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The Risk-Return Tradeoff and Leverage Effect in a Stochastic Volatility-in-Mean Model

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Abstract

We study the risk premium and leverage effect in the S&P500 market using the stochastic volatility-in-mean model of Barndorff-Nielsen & Shephard (2001). The Merton (1973, 1980) equilibrium asset pricing condition linking the conditional mean and conditional variance of discrete time returns is reinterpreted in terms of the continuous time model. Tests are performed on the risk-return relation, the leverage effect, and the overidentifying zero intercept restriction in the Merton condition. Results are compared across alternative volatility proxies, in particular, realized volatility from high-frequency (5-minute) returns, implied Black-Scholes volatility backed out from observed option prices, model-free implied volatility (VIX), and staggered bipower variation. Our results are consistent with a positive risk-return relation and a significant leverage effect, whereas an additional overidentifying zero intercept condition is rejected. We also show that these inferences are sensitive to the exact timing of the chosen volatility proxy. Robustness of the conclusions is verified in bootstrap experiments.

JEL Classifications: G13, L12.

Keywords: Financial leverage effect, implied volatility, realized volatility, risk-return relation, stochastic volatility, VIX

1 Introduction

The relation between risk and return is central to financial economics. Asset pricing theory predicts a positive risk-return tradeoff. Empirically, conditional first and second moments of asset returns are time-varying, and this must be accounted for when testing the risk-return relation. Empirical research also documents a strong financial leverage effect that potentially makes it more difficult to identify the risk-return relation. Thus, according to the Black (1976) explanation of the leverage effect, a price drop increases the debt-equity ratio and hence expected risk. The increase in risk would in turn increase expected returns in case of a positive risk-return relation. Depending on whether the empirical researcher associates the increase in risk with the initial price drop (negative return) or manages to link it to the higher subsequent (expected) returns, the apparent empirical risk-return relation may be of either sign. This highlights the identification issue, and may suggest why the empirical literature tends to show mixed results on the significance and sign of the risk-return relation. Evidently, further analysis should be carried out in a model that in addition to the risk-return relation explicitly accommodates a separate leverage effect.

Consider a standard representation of the risk-return relation given by

$$E_{t-1}r_t = \alpha + \gamma Var_{t-1}(r_t). \tag{1}$$

Merton (1973, 1980) derives this as an equilibrium condition. In the Merton model, the slope γ is an average relative risk aversion parameter across investors. Furthermore, the equilibrium conditions imply a zero restriction on the intercept α in the risk-return relation. Empirical testing requires a model for computing the conditional first and second moments of returns, and, as already argued, this model should allow for leverage. In the present paper, we use the Barndorff-Nielsen & Shephard (2001) (henceforth BNS) model since this includes a leverage effect, along with a risk-return relation and time-varying conditional moments, and is consistent with a number of additional stylized facts that characterize asset returns. These include volatility clustering, semi-heavy-tailed non-normal return distributions with skewness and excess kurtosis, and aggregational Gaussianity, i.e., the normal approximation improves as the observation frequency is reduced. Furthermore, the model allows explicit calculation of joint conditional and unconditional moments, thus facilitating explicit estimating equations, closed form parameter estimators, highly tractable option pricing formulas, etc.. In spite of this, the model has rarely been applied. Indeed, it has not been used in a systematic study of the risk premium and leverage effect.

The basic structure of the BNS continuous-time stochastic volatility-in-mean model is

$$dX_t = \left(\alpha^* + \gamma^* \sigma_t^2\right) dt + \text{error}, \tag{2}$$

where X_t is the log-price and σ_t is the stochastic volatility that is seen to enter into the drift of this continuous-time model, and where the error term in addition to stochastic volatility includes the leverage effect. Our interest is in testing for a positive risk-return relation, $\gamma > 0$, and the overidentifying equilibrium condition $\alpha = 0$ in (1). In this paper, we show that this is not quite equivalent to testing both hypotheses on the corresponding BNS model parameters γ^* and α^*. We derive the relevant hypotheses in terms of the stochastic volatility-in-mean model. We then present the explicit closed-form estimators and implement the relevant hypothesis tests in an application to Standard and Poor's 500 index futures, associated futures options, and the VIX index.

Recent literature documents strong persistence, to the point of possible long memory or fractional integration, in volatility. Christensen & Nielsen (2007) use a model with long memory in volatility and find a strong leverage effect in monthly data, using either realized volatility or option implied volatility, whereas the risk premium is positive and significant in some of the specifications. Since long memory in volatility would spill over into long memory in returns through a risk-return relation of the Merton type, and since long memory in returns is not empirically warranted, Christensen & Nielsen (2007) modify the risk-return relation to depend on changes in volatility rather than volatility levels, following Ang, Hodrick, Xing & Zhang (2006). The BNS model on the other hand is able to capture strong serial dependence in volatility without being of the long memory type. Hence, in the present paper we use the BNS model along with the unmodified Merton risk-return relation: The risk premium depends on volatility levels, not volatility changes.

The paper is laid out as follows. Section 2 describes the continuous-time stochastic volatility-in-mean model. Section 3 introduces the discrete time framework and provides the link between the stochastic volatility-in-mean model and the Merton asset pricing conditions. Section

4 presents the estimating equations. Section 5 discusses data and summary statistics. Estimation results are discussed in Section 6, and Section 7 concludes.

2 The Continuous-Time Stochastic Volatility-in-Mean Model

Let P_t be the price of a financial asset at time t and let $X_t = \log P_t$ denote the log price. We consider the continuous-time stochastic volatility-in-mean model given by

$$\begin{aligned} dX_t &= \left(\alpha^* + \gamma^* \sigma_t^2\right) dt + \sigma_t dZ_t^1 + \rho dZ_{\lambda t}^2 \\ d\sigma_t^2 &= \lambda(\zeta - \sigma_t^2)dt + dZ_{\lambda t}^2 \end{aligned} \tag{3}$$

where σ_t is the stochastic volatility, and the increments dZ^1 and dZ^2 to the two independent underlying stochastic processes are mean zero, stationary, and serially independent. The presence of the increment from the volatility equation in the return equation, with coefficient ρ, accommodates the leverage effect, the case $\rho < 0$. The BNS specification is that Z_t^1 is a standard Wiener process generating continuous sample path movements in the log price, and it is these Wiener increments that are scaled by the stochastic volatility σ_t. The specification allows that the instantaneous variance σ_t^2 enters the drift, with a slope coefficient γ^*, and the constant portion of the drift is denoted α^*. In short, the return equation is a stochastic volatility-in-mean specification, with a leverage effect.

Turning to the volatility specification, the second equation in (3), this is of mean-reverting (Ornstein-Uhlenbeck, or OU) type, with rate of mean reversion λ for the instantaneous variance σ_t^2, and unconditional mean or target for mean reversion given by ζ. The BNS specification is that of a non-Gaussian OU-process. Thus, Z^2 is not a standard Wiener process, but another time-homogeneous Levy process, also known as a subordinator, i.e., a process with independent and stationary increments. A Wiener process for Z^2 would imply negative variances σ_t^2 with positive probability. The subordinator specification implies that Z^2 has positive jumps, and although the drift of σ_t^2 can be negative, it turns positive when σ_t^2 gets sufficiently small, and volatility never becomes negative.[1] By running the subordinator according to time index λt

[1] We work with a zero mean process, i.e., $Z_t^2 = \widetilde{Z}_t^2 - \zeta t$, in terms of the standard subordinator (or background driving Levy process) $\widetilde{Z}_t^2$ from BNS, with positive mean ζt. The drift $-\zeta dt$ of Z_t^2 is offset by the first term in the drift of σ_t^2, and the second term in the latter, $-\lambda \sigma_t^2 dt$, becomes small with σ_t^2. Note that the unconditional

instead of just t (e.g., large λ means running the process faster), the OU structure implies that the unconditional or invariant distribution of σ_t^2 is independent of λ, and only depends on the choice of subordinator process Z^2. In fact, it is convenient to identify the subordinator Z^2 by the invariant distribution of σ_t^2. For example, this could be the Gamma or the inverse Gaussian (IG) distribution. If $\rho = 0$, the inverse Gaussian distribution for σ_t^2 implies a normal-inverse Gaussian (NIG) distribution for returns, which has proved empirically relevant in some cases. For other parameter values and subordinators, more general return distributions are obtained, all consistent with volatility clustering, non-normal returns, and leverage (if $\rho < 0$).

We consider the parameter vector $\theta = (\alpha^*, \gamma^*, \rho, \lambda, \zeta, \eta)$. Here, the final component η is a volatility-of-volatility parameter, $\eta = var(\sigma_t^2)$, which is part of the parametrization of Z^2. If σ_t^2 follows a Gamma process, then it has an unconditional Gamma distribution with mean ζ and variance η. If σ_t^2 follows an IG-OU process, then it has an unconditional IG distribution with mean ζ and variance η.

In this stochastic volatility-in-mean model we test three main hypotheses of financial interest: (i) Is γ^* positive and significant? We establish below that this is indeed a test on the conditional risk-return relation in the Merton (1973, 1980) specification (1). (ii) Is ρ negative and significant? This is testing the financial leverage effect. (iii) We also test the additional overidentifying zero intercept restriction on (1). We show below that this is not equivalent to testing a zeron condition on α^* in the BNS model (3). Instead, we derive the relevant test on θ.

3 The Discrete-Time Framework

In the model from the previous section, the discrete time return over a time interval of length $\Delta > 0$ is

$$r_{t,t+\Delta} = X_{t+\Delta} - X_t. \tag{4}$$

In our empirical work we use daily data. In early work, Merton (1980) regressed the one period (in his case one month) return on the realized variance from higher frequency returns over the same period. Realized variance is essentially an estimate (in the absence of jumps) of integrated

means of $\widetilde{Z}_t^2$ and σ_t^2 coincide, at ζ, in the BNS model.

variance

$$V_{t,t+\Delta} = \int_t^{t+\Delta} \sigma_s^2 ds. \tag{5}$$

From (1), the risk-return relation in asset pricing theory actually does not relate expected return to integrated variance, but to the conditional variance of return,

$$E_t r_{t,t+\Delta} = \alpha\Delta + \gamma Var_t r_{t+\Delta}. \tag{6}$$

To test for positive risk-return trade-off, $\gamma > 0$, and the overidentifying restriction $\alpha = 0$, within the BNS framework, we thus need the relation between the parameters (α, γ) from the risk-return relation (6) and the parameters θ of the BNS model (3). To this end, we develop the exact discrete time version of the BNS model and in this calculate the conditional mean and variance of the discrete time returns. We then seek conditions under which the two satisfy the equilibrium risk-return relation.

The exact discrete time model is

$$r_{t,t+\Delta} = \alpha^*\Delta + \gamma^* V_{t,t+\Delta} + \int_t^{t+\Delta} \sigma_s dZ_s^1 + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right). \tag{7}$$

Since Z^2 and the stochastic integral with respect to Z^1 are zero-mean processes, the conditional mean return is

$$E_t r_{t,t+\Delta} = \alpha^*\Delta + \gamma^* E_t(V_{t,t+\Delta}). \tag{8}$$

Thus, the conditional mean return is an affine function of the conditional mean of integrated volatility. Comparing (8) and (6), if conditional return variance were simply given by the conditional mean of integrated volatility, then the parameters γ and γ^* could be identified with one another, and so could α with α^*. Thus, the test for positive risk-return relation could be on the coefficient γ^* on the conditional mean of integrated volatility in the conditional mean return equation (8), and the test of the Merton zero intercept condition $\alpha = 0$ could be directly on the intercept α^* in (8). Of course, the conditional variance of the discrete time return does not in fact coincide with the conditional expectation of integrated continuous time volatility, and the tests must account for this. The following theorem provides the necessary results. The detailed proof is in the appendix.

Theorem 1 *The relation between the parameters (α, γ) from the risk-return relation (6) and the parameters $\theta = (\alpha^*, \gamma^*, \rho, \lambda, \zeta, \eta)$ of the continuous-time stochastic volatility-in-mean model (3) is given by*

$$\begin{aligned}\alpha &= \frac{(\gamma^*)^3\eta\left(3 - 4e^{-\lambda\Delta} + e^{-2\lambda\Delta} - 2\Delta\lambda\right)}{\lambda^2} + 4(\gamma^*)^2\left(-\Delta + \frac{1 - e^{-\lambda\Delta}}{\lambda}\right)\eta\rho - 2\gamma^*\Delta\eta\lambda\rho^2 + \Delta\alpha^* \\ \gamma &= \gamma^*\end{aligned}$$

Proof: By (7), the conditional return variance is

$$\begin{aligned}Var_t r_{t,t+\Delta} &= Var_t\left(\gamma^* V_{t,t+\Delta} + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right) + \int_t^{t+\Delta} \sigma_s dZ^1_s\right) \\ &= Var_t\left(\gamma^* V_{t,t+\Delta} + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right)\right) + Var_t\left(\int_t^{t+\Delta} \sigma_s dZ^1_s\right),\end{aligned}$$

where we have used that integrated variance (5) is driven by Z^2, and Z^1 is independent of this, so the two terms inside the variance operators are uncorrelated. Since Z^2 has independent increments, the first variance is a constant, not depending on state variables. The second is computed by conditioning:

$$\begin{aligned}Var_t r_{t,t+\Delta} &= Var_t\left(\gamma^* V_{t,t+\Delta} + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right)\right) + E_t Var_t\left(\int_t^{t+\Delta} \sigma_s dZ^1_s | \{\sigma^2_s\}_{s=t}^{t+\Delta}\right) \\ &\quad + Var_t\left(E_t\left(\int_t^{t+\Delta} \sigma_s dZ^1_s | \{\sigma^2_s\}_{s=t}^{t+\Delta}\right)\right) \\ &= Var_t\left(\gamma^* V_{t,t+\Delta} + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right)\right) + E_t\left(\int_t^{t+\Delta} \sigma^2_s ds\right),\end{aligned}$$

and from (5) the last term is recognized as $E_t(V_{t,t+\Delta})$, so

$$Var_t r_{t,t+\Delta} = Var_t\left(\gamma^* V_{t,t+\Delta} + \rho\left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t}\right)\right) + E_t(V_{t,t+\Delta}). \tag{9}$$

From this and (8), both the conditional mean and the conditional variance of return are affine in the conditional mean of integrated volatility, and the latter with unit slope. Inserting (9) and (8) in the the Merton condition (6) yields

$$\begin{aligned}\gamma &= \gamma^*, \\ \alpha &= \alpha^* - \frac{\gamma^*}{\Delta} Var_t \left(\gamma^* V_{t,t+\Delta} + \rho \left(Z^2_{\lambda(t+\Delta)} - Z^2_{\lambda t} \right) \right).\end{aligned}$$

Using the exact expressions for the relevant moments in the BNS model yields the result.

Consequently, in the continuous-time stochastic volatility-in-mean model, the equilibrium risk-return relation holds up to a constant intercept that may be calculated in terms of model parameters. Merton's overidentifying zero condition on the intercept may thus be tested as a cross-restriction on the model parameters, using the delta-rule to calculate the relevant asymptotic standard error. The proportionality parameter γ in the equilibrium risk-return relation (1) actually coincides with the slope parameter γ^* from the drift specification (2) of the continuous-time model. Thus, the search for sign and significance of the risk-return relation γ in asset pricing becomes a test on the γ^* parameter in the BNS model. Finally, the test for the leverage effect is on the parameter ρ, and the presence of this parameter facilitates an interpretation of any finding of a risk-return relation as being free of contamination by leverage.

4 The Estimating Equations

In the BNS model, it is possible to carry out explicit calculation of joint moments, thus facilitating explicit estimating equations and closed form estimators. We follow **?**. The estimation approach uses a martingale estimating function method based on daily returns and daily volatilities. Different proxies are used for the volatilities. We want to find an estimator for θ_0 using observations $X_1, \ldots, X_n, \sigma^2_1, \ldots, \sigma^2_n$. Here, $\Delta = 1$. We are interested in asymptotics as $n \to \infty$.

For that purpose let us consider the following martingale estimating functions:

$$
\begin{aligned}
G_n^1(\theta) &= \sum_{k=1}^n \big[\sigma_t^2 - f^1(\sigma_{t-1}^2, \theta)\big], & f^1(v,\theta) &= E_\theta[\sigma_1^2 | \sigma_0^2 = v] \\
G_n^2(\theta) &= \sum_{k=1}^n \big[\sigma_t^2\sigma_{t-1}^2 - f^2(\sigma_{t-1}^2, \theta)\big], & f^2(v,\theta) &= E_\theta[\sigma_1^2\sigma_0^2 | \sigma_0^2 = v] \\
G_n^3(\theta) &= \sum_{k=1}^n \big[(\sigma_t^2)^2 - f^3(\sigma_{t-1}^2, \theta)\big], & f^3(v,\theta) &= E_\theta[(\sigma_1^2)^2 | \sigma_0^2 = v] \\
G_n^4(\theta) &= \sum_{k=1}^n \big[X_t - f^4(\sigma_{t-1}^2, \theta)\big], & f^4(v,\theta) &= E_\theta[X_1 | \sigma_0^2 = v] \\
G_n^5(\theta) &= \sum_{k=1}^n \big[X_t\sigma_{t-1}^2 - f^5(\sigma_{t-1}^2, \theta)\big], & f^5(v,\theta) &= E_\theta[X_1\sigma_0^2 | \sigma_0^2 = v] \\
G_n^6(\theta) &= \sum_{k=1}^n \big[X_t\sigma_t^2 - f^6(\sigma_{t-1}^2, \theta)\big], & f^6(v,\theta) &= E_\theta[X_1\sigma_1^2 | \sigma_0^2 = v]
\end{aligned}
\tag{10}
$$

We have the explicit expressions

$$
\begin{aligned}
f^1(v,\theta) &= e^{-\lambda}v + (1-e^{-\lambda})\zeta \\
f^2(v,\theta) &= e^{-\lambda}v^2 + (1-e^{-\lambda})\zeta v \\
f^3(v,\theta) &= e^{-2\lambda}v^2 + 2e^{-\lambda}(1-e^{-\lambda})\zeta v + (1-e^{-\lambda})^2\zeta^2 + (1-e^{-2\lambda})\eta \\
f^4(v,\theta) &= \gamma(1-e^{-\lambda})\lambda^{-1}(v-\zeta) + \Delta(\xi + \gamma\zeta) \\
f^5(v,\theta) &= \big(\gamma(1-e^{-\lambda})\lambda^{-1}(v-\zeta) + \Delta(\xi+\gamma\zeta)\big)v \\
f^6(v,\theta) &= 2(1-e^{-\lambda})\lambda^{-1}\eta\lambda\rho + \Delta\xi\big(e^{-\lambda}(v-\zeta)+\zeta\big) + \gamma\big((1-e^{-\lambda})^2\lambda^{-1}\eta + (e^{-\lambda}(v-\zeta)+\zeta)((1-e^{-\lambda})\lambda^{-1}(v-\zeta) \cdots
\end{aligned}
\tag{11}
$$

Note that f^1 is the conditional mean in the OU process. f^2 is f^1 times the conditioning argument. f^3 is the conditional second non-central moment of the OU process. It is f^1 squared plus the last term, so the conditional second moment is the conditional mean squared plus conditional variance, i.e., f^1 squared plus unconditional variance η times $(1-\gamma_1^2)$ where the latter is the conditioning. f^4 is the conditional mean return given initial variance. f^5 is f^4 times the conditioning argument. f^6 is the conditional cross-second moment which is the conditional covariance plus the product of the conditional means. The estimator $\hat{\theta}_n$ is obtained by solving the estimating equation $G_n(\theta) = 0$. This equation has an explicit solution

$\hat{\theta}_n = (\lambda_n, \zeta_n, \eta_n, \gamma_n^*, \rho_n, \alpha_n^*)$ given by

$$\begin{aligned}
\zeta_n &= (\xi_n^1 - \gamma_{1n}\upsilon_n^1)/(1-\gamma_{1n}) \\
\eta_n &= -\frac{(-1+\gamma_{1n}^2)(\upsilon_n^1)^2 - \gamma_{1n}^2\upsilon_n^2 + \xi_n^3}{-1+\gamma_{1n}^2} \\
\lambda_n &= -\log((\xi_n^2 - \xi_n^1\upsilon_n^1)/(\upsilon_n^2 - (\upsilon_n^1)^2))/\Delta \\
\gamma_n^* &= (\xi_n^5 - \upsilon_n^1\xi_n^4)/(\epsilon_n(\upsilon_n^2 - (\upsilon_n^1)^2)) \\
\rho_n &= \big(-\gamma_n\epsilon_n(-(\upsilon_n^1)^2 + \epsilon_n\lambda_n(\eta_n + (\upsilon_n^1)^2 - \upsilon_n^2) + \upsilon_n^2) - \xi_n^1\xi_n^4 + \xi_n^6\big)/(2\epsilon_n\eta_n\lambda_n) \\
\alpha_n^* &= \big(-\gamma_n(\Delta\zeta_n + \epsilon_n(-\zeta_n + \upsilon_n^1)) + \xi_n^4\big)/\Delta
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\xi_n^1 = \tfrac{1}{n}\sum_{i=1}^n \sigma_i^2 \quad \xi_n^2 = \tfrac{1}{n}\sum_{i=1}^n \sigma_i^2\sigma_{i-1}^2 \quad \xi_n^3 = \tfrac{1}{n}\sum_{i=1}^n (\sigma_i^2)^2 \\
&\xi_n^4 = \tfrac{1}{n}\sum_{i=1}^n X_i \quad \xi_n^5 = \tfrac{1}{n}\sum_{i=1}^n X_i\sigma_{i-1}^2 \quad \xi_n^6 = \tfrac{1}{n}\sum_{i=1}^n X_i\sigma_i^2
\end{aligned} \tag{13}$$

and

$$\upsilon_n^1 = \tfrac{1}{n}\sum_{i=1}^n \sigma_{i-1}^2 \quad \upsilon_n^2 = \tfrac{1}{n}\sum_{i=1}^n (\sigma_{i-1}^2)^2 \tag{14}$$

The first three equations $G_n^j(\theta) = 0$, for $j = 1, 2, 3$ contain only the unknowns ζ, η, λ and are easily solved. The last three equations $G_n^j(\theta) = 0$, for $j = 4, 5, 6$ can be seen as a linear system for the unknowns α^*, γ^*, ρ, once the other parameters have been determined. The asymptotic distribution can be given explicitly.

5 Data and Descriptive Statistics

Our data span the period January 2, 1990, through March 30, 2007. The daily returns are calculated as the daily log price differences on the S&P 500 futures, the most heavily traded futures contract. Our daily realized volatilities are based on linearly interpolated five-minute observations (following Müller, Dacorogna, Olsen, Pictet, Schwarz & Morgenegg (1990), Dacorogna, Müller, Nagler, Olsen & Pictet (1993), and Barucci & Reno (2002), among others) on S&P 500 futures prices. There is open auction CME trading from 8:30 a.m. to 3:15 p.m. central time in S&P 500 futures, providing us with 81 high-frequency returns per day. The data are obtained from price-data.com, which is an affiliate of RC Research. In addition to realized volatility, we also consider staggered bipower volatility, a measure discussed in more detail below. We also consider implied volatilities calculated from daily closing prices of at-the-money futures options with short term to expiration. In particular, the contracts considered are calls with between 6 and 36 days to expiration. The options follow a monthly expiration cycle. Daily closing prices are obtained from the Commodity Research Bureau, along with strike and underlying prices, and expiration date. Riskfree interest rates (one-month LIBOR) are obtained from the Federal Reserve. From these data, implied volatility is computed by inverting the futures option formula. Finally, we also use the VIX volatility index, which is a model-free estimate of implied volatility with 30 days to expiration. Both implied volatility and VIX are measured at the close of the previous trading day, as they are considered forward-looking measures of volatility over the life of the option. This produces four daily times series on returns and the three volatility measures (realized, implied, and VIX), with 4,182 observations in each.

In order to describe our data precisely, write $M + 1$ for the number of evenly spaced intra-daily observations of the index level on day t, denoted by $X_{t,j}$. The M continuously compounded intra-daily returns for day t are

$$r_{t,j} = X_{t,j} - X_{t,j-1}, \quad j = 1, ..., M, . \tag{15}$$

Realized volatility (RV) for day t is given by the sum of squared intra-daily returns,

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2, \quad t = 1, ..., T, \tag{16}$$

where T is the number of days in the sample. In our application, $M = 81$ and $T = 4,182$. Some authors refer to the quantity (16) as realized variance and reserve the term realized volatility for the square root of (16), e.g. Barndorff-Nielsen & Shephard (2001, 2002*a*, 2002*b*), but we shall use the more conventional term realized volatility. By definition, RV converges to quadratic variation $QV(t)$, defined for any semimartingale by

$$QV(t) = p\lim \sum_{j=1}^{M} \left(p(s_j) - p(s_{j-1})\right)^2, \tag{17}$$

where $0 = s_0 < s_1 < ... < s_M = t$ and the limit is taken for $\max_j |s_j - s_{j-1}| \to 0$ as $M \to \infty$. Using (17), RV_t in (16) is by definition a consistent estimator of the monthly increment to the quadratic variation process as $M \to \infty$ (Andersen & Bollerslev (1998), Andersen, Bollerslev, Diebold & Labys (2001) and Barndorff-Nielsen & Shephard (2002*a*, 2002*b*)). An important quantity in this model is the integrated volatility (or integrated variance)

$$\sigma^{2*}(t) = \int_0^t \sigma^2(s)\,ds. \tag{18}$$

In option pricing, this is the relevant volatility measure, see Hull & White (1987). Estimation of integrated volatility is studied e.g. in Andersen & Bollerslev (1998). Integrated volatility is closely related to QV. They coincide if $\rho = 0$. Thus RV converges to integrated variance in this case. The nonparametric estimation of the separate continuous sample path and jump components of quadratic variation, following Barndorff-Nielsen & Shephard (2004, 2006), requires the related bipower variation measure. The (first lag) realized bipower variation is defined as

$$BV_t = \frac{1}{\mu_1^2} \sum_{j=2}^{M} |r_{t,j}|\,|r_{t,j-1}|, \quad t = 1, ..., T, \tag{19}$$

where $\mu_1 = \sqrt{2/\pi}$. BV_t is consistent for month t integrated volatility, the component of the increment to quadratic variation due to continuous sample path movements in the price process, i.e.,

$$BV_t \to_p \sigma_t^{2*} = \int_{t-1}^{t} \sigma^2(s)\,ds, \quad \text{as } M \to \infty, \tag{20}$$

as shown by Barndorff-Nielsen & Shephard (2004). In theory, a higher value of M improves the precision of the estimators, but in practice it also makes them more susceptible to market microstructure effects, such as bid-ask bounces, stale prices, measurement errors, etc., see e.g. Hansen & Lunde (2006) and Barndorff-Nielsen & Shephard (2007). These effects potentially introduce artificial (typically negative) serial correlation in returns. Huang & Tauchen (2005) show that the resulting bias in (19) is mitigated by considering the staggered (second lag, i.e. skip-one) realized bipower variation

$$\widetilde{BV}_t = \frac{1}{\mu_1^2(1-2M^{-1})}\sum_{j=3}^{M} |r_{t,j}|\,|r_{t,j-2}|\,, \quad t = 1, ..., T. \tag{21}$$

The staggered version avoids the sharing of the price $p_{t,j-1}$ which by (15) enters the definition of both $r_{t,j}$ and $r_{t,j-1}$ in the non-staggered version (19). The staggered quantity $\widetilde{BV}_t$ is asymptotically equivalent to its non-staggered counterpart BV_t, and the staggered version can be applied for robustness against market microstructure effects without sacrificing asymptotic results.

Summary statistics are presented in Table 1. From the first row of the table, the mean return is 14.56%, annualized, with a standard deviation of 16.11% for the daily returns r_t. There is very little serial dependence in returns (first order autocorrelation of $-.64\%$). The next line of the table shows that the average of the daily realized volatility measures RV_t is 16.10% annualized. From the following rows, average staggered bipower variation SBV_t is slightly less, at 15.39%, as it should be, since the jumps are removed. The implied volatility measures are higher, with VIX_t at 18.73% and the option-implied IV_t at 16.47%. This indicates a negative price of volatility risk, consistent with the value of the derivatives for incomplete market hedging purposes. The higher implied volatility from VIX than the ATM option-based measure is consistent with the volatility smile and the former measure including out-of-the money options. From the second column of the table, the realized measures exhibit higher variation than the implied measures in the time series dimension, consistent with implied volatility being a conditional expectation of subsequent realized volatility. Finally, all the volatility measures are strongly serially dependent, even though returns are not, and the implied measures are more strongly dependent than the realized. Among the realized measures, SBV_t shows stronger dependence than RV_t, consistent with the jumps being unpredictable. Among the imlied measures, VIX shows stronger dependence than the ATM measure, presumably reflecting smoothing in the former. All in all, the descriptive statistics make sense. In particular, there is no indication that there is anything unusual about our data.

6 Estimation results

Table 2 shows results of estimation using realized volatility RV_t as the volatility proxy. The results in the first column are for the case where RV_t is calculated over the course of the day covered by the return r_t. Asymptotic standard errors are reported in parentheses under each estimate. For some parameters, imposing $\rho = 0$ (no leverage effect) makes a difference for the calculation of the standard error, and in these cases, the standard error imposing the condition is reported in a separate set of square brackets underneath.

From the reported results in the first column of Table 2, γ^* is significantly positive and ρ significantly negative in this case. Indeed, γ^* remains significantly positive if $\rho = 0$ is imposed. These findings indicate a positive price of risk and simultaneously a significant leverage effect, and so are consistent with theory. From the last two rows of the table, the Merton zero intercept condition is rejected, and this is so whether $\rho = 0$ is imposed or not. Here, not only the standard error but also the point estimate of the intercept depends on whether leverage is allowed, and the test rejects in both cases. In particular, the estimated intercept is negative, suggesting that mean return is below its equilibrium value. In this case, the same conclusion would have been reached if simply (erroneously) testing directly on α^* (first line of table), and this is so whether allowing a leverage effect or not ($\rho = 0$ being imposed in the latter case). This shows that the results using the realized volatility proxy do not lead to a conflict between positive risk-return tradeoff and leverage, or to erroneous inference from tests directly on the intercept instead of correct tests of the Merton condition.

Turning to the Levy parameters, the reported results include parameters corresponding to a Gamme-OU parametrization of the volatility process. Thus, σ_t^2 has an unconditional $\Gamma(\nu, a)$ distribution. The corresponding background driving Levy process (BDLP) Z^2 is a compound Poisson process with jumps arriving at rate ν and exponentially distributed jump size of mean $1/a$. Thus, $E(\sigma_t^2) = \zeta = \nu/a$ and $var(\sigma_t^2) = \eta = \nu/a^2$. The results in the first column of Table 2 show a strong decay rate λ, a mean jump size $1/a$ of about .062, and a jump arrival rate ν indicating a $.42 \cdot \Delta = .12\%$ chance of a jump each day. All parameters are strongly significant. In the alternative parametrization, the squareroot of the estimated mean ζ in the stationary distribution of σ_t^2 translates into a standard deviation of about 16%, consistent with the summary statistics in Table 1. The unconditional variance η of σ_t^2 is estimated at .0016, corresponding to a standard deviation of about 4%, somewhat lower than that of the proxy in Table 1, at 7%. The mean $\lambda\zeta\Delta$ of the BDLP is estimated at .016, and the variance $2\lambda\eta\Delta$ at .0019.

Next, we investigate how fragile the results are. The second column of Table 2 shows the results of using lagged realized volatility RV_{t-1} as proxy instead. In this case, ρ changes sign and becomes significantly positive. This shows the sensitivity to the exact timing of the data. The third column results show that if leading rather than lagging the volatility proxy, it is γ^* rather than ρ that changes sign, and now becomes significantly negative. Together, the results show that inference on γ^* and ρ is delicate. Shifting the volatility proxy a single day in either direction makes either γ^* or ρ change sign and even turn significant in the opposite direction. From the last two lines, when γ^* changes sign, so does the Merton intercept, and the test rejects in the opposite direction. Even if the test should reject (as the results in the first column suggest), it is little consolation if it does so for the wrong reason, and the point estimate indicates that returns are too high rather than too low for equilibrium. Also, from the first line of the second column, the BNS intercept α^* changes sign and becomes significant in the opposite direction, even when γ and α do not, hence showing that in some cases the test directly on the intercept is not testing the Merton condition.

The last three columns of the table show the similar results for the case where the condition $\gamma^* = 0$ (risk is unpriced) is imposed in the estimation. The results confirm that ρ turns significantly positive when the lagged volatility proxy is used. Thus, inference on the leverage effect is highly sensitive to the timing of the volatility proxy, even if the analysis does not involve the risk-return tradeoff.[2]

Table 3 shows the results using VIX instead of realized volatility. Again, lagging the volatility proxy makes ρ turn significantly positive. The other cases have the expected pattern, with γ^* significantly positive and ρ significantly negative. With $\gamma^* = 0$ imposed, ρ turns significantly positive even when the volatility proxy is not lagged. This is consistent with the delicate relation between γ^* and ρ. When $\gamma^* = 0$ is imposed, ρ takes on part of the role of γ^* as market price of risk, and turns positive in the estimation.

Table 4 shows that the results from Table 2 based on realized volatility largely hold up when

[2] When the pricing of risk γ^* is not considered (last three columns of the table), the Merton test is also not reported (formally, it coincides with the test on the intercept α^* in the first row).

using instead the alternative staggered bipower variation measure that should avoid jumps and market microstructure noise. On the substantive side, the evidence is still against the Merton intercept condition. Nevertheless, the results show that the two volatility proxies (with and without jump correction) are not equivalent for the purposes of drawing economic inferences.

Table 5 show that the results from Table 3 based on VIX largely hold up when using instead implied volatility backed out from option prices. The main qualitative difference in inference is that the leverage effect disappears (indeed, ρ is significantly positive) for the contemporaneous (unlagged) volatility proxy IV_t, even allowing that risk is priced ($\gamma^* = 0$ is not imposed). Thus, smoothed VIX and direct option-based implied volatility proxies are also not equivalent for drawing economic inferences.

To investigate the robustness of the results, we conduct a bootstrap experiment. Following Hall (1992), studentized bootstrap confidence intervals generally work better than those based on the asymptotic theory. Originally introduced for i.i.d. observations by Efron (1979), the bootstrap also applies to Markov processes, see Horowitz (2003). Andrews (2005) shows that for stationary strong mixing Markov processes, the error in rejection probability of a parametric bootstrap t-test is essentially the same as that for the nonparametric i.i.d. bootstrap. Masuda (2004) shows in the context of Levy-driven Ornstein-Uhlenbeck processes that if the marginal distribution of the volatility process admits a finite absolute moment of some positive order, then the process is exponentially beta-mixing. We apply the parametric bootstrap to our stationary Markov model.

Table 6 reports bootstrap standard errors, and for comparison also includes the empirical point estimates and asymptotic standard errors (repeated from Tables 2 and 3). The bootstrap standard errors appear in square brackets and are based on 10,000 bootstrap samples. The parametric bootstrap is employed, i.e., the residuals are drawn from the assumed distribution. The residual formulas (using Euler approximation in the parametric Markov structure) for bootstrapping with and without $\rho = 0$ imposed are in Hubalek and Posedel (2010). Here, we assume the variance-gamma which is finite activity, in contrast to the IG case, even though this was not necessary for the estimation. The bootstrap standard errors in square brackets may be compared to the asymptotic standard errors in round parentheses in the line immediately above. In all cases, asymptotic and bootstrap standard errors are close, suggesting that the sample size is sufficiently large for the asymptotic distribution to be relevant. The biggest difference occurs for the standard error on the Merton intercept, possibly because this is based on the delta-rule, but the difference is not relevant for the qualitative inferences.

Since the higher order refinement of the distribution requires that a pivotal statistic be bootstrapped, we also consider p-values based on the bootstrap distribution of t-statistics, reported in round parentheses right next to the bootstrap standard errors, but only for those cases where testing for a zero value of the parameter in question is relevant. This is the case for gamma*, rho, CAPMalpha. Specifically, when drawing each of the 10,000 bootstrap samples, the relevant parameter is set equal to zero, all parameters (including that imposed equal to zero in drawing the sample) are estimated. The asymptotic standard error for the parameter in question is computed at the estimated parameter values, thus allowing the construction of the relevant t-statistic in each bootstrap sample. The reported p-values are calculated as the fractions of the resulting 10,000 bootstrap t-statistics exceeding the empirical t-statistic for the parameter in question. This procedure is repeated for each of the parameters for which a test of zero parameter value is of interest. Note that the bootstrap standard errors in square brackets are all obtained from the same set of 10,000 bootstrap samples, whereas each p-value in round parenthesis requires a fresh bootstrap sample, imposing the relevant zero restriction.

Finally, for robustness against possible departures from the assumed distribution, we also consider p-values constructed in a similar fashion, but using resampling with replacement from the empirical distribution of residuals, rather than drawing from the assumed distribution. These semiparametric bootstrap p-values (parametric with respect to the Markov structure, including how the null is imposed, nonparametric with respect to the residual distribution) are reported in curly braces, right next to the parametric bootstrap standard errors and p-values. We find that the semiparametric (resampling) p-values largely confirm the full parametric bootstrap p-values, but instead of 0% and 100% they are a little less extreme. This suggests that there is a certain deviation between the assumed distribution and the true data generating process, but that the difference is small enough to leave our empirical approach useful.

7 Conclusion

In this paper, we study the risk premium and leverage effect in the S&P500 market using the BNS model. The Merton (1973, 1980) asset pricing condition linking the conditional mean and conditional variance of discrete time returns is reinterpreted in terms of the continuous time model. Tests are performed on the risk-return relation, the leverage effect, and the overidentifying zero intercept restriction in the Merton equilibrium condition. Results are compared across alternative volatility proxies, in particular, realized volatility from high-frequency (5-minute) returns, implied Black-Scholes volatility backed out from observed option prices, model-free implied volatility (VIX), and staggered bipower variation. Our results are consistent with a positive risk-return relation and a significant leverage effect, whereas the Merton zero intercept condition is rejected. We also show that these inferences are sensitive to the exact timing of the chosen volatility proxy. Robustness of the conclusions is verified in bootstrap experiments.

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Table 1: Summary Statistics

Variable	Mean	Std. Deviation	Autocorr.
Return	0.1456	0.1611	−0.0064
Realized Volatility	0.1610	0.0745	0.7430
Staggered Bipower Variation	0.1539	0.0726	0.7782
VIX	0.1873	0.0628	0.9824
Implied Volatility	0.1647	0.0603	0.9432

Note: Summary statistics are reported for the daily returns and the alternative volatility proxies.

Table 2: Realized Volatility

Parameter	RV_t	RV_{t-1}	RV_{t+1}	RV_t	RV_{t-1}	RV_{t+1}
α^*	−22.01	2.411	109.9	53.33	53.33	53.33
	(3.907)	(0.8423)	(4.756)	(1.224)	(1.631)	(4.826)
	[0.7718]	[0.5245]	[0.5812]			
γ^*	2,906	1,964	−2,181	0	0	0
	(100.6)	(20.56)	(111.7)			
	[20.18]	[13.86]	[15.31]			
ρ	−11.48	2.126	−14.49	−4.922	6.559	−19.41
	(0.0227)	(0.1462)	(0.3237)	(0.1122)	(0.1494)	(0.4419)
λ	221.5	221.5	221.5	221.5	221.5	221.5
	(8.683)	(8.683)	(8.683)	(8.683)	(8.683)	(8.683)
a	16.23	16.23	16.23	16.23	16.23	16.23
	(0.9103)	(0.9104)	(0.9103)	(0.9103)	(0.9104)	(0.9103)
ν	0.4206	0.4208	0.4206	0.4206	0.4208	0.4206
	(0.0230)	(0.0230)	(0.0230)	(0.0230)	(0.0230)	(0.0230)
ζ	0.0259	0.0259	0.0259	0.0259	0.0259	0.0259
η	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016
α	−395.3	−77.07	1223			
	(35.92)	(6.724)	(116.9)			
α (ρ=0)	−77.48	−23.89	33.02			
	[6.597]	[2.038]	[2.790]			

Note: Estimation results based on the realized volatility proxy.

Table 3: Model-free VIX

Parameter	VIX_t	VIX_{t-1}	VIX_{t+1}	VIX_t	VIX_{t-1}	VIX_{t+1}
α^*	-36.41 (0.7522) [0.1471]	-24.35 (1.077) [0.1370]	-28.08 (25.53) [0.1399]	53.33 (4.175)	53.33 (4.602)	53.33 (10.62)
γ^*	$2,299$ (15.91) [3.743]	$1,991$ (22.65) [3.570]	$2,086$ (535.3) [3.622]	0	0	0
ρ	-12.54 (0.3126)	11.18 (0.3061)	-434.6 (0.2302)	122.6 (8.894)	135.2 (9.806)	-311.9 (22.63)
λ	8.495 (1.232)	8.489 (1.232)	8.493 (1.232)	8.495 (1.232)	8.489 (1.232)	8.493 (1.232)
a	49.78 (8.788)	49.74 (8.785)	49.72 (8.782)	49.78 (8.788)	49.74 (8.785)	49.72 (8.782)
ν	1.937 (0.3419)	1.935 (0.3418)	1.933 (0.3414)	1.937 (0.3419)	1.935 (0.3418)	1.933 (0.3414)
ζ	0.0389	0.0389	0.0389	0.0389	0.0389	0.0389
η	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
α	-7.782 (1.474)	-31.83 (5.786)	$-14,150$ (4,434)			
α (ρ=0)	-1.188 [0.1957]	-0.7725 [0.1270]	-0.8895 [0.1463]			

Note: Estimation results based on the VIX volatility proxy.

Table 4: Staggered Bipower Variation

Parameter	SBV_t	SBV_{t-1}	SBV_{t+1}	SBV_t	SBV_{t-1}	SBV_{t+1}
α^*	-21.40 (5.476) [0.6012]	-9.091 (0.6269) [0.5035]	145.7 (4.259) [0.7415]	53.33 (2.310)	53.33 (1.869)	53.33 (5.575)
γ^*	3,153 (148.1) [16.60]	2,634 (16.59) [13.99]	$-3,897$ (112.9) [20.36]	0	0	0
ρ	-22.77 (0.0271)	1.678 (0.2007)	-17.79 (0.4612)	-12.58 (0.2913)	10.182 (0.2358)	-30.38 (0.7029)
λ	154.9 (6.524)	154.9 (6.524)	154.8 (6.524)	154.9 (6.524)	154.9 (6.524)	154.8 (6.524)
a	19.02 (1.196)	19.02 (1.196)	19.02 (1.196)	19.02 (1.196)	19.02 (1.196)	19.02 (1.196)
ν	0.4507 (0.0279)	0.4509 (0.0280)	0.4508 (0.0279)	0.4507 (0.0279)	0.4509 (0.0280)	0.4508 (0.0279)
ζ	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237
η	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
α	$-1,217$ (124.9)	-72.86 (7.590)	2,103 (201.5)			
α (ρ=0)	-61.13 [5.670]	-35.61 [3.321]	115.7 [10.76]			

Note: Estimation results based on the staggered bipower variation volatility proxy.

Table 5: Implied Volatility

Parameter	IV_t	IV_{t-1}	IV_{t+1}	IV_t	IV_{t-1}	IV_{t+1}
α^*	-28.36 (0.2535) [0.2205]	-29.92 (0.2367) [0.2239]	-31.05 (17.28) [0.2265]	53.33 (2.490)	53.33 (2.412)	53.33 (8.048)
γ^*	2,655 (6.934) [6.166]	2,705 (6.567) [6.248]	2,742 (445.0) [6.310]	0	0	0
ρ	1.331 (0.2216)	-0.7600 (0.2201)	-179.0 (0.2510)	42.28 (1.626)	40.95 (1.575)	-136.7 (5.254)
λ	32.41 (2.488)	32.41 (2.488)	32.41 (2.488)	32.41 (2.488)	32.41 (2.488)	32.41 (2.488)
a	50.21 (4.737)	50.21 (4.737)	50.19 (4.737)	50.21 (4.737)	50.21 (4.737)	50.19 (4.736)
ν	1.545 (0.1457)	1.545 (0.1457)	1.544 (0.1456)	1.545 (0.1457)	1.545 (0.1457)	1.544 (0.1456)
ζ	0.0308	0.0308	0.0308	0.0308	0.0308	0.0308
η	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
α	-8.079 (1.019)	-3.686 (0.5138)	$-9,172$ (1,771)			
α (ρ=0)	-4.851 [0.5010]	-5.126 [0.5294]	-5.338 [0.5515]			

Note: Estimation results based on the implied volatility proxy.

Table 6: Bootstrap

Parameter	RV_t ($\rho = 0$)	RV_t ($\rho \neq 0$)	VIX_t ($\rho = 0$)	VIX_t ($\rho \neq 0$)
α^*	-22.01	-22.01	-36.41	-36.41
	(0.7718)	(3.907)	(0.1471)	(0.7522)
	[0.7657]	[3.920]	[0.1480]	[0.7629]
γ^*	$2,906$	$2,906$	$2,299$	$2,299$
	(20.18)	(100.6)	(3.743)	(15.91)
	[19.86](0.00%){0.00%}	[99.74](0.00%){0.01%}	[3.723](0.00%){0.00%}	[15.98](100.00%){0.00%}
ρ	$-$	-11.48	$-$	-12.54
		(0.0227)		(0.3126)
		[0.0225](100.00%){93.40%}		[0.3055](100.00%){98.69%}
λ	221.5	221.5	8.495	8.495
	(8.683)	(8.683)	(1.232)	(1.232)
	[8.608]	[8.675]	[1.254]	[1.237]
a	16.23	16.23	49.78	49.78
	(0.9103)	(0.9103)	(8.788)	(8.788)
	[0.9137]	[0.9079]	[9.115]	[9.078]
ν	0.4206	0.4206	1.937	1.937
	(0.0230)	(0.0230)	(0.3419)	(0.3419)
	[0.0236]	[0.0237]	[0.3503]	[0.3487]
α	-77.48	-395.3	-1.188	-7.782
	(6.597)	(35.92)	(0.1957)	(1.474)
	[2.744](100.00%){100.00%}	[30.88](99.87%){78.59%}	[0.1576](100.00%){100.00%}	[1.308](80.37%){69.54%}

Note: The table reports the results of parametric and nonparametric bootstrap experiments.

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