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**Modelling asset correlations during the recent financial
crisis: A semiparametric approach**

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Modelling asset correlations during the recent financial crisis: A semiparametric approach*

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Abstract

This article proposes alternatives to the Dynamic Conditional Correlation (DCC) model to study assets' correlations during the recent financial crisis. In particular, we adopt a semiparametric and nonparametric approach to estimate the conditional correlations for two interesting portfolios. The first portfolio consists of equity sectors SPDRs and the S&P 500 composite, while the second one contains major currencies that are actively traded in the foreign exchange market. Methodologically, our contribution is two-fold. First, we propose the Local Linear (LL) estimator for the correlations instead of the standard Nadaraya–Watson (NW) estimator used in Hafner et al. (2006) and Long et al. (2010). Second, we perform an extensive set of Monte Carlo experiments to compare the semiparametric and nonparametric models with the DCC specification. Unlike the aforementioned papers we also perform multivariate simulations in addition to the bivariate ones. Our simulation results show that the semiparametric and nonparametric models are best in DGPs with gradual changes or structural breaks in correlations. However, in DGPs with rapid changes or constancy in correlations the DCC delivers the best outcome. Finally, portfolio evaluation results show that the nonparametric model dominates its competitors, particularly in minimum variance weighted portfolios.

Keywords: *Semiparametric Conditional Correlation Model, Nonparametric Correlations, DCC, Local Linear Estimator, Portfolio Evaluation.*

JEL Classifications: C14; C58; G10.

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Introduction

The financial crisis beginning in September 2008 rattled the whole world and left a big global economical crisis which we are still feeling today. Given its recency, very little empirical work has been undertaken on the crisis.¹ It is well known that in many crises the degree of co-movement between assets returns changes rapidly, partly as a result of generally increased uncertainty. Therefore, an accurate assessment of the correlation between assets during the crisis period is of particular interest. For instance, rapid changes in correlation patterns call for an immediate adjustment of portfolios. Furthermore, policy makers are also interested in these links because of their implications for systemic risk.

With regard to the application, this paper estimates the conditional correlations of two interesting portfolios. The first portfolio consists of nine equity sectors SPDRs and the S&P 500 composite during 2004–2009. This portfolio is included in the analysis of the Volatility Institute of the New York University. Estimation results show a high correlation between the S&P 500 and most of the sectors. At the same time, however, there are considerable fluctuations especially in the correlation between the S&P 500 and the Materials, Utilities and Energy sectors. The second portfolio consists of the five major currencies plus two other currencies from emerging economies that are actively traded in the foreign exchange market. Correlations among the major currencies shift to a higher level in the period 2002–2005 possibly reflecting the “global savings glut” phenomenon. On the other hand, the correlations of the Japanese yen dropped around 2006 and even became negative in the period afterwards. All this information can be useful in the fields of trading strategies and portfolio diversification.

Methodologically, this article proposes alternatives to the Dynamic Conditional Correlation (DCC) model developed by Engle (2002). The DCC assumes that the correlation between assets evolves according to a simple GARCH-type structure. An attractive feature of this model is the so-called correlation targeting (substituting unconditional correlation by sample correlation), which reduces the number of estimated parameters. Other parametric methods available are the Smooth Transition Conditional Correlation (STCC) model proposed by Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005, 2009) and the Regime Switching Dynamic Correlation (RSDC) model of Pelletier (2006). The STCC method allows for the correlation of a Constant Conditional Correlation (CCC) model to change smoothly over time, while in the Pelletier’s model the correlation regimes depend on an unobserved Markov–Switching process. Note, however, that correlation targeting is not possible with the last two models.

Although easy to interpret, the parametric correlation models are restrictive in the sense that they impose a specific functional form to the correlation matrix and they assume that the data comes from a normal distribution. If the parametric model is misspecified in the functional

¹One of the exceptions is the paper by Fry et al. (2010) that use contagion tests to identify the transmission channels of the recent financial crisis.

form or the joint density function, parametric estimators will often be inconsistent. Another problem with these models is that correlation processes are restricted to have the same dynamic patterns. The only exception is the DCC model in its generalized version, which allows for correlation-specific dynamics.

Hence, we adopt a semiparametric and a nonparametric approach for the correlations that is flexible enough as the correlation matrix can a priori take any functional form and does not rely on the distributional assumption on the error term. Moreover, the models are easy to estimate even in high dimensions, which is usually not the case for parametric correlation models.

For the semiparametric method, we build on Hafner et al. (2006) and Long et al. (2010) who propose the Semiparametric Conditional Correlation (SPCC) model. This model combines a parametric estimation with a subsequent nonparametric estimation.² In particular, the estimation consists of two steps: In the first step, the conditional variance of each asset is estimated separately by using a parametric GARCH model. In the second step, the nonparametric model estimates the conditional covariance matrix of the standardized returns obtained in the first step. As noticed by Long et al. (2010) the key point behind the semiparametric estimator is that if the parametric estimation in the first step captures the main features of the volatility, the nonparametric estimation of the correlation in the second step will be easier, compared to the estimator of the whole covariance matrix. This 2-step estimation procedure is also adopted for the parametric DCC specification (Engle, 2002) so that the main difference between the SPCC and DCC models lies in the definition of the correlations in the second step of the estimation. As for the Nonparametric Correlation (NPC) model, it also consists of two steps. First the individual volatilities, and after the covariance matrix of the standardized returns are estimated with the Local Linear (LL).

We differ from Hafner et al. (2006) and Long et al. (2010), however, in the choice of the conditioning variable. Both these studies assume that the correlations depend on one of several exogenous (or predetermined) variables. They estimate the conditional correlation based on the previous information lying on the chosen dependent variables. This approach has two main drawbacks: 1) the choice of the exogenous variables is not always clear and 2) kernel smoothing methods are not feasible with many dependent variables (e.g., ‘curse of dimensionality’, difficulties of interpretation). Our solution to this problem is to model the correlations as a deterministic function of time. Time may be a good proxy for several economic factors expressing themselves mainly in the level of unconditional correlations. As explained further below, it is also necessary to scale this deterministic time trend by the total number of observations (conditioning variable ranges between 0 and 1) so the estimator depends on the sample size. The same rescaling device is commonly used in non-stationary processes (see Robinson, 1989; Drees

²Notice, however, the model of Long et al. (2010) is a semiparametric model for the conditional covariance matrix of raw returns where the nonparametric estimation serves as a correction for the parametric conditional covariance estimator.

and Stărică, 2002; Dahlaus and Rao, 2006, amongst others) to ensure the asymptotic behaviour of the estimator.

Another important difference with the aforementioned studies is that they use the Nadaraya–Watson (NW) estimator for the correlations, while we adopt the Local Linear (LL) estimator, which has smaller bias and behaves better at the boundaries (Fan and Gijbels (1996)). In addition, we present an automatic bandwidth selection through least squares cross-validation.

Moreover, we follow a more systematic procedure to compare the SPCC, NPC and DCC models by performing an extensive set of Monte Carlo simulations. In particular, we simulate bivariate processes and test the robustness of the results to a variety of misspecifications in volatility such as when the true variance is an asymmetric GARCH or when there are volatility spillovers. We find the following interesting points. In terms of mean absolute error (MAE) the semiparametric and nonparametric estimators are superior in DGPs with gradual changes or structural breaks in correlations. However, in DGP with rapid changes or constancy in parametric DCC model outperforms the SPCC and NPC models. This is observed throughout our simulations and is generally robust to misspecifications in the volatility processes.

In addition, we further perform a multivariate simulation experiment to show the performance of the models in higher dimensions. To our knowledge, the current paper is the first attempt on a multivariate simulation of this kind. Our results show that overall the SPCC and NPC models dominate the DCC specification. Furthermore, the performance of the semiparametric and nonparametric estimators is very similar.

As for the application, our results can be summarized as follows. Most SPDR sectors show high and constant correlation with the S&P 500 over the sample period. Exceptions are Utilities, Energy and Materials, whose correlations with the aggregate market experience notable drops in the fourth quarter of 2006 and in the third quarter of 2008. On the other hand, the correlations of the currencies show more variability over time which implies a more frequent rebalancing of portfolios. Portfolio evaluation results show that the nonparametric model dominates the semiparametric and DCC models, particularly in minimum variance weighted portfolios.

The outline of the paper is as follows. Section 2 presents the three correlation estimators. A detailed Monte Carlo experiment comparing their performance is shown in Section 3. Section 4 discusses the data and empirical results. Finally, in Section 5, we briefly summarize the main findings and give our conclusions.

1 Time-varying conditional correlations

Let r_t denote an N -dimensional vector time series (zero-mean asset returns) with time-varying conditional covariance matrix

$$\text{Var}[r_t|\mathfrak{S}_{t-1}] = E[r_t r_t'|\mathfrak{S}_{t-1}] = H_t \quad (1)$$

where \mathfrak{S}_{t-1} is the information set at time t . The conditional covariance matrix can be decomposed as

$$H_t = D_t R_t D_t \quad (2)$$

where $D_t = \text{diag}(\sqrt{h_{1,t}}, \sqrt{h_{2,t}}, \dots, \sqrt{h_{N,t}})$ is a diagonal matrix with the square root of the conditional variances $h_{i,t}$ for each asset i at time t on the diagonal. The matrix R_t , with the (i,j) -th element denoted as $\rho_{ij,t}$, is the possibly time-varying correlation matrix with $\rho_{ii,t} = 1$, $i, j = 1, \dots, N$ and $t = 1, \dots, T$. The standardized residuals are denoted by $\epsilon_t = D_t^{-1} r_t$. They are independent and identically distributed with $E(\epsilon_i) = 0$ and $\text{Var}(\epsilon_i) = 1$. The Constant Conditional Correlation (CCC) model assumes that R_t is constant over time, while the Semi-parametric Conditional Correlation (SPCC) and Dynamic Conditional Correlation (DCC) models allow distinct patterns of time-variation in R_t .

This paper compares the performance of the Dynamic Conditional Correlation (DCC) model, the Nonparametric Correlation model (NPC) and the Semiparametric Conditional Correlation (SPCC) models. The following three subsections describe in detail the three different methodologies.

1.1 Dynamic Conditional Correlation Model

Engle (2002) specifies the bivariate DCC model through the GARCH(1,1)-type process

$$Q_t^{\text{DCC}} = \Omega + \alpha \epsilon_{t-1} \epsilon_{t-1}' + \beta Q_{t-1}^{\text{DCC}} \quad (3)$$

where α is the news parameter and β is the decay parameter. A simple estimator for the intercept parameter matrix Ω is available through what is called correlation targeting. That is, using the estimator

$$\Omega = (1 - \alpha - \beta) \bar{Q} \quad (4)$$

where \bar{Q} is the sample unconditional correlation matrix between the standardized errors ϵ_t . Substituting (4) into (3) gives the basic form for the mean-reverting DCC model given by

$$Q_t^{\text{DCC}} = \bar{Q} + \alpha(\epsilon_{t-1} \epsilon_{t-1}' - \bar{Q}) + \beta(Q_{t-1} - \bar{Q}) \quad (5)$$

It is easy to see how this model behaves. The correlations evolve over time in response to new information on the asset returns. When returns are moving in the same direction — either they are both moving up or they are both moving down— the correlations will rise above the average

level and remain there for a while. Gradually this information will decay and correlations will fall back to the long-run average. Similarly, when assets move in opposite directions, the correlations will temporarily fall below the unconditional level. The two parameters (α, β) govern the speed of this adjustment.

If the estimators of the conditional variance $\hat{h}_{i,t}$, although consistent, are not very close to the true conditional variance at time t , then the diagonal of \hat{Q}_t^{DCC} will not be close to the unity vector $\mathbf{1}$. Therefore, the quantity \hat{Q}_t^{DCC} is typically rescaled using

$$R_t^{\text{DCC}} = (Q_t^{\text{DCC}*})^{-1} Q_t^{\text{DCC}} (Q_t^{\text{DCC}*})^{-1} \quad (6)$$

where $Q_t^{\text{DCC}*}$ is a diagonal matrix composed of the square roots of the diagonal elements of Q_t^{DCC} .

In a multivariate framework, the basic DCC specification may be too restrictive. In particular, note that the DCC model implies that all correlations pairs have the same dynamic pattern as implied by the parameters α and β . Cappiello et al. (2006) propose the Generalized DCC (G-DCC), which allows for correlation-specific news and decay parameters. The generalized DCC model is given by

$$Q_t^{\text{GDCC}} = (\bar{Q} + A'\bar{Q}A - B'\bar{Q}B) + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'Q_{t-1}^{\text{GDCC}}B \quad (7)$$

where A and B are defined to be $N \times N$ parameter diagonal matrices. So, the basic DCC is obtained as a special case of the G-DCC if the matrices A and B are replaced by scalars. However, the number of parameters in the G-DCC increases rapidly with the dimension of the model. In particular, for our portfolio application (consisting of 10 assets in total—nine equity sectors SPDRs and the S&P 500) we propose the more parsimonious Semigeneralized DCC (SG-DCC) initially studied by Hafner and Franses (2009). This model allows for correlation-specific news parameter α and restricts only the decay parameter β to be the same across correlation pairs. It is expected that the news parameters varies across correlation pairs more than the decay parameter. The SG-DCC equation is given by

$$Q_t^{\text{SGDCC}} = (\bar{Q} + A'\bar{Q}A - \beta\bar{Q}) + A'\epsilon_{t-1}\epsilon'_{t-1}A + \beta Q_{t-1}^{\text{SGDCC}} \quad (8)$$

A sufficient condition for Q_t to be positive definite for all possible realizations is that the intercept, $\bar{Q} - A'\bar{Q}A - \beta\bar{Q}$, is positive semi-definite and the initial covariance matrix Q_0 is positive definite (Cappiello et al., 2006). As before, we rescale the quantity Q_t to obtain a proper correlation matrix.

In summary, the (SG)-DCC conditional correlation estimator \hat{R}_t^{SGDCC} is obtained in three steps:

Step 1 Devolatilisation. In this step the data volatility of each return is estimated to obtain the

standardised residuals. Basically, \hat{D}_t is obtained by assuming a certain parametric model driving the volatility process, for example a GARCH(1,1). Therefore, $\hat{\epsilon}_t = \hat{D}_t^{-1}r_t$.

Step 2 The standardized residuals $\hat{\epsilon}_t$ are then used to estimate the (SG)–DCC correlations by Gaussian Maximum Likelihood (ML) obtaining \hat{Q}_t^{SGDCC} .

Step 3 Matrix scaling. The \hat{Q}_t^{SGDCC} is in general not a correlation matrix. Engle (2009) discusses how this is a technical problem that can be solved by rescaling as in equation (??).

1.2 Semiparametric Conditional Correlation Model

Correlations $\rho_{ij,t}$ in the DCC models are assumed to depend on certain parameters α and β (A and β is the case of SG-DCC). These parameters do not change with time and therefore the DCC models may be restrictive to describe time-varying correlations.

It is easy to show that $E(\epsilon_t \epsilon_t' | \mathfrak{S}_{t-1}) = R_t$ and therefore an estimator of the correlation can be obtained using nonparametric kernel regression methods. For instance, Hafner et al. (2006) and Long et al. (2010) use the classical Nadaraya–Watson estimator. Instead, we propose the Local Linear (LL) estimator, which has smaller bias and behaves better at the boundaries (Fan and Gijbels (1996)). Another difference with the Hafner et al. (2006) and Long et al. (2010) is in the choice of the conditioning variable. The aforementioned studies assume that the correlations depend on a single exogenous (or predetermined) variable. Instead, we model the correlations as a deterministic function of time. Time may capture several economic factors expressing themselves mainly in the level of unconditional correlations. Thus, this paper proposes the Local Linear (LL) estimator in Fan and Gijbels (1996) with time as the dependent variable.

Notice that the difference between using time as the “dependent ”variable instead of a stochastic variable is that each point in time is visited only once and not previous information is used. In addition, increasing T does not increase the density of information, and therefore asymptotically speaking, it is the same to estimate the correlation using a large or a short T . Therefore it is necessary, as in Robinson (1989), to assume the existence of a smooth function $\rho_{ij}(t)$ on $(0, 1)$ such that:

$$\rho_{ij,t} = \rho_{ij}\left(\frac{t}{T}\right) \quad \text{for } t = 1, 2, \dots, T.$$

This condition ensures that the amount of local information around a point $\frac{t}{T} \in (0, 1)$ increases as T increases and therefore the bias and variance of an estimator of $\rho_{ij}(\frac{t}{T})$ will decrease. Thus, $\hat{\rho}_{ij,t} = \hat{\rho}_{ij}(\frac{t}{T})$.

The SPCC estimator that we propose for a value $\tau \in (0, 1)$ is defined as:

$$\hat{Q}_\tau^{\text{SPCC}} = \sum_{t=1}^T \hat{\epsilon}_t \hat{\epsilon}_t' K_b\left(\frac{t - T\tau}{T}\right) \frac{s_2 - s_1\left(\frac{t - T\tau}{T}\right)}{s_0 s_2 - s_1^2} \quad (9)$$

where $K_b(\cdot) = (1/b)K(\cdot/b)$, K is a symmetric kernel function heavily concentrated around the origin, b is the bandwidth parameter and τ is the focal point. In addition, $s_j = \sum_{t=1}^T (\frac{t-T\tau}{T})^j K_b(\frac{t-T\tau}{T})$ for $j = 0, 1, 2$. Equation (9) displays the nonparametric point estimator of the covariance of $\{\hat{\epsilon}_\tau\}$. Drees and Stărică (2002) and Dahlaus and Rao (2006) have used this rescaling device to estimate the time-varying volatility. As before, we scale \hat{Q}_t^{SPCC} to obtain a proper correlation matrix \hat{R}_t^{SPCC} .

The bandwidth parameter b plays an essential role in nonparametric modelling. It is desirable to have a reliable data-driven and yet easily implemented bandwidth selection procedure. Hafner et al. (2006) adopt a local bandwidth estimate which has been obtained *ad hoc* for the particular data set under study. On the other hand, Long et al. (2010) set $b = c\hat{\sigma}T^{-1/6}$ and follow a grid search over $c \in [0.5, 5]$ with $\hat{\sigma}$ being the empirical standard deviation of the conditioning variable. This choice aims at finding the bandwidth that ensures the optimal rate of convergence which minimises the asymptotic mean integrated square error of the estimator. However, although choosing the bandwidth amongst a grid of values is practical, it is not always correct. For example, infinite is the appropriate bandwidth for a constant or linear correlation and this value is not included in the grid. Therefore, we propose finding the global bandwidth through least squares cross-validation as it is defined in (10). In practice, we use a Newton-type minimization algorithm. As the Newton minimisers are susceptible to the starting point, we do several numerical minimizations with different starting points and choose the most appropriate bandwidth.

$$b^{\text{SPCC}} = \arg \min_b \sum_{t=1}^T [\text{vecl}(\hat{\epsilon}_t \hat{\epsilon}_t') - \text{vecl}(\hat{Q}_{-t}^{\text{SPCC}})]^2 \quad (10)$$

where $\hat{Q}_{-t}^{\text{SPCC}}$ is the nonparametric estimator obtained when pair $(t, \hat{\epsilon}_t)$ is left out. The *vecl* of a matrix takes the lower diagonal matrix, excluding the diagonal.

In summary, the SPCC estimator proposed here consists of three steps:

Step 1 Devolatilisation. The conditional variances \hat{h}_t are obtained in this step in the same way than for the DCC model.

Step 2 Pseudo-correlation matrix. \hat{Q}_t^{SPCC} is obtained as in equation (9) for all $t = 1, \dots, T$.

Step 3 Matrix scaling. $\hat{R}_t^{\text{SPCC}} = (\hat{Q}_t^{\text{SPCC}*})^{-1} \hat{Q}_t^{\text{SPCC}} (\hat{Q}_t^{\text{SPCC}*})^{-1}$.

1.3 Nonparametric Correlation Model

It is inevitable to wonder how the nonparametric estimator performs in comparison with the parametric and semiparametric estimators. As before, we estimate the model in three steps.

First, as in Drees and Stărică (2002), the (unconditional) volatility of each individual asset is estimated using the LL:

$$\hat{h}_{i,\tau}^{\text{NPC}} = \sum_{t=1}^T r_{i,t}^2 K_b \left(\frac{t - T\tau}{T} \right) \frac{s_2 - s_1 \left(\frac{t - T\tau}{T} \right)}{s_0 s_2 - s_1^2}, i = 1, \dots, N \quad (11)$$

Here $s_j = \int x^j K(x) dx$. Basically, one must assume that $h(t)$ is a smooth deterministic function defined on the interval $(0,1)$ such that $h_{j,t} = h\left(\frac{t}{T}\right)$. The positive aspect of this approach is that $\hat{h}_{i,\tau}$ is a consistent estimator of the volatility at point τ if τ is not a boundary point and the volatility function is continuous there (see Robinson, 1989). This is not always the case for the previous two models if the volatility is not well-modelled by GARCH.

The second step consists on finding the (unconditional) pseudo-correlation:

$$\hat{Q}_\tau^{\text{NPC}} = \sum_{t=1}^T \hat{\epsilon}_t \hat{\epsilon}_t' K_b \left(\frac{t - T\tau}{T} \right) \frac{s_2 - s_1 \left(\frac{t - T\tau}{T} \right)}{s_0 s_2 - s_1^2}. \quad (12)$$

for $\tau \in (0,1)$. After, the appropriate matrix scaling the unconditional correlation matrix estimator is:

$$\hat{R}_t^{\text{NPC}} = (\hat{Q}_t^{\text{NPC}*})^{-1} \hat{Q}_t^{\text{NPC}} (\hat{Q}_t^{\text{NPC}*})^{-1} \quad (13)$$

The bandwidth is chosen by cross-validation as:

$$b^{\text{NPC}} = \arg \min_b \sum_{t=1}^T [\text{vecl}(\hat{\epsilon}_t \hat{\epsilon}_t') - \text{vecl}(\hat{Q}_{-t}^{\text{NPC}})]^2. \quad (14)$$

2 Monte Carlo simulations

In this section, we compare the sample performance of the three models by examining certain characteristics of conditional correlations when the true correlation processes are observable. We simulate bivariate processes of length $T = 1000$ corresponding to four financial years. The parameter values for the DGPs are chosen from Engle (2002). In particular, we consider four different scenarios (DGPs) for the correlations:

Scenario 1: constant correlation, $\rho_t = 0.9$,

Scenario 2: correlation with weak seasonality, $\rho_t = 0.5 + 0.4 \cos(2\pi t/200)$,

Scenario 3: correlation with strong seasonality, $\rho_t = 0.5 + 0.4 \cos(2\pi t/20)$, and,

Scenario 4: correlation with a structural break, $\rho_t = 0.9 - 0.5(t > 500)$.

2.1 Experiment 1

Two series of returns are simulated with volatility following a GARCH(1,1) and the innovations are distributed as a bivariate normal with a vector zero as mean and a correlation matrix which is the interest.

A total $M = 200$ experiments were conducted for each scenario with the same model specification as in Engle (2002) which is transcribed below:

$$\begin{aligned}
 r_{1,t} &= \sqrt{h_{1,t}}\epsilon_{1,t}, & r_{2,t} &= \sqrt{h_{2,t}}\epsilon_{2,t} \\
 h_{1,t} &= 0.01 + 0.05r_{1,t-1}^2 + 0.94h_{1,t-1} & h_{2,t} &= 0.5 + 0.2r_{2,t-1}^2 + 0.5h_{2,t-1} \\
 \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} &\sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_t \\ \rho_t & 0 \end{pmatrix} \right)
 \end{aligned} \tag{15}$$

The DGP $h_{1,t}$ is chosen such that the unconditional variance is lower but the persistence is higher than in DGP $h_{2,t}$. Each estimate's performance is measured by the mean absolute error which for each simulation N is defined by:

$$\text{MAE}_M = \frac{1}{T} \sum_{t=1}^T |\rho_t - \hat{\rho}_t| \tag{16}$$

where $\hat{\rho}_t$ are the elements of \hat{R}_t .

Figure 1 shows the estimator from each model chosen as the typical sample whose MAE is equal to the median of all the MAE_M , while Figure 2 shows the boxplots of the MAE_M for the 200 simulations. The boxes represent the 25% and 75% quantiles (interquartile range, IQR) of the MAE_M . The line in the middle of the box represents the median of the MAE_M and therefore the error corresponding to the estimator in Figure 1. The end of the whiskers represent the 1.5IQR of the lower quartile and upper quartiles. As seen, the DCC is the best model for constant correlations (Figures 1-2 (a)). In this scenario, the SPCC also performs quite well choosing a very large bandwidth which results in a very smooth estimator. On the other hand, for Scenarios 2 and 4 the semiparametric and nonparametric estimators improve substantially on the DCC. Also, their performance is quite similar in those two scenarios. For instance, the NPC slightly outperforms the SPCC for periodic correlations with a yearly frequency (gradual changes) such as in Figures 1-2 (b), while the SPCC is more accurate for correlations with a regime switch (structural break) like in Scenario 4 (Figures 1-2 (d)). However, in Scenario 3 (periodic correlations with large frequencies—Figures 1-2 (c)) SPCC and NPC imply correlations that are too smooth and, therefore, are less accurate than the DCC model.

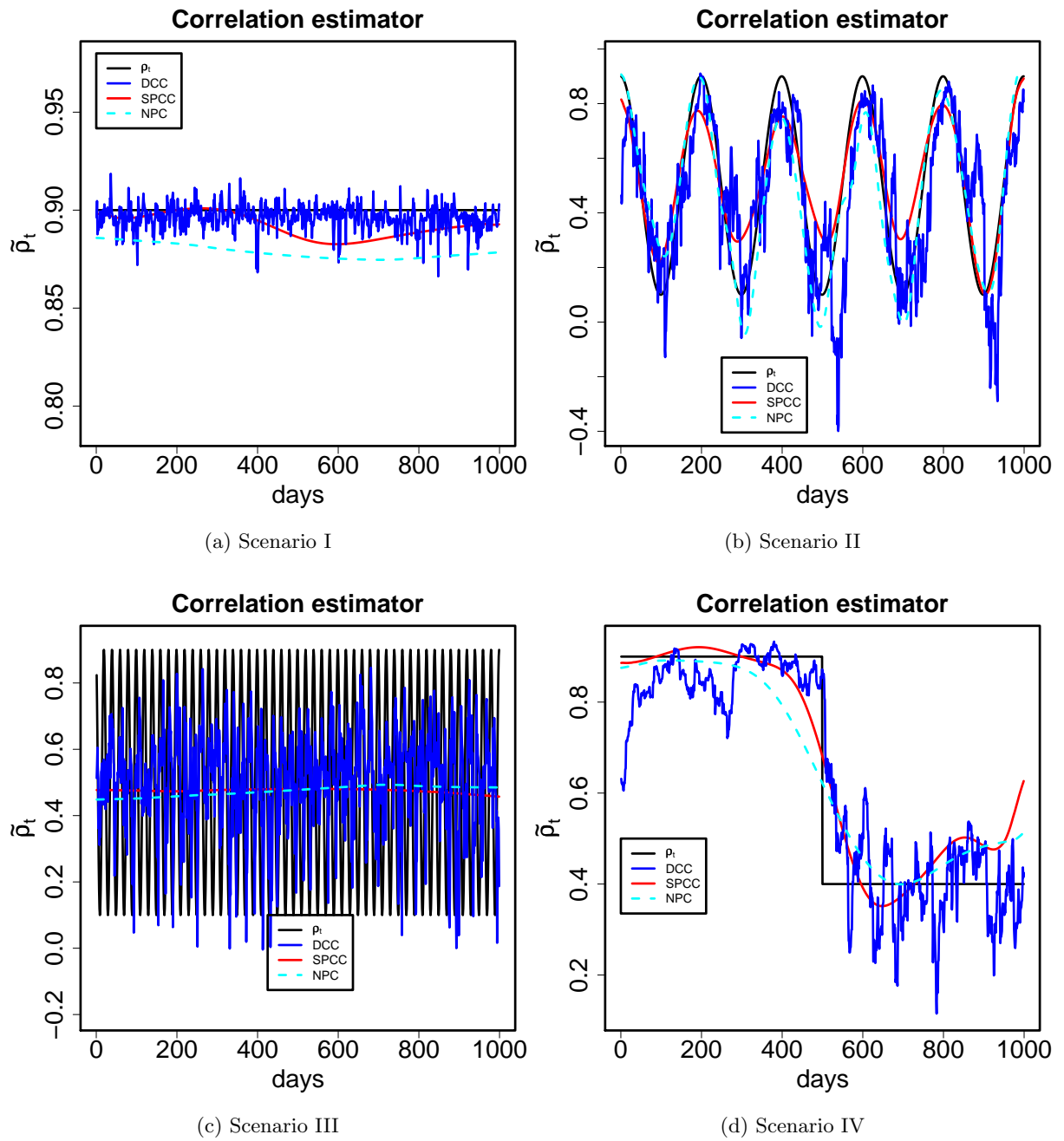


Figure 1: Correlation estimators for the different scenarios of Experiment 1.

2.2 Experiment 2

Typically, for stock returns negative shocks have a larger impact on volatility than positive shocks of the same magnitude (so-called leverage effect). In this experiment, we simulate DGPs from the following asymmetric GARCH(1,1) (Glosten et al., 1993) models:

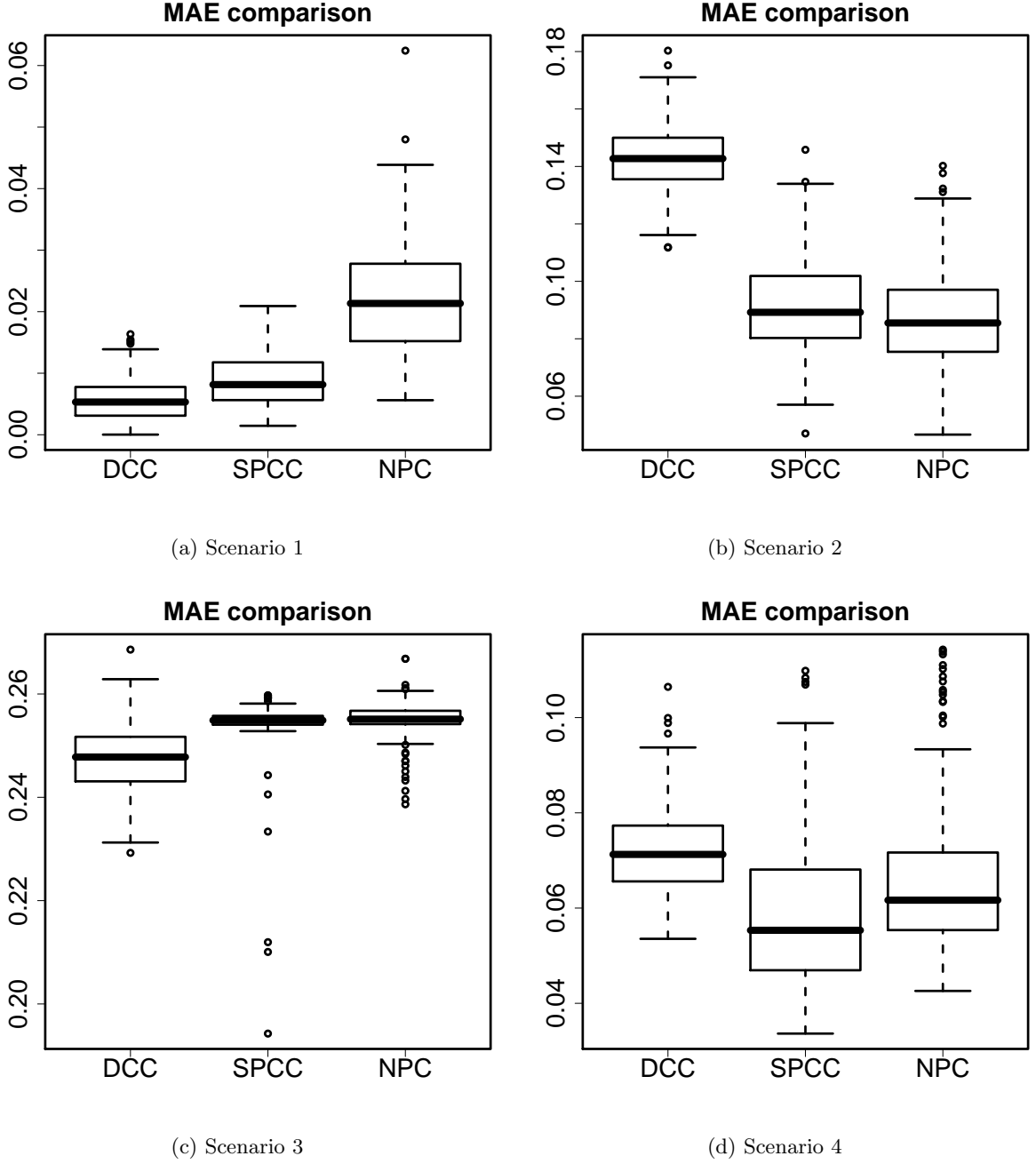


Figure 2: Distribution of MAE for the different scenarios of Experiment 1.

$$h_{1,t} = 0.01 + 0.025r_{1,t-1}^2(1 - \mathbb{I}_{\{r_{1,t-1} < 0\}}) + 0.075r_{1,t-1}^2\mathbb{I}_{\{r_{1,t-1} < 0\}} + 0.94h_{1,t-1} \quad (17)$$

$$h_{2,t} = 0.5 + 0.1r_{2,t-1}^2(1 - \mathbb{I}_{\{r_{2,t-1} < 0\}}) + 0.3r_{2,t-1}^2\mathbb{I}_{\{r_{2,t-1} < 0\}} + 0.5h_{1,t-1}$$

where $\mathbb{I}_{\{A\}} = 1$ if A is true and zero otherwise.

We choose the parameter values such that the effect of a negative lagged return on current volatility is three times larger than the effect of a positive lagged return. In practice, we test the robustness of the previous results by estimating symmetric GARCH(1,1) processes. As for the correlation, we assume the same four scenarios described previously.

Figures 3–4 show that for Scenarios 1–3 the results are qualitative similar to the ones reported for symmetric GARCH(1,1). The DCC is the best model for constant (Scenario 1) and rapid changes in correlations (Scenario 3), while the NPC is the dominant specification for gradual changes in correlations (Scenario 2). In contrast, the results for Scenario 4 with the SPCC becoming sensitive to misspecification in the conditional variance and performing the worst. As seen, the NPC outperforms its two competitors being in overall the best model in two out of four scenarios.

2.3 Experiment 3

The main drawback of the univariate (asymmetric) GARCH(1,1) processes simulated in Sections 2.1-2.2 is that they rule out potential feedback effects between the volatilities. In this experiment, we simulate DGPs from the following bivariate unrestricted GARCH(1,1) system (Conrad and Karanasos, 2009):

$$\begin{aligned} h_{1,t} &= 0.1 + 0.03r_{1,t-1}^2 + 0.02r_{2,t-1}^2 + 0.3h_{1,t-1} + 0.1h_{2,t-1} \\ h_{2,t} &= 0.2 + 0.2r_{1,t-1}^2 + 0.05r_{2,t-1}^2 - 0.15h_{1,t-1} + 0.8h_{2,t-1} \end{aligned} \quad (18)$$

Notice that sign of the volatility feedback is different across the two equations. The second asset has a positive volatility effect on the first variable, while the first asset has a negative volatility effect on the second asset (for volatility spillovers see (Baele, 2005; Diebold and K., 2010)). The parameter values for these DGPs are chosen from Conrad and Karanasos (2009).

Results in Figures 5 and 6 are very much in line with the results when the volatility is generated with a univariate symmetric GARCH(1,1) process. For instance, the NPC continues to outperform the DCC and only slightly the SPCC in Scenario 2 (gradual changes) while in Scenario 4 (structural break) the SPCC delivers the best outcome (again the performance of SPCC and NPC is very similar). The DCC, however, is the best specification in Scenario 1 (constant correlation) and Scenario 3 (rapid changes).

2.4 Experiment 4

In this part, we perform a multivariate simulation experiment to show the performance of the models in higher dimensions. Higher-dimensional models are of particular interest as portfolios are typically designed to include many assets. We decided to do an experiment with the returns of four variables. As before, the series have a length of $T = 1000$ and the number of simulations

is $M = 200$.

A fundamental issue for any multivariate model is how to guarantee positive definiteness of the conditional covariance matrix. One solution is to use real data to estimate conditional correlations and then consider these correlations as the true DGP. In practice, the correlations can be obtained by any conditional correlation method. Here, we consider just one scenario and allow for time—variability in the correlations obtained from a DCC specification. Thus, this way we can ensure that we have a positive definite matrix to generate the innovations $\epsilon_t \sim N(\mathbf{0}, R_t)$ for $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t})'$.

As for performance measure, we use an overall mean absolute error which for each simulation M is defined by:

$$\text{MAE}_M = \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T |\text{vecl}(R_t) - \text{vecl}(\hat{R}_t)|. \quad (19)$$

Figure 7 plots the correlation pairs obtained from each model chosen as the typical sample whose MAE is equal to the median of all the MAE_M , while Figure 8 shows the boxplot of the MAE_M for the 200 simulations. As seen, the SPCC and NPC models outperform substantially the DCC specification. Thus, while in bivariate models there are occasions where the DCC can perform well, at a multivariate framework the semiparametric and nonparametric models clearly delivers the best results. Once again, the performance of SPCC and NPC is quite similar. Figure 8 also shows that for all three models there is very little dispersion in MAE values.

3 Empirical Results

Empirical examples of the above three correlation models are presented for two interesting portfolios. The first, referred to as SPDR, consists of the nine Select Sector SPDRs Exchange Traded Funds (ETF) that divide the S&P 500 index into sector index funds. The second portfolio consists of five major currencies plus two from emerging economies that are actively traded in the foreign exchange market. These portfolios have been studied by the Volatility Institute of the New York University.

3.1 Portfolio of Equity SPDRs and the S&P 500

We estimate the cross-correlation of a portfolio containing nine equity SPDRs and the S&P 500. The data of interest is daily from January 5, 2004 until December 21, 2009 (T=1504 observations). This period includes some years of market stability followed by the last financial crisis years. We expect a high correlation between the SPDRs and the S&P 500 which is built with the most representative companies of each sector. Table 1 summarises the sectors and their market index.

Sector	Index
Materials Select Sector SPDR Fund	XLB
Energy Select Sector SPDR Fund	XLE
Financial Select Sector SPDR Fund	XLF
Industrial Select Sector SPDR Fund	XLI
Technology Select Sector SPDR Fund	XLK
Consumer Staples Select Sector SPDR Fund	XLP
Utilities Select Sector SPDR Fund	XLU
Health Care Select Sector SPDR Fund	XLV
Consumer Discretionary Select Sector SPDR Fund	XLY

Table 1: List of the nine sector indexes within the S&P 500.

Figure 9 shows the correlation estimates obtained from the three correlation models. In practice, we estimate 45 correlation pairs but for presentation purposes, we only plot the correlation between the S&P 500 and the sectors. The volatility is assumed to be driven by a GARCH(1,1) process.

It can be seen that for most of the sample period the Financial, the Industrial, the Technology, the Consumer Staples, the Health Care and the Consumer Discretionary sectors show a stable as well as high correlation with the S&P 500. High correlation, however, means that there are little diversification opportunities in a portfolio including the S&P 500 and the sector indexes mentioned above. It is interesting to see though that the Utilities, Energy and, to a lesser extent, Materials sectors display a different behaviour. Their correlation with the S&P 500 experiences notable drops. For instance in the fourth quarter of 2006 and in the third quarter of 2008, correlations for these sectors decrease to values close to 0.3. The first period might be linked to the background of the financial crisis. In particular, one of the main causes of the crisis was the bursting of the housing bubble which peaked in approximately 2006. On the other hand, the second period corresponds to the time when the crisis hits its most critical stage. This distinct behaviour of the aforementioned correlations may be expected given that energy and utilities are considered noncyclical sectors.

Let us investigate the changes in the behaviour of the correlation of the Financial sector with the rest. This is of interest given that the current economic crisis was initiated by irregularities in the financial sector. The Financial index has a similar behaviour to the S&P 500. Figure 10 shows how the decay in the Financial index at the beginning of 2007 was followed closely by a decay in the S&P 500 only a few weeks later. The correlations of the Financial index with the other indexes are plotted in Figure 11. We observe that when the financial crisis hit its peak in September and October 2008³, we observe that there was a strong drop in the correlation with

³During September–October 2008 several major financial institutions either failed, were acquired under duress, or were subject to government takeover. These included Lehman Brothers, Merrill Lynch, Fannie Mae, Freddie

the Energy, Materials and Utilities sectors. This pattern is the same as the one for the S&P500 vs. the sectors. Indeed, the correlation between Financials and S&P 500 is quite stable and around 0.9 during the whole period. Also, it is interesting to notice that the correlation between the financial sector and health care decreases later on in the sample during the second and third quarters of 2009.

3.2 Portfolio of Currencies

We consider daily US dollar exchange rates of the Australian dollar (AUS), Swiss franc (CHF), Euro (EUR), British pound (GBP), South African rand (RAND), Brazilian real (REALB), and Japanese yen (YEN) over the period from January 1, 1999 until May 7, 2010 (T=2856 observations).

Figures 12–14 plot the correlation estimates obtained from the three correlation models. As seen, correlations among the Australian dollar, Swiss franc, Euro and British pound shift to a higher level in the period 2002–2005. This may be related to the “global savings glut” a situation where during the first half of the decade industrial countries received large amounts of excess savings created in other parts of the world (e.g., South–East Asia).⁴ Notice also that the correlation of the Japanese yen with the other currencies started to decrease around 2006 and became even negative in the period afterwards. This may reflect the severe financial problems of the Japanese economy. We further observe that the correlations of the South African rand experience pronounced shifts, presumably linked to the efforts of the South African Reserve Bank to keep inflation within the target range. On the other hand, the correlation of the Brazilian real against the other currencies steadily increases (with the exception of the yen). This may reflect the improved macroeconomic stability of the Brazilian economy during this decade. Finally, we remark that the chf–eur pair is the most stable correlation typically approaching one, except for drops during the crisis period of 2008. Compared to the portfolio of the SPDRs, the correlations of the currencies show more variability over time implying also more frequent rebalancing of portfolios.

3.3 Evaluation of Models

We consider two types of criterion functions to evaluate the models in terms of portfolio’s certain characteristics. The first one is the mean square error (MSE) loss function and second one is based on the portfolio’s Value–at–Risk (VaR). Define the weighted in–sample portfolio’s returns

Mac, Washington Mutual, Wachovia and AIG.

⁴“Global savings glut” is a term coined by Ben Bernanke in his speech in 2005.

and variance as follows:

$$r_{p,t} = \omega_t' r_t \quad (20)$$

$$h_{p,t} = \omega_t' H_t \omega_t \quad (21)$$

where ω_t is a (possibly) time-varying weight vector and H_t is the in-sample covariance matrix. We consider two different portfolio weighting methods. First, the benchmark equally weighted portfolio (EWP) where the weights are constant and equal to $\omega_t = \omega = N^{-1}i$ and i is a $(N \times 1)$ vector of ones. Second, the minimum variance portfolio (MVP) where the weights are given by $\omega_t = H_t^{-1}i/(i'H_t^{-1}i)$. Note that the MVP weights are time-varying as they depend on the conditional covariance matrix. In practice, we allow the weight vector ω_t to change only after

every 20 observations, so that the portfolios are rebalanced approximately every month.

The MSE loss functions for the EWP and MVP are defined respectively as:

$$MSE^{j,EWP} = T^{-1} \sum_{t=1}^T \left(\omega' \hat{H}_t^j \omega - \omega' r_t r_t' \omega \right)^2 \quad (22)$$

$$MSE^{j,MVP} = T^{-1} \sum_{t=1}^T \left(\omega_{t_{20}}' \hat{H}_t^j \omega_{t_{20}} - \omega_{t_{20}}' r_t r_t' \omega_{t_{20}} \right)^2 \quad (23)$$

where \hat{H}_t^j is the covariance matrix estimate obtained from model j and $r_t r_t'$ is the matrix of the cross-product of the returns. The second loss functions is defined in terms of VaR. More specifically, the VaR values of the EWP and MVP for model j at the confidence level α are given by:

$$VaR_t^{j,EWP}(\alpha) = \Phi_\alpha \sqrt{\omega' \hat{H}_t^j \omega} \quad (24)$$

$$VaR_t^{j,MVP}(\alpha) = \Phi_\alpha \sqrt{\omega_{t_{20}}' \hat{H}_t^j \omega_{t_{20}}} \quad (25)$$

where Φ_α is the quantile of standard normal distribution at tail probability $\alpha \in (0, 1)$. The corresponding VaR loss functions for model j is calculated as follows:

$$Q^{j,EWP}(\alpha) = T^{-1} \sum_{t=1}^T \left(\alpha - \mathbb{I}_{\{r_{p,t} < VaR_t^{j,EWP}(\alpha)\}} \right) \left(r_{p,t} - VaR_t^{j,EWP}(\alpha) \right) \quad (26)$$

$$Q^{j,MVP}(\alpha) = T^{-1} \sum_{t=1}^T \left(\alpha - \mathbb{I}_{\{r_{p,t} < VaR_t^{j,MVP}(\alpha)\}} \right) \left(r_{p,t} - VaR_t^{j,MVP}(\alpha) \right). \quad (27)$$

In practice, we use $\alpha = 5\%$.

Table 2 shows the criterion functions of the estimated conditional correlation models. Over-

all, the results show that the NPC dominates the SPCC and DCC models. In particular, for the MVP case the NPC offers a significant improvement over its two competitors. On the other hand, for the EWP the performance of the different correlations models is similar, with the largest difference perhaps in the MSE loss function.

Panel A: MSE loss for in-sample estimation of the conditional covariance matrix.

	SPDR			Currencies		
	DCC	SPCC	NPC	DCC	SPCC	NPC
Equal weight	0.0346	0.0344	0.0287	0.4300	0.4276	0.3856
Minimum variance weight	0.0053	0.0051	0.0001	0.1725	0.1488	0.0347

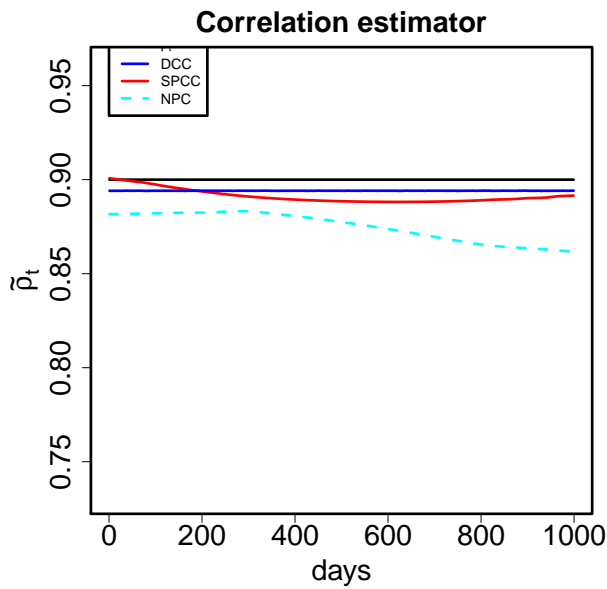
Panel B: VaR loss at 5% for in-sample estimation of the conditional covariance matrix.

	SPDR			Currencies		
	DCC	SPCC	NPC	DCC	SPCC	NPC
Equal weight	0.0173	0.0173	0.0170	0.8005	0.7993	0.7789
Minimum variance weight	0.0095	0.0094	0.0069	0.5961	0.5853	0.4829

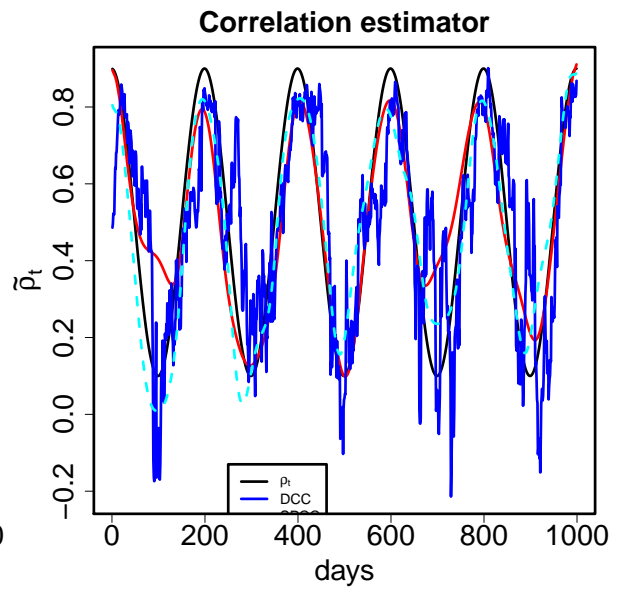
Table 2: Evaluation of empirical results of both portfolios.

4 Conclusion

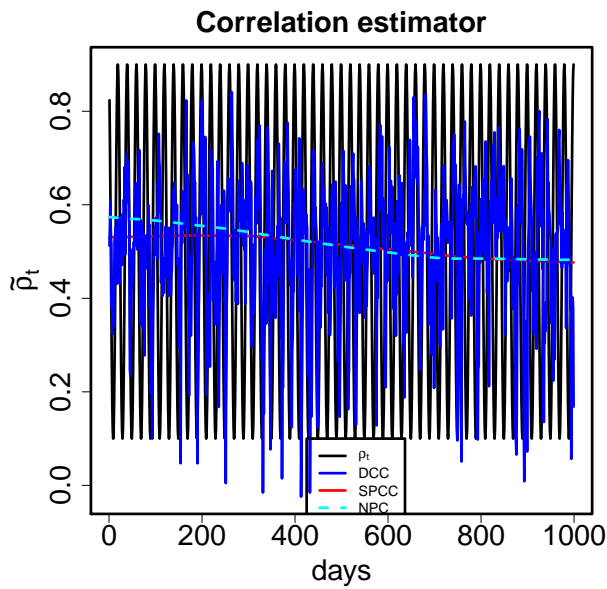
In this paper, we compare three promising methodologies of time-varying asset correlations. The popular parametric DCC model, a semiparametric model and a fully nonparametric approach. In terms of Monte Carlo simulations and bivariate processes the semiparametric and nonparametric models perform well when correlations experience gradual changes or a structural break. On the other hand, the DCC model is the best in DGPs with rapid changes or constancy in correlations. Moreover, in a multivariate framework our simulation exercise shows that the semiparametric and nonparametric methods are far better than the DCC. With regard to the application we consider two asset portfolios during the recent financial crisis. The first portfolio consists of equity sectors and the S&P 500, while the second one contains major currencies that are actively traded in the foreign exchange market. We carry out a portfolio evaluation exercise and show that the nonparametric model dominates its competitors, particularly in minimum variance weighted portfolios. However, our application considers portfolios based only on in-sample results. We feel this can be improved by considering out-of-sample forecasts of the conditional correlation/covariance matrix. Also, as a future research it would be interesting to look at other applications of the above models including bonds, international stock markets and commodities prices.



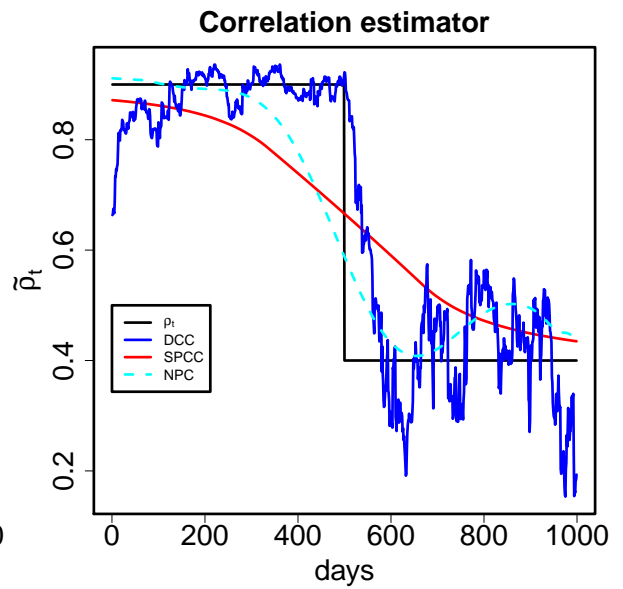
(a) Scenario I



(b) Scenario II

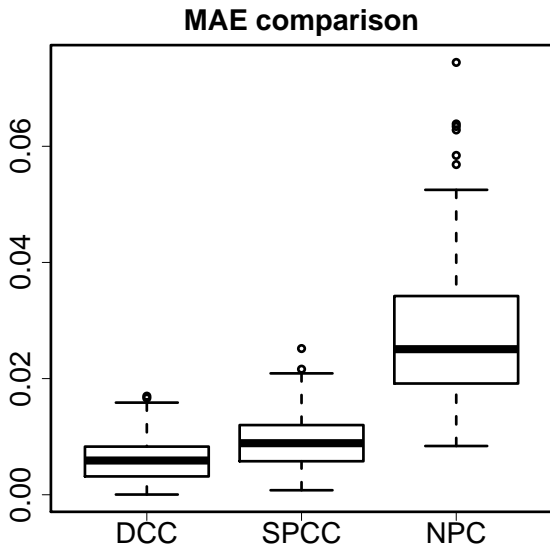


(c) Scenario III

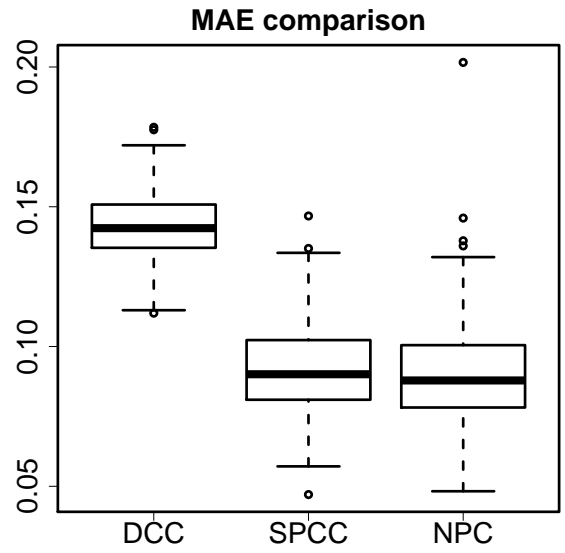


(d) Scenario IV

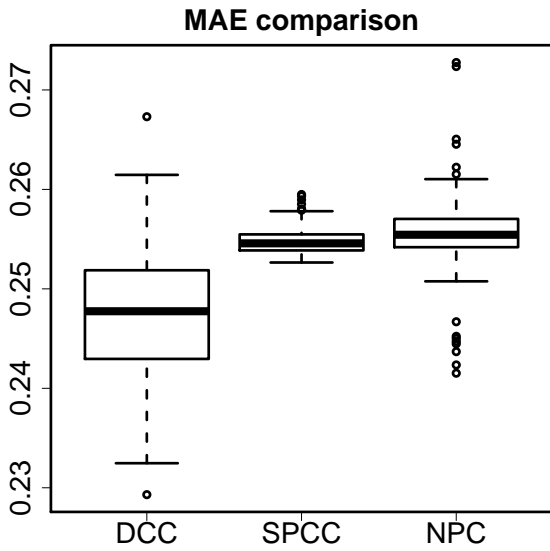
Figure 3: Correlation estimators for the different scenarios of Experiment 2.



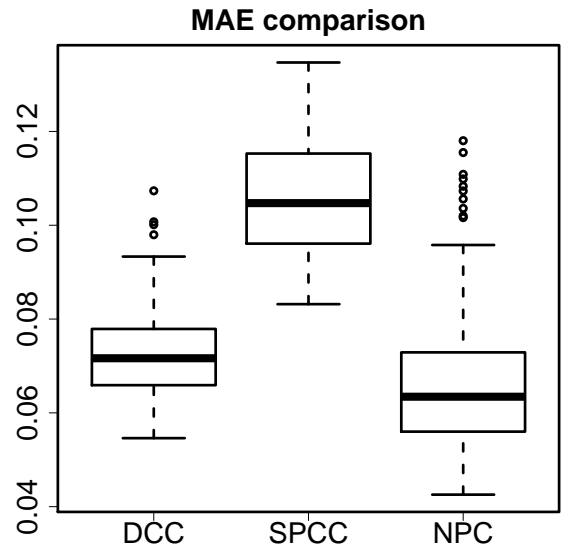
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 4: Distribution of MAE for the different scenarios of Experiment 2.

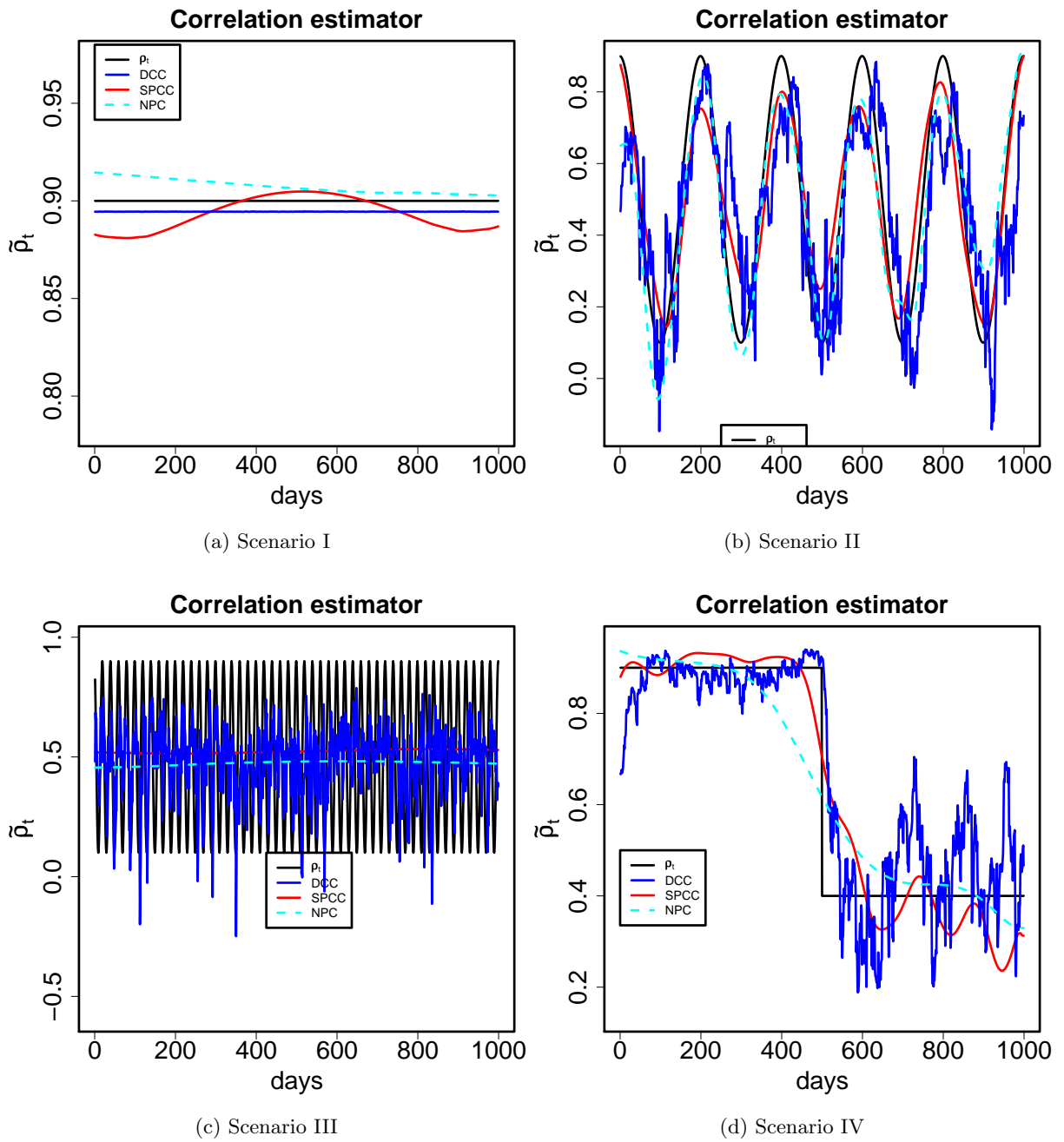
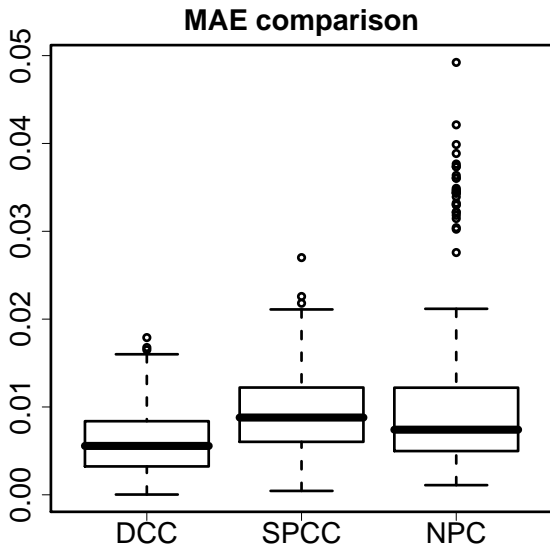
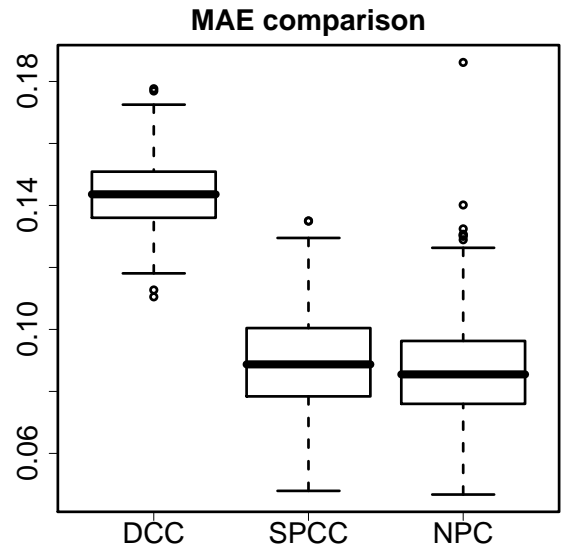


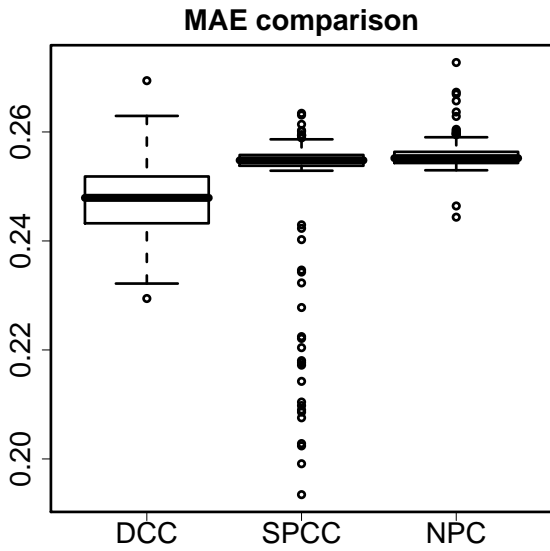
Figure 5: Correlation estimators for the different scenarios of Experiment 3.



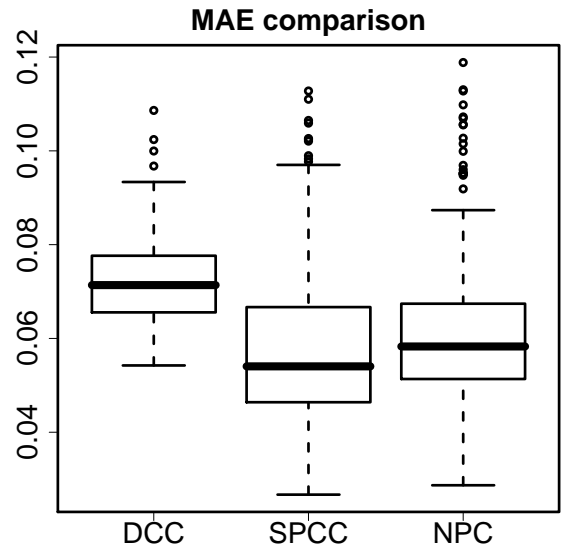
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 6: Distribution of MAE for the different scenarios of Experiment 3.

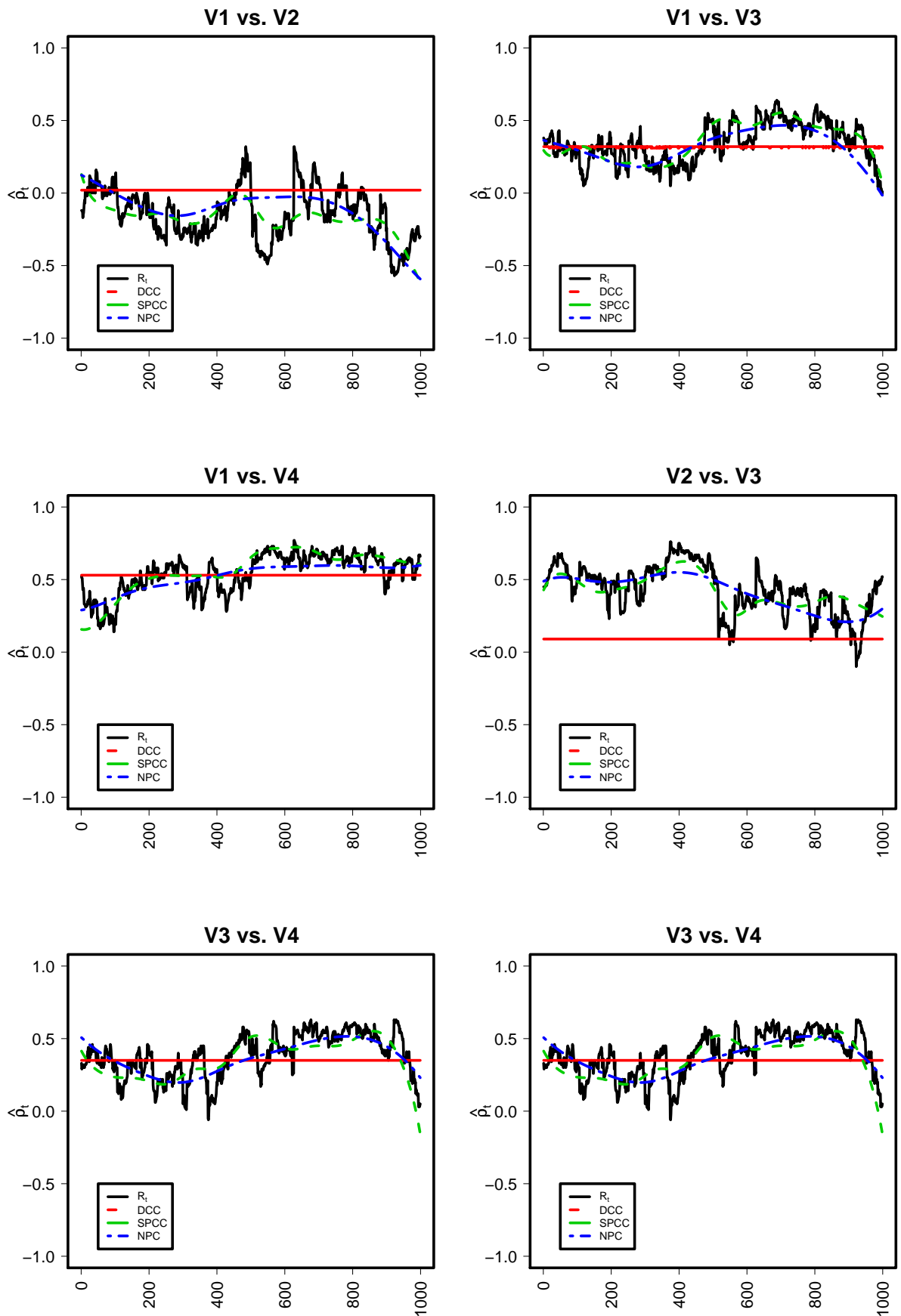


Figure 7: Correlation estimators for Experiment 4.

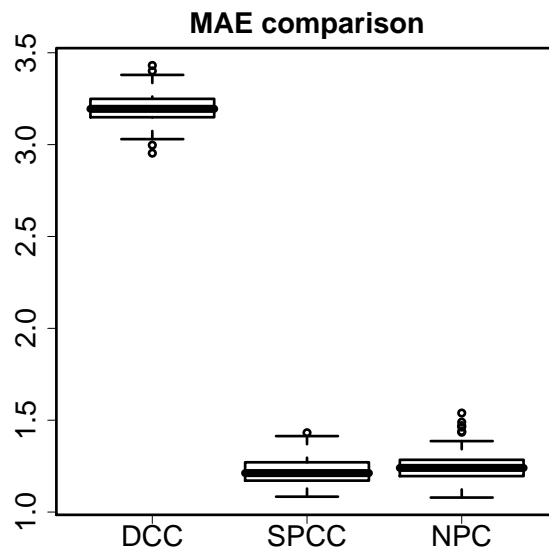


Figure 8: Distribution of MAE for Experiment 4.

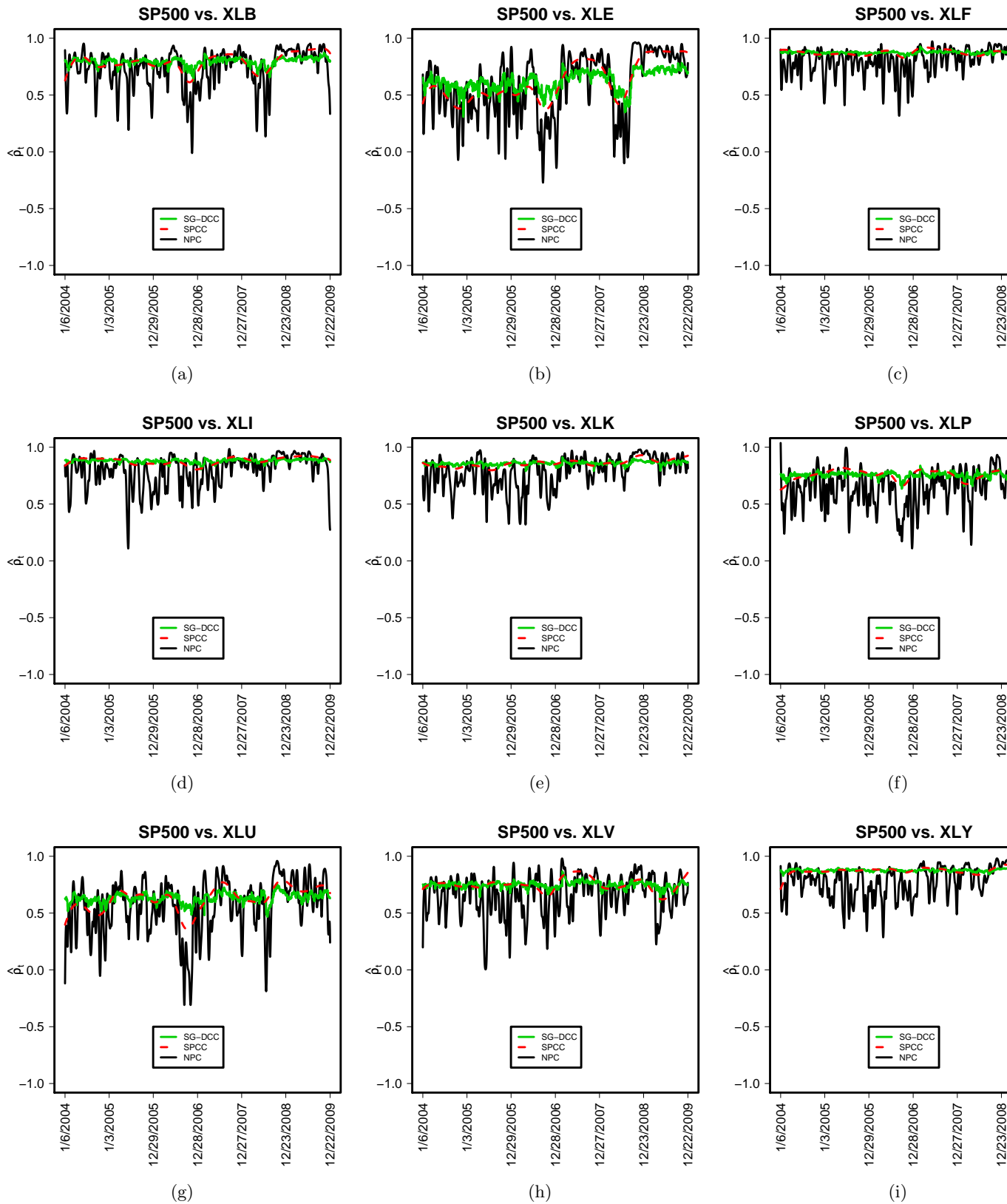


Figure 9: Correlations of S&P500 with the rest of the sectors when the volatility process is estimated with a GARCH(1,1) process.

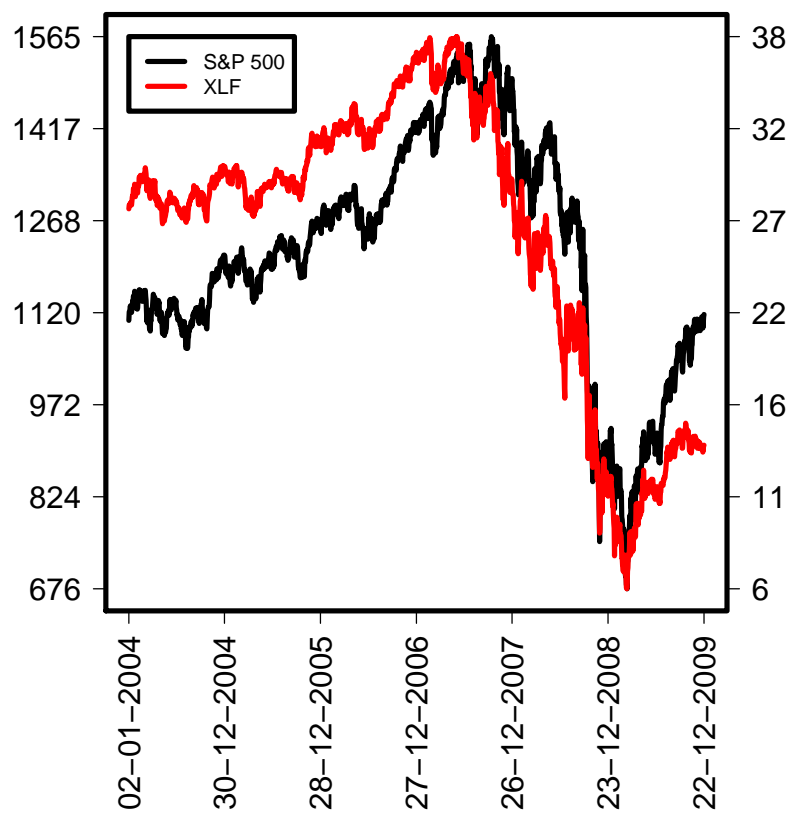


Figure 10: Comparison of the S&P 500 and the Financial sector index. The left axis displays the values of the S&P 500 and the right axis displays the values of the XLF.

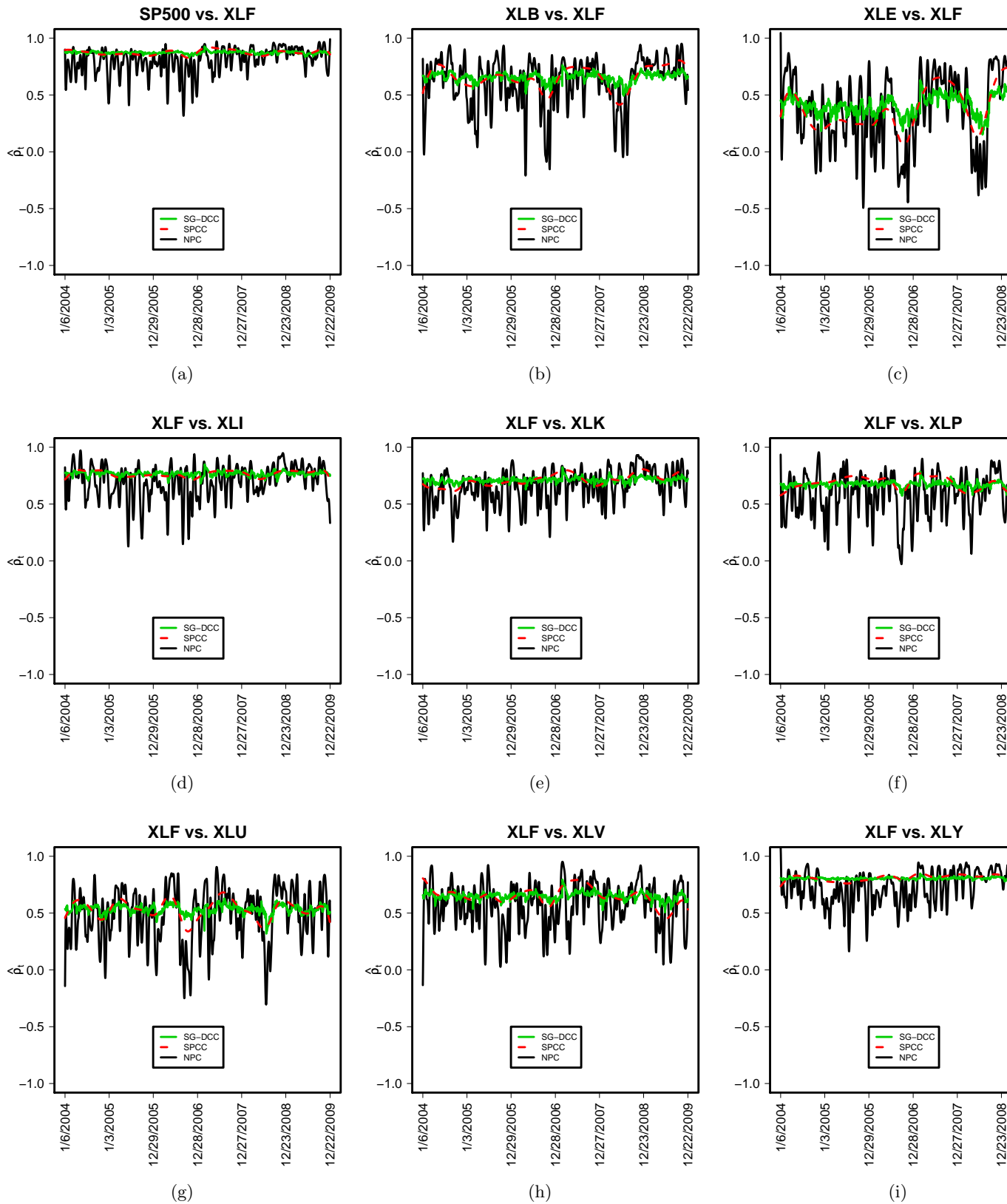


Figure 11: Correlations of XLF with the rest of the sectors when the volatility is estimated with a GARCH(1,1) process.

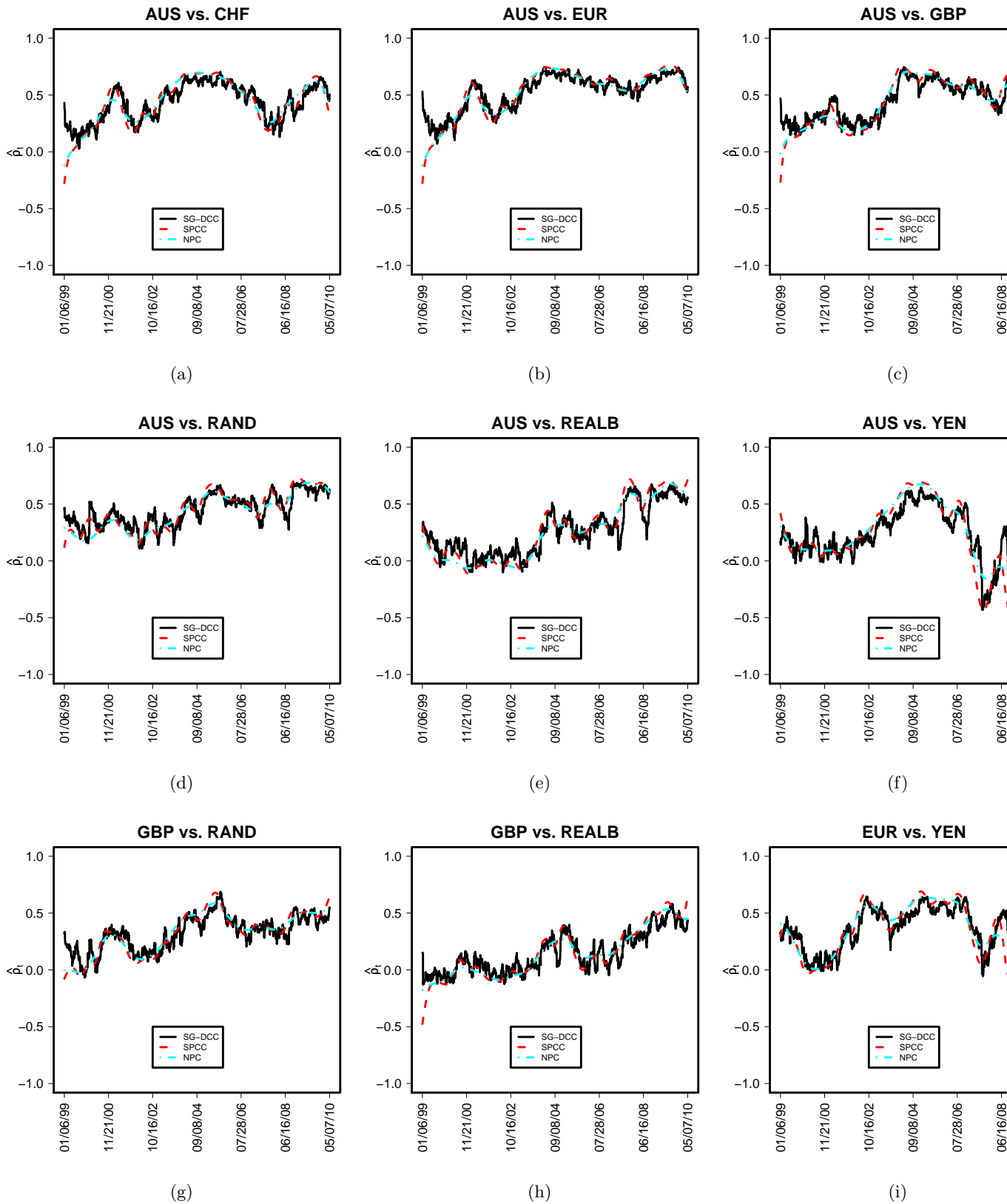


Figure 12: Correlations of AUS and GBP with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.

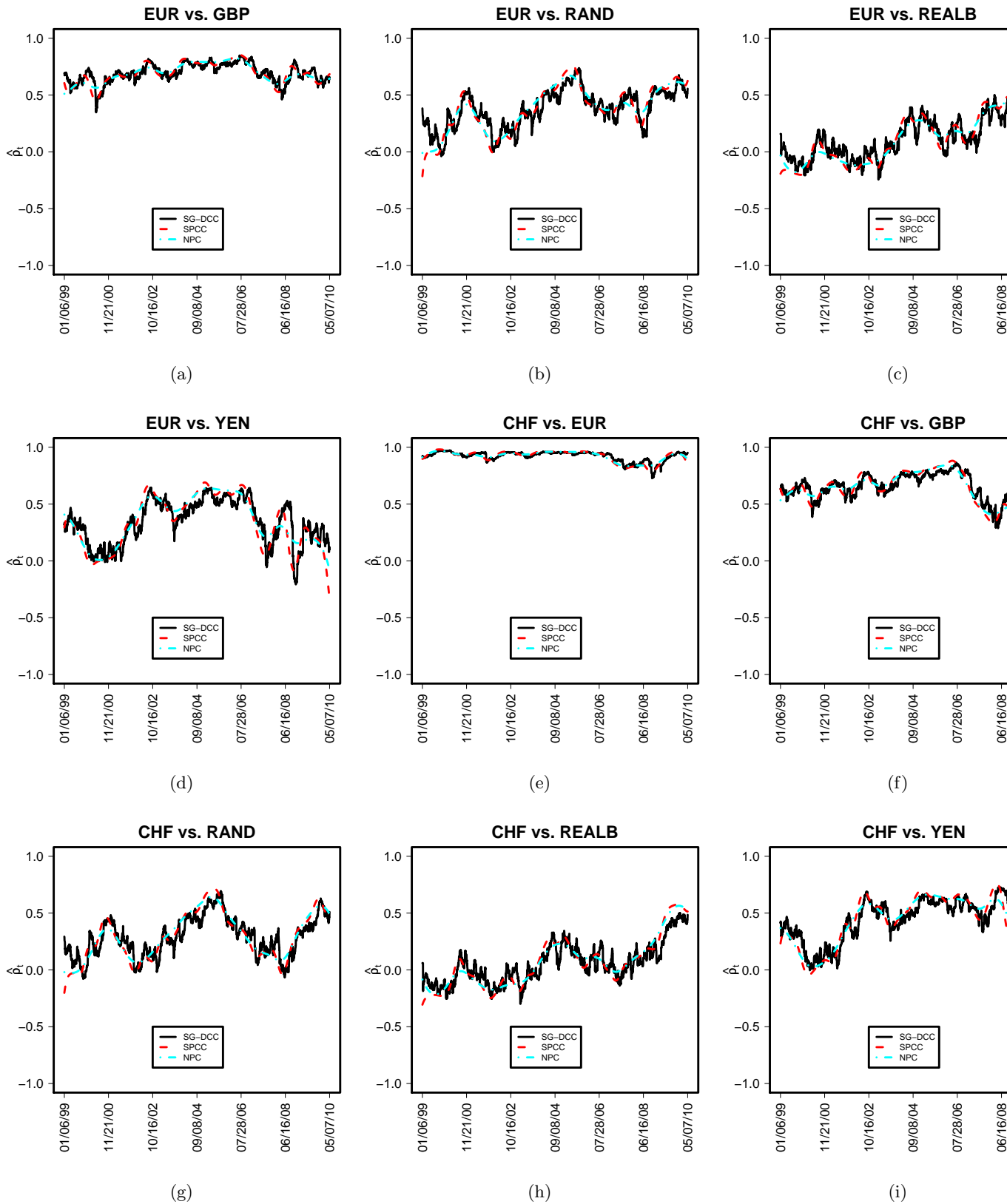
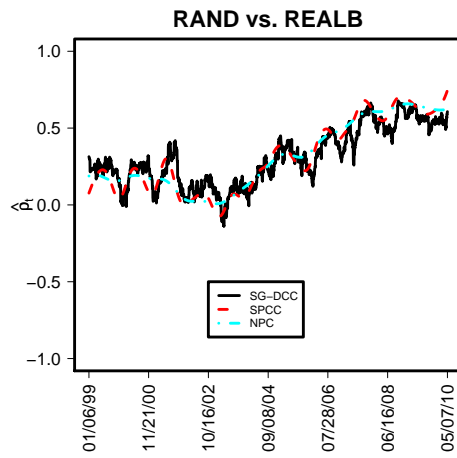
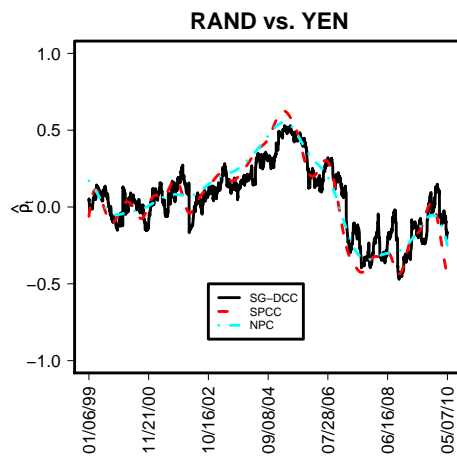


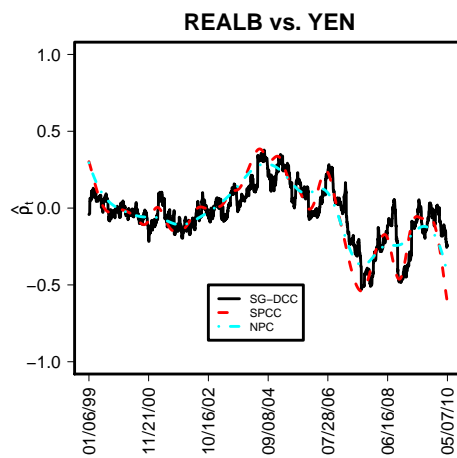
Figure 13: Correlations of EUR and CHF with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.



(a)



(b)



(c)

Figure 14: Correlations of RAND, REALB and YEN with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.

References

- L. Baele. Volatility spillover effects in european equity markets. *Journal of Financial and Quantitative Analysis*, pages 373–402, 2005.
- R.P. Berben and W.J. Jansen. Comovement in international equity markets: A sectoral view. *Journal of International Money and Finance*, 24:832–857, 2005.
- L. Cappiello, R.F. Engle, and K. Sheppard. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, pages 537–572, 2006.
- C. Conrad and M. Karanasos. Negative volatility spillovers in the unrestricted eccc-garch model. *Econometric Theory*, pages 1–25, 2009.
- R. Dahlaus and S.S. Rao. Statistical inference for time-varying arch processes. *The Annals of Statistics*, pages 1075–1114, 2006.
- F.X. Diebold and Yilmaz K. Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting*, 2010. Forthcoming.
- H. Drees and C. Stărică. A simple non-stationary model for stock returns. 2002. In print.
- R.F. Engle. Dynamic conditional correlation – a simple class of multivariate garch models. *Journal of Business and Economic Statistics*, pages 339–350, 2002.
- R.F. Engle. *Anticipating correlations*. Princeton University Press, 2009.
- J. Fan and I. Gijbels. *Local Polynomial Modelling and Its Applications*. Chapman and Hall/CRC, 1996.
- R. Fry, V.L. Martin, and C. Tang. A new class of tests of contagion with applications. *Journal of Business and Economic Statistics*, pages 423–437, 2010.
- L.R. Glosten, R. Jagannathan, and Runkle D.E. On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, pages 1779–1801, 1993.
- C.M. Hafner and P.H. Franses. A generalized dynamic conditional correlation model for many assets. *Econometric Reviews*, pages 612–631, 2009.
- C.M. Hafner, D.J.C. Van Dijk, and P.H. Franses. Semi-parametric modelling of correlation dynamics. *Advances in Econometrics*, pages 59–103, 2006.
- X.D. Long, L.J. Sun, and A. Ullah. Estimation of dynamic conditional covariance: A semiparametric multivariate model. *Journal of Business Economics and Statistics*, 2010. Forthcoming.

- D. Pelletier. Regime switching for dynamic correlations. *Journal of Econometrics*, pages 445–473, 2006.
- P.M. Robinson. Nonparametric estimation of time-varying parameters. *Statistics analysis and forecasting of economic structural change*, pages 253–264, 1989.
- A. Silvennoinen and T. Teräsvirta. Multivariate autoregressive conditional heteroskedasticity with smooth transitions in conditional correlations. Technical report, Working Paper Series in Economics and Finance No. 577, SSE/EFI, 2005.
- A. Silvennoinen and T. Teräsvirta. Modelling multivariate autoregressive conditional heteroskedasticity with the double smooth transition conditional correlation garch model. *Journal of Financial Econometrics*, pages 373–411, 2009.

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