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Jump Tails, Extreme Dependencies, and the Distribution of Stock Returns

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Abstract

We provide a new framework for estimating the systematic and idiosyncratic jump tail risks in financial asset prices. The theory underlying our estimates are based on in-fill asymptotic arguments for directly identifying the systematic and idiosyncratic jumps, together with conventional long-span asymptotics and Extreme Value Theory (EVT) approximations for consistently estimating the tail decay parameters and asymptotic tail dependencies. On implementing the new estimation procedures with a panel of high-frequency intraday prices for a large cross-section of individual stocks and the aggregate S&P 500 market portfolio, we find that the distributions of the systematic and idiosyncratic jumps are both generally heavy-tailed and not necessarily symmetric. Our estimates also point to the existence of strong dependencies between the market-wide jumps and the corresponding systematic jump tails for all of the stocks in the sample. We also show how the jump tail dependencies deduced from the high-frequency data together with the day-to-day temporal variation in the volatility are able to explain the "extreme" dependencies vis-a-vis the market portfolio.

Keywords: Extreme events, jumps, high-frequency data, jump tails, non-parametric estimation, stochastic volatility, systematic risks, tail dependence.

JEL classification: C13, C14, G10, G12.

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1 Introduction

Tail events and non-normal distributions are ubiquitous in finance. The earliest comprehensive empirical evidence for fat-tail marginal return distributions dates back more than half a century to the influential work of Mandelbrot (1963) and Fama (1965). It is now well recognized that the fat-tailed unconditional return distributions first documented in these, and numerous subsequent, studies may result from time-varying volatility and/or jumps in the underlying stochastic process governing the asset price dynamics. Intuitively, periods of high-volatility can result in seemingly "extreme" price changes, even though the returns are drawn from a normal distribution with light tails but one with an unusually large variance; see e.g., Bollerslev (1987), Mikosch and Starica (2000), and the empirical analyzes in Kearns and Pagan (1997) and Wagner and Marsh (2005) pertaining to the estimation of tail parameters in the presence of GARCH effects. On the other hand, the aggregation of multiple jump events over a fixed time interval will similarly result in fat-tailed asset return distributions, even for a pure Lévy-type jump processes with no dynamic dependencies; see, e.g., Carr et al. (2002). As such, while fundamentally different, these two separate mechanisms will both manifest themselves in the form of apparent "tail" events and leptokurtic marginal return distributions.[1]

These same general issues carry over to a multivariate context and questions related to "extreme" dependencies across assets. In particular, it is well documented that the correlations between equity returns, both domestically and internationally, tend to be higher during sharp market declines than during "normal" periods;[2] see e.g., Longin and Solnik (2001) and Ang and Chen (2002). Similarly, Starica (1999) documents much stronger dependencies for large currency moves compared to "normal-sized" changes. In parallel to the marginal effects, however, it is unclear whether these increased dependencies are coming from commonalities in time-varying volatilities across assets and/or common jumps. Poon et al. (2004), for instance, report that "devolatilizing" the daily returns for a set of international stock markets significantly reduces the joint tail dependence, while Bae et al. (2003) find that time-varying volatility and GARCH effects can not fully explain the counts of coincident "extreme" daily price moves observed across international equity markets. More closely related to the present paper, recent studies by Bollerslev et al. (2008), Jacod and Todorov (2009), and Gobbi and Mancini (2009), based on high-frequency data and nonparametric methods, have all argued for the presence of common jump arrivals across different assets, thus possibly inducing stronger dependencies in the "extreme."

[1]Importantly, these different mechanisms also have very different pricing implications and risk premia dynamics, as recently explored by Bollerslev and Todorov (2010b).

[2]The use of simple linear correlations as a measure of dependence for "extreme" observations has been called into question by Embrechts et al. (2002), among others.

In light of these observations, one of the goals of the present paper is to separate jumps from volatility to more directly assess the "extreme" dependencies inherent in the jump tails. Motivated by the basic idea from asset pricing finance that only non-diversifiable systematic jump risks should be compensated, we further dissect the jumps into their systematic and idiosyncratic components, as in Todorov and Bollerslev (2010). This decomposition in turn allows us to compare and contrast the behavior of the two different jump tails and how they impact the return distributions.[3]

Our estimation methodology is based on the idea that even though jumps and time-varying volatility may have similar implications for the distribution of the returns over coarser sampling frequencies, the two features manifest themselves very differently in high-frequency returns. Intuitively, treating the volatility as locally constant over short time horizons, it is possible to perfectly separate jumps from the price moves associated with the slower temporally varying volatility through the use of increasingly finer sampled observations. Empirically, this allows us to focus directly on the high-frequency "filtered" jumps. Relying on the insight from Bollerslev and Todorov (2010a) that regardless of any temporal variation in the jump intensity, the jump compensator for the "large" jumps behaves like a probability measure, we non-parametrically estimate the decay parameters for the univariate jump tails using a variant of the Peaks-Over-Threshold (POT) method.[4]

Going one step further, we characterize the extreme joint behavior of the "filtered" jump tails through non-parametric estimates of Pickands (1981) dependence function. This particular functional relationship provides a general framework for describing a bivariate extreme value distribution from its marginals.[5] Following the original suggestion by Sibuya (1960), and the more recent discussion of extreme dependence measures in Coles et al. (1999), we further assess the degree of asymptotic jump tail dependence through a plug-in estimate from Pickands dependence function for the probability of observing an "extreme" jump tail event given that the other jump tail is also "extreme." Together with the estimated decay parameters for each of the underlying univariate extreme distributions, this summary measure describes the key features of the bivariate joint tail behavior.

Our actual empirical analysis is based on high-frequency observations for fifty large capi-

[3]In a related context, Barigozzi et al. (2010) have recently explored a factor structure for disentangling the total realized variation for a large panel of stocks into a single systematic component and remaining idiosyncratic components.

[4]The POT method for characterizing extremes dates back to Fisher and Tippett (1928). It has been formalized more recently by Balkema and de Haan (1974) and Pickands (1975); for general textbook discussions see also Embrechts et al. (2001) and Jondeau et al. (2007).

[5]For a general textbook discussion, see, e.g., Coles (2001) and Beirlant et al. (2004). Existing applications of this idea have primarily been restricted to climatology and insurance. Steinkohl et al. (2010), for instance, have recently employed this approach to characterize the asymptotic dependence for high-frequency wind speeds across separate geographical locations.

talization stocks and the S&P 500 aggregate market portfolio spanning the period from 1997 through 2008. We find that the number of "filtered" idiosyncratic jumps exceed the number of systematic jumps for all of the stocks in the sample, and typically by quite a large margin. Nonetheless, the hypothesis of fully diversifiable individual jump risk is clearly not supported by the data, thus pointing to more complicated dependence structures in the tails than hitherto entertained in most of the existing asset pricing literature.[6] Even though the assumption of "light" Gaussian jump tails can not necessarily be rejected based on many of the individual estimates, the combined evidence for all of the stocks clearly support the hypothesis of heavy jump tails. Our estimates for the individual jump tail decay parameters also suggest that the tails associated with the systematic jumps are slightly thinner than those for the idiosyncratic jumps, albeit not uniformly so. Somewhat surprisingly, we also find that the right tail decay parameters for both types of jumps often exceed those for the left tail.

Our estimates of Pickands dependence function reveal a strong degree of asymptotic tail dependence between the market-wide jumps and the systematic jumps in the individual stocks. This therefore calls into question the assumption of normally distributed, and thereby asymptotic independent, jumps previously used in the literature.

Further, comparing our high-frequency based estimation results with those obtained from daily returns, we find that the latter indicate much weaker asymptotic tail dependencies. Intuitively, while the estimates based on the daily returns represent the tail dependence attributable to both systematic jumps and common volatility factors, both of which may naturally be expected to be associated with positive dependence, the idiosyncratic jumps when aggregated over time will tend to weaken the dependence. In contrast, by focussing directly on the high-frequency "filtered" systematic and idiosyncratic jumps, we are able to much more accurately assess the true extreme jump tail dependencies, and assess how the different effects impart the dependencies in the lower frequency daily returns.

The rest of the paper is organized as follows. Section 2 introduces the formal setup and assumptions. Section 3 outlines the statistical methodology and econometric procedures, beginning in Section 3.1 with the way in which we disentangle jumps from continuous prices moves, followed by a discussion of our univariate tail estimation procedures in Section 3.2, and the framework that we rely on for assessing the joint jump tail dependencies in Section 3.3. Section 4 summarizes our main empirical results, starting in Section 4.1 with a brief description of the data, followed by our findings pertaining to the individual jump tails in Section 4.2, and the bivariate jump tail dependencies in Section 4.3. Section 5 concludes.

[6]The mere existence of market-wide jumps, of course, refutes the hypothesis of fully diversifiable jump risk as in Merton (1976). The estimates reported in, e.g., Eraker et al. (2003), also suggest a large risk premium for systematic jump risk.

2 Formal Setup and Assumptions

We will work with a total of $M+1$ financial asset prices. The individual assets will be enumerated $1, ..., M$, while the aggregate market portfolio will be indexed by 0. The dynamics for the log-price for the j'th asset is assumed to follow the generic semimartingale process,

$$dp_t^{(j)} = \alpha_t^{(j)} dt + \sigma_t^{(j)} dW_t^{(j)} + \int_{\mathbb{R}} x \mu^{(j)}(dt, dx), \quad j = 0, ..., M, \tag{2.1}$$

where $\alpha_t^{(j)}$ and $\sigma_t^{(j)}$ are locally bounded processes, $W_t^{(j)}$ denote possibly correlated Brownian motions, and $\mu^{(j)}(ds, dx)$ are integer-valued random measures that capture the jumps in $p_t^{(j)}$ over time dt and size dx.[7]

Our main focus centers on the behavior of the jumps in the individual assets; i.e., the $\mu^{(j)}(ds, dx)$ measures for $j = 1, ..., M$. We will further categorize these jumps as being either systematic or idiosyncratic depending upon their association with the market wide jumps, or $\mu^{(0)}(ds, dx)$. As we show below, as long as the systematic market factor is assumed to be directly observable, such a decomposition can easily be formally justified and implemented empirically.[8]

To more rigorously set out our procedures, let

$$\mathcal{T}_{[0,T]}^{(j)} = \left\{ s \in [0, T] : \Delta p_s^{(j)} \neq 0 \right\}, \quad j = 0, ..., M,$$

where $\Delta p_s^{(j)} \equiv p_s^{(j)} - p_{s-}^{(j)}$, denote the set of jump times for asset j. The $\mathcal{T}_{[0,T]}^{(j)}$ sets may in theory be infinite, but countable, as the jump processes may be infinitely active.[9] Note also, that in a standard one-factor market model $\mathcal{T}_{[0,T]}^{(0)} \subset \mathcal{T}_{[0,T]}^{(j)}$, but in general this need not be the case.

Further denote with $\mu^{(j,0)}(ds, dx)$ the jump measure for asset j for the jumps that occur at the same time as the market-wide jumps; i.e., at times restricted to the intersection of $\mathcal{T}_{[0,T]}^{(j)}$ and $\mathcal{T}_{[0,T]}^{(0)}$. Similarly, let $\mu^{(j,j)}(ds, dx)$ denote the jump measure for the asset j jumps that occur at times restricted to the set $\mathcal{T}_{[0,T]}^{(j)} \setminus \left\{ \mathcal{T}_{[0,T]}^{(0)} \cap \mathcal{T}_{[0,T]}^{(j)} \right\}$. Then by definition

$$\mu^{(j)}(ds, dx) \equiv \mu^{(j,0)}(ds, dx) + \mu^{(j,j)}(ds, dx), \quad j = 1, ..., M.$$

In parallel, denote with $\mu^{(0,j)}(ds, dx)$ the jump measure for the aggregate market jumps that arrive at the same time as the jumps in asset j; i.e., the counting measure for the systematic jumps restricted to the subset $\mathcal{T}_{[0,T]}^{(0)} \cap \mathcal{T}_{[0,T]}^{(j)}$.

[7] Equation (2.1) implicitly assumes that the jumps are of finite variation. This assumption only restricts the behavior of the very small jumps, and has no practical implications for our subsequent analysis of the jump tails. We also implicitly assume that $\alpha_t^{(j)}$ and $\sigma_t^{(j)}$ both satisfy sufficient integrability conditions.

[8] The current analysis could also quite easily be extended to situations with more than one observable systematic risk factor, including e.g., the popular Fama-French portfolios.

[9] This has no practical implication for our statistical analysis, however, as we focus on the "large" jumps, of which there are always a finite number in a finite sample.

In addition, denote the compensators, or jump intensities, for $\mu^{(j,0)}(ds,dx)$, $\mu^{(j,j)}(ds,dx)$ and $\mu^{(0,j)}(ds,dx)$ by $dt \otimes \nu_t^{(j,0)}(dx)$, $dt \otimes \nu_t^{(j,j)}(dx)$ and $dt \otimes \nu_t^{(0,j)}(dx)$, respectively, where $\nu_t^{(j,0)}(dx)$, $\nu_t^{(j,j)}(dx)$ and $\nu_t^{(0,j)}(dx)$ are some nonnegative measures satisfying the condition

$$\int_{\mathbb{R}} (x^2 \wedge 1)\nu_t^{(j,0)}(dx) + \int_{\mathbb{R}} (x^2 \wedge 1)\nu_t^{(j,j)}(dx) + \int_{\mathbb{R}} (x^2 \wedge 1)\nu_t^{(0,j)}(dx) < \infty,$$

for any $t > 0$. The main goal of the paper then in essence amounts to estimating and succinctly characterizing the tail properties of $\nu_t^{(j,0)}(x)$, $\nu_t^{(j,j)}(x)$ and $\nu_t^{(0,j)}(x)$.

Rather than doing so directly, for theoretical reasons explained in Bollerslev and Todorov (2010a), we will do so for their images under the following mappings

$$\psi^+(x) = \begin{cases} e^x - 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \qquad \psi^-(x) = \begin{cases} 0, & x \geq 0 \\ e^{-x} - 1, & x < 0 \end{cases}. \tag{2.2}$$

This in effect transforms the logarithmic jumps Δp_s into $\frac{P_s - P_{s-}}{P_{s-}}$, or functions thereof, akin to switching from discrete-time logarithmic returns to arithmetic returns. In practice, of course, for the actually observed jumps, the difference between Δp_s and $\frac{P_s - P_{s-}}{P_{s-}}$ is very small.

For the implementation of our estimation strategy, we will further assume that the jump compensators $\nu_t^{(j,0)}(x)$ and $\nu_t^{(j,j)}(x)$ satisfy

$$\begin{aligned} \nu_t(dx)^{(j,d)} &= (\varphi_t^{+(j,d)} 1_{\{x>0\}} + \varphi_t^{-(j,d)} 1_{\{x<0\}})\nu^{(j,d)}(dx), \quad j = 1, ..., M, \quad d = 0, j, \\ \nu_t(dx)^{(0,j)} &= (\varphi_t^{+(j,0)} 1_{\{x>0\}} + \varphi_t^{-(j,0)} 1_{\{x<0\}})\nu^{(0,j)}(dx), \quad j = 1, ..., M, \end{aligned} \tag{2.3}$$

where $\varphi_t^{\pm(j,0)}$ and $\varphi_t^{\pm(j,j)}$ are nonnegative-valued stochastic processes with càdlàg paths.[10] The separability of the jump compensators into time and jump size in equation (2.3) is trivially satisfied for almost all of the parametric jump models hitherto analyzed in the literature, including the popular affine jump-diffusion class of models advocated by, e.g., Duffie et al. (2000).

Next, denote the tail jump intensities by $\overline{\nu}_\psi^{\pm(j,d)}(x) = \int_{\psi^{\pm}(u) \geq x} \nu^{\pm(j,d)}(du)$, for $x \in \mathbb{R}_+$. We will then assume that for some (and hence any) $x > 0$ and $u > 0$, the ratio[11]

$$\frac{\overline{\nu}_\psi^{\pm(j,d)}(u+x)}{\overline{\nu}_\psi^{\pm(j,d)}(x)}, \tag{2.4}$$

is in the domain of attraction of an extreme value distribution and satisfies a second-order condition as in, e.g., Smith (1987). Recall, see, e.g., Theorem 1.2.5 of de Haan and Ferreira

[10]Note that equation (2.3) implicitly assumes that the temporal variation in the jump intensities for asset j and the market portfolio constrained to the set $\mathcal{T}_{[0,T]}^{(0)} \cap \mathcal{T}_{[0,T]}^{(j)}$ are the same.

[11]Note that although the jump intensity $\overline{\nu}_\psi^{\pm(j,d)}(x)$ is not a distribution function, the ratio is.

(2006), that a distribution function F is defined to be in the domain of attraction of an extreme value distribution if and only if for some positive valued function f,

$$\lim_{u \uparrow x^*} \frac{1 - F(u + xf(u))}{1 - F(u)} = (1 + \xi x)^{-1/\xi}, \tag{2.5}$$

where $1+\xi x > 0$, and x^* denotes the endpoint of the distribution; i.e., $x^* = \sup\{x : F(x) < 1\}$. The case $\xi > 0$ corresponds to heavy-tail distributions, while $\xi = 0$ defines light tails.[12] In the heavy-tail case, the extreme value approximation amounts to assuming that $\overline{\nu}_{\psi}^{\pm(j,d)}(x)$ are regularly varying at infinity functions; i.e., $\overline{\nu}_{\psi}^{\pm(j,d)}(x) = x^{-\beta^{\pm(j,d)}} L^{\pm(j,d)}(x)$, where $\beta^{\pm(j,d)} > 0$ corresponds to ξ in (2.5), and $L^{\pm(j,d)}(x)$ are slowly varying at infinity functions.

To facilitate the discussion of our assumptions needed for the systematic jump tail dependencies, we will let $\nu_{\text{syst}}^{(j)}(\mathbf{x})$ for $\mathbf{x} \in \mathbb{R}^2 \setminus (0,0)$ denote a measure with marginals $\nu_{\text{syst}}^{(j)}([x \times (-\infty, +\infty)]) = \nu^{(j,0)}(x)$ and $\nu_{\text{syst}}^{(j)}([(-\infty, +\infty) \times x]) = \nu^{(0,j)}(x)$ for $x \in \mathbb{R}$, respectively. This measure will control the time-invariant part of the jump compensator of the jumps in asset j and the market constraint to the common set $\mathcal{T}_{[0,T]}^{(0)} \cap \mathcal{T}_{[0,T]}^{(j)}$. Generalizing the univariate tail measures to a vector $[x_1,\ x_2] \in \mathbb{R}_+^2 \setminus (0,\ 0)$, we denote the corresponding jump tail intensity by $\overline{\nu}_{\text{syst},\psi}^{\pm(j)}([x_1,\ x_2]) = \int_{\psi^{\pm}(u_1) \geq x_1,\ \psi^{\pm}(u_2) \geq x_2} \nu_{\text{syst}}^{\pm(j)}(d[u_1,\ u_2])$.[13]

We will then assume that for some (and hence any) $\mathbf{x} \in \mathbb{R}^2 \setminus (0,0)$ and $\mathbf{u} \in \mathbb{R}_+^2$, that the ratio

$$\frac{\overline{\nu}_{\text{syst},\psi}^{\pm(j)}(\mathbf{u} + \mathbf{x})}{\overline{\nu}_{\text{syst},\psi}^{\pm(j)}(\mathbf{x})}, \tag{2.6}$$

is in the domain of attraction of a mutlivariate extreme value distribution and satisfies certain second order conditions as in, e.g., Einmahl et al. (1997). Recall, see, e.g., Theorem 6.2.1 of de Haan and Ferreira (2006), that a bivariate distribution function F, with marginals F_i in the domain of attraction of $\exp(-(1 + \xi_i x)^{-1/\xi_i})$ for $i = 1, 2$, is defined to be in the domain of attraction of a multivariate extreme value distribution if and only if for every $x, y > 0$,

$$\lim_{u \to \infty} \frac{1 - F(U_1(u \cdot x), U_2(u \cdot y))}{1 - F(U_1(u), U_2(u))} = \int_0^{\pi/2} \left(\frac{1 \wedge \tan(\theta)}{x} \vee \frac{1 \wedge \cot(\theta)}{y} \right) \Phi(d\theta), \tag{2.7}$$

where $U_i(\cdot)$ for $i = 1, 2$ denote the inverse of the functions $x \to 1/(1-F_i(x))$ that standardize the marginals to belong to the domain of attraction of $exp(-1/x)$, and the distribution function $\Phi(\cdot)$ is concentrated on $[0, \pi/2]$ and satisfy the terminal condition $\int_0^{\pi/2}(1 \wedge \tan(\theta))\Phi(d\theta) = \int_0^{\pi/2}(1 \wedge \cot(\theta))\Phi(d\theta) = 1$. Following Einmahl et al. (1997), $\Phi(\cdot)$ is commonly referred to as

[12] The normal distribution, of course, implies $\xi = 0$.

[13] Formally, this definition only pertains to the quadrants of $\mathbb{R}^2$ for which the sign of the jumps coincide. It would be trivial, albeit notationally more cumbersome, to extended the analysis to jumps of opposite signs. However, those cases are practically irrelevant.

the spectral, or angular, measure of the extreme value distribution. It accounts for the tail dependence between the two components and together with the extreme value distributions for the marginals completely characterizes the bivariate extreme value distribution.

However, rather than directly estimating and interpreting the angular extreme value measure, empirically it is more convenient to characterize the tail dependencies through Pickands (1981) dependence function. This function is formally defined from $\Phi(\cdot)$ as

$$A(u) = \int_0^{\pi/2} \left((1-u)(1 \wedge \tan(\theta)) \vee u(1 \wedge \cot(\theta))\right) \Phi(d\theta), \quad u \in [0,1]. \tag{2.8}$$

The function $A(u)$ is convex and restricted to lie in the unit triangle; i.e., $u \vee (1-u) \leq A(u) \leq 1$, with endpoints $A(0) = A(1) = 1$. The lower bound of the triangle $u \vee (1-u)$ corresponds to perfect dependence, while the upper bound of unity obtains for asymptotically independent variables; see, e.g., the discussion in Coles (2001) and Beirlant et al. (2004). In particular, as first pointed out by Sibuya (1960), a bivariate normal distribution with correlation less than unity has asymptotically independent tails and implies $A(u) = 1$ for all $u \in [0,1]$.

The overall degree of asymptotic dependence may also be conveniently summarized in terms of the single tail-dependence parameter,

$$\chi = \lim_{u \to 1^-} \mathbb{P}\left(F_1(x) > u | F_2(y) > u\right). \tag{2.9}$$

originally proposed by Sibuya (1960); see also the more recent discussion in Coles et al. (1999). Intuitively, this measure gives the probability of observing an "extreme" observation in one of the series given that the other series is also "extreme." For two asymptotically independent series with $A(u) \equiv 1$ it follows that $\chi = 0$. More generally, it is possible to show that $\chi = 2\left(1 - A(\frac{1}{2})\right)$, so that the tail-dependence parameter is directly related to the value of Pickands dependence function at one-half.

Before we discuss the actual inference procedures that we rely in quantifying the different theoretical measures outline above, it is important to stress that all of these pertain to "large" jumps and corresponding "extreme" dependencies. We have essentially nothing to say about the dependencies inherent in the "smaller" jumps related to the pathwise properties of the process and the degree of jump activity. An empirical study of these features would require the use of entirely different statistical techniques from the ones that we discuss next.

3 Jump Tail Estimation from High-Frequency Data

We will assume the availability of equidistant price observations for each of the M+1 assets over the discrete time grid $0, \frac{1}{n}, \frac{2}{n}, ..., T$, where $n \in \mathbb{N}$ and $T \in \mathbb{N}$. We will denote the log-price increments over the corresponding discrete time-intervals $[\frac{i-1}{n}, \frac{i}{n}]$ by $\Delta_i^n p^{(j)} = p^{(j)}_{\frac{i}{n}} - p^{(j)}_{\frac{i-1}{n}}$.

Our estimation procedures will rely on both increasing sampling frequency and time span; i.e., $n \to \infty$ and $T \to \infty$. Intuitively, we will use in-fill asymptotics, or $n \to \infty$, to non-parametrically separate jumps from continuous price moves, and more conventional long-span asymptotics, or $T \to \infty$, and Extreme Value Theory (EVT) for our inference about the jump tails. We begin with a discussion of the former.

3.1 Separating Jumps from Volatility

In our separation of the price increments into jumps and continuous price moves we take into account both the strongly persistent day-to-day variation and the intraday diurnal patterns in the volatility; see, e.g, Andersen and Bollerslev (1997). In order to do so, for each day, $t = 1, ..., T$, and each asset, $j = 0, 1, ..., M$, in the sample, we first compute the Realized Variation (RV) and Bipower Variation (BV), defined by

$$RV_t^{(j)} = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p^{(j)}|^2, \qquad BV_t^{(j)} = \frac{\pi}{2} \sum_{i=tn+2}^{tn+n} |\Delta_i^n p^{(j)}||\Delta_{i-1}^n p^{(j)}|, \tag{3.1}$$

respectively. Under weak regularity conditions and $n \to \infty$, see e.g., Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004, 2006),

$$RV_t^{(j)} \xrightarrow{\mathbb{P}} \int_t^{t+1} (\sigma_s^{(j)})^2 ds + \int_t^{t+1} \int_{\mathbb{R}} x^2 \mu^{(j)}(ds, dx), \qquad BV_t^{(j)} \xrightarrow{\mathbb{P}} \int_t^{t+1} (\sigma_s^{(j)})^2 ds. \tag{3.2}$$

Note that the Bipower Variation consistently estimates only the part of the total variation due to continuous prices moves, or the so-called daily integrated variance.

Based on these daily realized variation measures, we subsequently estimate the Time-of-Day (TOD) volatility pattern for each of the stocks and the aggregate market by,[14]

$$TOD_i^{(j)} = \frac{n \sum_{t=1}^{T} |\Delta_{i_t}^n p|^2 1\left(|\Delta_{i_t}^n p^{(j)}| \leq \tau \sqrt{BV_t^{(j)} \wedge RV_t^{(j)}} n^{-\varpi}\right)}{\sum_{s=1}^{nT} |\Delta_s^n p|^2 1\left(|\Delta_s^n p^{(j)}| \leq \tau \sqrt{BV_{[s/n]}^{(j)} \wedge RV_{[s/n]}^{(j)}} n^{-\varpi}\right)}, \quad i_t = (t-1)n + i, \tag{3.3}$$

where $i = 1, ..., n$, and $\tau > 0$ and $\varpi \in (0, 0.5)$ are both constants. The truncation of the price increments implied by τ and ϖ in the definition of $TOD_i^{(j)}$ effectively remove the jumps. Hence $TOD_i^{(j)}$ measures the ratio of the diffusive variation over different parts of the day relative to its average value for the day. In the empirical analysis reported on below we set $\tau = 3$ and $\varpi = 0.49$. Intuitively, this means that we classify as jumps all of the high-frequency price increments that are beyond three standard deviations of a local estimator of the corresponding

[14]Note, the asymptotic limit of $BV_t^{(j)}$ is always below that of $RV_t^{(j)}$. The trimming $BV_{[i/n]}^{(j)} \wedge RV_{[i/n]}^{(j)}$ is merely a finite-sample adjustment.

stochastic volatility. We not report these estimates below, the resulting $TOD_i^{(j)}$'s generally exhibit the well-known U-shaped pattern as a function of i over the trading day; this same approach has also recently been used by Bollerslev and Todorov (2010b), who do provide a plot of the estimated $TOD_i^{(0)}$ for the aggregate market.

Relying on a similar approach, we estimate the Continuous Variation over the whole day using a modification of the truncated variation measure originally proposed by Mancini (2009),

$$CV_t^{(j)} = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p^{(j)}|^2 1\left(|\Delta_i^n p^{(j)}| \leq \alpha_i^{(j)} n^{-\varpi}\right). \tag{3.4}$$

Consistency and asymptotic normality of this estimator for $n \to \infty$ and appropriate choice of truncation level follows from Mancini (2009) and Jacod (2008). The truncation level $\alpha_i^{(j)}$ that we actually use in separating the "realized" jumps from from the continuous price moves is chosen adaptively based on our preliminary estimates of the stochastic volatility over the day together with the within-day volatility pattern. Specifically,

$$\alpha_i^{(j)} = \tau\sqrt{(BV_{[i/n]}^{(j)} \wedge RV_{[i/n]}^{(j)}) * TOD_{i-[i/n]n}^{(j)}}, \quad i = 1, ..., nT, \tag{3.5}$$

with τ and ϖ set to same values as discussed above. We rely on the difference between the continuous and previously defined realized variation measures,

$$JV_t^{(j)} \;=\; RV_t^{(j)} - CV_t^{(j)} \;\xrightarrow{\mathbb{P}}\; \int_t^{t+1}\int_{\mathbb{R}} x^2 \mu^{(j)}(ds, dx), \tag{3.6}$$

for consistently estimating the total variation attributable to jumps.

We also use the identical truncation approach to directly identify the sets of high-frequency jump increments for each of the assets,

$$\widehat{\mathcal{T}}_{[0,T]}^{(j)} = \left\{i \in [0, nT] : |\Delta_i^n p^{(j)}| \geq \alpha_i^{(j)} n^{-\varpi}\right\}, \quad j = 0, 1, ..., M. \tag{3.7}$$

Similarly, we define the sets of systematic and idiosyncratic jump times by,

$$\widehat{\mathcal{T}}_{[0,T]}^{(j,0)} = \widehat{\mathcal{T}}_{[0,T]}^{(j)} \cap \widehat{\mathcal{T}}_{[0,T]}^{(0)}, \qquad \widehat{\mathcal{T}}_{[0,T]}^{(j,j)} = \widehat{\mathcal{T}}_{[0,T]}^{(j)} \setminus \left\{\widehat{\mathcal{T}}_{[0,T]}^{(j)} \cap \widehat{\mathcal{T}}_{[0,T]}^{(0)}\right\}, \quad j = 1, ..., M. \tag{3.8}$$

Armed with these high-frequency based estimates for the times and actual "realized" jumps, we next show how to use these in our estimation of the jump tail characteristics. We begin with the univariate jump tails.

3.2 Univariate Jump Tails

To keep the notation simple, we will focus on the right tail and the systematic jumps. Our estimation of the parameters for the negative and/or idiosyncratic jumps proceed analogous.

The general assumptions about the jump tails set out in Section 2 imply that

$$1-\frac{\overline{\nu}_{\psi}^{+(j,0)}(u+x)}{\overline{\nu}_{\psi}^{+(j,0)}(x)} \overset{\text{appr}}{\sim} \begin{cases} 1-\left(1+\xi^{+(j,0)}u/\eta^{+(j,0)}\right)^{-1/\xi^{+(j,0)}}, & \xi^{+(j,0)}\neq 0, \\ e^{-u/\eta^{+(j,0)}}, & \xi^{+(j,0)}=0, \end{cases} \tag{3.9}$$

where $u>0$, $x>0$ is some "large" value, and $\eta^{+(j,0)}>0$;[15] for additional discussion of the approximating Generalized Pareto distribution, see, e.g., Embrechts et al. (2001). Now, denote the (re-scaled) scores associated with the log-likelihood function of the Generalized Pareto distribution by,

$$\begin{aligned} \phi_1^{+}(u,\xi^{+(j,0)},\eta^{+(j,0)}) &= \frac{1}{\eta^{+(j,0)}}\left(1-\left(1+\xi^{+(j,0)}\right)\left(1+\frac{\xi^{+(j,0)}u}{\eta^{+(j,0)}}\right)^{-1}\right), \\ \phi_2^{+}(u,\xi^{+(j,0)},\eta^{+(j,0)}) &= \log\left(1+\frac{\xi^{+(j,0)}u}{\eta^{+(j,0)}}\right)-\left(1+\xi^{+(j,0)}\right)\left\{1-\left(1+\frac{\xi^{+(j,0)}u}{\eta^{+(j,0)}}\right)^{-1}\right\}, \end{aligned} \tag{3.10}$$

where $i=1,2$ refer to the derivatives with respect to $\eta^{+(j,0)}$ and $\xi^{+(j,0)}$, respectively. Then, for truncation level $tr_T^{(j,0)}$ increasing to infinity with $T\to\infty$,

$$\int_0^t\int_{\mathbb{R}}\phi_i^{+}(\psi^{+}(x)-tr_T^{(j,0)},\xi^{+(j,0)},\eta^{+(j,0)})1\left(\psi^{+}(x)\geq tr_T^{(j,0)}\right)\mu^{(j,0)}(ds,dx) \qquad i=1,2,$$

behave approximately as martingales. Combined with our previously discussed procedures for directly "filtering" the "large" jumps from the high-frequency data, this in turn allows for the construction of consistent and asymptotically normal method-of-moments type estimators for the jump tail parameters.

In particular, following Bollerslev and Todorov (2010a) the simple-to-implement moment conditions defined from the two martingales above and the jump sets defined in equation (3.8),

$$\sum_{i\in\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}}\begin{pmatrix} \phi_1^{+}(\psi^{+}(\Delta_i^n p^{(j)})-tr_T^{(j,0)},\xi^{+},\eta^{+})1\left(\psi^{+}(\Delta_i^n p^{(j)})\geq tr_T^{(j,0)}\vee\psi^{+}(\alpha_i^{(j)}n^{-\varpi})\right) \\ \phi_2^{+}(\psi^{+}(\Delta_i^n p^{(j)})-tr_T^{(j,0)},\xi^{+},\eta^{+})1\left(\psi^{+}(\Delta_i^n p^{(j)})\geq tr_T^{(j,0)}\vee\psi^{+}(\alpha_i^{(j)}n^{-\varpi})\right) \end{pmatrix}, \tag{3.11}$$

should both be arbitrarily close to zero asymptotically under the joint fill-in and long-span asympotics. The precision of the resulting estimator for $\xi^{+(j,0)}$, determined by the asymptotic limiting variance of the moment conditions, may be conveniently expressed as,

$$\widehat{\text{Var}}\left(\widehat{\xi}^{+(j,0)}\right)=\frac{1}{M_T^{+(j,0)}}\left(1+\widehat{\xi}^{+(j,0)}\right)^2, \tag{3.12}$$

where

$$M_T^{+(j,0)}=\sum_{i\in\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}}1\left(\psi^{+}(\Delta_i^n p^{(j)})\geq tr_T^{(j,0)}\vee\psi^{+}(\alpha_i^{(j)}n^{-\varpi})\right), \tag{3.13}$$

[15]Note with fat tails and $\beta^{+(j,0)}>0$, as discussed in Section 2, $\eta^{+(j,0)}\equiv\frac{x}{\beta^{+(j,0)}}$ and $\xi^{+(j,0)}\equiv\frac{1}{\beta^{+(j,0)}}$.

denotes the actual number of jumps used in the estimation.

In order to actually implement these estimating equations, we obviously need to specify the truncation level $tr_T^{(j,0)}$ for each of the assets, $j = 0, 1, ..., M$. This choice must balance the two opposing effects associated with the use of more jumps in the estimation generally resulting in smaller sampling error, versus the use of more, and hence smaller, jumps resulting in poorer approximation by the EVT distribution in equation (3.9)). In the main empirical result reported on below, we set $tr_T^{+(j,0)}$ such that $M_T^{+(j,0)}/T = 0.02$, corresponding to jumps of that size or larger occurring $7-8$ times per year.[16]

We next turn to a discussion of our multivariate estimation procedures and the strategy that we use for assessing the "extreme" jump tail dependencies.

3.3 Jump Tail Dependencies

We will focus our discussion on the estimation of the tail dependencies between the jumps in the aggregate market and the systematic jumps in the individual stocks. However, the same basic estimation techniques may be applied to other bivariate series, and we do so for other pairs of returns and jump tails in the empirical section.

For now, consider the dependence between the jumps in $p^{(0)}$ and $p^{(j)}$ that arrive at the same time; i.e., the jumps that occur in the set $\mathcal{T}_{[0,T]}^{(0)} \cap \mathcal{T}_{[0,T]}^{(j)}$. As discussed in Section 2, the tail dependence between these is conveniently captured by Pickands dependence function, $A(\cdot)$. We follow the approach of Einmahl et al. (1997) by essentially first estimating the underlying spectral measure; see also Steinkohl et al. (2010). Specifically, for $i \in \widehat{\mathcal{T}}_{[0,T]}^{(j,0)}$, denote

$$\begin{aligned} \widehat{X}_{i,1} &= \left\{ \frac{\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}{M_T^{+(j,0)}} \left[1 + \frac{\widehat{\xi}^{+(j,0)}}{\widehat{\eta}^{+(j,0)}} (\psi^+(\Delta_i^n p^{(j)}) - tr_T^{(j,0)}) \right]^{1/\widehat{\xi}^{+(j,0)}} - 1 \right\} \\ &\quad \times 1\left(\psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)} \vee \psi^+(\alpha_i^{(j)}) \right) + 1, \\ \widehat{X}_{i,2} &= \left\{ \frac{\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}{M_T^{+(0,j)}} \left[1 + \frac{\widehat{\xi}^{+(0,j)}}{\widehat{\eta}^{+(0,j)}} (\psi^+(\Delta_i^n p^{(0)}) - tr_T^{(0,j)}) \right]^{1/\widehat{\xi}^{+(0,j)}} - 1 \right\} \\ &\quad \times 1\left(\psi^+(\Delta_i^n p^{(0)}) \geq tr_T^{(0,j)} \vee \psi^+(\alpha_i^{(0)}) \right) + 1, \end{aligned} \tag{3.14}$$

where $\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|$ refers to the number of elements in the set $\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}$, and we have suppressed the dependence on the index j for notational convenience. Also, let

$$\widehat{R}_i = \widehat{X}_{i,1} + \widehat{X}_{i,2}, \tag{3.15}$$

[16]This choice, of course, directly dictates the accuracy of the estimator for $\xi^{+(j,0)}$ according to the expression in equation (3.12). As noted below, however, we also experimented with the use of other truncation levels, resulting in qualitatively very similar point estimates.

denote the sum of the two marginals. An initial estimator for Pickand's dependence function is then naturally obtained by,

$$\widehat{A}^{j,0}(u) = \frac{2}{M_T^{+(j,0)}} \sum_{i \in \widehat{\mathcal{T}}^{(j,0)}_{[0,T]}} 1\left(\widehat{R}_i > \widehat{R}_{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right| - M_T^{+(j,0)},\ \left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}\right) \frac{\max\left\{(1-u)\widehat{X}_{i,1},\ u\widehat{X}_{i,2}\right\}}{\widehat{R}_i},\ \ u \in [0,1].$$

where $\widehat{R}_{i,\ \left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}$ denotes the i-th order statistics.

Following Beirlant et al. (2004), Section 9.4.1, we further modify this initial estimator,

$$\widetilde{A}^{j,0}(u) = \max\left\{u,\ 1-u,\ \widehat{A}^{j,0}(u) + 1 - (1-u)\widehat{A}^{j,0}(0) - u\widehat{A}^{j,0}(1)\right\},\quad u \in [0,1]. \tag{3.16}$$

so that it always stays within its lower asymptotic bound of $\max(1-u,u)$ and the upper bound of unity. Using the relationship discussed in Section 2, our final non-parametric estimate for the degree of asymptotic tail dependence is simply obtained by evaluating this function at one-half,[17]

$$\widehat{\chi}^{j,0} = 2(1 - \widetilde{A}^{j,0}(1/2)). \tag{3.17}$$

This completes our discussion of the different estimation procedures. We next summarize our empirical findings based on high-frequency intraday data for a large cross-section of individual stocks.

4 Empirical Results

4.1 Data

Our high-frequency data for the individual stocks was obtained from price-data.com. It consist of 5-minute transaction prices for the fifty largest capitalization stocks included in the S&P 100 index with continuous price records from mid 1997 until the end of 2008.[18] The price records cover the trading hours from 9:35 EST to 16:00 EST, for a total of 76 intraday return observations per day. Our proxy for the aggregate market portfolio is based on comparable 5-minute data for the S&P 500 futures index obtained from Tick Data Inc.

[17] χ may alternatively be estimated based on explicitly parameterized bivariate distributions, or copulas. In particular, $\chi = \lim_{u\to 1^-} C(u,u)$, for the copula $C(\cdot,\cdot)$ that links the two marginals; Smith et al. (2011), e.g., estimate the upper tail dependence in "spikes" in electricity prices for different geographical regions based on asymmetric skew t copulas. The estimation of the underlying copula, however, is generally based on all of the data and not just the "tail" observations, and as pointed out by Frahm et al. (2005), a misspecified copula that might fit well in the center of the distribution can give rise to very misleading estimates for the tail dependencies.

[18] The actual start date and number of complete trading days available for each of the stocks in the sample differ slightly, ranging from a low of 2,832 to a high of 2,922, with a medium of 2,918 days.

Table 1 provides key summary statistics for each of the stocks included in the sample as well as the S&P 500 futures index (SPFU). Not surprisingly, the average continuous variation (CV) for all of the individual stocks far exceed that of the market. Similarly, the variation attributable to jumps (JV) is also numerically much larger for each of the individual stocks than it is for the market. In terms of the total variation, the share due to jumps range from a low of 6.5% to a high of 19.0%, with a median value across all fifty stocks of 13.0%. In contrast, the corresponding number for the aggregate market index equals 9.4%, so that jump risk appears to be relatively more important at the individual stock level.

Table 1 about here

The last two columns in the table, which report the total number of systematic and idiosyncratic jumps detected for each of the stocks, further corroborates this idea. For almost all of the stocks the total number of jumps exceed the number of market-wide jumps for the S&P 500. These numbers also suggest very high overall jump intensities ranging from slightly more than one jump per day to about one jump every other day.[19] The finding that the individual stocks contain more jumps than the market is consistent with the hypothesis of diversifiable individual jump risk originally put forth by Merton (1976). Of course, the mere existence of jumps at the market level refutes the conjecture that jump risk is entirely firm specific.

Further to this effect, the number of so-called systematic jumps, or jumps in the individual stocks that occur at the same time as the market jumps, are clearly non-trivial. Still, it is obviously not the case that when a "large" market jump occurs, it automatically triggers "large" jumps in all of the individual stocks. As such, a simple linear one factor market model appears too simplistic to describe the relation between the individual and market-wide jumps, and in turn the joint dependencies in the jump tails.

Figure 1 about here

In order to more clearly visualize the different types of jump sets, we plot in Figure 1 the 5-minute logarithmic prices for three separate days for IBM, as a representative stock, and the S&P 500 market portfolio. For ease of comparison, we normalize the logarithmic price at the beginning of the day to zero across all of the panels. The top panel shows the intraday prices on October 29, 2002, a day where the aggregate market jumped but IBM did not. The jump

[19] With infinitely activity jumps, the total number of "significant" jumps will naturally be expected to increase to infinity for ever increasing sampling frequency, and these numbers need to be interpreted accordingly.

in the market obviously occurred at 10:00EST, and is readily associated with a disappointing reading of the Consumer Confidence Index released at that exact time.[20] The middle panel shows the prices on January 3, 2001, a day with a systematic jump in IBM. The timing of the systematic jumps is again readily associated with the surprise cut in the Federal Funds Rate announced at 13:10EST on that day. The final third panel shows February 26, 2008, when the board of directors for IBM announced at 11:00EST that they had authorized $15 billion in additional funds for stock repurchases, resulting in an idiosyncratic jump in IBM, but no discernable dis-continuities in the within day prices for the aggregate market.

We continue next with a discussion of our estimation results pertaining to these different idiosyncratic and systematic jump tail distributions.

4.2 Marginal Jump Tails

Our estimation results for the scalar tail decay parameter ξ for each of the marginal jump tail distributions are reported in Table 2. The relevant truncation levels for the different tails are determined by the equivalent of 0.02 times the daily sample sizes. Corresponding asymptotic standard errors for each of the individual estimates are immediately available from the formula in equation (3.12).

Looking first at the results for the S&P 500 market portfolio, both of the jump tails are heavy with the right tail decaying at a slower rate than the left, or $\widehat{\xi}^{+(0,0)} > \widehat{\xi}^{-(0,0)} > 0$. This is consistent with the empirical evidence reported in Bollerslev and Todorov (2010a), and directly refutes the popular compound Poisson jump model with normally distributed jump sizes that have been used extensively in the existing literature.

Turning to the results for the systematic jumps in the individual stocks, all of the point estimates for ξ are positive, again indicating heavy-tailed jump distributions. In parallel to the results for the market, for many of the stocks the estimate for the right tail appears larger than the left.[21] Of course, given the relatively low number of observations invariably available for the estimation of the jump tails, many of the estimates are not significantly different from zero when judged by their individual standard errors of approximately $M^{-1/2} \approx 0.131$ under the null hypothesis of light tails. Taken as a whole, however, the cross-sectional evidence clearly suggests that the systematic jumps are heavy-tailed.[22] At the same time, the dispersion in the

[20] Andersen et al. (2003) and Andersen et al. (2007), among many others, have previously studied the relationship between regularly scheduled macroeconomic news announcements and jumps and/or large price movements in asset prices.

[21] Related empirical evidence for overall larger right tails in half-hourly raw returns for various sector indexes has recently been reported by Straetmans et al. (2008).

[22] Related to this, Kelly (2010) has recently explored ways in which to increase the efficiency of tail index estimation by pooling across different stocks. His estimates, however, are based on coarser daily frequency returns and not the jump tails per se, and do not explicitly differentiate between the systematic and idiosyncratic

estimates again suggests that the relationship between the individual and market-wide jumps is not well described by a simple one factor market model, which would imply identical systematic jump tail decay parameters across all of the stocks.

Table 2 about here

The estimates for the idiosyncratic jump tails are reported in the last two columns of the table. Almost all of the point estimates are again positive, and for many of the stocks exceed those for the systematic jump tails. Also, in parallel to the systematic jump tails, the tail decay parameters for the right tails often dominate those for the left, indicative of greater upside potential than downside firm specific risks.

Meanwhile, as previously noted, given the relatively short time span and limited number of "tail" observations underlying the estimation, all of the point estimates are admittedly somewhat imprecise. To check the robustness of the results, we therefore redid the estimation for the idiosyncratic jump tails based on a truncation level equivalent to a total of 200 jump tail observations, implying a smaller asymptotic standard error under the null of $\xi = 0$ of approximately $M^{-1/2} \approx 0.071$. The resulting estimates are generally fairly close to the ones based on the larger truncation level, with medium estimates of 0.187 and 0.193 for the left and right tail decay parameters, respectively, compared to the values 0.185 and 0.155 reported in the table.[23]

Figure 2 about here

To more directly illustrate the estimation results, we plot in Figure 2 the relevant jump tail estimation for IBM together with the actually observed "moderate" to "large" sized jumps. To facilitate the visual comparisons, the tails are plotted on a double logarithmic scale.[24] As is evident from the figure, the overall magnitude of the idiosyncratic jump tails in the bottom two panels dominate the systematic ones depicted in the top two panels. At the same time,

parts of the tails.

[23]Further details concerning these robustness checks are available upon request. We also experimented with the use of lower truncation levels for the estimation of the systematic jump tails. However, the total number of systematic jumps for each of the stocks defined by the set in (3.8) and the relatively high threshold level in (3.5) naturally limits the total number of systematic jumps, as reported in Table 1. As such, the jumps identified as systematic are truly the "large" ones in a joint sense, and the ones actually used in the estimation much "deeper" in the tails than a naive comparison of their number relative to the total number of systematic jumps would suggest.

[24]The flat lines for the actually observed jumps at $-7.98 \approx log(1/2,922)$ correspond to the occurrence of one jump of that particular size in the sample. Similarly, for the other apparent lines at $log(j/2,922)$ for integer j.

the corresponding estimates for ξ are all quite similar, except for the right systematic jump tail shown in the top right panel, which decays at a somewhat slower rate. The generally excellent fits afforded by the estimated solid lines for the actually observed jump tails, also directly underscore the accuracy of the marginal EVT approximation underlying our estimation procedures.[25]

4.3 Systematic Jump Tail Dependencies

4.3.1 High-Frequency Dependencies

Before we discuss the general set of estimation results pertaining to all of the fifty stocks, it is instructive to again consider the jump tail dependencies that we are after by looking at IBM as a representative stock. To this end, we plot in Figure 3 the pairs of realized positive and negative systematic jumps for IBM and the S&P 500 market portfolio. The figure clearly reveals a strong positive association between the systematic jumps in the stock and the jumps in the market index. Visual inspection also suggests that for IBM this association might be slightly stronger for the negative than the positive jumps, although not overwhelmingly so.[26]

Figure 3 about here

Of course, we are primarily interested in the "extreme" tail dependencies, and the probability/intensity of observing a "large" jump in one of the individual stocks given that the market jumped by a "large" amount. As discussed above, this probability follows directly from Pickands dependence function. Our estimates of that function for the negative (solid line) and positive (dashed line) systematic IBM and market wide jumps are plotted in Figure 4. Both of the estimated curves are far below unity, as would be implied by independent tails, and much closer to the lower bound of perfect dependence as indicated by the triangle. Moreover, while simple visual inspection of the aforementioned scatter plot in Figure 3 seemingly point to somewhat stronger dependencies for the negative jump tails, the non-parametrically estimated "extreme" dependence functions are fairly close throughout most of the support. The corresponding estimates for the tail-dependence coefficients obtained by evaluating the functions at one-half together with the formula for χ in equation (3.17) equal 0.749 and 0.706 for the right and left tails, respectively. Hence, counter to the naive impression from Figure 3 and

[25]Importantly, the new procedures would also allow us to meaningfully extrapolate the behavior of the jump tails and corresponding "extreme" jump quantiles to levels which would be impossible to accurately estimate with standard parametric approaches and lower frequency, say daily, data; for further discussion along these lines see Bollerslev and Todorov (2010a).

[26]The simple linear correlations for the jump pairs depicted in two panels equal 0.902 and 0.752 for the negative and positive jumps, respectively.

many stories in the popular financial press about various "doomsday scenarios," our formal high-frequency based estimates actually suggest slightly stronger asymptotic tail dependencies during sharp market rallies, or positive jumps, than during steep market declines, or negative jumps.

Figure 4 about here

To further help gauge the magnitude of the estimated dependencies, we also include in Figure 4 estimates of Pickands dependence function based on the raw high-frequency 5-minute returns. These functions are systematically higher, and the resulting tail dependencies lower, than the ones based on the systematic jump tails. Intuitively, the dependence in the raw returns manifest several features in the underlying latent bivariate semimartingale process that describes the joint dynamics of the two price series. On the one hand, the presence of common, or systematic, jumps tends to produce strong tail dependencies, as directly evidenced by the previously discussed estimates. On the other hand, the presence of idiosyncratic jumps tends to weaken the tail dependencies. Similarly, pure diffusive price moves formally imply asymptotic independence. At the same time, however, the presence of time-varying stochastic volatility will tend to generate tail dependence through periods of high volatility. As further discussed below, the combination of all of these separate effects in turn accounts for the weaker tail dependencies observed with the raw high-frequency returns.

Table 3 about here

These specific results for IBM carry over to the rest of the stocks in the sample. In particular, turning to Table 3, the first two columns in the table show the estimated asymptotic tail dependencies for the raw 5-minute returns for each of the fifty stocks. These estimates are generally fairly low. The results also closely mirror those obtained by restricting the sample to only those 5-minute returns that are classified as jumps, or the set $\widehat{\mathcal{T}}^{(j)}_{[0,T]}$. In contrast, the estimated dependence coefficients for the systematic jump tails, or the returns in the set $\widehat{\mathcal{T}}^{(j,0)}_{[0,T]} = \widehat{\mathcal{T}}^{(j)}_{[0,T]} \cap \widehat{\mathcal{T}}^{(0)}_{[0,T]}$, are all very high ranging from 0.604 to 0.794. The estimates are also surprisingly close to symmetric for most of the stocks, and if anything slightly large for the right tails. As such, the results in the table clearly support the notion that most of the "extreme" dependencies reside in the systematic jumps tails. Building on this idea, we next show how to identify and isolate the effect of time-varying stochastic volatility as another separate source of tail dependence in lower frequency daily returns.

4.3.2 Daily Dependencies

We continue to rely on the high-frequency data for explicitly "filtering" out the jumps in the daily returns and variation measures. In particular, for each asset, $j = 0, 1, ..., M$, and day, $t = 1, ..., T$, in the sample, the part of the daily returns associated with continuous price moves are naturally estimated by the sum of the intraday high-frequency returns that are not classified as jumps,

$$\mathbf{z}_t^{(j)} = \sum_{i=(t-1)n+1}^{tn} \left[\begin{pmatrix} \Delta_i^n p^{(j)} \\ \Delta_i^n p^{(0)} \end{pmatrix} \mathbf{1} \begin{pmatrix} |\Delta_i^n p^{(j)}| \leq \alpha_i^{(j)} n^{-\varpi} \\ |\Delta_i^n p^{(0)}| \leq \alpha_i^{(0)} n^{-\varpi} \end{pmatrix} \right]. \tag{4.1}$$

Under the assumption of finite variation jumps and weak additional regularity conditions, it follows readily from the expression for the general semimartingale process in equation (2.1) that for $n \to \infty$,

$$\mathbf{z}_t^{(j)} \xrightarrow{\mathbb{P}} \begin{pmatrix} \int_{t-1}^t \alpha_s^{(j)} ds + \int_{t-1}^t \sigma_s^{(j)} dW_s^{(j)} ds \\ \int_{t-1}^t \alpha_s^{(0)} ds + \int_{t-1}^t \sigma_s^{(0)} dW_s^{(0)} ds \end{pmatrix}. \tag{4.2}$$

The first integrals on the right-hand-side associated with the drifts in the individual stock and aggregate market prices are both negligible, and will not affect the estimated daily tail dependencies. Further, assuming the diffusive volatilities to be constant and the Brownian motions not perfectly correlated, the terms associated with the second integrals would be jointly normally distributed and hence result in asymptotically independent tails. Consequently, any tail dependence between the two components in $\mathbf{z}_t^{(j)}$ is therefore directly attributable to time-varying stochastic volatility.

Going one step further, it is possible to non-parametrically "remove" the effect of the stochastic volatility by standardizing $\mathbf{z}_t^{(j)}$ with an estimator of its quadratic variation. Specifically,

$$\widetilde{\mathbf{z}}_t^{(j)} = \left\{ \sum_{i=(t-1)n+1}^{tn} \left[\begin{pmatrix} (\Delta_i^n p^{(j)})^2 & \Delta_i^n p^{(j)} \Delta_i^n p^{(0)} \\ \Delta_i^n p^{(j)} \Delta_i^n p^{(0)} & (\Delta_i^n p^{(0)})^2 \end{pmatrix} \mathbf{1} \begin{pmatrix} |\Delta_i^n p^{(j)}| \leq \alpha_i^{(j)} n^{-\varpi} \\ |\Delta_i^n p^{(0)}| \leq \alpha_i^{(0)} n^{-\varpi} \end{pmatrix} \right] \right\}^{-1/2} \mathbf{z}_t^{(j)}, \tag{4.3}$$

where the estimator for the daily quadratic variation is based on a multivariate version of the truncated Continuous Variation measure defined in equation (3.4).[27] Then, in analogy to the results discussed above, it follows that for $n \to \infty$,

$$\widetilde{\mathbf{z}}_t^{(j)} \xrightarrow{\mathbb{P}} \begin{pmatrix} \langle p_s^{(j)}, p_s^{(j)} \rangle_{(t-1,t]} & \langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]} \\ \langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]} & \langle p_s^{(0)}, p_s^{(0)} \rangle_{(t-1,t]} \end{pmatrix}^{-1} \begin{pmatrix} \int_{t-1}^t \alpha_s^{(j)} ds + \int_{t-1}^t \sigma_s^{(j)} dW_s^{(j)} ds \\ \int_{t-1}^t \alpha_s^{(0)} ds + \int_{t-1}^t \sigma_s^{(0)} dW_s^{(0)} ds \end{pmatrix}, \tag{4.4}$$

[27] Note, this estimator is guaranteed to be positive semi-definite by construction.

where $\langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]}$ refers to the continuous part of the quadratic covariation between $p^{(j)}$ and $p^{(0)}$ over the $(t-1, t]$ daily time interval. As before, the impact of the drift terms may be ignored, so that the non-parametrically "devolatilized" pairs of returns $\widetilde{\mathbf{z}}_t^{(j)}$ should be approximately bivariate standard normally distributed.[28]

Figure 5 about here

Motivated by these ideas, Figure 5 plots Pickands dependence functions for each of the bivariate IBM series $\mathbf{z}_t^{(IBM)}$ and $\widetilde{\mathbf{z}}_t^{(IBM)}$. Our estimates are based on exactly the same estimation procedures and truncation levels as the ones described for the jump tails in Sections 3.2 and 3.3. In light of the above discussion, we would expect the left and right tail functions corresponding to $\widetilde{\mathbf{z}}_t^{(IBM)}$ to be close to unity. The two curves in the figure confirm this, thus indirectly underscoring the accuracy of our empirical approximations, and the minimal influence imparted by the finite-sample measurement errors and "leverage effect."

Further elaborating on the results, the wedge between the estimated dependence functions for $\mathbf{z}_t^{(IBM)}$ and $\widetilde{\mathbf{z}}_t^{(IBM)}$ directly reveals the effect of the diffusive stochastic volatility on the overall tail dependence. As seen from the figure, this wedge is obviously non-trivial, and shows that time-varying volatility is indeed responsible for some of the asymptotic tail dependence between the daily individual stock returns and the return on the aggregate market portfolio. Interestingly, the figure also points to slightly weaker dependencies in the right (dashed line) than the left (solid line) jump-adjusted return tails. This slight difference and reversal vis-a-vis the earlier results for the high-frequency returns may in part be attributed to the within day "leverage effect." Meanwhile, comparing Figures 4 and 5 and the magnitudes therein, the systematic jump tails are clearly associated with much stronger "extreme" dependencies than the ones induced by the more slowly moving daily diffusive volatility.

Table 4 about here

To further corroborate these specific results for IBM, we report in Table 4 the estimated tail dependence coefficients χ for the raw daily returns, the jump-adjusted returns $\mathbf{z}_t^{(j)}$, and

[28]The approximation comes from the need to estimate the quadratic variation and a possible "leverage effect," or negative correlation between the within day stochastic volatility and price innovations. The latter should have only minimal effect, and if anything, result in slightly stronger negative tail dependencies. A related univariate standardization approach has been proposed by Andersen et al. (2007), and further explored empirically by Andersen et al. (2010), who confirm that the jump-adjusted "devolatilized" returns for a sample of individual stocks are approximately univariate standard normally distributed.

the jump-adjusted "devolatilized" returns $\widetilde{\mathbf{z}}_t^{(j)}$, for each of the fifty stocks in the sample. For comparison purposes, we also include the results where we ignore the temporal variation in the continuous covariation and only standardize the jump-adjusted returns by their respective univariate continuous variation measures; i.e., the series obtained by restricting the off-diagonal elements in the matrix in equation (4.3) to be zero.[29]

Looking first at the results for the raw daily returns, the estimated tail-dependence coefficients are generally quite close across all of the fifty stocks, with median values of 0.247 and 0.229 for the positive and negative tails, respectively. These numbers are, of course, somewhat larger than the median dependencies estimated with the raw 5-minute returns, but they are dwarfed by the estimates for the systematic jump tails. Removing all of the "large" jumps from the daily returns barely changes the average dependence-coefficient estimates. This is consistent with the aforemention results for the high-frequency "filtered" jumps reported in Table 3, which are similarly close to the results for the raw 5-minute returns. The results for the univariate "devolatilized" returns reported in the next pair of columns, confirm that some of the tail dependence may indeed be ascribed to time-varying volatility. For most of the stocks, the univariate standardization reduces $\widehat{\chi}^+$ and $\widehat{\chi}^+$ by a factor of roughly two relative to the estimates for the jump-adjusted returns $\mathbf{z}_t^{(j)}$.[30] Meanwhile, standardizing the jump-adjusted returns by the full realized continuous covariation *matrix* to explicitly account for the temporal variation in the diffusive covariance risk as well, effectively eliminates all of the remaining dependencies, and results in *i.i.d.* bivariate normal distributions and asymptotically independent tails.

All-in-all, the empirical results reported in the tables clearly show how the new high-frequency based procedures developed here allow us to "dissect" the generic semimartingale representation in equation (2.1), and assess the role of the different terms in generating asymptotic tail dependencies.

5 Conclusion

We propose a new set of statistical procedures for dissecting and estimating the distributional features of individual asset return jump tails. Our estimation techniques are based on in-fill and long-span asymptotics, together with extreme value type approximations. On applying the new estimation methods with a large panel of high-frequency data for fifty individual stocks and

[29] As previously noted, Andersen et al. (2010) have recently shown that a closely related univariate standardization scheme results in approximate univariate standard normal distributions empirically.

[30] This is also consistent with the earlier empirical evidence in Poon et al. (2004), who report that standardizing daily international equity index returns by simple univariate parametric GARCH models tend to reduce the estimated tail dependencies.

the S&P 500 market portfolio, we find that the idiosyncratic and systematic jumps are both generally heavy tailed, albeit typically less so than the tails for the market-wide jumps. We also find strong evidence for asymptotic tail dependence between the individual stocks and the market index, with most of it directly attributable to the systematic jump tails. Thus, there is not only commonality across jump arrivals in stocks, but also strong dependence between their sizes. Further building on the same techniques, we also document non-trivial tail dependencies in longer horizon daily returns, and show how some of that dependence may be directly ascribed to the effect of interdaily temporally varying stochastic volatility.

As such, our empirical findings directly highlight the importance of the new estimation framework for better understanding and more accurately modeling tail and systemic risk events, like the ones experienced during the recent financial crises. The estimation techniques develop here could also be usefully applied with high-frequency data from different countries in the study of "extreme" international market linkages and contagion type effects.[31] We leave further empirical investigations along these lines for future work.

[31] Related to this, Aït-Sahalia et al. (2010) have recently explored the use of mutually exciting jump processes for describing the propagation of stock market shocks, or jumps, around the world.

References

Aït-Sahalia, Y., J. Cacho-Diaz, and R. J. A. Laeven (2010). Modeling Financial Contagion Using Mutually Exciting Jump Processes. Working paper, Princeton University.

Andersen, T. G. and T. Bollerslev (1997). Intraday Periodicity and Volatility Persistence in Financial Markets. *Journal of Empirical Finance 4*, 115–158.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007). Roughing It Up: Disentangling Continuous and Jump Components in Measuring, Modeling and Forecasting Asset Return Volatility. *Review of Economics and Statistics 73*, 251–277.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2003). Modeling and Forecasting Realized Volatility. *Econometrica 71*, 579–625.

Andersen, T. G., T. Bollerslev, and D. Dobrev (2007). No-Arbitrage Semimartingale Restrictions for Continuous-Time Volatilities Models subject to Leverage Effects, Jumps and i.i.d. Noise: Theory and Testable Distributional Implications. *Journal of Econometrics 138*, 125–180.

Andersen, T. G., T. Bollerslev, P. Frederiksen, and M. O. Nielsen (2010). Continuous-Time Models, Realized Volatilities, and Distributional Implications for Daily Stock Returns. *Journal of Applied Econometrics 25*, 233–261.

Andersen, T. G., T. Bollerslev, D. F. X., and C. Vega (2003). Micro Effects of Macro Annoucements: Real-Time Price Discovery in Foreign Exchange. *American Economic Review 93*, 38–62.

Ang, A. and J. Chen (2002). Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics 63*, 443–494.

Bae, K. H., G. A. Karolyi, and R. M. Stulz (2003). A New Approach to Measuring Financial Contagion. *Review of Financial Studies 16*, 717–763.

Balkema, A. and L. de Haan (1974). Residual Life Time at Great Age. *Annals of Probability 2*, 792–804.

Barigozzi, M., C. T. Brownless, G. M. Gallo, and D. Veredas (2010). Disentangling Systematic and Idiosyncratic Risk for Large Panels of Assets. Working paper, ECARES, University of Brussels.

Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics 2*, 1–37.

Barndorff-Nielsen, O. E. and N. Shephard (2006). Econometrics of Testing for Jumps in Financial Economics using Bipower Variation. *Journal of Financial Econometrics 4*, 1–30.

Beirlant, J., Y. Goegebeur, J. Segers, and J. Teugels (2004). *Statistics of Extremes: Theory and Applications*. Chichester, UK: John Wiley and Sons, Ltd.

Bollerslev, T. (1987). A Conditional Heteroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics 69*, 542–547.

Bollerslev, T., T. Law, and G. Tauchen (2008). Risk, Jumps, and Diversification. *Journal of Econometrics 144*, 234–256.

Bollerslev, T. and V. Todorov (2010a). Estimation of Jump Tails. Working paper, Duke and Northwestern Universities.

Bollerslev, T. and V. Todorov (2010b). Tails, Fears and Risk Premia. Working paper, Duke and Northwestern Universities.

Carr, P., H. Geman, D. Madan, and M. Yor (2002). The Fine Structure of Asset Returns: An Empirical Investigation. *Journal of Business 75*, 305–332.

Coles, S. G. (2001). *An introduction to Statistical Modelling of Extreme Values*. Berlin: Springer-Verlag.

Coles, S. G., J. Heffernan, and J. A. Tawn (1999). Dependence Measures for Extreme Value Analysis. *Extremes 3*, 5–38.

de Haan, L. and A. Ferreira (2006). *Extreme Value Theory: An Introduction*. Berlin: Springer-Verlag.

Duffie, D., J. Pan, and K. Singleton (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica 68*, 1343–1376.

Einmahl, J., L. de Haan, and A. Sinha (1997). Estimating the Spectral Measure of an Extreme Value Distribution. *Stochastic Processes and their Applications 70*, 143–171.

Embrechts, P., C. Kluppelberg, and T. Mikosch (2001). *Modelling Extremal Events for Insurance and Finance* (3rd ed.). Berlin: Springer-Verlag.

Embrechts, P., A. J. McNeil, and D. Straumann (2002). Correlation and Dependence in Risk Management: Properties and Pitfalls. In M. Dempster (Ed.), *Risk Management: Value at Risk and Beyond*. Cambridge, UK: Cambridge University Press.

Eraker, B., M. Johannes, and N. Polson (2003). The Impact of Jumps in Volatility and Returns. *Journal of Finance 58*, 1269–1300.

Fama, E. (1965). The Behavior of Stock Market Prices. *Journal of Business 38*, 34–105.

Fisher, R. and L. Tippett (1928). Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample. *Proceedings of the Cambridge Philosophical Society 24*, 180–190.

Frahm, G., M. Junker, and R. Schmidt (2005). Estimating the Tail-Dependence Coefficient: Properties and Pitfalls. *Insurance: Mathematics and Economics 37*, 80–100.

Gobbi, F. and C. Mancini (2009). Identifying the Covariation between the Diffusive Parts and the Co-Jumps Given Discrete Observations. Working paper, Universita degli Studi di Firenze.

Jacod, J. (2008). Asymptotic Properties of Power Variations and Associated Functionals of Semimartingales. *Stochastic Processes and their Applications 118*, 517–559.

Jacod, J. and V. Todorov (2009). Testing for Common Arrivals of Jumps for Discretely Observed Multidimensional Processes. *Annals of Statistics 37*, 1792–1838.

Jondeau, E., S. Poon, and M. Rockinger (2007). *Financial Modeling Under Non-Gaussian Distributions*. Berlin: Spring-Verlag.

Kearns, P. and A. Pagan (1997). Estimating the Density Tail Index for Financial Time Series. *Review of Economics and Statistics 79*, 171–175.

Kelly, B. (2010). Risk Premia and the Conditional Tails of Stock Returns. Working paper, University of Chicago, Booth School of Business.

Longin, F. M. and B. Solnik (2001). Extreme Correlation of International Equity Markets. *Journal of Finance 56*, 649–676.

Mancini, C. (2009). Non-Parametric Threshold Estimation for Models with Stochastic Diffusion Coefficient and Jumps. *Scandinavian Journal of Statistics 36*, 270–296.

Mandelbrot, E. (1963). The Variation of Certain Speculative Prices. *Journal of Business 35*, 394–419.

Merton, R. (1976). Option Pricing when Underlying Asset Returns are Discontinuous. *Journal of Financial Economics 3*, 125–144.

Mikosch, T. and C. Starica (2000). Limit Theory for the Sample Autocorrelations and Extremes of a GARCH(1,1) Process. *Annals of Statistics 28*, 1427–1451.

Pickands, J. (1975). Statistical Inference Using Extreme Order Statistics. *Annals of Statistics 3*, 119–131.

Pickands, J. (1981). Multivariate Extreme Value Distributions. In *Bulletin of the International Statistical Institute: Proceedings of the 43rd Session*, Voorburg, Netherlands, pp. 859–878.

Poon, S., M. Rockinger, and J. Tawn (2004). Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications. *Review of Financial Studies 17*, 581–610.

Sibuya, M. (1960). Bivariate Extreme Statistics. *Annals of the Institute of Statistical Mathematics 11*, 195–210.

Smith, M., Q. Gan, and R. Kohn (2011). Modeling Dependence using Skew t Copulas: Bayesian Inference and Applications. *Journal of Applied Econometrics forthcoming*.

Smith, R. (1987). Estimating Tails of Probability Distributions. *Annals of Statistics 15*, 1174–1207.

Starica, C. (1999). Multivariate Extremes for Models with Constant Conditional Correlations. *Journal of Empirical Finance 6*, 513–553.

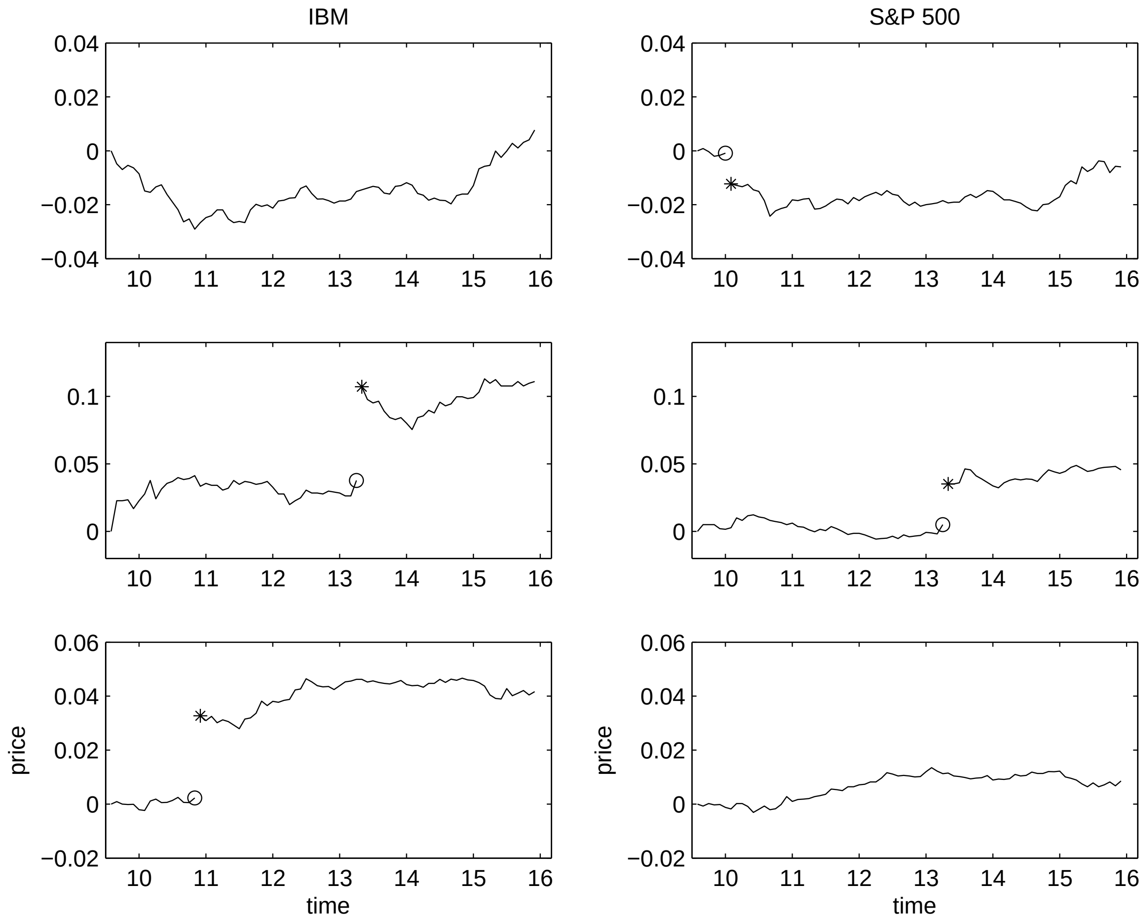
Steinkohl, C., R. A. Davis, and C. Kluppelberg (2010). Extreme Value Analysis of Multivariate High Frequency Wind Speed Data. Working paper, University of Munich and Columbia University.

Straetmans, S. T. M., W. F. C. Verschoor, and C. C. P. Wolff (2008). Extreme US Stock Market Fluctuation in the Wake of 9/11. *Journal of Applied Econometrics 23*, 17–42.

Todorov, V. and T. Bollerslev (2010). Jumps and Betas: A New Framework for Disentangling and Estimating Systematic Risks. *Journal of Econometrics 157*, 220–235.

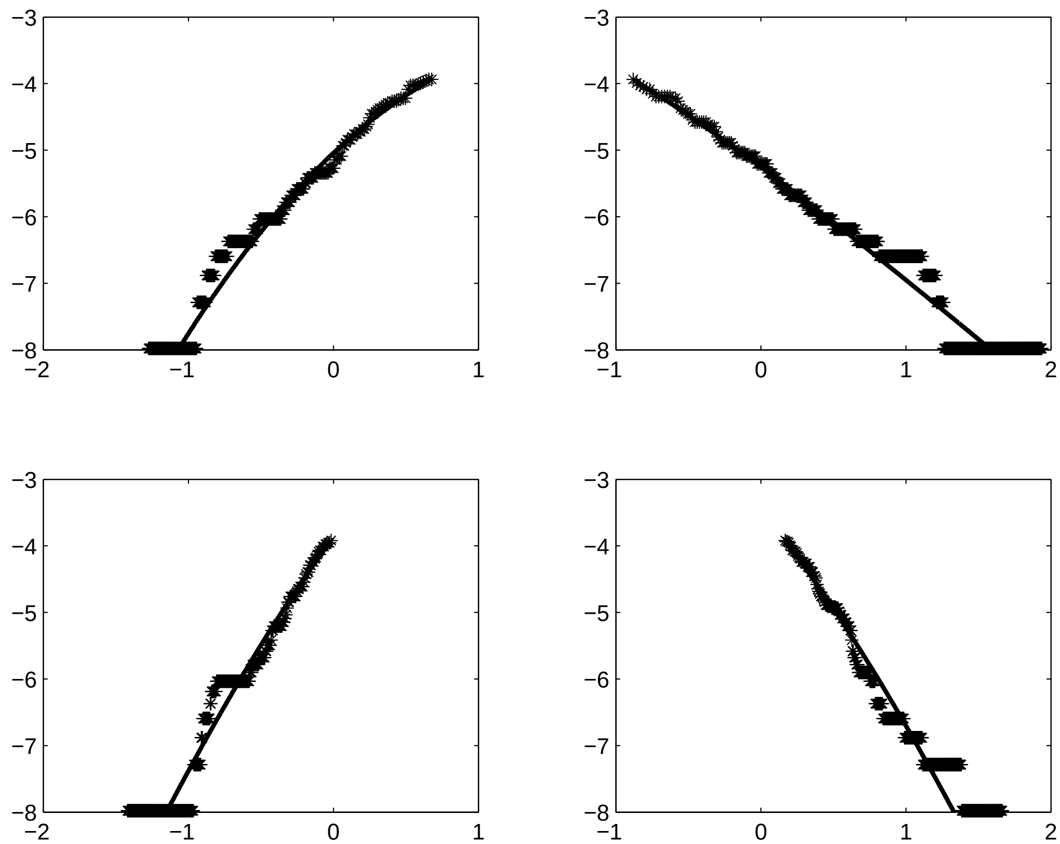
Wagner, N. and T. A. Marsh (2005). Measuring Tail Thickness under GARCH and an Application to Extreme Exchange Rate Changes. *Journal of Empirical Finance 12*, 165–185.

Figure 1: *IBM and Market-Wide Jumps*



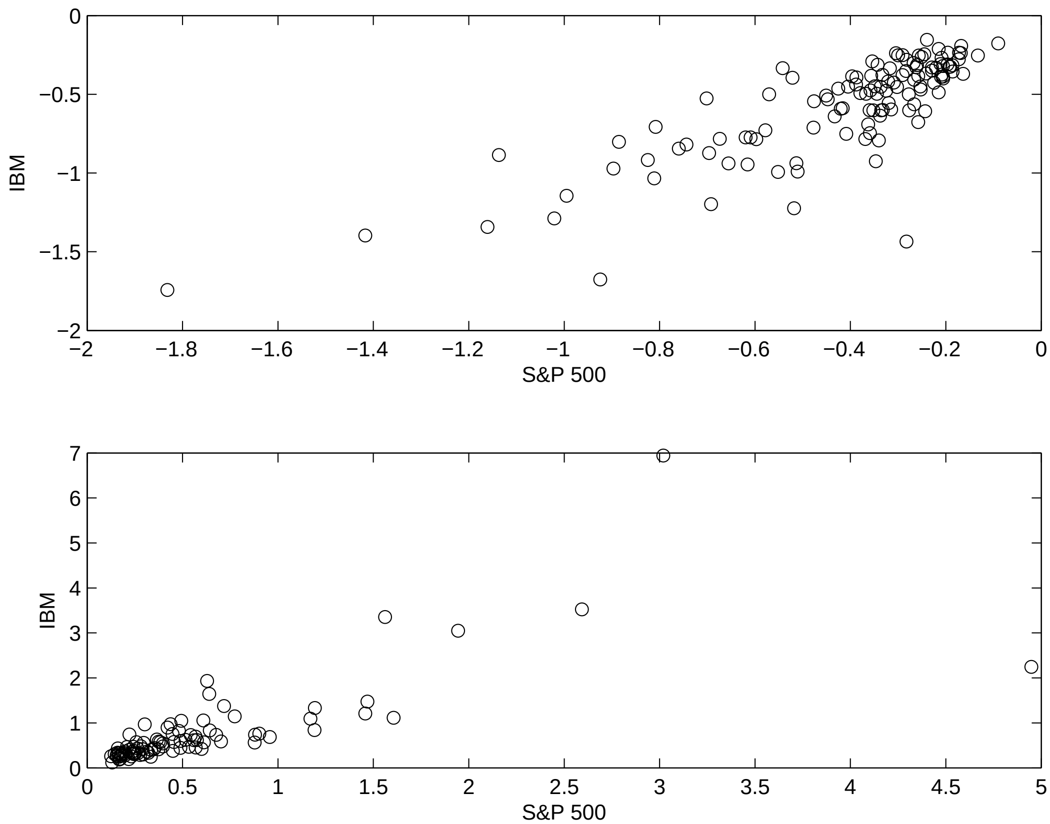
Note: The figure shows the 5-minute logarithmic prices for IBM and the S&P 500 futures index for October 29, 2002 (top panel), January 3, 2001 (middle panel), and February 26, 2008 (bottom panel). The logarithmic prices are normalized to zero at the beginning of each day.

Figure 2: *IBM Jump Tails*



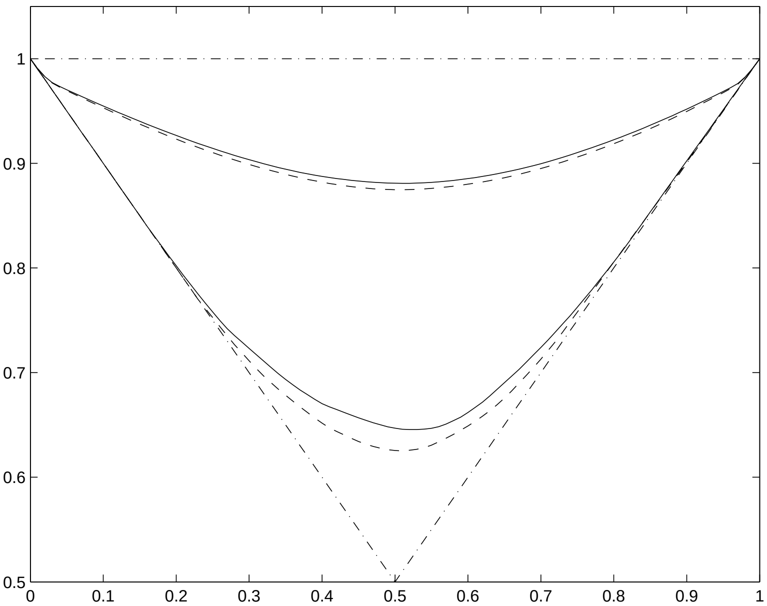
Note: The figure shows the estimated (solid line) and actually observed (stars) systematic (top two panels) and idiosyncratic (bottom two panels) negative (left two panels) and positive (right two panels) jump tails for IBM. The tails are plotted on a double logarithmic scale. All of the jumps are extracted from 5-minute returns spanning 1997 through 2008.

Figure 3: *IBM Systematic Jumps*



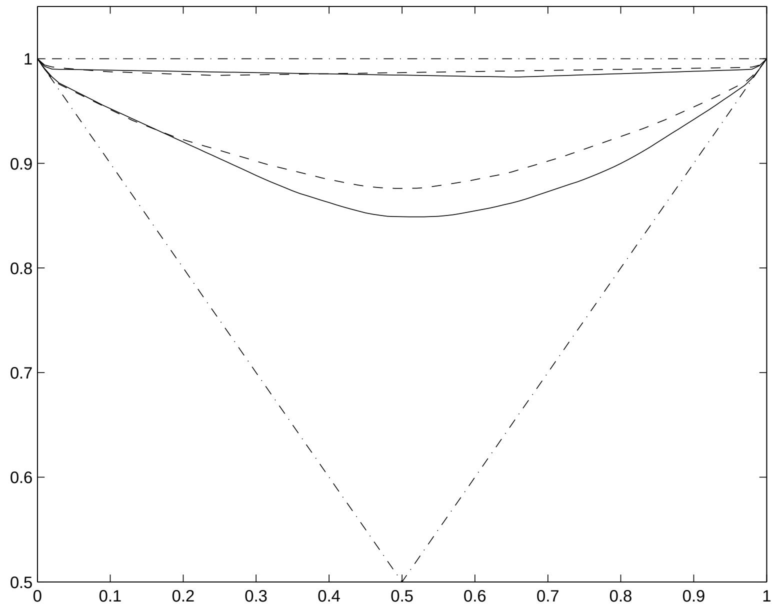
Note: The figure shows the scatter of systematic negative (top panel) and positive (bottom panel) jumps in IBM and the S&P 500 market portfolio. The jumps are extracted from 5-minute returns spanning 1997 through 2008.

Figure 4: *Pickands Dependence Functions for High-Frequency IBM Returns*



Note: The figure shows estimates of Pickands dependence function for IBM and the S&P 500 market portfolio based on 5-minute returns (top two curves) and the systematic jump tails (bottom two curves). The dashed (solid) lines correspond to the positive (negative) tails. The jumps are extracted from 5-minute returns spanning 1997 through 2008.

Figure 5: *Pickands Dependence Functions for Daily IBM Returns*



Note: The figure shows estimates of Pickands dependence function for IBM and the S&P 500 market portfolio based on daily returns (bottom two curves) and the jump-adjusted "devolatilized" daily returns (top two curves) denoted by $\widetilde{\mathbf{z}}_t^{(j)}$ in the main text. The dashed (solid) lines correspond to the positive (negative) tails. The sample spans the period from 1997 through 2008.

Table 1: *Summary Statistics*

	CV		JV		Jump Counts	
Ticker	Mean	st.dev.	Mean	st.dev.	Systematic	Idiosyncratic
PG	1.934	2.665	0.294	1.703	197	1572
GE	2.789	4.961	0.333	1.862	314	1320
T	3.371	4.481	0.469	1.520	163	1765
JNJ	1.643	2.417	0.226	0.679	191	1915
MSFT	3.060	3.916	0.292	0.949	191	1292
WMT	2.840	3.617	0.407	1.797	236	1508
PFE	2.543	3.036	0.373	1.076	182	1543
JPM	4.292	9.896	0.555	2.432	258	1529
IBM	2.461	3.303	0.311	1.488	222	1375
WFC	3.220	7.381	0.478	2.429	257	1726
CSCO	5.436	7.190	0.453	1.456	155	1276
KO	1.970	2.522	0.242	0.647	216	1635
HPQ	4.339	5.271	0.728	2.443	188	1826
PEP	2.300	2.837	0.330	1.398	166	1738
ABT	2.514	2.879	0.373	0.969	201	1796
INTC	4.877	5.960	0.340	0.966	168	1126
AAPL	7.539	7.882	1.069	2.636	146	1872
BAC	3.647	8.483	0.504	2.588	254	1488
ORCL	6.817	8.517	0.745	1.761	159	1669
MCD	2.615	3.346	0.378	0.958	174	1867
MRK	2.326	3.239	0.416	2.143	186	1704
AMGN	4.035	5.040	0.546	1.351	145	1846
QCOM	8.348	11.942	0.969	2.463	161	1616
UTX	2.601	3.768	0.408	1.255	187	1984
BMY	2.793	3.620	0.538	2.264	197	2087
USB	3.810	6.673	0.696	1.763	230	2187
DIS	3.313	4.273	0.494	1.792	175	1825
MMM	2.070	2.954	0.305	0.726	194	1865
C	5.029	19.785	0.566	2.613	240	1329
MDT	2.443	3.377	0.558	2.052	172	2292
CL	2.202	2.787	0.379	1.023	183	2065
BAX	2.198	3.451	0.480	1.881	139	2451
BK	4.544	13.167	0.821	4.984	275	1975
UNH	3.084	4.832	0.638	1.754	125	2414
LOW	3.929	5.042	0.650	1.693	169	1977
BA	3.046	3.937	0.423	1.032	152	1775
MO	2.517	2.995	0.564	2.827	158	1833
SO	2.400	2.795	0.304	1.083	151	1841
LMT	2.899	3.687	0.544	1.400	134	2162
SGP	3.786	4.767	0.724	3.456	145	1903
CAT	3.286	4.931	0.441	0.961	187	1800
WAG	3.071	3.700	0.524	1.380	170	2021
DD	2.996	4.105	0.352	0.794	192	1610
GD	2.221	2.949	0.531	1.186	183	3104
AXP	3.841	7.596	0.595	2.786	233	1767
TXN	6.442	7.136	0.751	2.269	155	1464
FDX	2.695	3.410	0.488	1.004	171	2350
ETR	2.291	3.805	0.469	1.066	153	2932
DOW	2.871	3.878	0.520	1.462	190	2110
AEP	2.443	6.549	0.397	1.532	167	2513
min	1.643	2.417	0.226	0.647	125	1126
max	8.348	19.785	1.069	4.984	314	3104
25th	2.448	3.314	0.374	1.069	160	1612
50th	2.947	3.927	0.479	1.526	183	1826
75th	3.833	6.402	0.562	2.234	197	2012
SPFU	1.122	2.19	0.116	0.723	1621	

Note: The sample period for the fifty stocks range from mid 1997 through December 2008, for a minimum of 2,832 to a maximum of 2,922 daily observations.

Table 2: *Jump Tail Decay Parameters*

	Systematic		Idiosyncratic	
Ticker	$\widehat{\xi}^{-}$	$\widehat{\xi}^{+}$	$\widehat{\xi}^{-}$	$\widehat{\xi}^{+}$
PG	0.240	0.138	0.307	0.322
GE	0.014	0.340	0.221	0.365
T	0.022	0.214	0.231	0.102
JNJ	0.017	0.012	0.316	0.003
MSFT	0.028	0.216	0.144	0.161
WMT	0.041	0.008	0.234	0.308
PFE	0.045	0.143	0.207	0.149
JPM	0.379	0.310	0.398	0.304
IBM	0.186	0.546	0.202	0.208
WFC	0.334	0.332	0.007	0.481
CSCO	0.061	0.236	0.314	0.236
KO	0.133	0.140	-0.004	0.160
HPQ	0.343	0.307	0.048	0.041
PEP	0.050	0.283	0.001	0.192
ABT	0.007	0.149	0.240	0.197
INTC	0.028	0.022	0.026	0.090
AAPL	0.123	0.009	0.061	0.402
BAC	0.304	0.171	0.228	0.369
ORCL	0.208	0.047	0.005	0.017
MCD	0.063	0.116	0.193	0.039
MRK	0.091	0.265	0.006	0.263
AMGN	0.009	0.082	0.066	0.035
QCOM	0.219	0.258	0.013	0.142
UTX	0.131	0.067	0.239	0.005
BMY	0.016	0.015	0.003	0.009
USB	0.163	0.011	0.010	0.101
DIS	0.348	0.203	0.378	0.316
MMM	0.225	0.176	0.023	0.339
C	0.332	0.379	0.186	0.136
MDT	0.008	0.102	0.004	0.130
CL	0.164	0.029	-0.006	0.312
BAX	0.013	0.006	0.445	0.002
BK	0.022	0.286	0.472	0.390
UNH	0.159	0.240	0.123	0.279
LOW	0.234	0.308	0.050	0.035
BA	0.284	0.011	0.343	0.103
MO	0.176	0.266	0.134	0.274
SO	0.022	0.015	0.387	0.362
LMT	0.367	0.066	0.217	0.332
SGP	0.015	0.008	0.320	0.006
CAT	0.076	0.197	0.277	0.062
WAG	0.249	0.295	0.260	-0.004
DD	0.047	0.011	0.132	0.136
GD	0.132	0.058	0.183	0.014
AXP	0.205	0.206	0.313	0.150
TXN	0.019	0.250	0.043	0.272
FDX	0.290	0.130	0.028	0.066
ETR	0.300	0.215	0.020	0.116
DOW	0.140	0.227	0.172	0.303
AEP	0.131	0.022	0.572	0.450
min	0.007	0.006	-0.006	-0.004
max	0.379	0.546	0.572	0.481
25th	0.028	0.033	0.027	0.063
50th	0.132	0.160	0.185	0.155
75th	0.232	0.256	0.273	0.307
SPFU	0.232	0.465		

Note: The estimated jump tail decay parameters are based on M = 0.02*T jump observations extracted from 5-minute returns spanning 1997 through 2008.

Table 3: *High-Frequency Tail-Dependence Coefficients*

	5-min Returns		All Jumps		Systematic Jumps	
Ticker	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$
PG	0.138	0.064	0.065	0.079	0.689	0.691
GE	0.188	0.106	0.149	0.174	0.676	0.691
T	0.132	0.042	0.071	0.118	0.754	0.688
JNJ	0.122	0.071	0.073	0.061	0.680	0.697
MSFT	0.076	0.103	0.103	0.111	0.773	0.732
WMT	0.114	0.055	0.069	0.088	0.690	0.631
PFE	0.134	0.079	0.090	0.054	0.723	0.678
JPM	0.113	0.056	0.106	0.144	0.704	0.679
IBM	0.083	0.133	0.104	0.143	0.749	0.706
WFC	0.192	0.100	0.122	0.065	0.664	0.634
CSCO	0.115	0.039	0.090	0.139	0.770	0.650
KO	0.121	0.092	0.112	0.092	0.733	0.671
HPQ	0.057	0.032	0.087	0.054	0.754	0.670
PEP	0.139	0.080	0.090	0.086	0.748	0.733
ABT	0.111	0.033	0.072	0.056	0.709	0.633
INTC	0.125	0.058	0.103	0.091	0.687	0.689
AAPL	0.065	0.074	0.038	0.064	0.738	0.657
BAC	0.150	0.079	0.086	0.105	0.656	0.653
ORCL	0.065	0.016	0.031	0.033	0.743	0.656
MCD	0.116	0.032	0.064	0.028	0.710	0.621
MRK	0.116	0.044	0.070	0.046	0.733	0.653
AMGN	0.103	0.009	0.067	0.032	0.777	0.610
QCOM	0.030	0.023	0.038	0.048	0.699	0.641
UTX	0.161	0.033	0.056	0.046	0.683	0.691
BMY	0.080	0.053	0.080	0.047	0.619	0.738
USB	0.160	0.075	0.098	0.064	0.713	0.698
DIS	0.154	0.084	0.090	0.087	0.780	0.695
MMM	0.137	0.057	0.076	0.055	0.634	0.621
C	0.073	0.087	0.106	0.118	0.742	0.625
MDT	0.030	0.001	0.029	0.017	0.661	0.670
CL	0.087	0.021	0.081	0.038	0.638	0.656
BAX	0.070	0.029	0.017	0.026	0.678	0.704
BK	0.227	0.038	0.143	0.125	0.683	0.656
UNH	0.179	0.069	0.073	0.023	0.736	0.701
LOW	0.132	0.075	0.062	0.080	0.708	0.741
BA	0.160	0.064	0.059	0.074	0.708	0.776
MO	0.066	0.050	0.052	0.022	0.722	0.655
SO	0.114	0.063	0.073	0.050	0.732	0.726
LMT	0.066	0.047	0.034	0.049	0.794	0.726
SGP	0.082	0.066	0.049	0.065	0.762	0.667
CAT	0.147	0.087	0.036	0.153	0.674	0.701
WAG	0.123	0.071	0.037	0.031	0.745	0.604
DD	0.140	0.085	0.097	0.056	0.731	0.651
GD	0.136	0.028	0.037	0.019	0.644	0.696
AXP	0.184	0.079	0.139	0.090	0.757	0.696
TXN	0.047	0.013	0.115	0.057	0.792	0.650
FDX	0.144	0.058	0.023	0.042	0.708	0.714
ETR	0.142	0.058	0.055	0.017	0.729	0.722
DOW	0.064	0.013	0.033	0.049	0.741	0.712
AEP	0.069	0.042	0.036	0.090	0.725	0.715
min	0.030	0.001	0.017	0.017	0.619	0.604
max	0.227	0.133	0.149	0.174	0.794	0.776
25th	0.077	0.035	0.050	0.046	0.684	0.653
50th	0.118	0.058	0.072	0.059	0.722	0.683
75th	0.142	0.078	0.095	0.090	0.745	0.703

Note: The estimated tail-dependence coefficients for each of the stocks with the S&P 500 market portfolio reported in the three pairs of columns are based on: all of the 5-minute returns; all of the jumps; and the systematic jumps only. The jumps are extracted from the 5-minute returns spanning 1997 through 2008.

Table 4: *Daily Tail-Dependence Coefficients*

	Daily Returns		Jump Adj. Returns		Univariate De-vol.		Multivariate De-vol.	
Ticker	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$
PG	0.248	0.214	0.202	0.188	0.083	0.095	0.036	0.061
GE	0.406	0.438	0.258	0.409	0.126	0.152	0.032	0.084
T	0.247	0.218	0.185	0.241	0.052	0.094	0.027	0.036
JNJ	0.198	0.255	0.176	0.256	0.056	0.098	0.072	0.024
MSFT	0.278	0.266	0.232	0.223	0.054	0.114	0.029	0.103
WMT	0.256	0.098	0.190	0.160	0.104	0.194	0.069	0.038
PFE	0.148	0.275	0.169	0.290	0.075	0.140	0.044	0.033
JPM	0.300	0.292	0.284	0.209	0.101	0.109	0.044	0.051
IBM	0.252	0.342	0.164	0.283	0.100	0.118	0.049	0.071
WFC	0.273	0.298	0.260	0.305	0.045	0.145	0.032	0.067
CSCO	0.203	0.189	0.254	0.169	0.142	0.105	0.067	0.045
KO	0.240	0.229	0.267	0.236	0.133	0.122	0.047	0.043
HPQ	0.255	0.217	0.201	0.174	0.057	0.096	0.031	0.051
PEP	0.259	0.186	0.236	0.206	0.110	0.137	0.055	0.053
ABT	0.195	0.229	0.240	0.250	0.126	0.203	0.073	0.071
INTC	0.273	0.217	0.260	0.172	0.084	0.124	0.154	0.054
AAPL	0.136	0.144	0.112	0.135	0.025	0.137	0.013	0.027
BAC	0.283	0.359	0.162	0.280	0.084	0.159	0.034	0.086
ORCL	0.169	0.185	0.145	0.139	0.050	0.101	0.039	0.037
MCD	0.214	0.227	0.223	0.221	0.085	0.137	0.071	0.050
MRK	0.242	0.332	0.229	0.325	0.085	0.069	0.053	0.032
AMGN	0.275	0.206	0.185	0.217	0.037	0.087	0.016	0.044
QCOM	0.141	0.136	0.147	0.131	0.067	0.112	0.049	0.044
UTX	0.342	0.256	0.315	0.203	0.125	0.107	0.050	0.080
BMY	0.197	0.232	0.187	0.260	0.085	0.131	0.064	0.025
USB	0.247	0.272	0.243	0.270	0.115	0.120	0.040	0.048
DIS	0.245	0.274	0.187	0.311	0.055	0.111	0.032	0.049
MMM	0.294	0.269	0.236	0.275	0.153	0.108	0.058	0.051
C	0.328	0.283	0.312	0.261	0.117	0.167	0.080	0.058
MDT	0.144	0.196	0.164	0.188	0.055	0.106	0.037	0.079
CL	0.212	0.202	0.235	0.164	0.085	0.078	0.098	0.047
BAX	0.202	0.220	0.184	0.202	0.099	0.066	0.073	0.046
BK	0.309	0.329	0.304	0.347	0.088	0.145	0.064	0.068
UNH	0.205	0.219	0.207	0.225	0.109	0.046	0.046	0.049
LOW	0.275	0.258	0.284	0.269	0.104	0.058	0.073	0.025
BA	0.268	0.243	0.278	0.250	0.091	0.099	0.049	0.066
MO	0.200	0.152	0.194	0.175	0.036	0.131	0.040	0.088
SO	0.207	0.174	0.190	0.124	0.082	0.088	0.038	0.063
LMT	0.191	0.199	0.188	0.188	0.060	0.182	0.030	0.073
SGP	0.256	0.302	0.218	0.228	0.083	0.115	0.052	0.060
CAT	0.271	0.290	0.217	0.286	0.071	0.073	0.017	0.027
WAG	0.277	0.190	0.200	0.171	0.050	0.159	0.029	0.067
DD	0.302	0.266	0.292	0.312	0.096	0.068	0.052	0.066
GD	0.219	0.214	0.202	0.194	0.057	0.130	0.028	0.092
AXP	0.337	0.396	0.344	0.388	0.065	0.121	0.023	0.078
TXN	0.194	0.172	0.204	0.204	0.083	0.100	0.057	0.036
FDX	0.227	0.196	0.207	0.194	0.126	0.115	0.064	0.067
ETR	0.235	0.229	0.211	0.222	0.075	0.110	0.043	0.043
DOW	0.258	0.277	0.237	0.278	0.134	0.143	0.064	0.054
AEP	0.205	0.261	0.207	0.259	0.084	0.157	0.039	0.107
min	0.136	0.098	0.112	0.124	0.025	0.046	0.013	0.024
max	0.406	0.438	0.344	0.409	0.153	0.203	0.154	0.107
25th	0.204	0.200	0.187	0.188	0.058	0.098	0.033	0.044
50th	0.247	0.229	0.209	0.224	0.084	0.115	0.047	0.052
75th	0.274	0.274	0.251	0.274	0.104	0.137	0.064	0.068

Note: The estimated tail-dependence coefficients for each of the stocks with the S&P 500 market portfolio reported in the four pairs of columns are based on: the raw daily returns; the daily returns with the jumps removed denoted by $\mathbf{z}_t^{(j)}$ in the main text; the jump-adjusted $\mathbf{z}_t^{(j)}$ "devolatilized" by the scalar continuous variation measures; and $\mathbf{z}_t^{(j)}$ "devolatilized" by the multivariate continuous covariation measure denoted by $\widetilde{\mathbf{z}}_t^{(j)}$ in the main text. The sample spans the period from 1997 through 2008.

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