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# Forecasting autoregressive time series under changing persistence

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**Abstract**

Changing persistence in time series models means that a structural change from non-stationarity to stationarity or vice versa occurs over time. Such a change has important implications for forecasting, as negligence may lead to inaccurate model predictions. This paper derives generally applicable recommendations, no matter whether a change in persistence occurs or not. Seven different forecasting strategies based on a biased-corrected estimator are compared by means of a large-scale Monte Carlo study. The results for decreasing and increasing persistence are highly asymmetric and new to the literature. Its good predictive ability and its balanced performance among different settings strongly advocate the use of forecasting strategies based on the Bai-Perron procedure.

Key Words: Forecasting; changing persistence; structural break; pre-testing; break-point estimation; bias-correction.

# 1 Introduction

Recent research in time series econometrics has paid a lot of attention to structural breaks in autoregressive time series models. Perron (2006) provides an excellent survey on structural change. This work is dedicated to changing persistence, which is

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defined as a change in the degree of integration of a time series. A simple example for changing persistence is a first-order autoregressive (AR) model in which the AR parameter equals one during the pre-break period and less than unity in absolute value during the post-break period, or vice versa. Such a break in persistence has direct consequences for forecasting with AR models, see Pesaran and Timmermann (2005). Following Clements and Hendry (1998), structural breaks are of great importance for forecasting performance in general. Ignoring structural breaks in the parameters of the forecasting model may result in poor and highly inaccurate forecasts.

As Perron (2006) points out, changing persistence has been an important feature of economic time series. Among these are inflation (e.g., Barsky, 1987, Burdekin and Siklos, 1999), interest rates (e.g., Mankiw et al., 1987), government budget deficits (e.g., Hakkio and Rush, 1991) and real output (e.g., Delong and Summers, 1988). An emerging strand of literature discusses mainly decreasing persistence with a special emphasis on US and European inflation rates, see O'Reilly and Whelan (2005), Kang et al. (2009), Kumar and Okimoto (2007) and Pivetta and Reis (2007), Halunga et al. (2008), Noriega and Ramos-Francia (2009) and Kejriwal (2009). Forecasts of these variables, and especially of inflation, are crucial to policy makers and they are also relevant for financial markets.

This paper derives generally applicable recommendations, no matter whether a change in persistence occurs or not. To this end, we investigate the behaviour of seven forecasting strategies under a variety of data generating processes (DGPs). We consider forecasting strategies which allow for such breaks or ignore them. In more detail, we analyze the forecasting performance of a stationary autoregressive model, the random walk and a pre-testing strategy which is inspired by the work of Diebold and Kilian (2000). These three approaches to forecast autoregressive time series assume a constant degree of integration. On the contrary, we include a forecasting strategy which assumes a decline in persistence to happen. A pre-test for constant versus changing persistence based on Leybourne et al. (2007b) is conducted as well. Additionally, we apply the popular procedure for testing and dating structural breaks suggested by Bai and Perron (1998, 2003) in order to assess its usefulness in the context of forecasting time series with changing persistence. In order to cope with potential small-sample bias of OLS, we use the Roy and Fuller (2001) estimator which proves to be of empirical usefulness. Kim (2003) discusses its properties in the context of forecasting without breaks and finds it to be important for empirical applications. Beside simple processes with constant and changing persistence, we also consider stable shifts as a robustness check. These are defined as structural changes of the AR parameter within the region of stationarity. Hence, they do not constitute a change in persistence as the time series is stationary throughout the entire sample.

As a theoretical comparison of the seven forecasting strategies is not directly applicable in samples of small and moderate size, we conduct an extensive Monte Carlo study. The simulations allow us to quantify the precise gains and losses which arise from the application of certain forecasting strategies in many different settings. The numerical results highlight a significant asymmetry in the forecasting performance under increasing and decreasing persistence. This result is new to the literature on forecasting and structural change. In the case of decreasing persistence, forecasters are advised to take this type of structural change into account. A promising way to do so, is to apply forecasting strategies based on the Bai-Perron procedure. Their accuracy is surprisingly close to the one which imposes a decline in persistence. Pre-testing for changing persistence should be avoided. If persistence increases, the random walk forecast is most precise in all settings. Hence, it is meaningful to ignore the break and to work with a non-stationary forecasting model. Interestingly, the loss in forecast precision obtained by using Bai-Perron-based forecasting strategies instead is relatively low. Its good predictive ability and its balanced performance (also for processes with constant persistence) strongly advocate the use of forecasting strategies based on the Bai-Perron procedure.

This article is organized as follows: section 2 discusses the autoregressive time series model under changing persistence, related unit root tests and the Bai-Perron technique. Section 3 covers a detailed description of all forecasting strategies, while bias-corrected estimation of the AR model is briefly reviewed in section 4. The Monte Carlo setup and discussions of numerical results are given in section 5. Practical recommendations are summarized in section 6.

# 2 Changing persistence and related statistics

## 2.1 AR model with changing persistence

We consider the following first-order autoregressive model that is subject to a change in persistence at some breakpoint $T_B = [\tau T]$ with $\tau \in (0, 1)$:

$$y_t = \beta_1 y_{t-1} + \varepsilon_t, \qquad \text{for } t = 1, ..., T_B \tag{1}$$

$$y_t = \beta_2 y_{t-1} + \varepsilon_t, \qquad \text{for } t = T_B + 1, ..., T\ . \tag{2}$$

The innovation process $\varepsilon_t$ is assumed to be a zero mean white noise process. In this model, persistence is determined through autoregressive parameters $|\beta_1| \leq 1$ and $|\beta_2| \leq 1$. As long as $\beta_1 \neq \beta_2$, a structural change occurs at time $T_B$. The special case of this particular break in which $|\beta_1| = 1$ and $|\beta_2| < 1$ hold, is called a decline in persistence because the AR model in equations (1)-(2) is non-stationary, that is $I(1)$, during time $t = 1, \ldots, T_B$ and stationary, that is $I(0)$, afterwards. Analogously, an increase in persistence takes place if $|\beta_1| < 1$ and $|\beta_2| = 1$ hold, i.e. the process switches from stationary to a unit root process. Chong (2001) shows that the OLS estimator for $\beta_1$ and $\beta_2$ are consistent and asymptotically normally distributed. It should be noted that stable shifts, i.e., $\beta_1 \neq \beta_2$ with $|\beta_1| < 1$ and $|\beta_2| < 1$, do not constitute a change in persistence as the process is stationary over the whole sample period. Such changes are considered in this study as well.

The breakpoint $T_B$ is treated as unknown and is therefore estimated. Furthermore, it may be beneficial to pre-test for constant versus changing persistence instead of assuming one or the other. If no actually no break occurs, then useful information for forecasting may be wasted when a change is assumed to happen. This can lead to reduced forecast precision. On the contrary, ignoring a change in persistence can be costly as well.

## 2.2 Unit root test against a change in persistence

The literature on testing constant against changing persistence considers the null hypothesis of a constant $I(1)$ or constant $I(0)$ process while the alternative is given by a deterministic change from $I(1)$ to $I(0)$ over time, or vice versa (see Kim 2000, Kim et al. 2002, Leybourne et al. 2003, Busetti and Taylor 2004, Harvey et al. 2006). In principle, these tests can be carried out as one-sided tests with a known direction of change or as two-sided tests with unknown direction. In this paper, we shall not assume that the direction of change is known a priori and therefore, two-sided tests are carried out.

One of the most recent contributions to this literature is the CUSUM of squares-based test by Leybourne et al. (2007b). The authors solve an important problem that is inherent in other tests for changing persistence: The asymptotic size equals one if both, the null and the alternative hypotheses are wrong. This situation means for a unit root test against changing persistence that the process is constantly $I(0)$ and for a stationarity test that it is constantly $I(1)$. The construction of the CUSUM of squares-based test by Leybourne et al. (2007b) results in a conservative test, i.e. the asymptotic size is equal to zero. This property can be of use in a forecasting context. The reason is that a spurious rejection of the null hypothesis (in the case of $y_t \sim I(0)$) would imply an unnecessary waste of important data points for estimation of the forecast model. Such spurious rejection do not occur when applying the Leybourne et al. (2007b) test. Therefore, the focus is on this test here.

It builds upon the test statistic $R$ which is given by

$$R = \frac{\inf_{\tau \in \Lambda} K^f(\tau)}{\inf_{\tau \in \Lambda} K^r(\tau)},$$

where $K^f(\tau)$ and $K^r(\tau)$ are CUSUM of squares-based statistics based on the forward and reversed residuals of the data generating process as given below. The relative breakpoint $\tau \in \Lambda = [\underline{\tau}, \overline{\tau}] = [0.15, 0.85]$ is assumed to be unknown and an estimator for $\tau$ is given below. In more detail, $K^f(\tau)$ and $K^r(\tau)$ are given by

$$K^f(\tau) = \frac{1}{[\tau T]^2 \hat{\gamma}_0^f(\tau)} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2$$

and

$$K^r(\tau) = \frac{1}{(T - [\tau T])^2 \hat{\gamma}_0^r(\tau)} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2.$$

Here, $\hat{v}_{t,\tau}$ are the residuals from the OLS regression of $y_t$ on a constant based on the observations up to $[\tau T]$. This is

$$\hat{v}_{t,\tau} = y_t - \bar{y}(\tau)$$

with $\bar{y}(\tau) = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} y_t$. Similarly $\tilde{v}_{t,\tau}$ is defined for the reversed series $z_t \equiv y_{T-t+1}$. In addition, $\hat{\gamma}_0^f(\tau)$ and $\hat{\gamma}_0^r(\tau)$ are OLS variance estimators for $\Delta\hat{v}_{t,\tau}$ and $\Delta\tilde{v}_{t,\tau}$, respectively. The null hypothesis of a constant unit root process is rejected for small or large values of $R$ in favor of increasing or decreasing persistence, respectively. Regarding the unknown breakpoint, Leybourne et al. (2007b) prove consistency of two breakpoint estimators which are given by

$$\begin{aligned} \hat{\tau}_f &= \arg\inf_{\tau \in \Lambda} \frac{1}{[\tau T]^2} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2 \qquad \text{for } y_t \sim I(0) \to I(1) \\ \hat{\tau}_r &= \arg\inf_{\tau \in \Lambda} \frac{1}{(T - [\tau T])^2} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2 \qquad \text{for } y_t \sim I(1) \to I(0). \end{aligned}$$

Note, that $\frac{1}{[\tau T]^2} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2$ and $\frac{1}{(T-[\tau T])^2} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2$ are equal to the unstandardized (excluding the long-run variance estimator) forward and backward statistics $K^f(\tau)$ and $K^r(\tau)$, respectively.

## 2.3 Bai-Perron approach for dating structural breaks

Bai and Perron (1998, 2003) suggested a technique for testing for structural breaks in linear regression models. Their approach also allows consistent dating of breaks and is widely applied in applied econometric research. In the following, we present the Bai and Perron (1998, 2003) approach for the case of one break, although it is more flexible in general. Given one structural break at $T_B$, the linear regression model with a lagged dependent variable as a regressor is given by

$$y_t = \mu_1 + \beta_1 y_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T_B \tag{3}$$

$$y_t = \mu_2 + \beta_2 y_{t-1} + \varepsilon_t \qquad t = T_B + 1, \ldots, T\ . \tag{4}$$

This model allows for a break in the mean through the parameters $(\mu_1, \mu_2)$ and a break in the autoregressive parameter through $(\beta_1, \beta_2)$, while the testing approach outlined in section 2.2 is more restrictive in the sense that it only permits restricted changes in the autoregressive parameters by construction. Please note, that the model (3)-(4) can be restricted so that breaks in the mean are ignored and only structural changes in the AR parameter are considered. Imposing this restriction is not recommendable in our setting.[1]

The Bai-Perron procedure allows consistent estimation of the breakpoint $T_B$ by applying a dynamic programming algorithm. We require to have at least a fraction of 0.15 data points in each data segment. The number breaks (zero or one) is selected with the Bayesian information criterion (BIC) for the following reasons: Perron (1997) simulates and discusses the properties of the BIC in comparison with the modified Schwarz criterion (LWZ) proposed by Liu et al. (1997). In the presence of a lagged dependent variable, as given here, the BIC and the LWZ show significantly different performance. If no break occurs, the BIC tends to select a break much more often than the LWZ, but only in the unit root case, see Tableau 3 in Perron (1997). If breaks are present, the LWZ approach is of less use, especially for highly persistent time series, as it underestimates the number of breaks dramatically (Tableaux 7A and 7B in Perron 1997). Our simulation results in section 5 show that the BIC indeed selects a break too often if no change occurs (in around 19% of the cases for a unit root and much less often for stationary AR processes). However, the effect

[1] We do not report results for the Bai-Perron procedure including this restriction in order to save space. They are, however, available upon request from the author. The data generating processes in the simulation study (section 5) do not contain deterministic terms. It turns out that the forecasting strategies based on the restricted model are less precise in terms of out-of-sample mean square forecast error (MSFE) than the ones using the unrestricted model for all considered DGPs.

**Table 1:** Forecasting strategies

| | |
|---|---|
| | No change in persistence |
| S1 | Stationary AR(1) model, full sample |
| S2 | Random walk model, full sample |
| S3 | Pre-testing for unit root (choose between S1 and S2), full sample |
| | Change in persistence |
| S4 | Stationary AR(1) model, post-break sample |
| S5 | Pre-testing for constant persistence (choose between S3 and S4) |
| | General structural breaks |
| S6 | Bai-Perron, pre-testing (choose between S3 and S4) |
| S7 | Bai-Perron, stationary AR(1) model (choose between S1 and S4) |

on forecasting performance is negligible. On the contrary, the strong ability of the BIC to detect breaks if they are actually present proves to be very useful in terms of forecasting, see section 5. Further details regarding the Bai-Perron approach can be found in Bai and Perron (1998, 2003).

# 3 Forecasting strategies

We consider a variety of forecasting strategies which may be divided into two groups: those accounting for structural breaks and ones permitting no breaks. Table 1 summarizes different forecasting strategies. The next subsections are dedicated to a more careful explanation of details regarding Strategies 1–7.

## 3.1 Constant parameters

**Strategy 1** A standard forecasting model for many economic time series is the stationary AR(1) model including a constant. This model is given by $y_t = \mu + \beta y_{t-1} + \varepsilon_t$, with the imposed stationary condition $|\beta| < 1$. Instead of the OLS estimator we consider an approximately median-unbiased estimator proposed by Roy and Fuller (2001). This estimator is particularly useful for forecasting purposes in small and moderate samples, see Kim (2003). This estimation technique is briefly described in section 4.

**Strategy 2** The second considered forecasting model is the random walk without drift,

$$y_t = y_{t-1} + \varepsilon_t \ .$$

The $h$-step ahead forecast from this difference-stationary model is given by $\widehat{y}^{S2}_{t+h|t} = y_t$. As the $h$-step ahead forecast is simply given by the $y_t$ and does not depend on any estimated parameters, it does not matter whether we use the entire in-sample period or just a fraction of it: the $h$-step forecast ahead forecast will be the same for all $h$. The further consequences of this circumstance for our Monte Carlo setup will be discussed in section 5.

**Strategy 3** As intensively discussed in Diebold and Kilian (2000), pre-testing offers often a significant improvement compared to S1 or S2 when a linear trend is included. The authors, however, do not provide any results for the case where just a constant is included in the stationary AR(1) model. Our conjecture is that the usefulness of unit root pre-testing is not limited to the case of a linearly trending AR(1) model. Hence, we include a similar pre-testing strategy under the assumption of constant persistence in our study. We choose the powerful DF-GLS test as the pre-test which has been used in a another study on pre-testing by Stock (1996) and which is also analyzed in Diebold and Kilian (2000). In particular, we run the DF-GLS test with a constant,

$$\Delta\widetilde{y}_t = \phi\widetilde{y}_{t-1} + u_t \ .$$

The GLS-demeaned data is given by $\widetilde{y}_t \equiv y_t - \widehat{\psi}z_t$ with $z_t = 1$. Define $(x^\gamma_0, x^\gamma_t) \equiv (x_0, (1-\gamma L)x_t)$ for $t = 1, \ldots, T$ where $\gamma = 1 + \bar{c}/T$. Eliott et al. (1996) suggest to use the value $\bar{c} = -7$ in order to ensure the limiting power function lies close to the local power envelope. $\widehat{\psi}$ is obtained by minimizing $(y^\gamma - \psi z^\gamma)'(y^\gamma - \psi z^\gamma)$. If the null hypothesis $H_0 : \phi = 0$ is rejected at some nominal significance level in favor of $H_1 : \phi < 0$, then the stationary AR forecast model is selected, i.e., S1. Otherwise, S2 is applied.

## 3.2 Change in persistence

**Strategy 4** assumes that a change in persistence occurs. This means implicitly that the size and power of the Leybourne et al. (2007b) pre-test is automatically one. Moreover, the breakpoint $\tau$ is always estimated even if no break occurs. It is expected that this strategy shows a relatively good performance if the alternative is true, i.e. a change in persistence takes place since the type II-error is zero. Under validity of the null hypothesis of a constant $I(1)$ process however, the accuracy may be significantly worse than the one of the random walk model which serves as the benchmark in this case. The effects are quantified in section 5. We only use the breakpoint estimator $\widehat{\tau}_r$ for decreasing persistence. This is motivated by the fact that $\widehat{\tau}_f$ is useless since increasing persistence means that the random walk model should be used. As already discussed, this forecast depends only on the last observation available but not on the number of observations. Hence, breakpoint estimation plays no role in this particular case.

**Strategy 5** A compromise between S1 to S4 is to pre-test for a unit root against a change in persistence. In case of a rejection in favor of a decrease in persistence from $I(1)$ to $I(0)$, $\widehat{\tau}_r$ is used to select the post-break window of data points for estimating the stationary AR(1) model. If the test rejects $H_0$ in favor of an increase in persistence, S2 is selected. The case of a non-rejection is a bit more complicated. Due to the properties of the CUSUM of squares-based pre-test, a non-rejection can be interpreted as evidence for constant persistence, but it is unclear whether the non-rejection is caused by a constant $I(0)$ or a constant $I(1)$ process. Therefore, S3 (pre-testing under constant persistence) is applied in this case. Depending on the outcome of the second pre-test, S1 or S2 is applied.

## 3.3 General structural breaks

**Strategy 6** Another way of coping with potential structural breaks in the parameters of the AR(1) model is to apply the Bai-Perron approach. As a first step, the optimal number of data segments is determined. In our study we allow for one or two segments which corresponds to no or one break, respectively. If a structural break is found, the second segment of data points is used for estimation of the stationary AR(1) model. Otherwise, S3 is applied in order to select an appropriate forecasting model.

**Strategy 7** A variant of S6 is to use the full sample and estimate the stationary AR(1) model in case of a non-rejection instead of conducting a DF-GLS pre-test. S7 and S6 are equivalent in case of a rejection.

## 4 Bias-corrected estimation

It is a well known fact that the OLS estimator for the persistence parameter of the AR(1) model is downward-biased in small samples. Due to the fact that we consider a sample size of $T = 150$ in our simulation study it seems to be reasonable to analyze the performance of forecasting strategies when a biased-corrected estimator is applied. Therefore, we consider the median-unbiased Roy-Fuller estimator which has been proven to be of empirical usefulness in Kim (2003) in a forecasting context without breaks.

The Roy-Fuller (2001) estimator provides a simple modification to the OLS estimator for the persistence parameter $\beta$. We briefly review some details of this estimator, denoted as $\widetilde{\beta}$. Let $\widetilde{\beta} = \min(\breve{\beta}, 1)$, where

$$\breve{\beta} = \widehat{\beta} + (C_p(\widehat{\lambda}_1) + C_{-p}(\widehat{\lambda}_{-1}))\widehat{\sigma}_1 \ .$$

Here, $\widehat{\beta}$ denotes the OLS estimator for $\beta$ in $\bar{y}_t = \beta\bar{y}_{t-1} + \varepsilon_t$, where $\bar{y}_t$ is the previously de-meaned time series $y_t$, i.e., $\bar{y}_t \equiv y_t - (1/T)\sum_{t=1}^{T} y_t$. Furthermore, $\widehat{\sigma}_1$ denotes the standard error of $\widehat{\beta}$ and $\widehat{\lambda}_1 = (\widehat{\beta} - 1)/\widehat{\sigma}_1$ is the usual Dickey-Fuller (1979) unit root test statistic, while $\widehat{\lambda}_{-1}$ is a similar statistic for the unit root hypothesis $H_0 : \beta = -1$. Related to the asymptotic bias of the OLS estimator, the two functions $C_p(\widehat{\lambda}_1)$ and $C_{-p}(\widehat{\lambda}_{-1})$ are constructed to make $\widetilde{\beta}$ approximately median-unbiased at $\beta = 1$ and $\beta = -1$, respectively. Further details can be found in Roy and Fuller (2001).

# 5 Monte Carlo study

This section deals with the Monte Carlo simulation setup, the presentation and discussion of numerical results. The performance of forecasting strategies 1–7, see Table 1, is evaluated for a collection of different data generating processes. The forecast horizon $h$ takes values from 1 to 25. This choice corresponds to short- and medium-term forecasting when having monthly and quarterly data in mind. The in-sample size is $T = 150$ which is usual sample size in macroeconomics. Time series are generated with a total of 275 observations, where the first 100 data points are discarded in order to reduce the impact of the initial condition. The remaining 175 observations are divided into an in-sample period of 150 and an out-of-sample period of 25 observations, respectively. If the data generating process contains a structural break, then the breakpoint is drawn from a uniform distribution, i.e., $\tau \sim U[0.3, 0.7]$. The mean and the lower and upper bound are usual choices in simulations with structural breaks. The choice of drawing the true breakpoint from a uniform distribution allows us to reduce the number of experiments and to summarize them into a single set of experiments.[2] The number of Monte Carlo repetitions is set equal to 5,000. All computations and simulations are carried out in the open source language R (2009). The following packages are used: *bootpr* developed by Kim (2009), *dynlm* and *strucchange* (Zeileis et al. 2002 and Zeileis et al. 2003).

We consider 14 different experiments, see Table 2 for an overview. Among these are a constant $I(1)$ process (Exp 1), constant $I(0)$ processes (Exps 2-4) and processes with a pure change in persistence (Exps 5–10). Moreover, we consider another type of structural changes which is related to the previous ones: experiments 11-14 consider stable shifts. This means that the process $y_t$ is stationary throughout the entire sample period, but the AR parameter changes within region where stationarity is

[2] If we would consider for example, three different breakpoints, say $\tau = \{0.3, 0.5, 0.7\}$, then the number of experiments are tripled. When using the uniform distribution we are able to consider all breakpoints between 0.3 and 0.7 without increasing the number of experiments. The number of Monte Carlo repetitions is set high enough in order to ensure that the whole range of possible breakpoints are actually considered in the simulations.

**Table 2:** Overview of experiments

| Experiments | $\beta_1$ | $\beta_2$ |
|---|---|---|
| Constant persistence | | |
| 1: constant $I(1)$ | 1.0 | 1.0 |
| 2: constant $I(0)$ | 0.9 | 0.9 |
| 3: constant $I(0)$ | 0.7 | 0.7 |
| 4: constant $I(0)$ | 0.5 | 0.5 |
| Change in persistence | | |
| 5: $I(1)$ to $I(0)$ | 1.0 | 0.9 |
| 6: $I(1)$ to $I(0)$ | 1.0 | 0.7 |
| 7: $I(1)$ to $I(0)$ | 1.0 | 0.5 |
| 8: $I(0)$ to $I(1)$ | 0.9 | 1.0 |
| 9: $I(0)$ to $I(1)$ | 0.7 | 1.0 |
| 10: $I(0)$ to $I(1)$ | 0.5 | 1.0 |
| Stable shifts | | |
| 11: constant $I(0)$ | 0.9 | 0.7 |
| 12: constant $I(0)$ | 0.9 | 0.5 |
| 13: constant $I(0)$ | 0.7 | 0.9 |
| 14: constant $I(0)$ | 0.5 | 0.9 |

**Notes:** The data generating process is given by$y_t = \beta_1 y_{t-1} + \varepsilon_t$ for $t = 1, ..., T_B$ and $y_t = \beta_2 y_{t-1} + \varepsilon_t$ for $t = T_B + 1, ..., T$. Innovations are standard normally distributed, $\varepsilon_t \sim N(0, 1)$.

ensured, i.e. $|\beta_1| < 1$ and $|\beta_2| < 1$. The exact parametrization can be found in Table 2. Parameters are chosen to mimic typical behaviour of economic time series which usually show a high degree of persistence. The data generating processes do not contain deterministic terms.

The specified loss function is the mean squared forecast error (MSFE) and the benchmark is S1 or S2 depending on the true degree of integration of $y_t$ in the out-of-sample period. This means that the random walk forecast is the benchmark in experiments 1 and 8-10, while the stationary AR(1) forecast using full sample information is the benchmark in experiments 2-7 and 11-14. MSFEs relative to the benchmark are reported in Figures 1–14 instead of tables for convenience. Whenever the relative MSFE ratio is below (above) one, the benchmark is outperformed (not outperformed). The

**Table 3:** Summary statistics for pre-tests and Bai-Perron pre-selection

| Experiment | DF-GLS | CUSUM 5L | CUSUM 5U | BP |
|---|---|---|---|---|
| Constant persistence | | | | |
| 1 | 0.135 | 0.046 | 0.041 | 0.188 |
| 2 | 0.904 | 0.000 | 0.000 | 0.052 |
| 3 | 0.999 | 0.000 | 0.000 | 0.020 |
| 4 | 1.000 | 0.000 | 0.000 | 0.010 |
| Change in persistence | | | | |
| 5 | 0.306 | 0.001 | 0.095 | 0.200 |
| 6 | 0.359 | 0.000 | 0.310 | 0.629 |
| 7 | 0.407 | 0.000 | 0.485 | 0.909 |
| 8 | 0.348 | 0.099 | 0.001 | 0.241 |
| 9 | 0.471 | 0.287 | 0.000 | 0.594 |
| 10 | 0.512 | 0.472 | 0.000 | 0.877 |
| Stable shifts | | | | |
| 11 | 0.843 | 0.000 | 0.027 | 0.378 |
| 12 | 0.886 | 0.000 | 0.087 | 0.730 |
| 13 | 0.993 | 0.002 | 0.000 | 0.453 |
| 14 | 0.934 | 0.076 | 0.000 | 0.652 |

**Notes:** For details on experiments, see Table 2. DF-GLS is the rejection frequency of the unit root pre-test by Elliott et al. (1996) at a significance level of ten percent, CUSUM 5L and CUSUM 5U are the rejection frequencies of the CUSUM unit root pre-test by Leybourne et al. (2007b) at a five percent level in favor of an increase or a decrease in persistence, respectively. BP is the frequency of selecting a break by using the Bai-Perron procedure on the basis of the BIC.

nominal significance levels for both unit root pre-tests (DF-GLS and CUSUM of squares-based) are set equal to ten percent. The CUSUM of squares-based unit root test is conducted as a two-sided test with five percent mass of probability in each tail.

The simulation study is limited in several aspects. First, the lag length of the autoregressive forecasting model is assumed to be known and equal to one. Although the choice of the lag length may have some effects on the forecasting performance, we do not expect the main conclusions to be changed. Second, we set the number of maximal changes in persistence equal to one. Reasons for this restrictions are the following: the main strand of theoretical and empirical literature on changing persistence deals with the possibility of a one-time change. Therefore, we follow this line closely although the multiple changes in persistence may occur in practice. The body of literature on multiple changes in persistence is growing but still small (see Leybourne et al. 2007a and Kejriwal et al. 2009). Third, we restrict our analysis to only one sample size ($T = 150$). This sample size is common for quarterly data in empirical macroeconomics in general. Moreover, it is also relevant for European monthly series starting in 1999 for exmaple. Given this relatively short sample, it makes sense to restrict the attention to at most one structural change in persistence.

## 5.1 Constant $I(1)$ and $I(0)$

Under an $I(1)$ data generating process, we find that all forecasting strategies are performing very similar, see Figure 1 for experiment 1. The benchmark is the random walk forecast (red line). The relative MSFE ratios of other forecasting strategies are slightly above but close to one. It can be observed that the performance of S4 is a bit worse. The reasons are that this strategy assumes a stationary AR(1) process and a decline in persistence to happen. Given that both assumptions are wrong, it is convincing that this strategy performs less well. Table 3 provides some summary statistics for the DF-GLS unit root pre-test, the CUSUM of squares-based pre-test for changing persistence and the Bai-Perron procedure. It can be seen, that the pre-tests are approximately correctly sized and that the Bai-Perron procedure selects a break in 18.8% of the cases. Although this number is relatively large, it does not affect the forecasting performance of S6 and S7 much.

The case of a highly persistent but stationary AR(1) model (experiment 2, Figure 2) shows a similar picture except the worse performance of the random walk forecast S2. Recall, that S1 is the benchmark. Table 3 shows that the DF-GLS test has decent power, that the CUSUM of squares-based test is conservative and that the Bai-Perron procedure selects a break only in 5.2% of the cases. For lower values of the AR(1) coefficient, forecasting strategies get indistinguishable except the poor random walk forecast. Its relative MSFE ratio is close to two, indicating that the loss is close to 100%.

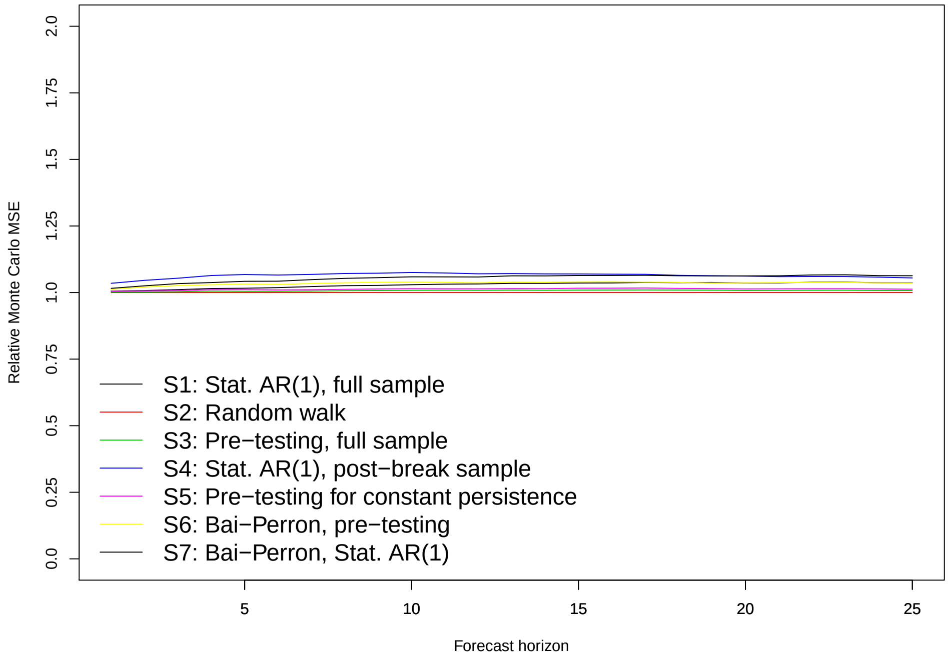


Figure 1: Constant $I(1)$, $\beta_1 = \beta_2 = 1$ (Exp 1)

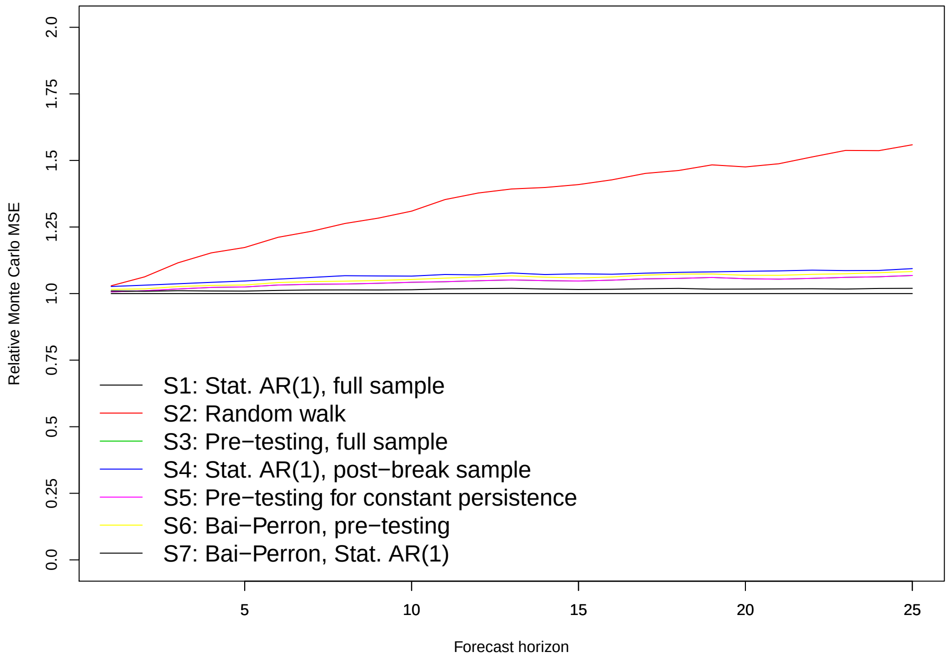


Figure 2: Constant $I(0)$, $\beta_1 = \beta_2 = 0.9$ (Exp 2)

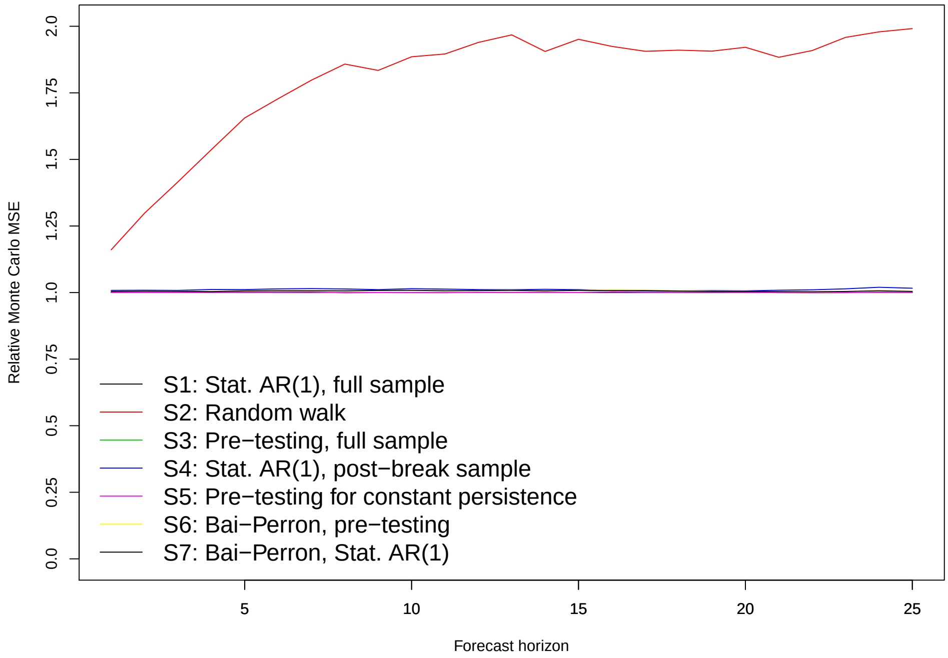


Figure 3: Constant $I(0)$, $\beta_1 = \beta_2 = 0.7$ (Exp 3)

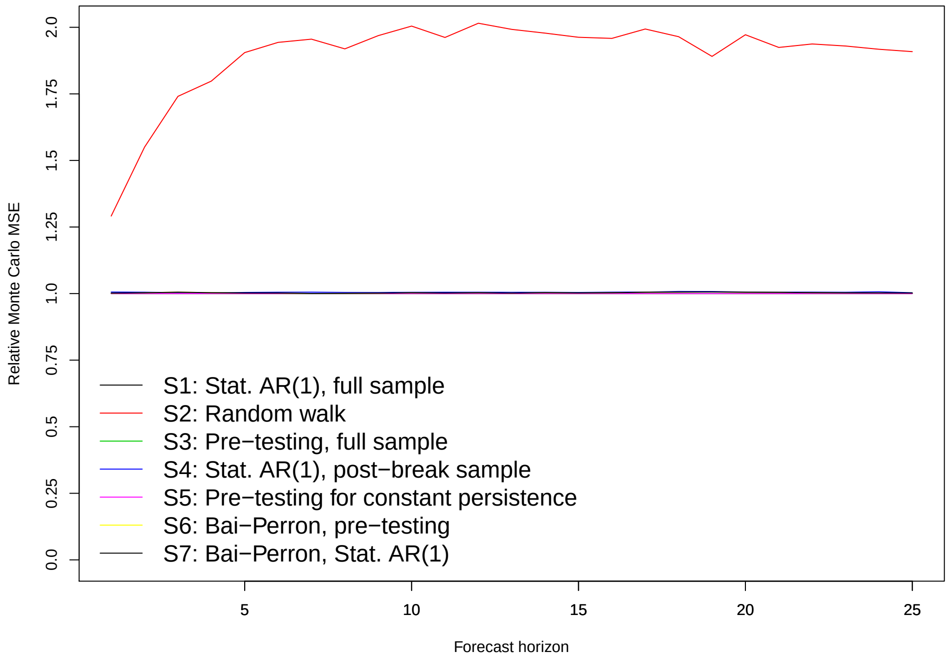


Figure 4: Constant $I(0)$, $\beta_1 = \beta_2 = 0.5$ (Exp 4)

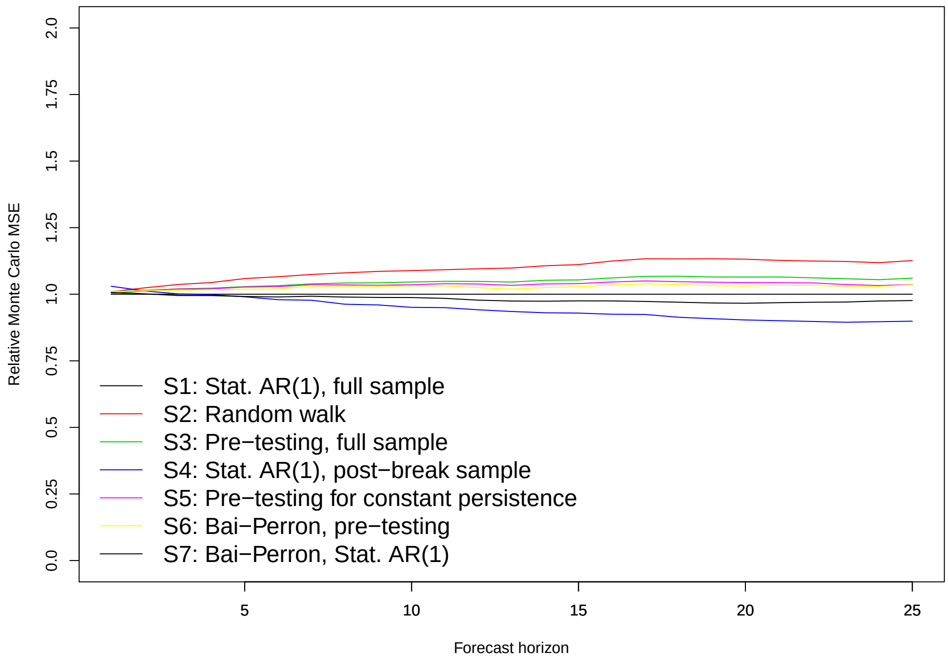


Figure 5: Change in persistence from $I(1)$ to $I(0)$, $\beta_1 = 1, \beta_2 = 0.9$ (Exp 5)

## 5.2 Change in persistence

### 5.2.1 Decreasing persistence

The case of decreasing persistence is covered in experiments 5–7 and results are shown in Figures 5–7. As the true degree of integration is zero in the out-of-sample period, S1 is the chosen as the benchmark. Firstly, results shown in Figure 5 are interpreted. For short forecast horizons up to $h = 5$, it is difficult to provide a ranking of forecasting strategies, but for higher values of $h$, we observe that S4 is dominating. This result is not surprising as this strategy assumes a decline in persistence to happen. The gains increase with $h$, but they are relatively small. Table 3 shows that the pre-test for a change in persistence has little chance to detect the change as opposed to the Bai-Perron procedure which is able to detect the structural change in 20% of the cases. This circumstances results in a relatively poor performance of strategy S5 (pre-testing for a change in persistence). Its relative MSFE ratio is above one, indicating that is performs worse then the benchmark which does not account for structural changes. Although the Bai-Perron procedure is able to detect the break sometimes,

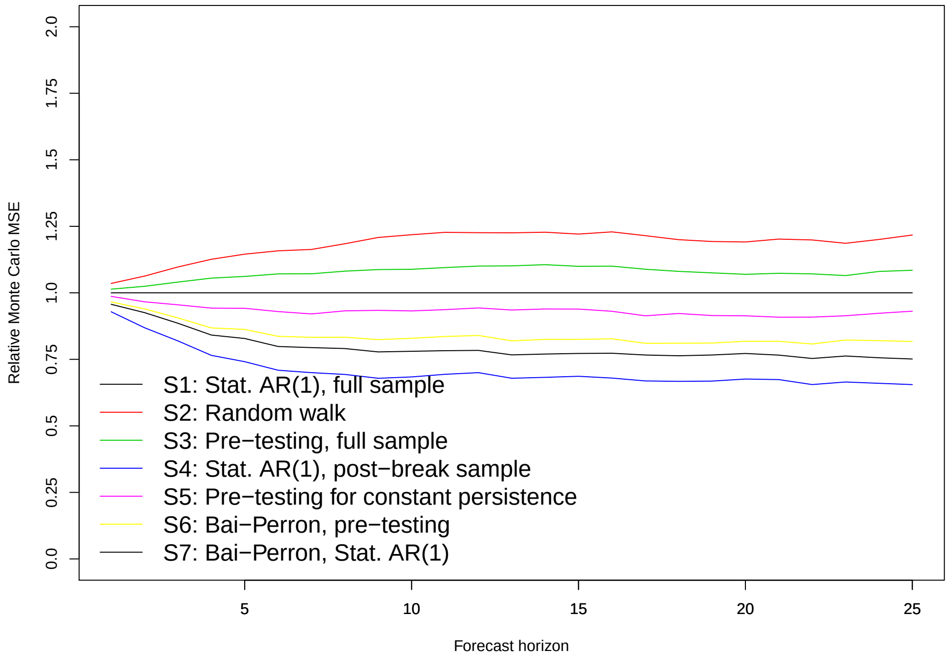


Figure 6: Change in persistence from $I(1)$ to $I(0)$, $\beta_1 = 1, \beta_2 = 0.7$ (Exp 6)

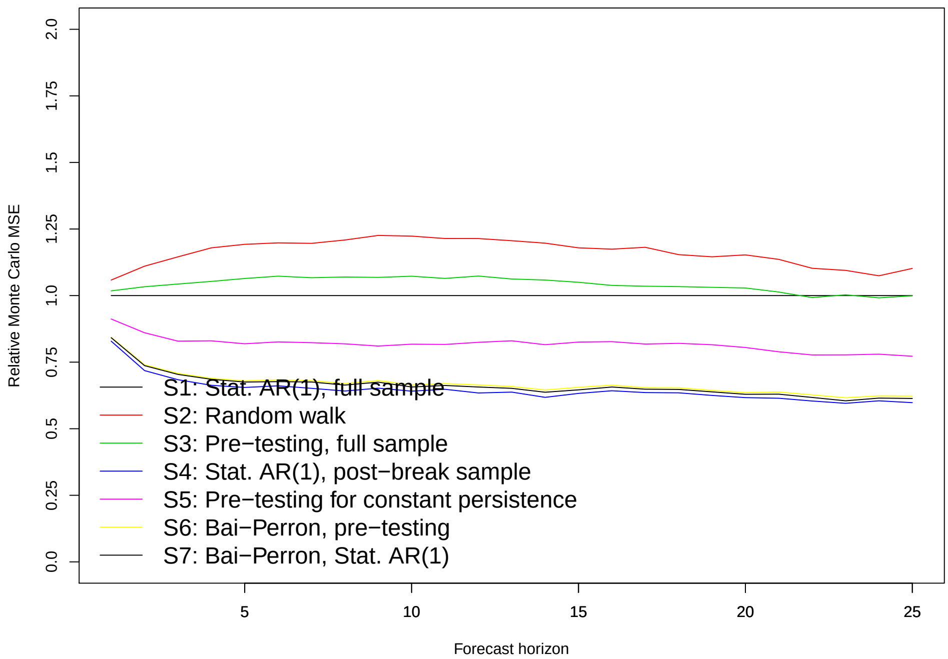


Figure 7: Change in persistence from $I(1)$ to $I(0)$, $\beta_1 = 1, \beta_2 = 0.5$ (Exp 7)

it is crucial whether S6 or S7 is applied. While the relative MSFE ratio of S6 is below one, the one S7 is above one, making S6 preferable over S7. These strategies differ when the Bai-Perron procedure does not select a break: S6 applies the DF-GLS test, while S7 directly uses a stationary AR(1) forecasting model. Strategy S6 suffers from the fact that the DF-GLS test gives mixed evidence (rejection rate is 30.6%).

When turning to experiment 6 (see Figure 6), where the change in persistence is moderate (from 1.0 to 0.7), a clear ranking across the forecast horizon can be given. S4 dominates other strategies from $h = 1$ to $h = 25$, followed by Bai-Perron-based forecasting approaches S7 and S6. The pre-test for a change in persistence is better performing than in the previous experiment as it is more powerful, but still not as good performing as the Bai-Perron procedure. Results in Table 3 show that the frequencies of a selected break for the Bai-Perron procedure are 31.0% and 62.9%, respectively. Gains in forecast precision increase up to a short forecast horizon of $h = 5$ and become constant over $h$ afterwards. Notably, the potential gain to be made is larger than 25% if S4 is applied instead of S1. The pre-testing strategy performs somewhat better but it is clearly dominated by S4, S6 and S7.

Finally, in experiment 7 (see Figure 7), we still observe the same ranking but S6 and S7 are now as equally good as S4. Table 3 shows that the simulated frequency of selecting a break is high (90.9%). The gains in forecast precision are around 35%. The results for decreasing persistence show that negligence is very costly and that the strategies S4 and S6 are able to provide relatively accurate forecasts.

### 5.2.2 Increasing persistence

When we consider the case of increasing persistence (experiments 8–10 and Figures 8–10), we find a striking asymmetry. As the results are not too distinct in the individual experiments 8–10, a more general analysis of the results is given here. The random walk forecast serves as the benchmark in these experiments. As this forecast does not depend on any parameter estimation, the choice of the sample (full or post-break only) does not matter. Effectively, only the last observation is crucial.

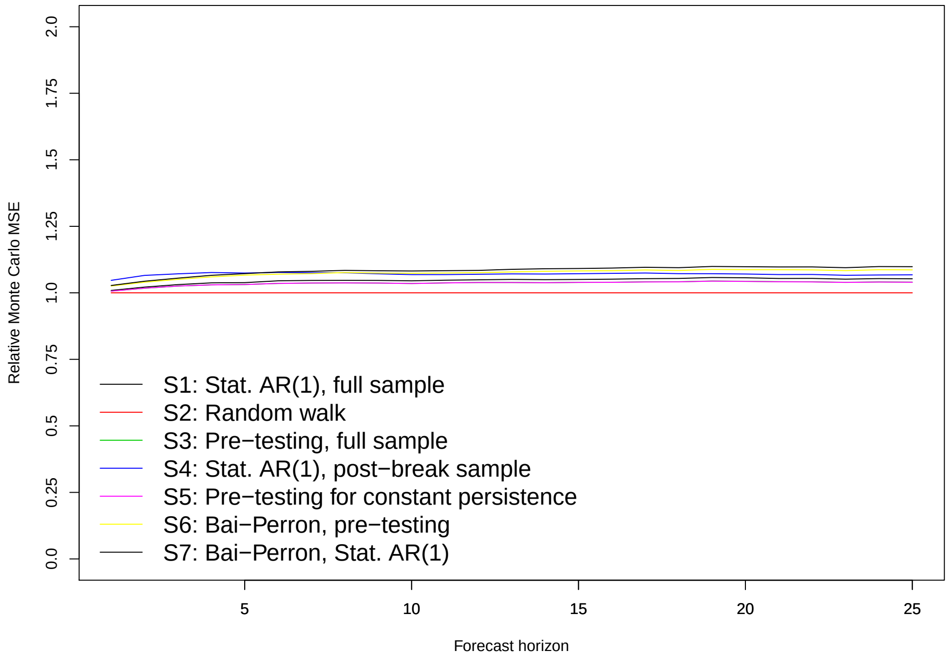


Figure 8: Change in persistence from $I(0)$ to $I(1)$, $\beta_1 = 0.9, \beta_2 = 1$ (Exp 8)

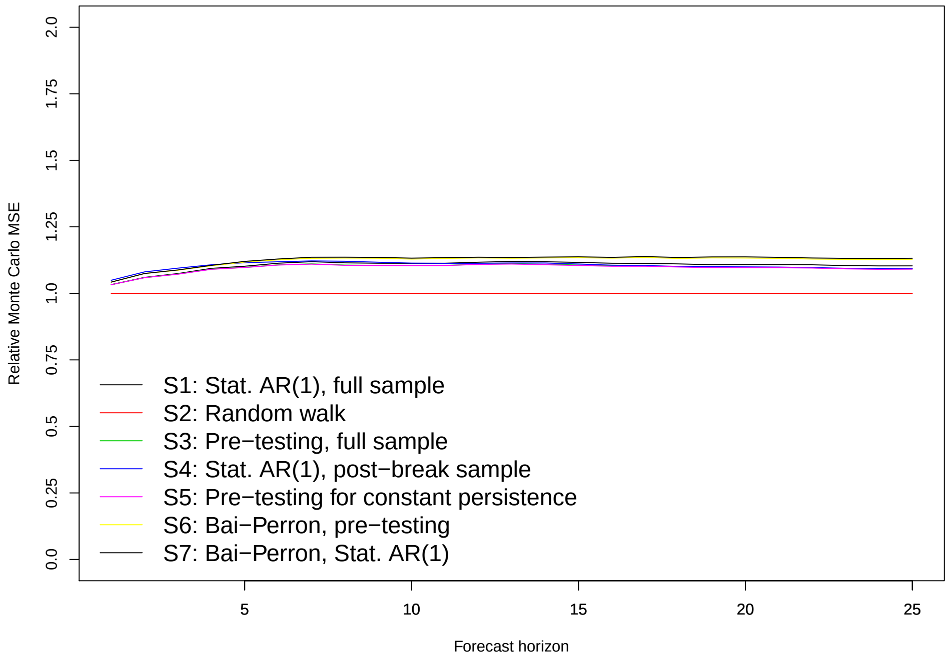


Figure 9: Change in persistence from $I(0)$ to $I(1)$, $\beta_1 = 0.7, \beta_2 = 1$ (Exp 9)

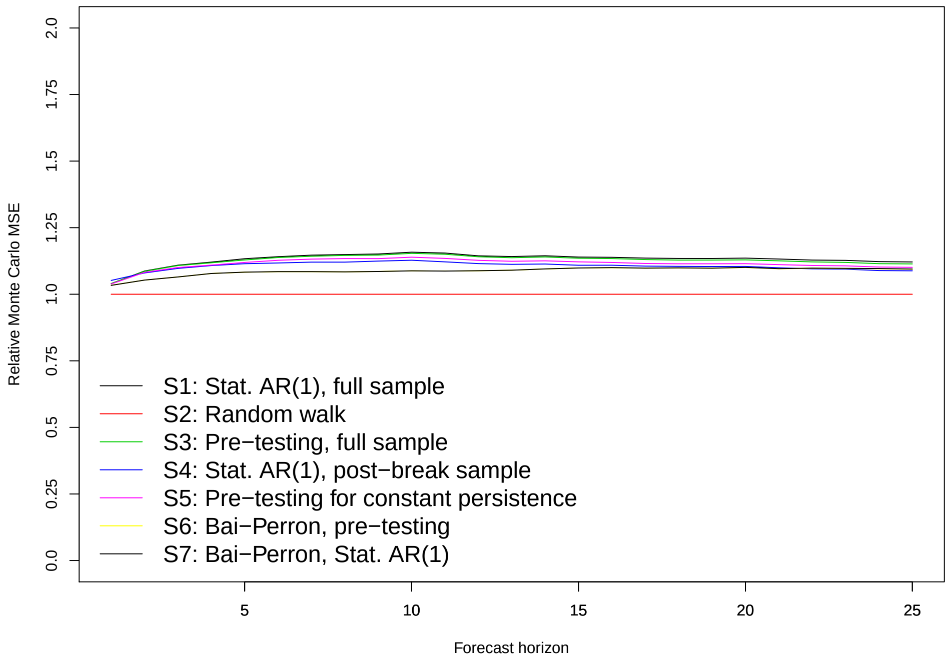


Figure 10: Change in persistence from $I(0)$ to $I(1)$, $\beta_1 = 0.5, \beta_2 = 1$ (Exp 10)

Even if the pre-tests would have hundred percent power against the alternative of an increase in persistence, the performance of these strategies would only as good as the random walk forecast, but not better. Therefore, it should not be surprising that S2 is not outperformed in experiments 8–10. The results in Figures 8–10 show that the random walk forecast is actually never outperformed by any of its competitors for all forecast horizons. Moreover, all other strategies are performing similarly with only minor differences among them. In addition, the relative MSFE ratios are more or less constant over $h$.

It is important to note, that the losses in forecast accuracy are not as high as the ones obtained for decreasing persistence. Even if the increase in persistence is large (experiment 10, Figure 10) where the AR(1) coefficient changes from 0.5 to 1.0, the loss of ignoring the structural break is relatively small (12.5%). Figure 10 shows that strategies S6 and S7 are the relatively well performing. Table 3 shows that the performance of pre-tests and the Bai-Perron procedure can be judged as symmetric,

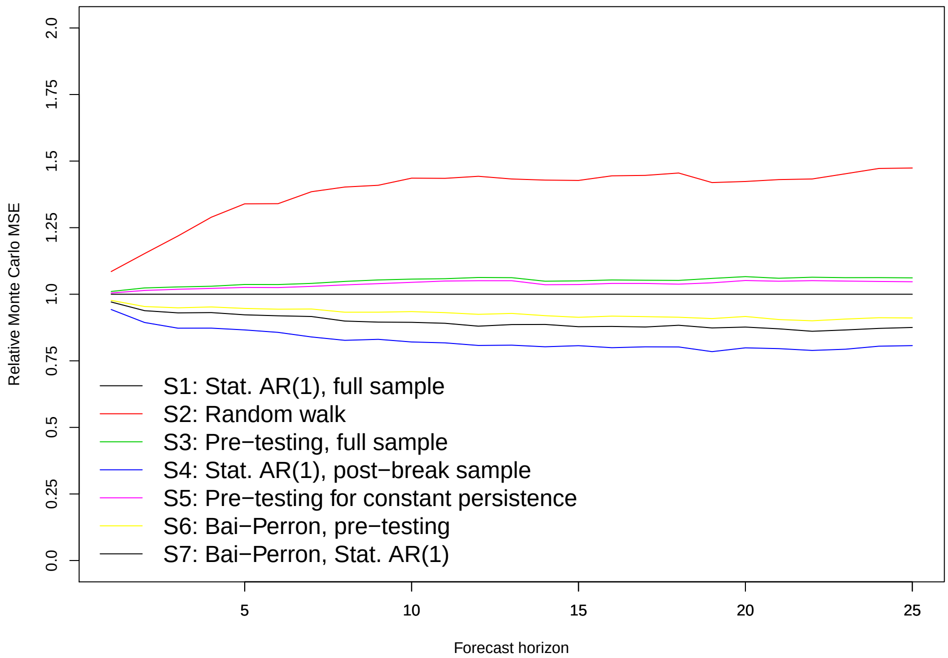


Figure 11: Stable shift constant $I(0)$, $\beta_1 = 0.9, \beta_2 = 0.7$ (Exp 11)

when compared to the case of decreasing persistence. Summing up, the results for increasing persistence show that negligence is better than accounting for it when a unit root (S2) is imposed. On the contrary, not much forecast precision is lost when any of the other six strategies is applied. This is in clear contrast to the results obtained for the case of decreasing persistence.

## 5.3 Stable shifts

The last four experiments 11–14 cover the case of stable shifts which means that the AR(1) coefficient is subject to a structural change which occurs within the parameter region of stationarity. Therefore, the data generating process is $I(0)$ throughout the whole sample. In experiments 11 and 12, the AR(1) coefficient is decreasing, while the opposite holds for experiments 13 and 14. The results for the former two experiments are reported in Figures 11 and 12, respectively. They show a similar ranking as in experiments 6 and 7, where S4 dominates and S6 and S7 follow closely. The pre-test for a change in persistence is useless since it is not able to detect a change within

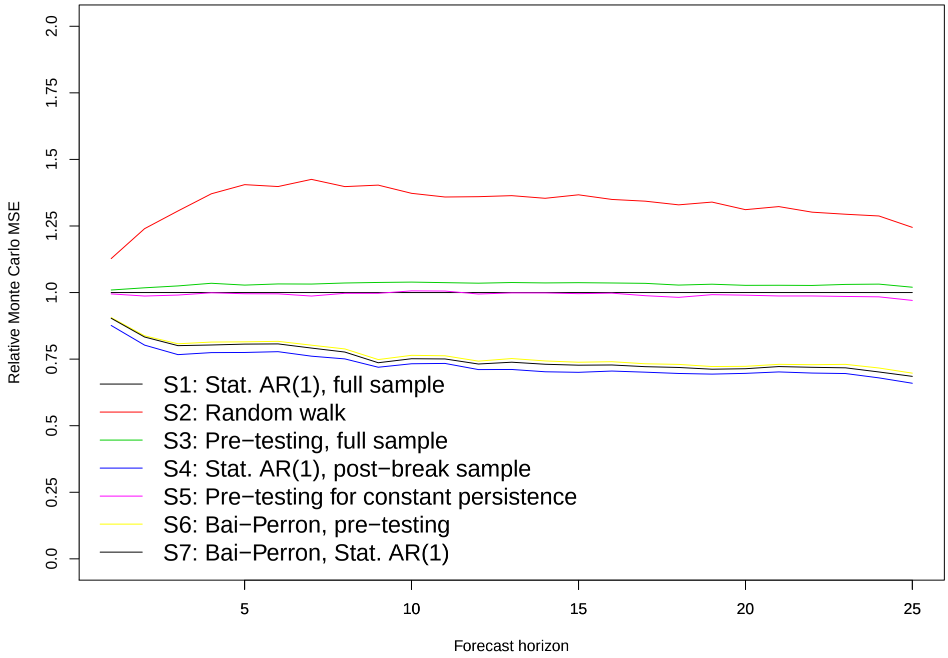


Figure 12: Stable shift constant $I(0)$, $\beta_1 = 0.9, \beta_2 = 0.5$ (Exp 12)

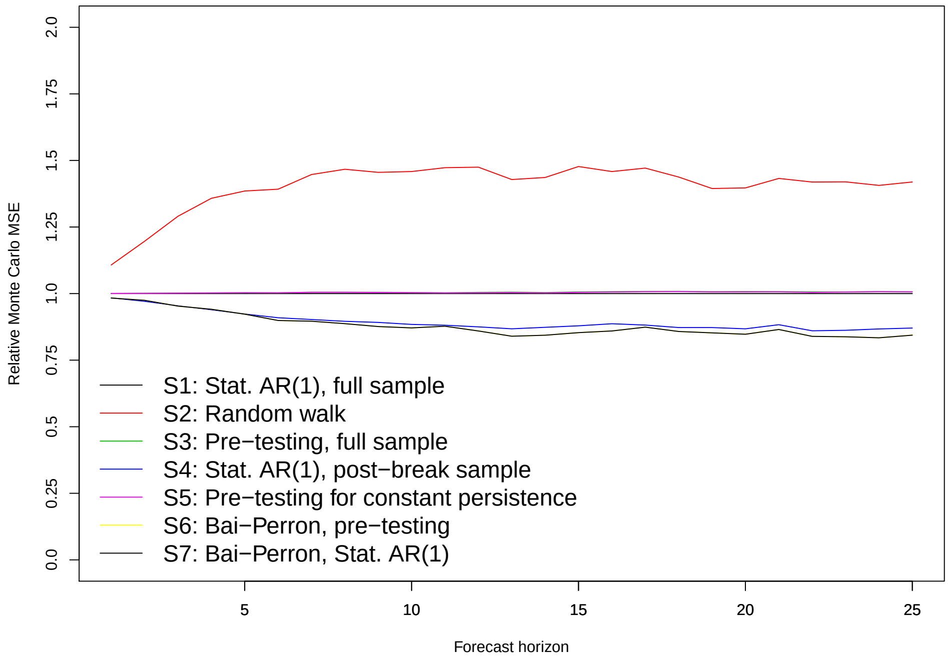


Figure 13: Stable shift constant $I(0)$, $\beta_1 = 0.7, \beta_2 = 0.9$ (Exp 13)

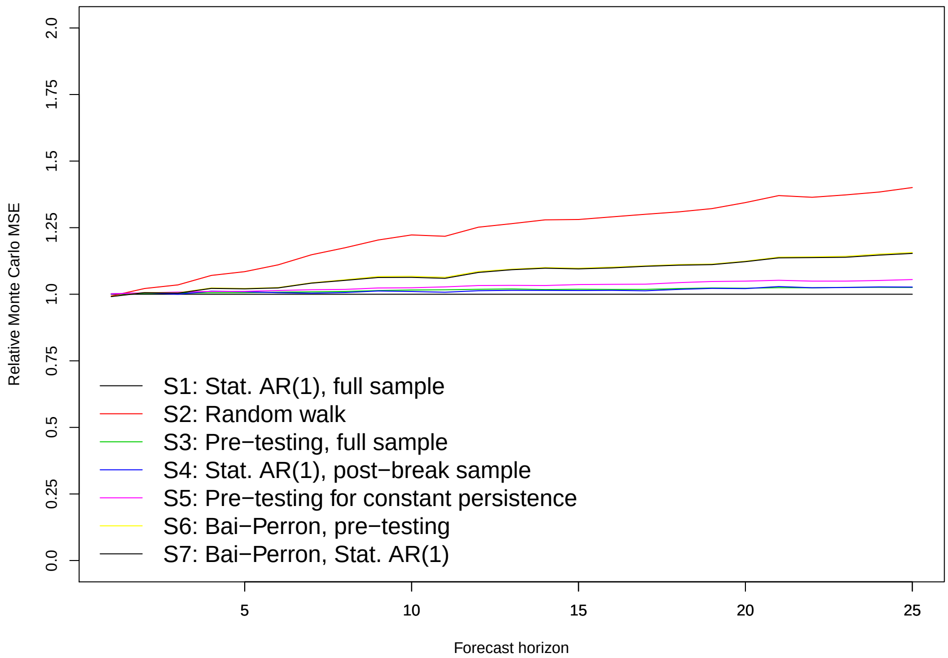


Figure 14: Stable shift constant $I(0)$, $\beta_1 = 0.5, \beta_2 = 0.9$ (Exp 14)

the region of stationarity, see Table 3. On the contrary, the Bai-Perron procedure shows a good performance as it selects a break in many cases (37.8% and 73.0%). Remarkable gains (around 25%) can be made in the case of $\beta_1 = 0.9$ and $\beta_2 = 0.5$, see Figure 12.

The case of an increasing AR(1) coefficient is interesting to compare with the outcomes of experiments 9 and 10. If the AR(1) coefficient changes mildly from 0.7 to 0.9, strategies S4, S6 and S7 are best performing monotonically over the forecast horizon $h$. The random walk forecast S2 performs poorly. All other strategies show a relative MSFE ratio around one for all $h$. If the size of the break increases (last experiment 14, Figure 14), we observe that the benchmark (S1) cannot be outperformed. S4 and the simple pre-testing strategy S3 are, however, extremely close to S1 in terms of forecast precision. The forecasting strategies based on the Bai-Perron procedure perform slightly worse for large values of $h$.

# 6 Conclusion

This paper considers out-of-sample forecasting of autoregressive time series in the context of potentially changing persistence. A change in persistence means that the time series process switches from stationarity to non-stationarity over time, or vice versa. This type of structural change is of substantial empirical interest. Therefore, we provide guidance for forecasting under uncertainty about the existence of a change in persistence. Neglecting such changes can be very costly in terms of out-of-sample MSFE. On the contrary, falsely imposing a change in persistence may lead to less precise forecasts. To this end, we study the empirical performance of seven forecasting strategies which cope with structural breaks in different ways or neglect them. Among these are pre-tests for unit roots and changing persistence as well as the Bai-Perron procedure for detecting and dating of structural breaks.

The outcomes of an extensive Monte Carlo study allow several important practical recommendations. First of all, the cases of decreasing and increasing persistence have to be treated separately. The striking asymmetry between the forecasting performance under increasing and decreasing persistence differs from the established literature on forecasting and structural change. If persistence decreases, forecasters should account for it. Secondly, the application of forecasting strategies based on the Bai-Perron procedure is promising: their accuracy is often very close to the one which imposes a decline in persistence. Third, pre-testing for changing persistence should be avoided. Fourth, if persistence increases, the random walk forecast produces most accurate forecasts. It proves to be beneficial to neglect the break and to impose a unit root prior to forecasting. However, the loss in forecast precision implied by Bai-Perron-based forecasting strategies is relatively low, especially when compared to the potential gains to be made under decreasing persistence.

Under constant persistence, modeled by a random walk or stationary autoregressive models, we find that falsely imposing a change in persistence leads to losses. Again, Bai-Perron-based forecasting strategies are highly recommendable. Finally, another empirically relevant possibility of a so-called stable shift, a structural change of the autoregressive parameter within the region of stationarity, is considered. The forecasting strategies based on the Bai-Perron procedure prove to be of usefulness, too.

Future research may consider interval and density forecasts under changing persistence. These are especially relevant for inflation series which are a leading empirical example for changing persistence. Moreover, persistence may also be measured by means of fractionally integrated processes instead of considering autoregressive roots.

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