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What do we know about real exchange rate non-linearities?*

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Abstract

This research points to the serious problem of potentially misspecified alternative hypotheses when testing for unit roots in real exchange rates. We apply a popular unit root test against nonlinear ESTAR and develop a Markov Switching unit root test. The empirical power of these tests against correctly and misspecified non-linear alternatives is analyzed by means of a Monte Carlo study. The chosen parametrization is obtained by real-life exchange rates. The test against ESTAR has low power against all alternatives whereas the proposed unit root test against a Markov Switching autoregressive model performs clearly better. An empirical application of these tests suggests that real exchange rates may indeed be explained by Markov-Switching dynamics.

Key Words: real exchange rates, unit root test, ESTAR, Markov Switching, PPP.
JEL numbers: C12, C22, F31

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1 Introduction

The debate about real exchange rate behavior suggests that purchasing power parity (PPP) may hold as a longer run concept (e.g. Rogoff, 1996, Sarno and Taylor, 2002). Our understanding of exchange rate dynamics that bring about longer run PPP, however, is much less clear. In particular, there are two competing approaches in wider use aiming for modeling a non-linear adjustment towards PPP, i.e. Exponential Smooth Transition Autoregressive (ESTAR) models (e.g. Michael et al. 1997) and Markov Switching Autoregressive (MSAR) models (e.g. Kanas, 2006). Both processes rely on a distinct economic intuition, since both, an ESTAR or a MSAR process may under certain conditions imply a global tendency towards PPP, respectively. As opposed to the above mentioned non-linear models, there is the alternative of a unit root process, implying missing adjustment towards PPP. In our context, tests for PPP mean testing the unit root hypothesis against the alternative of either stationary ESTAR or stationary MSAR. We will show that the implications of these tests heavily depend on the alternative chosen.

We argue that our understanding about real exchange rate non-linearities is weak because test results are not indicative. It is common to take the rejection of a unit root as evidence of a particular process because the latter is used as the alternative in the testing procedure. But can we really draw this conclusion if we do not consider further alternatives? A similar problem involving unit roots and trend breaks is discussed in Kilian and Ohanian (2002). For assessing the test results it is necessary to know the statistical power of the tests available in a setting that is close to reality. Our basic approach is applying unit root tests against ESTAR or MSAR under realistic conditions, including an application of these tests on simulated time series with correctly or misspecified alternatives. To this end, we develop a new unit root test where the MSAR model under the alternative is similarly specified as the ESTAR model. For the ESTAR model, a regime switch is driven by past and therefore observable values of the process itself, while an unobservable stochastic Markov process is the driving force in Markov Switching models.

A core ingredient of our procedure is that we examine the empirical power of different non-linear unit root tests under real world parameter constellations. Our simulation results show that standard tests perform surprisingly weak even if the data generating process equals the one in the alternative hypothesis of the test. Moreover, they suggest that typical parameters of real exchange rates are disadvantageous for the power of these tests, a fact that also explains why it is so hard to find evidence for PPP in exchange rates (Taylor et al. 2001). Parameters are obtained from Rapach and Wohar (2006) for ESTAR models who nearly similar results as Taylor et al. (2001). In contrast, coefficients for the MSAR model are retrieved by own calculations. We consider major monthly real exchange rates, namely to the German Mark (DEM), the Japanese Yen (JPY), the British Pound (GBP) against the US Dollar (USD). By Monte Carlo simulations, we generate realizations of these data generating processes under the assumption that PPP holds and separately for both models. We then compute the power of the Kapetanios et al. (2003) unit root test against the ESTAR alternative - in short: ESTAR test - as well as the power of a newly developed unit root test against a MSAR process. We study the power of these tests when the alternative is correctly specified (this is the true DGP is ESTAR/MSAR for the ESTAR/MSAR test) and when the alternative is misspecified (this is the true DGP is MSAR/ESTAR for the ESTAR/MSAR test).

We find that a non-rejection of an ESTAR test does not mean that we have to reject the purchasing power parity hypothesis. Furthermore, the power of the ESTAR test does not substantially depend on whether the DGP is really an ESTAR process or a MSAR process. While this means that the test is comparatively robust against misspecifications, it also implies that the rejection of the null by no means allow the premature conclusion that the non-linearity of the true underlying data generating process is really of an ESTAR-type. In explaining the low power of the ESTAR test, we show that it is due to the fact that the vast majorities of data points lie in the local-to-unity region. This generates a difficult situation for the test. Furthermore, we find that the approximation of the ESTAR process by third powers as used in the ESTAR test is also a good approximation for the MSAR processes. This explains that the test has comparable power against ESTAR and MSAR processes.

The results for our unit root test in a Markov Switching framework are clearly better: First, its power is substantially higher against the true alternative. Secondly, it appears to be more sensitive against the misspecification of the DGP. This means that it has quite a low power if the process is of an ESTAR-type, implying that the probability of confusing both processes is generally low when applying the proposed Markov Switching test.

As the last step, we apply the unit root tests to actual real exchange rates and find that the ESTAR test cannot reliably reject the unit root. This implies that either PPP does not hold or that the non-linear alternative to the unit root does not capture exchange rate properties well enough. It is consequently revealing that the Markov Switching test rejects the Null of a unit root in four out of the six major real exchange rates considered. This indicates that real exchange rate dynamics may be better characterized by Markov Switching processes.

Our research into the power of unit root tests has an obvious statistical motivation. In addition, it also has an intuitive economic motivation as ESTAR and Markov Switching processes imply a different understanding of the foreign exchange market. Therefore it does not only matter, *whether* real exchange rates are stationary, i.e. PPP is valid. It is also important which model describes real exchange rates best, i.e. *how* the real exchange rate adjusts to PPP.

Both models are based on different views of the foreign exchange market. ESTAR models, such as in Taylor et al. (2001), pick up the argument that due to tariffs and transportation costs goods are not traded internationally, as long as the price levels do not differ too much between countries (see the model in Dumas, 1992).[1] Therefore, the real exchange rate behaves like a random walk when it is close to its equilibrium value. As soon as the price

[1]ESTAR models build on the STAR model of Teräsvirta (1994). Applications to foreign exchange include for example Michael et al. (1997) who test this model with interwar data for several exchange rates and Taylor et al. (2001) who test the model at four exchanges rates against the USD during the post war period.

differences increase, a smooth transition process starts and arbitrage will adjust prices and thus the real exchange rate towards PPP. This behavior can be well described by an ESTAR model with a unit root regime switching to a stationary autoregressive regime if the process departs from its equilibrium.

In contrast, Markov Switching models, in particular the Markov Switching model as applied in Kanas (2006), break up the relation between regime switches and the deviation from PPP. Markov Switching models describe long swings in the exchange rate, as documented by Engel and Hamilton (1990). These swings refer to switching from one regime to another, with each regime lasting for years. The idea behind these models is that real exchange rates may be driven by various forces: stabilizing forces drive the exchange rate back to PPP, whereas destabilizing forces - one may think of short-term speculation - cause ongoing deviations from equilibrium. The Markov switching model is therefore often regarded as a model of bubbles (Hall et al. 1997, 1999).[2]

In short and somewhat overstating the point, the ESTAR view of real exchange rates emphasizes the tendency towards PPP and thus towards long run equilibrium, whereas the Markov Switching view stresses the importance of frictions in the market, that drive the real exchange rate away from its equilibrium and cause bubbles. De Grauwe and Grimaldi (2005), for example, link the reasoning behind these views to heterogeneous agents in this market, i.e. international goods arbitrage and short-term speculation.

Both views of real exchange rate behavior have a strong substantiation in international finance research. The ESTAR view is tentatively supported by long lasting research on PPP which has yielded the insight that forces towards PPP have been underestimated in earlier

[2]The Markov Switching model introduced by Hamilton (1989) was first applied to nominal exchange rates by Engel and Hamilton (1990) and in different settings by Engel (1994), Cheung and Erlandsson (2004) and Frömmel et al. (2005). The specific form of a Markov Switching error correction model we are interested in here, i.e. a Markov Switching error correction model applied to real exchange rates, has been introduced by Hall et al. (1997) and Psaradakis et al. (2004), first applied to our problem by Kanas and Genius (2005) and Kanas (2006).

studies (survey in Sarno and Taylor, 2002). However, also the Markov Switching view has remarkable economic substantiation in models of heterogeneous agents in the foreign exchange market, such as early Frankel and Froot (1990) or recently De Grauwe and Grimaldi (2006) and has recently become an increasingly popular alternative to ESTAR models (see e.g. Engle and Kim 2001, Kanas 2006). Although both views do not overwhelmingly imply a policy stance, the first view will clearly tend towards more benign neglect of the foreign exchange market than the latter view (further implications are discussed by Sarno, 2005, p.685f.).

The paper is organized as follows. Section 2 introduces the ESTAR model and the considered unit root test against ESTAR models in more detail. In section 3 we propose a new unit root test against a MSAR process. Section 4 contains our Monte Carlo study, section 5 applies the unit root tests introduced above on six real exchange rates. Section 6 summarizes and concludes.

2 Unit root test against ESTAR

In this section we review the ESTAR model and a popular test for a unit root against the ESTAR alternative (Kapetanios et al. 2003). The particular specification of the ESTAR model we consider, as used in several studies like Michael et al. (1997), Sarantis (1999), Taylor et al. (2001) and more recently, Rapach and Wohar (2006), is given by

$$y_t = \phi_t y_{t-1} + \varepsilon_t \tag{1}$$

where ε_t is assumed to be a zero mean white noise process and ϕ_t is a time-varying autoregressive parameter. Its dynamics are determined by the smooth transition function $\exp\{-\gamma(y_{t-1} - c)^2\}$ which is the source of nonlinearity in this model. The autoregressive parameter ϕ_t is bounded between zero and one and depends on a transition variable y_{t-1}, a smoothness parameter $\gamma > 0$ and a location parameter $c \in \mathbb{R}$.

The ESTAR model behaves locally like a random walk if the lagged real exchange rate (y_{t-1}) is exactly equal to c, since the autoregressive parameter ϕ_t equals one in this case.

If y_{t-1} departs from c, the process is stationary and therefore mean-reverting. Despite the local non-stationarity of y_t, the ESTAR model is globally stationary, see Kapetanios et al. (2003) for a proof. In the exponential smooth transition model, the degree of mean-reversion depends on the squared difference between y_{t-1} and c. In economic terms, if the real exchange rate was quite close to its long run equilibrium value in the last period then it behaves like a random walk. Furthermore, there are driving forces like arbitrage that lead to mean-reversion if the real exchange rate departs from its long run equilibrium. Moreover, arbitrage may not be profitable if departures are small as arbitrageurs face transaction costs. Therefore, the degree of mean-reversion is small as well. The parameter γ controls the shape of the exponential function and therefore influences the sensitivity of ϕ_t towards the deviation of y_{t-1} from c.

Kapetanios et al. (2003) suggest a modification of the Dickey-Fuller test for testing the random walk hypothesis against ESTAR in order to increase the power of the standard Dickey-Fuller test. Their test is based on the following test regression which is very similar to the original Dickey-Fuller regression:

$$\Delta y_t = \psi y_{t-1}^3 + u_t \tag{2}$$

with $\Delta y_t = y_t - y_{t-1}$. The nonlinear smooth transition function is approximated by the cubic power y_{t-1}. In this regression, the pair of hypotheses is now $H_0 : \psi = 0$ (unit root) versus $H_1 : \psi < 0$ (stationary ESTAR). Kapetanios et al. (2003) suggest a Dickey-Fuller-type test for this hypothesis given by

$$t_{\text{KSS}} \equiv \frac{\widehat{\psi}}{\sqrt{\text{var}(\widehat{\psi})}} = \frac{\sum_{t=1}^{T} y_{t-1}^3 \Delta y_t}{\sqrt{\widehat{\sigma}^2 \sum_{t=1}^{T} y_{t-1}^6}}, \tag{3}$$

where $\widehat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(\Delta y_t - \widehat{\psi} y_{t-1}^3)^2$ is the usual estimator of the error variance. The limiting distribution of the test statistic t_{KSS} is non-standard and asymptotic critical values can be found in Kapetanios et al. (2003). Still, we also provide small sample critical values in section 4.1. Deterministic components as a constant or a constant and a linear trend are removed in a first step, i.e. one applies the test to previously de-meaned or de-trended data. For further details, see Kapetanios et al. (2003).

3 Unit root test against Markov Switching

We consider a MSAR model which has similar properties to the ESTAR model presented in the previous section. The main difference between the ESTAR and the MSAR model is, at least from a statistical viewpoint, the regime switching mechanism. Within the ESTAR model, regime switches are determined by past observable values of y_{t-1}, while the MSAR model is based on an unobservable stochastic Markov process (s_t). Our specification of the MSAR model is given by

$$y_t = \phi_{s_t} y_{t-1} + \varepsilon_t \ , \tag{4}$$

where the autoregressive parameter ϕ_{s_t} depends on a first-order Markov chain (s_t) that takes the values one or two. Furthermore, it is assumed that s_t is irreducible and aperiodic, i.e., it is characterized by the transition probability matrix

$$\begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

with $p_{ii} = P(s_t = i | s_{t-1} = i)$ for $i = 1, 2$, being the probability that the process is in regime i in period t, given that it was in the same regime in the previous period.

Within the MSAR framework, Francq and Zakoïan (2001) establish a necessary and sufficient condition for global stationarity of the MSAR model. It is given by the following two inequalities $c_1 < 1$ and $c_2 < 2$, where

$$\begin{aligned} c_1 &= p_{11}\phi_1^2 + p_{22}\phi_2^2 + (1 - p_{11} - p_{22})\phi_1^2\phi_2^2 \ , \\ c_2 &= p_{11}\phi_1^2 + p_{22}\phi_2^2 \ . \end{aligned}$$

Similar to the ESTAR model, the MSAR model can also be locally non-stationary while maintaining global stationarity. Suppose that one of the autoregressive parameters is equal to one, while the other autoregressive parameter satisfies the condition for local stationarity, i.e., $\phi_1 = 1$ and $0 \leq \phi_2 < 1$. In this case, global stationarity is still given regardless of what values p_{11} and p_{22} are.

As no unit root test against this specific MSAR model exists, we newly develop such a test and make use of the results obtained by Francq and Zakoïan (2001). The non-linear

MSAR model becomes a random walk when $\phi_1 = \phi_2 = 1$. In this case, the condition for c_1 is violated. Like in the other applications of Markov Switching models, testing the linearity hypothesis is complicated by the presence of unidentified parameters in this model as well. Here, the probabilities p_{11} and p_{22} are unidentified parameters when testing the null hypothesis $H_0 : \phi_1 = \phi_2 = 1$. The treatment of unidentified parameters follows Hansen (1996) and Garcia (1998) which is explained below in detail.

Our test statistic is constructed similar to the one suggested in Caner and Hansen (2001) who construct a unit root test against a Threshold Autoregressive (TAR) model and also face the problem of unidentified parameters under H_0. For convenience, we re-write the specification of the MSAR model we use in a Dickey-Fuller style i.e.,

$$\Delta y_t = \psi_{s_t} y_{t-1} + \varepsilon_t \ ,$$

where $\psi_{s_t} = \phi_{s_t} - 1$. A one-sided Wald test statistic for $H_0 : \psi_1 = \psi_2 = 0$ (unit root) against $H_1 : \psi_1 < 0$ or $\psi_2 < 0$ (stationary MSAR) is given by

$$R = 1\left(\widehat{\psi}_1 < 0\right) t^2_{\psi_1=0} + 1\left(\widehat{\psi}_2 < 0\right) t^2_{\psi_2=0} \ ,$$

where $t_{\psi_i=0}$ denotes the conventional t-statistic for the null hypothesis that ψ_i equals zero. $1(\cdot)$ denotes the indicator function. Parameters are estimated jointly via maximum likelihood. As mentioned above, the transition probabilities p_{11} and p_{22} are unidentified under the validity of the null hypothesis. In order to tackle this problem, we follow Garcia (1998) and consider a sequence of test statistics $R(p_{11}, p_{22})$ where the transition probabilities take values of a bounded grid $\Pi = (0, 1) \times (0, 1)$. This means that we fix p_{11} and p_{22} at certain values and compute the test statistics R. We then proceed by using other values for p_{11} and p_{22} and so on until we consider all possible values in Π. As a next step, we consider the supremum of the random sequence of test statistics $R(p_{11}, p_{22})$, i.e.,

$$R^* = \sup_{p_{11}, p_{22} \in \Pi} R(p_{11}, p_{22}) \ .$$

In our Monte Carlo study we provide critical values for the R^* statistic. Regarding the deterministic terms, we follow the procedure suggested by Kapetanios et al. (2003), see our section 2.1. This means that data is de-meaned or de-trended before the unit root test is applied in order to cope with non-zero means or linear trends.

4 Monte Carlo study

4.1 General approach

The following Monte Carlo study is about the empirical power of the Dickey-Fuller test, the unit root test against ESTAR by Kapetanios et al. (2003) and the new unit root test against MSAR under situations which are realistic in practice when analyzing real exchange rates.

In a related study, Choi and Moh (2007) consider the behavior of various unit root tests against different non-linear alternatives. Among these tests is the Kapetanios et al. (2003) test, which are considered in this paper as well. Choi and Moh (2007) find that all unit root tests have power against various non-linear alternatives. Whether a test has power does not depend on the correct specification of the alternative but on the distance to the null of a unit root. However, Choi and Moh (2007) consider idealized parameter constellations and therefore obtain a satisfying power for each test. They do not consider real world parameter constellations which are the focus of this paper.

Our Monte Carlo study addresses the question whether this might be due to a lack of power of the developed tests under realistic situations rather than to a correct decision of the test by not rejecting the unit root hypothesis. We consider whether the unit root test against ESTAR has also power against Markov Switching processes and vice versa.

In general, unit root tests have good power properties in Monte Carlo studies relying on parameter constellations which do not appear in the analysis of real exchange rates. It is quite common to simulate processes with $N(0, 1)$ innovations, but we account for small standard deviations that are often found empirically, see Rapach and Wohar (2006). Another issue is that the location parameter c in ESTAR models is usually assumed to be equal to zero which is not correct in many practical situations either. Kruse (2009) shows that this assumption may lead to a substantial loss in power if it is wrong. In order to obtain realistic parameter settings, estimations are carried out using data from the International Financial

Table 1: Parameter estimation results

	DEM/USD
ESTAR	$\gamma = 0.264$, $c = -0.007$, $\sigma = 0.035$
MSAR	$[\psi_1, \psi_2] = [-0.074, 0.007]$, $[p_{11}, p_{22}] = [0.917, 0.945]$, $\sigma = 0.028$
	$c_1 = 0.995$, $c_2 = 1.744$
	GBP/USD
ESTAR	$\gamma = 0.449$, $c = 0.150$, $\sigma = 0.033$
MSAR	$[\psi_1, \psi_2] = [-0.310, 0.028]$, $[p_{11}, p_{22}] = [0.300, 0.860]$, $\sigma = 0.030$
	$c_1 = 0.971$, $c_2 = 1.052$
	JPY/USD
ESTAR	$\gamma = 0.165$, $c = 0.515$, $\sigma = 0.033$
MSAR	$[\psi_1, \psi_2] = [-0.233, 0.001]$ $[p_{11}, p_{22}] = [0.235, 0.953]$, $\sigma = 0.030$
	$c_1 = 0.982$, $c_2 = 1.093$

Remarks: Estimated parameter values for DEM/USD, GBP/USD and JPY/USD are taken from Rapach and Wohar (2006) for ESTAR models. Markov Switching models are estimated via conditional maximum likelihood in Gauss.

Statistics database from 1973:02 to 1996:12 for the DEM/USD, FRF/USD, GBP/USD and JPY/USD real exchange rates as done in Rapach and Wohar (2006) for ESTAR models. Their reported estimates are very close to those reported in Taylor et al. (2001). Due to the fact that the estimation results for the DEM/USD and the FRF/USD are quite similar, we do not consider the latter currency in our study. Since Markov Switching models are neither considered in Rapach and Wohar (2006) nor in Taylor et al. (2001), we fit the Markov Switching model described in section 3 to the same data set in order to achieve the highest degree of comparability.

The exact parameter constellations are given in Table 1 for the three considered pairs of currencies (JPY/USD, DEM/USD, GBP/USD). In each case we use first–order autoregressive models. An application of standard diagnostic tests[3] suggest that these models are correctly specified. Starting with the ESTAR specification, we observe that the smoothness parameter γ takes quite different values ranging from 0.165 (JPY/USD) to 0.449 (GBP/USD). Note that it is difficult to distinguish an ESTAR process that exhibits a

[3]Available upon request from the authors.

Table 2: Small sample critical values

de-meaning							
$T = 250$	t_{DF}	t_{KSS}	R^*	$T = 500$	t_{DF}	t_{KSS}	R^*
1%	-3.46	-3.46	27.42	1%	-3.44	-3.51	29.26
5%	-2.88	-2.91	18.65	5%	-2.87	-2.94	20.03
10%	-2.57	-2.63	15.02	10%	-2.57	-2.67	16.20
de-trending							
$T = 250$	t_{DF}	t_{KSS}	R^*	$T = 500$	t_{DF}	t_{KSS}	R^*
1%	-3.99	-3.99	31.53	1%	-3.98	-4.01	32.85
5%	-3.43	-3.49	22.62	5%	-3.42	-3.40	22.98
10%	-3.13	-3.12	18.40	10%	-3.13	-3.12	19.00

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR.

small value of γ from a unit root process as $y_t = y_{t-1} + \varepsilon_t$ for $\gamma \to 0$. Therefore, the expected power is low for the JPY/USD parameter constellation and somewhat higher for the GBP/USD parameters. However, one should also bear in mind that small changes of γ near zero do change the behavior of the process significantly. Therefore, we expect to find clear differences in the behavior of the tests for the different parameter constellations. The location parameter c varies also across currencies, while the estimated standard deviation of the error term σ is very low and far away from unity for each currency. The location parameter c is significantly different from zero in each case although it seems to be rather small for some currencies.

Regarding the Markov Switching model, we always find a stationary regime characterized by $\widehat{\psi}_1 < 0$ and a second regime with an autoregressive parameter which is very close to zero. The latter implies a random walk regime. The MSAR model is globally stationary for all pairs of currencies because the two conditions (c_1 and c_2 in Table 1) derived in Francq and Zakoïan (2001) are not violated, see Table 1. Therefore, the behavior of real exchange rates can be reproduced. The state probabilities are also close to one for the DEM/USD exchange rate. Consequently, the estimated MSAR model for the DEM/USD exchange rate is close to a random walk in both regimes which means that the expected

Table 3: Size experiments for the R^* test statistic

de-meaning							
$T = 250$	Exp1	Exp2	Exp3	$T = 500$	Exp1	Exp2	Exp3
$\alpha = 1\%$	0.7	0.9	1.2	$\alpha = 1\%$	0.8	0.9	1.1
$\alpha = 5\%$	4.8	5.3	5.5	$\alpha = 5\%$	4.8	5.1	5.3
$\alpha = 10\%$	9.5	10.2	10.2	$\alpha = 10\%$	9.6	10.3	10.4
de-trending							
$T = 250$	Exp1	Exp2	Exp3	$T = 500$	Exp1	Exp2	Exp3
$\alpha = 1\%$	0.6	1.1	1.4	$\alpha = 1\%$	0.8	1.2	1.3
$\alpha = 5\%$	4.7	5.2	5.6	$\alpha = 5\%$	4.9	4.9	5.4
$\alpha = 10\%$	9.7	10.4	10.1	$\alpha = 10\%$	9.6	9.8	10.3

Remarks: R^* is the unit root test against MSAR. The DGP is given by $y_t = y_{t-1} + u_t$ with $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0,1)$ and $\rho = \{-0.5, 0, 0.5\}$ in Exp1, Exp2 and Exp3, respectively.

power of the Markov Switching unit root test is low for this currency pair. This can also be seen by considering the values for c_1 and c_2. They imply that we can expect that the Markov Switching test has higher power when the estimated model for the British Pound is considered instead of the one for the German Mark. The estimated standard deviation is similar to that of the ESTAR models and thus again far away from unity for each currency.

We simulate 2,000 replications of each process and apply them to the standard Dickey-Fuller unit root test (denoted by DF) as a benchmark test, the unit root test versus ESTAR suggested by Kapetanios et al. (denoted by KSS) and the Markov Switching test proposed in section 3. The power is considered at the 5% level by using size adjusted small sample critical values obtained from 20,000 replications for sample sizes of $T = 250$ and $T = 500$ which corresponds approximately to 20 and 40 years of monthly data, respectively. Note, that we simulate processes of length $T + 100$ and delete the first hundred observations in order to reduce the effect of the starting value. We use simulated small sample critical values for all tests and not just for the Markov Switching test in order to obtain comparability of the results. The size-adjusted critical values for all tests are given in Table 2.

We conduct some size experiments for the proposed unit root test against MSAR at the nominal significance levels of one, five and ten percent. The data generating process is given by $y_t = y_{t-1} + u_t$ with $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, 1)$. The three different experiments (Exp1, Exp2 and Exp3) cover the cases $\rho = \{-0.5, 0, 0.5\}$. Results are reported in Table 3. The results show that the new test has accurate nominal size and that the distortions are small.

Regarding our power experiments, we consider both, correctly and misspecified models. To this end, we simulate MSAR and ESTAR models corresponding to the parameters we found for real data, see Table 1. It should be noted, that the alternative is misspecified for the standard Dickey-Fuller test for all considered models.

4.2 Main results

In this subsection, we discuss the power results for the non-linear unit root tests. Table 4 gives the power for a sample size of $T = 250$ observations. We consider all non-linear unit root tests after de-meaning as well as after de-trending as both deterministics can be reasonable for real exchange rate data. Note, that we include a constant or a constant and a linear trend term in the Dickey-Fuller test regression. However, the results are rather similar in both cases. The ESTAR test has no remarkable power against any of our models. Interestingly enough, the standard Dickey-Fuller test has higher power against ESTAR than the ESTAR test for the JPY/USD and the DEM/USD in the de-trended case.

However, the power of all tests is extremely low when the true DGP is an ESTAR model in any case. This also holds for the Markov Switching test. When the true DGP is ESTAR, the Markov Switching test proves to be conservative. In opposition to the ESTAR test, this is a rather convincing test property as a non-rejection of the test is the desired property for a correct model selection. Unfortunately, the ESTAR test has power against the Markov Switching model. In each case it is at least in the same region as the power against ESTAR models. For the GBP/USD it is far higher for the Markov Switching alternative than for the ESTAR alternative. Only for the DEM/USD exchange rate, the power of the tests is quite low. This was expected as the Markov Switching model is close to a unit root in

Table 4: Empirical power, $T = 250$

de-meaning	t_{DF}	t_{KSS}	R^*	de-trending	t_{DF}	t_{KSS}	R^*
JPY–ESTAR	10.5	10.1	2.6	JPY–ESTAR	8.2	7.0	9.2
JPY–MSAR	7.6	10.3	39.5	JPY–MSAR	6.1	6.7	37.7
DEM–ESTAR	11.2	12.9	2.7	DEM–ESTAR	9.1	7.8	9.3
DEM–MSAR	12.0	8.9	16.6	DEM–MSAR	7.5	5.3	13.4
GBP–ESTAR	14.3	15.7	4.8	GBP–ESTAR	10.2	10.5	10.5
GBP–MSAR	20.1	35.8	87.1	GBP–MSAR	11.5	24.5	79.7

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR. JPY-ESTAR is the simulated ESTAR model with parameters according to JPY/USD real exchange rate, see Table 1. The other entries are analogous.

this case. The Markov Switching test has satisfying power properties. Its power is quite high against a Markov Switching DGP except for the DEM/USD exchange rate where a low power was expected because of the near unit root structure of the DGP. On the other hand it has low power against ESTAR models. The DF test has similar power properties to the ESTAR test.

Similar results can be observed for $T = 500$ (see Table 5). As expected, the power is generally higher compared to $T = 250$ but the results are qualitatively the same as before. The results for the de-trending case are qualitatively similar to those of the de-meaning case although all tests have less power under de-trending. This was expected as another deterministic parameter has to be fitted under de-trending. Unfortunately, the Markov Switching test is no longer conservative under de-trending when the true DGP is ESTAR. However, its power is still low and within the range of the ESTAR test.

Altogether, we can say that there is no ESTAR test which dominates in terms of power. It is argued, however, that the ESTAR test has rather poor power against ESTAR with our parameter constellations which are realistic for real exchange rates. In some constellations the power of the ESTAR test is even better for the Markov Switching alternative. As a result, by not rejecting the Null, this ESTAR test do not allow us to conclude that the null hypothesis unit root is correct and therefore we cannot reject the purchasing power parity

Table 5: Empirical power, $T = 500$

de-meaning	t_{DF}	t_{KSS}	R^*	de-trending	t_{DF}	t_{KSS}	R^*
JPY–ESTAR	16.1	18.8	2.0	JPY–ESTAR	11.4	10.8	20.1
JPY–MSAR	16.4	19.6	74.6	JPY–MSAR	9.3	11.2	70.9
DEM–ESTAR	22.3	29.7	2.2	DEM–ESTAR	13.4	14.2	21.7
DEM–MSAR	22.8	11.6	42.3	DEM–MSAR	13.1	7.5	30.8
GBP–ESTAR	30.5	49.1	1.9	GBP–ESTAR	22.4	23.9	23.7
GBP–MSAR	50.1	63.5	92.3	GBP–MSAR	31.6	49.1	93.9

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR. JPY-ESTAR is the simulated ESTAR model with parameters according to JPY/USD real exchange rate, see Table 1. The other entries are analogous.

hypothesis. However, when rejecting the Null we cannot conclude that the true model is ESTAR either.

4.3 Discussion

A natural question which arises out of the results in section 4.2 is why especially the ESTAR test has so poor power properties. Figure 1 sheds light on this problem. In these graphs, the transition function of each estimated ESTAR process based on real data and parameters reported in Table 1 are depicted together with corresponding data points. Almost all data points are in the region where the transition function is close to its maximum. There are no data points at the tails of the function. Close to the maximum of the transition function the process behaves similarly to a unit root process or a highly persistent local-to-unity autoregressive process. The mean reverting property of the non-linear time series model has a strong effect only in the outer regimes away from the equilibrium. Therefore, for the vast majorities of data points, the process behaves like a linear unit root process. This makes it hard or almost impossible for the tests to detect the non-linear mean reverting behavior of the DGP, given the sample sizes we consider.

In addition to this, our simulation study shows that the ESTAR test has similar power properties against ESTAR as against MSAR models. To intuitively explain this finding, we

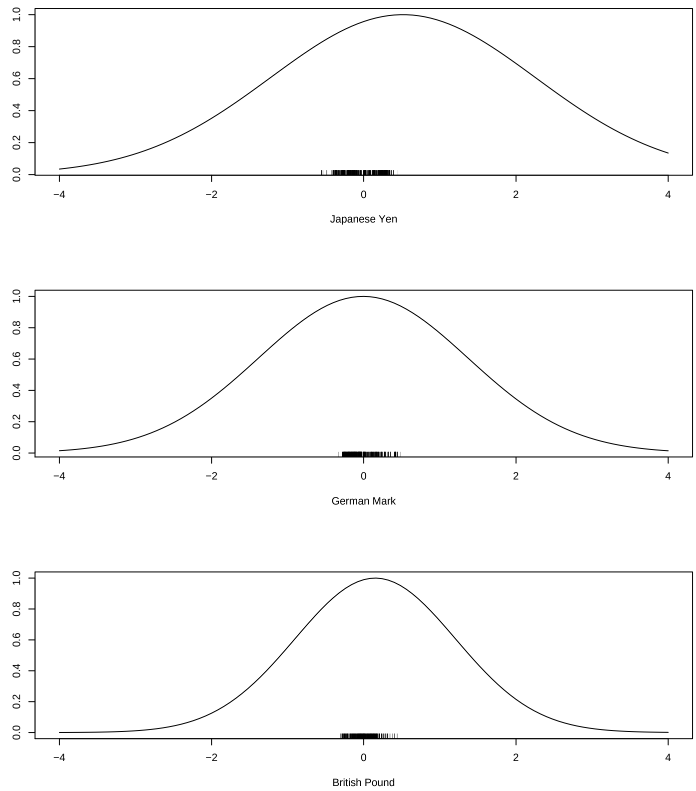

Figure 1: Estimated transition functions and data points.

generate plots of y_{t-1} against the first difference $\Delta y_t = y_t - y_{t-1}$ for ESTAR and MSAR simulated time series generated from our parameter constellations and estimate the functional relationship between Δy_t and y_{t-1} in a non-parametric way by using the Naradaya-Watson estimator, see Figure 2. If the ESTAR effect is strong, the estimated curve should be near a cubic function. If the time series process exhibits a unit root, it is flat and identical to zero. As we can see, the cubic behavior is clearly pronounced for the simulated DEM/USD and GBP/USD real exchange rate and less pronounced for the simulated JPY/USD real exchange rate which is in line with our parameter settings. Moreover, it is that both functions, the ESTAR and the MSAR function, are rather similar and quite close to each other. The MSAR process generates also a cubic shape for this function which is similar to the ESTAR model. Loosely speaking, the idea of the Kapetanios et al. test is to check whether this function has a cubic shape or not, it detects the cubic form also for the MSAR process. As both functions are close to each other, the power is similar for both models.

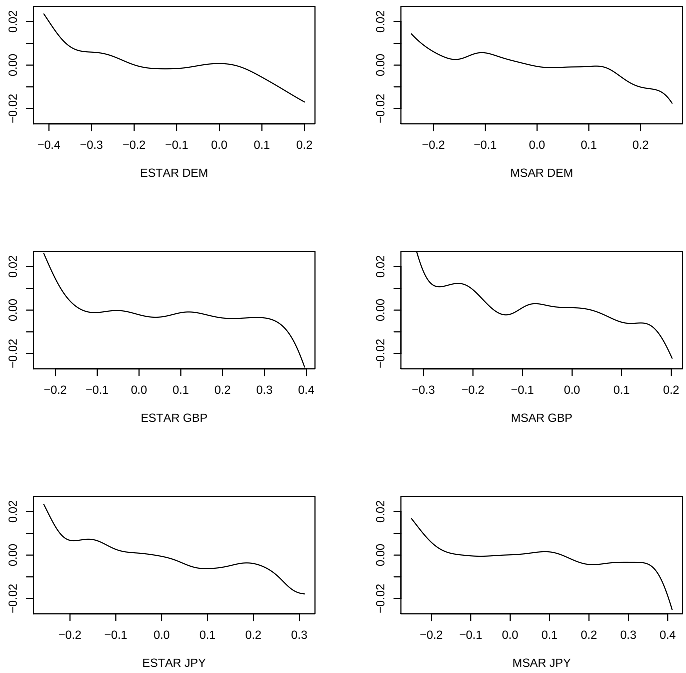

Figure 2: Nadaraya-Watson estimates for the functional relationship of ESTAR and MSAR processes.

This shows that the present tests are not able to detect ESTAR non-linearities as they are found in real exchange rates. Although the tests have convincing properties in many situations, they prove to have a lack of power under the very special parameter conditions which can be found in real exchange rates. It is rather difficult to draw any conclusion from the outcome of an ESTAR test under these conditions. Neither does a non-rejection of the Null mean that the true DGP which drives real exchange rates, is a linear unit root process nor does a rejection of the Null mean that the true DGP is actually an ESTAR process.

5 Application

This section applies the unit root tests studied in the Monte Carlo simulations to the G7 exchange rates. Thus, we examine non-linearities in the real exchange rates of the US

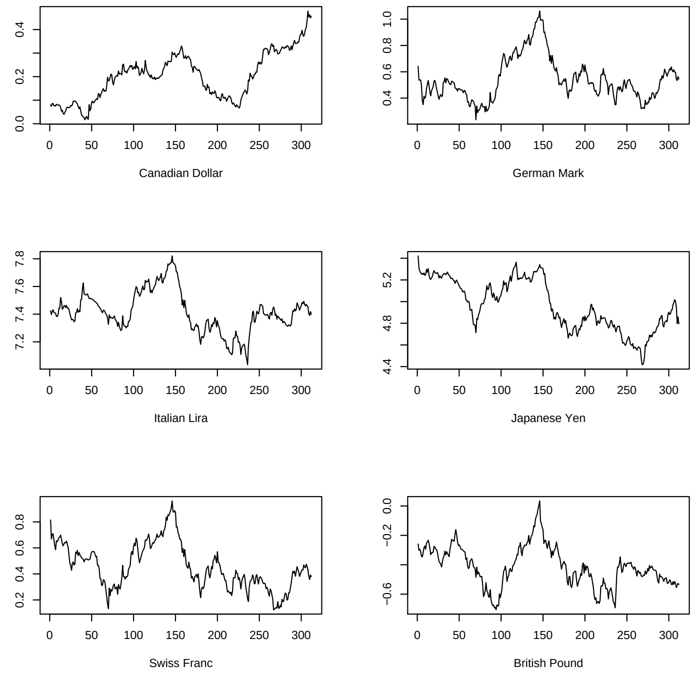

Figure 3: Logarithm of CPI-based real exchange rates against US Dollar.

Dollar against the Canadian Dollar (CAD), Swiss Franc (CHF), German Mark (DEM), British Pound (GBP), Italian Lira (ITL) and Japanese Yen (JPY). Data is taken from the IMF International Financial Statistics database. Price levels are measured by the consumer price index (CPI). The sample covers the post-Bretton Woods period from 1973.01 to the Euro introduction 1998.12 implying a sample size of $T = 312$. This data set is chosen to achieve comparability to other studies and has the advantage that potential structural breaks that might have occurred due to the introduction of the Euro are excluded and thus not biasing our analysis. All time series seem to be persistent and locally trending, see Figure 3. The estimated partial autocorrelation functions (graphs are available upon request) indicate that all time series can be modelled by first-order processes.

Next, we apply the standard Dickey-Fuller regression including a constant, i.e.,

$$\Delta y_t = c + \rho y_{t-1} + u_t$$

and test for linearity in the residuals $\widehat{u}_t$. Linearity is tested by (i) the neural network test

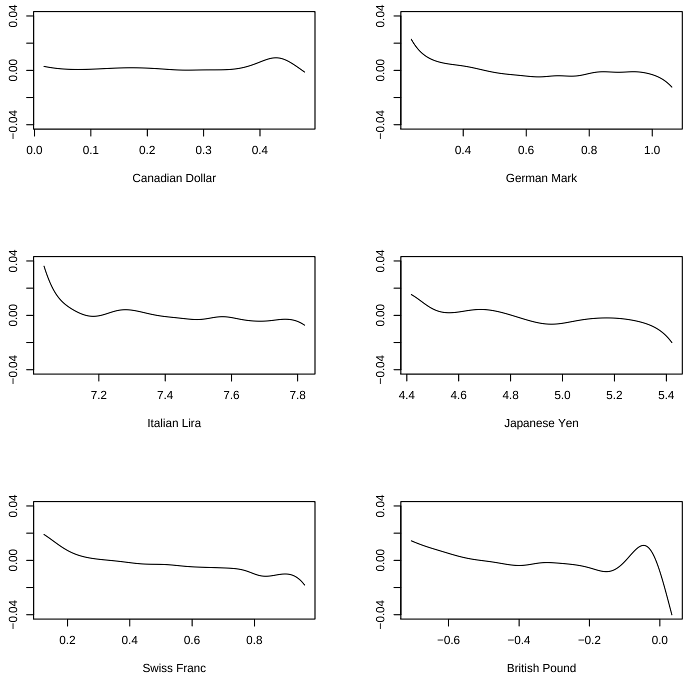

Figure 4: Nadaraya-Watson estimates for the functional relationship.

proposed by Lee et al. (1993), (*ii*) Ramsey's RESET test (1969) and (*iii*) the BDS test for independence by Broock et al. (1996). These tests assume stationarity which is crucial when applied to real exchange rates themselves but not when applied to residuals. Results can be found in Table 6. They show that the linearity hypothesis has to be rejected in many cases. This also means that the Dickey-Fuller test regression neglects important non-linearities and is therefore misspecified for testing PPP.

Moreover, we investigate the non-linearities by estimating the functional relationship between Δy_t and y_{t-1} in a non-parametric way by employing the Nadaraya-Watson estimator. Figure 4 shows these estimates. Only for the CAD/USD the estimated curve is very flat suggesting that there is no relationship between Δy_t and y_{t-1} which hints at a unit root. All other plots suggest mean-reversion and the functional relationship appears to be non-linear.

As the last step, we apply the previously analyzed unit root tests to the six real exchange rate series in order to test empirically for the validity of PPP. Since all time series appear

Table 6: Linearity and unit root test results

Test	CAD	CHF	DEM	GBP	ITL	JPY
Linearity tests (p-values)						
NN	0.007	0.008	0.028	0.535	0.745	0.038
RESET(2)	0.885	0.204	0.118	0.185	0.243	0.038
RESET(3)	0.727	0.356	0.053	0.147	0.288	0.228
RESET(4)	NA	0.344	0.025	0.065	0.014	0.005
BDS(2)	0.135	0.006	0.054	0.008	0.012	0.408
BDS(3)	0.033	0.016	0.178	0.013	0.001	0.216
BDS(4)	0.045	0.007	0.270	0.009	0.000	0.210
Unit root tests (test statistics)						
t_{DF}	-0.10	-2.47	-1.92	-2.17	-1.84	-1.94
t_{KSS}	0.08	-2.54	-1.36	-2.46	-2.10	-2.45
R^*	2.47	15.83	14.55	29.24	18.36	59.45

Notes: NN denotes the neural network test statistic by Lee et al. (1993). Hochberg's improved Bonferroni bound is used with one hundred draws to obtain reliable p-values for the neural network test, see Lee et al. (1993). RESET(m) is Ramsey's (1969) test statistic with terms up to power $m+1$. BDS(n) is the Broock et al. (1996) test statistic for independence with embedding dimension n. For unit root tests, see Table 2.

to be first-order processes, we do not include any lagged differences in the test regressions. The Dickey-Fuller regression contains a constant, while de-meaned data is used for the non-linear unit root tests.

The resulting test statistics are reported in the lower panel of Table 6. Neither the linear unit root test by Dickey and Fuller (1979) nor the non-linear unit root test by Kapetanios et al. (2003) are able to reject the null hypothesis of a unit root at the ten percent level of significance. These results contradict the validity of PPP since there is no mean-reversion when a unit root is present. On the contrary, the new test against MSAR rejects the Null in favor of a stationary MSAR model in four out of six cases. When having the outcomes of our preliminary analysis in mind (see Figure 4), it is not surprising that the unit root hypothesis cannot be rejected in the case of CAD/USD. In addition, we note that the R^* statistic for the DEM/USD is quite close to the critical value of 15.02 which means that the test decision is borderline. Due to the fact that the Markov Switching unit root test does not have substantial power against ESTAR, especially in the case of de-meaned data, it is legitimate to conclude that there is no evidence for ESTAR dynamics in the data. Markov Switching seems to be a more plausible model for explaining the dynamics of real exchange rates.

6 Conclusions

This paper provides a thorough empirical examination into the power of unit root tests against the alternatives of ESTAR or Markov switching processes. In particular, we investigate the power of unit root tests against ESTAR and Markov Switching. Moreover, we present evidence being consistent with the relevance of Markov Switching processes in real exchange rates.

Overall, we show that unit root tests against nonlinear alternatives exhibit very limited ability to identify the precise form of real exchange rate non-linearity. This is particularly the case for the ESTAR test, less for the Markov Switching test. Furthermore, we find that the parameter setting is crucial for the power of unit root tests against stationary ESTAR or Markov Switching models. Although these tests are powerful in general, under the specific conditions of currency markets, they seem to become clearly less powerful. This result - nonlinearities in combination with unfavorable parameter constellations - partly explains the difficulties in testing for PPP that are known from the empirical literature (Taylor et al. 2001).

As there is no simple unit root test against MSAR which is available in the form needed here, we propose a new test that builds upon inference techniques developed by Hansen (1996) and refined by Garcia (1998). This brings us to the core of this research, which is to compare ESTAR and MSAR tests in a broad simulation study showing that the ESTAR test has low power, whereas the MSAR test seems to be much more powerful. Moreover, we observe that ESTAR tests are not sensitive with respect to Markov Switching dynamics, i.e. misspecification of the alternative, while the opposite holds for our newly developed test. This means that a rejection of the unit root hypothesis by an ESTAR test, if any occurs, does not necessarily contain information about the type of non-linear adjustment to equilibrium.

Finally, when applying these tests to important real exchange rates, we find that the ESTAR test do not reject the unit root hypothesis for any currency pair. This indicates that either PPP does not hold - which is not very plausible from an economic point of view - or that is due to the low power of the ESTAR test under realistic parameter settings. Since the MSAR test is able to reject the null hypothesis in most cases, the MSAR model seems to be more plausible. The question arises whether the exchange rates are well described by an ESTAR process. Our results provide slight evidence against this.

The obvious economic implication stems from the properties of MSAR vs. ESTAR processes. Whereas ESTAR models are used in international finance to capture the working of international arbitrage in goods and services, the Markov Switching model fits more with the idea of currency markets with heterogeneous agents whose interaction can create temporary exchange rate bubbles. This suggests that real exchange rate dynamics may be influenced to a substantial degree by destabilizing forces.

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