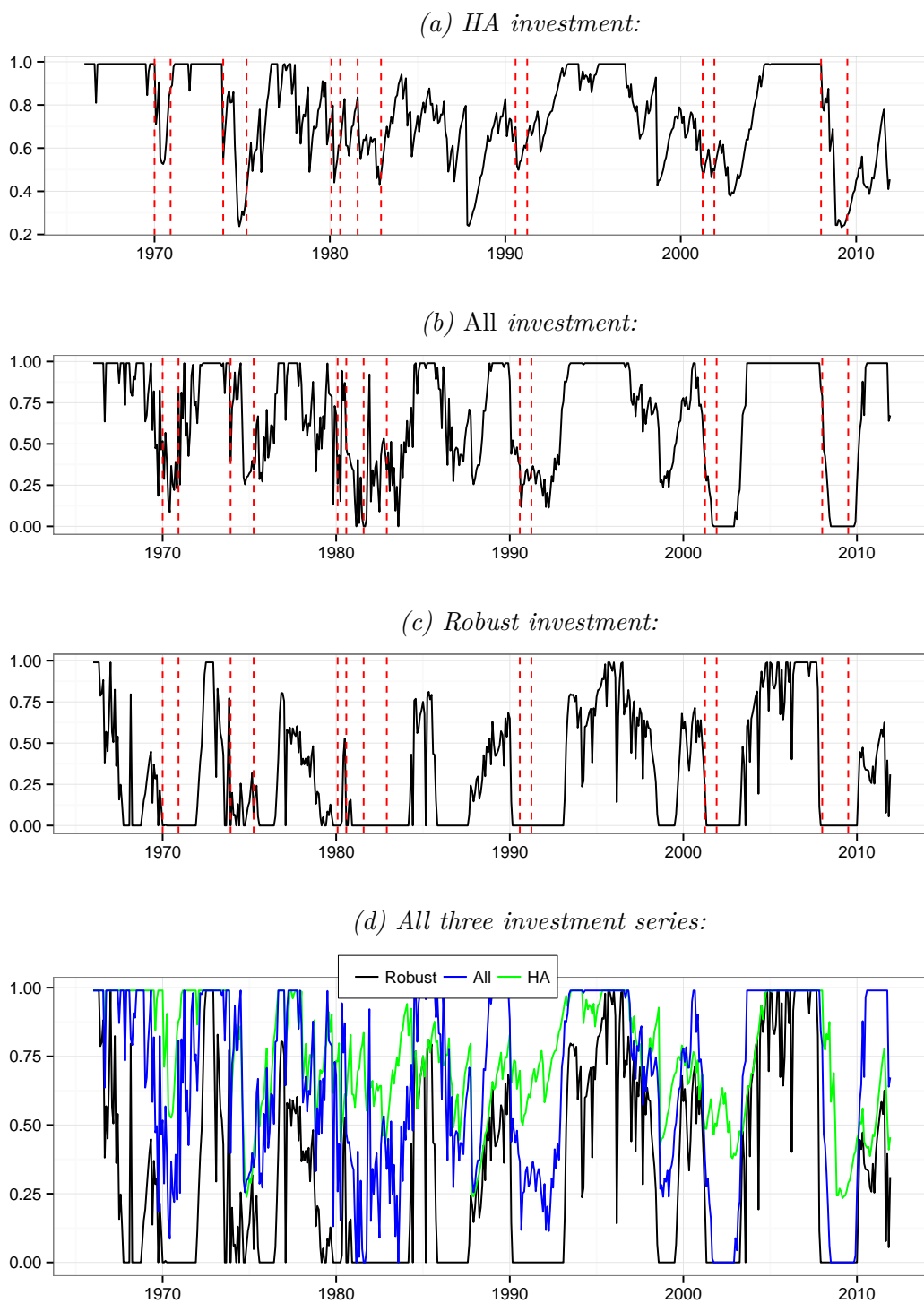
Figure 2: Investment in stocks, θ_t, for model with all variables, historical average HA, and robust investment. Robust investment for $\gamma = 5$. Dashed red lines indicate start- and end-dates of NBER recessions.

(a) CER confidence sets for HA investment:

(b) CER confidence sets for robust investment:
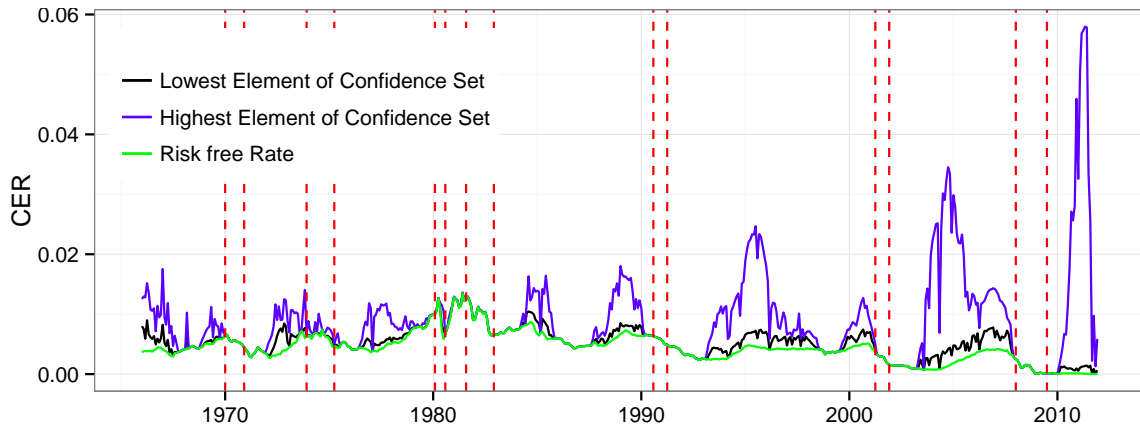
Figure 3: CER confidence sets and risk free rate. $\gamma = 5$. Confidence sets are constructed with rolling window of 120 observations. $\alpha = 0.25$. Dashed red lines indicate start- and end-dates of NBER recessions.
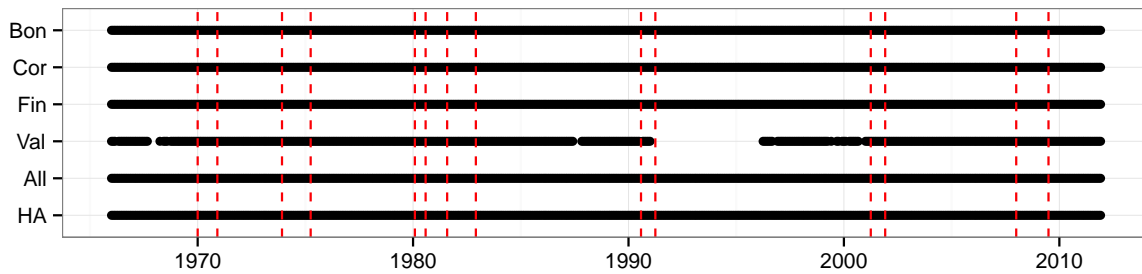
(a) Model inclusion:

(b) CER confidence sets:
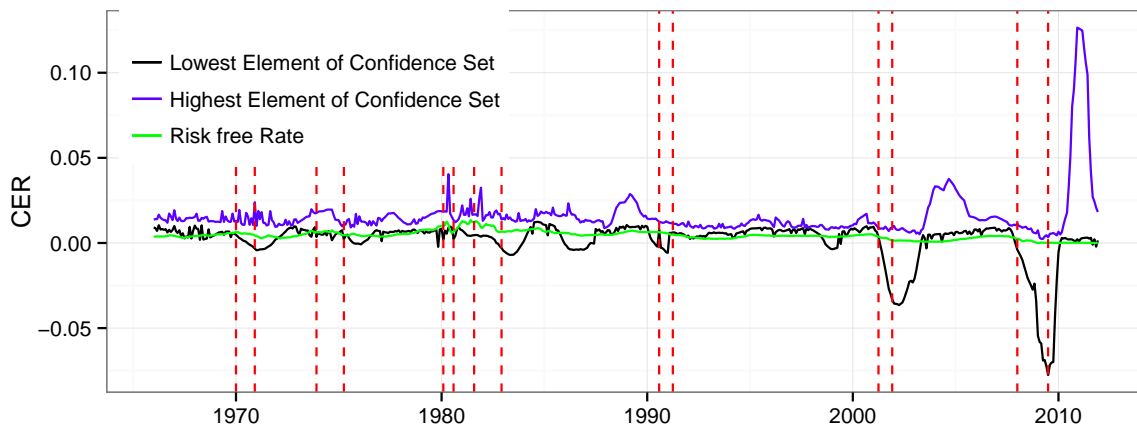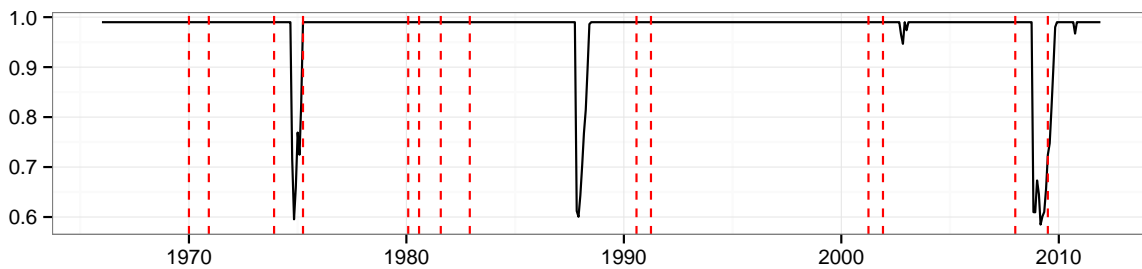
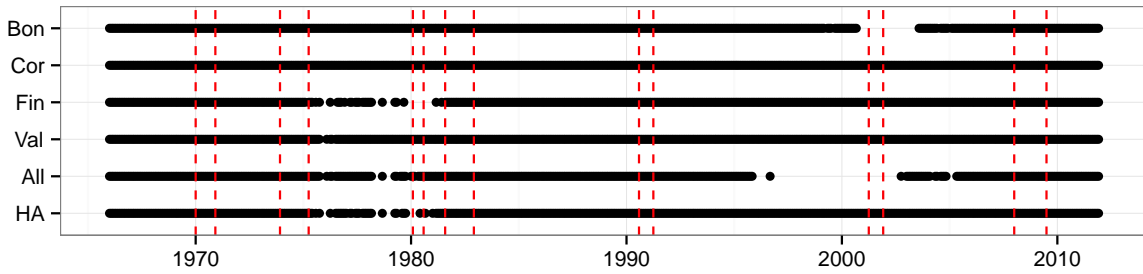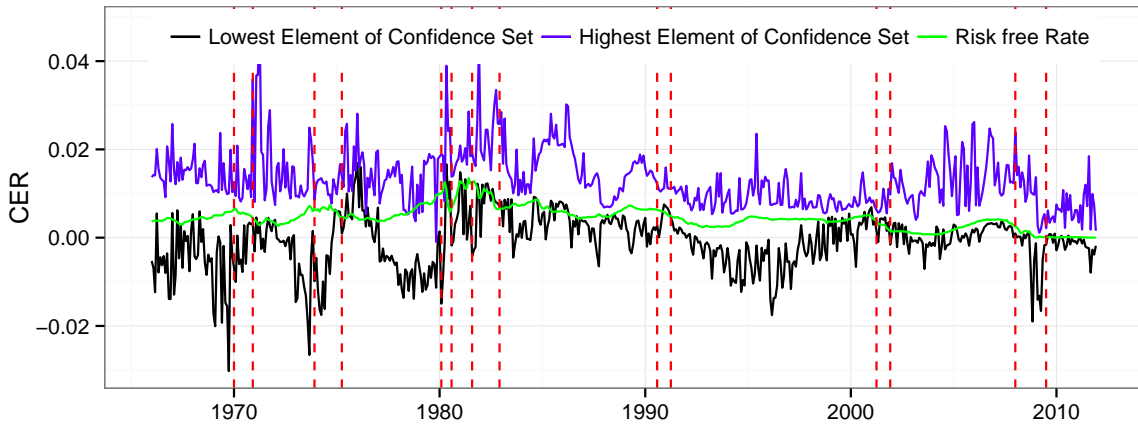*(c) HA investments:*

(d) Robust investments:
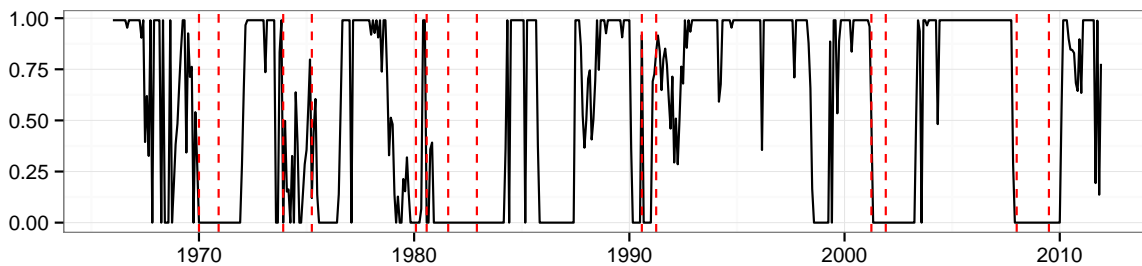
Figure 4: Results for lower risk aversion, $\gamma = 2$. MCS for $\alpha = 0.25$ and rolling window. Dashed red lines indicate start- and end-dates of NBER recessions.
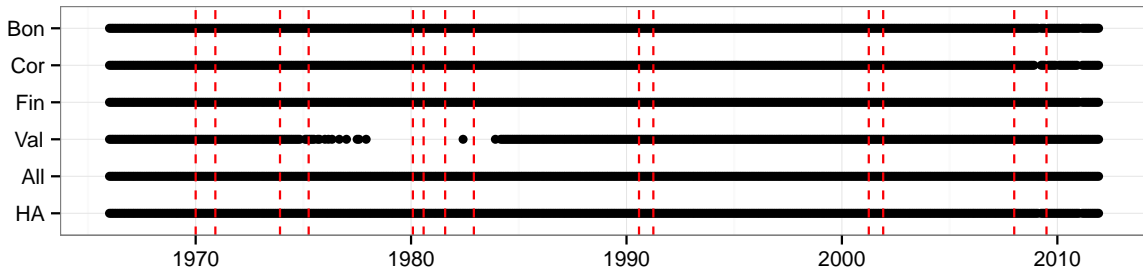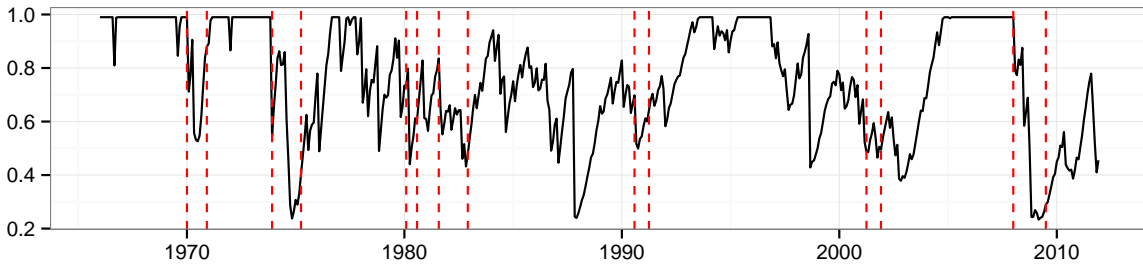
(a) Model inclusion:

(b) CER confidence sets:
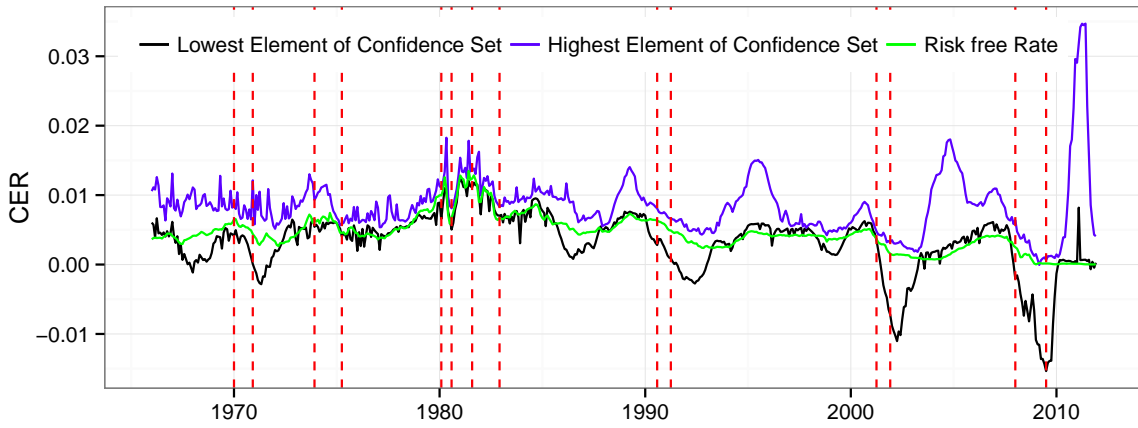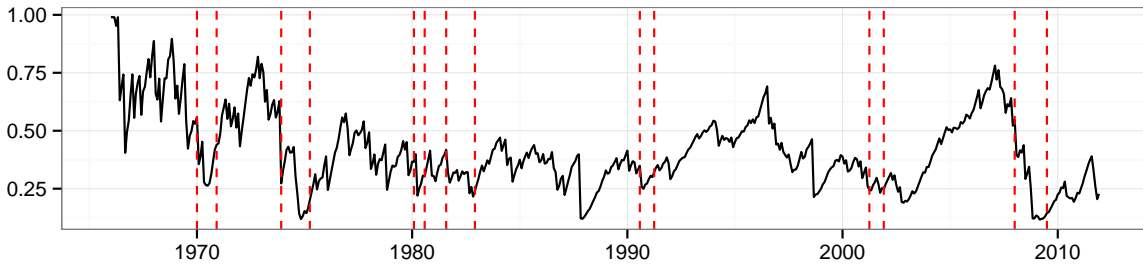
(c) HA investments:

(d) Robust investments:
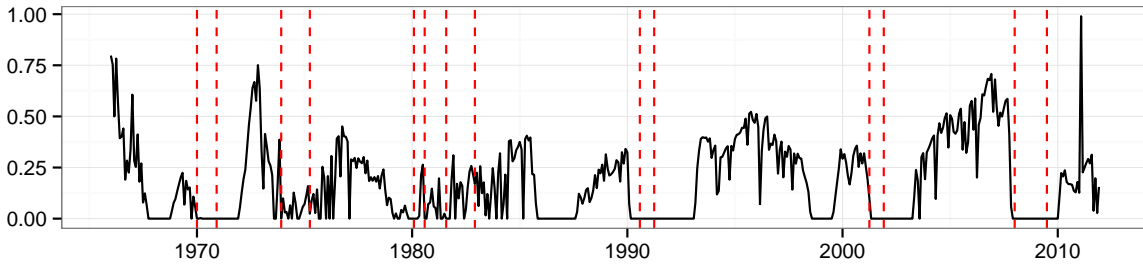
Figure 5:   Results for higher risk aversion, $\gamma = 10$. MCS for $\alpha = 0.25$ and rolling window. Dashed red lines indicate start- and end-dates of NBER recessions.

(a) Model inclusion:

(b) CER confidence sets:
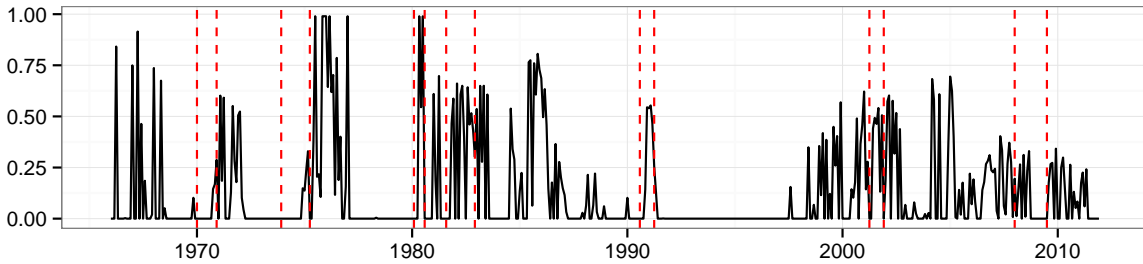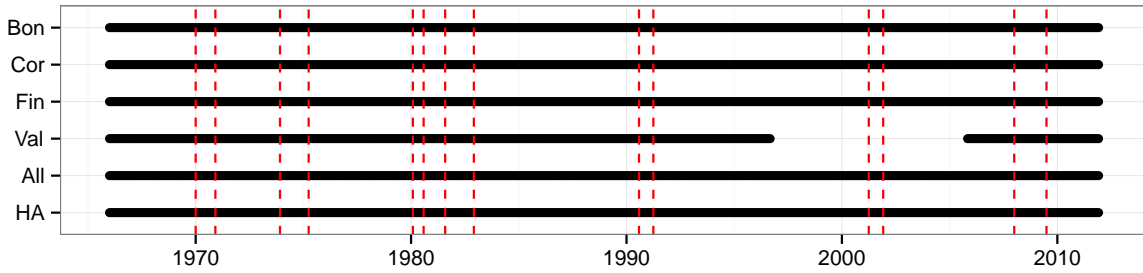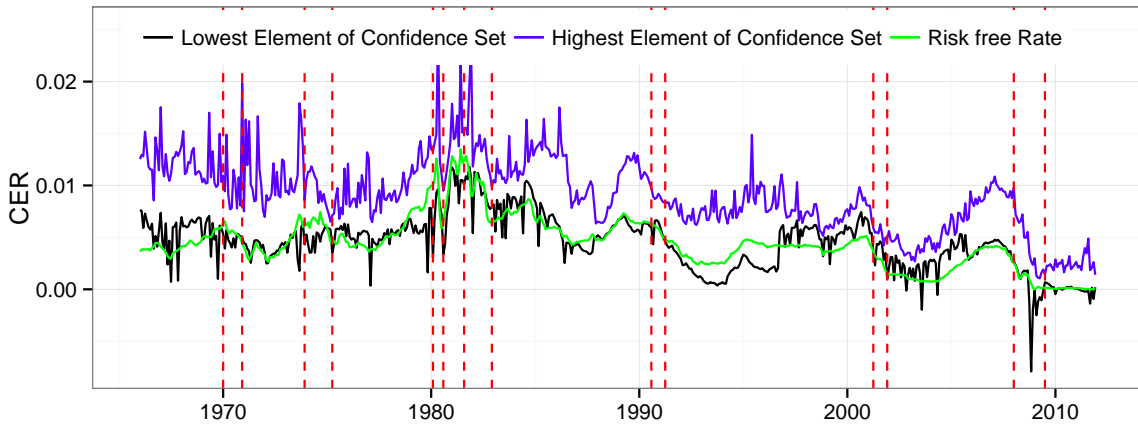
(c) HA investments:

(d) Robust investments:

Figure 6: Multiple regressions with for $\gamma = 5$ and $\alpha = 0.25$. Dashed red lines indicate start- and end-dates of NBER recessions.
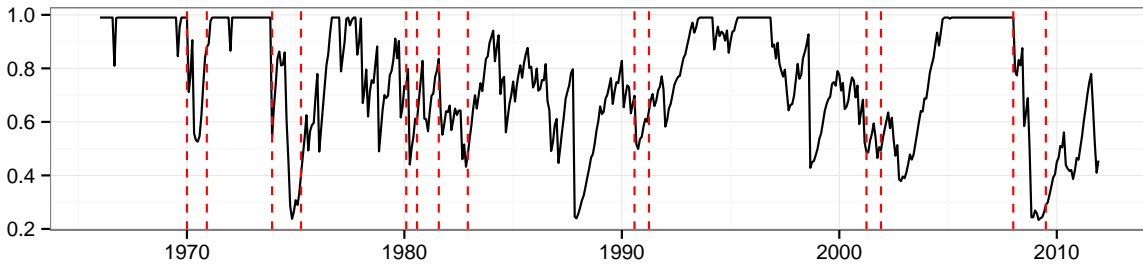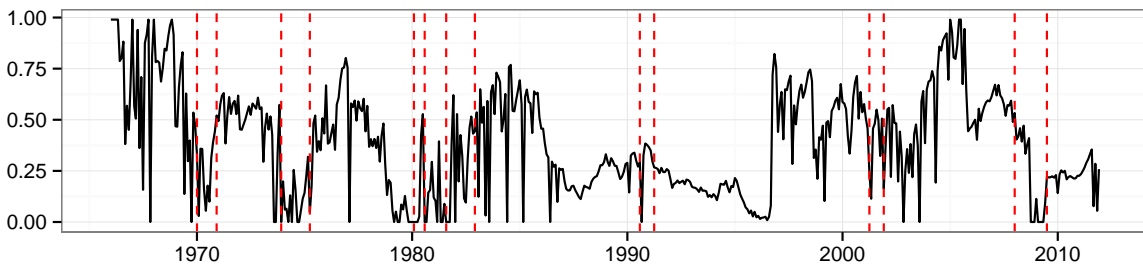
(a) Model inclusion:

(b) CER confidence sets:

(c) HA investments:

(d) Robust investments:

Figure 7: *Valuation variable (val) based forecast with average of univariate regression, following Rapach et al. (2010), instead of sum-of-parts approach. Risk aversion $\gamma = 5$, and MCS for $\alpha = 0.25$ with rolling window. Dashed red lines indicate start- and end-dates of NBER recessions.*

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