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Explaining Macroeconomic and Term Structure Dynamics Jointly in a Non-linear DSGE Model

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Explaining Macroeconomic and Term Structure Dynamics Jointly in a Non-linear DSGE Model*

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Abstract

This paper shows how a standard DSGE model can be extended to reproduce the dynamics in the 10 year yield curve for the post-war US economy with a similar degree of precision as in reduced form term structure models. At the same time, we are able to reproduce the dynamics of four key macro variables almost perfectly. Our extension of a standard DSGE model is to introduce three non-stationary shocks which allow us to explain interest rates with medium and long maturities without distorting the dynamics of the macroeconomy.

Keywords: Price stickiness, Stochastic and deterministic trends, Term structure model, The Central Difference Kalman Filter, Yield curve

JEL: E10, E32, E43, E44

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1 Introduction

In recent years much focus has been dedicated to setting up term structure models with macroeconomic variables in an attempt to give economic explanations to the dynamics of the yield curve. Typically in these models, the three well-known factors of level, slope, and curvature from reduced form term structure models are explained by various macroeconomic shocks such as monetary shocks, supply shocks, etc. Another appealing property of these new term structure models is that they introduce information from financial markets into macroeconomic models, and this may be useful when trying to discriminate between various models. For instance, the question of whether prices are flexible or sticky has been a subject of much debate in the macroeconomic literature, primarily because monetary policy is typically considered to have real effects due to the presence of nominal frictions (see for instance Erceg, Henderson & Levin (2000) and Christiano, Eichenbaum & Evans (2005)). This is partly in contrast to recent micro evidence by Bils & Klenow (2004) which indicates that many prices are quite flexible. In this context, the yield curve contains valuable additional information because it embodies the private sector's expectation of future interest rates and thus of future inflation rates. Hence, it is interesting to see whether the dynamics of the yield curve is best explained by sticky or flexible prices.

The reduced form term structure models of the type suggested by Duffie & Kan (1996) and Dai & Singleton (2000), among others, cannot answer questions about the relationship between interest rates and the macroeconomy, but only tell us that the "level factor" or the "slope factor" have changed. However, the ability of reduced form term structure models to reproduce the dynamics of the yield curve is indisputable, and various attempts have therefore been made to introduce macroeconomic variables into these models. In this branch of the literature an exogenous pricing kernel is used, and exogenous affine relations are specified to describe the dynamics in the macroeconomy. Examples of such models are Ang & Piazzesi (2003), Rudebusch & Wu (2004), Ang, Dong & Piazzesi (2005), Dewachter & Lyrio (2006), and Ang, Piazzesi & Wei (2006). As in the case of reduced form macroeconomic models, these term structure models struggle when it comes to identifying the structural shocks driving the economy and therefore also the yield curve. For this reason, another branch of the literature derives micro founded term structure models based on a structural description of the macroeconomy so that shocks to the economy are easy to identify. The papers by Bekaert, Cho & Moreno (2005) and Wu (2005) adopt this strategy for log-linearized DSGE models, and Ravenna & Seppälä (2005), Hördahl, Tristani & Vestin (2007), and Doh (2007) further develop this approach in DSGE models approximated up to second or higher order.

Although shocks in micro founded term structure models are easy to identify, the models often struggle when it comes to reproducing the dynamics of the yield curve, and this has so far limited their use. It is in this perspective that Diebold, Piazzesi & Rudebusch (2005) conclude that *"the goal of an estimated no-arbitrage macro-finance model specified in terms of underlying preferences and technology parameters (so the asset pricing kernel is consistent with the macrodynamics) remains a major challenge"*.

The present paper addresses this challenge by improving the ability of DSGE models to reproduce yield curve dynamics. More specifically, we reproduce the dynamics in the 10 year yield curve for the post-war US economy with a similar degree of precision as reduced form term structure models. At the same time, we are able to reproduce the dynamics of four key macro variables almost perfectly.

Our extension of a standard DSGE model is to introduce three non-stationary shocks into the model. The non-stationary shocks we consider have a stochastic trend and in some case also a deterministic trend.¹ Although the presence of non-stationary shocks is widely used in DSGE models following the work of King, Plosser & Rebelo (1988*a*) and King, Plosser & Rebelo (1988*b*), we use these shocks in two new ways. First, non-stationary preference shocks are introduced as described in Andreasen (2008*a*). This implies that preference shocks may affect the entire yield curve contrary to stationary preference shocks which typically only affect interest rates with short maturities. Second, we allow for both a stationary *and* a non-stationary component in labor productivity and investment shocks around a deterministic trend. Hence, we deviate from the typical approach of having either non-stationary processes or stationary processes for these shocks around a deterministic trend (see Altig, Christiano, Eichenbaum & Linde (2005), Smets & Wouters (2007), among others). We allow for the presence of both components, because the stationary component helps us explain the dynamics of interest rates with short maturities and the four macro variables, while the non-stationary component helps us explain the dynamics of interest rates with medium and long maturities without distorting the dynamics of the four macro variables.

To understand the role of the non-stationary component, recall that all interest rates in a micro founded term structure model are a function of i) future inflation rates and ii) future marginal utility of consumption expressed in relation to current marginal utility of consumption (Cochrane (2001)). Hence, stationary shocks to the economy will in general only have moderate effects on interest rates with medium and long maturities, because the effect of these shocks on future inflation and future marginal consumption is small. On the other hand, since the effect of non-stationary shocks to the economy lasts forever, these shocks significantly affect interest rates with medium and long maturities in our model. Of course, highly persistent stationary shocks may also affect interest rates with medium and long maturities, but these shocks are likely to distort the dynamics of macroeconomy if they alone must explain all interest rates (Rudebusch & Swanson (2008)).

A potential problem when shocks are assumed to have both stationary and non-stationary components is the separate identification of the two components. The inclusion of the yield curve effectively avoids this potential identification problem because the yield curve reacts quite differently to the two types of shocks, as we show in this paper.

We highlight the following results from the estimation of our model on post-war US data. First, our model reproduces almost perfectly the movements in the 10 year yield curve. The standard deviation in all ex post model/measurement errors is only between 2 and 16 annual basis points. Second, the model is simultaneously able to fit the inflation rate and surprisingly also the real growth rates in consumption, investments, and GDP. The ex post model errors are here only between 6 and 11 quarterly basis points. Third, our model partly avoids the bond premium puzzle in the sense that it generates a term premium with an average of 128 annual basis points during the post-war period, provided that we account for minor deviations from the assumption of zero mean values for the innovations to the model. Fourth, all these results are generated without relying on any substantial amount of price stickiness. That is, on average firms reoptimize their prices every 1.1 quarter in our model. We show that the presence of both a stationary and a non-stationary component in labor productivity and investment shocks

¹Stochastic trends are generated by processes with a unit root.

substitutes for the typical high degree of price stickiness found in this type of model.

The rest of this paper is organized as follows. Section 2 presents the macro part of our macro-finance model which consists of a standard DSGE model. The finance part is presented in section 3 where we derive the yield curve implied by the DSGE model. As an application, we estimate our model on post-war US data in section 4. Section 5 concludes.

We do emphasize that the decomposition of our macro-finance model into a macro part and a finance part should not be considered as though the model consists of two independent parts. On the contrary, the macro part and the finance part are intrinsically related due to the general equilibrium structure of our model. The decomposition is only introduced in order to structure the presentation.

2 A DSGE model: The macro part

This section presents a DSGE model which has three types of agents: i) households, ii) firms, and iii) a central bank. The economy is assumed to be driven by mutually independent structural shocks to i) investment technology, ii) labor productivity, iii) firms' fixed costs, iv) households' preferences, and v) the central bank's inflation rate target. Nominal frictions are introduced through sticky prices, and real frictions are added through i) adjustment costs related to new investments and ii) habit formation. For simplicity, we consider a cashless economy as described in Woodford (2003). However, money can easily be introduced in this setup (see for instance Schmitt-Grohé & Uribe (2004a) and Christiano et al. (2005)).

2.1 The households

We assume that the behavior of the households can be described by a representative household. The preferences of this household are specified by a utility function defined over real per capita consumption (c_t) and per capita labor supply (h_t)

$$U_t = E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l} u(c_{t+l} - bc_{t-1+l}, h_{t+l}). \quad (1)$$

Here, E_t denotes the conditional expectation given information available at time t , and $\beta \in [0, 1[$ is the household's discount factor. The function $u(\cdot, \cdot)$ is a period utility index which we assume has the standard form

$$u(c_t - bc_{t-1}, h_t) = \frac{\left[(c_t - bc_{t-1})^{1-\xi_1} (1 - h_t)^{\xi_1} \right]^{1-\xi_2} - 1}{1-\xi_2}, \quad (2)$$

where $b \in [0, 1]$, $\xi_1 \in]0, 1[$, and $\xi_2 \in]0, 1[\cup]1, \infty[$. The parameter b specifies the degree of the internal habit effect in the consumption good. We include habit formation because it improves the ability of this type of model to reproduce certain financial and macroeconomic moments (Campbell & Cochrane (1999), Fuhrer (2000)). Moreover, Hördahl, Tristani & Vestin (2005) show that habit formation can generate a positive term premium and positive correlation

in consumption growth by disentangling the otherwise tight relationship between the marginal rate of intertemporal substitution and the consumption growth rate. We also include preference shocks ($\varepsilon_{h,t}$) in the utility function because Primiceri, Schaumburg & Tambalotti (2006) show that these shocks explain a large fraction of the variation in labor supply, GDP, and investments. Contrary to Primiceri et al. (2006) who specify a stationary process for ε_h , we follow Andreasen (2008a) and specify a non-stationary process for these preference shocks. The process for $\varepsilon_{h,t}$ is specified by defining $\mu_{\varepsilon_h,t+1} \equiv \varepsilon_{h,t+1}/\varepsilon_{h,t}$ and assuming that

$$\ln(\mu_{\varepsilon_h,t+1}) = \rho_{\varepsilon_h} \ln(\mu_{\varepsilon_h,t}) + \epsilon_{\varepsilon_h,t+1}, \quad (3)$$

where $\rho_{\varepsilon_h} \in]-1, 1[$ and $\varepsilon_{h,0} \equiv 1$. The error terms $\{\epsilon_{\varepsilon_h,t}\}_{t=1}^{\infty}$ are assumed to be independent and normally distributed. We denote this by $\epsilon_{\varepsilon_h,t} \sim \mathcal{NID}(0, \sigma_{\varepsilon_h}^2)$. The assumed process for $\varepsilon_{h,t}$ implies that preference shocks may affect the entire yield curve in contrast to stationary preference shocks which typically only affect interest rates with short maturities.

The consumption good is constructed from a continuum of differentiated goods ($c_{i,t}, i \in [0, 1]$) and the aggregation function

$$c_t = \left[\int_0^1 c_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \quad (4)$$

Here, $\eta > 1$ is the intratemporal elasticity of substitution across the differentiated goods. The demand for $c_{i,t}$ with nominal price $P_{i,t}$ is found by solving the optimization problem

$$\underset{c_{i,t} \geq 0}{\text{Min Cost}} = \int_0^1 P_{i,t} c_{i,t} di \quad \text{st.} \quad \left[\int_0^1 c_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \geq c_t. \quad (5)$$

This implies that the demand for good i is given by

$$c_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} c_t, \quad (6)$$

where $P_t \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di \right]^{1/(1-\eta)}$ is the nominal price index in the economy. Thus, the inflation rate is given by $\pi_t \equiv P_t/P_{t-1}$.

The first constraint on the household is the law of motion for the capital stock (k_t) which is assumed to be owned by the household. We follow Christiano et al. (2005) and assume

$$k_{t+1} = (1 - \delta) k_t + i_t \left(1 - S \left(\frac{i_t}{i_{t-1}} \right) \right). \quad (7)$$

The parameter $\delta \in [0, 1]$ is the rate of depreciation of the capital stock, and i_t is gross investments. The function $S \left(\frac{i_t}{i_{t-1}} \right) = \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - \mu_{i,ss} \right)^2$ with $\kappa \geq 0$ adds investment adjustment costs to the economy. We use $\mu_{i,ss}$ to denote the gross growth rate for investments in the steady state.

The second constraint is the household's real period by period budget constraint

$$E_t r_{t,t+1} x_{t+1}^h + c_t + (e_t \Upsilon_t)^{-1} i_t = \frac{x_t^h}{\pi_t} + w_t h_t + \phi_t. \quad (8)$$

The left hand side of (8) is the household's total expenditures in period t which are spent on i) state-contingent claims $(E_t r_{t,t+1} x_{t+1}^h)$, ii) consumption (c_t) , and iii) investments $((e_t \Upsilon_t)^{-1} i_t)$. Here, we follow Greenwood, Hercowitz & Krusell (1997) and allow for a time-varying real price of investments in terms of the consumption good, $(e_t \Upsilon_t)^{-1}$. Changes in this price are often referred to as investment specific shocks or embodied technology shocks as changes in $e_t \Upsilon_t$ are embodied in the economy's capital stock. A key assumption in our model is that investment specific shocks have both a stationary and a non-stationary component. The stationary component is denoted by e_t , and we assume

$$\ln e_{t+1} = \rho_e \ln e_t + \epsilon_{e,t+1}, \quad (9)$$

where $\rho_e \in]-1, 1[$ and $\epsilon_{e,t} \sim \mathcal{NID}(0, \sigma_e^2)$. For the non-stationary component, we define $\mu_{\Upsilon,t} \equiv \Upsilon_t / \Upsilon_{t-1}$ and let

$$\ln \left(\frac{\mu_{\Upsilon,t+1}}{\mu_{\Upsilon,ss}} \right) = \rho_{\Upsilon} \ln \left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}} \right) + \epsilon_{\Upsilon,t+1}, \quad (10)$$

where $\Upsilon_0 \equiv 1$, $\rho_{\Upsilon} \in]-1, 1[$, and $\epsilon_{\Upsilon,t} \sim \mathcal{NID}(0, \sigma_{\Upsilon}^2)$. It is easy to show that the process for Υ_t is non-stationary because $\ln \Upsilon_t$ has a stochastic trend around the deterministic trend $\ln \mu_{\Upsilon,ss}$. Thus, we will henceforth refer to changes in e_t as stationary investment shocks, and they serve the purpose of capturing the short-run variation in the investment technology. Similarly, changes in Υ_t are referred to as non-stationary investment shocks, and they serve the purpose of modeling the long-run changes in the technology for producing new capital. We emphasize that e_t and $\mu_{\Upsilon,t}$ are mutually independent shocks, and so are all other shocks in our model.

The right hand side of (8) describes the household's total wealth in period t . It consists of i) pay-off from state-contingent assets purchased in period $t-1$ (x_t^h / π_t), ii) real labor income ($w_t h_t$), and iii) dividends received from the firms (ϕ_t). We restrict dividend payments to be zero in steady state.

The final constraints are a no-Ponzi-game condition and a no-arbitrage restriction on the gross one-period nominal interest rate, $R_{t,1} \geq 1$.

2.2 The firms

The production in the economy is assumed to be undertaken by a continuum of firms, indexed by $i \in [0, 1]$. Here, we adopt the standard assumption that each firm supplies a differentiable good $(y_{i,t}^s)$ to the goods market, which is characterized by monopolistic competition with no exit or entry. Furthermore, all firms have access to the same technology given by

$$y_{i,t}^s = \begin{cases} F(k_{i,t}, a_t z_t h_{i,t}) - \psi_t z_t^* & \text{if } F(k_{i,t}, a_t z_t h_{i,t}) - \psi_t z_t^* > 0 \\ 0 & \text{else} \end{cases} \quad (11)$$

where $F(\cdot) \equiv k_{i,t}^{\theta} (z_t h_{i,t})^{1-\theta}$ with $\theta \in]0, 1[$ and $\psi_t \geq 0$. Here, $k_{i,t}$ and $h_{i,t}$ denote the amount of capital and labor used by the i 'th firm, respectively. As for the investment specific shocks in the previous section, we also assume that labor productivity shocks have both a stationary and a non-stationary component. The variable a_t denotes the stationary component, and we assume

$$\ln a_{t+1} = \rho_a \ln a_t + \epsilon_{a,t+1}, \quad (12)$$

where $\rho_a \in]-1, 1[$ and $\epsilon_{a,t} \sim \mathcal{NID}(0, \sigma_a^2)$. Hence, changes in a_t represent temporary changes in labor productivity. On the other hand, the variable z_t in (11) denotes the non-stationary component of labor productivity. Letting $\mu_{z,t} \equiv z_t/z_{t-1}$, we assume

$$\ln \left(\frac{\mu_{z,t+1}}{\mu_{z,ss}} \right) = \rho_z \ln \left(\frac{\mu_{z,t}}{\mu_{z,ss}} \right) + \epsilon_{z,t+1}, \quad (13)$$

where $z_0 \equiv 1$, $\rho_z \in]-1, 1[$, and $\epsilon_{z,t} \sim \mathcal{NID}(0, \sigma_z^2)$. Thus, changes in z_t represent long-lasting changes in labor productivity.

Following Altig et al. (2005), we define z_t^* by the relation $z_t^* \equiv \Upsilon_t^{\theta/(1-\theta)} z_t$. So z_t^* is an overall measure of technological progress in the economy. To ensure that our economy has a balanced growth path, it is necessary to scale ψ_t in (11) by z_t^* . This implies that positive changes in the non-stationary components of labor productivity and/or investment shocks also induce increased fixed costs for the firms, and visa versa. We will think of these additional costs as costs the firms have to pay in order to benefit from a permanent improvement in either of the two types of technology.

Smets & Wouters (2003) introduce real supply shocks in a DSGE model similar to ours by letting firms' markup rates be subject to random shocks. With Calvo price contracts, such markup shocks prevent an exact recursive representation of the equilibrium conditions, which is needed for a non-linear approximation of the economy. Instead, we introduce real supply shocks by letting the firms' fixed costs be time-varying beyond the variation in z_t^* . The inclusion of these real supply shocks can be motivated by variation in firms' fixed costs due to changes in i) oil prices, ii) maintenance costs, iii) firms' subsidies, etc. We let

$$\ln \left(\frac{\psi_{t+1}}{\psi_{ss}} \right) = \rho_\psi \ln \left(\frac{\psi_t}{\psi_{ss}} \right) + \epsilon_{\psi,t+1}, \quad (14)$$

where $\epsilon_{\psi,t+1} \sim \mathcal{NID}(0, \sigma_\psi^2)$ and $\rho_\psi \in]-1, 1[$. Hence, changes in ψ_t represent temporary changes in firms' fixed costs, whereas changes in z_t^* model long-lasting changes in these costs.

All firms are assumed to maximize the present value of their nominal dividend payments, denoted $d_{i,t}$. That is, each firm maximizes

$$d_{i,t} \equiv E_t \sum_{l=0}^{\infty} D_{t,t+l} P_{t+l} \phi_{i,t+l}, \quad (15)$$

where $D_{t,t+l}$ is the nominal stochastic discount factor and the expression for the real dividend payments from the i 'th firm $\phi_{i,t}$ is given below in (17). The firms face four constraints when maximizing $d_{i,t}$. The first is related to the good produced by the i 'th firm. The total amount of good i is allocated to consumption and investments. We make the standard assumption that the aggregation function for the latter component coincides with the aggregation function for consumption in (4). With cost minimization in the production of aggregate investments, the restriction on aggregate demand can therefore be written as

$$y_t^d = c_t + (e_t \Upsilon_t)^{-1} i_t. \quad (16)$$

In addition, we make the standard assumption that firms satisfy demand, i.e. $y_{i,t}^s \geq y_{i,t}^d \forall i \in [0, 1]$.

The second constraint is the budget restriction, which gives rise to the expression for real dividends from firm i in period t

$$\phi_{i,t} = (P_{i,t}/P_t) y_{i,t}^d - r_t^k k_{i,t} - w_t h_{i,t} - E_t r_{t,t+1} x_{i,t+1}^f - \pi_t^{-1} \left(x_{i,t}^f \right). \quad (17)$$

The first term in (17) denotes the real revenue from sales of the i 'th good. The firm's expenditures are allocated to purchase of capital services ($r_t^k k_{i,t}$) and payments to the workers ($w_t h_{i,t}$). The final two terms in (17) are the change in the firm's real financial wealth.

The third constraint introduces sticky prices a la Calvo (1983). That is, in each period a fraction $\alpha \in [0, 1[$ of randomly picked firms are not allowed to set the optimal nominal price of the good they produce. Instead, these firms set the current prices equal to the prices from the previous period, i.e. $P_{i,t} = P_{i,t-1}$. The fourth constraint is a no-Ponze-game condition.

2.3 The central bank

We let the central bank determine the nominal interest rate according to a forward-looking Taylor rule of the form

$$\ln \left(\frac{R_{t,1}}{R_{ss,1}} \right) = \alpha_R \ln \left(\frac{R_{t-1,1}}{R_{ss,1}} \right) + \alpha_\pi E_t \ln \left(\frac{\pi_{t+1}}{\pi_{t+1}^*} \right) + \alpha_y E_t \ln \left(\frac{y_{t+1}^d / y_t^d}{\mu_y} \right), \quad (18)$$

where π_t^* is a time-varying inflation rate target. The parameters in this rule are subject to the constraints i) $\alpha_R \in [0, 1]$, ii) $\alpha_\pi \geq 0$, and iii) $\alpha_y \geq 0$. We use a standard specification of the inflation rate target (π_t^*) by letting this target be a weighted sum of all previous inflation rates and a noise component (see Bekaert et al. (2005))

$$\pi_t^* = (1 - \omega^*) \sum_{j=0}^{\infty} (\omega^*)^j \left(\pi_{t-1-j} + \frac{\epsilon_{\pi^*,t-j}}{1 - \omega^*} \right). \quad (19)$$

Thus, the law of motion for π_t^* is

$$\pi_t^* = (1 - \omega^*) \pi_{t-1} + \omega^* \pi_{t-1}^* + \epsilon_{\pi^*,t}. \quad (20)$$

We emphasize that this inflation rate target is partly endogenously determined through values of lagged inflation and partly exogenously determined through lagged values of $\epsilon_{\pi^*,t}$. From the derivation of (20), it follows that the parameter ω^* can be interpreted as the degree of backward looking-ness in the inflation rate target. We impose the restriction $\omega^* \in [0, 1)$ to get smooth changes in the target and assume $\epsilon_{\pi^*,t} \sim \mathcal{NID}(0, \sigma_{\pi^*}^2)$. Hence, for $\omega^* \rightarrow 1$ the inflation rate target in (20) evolves as a random walk as in Smets & Wouters (2003), and if we also have $\epsilon_{\pi^*,t} = 0$ for all t , then we get a constant inflation rate target as in Wu (2005), Ravenna & Seppälä (2005), Fernández-Villaverde & Rubio-Ramirez (2007b), among others.

2.4 Solving our DSGE model

It is straightforward to set up i) the market clearings conditions, ii) the first order conditions for the household, and iii) the first order conditions for the firms in our economy.² The presence of non-stationary shocks in the model implies that variables such as consumption (c_t) and the household's marginal value of income (λ_t) are non-stationary, and this fact must be taken into account when solving the model. We adopt the standard method to deal with this feature by approximating the model's solution around the economy's balanced growth path. This is done by scaling the non-stationary variables such that they become stationary. For instance, scaling c_t by $1/z_t^*$ and λ_t by $1/(z_t^{*(1-\phi_3)(1-\phi_4)-1}\varepsilon_{h,t})$ implies that $C_t \equiv \frac{c_t}{z_t^*}$ and $\Lambda_t \equiv \frac{\lambda_t}{z_t^{*(1-\phi_3)(1-\phi_4)-1}\varepsilon_{h,t}}$ are stationary variables. Based on this equivalent representation of our economy, standard solution methods for DSGE models can be applied.

The solution to our DSGE model is approximated by the perturbation method, and the pruning scheme is applied to this approximation. We will briefly summarize this approximation method in order to introduce some notation which will be useful below. First, the law of motion for the economy's control vector \mathbf{y}_t of dimension 7×1 and the state vector \mathbf{x}_t of dimension 13×1 are

$$\mathbf{y}_t = \mathbf{g}(\mathbf{x}_t, \sigma) \quad (21)$$

$$\mathbf{x}_{t+1} = \mathbf{h}(\mathbf{x}_t, \sigma) + \sigma\boldsymbol{\eta}\boldsymbol{\epsilon}_{t+1}. \quad (22)$$

The innovations to the structural shocks are denoted by $\boldsymbol{\epsilon}_{t+1}$ which has dimensions 7×1 . The parameter σ is the perturbation parameter scaling the covariance matrix ($\boldsymbol{\eta}$) for the structural shocks. The functions $\mathbf{g}(\mathbf{x}_t, \sigma)$ and $\mathbf{h}(\mathbf{x}_t, \sigma)$ are unknown and are therefore approximated up to second order as described by Schmitt-Grohé & Uribe (2004b).

Second, the pruning scheme implies that terms of higher order than two are ignored in the approximation when the model is iterated forward in time. We emphasize that Kim, Kim, Schaumburg & Sims (2003) strongly recommend the use of this pruning scheme based on two arguments. First, terms of higher order than two do not necessarily improve the accuracy of the second order approximation, because they do not correspond to the additional terms in a more accurate Taylor series expansion. Second, the pruning scheme generates non-explosive sample paths when simulating time series from the approximated model.

Thus, we get the following approximated law of motions

$$\mathbf{y}_t = \mathbf{g}_{\mathbf{x}}(\mathbf{x}_t^f + \mathbf{x}_t^s) + \frac{1}{2}\mathbf{G}_{\mathbf{xx}}\text{vec}\left(\mathbf{x}_t^f(\mathbf{x}_t^f)'\right) + \frac{1}{2}\mathbf{g}_{\sigma\sigma}\sigma^2 \quad (23)$$

$$\mathbf{x}_{t+1}^f = \mathbf{h}_{\mathbf{x}}\mathbf{x}_t^f + \sigma\boldsymbol{\eta}\boldsymbol{\epsilon}_{t+1} \quad (24)$$

$$\mathbf{x}_{t+1}^s = \mathbf{h}_{\mathbf{x}}\mathbf{x}_t^s + \frac{1}{2}\mathbf{H}_{\mathbf{xx}}\text{vec}\left(\mathbf{x}_t^f(\mathbf{x}_t^f)'\right) + \frac{1}{2}\mathbf{h}_{\sigma\sigma}\sigma^2 \quad (25)$$

$$\mathbf{x}_t \equiv \mathbf{x}_t^f + \mathbf{x}_t^s \quad (26)$$

$$\mathbf{G}_{\mathbf{xx}} \equiv \left[\text{vec}(\mathbf{g}_{\mathbf{xx}}^1) \quad \text{vec}(\mathbf{g}_{\mathbf{xx}}^2) \quad \dots \quad \text{vec}(\mathbf{g}_{\mathbf{xx}}^{n_y}) \right]' \quad (27)$$

$$\mathbf{H}_{\mathbf{xx}} \equiv \left[\text{vec}(\mathbf{h}_{\mathbf{xx}}^1) \quad \text{vec}(\mathbf{h}_{\mathbf{xx}}^2) \quad \dots \quad \text{vec}(\mathbf{h}_{\mathbf{xx}}^{n_x}) \right]' \quad (28)$$

²We refer to the paper's technical appendix for details on how this is done. The technical appendix is available from the author's homepage.

where n_y and n_x are the size of \mathbf{y}_t and \mathbf{x}_t , respectively. We use $vec(\cdot)$ to denote the operator stacking all columns in a matrix into a column vector, and the matrices \mathbf{g}_x , \mathbf{G}_{xx} , $\mathbf{g}_{\sigma\sigma}$, \mathbf{h}_x , \mathbf{H}_{xx} , and $\mathbf{h}_{\sigma\sigma}$ contain the first and second order derivatives of the two unknown functions. Notice finally that the state vector is decomposed into first order effects \mathbf{x}_t^f and second order effects \mathbf{x}_t^s .

It is important to realize that we do not need to include the entire yield curve when solving the macro part of our model. Only the value of the one-period interest rate is needed, and this property of our model is standard in term structure models based on a DSGE model (see for instance, Bekaert et al. (2005), Wu (2005), Doh (2007), and Hördahl et al. (2007)). The study conducted by Diebold, Rudebusch & Aruoba (2006) on the post-war US economy finds empirical support for such an assumption, because they conclude that changes in the yield curve have small effects on the macroeconomy. Hence, we can start by solving the macro part of our model and then derive the yield curve afterwards.

2.5 Ensuring finite objective functions

As pointed out by Andreasen (2008a), the normal requirement of having $\beta < 1$ may not be sufficient to ensure that the household's and the firms' objective functions are finite when stochastic and deterministic trends are included in a DSGE model. The model considered by Andreasen (2008a) has a similar structure as our model, and we can therefore use proposition 1(a) in Andreasen (2008a) to ensure that the objective functions of the household and the firms are finite. Since we have assumed that innovations to the log-transformed non-stationary shocks are normally distributed, this gives rise to the conditions

$$\exp \left\{ \frac{\sigma_{\varepsilon_h}^2}{2(1-\rho_{\varepsilon_h})^2} \right\} \beta < 1 \quad (29)$$

$$\exp \left\{ \frac{\sigma_{\varepsilon_h}^2}{2(1-\rho_{\varepsilon_h})^2} \right\} \exp \left\{ \frac{F_{\Upsilon}^2 \sigma_{\Upsilon}^2}{2(1-\rho_{\Upsilon})^2} \right\} \exp \left\{ \frac{F_z^2 \sigma_z^2}{2(1-\rho_z)^2} \right\} \beta \mu_{\Upsilon,ss}^{F_{\Upsilon}} \mu_{z,ss}^{F_z} < 1 \quad (30)$$

$$F_{\Upsilon} \equiv (1-\xi_1)(1-\xi_2) \frac{\theta}{1-\theta} \quad (31)$$

$$F_z \equiv (1-\xi_1)(1-\xi_2) \quad (32)$$

and a boundedness condition. Andreasen (2008a) shows that this boundedness condition is satisfied if we assume that all variables in the economy are never too far away from the economy's growth path. Given this assumption, the representative household and the firms have finite objective functions provided that (29) and (30) hold. We therefore impose (29) and (30) throughout.

3 The finance part

The content of this section is as follows. We start by deriving the yield curve implied by our DSGE model. This is done based on the micro founded pricing kernel and no-arbitrage arguments. We then compare our pricing kernel with the typical pricing kernel in reduced form term structure models from the finance literature.

3.1 The yield curve

The presence of state contingent claims in the DSGE model implies that we can price all financial assets in the economy based on no-arbitrage arguments. Hence, the price of a zero-coupon bond maturing k periods into the future and paying 1 dollar at maturity is

$$P_{t,k} = E_t \left[\beta^k \frac{\lambda_{t+k}}{\lambda_t} \prod_{j=1}^k \frac{1}{\pi_{t+j}} \right]. \quad (33)$$

The value of λ_t is determined from the household's first-order condition for consumption and is given by

$$\lambda_t = \varepsilon_{h,t} u_c(c_t - bc_{t-1}, 1 - h_t) - b\beta E_t \varepsilon_{h,t+1} u_c(c_{t+1} - bc_t, 1 - h_{t+1}). \quad (34)$$

With discrete compounded interest rates, we have $R_{t,k}^{-k} \equiv P_{t,k}$, where $R_{t,k}$ is the gross interest rate in period t for a bond maturing in k periods. Hence, all interest rates in our model are determined by the behavior of i) consumption (c_t), ii) labor supply (h_t), iii) inflation (π_t), and iv) preference shocks ($\varepsilon_{h,t}$).

A potential problem in relation to (33) and (34) might be that the endogenously determined process for consumption is non-stationary, as is the exogenous process for the preference shocks. On the other hand, the processes for h_t and π_t are stationary in our model. However, the restrictions in the DSGE model which generate a balanced growth path also ensure that the non-stationary part of the consumption process and the non-stationary part of the preference shocks cancel out in the yield curve. In other words, even though the household's marginal utility of income evolves as a non-stationary process, the processes for λ_{t+k} and λ_t co-move such that the ratio λ_{t+k}/λ_t forms a stationary relation. Hence, the yield curve from our model is a set of non-linear cointegrated relations. The following equivalent expression of the yield curve verifies this statement

$$\ln R_{t,k} = \frac{1}{-k} \ln E_t \left[\beta^k \frac{\Lambda_{t+k}}{\Lambda_t} \prod_{j=1}^k \left(\left(\mu_{\Upsilon,t+j}^{\frac{\theta}{1-\theta}} \mu_{z,t+j} \right)^{(1-\xi_1)(1-\xi_2)-1} \mu_{\varepsilon_{h,t+j}} \frac{1}{\pi_{t+j}} \right) \right], \quad (35)$$

because the yield curve is expressed solely in terms of stationary variables. Thus, our model reconciles the non-stationary behavior of consumption from the macro literature with the assumption of stationary interest rates in the finance literature.

In general, it is difficult to generate movements in the yield curve where the standard deviations at all maturities are only marginally different from the standard deviation of the short interest rate. The three stochastic trends in our yield curve (besides the stochastic trend in prices) are helpful in matching this feature because the non-stationary shocks are propagated almost equally through the yield curve.

3.2 The micro founded pricing kernel

This section compares our pricing kernel to the pricing kernel in a typical reduced form term structure model from the finance literature. For this purpose, recall that $D_{t,t+1}$ is the nominal stochastic discount factor in period t for payments in period $t+1$. As shown by Doh (2007), the law of motion for the state variables in (24) and (25) and the normality of the structural shocks

imply that we can evaluate the expectation $E_t [D_{t,t+1}]$ without further approximations. Based on this result, we get the following expression for the stochastic discount factor³

$$\ln D_{t,t+1} = -\ln R_{t,1} + \frac{1}{2} \ln (|\mathbf{I} - 2\boldsymbol{\lambda}_2|) - 0.5\boldsymbol{\lambda}_{1,t}' [I - 2\boldsymbol{\lambda}_2]^{-1} \boldsymbol{\lambda}'_{1,t} - \boldsymbol{\lambda}_{1,t}\boldsymbol{\epsilon}_{t+1} + \boldsymbol{\epsilon}'_{t+1}\boldsymbol{\lambda}_2\boldsymbol{\epsilon}_{t+1}, \quad (36)$$

where we have defined

$$\boldsymbol{\Gamma}_4 \equiv [\mathbf{g}_x^\Lambda - \mathbf{g}_x^\pi + H_\Upsilon \mathbf{e}_\Upsilon + H_z \mathbf{e}_z + \mathbf{e}_{\varepsilon_h}] \boldsymbol{\eta} \sigma \quad (37)$$

$$\boldsymbol{\Gamma}_5 \equiv 2\sigma \mathbf{h}'_x \left[\frac{1}{2} \mathbf{g}_{xx}^\Lambda - \frac{1}{2} \mathbf{g}_{xx}^\pi \right] \boldsymbol{\eta} \quad (38)$$

$$\boldsymbol{\lambda}_2 \equiv \sigma^2 \boldsymbol{\eta}' \left[\frac{1}{2} \mathbf{g}_{xx}^\Lambda - \frac{1}{2} \mathbf{g}_{xx}^\pi \right] \boldsymbol{\eta} \quad (39)$$

$$\boldsymbol{\lambda}_{1,t} \equiv -\boldsymbol{\Gamma}_4 - \left(\mathbf{x}_t^f \right)' \boldsymbol{\Gamma}_5 \quad (40)$$

We use the notation $\mathbf{e}_i \mathbf{x}_t^f = \mu_{i,t}$ for $i = \{\Upsilon, z, \varepsilon_h\}$. Expressions of the form \mathbf{g}_x^π and \mathbf{g}_{xx}^π refer to the first and second order effects of the state variables on inflation π_t , and similarly for Λ_t . Finally, $H_z \equiv (1 - \xi_2)(1 - \xi_1) - 1$ and $H_\Upsilon \equiv \frac{\theta}{1-\theta} H_z$. It turns out that all elements in the matrix $\boldsymbol{\lambda}_2$ are very small for realistic values of the structural parameters. Hence, when comparing our pricing kernel to the finance literature, we can for a moment disregard the effect of $\boldsymbol{\lambda}_2$ such that

$$\ln D_{t,t+1} \simeq -\ln R_{t,1} - 0.5\boldsymbol{\lambda}_{1,t}' \boldsymbol{\lambda}'_{1,t} - \boldsymbol{\lambda}_{1,t}\boldsymbol{\epsilon}_{t+1}. \quad (41)$$

Thus, when we ignore the effect of $\boldsymbol{\lambda}_2$, then $\boldsymbol{\lambda}_{1,t}$ can be interpreted as the market price of risk because $\boldsymbol{\lambda}_{1,t}$ is the additional compensation the representative household requires in order to accept an additional amount of uncertainty. Note also that this market price of risk is time-varying due to the second order terms in the approximation of our DSGE model.⁴ Actually, the market price of risk is affine in the state vector as assumed in many reduced form term structure models in the finance literature (see Duffee (2002), Dai & Singleton (2002)). Apart from this similarity, we do emphasize that the micro foundation in our model makes our pricing kernel very different from the pricing kernel in reduced form finance models, since the micro foundation introduces a large number of cross-restrictions between the elements in our pricing kernel. For instance, the parameters in the affine relation for the market price of risk are not free parameters but are determined by the deep structural parameters of our model.

Using the method by Doh (2007), it is straightforward to derive the following expression for the yield curve

$$\ln R_{t,k} = \frac{-1}{k} \left(a_k + \mathbf{b}_k^f \mathbf{x}_t^f + \mathbf{b}_k^s \mathbf{x}_t^s + \left(\mathbf{x}_t^f \right)' \mathbf{c}_k \left(\mathbf{x}_t^f \right) \right), \quad (42)$$

where the recursive formulas for the coefficients are given in the appendix.

³We refer to the paper's technical appendix for a proof of this statement.

⁴The method by Doh (2007) evaluates $E_t [D_{t,t+1}]$ directly to find the bond prices. As a result, the term $\left(\boldsymbol{\Gamma}_4 + \left(\mathbf{x}_t^f \right)' \boldsymbol{\Gamma}_5 \right) \boldsymbol{\epsilon}_{t+1}$ enters into the expression for the pricing kernel and generates the time-varying market price of risk. The method by Hördahl et al. (2005) relies on an approximated expression of the pricing kernel and the moments $E_t \left[\ln \left(\frac{D_{t,t+1}}{D_{ss}} \right) \right]$ and $Var_t \left[\ln \left(\frac{D_{t,t+1}}{D_{ss}} \right) \right]$ to derive bond prices. In Hördahl et al. (2005), the first moment is evaluated based on a second order approximation of the model, whereas the variance is evaluated based on a first order approximation. By applying these approximations, the term $\left(\boldsymbol{\Gamma}_4 + \left(\mathbf{x}_t^f \right)' \boldsymbol{\Gamma}_5 \right) \boldsymbol{\epsilon}_{t+1}$ does not appear in the pricing kernel and hence the method generates a constant market price of risk.

4 Application: Post-war US data

This section estimates our model on post-war US data. We start out by describing the data and our estimation methodology in subsection 4.1. Then estimation results are reported in subsection 4.2, and the model’s ability to reproduce the dynamics in four macro series and the yield curve is examined in subsection 4.3. The implied term premium from the model is studied in subsection 4.4. The section is closed with a specification analysis and a robustness analysis in subsections 4.5 and 4.6, respectively.

4.1 Data and the estimation methodology

We use quarterly data from the Federal Reserve Bank of St. Louis covering the period 1957:Q1-2006:Q4.⁵ The 10 year yield curve expressed in annually terms is represented by the 3 month, 1 year, 3 year, 5 year, and 10 year interest rates.⁶ The dynamics of the macroeconomy are represented by i) the quarterly inflation rate in consumer prices, and the real quarterly growth rates in ii) consumption, iii) investments, and iv) GDP.⁷ These nine series are placed in the vector \mathbf{y}_t^{obs} . For the subsequent estimation, we allow for measurement errors in the series for \mathbf{y}_t^{obs} and assume that these errors (\mathbf{v}_t) are of the form $\mathbf{v}_t \sim \mathcal{NID}(\mathbf{0}, \mathbf{R}_v)$, where \mathbf{R}_v is a diagonal matrix.

The set of structural parameters in our model is partitioned into two groups. The first group of parameters is not estimated but determined based on calibration arguments and previous findings in the literature. The second group consists of all the remaining structural parameters which are estimated. We emphasize that this partitioning of the structural parameters is the usual practice when taking large DSGE models to the data (see Christiano et al. (2005), Smets & Wouters (2007), Justiniano & Primiceri (2008), among others).

The parameter in the firms’ production function θ is calibrated based on the capital share in the postwar US economy. That is, we let $\theta = 0.36$, which is a standard benchmark for θ (see Altig et al. (2005), Christiano et al. (2005), among others). For a given value of θ , the mean values of the real growth rate in GDP (1.0052) and the real growth rate in investments (1.0070) can be used to find $\mu_{z,ss}$ and $\mu_{\Upsilon,ss}$. This follows from the fact that the real growth rate for GDP and investments in steady state are given by

$$\mu_{y,ss} = \mu_{\Upsilon,ss}^{\frac{\theta}{1-\theta}} \mu_{z,ss} \quad (43)$$

$$\mu_{i,ss} = \mu_{\Upsilon,ss}^{\frac{1}{1-\theta}} \mu_{z,ss}. \quad (44)$$

Thus, the calibration implies $\mu_{z,ss} = 1.0042$ and $\mu_{\Upsilon,ss} = 1.0018$. The depreciation rate is often hard to identify, so we follow Christiano et al. (2005), Smets & Wouters (2007), Justiniano

⁵The first two years of data are used to initialize our filter.

⁶The 3 month interest rate is measured by the rate in the secondary market (TB3MS). The remaining interest rates are measured by the 1 year, 3 year, 5 year, and 10 year treasury constant maturity rates (GS1, GS3, GS5, GS10).

⁷The growth rate in consumption is calculated from real consumption expenditures (PCECC96). We use the series for real private fixed investments (FPIC96) to calculate the growth rate in investments. Finally, the growth rate in GDP is calculated from real GDP (GDPC96). All growth rates are expressed in per capita based on the total population in the US.

& Primiceri (2008), among others and let $\delta = 0.025$. The inflation rate in the steady state is calibrated to the empirical average for the quarterly inflation rate in our sample, i.e. $\pi_{ss} = 1.0089$. Finally, the perturbation parameter σ turns out to be not identified and we therefore let $\sigma = 0.01$.

To further economize on the number of parameters to be estimated, we let the standard deviation in the measurement errors for inflation and the three real growth rates be equal to 20 basis points. The level of these measurement errors seems realistic and ensures a proper balance between fitting jointly the interest rates and the four macro variables. Lowering the level of these measurement errors would lead to a better fit of the four macro variables at the expense of a less precise fit to the interest rates, and vice versa.

For the variance of the measurement errors along the yield curve, the following parsimonious specification is adopted

$$\ln Var_k = \gamma_0 + \gamma_1 k + \gamma_2 k^2, \quad (45)$$

where k denotes the maturity of the interest rates. The Nelson-Siegel curve or extensions of this curve as presented in Björk & Christensen (1999) could also be used in this context. However, the results in Diebold et al. (2006) display a parabola form for the variances in the measurement errors along the yield curve corresponding to $\gamma_2 \geq 0$. Hence, we settle with the specification in (45) and impose the condition $\gamma_2 \geq 0$.

All the remaining parameters are estimated by Quasi-Maximum Likelihood (QML) based on the Central Difference Kalman Filter (CDKF) as developed by Norgaard, Poulsen & Ravn (2000). The CDKF is an extension of the standard Kalman Filter to non-linear and non-normal state space systems where the non-linear moments in the filtering equations are approximated at least up to second order accuracy. For DSGE models approximated up to second order, Andreasen (2008c) shows that this QML estimator can be expected to be consistent and asymptotically normal even if non-normal shocks are hitting the economy.

Several arguments motivate our choice of the QML estimator. First, calculating the quasi log-likelihood function by the CDKF only takes a fraction of a second, and this makes our estimation feasible. Second, Andreasen (2008c) shows in a Monte Carlo study that the CDKF delivers the same or even better performance compared to particle filters when more than three shocks are hitting the economy. Here, performance of filters is measured by the root mean squared errors for the state estimates. Third, the quasi log-likelihood function based on the CDKF is a smooth function of the structural parameters, and this makes optimization of the quasi log-likelihood function much easier than optimizing a non-smooth approximated log-likelihood function from a particle filter. A well-known disadvantage of any QML estimator is their possible lack of efficiency which we acknowledge. However, it remains to be shown for large non-linear DSGE models like ours that the efficiency of using Maximum Likelihood (ML) outweighs the approximation error in the log-likelihood function from a particle filter and thus leads to more precise ML estimates than our QML estimates.

4.2 Estimation results

The estimation results for our benchmark model are reported in table 1. Throughout this paper, a modified version of the CMA-ES routine is used to maximize the quasi log-likelihood function. Andreasen (2008b) shows that this modified version of the CMA-ES routine is very successful at optimizing likelihood functions for DSGE models.

We find the degree of the internal habit effect to be of a moderate size ($b = 0.31$). The household is seen to spend approximately 34% of their time working and the remaining 66% on leisure ($\xi_1 = 0.66$). As for the parameter governing the degree of curvature in the household's utility function, we find $\xi_2 = 3.31$. The size of the adjustment costs in investments is found to be low ($\kappa = 0.27$) and so is the degree of price stickiness ($\alpha = 0.09$). Hence, on average firms reoptimize their prices every 1.1 quarter, and it is thus obvious that the degree of price stickiness is low in our benchmark model. The intratemporal elasticity of substitution between goods is also low ($\eta = 1.11$) and this leads to an extremely large price markup which is hard to justify based on micro economic evidence. We examine the robustness of the latter two findings in subsection 4.6.

The central bank is seen to focus mostly on stabilizing inflation ($\alpha_\pi = 10.31$) compared to stabilizing aggregated demand ($\alpha_y = 8.10$). Note in this context that the parameter α_π is estimated with a high degree of uncertainty.⁸ For the inflation rate target, the parameter ω^* is estimated to be zero, meaning that $\pi_t^* = \pi_{t-1} + \epsilon_{\pi^*,t}$. Thus, expressing inflation and the inflation target in percentage deviation from the steady state, we find that the central bank reacts to the expected growth rate in inflation.

The growth rate in the non-stationary component of investment shocks displays a high degree of persistency ($\rho_\Upsilon = 0.85$). On the other hand, the growth rate in the non-stationary component of labor productivity shocks evolves almost as a random walk ($\rho_z = 0.01$). The three auto-correlated stationary shocks all display a fairly high degree of persistency. In particular, the stationary component of labor productivity a_t which has a persistency parameter of $\rho_a = 0.98$. These estimates of ρ_z and ρ_a imply that the impulse response patterns in labor productivity following a non-stationary and a stationary shock, respectively, are very similar. However, as shown in figure 1 the response in the yield curve is quite different for the two shocks, because a non-stationary productivity shock induces additional increases in firms' fixed costs. Such additional fixed costs are not present in the case of a stationary productivity shock.

< Figure 1 about here >

4.3 Evaluating the fit of the macro-finance model

Figure 2 displays the nine series in our sample. As we will demonstrate shortly, our model performs remarkably well in terms of reproducing the dynamics in the nine series, and this makes the difference between the actual series and the series implied by the model very small.⁹ Hence, we do not plot the actual series and the model implied series in the same diagram because

⁸A word of caution is in order in relation to the reported standard errors in table 1. Due to instability in the Hessian matrix, standard errors are calculated based on the outer product of the score function. Andreasen (2008c) shows in a Monte Carlo study that this procedure gives quite accurate estimates of the standard errors in the case where the shocks are normally distributed. However, our shocks display evidence of non-normality in the form of thicker tails than the normal distribution. In such a setting Andreasen (2008c) shows that calculating the standard errors by the outer product of the score function has a negative bias.

⁹All model implied series are calculated based on the filtered and not the smoothed state estimates. The reason being that the presence of growth rates in the set of observables induces an extended state space system where we estimate $\tilde{\mathbf{x}}_t = [\mathbf{x}'_t \quad \mathbf{x}'_{t-1}]'$, and in this state space system we cannot calculate the smoothing gain with a sufficient degree of precision. A small simulation study for the benchmark model shows that the total mean squared errors for the filtered state estimates are 0.0413, which is quite low given the seven shocks to the model (Andreasen (2008c)).

differences between the two are hard to locate. Instead, model/measurement errors are plotted next to each of the nine series in figure 2 where model errors are expressed in basis points.¹⁰

We first notice from figure 2 that our model fits all five interest rates extremely well and model errors are in general no larger than 20 basis points. The exceptions are for the 3 month and the 1 year interest rates around 1980 during the Fed experiment where larger model errors are present.

< Figure 2 about here >

The first four rows in table 2 report various statistics to evaluate the goodness of fit of our macro-finance model. Here, we notice that all model errors have mean values very close to zero (between 2 and -6 basis points). Also the standard deviation for the ex ante model errors are small, and the standard deviations in the ex post model errors are even smaller - all between 2 and 16 basis points.

The ability of our model to reproduce the dynamics of the 10 year yield curve is thus similar or even better than the performance of standard three factor affine term structure models. For instance, Cheridito, Filipovic & Kimmel (2007) report the standard deviations in model errors for these models to be between 10 and 25 basis points. Of course, a fair comparison of our macro-finance model to these models is difficult due to the very different nature of the two types of models. However, it is important to notice that our model is not automatically more flexible than three factor affine term structure models for the following two reasons. First, DSGE models typically use the same number of structural shocks as the number of observables. Hence, four out of our seven shocks could be taken to match the four macro series. This leaves us with three shocks to match the yield curve like in three factor affine term structure models. Second, the presence of a micro foundation imposes more cross restrictions in our model than in reduced form models.

Turning to the fit of the four macro series, figure 2 shows that our model is also able to reproduce the dynamics in the inflation series and the dynamics in the three real growth rates. In general, model errors are again seen to be no larger than 20 basis points. The mean value of the model errors are again very close to zero, and the ex post standard deviations in these errors are only between 6 and 11 basis points. This gives rise to correlation coefficients of 0.99 or higher between the actual series and the model implied series. Especially the high correlation coefficients for the three real growth rates are remarkable because each of these series displays a very low degree of persistency and is therefore difficult to fit.

Another way to measure the ability of our model to capture the dynamics in the yield curve is to examine how well it reproduces the dynamics of interest rates not included in the estimation. The 2 year and the 7 year interest rates as reported by the Federal Reserve Bank of St. Louis start in 1976Q2 and 1969Q3, respectively, and were therefore not included in the estimation. Figure 3 plots the two series and the corresponding series implied by our model. Again model errors are plotted next to each of the two graphs to better evaluate the performance of our model. Inspection of the graphs in figure 3 shows that our model is quite successful in also explaining these interest rates. The mean values of the model errors are 4 and 3 annual basis points,

¹⁰Recall that 1% corresponds to 100 basis points.

respectively, and the ex post standard deviations are only 5 and 7 basis points, respectively.¹¹ Thus, our model is also quite successful at fitting interest rates not included in the estimation.

< Figure 3 about here >

Summarizing, our benchmark model is able to reproduce the dynamics of the 10 year yield curve with the same degree of precision as reduced form models. This ability of our model is achieved while simultaneously fitting four key macro variables. Moreover, the model performs well in terms of reproducing interest rates not included in the estimation.

4.4 The term premium

In two recent papers, Rudebusch, Sack & Swansson (2007) and Rudebusch & Swanson (2008) show that a standard DSGE model is unable to generate a realistic term premium without distorting the dynamics of the macroeconomy, a phenomenon referred to as "the bond premium puzzle". As we have documented above, our model captures quite accurately the dynamics of the macroeconomy and also the dynamics of the yield curve. It is therefore interesting to analyze whether the model also suffers from the bond premium puzzle. We follow Rudebusch et al. (2007) and define the term premium for a zero-coupon bond with k periods to maturity as

$$TP(k)_t = R_{t,k} - \frac{1}{k} \sum_{i=0}^{k-1} E_t [R_{t+i,1}] \quad (46)$$

< Figure 4 about here >

Figure 4 shows the model implied term premium during our estimation period. The mean value of this premium is -16 annual basis points and its standard deviation is 64 annual basis points. The corresponding sample moments are in the neighborhood of 106 and 54 annual basis points, respectively, according to Rudebusch & Swanson (2008). Thus, our model is able to reproduce the standard deviation in the term premium, but not its level.

The inability of our model to match the level of the term premium is a bit surprising given the model's good performance as stated above. Note here that the term premium on average is equal to the slope of the yield curve (Rudebusch & Swanson (2008)), and our model reproduces quite accurately the slope of the yield curve during the estimation period. The correlation coefficient between the actual slope and the model implied slope is as high as 0.9893. Thus, the low level of the term premium calculated by (46) must imply that the estimated shocks in our model in some way deviate from the stated assumptions, and ignoring these deviations generates too high values of $E_t [R_{t+i,1}]$, which makes the term premium too low. As we will argue in the next section, all of the stated assumptions for the innovations (ϵ_t) are reasonably satisfied in our case. Nevertheless, it turns out that ignoring minor deviations from the zero-mean assumption in the innovations is what generates the low level of the term premium. That is, if we impose the same mean value for ϵ_t when generating $E_t [R_{t+i,1}]$ as we observe in the estimated innovations during our sample period, then we get a term premium with a mean value of 128 annual basis points and a

¹¹The corresponding correlation coefficients between actual series and model implied series are 0.9999 for the 2 year interest rate and 0.9997 for the 7 year interest rate.

standard deviation of 64 annual basis points. Figure 4 also plots the term premium as calculated under this assumption. Calculating the term premium under the alternative assumption that innovations have the same contemporaneous correlation and auto-correlation as found in the estimated innovation has only a minor effect on the level of the term premium. Thus, only the mean value of the innovation is important in this context.

To summarize, our model partly avoids the bond premium puzzle in the sense that it generates a realistic term premium provided that we impose the same mean value for the innovations as that observed in the sample period.

4.5 Specification analysis for the estimated innovations

The findings in the previous section indicate that the estimated innovations deviate from the assumptions that $\epsilon_t \sim \mathcal{NID}(\mathbf{0}, \mathbf{I})$. This subsection therefore examines the statistical properties of the estimated innovations in the benchmark model in further detail.

The first assumption about the innovations is that they should have a mean value of zero. Table 3 reports the mean values for the estimated innovation during our sample period. We see that most values are close to zero, except for innovations in the inflation rate target $\hat{\epsilon}_{\pi^*,t}$ and in preference shocks $\hat{\epsilon}_{\varepsilon_h,t}$ which have mean values of -0.42 and 0.25, respectively.

The second assumption analyzed is that of mutually independent innovations, and thus contemporaneous uncorrelated innovations. The contemporaneous correlation matrix in table 4 shows that most correlations are in fact rather small, as desired. The exceptions are three pairs, namely i) $(\hat{\epsilon}_{\pi^*,t}, \hat{\epsilon}_{\varepsilon_h,t})$, ii) $(\hat{\epsilon}_{\pi^*,t}, \hat{\epsilon}_{a,t})$, and iii) $(\hat{\epsilon}_{\Upsilon,t}, \hat{\epsilon}_{a,t})$, with correlation coefficients around 0.3 or slightly higher.

The third assumption about the innovations which we analyze is that they are independent across time and thus display no auto-correlation or cross auto-correlation. Table 5 examines this assumption by calculating the one period auto-correlation coefficients and the one period cross auto-correlation coefficients. Again, most coefficients are rather small. The exceptions are the auto-correlation coefficient for innovations in the inflation target which is 0.73 and the auto-correlation coefficient for innovations in preference shocks which is 0.36. The auto-correlation in innovations for the inflation target is surprising because the parameter ω^* is included in the law of motion for the inflation target in order to capture such dependency. Nevertheless, what the data is calling for is persistent monetary policy shocks which are unexpected by the household and the firms, contrary to persistent monetary policy shocks as specified by a high value of ω^* .

The estimated innovations for the entire sample period are plotted in figure 5, and the assumption of time-invariant variances is seen to be quite reasonable. An inspection of the distributions for the estimated innovations shows that they are approximately bell-shaped but with thicker tails than the normal distribution.

< Figure 5 about here >

To get a feel of the quantitative effect of these dependencies in the innovations, we simulate the key first and second unconditional moments in our model based on the assumptions $\epsilon_t \sim \mathcal{NID}(\mathbf{0}, \mathbf{I})$. The results are shown in table 6, and the corresponding sample moments and their standard errors are displayed in table 7. A bold number in table 6 indicates that the simulated

moment in our model does not lie within the 95% confidence interval for the corresponding sample moment. Our model is seen to do quite well in terms of matching the mean values for all nine series, but it generates slightly larger standard deviations for all interest rates and the inflation rate. As for the cross correlations, we see that all interest rates and the inflation rate are slightly counter-cyclical in the actual sample, whereas all interest rates and the inflation rate are slightly pro-cyclical in our model. Apart from these minor deviations, our model is thus seen to match the desired sample moments reasonably well, and the dependencies in the innovations do not seem to be important for the performance of the model along these dimensions.

Table 8 reports the sample moments when the simulated innovations have the same non-zero mean values as those found in the estimated innovations. As expected, this modification only affects the first moments, and notable changes are only present in the interest rates. Note in particular that the unconditional yield curve now has a rather steep positive slope.

We emphasize that a detailed specification analysis should always be performed when using filters to estimate DSGE models. This is true regardless of whether a Bayesian or a classical estimation principle is used. Note also that applying particle filters is not a way to avoid such a specification analysis because these filters evaluate the likelihood function given the stated assumptions for the innovations which are similar to our assumptions (see for instance Fernández-Villaverde & Rubio-Ramírez (2007a)). The only way to enforce the stated assumptions regarding the innovations in the estimation is to apply moment matching methods such as GMM, Simulated Method of Moment, or Indirect Inference. However, these estimation methods also have their shortcomings, the most notable being: which moments to match?

4.6 Robustness analysis

This subsection examines the robustness of the previous finding that our model can reproduce the dynamics of the yield curve and the four macro series. This robustness analysis is carried out by examining the effect of excluding one or more shocks in the model. During this analysis special attention is devoted to the surprising finding in our benchmark model that the degree of price stickiness is estimated to be low, even though wages are fully flexible in our model and no price indexation is possible for non-optimizing firms. This result is contrary to most findings in the literature, which emphasize the importance of nominal frictions (Smets & Wouters (2007) Justiniano & Primiceri (2008), among others).

A natural starting point for this robustness analysis is the moderate degree of persistency in the estimated innovations for the inflation target, because this persistency might substitute for the common finding of sticky prices. These considerations motivate a reestimation of our model where we exclude shocks to the inflation rate target ($\epsilon_{\pi^*,t} = 0$). Surprisingly, we still find that the degree of price stickiness is very low ($\alpha = 0.11$), but the intratemporal elasticity of substitution between goods is now $\eta = 3.57$, which is more reasonable. Table 2 shows that the model still fits the yield curve and the three real growth rates quite well. However, the model now has some trouble reproducing the inflation dynamics. This is reflected by an increase in the ex post standard deviation in model errors for inflation, from 6 to 23 basis points. Thus, even a version of our model without monetary policy shocks matches the data quite well.

The second point we examine is whether the inclusion of non-stationary preference shocks substitutes for a low degree of price stickiness and whether it is responsible for matching the yield curve in our model. Hence, we reestimate the model without the non-stationary preference shocks ($\epsilon_{\varepsilon_h,t} = 0$). Again, the degree of price stickiness is very low ($\alpha = 0.00002$), and the intratemporal elasticity of substitution between goods is also low ($\eta = 1.09$). Table 2 shows that the model now has some trouble matching interest rates with maturities of 3 month and 1 year, but the fit of this reduced model is otherwise similar to the fit in the benchmark model. Reestimating the model with preference shocks but without shocks to the stationary component of firms' fixed costs ($\epsilon_{\psi,t} = 0$) leads to the same implications, namely $\alpha = 0.003$ and $\eta = 1.11$, and we match all variables except for the 3 month and 1 year interest rates. Thus, preference shocks and/or shocks to the stationary component of firms' fixed costs are not essential for the overall performance of the model, but they do help the model match interest rates with short maturities.

Another explanation for the low degree of price stickiness could be that we allow for both a stationary and a non-stationary component in labor productivity and investment shocks. Hence, we next reestimate the model where we exclude stationary labor productivity shocks and stationary investment shocks ($\epsilon_{a,t} = \epsilon_{e,t} = 0$). In this case, the intratemporal elasticity of substitution between goods is $\eta = 1.07$, and the degree of price stickiness is found to be $\alpha = 0.25$. The latter means that firms reoptimize prices on average every 1.3 quarters, which is still quite low. For instance, Christiano et al. (2005) estimate α to be 0.60, and the corresponding estimate in Smets & Wouters (2007) is 0.66. Our model is now seen to have some trouble matching the real growth rates in consumption and in GDP with standard deviations in ex post model errors of 19 and 14 basis points, respectively. Interest rates with maturities of 3 month and 1 year also experience larger ex post model errors with standard deviations of 62 and 30 basis points, respectively.

Next, consider the opposite scenario where we exclude non-stationary investment shocks and non-stationary labor productivity shocks ($\epsilon_{\Upsilon,t} = \epsilon_{z,t} = 0$). Now, a high degree of price stickiness of $\alpha = 0.86$ is found, implying that firms on average reoptimize every 7.1 quarters. Thus, the model substitutes the missing exogenous persistency from shocks to $\mu_{\Upsilon,t}$ and $\mu_{z,t}$ with strong internal persistency through very sticky prices. Firms' intratemporal elasticity of substitution between goods is also estimated to be higher in this case ($\eta = 2.5$) than in the benchmark model. Table 2 shows that the reduced model now has great difficulty reproducing the dynamics of the inflation rate, and it also struggles with the dynamics of the real growth rates for consumption and GDP. Again, interest rates with short maturities also experience rather large model errors. Interestingly, the dynamics of this reduced version of the model is generated by extremely persistent stationary labor productivity shocks ($\rho_a = 0.9994$), meaning that data truly call for a unit root in the process for labor productivity.¹² This version of our model also illustrates our main point, namely that explaining interest rates with medium and long maturities without distorting the dynamics of the macroeconomy is difficult without a sufficient number of non-stationary shocks.

To summarize, the good performance of our model can be attributed to the fact that we allow for both a stationary and a non-stationary component in labor productivity and in investment shocks. When doing so, the degree of price stickiness is consistently found to be very low, whereas

¹²All estimated parameters in the different versions of our model can be found in the paper's technical appendix.

the low value of intratemporal substitution between goods is not robust. However, when we only allow for stationary labor productivity and investment shocks around a deterministic trend, we recover the standard finding in the literature of a substantial degree of price stickiness. In other words, different specifications of the real shocks driving the economy change the importance of nominal frictions dramatically. We emphasize that the model with low degree of price stickiness matches the data much better than a model with very sticky prices, and a low degree of price stickiness is also in accordance with recent micro evidence (Bils & Klenow (2004)).

5 Conclusion

This paper shows how a standard DSGE model can be extended to reproduce the dynamics of the 10 year yield curve for the post-war US economy and at the same time explain the dynamics of four key macro variables. We achieve these results by introducing three non-stationary shocks which allow us to explain interest rates with medium and long maturities without distorting the dynamics of the macroeconomy. Our model partly avoids the bond premium puzzle in the sense that it generates a realistic term premium provided that we account for minor deviations from the assumption of zero mean values for the innovations in the model.

All the mentioned results are generated without relying on any substantial amount of price stickiness. Thus, our model shows that the presence of sticky prices is not necessary in explaining the joint dynamics of the inflation rate *and* the dynamics of the entire 10 year yield curve. But most importantly, our analysis also shows that there no longer seems to be a trade-off between having a structural model and explaining the dynamics of the yield curve.

A The recursive coefficients in the yield curve

The expression for the yield curve is given by

$$\ln R_{t,k} = \frac{-1}{k} \left(a_k + \mathbf{b}_k^f \mathbf{x}_t^f + \mathbf{b}_k^s \mathbf{x}_t^s + \left(\mathbf{x}_t^f \right)' \mathbf{c}_k \mathbf{x}_t^f \right)$$

where $R_{t,k}$ is the gross interest rate in period t for a zero-coupon bond maturing in k periods. The recursion for the coefficients is as follows:

For $k = 1$:

$$a_1 = -0.5g_{\sigma\sigma}^R \sigma^2 - \ln R_{ss}$$

$$\mathbf{b}_1^f = -\mathbf{g}_x^R$$

$$\mathbf{b}_1^s = -\mathbf{g}_x^R$$

$$\mathbf{c}_1 = -\frac{1}{2}\mathbf{g}_{xx}^R$$

where R_{ss} denotes the value of $R_{t,1}$ in steady state.

For $k \geq 2$

$$a_k = a_{k-1} + \Gamma_0 + \frac{1}{2}\mathbf{b}_{k-1}^s \mathbf{h}_{\sigma\sigma} \sigma^2 + \frac{1}{2}\Gamma_4 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \Gamma_4' + \Gamma_4 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \sigma \boldsymbol{\eta}' \left(\mathbf{b}_{k-1}^f \right)' \\ + \frac{1}{2}\mathbf{b}_{k-1}^f \sigma \boldsymbol{\eta}' [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \sigma \boldsymbol{\eta}' \left(\mathbf{b}_{k-1}^f \right)' - \frac{1}{2} \ln |\mathbf{I} - 2\mathbf{C}_{k-1}|$$

$$\mathbf{b}_k^f = \Gamma_1 + \mathbf{b}_{k-1}^f \mathbf{h}_x + \Gamma_4 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \Gamma_5' + 2\Gamma_4 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \boldsymbol{\eta}' \sigma \mathbf{c}'_{k-1} \mathbf{h}_x \\ + \mathbf{b}_{k-1}^f \sigma \boldsymbol{\eta}' [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \Gamma_5' + 2\mathbf{b}_{k-1}^f \sigma \boldsymbol{\eta}' [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \boldsymbol{\eta}' \sigma \mathbf{c}'_{k-1} \mathbf{h}_x$$

$$\mathbf{b}_k^s = \Gamma_2 + \mathbf{b}_{k-1}^s \mathbf{h}_x$$

$$\mathbf{c}_k = \frac{1}{2} \sum_{i=1}^{n_x} \mathbf{b}_{k-1}^s (1, i) \mathbf{h}_{xx}^i + \Gamma_3 + \mathbf{h}'_x \mathbf{c}_{k-1} \mathbf{h}_x + \frac{1}{2}\Gamma_5 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \Gamma_5' \\ + \Gamma_5 [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \boldsymbol{\eta}' \sigma \mathbf{c}'_{k-1} \mathbf{h}_x \\ + \mathbf{h}'_x \mathbf{c}_{k-1} \sigma \boldsymbol{\eta}' [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \Gamma_5' + 2\mathbf{h}'_x \mathbf{c}_{k-1} \sigma \boldsymbol{\eta}' [\mathbf{I} - 2\mathbf{C}_{k-1}]^{-1} \boldsymbol{\eta}' \sigma \mathbf{c}'_{k-1} \mathbf{h}_x$$

$$\mathbf{C}_{k-1} \equiv \Gamma_6 + \sigma^2 \boldsymbol{\eta}' \mathbf{c}_{k-1} \boldsymbol{\eta}$$

where the constant coefficients are given by

$$\Gamma_0 \equiv \ln \beta + H_\gamma \ln \mu_{\gamma,ss} + H_z \ln \mu_{z,ss} - \ln \pi_{ss} - 0.5g_{\sigma\sigma}^\pi \sigma^2 + \frac{1}{2} [\mathbf{g}_x^\Lambda - \mathbf{g}_x^\pi] \mathbf{h}_{\sigma\sigma} \sigma^2$$

$$\Gamma_1 \equiv \left(\mathbf{g}_x^\Lambda - \mathbf{g}_x^\pi + H_\gamma \mathbf{e}_{\mu_\gamma} + H_z \mathbf{e}_{\mu_z} + \mathbf{e}_{\varepsilon_h} \right) \mathbf{h}_x - \mathbf{g}_x^\Lambda$$

$$\Gamma_2 \equiv \left(\mathbf{g}_x^\Lambda - \mathbf{g}_x^\pi \right) \mathbf{h}_x - \mathbf{g}_x^\Lambda$$

$$\Gamma_3 \equiv \frac{1}{2} \sum_{i=1}^{n_x} \mathbf{h}_{xx}^i [\mathbf{g}_x(\Lambda, i) - \mathbf{g}_x(\pi, i)] + \mathbf{h}'_x \left[\frac{1}{2}\mathbf{g}_{xx}^\Lambda - \frac{1}{2}\mathbf{g}_{xx}^\pi \right] \mathbf{h}_x - \frac{1}{2}\mathbf{g}_{xx}^\Lambda$$

$$\Gamma_4 \equiv \left[\mathbf{g}_x^\Lambda - \mathbf{g}_x^\pi + H_\gamma \mathbf{e}_{\mu_\gamma} + H_z \mathbf{e}_{\mu_z} + \mathbf{e}_{\varepsilon_h} \right] \sigma \boldsymbol{\eta}$$

$$\Gamma_5 \equiv 2\mathbf{h}'_x \left[\frac{1}{2}\mathbf{g}_{xx}^\Lambda - \frac{1}{2}\mathbf{g}_{xx}^\pi \right] \sigma \boldsymbol{\eta}$$

$$\Gamma_6 \equiv \sigma^2 \boldsymbol{\eta}' \left[\frac{1}{2}\mathbf{g}_{xx}^\Lambda - \frac{1}{2}\mathbf{g}_{xx}^\pi \right] \boldsymbol{\eta}$$

We use the notation $\mathbf{e}_i \mathbf{x}_t^f = \mu_{i,t}$ for $i = \{\Upsilon, z, \varepsilon_h\}$. Expressions of the form \mathbf{g}_x^π and \mathbf{g}_{xx}^π refer to the first and second order effects of the state variables on inflation π_t and similarly for Λ_t . Finally, $H_z \equiv (1 - \xi_2)(1 - \xi_1) - 1$ and $H_\Upsilon \equiv \frac{\theta}{1-\theta} H_z$.

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Table 1: Estimation results

The standard errors are calculated from the outer product of the score function. No standard errors are available for β and ω^* because the point estimates are on the boundary of the parameter space

| Label | | Estimates | SE |
|--|--------------------------|-----------|--------|
| Discount factor | β | 0.9999 | |
| Habit degree | b | 0.3098 | 0.0290 |
| Preference | ξ_1 | 0.6570 | 0.0058 |
| Preference | ξ_2 | 3.3090 | 0.0289 |
| Adj costs for investments | κ | 0.2672 | 0.0523 |
| Price elasticity | η | 1.1103 | 0.0061 |
| Price stickiness | α | 0.0882 | 0.0103 |
| Process for infl. target | ω^* | 0.0000 | |
| Interest rate rule | α_R | 0.2477 | 0.0658 |
| Interest rate rule | α_π | 10.3111 | 1.3127 |
| Interest rate rule | α_y | 8.1046 | 0.0590 |
| Non-stationary investment shock | ρ_Υ | 0.8482 | 0.0153 |
| Non-stationary labor prod. shock | ρ_z | 0.0065 | 0.0662 |
| Preference shock | ρ_{ε_h} | 0.5433 | 0.0288 |
| Firms' fixed costs | ρ_ψ | 0.9099 | 0.0076 |
| Stationary labor prod. shocks | ρ_a | 0.9787 | 0.0029 |
| Stationary investment shock | ρ_e | 0.8700 | 0.0106 |
| Std. inflation target shocks | σ_{π^*} | 0.7960 | 0.0488 |
| Std. for non-stationary investment shocks | σ_Υ | 0.0442 | 0.0046 |
| Std. for non-stationary labor prod. shocks | σ_z | 1.3626 | 0.1137 |
| Std. for preference shocks | σ_{ε_h} | 0.3205 | 0.0441 |
| Std. for shocks to firms' fixed costs | σ_ψ | 0.4194 | 0.0544 |
| Std. for stationary labor prod. shocks | σ_a | 0.6223 | 0.0674 |
| Std. for stationary investment shocks | σ_e | 1.0300 | 0.0795 |
| Parameter for measurement error in yield curve | γ_1 | -5.8368 | 0.0972 |
| Parameter for measurement error in yield curve | γ_2 | -0.2306 | 0.0212 |
| Parameter for measurement error in yield curve | γ_3 | 0.0050 | 0.0005 |

Table 2: Goodness of fit

\mathcal{L} denotes the value of the quasi log-likelihood function.

| | interest rates | | | | | growth rates in | | | | \mathcal{L} |
|---|----------------|---------|---------|---------|---------|-----------------|---------|---------|---------|---------------|
| | 3 md | 1 y | 3 y | 5 y | 10 y | infl | C | I | GDP | |
| Benchmark model | | | | | | | | | | |
| Mean in model errors | -5.9787 | -2.0511 | -0.0430 | -0.3643 | 1.9981 | 0.5326 | 0.8023 | 0.1563 | -0.6315 | |
| Ex ante std. in model errors | 23.2883 | 12.5733 | 3.7840 | 2.1687 | 9.0315 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 16.3215 | 10.1253 | 3.3558 | 1.8073 | 7.8868 | 6.3332 | 10.9770 | 8.7731 | 8.5341 | |
| Correlation | 0.9987 | 0.9994 | 0.9999 | 1.000 | 0.9995 | 0.9951 | 0.9905 | 0.9993 | 0.9952 | 6935 |
| No infl target shocks | | | | | | | | | | |
| Mean in model errors | -20.8146 | 1.8894 | 0.4945 | 0.0574 | -2.3535 | -0.1652 | 3.1677 | -1.2950 | -1.2448 | |
| Ex ante std. in model errors | 37.0047 | 16.5669 | 3.5531 | 1.8330 | 16.3048 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 28.3124 | 13.7930 | 3.1240 | 1.6760 | 16.1190 | 22.5254 | 12.8262 | 5.7972 | 11.3899 | |
| Correlation | 0.9957 | 0.9990 | 0.9999 | 1.0000 | 0.9983 | 0.9333 | 0.9864 | 0.9997 | 0.9910 | 6799 |
| No pref. shocks | | | | | | | | | | |
| Mean in model errors | -33.9323 | 0.1129 | 0.9158 | -0.3087 | 1.9724 | 0.9315 | -0.2730 | 1.1627 | 0.0760 | |
| Ex ante std. in model errors | 61.5929 | 26.7065 | 5.0824 | 2.2137 | 10.3751 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 57.4961 | 21.0495 | 3.6954 | 1.1504 | 10.0654 | 6.1728 | 8.7493 | 9.5017 | 7.2707 | |
| Correlation | 0.9859 | 0.9974 | 0.9999 | 1.0000 | 0.9992 | 0.9958 | 0.9961 | 0.9992 | 0.9964 | 6832 |
| No stationary fixed costs shocks | | | | | | | | | | |
| Mean in model errors | -32.2119 | -1.2924 | 0.6641 | -0.1663 | 0.5010 | 2.6198 | 1.3368 | -0.5046 | -0.2379 | |
| Ex ante std. in model errors | 57.2483 | 24.7156 | 4.7623 | 2.1746 | 13.3571 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 55.8674 | 18.5027 | 3.2065 | 1.0096 | 12.2382 | 7.0478 | 12.8535 | 7.4800 | 8.2806 | |
| Correlation | 0.9861 | 0.9980 | 0.9999 | 1.0000 | 0.9990 | 0.9950 | 0.9860 | 0.9995 | 0.9956 | 6794 |
| No stationary invest. and labor prod. shocks | | | | | | | | | | |
| Mean in model errors | -40.1748 | -2.6943 | 1.9104 | -0.5906 | -0.2103 | 4.0271 | 2.0022 | 0.2843 | -2.1788 | |
| Ex ante std. in model errors | 72.1740 | 34.6644 | 8.0821 | 3.8971 | 15.1181 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 61.8889 | 30.2434 | 6.6030 | 2.5461 | 13.2314 | 11.5939 | 18.7706 | 7.1448 | 14.2455 | |
| Correlation | 0.9801 | 0.9947 | 0.9997 | 1.0000 | 0.9988 | 0.9838 | 0.9642 | 0.9995 | 0.9860 | 6657 |
| No non-stationary invest. and labor prod. shocks | | | | | | | | | | |
| Mean in model errors | -37.9323 | -2.9301 | 0.2158 | -0.4686 | 0.2172 | 1.6282 | 1.6909 | -0.6743 | -1.8659 | |
| Ex ante std. in model errors | 64.8979 | 27.3478 | 4.9222 | 2.0886 | 10.4371 | 20 | 20 | 20 | 20 | |
| Ex post std. in model errors | 59.3024 | 22.5135 | 5.7192 | 4.4245 | 9.7692 | 30.6122 | 14.6669 | 6.9094 | 18.1414 | |
| Correlation | 0.9817 | 0.9971 | 0.9998 | 0.9999 | 0.9993 | 0.8974 | 0.9884 | 0.9996 | 0.9772 | 6621 |

Table 3: Mean values of the estimated innovations

| | $\hat{\epsilon}_{\pi^*,t}$ | $\hat{\epsilon}_{\Upsilon,t}$ | $\hat{\epsilon}_{z,t}$ | $\hat{\epsilon}_{\varepsilon_h,t}$ | $\hat{\epsilon}_{\psi,t}$ | $\hat{\epsilon}_a,t$ | $\hat{\epsilon}_e,t$ |
|-------------|----------------------------|-------------------------------|------------------------|------------------------------------|---------------------------|----------------------|----------------------|
| Mean values | -0.4295 | 0.0392 | 0.0246 | 0.2460 | -0.1212 | -0.0683 | 0.0515 |

Table 4: The contemporaneous correlation matrix for the estimated innovations

| | $\hat{\epsilon}_{\pi^*,t}$ | $\hat{\epsilon}_{\Upsilon,t}$ | $\hat{\epsilon}_{z,t}$ | $\hat{\epsilon}_{\varepsilon_h,t}$ | $\hat{\epsilon}_{\psi,t}$ | $\hat{\epsilon}_a,t$ | $\hat{\epsilon}_e,t$ |
|------------------------------------|----------------------------|-------------------------------|------------------------|------------------------------------|---------------------------|----------------------|----------------------|
| $\hat{\epsilon}_{\pi^*,t}$ | 1 | 0.2768 | 0.2183 | 0.3498 | -0.2337 | 0.4289 | 0.2603 |
| $\hat{\epsilon}_{\Upsilon,t}$ | | 1 | 0.0883 | -0.0090 | -0.1571 | 0.3454 | 0.0277 |
| $\hat{\epsilon}_{z,t}$ | | | 1 | 0.2582 | 0.0666 | -0.1467 | 0.2598 |
| $\hat{\epsilon}_{\varepsilon_h,t}$ | | | | 1 | 0.2950 | 0.0064 | -0.0716 |
| $\hat{\epsilon}_{\psi,t}$ | | | | | 1 | -0.0180 | -0.1239 |
| $\hat{\epsilon}_a,t$ | | | | | | 1 | -0.0302 |
| $\hat{\epsilon}_e,t$ | | | | | | | 1 |

Table 5: The one period auto-correlation matrix for the estimated innovations
The auto-correlations are shown along the diagonal and cross auto-correlations are shown in the off-diagonal elements.

| | $\hat{\epsilon}_{\pi^*,t}$ | $\hat{\epsilon}_{\Upsilon,t}$ | $\hat{\epsilon}_{z,t}$ | $\hat{\epsilon}_{\varepsilon_h,t}$ | $\hat{\epsilon}_{\psi,t}$ | $\hat{\epsilon}_a,t$ | $\hat{\epsilon}_e,t$ |
|------------------------------------|----------------------------|-------------------------------|------------------------|------------------------------------|---------------------------|----------------------|----------------------|
| $\hat{\epsilon}_{\pi^*,t}$ | 0.7340 | 0.2320 | 0.0147 | 0.2320 | -0.0404 | 0.2585 | 0.1169 |
| $\hat{\epsilon}_{\Upsilon,t}$ | | 0.0811 | 0.0229 | -0.0654 | 0.0004 | 0.1597 | 0.0819 |
| $\hat{\epsilon}_{z,t}$ | | | 0.0641 | 0.0634 | 0.1287 | -0.0808 | -0.0252 |
| $\hat{\epsilon}_{\varepsilon_h,t}$ | | | | 0.3620 | -0.0552 | 0.1184 | 0.0248 |
| $\hat{\epsilon}_{\psi,t}$ | | | | | -0.0879 | 0.0223 | -0.0376 |
| $\hat{\epsilon}_a,t$ | | | | | | 0.1565 | 0.1963 |
| $\hat{\epsilon}_e,t$ | | | | | | | 0.1724 |

Table 6: Simulated moments in DSGE model - innovations with zero mean
Standard deviations are shown along the diagonal and correlation coefficients are shown in the off-diagonal elements. Moments are calculated based on a simulated time series of 500,000 observations. Mean values of zero for the innovations are imposed when generating this time series. Bold numbers indicate that the moment is not in the 95 percentage confidence interval for the corresponding sample moment.

| | interest rates | | | | | growth rates in | | | |
|-------------|----------------|--------------|--------------|--------------|--------------|-----------------|--------------|--------------|--------------|
| | 3 md | 1 y | 3 y | 5 y | 10 y | infl | C | I | GDP |
| 3 md | 5.12% | 0.992 | 0.972 | 0.960 | 0.936 | 0.689 | 0.066 | -0.032 | 0.031 |
| 1 y | | 4.83% | 0.993 | 0.984 | 0.965 | 0.742 | 0.083 | 0.015 | 0.061 |
| 3 y | | | 4.45% | 0.998 | 0.987 | 0.749 | 0.084 | 0.034 | 0.067 |
| 5 y | | | | 4.17% | 0.995 | 0.745 | 0.080 | 0.036 | 0.063 |
| 10 y | | | | | 3.61% | 0.731 | 0.067 | 0.034 | 0.054 |
| infl | | | | | | 1.28% | 0.266 | 0.305 | 0.273 |
| C | | | | | | | 0.75% | 0.472 | 0.803 |
| I | | | | | | | | 2.08% | 0.604 |
| GDP | | | | | | | | | 1.06% |
| Mean values | 7.13% | 7.14% | 7.13% | 7.10% | 6.97% | 0.89% | 0.52% | 0.70% | 0.52% |

Table 7: Moments in post-war US data

Sample moments from the US economy, 1957Q1-2007Q4. Standard deviations are shown in the diagonal and correlation coefficients are shown in the off-diagonal elements. Numbers in parenthesis denote standard errors which are calculated by the circular block bootstrap with 10,000 blocks and a window of 60.

| | interest rates | | | | | growth rates in | | | |
|-------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| | 3 md | 1 y | 3 y | 5 y | 10 y | infl | C | I | GDP |
| 3 md | 2.76% (0.55) | 0.990 0.0028 | 0.957 (0.0131) | 0.933 (0.0254) | 0.896 (0.0511) | 0.634 (0.1196) | -0.236 (0.1306) | -0.234 (0.1064) | -0.183 (0.0881) |
| 1 y | | 2.92% (0.58) | 0.983 (0.0063) | 0.964 (0.0187) | 0.931 0.0469 | 0.619 (0.1210) | -0.210 (0.1198) | -0.208 (0.1034) | -0.173 (0.0897) |
| 3 y | | | 2.77% (0.57) | 0.995 (0.0044) | 0.978 (0.0248) | 0.565 (0.1284) | -0.155 (0.1034) | -0.154 (0.0956) | -0.133 (0.0861) |
| 5 y | | | | 2.68% (0.57) | 0.993 (0.0092) | 0.549 (0.1344) | -0.137 (0.1006) | -0.129 (0.0929) | -0.119 (0.0858) |
| 10 y | | | | | 2.60% (0.56) | 0.531 (0.1404) | -0.113 (0.0982) | -0.101 (0.0896) | -0.099 (0.0836) |
| infl | | | | | | 0.62% (0.13) | -0.296 (0.0737) | -0.196 (0.0682) | -0.198 (0.0595) |
| C | | | | | | | 0.69% (0.07) | 0.610 (0.0427) | 0.667 0.0363 |
| I | | | | | | | | 2.35% (0.23) | 0.722 0.0307 |
| GDP | | | | | | | | | 0.90% (0.11) |
| Mean values | 5.38% (0.98) | 5.97% (1.07) | 6.37% (1.07) | 6.55% (1.06) | 6.75% (1.06) | 0.89% (0.21) | 0.58% (0.04) | 0.70% (0.12) | 0.52% (0.05) |

Table 8: Simulated moments in DSGE model - innovations with non-zero mean
Standard deviations are shown along the diagonal and correlation coefficients are shown in the off-diagonal elements. Moments are calculated based on a simulated time series of 500,000 observations. Innovations have the same non-zero mean values as found in the estimated sample period when generating this time series. Bold numbers indicate that the moment is not in the 95 percentage confidence interval for the corresponding sample moment.

| | interest rates | | | | | growth rates in | | | |
|-------------|----------------|--------------|--------------|--------------|-------|-----------------|--------------|--------------|--------------|
| | 3 md | 1 y | 3 y | 5 y | 10 y | infl | C | I | GDP |
| 3 md | 5.13% | 0.992 | 0.973 | 0.959 | 0.935 | 0.689 | 0.066 | -0.034 | 0.033 |
| 1 y | | 4.84% | 0.993 | 0.984 | 0.964 | 0.742 | 0.084 | 0.014 | 0.063 |
| 3 y | | | 4.45% | 0.998 | 0.987 | 0.749 | 0.084 | 0.034 | 0.069 |
| 5 y | | | | 4.17% | 0.995 | 0.744 | 0.080 | 0.036 | 0.065 |
| 10 y | | | | | 3.61% | 0.730 | 0.067 | 0.034 | 0.057 |
| infl | | | | | | 1.28% | 0.270 | 0.306 | 0.277 |
| C | | | | | | | 0.75% | 0.473 | 0.800 |
| I | | | | | | | | 2.05% | 0.604 |
| GDP | | | | | | | | | 1.05% |
| Mean values | 5.97% | 6.49% | 6.91% | 7.10% | 7.25% | 0.95% | 0.56% | 0.75% | 0.56% |

Figure 1: Impulse response functions

Impulse response functions for: i) the 3 month interest rate (the unmarked line), ii) the 10 year interest rate (the thick red line), iii) the inflation rate (the dotted line), iv) consumption (the line marked with circles), v) aggregate demand (the line marked with stars), and vi) investments (the line marked with pluses).

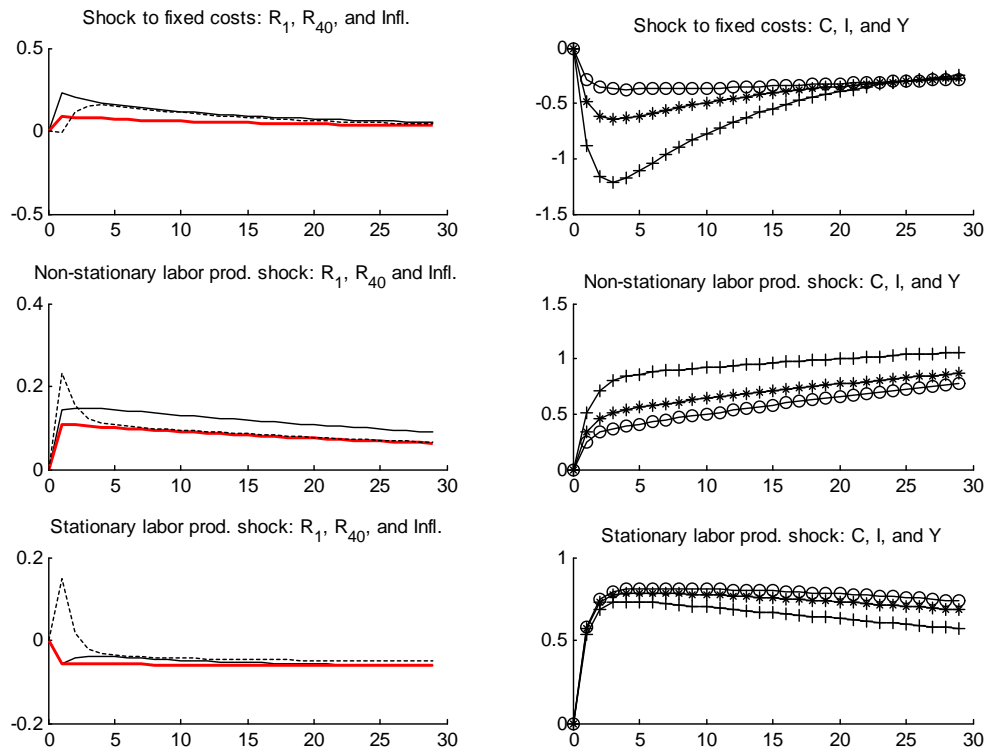


Figure 2: Model fit

Model errors are displayed in basis points for the interest rates and for the four macro variables.

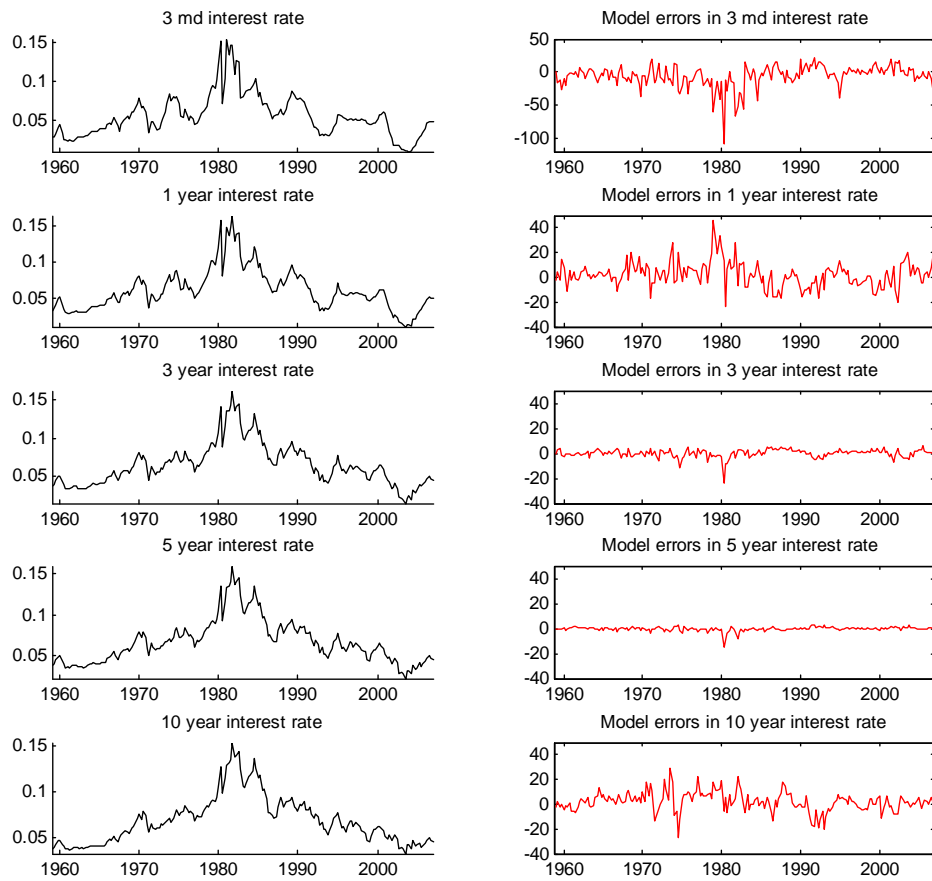


Figure 2: Model fit - continued

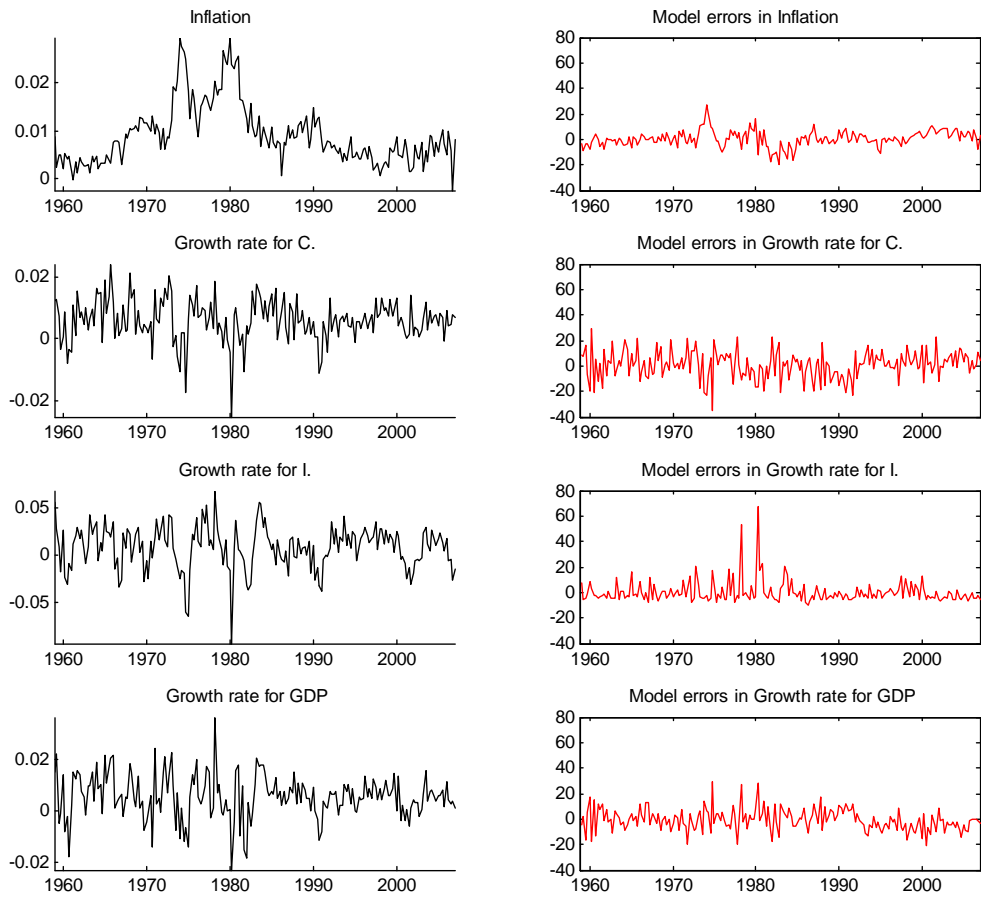


Figure 3: Model fit for interest rates not included in the estimation

For the graphs to the left, the black line denotes the actual interest rates and the red line denotes the model implied interest rates. For the graphs to the right, model errors are displayed in basis points.

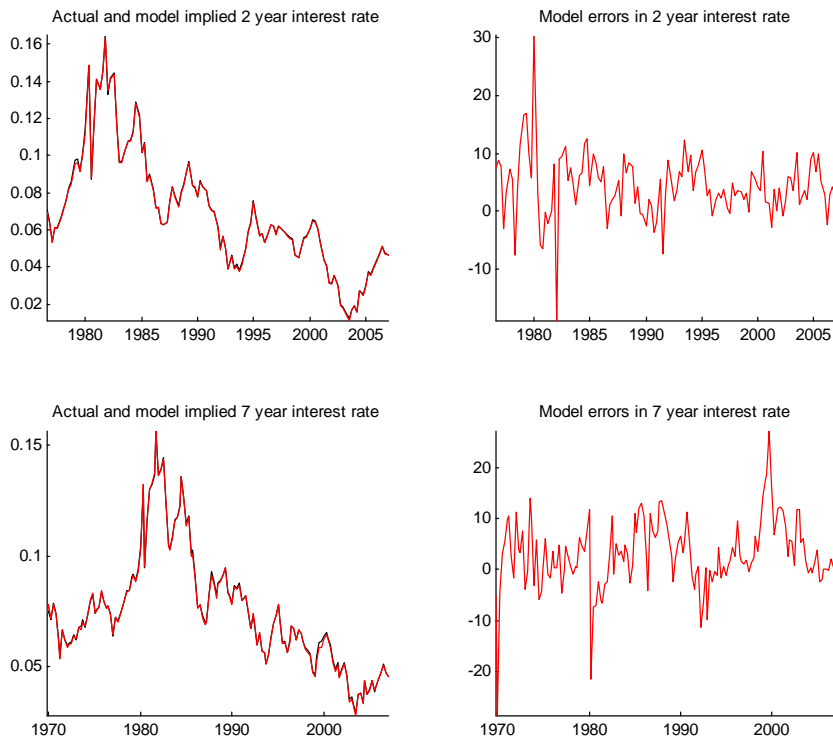


Figure 4: The term premium

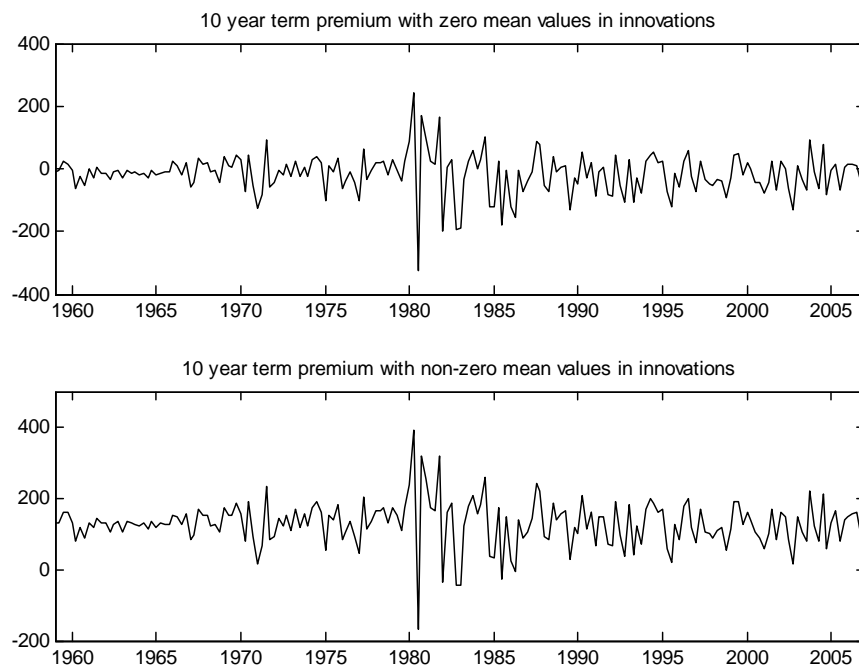
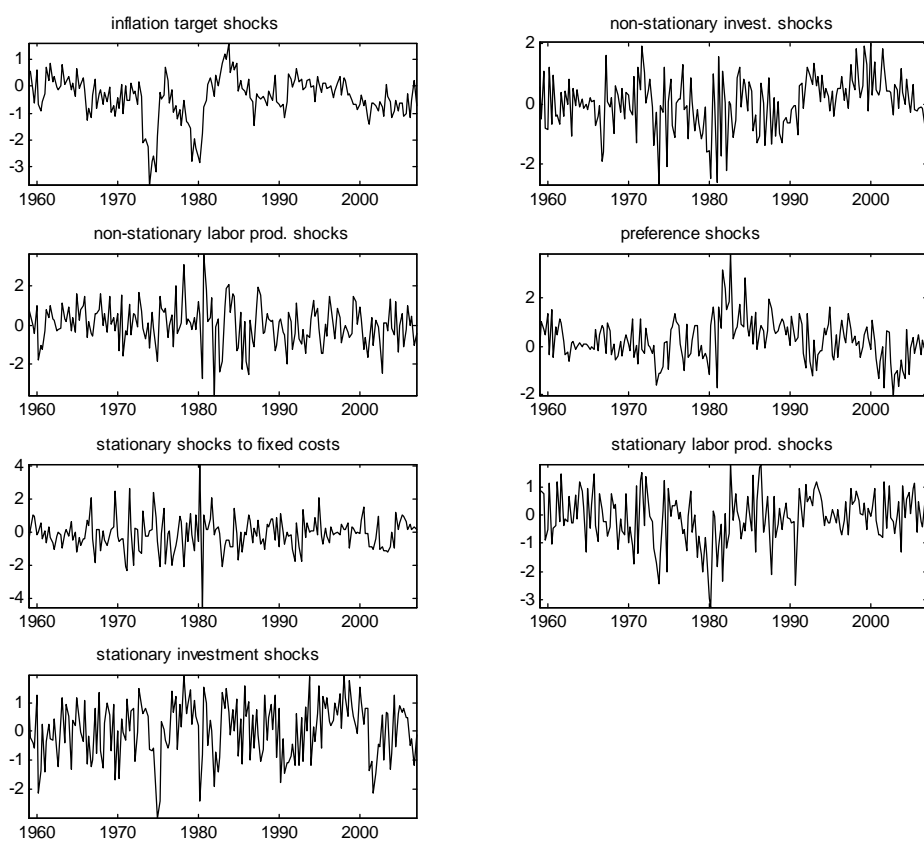


Figure 5: Time plots of estimated innovations



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