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The Effects of Interest Rate Movements on Assets' Conditional Second Moments

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Abstract

This paper investigates whether the short term interest rate may explain the movements observed in the conditional second moments of asset returns. The theoretical connections between these seemingly unrelated quantities are studied within the C-CAPM framework. Under the assumption that the product of the relative risk aversion coefficient and the marginal utility is monotonic in consumption, original results are derived that attest the existence of a relation between the risk-free rate and the conditional second moments. The empirical findings, involving 165 stock returns quoted at the NYSE, confirm that, at low frequencies, the interest rate is a determinant of the 165 conditional variances and 13530 conditional correlations.

Keywords: Conditional Variance, Conditional Correlations, Interest Rate, Capital Asset Pricing Model, Sequential Conditional Correlations.

JEL classification: G10, G19, C50.

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1 Introduction

Models of the conditional second moments successfully fit and predict the underlying unobservable variables by making use of the information contained in the corresponding time series. While these models accurately describe and replicate the phenomena, they do not provide further understanding of the latent mechanism and the possible determinants behind it. In particular, the fundamental question that remains unanswered is how time varying volatility and correlations relate to the state of the economy and financial markets.

Various studies have investigated the link between the conditional variance of financial returns and exogenous explanatory variables. Factors that may potentially influence stock volatility have been considered by Schwert (1989). In particular, with respect to the linkages between financial and macro volatility *"the puzzle highlighted by the results [..] is that stock volatility is not more closely related to other measures of economic volatility."* Nonetheless, economic recessions are found to be the primary factor that drives fluctuations in volatility, a finding that is consistent with a later study by Hamilton and Lin (1996). Economic recessions aside, King et al. (1994) find that *"...only a small proportion of the covariances between national stock markets and their time-variation can be accounted for by* observable *economic variables. Changes in correlations between markets are driven primarily by movements in* unobservable *variables."* Engle and Rangel (2008), separately model fast- and slow-moving components of equity volatilities and find that the latter *"...is greater when the macroeconomic factors of GDP, inflation and short-term interest rates are more volatile or when inflation is high and output growth is low."*

Other studies have considered the effects of news announcements on volatility. Cutler et al. (1990) find that contemporaneous news events explain only a fraction of volatility *ex post*. With respect to scheduled macroeconomic news announcements on interest rate and foreign futures markets, Ederington and Lee (1993) observe that *"...volatility remains substantially higher than normal for roughly fifteen minutes and slightly elevated for several hours."* Similarly, in Balduzzi et al. (2001) it is found that *"public news can explain a substantial fraction of price volatility in the aftermath of announcements..."* and that *"...volatility increase[s] immediately after the announcements and persist for up to 60 minutes after the announcements."* Fleming and Remolona (1999) distinguish between a first and a second stage after the release of a major macroeconomic announcement and find that, in the latter, the announcement induces price volatility persistence. Using intra-day data, Andersen and Bollerslev (1998) *"...conclude that these effects (release of public information and, in particular, certain macroeconomic announcements) are secondary when explaining overall volatility."*

This paper investigates whether the short term interest rate may explain the movements observed in assets' conditional variances and correlations. The theoretical connections between these seemingly different quantities are investigated within the Consumption Capital Asset Pricing Model (C-CAPM) framework. The results are derived under the unique assumption that the product of the relative risk aversion coefficient and the marginal utility is a monotonic function of consumption. A logical connection is showed to exist between the conditional expectation and the conditional variance of the stochastic discount factor. Through the well established inverse relationship between the risk-free rate and the expected value of the discount factor, it is possible then to characterize the stochastic discount factor's conditional variance as related to the interest rate. An analysis on the degree of association between asset returns conditional second moments and the conditional variance of the discount factor closes the sequence of effects beginning with interest rate variations and ending with movements of variances and correlations. Under the stated assumption, the model does not specify the sign of the response of the conditional moments to changes in the risk-free rate. However, it does specify the conditions under which these quantities will move in the same or opposite direction.

The parameterization of the conditional variance-covariance matrix as a function of the interest rate and more generally of macroeconomic variables, requires the specification of a Multivariate GARCH (MGARCH) model suited for the inclusion of exogenous variables. This feature, however, brings about a series of complications due to the necessity of preserving positive definiteness of the conditional variance-covariance matrix. As a consequence, in most MGARCH models, the vector of exogenous determinants must enter the dynamic equations through some quadratic form. Clearly, this amounts to the imposition of undesirable restrictions on the functional form and on the set of parameters.

The Sequential Conditional Correlations (SCC) methodology introduced by Palandri (2009) ensures positive definiteness of the conditional correlation matrix under the necessary and sufficient condition that each correlation and partial correlation is bounded between plus and minus one. The associated bivariate Autoregressive Conditional Correlation (ACC) model, based on the Fisher transformation of the correlations, satisfies the required bounds by construction and can therefore effortlessly accommodate the inclusion of exogenous variables.

The theoretical findings of the paper are confirmed by the empirical results on 165 stock returns quoted at the NYSE, involving 165 conditional variances and 13530 conditional correlations. Glosten et. al (1993) find that the short term interest rates positively forecast stock market volatility. Using a large number of stocks, observed for more than forty years, I find that the sign of the relationship between interest rate and volatility varies from one asset to the other, as allowed by the theoretical model. Similarly, the sign of the interest rate effects on the conditional correlations is found to vary from one pair of assets to the other. The interest rate is found to be a significant determinant of the assets' conditional second moments even at the daily frequency. However, it is at lower frequencies, and therefore over longer horizons, that its importance overwhelms that of the purely dynamic components such as the GARCH effects.

The paper is organized as follows. Sections 2 and 3 present the theoretical investigation of the C-CAPM, leading to the identification of the relationship between the risk-free rate and the conditional second moments of assets' returns. The dataset is described in Section 4 followed by the description of the econometric model specifying the conditional second moments as functions of exogenous variables in Section 5. Details on the estimation are contained in Section 6. Section 7 presents the results and Section 8 concludes. Appendix A contains the proof related to the covariance of monotonic functions and Appendix B lays out the conditions for uniform ergodicity of the Autoregressive Conditional Correlations model.

2 Stochastic Discount Factor and Risk-Free Rate

Consider a representative agent with preferences described by a differentiable utility function u with $u' > 0$ and $u'' < 0$. The asset pricing restrictions for the net return $r_{i,t+1}$ satisfy:

$$\mathbb{E}_t \left[m_{t+1} \left(1 + r_{i,t+1} \right) \right] = 1 \tag{1}$$

where $\mathbb{E}_t$ is the time t conditional expectation, $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor, c_{t+1} next period's consumption and β the inter-temporal discount factor.

The risk-free rate pays the net return $r_{f,t+1}$ with certainty and may therefore be pulled out of the conditional expectation in equation (1) from which it follows that the product of the expected value of the stochastic discount factor and the risk-free gross return $(1 + r_{f,t+1})$ equals one. Movements in $\mathbb{E}_t[m_{t+1}]$ imply movements in the opposite direction for the risk-free rate. Furthermore, since the first is not an observable quantity, its levels and variations may be inferred from observations of the latter. Equation (1) does not seem to suggest any other immediate relationship. In particular, between $r_{f,t+1}$ and higher order conditional moments of the random variable m_{t+1}.

To unravel possible links between the first and the second order moments of the stochastic discount factor and therefore the risk-free rate, a simple stylized economy is analyzed. The underlying assumptions are:

(i) *The log-consumption process at time $t+1$ has a stochastic growth rate with time t conditional mean μ_t, time t conditional variance σ_t^2 and independent and identically distributed standardized innovations z_{t+1} with support in $\mathbb{R}$ and density $p(z)$:*

$$c_{t+1} = c_t \cdot \exp\{\mu_t + \sigma_t z_{t+1}\} \tag{2}$$

(ii) *The product of the relative risk aversion $\gamma(c)$ and the marginal utility u' is a monotonic function of consumption:*

$$\frac{d}{dc}\left[\gamma(c) \cdot u'(c)\right] = v(c) \tag{3}$$

with either:

(iia) $v(c) \leq 0 \;\; , \;\; \forall c \in \mathbb{R}^+ \quad \longrightarrow \quad -\frac{u'''(c) \cdot c}{u''(c)} \geq 1 \;\; , \;\; \forall c \in \mathbb{R}^+$

(iib) $v(c) \geq 0 \;\; , \;\; \forall c \in \mathbb{R}^+ \quad \longrightarrow \quad -\frac{u'''(c) \cdot c}{u''(c)} \leq 1 \;\; , \;\; \forall c \in \mathbb{R}^+$

Assumption (i) is a general statement about the dynamic properties of consumption growth rate. Assumption (ii) allows to establish precise relationships between the conditional first two moments of the stochastic discount factor. For three times differentiable utility functions, the monotonicity assumption (ii) may be interpreted in terms of restrictions on the curvature of the utility function. Furthermore, assumption (ii) may be seen as an extension of the Constant Relative Risk Aversion (CRRA) utility function. Assumption (iia) admits relative risk aversion that is decreasing in consumption, constant and increasing at a rate bounded above by the decrease in the marginal utility. Relative risk aversion increasing at a rate bounded below by the decrease in the marginal utility is consistent with Assumption (iib).

A comparative statics approach may be used to conduct a qualitative analysis of the effects of variations in the first two conditional moments of consumption's growth rate on the first two conditional moments of the stochastic discount factor. Let ν_t and s_t^2 denote, respectively, the conditional expectation and variance of m_{t+1}. Differentiating these moments with respect to consumption's conditional mean μ_t and standard deviation σ_t yields:

$$\begin{aligned}
\frac{\partial \nu_t}{\partial \mu_t} &= \beta \cdot \mathbb{E}_t\left[\frac{-\gamma(c_{t+1})u'(c_{t+1})}{u'(c_t)}\right] &(4)\\
\frac{\partial \nu_t}{\partial \sigma_t} &= \beta \cdot \mathbb{COV}_t\left[\frac{-\gamma(c_{t+1})u'(c_{t+1})}{u'(c_t)}, z_{t+1}\right] &(5)\\
\frac{\partial s_t^2}{\partial \mu_t} &= 2\beta^2 \cdot \mathbb{COV}_t\left[\frac{-\gamma(c_{t+1})u'(c_{t+1})}{u'(c_t)}, \frac{u'(c_{t+1})}{u'(c_t)}\right] &(6)\\
\frac{\partial s_t^2}{\partial \sigma_t} &= 2\beta^2 \cdot \mathbb{COV}_t\left[\frac{-\gamma(c_{t+1})u'(c_{t+1})z_{t+1}}{u'(c_t)}, \frac{u'(c_{t+1})}{u'(c_t)}\right] &(7)
\end{aligned}$$

where $\mathbb{COV}_t$ is the time t conditional covariance.

Signing derivative (4) is straightforward as the random variable inside the expectation only takes negative values: $\partial\nu_t/\partial\mu_t < 0$. Derivatives (5) and (6) may be easily signed recalling that monotonic non-decreasing functions of the random variable X exhibit positive covariances as proved in Appendix A. In particular, by assumption (i) consumption is an increasing function of the innovation z_{t+1}. Together with assumption (ii) this implies that the first term inside the covariances of equations (5) and (6) are either increasing (iia) or decreasing (iib) functions of z_{t+1}. It follows immediately that either $\partial\nu_t/\partial\sigma_t > 0$ (iia) or $\partial\nu_t/\partial\sigma_t < 0$ (iib). The second term in equation (6) is decreasing in z_{t+1} and hence either $\partial s_t^2/\partial\mu_t < 0$ (iia) or $\partial s_t^2/\partial\mu_t > 0$ (iib). Derivative (7) may be signed in a similar manner by letting z^* be the value of the innovation z_{t+1} such that:

$$\gamma(c(z^*))u'(c(z^*))z^* = \mathbb{E}_t[\gamma(c_{t+1})u'(c_{t+1})z_{t+1}] \tag{8}$$

The right hand side of equation (8) is either negative (iia) or positive (iib): notice the opposite sign with respect to the derivative in (5). Since the coefficient of relative risk aversion and the marginal utility are positive, equation (8) implies that $z^* < 0$ (iia) or $z^* > 0$ (iib). Let's consider the case in which (iia) holds and rewrite the expected value of the deviations from the mean as the sum of three integrals over disjoint subsets:

$$\begin{aligned}
\int_{-\infty}^{z^*} [\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \quad &+ \qquad (9)\\
\int_{z^*}^{0} [\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \quad &+ \qquad (10)\\
\int_{0}^{+\infty} [\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \quad &= \quad 0 \qquad (11)
\end{aligned}$$

The derivative of $\gamma u' z$ with respect to z is equal to:

$$\frac{d\gamma u' z}{dz} = \frac{d(\gamma u')}{dz} \cdot z + \gamma u' \qquad (12)$$

The product $\gamma u'$ is positive and by assumptions (i) and (iia) is decreasing in z. Therefore for any $z \in (-\infty, 0)$ the derivative in (12) is positive. Hence, $\forall z \in (-\infty, z^*)$ the quantity $[\gamma u' z - \mathbb{E}_t (\gamma u' z)]$ is negative and so is integral (9). For $z \in (z^*, 0)$ the sign of the derivative in (12) is still positive, making the integrand $[\gamma u' z - \mathbb{E}_t (\gamma u' z)]$ as well as the integral (10) positive. Finally, the integral in (11) is positive as both terms of the integrand $\gamma u' z$ and $-\mathbb{E}_t[\gamma u' z]$ are positive. Collecting positive terms:

$$\int_{z^*}^{+\infty} [\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz = - \int_{-\infty}^{z^*} [\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \qquad (13)$$

Since u' is decreasing in z, if multiplied by the integrands of (13) it yields:

$$\begin{aligned}
\int_{z^*}^{+\infty} u'[\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \quad &< \quad - \int_{-\infty}^{z^*} u'[\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz\\
\int_{\mathbb{R}} u'[\gamma u' z - \mathbb{E}_t(\gamma u' z)] p(z) dz \quad &< \quad 0 \quad \Rightarrow\\
\mathbb{COV}_t[u', \gamma u' z] \quad &< \quad 0 \quad \Rightarrow\\
\frac{\partial s_t^2}{\partial \sigma_t} \quad &> \quad 0 \qquad (14)
\end{aligned}$$

It may be shown, *mutatis mutandis*, that the sign of derivative (7) is positive even under assumption (iib). Table I reports the signs of the derivatives of the conditional mean and variance of the stochastic discount factor with respect to the first two conditional moments of income growth rate under assumptions (i) and (ii).

From the signs of derivatives (4)-(7) it is possible to determine the directions of the adjustments of the first two conditional moments of the stochastic discount factor resulting from variations in the conditional mean and variance of consumption's growth rate. Under assumption (i) and (iia), a higher (lower) μ_t and a lower (higher)

σ_t^2 imply a lower (higher) $\mathbb{E}_t[m_{t+1}]$ and $\mathbb{V}_t[m_{t+1}]$. Since the risk-free rate moves in the opposite direction of the conditional expectation of m_{t+1}, it follows that an increase (decrease) in the level of the risk-free rate is associated to a decrease (increase) in the conditional variance of the stochastic discount factor. Under assumptions (i) and (iib), instead, higher (lower) μ_t and σ_t^2 imply lower (higher) $\mathbb{E}_t[m_{t+1}]$ and higher (lower) $\mathbb{V}_t[m_{t+1}]$. In turn, this implies that an increase (decrease) in the risk-free rate is associated with an increase (decrease) in the conditional variance of m_{t+1}.

3 Risky Assets and Stochastic Discount Factor

The next period return on any risky asset i can be decomposed into a part correlated with the stochastic discount factor and a conditionally orthogonal idiosyncratic part:

$$r_{i,t+1} = a_{i,t} + b_{i,t} \cdot m_{t+1} + \sigma_{i,t} \cdot \epsilon_{i,t+1} \tag{15}$$

with $\mathbb{E}_t\left[\epsilon_{i,t+1}\right] = 0$, $\mathbb{E}_t\left[\epsilon_{i,t+1}^2\right] = 1$ and $\mathbb{E}_t\left[m_{t+1} \cdot \epsilon_{i,t+1}\right] = 0$ by construction. Since non-linear relations between the risky returns and the stochastic discount factor may only be approximated by such one-factor model, as a result, the conditional variance $\sigma_{i,t}^2$ of the residuals will in general be a function of the conditional moments of m_{t+1}. Imposing the asset pricing restrictions (1) on the time t conditional projection (15) yields:

$$\begin{aligned} r_{i,t+1} &= \frac{1 - \nu_t - b_{i,t}\left(s_t^2 + \nu_t^2\right)}{\nu_t} + b_{i,t} \cdot m_{t+1} + \sigma_{i,t} \cdot \epsilon_{i,t+1} \\ &= \frac{1 - \nu_t - b_{i,t} s_t^2}{\nu_t} + b_{i,t}\left(m_{t+1} - \nu_t\right) + \sigma_{i,t} \cdot \epsilon_{i,t+1} \end{aligned} \tag{16}$$

where ν_t and s_t^2 are the conditional mean and variance of the stochastic discount factor. The conditional variance of $r_{i,t+1}$ is equal to:

$$\mathbb{V}_t\left[r_{i,t+1}\right] = b_{i,t}^2 \cdot s_t^2 + \sigma_{i,t}^2 \tag{17}$$

Thus, in general, changes in the first two conditional moments of the stochastic discount factor may be seen as affecting the conditional variance of a risky asset through three main channels: the factor loading $b_{i,t}$, the conditional variance s_t^2 of m_{t+1} and the conditional variance $\sigma_{i,t}^2$ of the projection residuals. Equation (17) outlines the transmission mechanism between the moments of m_{t+1} and $r_{i,t+1}$ while suggesting that the direction and magnitude of such co-movements need not to be the same across assets. There is no transmission mechanism when equation (17) does not depend on the moments of the stochastic discount factor. Specifically, this is the case either when the conditional variance of the idiosyncratic term adjusts to offset the effects of m_{t+1}:

$$r_{i,t+1} = a_{i,t+1} + b_{i,t} \cdot m_{t+1} + \left(\mathbb{V}_t\left[r_{i,t+1}\right] - b_{i,t}^2 s_t^2\right)^{1/2} \epsilon_{i,t+1} \tag{18}$$

or when the factor loading adjusts:

$$r_{i,t+1} = a_{i,t+1} \pm \left(\mathbb{V}_t\left[r_{i,t+1}\right] - \sigma_{i,t}^2\right)^{1/2} \left[\frac{m_{t+1} - \nu_t}{s_t}\right] + \sigma_{i,t} \cdot \epsilon_{i,t+1} \tag{19}$$

or when a combination of both effects takes place.

For the simple one-factor model with constant factor loading and $\sigma_{i,t}^2$ that does not depend on the conditional moments of m_{t+1}:

$$\mathbb{V}_t\left[r_{i,t+1}\right] = b_i^2 s_t^2 + \sigma_{i,t}^2 \tag{20}$$

from which it is possible to establish very precisely that an increase in the conditional variance of the stochastic discount factor produces an increase in the conditional variance of the risky asset. Therefore, under assumptions (i) and (iia), in periods of high (low) interest rates the stock market's volatility will be relatively low (high). On the contrary, under assumptions (i) and (iib), periods of high (low) interest rates should be accompanied by a relatively high (low) stock market's volatility.

Let $\mathring{r}_{i,t+1}$ be a risky return standardized by its conditional mean and variance:

$$\mathring{r}_{i,t+1} = \rho_{i,m,t} \cdot \mathring{m}_{t+1} + \left(1 - \rho_{i,m,t}^2\right)^{1/2} \cdot \mathring{\epsilon}_{i,t+1} \tag{21}$$

where $\mathring{m}_{t+1}$ is the standardized stochastic discount factor, $\mathring{\epsilon}_{i,t+1}$ is the standardized orthogonal residual and $\rho_{i,m,t,}$ is the conditional correlation between m_{t+1} and the return $r_{i,t+1}$. The conditional correlation $\rho_{i,j,t}$ between any two returns i and j is then given by:

$$\rho_{i,j,t} = \rho_{i,m,t}\rho_{j,m,t} + (1 - \rho_{i,m,t}^2)^{1/2}(1 - \rho_{j,m,t}^2)^{1/2}\rho_{i,j,t}^{\epsilon} \tag{22}$$

where $\rho_{i,j,t}^{\epsilon}$ is the conditional correlation between the components of the two returns that are orthogonal to the stochastic discount factor. In general, variations in the first two conditional moments of the stochastic discount factor translate into adjustments of the conditional correlations of the assets and m_{t+1} and therefore of weights being shifted between the standardized stochastic discount factor and the projection residual. This in turn will lead to a realignment of the conditional correlation between assets i and j. At the same time, the risk-free rate will also respond to the movements in ν_t and s_t^2 and react accordingly.

In the one-factor model, the conditional correlation between asset i and j equals:

$$\rho_{i,j,t} = \frac{b_i b_j s_t^2}{\left[b_i^2 s_t^2 + \sigma_{i,t}^2\right]^{1/2} \cdot \left[b_j^2 s_t^2 + \sigma_{j,t}^2\right]^{1/2}} \tag{23}$$

and the derivative of the conditional correlation with respect to the conditional variance of m_{t+1} is:

$$\begin{aligned} \frac{\partial \rho_{i,j,t}}{\partial s_t^2} &= \frac{1}{2}\rho_{i,j,t} \cdot \left\{ \frac{2}{s_t^2} - \frac{b_i^2}{\left[b_i^2 s_t^2 + \sigma_{i,t}^2\right]} - \frac{b_j^2}{\left[b_j^2 s_t^2 + \sigma_{j,t}^2\right]} \right\} \\ &= \frac{1}{2}\rho_{i,j,t} \cdot \left\{ \frac{b_i^2 s_t^2 \sigma_{j,t}^2 + b_j^2 s_t^2 \sigma_{i,t}^2 + 2\sigma_{i,t}^2 \sigma_{j,t}^2}{s_t^2 \cdot \left[b_i^2 s_t^2 + \sigma_{i,t}^2\right] \cdot \left[b_j^2 s_t^2 + \sigma_{j,t}^2\right]} \right\} \\ &\propto \rho_{i,j,t} \end{aligned} \tag{24}$$

Assets that pay a positive risk-premium have negative factor loadings ($b_i < 0$, $b_j < 0$) and are therefore positively correlated ($\rho_{i,j,t} > 0$). From equation (24) it follows that an increase (decrease) in the conditional variance of the stochastic discount factor results in an increase (decrease) in the correlation between assets. This effect is due to the fact that an increase (decrease) in s_t^2 is the same as an increase (decrease) in the magnitude of the loadings on the standardized factor $\mathring{m}_{t+1}$. The immediate consequence is that these assets will exhibit conditional correlations that move in the same direction of s_t^2 and therefore in the opposite direction of the risk-free rate under (i)+(iia) and in the same direction under (i)+(iib).

4 Data

The analysis is based on 165 asset returns quoted at the New York Stock Exchange[1] spanning from January 1962 through December 2006. Source of raw daily closing prices and factors to adjust for splits is the CRSP database. The returns characteristics have been studied at six different frequency levels beginning with the daily frequency which comprises 11,327 time series observations per asset. Lower frequency returns have been constructed by aggregating the daily returns over 5, 20, 40, 80 and 120 days corresponding roughly to 1 week, 1, 2, 4 and 6 months respectively. For each frequency, all possible aggregation schemes have been considered, resulting

[1]The list of ticker symbols may be found in Table III

in 5 subsamples of 2,264 observations, 20 subsamples of 565 observations, 40 subsamples of 282 observations, 80 subsamples of 140 observations and 120 subsamples of 93 observations. The index $a = 1, ..., A$ is used to refer to a particular subsample with A being the aggregation level: 1, 5, 20, 40, 80 or 120 days. In each subsample a, the non-overlapping aggregate time interval has been normalized to unity so that a time index may be defined as $t = 1, ..., T$ where T is the floor of $(11327 - A + 1)/A$. As for the corresponding time intervals, in each subsample returns do not overlap. The cross-sectional dimension is spanned using the index $i = 1, ..., 165$ and the returns are therefore denoted by $r_{i,a,t}$.

The 1-Year Treasury Constant Maturity Rate is used as a proxy for the risk-free rate. Observations are at the daily frequency over the same 45 year period. Instead of aggregating to generate lower frequency returns, only the most recent observation within each time interval is selected. In particular, the time t, a-th aggregate return of asset i over A days $r_{i,a,t} \equiv \left(r_{i,a+A(t-1)} + \cdots + r_{i,a+At-1}\right)$ is associated to the risk-free rate $r_{f,a+At-1}$. Thus, for the given aggregate risky return, lags of the constant maturity rate have been defined to be $r_{f,a,t-1} \equiv r_{f,a+A(t-1)-1}$ at one lag, $r_{f,a,t-2} \equiv r_{f,a+A(t-2)-1}$ at two lags, etc. In this way, rather than the cumulated interest rate, only the most recent and available level of r_f will be used as a covariate, consistently with the argument that it is the most informative about the current conditional moments of the stochastic discount factor.

The time dimension should be long enough to make the potential findings robust to spurious local relations. On the other hand, the size of the cross-section should ensure that the results do not pertain to few assets alone. Of the companies listed in the NYSE directory[2] 167 have been quoted continuously for the selected 45 year period. Among these, two have been excluded from this study due to corrupted price listings in the CRSP database at the time of the download[3]. Increasing the time dimension signifies reducing the cross-section in the same way as increasing the cross-section reduces the time dimension: fewer assets have been quoted for longer periods of time. 165 assets observed from 1962 to 2006 seem a reasonable compromise with the trade-off between the two dimensions.

[2]www.nyse.com

[3]Corrupted data files regard Exelon Corporation (EXC) and Ameren Corporation (AEE) which did not cover the whole sample period. Moreover, EXC contained fragments of price listings of Peco Energy Co. (PE) while AEE contained fragments of Union Electric Co. (UEP).

4.1 Polynomial Regressions

The one lag constant maturity rate, as previously defined, is used to construct a base of orthogonal polynomials for each of the aggregation levels. The risky returns are projected on such space by means of ordinary least squares. The optimal degree of the polynomial for each of the 165 assets is determined on the basis of the Schwarz Information Criterion (SIC):

$$r_{i,a,t} = \sum_{p=0}^{P} c_p x_{p,a,t-1} + \epsilon_{i,a,t} \tag{25}$$

where $x_{p,a,t-1}$ is the time $t-1$ projection residual of $r^p_{f,a,t-1}$ on $\{x_{l,a,t-1}\}_{l=0}^{p-1}$ and $x_{0,a,t-1}$ is equal to one. These regressions will ensure that the potential links between the assets' conditional second moments and the risk-free rate are not due to the incorrect exclusion of the interest rate from the specification of the conditional mean. The summary statistics of the polynomial regressions are displayed in Table II where for each series the degree of the polynomial has been determined using the SIC. For relatively high frequencies, ranging from 1-day to 2-months returns, the interest rate did not exhibit any economically significant explanatory power. At lower frequencies, 4- and 6-months, mean and median R-square are below 5%. Nevertheless, the behaviour of a few assets' returns seems to be captured quite accurately by a non linear function, here approximated by the polynomial expansion, of the lag-one interest rate. Whether the relatively high explanatory power of the interest rate, for some assets, is structural or it is due to noise fitting is beyond the scope of this study. Here the polynomial filtration is carried out to ensure that unaccounted first moment effects are not carried over to the conditional second moments and, therefore, mistakenly detected as second moment effects. Section 5 describes the models for the conditional variances and correlations used as filters of the returns' conditional variance-covariance matrix.

5 Multivariate Modeling

Let $H_{a,t}$ be the $(M \times M)$ conditional variance-covariance matrix of the time t M-dimensional vector of residuals $\epsilon_{a,t}$ of $a-$aggregated asset returns. Bollerslev (1990) and Engle (2002) have exploited the possibility of separating the conditional variances from the conditional correlations:

$$H_{a,t} = D_{a,t} R_{a,t} D_{a,t} \tag{26}$$

where $D_{a,t}$ is the $(M \times M)$ diagonal matrix of time-varying standard deviations and $R_{a,t}$ is the $(M \times M)$ matrix of conditional correlations. The Sequential Conditional Correlations methodology allows to further decompose the conditional correlation matrix into its constituting components: the K-matrices. In particular, any correlation matrix may be expressed as the product of a sequence of K-matrices and that the product of any sequence of K-matrices (with $|\rho| < 1$) is a correlation matrix[4]:

$$H_{a,t} = D_{a,t}\left(\prod_{i=1}^{M-1}\prod_{j=i+1}^{M} K_{i,j,a,t}\right)\left(\prod_{i=1}^{M-1}\prod_{j=i+1}^{M} K_{i,j,a,t}\right)' D_{a,t} \tag{27}$$

The $K_{i,j,a,t}$ matrices are lower triangular with generic element $[row, col]$ given by:

$$\mathbf{K_{i,j,a,t}}[\mathbf{row}, \mathbf{col}] = \begin{cases} \rho_{i,j,a,t} & \text{if } row = j \text{ and } col = i \\ (1-\rho^2_{i,j,a,t})^{1/2} & \text{if } row = j \text{ and } col = j \\ \mathbf{I}[row, col] & \text{otherwise} \end{cases}$$

where $\mathbf{I}$ is the identity matrix and the element $\rho_{i,j,a,t}$ is the time t correlation $(i = 1)$ or partial correlation $(i > 1)$ between the time t returns of assets i and j. Thus, SCC eliminates the dimensionality problem inherent to MGARCH models by making it possible to separately model and estimate the elements $\rho_{i,j,a,t}$ without violating positive definiteness and without imposing parametric restrictions.

SCC converts the high-dimensional and intractable optimization problem associated with MGARCH modeling into a series of simple and feasible estimations. This is done by working from the outside toward the inside of the specification in (27): model and estimate the elements of $D_{a,t}$ and use it to standardize the data, model and estimate the element of $K_{1,2,a,t}$ and use it to standardize the data, model and estimate the element of $K_{1,3,a,t}$ etc.

Thanks to the SCC's decomposition of the conditional correlation matrix, this methodology results to be tailor made for parallel computing. A natural parallelization consists in estimating the parameters of the correlations' models from left to right and top to bottom of the upper triangular part of the matrix $R_{a,t}$. Furthermore, the rather loose sequentiality of SCC reduces to a negligible amount the nodes' total waiting times. Specifically, given that the elements of the first row of $R_{a,t}$ may be

[4]The K-matrix decomposition is unique for given R, however a permutation of the order of the series implies a permutation of the elements of R, leading to a different sequence of K-matrices. Following Palandri (2009), in this study the 165 assets have been ordered in decreasing order of total square correlation: Euclidean norms of the columns of the unconditional correlation matrix.

estimated independently from each other as soon as any of the previously allocated nodes becomes available it may be immediately reallocated. The generic element $\rho_{i,j,a,t}$ (where $j > i$) on the i-th row and j-th column may be estimated as soon as $\rho_{i-1,i,a,t}$ and $\rho_{i-1,j,a,t}$ have been estimated. The estimations of the various specifications of the conditional correlation matrix have been carried out running parallel C++ code, using MPI[5], on 2 Quad Core processors. Due to the mentioned loose sequential structure of SCC, it has been possible to achieve a high efficiency of parallelization: the master and the 7 nodes performed as 1 single node running approximately 7.5 times faster, corresponding to a reduction of more than 86% in total computing time when benchmarked against a single node.

5.1 Models of the Conditional Variances

Section 3 establishes the existence of some type of relation between the conditional variances of risky assets and the risk-free rate. To model such connection and evaluate its statistical and economic significance, different univariate models are compared:

$$
\begin{aligned}
\mathbf{V0}: \quad h_{i,a,t} &= \omega_i \\
\mathbf{V1}: \quad h_{i,a,t} &= \omega_i + \beta_i h_{i,a,t-1} + \alpha_i \epsilon^2_{i,a,t-1} \\
\mathbf{V2}: \quad \ln h_{i,a,t} &= \omega_i + \gamma_{i,1} x_{1,a,t-1} + \gamma_{i,2} x_{2,a,t-1} \\
\mathbf{V3}: \quad \ln h_{i,a,t} &= \omega_i + \beta_i \ln h_{i,a,t-1} + \gamma_{i,1} x_{1,a,t-1}
\end{aligned}
$$

$$
\mathbf{V4}: \quad \begin{cases} h_{i,a,t} = & \nu_{i,a,t} \cdot s_{i,a,t} \\ \ln \nu_{i,a,t} = & \omega_i + \delta_i \ln \nu_{i,a,t-1} + \gamma_{i,1} x_{1,a,t-1} + \gamma_{i,2} x_{2,a,t-1} \\ s_{i,a,t} = & (1 - \beta_i - \alpha_i) + \beta_i s_{i,a,t-1} + \alpha_i \left(\epsilon^2_{i,a,t-1} / \nu_{i,a,t-1} \right) \end{cases}
$$

V0 describes constant conditional variances while specification V1 is the GARCH(1,1) of Bollerslev (1986). Here, the conditional variance $h_{i,a,t}$ of asset i, in the a-th aggregation scheme and at time t is a function of the squared residual $\epsilon^2_{i,a,t-1}$ of the same asset, same aggregation and at time $t-1$. V2 and V3 are functionally similar to the ARCH of Engle (1982) and the GARCH specifications but with a crucial difference: the lagged realizations, which do not enter the model, are replaced by functions of the interest rate. In V2, the relationship between the risk-free rate and asset i log-variance is approximated by a second degree polynomial. $x_{1,a,t-1}$ refers to the unaggregated and de-meaned interest rate observable just prior to time t in

[5] Message Passing Interface standard

the a-th aggregation. Similarly, $x_{2,a,t-1}$ refers to the component of the squared the interest rate that is orthogonal to a constant and $x_{1,a,t-1}$. In V3, instead, the asset's conditional log-variance is a linear function of the de-meaned interest rate, augmented by a GARCH-type memory parameter. The magnitude of β will reflect the degree of persistence in volatility as opposed to a specification like V2 in which any persistence in the variance may arise solely from that of its covariates. Furthermore, the memory parameter in V3 allows shocks to the interest rate to affect the conditional variances over time as opposed to V2 where such effects exhaust in a single period. Finally, V4 includes both GARCH and interest rate effects in a specification that nests V0-V3.

5.2 Models of the Conditional Correlations

Within the SCC methodology, the conditional correlation matrix is decomposed into its constituting elements: conditional correlations and partial correlations. It is convenient, in this setting, to parameterize the Fisher transformation $\chi_{i,j,a,t}$ of the conditional correlation $\rho_{i,j,a,t}$ between assets i and j:

$$\chi_{i,j,a,t} = \frac{1}{2} \ln \left(\frac{1 + \rho_{i,j,a,t}}{1 - \rho_{i,j,a,t}} \right) \tag{28}$$

Mapping the interval $(-1, 1)$ into $(-\infty, +\infty)$, this transformation allows $\chi_{i,j,a,t}$ to take any value on the real line. Thus, there will be no need to bound the process by imposing constraints on the model's parameters. These quantities are modeled by the bivariate specifications C0-C4 which closely resemble V0-V4 of the conditional variances:

$$\begin{aligned}
\mathbf{C0}: \quad \chi_{i,j,a,t} &= \omega_{i,j} \\
\mathbf{C1}: \quad \chi_{i,j,a,t} &= \omega_{i,j} + \theta_{i,j}\psi_{i,j,a,t-1} \\
\mathbf{C2}: \quad \chi_{i,j,a,t} &= \omega_{i,j} + \gamma_{i,j,1}x_{1,a,t-1} + \gamma_{i,j,2}x_{2,a,t-1} \\
\mathbf{C3}: \quad \chi_{i,j,a,t} &= \omega_{i,j} + \delta_{i,j}\chi_{i,j,a,t-1} + \gamma_{i,j,1}x_{1,a,t-1} \\
\mathbf{C4}: \quad \chi_{i,j,a,t} &= \omega_{i,j} + \delta_{i,j}\chi_{i,j,a,t-1} + \theta_{i,j}\psi_{i,j,a,t-1} + \gamma_{i,j,1}x_{1,a,t-1} + \gamma_{i,j,2}x_{2,a,t-1}
\end{aligned}$$

In C0, the conditional correlations are constant while in C1 their dynamics are described by the Autoregressive Conditional Correlations (0,1) model which closely resembles the GARCH(1,1)[6]. The variable $\psi_{i,j,a,t-1}$ is the Fisher transformation of the time $(t-1)$ *realized* correlation $\phi_{i,j,a,t-1}$. Since the latter is not observable it

[6]Similarities between the ACC(0,1) and GARCH(1,1) specifications are sketched in Appendix B.1

is estimated by means of an exponential smoothing, with parameter $\alpha_{i,j}$, of past realizations:

$$\psi_{i,j,a,t} = \frac{1}{2}\ln\left(\frac{1+\phi_{i,j,a,t}}{1-\phi_{i,j,a,t}}\right) \tag{29}$$

$$\phi_{i,j,a,t} = \frac{Q_{i,j,a,t}[1,2]}{\sqrt{Q_{i,j,a,t}[1,1]\cdot Q_{i,j,a,t}[2,2]}} \tag{30}$$

$$Q_{i,j,a,t} = \alpha_{i,j}Q_{i,j,a,t-1} + (1-\alpha_{i,j})\epsilon_{i,j,a,t}\epsilon'_{i,j,a,t} \tag{31}$$

$\epsilon_{i,j,a,t}$ is the bi-dimensional vector containing $\epsilon_{i,a,t}$ and $\epsilon_{j,a,t}$. The (2×2) matrix $Q_{i,j,a,t}$ is the exponential smoothing of the underlying variance-covariance matrix. It should be noticed that, as a consequence of the volatility filtration, the elements on the main diagonal have a conditional mean of one. From the smoothed variance-covariance matrix the corresponding correlation $\phi_{i,j,a,t}$ is computed and then its Fisher transformation $\psi_{i,j,a,t}$. The conditions for uniform ergodicity of the ACC model are derived in Appendix B.2. C2 and C3 are the transposition to the modeling of the conditional correlations of V2 and V3 respectively. Similarly to V2 and V3, shocks to the interest rate have an instantaneous effect on the conditional correlations in C2 while it is spread over time in C3. C4 includes both autoregressive and interest rate effects in a parameterization that nests C0-C3.

6 Estimation

The parameters of the models for the conditional variances and conditional correlations are estimated by GMM based on the scores of Gaussian likelihood functions. In particular, there are M scores pertaining to the variance models and $M(M-1)/2$ scores pertaining to the correlation models. The score-based moment conditions arising from the SCC methodology give rise to a block-triangular system of equations. Standard GMM estimates are obtained by setting all moment conditions to zero and simultaneously solving for the whole set of parameters. However, given the block-triangular structure of the moment conditions, this is the same as solving for one set of moments at the time. The equivalence between the simultaneous and the step-by-step solution allows the SCC estimation to fall within the GMM framework. Nevertheless, the theoretical convenience of standard GMM asymptotics does not solve the practical issues related to the computation of the parameters' variance-covariance matrix. Especially the estimation of the variance of the moments as the average of the outer-products of the moment conditions: given that its dimensions are of order M^2, this matrix is not full rank for large cross-sections $M > T^{1/2}$.

6.1 Score of the Variances

Let ϵ_i be the $(A \cdot T \times 1)$ vector of residuals from the polynomial regression of asset i: $\epsilon_i' = \left(\epsilon_{i,1}', \epsilon_{i,2}', ..., \epsilon_{i,A}'\right)$ with $\epsilon_{i,a}' = (\epsilon_{i,a,1}, \epsilon_{i,a,2}, ..., \epsilon_{i,a,T})$. Then, the concentrated log-likelihood L_c is:

$$L_c = -\ln|\Omega_i| - \epsilon_i'\Omega_i^{-1}\epsilon_i \tag{32}$$

where Ω_i is the conditional variance-covariance matrix:

$$\Omega_i = \begin{pmatrix} \Omega_{i,1,1} & \Omega_{i,1,2} & \dots & \Omega_{i,1,A} \\ \Omega_{i,1,2}' & \Omega_{i,2,2} & \dots & \Omega_{i,2,A} \\ \vdots & & \ddots & \vdots \\ \Omega_{i,1,A}' & \Omega_{i,2,A}' & \dots & \Omega_{i,A,A} \end{pmatrix}$$

The $\Omega_{i,a,a}$ are diagonal matrices of time-varying variances $h_{i,a,t}$ while the $\Omega_{i,a,l}$ matrices, with $a \neq l$, are bidiagonal and contain the time-varying covariances between residuals overlapping across aggregation schemes. These covariances arise as the byproduct of the different aggregation schemes and, with respect to this study, do not contain any useful information. Hence, the matrix Ω_i is replaced with the *misspecified* block diagonal matrix Ω_i^* which disregards the artificially induced covariances:

$$\Omega_i^* = \begin{pmatrix} \Omega_{i,1,1} & 0 & \dots & 0 \\ 0 & \Omega_{i,2,2} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \Omega_{i,A,A} \end{pmatrix}$$

Hence, the log-likelihood becomes:

$$\begin{aligned} L_c &= -\ln|\Omega_i^*| - \epsilon_i' \left[\Omega_i^*\right]^{-1} \epsilon_i \\ &= -\sum_{a=1}^{A} \left[\ln|\Omega_{i,a,a}| + \epsilon_{i,a}'\Omega_{i,a,a}^{-1}\epsilon_{i,a}\right] \\ &= -\sum_{a=1}^{A}\sum_{t=1}^{T} \left[\ln h_{i,a,t} + \epsilon_{i,a,t}^2 h_{i,a,t}^{-1}\right] \end{aligned} \tag{33}$$

6.2 Score of the Correlations

Let $\eta_i' \equiv \left(\epsilon_{i,1}'\Omega_{i,1,1}^{-1/2}, \epsilon_{i,2}'\Omega_{i,2,2}^{-1/2}, ..., \epsilon_{i,A}'\Omega_{i,A,A}^{-1/2}\right)$ be the $(A{\cdot}T\times1)$ vector of standardized residuals. Specifically, $\eta_{i,a,t} = h_{i,a,t}^{-1/2}\cdot\epsilon_{i,a,t}$ is homoscedastic with unit variance for any aggregation level a. Letting $\eta_{i,j}' = (\eta_i', \eta_j')$, the joint concentrated log-likelihood is:

$$L_c = -\ln|Q_{i,j}| - \eta_{i,j}'Q_{i,j}^{-1}\eta_{i,j} \tag{34}$$

where $Q_{i,j}$ is the conditional variance-covariance matrix:

$$Q_{i,j} = \begin{pmatrix} Q_{i,j,1,1} & Q_{i,j,1,2} \\ Q_{i,j,1,2}' & Q_{i,j,2,2} \end{pmatrix}$$

All elements on the main diagonals of $Q_{i,j,1,1}$ and $Q_{i,j,2,2}$ are equal to unity. The off-diagonal elements contain time-varying correlations between different aggregation schemes that would require modeling. However, these correlations are of no particular interest as they are artificially induced by the aggregation schemes. The main diagonal of $Q_{i,j,1,2}$ contains the contemporaneous conditional correlations on which this part of the analysis focuses while the off-diagonal elements of $Q_{i,j,1,2}$ contain lead and lag correlations induced by the aggregation of the data. Thus, $Q_{i,j}$ is replaced with the *misspecified* matrix $Q_{i,j}^*$ containing only the quantities of interest and ignoring all the artificial features:

$$Q_{i,j}^* = \begin{pmatrix} I & Q_{i,j,1,2}^* \\ Q_{i,j,1,2}^* & I \end{pmatrix}$$

where $Q_{i,j,1,2}^*$ is the diagonal matrix of the conditional correlations $\rho_{i,j,a,t}$ and its inverse is equal to:

$$\left[Q_{i,j}^*\right]^{-1} = \begin{pmatrix} Q_{i,j}^{*1,1} & Q_{i,j}^{*1,2} \\ Q_{i,j}^{*1,2} & Q_{i,j}^{*2,2} \end{pmatrix} = \begin{pmatrix} (I-Q_{i,j,1,2}^2)^{-1} & -Q_{i,j,1,2}(I-Q_{i,j,1,2}^2)^{-1} \\ -Q_{i,j,1,2}(I-Q_{i,j,1,2}^2)^{-1} & (I-Q_{i,j,1,2}^2)^{-1} \end{pmatrix}$$

The concentrated log-likelihood is then equal to:

$$\begin{aligned} L_c &= -\ln|Q_{i,j}^*| - \eta_{i,j}'\left[Q_{i,j}^*\right]^{-1}\eta_{i,j} \\ &= -\ln|I - Q_{i,j,1,2}^*\cdot Q_{i,j,1,2}^*| - \eta_i'Q_{i,j}^{*1,1}\eta_i - 2\eta_i'Q_{i,j}^{*1,2}\eta_j - \eta_j'Q_{i,j}^{*2,2}\eta_j \\ &= -\ln|I - Q_{i,j,1,2}^*\cdot Q_{i,j,1,2}^*| - \sum_{a=1}^{A}\sum_{t=1}^{T}\frac{\left(\eta_{i,a,t}^2 - 2\rho_{i,j,a,t}\eta_{i,a,t}\eta_{j,a,t} + \eta_{j,a,t}^2\right)}{\left(1-\rho_{i,j,a,t}^2\right)} \\ &= -\ln\prod_{a=1}^{A}\prod_{t=1}^{T}\left(1-\rho_{i,j,a,t}^2\right) - \sum_{a=1}^{A}\sum_{t=1}^{T}\frac{\left(\eta_{j,a,t} - \rho_{i,j,a,t}\eta_{i,a,t}\right)^2 + \eta_{i,a,t}^2}{\left(1-\rho_{i,j,a,t}^2\right)} \\ &= -\sum_{a=1}^{A}\sum_{t=1}^{T}\left[\ln\left(1-\rho_{i,j,a,t}^2\right) + \frac{\left(\eta_{j,a,t} - \rho_{i,j,a,t}\eta_{i,a,t}\right)^2}{1-\rho_{i,j,a,t}^2}\right] \end{aligned} \tag{35}$$

The sum of the $\eta^2_{i,a,t}$ terms, on the fourth line, is dropped from the concentrated log-likelihood as it does not depend on $\rho_{i,j,a,t}$.

6.3 Estimation Strategy

For a cross-section of dimensions M, the number of correlations and partial correlations to be estimated is $0.5 \cdot M(M-1)$, corresponding to $13,530$ elements for the 165 assets considered in this study. Given that some of these correlations may be constant, it would be appropriate to treat them as such rather than impose a dynamic specification to all the elements of R_t. The estimation strategy takes this into account pre- and post-estimation. Pre-estimation, a Lagrange Multiplier (LM) test is carried out to test the null hypothesis of constant correlations against the appropriate alternative. In particular, the LM tests are:

$$LM1: \begin{cases} H_0: & \rho_{i,j,a,t} = \omega_{i,j,a} \\ H_1: & \rho_{i,j,a,t} = f(\epsilon_{i,a,t-1} \cdot \epsilon_{j,a,t-1}) \end{cases}$$

$$LM2: \begin{cases} H_0: & \rho_{i,j,a,t} = \omega_{i,j,a} \\ H_1: & \rho_{i,j,a,t} = f(x_{1,a,t-1}) \end{cases}$$

The LM1 test is carried out when the model of choice is C1 while LM2 is carried out when the models of choice are C2 and C3. If, based on the outcome of the test, the null may not be rejected, $\rho_{i,j,a,t}$ is set to a constant while in case of rejection, model estimation is carried out. In the latter case, the estimated model is then compared to a constant correlation model in terms of the Schwartz Information Criteria (SIC) and the best specification is selected.

To select the best specification among C0-C4 both tests have been carried out and the following strategy adopted:

1) Estimate C0 if cannot reject LM1 and LM2
2) Estimate C1 if cannot reject LM2 but reject LM1
3) Estimate C2 and C3 if cannot reject LM1 but reject LM2
4) Estimate C1-C4 if reject LM1 and LM2

In 2), after C1 has been estimated, it is compared against C0 in terms of SIC and the best model selected. In 3), the best SIC specification is selected among C0, C2 and C3. Finally, in 4) all models are estimated and therefore the SIC-best is selected among C0-C4.

Such strategy results in parsimonious specifications that still may explain very well the variations in the conditional correlations. Furthermore, it should be noted that the LM tests operate a preliminary skimming of the models which are ultimately evaluated in terms of the SIC. This is the case due to the tendency of the LM tests to over-reject the null hypothesis (constant correlations) and the fact that, for nested models, the SIC corresponds to a Likelihood Ratio Test with very low significance levels. Therefore, the situation in which SIC would have selected any C1-C4 but LM testing turned in favor of C0 (and therefore inhibited further estimations according to the proposed strategy) is rather unlikely.

7 Results

7.1 Conditional Variances

Models V1-V3 have been fitted to the projection residuals of the 165 asset returns on the interest rate polynomial. For each asset and a given aggregation level, the models were ranked according to the SIC and the percentage of times that each model ranked first have been reported in Table IV. At the daily and weekly level, the GARCH effects dominate and for none of the assets included in the sample the conditional variance can be better explained in terms of interest rate movements. Nevertheless, at the one and two months frequencies, while still dominated by the GARCH effects, the interest effects slowly begin appearing. For the lowest frequencies considered, four and six months, the aggregation reduced the idiosyncratic noise and the associated GARCH effects. At the same time it allowed the relationship between the assets' volatilities and the risk-free rate to emerge in all their strength. What the numbers further suggest is that on average both GARCH and interest rate effects are present but while the first dominate at the high frequencies the second are stronger at the low frequencies mainly due to the idiosyncratic noise reduction resulting from the time aggregation of the returns.

The first three columns of Table IV draw a rough outline of which effect better describes the time varying variances at a given frequency. Introducing model V4 (which nests V1-V3) in the comparison, allows to determine whether the variance component explained by the interest rate is only a low-frequency phenomena or whether it is also relevant at higher frequencies even if weaker than the GARCH component. For daily and weekly data, a model that includes both effects is preferred to the pure GARCH V1 for nearly half and three quarters of the assets respectively. At the intermediate frequencies of one and two months, V4 is the preferred model for more than 90% of the assets. When the frequencies are lowered to four and six months and the GARCH effects mitigate, for 15% to 20% of the assets a purely interest rate based conditional variance model is preferred. Hence, both the GARCH and the risk-free rate induced conditional variance components appear to be present in the 165 asset returns considered.

The sign of the interest rate effects on the conditional variances are summarized in Table V and Table VI for the V2 and V4 models respectively. For the observed range, the functional form linking the conditional variance and the interest rate is approximated by a parabola ($\cup$), an upside down parabola ($\cap$), an increasing convex curve ($\lrcorner$), an increasing concave curve ($\ulcorner$), a decreasing convex curve ($\llcorner$) and a decreasing concave curve ($\urcorner$).

The upside down parabola ($\cap$) is the functional form that overall best approximates the response of the conditional variances to the interest rates. In particular, it is the shape taken by the polynomial in V2 for no less than two thirds of the assets. The polynomial in V4 reproduces such shape for approximately half of the assets at high frequencies and no less than three quarters of the assets at lower frequencies. Hence, for most of the returns considered, the conditional variances are lower, *ceteris paribus*, either when the interest rate is low or when it is high. It should be noted that the vertices of the parabolic approximations are not the same across assets. Therefore, an increase in the level of the interest rate will result in a higher conditional variance for some assets while lower for others.

7.2 Conditional Correlations

Tables VII-IX report the estimation results of the conditional correlation matrices for the 165 assets when the variances are modeled by V1-V3 respectively. Specifically, in Table VII, V1 is modeled for all the conditional variances. In the first panel are the results when the constant conditional correlation model C0 is estimated. In such case there are 13,530 parameters, corresponding to the number of correlations among the the 165 assets, at any frequency. The reported log-likelihood and SIC values are computed on the data standardized by the conditional variances estimated by V1. Panels 2-4 report the results for the conditional correlations modeled by C1-C3 respectively. According to the chosen information criteria, the purely dynamic model C1 is to be preferred at high frequencies (1-5 days), C2 at low frequencies (80-120 days) and C3 for medium frequencies (20-40 days). In Table VIII, the conditional variances are modeled by V2 and the resulting conditional correlations are best describer by C2 at low frequencies (40-120 days) and C3 at high frequencies (1-20 days). When the variances are estimated by V3, as in Table IX, C2 is generally the preferred model at low frequencies (1, 80 and 120 days) and C3 is preferred at high frequencies (5-40 days). Hence, the correlations' results are quite robust to the choice of the model for the conditional variances V1-V3 and point to the fact that the second degree polynomial of C2 is generally a better description of the time-varying correlations at low frequencies while the memory parameter of C3 helps to better fit the data at high frequencies. From the reported results, it is also clear that regardless of the choice of the model for the conditional variances, at low frequencies the interest rate is better at describing the evolution of the correlations than the purely dynamic GARCH-like model C1. Thus, Tables VII-IX clearly indicate that interest rate effects are present in the conditional correlations and that, at low frequencies, they provide a better description of the time-varying correlations than the dynamic model V1.

Table X reports the results of the model obtained selecting the best variance specification for each asset among V0-V4 and the best specification for each pair of conditional correlations and partial correlations among C0-C4, in terms of the SIC. The so obtained model is preferred to the constant correlation model C0 at any of the considered frequencies. It should be noted that, consistently with the findings in Tables VII-IX, the overall number of parameters of the preferred model increases as the frequency of the data decreases. Given that for each of the estimated models C0-C4 the number of parameters is fixed, it follows that at lower frequencies there are less correlations and partial correlations that are best described by the constant correlation specification C0. Hence, there are dynamic and interest rate effects in the correlations that may be detected at low but not at high frequencies.

For the specification determined by the SIC, the distributions of the selected conditional variance and correlation models are reported in Table IV and Table XI respectively. Column C0 confirms that the number of correlations that are best modeled as constant decreases as the frequency is also decreased. In particular, it is somewhat striking that while at the daily frequency more than 95% of the correlations and partial correlations are constant, at the 120-days frequency only 33% of them are constant. The relative distribution of the selected models that allow for time-varying correlations is reported in the lower panel of Table XI. At the daily frequency, more than half of the elements of the conditional correlation matrix are best modeled by a purely dynamic model and less than 1% of such elements exhibit both dynamic and interest rate effects as seen from column C4. At the weekly frequency, the dynamic specification C1 is preferred in roughly 25% of the cases while the most general specification C4 is preferred in no more than 2% of the cases. As the frequencies are further lowered, the number of elements best described by C1 stabilizes around 17%. Conversely, C4 is increasingly selected as the best model at lower frequencies. At 80 and 120 days more elements of the conditional correlation matrix exhibit both dynamic and interest rate effects than just dynamic effects. The purely exogenous models C2 and C3, in which the interest rate is the sole determinant of the level of the correlations, are each selected as the best specifications in roughly 30% of the cases. On the other hand, C2 exhibits a peak of preferences in the 5-20 days range, while C1 exhibits a significant low of 15.64% at the daily frequency. Relatively to the time varying components, the specifications C2-C4 which include interest rate effects are preferred to C1 in terms of the SIC in more than 83% of the cases at frequemcies of 20-120 days. At the higher frequencies of 1 and 5 days, C2-C4 are selected in 46.21% and 75.94% of the cases, respectively.

8 Conclusions

This paper extrapolates, within the C-CAPM framework, a theoretical connection between the risk-free rate and the first two conditional moments of the stochastic discount factor and through these, a connection with the conditional correlations among assets. Under the stated assumption, the model does not provide specific predictions about the reciprocal movements of the interest rate and the variances and correlations but it does identify the conditions under which these quantities will move in the same or in the opposite direction. A straightforward extension to the bivariate Autoregressive Conditional Correlations model is introduced. This allows for the inclusion of exogenous covariates in a way that is consistent with the natural bounds of the correlations. Furthermore, in association with the Sequential Conditional Correlations methodology, it will easily parameterize a positive definite conditional correlation matrix as a function of exogenous variables. The empirical analysis, conducted by modeling the 13539 correlations in the data set, confirms that the interest rate is a determinant of the conditional second moments. Empirically, the risk-free rate effects are found to be present at all frequencies and dominant at the low ones.

The inclusion of additional macroeconomic explanatory variables and further evaluations of the explanatory power of the interest rate are left as an area for future research.

A Appendix

Theorem A.1. *Let X be a continuous random variable and f, g be two non-constant, monotonic non-decreasing functions of X. Then:*

$$\mathbb{COV}\left[f(X), g(X)\right] > 0 \tag{36}$$

Proof of Theorem A.1: Define $\overset{\circ}{f} = f - \mathbb{E}f$ and $\overset{\circ}{g} = g - \mathbb{E}g$. Since f, g are increasing in X, $\exists!$ x_1 and x_2 such that $\overset{\circ}{f}(x_1) = 0$ and $\overset{\circ}{g}(x_2) = 0$. Without loss of generality, assume $x_1 \leq x_2$ and let $A = (-\infty, x_1]$, $B = (x_1, x_2]$ and $C = (x_2, +\infty)$. Then:

$$\begin{aligned}
\overset{\circ}{f}(x) \leq 0 \quad &; \quad \forall x \in A \\
\overset{\circ}{f}(x) > 0 \quad &; \quad \forall x \in B \cup C \\
\overset{\circ}{g}(x) \leq 0 \quad &; \quad \forall x \in A \cup B \\
\overset{\circ}{g}(x) > 0 \quad &; \quad \forall x \in C
\end{aligned}$$

The covariance between f, g may be rewritten as the expectation over disjoint sets:

$$\begin{aligned}
\mathbb{COV}\left[f(X), g(X)\right] &= \mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \mathbb{E}_B\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \mathbb{E}_C\left[\overset{\circ}{f}\overset{\circ}{g}\right] \\
&> \mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \overset{\circ}{f}(x_2)\mathbb{E}_B\left[\overset{\circ}{g}\right] + \mathbb{E}_C\left[\overset{\circ}{f}\overset{\circ}{g}\right] \\
&> \mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \overset{\circ}{f}(x_2)\mathbb{E}_B\left[\overset{\circ}{g}\right] + \overset{\circ}{f}(x_2)\mathbb{E}_C\left[\overset{\circ}{g}\right] \\
&> \mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \overset{\circ}{f}(x_2)\mathbb{E}_{B\cup C}\left[\overset{\circ}{g}\right] \\
&> \mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] + \overset{\circ}{f}(x_2)\left\{0 - \mathbb{E}_A\left[\overset{\circ}{g}\right]\right\} \\
&> 0
\end{aligned} \tag{37}$$

The last step is obtained by noticing that: in the set A, the random variable $\overset{\circ}{f}\overset{\circ}{g}$ is non-negative and therefore $\mathbb{E}_A\left[\overset{\circ}{f}\overset{\circ}{g}\right] > 0$; in the set A, the random variable $\overset{\circ}{g}$ is non-positive and therefore $-\mathbb{E}_A\left[\overset{\circ}{g}\right] > 0$.

B Appendix

B.1 Similarities between ACC and GARCH

Consider the ACC(1,1) model applied to the modeling of volatilities rather than correlations. The time t conditional variance h_t is then given by:

$$h_t = \omega + \delta h_{t-1} + \theta \psi_{t-1} \tag{38}$$

In this setting, the Fisher transformation of ACC may be simply replaced with the identity function: $\psi_{t-1} = \phi_{t-1}$. Furthermore, ϕ_{t-1} may be set equal to q_{t-1}, where the latter is the realization of the variance estimated by the exponential smoothing:

$$\psi_{t-1} = \alpha \psi_{t-2} + (1-\alpha)\epsilon_{t-1}^2 \tag{39}$$

Substituting equation (39) in (38) yields:

$$h_t = (1-\alpha)\omega + (\alpha+\delta)h_{t-1} - \alpha\delta h_{t-2} + (1-\alpha)\theta\epsilon_{t-1}^2 \tag{40}$$

Hence, the ACC(1,1) model is the correlations' analogue of the GARCH(1,2). In turn, the ACC(0,1) specification at C1, with $\delta = 0$, is the analogue of the GARCH(1,1).

B.2 Ergodicity of ACC

Consider the exponential smoothing with parameter $\alpha \in (0,1)$ associated to the ACC(1,1) specification:

$$q_t = \alpha \cdot q_{t-1} + (1-\alpha) \cdot \begin{pmatrix} u_{i,t}^2 \\ u_{i,t}u_{j,t} \\ u_{j,t}^2 \end{pmatrix} \tag{41}$$

where $q_t \equiv vech(Q_t)$ and $u_{j,t} = \rho_t u_{i,t} + (1-\rho_t^2)^{1/2}\epsilon_{j,t}$ with $\mathbb{E}_{t-1}(u_{i,t}^2) = 1$, $\mathbb{E}_{t-1}(\epsilon_{j,t}^2) = 1$ and $\mathbb{E}_{t-1}(u_{i,t}\epsilon_{j,t}) = 0$. Then:

$$q_t = \alpha \cdot q_{t-1} + (1-\alpha) \cdot \begin{pmatrix} 1 \\ \rho_t \\ 1 \end{pmatrix} + (1-\alpha) \cdot \begin{pmatrix} u_{i,t}^2 - 1 \\ u_{i,t}u_{j,t} - \rho_t \\ u_{j,t}^2 - 1 \end{pmatrix} \tag{42}$$

and therefore:

$$q_t = \alpha \cdot q_{t-1} + (1-\alpha) \cdot \begin{pmatrix} 1 \\ \rho_t \\ 1 \end{pmatrix} + Z_t \tag{43}$$

with $\mathbb{E}_{t-1}(Z_t) = 0$. Letting $IF(\bullet)$ denote the inverse Fisher transformation:

$$q_t = \alpha \cdot q_{t-1} + (1-\alpha) \cdot \begin{pmatrix} 1 \\ IF(\chi_t) \\ 1 \end{pmatrix} + Z_t \tag{44}$$

In the ACC(1,1):

$$\begin{aligned} \chi_t &= \omega + \delta\chi_{t-1} + \theta\psi_{t-1} \\ &= \omega + \delta\chi_{t-1} + \theta F(\phi_{t-1}) \\ &= \omega + \delta\chi_{t-1} + \theta F(C(q_{t-1})) \end{aligned} \tag{45}$$

where $F(\bullet)$ is the Fisher transformation and $C(\bullet)$ is the function that computes the correlation ϕ_{t-1} from the vector of variances and covariances q_{t-1}. Hence:

$$q_t = \alpha \cdot q_{t-1} + (1-\alpha) \cdot \begin{pmatrix} 1 \\ IF[\omega + \delta\chi_{t-1} + \theta F(C(q_{t-1}))] \\ 1 \end{pmatrix} + Z_t \tag{46}$$

From this and equation (45) it is possible to obtain the following nonlinear Markovian representation:

$$\begin{pmatrix} q_t \\ \chi_t \end{pmatrix} = f\begin{pmatrix} q_{t-1} \\ \chi_{t-1} \end{pmatrix} + \begin{pmatrix} Z_t \\ 0 \end{pmatrix} \tag{47}$$

The skeleton $f(\bullet)$ of the Markov chain has a unique fixed point.

Proof. Let q^*, ρ^*, ϕ^*, ψ^* and χ^* denote the fixed points of q_t, ρ_t, ϕ_t, ψ_t and χ_t respectively. From the exponential smoothing equation, it follows that $q_1^* = 1$, $q_2^* = \rho^*$ and $q_3^* = 1$. Then: $\phi^* = \rho^*$, $\psi^* = F(\phi^*) = F(\rho^*) = \chi^*$. Substituted in the ACC(1,1) equation it yields:

$$\begin{aligned} \chi^* &= \omega + \delta\chi^* + \theta\chi^* \\ &= \omega(1-\delta-\theta)^{-1} \end{aligned} \tag{48}$$

□

The mapping of the skeleton has the whole Euclidean space as domain of attraction.

Proof. Let the index n denote the mapping of the skeleton and consider, once again, the exponential smoothing equation:

$$\begin{aligned} q_{2,n} &= \alpha q_{2,n-1} + (1-\alpha)\rho_n \\ \psi_n &= F\left[\alpha q_{2,n-1} + (1-\alpha)\rho_n\right] \\ &= F\left[\alpha IF(\psi_{n-1}) + (1-\alpha) IF(\chi_n)\right] \end{aligned} \tag{49}$$

The Fisher transformation is an odd function, convex (concave) for positive (negative) arguments:

$$\begin{aligned} \psi_n &\leq (>) \quad \alpha F\left[IF(\psi_{n-1})\right] + (1-\alpha) F\left[IF(\chi_n)\right] \\ &\leq (>) \quad \alpha \psi_{n-1} + (1-\alpha)\chi_n \end{aligned} \tag{50}$$

It follows that when ψ_t is positive (negative) the skeleton is bounded above (below) by the following mapping:

$$\psi_n = \alpha \psi_{n-1} + (1-\alpha)\chi_n \tag{51}$$

Thus, the nonlinear mapping may be replaced by a linear mapping that bounds it. Using the lag operators, the χ and ψ components may be rewritten as:

$$(1-\delta L)\chi_n = \omega + \theta \psi_{n-1} \tag{52}$$

$$(1-\alpha L)\psi_{n-1} = (1-\alpha)\chi_{n-1} \tag{53}$$

from which it follows that:

$$\chi_n = (1-\alpha)\omega + \left[\alpha + \delta + (1-\alpha)\theta\right]\chi_{n-1} - \alpha\delta\chi_{n-2} \tag{54}$$

This is the skeleton of an AR(2). Under the following conditions, its domain of attraction is the whole Euclidean:

$$\begin{aligned} \alpha\delta &< 1 \\ \delta + \theta &< 1 \\ \delta + \left(\frac{1-\alpha}{1+\alpha}\right)\theta &> -1 \end{aligned} \tag{55}$$

□

The ACC(1,1) process is uniformly ergodic.

Proof. Being bounded by an AR(2), the ACC(1,1) process automatically satisfies the absorbing requirements of assumptions (A1)-(A5) of **Theorem 3.3.2** of Chan and Tong (2001) for uniform ergodicity. □

Conditions for uniform ergodicity may be easily generalized to the ACC(p,q) model even though they will not be available in an analytical form. In particular, the mapping of the skeleton of an ACC(p,q) is bounded by:

$$\chi_n = (1-\alpha)\omega + \sum_{i=1}^{m} \phi_i \chi_{n-i} \tag{56}$$

with $m = \max(p+1, q)$ and

$$\phi_i = \delta_i - \alpha\delta_{i-1} + (1-\alpha)\theta_i \tag{57}$$

where $\delta_0 = 0$, $\delta_i = 0\ \forall i > p$ and $\theta_i = 0\ \forall i > q$. Therefore, the ACC(p,q) is uniformly ergodic when its parameters satisfy the analogous conditions of the associated AR(m).

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Table I: Signs of the Derivatives

	(i)	(i)+(ii)	(i)+(iia)	(i)+(iib)
$\frac{d\nu_t}{d\mu_t}$	< 0			
$\frac{d\nu_t}{d\sigma_t}$			> 0	< 0
$\frac{ds_t^2}{d\mu_t}$			< 0	> 0
$\frac{ds_t^2}{d\sigma_t}$		> 0		

Table II: Polynomial Regressions Summary Statistics.

	Degree				R-square			
Freq.	min	mean	median	max	min	mean	median	max
1	0	0.01	0	1	0	0.00	0.00	0.00
5	0	0.24	0	2	0	0.00	0.00	0.00
20	0	0.97	0	8	0	0.00	0.00	0.03
40	0	1.52	1	9	0	0.01	0.00	0.04
80	0	2.42	2	16	0	0.02	0.01	0.21
120	0	3.30	3	18	0	0.03	0.02	0.29

Table III: Symbols of Included Stocks

AA	ABI	ABT	ADM	AEP	AME	AP	APD	AVA	AVT	AYE
BA	BAX	BC	BCO	BGG	BMY	BWS	CAT	CBE	CCK	CEG
CHG	CL	CLF	CMS	CNP	COP	CPB	CQB	CR	CSK	CSL
CTB	CVS	CVX	CW	DD	DE	DIS	DOV	DOW	DPL	DTE
DUK	EAS	ED	EDE	EGN	EIX	EK	EMR	EQT	ETN	ETR
F	FE	FL	FMC	FOE	FPL	GAM	GAP	GCO	GD	GE
GIS	GLW	GM	GMT	GNI	GR	GT	GXP	GY	HAL	HMX
HNZ	HP	HPQ	HRS	HSC	HSY	IBM	IDA	ILA	IP	IR
IRF	JCP	JNJ	K	KO	KR	LG	LMT	LTR	MDU	MEE
MHP	MMM	MO	MOT	MRK	MSB	MUR	NC	NEM	NFG	NL
NOC	OGE	OKE	OLN	PBI	PCG	PE	PEG	PEO	PEP	PFE
PG	PGN	PNW	PPG	PPL	PSD	PVH	R	ROH	ROK	RSH
RTN	SCG	SCX	SJI	SLE	SO	SP	STR	SUN	TKR	TPL
TR	TXN	TXT	TY	UEP	UGI	UIS	USG	UTX	UVV	VAR
WAG	WEC	WGL	WHR	WR	WWY	WYE	XEL	XOM	XRX	Y

Table IV: Models of the Conditional Variance.

Schwartz Information Criterion preferred model. In columns V1-V3 the corresponding models are SIC-compared and the relative percentages reported. Columns V1-V4 report SIC-comparison percentages when the set of competing models is extended to the nesting specification V4.

Freq.	V1	V2	V3	V1	V2	V3	V4
1	100.00%	0.00%	0.00%	54.55%	0.00%	0.00%	45.45%
5	100.00%	0.00%	0.00%	26.67%	0.00%	0.00%	73.33%
20	94.54%	3.64%	1.82%	9.70%	0.00%	0.00%	90.30%
40	77.58%	16.36%	6.06%	2.43%	4.24%	0.00%	93.33%
80	44.85%	36.97%	18.18%	1.21%	13.94%	1.21%	83.64%
120	27.88%	46.67%	25.45%	0.61%	20.00%	2.42%	76.97%

Table V: Interest Rate Effects in V2.

Summary statistics of the functional form linking interest rates and conditional variances in V2. Legend: parabola (∪), upside down parabola (∩), increasing convex curve (⌟), increasing concave curve (⌜), decreasing convex curve (⌞) and decreasing concave curve (⌝). Last column reports the number of times V2 is the Schwartz Information Criteria preferred model in the comparison V1-V4.

Freq.	∪	∩	⌟	⌜	⌞	⌝	Total
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0
5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0
20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0
40	28.57%	71.43%	0.00%	0.00%	0.00%	0.00%	7
80	8.70%	69.57%	0.00%	21.73%	0.00%	0.00%	23
120	12.12%	78.79%	3.03%	6.06%	0.00%	0.00%	33

Table VI: Interest Rate Effects in V4.

Summary statistics of the functional form linking interest rates and conditional variances in V4. Legend: parabola (∪), upside down parabola (∩), increasing convex curve (⌟), increasing concave curve (⌜), decreasing convex curve (⌞) and decreasing concave curve (⌝). Last column reports the number of times V4 is the Schwartz Information Criteria preferred model in the comparison V1-V4.

Freq.	∪	∩	⌟	⌜	⌞	⌝	Total
1	21.33%	48.00%	16.00%	14.67%	0.00%	0.00%	75
5	21.49%	57.02%	9.92%	11.57%	0.00%	0.00%	121
20	6.04%	79.87%	4.03%	6.71%	2.01%	1.34%	149
40	5.19%	85.07%	2.60%	5.19%	1.95%	0.00%	154
80	5.80%	88.41%	2.17%	0.72%	1.45%	1.45%	138
120	8.66%	80.32%	1.57%	4.72%	3.94%	0.79%	127

Table VII: Models of the Conditional Correlations

Panels C0-C3 report the number of parameters, log-likelihood and SIC values for the corresponding specifications of the whole conditional correlation matrix when all the conditional variances are filtered using specification V1.

	Correlations: C0			Correlations: C1		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	13,530	-717,375	1,561,050	14,002	-709,720	1,550,150
5	13,530	-574,216	1,274,720	14,540	-565,270	1,266,260
20	13,530	-361,317	848,987	16,368	-343,848	840,422
40	13,530	-195,129	516,456	16,784	-172,894	502,335
80	13,530	20,380	85,292	17,230	36,522	87,480
120	13,530	212,957	-299,960	17,490	216,039	-269,252

	Correlations: C2			Correlations: C3		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	14,046	-713,428	1,557,970	14,020	-714,046	1,558,970
5	15,622	-561,507	1,268,830	15,660	-560,781	1,267,730
20	21,724	-286,726	776,154	22,408	-282,330	773,745
40	25,096	-59,987	354,050	25,802	-54,449	349,559
80	27,430	210,482	-165,412	27,830	260,590	-153,902
120	28,210	415,177	-567,738	28,626	393,193	-519,898

Table VIII: Models of the Conditional Correlations

Panels C0-C3 report the number of parameters, log-likelihood and SIC values for the corresponding specifications of the whole conditional correlation matrix when all the conditional variances are filtered using specification V2.

	Correlations: C0			Correlations: C1		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	13,530	-690,873	1,508,050	13,940	-687,104	1,504,340
5	13,530	-559,592	1,245,470	14,736	-551,778	1,241,100
20	13,530	-353,250	832,745	16,430	-332,470	818,245
40	13,530	-187,707	501,611	16,964	-163,768	485,763
80	13,530	20,673	84,707	17,438	36,417	89,626
120	13,530	205,703	-285,451	17,698	207,453	-250,150

	Correlations: C2			Correlations: C3		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	14,312	-684,692	1,502,980	14,264	-684,899	<u>1,502,950</u>
5	16,248	-542,060	1,235,700	16,194	-539,873	<u>1,230,900</u>
20	22,232	-270,711	748,863	22,562	-267,486	<u>745,493</u>
40	25,300	-46,213	<u>328,406</u>	25,936	-43,864	329,693
80	27,208	218,968	<u>-184,453</u>	27,802	208,927	-158,836
120	28,116	423,935	<u>-586,130</u>	28,264	387,125	-511,133

Table IX: Models of the Conditional Correlations

Panels C0-C3 report the number of parameters, log-likelihood and SIC values for the corresponding specifications of the whole conditional correlation matrix when all the conditional variances are filtered using specification V3.

	Correlations: C0			Correlations: C1		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	13,530	-691,499	1,509,300	13,954	-688,028	1,506,310
5	13,530	-560,177	1,246,640	14,702	-552,499	1,242,220
20	13,530	-352,545	831,336	16,456	-332,324	818,195
40	13,530	-183,808	493,813	17,026	-159,991	478,788
80	13,530	33,948	58,156	17,528	52,147	59,005
120	13,530	227,194	-328,434	17,740	229,517	-293,887

	Correlations: C2			Correlations: C3		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	14,286	-685,520	1,504,400	14,248	-686,030	1,505,060
5	16,332	-541,600	1,235,640	16,212	-540,186	1,231,690
20	22,246	-271,390	750,352	22,594	-268,624	748,068
40	25,188	-46,057	327,048	25,730	-41,133	322,256
80	27,422	225,525	-195,572	27,876	224,679	-189,652
120	28,170	430,025	-597,808	28,696	415,328	-563,516

Table X: Models of the Conditional Correlations

For each asset the best SIC variance specification is selected among V0-V4. Left panel reports results on the whole conditional correlation matrix when its elements are set to constant. In the right panel, for each element of the conditional correlation matrix the best SIC specification is selected among C0-C4.

	Correlations: C0			Correlations: SIC best among C0-C4		
Freq.	Parameters	Log-Lik	SIC	Parameters	Log-Lik	SIC
1	13,530	-761,423	1,559,150	14,377	-706,583	1,547,370
5	13,530	-574,129	1,274,550	17,070	-550,275	1,259,880
20	13,530	-365,413	857,072	27,181	-244,842	743,305
40	13,530	-210,527	529,251	32,419	-1,873	306,126
80	13,530	6,027	113,997	36,261	260,981	-184,137
120	13,530	187,091	-248,228	37,048	424,251	-503,611

Table XI: Models of the Conditional Correlations
Schwartz Information Criterion preferred model. Top Panel: in columns C0-C4 the corresponding models are SIC-compared and the relative percentages reported. Bottom Panel: columns C1-C4 report SIC-comparison percentages relative to the time-varying elements of the correlation matrix.

Freq.	C0	C1	C2	C3	C4
1	96.87%	1.68%	0.95%	0.49%	0.01%
5	87.25%	3.07%	5.26%	4.20%	0.22%
20	54.71%	7.58%	17.74%	16.53%	3.44%
40	42.14%	9.56%	20.01%	20.33%	7.96%
80	34.62%	10.67%	20.90%	21.40%	12.41%
120	33.07%	11.11%	21.38%	21.12%	13.32%
1	-	53.79%	30.33%	15.64%	0.24%
5	-	24.06%	41.28%	32.93%	1.74%
20	-	16.74%	39.16%	36.50%	7.60%
40	-	16.53%	34.58%	35.14%	13.75%
80	-	16.32%	31.97%	32.73%	18.98%
120	-	16.60%	31.95%	31.56%	19.89%

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