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Abstract

We study the skewness premium (SK) introduced by Bates (1991) in a general context using Lévy Processes. Under a symmetry condition Fajardo and Mordecki (2006) have obtained that SK is given by the Bate's $x\%$ rule. In this paper, we study SK under the absence of that symmetry condition. More exactly, we derive sufficient conditions for the excess of SK to be positive or negative, in terms of the characteristic triplet of the Lévy Process under the risk neutral measure.

Keywords: Skewnes Premium; Lévy Processes.

JEL Classification: C52; G10

1 Introduction

The stylized facts of option prices have been studied by many authors in the literature. An important fact from option prices is that relative prices of out-of-the-money calls and puts can be used as a measure of symmetry or skewness of the risk neutral distribution. Bates (1991), called this diagnosis “skewness premium”, henceforth SK. He analyzed the behavior of SK using

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three classes of stochastic processes: Constant Elasticity of Variance (CEV), Stochastic Volatility and Jump-diffusion. He found conditions for the SK be positive or negative.

But, as many models in the literature have shown, the behavior of the assets underlying options is very complex, the structure of jumps observed is more complex than Poisson jumps. They have higher intensity, see for example Aït-Sahalia (2004). For that reason diffusion models cannot consider the discontinuous sudden movements observed on asset prices. In that sense, the use of more general process as Lévy processes have shown to provide a better fit with real data, as was reported in Carr and Wu (2004) and Eberlein, Keller, and Prause (1998). On the other hand, the mathematical tools behind these processes are very well established and known.

When the underlying follows a Geometric Lévy Process, Fajardo and Mordecki (2006) obtained a relationship between calls and puts, that they called *Put-Call duality* and obtain as a particular case the *Put-call symmetry*, and obtained, under a symmetry condition, that SK is given by the Bate's $x\%$ rule. Similar symmetry conditions are used in the literature to obtain important applications, as for example the construction of semi-static hedges for exotic options, as Carr, Ellis, and Gupta (1998) and Carr and Lee (2008) have shown.

Also, Bates (1997) verified that in many cases we have not empirical evidence of Put-Call symmetry, by constructing an hypothesis test that compare the observed relative prices and the theoretical ones, given by the Bates's rule. The absence of symmetry, have also been reported by Carr and Wu (2007), they found asymmetric implied volatility smiles in currency options, and by a result due to Fajardo and Mordecki (2006) and Carr and Lee (2008), we know that symmetric markets must present symmetric implied volatilities.

In this paper, we study the SK under absence of symmetry and obtain sufficient conditions for the excess of SK be positive or negative. The main idea behind the proofs is to exploit the monotonicity property of option prices with respect to some parameter of the Lévy measure. This monotonicity is not an easy task, monotonicity with respect to the intensity parameter of the jump have been recently address by Ekström and Tysk (2007), while the monotonicity with respect to the symmetry parameter have not been totally

addressed in previous works. A particular answer is given for the case of GH distributions in Bergenthum and Rüschendorf (2007).

The paper is organized as follows: in Section 2 we introduce the Lévy processes and we present the duality results. In Section 3 we discuss market symmetry and present our main results. In Section 4 we study the skewness premium. Section 5 discuss monotonicity with respect to the symmetry parameter and Section 6 concludes.

2 Lévy processes and Duality

Consider a real valued stochastic process $X = \{X_t\}_{t \geq 0}$, defined on a stochastic basis $\mathcal{B} = (\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t \geq 0}, \mathbb{Q})$, being càdlàg, adapted, satisfying $X_0 = 0$, and such that for $0 \leq s < t$ the random variable $X_t - X_s$ is independent of the σ -field $\mathcal{F}_s$, with a distribution that only depends on the difference $t - s$. Assume also that the stochastic basis $\mathcal{B}$ satisfies the usual conditions (see Jacod and Shiryaev (2002)). The process X is a Lévy process, and is also called a process with stationary independent increments (PIIS). For general reference on Lévy processes see Jacod and Shiryaev (2002), Skorokhod (1991), Bertoin (1996), Sato (1999). For Lévy process in Finance see Boyarchenko and Levendorskii (2002), Schoutens (2003) and Cont and Tankov (2004).

In order to characterize the law of X under $\mathbb{Q}$, consider, for $q \in \mathbb{R}$ the Lévy-Khinchine formula, that states

$$\mathbf{E} e^{iqX_t} = \exp \left\{ t \left[iaq - \frac{1}{2} \sigma^2 q^2 + \int_{\mathbb{R}} (e^{iqy} - 1 - iqh(y)) \Pi(dy) \right] \right\}, \quad (1)$$

with

$$h(y) = y \mathbf{1}_{\{|y| < 1\}}$$

a fixed truncation function, a and $\sigma \geq 0$ real constants, and Π a positive measure on $\mathbb{R} \setminus \{0\}$ such that $\int (1 \wedge y^2) \Pi(dy) < +\infty$, called the *Lévy measure*. The triplet (a, σ^2, Π) is the *characteristic triplet* of the process, and completely determines its law.

Consider the set

$$\mathbb{C}_0 = \left\{ z = p + iq \in \mathbb{C} : \int_{\{|y|>1\}} e^{py} \Pi(dy) < \infty \right\}. \quad (2)$$

The set $\mathbb{C}_0$ is a vertical strip in the complex plane, contains the line $z = iq$ ($q \in \mathbb{R}$), and consists of all complex numbers $z = p + iq$ such that $\mathbf{E} e^{pX_t} < \infty$ for some $t > 0$. Furthermore, if $z \in \mathbb{C}_0$, we can define the *characteristic exponent* of the process X , by

$$\psi(z) = az + \frac{1}{2} \sigma^2 z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zh(y)) \Pi(dy) \quad (3)$$

this function ψ is also called the *cumulant* of X , having $\mathbf{E} |e^{zX_t}| < \infty$ for all $t \geq 0$, and $\mathbf{E} e^{zX_t} = e^{t\psi(z)}$. The finiteness of this expectations follows from Theorem 21.3 in Sato (1999). Formula (3) reduces to formula (1) when $\text{Re}(z) = 0$.

2.1 Lévy market

By a *Lévy market* we mean a model of a financial market with two assets: a deterministic savings account $B = \{B_t\}_{t \geq 0}$, with

$$B_t = e^{rt}, \quad r \geq 0,$$

where we take $B_0 = 1$ for simplicity, and a stock $S = \{S_t\}_{t \geq 0}$, with random evolution modelled by

$$S_t = S_0 e^{X_t}, \quad S_0 = e^x > 0, \quad (4)$$

where $X = \{X_t\}_{t \geq 0}$ is a Lévy process.

In this model we assume that the stock pays dividends, with constant rate $\delta \geq 0$, and that the given probability measure $\mathbb{Q}$ is the chosen equivalent martingale measure. In other words, prices are computed as expectations with respect to $\mathbb{Q}$, and the discounted and reinvested process $\{e^{-(r-\delta)t} S_t\}$ is a $\mathbb{Q}$ -martingale.

In terms of the characteristic exponent of the process this means that

$$\psi(1) = r - \delta, \quad (5)$$

based on the fact, that $\mathbf{E} e^{-(r-\delta)t+X_t} = e^{-t(r-\delta+\psi(1))} = 1$, and condition (5) can also be formulated in terms of the characteristic triplet of the process X as

$$a = r - \delta - \sigma^2/2 - \int_{\mathbb{R}} (e^y - 1 - h(y)) \Pi(dy). \quad (6)$$

In the market model considered, we introduce some derivative assets. More precisely, we consider call and put options, of both European and American types. Denote by $\mathcal{M}_T$ the class of stopping times up to a fixed constant time T , i.e:

$$\mathcal{M}_T = \{\tau : 0 \leq \tau \leq T, \tau \text{ stopping time w.r.t } \mathbf{F}\}.$$

Then, for each stopping time $\tau \in \mathcal{M}_T$ we introduce

$$c(S_0, K, r, \delta, \tau, \psi) = \mathbf{E} e^{-r\tau} (S_\tau - K)^+, \quad (7)$$

$$p(S_0, K, r, \delta, \tau, \psi) = \mathbf{E} e^{-r\tau} (K - S_\tau)^+. \quad (8)$$

In our analysis (7) and (8) are auxiliary quantities, anyhow, they are interesting by themselves as random maturity options, as considered, for instance, in Schroder (1999) and Detemple (2001). If $\tau = T$, formulas (7) and (8) give the price of the European call and put options respectively.

2.2 Put Call duality and dual markets

Lemma 2.1 (Fajardo and Mordecki (2006)). *Consider a Lévy market with driving process X with characteristic exponent $\psi(z)$, defined in (3), on the set $\mathbb{C}_0$ in (2). Then, for the expectations introduced in (7) and (8) we have*

$$c(S_0, K, r, \delta, \tau, \psi) = p(K, S_0, \delta, r, \tau, \tilde{\psi}), \quad (9)$$

where

$$\tilde{\psi}(z) = \tilde{a}z + \frac{1}{2}\tilde{\sigma}^2 z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zh(y)) \tilde{\Pi}(dy) \quad (10)$$

is the characteristic exponent (of a certain Lévy process) that satisfies

$$\tilde{\psi}(z) = \psi(1 - z) - \psi(1), \quad \text{for } 1 - z \in \mathbb{C}_0,$$

and in consequence,

$$\begin{cases} \tilde{a} &= \delta - r - \sigma^2/2 - \int_{\mathbb{R}} (e^y - 1 - h(y)) \tilde{\Pi}(dy), \\ \tilde{\sigma} &= \sigma, \\ \tilde{\Pi}(dy) &= e^{-y} \Pi(-dy). \end{cases} \quad (11)$$

Now given a Lévy market with driving process characterized by ψ in (3), consider a market model with two assets, a deterministic savings account $\tilde{B} = \{\tilde{B}_t\}_{t \geq 0}$, given by

$$\tilde{B}_t = e^{\delta t}, \quad \delta \geq 0,$$

and a stock $\tilde{S} = \{\tilde{S}_t\}_{t \geq 0}$, modelled by

$$\tilde{S}_t = K e^{\tilde{X}_t}, \quad \tilde{S}_0 = K > 0,$$

where $\tilde{X} = \{\tilde{X}_t\}_{t \geq 0}$ is a Lévy process with characteristic exponent under $\tilde{\mathbb{Q}}$ given by $\tilde{\psi}$ in (10). The process $\tilde{S}_t$ represents the price of $K S_0$ dollars measured in units of stock S .

3 Market Symmetry

Here we use the symmetry concept introduced in Fajardo and Mordecki (2006). We define a Lévy market to be *symmetric* when the following relation holds:

$$\mathcal{L}(e^{-(r-\delta)t+X_t} \mid \mathbb{Q}) = \mathcal{L}(e^{-(\delta-r)t-X_t} \mid \tilde{\mathbb{Q}}), \quad (12)$$

meaning equality in law. Otherwise we call the Lévy market *Asymmetric*. As Fajardo and Mordecki (2006) pointed out, a necessary and sufficient condition for (12) to hold is

$$\Pi(dy) = e^{-y} \Pi(-dy). \quad (13)$$

This ensures $\tilde{\Pi} = \Pi$, and from this follows $a - (r - \delta) = \tilde{a} - (\delta - r)$, giving (12), as we always have $\tilde{\sigma} = \sigma$.

3.1 More Evidence of Absence of Symmetry

In Fajardo and Mordecki (2006) several concrete models proposed in the literature are reviewed. More exactly, Lévy markets with jump measure of the form

$$\Pi(dy) = e^{\beta y} \Pi_0(dy), \quad (14)$$

where $\Pi_0(dy)$ is a symmetric measure, i.e. $\Pi_0(dy) = \Pi_0(-dy)$, everything with respect to the risk neutral measure $\mathbb{Q}$.

As a consequence of (13), Fajardo and Mordecki (2006) found that the market is symmetric if and only if $\beta = -1/2$. Then, as we have seen when the market is symmetric, the skewness premium is obtained using the $x\%$ -rule.

Although from the theoretical point of view the assumption (14) is a real restriction, most models in practice share this property, and furthermore, they have a jump measure that has a Radon-Nikodym density. In this case, we have

$$\Pi(dy) = e^{\beta y} p(y) dy, \quad (15)$$

where $p(y) = p(-y)$, i.e. the function $p(y)$ is even. More precisely, all parametric models that we found in the literature, in what concerns Lévy markets, including diffusions with jumps, can be reparametrized in the form (15): The Generalized Hyperbolic model proposed by Eberlein and Prause (2000), The Meixner model proposed by Schoutens (2001) and The CGMY model proposed by Carr, Geman, Madan, and Yor (2002). Recently, Fajardo and Mordecki (2008) shows that under some conditions the Time Changed Brownian motion with drift is also included in this class. Then, they show that the resulting processes will satisfy the above symmetry if and only if the drift equal $-1/2$.

Using the risk neutral market measure and the Esscher transform measure as EMM, Fajardo and Mordecki (2006) obtain evidence that empirical risk-neutral markets are not symmetric. Then, the question naturally arises: How to obtain a Put-Call relationship, under absence of symmetry? In what follows we answer this question. It is worth noting that Carr and Lee (2008), have obtained a modified Put-Call symmetry, for certain classes of asymmetric dynamics. But, here we are interested with deviations from the original

SK, obtained under a symmetry condition.

Henceforth take $r = \delta$. We need the following assumption

Assumption 1. *Option prices are monotonic with respect to the symmetry parameter β .*

Our main result is stated as follows.

Theorem 3.1. *Consider Lévy measures given by (14). Under Assumption 1, if $\beta \geq -1/2$ then*

$$C(F_0, K_c, r, \tau, \psi) \geq (1+x) P(F_0, K_p, r, \tau, \psi), \quad (16)$$

where $K_c = (1+x)F_0$ and $K_p = F_0/(1+x)$, with $x > 0$.

Proof. We have that

$$\beta \geq -1/2 \iff \beta \geq \tilde{\beta} := -\beta - 1.$$

Then, $\Pi(dy) = e^{\beta y} \Pi_0(dy)$ has $\beta \geq \tilde{\beta}$ of $\tilde{\Pi} = e^{-(1+\beta)y} \Pi_0(dy)$. By monotonicity

$$\begin{aligned} C(F_0, K_c, r, \tau, a, \sigma, \Pi) &\geq C(F_0, K_c, r, \tau, a, \sigma, \tilde{\Pi}) \\ &= (1+x)P(F_0, K_c, r, \tau, a, \sigma, \Pi), \end{aligned}$$

where the last equality is obtained from duality and the fact that $\tilde{\tilde{\Pi}} = \Pi$. $\square$

In the next sections we present some particular cases.

3.2 Generalized Hyperbolic distributions (GH)

It is very well known that GH distributions allow for a more realistic description of asset returns (see Eberlein and Prause (2000) and Eberlein, Keller, and Prause (1998)). This model, under $\mathbb{P}$, has $\sigma = 0$, and a Lévy measure given by (14), with

$$p(y) = \frac{1}{|y|} \left(\int_0^\infty \frac{\exp(-\sqrt{2z + \alpha^2}|y|)}{\pi^2 z (J_\lambda^2(\delta\sqrt{2z}) + Y_\lambda^2(\delta\sqrt{2z}))} dz + \mathbf{1}_{\{\lambda \geq 0\}} \lambda e^{-\alpha|y|} \right),$$

where $\alpha, \beta_{\mathbb{P}}, \lambda, \delta$ are the historical parameters that satisfy the conditions $0 \leq |\beta_{\mathbb{P}}| < \alpha$, and $\delta > 0$; and J_λ, Y_λ are the Bessel functions of the first and second kind (for details see Eberlein and Prause (2000)). Particular cases are the hyperbolic distribution, obtained when $\lambda = 1$; and the normal inverse gaussian (NIG) when $\lambda = -1/2$.

In this particular case of the GH distributions, Assumption 1 can be guaranteed by the following theorem

Theorem 3.2 (Bergenthum and Rüschendorf (2007)).

Let S^i be $GH(d, \lambda^i, \alpha^i, \beta^i, \delta^i, \mu^i)$ distributed. If

$$\lambda^1 \leq \lambda^2, \quad \delta^1 \leq \delta^2, \quad \alpha^1 \geq \alpha^2, \quad \mu^1 \leq \mu^2$$

and β^i and Δ^i satisfy one of the below eq. Then $S^1 \leq_{icx} S^2$, where the order $\leq_i$ cx, means $Ef(S^1) \leq Ef(S^2)$ for all f increasing and convex, such that $f(S^1)$ and $f(S^2)$ are integrable.

$$0 \leq \beta^1 \leq \beta^2, \quad \Delta^i = I, \tag{17}$$

$$\beta^i = 0, \quad \Delta^1 \leq_{psd} \Delta^2, \tag{18}$$

$$0 \leq \beta^1 \leq \beta^2, \quad \Delta^1 \leq_{psd} \Delta^2, \quad 0 \leq \Delta_{ij}^1 \leq \Delta_{ij}^2, \quad \forall i, j \leq d, \tag{19}$$

where $\Delta^1 \leq_{psd} \Delta^2$ means $x'(\Delta^2 - \Delta^1)x \geq 0, \quad \forall x \in \mathbb{R}^d$

Now we can use the above result for GH distributions to state the following result.

Proposition 1. In the particular case of the GH distributions, if $\beta \geq -1/2$ then

$$C(F_0, K_c, r, \tau, \psi) \geq (1+x) P(F_0, K_p, r, \tau, \psi), \tag{20}$$

where $K_c = (1+x)F_0$ and $K_p = F_0/(1+x)$, with $x > 0$.

Proof. As our payoff is a call and all GH parameters are keep constant, except $\beta_1 = \beta$ and $\beta_2 = \tilde{\beta}$ and our $\Delta_i = 1, \quad i = 1, 2$. We satisfy eq. (17). Then, by Th. 3.2, assumption 1 above is satisfied and the result follows from Th. 3.1. $\square$

3.3 Diffusions with jumps

Consider the jump - diffusion model proposed by Merton (1976). The driving Lévy process in this model has Lévy measure given by

$$\Pi(dy) = \lambda \frac{1}{\delta \sqrt{2\pi}} e^{-(y-\mu)^2/(2\delta^2)} dy,$$

and is direct to verify that condition (13) holds if and only if $2\mu + \delta^2 = 0$. This result was obtained by Bates (1997) for future options, that result is obtained as a particular case.

Note that in that model $\beta = \frac{\mu}{\delta^2}$, so we obtain that sufficient conditions can be replaced by $\mu + \delta^2/2 \geq 0$, as also Bates (1997) found.

4 Skewness Premium

In order to have an intuition about the behavior of the sign of SK, let's analyze the following data on S&P500 American options in 08/31/2006 that matures in 09/15/2006 with future price $F = 1303.82$. To see if the Bates' rule holds we need to interpolate some non-observed option prices. To this end we use a cubic spline, as we can see in Fig. 1.

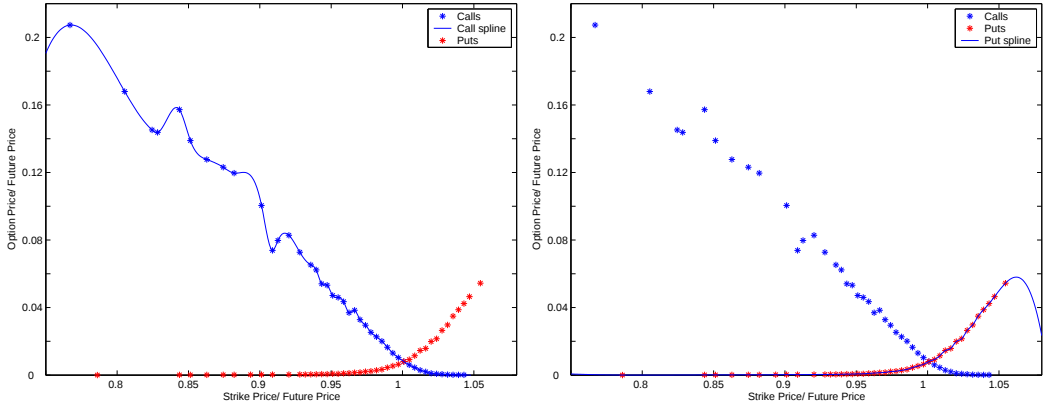


Figure 1: Observed Call and Put prices on S&P500 in 08/31/2006

The $x\%$ Skewness Premium is defined as the percentage deviation of $x\%$ OTM call prices from $x\%$ OTM put prices. The interpolating calls and put prices for the non-observed strikes are presented in Tables 1 and 2 at the end. We can see in both tables that this rule does not hold for this sample. Of course, a more rigorous statistical test with more sample data must be constructed to have stronger conclusions, as Bates (1997) did. Moreover, for OTM options usually $x_{obs} < x$, what implies $\frac{c}{p} - 1 < x$ and for ITM options, $x_{obs} > x$, implying $\frac{c}{p} - 1 > x$.

Then we want to know for what distributional parameter values we can capture the observed vies in these option price ratios. To this end we use the following definition introduced by Bates (1991).

$$SK(x) = \frac{c(S, T; X_c)}{p(S, T; X_p)} - 1, \text{ for European Options,} \quad (21)$$

$$SK(x) = \frac{C(S, T; X_c)}{P(S, T; X_p)} - 1, \text{ for American Options,}$$

where $X_p = \frac{F}{(1+x)} < F < F(1+x)$, $x > 0$.

The SK was addressed for the following stochastic processes: Constant Elasticity of Variance (CEV), include arithmetic and geometric Brownian motion. Stochastic Volatility processes, the benchmark model being those for which volatility evolves independently of the asset price. And the Jump-diffusion processes, the benchmark model is the Merton's (1976) model. For that classes Bates (1997) obtained the following result.

Proposition 2 (Bates (1997)). *For European options in general and for American options on futures, the SK has the following properties for the above distributions.*

- i) $SK(x) \leq x$ for CEV processes with $\rho \leq 1$.
- ii) $SK(x) \leq x$ for jump-diffusions with log-normal jumps depending on whether $2\mu + \delta^2 \leq 0$.
- iii) $SK(x) \leq x$ for Stochastic Volatility processes depending on whether $\rho_{S\sigma} \leq 0$.

Now in equation (21) consider

$$X_p = F(1 - x) < F < F(1 + x), \quad x > 0.$$

Then,

iv) $SK(x) < 0$ for CEV processes only if $\rho < 0$.

v) $SK(x) \geq 0$ for CEV processes only if $\rho \geq 0$.

When x is small, the two SK measures will be approx. equal. For in-the-money options ($x < 0$), the propositions are reversed. Calls $x\%$ in-the-money should cost $0\% - x\%$ less than puts $x\%$ in-the-money.

Now based on the results presented in the last section, we can extend Bates' result to Lévy processes by assuming the following.

Corollary 4.1. *If $\beta \gtrless -1/2$ (absence of symmetry),*

i) and Lévy measure is given by (14). Under Assumption 1, we have

$$SK(x) \gtrless x, \quad x > 0. \tag{22}$$

ii) and the underlying is driven by a GH distribution. Then,

$$SK(x) \gtrless x, \quad x > 0. \tag{23}$$

Proof. i) Follows from Th. 3.1.

ii) Follows from Proposition 1.

□

In the next section we present a discussion about Assumption 1.

5 Monotonicity and Symmetry Parameter

As we have seen in the last section we need the monotonicity of option prices with respect to the symmetry parameter to obtain our main result. The literature had study extensively the monotonicity properties of option prices.

The main idea is to exploit the *convexity preserving property*¹, to obtain the monotonicity of option prices with respect to certain parameter of the model. See Bergman, Grundy, and Wiener (1996), El Karoui, Jeanblanc-Picque, and Shreve (1998) and Ekström and Tysk (2007).

But we are interested in the possible misspecifications in the models when using a fixed equivalent martingale measure. That is, if we change the parameter β on the Lévy measure described by (14) what happens with the option price. Unfortunately, we have not a result that guarantees the validity of Assumption 1. In that sense the results obtained by Bergenthum and Rüschendorf (2007) for the GH distributions can bring some insights.

6 Conclusions

Under a given risk neutral probability measure, we use a measure of *symmetry* of a Lévy market model introduced by Fajardo and Mordecki (2006), to address the skewness premium under absence of symmetry. In that case we derive sufficient conditions for the excess of SK to be positive, in terms of the structure of the Lévy measure. In particular on the symmetry parameter.

Interesting issue to study in a future work is the monotonicity of option prices with respect to the symmetry parameter.

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¹We say that a model is *convexity preserving*, if for any convex contract function, the corresponding price is convex as a function of the price of the underlying asset at all times prior to maturity. Many models do not satisfy this property as for example general stochastic volatility models.

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K_c	$K_p = F^2/K_c$	$x = K_c/F - 1$	$x_{obs} = c_{obs}/p_{int} - 1$	$x - x_{obs}$
1230	1382.07	-0.05662	0.050681	-0.1073
1235	1376.475	-0.05278	0.13642	-0.1892
1240	1370.925	-0.04895	0.115006	-0.16395
1245	1365.419	-0.04511	0.197696	-0.24281
1250	1359.957	-0.04128	0.277944	-0.31922
1255	1354.539	-0.03744	0.280729	-0.31817
1260	1349.164	-0.03361	0.536286	-0.5699
1265	1343.831	-0.02977	0.574983	-0.60476
1270	1338.541	-0.02594	0.606719	-0.63266
1275	1333.291	-0.0221	0.675372	-0.69748
1280	1328.083	-0.01827	0.691325	-0.70959
1285	1322.916	-0.01443	0.966306	-0.98074
1290	1317.788	-0.0106	0.904839	-0.91544
1295	1312.7	-0.00676	0.794059	-0.80082
1300	1307.651	-0.00293	0.78018	-0.78311
1305	1302.641	0.000905	0.614561	-0.61366
1310	1297.669	0.00474	0.532798	-0.52806
1315	1292.735	0.008575	0.427299	-0.41872
1320	1287.838	0.01241	0.108911	-0.0965
1325	1282.979	0.016245	-0.11658	0.132826
1330	1278.155	0.020079	-0.45097	0.471053
1335	1273.368	0.023914	-0.50378	0.527697
1340	1268.617	0.027749	-0.61306	0.640807
1345	1263.901	0.031584	-0.73872	0.770305
1350	1259.22	0.035419	-0.81448	0.849896
1355	1254.573	0.039254	-0.80297	0.842224
1360	1249.961	0.043089	-0.82437	0.867454

Table 1: Options prices Interpolating Put prices

K_p	$K_c = F^2/K_p$	$x = F/K_p - 1$	$x_{obs} = c_{int}/p_{obs} - 1$	$x - x_{obs}$
1250	1359.957	0.043056	-0.88837	0.931421
1255	1354.539	0.0389	-0.86897	0.907873
1260	1349.164	0.034778	-0.85655	0.891331
1265	1343.831	0.030688	-0.78107	0.81176
1270	1338.541	0.02663	-0.70531	0.731941
1275	1333.291	0.022604	-0.63926	0.661869
1280	1328.083	0.018609	-0.51726	0.535865
1285	1322.916	0.014646	-0.31216	0.326801
1290	1317.788	0.010713	-0.20329	0.214005
1295	1312.7	0.006811	-0.03659	0.043397
1300	1307.651	0.002938	0.090739	-0.0878
1305	1302.641	-0.0009	0.130843	-0.13175
1310	1297.669	-0.00472	0.252541	-0.25726
1315	1292.735	-0.0085	0.261905	-0.27041
1320	1287.838	-0.01226	0.242817	-0.25507
1325	1282.979	-0.01598	0.346419	-0.3624
1330	1278.155	-0.01968	0.183207	-0.20289
1335	1273.368	-0.02336	0.237999	-0.26135
1340	1268.617	-0.027	0.145858	-0.17286
1345	1263.901	-0.03062	0.152637	-0.18325
1350	1259.22	-0.03421	0.101211	-0.13542
1355	1254.573	-0.03777	-0.03964	0.001869
1360	1249.961	-0.04131	0.028337	-0.06965
1365	1245.382	-0.04482	-0.0101	-0.03472
1375	1236.325	-0.05177	-0.0451	-0.00667

Table 2: Options prices Interpolating Call prices

- 2008-60: Thomas Q. Pedersen: Intertemporal Asset Allocation with Habit Formation in Preferences: An Approximate Analytical Solution
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