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Abstract: This paper re-examines the joint distribution of equity and bond returns using high frequency data. In particular, we analyze the weekly realized stock bond correlation calculated from 5-minute returns of the futures prices of the S&P 500 and the 10-year Treasury Note. A potentially gradual transition in the realized correlation is accommodated by regime switching smooth transition regressions. The regimes are defined by the VIX/VXO volatility index and the model includes additional economic and financial explanatory variables. The empirical results show that the smooth transition model has a better fit than a linear model at forecasting in sample, whereas the linear model is more accurate for out-of-sample forecasting. It is also shown that it is important to account for differences between positive and negative realized stock bond correlations.

Keywords: realized correlation; smooth transition regressions; stock bond correlation; VIX index

JEL Classifications: C22; G11; G17

1 Introduction

This paper investigates the nature of the realized stock bond correlation using high frequency (5-minute) returns. So far, little attention has been given to high frequency data in the stock bond correlation literature. We put forward a smooth transition regression (STR) for the correlation with two extreme regimes broadly corresponding to low volatility and high volatility states. This specification is attractive as it allows for a continuum of states between the two extreme correlation regimes. We then analyze how well the model fits stock bond comovements by characterizing how much of the correlation is ascribed to economic variables. Up to now, a number of methods have had a modest degree of success in modelling the correlation using economic data, e.g. Baele, Bekaert and Inghelbrecht (forthcoming). The STR model is new to the stock bond correlation literature and we show it provides a promising methodology to explore.

Understanding the nature of the stock bond correlation has crucial implications for asset allocation, risk management, and option pricing as these are the two main asset classes. In particular, it is important to know whether stock and bond returns are correlated and if so whether the correlation is positive or negative and whether the correlation is strong or weak. For the same reasons it is of interest to analyze the economic forces driving the time varying stock bond correlation.

Most studies on high frequency data have focused on realized volatility with only few recent papers analyzing the realized correlations between asset returns. High frequency data are appealing in that they contain as much information as possible and, therefore, may provide a more accurate correlation measure compared to correlations from rolling windows based on historical data or those from multivariate GARCH models using data of lower frequencies. The studies by Audrino and Corsi (2008) and Christiansen and Ranaldo (2007) have recently used high frequency data in the analysis of the stock bond correlation. The first paper adopts a heterogeneous autoregressive model and shows that its out-of-sample forecasts are more accurate than those of standard autoregressive models (AR or ARMA). On the other hand, Christiansen and Ranaldo (2007) look at how the stock bond correlation changes when (surprises to) scheduled macroeconomic news are announced.

Recent years have seen a growing literature exploring the economic determinants of the time varying stock bond correlations. For instance, Li (2002) shows that the unexpected inflation is the most important determinant of the stock bond correlation and addresses the welfare effects of correlation changes for investors. Ilmanen (2003) argues that stock bond correlations calculated by rolling windows of historical data depend upon the business cycle of the macro economy as well as upon the inflation rate. Pastor and Stambaugh (2003) find that changes in stock bond correlations are related to different levels of liquidity. By advocating the use of regime switching models, Guidolin and Timmermann (2006) argue for the role of the macro economy in determining correlation regimes. In a similar spirit, Bansal, Connolly and Stivers (forthcoming) use a Hamilton (1988) regime switching model and find regime shifts in the stock bond correlation. They argue that the state of the regime switching model may be linked to the VIX volatility index. Baur and Lucey (2009) uses the DCC model of Engle (2002) and document significant time variation in stock bond correlations. The above analyses are mainly based upon daily data. At lower frequencies (monthly data) and from a historical perspective, Yang, Zhou and Wang (2009) investigate the correlations over the last 150 years and document significant differences across the business cycle. On the other hand, Connolly, Stivers and Sun (2007) and Connolly, Stivers and Sun (2005) reveal the importance of stock market volatility as a major determinant for correlations. Finally, using quarterly data, Baele et al. (forthcoming) investigate various possible economic sources of the stock bond correlation. Macroeconomic factors are found to play only a minor role and, therefore, they conclude that the debate remains open on how the time variation in the stock bond correlation is driven by changing macroeconomic conditions.

The present paper contributes to the existing literature as follows. We test for the significance of various economic determinants of the realized stock bond correlation. Unlike other studies in the literature, we augment the set of determinants with the realized stock and bond returns and their corresponding volatilities. We propose a STR model that identifies two extreme regimes for the realized stock bond correlation. These correspond to low volatility and high volatility with a gradual change between the two volatility regimes. We further find that the lagged realized stock bond correlation, the realized stock and bond volatilities and the inflation rate come out as important determinants of the realized correlation across the regimes. These results are robust to different forecast horizons and to the effects of the realized correlation being positive and negative. Nevertheless, although our STR model improves the fit of its linear counterpart, it provides less accurate out-of-sample forecasts.

The remaining part of the paper is structured as follows. Section 2 introduces the econometric framework. Section 3 contains the data description. Section 4 provides the empirical results based upon in sample estimations. Section 5 considers out-of-sample results. Section 6 concludes.

2 The Smooth Transition Regression Model

One of the most prominent among the regime switching models in the macroeconomics area has been the smooth transition regression (STR) class of models promoted by Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993), and Teräsvirta (1994). Modelling the realized stock bond correlation within the STR context can be motivated by the fact that the regime switching mechanism can be controlled by an observable economic determinant of the correlation. For example, we can differentiate between the impact of the stock market during periods when volatility is large and its impact on correlations during periods when market volatility is low. In particular, the equation of interest is the 2-regime STR model given by

$$FRC_t = \beta x_t + (\delta x_t) F(\gamma, c; s_t) + \varepsilon_t \tag{1}$$

where FRC_t is the realized correlation, β and δ are parameters vectors, and x_t is a vector of predictor variables.[1] The function $F(\gamma, c; s_t)$ is the transition function, which is assumed to be continuous and bounded by zero and unity. The variable s_t acts as the transition variable and is chosen from the vector of predictor variables.

By writing Eq. (1) as

$$FRC_t = (\beta + \delta F(\gamma, c; s_t)) x_t + \varepsilon_t \tag{2}$$

we can see that the model is locally linear in x_t and that the combined parameter vector $(\beta + \delta F(\gamma, c; s_t))$ is a function of the transition variable s_t. As $F()$ is bounded between zero and one, the combined parameter fluctuates between β and $(\beta + \delta)$. δ is the addition to the coefficients when we are in the second regime. Values of zero of the transition function identify regime one, and values of unity identify the alternative regime. Then, values of $F()$ between 0 and 1 would define situations where the relationship is a mixture of the two regimes. In the analysis of the realized stock bond correlation, this property makes it possible to study how the correlation responds asymmetrically (possibly in a smooth way) to changing macroeconomic conditions.

The practical applicability of the above specification depends on how $F()$

[1] FRC_t is the Fisher transformation of the realized correlation. The exact definition is provided below.

is defined. One type of transition function used a lot in the literature is the logistic function

$$F(\gamma, c; s_t) = \frac{1}{1 + \exp(-\gamma(s_t - c))}, \ \gamma > 0 \tag{3}$$

where the parameter c is the threshold between the two regimes, that is the location of the transition function. The parameter γ is the slope of the function and determines the smoothness of the change in the value of the logistic function and thus the speed of the transition from one regime to the other. When $\gamma \to \infty$, $F()$ becomes a step function and the transition between the regimes is abrupt, ($F() = 0$ if $s_t < c$ and $F() = 1$ if $s_t > c$). In this case, the model approaches a threshold model. Finally, identification requires $\gamma > 0$.

A smooth transition between the two extremes may be an attractive parameterization because, from a theoretical point of view, the assumption of two discrete regimes (like in threshold models) may sometimes be too restrictive compared to the STR alternative where there is a continuum of states between the two extremes. Nevertheless, the two viewpoints are not in conflict since an abrupt switch is a special case of the STR model and can therefore be treated within the STR framework.

Estimation of the STR model in Eq. (1) is carried out by nonlinear least squares (NLS), which is equivalent to the maximum likelihood estimation in the case of normal errors. As pointed out by Granger and Teräsvirta (1993) and Lundbergh and Teräsvirta (1998) STR models may not have a good fit if there is heteroskedasticity in the data. Therefore, we use Newey and West (1987) standard errors that allow for heteroskedasticity and autocorrelation. Another issue that deserves attention is the selection of starting values for the NLS estimation. Starting values for the STR are based on a parsimonious STR with only the autoregressive component as regressor and the volatility index VXO/VIX acting as the transition variable.[2]

3 Data Description

For the empirical analysis we use a number of time series all recorded at a weekly frequency on Fridays. The sample covers the period January 1986 to May 2009 which gives us a total of 1,222 observations.

We obtain trade data from Tick Data on the futures contracts on the S&P 500 and the 10-year Treasury Note. They have the symbols SP and TY and

[2] All reported results have been obtained using EViews.

trade at the Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT), respectively. We use the SP and TY to calculate the bond and stock realized returns, the bond and stock realized volatilities, and the stock bond realized correlation. The CME is open 9.00-15.15 (Eastern Standard Time) whereas the CBOT is open 8.00-14.00. The CME and CBOT have overlapping trading sessions from 9.00-14.00. Therefore, we calculate the 5-minute returns on the SP and TY during 9.00-14.00 each day. Thus, we leave out the returns during periods when both exchanges are not open (including overnight and weekend returns).

The weekly realized stock return on the Friday in week t is denoted $RRSP_t$ and it is calculated as the sum of all the 5-minute stock returns during that week. Similarly, the weekly realized bond return on week t is denoted $RRTY_t$. The weekly realized stock variance on a given Friday is then the sum of all the squared stock returns during that week. We use the square root of the realized variance, the realized stock volatility. We let the symbol $RVSP_t$ indicate the realized stock volatility in week t. The realized bond volatility for week t is denoted $RVTY_{t.}$

The realized stock bond correlation for week t is noted RC_t. First, we calculate the realized covariance for that week as the sum of the cross multiplied 5-minute stock and bond returns. Then, the realized correlation is the realized covariance divided by the product of the realized bond and stock volatilities. We make use of the Fisher transformation of the realized correlation which is a continuous variable which is not bounded between -1 and 1. The Fisher transform is given as:

$$FRC_t = \frac{1}{2} \ln \left(\frac{1 + RC_t}{1 - RC_t} \right). \tag{4}$$

We make use of some further explanatory variables, namely the VXO_t, $DTBILL_t$, INF_t, and GDP_t. The VXO_t is the CBOE (Chicago Board of Options Exchange) volatility index that is based upon the trading of options on the S&P100 index. The launch of the VXO determines the starting point of our sample. Before 2003 the VXO was denoted the VIX index, now the VIX index measures the volatility of options on the S&P 500 index. The VXO plays an important role in describing the relationship between bond and stock returns, cf. Connolly et al. (2005). We use the short rate as explanatory variable, similarly to Baele et al. (forthcoming). We use the first differences of the 3-month US Treasury Bill middle rate from the secondary market. The T-bill rates are available from DataStream. This series of short rates is denoted $DTBILL_t$. We obtain a series of weekly inflation rates, INF_t, using US Core

CPI available from the Bureau of Labor Statistics. The CPI data area available monthly. From these we calculate the monthly inflation rates as the log-returns. Then for each week we use the most recent monthly inflation observation. This means that the inflation variable will be constant for (most often) four weeks in a row. We obtain a series of weekly GDP growth rates, GDP_t. Again, these are obtained from the monthly GDP figures and are calculated in the same way as the inflation data.

All variables except the FRC_t are standardized by subtracting their mean and dividing by their standard deviation. Hereby they have mean zero and unit variance. All variables are of the same magnitude and will ease the interpretation of the results, in particular the size of the coefficients. Still, other features of the variables are intact.

Figure 1 shows the graph of the realized correlation. The series starts out being positive, at about 0.4, and then turns negative in the middle of the sample period. Although the realized correlation is somewhat erratic, there are still clear trends to be seen. It is noticeable that the realized correlation is both positive and negative during the sample period.

Table 1 shows the descriptive statistics. The FRC has negative skewness which implies that is has a long left tail, and its distribution is flat (mesokurtic). The remaining variables all have positive skewness and are leptokurtic.

4 In Sample Results

Here we consider the results arising from estimating the STR model in sample. First, we consider the so-called simple model. Second, we take into account the differences between positive and negative realized correlations in what we denote the extended model. Third, we consider different forecast horizons.

4.1 Simple STR Model

In the simple model we predict four weeks ahead. We use the volatility index lagged four weeks as the transition variable

$$s_t = VXO_{t-4}. \tag{5}$$

The VXO index represents the market expectations of 30-day volatility and is constructed using the implied volatilities of a wide range of S&P 100 index options. It is a commonly used measure of market risk and economic uncertainty. We get the following classification of regimes for the realized stock bond correlation: When the VXO is low, we are in the low stock market volatility regime and the parameters of the model are β. On the other hand, when the VXO is high, we are in the high volatility regime and the parameters change smoothly to $(\beta + \delta)$. In particular, an alternative way of writing Eq. (1) is

$$FRC_t = \begin{cases} \beta x_t + \varepsilon_t & F() = 0 \quad \text{low volatility regime} \\ (\beta + \delta) x_t + \varepsilon_t & F() = 1 \quad \text{high volatility regime} \end{cases} \tag{6}$$

The explanatory variables in the simple model are given as 4 week lagged observations of a number of financial and macroeconomic variables:

$$x_t = \left\{ \begin{array}{c} 1, FRC_{t-4}, VXO_{t-4}, RRSP_{t-4}, RRTY_{t-4}, \\ RVSP_{t-4}, RVTY_{t-4}, DTBILL_{t-4}, INFL_{t-4}, GDP_{t-4} \end{array} \right\} \tag{7}$$

The estimation results are shown in Table 2 (middle columns).

In the simple STR model, $\widehat{\gamma} = 0.9$. This is a small value which implies that the the transition from the low to the high volatility regime is fairly gradual (in contrast to being abrupt).[3] The parameter c is fairly large ($\widehat{c} = 4.2$) which implies that it takes large values of the transition variable to move from the low volatility regime to the high volatility regime. Remember that all variables are standardized to have zero mean and unit variance. Figure 2 plots the estimated transition function versus the transition variable VXO_{t-4}. Notice the very gradual change between the two volatility regimes. To assist with the economic interpretation of these regimes, Figure 3 (Panel A) shows the time series plot of the transition function together with the NBER business cycle dates. As seen, the high volatility regime picks up mainly the October 1987 crash and the recession of 2008. Although the matching with NBER recession dates is by no means direct, there is clearly an association during the recent recession.

In the low volatility regime, the autoregressive component, the realized stock volatility, and the inflation are all significant in explaining the realized correlation. For these variables there is a positive relationship so that the higher the explanatory variable, the stronger is the stock bond correlation. It is also clear that the autoregressive component is much stronger than the effects from realized stock volatility and from inflation. In particular, the autoregressive component is large but smaller than unity (0.86) so it has a positive and strong autoregressive component. A large correlation today will then tend to be associated with a large correlation four weeks from now when we are in the low volatility regime. Large in this context means both large positive and large neg-

[3] The parameter γ has a large standard error which is a general problem arising in estimating STR models. For more details, cf. Teräsvirta (1994), p. 213.

ative. Ferland and Lalancette (2006) and Audrino and Corsi (2008) also find a strong temporal dependence for the realized stock bond correlation.

On the other hand, in the high volatility regime the effective slope coefficients are $(\beta + \delta)$. None of the individual δ coefficients are significant, but jointly they are just significant (Wald test gives rise to p-value of 9.9%).

Next, we conduct a number of joint Wald tests for the significance of a given explanatory variables in the low and high volatility regime simultaneously, $H_0 : \beta_j = \delta_j = 0$ (see Table 3). The lagged realized correlation, the realized stock volatility, the realized bond volatility, and the inflation come out as significant explanatory variables. From this we also learn that the realized returns (both stock and bond), short rates, and GDP growth rates are unimportant for the realized stock bond correlation.

A similar effect for inflation is found by Yang et al. (2009), but in addition they find a significant positive effect for the short rate. Note, however, they adopt a different procedure than ours: They use a multivariate GARCH model with a smooth transition conditional correlation (STCC) to estimate the stock bond correlation with the short rate and inflation as the transition variables (one each time). Instead, we calculate the realized stock bond correlation directly from intradaily data. Another important difference to Yang et al. (2009) is that they use monthly data while we look at higher frequency data. The monthly data interval may be too long for stock and bond prices as news is quickly incorporated into these two types of liquid assets.

For sake of comparison, Table 2 (left columns) also shows the results from the linear model estimated using OLS. In the linear model only the autoregressive component and the inflation are significant in explaining the realized stock bond correlation. Nevertheless, the explanatory power is large, in that the adjusted R-squared is 0.685. Also, going from the linear to the STR model provides some improvement in model fit, the adjusted R-squared increases slightly. However, the improvement is small, which is also seen from the fact that the individual δ coefficients are not significant in the STR model. Finally, the root mean squared error (RMSE) and the mean absolute error (MAE) of the fitted values are slightly smaller for the simple STR model than for the linear model, which implies a better in sample fit for the STR model.

4.2 Extended STR Model with Sign Dummies

We extend the simple STR model by considering the effects of the realized correlation being positive and negative. In particular, we introduce the dummy variable which is equal to 1 when the realized correlation is positive, $PosD_t = 1$

if $FRC_t > 0$ and 0, otherwise. We allow all coefficients of the explanatory variables in the STR model to depend upon the sign of the realized correlation at the same time as the explanatory variable is dated. So, now the explanatory variables are as follows:

$$x_t = \left\{ \begin{array}{c} 1, PosD_{t-4}, FRC_{t-4}, FRC_{t-4}PosD_{t-4}, \\ VXO_{t-4}, PosD_{t-4}VXO_{t-4}, RRSP_{t-4}, PosD_{t-4}RRSP_{t-4}, \\ RRTY_{t-4}, PosD_{t-4}RRTY_{t-4}, RVSP_{t-4}, PosD_{t-4}RVSP_{t-4} \\ RVTY_{t-4}, PosD_{t-4}RVTY_{t-4}, DTBILL_{t-4}, PosD_{t-4}DTBILL_{t-4}, \\ INFL_{t-4}, PosD_{t-4}INFL_{t-4}, GDP_{t-4}, PosD_{t-4}GDP_{t-4} \end{array} \right\} \tag{8}$$

The results from estimating the sign dummy extended STR model are reported in Table 2 (right columns). The explanatory power is higher for the sign dummy extended model than for the simple model; the R-squared is higher and the RMSE and MAE are smaller. The estimated slope parameter $\widehat{\gamma} = 34$, so the shift between regimes is rather fast. Moreover, now only a small value of the VXO is necessary to shift regime ($\widehat{c} = 0.4$). Still, the low volatility regime is the most frequently visited regime in our sample. All this is clearly seen in Figure 2, which plots the transition function versus the transition variable VXO_{t-4}. It is also interesting to relate these regimes to the underlying business cycle. Figure 3 (Panel B) plots the time series of the transition function with the NBER business cycle dates. Although erratic, the high volatility regime occurs around the three official recession periods of 1991, 2001, and 2008. Other historical episodes that trigger regime switches to the high volatility regime include the October 1987 crash and the 'Asian flu' of 1997.

Moreover, for the extended STR model we find significant differences between the dependence structure when the stock bond correlation is positive and negative; the joint Wald test has associated a p-value below 1% (results not tabulated). So, we conclude that it is important to take the sign of the current correlation into account when predicting future correlation.

We next test for the joint significance of the various explanatory variables and report the p-values of the Wald tests in Table 3. More specifically, for each variable x_j we test the following three hypotheses: $\{\beta_j = \beta_{jD} = 0\}$, $\{\delta_j = \delta_{jD} = 0\}$, and $\{\beta_j = \beta_{jD} = \delta_j = \delta_{jD} = 0\}$. The same variables as in the simple STR model are overall significant (lagged realized correlation, VXO, realized stock volatility, realized bond volatility) with two exceptions: Interestingly, in the extended model the VXO volatility turns out to be significant and the GDP growth rate is now significant in place of inflation. In addition, these variables are significant both in the low and high volatility regimes.

As in the simple model, the lagged correlation is most important for explaining the current correlation in the low volatility regime. The autoregressive behavior is stronger when the realized correlation is positive than when it is negative (0.73 compared to 0.57). Moreover, the addition to the parameters in the high volatility regime is statistically significant (the joint Wald test gives rise to a p-value below 1%). As in the simple model, the addition in the high volatility regime to the lagged realized correlation parameter is negative, yet the point estimate is much smaller, and it does not change the sign of the coefficient.

4.3 Different Forecast Horizons

In this subsection we access the robustness of the results to different forecast horizons. In addition to the base case of $k = 4$ we also estimate the model considering $k = 1$ and $k = 8$ weeks ahead. For the simple STR model the results are reported in Table 4.

It can be seen the results are qualitatively similar across the different horizons. The upper regime continues to pick up episodes of high market volatility while the most important element comes from the autoregressive component. As wit the base case of $k = 4$, the lagged VXO, realized stock volatility, realized bond volatility, and inflation turn out to be the most significant explanatory variables. As expected, the explanatory power decreases as the horizon becomes longer. Moreover, the change in the regime is more abrupt for the short horizon ($k = 1$) than for the middle ($k = 4$) and long ($k = 8$) horizons. This is perhaps not surprising given that in short horizons news is quickly incorporated into assets prices in short horizons making the change in the regime quick as well.

Table 5 reports the results from the STR model with sign dummies for all three horizons. Similar to the model without sign dummies the main findings continue to hold across the different forecast horizons. As before, the explanatory power decreases with the horizon. Finally, as expected the sign dummy model has a better fit to the data than the simple STR specification.

5 Out-of-Sample Results

We now examine the out-of-sample forecasting ability of the STR models. We use an expanding window for the out-of-sample estimation. The first window covers the period January 1986 to March 2005. Using this window of observations we estimate the linear, simple STR, and extended STR models using lagged explanatory variables with horizons of $k = \{1, 4, 8\}$. From these estimated models we make an out-of-sample forecast of the realized correlation.

Subsequently, the estimation window is expanded with one further observation and the out-of-sample forecasting is repeated. So, the out-of-sample forecast period runs from March 2005 to May 2009, thus providing 219 observations.

Table 6 shows the RMSE and the MAE values arising from the out-of-sample forecasts of the three models and for the three forecast horizons. Interestingly, the STR models do not improve on the linear benchmark Actually, the linear model has the lowest RMSE and MAE values followed by extended STR model, while the simple STR specification delivers the least accurate forecasts. This holds for all the forecast horizons. Similar results are obtained when focusing on the recent financial crisis period (results not reported).

To provide more insight, Figure 4 plots the out-of-sample forecasts and the actual realized correlation for the forecast horizon $k = 4$. In general, the linear and the extended STR forecasts both track the actual realized stock bond correlation quite well during the entire period. In fact, there are not big differences between the linear and the extended STR forecasts. In contrast, the simple STR forecast lie well below the actual correlation at the end of the sample period. Thus, it is mainly in the last part of the sample period that the simple STR model provides poor forecasts.

6 Conclusion

This study documents time-varying patterns for the stock bond correlation over macroeconomic conditions using high frequency data. High frequency data are appealing in that they provide a more accurate correlation measure compared to correlations obtained from rolling windows based on historical data or from multivariate GARCH models using data of lower frequencies. The realized stock bond correlation is described by smooth transition regressions (STR) with two extreme regimes broadly corresponding to low volatility and high volatility states. Unlike other studies in the literature, we augment the set of determinants of the realized stock bond correlation with the realized stock and bond returns and their corresponding volatilities.

Our results show that there is a rather gradual change between the low and high volatility regimes. We further find that the lagged realized stock bond correlation, the realized stock and bond volatilities, and the inflation rate come out as important determinants of the realized correlation across regimes. These results are robust to different forecast horizons and to the effects of the realized correlation being positive and negative. Nevertheless, although the STR model improves the fit of its linear counterpart, it provides less accurate out-of-sample forecasts.

As shown in the present paper, it is important to account for differences between positive and negative realized stock bond correlations in an STR framework. In future work, we believe it would be interesting to investigate further the causes of the realized correlation being positive or negative even further.

References

Audrino, F. and Corsi, F.: 2008, Modeling Tick-by-Tick Realized Corrlations, *Working paper*, Univerisity of St. Gallen.

Baele, L., Bekaert, G. and Inghelbrecht, K.: forthcoming, The Determinants of Stock and Bond Return Comovements, *Review of Financial Studies* .

Bansal, N., Connolly, R. A. and Stivers, C.: forthcoming, Regime-Switching in Stock and T-Bond Futures Returns and Measures of Stock Market Stress, *Journal of Futures Markets* .

Baur, D. G. and Lucey, B. M.: 2009, Flughts and Contagion - An Empirical Analysis of Stock-Bond Correlation, *Journal of Financial Stability* **5**, 339–352.

Christiansen, C. and Ranaldo, A.: 2007, Realized Bond-Stock Correlation: Macroeconomic Announcement Effects, *Journal of Futures Markets* **27**(5), 439–469.

Connolly, R. A., Stivers, C. and Sun, L.: 2007, Commonality in the Time-Variation of Stock-Stock and Stock-Bond Return Comovements, *Journal of Financial Markets* **10**, 192–218.

Connolly, R., Stivers, C. and Sun, L.: 2005, Stock Market Uncertainty and the Stock-Bond Return Relation, *Journal of Financial and Quantitative Analysis* **40**(1), 161–194.

Engle, R.: 2002, Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, *Journal of Business & Economic Statistics* **20**(3), 339–350.

Ferland, R. and Lalancette, S.: 2006, Dynamics of Realized Volatilities and Correlations: An Empirical Study, *Journal of Banking and Finance* **30**, 2109–2130.

Granger, C. W. J. and Teräsvirta, T.: 1993, *Modelling Nonlinear Economic Relationships*, Oxford University Press.

Guidolin, M. and Timmermann, A.: 2006, An Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns, *Journal of Applied Econometrics* **21**, 1–22.

Hamilton, J. D.: 1988, Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates, *Journal of Economics Dynamics and Control* **12**, 385–423.

Ilmanen, A.: 2003, Stock-Bond Correlations, *Journal of Fixed Income* **13**(2), 55–66.

Li, L.: 2002, Macroeconomic Factors and the Correlation of Stock and Bond Returns, *Working paper*, Yale International Center for Finance.

Lundbergh, S. and Teräsvirta, T.: 1998, Modeling Economic High-Frequency Time Series withe STAR-STGARCH, *Working paper*, Stockholm School of Economics.

Newey, W. and West, K.: 1987, A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* **55**, 703–708.

Pastor, L. and Stambaugh, R. F.: 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* **111**, 642–685.

Teräsvirta, T.: 1994, Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models, *Journal of American Statistical Association* **89**, 208–218.

Teräsvirta, T. and Anderson, H. M.: 1992, Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models, *Journal of Applied Econometrics* **7**, 119–136.

Yang, J., Zhou, Y. and Wang, Z.: 2009, The Stock-Bond Correlation and Macroeconomic Conditions: One and a Half Centuries of Evidence, *Journal of Banking and Finance* **33**, 670–680.

Table 1: Descriptive Statistics

	Mean	St.dev.	Min	Max	Skew	Kurt	St min	St max
RC	0.04	0.39	-0.81	0.78	-0.37	1.89	-	-
FRC	0.03	0.43	-1.12	1.05	-0.42	2.18	-	-
VXO	21.21	9.05	9.04	98.81	2.20	12.27	-1.35	8.58
RRSP	0.00	0.03	-0.28	0.18	-1.58	21.59	-11.53	-10.82
RRTY	0.00	0.01	-0.11	0.08	-1.20	20.00	7.55	6.81
RVSP	0.02	0.01	0.00	0.26	10.45	211.85	-1.01	22.41
RVTY	0.01	0.00	0.00	0.03	2.45	15.53	-1.68	8.85
DTBILL	-0.01	0.13	-1.64	0.69	-2.25	28.61	-12.40	5.28
INF	0.00	0.00	0.00	0.01	0.54	3.52	-2.14	3.43
GDP	0.01	0.01	-0.02	0.02	-1.01	5.46	-3.83	2.08

The table (left columns) shows the descriptive statistics (mean, standard deviation, minimum, maxium, skewness, and kurtosis) for the following variables: realized correlation (RC), Fisher transform of RC (FRC), VXO index, realised stock return (RRSP), realized bond return (RRTY), realized stock volatility (RRSP), realized bond volatility (RVTY), short rate changes (DTBILL), inflation rate (INF), and the GDP growth rate (GDP). The table (right columns) shows the minimum and maximum of the standardized variables.

Table 2: Estimated STR Models

		Linear Model			Simple STR			Extended STR		
		Coef	T-stat	Prob	Coef	T-stat	Prob	Coef	T-stat	Prob
β	Cons	0.004	0.44	0.66	-0.032	-0.32	0.75	-0.208	-4.19	0.00
	PosD							0.247	4.33	0.00
	FRC(-4)	0.807	27.03	0.00	0.857	13.55	0.00	0.568	5.64	0.00
	FRC(-4)*PosD							0.166	1.20	0.23
	VXO(-4)	-0.021	-1.26	0.21	-0.088	-1.52	0.13	-0.357	-6.62	0.00
	VXO(-4)*PosD							0.427	6.18	0.00
	RRSP(-4)	-0.009	-1.41	0.16	-0.006	-0.62	0.53	0.004	0.18	0.85
	RRSP(-4)*PosD							-0.019	-0.71	0.48
	RRTY(-4)	-0.002	-0.24	0.81	-0.002	-0.22	0.83	0.004	0.21	0.83
	RRTY(-4)*PosD							0.000	0.02	0.99
	RVSP(-4)	0.036	1.61	0.11	0.099	2.80	0.01	0.380	5.54	0.00
	RVSP(-4)*PosD							-0.508	-5.96	0.00
	RVTY(-4)	-0.015	-1.48	0.14	-0.018	-1.44	0.15	-0.067	-2.73	0.01
	RVTY(-4)*PosD							0.075	2.51	0.01
	DTBILL(-4)	-0.003	-0.48	0.63	0.000	0.01	1.00	0.000	-0.02	0.99
	DTBILL(-4)*PosD							-0.006	-0.20	0.84
	INF(-4)	0.025	2.52	0.01	0.026	2.40	0.02	-0.008	-0.32	0.75
	INF(-4)*PosD							0.013	0.45	0.65
	GDP(-4)	0.013	1.42	0.16	0.016	1.24	0.22	0.090	4.35	0.00
	GDP(-4)*PosD							-0.105	-4.20	0.00
δ	Cons				1.146	0.52	0.60	0.052	0.73	0.47
	PosD							0.156	1.36	0.17
	FRC(-4)				-1.178	-1.26	0.21	-0.045	-0.38	0.71
	FRC(-4)*PosD							-0.153	-0.77	0.44
	VXO(-4)				-0.128	-0.41	0.68	0.387	6.07	0.00
	VXO(-4)*PosD							-0.601	-6.89	0.00
	RRSP(-4)				-0.003	-0.13	0.89	-0.009	-0.38	0.70
	RRSP(-4)*PosD							0.008	0.19	0.85
	RRTY(-4)				0.024	0.36	0.72	-0.029	-1.14	0.26
	RRTY(-4)*PosD							0.034	0.78	0.44
	RVSP(-4)				-0.081	-1.36	0.17	-0.382	-5.36	0.00
	RVSP(-4)*PosD							0.523	5.24	0.00
	RVTY(-4)				-0.096	-0.96	0.34	0.089	2.60	0.01
	RVTY(-4)*PosD							-0.100	-2.40	0.02
	DTBILL(-4)				-0.066	-0.92	0.36	-0.004	-0.15	0.88
	DTBILL(-4)*PosD							0.007	0.18	0.85
	INF(-4)				0.078	0.52	0.60	0.056	1.44	0.15
	INF(-4)*PosD							-0.025	-0.56	0.57
	GDP(-4)				0.045	0.26	0.79	-0.061	-2.17	0.03
	GDP(-4)*PosD							0.120	3.06	0.00
γ					0.914	1.11	0.27	34.272	1.16	0.25
c					4.152	2.08	0.04	0.380	12.67	0.00
	R-squared		0.687			0.695			0.723	
	Adjusted R-squared		0.685			0.690			0.714	
	Akike IC		0.024			0.018			-0.046	
	Schwarz IC		0.066			0.110			0.130	
	RMSE		0.243			0.240			0.228	
	MAE		0.180			0.177			0.169	

The table shows the results from estimating the linear, simple STR, and extended STR models.

Table 3: Wald Tests for Estimated STR Models

Hypothesis	Simple	Extended		
	$\beta=\delta=0$	$\beta=0$	$\delta=0$	$\beta=\delta=0$
FRC(-4)	0.00	0.00	0.41	0.00
VXO(-4)	0.27	0.00	0.00	0.00
RRSP(-4)	0.62	0.62	0.93	0.83
RRTY(-4)	0.94	0.89	0.52	0.64
RVSP(-4)	0.01	0.00	0.00	0.00
RVTY(-4)	0.04	0.02	0.03	0.07
DTBILL(-4)	0.50	0.87	0.98	0.97
INF(-4)	0.01	0.88	0.13	0.13
GDP(-4)	0.13	0.00	0.01	0.00

The table shows the p-values associated with joint Wald tests for the reported hypotheses in the simple STR and extended STR models.

Table 4: Estimated Simple STR Models across Horizons

		Horizon k=1			Horizon k=4			Horizon k=8		
		Coef	T-stat	Prob	Coef	T-stat	Prob	Coef	T-stat	Prob
β	Cons	0.006	0.94	0.35	-0.032	-0.32	0.75	-0.023	-0.22	0.82
	FRC(-k)	0.887	58.11	0.00	0.857	13.55	0.00	0.930	10.65	0.00
	VXO(-k)	-0.033	-1.80	0.07	-0.088	-1.52	0.13	-0.132	-1.48	0.14
	RRSP(-k)	-0.014	-1.63	0.10	-0.006	-0.62	0.53	-0.021	-1.38	0.17
	RRTY(-k)	0.001	0.11	0.91	-0.002	-0.22	0.83	0.033	2.43	0.02
	RVSP(-k)	0.051	2.19	0.03	0.099	2.80	0.01	0.143	2.25	0.02
	RVTY(-k)	-0.015	-2.23	0.03	-0.018	-1.44	0.15	-0.014	-0.63	0.53
	DTBILL(-k)	-0.002	-0.28	0.78	0.000	0.01	1.00	0.006	0.44	0.66
	INF(-k)	0.023	4.93	0.00	0.026	2.40	0.02	0.015	0.75	0.45
	GDP(-k)	0.012	1.84	0.07	0.016	1.24	0.22	0.003	0.13	0.90
δ	Cons	-0.224	-3.35	0.00	1.146	0.52	0.60	-0.006	-0.01	0.99
	FRC(-k)	-0.292	-3.59	0.00	-1.178	-1.26	0.21	-0.594	-3.03	0.00
	VXO(-k)	0.096	3.27	0.00	-0.128	-0.41	0.68	0.205	2.68	0.01
	RRSP(-k)	0.027	2.25	0.02	-0.003	-0.13	0.89	0.026	1.20	0.23
	RRTY(-k)	-0.001	-0.05	0.96	0.024	0.36	0.72	-0.047	-1.57	0.12
	RVSP(-k)	-0.084	-3.09	0.00	-0.081	-1.36	0.17	-0.146	-1.81	0.07
	RVTY(-k)	0.017	0.99	0.32	-0.096	-0.96	0.34	-0.058	-1.26	0.21
	DTBILL(-k)	-0.019	-1.21	0.23	-0.066	-0.92	0.36	-0.014	-0.43	0.66
	INF(-k)	0.063	2.65	0.01	0.078	0.52	0.60	0.115	1.80	0.07
	GDP(-k)	-0.017	-1.30	0.19	0.045	0.26	0.79	0.030	0.70	0.48
γ		7.854	1.07	0.29	0.914	1.11	0.27	1.000		
c		1.179	7.97	0.00	4.152	2.08	0.04	1.302	1.69	0.09
R-squared			0.807			0.695			0.646	
Adjusted R-squared			0.804			0.690			0.640	
Akike IC			-0.440			0.018			0.164	
Schwarz IC			-0.347			0.110			0.253	

The table shows the results from estimating the simple STR model at three different forecast horizons.

Table 5: Estimated Extended STR Models across Horizons

		Horizon k=1			Horizon k=4			Horizon k=8		
		Coef	T-stat	Prob	Coef	T-stat	Prob	Coef	T-stat	Prob
β	Cons	-0.106	-2.90	0.00	-0.208	-4.19	0.00	-0.700	-0.86	0.39
	PosD	0.160	3.47	0.00	0.247	4.33	0.00	0.260	0.36	0.72
	FRC(-k)	0.728	10.31	0.00	0.568	5.64	0.00	0.449	1.48	0.14
	FRC(-k)*PosD	0.051	0.56	0.57	0.166	1.20	0.23	0.801	1.01	0.31
	VXO(-k)	0.005	0.31	0.76	0.004	0.18	0.85	-0.018	-0.41	0.68
	VXO(-k)*PosD	-0.036	-1.62	0.11	-0.019	-0.71	0.48	-0.100	-0.83	0.41
	RRSP(-k)	0.010	0.77	0.44	0.004	0.21	0.83	0.074	1.00	0.32
	RRSP(-k)*PosD	-0.001	-0.08	0.94	0.000	0.02	0.99	-0.012	-0.19	0.85
	RRTY(-k)	-0.207	-4.96	0.00	-0.357	-6.62	0.00	-0.964	-1.32	0.19
	RRTY(-k)*PosD	0.253	4.60	0.00	0.427	6.18	0.00	0.969	1.19	0.23
	RVSP(-k)	0.202	3.95	0.00	0.380	5.54	0.00	0.637	1.81	0.07
	RVSP(-k)*PosD	-0.264	-4.01	0.00	-0.508	-5.96	0.00	-0.976	-1.58	0.11
	RVTY(-k)	-0.033	-1.89	0.06	-0.067	-2.73	0.01	0.069	0.52	0.60
	RVTY(-k)*PosD	0.044	2.06	0.04	0.075	2.51	0.01	0.009	0.09	0.93
	DTBILL(-k)	-0.020	-1.07	0.28	0.000	-0.02	0.99	-0.032	-0.53	0.60
	DTBILL(-k)*PosD	0.024	1.10	0.27	-0.006	-0.20	0.84	0.029	0.39	0.70
	INF(-k)	0.010	0.53	0.60	-0.008	-0.32	0.75	-0.046	-0.60	0.55
	INF(-k)*PosD	0.001	0.06	0.95	0.013	0.45	0.65	-0.045	-0.43	0.67
	GDP(-k)	0.032	2.25	0.02	0.090	4.35	0.00	0.048	0.86	0.39
	GDP(-k)*PosD	-0.022	-1.23	0.22	-0.105	-4.20	0.00	-0.046	-0.57	0.57
δ	Cons	0.026	0.41	0.68	0.052	0.73	0.47	0.917	0.80	0.43
	PosD	0.187	1.57	0.12	0.156	1.36	0.17	0.083	0.06	0.95
	FRC(-k)	0.013	0.13	0.89	-0.045	-0.38	0.71	0.181	0.41	0.68
	FRC(-k)*PosD	-0.410	-2.24	0.03	-0.153	-0.77	0.44	-1.607	-1.65	0.10
	VXO(-k)	-0.007	-0.38	0.71	-0.009	-0.38	0.70	0.007	0.13	0.90
	VXO(-k)*PosD	0.068	1.43	0.15	0.008	0.19	0.85	0.190	1.26	0.21
	RRSP(-k)	-0.029	-1.31	0.19	-0.029	-1.14	0.26	-0.106	-1.17	0.24
	RRSP(-k)*PosD	-0.002	-0.03	0.97	0.034	0.78	0.44	0.025	0.24	0.81
	RRTY(-k)	0.225	4.77	0.00	0.387	6.07	0.00	0.990	1.42	0.16
	RRTY(-k)*PosD	-0.418	-6.02	0.00	-0.601	-6.89	0.00	-1.201	-1.63	0.10
	RVSP(-k)	-0.210	-3.87	0.00	-0.382	-5.36	0.00	-0.600	-1.65	0.10
	RVSP(-k)*PosD	0.312	2.87	0.00	0.523	5.24	0.00	1.086	1.67	0.10
	RVTY(-k)	0.048	1.91	0.06	0.089	2.60	0.01	-0.185	-1.12	0.26
	RVTY(-k)*PosD	-0.072	-2.09	0.04	-0.100	-2.40	0.02	0.049	0.31	0.75
	DTBILL(-k)	0.023	1.00	0.32	-0.004	-0.15	0.88	0.038	0.49	0.62
	DTBILL(-k)*PosD	-0.072	-1.56	0.12	0.007	0.18	0.85	-0.012	-0.10	0.92
	INF(-k)	0.021	0.61	0.54	0.056	1.44	0.15	0.102	1.02	0.31
	INF(-k)*PosD	0.021	0.47	0.64	-0.025	-0.56	0.57	0.140	0.95	0.34
	GDP(-k)	-0.012	-0.65	0.51	-0.061	-2.17	0.03	-0.003	-0.04	0.97
	GDP(-k)*PosD	-0.002	-0.06	0.95	0.120	3.06	0.00	-0.011	-0.09	0.93
γ		9.175	1.57	0.12	34.272	1.16	0.25	1.010	2.22	0.03
c		0.378	4.54	0.00	0.380	12.67	0.00	-0.198	-0.18	0.86
R-squared		0.817			0.723			0.679		
Adjusted R-squared		0.811			0.714			0.668		
Akike IC		-0.460			-0.046			0.101		
Schwarz IC		-0.284			0.130			0.278		

The table shows the results from estimating the extended STR model at three different forecast horizons.

Table 6: Out-of-Sample Results

Horizon	k=1	k=4	k=8
RMSE Linear Model	0.226	0.290	0.309
RMSE Simple STR	0.256	0.366	0.335
RMSE Extended STR	0.236	0.306	0.355
MAE Linear Model	0.172	0.214	0.234
MAE Simple STR	0.196	0.269	0.250
MAE Extended STR	0.183	0.227	0.263

The table shows the RMSE and MAE for the linear, simple STR, and extended STR models at three different out-of-sample forecast horizons.

Figure 1: The Weekly Realized Stock Bond Correlation

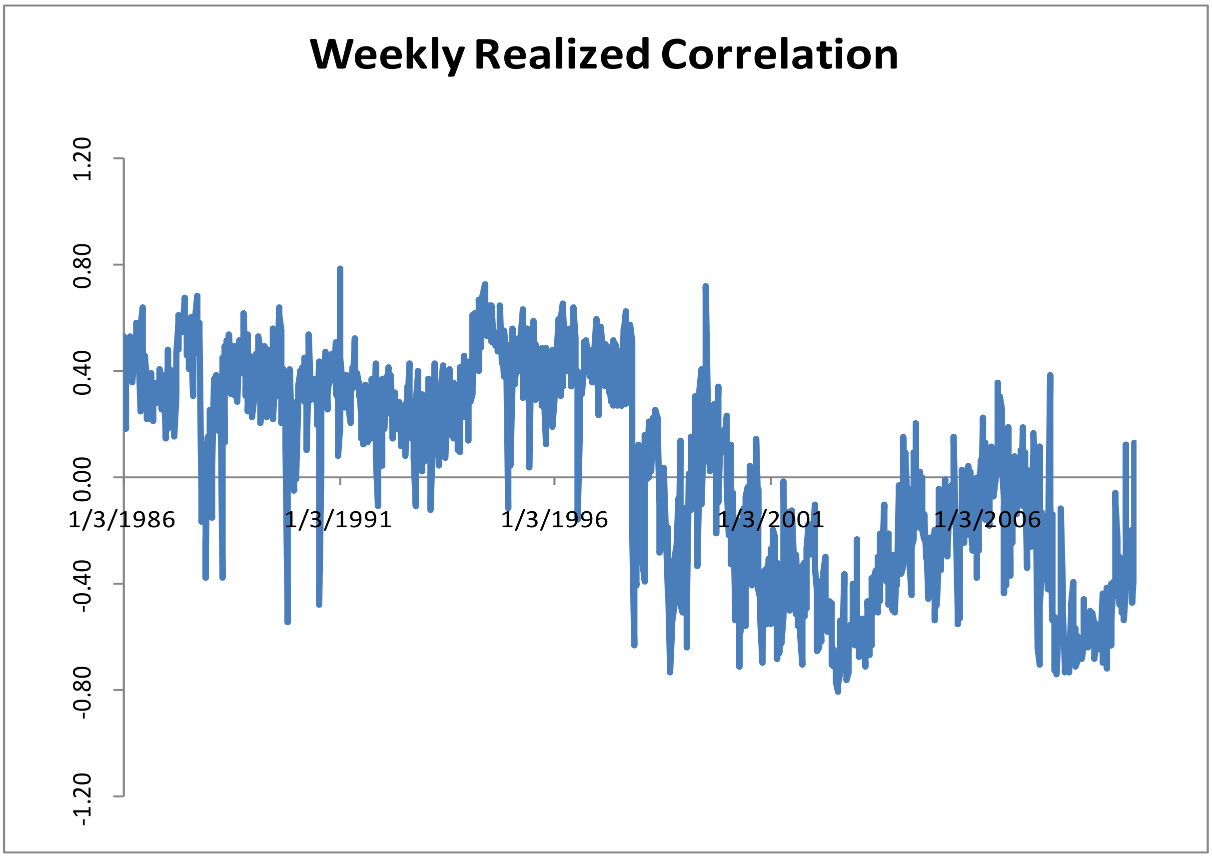


Figure 2: Estimated Transition Function against Transition Variable (VXO)

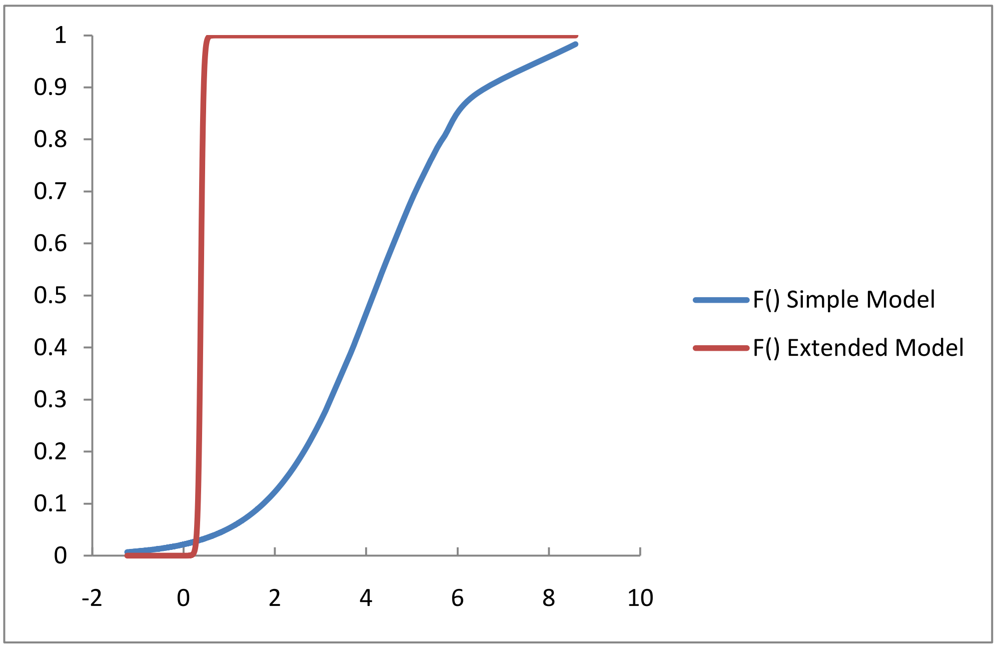
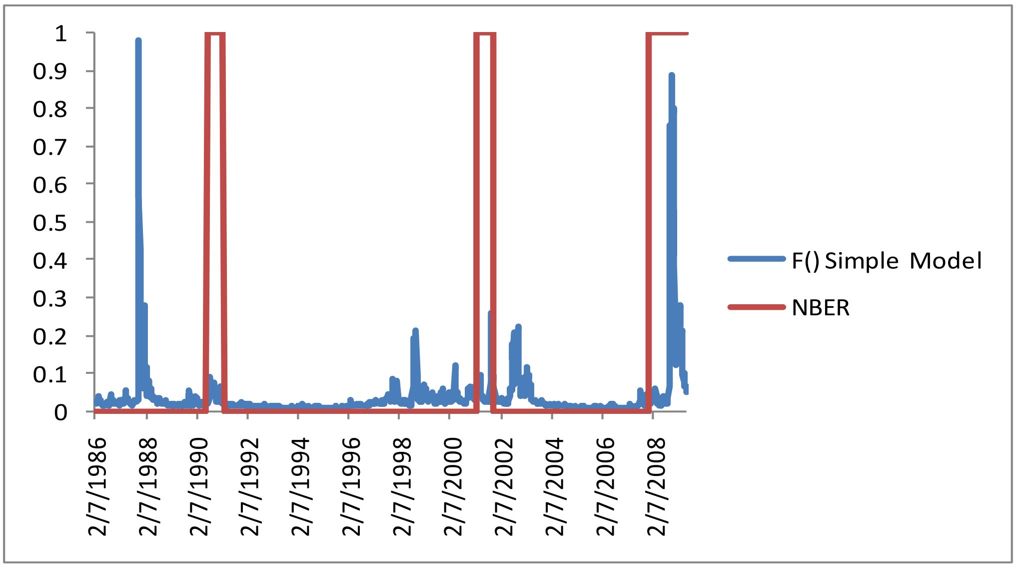


Figure 3: Estimated Transition Functions

Panel A: Simple STR Model



Panel B: Extended STR Model

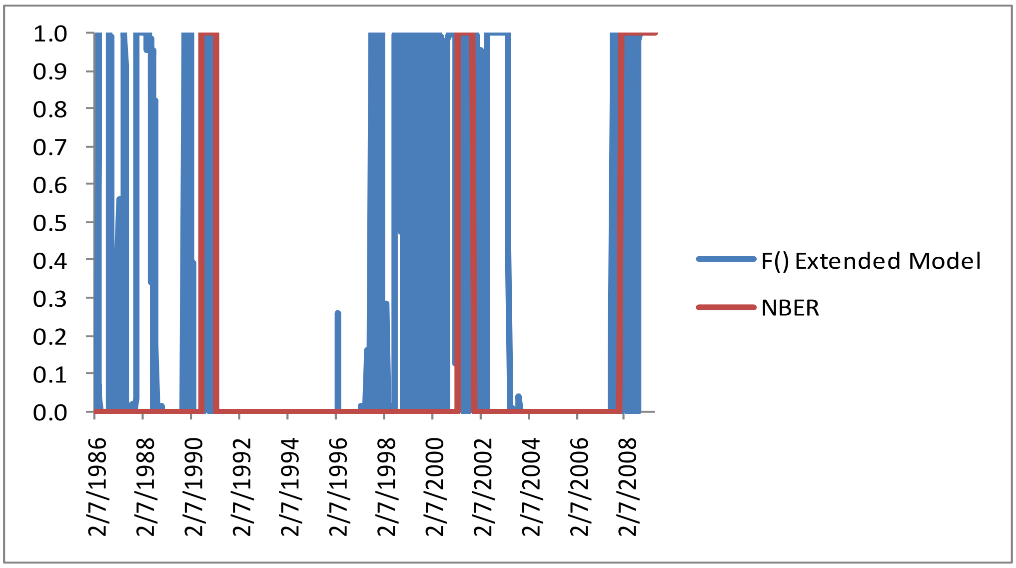
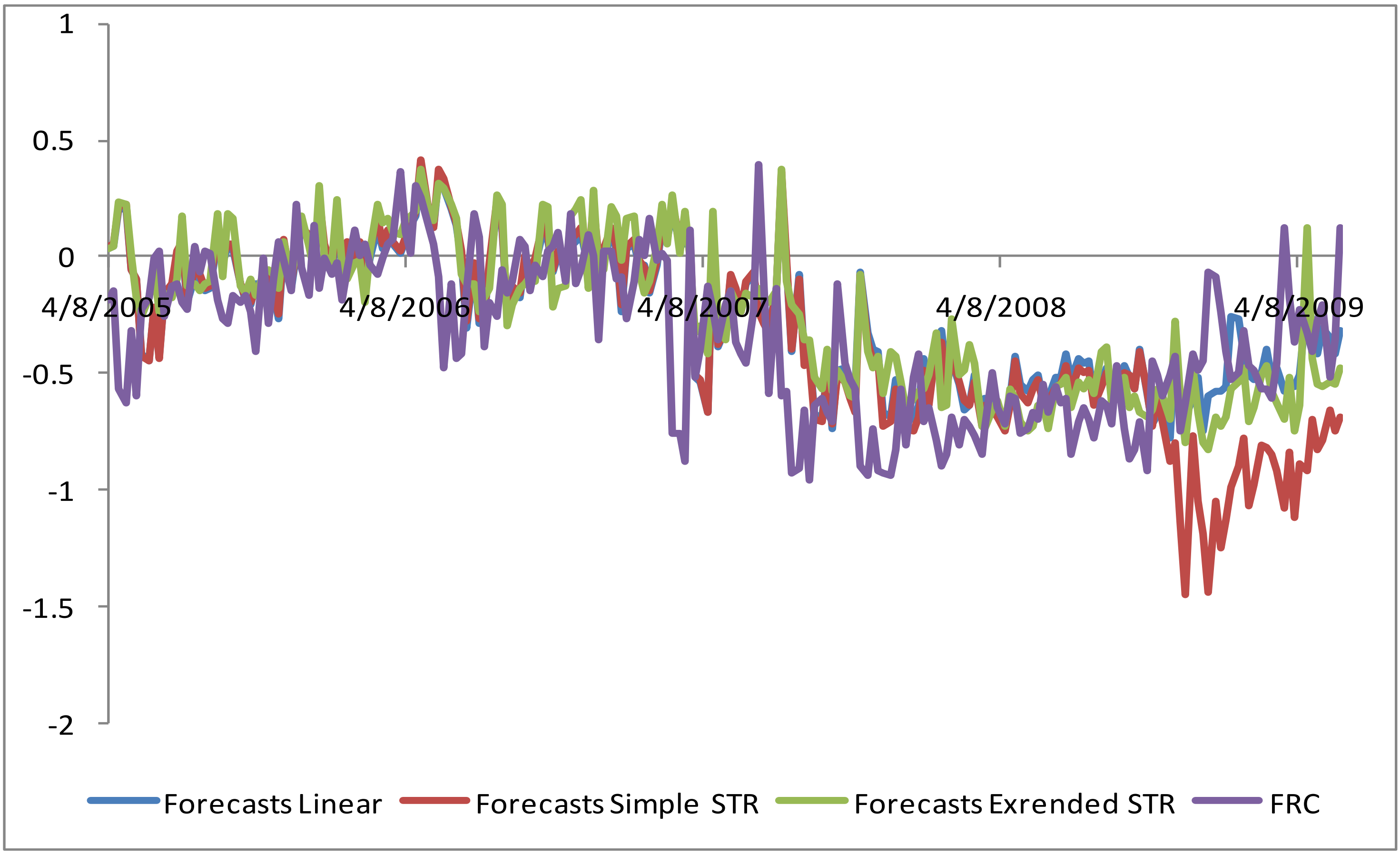


Figure 4: Out-of-Sample Forecasting



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