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Mean Reversion in US and International Short Rates

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Abstract: In this paper we extend the CKLS one factor short rate model to include extreme value nonlinear mean reversion. Similarly to a recent stock market study, we include the smallest short rate during the previous year in the mean equation. We investigate the US and five other major markets (Canada, Germany, Japan, Switzerland, and the UK). There is extreme value mean reversion in the US short rate. For Japan there is both linear and nonlinear mean reversion. For the remaining short rates there is no evidence of mean reversion.
Keywords: Short term interest rate; Mean reversion; Extreme value; Nonlinearity
JEL Classifications: G12; G15; E43; C13

1 Introduction

Based on theoretical considerations, we expect short rates to be mean reverting. However, empirically it is harder to demonstrate mean reversion in short rates. This is most likely because short rates are strongly autocorrelated and thereby have near unit root behavior. In this paper, we suggest a new one factor short rate model with nonlinear mean reversion. The empirical findings based on this new short rate model are consistent with the US short rate being mean reverting. Apart from the US short rate, we also investigate short rates from five other major industrialized countries.

In the popular one factor short rate model of Chan, Karolyi, Longstaff and Sanders (1992) (known as the CKLS model), the linear mean reversion parameter is typically found to be negative, which is consistent with mean reversion, but it is typically found to be statistically insignificant. This means that the mean reversion evidence is only weak. Moreover, the mean reversion parameter is measured with great imprecision, see Faff and Gray (2006).

In general, analysis of one factor short rate models provides mixed empirical evidence of mean reversion and it is not unambiguous whether this possible mean reversion is linear or nonlinear.

Most empirical studies of one factor short rate models are concerned with US interest rates: Aït-Sahalia (1996) uses nonparametric methods and introduces nonlinear mean reversion by extending the CKLS model to have nonlinear terms in the mean equation. Takamizawa (2008) uses cross sectional relations to estimate the Aït-Sahalia (1996) model, but finds that there is not generally nonlinear mean reversion. Jones (2003) uses a Bayesian method to analyze short rate models and he finds that there is not generally evidence of nonlinear mean reversion, only when some specific prior distributions are assumed. Durham (2003) finds that mean reversion is insignificant. Instead, it is the volatility specification that is important for the model fit. Chapman and Pearson (2000) use Monte Carlo studies to show that there is not nonlinear mean reversion in short rates. Arapis and Gao (2006) use nonparametric methods to show that the short rate drift is nonlinear. In contrast to Durham (2003), they find that the drift is important for the model fit. Bali and Wu (2006) contain an empirical analysis of an extended version of the Aït-Sahalia (1996) short rate model. They find that the extend to which there is nonlinear mean reversion depends on which US short rate is analyzed; there is only weak nonlinearity in the drift of the US Treasury yield, but much stronger nonlinearity in the US federal funds rate and the Eurodollar rate.

Treepongkaruna and Gray (2006) estimate the CKLS and the Aït-Sahalia

(1996) short rate models for 11 countries and find linear mean reversion for some countries, nonlinear mean reversion for others, and no mean reversion for others still. Wu and Chen (2001) reject a unit root in short rates by conducting a joint test across the eurocurrency short rates of seven countries. This result is in contrast with the findings when they test for a unit root separately in each of the series. So their results are in favor of short rates being mean reverting instead of exploding. Similar findings are documented by Wu and Zhang (1996).

In the present paper, we extend the CKLS model to include nonlinear mean reversion in a new way. In addition to this, we include the Aït-Sahalia (1996) nonlinear terms in the mean equation. We include the smallest short rate during the previous, say, last year in the mean equation. Accounting for mean reversion in this way has been applied to stock market returns by Bali, Demirtas and Levy (2008). We use daily data for the short rates for six major developed countries (Canada, Germany, Japan, Switzerland, the UK, and the US) for the period 1985-2008. In this manner, we detect nonlinear mean reversion for the US short rate. For Canada, Germany, Switzerland, and the UK we find no signs of mean reversion - neither linear nor nonlinear. For Japan there is evidence of mean reversion (both linear and nonlinear) in the short rate.

The remaining part of the paper is structured as follows. In Section 2 we describe the short rate models. Subsequently, we introduce the data in Section 3. In Section 4 we describe the empirical findings. Section 5 concludes.

2 Short Rate Model

2.1 Continuous Time One Factor Short Rate Models

One factor short rate models can be described by the following stochastic differential equation:

$$dr_t = \mu(r_t)dt + \sqrt{\sigma(r_t)}dW_t$$

The specifications of the drift function - $\mu(r_t)$ - and the variance function - $\sigma(r_t)$ - differ across various short rate models.

Chan et al. (1992) suggest a general one factor short rate model that includes linear mean reversion and where the volatility is proportional to the γs power of the level of the short rate; this is denoted the CKLS model:

$$dr_t = (\alpha_0 + \alpha_1 r_t)\, dt + (\sigma r_t^{\gamma})\, dW_t$$

When $\alpha_1 < 0$, there is mean reversion in the CKLS model. The mean reversion in the CKLS model is linear.

Aït-Sahalia (1996) suggests the following extension of the CKLS model to include nonlinear mean reversion. The Aït-Sahalia (1996) model has nonlinear mean reversion in that the short rate changes depend not only on the short rate itself but also on the square of the short rate and the inverse of the short rate.

$$dr_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1}\right) dt + \sqrt{\sigma(r_t)} dW_t$$

Here we suggest to extend the Aït-Sahalia (1996) model by another kind of nonlinear mean reversion similarly to how Bali et al. (2008) describe the mean reversion of stock returns. We include the lowest short rate during the last n days in the mean equation ($r_{n,t}^{\min}$)

$$dr_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} + \alpha_4 r_{n,t}^{\min}\right) dt + \sqrt{\sigma(r_t)} dW_t$$

This we denote extreme value mean reversion.

2.2 Empirical Short Rate Model

In order to conduct the empirical analysis, we specify the short rate model in discrete time:

$$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-1}^2 + \alpha_3 r_{t-1}^{-1} + \alpha_4 r_{n,t-1}^{\min} + \varepsilon_t$$

$r_{n,t-1}^{\min}$ is the minimum short rate observed the last n days. We use $n = \{20, 125, 250, 500\}$ in the estimations, which corresponds to about 1 month, 6 months, 1 year, and 2 year horizons. So the empirical analysis will also be able to give us information about at which horizon we have extreme value mean reversion.

The residual ε_t has mean 0, and its variance is given by

$$\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$$

The variance equation is identical to Chan et al. (1992) so that the current volatility is proportional to the γs power of the lagged short rate.

Below, we estimate four different specifications of mean equation. The first specification is the classical CKLS model. The second specification has extreme value mean reversion in addition to the CKLS linear mean reversion. The third model has the Aït-Sahalia (1996) mean specification. The fourth model has both Aït-Sahalia (1996) nonlinear mean reversion as well as extreme value mean reversion from Bali et al. (2008).

- CKLS model, where $\alpha_2 = \alpha_3 = \alpha_4 = 0$
- Extreme value model, where $\alpha_2 = \alpha_3 = 0$
- Aït-Sahalia (1996) model, where $\alpha_4 = 0$
- General nonlinear model (no restrictions)

3 Short Rate Data

We use daily data for the 1-month Eurocurrency offered interest rates for the following six countries: Canada, Germany, Japan, Switzerland, UK, and US. The countries are all large developed countries. The sample covers the period from 1985 to 2008, providing us with 6,066 daily observations.[1] The data are obtained from DataStream.

Table 1 contains descriptive statistics for the short rate series. On average the short rate is lowest in Japan (average of 2.40%) and highest in the UK (7.61%). The standard deviations are of about the same size, namely between 2.15% (Germany) and 3.31% (the UK). The short rates are slightly skewed and have thin tails. Figure 1 shows the time series evolution of the short rates, which are seen to be highly volatile.

The left hand side of the mean equation of the short rate model concerns the first differences of the short rate. Therefore, the second part of Table 1 concerns the first differences of the short rates. On average the short rate changes are close to zero. The standard deviation of the short rate changes are much larger than their means. The short rate changes have very thick tails.

To get an indication of the mean reversion which is not based on any model, we show scatter plots of Δr_t against r_{t-1}. A negative relation indicates mean reversion. Figures 2-7 show the scatter plots for each of the short rates. In the scatter plots we see almost no relation between the short rate changes and the lagged level of the short rate. This is also highlighted in the last row of Table 1 which shows the correlation coefficient between the short rate changes and the lagged short rate, which is negative but low, namely about -0.02 for all currencies. So there are only slight indications of mean reversion from the data description.

The short rates are strongly autocorrelated, the 10-lag autocorrelation coefficients are above 0.99. For the short rate changes, the autocorrelation is not

[1] For Switzerland the sample period ends in December 2007 due to data availability.

very strong, the 1-day autocorrelation coefficient is negative and between -0.15 (Japan) and -0.04 (Switzerland).[2]

4 Results

The estimation is done in GAUSS using QML estimation. This is similar to e.g. Bali and Wu (2006). Duffee (2002) contains a discussion of the advantages of using QML estimation for short rate model estimation. We use a 5% level of significance.

Below, we discuss the estimation results for each of the four models separately.

4.1 CKLS Model

Table 2 shows the parameter estimates from the CKLS short rate model estimated for each of the six countries' short rates.

The US short rate is the most thoroughly investigated short rate. We see that the linear mean reversion parameter, α_1, is negative but that it is insignificant. So, there are only weak signs of mean reversion for the US short rate.[3] This is similar to previous findings, see e.g. Chan et al. (1992), Bali and Wu (2006), and Faff and Gray (2006).

The linear mean reversion parameter is negative for all six countries, and only for Japan and Switzerland is it significantly negative. So the overall picture is that there are some but not strong signs of linear mean reversion.

The level effect for the volatility is measured by the γ parameter and differs across the countries. It is strongest for the UK where it exceeds unity, and for the US, Canada, and Germany the level parameter is just below unity. For these four countries, we cannot reject that the level parameter is equal to unity, which is in accordance with the Brennan and Schwartz (1980) short rate model. For Switzerland the level parameter is insignificantly different from $\frac{1}{2}$, which is in accordance with the square root model of Cox, Ingersoll and Ross (1985). For Japan the level effect is even lower (at 0.21), which is significantly smaller than $\frac{1}{2}$ and significantly larger than zero.

[2] According to Wu and Chen (2001) and Wu and Zhang (1996), ordinary unit root tests are not suitable for short rate data. Still, to make our data comparable to previous studies, we report the results of using the Augmented Dickey-Fuller unit root test with a constant and 5 lags, see Dickey and Fuller (1979): It cannot be rejected that all of the short rate series have a unit root whereas, there are no indications of unit roots in the short rate changes.

[3] In the CKLS model, the ordinary t-test is not valid, see Koedjik, Nissen, Schotman and Wolff (1997).

It is noticeable that it is in the short rates for the same two countries (Japan and Switzerland) that we see significant mean reversion and lower than unity level effect.

4.2 Extreme Value Model

Table 3 shows the results of estimating the extreme value model, where the CKLS mean equation has been extended with the lowest short rate the last $n = 250$ days. This shows how the current short rate changes depend on the smallest short rate within the previous year.

For the US the dependence of the smallest short rate (α_4) is negative and significant, which indicates mean reversion. In contrast, we see no linear mean reversion for the US; now α_1 is positive and significant, indicating mean aversion. So there are two effects at play, namely nonlinear mean reversion (α_4) and linear mean aversion (α_1). As the nonlinear parameter is largest in absolute size (-0.41 compared to 0.32), the mean reversion dominates.

For Canada, Germany, Switzerland, and the UK, the dependence upon the smallest short rate is also negative, but the parameter is not significant. For Canada, Germany, and the UK, the linear mean reversion parameter has turned positive similarly to the US. For these three countries, the mean reversion dominated. For Switzerland the linear mean reversion parameter is still negative but it is not significant. So also for Switzerland do we see mean reversion.

For Japan both the linear and the nonlinear parameters are significant, but the signs are opposite those of the US. For Japan there appears to be mean aversion. So the results for Japan are very different from the other countries. This is perhaps also caused by the fact that there are some negative short rates observed for Japan during the sample period.

Table 4 shows the results from estimating the extreme value model where the period for observing the smallest short rate mow is increased to $n = 500$. Qualitatively, the results are identical to the model where the mean reversion period is 250 days. The nonlinear mean reversion effect is relatively stronger than the linear mean aversion effect for $n = 500$ compared to $n = 250$. For the US the coefficients are $\widehat{\alpha}_1 = 0.17$ and $\widehat{\alpha}_4 = -0.27$.

Table 5 shows the results of shortening the period for mean reversion, namely when $n = 125$. For the US the linear mean reversion parameter α_1 is no longer significant, but it is still positive. The nonlinear mean reversion parameter α_4 is significantly negative. So for the shorter horizon, there are only signs of mean reversion, not mean aversion. For Canada, Germany, Japan, Switzerland, and the UK, the linear mean reversion parameter is negative, but insignificant, except for Japan. The nonlinear mean reversion parameter is positive, but insignificant for Canada and Switzerland, and negative and insignificant for Germany and the UK. For Japan the nonlinear mean reversion parameter is significantly positive.

For a 1-month horizon with $n = 20$, the picture is very different. The linear mean reversion parameter is significantly negative for all countries (except for Germany), and the nonlinear mean reversion parameter is significantly positive (again, expect for Germany). So in the short run, there is linear mean reversion and nonlinear mean aversion.

For all the estimated nonlinear models for different values of n, it is the case that the level effect is very similar to what was found for the CKLS model. So, the specification of the mean equation does not appear to affect the estimates of the variance equation.

4.3 Aït-Sahalia (1996) Model

Table 7 shows the results of estimating the Aït-Sahalia (1996) model with the Aït-Sahalia (1996) mean equation. For all countries except Japan, none of the parameters in the mean equation are significant. Thus, extending the CKLS model with the square of the short rate and the inverse of the short rate does not alter the conclusion that there are not strong evidence of mean reversion.

For Japan the picture is different in that there is significant mean reversion from the spot rate (α_1 is negative and significant). The Japanese short rate changes also depend upon the squared short rate (positively) and the inverse of the short rate (negatively). The positive dependence on the squared short rate is similar to the results for the US in Bali and Wu (2006).

4.4 General Nonlinear Model

Table 8 shows the results from estimating the general nonlinear model which includes the extreme value, the short rate, the square of the short rate, and the inverse of the short rate.

For Canada, Switzerland, and the UK, neither of the explanatory variables are significant, so there are at best no signs of mean reversion for these countries. For Germany we have significant mean reversion in that the minimum short rate is significant in explaining the short rate changes (and the relation is negative as expected when mean reversion prevails).

For Japan the short rate shows significant linear mean reversion, and the nonlinear mean reversion parameter is significantly positive, so it is the same as in the extreme value model. The Japanese short rate depends significantly on both the squared short rate and the inverse short rate; the relation with the squared short rate is positive, and with the inverse short rate it is negative.

For the US short rate changes depend significantly on the short rate (positive dependence), the squared short rate (negative dependence), and the smallest short rate the previous year (negative dependence).

5 Conclusion

In this paper, we suggest a new one-factor short rate model that includes non-linear mean reversion in a new way for short rate models. The new model is the CKLS model extended so that the short rate changes depend upon not only on the short rate itself but also the smallest short rate the previous period (e.g. the year). We also include the terms from the Aït-Sahalia (1996) mean specification, namely the squared short rate and the inverse of the short rate.

Empirically, it is shown that the only model that is consistent with mean reverting US short rates includes the minimum short rate as explanatory variable. For Canada, Germany, Switzerland, and the UK, neither of the suggested models provide evidence of mean reversion. For Japan all the models are consistent with mean reverting short rates.

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Figure 1: Time Series of Spot Rates for US, UK, Germany (GE), Japan (JA), Switzerland (SW), and Canada (CA)

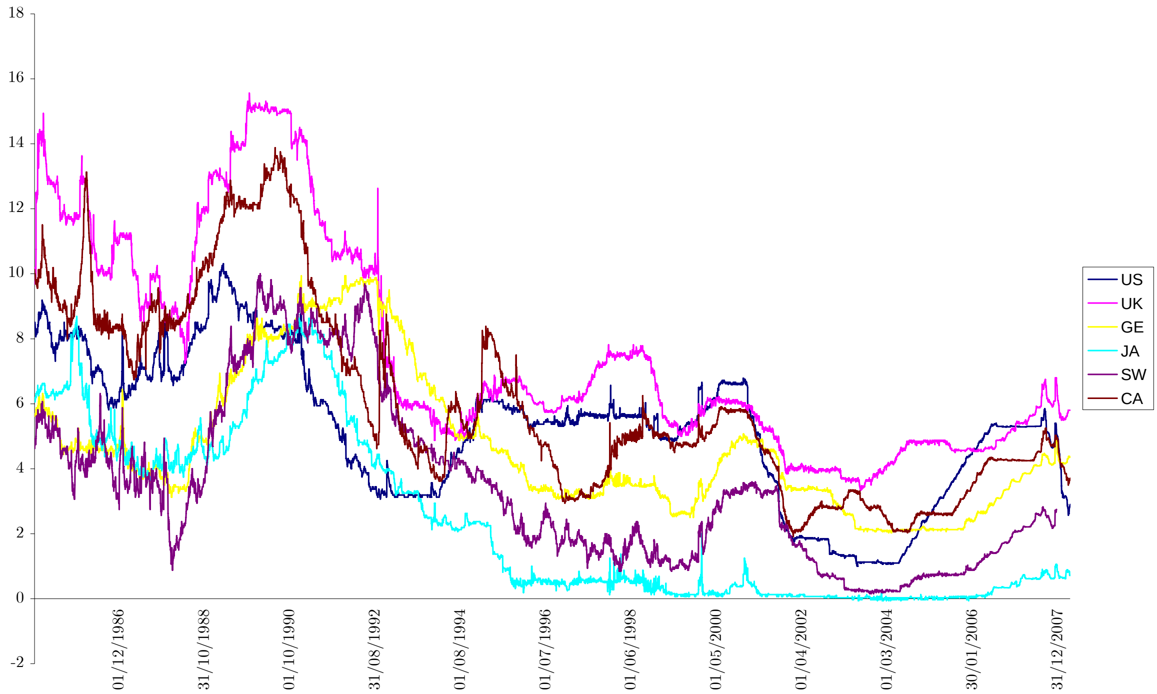

Figure 2: Scatter Plot for Canada of Short Rate Changes against Lagged Short Rate

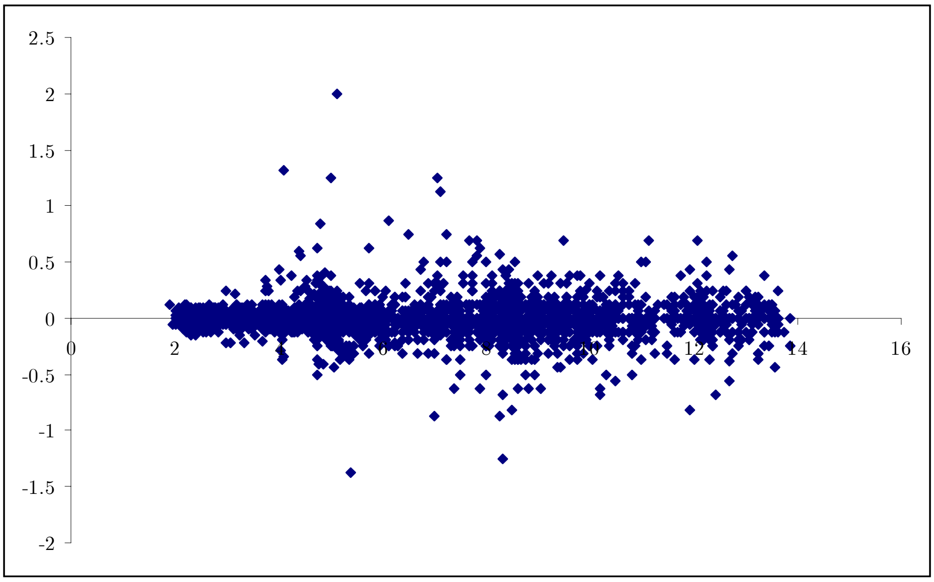

Figure 3: Scatter Plot for Germany of Short Rate Changes against Lagged Short Rate

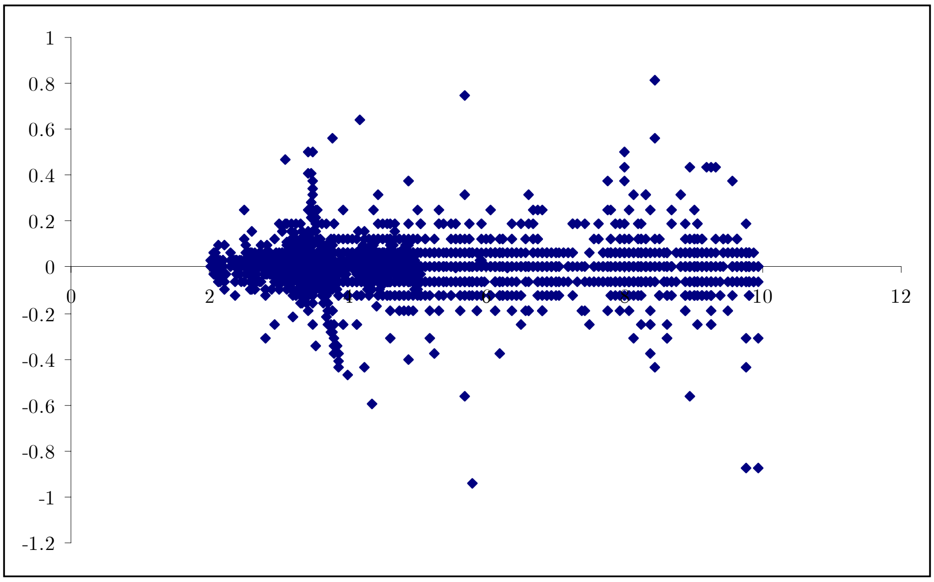

Figure 4: Scatter Plot for Japan of Short Rate Changes against Lagged Short Rate

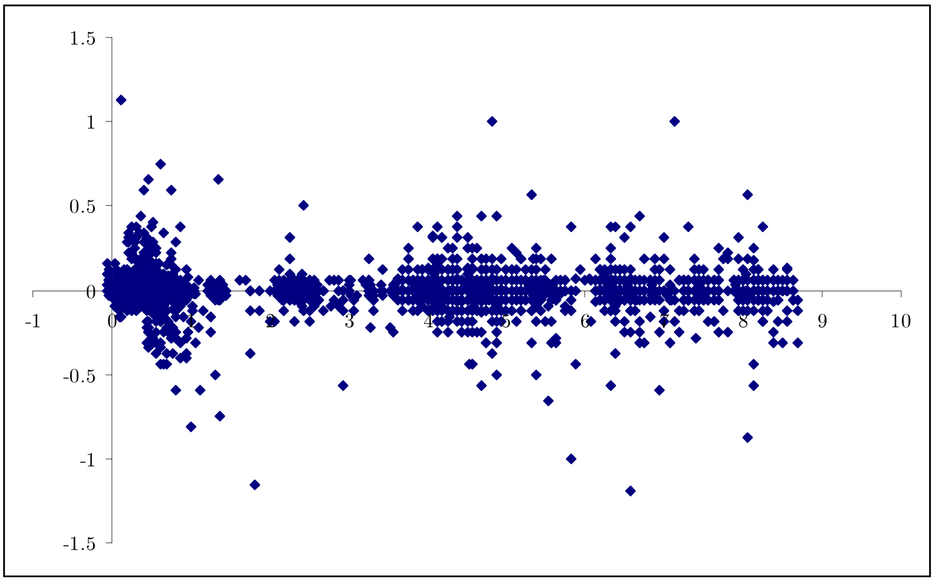

Figure 5: Scatter Plot for Switzerland of Short Rate Changes against Lagged Short Rate

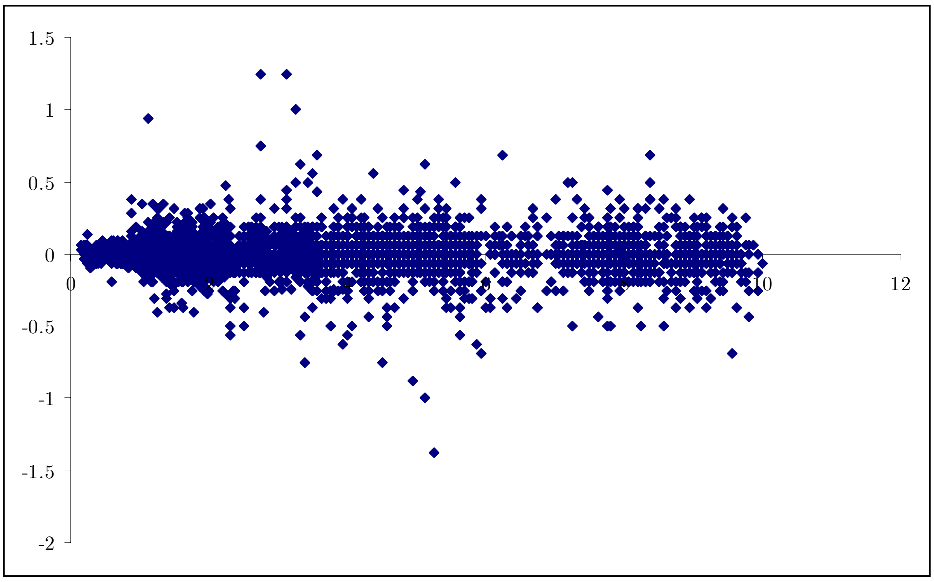

Figure 6: Scatter Plot for the UK of Short Rate Changes against Lagged Short Rate

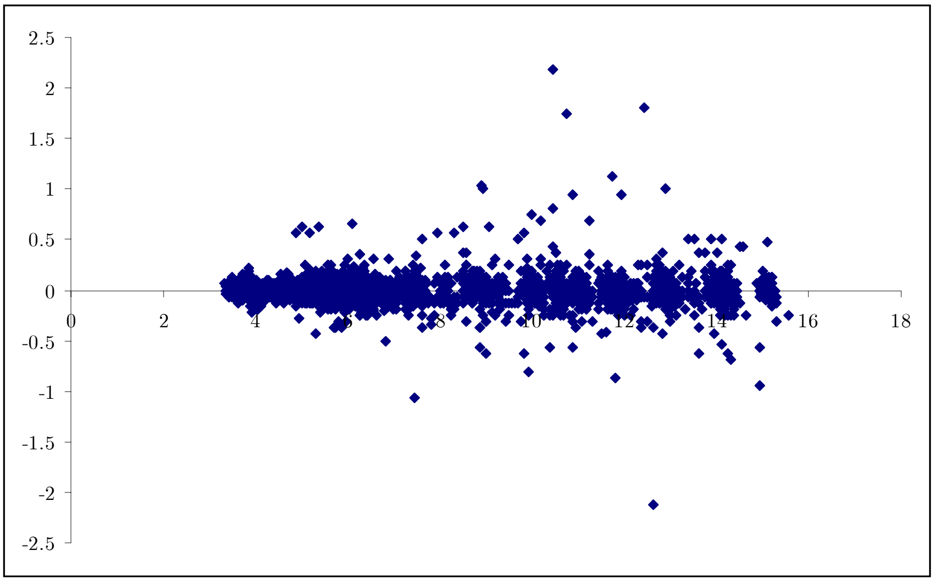

Figure 7: Scatter Plot for the US of Short Rate Changes against Lagged Short Rate

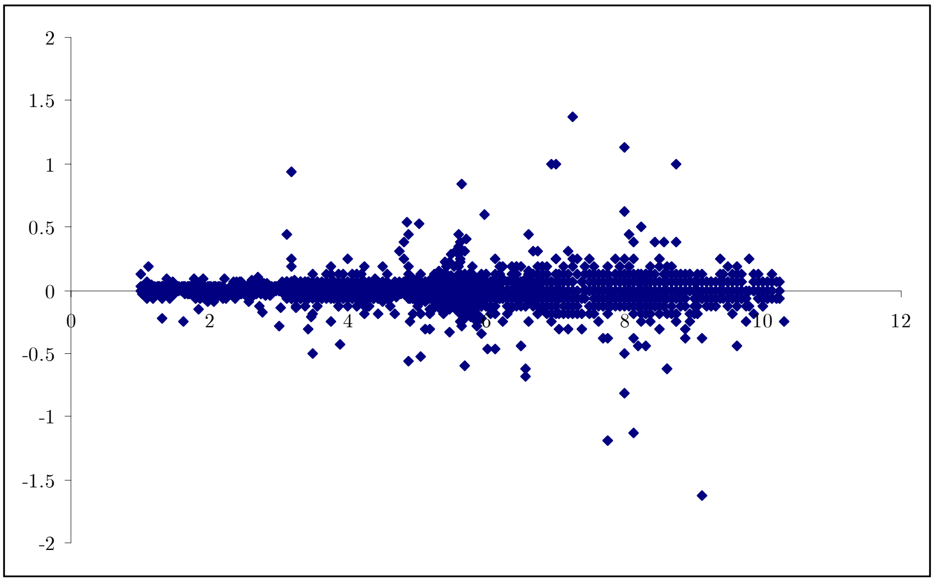

Table 1: Descriptive Statistics

	Canada	Germany	Japan	Switzerland	UK	USA
Start date	Jan. 1, 1985	Jan. 1, 1985	Jan. 1, 1985	Jan. 1, 1985	Jan. 1, 1985	Jan. 1, 1985
End date	Apr. 1, 2008	Apr. 1, 2008	Apr. 1, 2008	Dec. 12, 2007	Apr. 1, 2008	Apr. 1, 2008
# of obs	6066	6066	6066	5987	6066	6066
Sport rate (%)						
Mean	6.06	4.69	2.40	3.42	7.61	5.24
Std. Dev	2.99	2.15	2.64	2.55	3.31	2.18
Maximum	13.88	9.94	8.69	10.00	15.56	10.31
Minimum	1.91	2.03	-0.06	0.16	3.34	1.00
Skewness	0.77	0.97	0.80	0.85	0.86	-0.13
Excess kurtosis	-0.38	-0.10	-0.77	-0.27	-0.51	-0.58
Rho(1)	0.999	0.999	0.999	0.999	0.999	0.999
Rho(10)	0.993	0.996	0.995	0.993	0.996	0.992
First differences (%)						
Mean	-0.0010	-0.0002	-0.0009	-0.0003	-0.0007	-0.0009
Std. Dev	0.11	0.07	0.09	0.11	0.10	0.08
Maximum	2.00	0.81	1.13	1.25	2.19	1.38
Minimum	-1.38	-0.94	-1.19	-1.38	-2.13	-1.63
Skewness	1.45	-0.35	-0.66	0.04	2.41	-0.21
Excess kurtosis	46.52	32.01	40.78	21.88	105.83	83.17
Rho(1)	-0.062	-0.145	-0.151	-0.042	-0.132	-0.117
Rho(10)	0.002	0.007	-0.004	0.039	0.055	0.009
Correlation(lagged spot rate, first differences)						
	-0.021	-0.017	-0.020	-0.021	-0.016	-0.021

The table shows descriptive statistics for the spot rates and the first differences of the spot rates.

Table 2: CKLS Model

	Canada	Germany	Japan	Switzerland	UK	USA
$\boldsymbol{\alpha_0}$	0.0034 *	0.0025 *	0.0024 ***	0.0027 **	0.0046 *	0.0012
	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
$\mathbf{100}\boldsymbol{\alpha_1}$	-0.0724	-0.0552	-0.1463 ***	-0.0899 **	-0.0786 *	-0.0394
	(0.048)	(0.041)	(0.045)	(0.050)	(0.047)	(0.035)
$\mathbf{100}\boldsymbol{\sigma}$	0.0346 ***	0.0304 ***	0.6830 ***	0.3646 ***	0.0090 **	0.0328 ***
	(0.010)	(0.007)	(0.061)	(0.033)	(0.004)	(0.009)
$\boldsymbol{\gamma}$	0.9720 ***	0.8842 ***	0.2096 ***	0.4871 ***	1.1226 ***	0.8710 ***
	(0.067)	(0.079)	(0.031)	(0.032)	(0.121)	(0.079)

The table shows the parameter estimates and the robust standard errors in parentheses of the CKLS short rate model, $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t$ and $\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 3: Extreme Value Model with n=250

	Canada	Germany	Japan	Switzerland	UK	USA
α_0	0.0036 *	0.0033 **	0.0038 ***	0.0027 **	0.0055 **	0.0012
	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
$100\alpha_1$	0.1101	0.1408	-1.4507 ***	-0.0339	0.0896	0.3168 ***
	(0.196)	(0.129)	(0.416)	(0.156)	(0.139)	(0.102)
$100\alpha_4$	-0.2215	-0.2454 *	1.5065 ***	-0.0738	-0.2007	-0.4086 ***
	(0.215)	(0.142)	(0.470)	(0.181)	(0.158)	(0.105)
100σ	0.0345 ***	0.0303 ***	0.6816 ***	0.3647 ***	0.0091 **	0.0326 ***
	(0.010)	(0.007)	(0.061)	(0.033)	(0.004)	(0.009)
γ	0.9730 ***	0.8841 ***	0.2114 ***	0.4870 ***	1.1219 ***	0.8729 ***
	(0.067)	(0.079)	(0.031)	(0.032)	(0.121)	(0.080)

The table shows the parameter estimates and the robust standard errors in parentheses of the extreme value model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_4 r^{\min}_{n,t-1} + \varepsilon_t$ and $\sigma^2_t = \sigma r^{2\gamma}_{t-1}$. $r^{\min}_{n,t-1}$ is the minimum short rate observed the last $n = 250$ days. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 4: Extreme Value Model with n=500

	Canada	Germany	Japan	Switzerland	UK	USA
α_0	0.0039 **	0.0040 ***	0.0024 ***	0.0026 **	0.0058 *	0.0016
	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
$100\alpha_1$	0.0497	0.0755	-0.4205 **	0.0114	0.0122	0.1715 **
	(0.124)	(0.078)	(0.189)	(0.069)	(0.118)	(0.058)
$100\alpha_4$	-0.1743	-0.2096 **	0.4172 *	-0.1612 *	-0.1308	-0.2733 ***
	(0.154)	(0.095)	(0.244)	(0.096)	(0.154)	(0.073)
100σ	0.0351 ***	0.0305 ***	0.6342 ***	0.3567 ***	0.0104 **	0.0320 ***
	(0.010)	(0.007)	(0.055)	(0.033)	(0.005)	(0.008)
γ	0.9674 ***	0.8842 ***	0.1844 ***	0.4663 ***	1.0812 ***	0.8865 ***
	(0.072)	(0.079)	(0.032)	(0.032)	(0.140)	(0.080)

The table shows the parameter estimates and the robust standard errors in parentheses of the extreme value model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_4 r^{\min}_{n,t-1} + \varepsilon_t$ and $\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$. $r^{\min}_{n,t-1}$ is the minimum short rate observed the last $n = 500$ days. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 5: Extreme Value Model with n=125

	Canada	Germany	Japan	Switzerland	UK	USA
α_0	0.0034 *	0.0027 *	0.0053 ***	0.0029 ***	0.0048 *	0.0010
	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
$100\alpha_1$	-0.0770	-0.0537	-3.4816 ***	-0.2850	-0.0583	0.3754 *
	(0.404)	(0.157)	(0.794)	(0.275)	(0.193)	(0.200)
$100\alpha_4$	0.0037	-0.0090	3.6677 ***	0.2214	-0.0241	-0.4438 **
	(0.435)	(0.185)	(0.855)	(0.302)	(0.198)	(0.201)
100σ	0.0353 ***	0.0304 ***	0.6792 ***	0.3656 ***	0.0099 **	0.0330 ***
	(0.010)	(0.007)	(0.060)	(0.033)	(0.004)	(0.009)
γ	0.9654 ***	0.8815 ***	0.2137 ***	0.4876 ***	1.0955 ***	0.8658 ***
	(0.067)	(0.079)	(0.031)	(0.031)	(0.119)	(0.079)

The table shows the parameter estimates and the robust standard errors in parentheses of the extreme value model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_4 r^{\min}_{n,t-1} + \varepsilon_t$ and $\sigma^2_t = \sigma r^{2\gamma}_{t-1}$. $r^{\min}_{n,t-1}$ is the minimum short rate observed the last $n = 125$ days. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 6: Extreme Value Model with n=20

	Canada	Germany	Japan	Switzerland	UK	USA
$\boldsymbol{\alpha_0}$	0.0031 *	0.0019	0.0072 ***	0.0035 ***	0.0041 *	0.0016
	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
$\mathbf{100}\boldsymbol{\alpha_1}$	-0.9625	-3.4401 ***	-10.5500 ***	-1.8466 **	-3.5049 ***	-2.4761 **
	(1.069)	(1.003)	(2.030)	(0.861)	(1.120)	(1.185)
$\mathbf{100}\boldsymbol{\alpha_4}$	0.9226	3.4833 ***	10.7301 ***	1.8402 **	3.5050 ***	2.4902 **
	(1.091)	(1.021)	(2.079)	(0.888)	(1.130)	(1.196)
$\mathbf{100}\boldsymbol{\sigma}$	0.0355 ***	0.0304 ***	0.6550 ***	0.3638 ***	0.0096 **	0.0337 ***
	(0.010)	(0.007)	(0.058)	(0.033)	(0.004)	(0.009)
$\boldsymbol{\gamma}$	0.9624 ***	0.8785 ***	0.2110 ***	0.4916 ***	1.1026 ***	0.8567 ***
	(0.066)	(0.077)	(0.032)	(0.031)	(0.110)	(0.078)

The table shows the parameter estimates and the robust standard errors in parentheses of the extreme value model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_4 r_{n,t-1}^{\min} + \varepsilon_t$ and $\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$. $r_{n,t-1}^{\min}$ is the minimum short rate observed the last $n = 20$ days. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 7: Aït-Sahalia (1996) Model

	Canada	Germany	Japan	Switzerland	UK	USA
α_0	0.0051	0.0332	0.0097 ***	-0.0008	-0.0071	-0.0167
	(0.137)	(0.032)	(0.001)	(0.004)	(0.023)	(0.011)
$100\alpha_1$	-0.1005	-0.7857	-0.8309 ***	0.0721	0.0268	0.5485
	(2.720)	(0.766)	(0.156)	(0.217)	(0.321)	(0.349)
$100\alpha_2$	0.0012	0.0497	0.0915 ***	-0.0148	-0.0018	-0.0517 *
	(0.149)	(0.052)	(0.024)	(0.023)	(0.013)	(0.031)
$100\alpha_3$	-0.2745	-3.7944	-0.0470 ***	0.0963	3.2910	1.4057
	(19.691)	(3.943)	(0.005)	(0.096)	(5.258)	(0.972)
$100\beta_0$	0.0346 ***	0.0303 ***	0.6789 ***	0.3641 ***	0.0090 **	0.0328 ***
	(0.010)	(0.007)	(0.061)	(0.034)	(0.004)	(0.009)
β_1	0.9720 ***	0.8849 ***	0.2196 ***	0.4879 ***	1.1226 ***	0.8712 ***
	(0.068)	(0.079)	(0.033)	(0.032)	(0.121)	(0.079)

The table shows the parameter estimates and the robust standard errors in parentheses of the Aït-Sahalia (1996) model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-1}^2 + \alpha_3 r_{t-1}^{-1} + \varepsilon_t$ and $\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

Table 8: Genral Nonlinear Model with n=250

	Canada	Germany	Japan	Switzerland	UK	USA
α_0	0.0034	0.0446	0.0127 ***	-0.0008	-0.0156	-0.0222 *
	(0.006)	(0.033)	(0.001)	(0.004)	(0.062)	(0.012)
$100\alpha_1$	0.1143	-0.8368	-3.0155 ***	0.1317	0.3402	1.0801 ***
	(0.292)	(0.804)	(0.620)	(0.278)	(0.907)	(0.376)
$100\alpha_2$	-0.0002	0.0700	0.1379 ***	-0.0138	-0.0082	-0.0621 **
	(0.014)	(0.054)	(0.029)	(0.026)	(0.038)	(0.031)
$100\alpha_3$	0.0378	-4.9709	-0.0503 ***	0.0978	5.1400	1.8864 *
	(0.355)	(4.095)	(0.005)	(0.103)	(12.980)	(1.007)
$100\alpha_4$	-0.2216	-0.2793 **	2.1874 ***	-0.0856	-0.2035	-0.4447 ***
	(0.213)	(0.141)	(0.542)	(0.180)	(0.155)	(0.110)
100σ	0.0345 ***	0.0303 ***	0.6759 ***	0.3641 ***	0.0091 **	0.0325 ***
	(0.010)	(0.007)	(0.061)	(0.034)	(0.004)	(0.009)
γ	0.9730 ***	0.8849 ***	0.2224 ***	0.4878 ***	1.1219 ***	0.8734 ***
	(0.067)	(0.079)	(0.033)	(0.032)	(0.121)	(0.079)

The table shows the parameter estimates and the robust standard errors in parentheses of the general nonlinear model , $\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-1}^2 + \alpha_3 r_{t-1}^{-1} + \alpha_4 r_{n,t-1}^{\min} + \varepsilon_t$ and $\sigma_t^2 = \sigma r_{t-1}^{2\gamma}$. $r_{n,t-1}^{\min}$ is the minimum short rate observed the last $n = 250$ days. ***/**/* indicates parameter significance at a 1%/5%/10% level of significance.

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