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Abstract

In this paper we investigate the effects of careful modelling the long-run dynamics of the volatilities of stock market returns on the conditional correlation structure. To this end we allow the individual unconditional variances in Conditional Correlation GARCH models to change smoothly over time by incorporating a nonstationary component in the variance equations. The modelling technique to determine the parametric structure of this time-varying component is based on a sequence of specification Lagrange multiplier-type tests derived in Amado and Teräsvirta (2011). The variance equations combine the long-run and the short-run dynamic behaviour of the volatilities. The structure of the conditional correlation matrix is assumed to be either time independent or to vary over time. We apply our model to pairs of seven daily stock returns belonging to the S&P 500 composite index and traded at the New York Stock Exchange. The results suggest that accounting for deterministic changes in the unconditional variances considerably improves the fit of the multivariate Conditional Correlation GARCH models to the data. The effect of careful specification of the variance equations on the estimated correlations is variable: in some cases rather small, in others more discernible. As a by-product, we generalize news impact surfaces to the situation in which both the GARCH equations and the conditional correlations contain a deterministic component that is a function of time.

JEL classification: C12; C32; C51; C52.

Key words: Multivariate GARCH model; Time-varying unconditional variance; Lagrange multiplier test; Modelling cycle; Nonlinear time series.

1 Introduction

Many financial issues, such as hedging and risk management, portfolio selection and asset allocation rely on information about the covariances or correlations between the underlying returns. This has motivated the modelling of volatility using multivariate financial time series rather than modelling individual returns separately. A number of multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models have been proposed, and some of them have become standard tools for financial analysts. For recent surveys of Multivariate GARCH models see Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009b).

In the univariate setting, volatility models have been extensively investigated. Many modelling proposals of univariate financial returns have suggested that nonstationarities in return series may be the cause of the extreme persistence of shocks in estimated GARCH models. In particular, Mikosch and Stărică (2004) showed how the long-range dependence and the 'integrated GARCH effect' can be explained by unaccounted structural breaks in the unconditional variance. Previously, Diebold (1986) and Lamoureux and Lastrapes (1990) also argued that spurious long memory may be detected from a time series with structural breaks.

The problem of structural breaks in the conditional variance can be dealt with by assuming that the ARCH or GARCH model is piecewise stationary and detecting the breaks; see for example Berkes, Gombay, Horváth, and Kokoszka (2004), or Lavielle and Teyssière (2006) for the multivariate case. It is also possible to assume, as Dahlhaus and Subba Rao (2006) recently did, that the parameters of the model change smoothly over time such that the conditional variance is locally but not globally stationary. These authors proposed a locally time-varying ARCH process for modelling the nonstationarity in variance. van Bellegem and von Sachs (2004), Engle and Gonzalo Rangel (2008) and, independently, Amado and Teräsvirta (2011) assumed global nonstationarity and, among other things, developed an approach in which volatility is modelled by a multiplicative decomposition of the variance to a nonstationary and stationary component. The stationary component is modelled as a GARCH process, whereas the nonstationary one is a deterministic time-varying component. In van Bellegem and von Sachs (2004) this component is estimated nonparametrically using kernel estimation, whereas in Engle and Gonzalo Rangel (2008), it is an exponential spline. Amado and Teräsvirta (2011) described the nonstationary component by a linear combination of logistic functions of time and their generalisations and developed a data-driven specification technique for determining the parametric structure of the time-varying component. The parameters of both the unconditional and the conditional component were estimated jointly.

Despite the growing literature on multivariate GARCH models, little attention has been devoted to modelling multivariate financial data by explicitly allowing for nonstationarity in variance. Recently, Hafner and Linton (2010) proposed what they called a semiparametric generalisation of the scalar multiplicative model of Engle and Gonzalo Rangel (2008). Their multivariate GARCH model is a first-order BEKK model with a deterministic nonstationary or 'long run' component. In fact, their model is closer in spirit to that of van Bellegem and von Sachs (2004), because they estimate the nonstationary component nonparametrically. The authors suggested an estimation procedure for the parametric and nonparametric components and established semiparametric efficiency of their estimators.

In this paper, we consider a parametric extension of the univariate multiplicative GARCH model of Amado and Teräsvirta (2011) to the multivariate case. We investigate the effects of careful modelling of the time-varying unconditional variance on the correlation structure of Conditional Correlation GARCH (CC-GARCH) models. To this end, we allow the individual unconditional variances in the multivariate GARCH models to change smoothly over time by incorporating a nonstationary component in the variance equations. The empirical analysis consists of fitting bivariate conditional correlation GARCH models to pairs of daily return series and comparing the results from models with the time-varying unconditional variance component to models without such a component. As a by-product, we extend the concept of news impact surfaces of Kroner and Ng (1998) to the case where both the variances and conditional correlations are fluctuating deterministically over time. These surfaces illustrate how the impact of news on covariances between asset returns depends both on the state of the market and the time-varying dependence between the returns.

This paper is organised as follows. In Section 2 we describe the Conditional Correlation GARCH model in which the individual unconditional variances change smoothly over time. Estimation of parameters of these models is discussed in Section 3 and specification of the unconditional variance components in Section 4. Section 5 contains the empirical results of fitting bivariate CC-GARCH models to the 21 pairs of seven daily return series of stocks belonging to the S&P 500 composite index. Conclusions can be found in Section 6.

2 The model

2.1 The general framework

Consider a $N \times 1$ vector of return time series $\{\mathbf{y}_t\}$, $t = 1, ..., T$, described by the following vector process:

$$\mathbf{y}_t = \mathsf{E}(\mathbf{y}_t|\mathcal{F}_{t-1}) + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\mathcal{F}_{t-1}$ is the sigma-algebra generated by the available information up until $t-1$. For simplicity, we assume $\mathsf{E}(\mathbf{y}_t|\mathcal{F}_{t-1}) = \mathbf{0}$. The N-dimensional vector of innovations (or now, returns) $\{\boldsymbol{\varepsilon}_t\}$ is defined as

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma}_t^{1/2}\boldsymbol{\zeta}_t \tag{2}$$

where the conditional covariance matrix $\boldsymbol{\Sigma}_t = [\sigma_{ijt}]$ of $\boldsymbol{\varepsilon}_t$ given the information set $\mathcal{F}_{t-1}$ is a positive-definite $N \times N$ matrix. The error vector $\boldsymbol{\zeta}_t$ form a sequence of independent and identically distributed variables with mean zero and a positive definite correlation matrix $\mathbf{P}_t$. This implies $\mathbf{P}_t^{-1/2}\boldsymbol{\zeta}_t \sim iid(\mathbf{0}, \mathbf{I}_N)$. Under these assumptions, the error vector $\boldsymbol{\varepsilon}_t$ satisfies the following moments conditions:

$$\begin{aligned}
&\mathsf{E}(\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}) = \mathbf{0} \\
&\mathsf{E}(\boldsymbol{\varepsilon}_t|\boldsymbol{\varepsilon}_t'|\mathcal{F}_{t-1}) = \boldsymbol{\Sigma}_t = \mathbf{D}_t\mathbf{P}_t\mathbf{D}_t'
\end{aligned} \tag{3}$$

where $\mathbf{D}_t$ is a diagonal matrix of time-varying standard deviations. It is now assumed that $\mathbf{D}_t$ consists of a conditionally heteroskedastic component and a deterministic time-dependent one such that

$$\mathbf{D}_t = \mathbf{S}_t\mathbf{G}_t \tag{4}$$

where $\mathbf{S}_t = \mathrm{diag}(h_{1t}^{1/2}, ..., h_{Nt}^{1/2})$ contains the conditional standard deviations $h_{it}^{1/2}, i = 1, ..., N$, and $\mathbf{G}_t = \mathrm{diag}(g_{1t}^{1/2}, ..., g_{Nt}^{1/2})$. The elements g_{it}, $i = 1, ..., N$, are positive-valued deterministic functions of rescaled time, whose structure will be defined in a moment. Equations (3) and (4) jointly define the time-varying covariance matrix

$$\boldsymbol{\Sigma}_t = \mathbf{S}_t\mathbf{G}_t\mathbf{P}_t\mathbf{G}_t\mathbf{S}_t. \tag{5}$$

It follows that

$$\sigma_{ijt} = \rho_{ijt}(h_{it}g_{it})^{1/2}(h_{jt}g_{jt})^{1/2},\ i \neq j \tag{6}$$

and that

$$\sigma_{iit} = h_{it}g_{it},\ i = 1, ..., N. \tag{7}$$

From (7) it follows that $h_{it} = \sigma_{iit}/g_{it} = \mathsf{E}(\varepsilon^*_{it}\varepsilon^{*\prime}_{it}|\mathcal{F}_{t-1})$, where $\varepsilon^*_{it} = \varepsilon_{it}/g_{it}^{1/2}$. When $\mathbf{G}_t \equiv \mathbf{I}_N$ and the conditional correlation matrix $\mathbf{P}_t \equiv \mathbf{P}$, one obtains the Constant Conditional Correlation (CCC-) GARCH model of Bollerslev (1990). More generally, when $\mathbf{G}_t \equiv \mathbf{I}_N$ and $\mathbf{P}_t$ is a time-varying correlation matrix, the model belongs to the family of Conditional Correlation GARCH models.

Following Amado and Teräsvirta (2011), the diagonal elements of the matrix $\mathbf{G}_t$ are defined as follows:

$$g_{it} = 1 + \sum_{l=1}^{r} \delta_{il} G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il}) \tag{8}$$

where $\gamma_{il} > 0$, $i = 1, ..., N$, $l = 1, ..., r$. Each g_{it} varies smoothly over time satisfying the conditions $\inf_{t=1,...,T} g_{it} > 0$, and $\delta_{il} \leq M_\delta < \infty$, $l = 1, ..., r$, for $i = 1, ..., N$. The function $G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il})$ is a generalized logistic function, that is,

$$G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il}) = \left(1 + \exp\left\{-\gamma_{il} \prod_{j=1}^{k} (t/T - c_{ilj})\right\}\right)^{-1}, \ \gamma_{il} > 0, \ c_{il1} \leq ... \leq c_{ilk}. \tag{9}$$

Function (9) is by construction continuous for $\gamma_{il} < \infty$, $i = 1, ..., r$, and bounded between zero and one. The parameters, c_{ilj} and γ_{il} determine the location and the speed of the transition between regimes.

The parametric form of (8) with (9) is very flexible and capable of describing smooth changes in the amplitude of volatility clusters. Under $\delta_{i1} = ... = \delta_{ir} = 0$ or $\gamma_{i1} = ... = \gamma_{ir} = 0$, $i = 1, ..., N$, in (8), the unconditional variance of ε_t becomes constant, otherwise it is time-varying. Assuming either $r > 1$ or $k > 1$ or both with $\delta_{il} \neq 0$ adds flexibility to the unconditional variance component g_{it}. In the simplest case, $r = 1$ and $k = 1$, g_{it} increases monotonically over time when $\delta_{i1} > 0$ and decreases monotonically when $\delta_{i1} < 0$. The slope parameter γ_{i1} in (9) controls the degree of smoothness of the transition: the larger γ_{i1}, the faster the transition is between the extreme regimes. As $\gamma_{i1} \to \infty$, g_{it} approaches a step function with a switch at c_{i11}. For small values of γ_{i1}, the transition between regimes is very smooth.

In this work we shall account for potentially asymmetric responses of volatility to positive and negative shocks or returns by assuming the conditional variance components to follow the GJR-GARCH process of Glosten, Jagannathan, and Runkle (1993). In the present context,

$$h_{it} = \omega_i + \sum_{j=1}^{q} \alpha_{ij} \varepsilon^{*2}_{i,t-j} + \sum_{j=1}^{q} \kappa_{ij} I(\varepsilon^*_{i,t-j} < 0) \varepsilon^{*2}_{i,t-j} + \sum_{j=1}^{p} \beta_{ij} h_{i,t-j}, \tag{10}$$

where the indicator function $I(A) = 1$ when A is valid, otherwise $I(A) = 0$. The assumption of a discrete switch at $\varepsilon^*_{i,t-j} = 0$ can be generalised following Hagerud (1997), but this extension is left for later work.

2.2 The structure of the conditional correlations

The purpose of this work is to investigate the effects of modelling changes in the unconditional variances on conditional correlation estimates. The idea is to compare the standard approach, in which the nonstationary component is left unmodelled, with the one relying on the decomposition (5) with $\mathbf{G}_t \neq \mathbf{I}_N$. As to modelling the time-variation in the correlation matrix $\mathbf{P}_t$, several choices exist. As already mentioned, the simplest multivariate correlation model is the CCC-GARCH model in which $\mathbf{P}_t \equiv \mathbf{P}$. With h_{it} specified as in (10), this model will be called the CCC-TVGJR-GARCH model. When $g_{it} \equiv 1$, (10) defines the ith conditional variance of the CCC-GJR-GARCH model.

The CCC-GARCH model has considerable appeal due to its computational simplicity, but in many studies the assumption of constant correlations has been found to be too restrictive. There are several ways of relaxing this assumption using parametric representations for the correlations. Engle (2002) introduced the so-called Dynamic CC-GARCH (DCC-GARCH) model in which the conditional correlations are defined through GARCH(1,1) type equations. Tse and Tsui (2002) presented a rather similar model. In the DCC-GARCH model, the coefficient of correlation ρ_{ijt} is a typical element of the matrix $\mathbf{P}_t$ with the dynamic structure

$$\mathbf{P}_t = \{\text{diag}\mathbf{Q}_t\}^{-1/2}\mathbf{Q}_t\{\text{diag}\mathbf{Q}_t\}^{-1/2} \tag{11}$$

where

$$\mathbf{Q}_t = (1-\theta_1-\theta_2)\overline{\mathbf{Q}} + \theta_1\boldsymbol{\zeta}_{t-1}\boldsymbol{\zeta}'_{t-1} + \theta_2\mathbf{Q}_{t-1} \tag{12}$$

such that $\theta_1 > 0$ and $\theta_2 \geq 0$ with $\theta_1 + \theta_2 < 1$, $\overline{\mathbf{Q}}$ is the unconditional correlation matrix of the standardised errors ζ_{it}, $i = 1, ..., N$, and $\boldsymbol{\zeta}_t = (\zeta_{1t}, ..., \zeta_{Nt})'$. In the model of Tse and Tsui (2002), $\mathbf{Q}_t$ has a definition that is slightly different from (12). In our case, each $\zeta_{it} = \varepsilon_{it}/(h_{it}g_{it})^{1/2}$, and this version of the model will be called the DCC-TVGJR-GARCH model. Accordingly, when $g_{it} \equiv 1$, the model becomes the DCC-GJR-GARCH model.

Another way of introducing time-varying correlations is to assume that the conditional correlation matrix $\mathbf{P}_t$ varies smoothly over time between two extreme states of correlations $\mathbf{P}_{(1)}$ and $\mathbf{P}_{(2)}$; see Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005, 2009a). The correlation matrix is a convex combination of these two matrices:

$$\mathbf{P}_t = \{1 - G(s_t;\gamma,c)\}\mathbf{P}_{(1)} + G(s_t;\gamma,c)\mathbf{P}_{(2)} \tag{13}$$

where $\mathbf{P}_{(1)}$ and $\mathbf{P}_{(2)}$ are positive definite $N \times N$ matrices with ones on the main diagonal and $\mathbf{P}_{(1)} \neq \mathbf{P}_{(2)}$. $G(s_t;\gamma,c)$ is a monotonic function bounded between zero and one, in which the stochastic or deterministic transition variable s_t controls the correlations. More specifically,

$$G(s_t;\gamma,c) = (1+\exp\{-\gamma(s_t-c)\})^{-1}, \ \gamma > 0 \tag{14}$$

where, as in (9), the parameter γ determines the smoothness and c the location of the transition between the two correlation regimes. In this work, $s_t = t^* = t/T$, and we call the resulting model the Time-Varying CC-TVGJR-GARCH (TVCC-TVGJR-GARCH) model when the equations for h_{it} are parameterized as in (10). Defining s_t to be an observable random variable is also possible. When $g_{it} \equiv 1$, (13) reduces to the conditional covariance of the TVCC-GJR-GARCH model.

3 Estimation of parameters

3.1 Estimation of DCC-TVGJR-GARCH models

In this section, we assume that $\omega_i = 1$, $i = 1, ..., N$, in (10) and that (8) has the form

$$g_{it} = \delta_{i0} + \sum_{l=1}^{r}\delta_{il}G_{il}(t/T;\gamma_{il},\mathbf{c}_{il})$$

where $\delta_{i0} > 0$. This facilitates the notation but does not change the argument. Under the assumption of normality, $\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1} \sim N(\mathbf{0},\boldsymbol{\Sigma}_t)$, the conditional log-likelihood function for observation t is defined as

$$\begin{aligned}
\ell_t(\boldsymbol{\theta}) &= -(N/2)\ln(2\pi) - (1/2)\ln|\boldsymbol{\Sigma}_t| - (1/2)\boldsymbol{\varepsilon}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\varepsilon}_t \\
&= -(N/2)\ln(2\pi) - (1/2)\ln|\mathbf{S}_t\mathbf{G}_t\mathbf{P}_t\mathbf{G}_t\mathbf{S}_t| - (1/2)\boldsymbol{\varepsilon}_t'\mathbf{S}_t^{-1}\mathbf{G}_t^{-1}\mathbf{P}_t^{-1}\mathbf{G}_t^{-1}\mathbf{S}_t^{-1}\boldsymbol{\varepsilon}_t \\
&= -(N/2)\ln(2\pi) - \ln|\mathbf{S}_t\mathbf{G}_t| - (1/2)\ln|\mathbf{P}_t| - (1/2)\boldsymbol{\zeta}_t'\mathbf{P}_t^{-1}\boldsymbol{\zeta}_t \\
&= -(N/2)\ln(2\pi) - \ln|\mathbf{G}_t| - (1/2)\widetilde{\boldsymbol{\varepsilon}}_t'\mathbf{G}_t^{-2}\widetilde{\boldsymbol{\varepsilon}}_t - \ln|\mathbf{S}_t| - (1/2)\boldsymbol{\varepsilon}_t^{*\prime}\mathbf{S}_t^{-2}\boldsymbol{\varepsilon}_t^* \\
&\quad +\boldsymbol{\zeta}_t'\boldsymbol{\zeta}_t - (1/2)\ln|\mathbf{P}_t| - (1/2)\boldsymbol{\zeta}_t'\mathbf{P}_t^{-1}\boldsymbol{\zeta}_t
\end{aligned} \tag{15}$$

where $\boldsymbol{\theta}=(\boldsymbol{\psi}',\boldsymbol{\varphi}',\boldsymbol{\phi}')'$ is the vector of all parameters of the model, and

$$\begin{aligned}
\widetilde{\boldsymbol{\varepsilon}}_t &= \mathbf{S}_t^{-1}\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}/\{h_{1t}(\boldsymbol{\psi}_1,\boldsymbol{\varphi}_1)\}^{1/2},...,\varepsilon_{Nt}/\{h_{Nt}(\boldsymbol{\psi}_N,\boldsymbol{\varphi}_N)\}^{1/2})' \\
\boldsymbol{\varepsilon}_t^* &= \mathbf{G}_t^{-1}\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}/\{g_{1t}(\boldsymbol{\psi}_1)\}^{1/2},...,\varepsilon_{Nt}/\{g_{Nt}(\boldsymbol{\psi}_N)\}^{1/2})' \\
\boldsymbol{\zeta}_t &= \mathbf{G}_t^{-1}\mathbf{S}_t^{-1}\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}/\{g_{1t}(\boldsymbol{\psi}_1)h_{1t}(\boldsymbol{\psi}_1,\boldsymbol{\varphi}_1)\}^{1/2},...,\varepsilon_{Nt}/\{g_{Nt}(\boldsymbol{\psi}_N)h_{Nt}(\boldsymbol{\psi}_N,\boldsymbol{\varphi}_N)\}^{1/2})'.
\end{aligned}$$

Equation (15) implies the following decomposition of the log-likelihood function for observation t :

$$\ell_t(\boldsymbol{\psi},\boldsymbol{\varphi},\boldsymbol{\phi}) = \ell_t^U(\boldsymbol{\psi}) + \ell_t^V(\boldsymbol{\psi},\boldsymbol{\varphi}) + \ell_t^C(\boldsymbol{\psi},\boldsymbol{\varphi},\boldsymbol{\phi})$$

where first, $\boldsymbol{\psi} = (\boldsymbol{\psi}_1',...,\boldsymbol{\psi}_N')'$, and

$$\ell_t^U(\boldsymbol{\psi}) = \sum_{i=1}^N \ell_{it}^U(\boldsymbol{\psi}_i) \tag{16}$$

with $\boldsymbol{\psi}_i = (\delta_{i0},\boldsymbol{\delta}_i',\boldsymbol{\gamma}_i',\mathbf{c}_i')'$, $\boldsymbol{\delta}_i = (\delta_{i1},...,\delta_{ir})'$, $\boldsymbol{\gamma}_i = (\gamma_{i1},...,\gamma_{ir})'$, $\mathbf{c}_i = (\mathbf{c}_{i1}',...,\mathbf{c}_{ir}')'$, $i = 1,...,N$, and

$$\ell_{it}^U(\boldsymbol{\psi}_i) = -(1/2)\{\ln g_{it}(\boldsymbol{\psi}_i) + \widetilde{\varepsilon}_{it}^2/g_{it}(\boldsymbol{\psi}_i)\}.$$

Second,

$$\ell_t^V(\boldsymbol{\psi},\boldsymbol{\varphi}) = \sum_{i=1}^N \ell_{it}^V(\boldsymbol{\psi}_i,\boldsymbol{\varphi}_i) \tag{17}$$

where $\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1',...,\boldsymbol{\varphi}_N')'$, and

$$\ell_{it}^V(\boldsymbol{\psi}_i,\boldsymbol{\varphi}_i) = -(1/2)\{\ln h_{it}(\boldsymbol{\psi}_i,\boldsymbol{\varphi}_i) + \widetilde{\varepsilon}_{it}^2/h_{it}(\boldsymbol{\psi}_i,\boldsymbol{\varphi}_i)\}.$$

with $\boldsymbol{\varphi}_i = (\alpha_{i1},...,\alpha_{iq},\kappa_{i1},...,\kappa_{iq},\beta_{i1},...,\beta_{ip})'$, $i = 1,...,N$. Finally,

$$\ell_t^C(\boldsymbol{\psi},\boldsymbol{\varphi},\boldsymbol{\phi}) = -(1/2)\{\ln|\mathbf{P}_t(\boldsymbol{\psi},\boldsymbol{\varphi},\boldsymbol{\phi})| + \boldsymbol{\zeta}_t'\mathbf{P}_t^{-1}(\boldsymbol{\psi},\boldsymbol{\varphi},\boldsymbol{\phi})\boldsymbol{\zeta}_t - 2\boldsymbol{\zeta}_t'\boldsymbol{\zeta}_t\}. \tag{18}$$

The GARCH equations are estimated separately using maximization by parts. The first iteration consists of the following:

1. Reparameterise the deterministic component (8) as follows:

$$g_{it}^* = \delta_{i0}^* + \sum_{l=1}^r \delta_{il}^* G_{il}(t/T;\gamma_{il},\mathbf{c}_{il}).$$

   and set $\boldsymbol{\psi}_i^* = (\delta_{i0}^*,\boldsymbol{\delta}_i^{*\prime},\boldsymbol{\gamma}_i',\mathbf{c}_i')'$, where $\delta_{i0}^* > 0$ and $\boldsymbol{\delta}_i^* = (\delta_{i1}^*,...,\delta_{ir}^*)'$. Maximize

$$L_{iT}^U(\boldsymbol{\psi}^*) = \sum_{t=1}^T \ell_{it}^U(\boldsymbol{\psi}^*) = -(1/2)\sum_{t=1}^T\{\ln g_{it}^*(\boldsymbol{\psi}_i^*) + \widetilde{\varepsilon}_{it}^2/g_{it}^*(\boldsymbol{\psi}_i^*)\}$$

   for each i, $i = 1,...,N$, separately, assuming $\tilde{\varepsilon}_{it} = \varepsilon_{it}$, that is, setting $h_{it}(\boldsymbol{\psi}_i,\boldsymbol{\varphi}_i) \equiv 1$. The resulting estimators are $\widehat{\boldsymbol{\psi}}_i^{*(1)} = (\widehat{\delta}_{i0}^{*(1)},\widehat{\boldsymbol{\delta}}_i^{*(1)\prime},\widehat{\boldsymbol{\gamma}}_i^{(1)\prime},\widehat{\mathbf{c}}_i^{(1)\prime})'$, $i = 1,...,N$. Obtain $\widehat{\boldsymbol{\delta}}_i^{(1)}$ as follows: $\widehat{\boldsymbol{\delta}}_i^{(1)} = (\widehat{\delta}_{i0}^{*(1)})^{-1}\widehat{\boldsymbol{\delta}}_i^{*(1)}$ so that $\widehat{\boldsymbol{\psi}}_i^{(1)} = (\widehat{\boldsymbol{\delta}}_i^{(1)\prime},\widehat{\boldsymbol{\gamma}}_i^{(1)\prime},\widehat{\mathbf{c}}_i^{(1)\prime})'$. Note that $\widehat{\delta}_{i0}^{*(1)} = \widehat{\omega}_i^{(1)}$.

2. Setting $\boldsymbol{\psi}_i = \widehat{\boldsymbol{\psi}}_i^{(1)}$, $i = 1, ..., N$, in (17), maximize

$$L_{iT}^V(\widehat{\boldsymbol{\psi}}_i^{(1)}, \boldsymbol{\varphi}_i) = \sum_{t=1}^{T} \ell_{it}^V(\widehat{\boldsymbol{\psi}}_i^{(1)}, \boldsymbol{\varphi}_i) = -(1/2)\sum_{t=1}^{T}\{\ln h_{it}(\widehat{\boldsymbol{\psi}}_i^{(1)}, \boldsymbol{\varphi}_i) + \varepsilon_{it}^{*2}/h_{it}(\widehat{\boldsymbol{\psi}}_i^{(1)}, \boldsymbol{\varphi}_i)\}$$

with respect to $\boldsymbol{\varphi}_i$ assuming $\varepsilon_{it}^* = \varepsilon_{it}/g_{it}^{1/2}(\widehat{\boldsymbol{\psi}}_i^{(1)})$, for each i, $i = 1, ..., N$, separately. Call the ith resulting estimators $\widehat{\boldsymbol{\varphi}}_i^{(1)}$.

The second iteration is as follows:

1. Maximize

$$L_{iT}^U(\boldsymbol{\psi}) = \sum_{t=1}^{T} \ell_{it}^U(\boldsymbol{\psi}_i) = -(1/2)\sum_{t=1}^{T}\{\ln g_{it}(\boldsymbol{\psi}_i) + \widetilde{\varepsilon}_{it}^2/g_{it}(\boldsymbol{\psi}_i)\}$$

assuming $\widetilde{\varepsilon}_{it} = \varepsilon_{it}/h_{it}^{1/2}(\widehat{\boldsymbol{\psi}}_i^{(1)}, \widehat{\boldsymbol{\varphi}}_i^{(1)})$, for each i, $i = 1, ..., N$. Call the ith resulting estimator $\widehat{\boldsymbol{\psi}}_i^{(2)}$. The important thing is that $\boldsymbol{\varphi}_i = \widehat{\boldsymbol{\varphi}}_i^{(1)}$ (fixed) in the definition of ε_{it}^*.

2. Maximize

$$L_{iT}^V(\widehat{\boldsymbol{\psi}}_i^{(2)}, \boldsymbol{\varphi}_i) = \sum_{t=1}^{T} \ell_{it}^V(\widehat{\boldsymbol{\psi}}_i^{(2)}, \boldsymbol{\varphi}_i) = -(1/2)\sum_{t=1}^{T}\{\ln h_{it}(\widehat{\boldsymbol{\psi}}_i^{(2)}, \boldsymbol{\varphi}_i) + \varepsilon_{it}^{*2}/h_{it}(\widehat{\boldsymbol{\psi}}_i^{(2)}, \boldsymbol{\varphi}_i)\}$$

with respect to $\boldsymbol{\varphi}_i$ for each i, $i = 1, ..., N$, separately, assuming $\varepsilon_{it}^* = \varepsilon_{it}/g_{it}(\widehat{\boldsymbol{\psi}}_i^{(2)})$. This yields $\widehat{\boldsymbol{\varphi}}_i^{(2)}$, $i = 1, ..., N$.

Iterate until convergence. Call the resulting estimators $\widehat{\boldsymbol{\psi}}_i$ and $\widehat{\boldsymbol{\varphi}}_i$, $i = 1, ..., N$, and set $\widehat{\boldsymbol{\psi}} = (\widehat{\boldsymbol{\psi}}_1', ..., \widehat{\boldsymbol{\psi}}_N')'$ and $\widehat{\boldsymbol{\varphi}} = (\widehat{\boldsymbol{\varphi}}_1', ..., \widehat{\boldsymbol{\varphi}}_N')'$.

Maximization is carried out in the usual fashion by solving the equations

$$\frac{\partial}{\partial \boldsymbol{\psi}_i} L_{iT}^U(\boldsymbol{\psi}_i) = (1/2)\sum_{t=1}^{T}(\frac{\widetilde{\varepsilon}_{it}^2}{g_{it}(\boldsymbol{\psi}_i)} - 1)\frac{1}{g_{it}(\boldsymbol{\psi}_i)}\frac{\partial g_{it}(\boldsymbol{\psi}_i)}{\partial \boldsymbol{\psi}_i} = \mathbf{0}$$

for ψ_i

$$\frac{\partial}{\partial \boldsymbol{\varphi}_i} L_{iT}^V(\boldsymbol{\varphi}_i) = (1/2)\sum_{t=1}^{T}(\frac{\varepsilon_{it}^{*2}}{h_{it}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i)} - 1)\frac{1}{h_{it}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i)}\frac{\partial h_{it}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i)}{\partial \boldsymbol{\varphi}_i} = \mathbf{0}$$

for φ_i in the nth iteration. Writing $G_{ilt} = G(t^*, \gamma_{il}, \mathbf{c}_{il})$, we have

$$\frac{\partial g_{it}(\boldsymbol{\psi}_i)}{\partial \boldsymbol{\psi}_i} = (1, G_{i1t}, G_{i1t}^{(\gamma)}, G_{i1t}^{(c)}, ..., G_{irt}, G_{irt}^{(\gamma)}, G_{irt}^{(c)})'$$

where, for $k = 1$ in (9),

$$\begin{aligned} G_{ilt}^{(\gamma)} &= \frac{\partial G_{ilt}}{\partial \gamma_{il}} = \delta_{il} G_{ilt}(1 - G_{ilt})(t^* - c_{il}),\ l = 1, ..., r \\ G_{ilt}^{(c)} &= \frac{\partial G_{ilt}}{\partial c_{il}} = -\gamma_{il}\delta_{il} G_{ilt}(1 - G_{ilt}),\ l = 1, ..., r \end{aligned}$$

and

$$\frac{\partial h_{it}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i)}{\partial \boldsymbol{\varphi}_i} = (1, \varepsilon_{i,t-1}^{*2}, ..., \varepsilon_{i,t-q}^{*2}, \varepsilon_{i,t-1}^{*2} I(\varepsilon_{i,t-1}^{*} < 0), ..., \varepsilon_{i,t-q}^{*2} I(\varepsilon_{i,t-q}^{*} < 0),$$
$$h_{i,t-1}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i), ..., h_{i,t-p}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i))' + \sum_{j=1}^{p} \beta_{ij} \frac{\partial h_{i,t-j}(\widehat{\boldsymbol{\psi}}_i^{(n)}, \boldsymbol{\varphi}_i)}{\partial \boldsymbol{\varphi}_i}$$

when the conditional variance h_{it} is defined in (10).

After estimating the TVGARCH equations, estimate $\boldsymbol{\phi}$ given $\widehat{\boldsymbol{\psi}}_i$ and $\widehat{\boldsymbol{\varphi}}_i$ by maximizing

$$L_T^C(\boldsymbol{\phi}) = \sum_{t=1}^{T} \ell_t^C(\boldsymbol{\phi}) = -(1/2) \sum_{t=1}^{T} \{\ln |\mathbf{P}_t(\boldsymbol{\phi})| + \boldsymbol{\zeta}_t' \mathbf{P}_t^{-1}(\boldsymbol{\phi}) \boldsymbol{\zeta}_t - 2\boldsymbol{\zeta}_t'\boldsymbol{\zeta}_t\}$$

where $\boldsymbol{\zeta}_t = (\zeta_{1t}, ..., \zeta_{Nt})'$ with $\zeta_{it} = \varepsilon_{it}/\{h_{it}(\widehat{\boldsymbol{\psi}}_i, \widehat{\boldsymbol{\varphi}}_i) g_{it}(\widehat{\boldsymbol{\psi}}_i)\}^{1/2}$, $i = 1, ..., N$, and

$$\frac{\partial}{\partial \boldsymbol{\phi}} L_T^C(\boldsymbol{\phi}) = -(1/2) \sum_{t=1}^{T} \frac{\partial \mathsf{vec}(\mathbf{P}_t)'}{\partial \boldsymbol{\phi}} \mathsf{vec}(\mathbf{P}_t^{-1} - \mathbf{P}_t^{-1} \boldsymbol{\zeta}_t \boldsymbol{\zeta}_t' \mathbf{P}_t^{-1}).$$

All computations in this paper have been performed using Ox, version 3.40, see Doornik (2002), and a modified version of Matteo Pelagatti's source code[1].

This approach is computationally feasible. Engle and Sheppard (2001) only estimate the GARCH equations once and show that for $\mathbf{G}_t = \mathbf{I}_N$, the maximum likelihood estimators $\widehat{\boldsymbol{\varphi}}_i$, $i = 1, ..., N$, (in their framework $g_{it}(\boldsymbol{\psi}_i) \equiv 1$) are consistent. The two-step estimator is, however, asymptotically less efficient than the full maximum likelihood estimator. Further iteration in order to obtain efficient estimators is possible, see Fan, Pastorello, and Renault (2007) for discussion, but it has not been undertaken here. Under regularity conditions, the maximum likelihood estimators of the TVGJR-GARCH equations are consistent and asymptotically normal; see Amado and Teräsvirta (2011).

3.2 Estimation of TVCC-TVGJR-GARCH models

The maximum likelihood estimation of the parameters of the model TVCC-GJR-GARCH model can be carried out in three steps as in Silvennoinen and Teräsvirta (2005, 2009a). The log-likelihood function can be decomposed as before. The components (16) and (17) remain the same, whereas (18) becomes

$$\ell_t^C(\boldsymbol{\psi}, \boldsymbol{\varphi}, \boldsymbol{\xi}) = -(1/2)\{\ln |\mathbf{P}_t(\boldsymbol{\xi})| + \boldsymbol{\zeta}_t' \mathbf{P}_t^{-1}(\boldsymbol{\xi}) \boldsymbol{\zeta}_t - 2\boldsymbol{\zeta}_t'\boldsymbol{\zeta}_t\}$$

where the $\{N(N-1)+2\} \times 1$ vector $\boldsymbol{\xi} = (\mathsf{vecl}(\mathbf{P}_{(1)})', \mathsf{vecl}(\mathbf{P}_{(2)})', \gamma, c)'$. (The vecl operator stacks the columns below the main diagonal into a vector.) In their scheme, the parameter vectors $\boldsymbol{\psi}$ and $\boldsymbol{\varphi}$ of the GARCH equations are estimated first, followed by the conditional correlations in $\mathbf{P}_{(1)}$ and $\mathbf{P}_{(2)}$, given the transition function parameters γ and c in (14). Finally, γ and c are estimated given $\boldsymbol{\psi}$, $\boldsymbol{\varphi}$, $\mathbf{P}_{(1)}$ and $\mathbf{P}_{(2)}$. The next iteration begins by re-estimating $\boldsymbol{\varphi}$ given the previous estimates of $\mathbf{P}_{(1)}$, $\mathbf{P}_{(2)}, \gamma$ and c. The only modification required for the estimation of TVCC-TVGJR-GARCH models compared to Silvennoinen and Teräsvirta (2005) is that for each main iteration there is an inside loop for iterative estimation (maximisation by parts) of $\boldsymbol{\psi}$ and $\boldsymbol{\varphi}$. In practice, compared to the two-step estimates, the extra iterations do not change the estimates much, but the estimators become fully efficient.

[1] The Ox estimation package is freely available at http://www.statistica.unimib.it/utenti/p_matteo/Ricerca/research.html

Asymptotic properties of the maximum likelihood estimators of the TVCC-TVGJR-GARCH model are not yet known. The existing results only cover the CCC-GARCH model; see Ling and McAleer (2003). Due to a time-varying correlation matrix, deriving corresponding asymptotic results for the TVCC-TVGJR-GARCH model is a nontrivial problem and beyond the scope of the present paper. Note that asymptotic normality has been proven for maximum likelihood estimates of the parameters of the TVGJR-GARCH model: our univariate GARCH components are of this form.

4 Modelling with TVGJR-GARCH models

4.1 Specifying the unconditional variance component

In applying a model belonging to the family of CC-TVGJR-GARCH models, there are two specification problems. First, one has to determine p and q in (10) and r in (8). Furthermore, if $r \geq 1$, one also has to determine k for each transition function (9). Second, at least in principle one has to test the null hypothesis of constant conditional correlations against either the DCC- or TVCC-GARCH model. We shall concentrate on the first set of issues. It appears that in applications involving DCC-GARCH models the null hypothesis of constant conditional correlations is never tested, and we shall adhere to that practice. In applications of the STCC-GARCH model, constancy of correlations is always tested before applying the larger model, see Silvennoinen and Teräsvirta (2005, 2009a). The test can be extended to the current situation in which the GARCH equations are TVGJR-GARCH ones instead of plain GJR-GARCH ones. Nevertheless, in this work we assume that the correlations do vary over time as is done in the context of DCC-GARCH models and apply the TVCC-GARCH model without carrying out a correlation constancy test.

We shall thus concentrate on the first set of specification issues. We choose $p = q = 1$ and test for higher orders at the evaluation stage. As to selecting r and k, we shall follow Amado and Teräsvirta (2011) and briefly review their procedure. The conditional variances are estimated first, assuming $g_{it} \equiv 1$, $i = 1, ..., N$. The number of deterministic functions g_{it} is determined thereafter equation by equation by sequential testing. For the ith equation, the first hypothesis to be tested is H_{01}: $\gamma_{i1} = 0$ against H_{11}: $\gamma_{i1} > 0$ in

$$g_{it} = 1 + \delta_{i1} G_{i1}(t/T; \gamma_{i1}, \mathbf{c}_{i1}).$$

The standard test statistic has a non-standard asymptotic distribution because δ_{i1} and $\mathbf{c}_{i1}$ are unidentified nuisance parameters when H_{01} is true. This lack of identification may be circumvented by following Luukkonen, Saikkonen, and Teräsvirta (1988). This means that $G_{i1}(t/T; \gamma_{i1}, \mathbf{c}_{i1})$ is replaced by its mth-order Taylor expansion around $\gamma_{i1} = 0$. Choosing $m = 3$, this yields

$$g_{it} = \alpha_0^* + \sum_{j=1}^{3} \delta_{ij}^* (t/T)^j + R_3(t/T; \gamma_{i1}, \mathbf{c}_{i1}) \tag{19}$$

where $\delta_{ij}^* = \gamma_{i1}^j \widetilde{\delta}_{ij}^*$ with $\widetilde{\delta}_{ij}^* \neq 0$, and $R_3(t/T; \gamma_{i1}, \mathbf{c}_{i1})$ is the remainder. The new null hypothesis based on this approximation is H_{01}': $\delta_{i1}^* = \delta_{i2}^* = \delta_{i3}^* = 0$ in (19). In order to test this null hypothesis, we use the Lagrange multiplier (LM) test. Furthermore, $R_3(t/T; \gamma_{i1}, \mathbf{c}_{i1}) \equiv 0$ under H_{01}, so the asymptotic distribution theory is not affected by the remainder. As discussed in Amado and Teräsvirta (2011), the LM-type test statistic has an asymptotic χ^2-distribution with three degrees of freedom when H_{01} holds.

If the null hypothesis is rejected, the model builder also faces the problem of selecting the order $k \leq 3$ in the exponent of $G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il})$. It is solved by carrying out a short test sequence within (19); for details see Amado and Teräsvirta (2011). The next step is then to estimate the alternative with the chosen k, add another transition, and test the hypothesis $\gamma_{i2} = 0$ in

$$g_{it} = 1 + \delta_{i1}^{*} G_{i1}(t/T; \gamma_{i1}, \mathbf{c}_{i1}) + \delta_{i2}^{*} G_{i1}(t/T; \gamma_{i2}, \mathbf{c}_{i2})$$

using the same technique as before. Testing continues until the first non-rejection of the null hypothesis. The LM-type test statistic still has an asymptotic χ^2-distribution with three degrees of freedom under the null hypothesis.

4.2 The modelling cycle

After specifying the model, its parameters are estimated and the estimated model evaluated. In short, building TVGJR-GARCH models for the elements of $\mathbf{D}_t = \mathbf{S}_t \mathbf{G}_t$ of the CC-GARCH model defined by equations (3) and (4) proceeds as follows:

1. Estimate the conditional variance h_t as in (10) with $p = q = 1$. This may be preceded by testing the null hypothesis of no ARCH. In applications to financial return series of sufficiently high frequency, the test may be omitted as it is clear that volatility clustering occurs.

2. After specifying and estimating h_{it}, test H_{01}: $g_{it} \equiv 1$ against H_{11}: $g_{it} = 1 + \delta_{i1} G_{i1}(t/T; \gamma_{i1}, \mathbf{c}_{i1})$, for $i = 1, ..., N$, at the significance level $\alpha^{(1)}$. In case of a rejection, select k and test H_{02}: $g_{it} = 1 + \delta_{i1} G_{i1}(t/T; \gamma_{i1}, \mathbf{c}_{i1})$ against H_{12}: $g_{it} = 1 + \sum_{l=1}^{2} \delta_{il} G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il})$ at the significance level $\alpha^{(2)} = \tau\alpha^{(1)}$, where $\tau \in (0, 1)$. More generally, $\alpha^{(j)} = \tau\alpha^{(j-1)}$, $j = 2, 3, ...$ The significance level is lowered at each stage for reasons of parsimony. In our application we choose $\tau = 1/2$, but the results are quite robust to the choice of τ in the sense that a wide range of the discount factor values yield the same r.

3. Evaluate the estimated individual TVGJR-GARCH equations by means of LM and LM-type diagnostic tests. For relevant misspecification tests for TV-GARCH models (they are directly applicable to testing TVGJR-GARCH models), see Amado and Teräsvirta (2011). This includes testing for higher orders of p and q in (10). If the models pass the tests, they will be incorporated in multivariate CC-GARCH models. If the multivariate model is the TVCC-GJR-GARCH model, the GARCH equations will be re-estimated as described in Section 3.

We shall now apply the modelling cycle to individual daily return series. As already indicated, the interest lies in how careful modelling of nonstationarity in return series affects correlation estimates. This will be investigated by a set of bivariate CC-GARCH models.

5 Empirical analysis

5.1 Data

The effects of modelling the nonstationarity in return series on the conditional correlations are studied with price series of seven stocks of the S&P 500 composite index traded at the New York Stock Exchange. The time series are available at the website *Yahoo! Finance*. They consist of daily closing prices of American Express (AXP), Boeing Company (BA), Caterpillar (CAT), Intel Corporation (INTC), JPMorgan Chase & Co. (JPM), Whirlpool (WHR) and Exxon Mobil Corporation (XOM). The seven companies belong to different industries that are consumer finance (AXP), aerospace and defence (BA), machines (CAT), semiconductors (INTC), banking (JPM), consumption durables (WHR) and energy (XOM). A bivariate analysis of returns of these companies may give some idea of how different the correlations between firms representing different industries can be. The observation period starts in September 29, 1998 and ends in October 7, 2008, yielding a total of 2521 observations. All stock prices are converted into continuously compounded rates of returns, whose values are plotted in Figure 1.

A common pattern is evident in the seven return series. There is a volatile period from the beginning until the middle of the observation period and a less volatile period starting around 2003 that continues almost to the end of the sample. At the very end, it appears that volatility increases again. Moreover, as expected, all seven return series exhibit volatility clustering, but the amplitude of the clusters varies over time.

Descriptive statistics for the individual return series can be found in Table 1. Conventional measures for skewness and kurtosis and also their robust counterparts are provided for all series. The conventional estimates indicate both non-zero skewness and excess kurtosis: both are typically found in financial asset returns. However, conventional measures of skewness and kurtosis are sensitive to outliers and should therefore be viewed with caution. Kim and White (2004) suggested to look at robust estimates of these quantities. The robust measures for skewness are all positive but very close to zero indicating that the return distributions show very little skewness. All robust kurtosis measures are positive, AXP and JPM being extreme examples of this, which suggests excess kurtosis (the robust kurtosis measure equals zero for normally distributed returns) but less than what the conventional measures indicate. The estimates are strictly univariate and any correlations between the series are ignored.

5.2 Modelling the conditional variances and testing for the nonstationary component

We first construct an adequate GJR-GARCH(1,1) model for the conditional variance of each of the seven return series. The models are tested against misspecification applying the tests in Lundbergh and Teräsvirta (2002) to the GJR-GARCH model. The estimation results can be found in Table 6. The estimated models show a distinct IGARCH effect: some of the estimates of $\alpha_{i1} + \kappa_{i1}/2 + \beta_{i1}$ even exceed unity. In a majority of cases, the asymmetry term $I(\varepsilon^*_{i,t-1} < 0)\varepsilon^{*2}_{i,t-1}$ dominates the term $\varepsilon^{*2}_{i,t-1}$. The misspecification test results in Table 8 do not reveal major inadequacies, except that the GJR-GARCH model seems to get rejected when tested against the model containing also a nonlinear STAR component. This situation changes when the GJR-GARCH model is tested against the TV-GJR-GARCH: the results appear in Table 3 under the heading 'single transition'. The null model is strongly rejected in all seven cases. From the same table it is seen when the single transition model is tested against two transitions ('double transition') that one transition is enough in all cases. The test sequence for selecting the type of transition shows that not all rejections imply a monotonically increasing function g_{it}.

The estimated TV-GJR-GARCH models can be found in Tables 4 and 5. Table 5 shows how the persistence measure $\widehat{\alpha}_{i1} + \widehat{\kappa}_{i1}/2 + \widehat{\beta}_{i1}$ is dramatically smaller in all cases than it is when $g_{it} \equiv 1$. In two occasions, remarkably low values, 0.782 for CAT and 0.888 for WHR, are obtained. For the remaining series the reduction in persistence is smaller but the values are still distinctly different from the corresponding ones in Table 6. From Table 4 it can be seen that $\widehat{g}_{it}$ changes monotonically only for BA, whereas for the other series this component first decreases and then increases again. In INTC and WHR, however, there is an increase very early on, after which the pattern is similar to that of the other four series. This is also clear from Figure 2 that contains the graphs of $\widehat{g}_{it}$ for the seven estimated models. The misspecification tests of Amado and Teräsvirta (2011) in Table 7 suggest that most of the estimated models are adequate. Exceptions include the models of BA and INTC returns for which the tests suggest that the GJR-GARCH type asymmetry is not enough, and a more flexible asymmetry component would be required. For WHR returns, a higher-order GARCH model might be a useful extension.

The effects of modelling the nonstationarity in variance is also illustrated by Figures 3, 4, 5 and 6. Figure 3 shows the estimated conditional standard deviations from the GJR-GARCH models. The behaviour of these series looks rather nonstationary. The conditional standard deviations from the TVGJR-GARCH model can be found in Figure 5. These plots, in contrast to the ones in Figure 3, do not show signs of nonstationarity. The deterministic component g_{it} is able to absorb the changing 'baseline volatility', and only volatility clustering is left to be parameterized by h_{it}. This is clearly seen from the graphs in Figure 5 as they look flatter but retain the peaks visible of those in Figure 3. Moreover, the sample autocorrelations of $|\varepsilon_{it}|$ shown in Figure 4 decay very slowly as a function of the lag length, whereas the decay rate of the autocorrelogram of $|\varepsilon^*_{it}|$ (or $|\varepsilon_{it}|/\widehat{g}_{it}^{1/2}$) in Figure 6 is considerably faster and appears close to the exponential. This is what we would expect after the unconditional variance component has absorbed the long-run movements in the series. These findings justify at an empirical level that the low lever of persistence is exclusively due to the modelling of the unconditional variance.

Furthermore, Table 2 contains the same statistics as Table 1, but now for the standardised returns $\varepsilon_{it}/\widehat{g}_t^{1/2}$. As may be expected, these returns are less leptokurtic than the original ones. Interestingly, there are three series, BA, INTC and XOM, for which the robust kurtosis measure indicates that the marginal distribution of the standardised returns is close to a normal distribution.

5.3 Effects of modelling the long-run dynamics of volatility on correlations

We now examine the effects of modelling nonstationary volatility equations on the correlations between pairs of stock returns. For that purpose, we consider three bivariate Conditional Correlation GARCH models, the CCC-, the DCC-, and the TVCC-GJR-GARCH(1,1) model defined in Section 2.2. For each model, two specifications will be estimated for modelling the univariate volatilities. One is the first-order GJR-GARCH model that corresponds to $\mathbf{G}_t \equiv \mathbf{I}_2$, whereas the other one is the GJR-GARCH model for which $\mathbf{G}_t \neq \mathbf{I}_2$ in (5).

We shall first compare the rolling correlation estimates for $(\varepsilon_{it}, \varepsilon_{jt})$ and $(\varepsilon_{it}/\widehat{g}_{it}^{1/2}, \varepsilon_{jt}/\widehat{g}_{jt}^{1/2})$ pairs. Figure 7 contains the pairwise correlations between the former and the latter computed over 100 trading days. The differences are sometimes quite remarkable in the first half of the series where the correlations of rescaled returns are often smaller than those of the original returns. In a few cases, this is true for the whole series.

This might suggest that there are also differences in conditional correlations between models based on GJR-GARCH type variances and their TVGJR-GARCH counterparts. A look at Figure 8 suggests, perhaps surprisingly, that when one compares DCC-GJR-GARCH models with DCC-TVGJR-GARCH ones, this is not the case. The figure depicts the differences between the conditional correlations over time for the 21 bivariate models. They are generally rather small, and it is difficult to find any systematic pattern in them. The AXP-CAT and CAT-WHR pairs are the two exceptions: for the former pair, the difference lies in the range $(-0.27, 0.32)$ whereas for the latter pair the difference on the correlations moves within the interval $(-0.22, 0.30)$. Thus, one may conclude that if the focus of the modeller is on conditional correlations, taking nonstationarity in the variance into account is not particularly important.

Figure 9 shows the estimated time-varying correlations for the TVCC-GJR-GARCH and TVCC-TVGJR-GARCH models. The parameter estimates can be found in Tables 10 and 11, respectively. For the majority of the estimated models, the estimate of the slope transition parameter γ attains its upper bound of 500. For these cases, the transition function is close to a step function. The differences in correlations have to do with the smoothness of the increase in correlations during the first quarter of the observations. These differences are not systematic: in some cases the increase is smoother in the former model, in others in the latter. In a few cases, the differences are very small. There is only one pair (WHR-XOM) for which the level of the correlations is different after the change is completed. The main conclusion from these comparisons would be rather similar to that obtained from considering DCC-GARCH models. It would also be possible to consider other transition variables than time, but it has not been done here.

Nevertheless, the fit of the models considerably improves when the unconditional variance component is properly modelled. The log-likelihood values of the estimated models for the 21 pairs of return series are reported in Table 9. The maxima of the log-likelihood functions are substantially larger when g_{it} is estimated than when it is ignored. A comparison between DCC- and TVCC-GARCH models suggests that both fit the time series equally well. The TVCC-TVGJR-GARCH model has the highest maximum in 11 cases out of 21, so the ordering between the two is completely random. The constant conditional correlation (CCC-) GARCH model fits the data less well than the other two, which is in line with the previous literature on conditional correlations.

5.4 Time-varying news impact surfaces

In this section we consider the impact of unexpected shocks to the asset returns on the estimated covariances. This is done by employing a generalization of the univariate *news impact curve* of Engle and Ng (1993) to the multivariate case introduced by Kroner and Ng (1998). The so-called *news impact surface* is the plot of the conditional covariance against a pair of lagged shocks, holding the past conditional covariances constant at their unconditional sample mean levels. The news impact surfaces of the multivariate correlation models with the volatility equations modelled as TVGJR-GARCH models are time-varying because they depend on the component $g_{i,t-1}$. They will be called *time-varying news impact surfaces.* The time-varying news impact surface for h_{ijt} is the three-dimensional graph of the function

$$h_{ijt} = f(\varepsilon_{i,t-1}/g_{i,t-1}, \varepsilon_{j,t-1}/g_{j,t-1}, \rho_{ij,t-1}; \mathbf{h}_{t-1})$$

where $\mathbf{h}_{t-1}$ is a vector of conditional covariances at time $t-1$ defined at their unconditional sample means. As an example, Figure 10 contains the time-varying news impact surface for the covariance generated by the CCC-TVGJR-GARCH model for the pair BA-XOM. The choice of this single pair of assets is merely illustrative, but the same shapes of the surfaces can be found for other pairs as well. From the figure we see how the surface can vary over time due to the nonstationary component $g_{i,t-1}$. We are able to distinguish different reaction levels of covariance estimates to past shocks during tranquil and turbulent times. It shows that the response to the news of a given size on the estimated covariances is clearly stronger during periods of calm in the market ('lower regime') than it is during periods of high turbulence. According to the results, when calm prevails a minor piece of 'bad news' (unexpected negative shock) is rather big news compared to a big piece of 'good news' (unexpected positive shock) during turbulent periods. This is seen from the asymmetric bowl-shaped impact surface.

Figure 11 contains the time-varying news impact surfaces under low and high volatility from the CCC-TVGJR-GARCH model for the conditional variance of BA when there is no shock to XOM. Figure 12 contains a similar graph for XOM when there is no shock to BA. The asymmetric shape shows that a negative return shock has a greater impact than a positive return shock of the same size. Furthermore, as already obvious from Figure 10, a piece of news of a given size has a stronger effect on the conditional variance when volatility is low than when it is high.

Estimated news impact surfaces from the TVCC-TVGJR-GARCH model of the BA-XOM pair are plotted in Figure 13. These news impact surfaces are able to distinguish between responses during low and high variance as well as low and high correlation levels. It is seen from

Figure 13 that both the degree of turbulence in the market and the level of the correlations affect the impact of past shocks on the covariances. This indicates that both factors play an important role in assessing the effect of shocks on the covariances according to the TVCC-TVGJR-GARCH model. It is evident from the figure that high covariance estimates are related to strong correlations and a high degree of turbulence in the market.

6 Conclusions

In this paper, we extend the univariate multiplicative TV-GARCH model of Amado and Teräsvirta (2011) to the multivariate CC-GARCH framework. The model allows the individual variances to vary smoothly over time according to the logistic transition function and its generalizations. We develop a modelling technique for specifying the parametric structure of the deterministic time-varying component that involves a sequence of Lagrange multiplier-type tests. In this respect, our model differs from the semiparametric model of Hafner and Linton (2010).

We consider a set of CC-GARCH models to investigate the effects of nonstationary variance equations on the conditional correlation matrix. The models are applied to pairs of seven daily stock returns belonging to the S&P 500 composite index. We find that in our examples modelling the time-variation of the unconditional variances considerably improves the fit of the CC-GARCH models. The results show that multivariate correlation models combining both time-varying correlations and time-varying unconditional variances provide the best in-sample fit. They also indicate that modelling the nonstationary component in the variance has relatively little effect on correlation estimates when the conditional correlation model is the DCC-GARCH model, whereas the results are different for the STCC-GARCH model of Silvennoinen and Teräsvirta (2005, 2009a). In a number of occasions, the correlations estimated from this model with time as the sole transition variable (TVCC-GARCH) are quite different from what they are when the GJR-GARCH equations are implicitly assumed stationary. The most conspicuous difference between the estimated TVCC-TVGJR-GARCH and TVCC-GJR-GARCH models can be found in the degree of smoothness of the change in the correlations, but the direction of the change is not systematic.

With the TVGJR-GARCH equations we are also able to consider the effect of the nonstationary variance component on the moving correlations. For many pairs of returns, the fact that correlations between raw returns increase over time can be attributed to increasing volatility. This conclusion is based on the observation that the same moving correlations computed from returns with constant unconditional variance do not increase over time.

Furthermore, the TVGJR-GARCH approach gives us the opportunity to generalize the news impact surfaces introduced by Kroner and Ng (1998) such that they can vary over time. In the TVCC-TVGJR-GARCH model, the impact of news (shocks) on the covariances between returns is a function of both time-varying variances and time-varying correlations. As in the univariate case already considered in Amado and Teräsvirta (2011), it is seen that the impact of a piece of news of a given size is larger when the market is calm than when it is during periods of high volatility. In the multivariate case we can also conclude that high conditional correlation between two returns adds to the impact as compared to the situation in which the correlation is low. We also reproduce the old result that negative shocks or news have a stronger effect on volatility than positive news of the same size.

An extension of this methodology to the case in which the conditional correlations are also controlled by a stochastic variable is available through the Double STCC-GJR-GARCH model. This makes it possible to model for example asymmetric responses of conditional correlations to functions of past returns. This, however, is a topic left for future research.

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Appendix A: Tables

Table 1: Descriptive statistics of the asset returns

Asset	Min	Max	Mean	Std.dev.	Skew	Ex.Kurt	Rob.Sk.	Rob.Kr.
AXP	-19.35	13.23	0.011	2.280	-0.265	4.864	0.004	0.418
BA	-19.39	9.513	0.019	2.069	-0.611	7.215	-0.003	0.106
CAT	-11.51	9.067	0.037	2.073	-0.260	3.945	-0.022	0.108
INTC	-24.87	18.32	-0.009	2.896	-0.470	6.197	-0.001	0.160
JPM	-19.97	15.47	0.025	2.524	0.282	6.901	-0.010	0.396
WHR	-13.30	12.95	0.022	2.254	0.183	3.516	0.004	0.272
XOM	-8.83	9.29	0.039	1.579	-0.136	2.334	-0.058	0.060

Notes: The table contains summary statistics for the raw returns of the seven stocks of the S&P 500 composite index. The sample period is from September 29, 1998 until October 7, 2008 (2521 observations). Rob.Sk. denotes the robust measure for skewness based on quantiles proposed by Bowley (see Kim and White (2004)) and the Rob.Kr. denotes the robust centred coefficient for kurtosis proposed by Moors (see Kim and White (2004)).

Table 2: Descriptive statistics of the standardised returns

Asset	Min	Max	Mean	Std.dev.	Skew	Ex.Kurt	Rob.Sk.	Rob.Kr.
AXP	-8.36	6.196	0.015	1.171	-0.071	2.756	0.018	0.122
BA	-11.45	6.771	0.019	1.512	-0.404	3.852	-0.012	0.032
CAT	-15.68	8.613	0.042	1.743	-0.280	4.354	-0.005	0.083
INTC	-19.35	13.23	-4×10^{-4}	1.738	-0.491	4.238	0.016	0.028
JPM	-7.356	5.700	0.019	1.122	0.183	3.236	0.017	0.192
WHR	-8.949	9.808	0.027	1.786	0.261	2.566	4×10^{-5}	0.194
XOM	-6.051	6.365	0.038	1.221	-0.172	1.430	-0.044	0.001

Notes: The table contains summary statistics for the standardised returns of the seven stocks of the S&P 500 composite index. The standardised returns are obtained dividing the raw returns by the estimate of the function $g_t^{1/2}$. The sample period is from September 29, 1998 until October 7, 2008 (2521 observations). Rob.Sk. denotes the robust measure for skewness based on quantiles proposed by Bowley (see Kim and White (2004)) and the Rob.Kr. denotes the robust centred coefficient for kurtosis proposed by Moors (see Kim and White (2004)).

Table 3: Sequence of tests of the GJR-GARCH model against a TV-GJR-GARCH model

Transitions	H_0	H_{03}	H_{02}	H_{01}
Single transition				
AXP	0.0184	0.1177	0.0071	0.5722
BA	0.0021	0.0616	0.0461	0.0072
CAT	0.0044	0.0260	0.0107	0.1971
INTC	5×10^{-5}	9×10^{-5}	0.1600	0.0197
JPM	9×10^{-4}	0.0073	0.0023	0.8500
WHR	6×10^{-5}	7×10^{-4}	0.0011	0.9401
XOM	0.0018	0.0836	7×10^{-4}	0.4271
Double transition				
AXP	0.0826	0.1953	0.0378	0.4032
BA	0.1208	0.1480	0.0547	0.8419
CAT	0.4011	0.1719	0.4961	0.4347
INTC	0.4307	0.8757	0.1050	0.7458
JPM	0.0947	0.0144	0.8678	0.5484
WHR	0.3059	0.8856	0.1450	0.2249
XOM	0.1111	0.1526	0.4198	0.0685

Notes: The entries are the p-values of the LM-type tests of constant unconditional variance applied to the seven stock returns of the S&P 500 composite index. The appropriate order k in (9) is chosen from the short sequence of hypothesis as follows: If the smallest p-value of the test corresponds to H_{02}, then choose $k = 2$. If either H_{01} or H_{03} are rejected more strongly than H_{02}, then select either $k = 1$ or $k = 3$. See Amado and Teräsvirta (2011) for further details.

Table 4: Estimation results for the univariate TV-GJR-GARCH models

Asset	$\widehat{\delta}_1$	$\widehat{\gamma}_1$	$\widehat{c}_{11}$	$\widehat{c}_{12}$	$\widehat{c}_{13}$	r
			g_t component			
AXP	4.3601	300	0.4825	0.9034	–	1
	(0.1989)	(–)	(0.0015)	(0.0021)		
BA	−0.651	300	0.4686	–	–	1
	(0.0135)	(–)	(0.0011)			
CAT	1.2366	300	0.3021	0.9726	–	1
	(0.1102)	(–)	(0.0011)	(0.0028)		
INTC	2.9973	300	0.0262	0.4775	0.9127	1
	(0.1553)	(–)	(0.0004)	(0.0031)	(0.0039)	
JPM	6.3688	300	0.4821	0.9042	–	1
	(0.2737)	(–)	(0.0012)	(0.0020)		
WHR	1.2272	300	0.0892	0.4195	0.8497	1
	(0.0917)	(–)	(0.0008)	(0.0072)	(0.0024)	
XOM	1.1063	300	0.4106	0.8672	–	1
	(0.0809)	(–)	(0.0029)	(0.0037)		

Notes: The table contains the parameter estimates of the g_{it} component from the TV-GJR-GARCH(1,1) model for the seven stocks of the S&P 500 composite index, over the period September 29, 1998 - October 7, 2008. The estimated model has the form $g_{it} = 1 + \sum_{l=1}^{r} \delta_{il} G_{il}(t/T; \gamma_{il}, c_{il})$, where $G_{il}(t/T; \gamma_{il}, c_{il})$ is defined in (9) for all i. The numbers in parentheses are the standard errors.

Table 5: Estimation results for the univariate TV-GJR-GARCH models

Asset	$\widehat{\omega}$	$\widehat{\alpha}_1$	$\widehat{\kappa}_1$	$\widehat{\beta}_1$	$\widehat{\alpha}_1 + \frac{\widehat{\kappa}_1}{2} + \widehat{\beta}_1$
			$\underline{h_t \text{ component}}$		
AXP	0.0477	–	0.1309	0.9045	0.9699
	(0.0123)		(0.0205)	(0.0146)	
BA	0.1050	–	0.0899	0.9103	0.9552
	(0.0463)		(0.0330)	(0.0326)	
CAT	0.6641	0.0477	–	0.7340	0.7817
	(0.4611)	(0.0290)		(0.1696)	
INTC	0.1203	0.0450	–	0.9155	0.9605
	(0.0498)	(0.0165)		(0.0269)	
JPM	0.0474	0.0213	0.1135	0.8890	0.9670
	(0.0152)	(0.0110)	(0.0262)	(0.0229)	
WHR	0.3569	0.0736	–	0.8141	0.8877
	(0.2392)	(0.0326)		(0.1009)	
XOM	0.0644	0.0272	0.0578	0.9008	0.9568
	(0.0222)	(0.0113)	(0.0215)	(0.0235)	

Notes: The table contains the parameter estimates of the h_{it} component from the TV-GJR-GARCH(1,1) model for the seven stocks of the S&P 500 composite index, over the period September 29, 1998 - October 7, 2008. The estimated model has the form $h_{it} = \omega_i + \alpha_{i1}\varepsilon^{*2}_{it-1} + \kappa_{i1}I_{it-1}(\varepsilon^*_{it-1})\varepsilon^{*2}_{it-1} + \beta_{i1}h_{it-1}$, where $\varepsilon^*_{it} = \varepsilon_{it}/g^{1/2}_{it}$ and $I_{it}(\varepsilon^*_{it}) = 1$ if $\varepsilon^*_{it} < 0$ (and 0 otherwise) for all i. The numbers in parentheses are the Bollerslev-Wooldridge robust standard errors.

Table 6: Estimation results for the univariate GJR-GARCH models

Asset	$\widehat{\omega}$	$\widehat{\alpha}_1$	$\widehat{\kappa}_1$	$\widehat{\beta}_1$	$\widehat{\alpha}_1 + \frac{\widehat{\kappa}_1}{2} + \widehat{\beta}_1$
AXP	0.0160	–	0.1199	0.9432	1.003
	(0.0061)		(0.0223)	(0.0094)	
BA	0.0247	–	0.0618	0.9647	0.996
	(0.0164)		(0.0304)	(0.0167)	
CAT	0.0151	0.0156	–	0.9810	0.994
	(0.0153)	(0.0081)		(0.0107)	
INTC	0.0471	0.0514	–	0.9444	0.997
	(0.0352)	(0.0233)		(0.0237)	
JPM	0.0122	0.0229	0.0987	0.9331	1.005
	(0.0062)	(0.0095)	(0.0232)	(0.0136)	
WHR	0.0338	–	0.0414	0.9744	0.995
	(0.0235)		(0.0146)	(0.0104)	
XOM	0.0406	0.0323	0.0526	0.9256	0.984
	(0.0174)	(0.0107)	(0.0194)	(0.0177)	

Notes: The table contains the parameter estimates from the GJR-GARCH(1,1) model for the seven stocks of the S&P 500 composite index, over the period September 29, 1998 - October 7, 2008. The estimated model has the form $h_{it} = \omega_i + \alpha_{i1}\varepsilon^2_{it-1} + \kappa_{i1}I_{it-1}(\varepsilon_{it-1})\varepsilon^2_{it-1} + \beta_{i1}h_{it-1}$, where $I_{it}(\varepsilon_{it}) = 1$ if $\varepsilon_{it} < 0$ (and 0 otherwise) for all i. The numbers in parentheses are the Bollerslev-Wooldridge robust standard errors.

Table 7: Misspecification tests for the TV-GJR-GARCH models

(a) LM test of no ARCH in the standardised residuals

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
$r = 1$	0.513	0.099	0.695	0.558	0.884	0.0376	0.494
$r = 5$	0.328	0.391	0.955	0.927	0.975	0.2107	0.326
$r = 10$	0.137	0.438	0.997	0.956	0.897	0.2649	0.145

(b) LM test of GJR-GARCH(1,1) vs. GJR-GARCH(1,2) model

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.532	0.134	0.853	0.104	0.269	0.0218	0.496

(c) LM test of GJR-GARCH(1,1) vs. GJR-GARCH(2,1) model

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.241	0.210	0.975	0.652	0.607	0.0161	0.972

(d) LM type test of no STGJR-GARCH model of order 1

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.429	2×10^{-5}	0.562	4×10^{-4}	0.403	0.1581	0.468

Notes: The entries are the p-values of the LM-type misspecification tests in Amado and Teräsvirta (2011). The diagnostic tests are the following: (a) test of no ARCH-in-GARCH against remaining ARCH of order r in the standardised residuals; (b) test of a GJR-GARCH(1,1) model against a GJR-GARCH(1,2) model; (c) test of a GJR-GARCH(1,1) model against a GJR-GARCH(2,1) model; (d) test of no remaining nonlinearity against a Smooth Transition GJR-GARCH (STGJR-GARCH) of order $k = 1$.

Table 8: Misspecification tests for the GJR-GARCH models

(a) LM test of no ARCH in the standardized residuals

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
$r = 1$	0.557	0.103	0.296	0.695	0.472	2×10^{-4}	0.194
$r = 5$	0.281	0.147	0.744	0.931	0.813	0.009	0.386
$r = 10$	0.338	0.324	0.976	0.971	0.772	0.036	0.231

(b) LM test of GJR-GARCH(1,1) vs. GJR-GARCH(1,2) model

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.478	0.167	0.168	0.819	0.693	0.749	0.519

(c) LM test of GJR-GARCH(1,1) vs. GJR-GARCH(2,1) model

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.386	0.026	0.295	0.992	0.968	0.100	0.871

(d) LM type test of no STGJR-GARCH model of order 1

Return	AXP	BA	CAT	INTC	JPM	WHR	XOM
	0.538	0.008	1×10^{-5}	0.023	0.498	0.007	0.180

Notes: The entries are the p-values of the LM-type misspecification tests in Amado and Teräsvirta (2011). The diagnostic tests are the following: (a) test of no ARCH-in-GARCH against remaining ARCH of order r in the standardised residuals; (b) test of a GJR-GARCH(1,1) model against a GJR-GARCH(1,2) model; (c) test of a GJR-GARCH(1,1) model against a GJR-GARCH(2,1) model; (d) test of no remaining nonlinearity against a Smooth Transition GJR-GARCH (STGJR-GARCH) of order $k = 1$.

Table 9: Log-likelihood values from the bivariate normal density for the CC-GJR-GARCH estimated models

Pairs of Assets	CCC		DCC		TVCC	
	GJR	TV-GJR	GJR	TV-GJR	GJR	TV-GJR
AXP−BA	−10153.3	−8214.5	−10127.1	−8194.3	−10120.2	**−8190.2**
AXP−CAT	−10207.7	−8563.1	−10187.2	−8544.1	−10183.8	**−8542.9**
AXP−INTC	−10809.2	−8533.0	−10780.2	**−8503.9**	−10779.8	−8510.3
AXP−JPM	−9707.6	−6911.8	−9688.0	**−6892.1**	−9702.6	−6907.4
AXP−WHR	−10415.9	−8629.3	−10378.7	**−8588.2**	−10385.6	−8594.9
AXP−XOM	−9532.4	−7699.4	−9483.3	**−7653.1**	−9490.5	−7664.2
BA−CAT	−10390.1	−9356.9	−10373.1	−9346.6	−10365.2	**−9336.3**
BA−INTC	−11015.0	−9349.6	−10994.8	−9333.9	−10986.7	**−9325.9**
BA−JPM	−10275.1	−8092.4	−10256.9	−8080.2	−10252.4	**−8078.6**
BA−WHR	−10586.4	−9410.7	−10566.1	−9393.5	−10561.9	**−9388.1**
BA−XOM	−9659.0	−8442.8	−9630.5	−8421.8	−9629.6	**−8420.0**
CAT−INTC	−11114.9	−9750.3	−11082.2	−9720.7	−11073.6	**−9715.6**
CAT−JPM	−10341.4	−8451.0	−10304.6	**−8419.3**	−10309.7	−8425.8
CAT−WHR	−10614.0	−9730.6	−10580.6	**−9696.8**	−10594.8	−9708.3
CAT−XOM	−9737.9	−8812.9	−9698.3	−8781.2	−9693.2	**−8776.6**
INTC−JPM	−10921.4	−8403.0	−10910.3	−8390.3	−10900.7	**−8388.9**
INTC−WHR	−11307.3	−9790.0	−11283.3	−9763.8	−11282.9	**−9762.5**
INTC−XOM	−10412.6	−8858.8	−10373.7	**−8816.2**	−10374.9	−8824.1
JPM−WHR	−10494.8	−8456.6	−10470.0	**−8430.9**	−10475.9	−8432.8
JPM−XOM	−9670.9	−7596.9	−9629.9	**−7558.1**	−9634.8	−7565.0
WHR−XOM	−9982.3	−8913.8	−9961.5	**−8892.8**	−9969.9	−8900.8

Notes: The table contains the log-likelihood values for each of the bivariate CC-GJR-GARCH model. The conditional variances are modelled as GJR-GARCH(1,1). The GJR-GARCH column indicates that the unconditional variances are time-invariant functions. The TV-GJR-GARCH column indicates that the unconditional variances vary over time according to function (8). The maximised values for the log-likelihood are shown in boldface.

Table 10: Estimation results for the bivariate TVCC-GJR-GARCH models

Pairs of assets	$\rho_{(1)}$	$\rho_{(2)}$	γ	c
AXP−BA	0.1172 (0.0407)	0.4203 (0.0109)	500 (−)	0.2391 (0.0012)
AXP−CAT	0.2264 (0.0392)	0.4719 (0.0086)	500 (−)	0.2321 (0.0007)
AXP−INTC	0.1922 (0.0407)	0.4695 (0.0087)	500 (−)	0.2359 (0.0007)
AXP−JPM	0.5658 (0.0250)	0.6427 (0.0055)	500 (−)	0.2324 (0.0006)
AXP−WHR	0.1473 (0.0452)	0.4495 (0.0275)	33.43 (22.67)	0.2139 (0.0215)
AXP−XOM	0.0790 (0.0399)	0.4243 (0.0097)	500 (−)	0.2650 (0.0009)
BA−CAT	0.1412 (0.0474)	0.3985 (0.0104)	500 (−)	0.2891 (0.0007)
BA−INTC	0.0718 (0.0453)	0.3587 (0.0104)	500 (−)	0.2624 (0.0009)
BA−JPM	0.1450 (0.0377)	0.3987 (0.0099)	500 (−)	0.2399 (0.0011)
BA−WHR	0.0870 (0.0423)	0.3608 (0.0129)	500 (−)	0.2336 (0.0014)
BA-XOM	0.0645 (0.0429)	0.3663 (0.0270)	17.25 (8.07)	0.2653 (0.0504)
CAT−INTC	0.0471 (0.0420)	0.4119 (0.0333)	32.89 (21.16)	0.2338 (0.0164)
CAT−JPM	0.1538 (0.0402)	0.4634 (0.0085)	500 (−)	0.2017 (0.0003)
CAT−WHR	0.1669 (0.0481)	0.4386 (0.0088)	500 (−)	0.1398 (0.0015)
CAT-XOM	0.0746 (0.0463)	0.4486 (0.0284)	11.21 (3.51)	0.2606 (0.0260)
INTC−JPM	0.2124 (0.0503)	0.4555 (0.0084)	500 (−)	0.2091 (0.0008)
INTC−WHR	0.0893 (0.0354)	0.3647 (0.0105)	500 (−)	0.2538 (0.0007)
INTC−XOM	−0.0184 (0.0364)	0.3323 (0.0116)	500 (−)	0.2750 (0.0012)
JPM-WHR	0.1689 (0.0954)	0.4668 (0.0310)	5.41 (1.83)	0.1825 (0.0778)
JPM−XOM	0.0490 (0.0421)	0.3871 (0.0107)	500 (−)	0.2352 (0.0005)
WHR-XOM	0.1053 (0.0390)	0.3091 (0.0311)	15.86 (11.62)	0.2881 (0.0490)

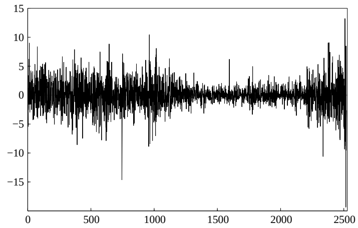
Notes: The table contains the estimation results for each of the bivariate TVCC-GJR-GARCH model. The conditional variances are modelled as GJR-GARCH(1,1) and the unconditional variances are time-invariant functions. The numbers in parentheses are the standard errors.

Table 11: Estimation results for the bivariate TVCC-TVGJR-GARCH models

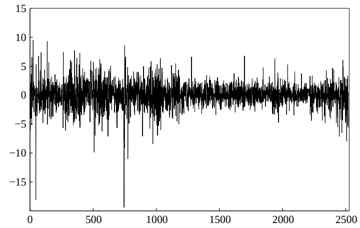
Pairs of assets	$\rho_{(1)}$	$\rho_{(2)}$	γ	c
AXP−BA	0.1349 (0.0421)	0.4073 (0.0104)	500 (−)	0.2393 (0.0011)
AXP−CAT	0.2360 (0.0406)	0.4695 (0.0084)	500 (−)	0.2322 (0.0007)
AXP−INTC	0.2029 (0.0450)	0.4580 (0.0251)	42.29 (14.53)	0.2305 (0.0210)
AXP−JPM	0.6557 (0.0064)	0.5953 (0.0078)	500 (−)	0.5189 (0.0003)
AXP−WHR	0.1378 (0.0453)	0.4538 (0.0252)	34.24 (15.56)	0.2162 (0.0187)
AXP−XOM	0.0723 (0.0677)	0.4110 (0.0327)	35.58 (125.43)	0.2481 (0.1146)
BA−CAT	0.1343 (0.0502)	0.3919 (0.0254)	93.48 (98.54)	0.2323 (0.0221)
BA−INTC	0.0787 (0.0458)	0.3518 (0.0102)	500 (−)	0.2503 (0.0010)
BA−JPM	0.1905 (0.0405)	0.3911 (0.0103)	500 (−)	0.2749 (0.0005)
BA−WHR	0.0984 (0.0445)	0.3611 (0.0115)	500 (−)	0.2449 (0.0009)
BA−XOM	0.0632 (0.0420)	0.3420 (0.0106)	500 (−)	0.2287 (0.0007)
CAT−INTC	0.0550 (0.0462)	0.4006 (0.0346)	33.03 (25.81)	0.2363 (0.0173)
CAT−JPM	0.1725 (0.0431)	0.4621 (0.0084)	500 (−)	0.2014 (0.0004)
CAT−WHR	0.1381 (0.0818)	0.4556 (0.0328)	9.94 (2.53)	0.1488 (0.0450)
CAT−XOM	0.0855 (0.0460)	0.4274 (0.0092)	500 (−)	0.2305 (0.0003)
INTC−JPM	0.2407 (0.0519)	0.4455 (0.0086)	500 (−)	0.2184 (0.0010)
INTC−WHR	0.0892 (0.0333)	0.3736 (0.0105)	500 (−)	0.2538 (0.0007)
INTC−XOM	−0.0288 (0.0375)	0.3189 (0.0115)	500 (−)	0.2688 (0.0009)
JPM−WHR	0.2219 (0.0377)	0.4745 (0.0091)	500 (−)	0.2141 (0.0007)
JPM−XOM	0.0479 (0.0452)	0.3795 (0.0106)	500 (−)	0.2353 (0.0005)
WHR−XOM	0.0859 (0.0381)	0.2951 (0.0120)	500 (−)	0.2222 (0.0008)

Notes: The table contains the estimation results for each of the bivariate TVCC-TVGJR-GARCH model. The conditional variances are modelled as GJR-GARCH(1,1) and the unconditional variances vary over time according to function (8). The numbers in parentheses are the standard errors.

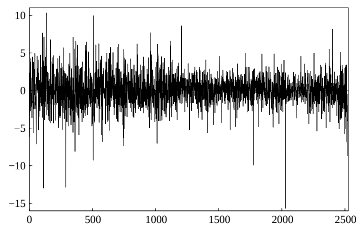
Appendix B: Figures



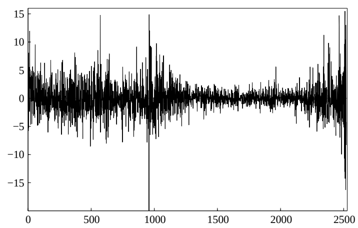
(a) AXP returns



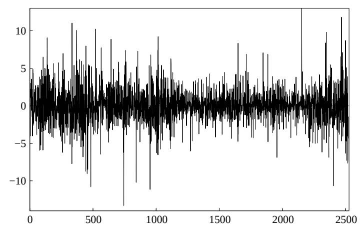
(b) BA returns



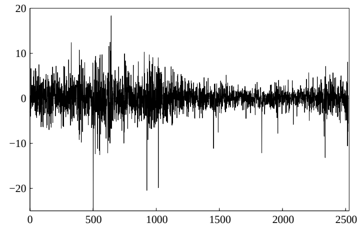
(c) CAT returns



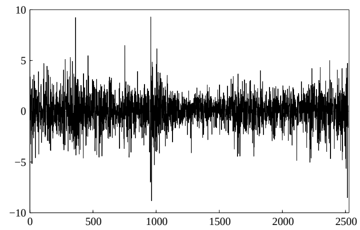
(d) JPM returns



(e) WHR returns

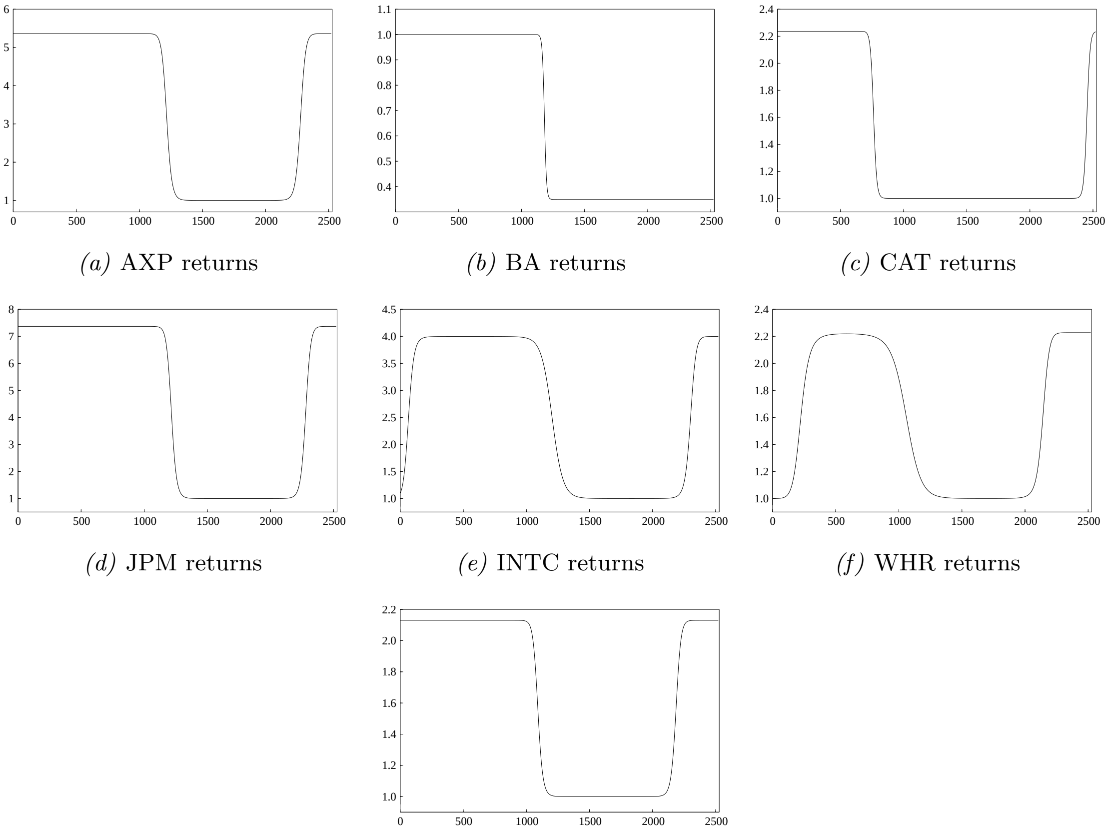


(f) INTC returns



(g) XOM returns

Figure 1: The seven stock returns of the S&P 500 composite index from September 29, 1998 until October 7, 2008 (2521 observations).

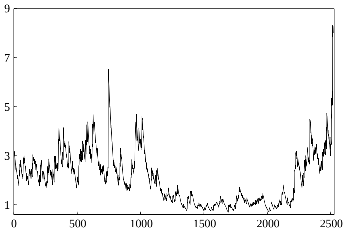


(a) AXP returns *(b)* BA returns *(c)* CAT returns

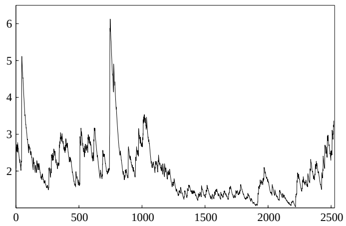
(d) JPM returns *(e)* INTC returns *(f)* WHR returns

(g) XOM returns

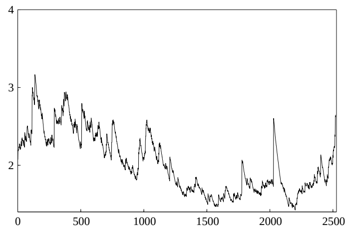
Figure 2: Estimated g_t functions for the seven stock returns of the S&P 500 composite index.



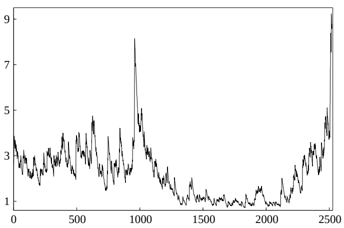
(a) AXP returns



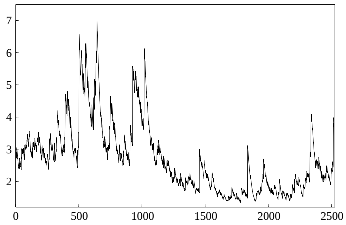
(b) BA returns



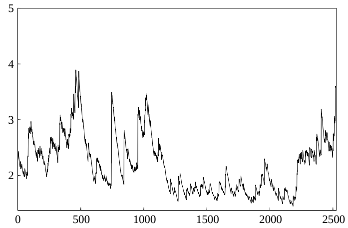
(c) CAT returns



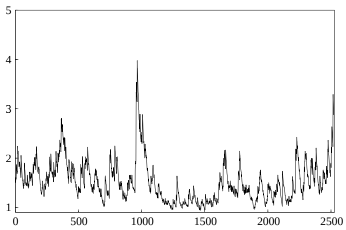
(d) JPM returns



(e) INTC returns

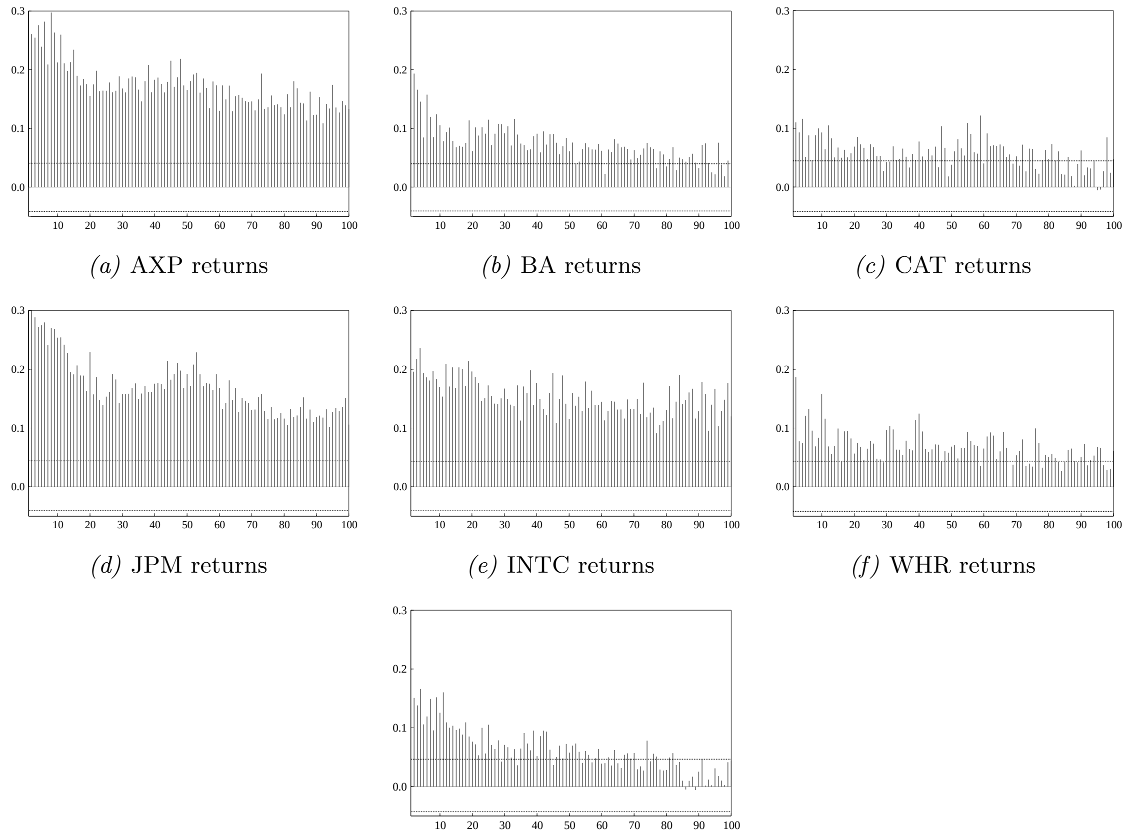


(f) WHR returns



(g) XOM returns

Figure 3: Estimated conditional standard deviations from the GJR-GARCH(1,1) model for the seven stock returns of the S&P 500 composite index.



(a) AXP returns

(b) BA returns

(c) CAT returns

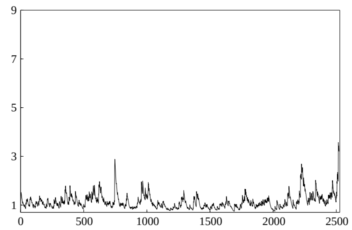
(d) JPM returns

(e) INTC returns

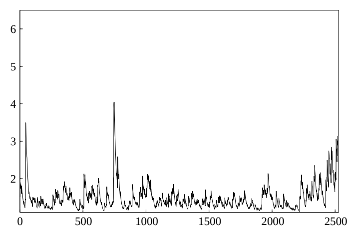
(f) WHR returns

(g) XOM returns

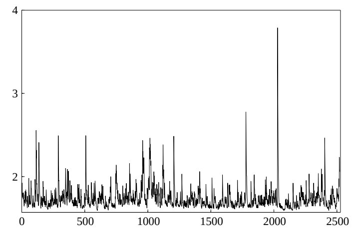
Figure 4: Sample autocorrelation functions of the absolute value for the seven stock returns of the S&P 500 composite index. The horizontal lines are the corresponding 95% confidence interval under the iid normality assumption.



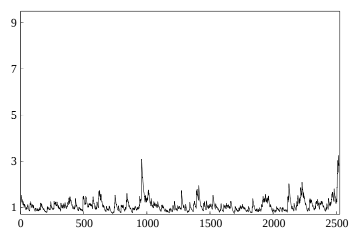
(a) AXP returns



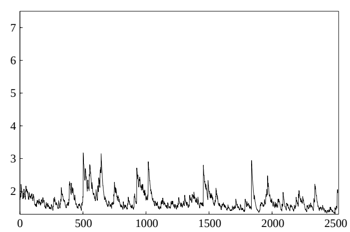
(b) BA returns



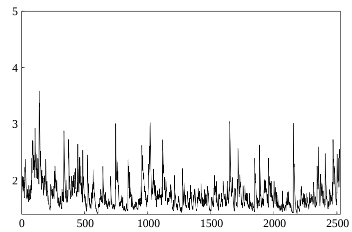
(c) CAT returns



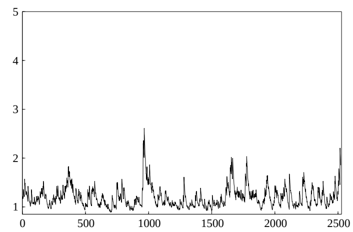
(d) JPM returns



(e) INTC returns

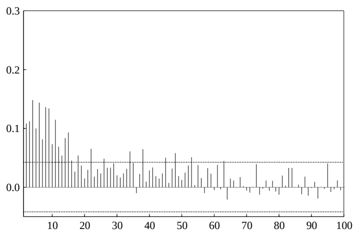


(f) WHR returns

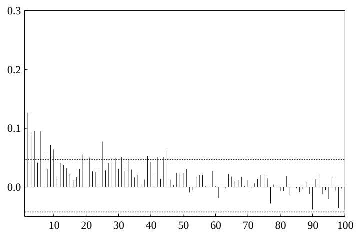


(g) XOM returns

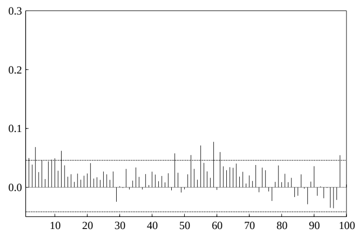
Figure 5: Estimated conditional standard deviations from the GJR-GARCH(1,1) model for the standardised variable $\varepsilon_t/\hat{g}_t^{1/2}$ for the seven stock returns of the S&P 500 composite index.



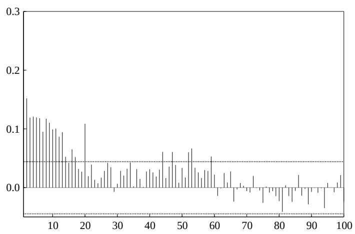
(a) AXP returns



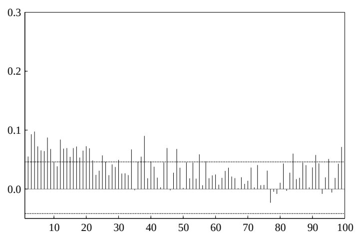
(b) BA returns



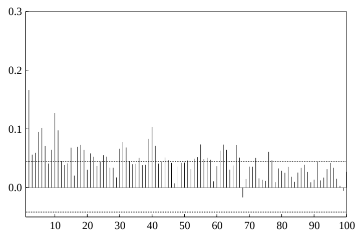
(c) CAT returns



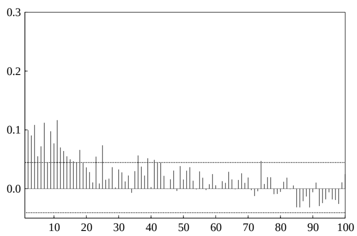
(d) JPM returns



(e) INTC returns



(f) WHR returns



(g) XOM returns

Figure 6: Sample autocorrelation functions of the absolute value of the standardised variable $\varepsilon_t/\hat{g}_t^{1/2}$ for the seven stock returns of the S&P 500 composite index. The horizontal lines are the corresponding 95% confidence interval under the iid normality assumption.

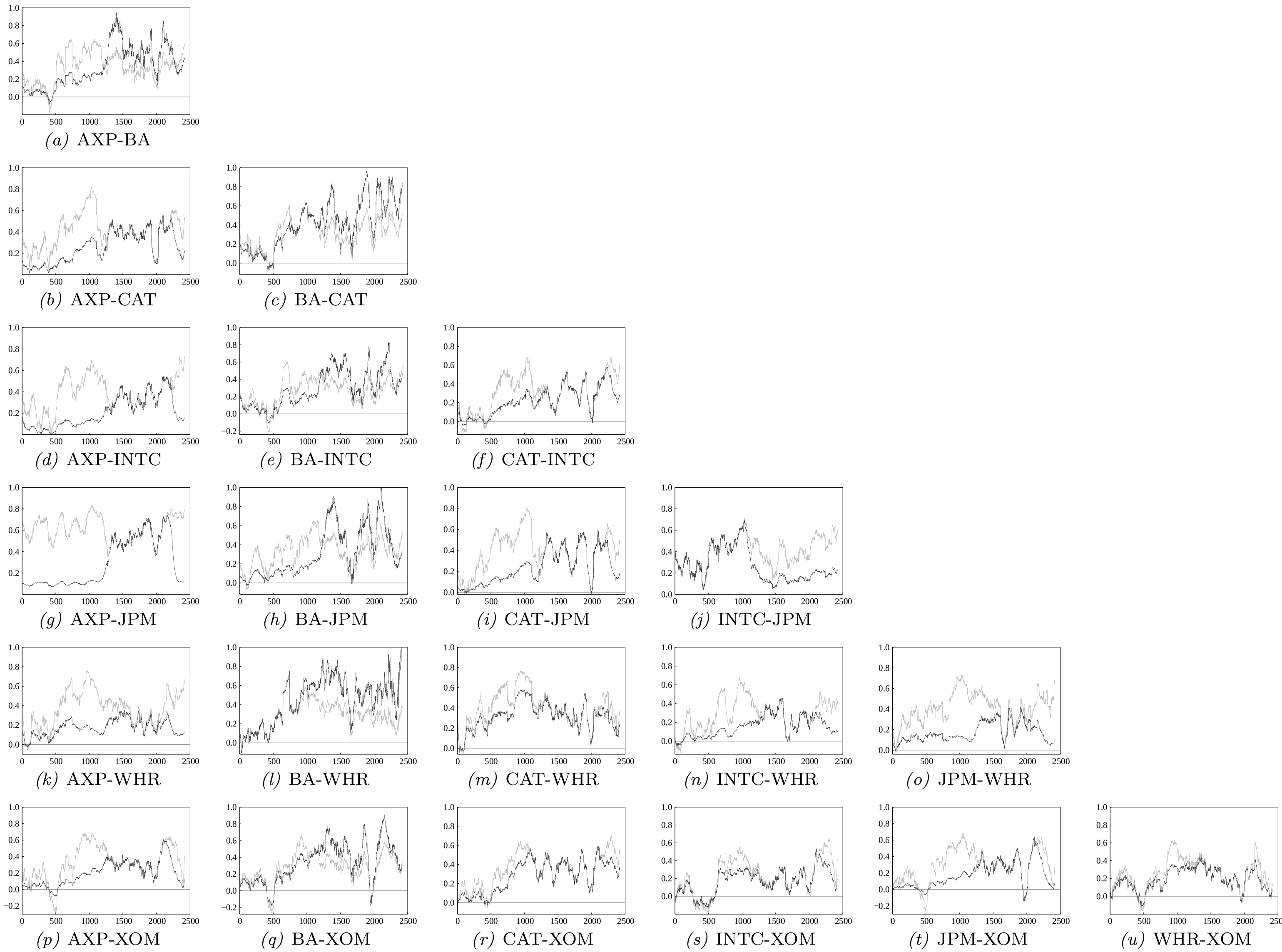
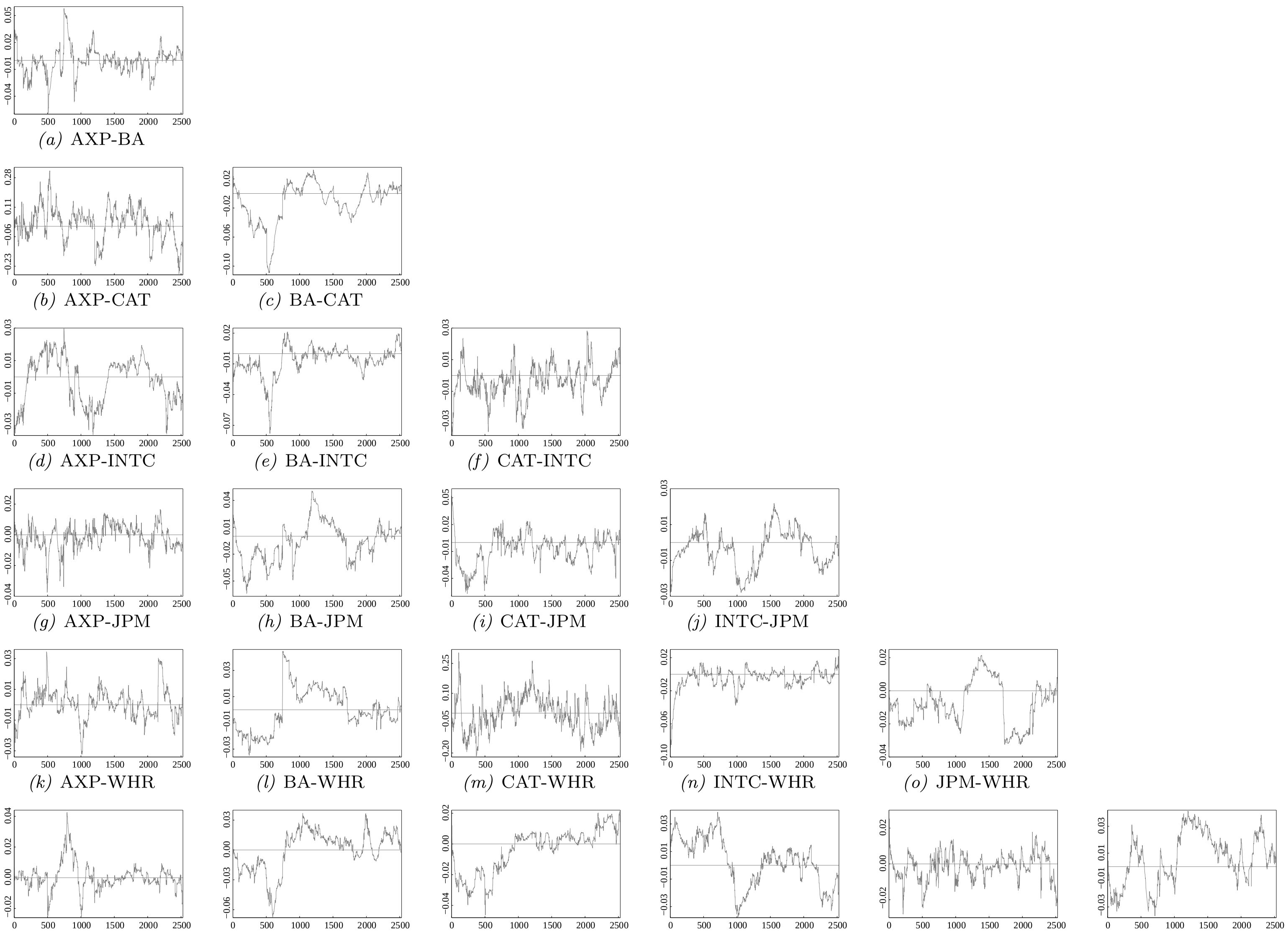


Figure 7: Difference between the estimated rolling correlation coefficients for pairs of the raw returns (grey dotted curve) and pairs of the standardised returns (black solid curve).



(a) AXP-BA
(b) AXP-CAT (c) BA-CAT
(d) AXP-INTC (e) BA-INTC (f) CAT-INTC
(g) AXP-JPM (h) BA-JPM (i) CAT-JPM (j) INTC-JPM
(k) AXP-WHR (l) BA-WHR (m) CAT-WHR (n) INTC-WHR (o) JPM-WHR
(p) AXP-XOM (q) BA-XOM (r) CAT-XOM (s) INTC-XOM (t) JPM-XOM (u) WHR-XOM

Figure 8: Difference between the estimated conditional correlations obtained from the bivariate DCC-GJR-GARCH and the bivariate DCC-TVGJR-GARCH models for the asset returns.

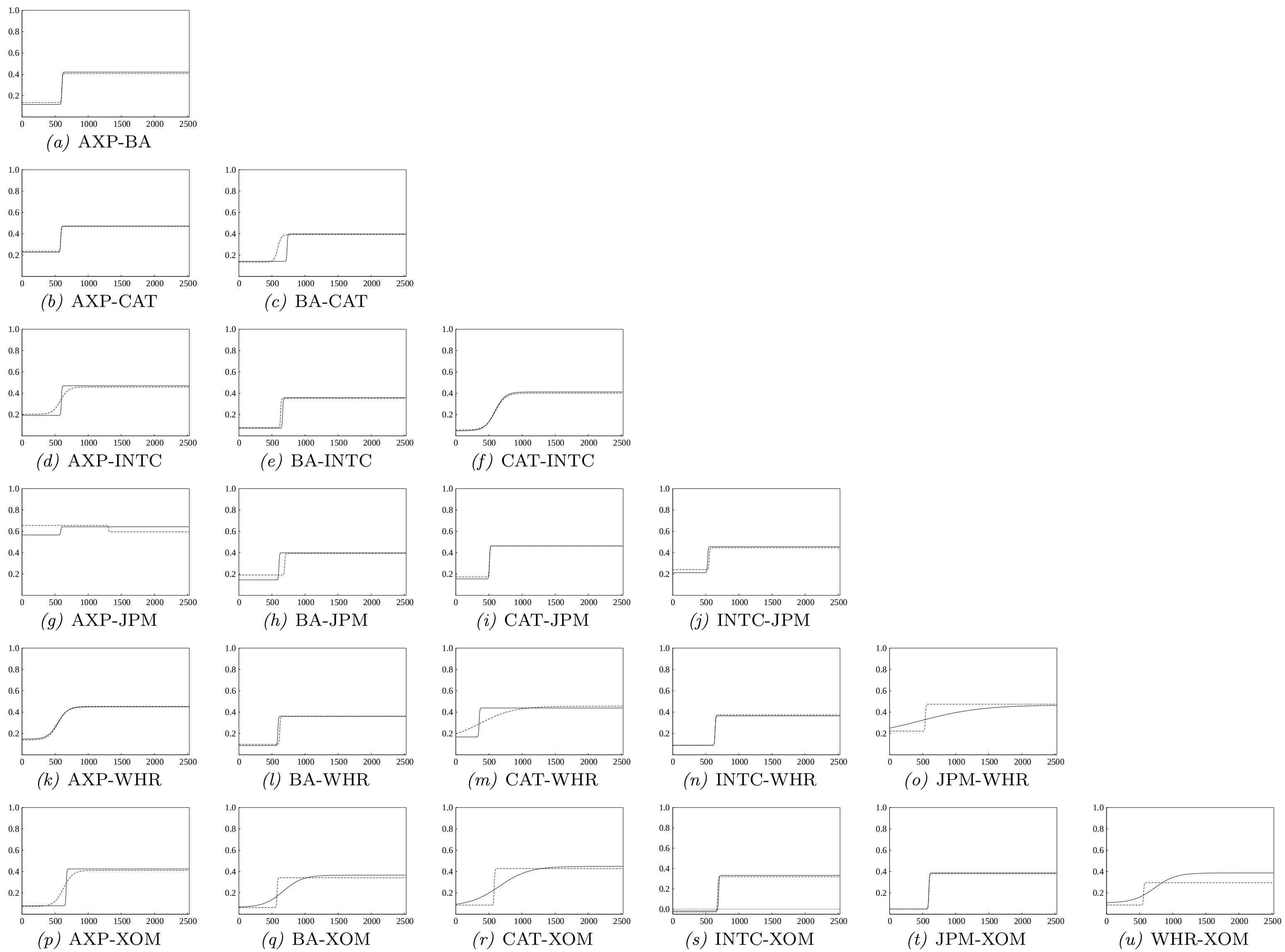
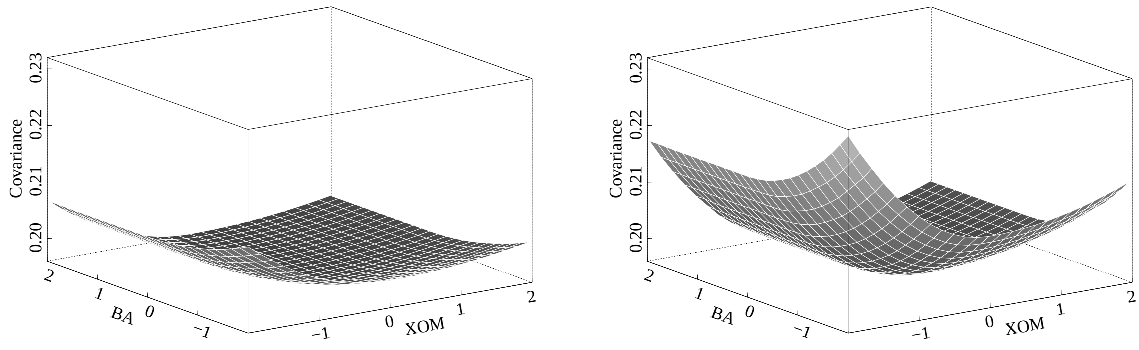


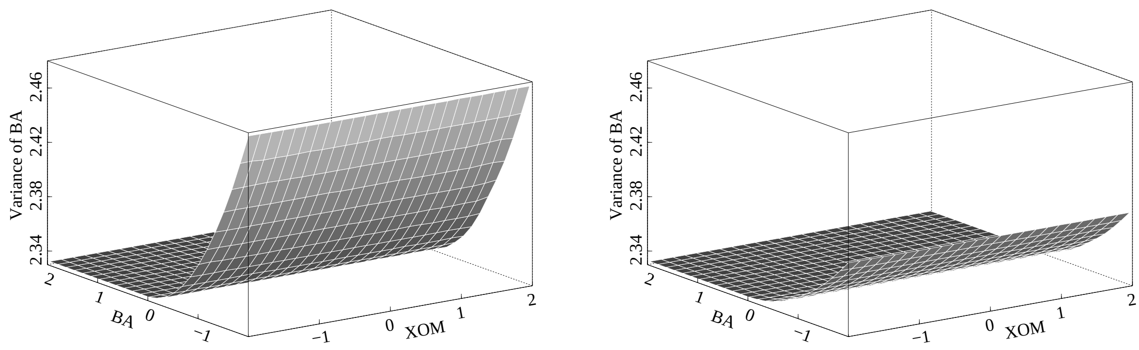
Figure 9: The estimated conditional correlations obtained from the bivariate TVCC-GJR-GARCH (solid curve) and the bivariate TVCC-TVGJR-GARCH (dotted curve) models for the asset returns.



(a) lower regime

(b) upper regime

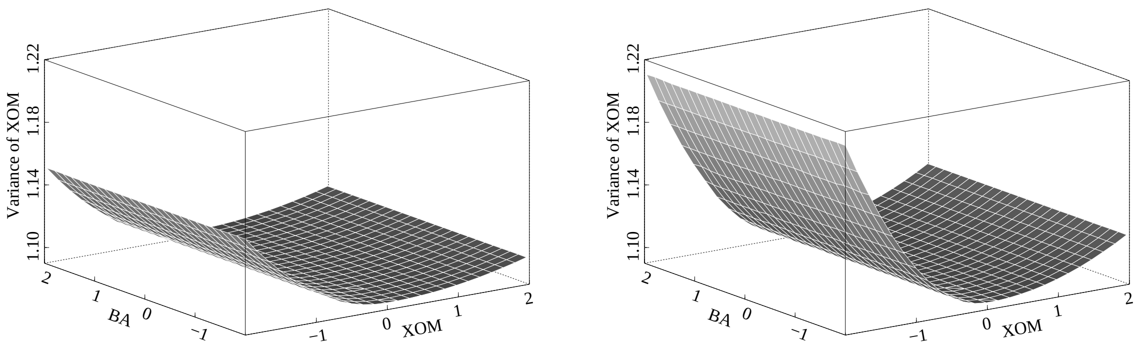
Figure 10: Estimated estimated time-varying news impact surfaces for the covariance between the BA and XOM returns under the CCC-TVGJR-GARCH model in the (a) lower regime and in the (b) upper regime of volatility.



(a) lower regime

(b) upper regime

Figure 11: Estimated time-varying news impact surfaces for the conditional variance of the BA returns under the CCC-TVGJR-GARCH model in the (a) lower regime and (b) upper regime of volatility.

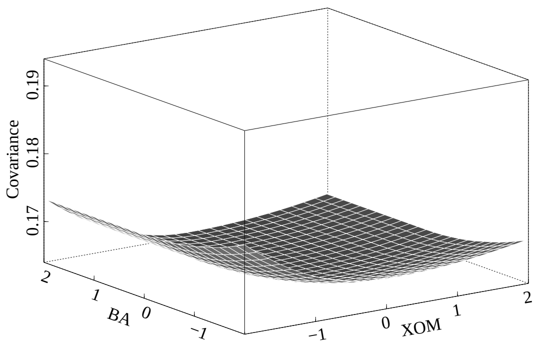


(a) lower regime

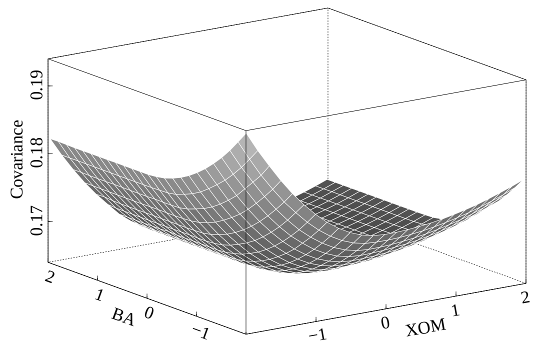
(b) upper regime

Figure 12: Estimated time-varying news impact surfaces for the conditional variance of the XOM returns under the CCC-TVGJR-GARCH model in the (a) lower regime and (b) upper regime of volatility.

Lower regime for the correlations

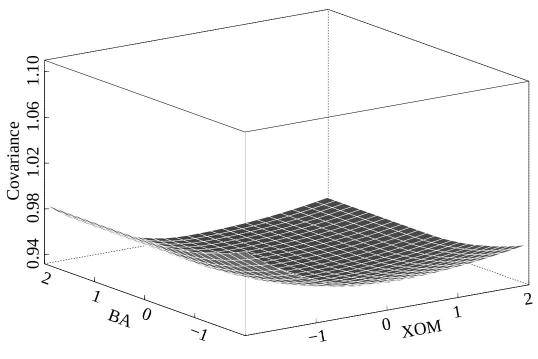


(a)

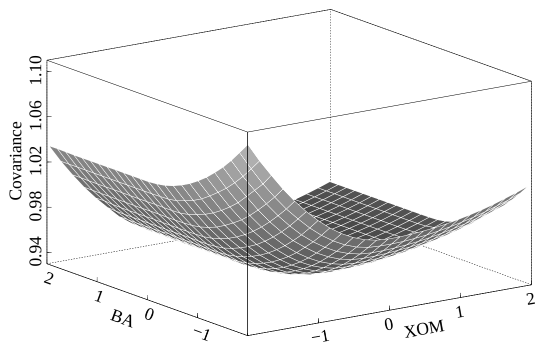


(b)

Upper regime for the correlations



(a)



(b)

Figure 13: Estimated time-varying news impact surfaces for the covariance between the BA and XOM returns under the TVCC-TVGJR-GARCH model in the (a) lower regime and in the (b) upper regime of volatility.

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