



SCHOOL OF ECONOMICS AND MANAGEMENT
FACULTY OF SOCIAL SCIENCES
AARHUS UNIVERSITY



CREATES
Center for Research in Econometric
Analysis of Time Series

CREATES Research Paper 2010-39

**The Role of Dynamic Specification in Forecasting
Volatility in the Presence of Jumps and Noisy High-
Frequency Data**

Rasmus Tangsgaard Varneskov

School of Economics and Management
Aarhus University
Bartholins Allé 10, Building 1322, DK-8000 Aarhus C
Denmark

The Role of Dynamic Specification in Forecasting Volatility in the Presence of Jumps and Noisy High-Frequency Data*

Rasmus Tangsgaard Varneskov[†]

Aarhus University

CREATES

2010-08-19

Abstract

This paper considers the performance of different long-memory dynamic models when forecasting volatility in the stock market using implied volatility as an exogenous variable in the information set. Observed volatility is separated into its continuous and jump components in a framework that allows for consistent estimation in the presence of market microstructure noise. A comparison between a class of HAR- and ARFIMA models is facilitated on the basis of out-of-sample forecasting performance. Implied volatility conveys incremental information about future volatility in both specifications, improving performance both in- and out-of-sample for all models. Furthermore, the ARFIMA class of models dominates the HAR specifications in terms of out-of-sample performance both with and without implied volatility in the information set. A vectorized ARFIMA (vecARFIMA) model is introduced to control for possible endogeneity issues. This model is compared to a vecHAR specification, re-enforcing the results from the single equation framework.

JEL classification: C14, C22, C32, C53, G10

Keywords: ARFIMA, HAR, Implied Volatility, Jumps, Market Microstructure Noise, VecARFIMA, Volatility Forecasting.

*I would especially like to thank Valeri Voev for his valuable time and useful considerations. I would also like to thank Bent Jesper Christensen and Morten Ørregaard Nielsen for useful comments. All errors are of course mine.

[†]School of Economics and Management, Aarhus University, 8000 Aarhus C, Denmark. Phone +45-2367-2009, email: rasmus.tangsgaard.varneskov@post.au.dk. Financial support from the Faculty of Social Sciences, Aarhus University, and from CREATES funded by the Danish National Research Foundation is gratefully acknowledged.

1 Introduction

Volatility modeling has been one of the most heavily researched topics of financial econometrics since the introduction of the ARCH and GARCH models by Engle (1982) and Bollerslev (1986), respectively. Since then, a lot of progress has been made by incorporating stylized facts about volatility such as the leverage effect, volatility clustering, and strong persistence into these models. Another important improvement has been the availability of high-frequency observations that has permitted more precise measures of volatility as input to these models. A topic that is still relevant to examine is which variables can be used together with different dynamic specifications to improve upon the forecast performance.

This paper considers the role of implied volatility together with two different dynamic specifications, the HAR model of Corsi (2009) and the ARFIMA model introduced in Hosking (1981), to investigate how these models perform when forecasting volatility out-of-sample in the stock market. The role of implied volatility with a class of HAR models has already been investigated in Busch, Christensen & Nielsen (2009). Hence, the main contributions of this paper come from extending their forecasting framework in three directions. First, the methodology applied to separate volatility and jumps is robust to the presence of both jumps and market microstructure noise, which allows for an investigation of the dynamic properties of both jumps and volatility. Secondly, their forecasting framework is extended to a class of single equation ARFIMA models that does not suffer from the limitations of a simple approximate long-memory structure. Finally, a vectorized ARFIMA model (vecARFIMA) is introduced as an alternative to the vecHAR model.

Recent advances in volatility forecasting such as Andersen, Bollerslev, Diebold & Labys (2003) and Chiriac & Voev (2009) show that simple time series models outperform the popular class of GARCH models when forecasting in both a univariate and a multivariate framework. Furthermore, Andersen, Bollerslev & Diebold (2007) show that a separation of the continuous and jump parts of realized volatility, and using both components in the information set, leads to significant gains when forecasting future volatility. The two components play distinctly different roles in forecasting since the two quantities possess just as different time series properties. The continuous part is highly persistent, while the jump component is less serially correlated. Estimation of volatility in the presence of both jumps and market microstructure noise is still an intensely researched field. Recent advances include a "Swap Variance" approach by

Jiang & Oomen (2008) and the modulated realized volatility approach by Podolskij & Vetter (2009a, 2009b), which is the methodology applied in this paper. This methodology offers a consistent estimator of volatility both when jumps are present, the modulated bipower variation estimator, and when jumps are absent, the modulated realized volatility estimator, together with a jump testing framework. This framework is applied to construct separate series of diffusive volatility and jump variation that are used as input in the dynamic models together with a series of implied volatility. Hence, the empirical analysis in this paper exploits the properties of volatility and jumps fully since both components are guaranteed to be estimated consistently.

Following Busch et al. (2009), implied volatility is added as an exogenous regressor to different specifications of both the HAR- and ARFIMA model to examine the effect of adding implied volatility in both frameworks, and to determine how well these two different dynamic specifications fare when forecasting modulated realized volatility out-of-sample. These two models are supposed to capture the long-memory properties of volatility, making it relevant to determine how well the simple approximate long-memory HAR model compares to the more complicated ARFIMA dynamics.

The results from the single equation forecasting framework are clear-cut. Implied volatility improves model performance both in- and out-of-sample for all HAR and ARFIMA specifications. Implied volatility conveys incremental information about future volatility relative to the continuous and jump components of volatility. This adds to the work of Busch et al. (2009) and Christensen & Prabhala (1998) by showing that the incremental information of implied volatility carry over to a highly non-linear long-memory model. Another important finding in this paper is that the ARFIMA models dominate the HAR specifications when forecasting volatility over the course of a month both with and without implied volatility in the information set. Implied volatility does not completely subsume the explanatory effect of the other variables in the ARFIMA framework, showing the importance of allowing for a richer set of dynamics.

The conclusions are supported by the results of the simultaneous equation vecARFIMA and vecHAR models. Implied volatility plays a highly significant role in forecasting future volatility. Furthermore, the dynamics of the continuous volatility component is better captured by the ARFIMA structure compared to the HAR specification. These simultaneous equation systems solve possible misspecifications due to endogeneity issues in the single equation framework. Hence, the conclusions from these models add robustness to the results of the single equation framework.

The outline of the paper is as follows. In the next section, the methodology of Podolskij & Vetter (2009a, 2009b) for noise-robust separation of the continuous and jump volatility components is described. Section 3 discusses VIX as a measure of implied volatility. The data is described in Section 4. Section 5 presents the different dynamic specifications used in the paper. Section 6 presents and discusses the results of the empirical analysis, and Section 7 concludes.

2 Basic Notation and Definitions

The observable logarithmic asset price is assumed to follow an Itô diffusion process which can be decomposed into three components. The first component is the efficient price process

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t \sigma_s dW_s \quad (1)$$

where W denotes the driving standard Brownian Motion, a_s is a locally bounded predictable drift term, and $\sigma_s > 0$ is a cadlag volatility process. The efficient price is thus a Brownian semimartingale, and it is defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$.

The traditional realized variance estimator is shown by Merton (1980) to perfectly estimate integrated volatility, $\Omega = \int_0^1 \sigma_s^2 ds$, which is the quantity of interest, in a setting where prices are observed continuously and without measurement error. However, when the efficient price process is no longer directly observable due to market microstructure noise, Hansen & Lunde (2006) show that the realized variance estimator is severely biased even in a simple setting with an i.i.d. noise component. Furthermore, the price is not observed continuously, but only at the times the asset is traded.

Hence, the price Y is defined to be observed at a grid $t_i = i/n$, $i = 0, \dots, n$ for each day where the increments are assumed to be evenly spaced. Furthermore, Y contains a noise process, which constitutes the second component of the observable logarithmic asset price. Hence, Y can be written

$$Y_{i/n} = X_{i/n} + U_i \quad (2)$$

where X is the efficient price process in (1), and $(U_i)_{0 \leq i \leq n}$ is assumed to be an i.i.d. noise process which can be defined following the procedure of Podolskij & Vetter

(2009b). It is assumed that X and U are independent and

$$E(U_i) = 0, \quad E(U_i^2) = \omega^2. \quad (3)$$

Motivation for the final component is given in Barndorff-Nielsen & Shephard (2006), who show that realized variance can be separated into a continuous and a jump part. Following the methodology of Podolskij & Vetter (2009b), the model considered when both market microstructure noise and jumps are present is defined as

$$Z = Y + J \quad (4)$$

where Y is the noisy diffusion process in (2) and J is assumed to be a finite activity jump process. An example of a finite activity jump process is the compound Poisson process.

As shown in Barndorff-Nielsen & Shephard (2006), it is critically important to test for the presence of jumps when making the separation of the continuous and jump components, which will be used in the dynamic analysis in later sections. Hence, the following four subsections review estimation both with and without jumps in the presence of market microstructure noise together with a testing framework for jumps based on Podolskij & Vetter (2009a, 2009b).

2.1 The General Class of Estimators

The general class of estimators is based on a pre-averaged generalization of the realized variance- and the bipower variation framework. It can be used to robustly estimate arbitrary powers of integrated volatility in the presence of market microstructure noise and a finite activity jump process. The class of modulated bipower variation statistics is represented as

$$MBV(Y, r, l)_n = n^{(r+l)/4-1/2} \sum_{m=1}^M |\bar{Y}_m^{(K)}|^r |\bar{Y}_{m+1}^{(K)}|^l, \quad r, l \geq 0, \quad (5)$$

$$\bar{Y}_m^{(K)} = \frac{1}{n/M - K + 1} \sum_{i=(m-1)n/M}^{mn/M-K} (Y_{(i+K)/n} - Y_{i/n}), \quad (6)$$

where

$$K = c_1 n^{1/2}, \quad M = \frac{n^{1/2}}{c_1 c_2} \quad (7)$$

for some constants c_1 and c_2 that have been chosen to minimize the variance of the estimator

$$c_1 = \sqrt{\frac{18}{(c_2 - 1)(4 - c_2)} \cdot \frac{\omega^2}{\Omega}}, \quad c_2 = \frac{8}{5} \quad (8)$$

where Ω is the integrated volatility from the process in (1)¹. The intuition behind the pre-averaged quantities $\bar{Y}_m^{(K)}$ is that the stochastic order of the noise component in (2) is smaller than that of the diffusion process X , and hence, the variance of the noise component is averaged out. This means that $\bar{Y}_m^{(K)}$ behaves like an increment in X and, as a result, it contains information about Ω . However, the class of statistics $MBV(Y, r, l)_n$ has a bias that must be corrected.

2.2 Modulated Realized Volatility

Assuming no jumps in the underlying diffusion process with noise in (2), the modulated realized volatility estimator given by Podolskij & Vetter (2009b) consistently estimates volatility. It is expressed as

$$MRV(Y)_n := \frac{c_1 c_2 MBV(Y, 2, 0)_n - \nu_2 \hat{\omega}^2}{\nu_1} \xrightarrow{P} \int_0^1 \sigma_u^2 du \quad (9)$$

where the constants ν_1 and ν_2 are given by

$$\nu_1 = \frac{c_1 \max[3c_2 - 4 + (2 - c_2)^3, 0]}{3(c_2 - 1)^2}, \quad \nu_2 = \frac{2 \min[c_2 - 1, 1]}{c_1 (c_2 - 1)^2}. \quad (10)$$

The noise variance ω^2 can be found consistently by an estimator proposed by Zhang, Mykland & Aït-Sahalia (2005)

$$\hat{\omega}^2 = \frac{1}{2n} \sum_{i=1}^n |Y_{i/n} - Y_{(i-1)/n}|^2. \quad (11)$$

When c_1 and c_2 are chosen optimally as in (8), the maximum and the minimum of the two functions in (10) will be determined straightforwardly since c_2 is constant. The modulated realized volatility estimator avoids the bias of the realized variance estimator by pre-averaging and bias correcting.

¹This complicates estimation since the quantity of interest has to be used as a part of the estimator. A solution is to estimate $\sqrt{\Omega}$ and use this estimate as input in the modulated bipower variation class of statistics. This is discussed in Section 4.

2.3 Modulated Bipower Variation

According to Cont & Tankov (2004), a striking feature from observing price movements for the stock market is that almost a third of the downward movements in the stock price can be characterized as being discontinuous. Since one of the properties of the Brownian motion is continuity, improvements can be made by including jump processes when modeling logarithmic asset prices.

Assuming that there is a finite activity jump process in addition to the noisy diffusion model as in (4), Podolskij & Vetter (2009b) propose the modulated bipower variation estimator

$$MBV(Z)_n := \frac{(c_1 c_2 / \mu_1^2) MBV(Z, 1, 1)_n - \nu_2 \hat{\omega}^2}{\nu_1} \xrightarrow{P} \int_0^1 \sigma_u^2 du \quad (12)$$

where

$$\mu_r = 2^{r/2} \frac{\Gamma(\frac{1}{2}(r+1))}{\Gamma(\frac{1}{2})} \quad (13)$$

which is robust even in the presence of market microstructure noise and a finite activity jump process. The set (c_1, c_2, ν_1, ν_2) is the same as in (8) and (10), and the estimator of $\hat{\omega}$ is the same as (11) since it is robust to jumps. It is interesting to note that the presence of jumps invalidates the consistency of the estimator $MRV(Z)_n$.

2.4 Testing for the Presence of Jumps in a Noisy Diffusion Setting

In order to separate the continuous and jump components of $MRV(Y)_n$, a test for the significance of jumps must be defined. Podolskij & Vetter (2009a) propose a test for jumps for more generalized versions of the modulated bipower variation class of statistics. They allow $\bar{Y}_m^{(K)}$ in (6) to be weighted using a simple kernel function $g(x) = \min[x, 1-x]$ for $x \in (0, 1)$.

However, in order to draw inference on the quadratic jump component of the $MRV(Y)_n$, both $MBV(Y, 2, 0)_n$ and $MBV(Z, 1, 1)_n$ must be estimated using equal weights as in Podolskij & Vetter (2009b). These estimators are consistent in the generalized jump testing setting whenever $l+r$ is an even number, and consequently, these estimators can be used to test the null hypothesis of no jumps. The distribution of the test can

be written as

$$n^{1/4}(MBV(Y, 2, 0)_n - \mu_1^{-2}MBV(Z, 1, 1)_n) \xrightarrow{L} MN(0, \tau^2). \quad (14)$$

This distribution can be used to derive a test for jumps in the underlying noisy diffusion process Y in (2). τ^2 is estimated robustly to both jumps and market microstructure noise as

$$\hat{\tau}_n^2 = \hat{w}_{11} - 2\mu_1^{-2}\hat{w}_{12} + \mu_1^{-4}\hat{w}_{22} \quad (15)$$

where an estimator of each \hat{w}_{pq} is based on the pre-averaging methodology and is given in Podolskij & Vetter (2009a), and μ_1 is given in (13). A test statistic follows from stable convergence in law.

$$S^n = n^{\frac{1}{4}} \frac{MBV(Y, 2, 0)_n - \mu_1^{-2}MBV(Z, 1, 1)_n}{\hat{\tau}_n} \xrightarrow{L} MN(0, 1). \quad (16)$$

Under the alternative hypothesis, $(MBV(Y, 2, 0)_n - \mu_1^{-2}MBV(Z, 1, 1)_n)$ converges to a strictly positive quantity because the estimated jump component is quadratic. Since $\hat{\tau}_n^2$ robustly estimates τ^2 in the presence of market microstructure noise and jumps, S^n tends to infinity as the realizations of Z have discontinuous paths. However, S^n may be negative in finite samples due to sample variation. This necessitates the need for a notion of a significant jump component.

The significant jump component of $MRV(Y)_n$ can be found as

$$J_n = I_{\{S^n > \Phi_{1-\alpha}^{-1}\}}(MRV(Y)_n - MBV(Z)_n) \quad (17)$$

where I_A is an indicator for the event A , $\Phi_{1-\alpha}^{-1}$ defines the $(1 - \alpha)$ quantile in the standard normal distribution, and α is the significance level (chosen to be 0.05). The continuous volatility component can similarly be defined as

$$C_n = MRV(Y)_n - J_n. \quad (18)$$

Since the significance level is chosen to be below 1/2, J_n is non-negative by construction. This follows despite neither $MRV(Y)_n$ in (9) nor $MBV(Z)_n$ in (12) are guaranteed to give a positive estimate of integrated volatility, and as a result C_n is not guaranteed to be positive. This is a huge drawback of the pre-averaging methodology. However, for the sample period of 1997-2007, C_n is estimated to be positive on every

single trading day.²

Using the framework in Section 2, it is possible to construct a time series of period-by-period MRV_t , J_t and C_t for $t = 1, \dots, T$ using 11 years of high-frequency data. These time series can be used as input in the dynamic models specified in Section 5 of the paper.

3 VIX

The important role of implied volatility in forecasting realized volatility over the span of one month is shown in Busch et al. (2009). They use implied volatility derived from future options on \$/DM spot exchange rates, the S&P 500, and T-bonds using the traditional Black & Scholes (1973) and Merton (1973) framework. These parametric estimates have been shown to dominate model-free estimates by Andersen & Bondarenko (2007), when they are used for forecasting.

This paper considers the role of implied volatility by using the VIX, which was introduced by the Chicago Board Options Exchange in 1993. It is a measure of the implied volatility on S&P 500 index options, and According to CBOE (2003) it has quickly become a benchmark of stock market volatility. The VIX reflects the markets expectations of volatility over the next 30 calendar days, making it a forward looking measure, and for this reason it is well-suited for forecasting.

The VIX is a nice measure of implied volatility for this applications, since the empirical analysis is based on data on the Standards & Poor's Depository Receipts (SPY), which is an exchange-traded fund that tracks the performance of the S&P 500. Hence, it is assumed that the SPY fund has the same implied volatility as the index it tracks.

In 2003 there were significant changes in the way the VIX is calculated. The most noteworthy change is that the new VIX is found using prices of stock index options with a variety of strike prices, and no longer just at-the-money strikes. Furthermore, the original VIX was calculated using the Black-Scholes option pricing formula, whereas the new VIX is independent of any model. This makes it more robust since it uses information from option prices over the whole volatility skew, and parametric misspecification is avoided as well. Historical data on the VIX has been calculated for each trading day dating back to 1986. This means that a similar time series of implied volatility can be defined as IV_t where $t = 1, \dots, T$, for the sample 1997-2007.

²But using data from 1995 this is not the case. This can be attributed to too few observations. There was as little as 30 observations on some trading days.

4 The Data

The data consists of high-frequency observations of trades on the SPY fund for the eleven year period 1997-2007 during the trading hours from 9.30 a.m. until 16.00 p.m.³ For the period 1997-2002 the data is sampled from the American Stock Exchange (AMEX), while the remaining five years of data are sampled from the Pacific Exchange (PACIF). This means that there are $T = 2767$ days in the sample. Modulated realized volatility and modulated bipower variation are computed using tick-by-tick data and the pre-averaging methodology described in section 2 for each day in the sample. As mentioned in section 2.1, there is an issue with the integrated volatility used in the constant c_1 . To solve this issue, a GMM type estimator is applied with two iterations. In the first step, the realized variance estimator with subsampling and averaging as demonstrated by Zhang et al. (2005) is used to estimate the integrated volatility as input. In the second step, the integrated volatility used as input is estimated using the modulated bipower variation estimator from the first step to ensure robustness even in the presence of jumps. A significance level of $\alpha = 0.05$ is chosen to detect jumps in volatility and to construct the separate series of C and J . Significant jumps are observed on 62.1% of the days in the sample. Thus, jumps are non-negligible for the SPY fund. Similarly, a daily series of closing values on the new VIX are found on the CBOE homepage.

The time series properties of the data are examined for 2002, the median year in the sample. Figure 1 shows the autocorrelation for modulated realized volatility, implied volatility and the continuous and jump components separately. As expected, modulated realized volatility and its continuous component have very similar autocorrelation structures. Both are highly persistent and their autocorrelation does not die out even after 20 lags. This illustrates the need for a long-memory dynamic specification to capture the persistence of the two variables. Implied volatility is seen to be extremely persistent. One explanation for this is that daily observations of implied volatility are measures of overlapping intervals. Thus, these expectations are unlikely to change dramatically on a daily basis. Lastly, the jump component lacks persistence as the autocorrelation dies out quickly. Summary statistics of daily estimates of the four quantities using 2002 data are seen in Table A1 of Appendix A.1. Similar to the results of Andersen, Bollerslev, Diebold & Ebens (2001), modulated realized volatility and its continuous and jump components are extremely right skewed and leptokurtic.

³I would like to thank Valeri Voev for providing me with cleaned data.

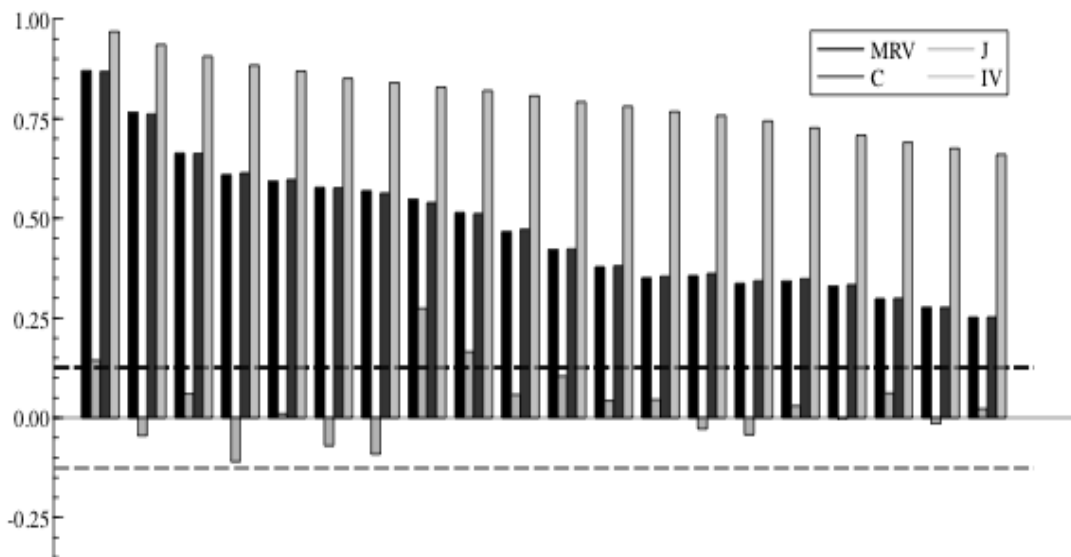


Figure 1: The autocorrelation function of the four quantities MRV , C , J , and IV using daily data from the median year in the sample, 2002.

The main emphasis in this paper is on forecasting monthly volatility. The series of modulated realized volatility, and its continuous and jump components are aggregated over the course of a month, which is defined as $h = 21$ (trading) days.⁴ The observed value of the VIX is selected for the last day of the previous month, so that the VIX and the aggregated data represent volatility for the same month. The first month of the sample is disregarded since the observed value of the VIX is missing on January 31st 1997. Starting from February 1997 there are $n = 130$ non-overlapping intervals in the data.⁵ The first observation is used as initialization and the last observation is used for forecasting, as explained in Section 6. This effectively leaves $n = 128$ observations for estimation of the dynamic models. Summary statistics of monthly aggregated modulated realized volatility, its continuous and jumps components, and implied volatility is shown in Table A1 of Appendix A.1 for the full sample. Similar to the daily estimates, the four variables are right skewed and exhibit excess kurtosis, but the skewness and kurtosis is less pronounced for implied volatility.

The time series properties of the monthly data are examined in Figure 2, which displays the autocorrelation structure and the monthly series themselves. Similar to the data

⁴Note that $h = 21$ is chosen as a convention for a month since the most frequent number of trading days in a year is 252, and $252/12 = 21$.

⁵Non-overlapping intervals were shown by Christensen & Prabhala (1998) to mitigate problems with serial correlation.

on a daily frequency, both modulated realized volatility and its continuous part are highly persistent, and implied volatility is the most persistent variable.

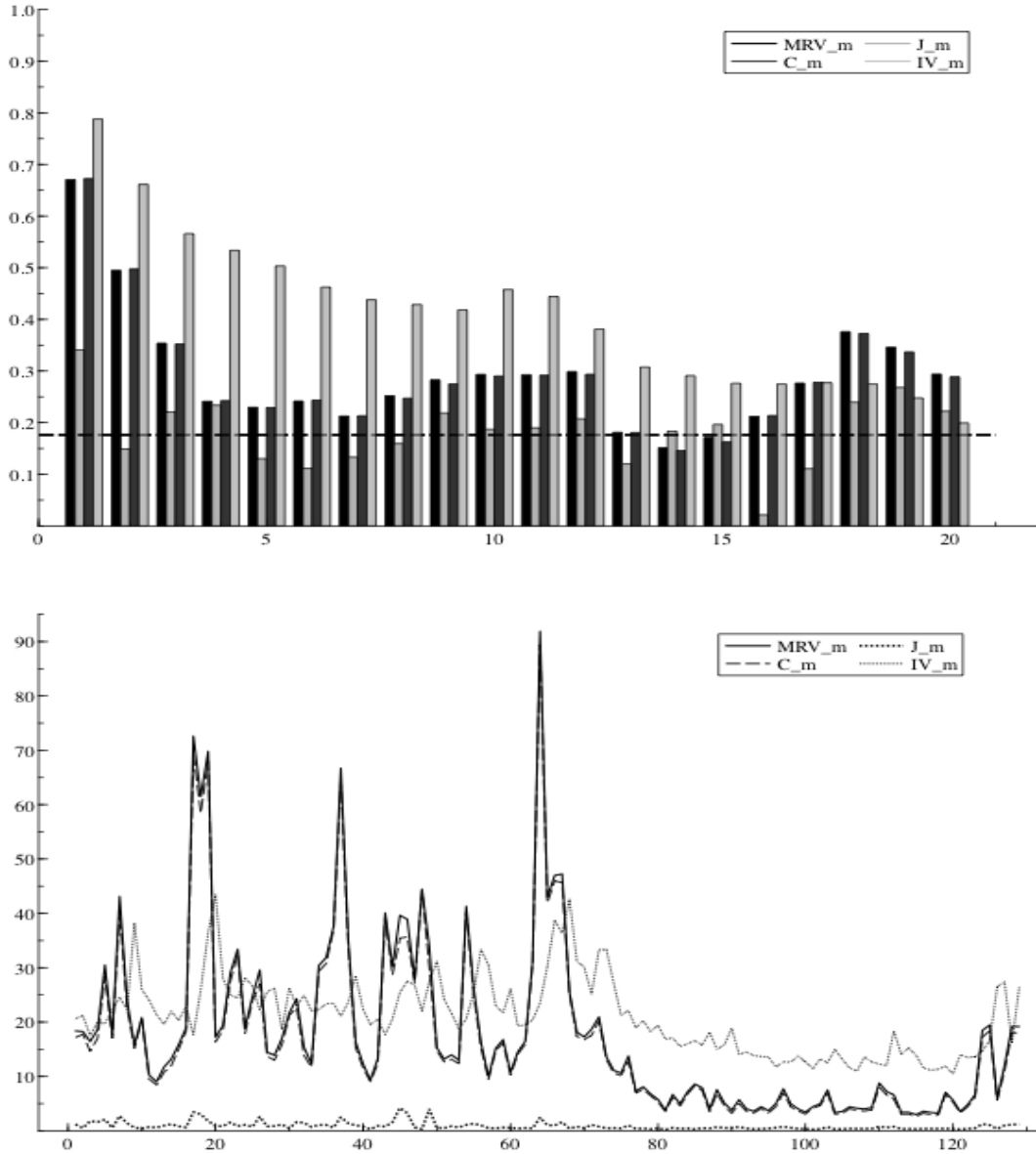


Figure 2: The autocorrelation function and a graphical representation of the four time series MRV , C , J , and IV using aggregated monthly data for the full sample.

Compared to data on a daily frequency, it looks as if aggregation of the jump process has added memory to the series. However, Chambers (1998) shows that the variables retain the same order of integration after temporal aggregation both in a discrete and continuous time setting. Table A.2 of Appendix A.1 shows the Gaussian semiparametric estimates of the order of integration based on Robinson (1995) of both the data

from 2002 and the full sample of monthly data. When considering the evidence on jump process, the order of integration seems to have increased. However, Chambers (1998) explains differences by small sample variation and sensitivity to the choice of bandwidth parameter.

As seen in the graphical depiction of the time series of the four variables, the jump process has a different variation pattern compared to the three other variables. Modulated realized volatility and its continuous component are closely linked, and implied volatility is similar to the two series, but its variation pattern does not exhibit the same degree of fluctuation. The jump component accounts for a numerically small part of the monthly volatility, and its variation pattern does not mimic that of the other variables. Due to the different time series properties, and the different variation patterns, it is relevant to specify forecasting models for the continuous and jump components separately.

5 Long-Memory Specifications

As seen in the data section, one of the main properties of volatility is its long memory. Two main types of long-memory models are presented in this paper. Firstly, the methodology of Busch et al. (2009) is followed. They use different specifications of the heterogeneous autoregressive (HAR) model of Corsi (2009), which is a simple approximate long-memory model. To allow for a richer set of dynamics, the methodology is extended to the ARFIMA model presented in Hosking (1981).

5.1 HAR Model for Modulated Realized Volatility

Persistence of volatility is not captured by standard GARCH models, and according to Corsi (2009) there are some difficulties using fractionally integrated long-memory models since they lack clear economic intuition, and they are non-trivial to estimate. This necessitates a need for a simpler model that captures much of the same dynamics and performs well when forecasting out of sample. The idea behind the HAR model is to place more weight on recent estimates of volatility than those from the distant past in a simple regression based setting.

Setting up the traditional model includes defining the term

$$MRV_{t-h,t} = \frac{1}{h} (MRV_t + MRV_{t-1} + \dots + MRV_{t-h+1}) \quad (19)$$

and using it to specify HAR model as

$$MRV_{t+1} = c + \beta_1 MRV_t + \beta_2 MRV_{t-5,t} + \beta_3 MRV_{t-21,t} + v_{t+1}, \quad t = 1, \dots, T \quad (20)$$

which is used to estimate one-step-ahead volatility. The term $MRV_{t,t-21}$ contains information about modulated realized volatility covering the last month, while $MRV_{t,t-5}$ and MRV_t are containing information about modulated realized volatility over the span of a week and a day, respectively.

A nice feature of the HAR model is that it allows extensions in a number of different ways. The first extension considered by Busch et al. (2009) is to replace the one-step-ahead dependent variable with a monthly modulated realized volatility term

$$MRV_{t,t+h} = \frac{21}{h} (MRV_{t+1} + MRV_{t+2} + \dots + MRV_{t+h}) \quad (21)$$

which is computed using non-overlapping time intervals in order to mitigate problems with serial correlation in the error terms.⁶ The model is denoted HAR-RV following the notation of Busch et al. (2009). A second extension is made by splitting MRV_t into C_t and J_t , and adding these as separate regressors instead of MRV_t . This model is similarly denoted HAR-RV-CJ. These two components are found using the framework in section 2 and are aggregated similarly to (21) when used as regressors. The third extension is to add IV_t as an additional regressor and to abbreviate the model HAR-IV-CJIV.

The HAR-RV-CJIV model is a Mincer & Zarnowitz (1969) type regression, and can be written as

$$MRV_{t,t+21} = \alpha + \gamma_1 x_t + \gamma_2 x_{t-5,t} + \gamma_3 x_{t-21,t} + \beta IV_t + \epsilon_{t,t+21}, \quad t = 21, 42, \dots, 21n \quad (22)$$

where $x_{t-h,t}$ is either the variable $MRV_{t-h,t}$ or the vector of separated components $(C_{t-h,t}, J_{t-h,t})$, and n is the number of months. The impact of the different explanatory variables is found by imposing exclusion restrictions on the parameters.

⁶Note that instead of using the average, the term is scaled to fit the monthly implied volatility measure. The lagged terms defined for the traditional model in (19) are scaled similarly. This has no effect on statistical inference, only for the numerical size of the coefficients.

5.2 HAR Model for the Continuous and Jump Components

The regressand of the HAR-RV-CJIV model in (22) is now split into its continuous and jump components, $C_{t,t+21}$ and $J_{t,t+21}$ respectively. It is obvious from looking at the data in section 4 that these two components exhibit different time series properties. Furthermore, according to Busch et al. (2009), the adjusted R^2 is much smaller when modeling jumps compared to its continuous counterpart, and the error terms exhibit strong serial correlation indicating a misspecified model. Hence, it is interesting to conduct the forecasting analysis of the continuous and jump components separately and compare the results with similar ARFIMA specifications. Given the different time series properties of the two components, their forecasts and model specifications are expected to yield different results.

A HAR-C-CJIV model is defined for the continuous component as

$$C_{t,t+21} = \alpha + \gamma_1 x_t + \gamma_2 x_{t-5,t} + \gamma_3 x_{t-21,t} + \beta IV_t + \epsilon_{t,t+21}, \quad t = 21, 42, \dots, 21n \quad (23)$$

and similarly for the jump component a HAR-J-CJIV model is defined as

$$J_{t,t+21} = \alpha + \gamma_1 x_t + \gamma_2 x_{t-5,t} + \gamma_3 x_{t-21,t} + \beta IV_t + \epsilon_{t,t+21}, \quad t = 21, 42, \dots, 21n \quad (24)$$

where, for both specifications, $x_{t-h,t}$ now contains either $C_{t-h,t}$ for the HAR-C-CJIV models or $J_{t-h,t}$ for the HAR-J-CJIV models, or the vector $(C_{t-h,t}, J_{t-h,t})$.

5.3 The Vector Heterogeneous Autoregressive (vecHAR) Model

The main contribution of Busch et al. (2009) is the specification of a simultaneous equation system for a joint analysis of C , J and IV . This system is denoted as the vecHAR model in this paper following the authors' abbreviation. Several factors necessitate this model. Firstly, the regression equations in (23) and (24) are not independent. Secondly, IV may contain measurement errors from non-synchronous option prices, and even though the new VIX is independent of model, the new measure may still be misspecified. Furthermore, the new VIX is a measure of implied volatility on the S&P 500. Thus, there can be measurement issues when it acts as the implied volatility on the SPY fund. These problems can generate correlation between the regressors and the error terms in (23) and (24) and thereby create an endogeneity problem.

The vecHAR model is proposed as a solution to these problems. It is defined as

$$\begin{pmatrix} 1 & 0 & -\beta_1 \\ 0 & 1 & -\beta_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{t,t+21} \\ J_{t,t+21} \\ IV_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} A_{11m} & A_{12m} & 0 \\ A_{21m} & A_{22m} & 0 \\ A_{31m} & A_{32m} & A_{33m} \end{pmatrix} \begin{pmatrix} C_{t-21,t} \\ J_{t-21,t} \\ IV_{t-1} \end{pmatrix} \quad (25)$$

$$+ \begin{pmatrix} A_{11w} & A_{12w} \\ A_{21w} & A_{22w} \\ A_{31w} & A_{32w} \end{pmatrix} \begin{pmatrix} C_{t-5,t} \\ J_{t-5,t} \end{pmatrix} + \begin{pmatrix} A_{11d} & A_{12d} \\ A_{21d} & A_{22d} \\ A_{31d} & A_{32d} \end{pmatrix} \begin{pmatrix} C_t \\ J_t \end{pmatrix} + \begin{pmatrix} \epsilon_{t,t+21}^1 \\ \epsilon_{t,t+21}^2 \\ \epsilon_{t,t+21}^3 \end{pmatrix}.$$

Equation (23) and (24) for forecasting the continuous and jump components, respectively, are included together with a third equation that endogenizes IV . This system exhibits two sources of simultaneity. First, the dependence of $C_{t,t+21}$ and $J_{t,t+21}$ on the endogenous variable is accommodated through β_1 and β_2 . Secondly, the error terms may be contemporaneously correlated. The third equation reflects the fact that the volatility over the past month may affect the IV estimate over the next month. Furthermore, the inclusion of IV_{t-1} is similar to using it as an instrument for IV_t . However, defining a simultaneous equation system is more general and efficient than simply using instrumental variable estimation.

5.4 ARFIMA Model for Modulated Realized Volatility

A problem with the HAR model is that it is only an approximate long-memory model, and as a result it might not be able to capture the dynamics of volatility properly. As seen in Figure 2 and Table A.2 of Appendix A.2, the diffusive volatility exhibit persistence both before and after being aggregated, while the jump component has much shorter memory. Both these series can be modeled by a class of ARFIMA models, since these can accommodate different degrees of fractionally integrated processes. The general class of ARFIMA(p, d, q) models are defined in Doornik & Ooms (2004) for the univariate case as

$$\Phi(L)(1-L)^d(y_t - \mu_t) = \Theta(L)\epsilon_t \quad t = 1, \dots, T \quad (26)$$

where $\Phi(L) = (1 - \phi_1 L - \dots - \phi_p L^p)$ and $\Theta(L) = (1 - \theta_1 L - \dots - \theta_q L^q)$ are the autoregressive and moving average lag polynomials with order p and q respectively, the error term is assumed to be normally distributed $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, μ_t is a function of a constant and other exogenous variables to be specified, and $(1-L)^d$ is the fractional

difference factor

$$(1 - L)^d = \sum_{j=0}^{\infty} \delta_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-L)^j \quad (27)$$

where d is a real number. The ARMA-part is assumed to be stationary and invertible. Furthermore, assuming $\Phi(z) = 0$ and $\theta(z) = 0$ do not have any common roots, then $(y_t - \mu_t)$ is said to be integrated of order d . The properties of the series $(y_t - \mu_t)$ hinge crucially on the value of d . As shown by Hosking (1981), it is covariance stationary if $-0.5 < d < 0.5$ with long memory if $d > 0$. When $0 < d < 0.5$ the series exhibits a hyperbolic decay, and it is thus characterized by long-term persistence. If $-0.5 < d < 0$ the process is said to have intermediate-memory and the inverse autocorrelations decay hyperbolically towards zero. For the special case of $d = 0$, the ARFIMA(p, d, q) model is equivalent to a ARMA(p, q) model since the fractional difference factor is negligible.

The empirical application in this paper is based on an ARFIMA($1, d, 1$) specification. This limits the number of parameters to be estimated which is especially important for the vecARFIMA model discussed in Section 5.6. Furthermore, as Chiriac & Voev (2009) argue, the reason to consider this restricted specification is that highly parameterized models lead to poor out-of-sample forecasts. Hence, the ARFIMA($1, d, 1$) model remains simple and is still able to capture the persistence required of a long-memory model. For this application it is used to capture the dynamics of (21) using the following specification

$$(1 - \phi L)(1 - L)^d(MRV_{t,t+21} - \alpha - \beta IV_t) = (1 - \theta L)\epsilon_t \quad t = 21, 42, \dots, 21n \quad (28)$$

where IV_t is added to the model as an exogenous variable to resemble its role as a regressor in the HAR specification in (22). The model is abbreviated ARFIMA-IV mimicking the HAR framework. The impact of implied volatility is found by using exclusion restrictions on β .

5.5 ARFIMA Model the Continuous and Jump Components

Utilizing the same arguments as in Section (5.2), modulated realized volatility is split into a continuous and a jump component. Both are modeled using long-memory specifications since the aggregation of the jump component has changed its dynamics. Even though the series of aggregated jumps exhibit longer memory, Figure 2 still shows that the two variables have different variation patterns and correlation structures,

making the separation relevant. The ARFIMA(1, d , 1) model, where these components are separated, is supposed to be an analogue of the HAR models in (23) and (24). The model for the continuous part can be written as

$$(1 - \phi L)(1 - L)^d(C_{t,t+21} - \alpha - \beta IV_t - \gamma J_{t-21,t}) = (1 - \theta L)\epsilon_t \quad t = 21, 42, \dots, 21n \quad (29)$$

and similarly for the jump component

$$(1 - \phi L)(1 - L)^d(J_{t,t+21} - \alpha - \beta IV_t - \gamma C_{t-21,t}) = (1 - \theta L)\epsilon_t \quad t = 21, 42, \dots, 21n \quad (30)$$

where lagged variables of the jump or the continuous component are added as an exogenous variable to the dynamics of the other component. These models are denoted as ARFIMA-C-IV and ARFIMA-J-IV respectively. The idea behind adding these lagged exogenous variables is to create a dynamic structure similar to the HAR framework, where the highest weight is placed on the most recent observations of the lagged variable and then decaying with time. Exclusion restrictions are used to investigate the significance of adding exogenous variables to the ARFIMA(1, d , 1) structure, and their impact on model specifications.

5.6 The Vector Autoregressive Fractional Integrated Moving Average (vecARFIMA) Model

The motivation for the vecARFIMA model is the same as for the vecHAR model, since the reasons to suspect the models in (29) and (30) of having endogeneity problems are the same as in the HAR framework. In addition to the endogeneity issue, the motivation of the vecARFIMA model is to fully capture the persistence in the data. Hence, the main contribution of this paper is to formulate a similar vectorized model that solves the endogeneity problems while not being limited by a simple approximate long-memory structure. The vecARFIMA model allows for dependency between C and J and possible measurement issues in IV , while capturing the long-memory properties of volatility. The unrestricted vecARFIMA is defined as

$$\begin{pmatrix} (1 - \phi_1 L) & 0 & 0 \\ 0 & (1 - \phi_2 L) & 0 \\ 0 & 0 & (1 - \phi_3 L) \end{pmatrix} \times \begin{pmatrix} (1 - L)^{d_1} & 0 & 0 \\ 0 & (1 - L)^{d_2} & 0 \\ 0 & 0 & (1 - L)^{d_3} \end{pmatrix} \quad (31)$$

$$\begin{aligned}
& \times \left[\begin{pmatrix} 1 & 0 & -\beta_1 \\ 0 & 1 & -\beta_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{t,t+21} \\ J_{t,t+21} \\ IV_t \end{pmatrix} - \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} - \begin{pmatrix} 0 & A_{12m} \\ A_{21m} & 0 \\ A_{31m} & A_{32m} \end{pmatrix} \begin{pmatrix} C_{t-21,t} \\ J_{t-21,t} \end{pmatrix} \right] \\
& = \begin{pmatrix} (1 - \theta_1 L) & 0 & 0 \\ 0 & (1 - \theta_2 L) & 0 \\ 0 & 0 & (1 - \theta_3 L) \end{pmatrix} \begin{pmatrix} \epsilon_{t,t+21}^1 \\ \epsilon_{t,t+21}^2 \\ \epsilon_{t,t+21}^3 \end{pmatrix}
\end{aligned}$$

where the interpretation of the simultaneous equation system is similar to that of the vecHAR model. The forecasting equations (29) and (30) are included in the model, where their dependency on the endogenous variable is accommodated through β_1 and β_2 , together with an equation that captures the dynamic long-memory structure of IV_t , which is seen in figure 2 to possess an extreme degree of persistence. The error terms may still be contemporaneously correlated. The vecARFIMA model solves the endogeneity problem by allowing the endogenous variable IV_t to be dependent on lagged values of itself, C , and J through a long-memory structure. The properties of this system are examined by using exclusion restrictions on the elements of the \mathbf{A} matrix. This system shows the convenience of the ARFIMA(1, d , 1) specification. If the lags of the autoregressive and moving average polynomials were of order p and q , respectively, the number of parameters to be estimated would have been immense, complicating the estimation procedure.

6 Empirical Evaluation

In this section the role of implied volatility in forecasting volatility using two different dynamic specifications is studied to evaluate the impact of implied volatility in both settings and to evaluate the benefits of using a long-memory structure that allows for a richer set of dynamics. The HAR model is used as a simple approximate long memory model, while the ARFIMA model is its more complicated counterpart.

6.1 Results for the HAR Framework

The estimation procedure for the class of HAR models is based on QML estimation since the simple structure of the models allows for a regression based approach, similar to that of Mincer & Zarnowitz (1969), where IV_t is added as an additional regressor,

but QML is applied for robust inference.⁷

Table 1 shows the results for the entire class of single equation HAR models. The coefficient estimates are reported with their t -statistics in parentheses together with an adjusted (adj) R^2 , and a Breusch-Godfrey LM test for serial correlation up to lag 12 (AR_{12}).⁸ The Breusch-Godfrey test is computed due to possible endogeneity issues. The test statistics are χ^2 distributed with 12 degrees of freedom under the null hypothesis of no serial correlation. The dynamic models are compared based on their performance forecasting out-of-sample. The mean absolute forecast errors (MAFE)⁹ are computed for 24 rolling one-step-ahead forecasts starting from $n - 24$ observations.¹⁰ One or two asterisks are used to denote significance at a 5% or 1% level, respectively.

In Panel A of table 1, the results are presented for the HAR-RV-CJIV framework for different exclusion restrictions. From the first line of the Panel, it is evident that without IV_t as an explanatory variable the lagged monthly modulated realized volatility term is the most significant variable when forecasting volatility. This changes when IV_t is added to the equation and subsumes its explanatory effect. By adding IV_t the adjusted R^2 increases by 7 percentage points and the MAFE decreases, confirming the results of Busch et al. (2009). Applying the vector $x = (C, J)$ as explanatory variables instead of modulated realized volatility in line 3 and 4 of Panel A, the results are similar. Without IV_t the weekly continuous and jump components are the only significant variables. By including IV_t the adjusted R^2 increases and the MAFE decreases, while it once again assumes the role of the most significant explanatory

⁷Robinson (1994) and Christensen & Nielsen (2006) show that if the variables are integrated of order $0 \leq d < 0.5$, OLS (and equivalently QML) will no longer be consistent, and a narrow-band frequency domain least squares (FDLS) estimator is necessary for consistency. The order of integration for the four variables are shown in Table A.2 to be around 0.5 for $MRV_{t,t+21}$, $C_{t,t+21}$, below 0.5 for $J_{t,t+21}$, and above 0.5 for IV_t . Hence, QML is applied in this paper. An interesting direction for further research would be to implement the FDLS estimator.

⁸All adjusted R^2 coefficients are pseudo R^2 , where the residuals from the different models have been compared to those of a null model with only a constant.

⁹According to Patton (2009), the only two evaluation criteria that is robust to noise in the volatility proxy are the MSE and QLIKE, where the latter has better power properties. This theoretical setup assumes that the returns are generated as a zero mean Brownian motion with constant volatility within each trading day, and that no jumps occur in the stock price. However, he shows that the forecast error for the MAFE criteria disappears asymptotically when the realized variance used as volatility proxy is computed using a sampling frequency denser than every 5 minutes. The MAFE are computed in this application for comparability with Busch et al. (2009), and since the estimators of volatility are based on much denser sampling than every 5 minutes. Using QLIKE as a evaluation criteria indicates a similar effect of implied volatility.

¹⁰The reported coefficients are from the last one-step-ahead forecast using the entire estimation sample of $n = 128$ observations. This is the case for table 1, 2 and 3.

Panel A: Modulated Realized Volatility													
Const.	$MRV_{t-21,t}$	$MRV_{t-5,t}$	MRV_t	$C_{t-21,t}$	$C_{t-5,t}$	C_t	$J_{t-21,t}$	$J_{t-5,t}$	J_t	IV_t	Adj. R ²	AR ₁₂	MAFE
6.153** (4.72)	0.4865** (3.12)	0.0656 (0.377)	0.0913 (0.895)	-	-	-	-	-	-	-	0.474	15.25	5.197
-10.13* (-2.55)	0.1859 (1.31)	-0.1186 (-0.858)	0.1528* (2.02)	-	-	-	-	-	-	1.1325** (3.71)	0.544	20.57	4.030
7.927** (5.28)	-	-	-	0.1953 (0.920)	0.5772* (1.97)	0.0664 (0.572)	0.4933 (0.228)	-5.029* (-2.20)	-0.6081 (-0.869)	-	0.529	24.30*	5.071
-7.028 (-1.92)	-	-	-	-0.0742 (-0.379)	0.4460 (1.75)	0.0633 (0.617)	0.6718 (0.331)	-5.247* (-2.52)	-0.0582 (-0.087)	1.016** (3.63)	0.583	20.74	3.673
Panel B: The Continuous Component													
5.808** (4.71)	-	-	-	0.4738** (2.90)	0.0770 (0.400)	0.0942 (0.836)	-	-	-	-	0.478	23.52*	4.929
-9.565* (-2.49)	-	-	-	0.1898 (1.29)	-0.1122 (-0.693)	0.1574 (1.82)	-	-	-	1.060** (3.63)	0.544	24.03*	3.777
7.400** (5.13)	-	-	-	0.1897 (0.933)	0.5683* (2.02)	0.0579 (0.524)	0.4163 (0.205)	-4.934* (-2.25)	-0.5806 (-0.880)	-	0.527	22.58*	4.840
-7.147* (-2.01)	-	-	-	-0.0724 (-0.390)	0.4408 (1.80)	0.0548 (0.563)	0.5903 (0.310)	-5.148** (-2.58)	-0.0444 (-0.070)	0.9886** (3.64)	0.582	18.72	3.525
Panel C: The Jump Component													
0.5755** (5.61)	-	-	-	-	-	-	0.2815* (2.18)	0.0181 (0.406)	0.0462** (3.39)	-	0.130	23.66*	0.3395
-0.1934 (-1.25)	-	-	-	-	-	-	0.0810 (0.513)	-0.0441 (-0.933)	0.0535** (4.23)	0.0475** (3.65)	0.260	15.03	0.1836
0.5281** (6.09)	-	-	-	0.0056 (0.494)	0.0089 (0.459)	0.0085 (0.770)	0.0760 (0.372)	-0.0959 (-0.682)	-0.0277 (-0.438)	-	0.275	13.35	0.2552
0.1231 (0.726)	-	-	-	-0.0017 (-0.148)	0.0053 (0.283)	0.0084 (0.777)	0.0808 (0.401)	-0.1014 (-0.745)	-0.0128 (-0.203)	0.0275* (2.33)	0.289	15.51	0.1884

Table 1: The coefficient estimates and robust t -statistics for the class of single equation HAR models. Adj. R² denotes the adjusted R². AR₁₂ denote the LM statistic with 12 lags. MAFE is the mean absolute forecast error for 24 rolling one-step-ahead forecasts using $n - 24$ observations. (*) and (**) denotes significance at a 5% and 1% level, respectively.

variable. This clearly confirms the relevance of including the information kept in implied volatility when forecasting volatility. The partition of MRV into C and J improves both the in- and out-of-sample performance. When moving from line 1 to 3 and 2 to 4, respectively, the model performs better in all categories except the Breusch-Godfrey test for serial correlation in line 3, which shows signs of misspecification.

The results of the HAR-C-CJIV framework are displayed in Panel B. When modeling the continuous part of modulated realized volatility, the results are highly similar to those of Panel A, as expected, because of the similar time series properties. As seen in line 2 the model greatly improves with the inclusion of IV_t both in- and out-of-sample. Implied volatility is the most significant regressor, and it subsumes the role of the realized measure. Furthermore, the model improves with the inclusion of jumps, though the improvements are mostly in-sample. The weekly jump variable is significant in both line 3 and 4. There are signs of misspecification of the model since three of the four Breusch-Godfrey tests statistics are significant at a 5% significance level.

The predictability of the jump component of modulated realized volatility is displayed in Panel C. The results of this analysis differ from the those of the previous two Panels. The models do a much poorer job forecasting in-sample as showed by the lower adjusted R^2 coefficients. It is difficult to compare the MAFE for the HAR-J-CJIV models with the other two Panels since the quantity they are forecasting is much smaller, as seen in Figure 2. The differences can be explained by the very different time series properties of jumps. With the above mentioned reservations in mind, it is still possible to gain valid insights into the forecasting of jumps. As seen from the first two lines in Panel C, the daily jump component is significant, and the model greatly improves with the inclusion of IV_t . This improvement is also evident by comparing the performance of the models in line 3 and 4. It is worth noting from line 2 and 4 that there are no gains from adding the continuous component in terms of out-of-sample performance when IV_t is already included in the model. Consequently, implied volatility carries significant information about future jumps as well as volatility.

6.2 Results for the ARFIMA Framework

As mentioned in Section 5.1, one of the motivations of the Corsi (2009) HAR model was the computational difficulties of fractionally integrated models. The ARFIMA models are estimated using the conditional maximum likelihood (ML) methodology

developed in Beran (1995) and Doornik & Ooms (2004), which is also applicable for non-stationary processes with $d > 0.5$. The standard errors are computed robustly against heteroskedasticity. Due to the highly non-linear structure, the test for autocorrelation is based on a simple AR_{12} structure to give an impression of possible misspecification of the models, knowing that this is not as strict as the Breusch-Godfrey test, and that the test may suffer from an endogenous regressor problem.¹¹

Table 2 shows the results for the class of single equation ARFIMA models. Panel A considers the ARFIMA-IV framework for two specifications. In the first line IV_t is excluded from the model, making it a simple ARFIMA(1, d , 1) specification with a constant. As expected $d = 0.2785$ and is highly significant reflecting a long-memory structure of the data, which confirms the autocorrelation plot in Figure 2. Furthermore, the AR coefficient is positive and highly significant as well. Both the AR and MA coefficients are well within the stationary range, and the model seems to be well-specified. Adding IV_t as an exogenous regressor has an effect similar to that of the HAR framework. IV_t assumes the role of the most significant forecasting variable. The variable controlling the memory structure d becomes insignificant, while the AR term remains significant. Hence, lagged values of the endogenous variable still convey information about future volatility. The adjusted R^2 increases with almost 10 percentage points and the MAFE decreases. By imposing a richer dynamic specification, the addition of IV_t still improves the model greatly, but there is additional information to be collected from the AR term. When the restriction $d = 0$ is imposed in line 3, the model performs slightly better out-of-sample. The results of Panel A in Table 2 are comparable with those of line 1 and 2 of Panel A in Table 1. The ARFIMA-IV and HAR-RV-IV models seem to perform similarly in-sample, with the HAR framework slightly better when IV_t is not included as an exogenous variable. When comparing the models on out-of-sample performance, the ARFIMA-IV specification dominates the HAR-RV-IV. The improvements is most significant when IV_t is excluded from the model. These results reflect the gains of using a dynamic model that allows for a richer long-memory structure.

¹¹The Ljung-Box test for lag 12 serial correlation is also computed for all ARFIMA and vecARFIMA models to give an indication of problems with serial correlation even though the models have stochastic regressors. The conclusions are the same as for the AR_{12} test. None of the models reject the null hypothesis of no serial correlation.

Panel A: Modulated Realized Volatility									
Const.	ϕ	θ	d	IV_t	$C_{t-21,t}$	$J_{t-21,t}$	Adj. R ²	AR ₁₂	MAFE
17.58** (2.85)	0.4388* (2.12)	0.1132 (0.601)	0.2785** (2.69)	-	-	-	0.448	7.500	3.990
-11.44* (-2.05)	0.5779** (3.31)	0.2012 (1.36)	-0.0489 (-0.243)	1.384** (4.60)	-	-	0.546	10.36	3.749
-11.01* (-2.42)	0.5368** (3.65)	0.2048 (1.40)	-	1.363** (5.26)	-	-	0.546	10.27	3.681
Panel B: The Continuous Component									
16.68** (2.81)	0.4486* (2.18)	0.1169 (0.639)	0.2752** (2.65)	-	-	-	0.451	6.876	3.814
-11.34* (-2.00)	0.5824** (3.29)	0.1905 (1.29)	-0.0629 (-0.295)	1.339** (4.43)	-	-	0.546	10.47	3.608
-10.78* (-2.40)	0.5307** (3.53)	0.1958 (1.34)	-	1.310** (5.15)	-	-	0.546	10.36	3.521
18.67* (2.56)	0.4597* (2.41)	-0.1927 (-1.49)	0.2750** (2.66)	-	-	-2.411 (-1.42)	0.457	5.529	3.743
-9.708 (-1.66)	0.5790** (3.60)	0.1638 (1.29)	0.0190 (0.109)	1.405** (4.23)	-	-3.557 (-1.87)	0.561	7.564	3.259
-9.852 (-1.89)	0.5945** (4.49)	0.1621 (1.33)	-	1.412** (4.519)	-	-3.544 (-1.876)	0.560	7.613	3.207
Panel C: The Jump Component									
1.207** (3.17)	0.2637* (1.25)	0.9050** (28.92)	0.8702** (5.45)	-	-	-	0.163	5.174	0.2037
-0.1335 (-0.404)	0.3869 (0.758)	0.5940 (0.997)	0.3263 (0.642)	0.0505** (4.07)	-	-	0.261	5.690	0.1857
-0.2172 (-1.01)	0.9165** (13.06)	0.8273** (7.32)	-	0.0529** (4.13)	-	-	0.264	6.346	0.1940
0.7668* (2.38)	0.1494 (0.550)	0.8857** (10.00)	0.7683** (2.79)	-	0.0188** (3.03)	-	0.243	8.170	0.2094
0.0422 (0.119)	0.3484 (0.786)	0.6135 (1.20)	0.3401 (0.647)	0.0361* (2.02)	0.077 (0.808)	-	0.262	6.875	0.1815
-0.0408 (-0.170)	0.9295** (16.90)	0.8604** (9.42)	-	0.0387* (2.18)	0.0072 (0.871)	-	0.264	7.448	0.1863

Table 2: The coefficient estimates and robust t -statistics for the class of single equation ARFIMA models. Adj. R² denotes the adjusted R². AR₁₂ denote the LM statistic with 12 lags. MAFE is the mean absolute forecast error for 24 rolling one-step-ahead forecasts using $n - 24$ observations. (*) and (**) denotes significance at a 5% and 1% level, respectively.

The ARFIMA-C-IV specifications are considered in Panel B of Table 2. Due to the similarity between modulated realized volatility and its continuous component, the results in the first two lines of Panel B are almost equivalent to those of Panel A. The role of implied volatility is central when forecasting its realized counterpart. The two models in Panel B seem to do slightly better in all evaluation criteria as expected since the shorter memory jump component has been removed. The impact of IV_t is equally significant when the lagged jump component is included as an exogenous variable in

line 4, 5, and 6. The addition of IV_t significantly improves the model both in- and out-of-sample while it subsumes the explanatory effects of the long-memory parameter d . In fact, the model improves slightly in terms of out-of-sample performance when d is restricted to 0 in line 3 and 6. The impact of including the jump component is seen by comparing line 2 and 5. There are small gains both in- and out-of-sample, and the jump component is marginally insignificant. Generally, the models seem to be well-specified and good at capturing the dynamics of volatility. From Panel B of Table 1 and 2, respectively, it is evident that the class of ARFIMA dominates the HAR specifications when forecasting volatility out-of-sample, elaborating on the conclusions from Panel A. The ARFIMA-C-IV model with jumps included (and $d = 0$ imposed) is the counterpart to the HAR-C-CJIV where both C and J are included. These two models represent the superior specifications for their respective frameworks both in- and out-of-sample. It is noteworthy that the HAR-C-CJIV has a higher adjusted R^2 of 58.2% compared to 56.0% showing better in-sample fit. However, the main emphasis of this paper is on out-of-sample performance, and the ARFIMA-C-IV dominates the HAR-C-CJIV specification with a MAFE of 3.207 compared to 3.525. The non-linear dynamic models are better at forecasting volatility, but the addition of IV_t makes d superfluous.

Panel C presents the results for the jump component. Similar to the HAR framework, the class of ARFIMA-J-IV models does a much poorer job in terms of in-sample fit. The adjusted R^2 coefficients are much smaller than for the continuous counterpart. The predictability of the jump component is difficult to compare with the continuous component since the two quantities are numerically very different. However, there are still some important insights to be gained from Panel C. The first line shows the simple ARFIMA(1, d , 1) specification with no exogenous variables. The model seems to do a poor job of capturing the dynamics of J . A significant d of 0.8702 approaching the unit root limit suggests that the jump component is more persistent than volatility which is seen in Figure 2 to be incorrect. Furthermore, the value of θ is close to unity as well. These are noteworthy observations since the AR_{12} test shows no sign of misspecification. When IV_t is included in line two, neither the d nor θ remain significant. IV_t is the only significant variable confirming the results from the HAR framework that implied volatility conveys information about jumps as well as volatility. When $d = 0$ is imposed for the jump component in line 3 and 6, the model performs slightly poorer in terms of out-of-sample performance, and the estimates of the AR and MA parameters approach the unit root limit. Another noteworthy

observation is found by comparing the models in line 2 and 5. There are no gains from adding the lagged continuous component to the model when IV_t is already included. Panel C of Table 1 and 2 shows that the results for the two dynamic specifications are very similar, with the ARFIMA-J-IV performing better out-of-sample in two models and in-sample in one model, while HAR-J-CJIV performs better in-sample in two model. However, this comparison is facilitated by the notion of possible specification difficulties of the ARFIMA-J-IV model.

Up to this point, the findings of this paper show that implied volatility conveys incremental information about future jumps as well as volatility in both dynamic specifications. The impact of implied volatility is seen both in- and out-of-sample for all specifications using MRV and the non-parametric separation of C and J . Furthermore, modeling implied volatility together with a richer long-memory ARFIMA specification leads to better out-of-sample performance compared to the simple approximate class of HAR models. These findings are similar to those of Chiriac & Voev (2009) who show that the precision of the forecasts of ARFIMA models increases relative to those of HAR models when the forecasting horizon increases from 1 to 10 days. Their paper is based on forecasts of portfolio covariances, and does not consider the role of implied volatility in those models.

6.3 Results for the Vector Specifications

To solve the possible endogeneity issues described in sections 5.3 and 5.6 of the single equation models, simultaneous equation systems are introduced for both the HAR and the ARFIMA framework. The vecHAR and the vecARFIMA models are estimated by the same conditional ML approach as the single equation ARFIMA models. The standard errors are computed robustly against heteroskedasticity. The Breusch-Godfrey test for serial correlation is used for the vecHAR model due to its linear structure, while a simple AR_{12} test is conducted for the vecARFIMA system. The results are presented in Table 3.

The results of the vecHAR model are presented in Panel A. Implied volatility is significant in the forecasting equations for both C and J , similar to the results of Busch et al. (2009). The precision of the forecasts are very similar to the single equation framework. The vecHAR system is introduced to correct for possible endogeneity issues, and the Breusch-Godfrey test statistics improve slightly for both C and J . This equations system reinforces the conclusions made earlier about the role of implied

Panel A: vecHAR											
Dep. Var.	Const.	$C_{t-21,t}$	$C_{t-5,t}$	C_t	$J_{t-21,t}$	$J_{t-5,t}$	J_t	IV_t	IV_{t-1}	AR ₁₂	MAFE
$C_{t,t+21}$	-7.065*	-0.0847	0.4960*	0.0390	0.2898	-5.774**	0.0758	0.9907**	-	17.83	3.526
	(-1.99)	(-0.453)	(1.99)	(0.407)	(0.148)	(-2.87)	(0.125)	(3.65)			
$J_{t,t+21}$	0.1245	-0.0023	0.0079	0.0077	0.0673	-0.1302	-0.0072	0.0278*	-	15.25	0.1872
	(0.683)	(-0.145)	(0.360)	(0.699)	(0.259)	(-0.793)	(-0.124)	()			
IV_t	7.026**	0.0706	0.1514	0.0138	-0.4354	0.1878	-0.4205*	-	0.5012**	17.56	2.591
	(7.76)	(0.954)	(1.54)	(0.343)	(-0.814)	(0.277)	(-1.99)	(9.05)			
Panel B: vecARFIMA Unrestricted											
Dep. Var.	Const.	ϕ	θ	d	IV_t	$C_{t-21,t}$	$J_{t-21,t}$	-	-	AR ₁₂	MAFE
$C_{t,t+21}$	-9.708	0.5790**	0.1638	0.0190	1.405**	-	-3.557	-	-	7.564	3.259
	(-1.65)	(3.60)	(1.29)	(0.109)	(4.23)		(-1.87)				
$J_{t,t+21}$	-0.1070	0.9794**	0.1097	-0.7960	0.0424*	0.0066	-	-	-	8.718	0.1882
	(-0.474)	(34.82)	(0.159)	(-1.88)	(2.45)	(-0.629)					
IV_t	16.27**	-0.1366	0.1674	0.6172**	-	0.2483**	0.3709	-	-	11.76	2.714
	(7.48)	(-0.285)	(0.258)	(2.61)		(6.55)	(0.665)				
Panel C: vecARFIMA, $A_{21m} = A_{31m} = A_{12m} = A_{32m} = 0$											
$C_{t,t+21}$	-11.36*	0.5847**	0.1869	-0.0686	1.340*	-	-	-	-	10.88	3.615
	(-1.98)	(3.21)	(1.27)	(-0.309)	(4.43)						
$J_{t,t+21}$	-0.1828	-0.4592	-0.5019*	0.1218	0.0511**	-	-	-	-	6.117	0.1786
	(-1.02)	(-1.42)	(-1.96)	(1.30)	(4.46)						
IV_t	20.81**	0.9785**	-0.1759	-0.4418**	-	-	-	-	-	11.77	2.737
	(10.68)	(50.29)	(-1.64)	(-3.48)							
Panel D: vecARFIMA, $d_1 = 0$											
$C_{t,t+21}$	-9.853	0.5944**	0.1621	-	1.412**	-	-3.544	-	-	7.613	3.207
	(-1.88)	(4.49)	(1.32)		(4.519)		(-1.88)				
$J_{t,t+21}$	0.0325	0.3491	0.6066	0.3329	0.0325	0.0076	-	-	-	6.839	0.1817
	(0.081)	(0.762)	(1.19)	(0.712)	(1.72)	(0.754)					
IV_t	16.27**	-0.1365	0.1676	0.6173**	-	0.2483**	0.3708	-	-	11.76	2.715
	(7.49)	(-0.286)	(0.260)	(2.62)		(6.55)	(0.665)				

Table 3: The coefficient estimates and robust t -statistics for the simultaneous equation vecHAR and vecARFIMA model along with two nested vecARFIMA models. Adj. R² denotes the adjusted R². AR₁₂ denote the LM statistic with 12 lags. MAFE is the mean absolute forecast error for 24 rolling one-step-ahead forecasts using $n - 24$ observations. (*) and (**) denotes significance at a 5% and 1% level, respectively.

volatility. It has a significant role in forecasting future volatility, and it helps forecast jumps as well. Hence, option prices are calibrated to encompass information about future jumps.

The unrestricted version of the vecARFIMA model is presented in Panel B of Table 3. The forecasting equation for the continuous component is similar to the ARFIMA-C-IV model in terms of coefficient estimates and of out-of-sample performance. Implied volatility and the AR coefficient are significant, while the memory parameter d is insignificant. The AR_{12} statistic has decreased, indicating the vecARFIMA model is better specified compared to its single equation counterpart. The forecasting equation for J in line 2 confirms the significant role of implied volatility in the ARFIMA class of models. However, it is important to emphasize both the numerical size of ϕ and d . The long memory coefficient d , though only marginally insignificant, is -0.7960 indicating a non-stationary intermediate-memory process. The AR coefficient ϕ is 0.9794 which is dangerously close to a unit root. Both of these coefficients indicate that the model is misspecified. The forecasting equation of IV_t is considered in the third line of Table 3. Past volatility has a significant impact on future implied volatility. Furthermore, implied volatility has a memory-parameter of $d = 0.6172$ indicating a very high degree of persistence in the non-stationary range. As seen in Figure 2, implied volatility is the most persistent process. Thus, the estimated coefficient is not unrealistic.

Due to questionable coefficient estimates for the jump component, the specification of the vecARFIMA model is examined by considering a benchmark case in Panel C where the coefficient matrix \mathbf{A} from (31) is restricted to zero. The impact on the coefficients of the forecasting equation for the continuous component is minimal, indicating that this equation is very well-specified. However, the equations for J and IV_t change dramatically. The numerical value of ϕ and d for the jump component changes sign, and the forecasting equation describes a stationary long-memory process, though d is still insignificant. The coefficients on ϕ and d undertake an equally dramatic change in the equation for implied volatility. The equation describes an intermediate memory process, since d is significantly smaller than zero. Furthermore, ϕ is close to unity. The combined results of the benchmark vecARFIMA model indicate that this is poorly specified as well.

Motivated by the fact that the forecasting equation for the continuous component and implied volatility are well-specified in Panel B, a third version of the vecARFIMA model is proposed in Panel D of Table 3. As seen in Panel B of Table 2, when the restriction $d = 0$ is imposed on the memory parameter, the model improves slightly

in terms of out-of-sample performance and causes minimal changes to the coefficient estimates. Imposing $d_1 = 0$ on the vecARFIMA model leads to a well-specified forecasting equations for all three variables. The estimated parameters in the forecasting equations for C and IV are similar to those of Panel B. The forecasting equation for J does no longer contain any significant variables, i.e. implied volatility has become an insignificant forecasting variable of jumps. Furthermore, neither ϕ or d is close to the unreasonable values in Panel B. This third vecARFIMA model is very well-specified, and it is the superior specification for forecasting out-of-sample volatility. A comparison between the vecHAR model, and the vecARFIMA models shows that the time series properties of diffusive volatility is better captured by the ARFIMA dynamics, which delivers more precise forecasts. Furthermore, the vecHAR model is marginally better at forecasting both the jump component and implied volatility.

The implications of different parameter restrictions on the vecARFIMA model is investigated, in particular three different cases are considered $A_{21m} = 0$, $d_2 = 0$, and $d_1 = d_2 = 0$. The results of the nested models are presented in Table A.3 of Appendix A1. The estimated parameters in the forecasting equations for C and IV for all three restrictions are similar to those of Panel B and D of Table 3, adding robustness to the results. Similarly, all the models perform better than the vecHAR model out-of-sample. The parameter estimates in the forecasting equation for J is similar for the three restrictions. The values of ϕ and θ approach the unit root limit, which is the border for stationarity and invertibility of the ARMA polynomial.. This shows that the vecARFIMA model, and especially the forecasting equation for J , must be carefully specified.

The dynamics of the continuous part of modulated realized volatility are better captured by the ARFIMA specification compared to the simple dynamic structure of the HAR model. Implied volatility is highly significant when used in both of these dynamic settings, elaborating on the results of Busch et al. (2009) and Christensen & Prabhala (1998), since their results translate to a non-linear forecasting framework. When using a vectorized model to correct errors due to possible endogeneity issues, the vecARFIMA model dominates the vecHAR specification when forecasting volatility out-of-sample. Thus, in addition to the incremental information in implied volatility, there is important information in the richer long-memory structure of the ARFIMA models. However, these models must be carefully specified, especially for the jump component.

7 Conclusion

This paper has examined the role of implied volatility in forecasting future volatility for two different dynamic models, the HAR model of Corsi (2009) and the ARFIMA model suggested by Hosking (1981), in a noisy diffusion setting with jumps. Busch et al. (2009) have examined the incremental information of implied volatility for different HAR specifications. This paper extended their framework by estimating and separating the continuous and jump components of volatility robustly to the presence of market microstructure noise, and by examining the role of implied volatility in both single equation and simultaneous equation dynamic models that is not limited by a simple approximate long-memory structure.

The methodology of Podolskij & Vetter (2009a, 2009b) is applied to estimate volatility by the means of modulated realized volatility in the stock market using observations on the SPY fund, and to make a non-parametric separation of its continuous and jump parts. Their methodology is the cutting edge within estimation of volatility in the presence of jumps and noisy high-frequency data. Furthermore, the new VIX is used as a measure of implied volatility on the SPY fund, since the SPY tracks the performance of the S&P 500. Time series of these components are then used as input to both the single equation dynamic models and to the simultaneous equation systems applied in the paper.

The results from the HAR model resemble those of Busch et al. (2009) and Christensen & Prabhala (1998). Implied volatility carries significant information about future volatility, improving model specifications both in- and out-of-sample. Furthermore, it subsumes the explanatory effects of most of the other variables. When using implied volatility together with ARFIMA specifications, the results are similar. Implied volatility improves model performance significantly. Thus, the important role of implied volatility translates to a non-linear forecasting framework. However, the *AR* coefficient conveys important information about future volatility in the ARFIMA setting as well. The ARFIMA class of models dominates the HAR models when forecasting out-of-sample, showing the benefits of using the richer dynamic structure of these models to fully capture the dynamics of volatility.

The forecasting framework is extended to simultaneous equation systems, since there are reasons to suspect the single equation models of suffering from endogeneity issues. The vecARFIMA model is introduced in this paper as an alternative to the vecHAR specification that is not limited by a simple approximate long-memory structure. The

vecARFIMA model outperforms the vecHAR model when forecasting volatility out-of-sample. However, the vecARFIMA model must be carefully specified.

Implied volatility conveys important information about future volatility. This paper shows that the role of implied volatility can be extended to a set of richer dynamic models. These are better at capturing the dynamics of volatility, making it an interesting complement to the existing literature on the role of implied volatility using simple long-memory specifications.

References

- Andersen, T. G., Bollerslev, T. & Diebold, F. X. (2007), ‘Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility’, *The Review of Economics and Statistics* **89**(4), 701–720.
- Andersen, T. G., Bollerslev, T., Diebold, F. X. & Ebens, H. (2001), ‘The distribution of realized stock return volatility’, *Journal of Financial Economics* **61**, 43–76.
- Andersen, T. G., Bollerslev, T., Diebold, F. X. & Labys, P. (2003), ‘Modeling and forecasting realized volatility’, *Econometrica* **71**, 579–625.
- Andersen, T. G. & Bondarenko, O. (2007), Construction and interpretation of model-free implied volatility. NBER Working Papers-13449.
- Barndorff-Nielsen, O. E. & Shephard, N. (2006), ‘Econometrics of testing for jumps in financial economics using bipower variation’, *Journal of Financial Econometrics* **4**(1), 1–30.
- Beran, J. (1995), ‘Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models’, *Journal of the Royal Statistical Society* **57**, 659–672.
- Black, F. & Scholes, M. S. (1973), ‘The pricing of options and corporate liabilities’, *Journal of Political Economy* **81**(3), 637–54.
- Bollerslev, T. (1986), ‘Generalized autoregressive conditional heteroskedasticity’, *Journal of Econometrics* **31**(3), 307–327.
- Busch, T., Christensen, B. J. & Nielsen, M. Ø. (2009), ‘The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets’, *Journal of Econometrics* . forthcoming.
- CBOE (2003), Vix - cboe volatility index. Chicago Board of Exchange working paper series.
- Chambers, M. J. (1998), ‘Long memory and aggregation in macroeconomic time series’, *International Economic Review* **39**(4), 1053–1072.
- Chiriac, R. & Voev, V. (2009), ‘Modelling and forecasting multivariate realized volatility’, *Journal of Applied Econometrics* . forthcoming.

- Christensen, B. J. & Nielsen, M. O. (2006), ‘Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting’, *Journal of Econometrics* **133**(1), 343–371.
- Christensen, B. J. & Prabhala, N. R. (1998), ‘The relation between implied and realized volatility’, *Journal of Financial Economics* **50**(2), 125–150.
- Cont, R. & Tankov, P. (2004), *Financial Modelling With Jump Processes*, Chapman & Hall/CRC Financial Mathematics Series, Boca Raton.
- Corsi, F. (2009), ‘A simple approximate long-memory model of realized volatility’, *Journal of Financial Econometrics* **7**, 174–196.
- Doornik, J. A. & Ooms, M. (2004), ‘Inference and forecasting for arfima models with an application to us and uk inflation’, *Studies in Nonlinear Dynamics & Econometrics* **8**, Article 14.
- Engle, R. (1982), ‘Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation’, *Econometrica* **50**, 987–1007.
- Hansen, P. R. & Lunde, A. (2006), ‘Realized variance and market microstructure noise’, *Journal of Business and Economic Statistics* **24**, 127–161.
- Hosking, J. R. M. (1981), ‘Fractional differencing’, *Biometrika* **68**(1), 165–176.
- Jiang, G. J. & Oomen, R. C. A. (2008), ‘Testing for jumps when asset prices are observed with noise – a ”swap variance” approach’, *Journal of Econometrics* **144**, 352–370.
- Merton, R. C. (1973), ‘Theory of rational option pricing’, *Bell Journal of Economics* **4**(1), 141–183.
- Merton, R. C. (1980), ‘On estimating the expected return on the market: An exploratory investigation’, *Journal of Financial Economics* **8**(4), 323–361.
- Mincer, J. A. & Zarnowitz, V. (1969), The evaluation of economic forecasts, in ‘Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance’, NBER Chapters, National Bureau of Economic Research, Inc, pp. 1–46.
- Patton, A. (2009), ‘Volatility forecast comparison using imperfect volatility proxies’, *Journal of Econometrics* . forthcoming.

- Podolskij, M. & Vetter, M. (2009a), ‘Bipower-type estimation in a noisy diffusion setting’, *Stochastic Processes and their Applications* **119**, 2803–2831.
- Podolskij, M. & Vetter, M. (2009b), ‘Estimation of volatility functionals in the simultaneous presence of microstructure noise and jumps’, *Bernoulli* **15**(3), 634–658.
- Robinson, P. M. (1994), ‘Semiparametric analysis of long-memory time series’, *The Annals of Statistics* **22**(1), 515–539.
- Robinson, P. M. (1995), ‘Gaussian semiparametric estimation of long range dependence’, *The Annals of Statistics* **23**(5), 1630–1661.
- Zhang, L., Mykland, P. A. & Aït-Sahalia, Y. (2005), ‘A tale of two time scales: Determining integrated volatility with noisy high frequency data’, *Journal of the American Statistical Association* **100**, 1394–1411.

A Appendix

A.1 Supplementary Tables

Panel A: 2002 Daily Data						
	Mean	Max	Min	SD	Skewness	Kurtosis
MRV_t	1.494	9.292	0.2544	1.319	2.652	9.665
C_t	1.454	9.292	0.2544	1.289	2.658	9.917
J_t	0.0405	0.8520	0.0000	0.0894	5.415	38.03
IV_t	27.28	45.08	17.40	6.896	0.5127	-0.8003
Panel B: Full Sample Monthly Data						
$MRV_{t,t+21}$	17.65	91.89	2.981	15.84	1.960	4.649
$C_{t,t+21}$	16.77	89.44	2.650	15.33	1.985	4.803
$J_{t,t+21}$	0.8854	4.199	0.2124	0.7408	2.366	6.004
IV_t	21.02	43.48	10.44	6.984	0.7661	0.5086

Table A.1: Summary statistics for the estimates MRV , C , J , and IV for both daily data from the median year in the sample, 2002, and for the full sample of monthly data.

Panel A: 2002 Daily Data				
	MRV_t	C_t	J_t	IV_t
Ljung-Box (lags)				
(L=12)	1097.4**	1094.2**	44.208**	2285.5**
GSP (bandwidth)				
$\hat{d} (m = 42)$	0.5777 (0.077)	0.5877 (0.077)	0.0337 (0.077)	0.8122 (0.077)
$\hat{d} (m = 63)$	0.6591 (0.063)	0.6434 (0.063)	0.0425 (0.063)	0.8218 (0.063)
$\hat{d} (m = 126)$	0.6933 (0.063)	0.6906 (0.045)	0.1056 (0.045)	0.8469 (0.045)
ADF (lags)				
$t(L=0)$	-4.142**	-4.166**	-13.52**	-2.047
$t(L=4)$	-2.702	-2.700	-6.936**	-1.610
Panel B: Full Sample Monthly Data				
	$MRV_{t,t+21}$	$C_{t,t+21}$	$J_{t,t+21}$	IV_t
Ljung-Box (lags)				
(L=12)	192.77**	191.78**	64.315**	434.76**
GSP (bandwidth)				
$\hat{d} (m = 21)$	0.4141 (0.109)	0.4158 (0.109)	0.3330 (0.109)	0.6670 (0.109)
$\hat{d} (m = 32)$	0.4959 (0.088)	0.4961 (0.088)	0.2870 (0.088)	0.5966 (0.088)
$\hat{d} (m = 64)$	0.5150 (0.063)	0.5168 (0.063)	0.2639 (0.063)	0.5906 (0.063)
ADF (lags)				
$t(L=0)$	-4.927**	-4.905**	-7.896**	-3.733**
$t(L=4)$	-3.142*	-3.554**	-3.141*	-2.227

Table A.2: Ljung-Box test of the significance of the autocorrelation function. GSP are Gaussian semiparametric estimates of the fractional integration order described in Robinson (1995). ADF are augmented Dickey-Fuller tests of the null hypothesis of a unit root. (*) and (**) denotes significance at a 5% and 1% level, respectively.

Panel A: vecARFIMA, $A_{21m} = 0$									
Dep. Var.	Const.	ϕ	θ	d	IV_t	$C_{t-21,t}$	$J_{t-21,t}$	AR ₁₂	MAFE
$\hat{C}_{t,t+21}$	-9.707 (-1.66)	0.5790** (2.62)	0.1638 (1.29)	0.0190 (0.109)	1.405** (4.23)	-	-3.557 (-1.87)	7.564	3.259
$J_{t,t+21}$	-0.2090 (-0.933)	0.9155** (12.55)	0.8249** (7.18)	-0.0016 (-0.047)	0.0526** (4.02)	-	-	6.364	0.1936
IV_t	16.27** (7.48)	-0.1368 (-0.285)	0.1670 (0.258)	0.6170** (2.62)	-	0.2483** (6.55)	0.3708 (0.507)	11.77	2.715
Panel B: vecARFIMA, $d_2 = 0$									
$\hat{C}_{t,t+21}$	-9.708 (-1.646)	0.5790** (3.49)	0.1638 (1.29)	0.0190 (0.103)	1.405** (4.22)	-	-3.557 (-1.86)	7.564	3.259
$J_{t,t+21}$	-0.0446 (-0.097)	0.9290** (12.91)	0.8597** (7.91)	-	0.0389 (1.58)	0.0072 (0.817)	-	7.441	0.1864
IV_t	16.27** (7.48)	-0.1366 (-0.286)	0.1673 (0.259)	0.6171** (2.62)	-	0.2483** (6.55)	0.3709 (0.664)	11.76	2.715
Panel C: vecARFIMA, $d_1 = d_2 = 0$									
$\hat{C}_{t,t+21}$	-9.853 (-1.89)	0.5944** (4.49)	0.1620 (1.324)	-	1.412** (4.52)	-	-3.5439 (-1.88)	7.613	3.207
$J_{t,t+21}$	-0.0431 (-0.331)	0.9292** (17.52)	0.8600** (10.31)	-	0.0389** (3.26)	0.0072 (0.992)	-	7.444	0.1865
IV_t	16.27** (7.47)	-0.1366 (-0.282)	0.1674 (0.255)	0.6172** (2.59)	-	0.2483** (6.55)	0.3709 (0.667)	11.76	2.715

Table A.3: The coefficient estimates and robust t -statistics for nested simultaneous equation models. Adj. R^2 denotes the adjusted R^2 . AR₁₂ denote the LM statistic with 12 lags. MAFE is the mean absolute forecast error for 24 rolling one-step-ahead forecasts using $n - 24$ observations. (*) and (**) denotes significance at a 5% and 1% level, respectively.

Research Papers 2010



- 2010-26: Robinson Kruse: On European monetary integration and the persistence of real effective exchange rates
- 2010-27: Sanne Hiller and Robinson Kruse: Milestones of European Integration: Which matters most for Export Openness?
- 2010-28: Robinson Kruse: Forecasting autoregressive time series under changing persistence
- 2010-29: Nikolaus Hautsch and Mark Podolskij: Pre-Averaging Based Estimation of Quadratic Variation in the Presence of Noise and Jumps: Theory, Implementation, and Empirical Evidence
- 2010-30: Martin M. Andreasen: Non-linear DSGE Models and The Central Difference Kalman Filter
- 2010-31: Morten Ørregaard Nielsen and Per Frederiksen: Fully Modified Narrow-Band Least Squares Estimation of Weak Fractional Cointegration
- 2010-32: Mogens Bladt and Michael Sørensen: Simple simulation of diffusion bridges with application to likelihood inference for diffusions
- 2010-33: Fernando Baltazar-Larios and Michael Sørensen: Maximum likelihood estimation for integrated diffusion processes
- 2010-34: Leonidas Tsiaras: The Forecast Performance of Competing Implied Volatility Measures: The Case of Individual Stocks
- 2010-35: Leonidas Tsiaras: Dynamic Models of Exchange Rate Dependence Using Option Prices and Historical Returns
- 2010-36: Robinson Kruse and Rickard Sandberg: Linearity Testing in Time-Varying Smooth Transition Autoregressive Models under Unknown Degree of Persistency
- 2010-37: Tom Engsted and Thomas Q. Pedersen: The log-linear return approximation, bubbles, and predictability
- 2010-38: Thomas Q. Pedersen: Predictable return distributions
- 2010-39: Rasmus Tangsgaard Varneskov: The Role of Dynamic Specification in Forecasting Volatility in the Presence of Jumps and Noisy High-Frequency Data