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Dynamic Models of Exchange Rate Dependence Using Option Prices and Historical Returns

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Abstract

Models for the conditional joint distribution of the U.S. Dollar/Japanese Yen and Euro/Japanese Yen exchange rates, from November 2001 until June 2007, are evaluated and compared. The conditional dependency is allowed to vary across time, as a function of either historical returns or a combination of past return data and option-implied dependence estimates.

Using prices of currency options that are available in the public domain, risk-neutral dependency expectations are extracted through a copula representation of the bivariate risk-neutral density. For this purpose, we employ either the one-parameter “Normal” or a two-parameter “Gumbel Mixture” specification. The latter provides forward-looking information regarding the overall degree of covariation, as well as, the level and direction of asymmetric dependence. Specifications that include option-based measures in their information set are found to outperform, in-sample and out-of-sample, models that rely solely on historical returns.

JEL Classifications: F31, F37, G14, G15, G17

Keywords: Exchange Rates; Implied Correlation; Copula; Forecasting; Options

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1 Introduction

Obtaining insights regarding the dependence pattern of exchange rate returns is of great importance for many economic agents (such as international investors, risk managers and policymakers) because the value of investments in foreign assets is affected by performance of the foreign currencies relative to the domestic one. For the risk-averse investor, accurate estimates of dependence are of value because the return distribution of his international portfolio is affected by the comovement of the involved exchange rates. An appropriate model must at least allow for time-varying (conditional) dependency, since the latter is usually not constant across time (see Andersen et al (2001), among others) and in certain cases exhibits apparent structural changes (Engle, 2002; Andreou and Ghysels, 2003). Furthermore, some empirical studies suggest that exchange rate dependence is deviating from the Gaussian paradigm¹ where linear correlation measures are conveniently couched². Sudden changes and/or neglected non-linearities in the dependence pattern can lead to a serious underestimation of risk, which poses an important challenge to researchers and practitioners alike.

Time-series models of conditional dependency generally rely on the historical record of financial returns in order to update their next period forecasts. In essence, the aim of these models is to provide conditional dependency estimates that are consistent with past return data. Although one cannot dispute the novelty of this approach given this limited information set, it must be noted that these models are essentially backward-looking. Whenever a change in the dependence pattern occurs, it may take a considerable amount of time until enough return data reflecting this change accumulate.

One tempting approach that can be used to enhance the performance of historical return models is to include option-implied dependence measures in the analysis. If markets are efficient, then the prices of multi-asset options must reflect market expectations regarding the future, risk-neutral, association of asset returns. Notably, these option-implied dependency expectations are formed by aggregating every piece of relevant information which includes, but far exceeds, historical return data. Moreover, these measures refer to the maturity of the option at some future date, so they have forward-looking features by construction.

Numerous authors have utilized distributional information from the prices of

¹See, for instance, Patton (2006) or Dias and Embrechts (2009).

²Embrechts, McNeil and Straumann (1999) explain why the use of the linear correlation coefficient is inappropriate outside the world of elliptical distributions.

derivatives, though the main strand of this literature considers exclusively implied volatilities. Indeed, it is now well-established that the latter contain information about future volatility levels that is absent from historical return data³. Besides implied volatilities, there are additional option-based measures that have been found to be constructive for forecasting purposes. Liu et al (2007) and Shackleton, Taylor and Yu (2007) find that option-implied forecast densities either outperform forecast densities obtained by ARCH processes or contribute significantly in mixture models that combine both sources of information. Christoffersen, Jacobs and Vainberg (2008) extract forward-looking betas, using the prices of individual and stock index options, and find that they contain incremental information relative to historical betas. Kostakis, Panigirtzoglou and Skiadopoulos (2009) consider a static asset allocation problem using either historical returns or adjusted risk-neutral densities and find that the latter deliver superior results.

Some work has also been done for the case of option-implied exchange rate dependence, which relates closely to this study. Siegel (1997) was the first to formally show, in a Black-Scholes setting, how the implied correlation of two exchange rates that have the same currency as a numeraire can be recovered from the prices of options written on these exchange rates, as well as, the cross-rate. This and subsequent empirical studies have generally reached to the same conclusion, namely that implied correlation forecasts contain incremental information with respect to those obtained from historical return models. In particular, Bodurtha and Shen (1995) find that implied correlations provide incremental explanatory power over return-based forecasts, while the results of Campa and Chang (1998) clearly favor the option-implied measures. Lopez and Walter (2000), who examine more than one currency trio, report that implied correlation forecasts, although less useful in certain cases, provide constructive information for the majority of the exchange rate pairs that they consider. Similarly, Castren and Mazzotta (2005) find that, although option-implied correlation forecasts are not always superior to those of historical return models, they tend to provide the most consistent results. In addition, their empirical results show that models that combine option-implied and return-based correlation measures deliver the most accurate forecasts.

While the aforementioned papers differ with respect to their data sources or the number of exchange rate pairs that they consider, they essentially share the same scope and methodology. Correlation forecasts obtained from historical return models (such as rolling window, RiskMetrics or bivariate GARCH models) and Black-Scholes implied correlations are compared, in a regression setting, using re-

³See the comprehensive literature reviews of Poon and Granger (2003) and Taylor (2005).

alized correlation as the target quantity. For the needs of both practitioners and regulators, however, it appears more suitable to evaluate the competing alternatives in terms of the conditional joint distribution that they define, and not just the covariance/correlation estimates. Unlike other financial applications where the challenge lies in the robust estimation of large variance/covariance matrices (such as the CAPM for instance), measuring and managing exchange rate risk requires an accurate description of the (relatively low-dimensional) joint density that provides the basis for the computation of risk measures such as Value-At-Risk or Expected Shortfall. Moreover, correlation is not a very operational input for typical risk management applications, which usually involve non-elliptical distributions, since it does not provide the necessary information to properly define a joint density⁴. Problems also arise when the dependence pattern exhibits non-linear characteristics, such as asymmetric dependency for joint positive and joint negative returns, as correlation is limited to reflect linear association. Generally, correlation is a scalar measure of dependency and, as such, it does not provide an adequate description of how the univariate distributions are associated, except for very special cases.

With these considerations in mind, we employ a completely different econometric methodology that is based on copula theory⁵. Firstly, we use the copula representation of a multivariate risk-neutral density derived in Rosenberg (2003) and Taylor and Wang (forthcoming), and find the copula's dependency parameter(s) that provides the closest fit to the observed prices of exchange rate options. Compared to the correlation-based studies, this approach allows the researcher to extract more information about the association of exchange rate returns. For example, one of our parametric (bivariate) copula specifications has two separate dependence parameters (instead of only one as with the implied correlation case), each controlling for either upper or lower tail dependency. In this way, we can study changes in either the degree of association or level of asymmetry.

Subsequently, we incorporate this information in the time-varying conditional copula model of Patton (2006b). There are at least three important reasons that justify this selection. First of all, this choice appears natural given that copula functions have been used in order to extract the dependency information from the prices of options. Second, this model offers great benefits from a goodness-of-fit

⁴Combining two marginal distributions together with a correlation coefficient does not generally lead to a valid or unique bivariate distribution. See Embrechts, McNeil and Straumann (1999) for further details.

⁵Copula functions, discussed in Section 2.1, are functions that describe the dependence structure of two or more random variables.

perspective; a well-specified joint density can always be constructed by combining the conditional copula with the (arbitrary) marginals, as long as the latter are not rejected by the data. This property is particularly appealing for the problem at hand since different, fat-tailed, distributions are usually required for the (conditional) marginal distributions of exchange rate returns⁶. Finally, the flexibility of the conditional copula model preserves the applicability of our approach to situations where the economic agent is already using a particular univariate model, or has a strong preference for a given marginal specification, as it is possible to make minor adjustments to the methodology presented in this paper and proceed with the modeling of the dependence process. This facilitates the implementation of integrated risk-management policies⁷, since individual and joint risks can be assessed in a consistent fashion, while still allowing for the possibility to exploit the dependency information embedded in the prices of exchange rate options. For the above reasons, we believe that model comparisons within this laboratory environment are more appropriate than those conducted in a simple regression setting.

In our empirical exercise we consider dynamic bivariate density models for the U.S. Dollar/Japanese Yen and Euro/Japanese Yen exchange rates. Two parametric copula functions, i.e. the (one-parameter) Normal and a (two-parameter) Gumbel mixture specification, are used in order to extract dependency information from option prices, as well as to define the time-varying conditional copula models. The univariate price processes of the aforementioned exchange rates are modeled using ARCH specifications that are common for all bivariate models. The dependence parameter of the Normal copula model is allowed to vary across time as a function of either past return data or a combination of historical returns and option-implied measures. In a similar fashion, we estimate two conditional copula models that employ the Gumbel mixture copula specification. One where upper and lower tail dependency are modeled using historical returns, and one where option-implied upper and lower tail dependency measures are included in the analysis.

Our empirical results indicate that the performance of the conditional copula models is enhanced when option-implied dependency measures are used to augment the return-based dynamic bivariate density specifications. For both the Normal and the Gumbel mixture copulas, the augmented models are preferred relative to their historical counterparts, using either in-sample or out-of-sample model selection cri-

⁶See, for example, Baillie and Bollerslev (1989), Wang et al (2001) or Alexander and Lazar (2006).

⁷For an unconditional copula approach concerning the problem of aggregating individual risks into an integrated risk management system see Rosenberg and Schuermann (2006).

teria. However, it is difficult to draw a firm conclusion regarding which of the four models performs best overall. The augmented Normal copula model ranks favorably in-sample, comes off second best out-of-sample, although some mild evidence of misspecification surface in both settings. Conversely, the augmented Gumbel mixture copula delivers the best results out-of-sample but comes off second best in-sample, as it is surpassed by the augmented Normal copula. Notably, the worst results are obtained from the Normal copula that utilizes only historical return information.

The remainder of the paper is organized as follows. Section 2 explains the methodology adopted in this study, i.e. the application of copula theory as a means to extract option-implied dependency measures and construct bivariate time-series models. Data issues and practical estimation of the univariate and bivariate risk-neutral densities are discussed in Section 3. Section 4 contains the empirical results of the paper, including the evaluation of the competing alternatives. The conclusions are set out in Section 5.

2 Methodology

2.1 Copula Functions

Copula functions, firstly introduced by Sklar (1959), are multivariate cumulative distribution functions whose marginals are uniform on the interval from 0 to 1. According to the theorem of Sklar, any multivariate distribution function can be decomposed into its marginal distributions and a copula function⁸. When all the marginal distributions are continuous, then this copula is unique.

As an example, consider the case of two random variables X and Y with cumulative distributions functions $F(x)$ and $G(y)$, respectively, and joint distribution $K(x, y)$. Also denote $f(x)$, $g(y)$, and $k(x, y)$ the corresponding probability density functions. According to Sklar's theorem we have that:

$$K(x, y) = C(F(x), G(y)) \quad \text{and} \quad (1)$$

$$k(x, y) = c(F(x), G(y)) \cdot f(x) \cdot g(y) \quad , \quad (2)$$

where $C(\cdot, \cdot)$ and $c(\cdot, \cdot)$ are the cumulative distribution and probability density function of the copula, respectively.

⁸For a detailed treatment of copula functions, see the textbooks of Joe (1997) and Nelsen (1999). Applications of copula theory in the modeling of financial time series are reviewed in Patton (2009).

In this paper, we consider two distinct copula functions for the joint distribution of the uniform variables $u = F(x)$ and $v = G(y)$. The first one is the *Normal* copula which is derived from the normal distribution. Denoting $\Phi_\rho(\cdot, \cdot)$ the bivariate cumulative distribution function (c.d.f.) of the standard normal with correlation coefficient ρ and $\Phi(\cdot)$ the univariate c.d.f. of the standard normal, the c.d.f. corresponding to the Normal copula with dependence parameter ρ is:

$$C_N(u, v; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \quad , \quad \rho \in [-1, 1] \quad (3)$$

The structural form of the Normal copula implies symmetric dependence for joint positive and joint negative innovations and no asymptotic tail dependency (except for the boundary case where $\rho = 1$).

The second copula used in this paper is a mixture of the Gumbel⁹ and Rotated Gumbel copulas, both of which belong to the Archimedean family. The c.d.f. of the Gumbel copula is:

$$C_G(u, v; \delta_1) = \exp \left\{ - \left((-\log u)^{\delta_1} + (-\log v)^{\delta_1} \right)^{1/\delta_1} \right\} \quad , \quad \delta_1 \in [1, \infty) \quad (4)$$

The Gumbel copula implies stronger dependence for joint positive innovations. The lower tail dependency of this copula is zero, while the upper tail dependency is positive for any $\delta_1 > 1$ and is equal to:

$$\lambda_U = \lim_{u \rightarrow 1^-} \frac{1 - 2u + C_G(u, u; \delta_1)}{1 - u} = 2 - 2^{1/\delta_1} \quad (5)$$

The Rotated Gumbel copula is the mirror image of the Gumbel copula, and its c.d.f. is:

$$C_{RG}(u, v; \delta_2) = u + v - 1 + C_G(1 - u, 1 - v; \delta_2) \quad , \quad \delta_2 \in [1, \infty) \quad (6)$$

The Rotated Gumbel copula is characterized by zero upper tail dependency and positive lower tail dependency for any $\delta_2 > 1$, i.e.

$$\lambda_L = \lim_{u \rightarrow 0^+} \frac{C_{RG}(u, u; \delta_2)}{u} = 2 - 2^{1/\delta_2} \quad (7)$$

⁹The Gumbel copula was introduced by Gumbel (1960), but since it was also discussed in Hougaard (1986), it is sometimes referred to as Gumbel-Hougaard copula.

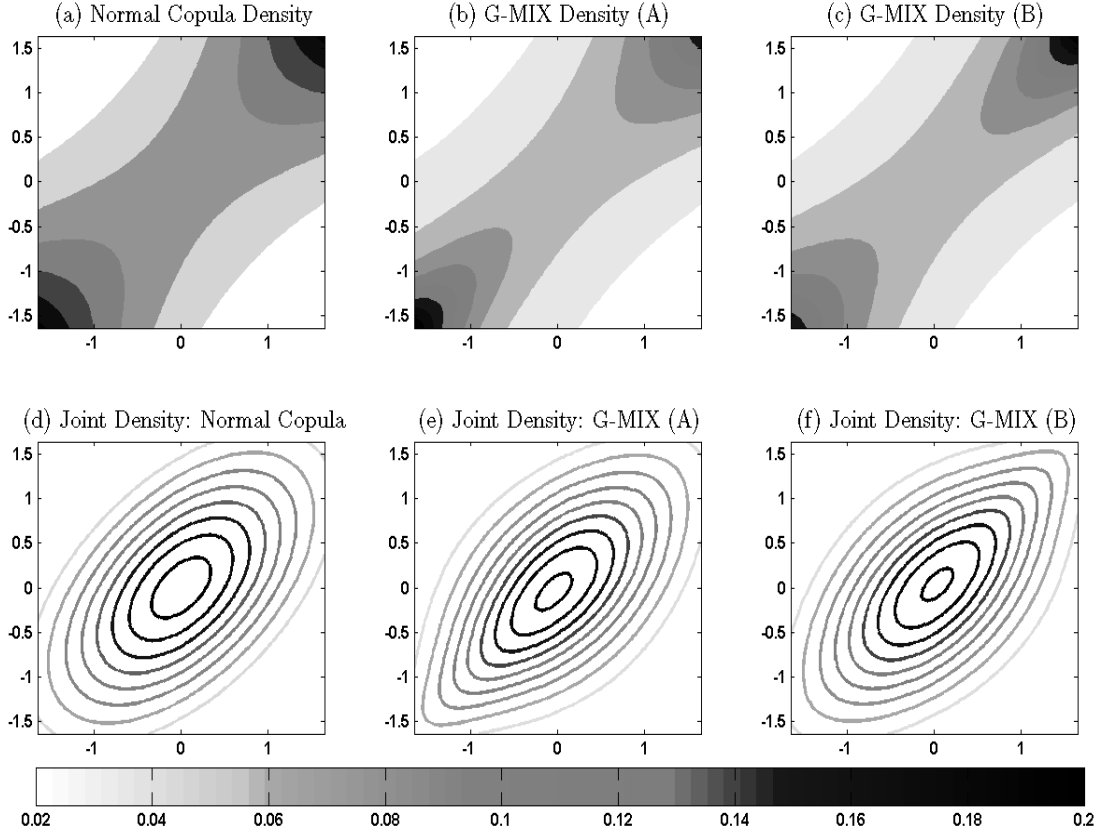


Figure 1: *Contour plots on 9/17/2003 (A) and 11/8/2006 (B), using standard normal margins*

Using the aforementioned properties of the Gumbel and Rotated Gumbel copulas we construct a, two-parameter, *Gumbel Mixture* copula defined as:

$$C_{GMIX}(u, v; \tau^U, \tau^L) = 0.5C_G(u, v; \delta(\tau^U)) + 0.5C_{RG}(u, v; \delta(\tau^L)) \quad (8)$$

where $\delta(\tau) = \log(2)/\log(2 - \tau)$ and $\tau^U \in [0, 1)$, $\tau^L \in [0, 1)$

The Gumbel Mixture copula can accommodate asymmetric dependence in either direction, i.e. the dependence is stronger for joint positive innovations whenever $\tau^U > \tau^L$ while the opposite is true if $\tau^U < \tau^L$. In the special case where $\tau^U = \tau^L$ the dependence pattern is symmetric, while when $\tau^U = \tau^L = 0$ the Gumbel Mixture

copula implies independence. Note that, since this copula is an equally weighted mixture of the Gumbel and the Rotated Gumbel specifications, its asymptotic upper and lower tail dependency are equal to $\tau^U/2$ and $\tau^L/2$, respectively.

Figure 1 illustrates how different copulas affect the shape of the bivariate density. The particular parameters of the Normal and Gumbel Mixture copulas were estimated from the prices of exchange rate options, using the methods discussed in Section 2.2 and Section 3.2. We have chosen two dates (9/17/2003 and 11/8/2006) where the dependence parameter of the Normal copula was approximately equal to 0.55. The respective parameters of the Gumbel Mixture copula were $\tau^U = 0.29$ and $\tau^L = 0.56$ for the first date (9/17/2003), while on the second date (11/8/2006) the estimated parameters were $\tau^U = 0.54$ and $\tau^L = 0.33$.

Figures 1.a to 1.c show the contours of the copula densities with the aforementioned parameters, while Figures 1.d to 1.f show the contours of the respective bivariate densities. We use standard normal margins in all cases, so as to isolate the effects of the different dependence patterns¹⁰. While the graphs would be identical for the case of the Normal copula, the plots corresponding to the Gumbel Mixture specification differ substantially, reflecting the opposite direction of asymmetry. As it can be seen from Figure 1.b (where the darker colors correspond to regions where the copula assigns higher probability) there is stronger dependence for joint negative than joint positive returns in first case, while the opposite case is depicted in Figure 1.c. Similarly, the bivariate density plots in Figure 1.e and 1.f illustrate how asymmetric dependence introduces skewness, whose direction depends on the level of the upper and lower tail dependence coefficients.

2.2 Extracting Dependence Information from Options

Similarly to the Black-Scholes implied volatilities, which can be computed using the prices of plain vanilla options, forward-looking measures of (risk-neutral) dependency involving two or more assets can sometimes be recovered from the prices of multi-asset and plain vanilla options. This is indeed true for the case of two exchange rates that are denominated in the same currency, as long as option prices on all three involved exchange rates (i.e. the two exchange rates with the common numeraire and the cross-rate) are observed and the law of one price holds. Within the copula framework adopted in this paper, this can be achieved using the results of

¹⁰Note that the parameter estimates of each copula were obtained using the risk-neutral GB2 densities (see Section 3.2 for details). The standard normal margins are applied only in these graphs, purely for exposition purposes.

Rosenberg (2003) and Taylor and Wang (forthcoming) who combine the triangular no-arbitrage condition, that links the three exchange rates, with copula theory and construct multivariate risk-neutral densities that are written as a product of the univariate risk-neutral densities and a risk-neutral dependence function.

To facilitate exposition of how this risk-neutral density representation can be used to extract dependency information, we discuss the construction of a copula-based multivariate risk-neutral density applied to our Dollar-Yen-Euro currency trio data. In what follows, we denote the time t price of €1 in Japanese Yen $S_t^{\text{¥}/\text{€}}$, the time t price of \$1 in Japanese Yen $S_t^{\text{¥}/\$}$ and the time t dollar price of €1 (i.e. the cross-rate) $S_t^{\$/\text{€}}$. The domestic risk-free rates corresponding to the Japanese Yen, U.S. Dollar and Euro currencies are $r_{\text{¥}}, r_{\$}$ and $r_{\text{€}}$, respectively.

Assume that markets are complete so that it is possible to define three unique risk-neutral densities $f_{\text{¥}}(x)$, $g_{\text{¥}}(y)$ and $m_{\$}(z)$, that can be used to price European options written on $S_t^{\text{¥}/\$}$, $S_t^{\text{¥}/\text{€}}$ and $S_t^{\$/\text{€}}$ respectively. The subscripts ¥ and \$ indicate whether the numeraire is the Japanese Yen or the U.S. Dollar. Furthermore, denote $k_{\text{¥}}(x, y)$ the bivariate risk-neutral density that can be used to price Yen payoffs that depend on $S_t^{\text{¥}/\text{€}}$ and $S_t^{\text{¥}/\$}$.

The fair dollar price of a European call option to buy €1 for \$ K at some future time T can be computed by taking expectations with respect to the univariate risk-neutral density $m_{\$}(z)$, i.e.

$$CALL_{\$}(K) = e^{-r_{\$}T} \int_K^{\infty} (z - K)m_{\$}(z)dz \quad (9)$$

From the viewpoint of a Japanese investor, the same contract can be viewed as an option to exchange one foreign currency asset for another foreign currency asset. Specifically, it is the same as an option to exchange $KS_T^{\text{¥}/\$}$ for $S_T^{\text{¥}/\text{€}}$ at time T , so that its fair price in Yen at time $t = 0$ is given by the bivariate risk-neutral density $k_{\text{¥}}(x, y)$ as follows:

$$CALL_{\text{¥}}(K) = e^{-r_{\text{¥}}T} E^Q [\max(y - Kx, 0)] \quad (10)$$

$$= e^{-r_{\text{¥}}T} \int_0^{\infty} \int_0^{\infty} \max(y - Kx, 0) k_{\text{¥}}(x, y) dx dy \quad , \quad (11)$$

while the fair price of this option in \$ is:

$$CALL_{\$}(K) = \frac{e^{-r_{\text{¥}}T}}{S_0^{\$/\text{¥}}} \int_0^{\infty} \int_0^{\infty} \max(y - Kx, 0) k_{\text{¥}}(x, y) dx dy \quad (12)$$

Rosenberg (2003) and Taylor and Wang (forthcoming) use equation (2) and decompose the bivariate risk-neutral density $k_{\text{¥}}(x, y)$ into a product of the marginal risk-neutral densities and the density of the risk-neutral copula, which leads to the following pricing formula:

$$CALL_{\$}(K) = \frac{e^{-r_{\text{¥}}T}}{S_0^{\$/\text{¥}}} \int_0^{\infty} \int_0^{\infty} \max(y - Kx, 0) f_{\text{¥}}(x) g_{\text{¥}}(y) c(F_{\text{¥}}(x), G_{\text{¥}}(y)) dx dy \quad , \quad (13)$$

where the $F_{\text{¥}}(x)$, $G_{\text{¥}}(y)$ are the cumulative distributions functions corresponding to $f_{\text{¥}}(x)$ and $g_{\text{¥}}(y)$, while $c(\cdot, \cdot)$ is the probability density function of the copula.

When all marginal risk-neutral densities can be recovered from the prices of traded options, this dual pricing representation, i.e. the pricing of a cross-rate option either as a plain vanilla or a multi-asset product (see equation (9) and equation (13), respectively) can provide insights regarding the covariation of exchange rate returns. Specifically, this can be accomplished by finding the copula that provides consistent prices for the cross-rate options. This “implied” information is reflective of the dependency of one-period exchange rate returns, where the length of the period corresponds to the maturity date of the observed options.

Estimating the dependence function from option data raises the important question of which copula to select¹¹. Since there is no obvious choice, the answer usually depends on the problem at hand. If the estimation of all marginal densities can be done in a reliable fashion and the goodness-of-fit to the observed options data is the primary concern, then a flexible specification seems appropriate. Following this route, Bennett and Kennedy (2004) rely on a copula constructed by a Normal copula that is perturbed by a cubic spline, while Salmon and Schleicher (2006) prefer the Bernstein copula of Sancetta and Satchell (2004) that can approximate any dependence function. Since the interest of this paper is to incorporate option-based information in a time-series model, these flexible alternatives were discarded since they are not very operational from a modeling perspective. Instead, we have decided

¹¹Taylor and Wang (forthcoming) investigate, empirically, which copula to choose when limited information is available from the prices of cross-rate options.

to employ the (one-parameter) Normal copula, whose time-varying version is well-studied in the literature, as well as the (two-parameter) Gumbel Mixture copula which allows us to consider more complex dependence patterns.

2.3 Conditional Copula Models

The copula representation of a bivariate density provides a flexible way of linking stationary distributions. However, a stylized fact of financial return data is that their distributions vary across time, for instance they are heteroskedastic. A direct application of Sklar's theorem to time-series data is therefore inappropriate, as it is inherently static.

Towards this purpose, Patton (2006b) has extended Sklar's theorem to conditional distributions. Denoting W the conditioning set, the copula representation a bivariate conditional cumulative distribution function (c.d.f.) $K(x, y|W)$ and conditional probability distribution function (p.d.f.) $k(x, y|W)$ is:

$$K(x, y|W) = C(F(x|W), G(y|W) |W) \quad \text{and} \quad (14)$$

$$k(x, y|W) = c(F(x|W), G(y|W) |W) \cdot f(x|W) \cdot g(y|W) \quad (15)$$

Patton (2006b) uses the above decomposition in order to estimate the conditional joint distribution of exchange rate returns. In his approach, the dynamics of the univariate return series $X = (r_{x,1}, r_{x,2}, \dots, r_{x,n})'$ and $Y = (r_{y,1}, r_{y,2}, \dots, r_{y,n})'$ are modeled using separate GARCH specifications, with parameter vectors θ_x and θ_y , respectively. The degree of association imposed by the conditional copula is also allowed to vary across time, according to the parameter vector θ_c and past return data of both X and Y . The estimation of the parameters θ_x , θ_y and θ_c is carried out using maximum-likelihood, i.e. the following quantity is maximized:

$$\begin{aligned} \log k_t(r_{x,t}, r_{y,t}|W_{t-1}) &= \sum_{t=1}^n \log c_t(F_t(r_{x,t}|W_{t-1}, \theta_x), G_t(r_{y,t}|W_{t-1}, \theta_y) |W_{t-1}, \theta_c) \\ &\quad + \sum_{t=1}^n \log f_t(r_{x,t}|W_{t-1}, \theta_x) \\ &\quad + \sum_{t=1}^n \log g_t(r_{y,t}|W_{t-1}, \theta_y) \end{aligned} \quad (16)$$

If the parameters of the marginal distributions can be separated from each other and from those of the copula, then it is possible to estimate the model in two

steps, i.e. find the parameters that maximize the log-likelihood contribution of the marginal densities and then estimate the copula parameters by maximizing the first summation in equation (16). In the case of multi-stage estimation, the standard errors of the estimated parameters need to be adjusted accordingly, as thoroughly discussed in Patton (2006a).

Models for the marginal distributions

Commencing with Bollerslev (1987) a t -GARCH(1,1) model has emerged as a parsimonious, yet credible, specification that can capture the essential features of observed exchange rate return data, namely heteroskedasticity of raw returns and excess kurtosis of conditional densities. Along these lines, we model the mean and conditional variance of each logarithmic return series as follows:

$$r_t = \mu + e_t, \quad (17)$$

$$e_t = \sqrt{h_t} z_t, \quad z_t \sim i.i.d. t_\nu \quad (18)$$

$$h_t = \omega + \alpha e_{t-1}^2 + \beta h_{t-1} \quad , \quad (19)$$

where t_ν is the (zero mean unit variance) standardized Student's t distribution with ν degrees of freedom.

Maximizing the log-likelihood of the model for each of the return series (i.e. setting $r_t = r_{x,t}$ or $r_t = r_{y,t}$) provides the estimated parameter vectors $\theta_x = (\mu_x, \omega_x, \alpha_x, \beta_x, \nu_x)'$ and $\theta_y = (\mu_y, \omega_y, \alpha_y, \beta_y, \nu_y)'$ that define the time t conditional probability density functions of the daily return innovations $f_t(r_{x,t}|W_{t-1}, \theta_x)$ and $g_t(r_{y,t}|W_{t-1}, \theta_y)$, as well as the corresponding conditional cumulative distribution functions $F_t(r_{x,t}|W_{t-1}, \theta_x)$ and $G_t(r_{y,t}|W_{t-1}, \theta_y)$.

It is important to clarify that the conditioning set W_{t-1} must be common for both series, i.e. the mean and variance specifications of $r_{x,t}$ must be conditioned on both $r_{x,t-1}, r_{x,t-2}, r_{x,t-n}$, as well as, $r_{y,t-1}, r_{y,t-2}, r_{y,t-n}$ and vice versa. Otherwise, as emphasized in Patton (2006b), the conditional copula will not generally define a valid joint distribution function. However, tests discussed in Section 4.2 suggested that the dynamic distributions of either $r_{x,t}$ and $r_{y,t}$ depended only on the past values of their own series, which justifies the exclusion of irrelevant variables and the application of the t -GARCH(1,1) specifications described earlier.

Another critical assumption that is required for the application of conditional copula theory is that the models for margins are correctly specified. If any of the marginal models is not correct, then the conditional probability integral transforms

(either $F_t(r_{x,t}|W_{t-1}, \theta_x)$ or $G_t(r_{y,t}|W_{t-1}, \theta_y)$) will not be i.i.d. *Uniform* $(0,1)$, so any copula model will automatically be misspecified. For this reason this assumption should always be tested, and we do so in Section 4.2.

Models for the joint distribution

Given the empirical evidence indicating that the dependence between financial returns is changing across time, it is necessary to allow the dependence parameters of the conditional copulas to be time-varying as well. Following Granger, Teräsvirta and Patton (2006), Patton (2006b) and Bartram et al (2007) we use lagged values¹² of the variable $|u_i - v_i|$ in order to capture changes in the degree of conditional dependency¹³. The intuition behind this selection is that the quantity $|u_i - v_i|$ becomes smaller (higher) in expectation as the dependence between u and v increases (decreases). Furthermore, we consider the case where option-implied measures are included as additional explanatory variables in the equations that determine the conditional dependency parameters of the alternative copula specifications. Specifically, for the case of the Normal copula the dependence parameter ρ_t evolves as follows:

$$\rho_t = \Lambda \left(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha_\rho \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}| + \gamma_\rho \rho_{Q,t-1} \right), \quad (20)$$

where $u_t = F_t(r_{x,t}|W_{t-1}, \theta_x)$ and $v_t = G_t(r_{y,t}|W_{t-1}, \theta_y)$ are the univariate cumulative probabilities given by the marginal models, $\rho_{Q,t}$ is the option-implied “correlation” coefficient at time t and $\Lambda(x) \equiv (1 + e^{-x})(1 - e^{-x})^{-1}$ is a modified logistic transformation that ensures that ρ_t is always between -1 and 1. Dropping the coefficient γ_ρ from this model results in a more “standard” conditional copula specification, in that only historical return information is utilized.

Concerning the Gumbel Mixture copula, the upper and lower dependency parameters τ_t^U and τ_t^L are modeled as follows:

$$\tau_t^U = \bar{\Lambda} \left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}| + \gamma_U \tau_{Q,t-1}^U \right) \quad (21)$$

¹²The selection of the number of lags corresponding to the $|u_i - v_i|$ observations is somewhat arbitrary. We have estimated the “simple” (i.e. without option-based information) Normal and Gumbel mixture models with different lag lengths (5, 10 and 15) and found that 10 lags provided the best results in terms of log-likelihood levels. For this reason we have decided to use 10 lags for all models, which is the same as in Patton (2006b).

¹³For the case of the Normal copula we have also used the variable $\Phi^{-1}(u_{t-i}) \cdot \Phi^{-1}(v_{t-i})$ but we have obtained worst results compared to the forcing variable $|u_i - v_i|$.

$$\tau_t^L = \bar{\Lambda} \left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}| + \gamma_L \tau_{Q,t-1}^L \right) , \quad (22)$$

where $\bar{\Lambda}(x) \equiv (1 + e^{-x})^{-1}$ is a logistic transformation that keeps the values of τ_t^U and τ_t^L between 0 and 1. The variables $\tau_{Q,t}^U$ and $\tau_{Q,t}^L$ represent upper and lower tail dependence indicators, computed from the prices of exchange rate options at time t . Again, dropping the last coefficients in the two equations defines a model where only past return data are used for the estimation of the copula's dependence parameters.

3 Practical Implementation

3.1 Data Issues

Spot rates of the U.S. Dollar/Japanese Yen, Euro/Japanese Yen and Euro/U.S. Dollar exchange rates are obtained from the Bank of England. They correspond to the exchange rates observed by the Bank's Foreign Exchange Desk in the London inter-bank market around 4 PM every day. We collect these spot rates from 1/11/2001 until 29/6/2007, which corresponds to 1430 daily observations for each exchange rate.

For the construction of option-based dependence measures, implied volatilities are collected from the British Bankers Association (BBA) website. The BBA publishes 25-delta "risk reversals" and 25-delta "butterfly spreads", as well as the "at-the-money" implied volatilities¹⁴, corresponding to options that have a constant expiration of one month¹⁵. These measures are computed by aggregating information from the over-the-counter market in London. Every day, approximately 12 major participants in the FX option market provide their volatility quotes which are then averaged by the BBA, after removing the two highest and lower rates¹⁶. All contributors have to supply their rates between 3:30 and 3:50 PM, which is slightly before the spot rates are recorded by the Bank of England. The BBA is also the source of the interest rate data required for converting the implied volatility data into option prices. Specifically, the BBA London InterBank Offer Rate is calculated

¹⁴These measures simply reflect the standard market quotation practice; it is straightforward to convert them into option prices with strike prices corresponding to "deltas" of 0.25, 0.5 and 0.75. See Section 3.2 for further details.

¹⁵The BBA source also provides data for other maturities but these have more missing observations.

¹⁶If fewer than 5 rates are received by the contributors, then the benchmark is not published.

using a panel of major banks and represents an indicative interest rate at which these banks can borrow from each other in a particular currency. These rates are distributed daily and reflect prevailing interest rates around 11 AM. In the case where some of the aforementioned inputs, which are essential for the computation of option prices, are missing we simply set the option-implied measures to their last known values.

As this paper discusses the informational content of option-implied and return-based dependency measures, it is important to highlight that the timing at which the required inputs are recorded does not offer an a priori advantage to the option-based estimates. This is because both implied volatilities and LIBOR rates are recorded before the corresponding daily spot rates, so option prices are computed using information that is already known to the market at the time at which spot rates are observed. On the contrary, the fact that the inputs required for the estimation of option prices (spot rates, implied volatilities and interest rates) are collected at different times, introduces measurement errors in the option-implied measures and, hence, impairs their performance. Nevertheless, we do not expect this non-synchronicity problem to cause severe distortions, since the time at which the implied volatilities are recorded is quite close to that of the spot rates, while interest rates exhibit far less intraday fluctuations.

3.2 Risk-Neutral Density Estimation

The BBA data, in conjunction with equations (9) and (13), provide the necessary information to extract the “implied” dependence parameter(s) of a given copula. The implied volatility data provided by the BBA follow the over-the-counter FX market conventions where, contrary to exchange-listed options, the quotes are in terms of implied volatilities (instead of option prices) at fixed “deltas” (instead of fixed strike prices). The “deltas” are those of the Garman-Kohlhagen (1983) option pricing formula, where the delta of a call option (Δ_c) at time $t = 0$ is defined as:

$$\Delta_c = e^{-r_f \tau} \Phi(d_1) \quad , \quad d_1 = \frac{\ln(S_t/K) + (r_d - r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad , \quad (23)$$

with S_t the current spot price, σ the volatility parameter, r_d and r_f the continuously compounded domestic and foreign risk-free rates, T the time to maturity and $\Phi(\cdot)$ the c.d.f. of the standard normal density.

For each of the exchange rates, the options data are in the form of a delta-neutral

straddle implied volatility (ATMIV), a 25-delta risk reversal (RR_{25}) and a 25-delta butterfly spread (BF_{25}) defined as:

$$ATMIV = \sigma(0.5) \quad (24)$$

$$RR_{25} = \sigma(0.25) - \sigma(0.75) \quad (25)$$

$$BF_{25} = 0.5\sigma(0.25) + 0.5\sigma(0.75) - ATMIV \quad , \quad (26)$$

where $\sigma(\Delta_c)$ is the implied volatility of a call option with delta equal to Δ_c . Note that the delta-neutral straddle implied volatility corresponds to the case where $d_1 = 0$, so that the strike price is very close to either the spot or the forward price. For this reason we refer to this quote as the at-the-money implied volatility quote (ATMIV).

Following Malz (1997), a continuous implied volatility “smile” can be constructed using the following functional form:

$$\hat{\sigma}(\Delta_c) = b_0 ATMIV + b_1 RR_{25}(\Delta_c - 0.5) + b_2 BF_{25}(\Delta_c - 0.5)^2 \quad (27)$$

As discussed in Malz (1997), setting $b_0 = 1$, $b_1 = -2$ and $b_2 = 16$ ensures that the implied volatility function perfectly matches the observed market data (i.e. the ATMIV, RR_{25} and BF_{25}) and provides a reasonable approximation to the implied volatility “smile”, especially for “deltas” that are roughly between 0.15 and 0.85.

For each of the three exchange rates, we use the aforementioned functional form to generate implied volatilities at equally spaced deltas, ranging from 0.15 and 0.85, with the stepsize set to 0.01. Each implied volatility-delta pair is then converted to the (call) option price-strike space creating, thus, a series of call option prices $CALL_j(K_{j,1}), CALL_j(K_{j,2}), \dots, CALL_j(K_{j,N})$, with j denoting the exchange rate under study (i.e. the Dollar-Euro (DE), Dollar-Yen (DY) or Euro-Yen (EY) rate) and $K_{j,1}, K_{j,2}, \dots, K_{j,N}$ the respective strike prices.

We treat these prices as if they were observed in the market, and then fit the Generalized Beta distribution of the second kind¹⁷ (GB2), proposed by Bookstaber and MacDonald (1987), to estimate the risk-neutral densities of the Yen denominated currencies¹⁸, i.e. for $j = DY$ or $j = EY$. The parameter vector of each GB2

¹⁷For some useful properties of the GB2 density see also Taylor (2005) and Liu et al (2007)

¹⁸It is important to clarify that, since all risk-neutral densities can be recovered by numerical differentiation of the call price functions, the application of the GB2 density is not technically required. However, the GB2 density has closed form expressions for both the cumulative distribution function and probability density function which, in light of equation (13), is a very appealing property from a computational perspective. We do not expect significant estimation errors from this simplification, since experimental results showed that the flexible GB2 density generally provides a very close fit to the implied volatility function produced by the Malz (1997) method.

density is estimated by minimizing the average of the squared pricing errors, i.e.

$$\frac{1}{N} \sum_{i=1}^N (CALL_j(K_{j,i}) - CALL_{GB2,j}(K_{j,i}|\theta_{GB2,j}))^2, \quad (28)$$

where $CALL_{GB2,j}(K_{j,i}|\theta_{GB2,j})$ is the theoretical (fitted) price of a call option given by a GB2 density with parameter vector $\theta_{GB2,j}$.

The univariate risk-neutral densities of the Yen denominated exchange rates (defined by $\theta_{GB2,DY}$ and $\theta_{GB2,EY}$) and a copula function (i.e. either the Normal or the Gumbel Mixture specification) with parameter vector θ_c^Q (containing either ρ_Q or τ_Q^U and τ_Q^L) can be used generate prices of cross-rate call options, say $CALL_{DY}^C(K_{DY,i})$, through equation (13). Conversely, the ‘‘observed’’ cross-rate option prices, denoted as $CALL_{DY}(K_{DY,i})$, can be used to estimate the copula’s dependence parameter vector θ_c^Q , by minimizing the average of the squared pricing errors, i.e.

$$\frac{1}{N} \sum_{i=1}^N (CALL_{DY}(K_{DY,i}) - CALL_{DY}^C(K_{DY,i}|\theta_c^Q))^2, \quad (29)$$

Repeating the same procedure for both copula specifications and for all days in the sample, we obtain the corresponding degree of dependence estimates, which we can then introduce as explanatory variables in the equations that determine the evolution of dependence in the conditional copula models.

It should be noted that these implied dependence estimates are reflective of the dependence pattern of one month return innovations (since we use options with one month to maturity), which is not the same as the daily return innovations used in the time-varying conditional copula models. In any case, the empirical results will indicate whether these option-implied estimates are informative despite the distortions caused by this simplification.

4 Empirical Results

4.1 Risk-Neutral Dependency Estimates

Figure 2.a and Figure 2.b show the option-implied parameters of the two copulas. In the first graph, the black line corresponds to the implied parameter of the Normal

copula (Rho Q), while the grey line corresponds to the conditional copula parameter that is estimated using past return data (Rho P)¹⁹. Not surprisingly, the two parameter estimates move in tandem across time. The, implied, upper and lower tail dependence parameters (denoted as UTD and LTD, respectively) of the Gumbel Mixture copula are displayed in Figure 2.b²⁰. Excluding a few observations, where the lower tail dependence parameter is slightly jagged, both series appear to be well-behaved. Upper tail dependence is generally higher than lower tail dependence at the beginning of the sample, while the opposite is true from the second quarter of 2003 until the second quarter of 2004. Subsequently, the dependence becomes roughly symmetric, i.e. the difference between upper and lower tail dependency (see Figure 2.c) evolves around zero until the end of the dataset.

Patton (2006b) suggests that central bank interventions can introduce asymmetric dependence of exchange rate returns. For the dataset that we examine, the Bank of Japan (BoJ) was the only monetary authority that has actively tried to influence the involved exchange rates²¹. Figure 2.d displays the daily volume (in billion Yen) of the BoJ interventions from the end of 2001 until the middle of 2007. These correspond to U.S. Dollar purchases using Japanese Yen, which represent the vast majority of the BoJ interventions²². According to Ito (2007), three distinct BoJ intervention regimes took place between November 2001 and June 2007. The first regime (21 June 1995 - 14 January 2003) was characterized by infrequent large-scale interventions, while during the second regime (15 January 2003 - 1 July 2004) the interventions were both frequent and large scale. In the last regime (from 2 July 2004 until the end of the dataset) BoJ adopted a no-intervention policy.

The two solid vertical lines (in grey) spanning all subfigures, indicate the end of the first and second regime respectively. The two dashed vertical lines (also in grey) indicate the dates when the public became officially aware about the changes in the intervention activities of the BoJ. Specifically, on May 8, 2003 (first line) it became known that there was a switch to a more frequent intervention regime,

¹⁹A detailed discussion concerning the results of the conditional copula models is provided in Sections 4.3-4.4.

²⁰As noted in Section 2.1, the actual asymptotic upper or lower tail dependence of the Gumbel Mixture copula is equal to the half of its corresponding parameter value.

²¹The BoJ intervention dates and respective amounts are published by the Japanese Ministry of Finance.

²²From June 2002 until May 2003 BoJ also intervened in the Euro/Yen exchange rate (buying Euros with Japanese Yen), however these actions are not comparable, in terms of either frequency or scale, with the U.S. Dollar interventions. For reasons of clarity, we only include the U.S. Dollar activities in Figure 2.d

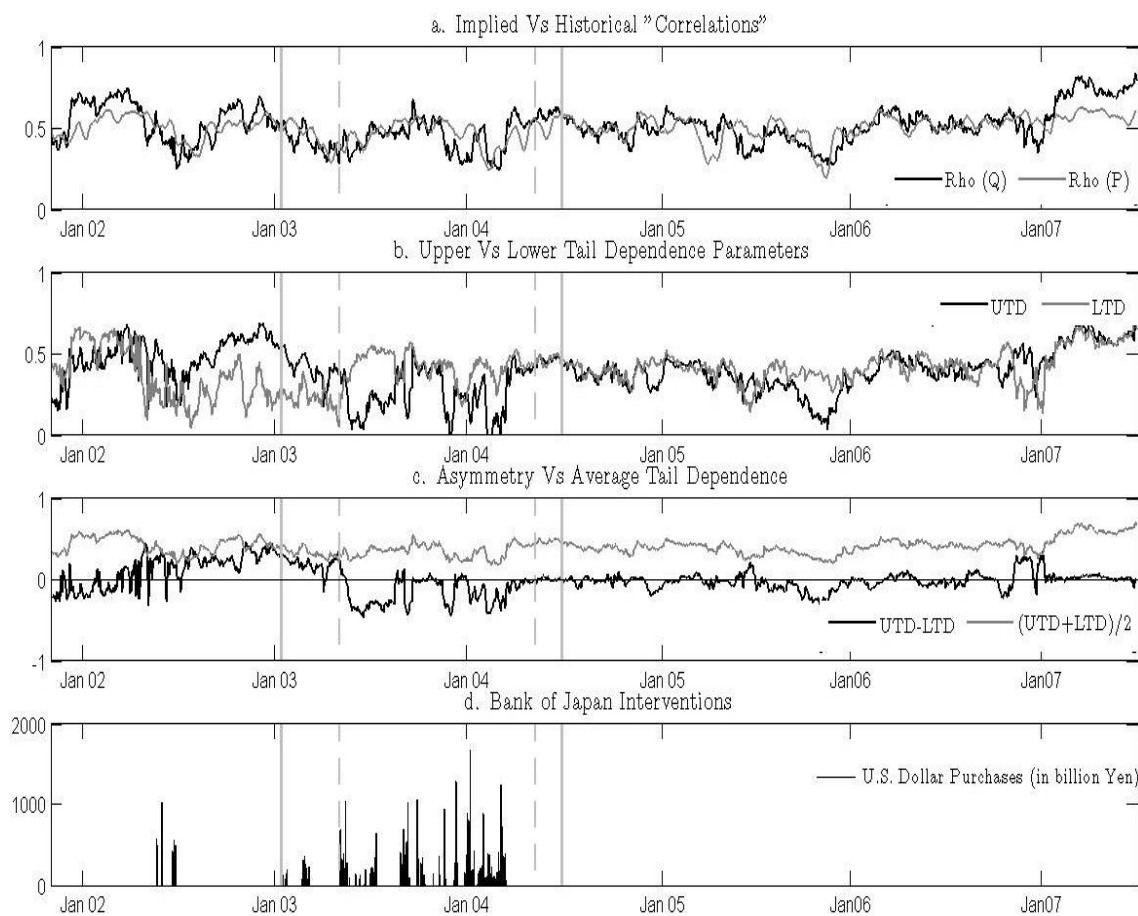


Figure 2: Option-Implied Dependence Expectations and BoJ Interventions

while on May 12, 2004 (second line) it was announced that no interventions were conducted for almost two months²³. We have highlighted the aforementioned dates because, during this particular regime, the BoJ has employed the, so called, “stealth intervention” tactic, i.e. interventions were conducted without announcement or an unintentional or intentional leak (Ito, 2005).

It is interesting to note that the asymmetry changes direction after the transition

²³At the end of April 2004 the market already knew that no interventions took place within that month, but there was still speculation about the details of the March 2004 activities. See Ito (2005) for a detailed discussion regarding these events.

to the second regime, especially after May 8, 2003 when the intervention details became publicly known. Furthermore, the difference between upper and lower tail dependency stabilizes close to zero around the time when the market recognized that the BoJ has stopped its interventions activities. The fact that some changes in the BoJ intervention policies appear to roughly coincide with changes in asymmetric dependence, is in line the explanation proffered in Patton (2006b)²⁴. Moreover, it is in agreement with Andreou and Ghysels (2003) and Castren and Mazzotta (2005), who find the BoJ interventions are related with changes in exchange rate correlations, as well as Castren (2004), who reports that the three risk-neutral densities of the Dollar-Yen-Euro currency trio exhibit systematic changes during episodes of BoJ interventions. Establishing robust links between central bank interventions and asymmetric dependence would be an interesting study, but it is beyond the scope of this paper.

4.2 Models for the Marginal Distributions

Table 1 shows the estimation results of the t -GARCH(1,1) specifications applied to the U.S. Dollar/Japanese Yen (DY) and Euro/Japanese Yen (EY) exchange rate return series. Panel A contains the parameter estimates along with the t-statistics corresponding to the null that the parameter in question is zero²⁵. Both series exhibit similar volatility persistence, with the sum of the ARCH (α) and GARCH (β) parameters being equal to 0.98 and 0.99, for the DY and EY series respectively. Furthermore, the hypothesis that the distribution of the standardized innovations is standard normal is strongly rejected in both cases. A distributional specification that is characterized by fatter tails is required to capture the essential features of the data, as indicated by the degrees of freedom estimates which are equal to 6.42 (DY) and 9.39 (EY).

Panel B contains some familiar misspecification tests applied to the standardized residual series, namely the Ljung-Box (1978) and Engle's (1982) ARCH LM test (for any remaining autocorrelation and heteroskedasticity respectively), the Kolmogorov-Smirnov test (for the adequacy of the distributional assumptions) and the LR3 test of Berkowitz (2001) which assesses whether $u_t = F_t(r_{x,t}|W_{t-1}, \theta_x)$ and $v_t = G_t(r_{y,t}|W_{t-1}, \theta_y)$ are i.i.d. Uniform (0,1). None of these tests suggest that either marginal model is misspecified, since all p-values are higher than conventional

²⁴We note that, since our option-implied estimates are computed by taking expectations with respect to the corresponding risk-neutral measures, a risk-premium effect could also be at play.

²⁵Except for the degrees of freedom parameter ν where the null is that $1/\nu$ is equal to zero.

TABLE 1. MARGINAL MODELS

<i>Panel A. GARCH Estimates</i>										
	DY					EY				
	μ_x	ω_x	α_x	β_x	ν_x	μ_y	ω_y	α_y	β_y	ν_y
Estimate	0.014	0.008	0.028	0.95	6.42	0.043	0.003	0.027	0.96	9.39
T-Stat.	0.99	1.89	2.88	57.6	6.14	3.21	1.38	2.95	72.54	4.56
Log-Lik.	-1184.32					-1114.58				

<i>Panel B. Diagnostics on standardized residuals</i>								
	DY				EY			
	L-B	ARCH	K-S	LR3	L-B	ARCH	K-S	LR3
T-Stat.	4.2	7.53	0.026	0.31	10.01	10.86	0.027	1.67
P-Value	0.94	0.68	0.29	0.86	0.44	0.37	0.26	0.44

<i>Panel C. "Hit" Tests</i>								
	DY				EY			
Region	LL_{UNR}	LL_{RE}	LR Stat.	P-Val.	LL_{UNR}	LL_R	LR Stat.	P-Val.
R1	-301.84	-302.91	2.14	0.71	-307.81	-308.8	1.99	0.74
R2	-429.45	-431.7	4.5	0.34	-447.47	-449.28	3.62	0.46
R3	-794.32	-795.32	2.01	0.73	-780.63	-782.14	3.03	0.55
R4	-695.85	-699.88	8.06	0.09	-693.07	-697.11	8.07	0.09
R5	-819.92	-821.69	3.54	0.47	-835.1	-838.17	6.15	0.19
R6	-508.95	-510.8	3.7	0.45	-485.56	-488.83	6.53	0.16
R7	-245.67	-246.97	2.59	0.63	-241.17	-244.02	5.71	0.22

NOTE: **Panel A** contains the estimation results of the t -GARCH(1,1) models. A t -statistic higher than 1.96 (1.65) in absolute value indicates that the coefficient is significant at the 5% (10%) level. **Panel B** presents the results of some diagnostic checks performed on the standardized residuals, i.e. the Ljung-Box (with 10 lags), Engle's (1982) ARCH LM (with 10 lags) and Kolmogorov-Smirnov (K-S) tests, as well as the likelihood ratio test (LR3) of Berkowitz (2001). **Panel C** exhibits the results of the univariate "hit" tests applied for the seven regions of the probability density (R1-R7). See the Appendix for further details.

rejection thresholds.

Since the assumption that the marginal densities are well-specified is crucial for the application of conditional copula models, additional scrutiny on the adequacy of the univariate models is required. For this reason, we apply "hit" tests (described in the Appendix) to investigate whether the marginal distributions are well-specified,

not only in their entirety, but also in various segments of the density support²⁶. We consider seven regions in total. Region 1 (region 7) corresponds to the lower (upper) 5% tail, region 2 (region 6) to the interval from the 5th to the 15th (85th to the 95th) quantile, while region 3 (region 5) is referring to observations falling within the 15th and 40th (60th and 85th) quantiles. Finally, region 3 relates to the central part of the density, as it contains observations from the 40th until the 60th quantile. As shown in Panel C of Table 1, both models pass the test for all regions at the 5% level.

In all the aforementioned tests the i.i.d. assumption was examined using data from a single return series, i.e. data from the DY return series were used for the misspecification tests of the DY marginal model, and similarly for the EY exchange rate. The application of conditional copulas requires that the i.i.d. property holds with respect to the entire conditioning set, so that “cross-variable interactions” should also be taken into account.

Along these lines we conduct two last tests. In the first one, we examine whether the observations u_t and v_t are independent with respect to the lagged values of both series. Note that if u_t (v_t) can be forecasted using past values of either u_t or v_t then u_t (v_t) is not an observation from an i.i.d. series, which in turn implies that the estimated conditional distributions of the DY (EY) return series do not coincide with the true ones. Using a test very similar to those applied in Diebold et al (1998), Christoffersen and Mazzota (2005) and Patton (2006b), we attempt to uncover any dependence operative through the conditional mean, conditional variance, conditional skewness, or conditional kurtosis. Specifically, we use the inverse of the standard normal density $\Phi(\cdot)^{-1}$ to create two new series $z_{u,t} = \Phi(u_t)^{-1}$ and $z_{v,t} = \Phi(v_t)^{-1}$ and subsequently regress $(z_{u,t} - \bar{z}_{u,t})^q$ and $(z_{v,t} - \bar{z}_{v,t})^q$ on 5 or 10 lags of both variables for $q = 1, 2, 3, 4$. We then test whether the respective coefficients are jointly zero. As shown in Table 2, both series pass the “moment” tests, so the i.i.d. hypothesis can not be rejected.

As a final test we examine whether the univariate models can be adequately modeled using only their own past returns. Recall that in our standard GARCH specifications the marginal model for the DY (EY) exchange rate is estimated using past DY (EY) returns only, while conditional copula theory requires that the same information set is used for both the margins and the conditional copula. Since the latter is estimated using past returns of both exchange rates and, in certain cases, option-implied dependency measures, it is necessary to check whether the reduction

²⁶Particularly, we use the “hit” test methodology proposed in Patton (2006b), which is an extension of the “hit” regressions of Christoffersen (1998) and Engle and Manganelli (2004).

TABLE 2. CROSS-VARIABLE INTERACTIONS

Panel A. Moment Independence Tests

Dollar-Yen Margin									
	1 st Moment		2 nd Moment		3 rd Moment		4 th Moment		
Lags	5	10	5	10	5	10	5	10	
P-val.	0.43	0.87	0.23	0.31	0.86	0.48	0.77	0.78	

Euro-Yen Margin									
	1 st Moment		2 nd Moment		3 rd Moment		4 th Moment		
Lags	5	10	5	10	5	10	5	10	
P-val.	0.28	0.43	0.78	0.49	0.39	0.5	0.81	0.67	

Panel B. Wald Tests

Normal Copula				
	Regression Equation		Restrictions	P-Val.
<i>Conditional Mean</i>				
DY	$e_{x,t} = \beta_1 e_{y,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \rho_{Q,t-1} + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = 0$	0.98
EY	$e_{y,t} = \beta_1 e_{x,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \rho_{Q,t-1} + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = 0$	0.36
<i>Conditional Variance</i>				
DY	$z_{x,t}^2 = \beta_0 + \beta_1 e_{y,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \rho_{Q,t-1} + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = 0$	0.34
EY	$z_{y,t}^2 = \beta_0 + \beta_1 e_{x,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \rho_{Q,t-1} + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = 0$	0.08

Gumbel Mixture Copula				
	Regression Equation		Restrictions	P-Val.
<i>Conditional Mean</i>				
DY	$e_{x,t} = \beta_1 e_{y,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \tau_{Q,t-1}^U + \beta_4 \tau_{Q,t-1}^L + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.94
EY	$e_{y,t} = \beta_1 e_{x,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \tau_{Q,t-1}^U + \beta_4 \tau_{Q,t-1}^L + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.24
<i>Conditional Variance</i>				
DY	$z_{x,t}^2 = \beta_0 + \beta_1 e_{y,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \tau_{Q,t-1}^U + \beta_4 \tau_{Q,t-1}^L + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.48
EY	$z_{y,t}^2 = \beta_0 + \beta_1 e_{x,t-1}^2 + \beta_2 FV_{t-1} + \beta_3 \tau_{Q,t-1}^U + \beta_4 \tau_{Q,t-1}^L + \eta_t$		$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.13

NOTE: **Panel A** displays the results of the Lagrange Multiplier tests of serial independence in the first four moments of the variables u_t and v_t , described in Section 4.2. We regress $(z_{u,t} - \bar{z}_{u,t})^q$ and $(z_{v,t} - \bar{z}_{v,t})^q$ on 5 lags or 10 of both variables for $q = 1, 2, 3, 4$. The p-values correspond to the null that the coefficients are jointly zero. **Panel B** contains the results for the regressions that examine whether mean and variance models of the DY and EY series would improve if additional conditioning variables were used. This test is also discussed at Section 4.2. The time t residuals and standardized residuals of the GARCH model for DY (EY) are denoted as $e_{x,t}$ ($e_{y,t}$) and $z_{x,t}$ ($z_{y,t}$), respectively, $\rho_{Q,t}$, $\tau_{Q,t}^U$ and $\tau_{Q,t}^L$ represent the option-implied parameters of the Normal and Gumbel mixture copulas, while $FV_t = \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}|$ is the forcing variable that is included in all conditional copula models.

of the conditioning set used in the univariate models is innocuous. For this reason we examine whether the mean and variance specifications for the DY (EY) margin require the inclusion of the conditioning information used in the EY (DY) margin and the conditional copula.

Following Patton (2006b) we test for misspecification in the conditional mean of the DY (EY) series by regressing $e_{x,t}$ ($e_{y,t}$) on $e_{y,t-1}^2$ ($e_{x,t-1}^2$), the conditional copula's implied dependence parameter(s) and the "forcing variable" $\frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}|$. A Wald test is then used to check if the respective coefficients are jointly zero. To examine whether the model for the conditional variance of DY (EY) is adequate, we regress the squared standardized residuals $z_{x,t}^2$ ($z_{y,t}^2$) on the same explanatory variables plus a constant, and test if the coefficients of the explanatory variables are jointly zero. Since we will estimate two separate conditional copula models (Normal copula and Gumbel mixture copula) that have different conditioning sets (i.e. with or without option-implied dependency measures), we run eight regressions in total. As the results displayed in Table 2 Panel B indicate, we cannot reject the null at the 5% level in any these tests²⁷

4.3 Bivariate Models: In-Sample Comparisons

Preliminary Remarks

Since no evidence indicating misspecification of the univariate models were found, we can now proceed to the estimation of the Normal or Gumbel Mixture conditional copula models that will define the time-varying bivariate distributions of u and v . For each of these copula functions, time-variation in the degree of dependence parameter(s) is introduced using either historical return data or a combination of historical return and option-based information. To distinguish between the different information sets, we will refer to these models as *historical return* and *augmented* model, respectively. The main focus of the discussion that follows is whether the inclusion of option-implied dependence estimates enhances the performance of the models that rely solely on historical returns.

Along these lines we make comparisons in an in-sample as well as an out-of-sample setting. In-sample, we initially examine the significance of the coefficients

²⁷The regression corresponding to the conditional variance model of the Euro/Yen (EY) exchange rate suggest a rejection of the null at the 10% level, however, none of the coefficients is significantly different from zero at the 5% level when the robust standard errors of Newey and West (1987) are used. The detailed regression results are available from the author upon request.

referring to the option-implied measures and then compare the models using the Schwarz Information Criterion (SIC). Furthermore we perform “hit” tests to evaluate the goodness-of-fit of the competing bivariate models²⁸. To carry out these tests, we select seven economically meaningful segments of the joint density. Region 1 (region 5) refers to the case where both u and v are below 0.1 (above 0.9), while region 2 (Region 4) contains the u, v realizations that are between their 10th and the 25th (75th and the 90th) quantile. Region 3 relates to the central part of each distribution, as it consists of observations falling within the 25th and 75th quantiles. Finally, region 6 corresponds to the case where u is above its 75th quantile and v below its 25th quantile, while the reverse is true for Region 7.

For reasons of clarity, we will generally discuss in terms of the log-likelihood of the corresponding to the copula density, i.e.

$$Log-Like_c = \sum_{t=1}^n \log c_t(F(r_{x,t}|W_{t-1}, \hat{\theta}_x), G(r_{y,t}|W_{t-1}, \hat{\theta}_y) | W_{t-1}, \hat{\theta}_c) \quad (30)$$

instead of the log-likelihood of the full model. Likewise, we will refer to the SIC criterion corresponding to the copula, i.e.

$$SIC_c = -2 \cdot \log(Log-Like_c) + p \cdot \log(n) \quad , \quad (31)$$

where p is the number of estimated parameters in the reference model and n is the number of observations. The implications of any model comparison using this metric are the same as if the full bivariate model was used, since all models share the same specification for the marginal dynamics.

Results

It is useful to recall that the models of the time-varying Normal copula densities, presented in Section (2.3), have three free parameters for the “historical” and four for the “augmented” specification. Specifically, ω_ρ corresponds to the constant term, α_ρ is the coefficient of the forcing variable $\frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}|$, while β_ρ refers to the lagged degree of dependence. The coefficient γ_ρ that controls for the effect of the option-implied dependency “forecasts” is only present in the augmented model. In an analogous fashion, the parameters of the Gumbel mixture models are $\omega_U, \alpha_U, \beta_U, \gamma_U$ for the upper tail and $\omega_L, \alpha_L, \beta_L, \gamma_L$ for the lower tail dependence case. We begin our discussion with the Normal copula models. Table 3 contains the parameter

²⁸The details of these tests are provided in the Appendix

TABLE 3. CONDITIONAL NORMAL COPULA RESULTS

<i>Historical Return Model</i>				
	ω_ρ	α_ρ	β_ρ	γ_ρ
Estimate	0.152	-0.611	2.192	-
T-Stat.	0.634	-1.337	8.014	-
P-Value	0.263	0.091	0	-
Log-Lik.				209.123
SIC				-396.45
<i>Augmented Model</i>				
	ω_ρ	α_ρ	β_ρ	γ_ρ
Estimate	-1.433	2.039	-1.812	5.869
T-Stat.	-2.343	1.532	-9.337	7.376
P-Value	0.01	0.063	0	0
Log-Lik.				230.93
SIC				-432.8

NOTE: Estimation results of the conditional Normal copula models, described in Section 2.3.

The p-values and t-statistics correspond to the null that the respective coefficients are equal to zero. The asymptotic variance-covariance matrix of the estimated parameters is computed using the methodology discussed in Patton (2006a). The log-likelihoods and SIC statistics refer to the densities of the conditional copulas.

estimates, as well as the t-statistics and respective p-values corresponding to the null that the coefficient in question is equal to zero. All coefficients are significant at the 10% level, except for ω_ρ in the “historical” model. The γ_ρ coefficient is highly significant, indicating that options contain useful information regarding the evolution of exchange rate dependence across time. It is worth noting that the α_ρ coefficient, that has a p-value of 9% at the simple model, remains statistically significant at the (at the 7% level) in the augmented specification, suggesting that the information contained in the record of historical returns complements that of option-based, risk-neutral, “forecasts”. The enhancement over the simple model is also evident by the non-trivial difference in the SIC values of the two models. In particular, the SIC statistic of the augmented specification is equal to -432.8 which is smaller, and hence more preferable, than that of the historical return model (-396.45).

The results of the “hit” tests, presented in Table 5, indicate that both models appear somewhat problematic in Region 7 (where u is smaller than 0.25 and v is

TABLE 4. CONDITIONAL GUMBEL MIXTURE COPULA RESULTS

<i>Historical Return Model</i>								
	ω_U	α_U	β_U	γ_U	ω_L	α_L	β_L	γ_L
Estimate	-1.497	-2.058	3.864	-	0.214	5.039	-3.714	-
T-Stat.	-1.59	-0.929	4.446	-	0.133	1.31	-15.58	-
P-Value	0.056	0.176	0	-	0.447	0.095	0	-
Log-Lik.	227.37							
SIC	-411.14							
<i>Augmented Model</i>								
	ω_U	α_U	β_U	γ_U	ω_L	α_L	β_L	γ_L
Estimate	-2.218	-0.545	3.629	1.11	-2.365	4.909	-3.833	7.065
T-Stat.	-2.307	-0.095	1.01	0.24	-1.279	0.689	-8.279	1.67
P-Value	0.011	0.462	0.156	0.407	0.101	0.246	0	0.046
Log-Lik.	243.38							
SIC	-428.65							

NOTE: Estimation results of the conditional Gumbel Mixture copula models, described in Section 2.3. The p-values and t-statistics correspond to the null that the respective coefficients are equal to zero. The asymptotic variance-covariance matrix of the estimated parameters is computed using the methodology discussed in Patton (2006a). The log-likelihoods and SIC statistics refer to densities of the conditional copulas.

larger than 0.75) of their estimated bivariate densities. The corresponding p-values are approximately 0.05 for the historical and 0.07 for the augmented model, so that the null would be rejected at the 5% and 10% level, respectively. No evidence of misspecification can be found for rest of the regions, since the null is never rejected for either model, while an interesting pattern is that the p-values are always (slightly) higher for the model that includes option-based dependence measures.

Estimation results for the conditional copula models that assume a Gumbel Mixture dependence structure for the DY, EY exchange rate return series are displayed in Table 4. In the “historical return” model the autoregressive parameters β_U and β_L are highly significant, while ω_U and α_L are significant at the 10% level. On the contrary, α_U and ω_L are not significantly different from zero, with p-values of 0.18 and 0.45, respectively. For the “augmented” Gumbel Mixture specification, the coefficient α_L is no longer significant (p-value of 0.25), while ω_L and β_L are significant at

the 11% and 1% level respectively. Interestingly, γ_L has a p-value of 0.046, indicating that the lower tail dependence measures extracted from option prices are helpful in describing the dynamics of lower tail dependency across time. For the upper tail dependence case, the only coefficient that is statistically significant is ω_U . The rest of the estimated coefficients, i.e. α_U , β_U and γ_U , are not significantly different from zero, as their p-values are 0.46, 0.16 and 0.41 respectively.

It is worth noting that the p-value of α_U is increasing when option-implied estimates are included in the model and is the highest amongst all other coefficients in the augmented specification. Hence, one plausible explanation for the fact that the γ_U coefficient is not significant could be that the information provided by historical returns and option based estimates is of a similar nature²⁹. Maximizing the log-likelihood of a model that excludes both γ_U and α_U delivers a SIC value of -425.42, which is higher (and therefore less preferable) than that of the full model whose SIC value is -428.65. Since the exclusion of the corresponding variables appears to worsen the performance of the full model, it is not unreasonable to conjecture that these variables have a useful role in the modeling of the corresponding bivariate density.

The hit-tests for the Gumbel Mixture copula models are presented in Table 5. As it can be seen therein, neither the historical nor the augmented specification have problems passing these misspecification tests for any of the seven regions. The p-values are in most cases higher than those of the Normal copula models, but such comparisons are probably misleading since the less parsimonious Gumbel Mixture models are generally expected to provide a better in-sample fit.

Model selection using the Schwarz Information Criterion favors the augmented Normal copula specification that has the lowest value (-432.8) amongst all other alternatives, although the augmented Gumbel Mixture model achieves a comparable SIC statistic (-428.65). There is sizable difference between the aforementioned SIC values and those of the models that rely exclusively on historical return information, which underscores the benefits of including option-implied measures of covariation. In particular, the simple Gumbel Mixture has a SIC value of -411.14, while the

²⁹Of course, several other explanations can be proffered. For instance, the presence of “correlation” risk-premiums, i.e. the difference between expected “correlations” under the Q (risk-neutral) and P (real-world) measures, will impair the “forecast” performance of option-based dependence estimates. Such a wedge between the expectations under the Q and P measure can occur in realistic situations, such as when the price processes includes jump components (Branger and Schlag, 2004) or when correlations follow diffusion processes (Driessen et al, 2009). One can also consider model misspecification errors when extracting the dependence measures from option prices, or insufficient number of observation in the time-series model (since the standard errors are asymptotic).

TABLE 5. IN-SAMPLE HIT TESTS, JOINT DENSITY

NORMAL COPULA								
Region	<i>Historical Returns</i>				<i>Augmented Model</i>			
	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.
R1	-247.16	-250.29	6.27	0.18	-249.11	-252.16	6.09	0.19
R2	-220.77	-222.22	2.9	0.58	-219.06	-220.48	2.83	0.59
R3	-847.63	-848	0.73	0.95	-848.37	-848.55	0.37	0.99
R4	-238.42	-239.91	2.99	0.56	-237.33	-238.72	2.77	0.6
R5	-210.83	-212.9	4.13	0.39	-208.16	-210.15	3.98	0.41
R6	-150.1	-150.8	1.4	0.84	-152.89	-153.42	1.06	0.9
R7	-182.87	-187.67	9.6	0.05	-177.82	-182.25	8.85	0.07

GUMBEL MIXTURE COPULA								
Region	<i>Historical Returns</i>				<i>Augmented Model</i>			
	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.
R1	-247.03	-248.8	3.53	0.47	-248.86	-250.56	3.41	0.49
R2	-221	-222.42	2.84	0.58	-218.23	-219.76	3.07	0.55
R3	-847.12	-847.32	0.4	0.98	-847.34	-847.6	0.52	0.97
R4	-238.22	-239.3	2.16	0.71	-237.45	-238.41	1.93	0.75
R5	-210.72	-212.63	3.82	0.43	-208.94	-210.92	3.95	0.41
R6	-149.94	-150.05	0.22	0.99	-149.77	-149.9	0.25	0.99
R7	-181.91	-184.61	5.39	0.25	-179.41	-182.1	5.38	0.25

NOTE: Table 5 presents the estimation results of the “hit” tests applied to joint density defined by the conditional Normal or Gumbel Mixture copula models. LL_{UNR} and LL_{RE} correspond to the unrestricted and restricted likelihood of the test described in the Appendix. The p-values and t-statistics refer to the null that the model is correctly specified for regions R1 to R7. For the definition of these regions see Section 4.3).

simple Normal copula model is the worst performer with a SIC statistic of -396.45.

4.4 Bivariate Models: Out-of-Sample Comparisons

Preliminary Remarks

To assess the out-of-sample performance of the conditional copula models, we divide the sample into two parts. The first part contains all data from 31/10/2001 up to

12/30/2005 (1053 daily observations), and is used to estimate the parameters of the marginal models (θ_x^*, θ_y^*) and those of the copula density θ_c^* , for each of the four alternative copula models. These parameters are then kept fixed and the remaining data (18 months, 376 daily observations) are used to evaluate the competing specifications.

To illustrate how the out-of-sample exercise is carried out, consider the concrete example where θ_c^* corresponds to the “augmented” Normal copula specification. Denoting s the last in-sample observation, the raw returns $r_{x,s}$ and $r_{y,s}$ already provide the conditional mean and variance estimates one period ahead, i.e. $\mu_{x,s+1}$, $h_{x,s+1}$ and $\mu_{y,s+1}$, $h_{y,s+1}$ which (together with the degrees of freedom parameters ν_x^* and ν_y^*) define the conditional marginal densities $f_{s+1}(\cdot|W_s, \theta_x^*)$ and $g_{s+1}(\cdot|W_s, \theta_y^*)$. Similarly, the value of the copula’s dependence parameter ρ_{s+1} is also computable, since the conditioning variables $u_s, u_{s-1}, \dots, u_{s-9}$ and $v_s, v_{s-1}, \dots, v_{s-9}$ (as well as ρ_s and $\rho_{Q,s}$) are all known at time s , so that we also have the estimate of the conditional copula density corresponding to time $s+1$, i.e. $c_{s+1}(F_{s+1}(\cdot|W_s, \theta_x^*), G_{s+1}(\cdot|W_s, \theta_y^*) |W_s, \theta_c^*)$.

The first out-of-sample return observations $r_{x,s+1}$ and $r_{y,s+1}$, can then be used to evaluate the quantities $\tilde{u}_{s+1} = F_{s+1}(r_{x,s+1}|W_s, \theta_x^*)$ and $\tilde{v}_{s+1} = G_{s+1}(r_{y,s+1}|W_s, \theta_y^*)$, as well as the out-of-sample log-likelihood of the copula density, i.e.

$$OLL_{c,s+1} = \log c_{s+1}(F_{s+1}(r_{x,s+1}|W_s, \theta_x^*), G_{s+1}(r_{y,s+1}|W_s, \theta_y^*) |W_s, \theta_c^*)$$

The observations $r_{x,s+1}$ and $r_{y,s+1}$ also provide the necessary information to update the conditional mean and variance equations, and hence obtain $f_{s+2}(\cdot|W_{s+1}, \theta_x^*)$ and $g_{s+2}(\cdot|W_{s+1}, \theta_y^*)$. Likewise, the numerical values of \tilde{u}_{s+1} , \tilde{v}_{s+1} , ρ_{s+1} and $\rho_{Q,s+1}$ enable the computation of ρ_{s+2} which, in turn, defines the density of the copula one period ahead, i.e. $c_{s+2}(F_{s+2}(\cdot|W_{s+1}, \theta_x^*), G_{s+2}(\cdot|W_{s+1}, \theta_y^*) |W_{s+1}, \theta_c^*)$.

Proceeding in the same fashion until the end of the dataset (i.e. for $j = s+1, \dots, n$) produces a series of \tilde{u}_j and \tilde{v}_j realizations that can be used to examine whether the conditional copula model under study shows evidence of misspecification. We do so by applying the same “hit” tests methodology described in Section 4.3 and the Appendix, with the key difference that \tilde{u}_j and \tilde{v}_j represent out-of-sample observations.

Furthermore, we calculate the out-of-sample log-likelihood (OLL_c) statistic of the copula density, defined as:

$$OLL_c = \sum_{t=s+1}^n \log c_t(F(r_{x,t}|W_{t-1}, \theta_x^*), G(r_{y,t}|W_{t-1}, \theta_y^*) |W_{t-1}, \theta_c^*) \quad (32)$$

Following Amisano and Giacomini (2007), Bao, Lee and Saltoglu (2007) and Lee and Long (2009), among others, out-of-sample log-likelihood levels are used as a ranking criterion between competing density forecast methods. As these authors discuss, if one of the methods correctly specifies the forecast densities then it will have the highest expected out-of-sample log-likelihood, while in the case where all forecast methods are imperfect, the highest statistic indicates the specification that is closer to the true target density using the Kullback-Leibler Information Criterion as a loss function. Accordingly, we prefer the conditional copula model that yields the highest OLL_c , which is the same as if the out-of-sample log-likelihood of the entire bivariate density was used, since all forecast methods have identical out-of-sample log-likelihood statistics for the two margins.

As the earlier discussion regarding the construction of this out-of-sample experiment reveals, the competing models are essentially evaluated at their entirety, i.e. forecasts concerning both the marginals and the copula density are involved. This is because the \tilde{u}_j and \tilde{v}_j are computed by evaluating the univariate forecast densities and, consequently, any errors will be automatically transmitted to the bivariate forecast distributions. While the marginal models, and hence \tilde{u}_j and \tilde{v}_j , are common for all bivariate specifications, it can be argued that these will affect the historical models to greater extent since, in this case, the dynamics of the copulas' degree of dependence parameters are driven solely by past return observations (i.e. lagged values of \tilde{u}_j and \tilde{v}_j).

Results

Panel A of Table 6 summarizes the out-of-sample log-likelihood statistics for the four conditional copula models. The augmented Gumbel Mixture specification delivers the highest OLL_c statistic (90.26), followed by the augmented Normal copula model (86.15). Regarding the historical return models, the Gumbel Mixture copula model has a OLL_c statistic of 85.45, which is comparable to that of the augmented Normal copula specification, while the worst performer is the historical Normal copula model where the corresponding statistic is equal to 76.65.

The relative ranking of the competing alternatives in terms of their predictive accuracy is very similar to that suggested by the in-sample log-likelihood selection criterion (i.e. the SIC). The only discrepancy is that the augmented Gumbel Mixture model emerges as the best performer out-of-sample and second best in-sample, while the reverse is true for the augmented Normal copula specification. Notably, the differences between the historical and augmented specifications for either the Normal

TABLE 6. OUT-OF-SAMPLE RESULTS

<i>Panel A. Out-of-Sample Log-likelihoods</i>								
	NORMAL COPULA				GUMBEL MIXTURE COPULA			
	<i>Historical</i>	<i>Augmented</i>			<i>Historical</i>	<i>Augmented</i>		
OLL_c	76.649	86.148			85.451	90.261		

<i>Panel B. Hit Tests, Joint Density</i>								
NORMAL COPULA								
	<i>Historical Returns</i>				<i>Augmented Model</i>			
Region	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.
R1	-62.81	-65.96	6.3	0.18	-62.67	-65.37	5.39	0.25
R2	-49.9	-52.75	5.7	0.22	-49.76	-52.66	5.79	0.22
R3	-233.98	-237.63	7.29	0.12	-233.76	-236.63	5.76	0.22
R4	-80.95	-85.67	9.43	0.05	-80.32	-84.47	8.3	0.08
R5	-52.26	-52.73	0.94	0.92	-52.15	-52.71	1.12	0.89
R6	-12.07	-14.1	4.06	0.4	-11.71	-13.05	2.68	0.61
R7	-38.01	-38.5	0.98	0.91	-37.86	-38.86	1.98	0.74

GUMBEL MIXTURE COPULA								
	<i>Historical Returns</i>				<i>Augmented Model</i>			
Region	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.	LL_{UNR}	LL_{RE}	$LRStat$	P-Val.
R1	-62.65	-65.41	5.51	0.24	-63.1	-65.63	5.06	0.28
R2	-49.91	-52.81	5.8	0.21	-49.26	-52.22	5.92	0.2
R3	-234.1	-236.62	5.06	0.28	-233.66	-235.71	4.1	0.39
R4	-80.6	-84.24	7.28	0.12	-79.53	-82.73	6.4	0.17
R5	-52.36	-52.95	1.18	0.88	-52.73	-53.42	1.38	0.85
R6	-12.18	-15.48	6.61	0.16	-11.57	-14.45	5.76	0.22
R7	-38.39	-38.6	0.42	0.98	-39.33	-39.6	0.52	0.97

Panel A displays the out-of-sample log-likelihood statistics of the alternative conditional copula models. **Panel B** presents the results of the (out-of-sample) “hit” tests applied to the joint density constructed using the Normal or Gumbel Mixture copula models. LL_{UNR} and LL_{RE} refer to the unrestricted and restricted likelihood of the test. The t-statistics and p-values correspond to the null that the model is correctly specified for the regions R1 to R7. Details about the test can be found in the Appendix, while the seven regions are defined in Section 4.3.

and or the Gumbel Mixture copula remain discernible, while the historical Normal copula is the least preferred model in either setting.

Further evidence concerning the out-of-sample performance of the competing conditional copula models are provided by the “hit” tests, presented in the second Panel of Table 6, that examine the performance of the alternative bivariate distribution forecasts in a piece-wise fashion. Using the same region definitions as those of the in-sample case, the test attempts to uncover potential shortcomings of the forecast models by checking whether they imply the correct proportion of “hits”, as well as by inspecting whether the “hit” observations are autocorrelated.

Tests conducted at the 5% level indicate that the historical Normal copula model would be (marginally) rejected for Region 4, which corresponds to the case where \tilde{u} and \tilde{v} are between their 75th and the 90th quantile. Similarly, the augmented Normal copula model has a p-value of 8% for same region, so that the null would be rejected at the 10% level. On the contrary, the respective p-values for the Gumbel Mixture copula models are 12%, for the historical, and 17% for the augmented specification. No problems surface for the remaining regions, since all models pass the test at conventional significance levels. The only relatively low p-values are those of Region 3 (12%) and Region 1 (18%) for the historical Normal copula, as well as that of Region 6 (12%) in the historical Gumbel Mixture specification.

Although the inclusion of option-implied dependency measures in the Normal copula model provides clear benefits in terms of the likelihood-based criteria SIC and OLL_c , it fails to correct the shortcomings revealed by the “hit” tests. Specifically, neither the historical nor the augmented model would pass a 10% test for Region 7 (in-sample) and Region 4 (out-of-sample) and the source of this rejection is that they assign incorrect probability mass in the corresponding segment of the bivariate density³⁰. Considering the above, it appears that option-implied information is a very helpful indicator regarding the (concurrent or future) *degree* of dependence, whereas the remaining problems simply reflect deficiencies in the *type* of dependence, i.e. the structural form of association imposed by the Normal copula.

5 Conclusion

This paper examines whether option prices provide useful information regarding the conditional dependency of the U.S. Dollar/Japanese Yen - Euro/Japanese Yen

³⁰The detailed logit regression results, through which the hit tests are constructed, were omitted for reasons of brevity, but they are available from the author upon request.

exchange rate returns, for the period between November 2001 and June 2007. Bivariate time-series models that rely exclusively on historical return information are compared with “augmented” specifications, where option-implied dependence measures are included in the analysis.

We resort to copula theory in order to construct dynamic bivariate densities, as well as to extract forward-looking dependence indicators from the prices of options. In the first case, the univariate price dynamics are modeled using standard ARCH models and, subsequently, the conditional copula of Patton (2006b) is employed in order to link the marginal return innovations. We focus on two copula functions; the one-parameter “Normal” and a two-parameter “Gumbel Mixture” specification. The two-parameter copula permits a separate treatment of upper and lower tail dependency, so that asymmetry in either direction can be accommodated. This allows us to study, for the first time in the literature, option-implied dependency estimates that go beyond a scalar measure of association.

Our empirical results indicate that the information embedded in the prices of exchange rate options enhances the performance of the time-series specifications that we consider. For both the “Normal” and the “Gumbel Mixture” copulas, in-sample model selection criteria favor the specifications that include option-implied dependence measures in their information set. The same result holds true in the out-of-sample setting; the augmented models are again preferable relative to their historical counterparts, as they deliver the highest out-of-sample log-likelihood statistics. Pair-wise comparisons of the competing specifications indicate that the augmented Normal copula model exhibits the best performance in-sample, followed by the augmented Gumbel Mixture specification. The reverse is true in the out-of-sample exercise, so that it becomes difficult to distinguish a clear winner. Notably, however, the Normal copula model that relies solely on past return data delivers the worst results in all pair-wise comparisons.

This paper adds to the active literature that uses the prices of observed options in order to extract useful forward-looking information, beyond the well-established case of implied volatilities. As with implied correlation coefficients (Bodurtha and Shen, 1995; Campa and Chang, 1998; Lopez and Walter, 2000; Castren and Mazzotta, 2005), implied forecast densities (Liu et al, 2007; Shackleton et al, 2007; Kostakis et al, 2009) and implied betas (Christoffersen et al, 2008), we find that option-implied expectations can improve the performance of historical return models. One distinctive feature of this study is that the setting is no longer univariate. Moreover, our approach recognizes that the dependency of exchange rate returns may display non-linear characteristics and, along these lines, we suggest a simple

method that can be used to extract option-based information regarding both the degree of dependency, as well as, the direction of asymmetric dependence.

Overall, our findings support the notion that derivative prices provide a rich source of information that shouldn't be ignored when building time-series models. It is worth noting, however, that our approach can be refined in several respects. For instance, one can use the entire cross-section of exchange rate options and estimate the parameters of the risk-neutral price process for all the involved exchange rates. Since the corresponding risk-neutral densities can be then derived for any maturity date, one can find the dependence pattern that has the same horizon as that of the corresponding historical return model (for instance daily or weekly). Furthermore, one can also attempt to directly adjust the bivariate risk-neutral density using a risk-transformation method, as it is done by Liu et al (2007) or Shackleton et al (2007) in the univariate case. Including other copulas in the analysis might also be of interest. For example, the popular Student's t copula also has two parameters, one controlling for the degree of dependence and one determining the fat-tailness of the bivariate density.

As a final remark, we emphasize that in this study we only use three, publicly accessible, implied volatilities for the computation of each risk-neutral density. Outside the public domain, data providers currently supply implied volatilities for several strikes, so that the accuracy of option-implied dependency measures can be further improved.

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Appendix

To investigate whether the (univariate or bivariate) conditional density models are correctly specified in particular segments of the density we employ the “hit” test methodology suggested by Patton (2006b), which is a simple extension of the original test proposed in Christoffersen (1998) and refined in Engle and Manganelli (2004).

Let X_t be the (possibly multivariate) random variable under study, $R_{i,t}$ the relevant density region and p_{it} the probability according to the forecast model that X_t will belong to that region, i.e. $X_t \in R_{i,t}$. The hit variable H_{it} is defined as

$$H_{it} = \begin{cases} 1 & \text{if } X_t \in R_{i,t} \\ 0 & \text{otherwise.} \end{cases}$$

For correctly specified forecasts we have that we have that $p_{it} = \pi_{it}$ and $H_{it} \sim i.n.i.d.$ Bernoulli(p_i)³¹, where π_{it} denotes the true probability that $X_t \in R_{i,t}$. Using this property, we can examine if a density forecast model shows evidence of misspecification.

We follow Patton (2006b) and estimate the following logit model

$$\pi_{i,t} = \pi_i(Z_{it}, \beta_i, p_{it}) = \Lambda \left(Z_{it}\beta_i + \ln \left[\frac{p_{it}}{1 - p_{it}} \right] \right) \quad , \quad (33)$$

where $\Lambda(x) \equiv (1 + e^{-x})^{-1}$ is the logistic transformation, Z_{it} is a matrix containing elements from the time $t-1$ information set that could reveal potential shortcomings of the forecast model and β_i is the parameter vector to be estimated. In this way $H_0 : \beta_i = 0$ ($H_1 : \beta_i \neq 0$) corresponds to $H_0 : \pi_{it} = p_{it}$ ($H_1 : \pi_{it} \neq p_{it}$).

As in Patton (2006b), we include the following four variables in the Z_{it} matrix: a constant, to check whether the model has the correct proportion of hits, and three variables that count the number of hits in that region in the last 1, 5 and 10 observations.

If the forecast model produces well specified forecasts for the $R_{i,t}$ region, then all four elements of the β_i vector must be zero. Denoting LL_{RE} the restricted log-likelihood of the model and LL_{UNR} the unrestricted log-likelihood of the model

³¹The variables H_{it} are independent but not necessarily identically distributed (i.n.i.d) because the density mass of the target region is not restricted to be constant across time. If the target region of the univariate density is constant, then H_{it} would be *i.i.d.*. Regarding the multivariate case, the probability of a hit is not constant and depends on the amount of probability mass assigned by the copula in that region.

corresponding to equation (33), a likelihood ratio test can be conducted using the test statistic $LR = -2(LL_{RE} - LL_{UNR})$ which approximately follows a χ^2 distribution with 4 degrees of freedom under the null of correctly specified forecasts.

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