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CREATES Research Paper 2008-31

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Estimating High-Frequency Based (Co-) Variances: A Unified Approach*

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Abstract

We propose a unified framework for estimating integrated variances and covariances based on simple OLS regressions, allowing for a general market microstructure noise specification. We show that our estimators can outperform, in terms of the root mean squared error criterion, the most recent and commonly applied estimators, such as the realized kernels of Barndorff-Nielsen, Hansen, Lunde & Shephard (2006), the two-scales realized variance of Zhang, Mykland & Aït-Sahalia (2005), the Hayashi & Yoshida (2005) covariance estimator, and the realized variance and covariance with the optimal sampling frequency derived in Bandi & Russell (2005a) and Bandi & Russell (2005b). For a realistic trading scenario, the efficiency gains resulting from our approach are in the range of 35% to 50%.

JEL classification: $G10, F31, C32$

Keywords: High frequency data, Realized volatility and covariance, Market microstructure

*The work is supported in part by the European Community's Human Potential Program under contract HPRN-CT-2002-00232, Microstructure of Financial Markets in Europe; by the Fritz Thyssen Foundation through the project "Dealer-Behavior and Price-Dynamics on the Foreign Exchange Market"; and by the German Research Foundation through the project "Price-, Liquidity-, and Credit Risks: Measurement and Allocation". The paper benefited from several discussions with Sandra Nolte and Winfried Pohlmeier. We would like to thank Jan Beran, Tim Bollerslev, Wolfgang Bühler, Fulvio Corsi, Günter Franke, Ramazan Gençay and Jens Jackwerth for helpful comments.

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1 Introduction

This paper presents a unified approach for estimating the covariance of a multivariate diffusion process in the presence of market microstructure noise. Recently, the literature on estimating the variance of irregularly observed high-frequency financial prices has experienced a substantial development with respect to relaxing the usual i.i.d. noise assumption and constructing consistent estimators for the variance of the underlying efficient price process. Leading examples in this field are the realized kernels of Barndorff-Nielsen et al. (2006) and the two-scales realized volatility of Zhang et al. (2005).

The progress in estimating the covariance between two randomly observed diffusion processes has been somewhat more cumbersome, due to the additional complication of non-synchronicity. Nevertheless, recent contributions such as Hayashi & Yoshida (2005) and Corsi & Audrino (2007) have introduced consistent and unbiased estimators for non-synchronously observed processes, when there are no market frictions. Based on these approaches, Griffin & Oomen (2006), Voev & Lunde (2007) and Zhang (2006b), among others, consider the properties of such estimators in the presence of measurement noise and propose certain extensions in order to correct for the impact of the noise.

There still does not exist, however, a unified methodology for estimating both the variance and covariance of high-frequency noisy prices, thus allowing for estimation of the whole variance-covariance matrix, which accounts for a wide range of possible noise specifications and non-synchronicity. Our work is intended to fill this gap by providing precise and unbiased estimators, which are also easy to apply in practice. Furthermore, we obtain estimates for the dependence structure of the noise process, leading to a better understanding of market microstructure frictions on the transaction level. The power of our methodology lies in the ability to separate the variation of the efficient price from the variation of the noise process, which jointly contribute to the variation of the observed noisy price process. This identification is possible since the effect of the noise accumulates (up to a certain extent) by sampling more frequently, while the variation of the true process is constant. To illustrate our approach more intuitively, consider the so-called volatility signature plots introduced by Andersen, Bollerslev, Diebold & Labys (2001) as a graphical tool to study the effects of market microstructure noise on the properties of the realized variance estimator. These plots are an illustration of a particular relationship between the realized variance and the number of sampling points. For each sampling frequency, the expectation of the resulting realized volatility has a component which is constant across the range of sampling frequencies (the integrated variance of the underlying process), and a component which varies with the number of points on the grid (the noise variance which accumulates linearly with the number of returns on the grid). Since the number of sampling points is observable, the true integrated variance can be obtained as the constant of a projection of the realized volatility on the number of returns used to compute it. Based on this simple idea, our approaches lead to efficiency gains of 35% to 50% in a realistic trading scenario.

The paper is structured as follows: in Section 2 we introduce the notation and the theoretical framework we are working under, Section 3 presents our estimation methodology, Section 4 contains the results of a simulation study in which we compare our approach to other existing approaches, and Section 5 concludes. The Appendix contains detailed results of the simulation study.

2 Theoretical Setup

Our basic assumption is that we have irregularly spaced, non-synchronous observations of an n-dimensional continuous time process $\mathbf{p}(t)$, $t \geq 0$, which is a noisy signal for an underlying process $\mathbf{p}^*(t)$:

$$\mathbf{p}(t) = \mathbf{p}^*(t) + \mathbf{u}(t),$$

where $\mathbf{u}(t)$ is the noise term. The elements of $\mathbf{p}$, $\mathbf{p}^*$ and $\mathbf{u}$ are denoted by p^k, p^{*k} and u^k, for $k = 1, \ldots, n$, respectively. As in Barndorff-Nielsen & Shephard (2004), the process $\mathbf{p}^*(t)$ satisfies the following assumption:

Assumption 1. *The process $\mathbf{p}^*(t)$ is a multivariate martingale process with stochastic volatility satisfying*

$$\mathbf{p}^*(t) = \int_0^t \mathbf{\Theta}(u) d\mathbf{W}(u)$$

where $\mathbf{\Theta}$ is the spot covolatility process and $\mathbf{W}$ is a vector standard Brownian motion of dimension m. All the elements of $\mathbf{\Theta}(t)$ are cádlág.[1]

Defining the spot covariance as $\mathbf{\Sigma}(t) = \mathbf{\Theta}(t)\mathbf{\Theta}(t)'$, the integrated covariation process

[1] The acronym cádlág stands for "continue à droite, limite à gauche". This condition ensures that the integral with respect to $\mathbf{W}$ exists.

of $\mathbf{p}^*$ is given by

$$\mathbf{IC}(t) = \int_0^t \mathbf{\Sigma}(u) du.$$

The diagonal elements of $\mathbf{\Sigma}(t)$ are assumed to be integrable.
Our aim is to estimate the increment of integrated covariance

$$\mathbf{IC}(a,b) = \int_a^b \mathbf{\Sigma}(u) du = \mathbf{IC}(b) - \mathbf{IC}(a).$$

for some predetermined choice of (a, b), e.g., a trading day. Henceforth, we assume that the period of interest is a trading day with $a = 0$ and $b = 1$, and we will omit a and b in the notation.
With respect to the market microstructure noise process, we make the following assumption:

Assumption 2.

(*i*) $\mathbf{p}^*(s) \perp\!\!\!\perp \mathbf{u}(t)$, *for all* s *and* t*;*

(*ii*) $\mathrm{E}[\mathbf{u}(t)] = 0$ *for all* t*;*

Under this assumption the noise process can be serially correlated, but is exogenous to the true price process $\mathbf{p}^*$. In the univariate case a similar assumption can be found in Barndorff-Nielsen et al. (2006) and Aït-Sahalia, Mykland & Zhang (2006), among others, who relax the standard assumption of an i.i.d. noise process used in the early realized volatility literature.
In the univariate case, the i.i.d. assumption can be relaxed by postulating serial dependence either in calendar (physical) or in tick time, the latter being more intuitive and also easier to work with. In the multivariate case, i.i.d. noise is still the working assumption used, by e.g. Griffin & Oomen (2006), Bandi & Russell (2005b). Recently, Voev & Lunde (2007) have shown that the i.i.d. assumption cannot be sustained empirically, which can be explained by staggered information assimilation in asset prices. While securities, which are more closely followed by analysts and more frequently traded, react faster to new information, slower trading assets take time to adjust to the news, causing lagged correlations across assets.
Voev & Lunde (2007) discuss certain problems of adopting the univariate tick-time dependence assumption in the multivariate case which render the direct implementation of tick-time dependence infeasible, and therefore assume serial cross-dependence

in calendar time. In this paper we will follow this approach since it seems to be the most reasonable way to achieve an unified framework for modelling dependent noise processes in the multivariate framework with non-synchronicity. Thus, we complete Assumption 2 as follows

Assumption 2. *(continued)*

(*iii*) *The noise process* $\mathbf{u}$ *is covariance stationary with autocovariance function given by* $\mathbf{\Gamma}(q) = \mathrm{E}\left[\mathbf{u}(t)\mathbf{u}'(t-q)\right]$.

The (k,l)*-element of* $\mathbf{\Gamma}(q)$, $k,l = 1,\ldots,n$ *is denoted by* $\gamma_{k,l}(q)$.

Since it might be of concern that the given assumption does not take trading activity and diurnal seasonality into account, it is worth mentioning that there are other possibilities for defining dependence across assets, which to a certain extent address this issue, but bear some complications. One approach would be to use the pooled arrival process of any two assets as the time scale for defining the cross-correlations between them. This has the disadvantage that for a given asset k, its time-series properties will depend whether we consider it in combination with an asset l or an asset l'. Furthermore, with this approach, we would not be able to define the usual matrix autocorrelation function for multivariate processes, which should satisfy $\gamma_{k,l}(q) = \gamma_{l,k}(-q)$. A partial solution to this problem could be to consider the pooled process of the whole universe of assets under consideration. Such a structure, however, will still depend on the assets included in the universe, and will change from application to application. Apart from this, in the limit, when we include more and more assets, the pooled arrival process will converge to a regular time grid measured in the smallest available time unit (e.g., a second), and hence would be identical to our setup in which we implicitly assume that time is a discrete multiple of a fixed time unit.

While until now we have considered the noise process as a continuous time process, the the noise contamination is actually manifested when transactions or quote updates occur, for example the contamination through the bid-ask bounce effect can be considered to perturb the true price process only at those times when a buy or a sell transaction is carried out. Therefore, it is important to consider its properties at the event times and we will use the notation $u^k(t^k_j) = u^k_{t^k_j}$ and $p^k(t^k_j) = p^k_{t^k_j}$, where t^k_j, $j = 1,\ldots,N^k$ denotes the event (transaction, quotes, etc.) arrival times and N^k is the total number of events for asset $k = 1,\ldots,n$. Under Assumption 2, we have, e.g., that $\mathrm{E}\left[u^k_{t^k_j} u^l_{t^l_{j'}}\right] = \gamma_{k,l}(q)$, whenever $t^k_j - t^l_{j'} = q$.

The estimation approach presented in this paper is also applicable under the alternative assumption of dependence defined on the pooled arrival process discussed above, with slight modifications. In particular, if one is only interested in estimating variances, the assumption of tick-time dependence considerably simplifies the estimation.

3 Estimation Procedures

If the process p^* were observed directly, a simple and asymptotically error-free estimator for $\mathbf{IC}(a, b)$ is the so-called realized covariance which is the sum of the squares of the increments of the process p^* at the highest available frequency over the interval (a, b). The properties of this estimator under such ideal conditions are derived in Barndorff-Nielsen & Shephard (2004). Two main issues arise for this estimator when used in practice. Firstly, when the separate univariate processes are not observed simultaneously, one has to resort to synchronization techniques in order to define joint observation times for the multivariate process. Such techniques lead to biases in the estimated covariances, which are known as the Epps effect (Epps (1979)). Secondly, the presence of noise leads to biases and inconsistency. The properties of the last-tick interpolation based realized covariances are studied by Zhang (2006b), Griffin & Oomen (2006) and Martens (2004), among others. Based on the results of these studies, different approaches are proposed to make the realized covariance robust to market microstructure noise such as calculation of optimal sampling frequencies and lead-lag corrections. More recently, researchers have concentrated on developing sophisticated models which are specifically designed to estimate only the variance of a given asset (variance models) or a single covariance between two assets (covariance models). Concerning the variance models, recent advances include the two-scales realized variance by Zhang et al. (2005), the realized kernels of Barndorff-Nielsen et al. (2006), and the realized range-based variance which has newly been revived by Christensen & Podolskij (2007). With respect to covariance estimation Hayashi & Yoshida (2005) and Corsi & Audrino (2007) propose an estimator which does not require synchronization of observations and thus accounts for the Epps effect. Griffin & Oomen (2006) study the properties of this estimator under i.i.d. noise, while Voev & Lunde (2007) propose extensions to the Hayashi-Yoshida estimator to make it robust to market microstructure frictions of a general nature.

In our methodology, the variances and covariances are estimated separately, but within the same theoretical framework. Advantages of our estimation procedure are its efficiency, straightforward implementation and robustness to misspecifications of the noise process.

3.1 Variance Estimation

We first focus on estimating the integrated variance of a single asset and then we turn to covariance estimation. To separate the variance of the unobservable price process from the variance of the noise component, we use the idea of the volatility signature plot introduced by Andersen et al. (2001), which is the graphical representation of the realized variance against the sampling frequency at which it was computed. The volatility signature plot depicts the relationship between the realized variance computed with returns sampled on a certain grid and the number of sampling points on the grid for a set of predetermined grids. To gain an intuitive understanding for our estimation procedure, consider the i.i.d. noise case, under which theoretically the noise variance accumulates linearly with the number of sampling points, whereas the integrated variance is constant. Thus, an estimate of the integrated variance can simply be obtained as the intercept of the regression of the realized variances on the number of sampling points on the grid. Under a more general specification of the noise process, as in Assumption 2, the realized variances are further affected by the noise autocorrelations, which have to be taken into account in the regression by including appropriate additional regressors.

More formally, consider a given asset k with N^k observations (ticks, transactions, quote updates) within the period of interest. To this end, the grid of observations $\{t_j^k\}_{j=1,\dots,N^k}$ is subdivided into subgrids $\{t_{js+h}^k\}_{j=0,\dots,\left\lfloor\frac{N^k-h}{s}\right\rfloor}$, where $s = 1, \dots, S$ and $h = 1, \dots, s$, which denotes the h-th subgrid for a sampling frequency of s ticks (e.g., with $s = 2$ we can have two subgrids, the first one comprising the ticks $\{t_1^k, t_3^k, t_5^k, \dots\}$ and the second – the ticks $\{t_2^k, t_4^k, t_6^k, \dots\}$). For each subgrid, we can define the corresponding observed and efficient s-tick returns as

$$
\begin{aligned}
r_{t_{js+h}}^k &= p_{t_{(j-1)s+h}}^k - p_{t_{js+h}}^k, \quad j = 1, \dots, \left\lfloor \frac{N^k - h}{s} \right\rfloor \\
r_{t_{js+h}}^{*k} &= p_{t_{(j-1)s+h}}^{*k} - p_{t_{js+h}}^{*k}, \quad j = 1, \dots, \left\lfloor \frac{N^k - h}{s} \right\rfloor,
\end{aligned}
$$

and the noise returns as

$$
e_{t_{js+h}}^k = u_{t_{(j-1)s+h}}^k - u_{t_{js+h}}^k, \quad j = 1, \dots, \left\lfloor \frac{N^k - h}{s} \right\rfloor.
$$

Denote the number of returns for the h-th s-subgrid as $N_{h,s}^k = \left\lfloor \frac{N^k-h}{s} \right\rfloor - 1$. The realized variance of asset k based on this subgrid is defined explicitly as a function of the number of returns on the subgrid:

$$RV^k(N^k_{h,s}) = \sum_{j=1}^{N^k_{h,s}} \left(r^k_{t_{js+h}} \right)^2 .$$

To estimate the integrated variance we will exploit the following relationship, which holds under Assumptions 1 and 2:

$$\begin{aligned}
\mathrm{E}\left[RV^k(N^k_{h,s})\right] &= \mathrm{IV}^k + \sum_{j=1}^{N^k_{h,s}} \mathrm{Var}\left[e^k_{t_{js+h}}\right] \\
&= \mathrm{IV}^k + 2\sum_{q=1}^{\infty} N^k_{h,s}(q)\left(\gamma_{k,k}(0) - \gamma_{k,k}(q)\right) \\
&\approx \mathrm{IV}^k + 2N^k_{h,s}\gamma_{k,k}(0) - 2\sum_{q=1}^{Q} N^k_{h,s}(q)\gamma_{k,k}(q), \qquad (1)
\end{aligned}$$

where IV^k is the integrated variance of the true price process of asset k, i.e. element (k,k) of the matrix **IC** and $N^k_{h,s} = \sum_q N^k_{h,s}(q)$. Thereby, $N^k_{h,s}(q)$ counts the number of q-second returns of asset k for the (h,s)-subgrid given by

$$N^k_{h,s}(q) = \sum_j \mathbb{1}_{\{t^k_{js+h} - t^k_{(j-1)s+h} = q\}}.$$

Note, that these counts need to be considered because we work with irregularly-spaced returns, which under the assumption of an autocovariance function defined on the smallest regular time grid (each second), imply that each $\mathrm{Var}\left[e^k_{t_{js+h}}\right]$ depends on the length of the return and thus consists of two elements, namely $\gamma_{k,k}(0)$ and the q-second autocovariance $\gamma_{k,k}(q)$. The approximation in Equation (1) results from truncating the autocorrelation function at lag Q. This is reasonable, since for a covariance stationary process the autocovariance function tends to zero for large lags, so that Q has to be chosen appropriately. As we will see below, letting Q be too large leads to more estimation noise, because for large Q's there are relatively few counts $N^k_{h,s}(Q)$. Furthermore, since $N^k_{h,s} = \sum_q N^k_{h,s}(q)$, choosing Q too large yields a singular regressor matrix.

Under the assumption of an i.i.d. noise process we obtain from Equation (1) the standard result (as in e.g., Hansen & Lunde (2006)):

$$\mathrm{E}\left[RV^k(N^k_{h,s})\right] = \mathrm{IV}^k + 2N^k_{h,s}\gamma_{k,k}(0).$$

Equation (1) differs to the extent that we have to consider the q-th order autocorrelation of the noise process and we have to count the number of occurrences.

On the basis of the theoretical relationship in Equation (1) and the above assumptions, we can easily derive the corresponding pooled OLS regression

$$y_{h,s} = c + \beta' x_{h,s} + \varepsilon_{h,s}, \quad s = 1, \dots, S, \quad h = 1, \dots, s \tag{2}$$

where $y_{h,s} = RV^k(N^k_{h,s})$ and $x_{h,s}$ is the Q-dimensional vector given by $x_{h,s} = (N^k_{h,s}, N^k_{h,s}(1), \dots, N^k_{h,s}(Q))'$. In practice, Q has to be chosen appropriately, to reflect the degree of persistence of the noise process in the particular application. In the above regression, one simply regresses the realized variances on $N^k_{h,s}$ and the q-counts $N^k_{h,s}(q)$. The estimated constant $\hat{c}$ is an estimate of the integrated variance IV^k, while $\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_Q$ are estimates of $2\gamma_{k,k}(0), -2\gamma_{k,k}(1), \dots, -2\gamma_{k,k}(Q)$. Hence, as a byproduct of this estimation we can obtain the autocovariance function of the noise process, which can be identified under the assumption that the autocovariance $\gamma_{k,k}(Q)$ vanishes for a large enough Q. For a particular application, one could choose Q in an iterative manner starting from a relatively small value which is increased in each step. The optimal value of Q is the smallest value at which a given criterion (e.g. the gradient of the estimates) no longer changes considerably.

3.2 Covariance Estimation

Covariance estimation based on high-frequency data is inherently more challenging than variance estimation, since there is the additional complication of non-synchronicity. As mentioned already, non-synchronicity poses the problem of defining common event times for multiple assets. Typically, last-tick interpolation is employed, in which the last recorded price before a pre-defined observation time is taken as the observed price at that point of time. This leads to a bias towards zero in the estimated realized covariance as the sampling frequency increases. A solution to this problem is proposed by Hayashi & Yoshida (2005). Considering two assets k and l, the Hayashi-Yoshida (HY) estimator based on all observations is defined as

$$HY^{k,l} = \sum_{j=1}^{N^k} \sum_{j'=1}^{N^l} r^k_{t^k_j} r^l_{t^l_{j'}} \mathbb{1}_{\{(t^k_{j-1}, t^k_j] \cap (t^l_{j'-1}, t^l_{j'}]\}}.$$

As can be seen from the definition, this estimator sums all cross products of overlapping returns of the assets under consideration. We can also base the estimation on the (h, s)- subgrid of asset k in combination with the (h', s')-subgrid of asset l, which we denote by

$$HY^{k,l}(h, s, h', s') = \sum_{j=1}^{N^k_{h,s}} \sum_{j'=1}^{N^l_{h',s'}} r^k_{t^k_{js+h}} r^l_{t^l_{j's'+h'}} \mathbb{1}_{\{(t^k_{(j-1)s+h}, t^k_{js+h}] \cap (t^l_{(j'-1)s'+h'}, t^l_{j's'+h'}]\}}. \tag{3}$$

In practice, it is convenient to implement this estimator by picking one of the assets, say k, and determining for each of its tick returns $r^k_{t^k_{js+h}}$, the corresponding return of the other asset which envelops it, i.e. starts before or at $t^k_{(j-1)s+h}$ and spans over at least to t^k_{js+h}. Of course, if one interchanges the assets, the estimator is numerically identical, but with respect to speed of execution, we recommend using the slower trading asset to determine the corresponding enveloping returns of the faster asset. In the following exposition we set the slower asset to be asset k. While the HY estimator is defined using all returns of both assets, effectively, there are at most $\min(N^k_{h,s}, N^l_{h',s'})$ different pairs of returns which contribute to the sum. This arises, because two or more neighboring returns of asset k may happen to be enveloped by the same return of asset l. Due to the summability of log returns, this effectively amounts to only one return pair in the sum of the HY estimator and the noise contaminations cancel against each other. Thus, the amount of noise which accumulates in the sum is a function of such effective pairs, while some of the ticks t^k_{js+h} play no role and are hence irrelevant. In order to determine the number of effective pairs, we introduce the right- and left-continuous counting functions $N^k_{h,s}(t) = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}_{\{t^k_{js+h} \leq t\}}$ and $\breve{N}^k_{h,s}(t) = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}_{\{t^k_{js+h} < t\}}$, $k = 1, \ldots, n$, $s = 1, \ldots, S$, and $h = 1, \ldots, s$. The number of irrelevant ticks t^k_{js+h} can be computed as follows

$$NI^k(h,s,h',s') = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}\left\{t^k_{js+h} \neq t^l_{N^l_{h',s'}\left(t^k_{js+h}\right)}\right\} \mathbb{1}\left\{t^k_{(j+1)s+h} \leq t^l_{N^l_{h',s'}\left(t^k_{js+h}\right)+1}\right\} \mathbb{1}\left\{t^k_{(j-1)s+h} \geq t^l_{\breve{N}^l_{h',s'}\left(t^k_{js+h}\right)}\right\} \quad (4)$$

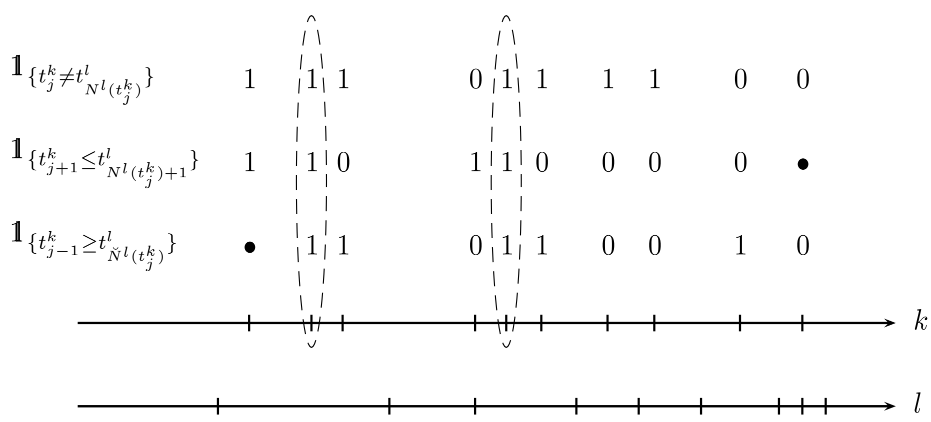

Figure 1: A graphical illustration for the identification of irrelevant ticks as described in equation (4). The indices h, s, h', s' have been suppressed.

Figure 1 illustrates graphically how such irrelevant ticks are obtained. A tick t^k_{js+h} is irrelevant if it fulfills three conditions: i.) it is not synchronous with any tick of the other asset, ii.) the next arrival on the pooled process is generated by the same asset, and iii.) the previous arrival on the pooled process is generated by the same asset. If these conditions are satisfied simultaneously, then the impact of the noise at this tick cancels out in the summation of the HY estimator. The number of effective pairs is then given by

$$\tilde{N}^k(h,s,h',s') = N^k_{h,s} - NI^k(h,s,h',s').$$

The HY estimator can then be rewritten as

$$\begin{aligned} HY^{k,l}(h,s,h',s') &= \sum_{j=1}^{N^k_{h,s}} r^k_{t^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(t^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(t^k_{js+h}\right)+1}} \\ &= \sum_{j=1}^{\tilde{N}^k(h,s,h',s')} r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}}, \end{aligned}$$

where $r^l_{t^l_{j'}:t^l_{i'}}$ denotes the (possibly multiple-tick) return of asset l over the interval $(t^l_{j'}, t^l_{i'})$, and the $\tilde{t}^k_{js+h}$'s denote the relevant ticks of asset k on the (h,s)-subgrid, i.e., the set of all (h,s)-ticks minus the set of ticks fulfilling the condition in equation (4). Each pair $r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}}$ can be decomposed as

$$\begin{aligned} r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} &= r^{*k}_{\tilde{t}^k_{js+h}} r^{*l}_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} \\ &\quad + e^k_{\tilde{t}^k_{js+h}} e^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}}. \end{aligned} \tag{5}$$

The first product on the right-hand side of equation (5) contributes to the estimation of the integrated covariance, which we would like to measure. The second one is due to noise and we examine it further:

$$\begin{aligned} e^k_{\tilde{t}^k_{js+h}} e^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} &= \left(u^k_{\tilde{t}^k_{js+h}} - u^k_{\tilde{t}^k_{(j-1)s+h}}\right)\left(u^l_{t_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} - u^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}}\right) \\ &= \gamma_{k,l}\left(\overleftarrow{q}\right) + \gamma_{k,l}\left(\overrightarrow{q}\right) - \gamma_{k,l}\left(\overleftarrow{\bar{q}}\right) - \gamma_{k,l}\left(\overrightarrow{\bar{q}}\right), \end{aligned}$$

where

$$
\begin{aligned}
\overleftarrow{q} &= \tilde{t}^k_{(j-1)s+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)} \\
\overrightarrow{q} &= \tilde{t}^k_{js+h} - t_{\breve{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1} \\
\overset{\dashleftarrow}{q} &= \tilde{t}^k_{js+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)} \\
\overset{\dashrightarrow}{q} &= \tilde{t}^k_{(j-1)s+h} - t_{\breve{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}
\end{aligned}
$$

are the time spans in seconds between the four returns' endpoints and therefore the cross-correlation orders in the autocorrelation function $\gamma_{k,l}(q)$.

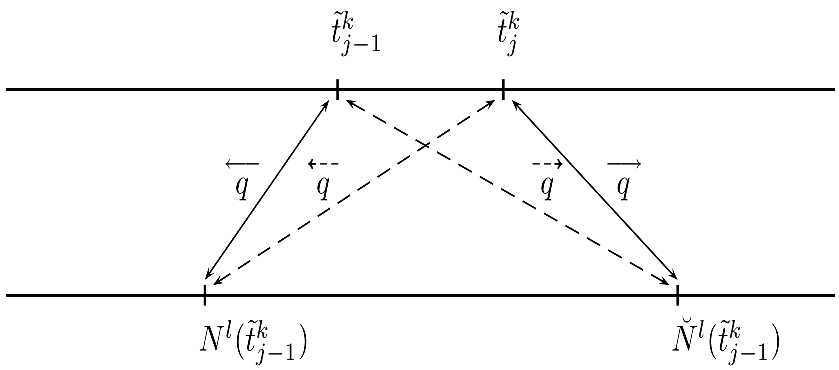

Figure 2: Illustration of the definition of $\overleftarrow{q}$, $\overrightarrow{q}$, $\overset{\dashleftarrow}{q}$ and $\overset{\dashrightarrow}{q}$. The indices h, s, h', s' have been suppressed.

Note that we always have $\overleftarrow{q} \geq 0$, $\overrightarrow{q} \leq 0$, $\overset{\dashleftarrow}{q} > 0$ and $\overset{\dashrightarrow}{q} < 0$. In Figure 2 we illustrate graphically the definition of $\overleftarrow{q}$, $\overrightarrow{q}$, $\overset{\dashleftarrow}{q}$ and $\overset{\dashrightarrow}{q}$.
The number of $\overleftarrow{q}$ which are of a given length q is given by

$$
\overleftarrow{N}^k_{h,s,h',s'}(q) = \sum_{\overleftarrow{q}} \mathbb{1}_{\left\{\overleftarrow{q}=q\right\}} = \sum_j \mathbb{1}_{\left\{\tilde{t}^k_{(j-1)s+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)} = q\right\}}.
$$

Similarly, we can define $\overrightarrow{N}^k_{h,s,h',s'}(q)$, $\overset{\dashleftarrow}{N}^k_{h,s,h',s'}(q)$ and $\overset{\dashrightarrow}{N}^k_{h,s,h',s'}(q)$. Finally, the number of occurrences of $\gamma_{k,l}(q)$ in the expectation of the HY estimator will be

$$
N^k_{h,s,h',s'}(q) = \overleftarrow{N}^k_{h,s,h',s'}(q) + \overrightarrow{N}^k_{h,s,h',s'}(q) - \overset{\dashleftarrow}{N}^k_{h,s,h',s'}(q) - \overset{\dashrightarrow}{N}^k_{h,s,h',s'}(q).
$$

Then, under Assumptions 1 and 2 it holds that

$$\begin{aligned} \mathrm{E}\left[HY^{k,l}(h,s,h',s')\right] &= \mathrm{IC}^{k,l} + \sum_{q=-\infty}^{\infty} N^k_{h,s,h',s'}(q)\gamma_{k,l}(q) \\ &\approx \mathrm{IC}^{k,l} + \sum_{q=-Q}^{Q} N^k_{h,s,h',s'}(q)\gamma_{k,l}(q), \end{aligned}$$

where $\mathrm{IC}^{k,l}$ is the integrated covariance of the price processes of assets k and l, i.e. element (k,l) of $\mathbf{IC}$. The corresponding pooled OLS regression is

$$y_{h,s,h',s'} = c + \beta' x_{h,s,h',s'} + \varepsilon_{h,s,h',s'}, \qquad \begin{array}{ll} s = 1,\dots,S, & h = 1,\dots,s, \\ s' = 1,\dots,S', & h' = 1,\dots,s', \end{array} \tag{6}$$

where $y_{h,s,h',s'} = HY^{k,l}(h,s,h',s')$ and $x_{h,s,h',s'}$ is the $(2Q+1)$-dimensional vector given by $x_{h,s} = (N^k_{h,s,h',s'}(-Q), N^k_{h,s,h',s'}(-Q+1), \dots, N^k_{h,s,h',s'}(0), \dots, N^k_{h,s,h',s'}(Q-1), N^k_{h,s,h',s'}(Q))'$, and Q is chosen suitably.

4 Monte Carlo Study

In this section we present the results of a Monte Carlo experiment designed to compare the bias and variance of a number of high-frequency volatility and covolatlity estimators for a broad set of different trading scenarios.

4.1 Simulation Setup

We simulate two univariate price processes $p^{*k}(t)$ and $p^{*l}(t)$ with the following stochastic differential equations:

$$dp^{*k}(t) = \sigma_k(t)dW_k, \qquad dp^{*l}(t) = \sigma_l(t)dW_l, \tag{7}$$

where $\sigma_k(t)$ and $\sigma_l(t)$ follow GARCH diffusion processes given below and $\langle W_l, W_k\rangle_t = \rho$, i.e., the efficient price processes have stochastic volatility but constant correlation and hence the covariation process is also stochastic. While this setup can be extended by allowing for stochastic correlation, we find this unnecessary here.[2] The volatility processes are modelled as in Andersen & Bollerslev (1998) by

$$\begin{aligned} d\sigma_k^2(t) &= \theta_k(\omega_k - \sigma_k^2(t))dt + \sqrt{2\lambda_k\theta_k}\sigma_k^2(t)dW_k^\sigma(t) \\ d\sigma_l^2(t) &= \theta_l(\omega_l - \sigma_l^2(t))dt + \sqrt{2\lambda_l\theta_l}\sigma_l^2(t)dW_l^\sigma(t) \end{aligned}$$

[2] Voev & Lunde (2007) find in their simulation study that stochastic vs. constant correlation, in the case of stochastic volatility, does not influence the performance of the covariance estimators.

where $\lambda_k > 0$, $\lambda_l > 0$, $\omega_k > 0$, $\omega_l > 0$, $0 > \theta_k > 1$, $0 > \theta_l > 1$ and the Brownian motions W_k^σ and W_l^σ are independent and also independent of W_k and W_l. Within this framework the integrated covariation matrix is given by

$$\mathbf{IC} = \int_0^1 \begin{pmatrix} \sigma_k^2(t) & \bullet \\ \rho\sigma_k(t)\sigma_l(t) & \sigma_l^2(t) \end{pmatrix} dt$$

The price and volatility processes are generated on a one-second grid for a total of 23400 seconds, corresponding to a typical trading session of 6.5 hours. The parameter values we use are as follows: $\lambda_k = 0.296$, $\lambda_l = 0.480$, $\omega_k = 0.636$, $\omega_l = 0.476$, $\theta_k = 0.035$, $\theta_l = 0.054$. These values have been obtained by Andersen & Bollerslev (1998) for the DM/USD and JPY/USD exchange rates. A similar simulation setup is employed by Barndorff-Nielsen & Shephard (2004), Christensen & Podolskij (2007) and Renò (2003), among others.

The noise processes u_t^k and u_t^l are generated as a bivariate VAR(1) process on the same grid as the price processes:

$$\begin{pmatrix} u_t^k \\ u_t^l \end{pmatrix} = \Phi \begin{pmatrix} u_{t-1}^k \\ u_{t-1}^l \end{pmatrix} + \begin{pmatrix} \varepsilon_t^k \\ \varepsilon_t^l \end{pmatrix},$$

where the matrix $\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$ fulfils the conditions for stationarity of a VAR(1) process and

$$\varepsilon_t \equiv \begin{pmatrix} \varepsilon_t^k \\ \varepsilon_t^l \end{pmatrix} \sim N(0, \Sigma_\varepsilon)$$

is a bivariate white noise process. Obviously, the i.i.d. noise case is obtained for $\Phi = 0$, and the two noise processes are i.i.d. and uncorrelated across assets if, in addition to $\Phi = 0$, Σ_ε is diagonal. An alternative specification to simulate a serially dependent noise process is to model it as an Ornstein-Uhlenbeck process. This possibility is discussed in Aït-Sahalia, Mykland & Zhang (2005), who note that under such an assumption, the noise variance depends on the particular time interval between two observations, i.e., it is of the same order as the integrated variance of the efficient price process $\mathbf{p}^*$. Empirically, however, the market microstructure noise accumulates with the number of observations within a given time interval, and its variance does not go to zero as the time interval shrinks, but rather stays constant. To achieve this one could combine an Ornstein-Uhlenbeck process with an i.i.d. process with constant variance. In this study, instead of adopting this approach which we think will make the

simulation intransparent, we approximate this continuous-time structure by a discrete time VAR(1) process on the finest time grid, which we consider a good description of the empirically observed properties of the noise process.
After both the price and the noise processes are generated we obtain the noisy prices

$$\mathbf{p}_t = \mathbf{p}_t^* + \mathbf{u}_t$$

for each $t = 1, \ldots, 23400$ on the second-by-second grid. To obtain different scenarios in terms of trading activity we generate random Poisson sampling times with constant intensities η_k and η_l for asset k and l, respectively.
In our simulation study, we keep the parameters pertaining to the volatility specification fixed, while we vary the parameters Φ, Σ_ε, η_k and η_l to reproduce various noise and trading intensity scenarios. The parameter constellations we consider are presented in Table 1. We consider three types of noise: i.i.d., dependent with low persistence, and dependent with high persistence. Each of these specifications is combined with large, moderate and low variances (diagonal elements of Σ_ε) of the white noise process ε_t. Furthermore, we generate observation arrival processes with very high, moderate and low intensities, to obtain a total of 27 scenarios.

Scenario	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	Var(ε_k)	Var(ε_l)	η_k	η_l
iid, low var., high int.	0	0	0	0	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
iid, low var., mod. int.	0	0	0	0	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
iid, low var., low int.	0	0	0	0	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
iid, mod. var., high int.	0	0	0	0	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
iid, mod. var., mod. int.	0	0	0	0	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
iid, mod. var., low int.	0	0	0	0	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
iid, high var., high int.	0	0	0	0	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
iid, high var., mod. int.	0	0	0	0	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
iid, high var., low int.	0	0	0	0	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$
low pers., low var., high int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
low pers., low var., mod. int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
low pers., low var., low int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
low pers., mod. var., high int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
low pers., mod. var., mod. int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
low pers., mod. var., low int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
low pers., high var., high int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
low pers., high var., mod. int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
low pers., high var., low int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$
high pers., low var., high int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
high pers., low var., mod. int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
high pers., low var., low int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
high pers., mod. var., high int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
high pers., mod. var., mod. int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
high pers., mod. var., low int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
high pers., high var., high int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
high pers., high var., mod. int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
high pers., high var., low int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$

Table 1: Monte Carlo Simulation Scenarios. We use the following abbreviations: "var." stands for variance, "mod." stands for moderate, "pers." stands for persistence, "int." stands for intensity. The correlation between ε_k and ε_l is set in all scenarios equal to -0.1.

4.2 Estimators

In order to compare the performance of our estimation approach against other techniques proposed in the literature, we include a large set of estimators as alternatives. For the univariate case we consider the standard realized volatility at different sampling frequencies, including the optimal sampling frequency derived in Bandi & Russell (2005a), realized volatility with lag correction, the realized kernels of Barndorff-Nielsen et al. (2006) and the two-scales estimator of Zhang et al. (2005). For the estimation of the integrated covariance we consider the realized covariance computed at different sampling frequencies, including the optimal sampling frequency derived in Bandi & Russell (2005b), realized covariance with lead/lag correction, and the HY estimator along with its subsampled version proposed by Voev & Lunde (2007). Our estimators are the estimated constants in the OLS regressions in equations (2) and (6) for the integrated variance and covariance, respectively. In our approach there are two parameters that need to be chosen: Q – the number of lags for the (cross) autocovariance function of the noise processes, and S – the number of subsamples. We discuss the choice of these parameters after we setup the notation for the estimators.

The standard realized variance is denoted by $RV(\delta)$, where δ is sampling frequency in seconds, while by $RV_L(\delta)$ we denote the realized variance with a lag correction of L lags. For the realized kernels we use the notation $K^{TH2}(\delta)$ for the modified Tukey-Hanning kernel as described in Barndorff-Nielsen et al. (2006). The two scales realized variance of Zhang et al. (2005) is a combination of an averaged subsampled realized variance at moderate frequencies combined with a very high frequency realized variance correction term and is denoted by $TSRV$. The multi-scale realized variance of Zhang (2006a) is not included as Barndorff-Nielsen et al. (2006) show that its asymptotic distribution is the same as for a realized kernel with a cubic kernel function, which is outperformed by the modified Tukey-Hanning kernel used in this paper.

The usual last-tick interpolation realized covariance is denoted by $RC(\delta)$, while its biased corrected version, with L^+ leads and L^- lags, is denoted by $RC_{L^+,L^-}(\delta)$. The Hayashi-Yoshida estimator is denoted as above by HY and its subsampled version based on S subsamples by $HY(S)$. Finally, we denote our estimators by $NV(S,Q)$ in the univariate case, and $NV(S,S',Q^+,Q^-)$ in the multivariate case. In our Monte Carlo study, we set $S'=1$ for simplicity, i.e., we only consider subsamples of the first asset. The estimators could be improved by subsampling the second asset as well.

For many of the estimators listed above, in order to determine optimal sampling frequencies or optimal numbers of subgrids, one needs to estimate in a first step the second moments of the noise process, as well as the integrated quarticity of the efficient price process, which we denote by IQ^k for asset k. Although there are various estimators for these quantities, we adopt a simple approach following Barndorff-Nielsen et al. (2006). Thus, the noise variances $\gamma_{k,k}(0)$, $\gamma_{l,l}(0)$ are obtained by averaging over subsampled realized variances computed at 60-second grids and dividing by twice the number of returns, while the integrated quarticity is obtained as the square of the average over realized variance computed at sampling frequency of 20 minutes. While there are currently better estimators for these quantities, this methodology is robust to a fairly large range of noise specifications and delivers reasonable estimates. In order to check whether the results are affected by the fact that we estimate these quantities, we also consider the infeasible versions of the estimators in which we use the true noise variances and integrated quarticity. In the multivariate case, we need to estimate $\gamma_{k,l}(0)$ and a quantity which corresponds to the integrated quarticity, which we denote by $\mathrm{IQ}^{k,l}$ and is given by

$$\mathrm{IQ}^{k,l} = \int_0^1 \sigma_{k,k}(t)\sigma_{l,l}(t) + \sigma_{k,l}^2(t)dt,$$

where $\sigma_{k,l}(t)$ is the (k,l)-element of $\mathbf{\Sigma}(t)$.[3] To estimate this quantity we rely on the approach proposed by Bandi & Russell (2005b). Having estimated $\gamma_{k,k}(0)$, $\gamma_{l,l}(0)$, IQ^k and $\mathrm{IQ}^{k,l}$, the optimal sampling frequency is given by $\delta^* = \left\lceil \frac{23400}{N^*} \right\rceil$, where N^* is determined by

$$N^* = \begin{cases} \left(\frac{\mathrm{IQ}^k}{\gamma_{k,k}^2(0)}\right)^{\frac{1}{3}}, & \text{in the univariate case} \\ \left(\frac{\mathrm{IQ}^{k,l}}{2\gamma_{k,l}^2(0)}\right)^{\frac{1}{3}}, & \text{in the multivariate case.} \end{cases} \tag{8}$$

For the optimal number of subgrids, we rely on results derived in Zhang et al. (2005) in the univariate case and Voev & Lunde (2007) in the multivariate case. Thus we determine

$$S^* = c^{1/3}N^{2/3}, \tag{9}$$

where N is the total number of observations of the asset under consideration in the univariate case, while in the multivariate case, N is the number of observations of the slower asset and the optimal constant c is given by

$$c = \begin{cases} \frac{12\gamma_{k,k}^2(0)}{\mathrm{IQ}^k}, & \text{in the univariate case} \\ \frac{12\left(\gamma_{k,k}^2(0)\gamma_{l,l}^2(0)+\gamma_{k,l}^2(0)\right)}{\mathrm{IQ}^{k,l}}, & \text{in the multivariate case.} \end{cases}$$

4.3 Simulation Results

The results of the Monte Carlo study are collected in the Appendix. The main message is that our estimators clearly outperform all other considered estimators both in the univariate, as well as in the multivariate case! Our main competitors are as expected the realized kernel and the TSRV in the univariate case, and the Bandi & Russell

[3]Barndorff-Nielsen & Shephard (2004) show that this quantity is the asymptotic variance of the realized covariance estimator.

(2005b) realized covariance as well as the HY-type estimators (in some very particular scenarios) in the multivariate case.

The considered realized kernel is the only other estimator, apart from our estimator, that delivers unbiased estimates across the range of Monte Carlo scenarios. It is, however, clearly outperformed in the i.i.d. case by the TSRV, while our estimator is not. In order to check whether it could be that the inputs required for the construction of the realized kernels and the TSRV impairs their performance, we computed their infeasible versions by setting the unknown quantities (e.g., the integrated quarticity or noise variance) to their true values. Overall, this did not qualitatively influence the results, implying that the estimates we use to construct the feasible estimators are reasonable.

In the covariance estimation, the subsampled HY estimator performs better than our estimator only in the case of i.i.d. noise with low variance and moderate or low trading intensity. In all other cases, it is severely biased and hence not competitive. The Bandi & Russell (2005b) estimator with a first-order lead/lag correction performs quite well and is second only to our estimator the best alternative.

A very nice feature of our approach is that the proposed estimator is very robust and not too sensitive to the choice of the number of subsamples S. What is important, however, is that Q is chosen reasonably, which on the one hand means that it should not be too low (omitted variable problem) in the case of highly persistent noise, and on the other hand not too close to $S(S+1)/2$ (the number of observations in the pooled OLS regression) to assure that the X matrix is not close to being singular. As we do not have a rule to determine S and/or Q according to a theoretically based optimality criterion, we employ a data-driven model selection. In particular we estimate our models for some predetermined set of S and Q values and then select a specification based on an information criterion. In this study we consider the set of values $Q = \{0, 10, 20\}$ (also for the Q^+ and Q^- in the multivariate case) and $S = \{S^*, 2S^*, 3S^*\}$, thus obtaining a set of nine estimators for each scenario. It is important to note, that these values are arbitrarily chosen and can lead to certain ill-specified X matrices. In particular, when we have low intensity specifications, the number of observations is small, and hence S^* is small. If Q is then chosen large, then the X matrix is near-singular (number of cases in the OLS regression close to the number of regressors). In our simulations this occurs for the models $NV(S^*, 10)$, $NV(S^*, 20)$ (in the univariate case) and $NV(S^*, 1, 10, 10)$, $NV(S^*, 1, 20, 20)$ (in the multivariate case) in combination with very low trading intensities (low intensity specification for asset 2, corresponding to a trade every 30 seconds on average) for which the S^* is too small.

Such S, Q-combinations should be avoided and can be identified by a nearly perfect fit in the OLS regression (R-squared very close to one and sum of squared residuals almost equal to zero). While we report results for these estimators in the Appendix, we exclude them from the set of models from which we select the optimal model.

For practical purposes, we propose two criteria for choosing the proper combination of the parameters S and Q, both based on the goodness-of-fit of the regression, given by

$$BIC = \ln\left(\frac{1}{n}\sum_{h,s}\hat{\varepsilon}^2_{h,s}\right) + \frac{p\ln n}{n}, \quad \text{with } n = \frac{S(S+1)}{2} \tag{10}$$

$$BIC^* = \ln\left(\frac{1}{n}\sum_{h,s}\frac{\hat{\varepsilon}^2_{h,s}}{s}\right) + \frac{p\ln n}{n}, \quad \text{with } n = S, \tag{11}$$

where $\hat{\varepsilon}_{h,s}$ is the pooled OLS regression residual and p is the number of parameters in the regression. The first criterion is the usual Bayesian Information Criterion (BIC) for the pooled regression, while the second one is a modified BIC for a regression over $s = 1, \ldots, S$, where the squared residual for each s is the mean squared residual of the s-block. The modified BIC is motivated by the fact that the number of elements in the s-block, which equally contribute to the estimation, is linearly increasing with s. It accounts for the fact that as s increases, a single (h, s)-observation becomes more noisy and therefore should be counted with an accordingly smaller weight.

Let us have a closer look at the "average" scenario (Table 17 in the Appendix) with moderate noise variances (0.001 and 0.002), low noise persistence and moderate trading intensities (1/5 and 1/10), which we believe is rather realistic. In this case the modified BIC criterion selects $NV(3S^*, 10)$ for the variance of the first asset, $NV(3S^*, 0)$ for the variance of the second asset, and $NV(3S^*, 1, 0, 0)$ for the covariance. We note that for this scenario, the modified BIC selects suboptimal models for the first variance, for which the selected model ranks 5th among the NV models with respect to the root mean squared error (RMSE), and for the covariance for which the selected model ranks 3rd among the NV models. Despite this suboptimal choice, the efficiency gain of the selected models compared to the best alternative outside the NV class, in terms of the RMSE, is 40%, 37%, and 47%, for the variance of the first asset, the variance of the second asset, and the covariance, respectively.

In Tables 2 and 3 we report summary statistics of the ranks of all considered estimators, according to their RMSE, across simulation scenarios. Additionally, the tables contain statistics on the RMSE of all estimators relative to the RMSE of the estimator selected by the BIC (in Table 2) and by the BIC^* (in Table 3).

	Model Ranking				Relative RMSE			
Model	Mean	Median	Min	Max	Mean	Median	Min	Max
				IV¹				
$RV(5)$	13.9	14	12	14	324.00	102.76	5.67	1587.26
$RV(300)$	7.7	7	2	12	10.93	3.67	1.77	48.39
$RV(900)$	7.2	6	3	11	5.13	3.47	2.29	16.14
$RV(1800)$	7.9	9	4	11	4.55	3.89	2.45	8.63
$RV_1(5)$	11.8	13	3	13	146.71	32.34	1.59	988.54
$RV_1(300)$	5.7	6	3	8	3.81	3.47	2.15	7.96
$RV_1(900)$	8.8	10	3	13	4.82	4.33	2.85	10.41
$RV_1(1800)$	10.3	12	3	14	6.13	5.20	3.47	14.33
$RV(\delta^*)$	6.4	5	3	10	3.85	2.89	1.83	9.30
$RV_1(\delta^*)$	6.7	7	3	9	3.73	3.52	2.64	6.25
$RV_2(\delta^*)$	8.8	9	5	10	4.40	4.29	3.14	7.52
$K^{TH2}(60)$	3.3	3	2	6	2.37	2.06	1.65	4.61
$TSRV$	5.3	2	1	12	7.03	1.39	0.80	63.48
$NV(BIC)$	1.2	1	1	2	1.00	1.00	1.00	1.00
				IV²				
$RV(5)$	14.0	14	13	14	379.89	111.34	4.64	2113.11
$RV(300)$	9.1	12	3	12	16.12	5.66	1.54	63.74
$RV(900)$	7.8	6	4	11	6.36	3.53	1.58	21.51
$RV(1800)$	7.8	8	4	10	4.53	3.60	2.17	11.25
$RV_1(5)$	12.9	13	11	13	205.10	70.72	3.91	1241.08
$RV_1(300)$	6.2	7	3	9	4.04	2.87	1.65	10.52
$RV_1(900)$	8.0	9	4	12	4.18	3.92	2.67	8.01
$RV_1(1800)$	8.9	11	3	14	5.00	4.55	3.26	11.37
$RV(\delta^*)$	7.0	8	3	10	4.49	3.35	1.42	11.62
$RV_1(\delta^*)$	6.6	7	3	8	3.47	3.30	1.89	5.42
$RV_2(\delta^*)$	8.3	9	5	10	4.02	3.87	2.21	6.37
$K^{TH2}(60)$	3.1	3	2	5	2.07	2.03	1.46	3.68
$TSRV$	4.2	2	1	12	4.58	1.42	0.85	41.04
$NV(BIC)$	1.2	1	1	2	1.00	1.00	1.00	1.00
				IC				
$RC(5)$	10.8	12	1	14	32.04	4.63	0.96	354.59
$RC(300)$	6.1	7	2	10	4.04	2.22	0.69	21.84
$RC(900)$	6.2	6	2	10	3.35	2.97	0.95	9.37
$RC(1800)$	7.4	8	2	12	3.51	3.27	1.22	6.82
$RC_{1,1}(5)$	10.9	12	4	14	46.91	6.70	0.90	549.92
$RC_{1,1}(300)$	7.2	7	4	11	3.49	3.10	0.95	8.56
$RC_{1,1}(900)$	8.9	11	3	13	4.06	3.79	1.46	8.27
$RC_{1,1}(1800)$	10.1	12	2	14	5.10	4.62	2.03	11.74
$RC(\delta^*)$	7.6	8	2	12	4.64	3.02	0.78	19.38
$RC_{1,1}(\delta^*)$	5.0	4	2	10	3.22	2.10	0.75	8.34
$RC_{2,2}(\delta^*)$	6.1	6	3	10	3.33	2.23	0.76	8.10
HY	10.4	12	2	14	27.13	7.97	0.55	159.16
$HY(S^*)$	6.3	7	1	11	7.67	3.14	0.44	53.74
$NV(BIC)$	2.0	1	1	9	1.00	1.00	1.00	1.00

Table 2: Mean, median, maximum and minimum of the root mean squared error (RMSE) rankings and of the relative RMSE across simulation scenarios. The relative RMSE is computed relative to the $NV(BIC)$ estimator, which is for each scenario the model with the smallest Bayesian Information Criterion (Equation (10)) across all NV estimators.

	Model Ranking				Relative RMSE			
Model	Mean	Median	Min	Max	Mean	Median	Min	Max
	IV[1]							
$RV(5)$	13.9	14	12	14	322.98	102.76	5.67	1383.74
$RV(300)$	7.7	7	2	12	10.80	3.67	1.07	45.97
$RV(900)$	7.2	6	3	11	5.06	3.47	1.38	15.64
$RV(1800)$	7.9	9	4	11	4.48	3.89	1.95	8.63
$RV_1(5)$	11.8	13	3	13	142.67	32.34	1.59	891.90
$RV_1(300)$	5.7	6	3	8	3.76	3.47	1.42	7.93
$RV_1(900)$	8.8	10	3	13	4.75	4.57	2.38	10.41
$RV_1(1800)$	10.3	12	3	14	6.04	5.20	3.19	14.33
$RV(\delta^*)$	6.4	5	3	10	3.82	2.89	1.10	9.25
$RV_1(\delta^*)$	6.7	7	3	9	3.69	3.51	1.59	6.25
$RV_2(\delta^*)$	8.8	9	5	10	4.35	4.28	1.89	7.52
$K^{TH2}(60)$	3.3	3	2	6	2.33	2.11	1.23	4.61
$TSRV$	5.3	2	1	12	6.70	1.08	0.80	55.34
$NV(BIC^*)$	1.2	1	1	2	1.00	1.00	1.00	1.00
	IV[2]							
$RV(5)$	14.0	14	13	14	385.82	123.71	4.64	2086.80
$RV(300)$	9.1	12	3	12	16.27	5.72	1.54	62.95
$RV(900)$	7.8	6	4	11	6.42	3.53	1.58	21.24
$RV(1800)$	7.8	8	4	10	4.57	3.60	2.17	11.11
$RV_1(5)$	12.9	13	11	13	206.57	71.81	3.91	1225.62
$RV_1(300)$	6.2	7	3	9	4.08	2.87	1.65	10.39
$RV_1(900)$	8.0	9	4	12	4.22	3.92	2.67	8.01
$RV_1(1800)$	8.9	11	3	14	5.05	4.89	3.26	11.37
$RV(\delta^*)$	7.0	8	3	10	4.53	3.35	1.42	11.47
$RV_1(\delta^*)$	6.6	7	3	8	3.51	3.30	1.89	5.35
$RV_2(\delta^*)$	8.3	9	5	10	4.07	3.87	2.21	6.37
$K^{TH2}(60)$	3.1	3	2	5	2.09	2.03	1.46	3.68
$TSRV$	4.2	2	1	12	4.59	1.42	0.85	40.53
$NV(BIC^*)$	1.2	1	1	2	1.00	1.00	1.00	1.00
	IC							
$RC(5)$	10.8	12	1	14	30.22	4.63	0.96	362.70
$RC(300)$	6.2	7	2	10	3.87	2.22	1.37	22.34
$RC(900)$	6.2	6	2	10	3.32	2.97	1.91	9.58
$RC(1800)$	7.4	8	2	12	3.55	3.22	2.20	6.82
$RC_{1,1}(5)$	10.9	12	4	14	42.79	6.24	1.34	562.50
$RC_{1,1}(300)$	7.2	7	4	11	3.46	3.10	1.91	8.75
$RC_{1,1}(900)$	8.9	11	3	13	4.12	3.79	2.60	8.27
$RC_{1,1}(1800)$	10.1	12	2	14	5.24	4.88	2.58	11.74
$RC(\delta^*)$	7.7	8	3	12	4.54	2.91	1.04	19.83
$RC_{1,1}(\delta^*)$	5.2	4	2	10	3.17	2.21	1.19	8.53
$RC_{2,2}(\delta^*)$	6.3	6	3	10	3.29	2.33	1.42	8.13
HY	10.5	12	2	14	24.63	7.97	0.80	118.20
$HY(S^*)$	6.3	7	1	11	7.17	3.14	0.79	54.97
$NV(BIC^*)$	1.3	1	1	3	1.00	1.00	1.00	1.00

Table 3: Mean, median, maximum and minimum of the root mean squared error (RMSE) rankings and of the relative RMSE across simulation scenarios. The relative RMSE is computed relative to the $NV(BIC^*)$ estimator, which is for each scenario the model with the smallest modified Bayesian Information Criterion (Equation (11)) across all NV estimators.

An alternative model selection strategy could rely on a procedure, in which one starts from a high value of Q which is sequentially reduced. Following this procedure, it can be detected whether the estimates change significantly as Q becomes smaller. If this is the case, there is an indication for the presence of persistence in the noise processes and consequently Q should be chosen preferably a bit too high rather than too low. Whenever Q is chosen to be large, it is beneficial to choose S large as well, since as mentioned above and explicit in the simulation results, one cannot choose S too poorly by choosing it too large, which also alleviates the discussed near-singularity problem.

From Tables 2 and 3 it is clear that our models are dominating and that the proposed selection criteria, although not perfect are doing a very good job. In particular, the modified BIC, which is motivated by the pooled form of the regression, is more robust and selects better models. Considering the model ranking for all 28 univariate models, including all of our nine specifications, we observe that there is at least one estimator from the NV class that outperforms all others in *each* Monte Carlo scenario in the univariate case, although it is not always selected by the BIC or the modified BIC! This presents possibilities for improvement of the selection criteria, which we consider to be a possible area of further research.

In the multivariate case we observe a similar pattern with three exceptions: the $HY(S^*)$ ranks first in the iid., low variance, moderate intensity and iid., low variance, low intensity scenarios, while the $RC(5)$ ranks first in the low persistence, low variance and high intensity scenario. The last case is a coincidence, in which the noise induced positive bias cancels almost exactly against the negative bias caused by the Epps effect.

An interesting issue which has to be addressed is whether the separate unconstrained estimation of variances and covariances leads to a well-defined covariance matrix. In the Appendix, we have added in the last row of the tables for the covariance estimators the number of cases (out of the 1000 replications) for which the matrix resulting from the particular covariance estimator combined with the two corresponding variance estimators was non-positive definite. In most scenarios and for most estimators this never happens. We observe, however, that for the estimators for which the choice of S and Q leads to singularity problems, which we discussed above, in the worst case this occurs up to 35% of the cases, which is caused by the very large variance of the particular covariance and/or variance estimators. For the estimators chosen by the BIC and modified BIC criteria, though, a non-positive definite matrix is obtained only in one case in only one scenario, i.e., once out of 27000 cases.

5 Conclusion

The paper introduces a unified framework for the estimation of integrated second moments of irregularly observed asset prices contaminated by market microstructure noise. The estimation is performed under fairly weak assumptions on the dependence structure of the noise processes in a simple OLS regression framework. This approach allows for a robust estimation of the whole covariance matrix of asset returns in applications with large number of assets. Moreover, we can identify the dependence structure of the noise process, which sheds light on market microstructure properties. We derive the OLS regressions theoretically for the univariate and multivariate case and perform an extensive Monte Carlo study to compare the performance of our estimators against the most recent and commonly used approaches in the extant literature. The results are unequivocal: our estimators clearly dominate the other approaches across a comprehensive range of trading scenarios.

Promising directions for further research are on the one hand a more in-depth analysis of a model selection criterion based on the statistical properties of our estimators, relaxing the assumption of noise exogeneity, and on the other hand an empirical application with a large number of assets, e.g., in the field of asset pricing or risk management.

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Appendix

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0002	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	1.5354 (0.0338)	1.9357 (0.0522)						
$RV(300)$	0.6326 (0.1027)	0.5466 (0.0919)						
$RV(900)$	0.6056 (0.1657)	0.5089 (0.1393)						
$RV(1800)$	0.5858 (0.2383)	0.4867 (0.1952)						
$RV_1(5)$	0.6527 (0.0373)	0.8640 (0.0456)						
$RV_1(300)$	0.6134 (0.1741)	0.5153 (0.1444)						
$RV_1(900)$	0.6038 (0.2910)	0.4997 (0.2412)						
$RV_1(1800)$	0.5793 (0.3989)	0.5003 (0.3431)						
$RV(\delta^*)$	0.6262 (0.1082)	0.5342 (0.0945)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_1(\delta^*)$	0.5990 (0.1728)	0.5051 (0.1496)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_2(\delta^*)$	0.6053 (0.2097)	0.5101 (0.1921)	0.0009 (0.0000)			0.0009 (0.0000)		
$K^{TH2}(60)$	0.6147 (0.1286)	0.5144 (0.1110)	0.0009 (0.0000)			0.0009 (0.0000)		
$TSRV$	0.6238 (0.0301)	0.5215 (0.0320)	0.0009 (0.0000)			0.0009 (0.0000)		
$NV(S^*, 0)$	0.6256 (0.0279)	0.5231 (0.0302)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*, 0)$	0.6255 (0.0362)	0.5224 (0.0372)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(3S^*, 0)$	0.6251 (0.0425)	0.5218 (0.0430)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(S^*, 10)$	0.6247 (0.0411)	0.5217 (0.0445)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0002 (0.0000)	0.0000 (0.0007)	-0.0001 (0.0019)
$NV(2S^*, 10)$	0.6254 (0.0481)	0.5217 (0.0476)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0002 (0.0000)	0.0000 (0.0007)	-0.0001 (0.0020)
$NV(3S^*, 10)$	0.6247 (0.0537)	0.5210 (0.0530)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0007)	-0.0001 (0.0020)
$NV(S^*, 20)$	0.6251 (0.0720)	0.5249 (0.0856)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0001)	0.0000 (0.0011)	-0.0001 (0.0034)
$NV(2S^*, 20)$	0.6255 (0.0541)	0.5217 (0.0524)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0011)	-0.0001 (0.0034)
$NV(3S^*, 20)$	0.6245 (0.0590)	0.5208 (0.0574)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0011)	-0.0001 (0.0036)

Table 4: Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0897 (0.0269)						
$RC(300)$	0.1962 (0.0743)						
$RC(900)$	0.1870 (0.1226)						
$RC(1800)$	0.1837 (0.1672)						
$RC_{1,1}(5)$	0.1767 (0.0288)						
$RC_{1,1}(300)$	0.1960 (0.1236)						
$RC_{1,1}(900)$	0.1943 (0.2037)						
$RC_{1,1}(1800)$	0.1951 (0.2892)						
$RC(\delta^*)$	0.1493 (0.0364)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1974 (0.0356)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1988 (0.0418)			0.0000 (0.0000)			
HY	0.1196 (0.0303)						
$HY(S^*)$	0.1769 (0.0193)						
$NV(S^*, 1, 0, 0)$	0.2008 (0.0246)			-0.0000 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2006 (0.0302)			-0.0000 (0.0000)			0
$NV(3S^*, 1, 0, 0)$	0.2003 (0.0347)			-0.0000 (0.0000)			0
$NV(S^*, 1, 10, 10)$	0.2015 (0.0368)	-0.0000 (0.0020)	0.0000 (0.0019)	-0.0000 (0.0001)	-0.0000 (0.0020)	-0.0000 (0.0021)	0
$NV(2S^*, 1, 10, 10)$	0.2003 (0.0388)	0.0000 (0.0016)	0.0001 (0.0016)	-0.0000 (0.0001)	-0.0000 (0.0016)	-0.0000 (0.0016)	0
$NV(3S^*, 1, 10, 10)$	0.2001 (0.0429)	0.0000 (0.0015)	0.0000 (0.0015)	-0.0000 (0.0000)	-0.0000 (0.0015)	-0.0000 (0.0015)	0
$NV(S^*, 1, 20, 20)$	0.2010 (0.1128)	0.0001 (0.0057)	0.0001 (0.0061)	-0.0000 (0.0001)	-0.0001 (0.0057)	-0.0001 (0.0064)	1
$NV(2S^*, 1, 20, 20)$	0.2000 (0.0428)	0.0001 (0.0033)	0.0002 (0.0034)	-0.0000 (0.0001)	-0.0002 (0.0034)	-0.0001 (0.0034)	0
$NV(3S^*, 1, 20, 20)$	0.1999 (0.0468)	0.0001 (0.0029)	0.0001 (0.0030)	-0.0000 (0.0001)	-0.0001 (0.0029)	-0.0001 (0.0030)	0

Table 4 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0002	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	1.2624	1.2699						
	(0.0350)	(0.0476)						
$RV(300)$	0.6312	0.5484						
	(0.1035)	(0.0910)						
$RV(900)$	0.6058	0.5091						
	(0.1647)	(0.1415)						
$RV(1800)$	0.5854	0.4879						
	(0.2356)	(0.1959)						
$RV_1(5)$	0.8268	0.9673						
	(0.0364)	(0.0457)						
$RV_1(300)$	0.6133	0.5160						
	(0.1723)	(0.1456)						
$RV_1(900)$	0.6031	0.5002						
	(0.2898)	(0.2422)						
$RV_1(1800)$	0.5805	0.5000						
	(0.3982)	(0.3424)						
$RV(\delta^*)$	0.6276	0.5341	0.0009			0.0009		
	(0.1035)	(0.0956)	(0.0000)			(0.0000)		
$RV_1(\delta^*)$	0.6026	0.5044	0.0009			0.0009		
	(0.1745)	(0.1560)	(0.0000)			(0.0000)		
$RV_2(\delta^*)$	0.6048	0.5100	0.0009			0.0009		
	(0.2098)	(0.1862)	(0.0000)			(0.0000)		
$K^{TH2}(60)$	0.6140	0.5138	0.0009			0.0009		
	(0.1351)	(0.1165)	(0.0000)			(0.0000)		
$TSRV$	0.6235	0.5201	0.0009			0.0009		
	(0.0380)	(0.0409)	(0.0000)			(0.0000)		
$NV(S^*, 0)$	0.6259	0.5234	0.0001			0.0002		
	(0.0363)	(0.0398)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6248	0.5220	0.0001			0.0002		
	(0.0454)	(0.0474)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6235	0.5213	0.0001			0.0002		
	(0.0522)	(0.0543)	(0.0000)			(0.0000)		
$NV(S^*, 10)$	0.6249	0.5227	0.0001	-0.0000	0.0001	0.0002	-0.0014	-0.0031
	(0.0538)	(0.0770)	(0.0000)	(0.0011)	(0.0066)	(0.0001)	(0.0980)	(0.1623)
$NV(2S^*, 10)$	0.6235	0.5206	0.0001	-0.0000	-0.0000	0.0002	-0.0008	-0.0005
	(0.0592)	(0.0617)	(0.0000)	(0.0012)	(0.0069)	(0.0001)	(0.0838)	(0.1499)
$NV(3S^*, 10)$	0.6220	0.5202	0.0001	-0.0001	0.0001	0.0002	-0.0005	-0.0002
	(0.0649)	(0.0677)	(0.0000)	(0.0014)	(0.0070)	(0.0001)	(0.0807)	(0.1622)
$NV(S^*, 20)$	0.6213	0.5229	0.0001	0.0000	0.0003	0.0002	0.0032	0.0132
	(0.0931)	(0.6583)	(0.0001)	(0.0032)	(0.0174)	(0.0009)	(0.8472)	(1.3007)
$NV(2S^*, 20)$	0.6228	0.5205	0.0001	0.0000	0.0000	0.0002	0.0146	-0.0066
	(0.0638)	(0.0687)	(0.0000)	(0.0029)	(0.0158)	(0.0001)	(0.5751)	(0.4042)
$NV(3S^*, 20)$	0.6213	0.5201	0.0001	0.0000	0.0002	0.0002	0.0121	-0.0087
	(0.0690)	(0.0734)	(0.0001)	(0.0029)	(0.0146)	(0.0001)	(0.5555)	(0.4068)

Table 5: Monte Carlo simulation results for scenario: **iid, low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0501 (0.0193)						
$RC(300)$	0.1930 (0.0752)						
$RC(900)$	0.1875 (0.1229)						
$RC(1800)$	0.1841 (0.1674)						
$RC_{1,1}(5)$	0.1228 (0.0262)						
$RC_{1,1}(300)$	0.1968 (0.1240)						
$RC_{1,1}(900)$	0.1932 (0.2025)						
$RC_{1,1}(1800)$	0.1945 (0.2880)						
$RC(\delta^*)$	0.1448 (0.0362)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1949 (0.0431)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1965 (0.0523)			0.0000 (0.0000)			
HY	0.1882 (0.0305)						
$HY(S^*)$	0.1950 (0.0274)						
$NV(S^*, 1, 0, 0)$	0.2005 (0.0340)			-0.0000 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2002 (0.0382)			-0.0000 (0.0001)			0
$NV(3S^*, 1, 0, 0)$	0.2001 (0.0431)			-0.0000 (0.0001)			0
$NV(S^*, 1, 10, 10)$	0.2001 (0.0526)	0.0001 (0.0026)	-0.0000 (0.0013)	-0.0000 (0.0006)	-0.0000 (0.0033)	0.0001 (0.0017)	0
$NV(2S^*, 1, 10, 10)$	0.1999 (0.0464)	0.0000 (0.0014)	0.0000 (0.0013)	-0.0000 (0.0004)	-0.0000 (0.0014)	-0.0000 (0.0017)	0
$NV(3S^*, 1, 10, 10)$	0.1999 (0.0511)	-0.0000 (0.0010)	-0.0000 (0.0010)	-0.0000 (0.0003)	0.0000 (0.0010)	0.0000 (0.0011)	0
$NV(S^*, 1, 20, 20)$	0.2001 (0.0526)	0.0001 (0.0026)	-0.0000 (0.0013)	-0.0000 (0.0006)	-0.0000 (0.0033)	0.0001 (0.0017)	63
$NV(2S^*, 1, 20, 20)$	0.1996 (0.0537)	0.0002 (0.0071)	0.0001 (0.0068)	0.0000 (0.0007)	-0.0003 (0.0089)	-0.0001 (0.0079)	0
$NV(3S^*, 1, 20, 20)$	0.1998 (0.0569)	-0.0001 (0.0023)	-0.0001 (0.0022)	-0.0000 (0.0003)	0.0001 (0.0024)	0.0001 (0.0023)	0

Table 5 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0001	0.0000	0.0000	0.0002	0.0000	0.0000
$RV(5)$	0.9066	0.8053						
	(0.0399)	(0.0522)						
$RV(300)$	0.6309	0.5483						
	(0.1029)	(0.0918)						
$RV(900)$	0.6041	0.5097						
	(0.1681)	(0.1413)						
$RV(1800)$	0.5847	0.4868						
	(0.2396)	(0.1947)						
$RV_1(5)$	0.8229	0.7588						
	(0.0403)	(0.0523)						
$RV_1(300)$	0.6130	0.5158						
	(0.1738)	(0.1449)						
$RV_1(900)$	0.6006	0.4995						
	(0.2929)	(0.2410)						
$RV_1(1800)$	0.5799	0.4982						
	(0.3986)	(0.3410)						
$RV(\delta^*)$	0.6268	0.5386	0.0009			0.0008		
	(0.1040)	(0.0933)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.5987	0.5010	0.0009			0.0008		
	(0.1723)	(0.1500)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6021	0.5063	0.0009			0.0008		
	(0.2134)	(0.1843)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6142	0.5154	0.0009			0.0008		
	(0.1455)	(0.1253)	(0.0001)			(0.0001)		
$TSRV$	0.6227	0.5165	0.0009			0.0008		
	(0.0517)	(0.0617)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6263	0.5226	0.0001			0.0002		
	(0.0496)	(0.0617)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6249	0.5212	0.0001			0.0002		
	(0.0582)	(0.0667)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6231	0.5212	0.0001			0.0002		
	(0.0659)	(0.0737)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6185	0.5211	0.0001	0.0007	0.0004	0.0002	-0.0003	0.0004
	(0.1404)	(0.0961)	(0.0002)	(0.0142)	(0.0110)	(0.0001)	(0.0278)	(0.0291)
$NV(2S^*, 10)$	0.6228	0.5199	0.0001	0.0005	-0.0002	0.0002	-0.0009	0.0007
	(0.0755)	(0.0818)	(0.0001)	(0.0096)	(0.0339)	(0.0001)	(0.0305)	(0.0222)
$NV(3S^*, 10)$	0.6207	0.5208	0.0001	0.0009	-0.0001	0.0002	0.0001	0.0003
	(0.0820)	(0.0876)	(0.0001)	(0.0088)	(0.0068)	(0.0001)	(0.0430)	(0.0273)
$NV(S^*, 20)$	0.6185	0.5211	0.0001	0.0007	0.0004	0.0002	-0.0003	0.0004
	(0.1404)	(0.0961)	(0.0002)	(0.0142)	(0.0110)	(0.0001)	(0.0278)	(0.0291)
$NV(2S^*, 20)$	0.6228	0.5199	0.0001	0.0005	-0.0002	0.0002	-0.0009	0.0007
	(0.0755)	(0.0818)	(0.0001)	(0.0096)	(0.0339)	(0.0001)	(0.0305)	(0.0222)
$NV(3S^*, 20)$	0.6207	0.5208	0.0001	0.0009	-0.0001	0.0002	0.0001	0.0003
	(0.0820)	(0.0876)	(0.0001)	(0.0088)	(0.0068)	(0.0001)	(0.0430)	(0.0273)

Table 6: Monte Carlo simulation results for scenario: **iid, low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0196 (0.0128)						
$RC(300)$	0.1798 (0.0732)						
$RC(900)$	0.1824 (0.1229)						
$RC(1800)$	0.1810 (0.1671)						
$RC_{1,1}(5)$	0.0555 (0.0204)						
$RC_{1,1}(300)$	0.1961 (0.1224)						
$RC_{1,1}(900)$	0.1922 (0.2034)						
$RC_{1,1}(1800)$	0.1927 (0.2879)						
$RC(\delta^*)$	0.1290 (0.0449)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1862 (0.0605)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1930 (0.0740)			0.0001 (0.0000)			
HY	0.1990 (0.0419)						
$HY(S^*)$	0.1998 (0.0416)						
$NV(S^*,1,0,0)$	0.2012 (0.0524)			-0.0000 (0.0004)			0
$NV(2S^*,1,0,0)$	0.2004 (0.0580)			-0.0000 (0.0005)			0
$NV(3S^*,1,0,0)$	0.2002 (0.0637)			-0.0000 (0.0006)			0
$NV(S^*,1,10,10)$	0.2014 (0.0521)		0.0001 (0.0078)	0.0000 (0.0016)	-0.0002 (0.0049)		1
$NV(2S^*,1,10,10)$	0.1996 (0.0759)	0.0048 (0.0778)	-0.0045 (0.0936)	-0.0001 (0.0267)	0.0022 (0.1040)	0.0034 (0.0682)	0
$NV(3S^*,1,10,10)$	0.1998 (0.0713)	-0.0001 (0.0051)	0.0000 (0.0041)	-0.0001 (0.0022)	-0.0000 (0.0039)	0.0001 (0.0064)	0
$NV(S^*,1,20,20)$	0.2014 (0.0521)		0.0001 (0.0078)	0.0000 (0.0016)	-0.0002 (0.0049)		1
$NV(2S^*,1,20,20)$	0.1996 (0.0759)	0.0048 (0.0778)	-0.0045 (0.0936)	-0.0001 (0.0267)	0.0022 (0.1040)	0.0034 (0.0682)	0
$NV(3S^*,1,20,20)$	0.1963 (0.3202)	-0.0004 (0.0158)	0.0004 (0.0230)	0.0001 (0.0439)	0.0001 (0.0960)	-0.0006 (0.0367)	127

Table 6 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).#NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000
$RV(5)$	9.7135	14.6443						
	(0.2364)	(0.4213)						
$RV(300)$	0.7718	0.8233						
	(0.1253)	(0.1399)						
$RV(900)$	0.6494	0.5950						
	(0.1775)	(0.1627)						
$RV(1800)$	0.6068	0.5280						
	(0.2462)	(0.2103)						
$RV_1(5)$	0.8907	3.9251						
	(0.2008)	(0.3271)						
$RV_1(300)$	0.6124	0.5190						
	(0.1855)	(0.1715)						
$RV_1(900)$	0.6080	0.5050						
	(0.2974)	(0.2577)						
$RV_1(1800)$	0.5797	0.5023						
	(0.4037)	(0.3511)						
$RV(\delta^*)$	0.7024	0.6473	0.0018			0.0027		
	(0.1556)	(0.1675)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.6073	0.5111	0.0018			0.0027		
	(0.2190)	(0.2187)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6169	0.5207	0.0018			0.0027		
	(0.2720)	(0.2724)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6122	0.5121	0.0018			0.0027		
	(0.1464)	(0.1442)	(0.0001)			(0.0001)		
$TSRV$	0.6212	0.5174	0.0018			0.0027		
	(0.0405)	(0.0495)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6248	0.5224	0.0010			0.0020		
	(0.0369)	(0.0451)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6248	0.5216	0.0010			0.0020		
	(0.0441)	(0.0504)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6239	0.5210	0.0010			0.0020		
	(0.0514)	(0.0584)	(0.0000)			(0.0000)		
$NV(S^*, 10)$	0.6254	0.5215	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0503)	(0.0577)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0129)
$NV(2S^*, 10)$	0.6248	0.5210	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0553)	(0.0601)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0129)
$NV(3S^*, 10)$	0.6235	0.5204	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0621)	(0.0681)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0128)
$NV(S^*, 20)$	0.6263	0.5239	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0002
	(0.0639)	(0.0674)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0065)	(0.0197)
$NV(2S^*, 20)$	0.6247	0.5212	0.0010	-0.0000	0.0000	0.0020	-0.0000	-0.0002
	(0.0607)	(0.0646)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0064)	(0.0192)
$NV(3S^*, 20)$	0.6230	0.5205	0.0010	0.0000	0.0000	0.0020	-0.0000	-0.0001
	(0.0672)	(0.0724)	(0.0001)	(0.0001)	(0.0003)	(0.0001)	(0.0063)	(0.0186)

Table 7: Monte Carlo simulation results for scenario: **iid, mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	-0.1145 (0.2056)						
$RC(300)$	0.1931 (0.0952)						
$RC(900)$	0.1851 (0.1347)						
$RC(1800)$	0.1835 (0.1753)						
$RC_{1,1}(5)$	0.1473 (0.1790)						
$RC_{1,1}(300)$	0.1949 (0.1365)						
$RC_{1,1}(900)$	0.1956 (0.2117)						
$RC_{1,1}(1800)$	0.1964 (0.2958)						
$RC(\delta^*)$	0.1042 (0.1437)			-0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1854 (0.1292)			-0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1940 (0.1254)			-0.0000 (0.0000)			
HY	-0.6116 (0.2206)						
$HY(S^*)$	0.0365 (0.0460)						
$NV(S^*, 1, 0, 0)$	0.2010 (0.0348)			-0.0001 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2004 (0.0385)			-0.0001 (0.0000)			0
$NV(3S^*, 1, 0, 0)$	0.2003 (0.0440)			-0.0001 (0.0000)			0
$NV(S^*, 1, 10, 10)$	0.2012 (0.0476)	0.0001 (0.0075)	0.0002 (0.0074)	-0.0001 (0.0002)	-0.0002 (0.0075)	-0.0001 (0.0076)	0
$NV(2S^*, 1, 10, 10)$	0.2002 (0.0467)	-0.0000 (0.0058)	0.0000 (0.0056)	-0.0001 (0.0001)	-0.0001 (0.0057)	0.0000 (0.0056)	0
$NV(3S^*, 1, 10, 10)$	0.2001 (0.0518)	-0.0001 (0.0048)	-0.0000 (0.0047)	-0.0001 (0.0001)	0.0000 (0.0048)	0.0001 (0.0047)	0
$NV(S^*, 1, 20, 20)$	0.2000 (0.0584)	0.0003 (0.0128)	0.0004 (0.0131)	-0.0001 (0.0002)	-0.0004 (0.0129)	-0.0003 (0.0132)	0
$NV(2S^*, 1, 20, 20)$	0.1998 (0.0502)	-0.0000 (0.0080)	0.0000 (0.0080)	-0.0001 (0.0001)	-0.0000 (0.0080)	0.0000 (0.0080)	0
$NV(3S^*, 1, 20, 20)$	0.2000 (0.0553)	-0.0001 (0.0057)	-0.0001 (0.0057)	-0.0001 (0.0001)	0.0001 (0.0057)	0.0001 (0.0056)	0

Table 7 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000
$RV(5)$	6.9911	7.9765						
	(0.2161)	(0.3151)						
$RV(300)$	0.7660	0.8286						
	(0.1275)	(0.1383)						
$RV(900)$	0.6504	0.5974						
	(0.1734)	(0.1707)						
$RV(1800)$	0.6044	0.5334						
	(0.2399)	(0.2141)						
$RV_1(5)$	2.6245	4.9516						
	(0.1706)	(0.2647)						
$RV_1(300)$	0.6143	0.5198						
	(0.1831)	(0.1749)						
$RV_1(900)$	0.6056	0.5095						
	(0.2932)	(0.2569)						
$RV_1(1800)$	0.5820	0.5010						
	(0.4030)	(0.3507)						
$RV(\delta^*)$	0.7104	0.6468	0.0018			0.0027		
	(0.1534)	(0.1750)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.6072	0.5082	0.0018			0.0027		
	(0.2168)	(0.2152)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6125	0.5232	0.0018			0.0027		
	(0.2714)	(0.2745)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6125	0.5148	0.0018			0.0027		
	(0.1526)	(0.1483)	(0.0001)			(0.0001)		
$TSRV$	0.6193	0.5137	0.0018			0.0027		
	(0.0507)	(0.0633)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6251	0.5229	0.0010			0.0020		
	(0.0473)	(0.0594)	(0.0000)			(0.0001)		
$NV(2S^*, 0)$	0.6235	0.5214	0.0010			0.0020		
	(0.0534)	(0.0632)	(0.0000)			(0.0001)		
$NV(3S^*, 0)$	0.6222	0.5207	0.0010			0.0020		
	(0.0617)	(0.0709)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6251	0.5218	0.0010	-0.0001	0.0007	0.0020	-0.0088	-0.0118
	(0.0666)	(0.0828)	(0.0001)	(0.0057)	(0.0306)	(0.0001)	(0.3847)	(0.7132)
$NV(2S^*, 10)$	0.6223	0.5204	0.0010	-0.0002	0.0007	0.0020	-0.0039	-0.0129
	(0.0664)	(0.0759)	(0.0001)	(0.0056)	(0.0292)	(0.0001)	(0.3653)	(0.7181)
$NV(3S^*, 10)$	0.6210	0.5199	0.0010	-0.0002	0.0006	0.0020	-0.0047	-0.0129
	(0.0741)	(0.0829)	(0.0001)	(0.0055)	(0.0282)	(0.0001)	(0.3611)	(0.7237)
$NV(S^*, 20)$	0.6227	0.5245	0.0010	-0.0003	0.0029	0.0020	0.0985	-0.0201
	(0.0853)	(0.1137)	(0.0001)	(0.0131)	(0.0654)	(0.0002)	(2.2512)	(1.7499)
$NV(2S^*, 20)$	0.6214	0.5206	0.0010	-0.0003	0.0021	0.0020	0.0715	-0.0334
	(0.0708)	(0.0813)	(0.0001)	(0.0122)	(0.0570)	(0.0001)	(2.1232)	(1.5831)
$NV(3S^*, 20)$	0.6203	0.5199	0.0010	-0.0001	0.0016	0.0020	0.0706	-0.0395
	(0.0783)	(0.0879)	(0.0001)	(0.0119)	(0.0531)	(0.0002)	(2.0800)	(1.6079)

Table 8: Monte Carlo simulation results for scenario: **iid, mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0048 (0.1204)						
$RC(300)$	0.1919 (0.0987)						
$RC(900)$	0.1882 (0.1370)						
$RC(1800)$	0.1847 (0.1766)						
$RC_{1,1}(5)$	0.1087 (0.1316)						
$RC_{1,1}(300)$	0.1954 (0.1404)						
$RC_{1,1}(900)$	0.1944 (0.2085)						
$RC_{1,1}(1800)$	0.1949 (0.2934)						
$RC(\delta^*)$	0.1159 (0.1205)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1861 (0.1132)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1938 (0.1146)			0.0000 (0.0000)			
HY	0.0745 (0.1325)						
$HY(S^*)$	0.1611 (0.0509)						
$NV(S^*, 1, 0, 0)$	0.2005 (0.0461)			-0.0001 (0.0002)			0
$NV(2S^*, 1, 0, 0)$	0.2004 (0.0489)			-0.0001 (0.0002)			0
$NV(3S^*, 1, 0, 0)$	0.2003 (0.0551)			-0.0001 (0.0002)			0
$NV(S^*, 1, 10, 10)$	0.1999 (0.0596)	-0.0001 (0.0051)	0.0001 (0.0041)	-0.0002 (0.0016)	0.0001 (0.0044)	0.0003 (0.0049)	0
$NV(2S^*, 1, 10, 10)$	0.2002 (0.0568)	-0.0000 (0.0022)	-0.0000 (0.0023)	-0.0001 (0.0005)	0.0001 (0.0021)	0.0001 (0.0026)	0
$NV(3S^*, 1, 10, 10)$	0.2001 (0.0623)	-0.0000 (0.0017)	0.0000 (0.0018)	-0.0001 (0.0004)	0.0000 (0.0017)	0.0000 (0.0020)	0
$NV(S^*, 1, 20, 20)$	0.2010 (0.0968)	-0.0003 (0.0330)	0.0000 (0.0274)	0.0000 (0.0047)	-0.0004 (0.0355)	0.0003 (0.0363)	1
$NV(2S^*, 1, 20, 20)$	0.2005 (0.0631)	-0.0001 (0.0049)	-0.0001 (0.0047)	-0.0001 (0.0006)	0.0001 (0.0049)	0.0001 (0.0051)	0
$NV(3S^*, 1, 20, 20)$	0.2001 (0.0677)	0.0000 (0.0034)	0.0000 (0.0033)	-0.0002 (0.0004)	0.0000 (0.0033)	-0.0000 (0.0035)	0

Table 8 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000
$RV(5)$	3.4362	3.3642						
	(0.1589)	(0.2099)						
$RV(300)$	0.7685	0.8294						
	(0.1241)	(0.1387)						
$RV(900)$	0.6494	0.6009						
	(0.1817)	(0.1699)						
$RV(1800)$	0.6052	0.5281						
	(0.2494)	(0.2086)						
$RV_1(5)$	2.5907	2.8987						
	(0.1430)	(0.1984)						
$RV_1(300)$	0.6133	0.5172						
	(0.1872)	(0.1751)						
$RV_1(900)$	0.6018	0.5058						
	(0.3010)	(0.2531)						
$RV_1(1800)$	0.5803	0.5027						
	(0.4020)	(0.3484)						
$RV(\delta^*)$	0.6977	0.6604	0.0018			0.0024		
	(0.1501)	(0.1692)	(0.0001)			(0.0002)		
$RV_1(\delta^*)$	0.6083	0.5126	0.0018			0.0024		
	(0.2116)	(0.2119)	(0.0001)			(0.0002)		
$RV_2(\delta^*)$	0.6168	0.5246	0.0018			0.0024		
	(0.2641)	(0.2609)	(0.0001)			(0.0002)		
$K^{TH2}(60)$	0.6127	0.5139	0.0018			0.0024		
	(0.1632)	(0.1565)	(0.0001)			(0.0002)		
$TSRV$	0.6174	0.5102	0.0018			0.0024		
	(0.0671)	(0.0852)	(0.0001)			(0.0002)		
$NV(S^*, 0)$	0.6252	0.5196	0.0010			0.0020		
	(0.0653)	(0.0867)	(0.0001)			(0.0001)		
$NV(2S^*, 0)$	0.6227	0.5208	0.0010			0.0020		
	(0.0701)	(0.0831)	(0.0001)			(0.0001)		
$NV(3S^*, 0)$	0.6213	0.5208	0.0010			0.0020		
	(0.0775)	(0.0901)	(0.0001)			(0.0002)		
$NV(S^*, 10)$	0.6237	0.5195	0.0010	-0.0009	-0.0061	0.0020	0.0058	0.0006
	(0.1026)	(0.1281)	(0.0001)	(0.0260)	(0.0494)	(0.0003)	(0.1047)	(0.0831)
$NV(2S^*, 10)$	0.6204	0.5217	0.0010	0.0005	0.0017	0.0020	0.0046	-0.0004
	(0.0880)	(0.0971)	(0.0001)	(0.0235)	(0.0207)	(0.0002)	(0.0958)	(0.0779)
$NV(3S^*, 10)$	0.6195	0.5211	0.0010	0.0047	0.0338	0.0020	0.0037	0.0000
	(0.0933)	(0.1032)	(0.0001)	(0.0360)	(0.1024)	(0.0003)	(0.1109)	(0.0831)
$NV(S^*, 20)$	0.6237	0.5195	0.0010	-0.0009	-0.0061	0.0020	0.0058	0.0006
	(0.1026)	(0.1281)	(0.0001)	(0.0260)	(0.0494)	(0.0003)	(0.1047)	(0.0831)
$NV(2S^*, 20)$	0.6204	0.5217	0.0010	0.0005	0.0017	0.0020	0.0046	-0.0004
	(0.0880)	(0.0971)	(0.0001)	(0.0235)	(0.0207)	(0.0002)	(0.0958)	(0.0779)
$NV(3S^*, 20)$	0.6195	0.5211	0.0010	0.0047	0.0338	0.0020	0.0037	0.0000
	(0.0933)	(0.1032)	(0.0001)	(0.0360)	(0.1024)	(0.0003)	(0.1109)	(0.0831)

Table 9: Monte Carlo simulation results for scenario: **iid, mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0141 (0.0505)						
$RC(300)$	0.1789 (0.0969)						
$RC(900)$	0.1807 (0.1361)						
$RC(1800)$	0.1792 (0.1751)						
$RC_{1,1}(5)$	0.0517 (0.0763)						
$RC_{1,1}(300)$	0.1944 (0.1372)						
$RC_{1,1}(900)$	0.1908 (0.2111)						
$RC_{1,1}(1800)$	0.1923 (0.2942)						
$RC(\delta^*)$	0.1165 (0.1009)			0.0001 (0.0001)			
$RC_{1,1}(\delta^*)$	0.1682 (0.1038)			0.0001 (0.0001)			
$RC_{2,2}(\delta^*)$	0.1830 (0.1082)			0.0001 (0.0001)			
HY	0.1830 (0.0941)						
$HY(S^*)$	0.1919 (0.0629)						
$NV(S^*, 1, 0, 0)$	0.2017 (0.0719)			-0.0002 (0.0010)			0
$NV(2S^*, 1, 0, 0)$	0.2004 (0.0678)			-0.0001 (0.0009)			0
$NV(3S^*, 1, 0, 0)$	0.2000 (0.0723)			-0.0001 (0.0011)			0
$NV(S^*, 1, 10, 10)$	0.2021 (0.0830)	-0.0001 (0.0290)	-0.0003 (0.0182)	-0.0001 (0.0067)	0.0001 (0.0089)	0.0003 (0.0163)	1
$NV(2S^*, 1, 10, 10)$	0.2000 (0.0763)	-0.0004 (0.0165)	0.0001 (0.0141)	-0.0001 (0.0061)	0.0002 (0.0159)	0.0001 (0.0147)	0
$NV(3S^*, 1, 10, 10)$	0.1998 (0.0786)	-0.0000 (0.0062)	-0.0000 (0.0061)	-0.0001 (0.0026)	-0.0002 (0.0060)	0.0001 (0.0077)	0
$NV(S^*, 1, 20, 20)$	0.2021 (0.0830)	-0.0001 (0.0290)	-0.0003 (0.0182)	-0.0001 (0.0067)	0.0001 (0.0089)	0.0003 (0.0163)	1
$NV(2S^*, 1, 20, 20)$	0.2055 (0.1445)	-0.0106 (0.1828)	-0.0064 (0.1093)	0.0059 (0.1025)	0.0069 (0.1084)	0.0070 (0.1149)	8
$NV(3S^*, 1, 20, 20)$	0.1992 (0.0873)	0.0005 (0.0176)	-0.0002 (0.0128)	-0.0001 (0.0038)	-0.0002 (0.0121)	0.0002 (0.0166)	0

Table 9 ***(cont'd)*:** Monte Carlo simulation results for scenario: **iid, mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0100	0.0000	0.0000	0.0200	0.0000	0.0000
$RV(5)$	91.5032	141.7244						
	(2.2836)	(4.1277)						
$RV(300)$	2.1649	3.5957						
	(0.3988)	(0.6507)						
$RV(900)$	1.0922	1.4731						
	(0.3210)	(0.4433)						
$RV(1800)$	0.8231	0.9514						
	(0.3432)	(0.3940)						
$RV_1(5)$	3.2787	34.5314						
	(1.8989)	(3.1807)						
$RV_1(300)$	0.6176	0.5542						
	(0.3739)	(0.5716)						
$RV_1(900)$	0.6397	0.5504						
	(0.3899)	(0.4704)						
$RV_1(1800)$	0.5923	0.5304						
	(0.4632)	(0.4680)						
$RV(\delta^*)$	0.9939	1.1298	0.0108			0.0207		
	(0.3376)	(0.4529)	(0.0002)			(0.0005)		
$RV_1(\delta^*)$	0.6265	0.5728	0.0108			0.0207		
	(0.4159)	(0.4804)	(0.0002)			(0.0005)		
$RV_2(\delta^*)$	0.6282	0.5645	0.0108			0.0207		
	(0.4740)	(0.5366)	(0.0002)			(0.0005)		
$K^{TH2}(60)$	0.6159	0.5124	0.0108			0.0207		
	(0.2032)	(0.2229)	(0.0002)			(0.0005)		
$TSRV$	0.6077	0.4981	0.0108			0.0207		
	(0.0805)	(0.1183)	(0.0002)			(0.0005)		
$NV(S^*, 0)$	0.6241	0.5224	0.0100			0.0200		
	(0.0671)	(0.0930)	(0.0001)			(0.0004)		
$NV(2S^*, 0)$	0.6223	0.5212	0.0100			0.0200		
	(0.0713)	(0.0859)	(0.0001)			(0.0004)		
$NV(3S^*, 0)$	0.6207	0.5196	0.0100			0.0200		
	(0.0815)	(0.0946)	(0.0001)			(0.0004)		
$NV(S^*, 10)$	0.6255	0.5223	0.0100	-0.0000	0.0001	0.0200	-0.0004	-0.0015
	(0.0740)	(0.0973)	(0.0001)	(0.0006)	(0.0027)	(0.0004)	(0.0416)	(0.1188)
$NV(2S^*, 10)$	0.6220	0.5208	0.0100	-0.0000	0.0001	0.0200	-0.0005	-0.0008
	(0.0806)	(0.0930)	(0.0001)	(0.0006)	(0.0027)	(0.0004)	(0.0411)	(0.1150)
$NV(3S^*, 10)$	0.6201	0.5189	0.0100	-0.0000	0.0001	0.0200	-0.0004	-0.0009
	(0.0912)	(0.1027)	(0.0002)	(0.0006)	(0.0027)	(0.0004)	(0.0406)	(0.1119)
$NV(S^*, 20)$	0.6256	0.5266	0.0100	-0.0000	0.0001	0.0200	-0.0005	0.0004
	(0.0800)	(0.1032)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0584)	(0.1712)
$NV(2S^*, 20)$	0.6215	0.5217	0.0100	-0.0000	0.0001	0.0200	-0.0007	0.0012
	(0.0852)	(0.0969)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0557)	(0.1578)
$NV(3S^*, 20)$	0.6196	0.5192	0.0100	-0.0000	0.0001	0.0200	-0.0005	0.0008
	(0.0957)	(0.1067)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0539)	(0.1487)

Table 10: Monte Carlo simulation results for scenario: **iid, high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	-2.1670 (1.9966)						
$RC(300)$	0.1559 (0.3476)						
$RC(900)$	0.1718 (0.2727)						
$RC(1800)$	0.1795 (0.2708)						
$RC_{1,1}(5)$	-0.1359 (1.6998)						
$RC_{1,1}(300)$	0.1872 (0.3165)						
$RC_{1,1}(900)$	0.1945 (0.3062)						
$RC_{1,1}(1800)$	0.2021 (0.3554)						
$RC(\delta^*)$	0.0466 (0.6606)			-0.0002 (0.0002)			
$RC_{1,1}(\delta^*)$	0.1853 (0.5640)			-0.0002 (0.0002)			
$RC_{2,2}(\delta^*)$	0.1863 (0.5778)			-0.0002 (0.0002)			
HY	-7.9279 (2.1306)						
$HY(S^*)$	-0.6357 (0.1918)						
$NV(S^*, 1, 0, 0)$	0.2009 (0.0654)			-0.0014 (0.0003)			0
$NV(2S^*, 1, 0, 0)$	0.2004 (0.0630)			-0.0014 (0.0003)			0
$NV(3S^*, 1, 0, 0)$	0.1999 (0.0713)			-0.0014 (0.0003)			0
$NV(S^*, 1, 10, 10)$	0.2013 (0.0745)	0.0008 (0.0420)	0.0010 (0.0410)	-0.0014 (0.0010)	-0.0011 (0.0412)	-0.0007 (0.0406)	0
$NV(2S^*, 1, 10, 10)$	0.2006 (0.0704)	0.0005 (0.0248)	0.0007 (0.0238)	-0.0014 (0.0007)	-0.0007 (0.0239)	-0.0005 (0.0233)	0
$NV(3S^*, 1, 10, 10)$	0.1998 (0.0789)	0.0001 (0.0167)	0.0003 (0.0158)	-0.0014 (0.0005)	-0.0003 (0.0160)	-0.0001 (0.0154)	0
$NV(S^*, 1, 20, 20)$	0.2005 (0.0756)	0.0015 (0.0452)	0.0017 (0.0444)	-0.0014 (0.0009)	-0.0018 (0.0447)	-0.0015 (0.0443)	0
$NV(2S^*, 1, 20, 20)$	0.2004 (0.0728)	0.0009 (0.0206)	0.0011 (0.0200)	-0.0014 (0.0006)	-0.0011 (0.0204)	-0.0009 (0.0200)	0
$NV(3S^*, 1, 20, 20)$	0.1996 (0.0821)	0.0003 (0.0130)	0.0004 (0.0126)	-0.0014 (0.0005)	-0.0004 (0.0130)	-0.0003 (0.0126)	0

Table 10 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0100	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0200	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	64.2994 (2.0523)	75.0304 (3.0137)						
$RV(300)$	2.1242 (0.3903)	3.6174 (0.6675)						
$RV(900)$	1.0926 (0.3024)	1.4931 (0.4988)						
$RV(1800)$	0.8106 (0.3215)	0.9815 (0.4201)						
$RV_1(5)$	20.5954 (1.5388)	44.7809 (2.4785)						
$RV_1(300)$	0.6379 (0.3641)	0.5499 (0.5726)						
$RV_1(900)$	0.6295 (0.3745)	0.5710 (0.4620)						
$RV_1(1800)$	0.5991 (0.4650)	0.5254 (0.4717)						
$RV(\delta^*)$	0.9938 (0.3440)	1.1267 (0.4259)	0.0108 (0.0003)			0.0206 (0.0009)		
$RV_1(\delta^*)$	0.6301 (0.4194)	0.5519 (0.4845)	0.0108 (0.0003)			0.0206 (0.0009)		
$RV_2(\delta^*)$	0.6241 (0.4665)	0.5648 (0.5218)	0.0108 (0.0003)			0.0206 (0.0009)		
$K^{TH2}(60)$	0.6121 (0.2152)	0.5190 (0.2360)	0.0108 (0.0003)			0.0206 (0.0009)		
$TSRV$	0.6023 (0.0953)	0.4897 (0.1458)	0.0108 (0.0003)			0.0206 (0.0009)		
$NV(S^*, 0)$	0.6223 (0.0824)	0.5214 (0.1264)	0.0100 (0.0002)			0.0200 (0.0006)		
$NV(2S^*, 0)$	0.6203 (0.0840)	0.5201 (0.1059)	0.0100 (0.0002)			0.0200 (0.0006)		
$NV(3S^*, 0)$	0.6179 (0.0963)	0.5168 (0.1119)	0.0100 (0.0002)			0.0200 (0.0006)		
$NV(S^*, 10)$	0.6220 (0.0947)	0.5197 (0.1380)	0.0100 (0.0003)	-0.0005 (0.0455)	0.0054 (0.2289)	0.0200 (0.0007)	-0.0460 (3.1439)	-0.1303 (6.2220)
$NV(2S^*, 10)$	0.6195 (0.0952)	0.5190 (0.1138)	0.0100 (0.0003)	-0.0004 (0.0409)	0.0039 (0.1970)	0.0200 (0.0007)	-0.0412 (3.1010)	-0.1209 (6.2363)
$NV(3S^*, 10)$	0.6167 (0.1081)	0.5153 (0.1210)	0.0100 (0.0003)	-0.0004 (0.0384)	0.0024 (0.1821)	0.0200 (0.0007)	-0.0457 (3.0999)	-0.1165 (6.2745)
$NV(S^*, 20)$	0.6189 (0.1002)	0.5217 (0.1492)	0.0100 (0.0003)	-0.0027 (0.1013)	0.0210 (0.4409)	0.0200 (0.0007)	0.4736 (17.6293)	-0.3382 (13.4039)
$NV(2S^*, 20)$	0.6185 (0.0992)	0.5195 (0.1186)	0.0100 (0.0003)	-0.0019 (0.0962)	0.0154 (0.3882)	0.0200 (0.0007)	0.4898 (17.4376)	-0.3224 (13.3130)
$NV(3S^*, 20)$	0.6160 (0.1124)	0.5151 (0.1256)	0.0100 (0.0003)	-0.0016 (0.0939)	0.0123 (0.3702)	0.0200 (0.0007)	0.4439 (17.3787)	-0.4288 (13.3842)

Table 11: Monte Carlo simulation results for scenario: **iid, high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	-0.4425 (1.1278)						
$RC(300)$	0.1726 (0.3597)						
$RC(900)$	0.1877 (0.2868)						
$RC(1800)$	0.1856 (0.2821)						
$RC_{1,1}(5)$	-0.0289 (1.1978)						
$RC_{1,1}(300)$	0.1893 (0.3415)						
$RC_{1,1}(900)$	0.2002 (0.3039)						
$RC_{1,1}(1800)$	0.1929 (0.3543)						
$RC(\delta^*)$	0.0829 (0.6716)			-0.0001 (0.0003)			
$RC_{1,1}(\delta^*)$	0.2038 (0.5936)			-0.0001 (0.0003)			
$RC_{2,2}(\delta^*)$	0.1919 (0.5873)			-0.0001 (0.0003)			
HY	-1.0662 (1.1969)						
$HY(S^*)$	-0.0045 (0.1781)						
$NV(S^*, 1, 0, 0)$	0.2009 (0.0851)			-0.0014 (0.0012)			0
$NV(2S^*, 1, 0, 0)$	0.2000 (0.0788)			-0.0014 (0.0011)			0
$NV(3S^*, 1, 0, 0)$	0.1985 (0.0863)			-0.0014 (0.0012)			0
$NV(S^*, 1, 10, 10)$	0.1999 (0.0925)	-0.0002 (0.0114)	-0.0003 (0.0119)	-0.0014 (0.0024)	0.0004 (0.0103)	0.0007 (0.0126)	0
$NV(2S^*, 1, 10, 10)$	0.1995 (0.0852)	-0.0002 (0.0072)	-0.0001 (0.0077)	-0.0014 (0.0017)	0.0002 (0.0067)	0.0004 (0.0079)	0
$NV(3S^*, 1, 10, 10)$	0.1979 (0.0930)	-0.0002 (0.0057)	-0.0001 (0.0058)	-0.0013 (0.0014)	0.0001 (0.0052)	0.0003 (0.0060)	0
$NV(S^*, 1, 20, 20)$	0.2012 (0.1024)	0.0001 (0.0218)	0.0000 (0.0210)	-0.0014 (0.0025)	0.0002 (0.0211)	0.0003 (0.0221)	1
$NV(2S^*, 1, 20, 20)$	0.1994 (0.0904)	-0.0002 (0.0128)	-0.0000 (0.0130)	-0.0014 (0.0016)	0.0002 (0.0122)	0.0003 (0.0131)	0
$NV(3S^*, 1, 20, 20)$	0.1976 (0.0980)	-0.0003 (0.0103)	-0.0002 (0.0102)	-0.0013 (0.0014)	0.0002 (0.0098)	0.0004 (0.0103)	0

Table 11 ***(cont'd)*:** Monte Carlo simulation results for scenario: **iid, high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0100	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0200	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	28.7558 (1.3774)	28.9571 (1.8439)						
$RV(300)$	2.1509 (0.3879)	3.6389 (0.6500)						
$RV(900)$	1.1036 (0.3295)	1.5145 (0.4746)						
$RV(1800)$	0.8178 (0.3449)	0.9497 (0.3918)						
$RV_1(5)$	20.2770 (1.2090)	24.2977 (1.7085)						
$RV_1(300)$	0.6247 (0.3846)	0.5372 (0.5959)						
$RV_1(900)$	0.6149 (0.3997)	0.5473 (0.4457)						
$RV_1(1800)$	0.5898 (0.4533)	0.5454 (0.4638)						
$RV(\delta^*)$	0.9936 (0.3554)	1.1847 (0.4470)	0.0107 (0.0006)			0.0179 (0.0013)		
$RV_1(\delta^*)$	0.6340 (0.4016)	0.5479 (0.4682)	0.0107 (0.0006)			0.0179 (0.0013)		
$RV_2(\delta^*)$	0.6276 (0.4728)	0.5756 (0.5260)	0.0107 (0.0006)			0.0179 (0.0013)		
$K^{TH2}(60)$	0.6125 (0.2306)	0.5210 (0.2467)	0.0107 (0.0006)			0.0179 (0.0013)		
$TSRV$	0.5995 (0.1205)	0.4839 (0.1896)	0.0107 (0.0006)			0.0179 (0.0013)		
$NV(S^*, 0)$	0.6210 (0.1136)	0.5184 (0.1775)	0.0100 (0.0004)			0.0200 (0.0012)		
$NV(2S^*, 0)$	0.6166 (0.1097)	0.5206 (0.1352)	0.0100 (0.0004)			0.0200 (0.0011)		
$NV(3S^*, 0)$	0.6140 (0.1228)	0.5166 (0.1382)	0.0100 (0.0004)			0.0200 (0.0011)		
$NV(S^*, 10)$	0.6196 (0.1330)	0.5263 (0.2106)	0.0100 (0.0005)	-0.0070 (0.1816)	-0.0026 (0.1463)	0.0200 (0.0014)	0.0373 (0.7124)	-0.0043 (0.6175)
$NV(2S^*, 10)$	0.6146 (0.1252)	0.5229 (0.1468)	0.0100 (0.0005)	-0.0047 (0.1828)	-0.0011 (0.1475)	0.0200 (0.0012)	0.0287 (0.6685)	0.0002 (0.6015)
$NV(3S^*, 10)$	0.6122 (0.1381)	0.5167 (0.1508)	0.0100 (0.0005)	-0.0029 (0.1876)	0.0001 (0.1501)	0.0200 (0.0012)	0.0122 (0.6757)	0.0089 (0.6024)
$NV(S^*, 20)$	0.6196 (0.1330)	0.5263 (0.2106)	0.0100 (0.0005)	-0.0070 (0.1816)	-0.0026 (0.1463)	0.0200 (0.0014)	0.0373 (0.7124)	-0.0043 (0.6175)
$NV(2S^*, 20)$	0.6146 (0.1252)	0.5229 (0.1468)	0.0100 (0.0005)	-0.0047 (0.1828)	-0.0011 (0.1475)	0.0200 (0.0012)	0.0287 (0.6685)	0.0002 (0.6015)
$NV(3S^*, 20)$	0.6122 (0.1381)	0.5167 (0.1508)	0.0100 (0.0005)	-0.0029 (0.1876)	0.0001 (0.1501)	0.0200 (0.0012)	0.0122 (0.6757)	0.0089 (0.6024)

Table 12: Monte Carlo simulation results for scenario: **iid, high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	-0.0373 (0.4276)						
$RC(300)$	0.1720 (0.3781)						
$RC(900)$	0.1826 (0.2885)						
$RC(1800)$	0.1748 (0.2648)						
$RC_{1,1}(5)$	0.0120 (0.6372)						
$RC_{1,1}(300)$	0.1910 (0.3323)						
$RC_{1,1}(900)$	0.1847 (0.2999)						
$RC_{1,1}(1800)$	0.1911 (0.3578)						
$RC(\delta^*)$	0.1315 (0.5130)			0.0000 (0.0005)			
$RC_{1,1}(\delta^*)$	0.1704 (0.4795)			0.0000 (0.0005)			
$RC_{2,2}(\delta^*)$	0.2003 (0.4771)			0.0000 (0.0005)			
HY	0.0178 (0.6799)						
$HY(S^*)$	0.1468 (0.1898)						
$NV(S^*, 1, 0, 0)$	0.1997 (0.1290)			-0.0015 (0.0056)			7
$NV(2S^*, 1, 0, 0)$	0.1999 (0.1041)			-0.0015 (0.0051)			0
$NV(3S^*, 1, 0, 0)$	0.1985 (0.1065)			-0.0014 (0.0050)			0
$NV(S^*, 1, 10, 10)$	0.2017 (0.1420)	-0.0001 (0.0334)	0.0007 (0.0326)	-0.0009 (0.0138)	-0.0021 (0.0325)	-0.0009 (0.0434)	19
$NV(2S^*, 1, 10, 10)$	0.2000 (0.1096)	-0.0001 (0.0144)	-0.0003 (0.0163)	-0.0013 (0.0065)	-0.0008 (0.0159)	0.0002 (0.0159)	0
$NV(3S^*, 1, 10, 10)$	0.1982 (0.1122)	-0.0002 (0.0113)	-0.0004 (0.0119)	-0.0013 (0.0052)	-0.0003 (0.0123)	-0.0000 (0.0120)	0
$NV(S^*, 1, 20, 20)$	0.1865 (0.3398)	0.0538 (0.9342)	-0.0113 (0.2582)	-0.0151 (0.1893)	0.0385 (0.5224)	0.0013 (0.2168)	164
$NV(2S^*, 1, 20, 20)$	0.2005 (0.1146)	0.0003 (0.0168)	-0.0004 (0.0183)	-0.0013 (0.0067)	-0.0007 (0.0170)	0.0000 (0.0174)	0
$NV(3S^*, 1, 20, 20)$	0.1984 (0.1169)	-0.0001 (0.0119)	-0.0003 (0.0122)	-0.0014 (0.0052)	-0.0001 (0.0124)	-0.0002 (0.0126)	0

Table 12 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0001	0.0001	0.0000	0.0003	0.0001	0.0001
$RV(5)$	1.6978	2.3404						
	(0.0370)	(0.0624)						
$RV(300)$	0.6369	0.5592						
	(0.1037)	(0.0948)						
$RV(900)$	0.6068	0.5131						
	(0.1663)	(0.1406)						
$RV(1800)$	0.5859	0.4890						
	(0.2383)	(0.1956)						
$RV_1(5)$	0.7057	1.0850						
	(0.0411)	(0.0583)						
$RV_1(300)$	0.6131	0.5162						
	(0.1743)	(0.1460)						
$RV_1(900)$	0.6033	0.4996						
	(0.2914)	(0.2422)						
$RV_1(1800)$	0.5800	0.4995						
	(0.3993)	(0.3430)						
$RV(\delta^*)$	0.6282	0.5448	0.0009			0.0009		
	(0.1096)	(0.1014)	(0.0000)			(0.0000)		
$RV_1(\delta^*)$	0.6061	0.5022	0.0009			0.0009		
	(0.1768)	(0.1578)	(0.0000)			(0.0000)		
$RV_2(\delta^*)$	0.6078	0.5102	0.0009			0.0009		
	(0.2162)	(0.1901)	(0.0000)			(0.0000)		
$K^{TH2}(60)$	0.6146	0.5150	0.0009			0.0009		
	(0.1295)	(0.1138)	(0.0000)			(0.0000)		
$TSRV$	0.6761	0.5889	0.0009			0.0009		
	(0.0331)	(0.0382)	(0.0000)			(0.0000)		
$NV(S^*, 0)$	0.6801	0.5982	0.0001			0.0002		
	(0.0287)	(0.0327)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6482	0.5532	0.0001			0.0002		
	(0.0365)	(0.0382)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6387	0.5401	0.0001			0.0002		
	(0.0427)	(0.0435)	(0.0000)			(0.0000)		
$NV(S^*, 10)$	0.6250	0.5217	0.0001	0.0001	0.0000	0.0003	0.0002	-0.0000
	(0.0423)	(0.0487)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0007)	(0.0022)
$NV(2S^*, 10)$	0.6254	0.5215	0.0001	0.0001	0.0000	0.0003	0.0002	-0.0000
	(0.0485)	(0.0485)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)	(0.0023)
$NV(3S^*, 10)$	0.6248	0.5209	0.0001	0.0001	0.0000	0.0003	0.0002	-0.0000
	(0.0539)	(0.0532)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0008)	(0.0023)
$NV(S^*, 20)$	0.6262	0.5245	0.0001	0.0001	0.0000	0.0003	0.0002	-0.0001
	(0.0765)	(0.0986)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0013)	(0.0040)
$NV(2S^*, 20)$	0.6255	0.5214	0.0001	0.0001	0.0000	0.0003	0.0002	-0.0000
	(0.0546)	(0.0533)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0013)	(0.0040)
$NV(3S^*, 20)$	0.6245	0.5207	0.0001	0.0001	0.0000	0.0003	0.0002	0.0000
	(0.0592)	(0.0576)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0013)	(0.0041)

Table 13: Monte Carlo simulation results for scenario: **low pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.1970 (0.0311)						
$RC(300)$	0.2002 (0.0753)						
$RC(900)$	0.1883 (0.1236)						
$RC(1800)$	0.1843 (0.1681)						
$RC_{1,1}(5)$	0.2462 (0.0337)						
$RC_{1,1}(300)$	0.1962 (0.1249)						
$RC_{1,1}(900)$	0.1943 (0.2047)						
$RC_{1,1}(1800)$	0.1949 (0.2895)						
$RC(\delta^*)$	0.2099 (0.0326)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1997 (0.0424)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1978 (0.0513)			0.0000 (0.0000)			
HY	0.2670 (0.0344)						
$HY(S^*)$	0.2467 (0.0211)						
$NV(S^*,1,0,0)$	0.2204 (0.0258)			0.0000 (0.0000)			0
$NV(2S^*,1,0,0)$	0.2087 (0.0307)			0.0000 (0.0000)			0
$NV(3S^*,1,0,0)$	0.2052 (0.0350)			0.0000 (0.0000)			0
$NV(S^*,1,10,10)$	0.2013 (0.0387)	0.0000 (0.0022)	0.0001 (0.0022)	0.0000 (0.0001)	0.0000 (0.0022)	-0.0000 (0.0023)	0
$NV(2S^*,1,10,10)$	0.2003 (0.0393)	0.0001 (0.0017)	0.0001 (0.0017)	0.0000 (0.0001)	-0.0000 (0.0018)	-0.0000 (0.0018)	0
$NV(3S^*,1,10,10)$	0.2000 (0.0430)	0.0001 (0.0016)	0.0001 (0.0016)	0.0000 (0.0001)	-0.0000 (0.0016)	-0.0000 (0.0016)	0
$NV(S^*,1,20,20)$	0.1990 (0.1256)	0.0002 (0.0061)	0.0003 (0.0066)	0.0000 (0.0001)	-0.0001 (0.0062)	-0.0002 (0.0069)	1
$NV(2S^*,1,20,20)$	0.2002 (0.0432)	0.0002 (0.0036)	0.0002 (0.0037)	0.0000 (0.0001)	-0.0002 (0.0036)	-0.0001 (0.0037)	0
$NV(3S^*,1,20,20)$	0.1999 (0.0468)	0.0001 (0.0032)	0.0001 (0.0032)	0.0000 (0.0001)	-0.0001 (0.0032)	-0.0001 (0.0032)	0

Table 13 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0001	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0003	$\gamma_2(1)$ 0.0001	$\gamma_2(2)$ 0.0001
$RV(5)$	1.3783 (0.0389)	1.5131 (0.0582)						
$RV(300)$	0.6352 (0.1041)	0.5609 (0.0942)						
$RV(900)$	0.6074 (0.1651)	0.5131 (0.1428)						
$RV(1800)$	0.5858 (0.2356)	0.4895 (0.1964)						
$RV_1(5)$	0.8934 (0.0390)	1.1515 (0.0551)						
$RV_1(300)$	0.6135 (0.1728)	0.5168 (0.1474)						
$RV_1(900)$	0.6028 (0.2907)	0.4994 (0.2427)						
$RV_1(1800)$	0.5810 (0.3988)	0.4993 (0.3424)						
$RV(\delta^*)$	0.6267 (0.1087)	0.5441 (0.0994)	0.0009 (0.0000)			0.0009 (0.0001)		
$RV_1(\delta^*)$	0.6001 (0.1743)	0.5011 (0.1571)	0.0009 (0.0000)			0.0009 (0.0001)		
$RV_2(\delta^*)$	0.6072 (0.2139)	0.5095 (0.1926)	0.0009 (0.0000)			0.0009 (0.0001)		
$K^{TH2}(60)$	0.6137 (0.1353)	0.5139 (0.1182)	0.0009 (0.0000)			0.0009 (0.0001)		
$TSRV$	0.6437 (0.0395)	0.5455 (0.0445)	0.0009 (0.0000)			0.0009 (0.0001)		
$NV(S^*, 0)$	0.6472 (0.0369)	0.5532 (0.0418)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*, 0)$	0.6334 (0.0456)	0.5341 (0.0480)	0.0001 (0.0000)			0.0003 (0.0000)		
$NV(3S^*, 0)$	0.6286 (0.0523)	0.5283 (0.0546)	0.0001 (0.0000)			0.0003 (0.0000)		
$NV(S^*, 10)$	0.6248 (0.0555)	0.5226 (0.0872)	0.0001 (0.0000)	-0.0000 (0.0012)	0.0003 (0.0069)	0.0003 (0.0001)	-0.0026 (0.1112)	-0.0009 (0.1879)
$NV(2S^*, 10)$	0.6234 (0.0595)	0.5206 (0.0625)	0.0001 (0.0000)	-0.0000 (0.0013)	0.0003 (0.0071)	0.0003 (0.0001)	-0.0011 (0.0944)	0.0009 (0.1750)
$NV(3S^*, 10)$	0.6220 (0.0649)	0.5201 (0.0680)	0.0001 (0.0000)	-0.0001 (0.0014)	0.0004 (0.0072)	0.0003 (0.0001)	-0.0009 (0.0911)	0.0016 (0.1869)
$NV(S^*, 20)$	0.6197 (0.0965)	0.5217 (0.7495)	0.0001 (0.0001)	0.0000 (0.0034)	0.0007 (0.0181)	0.0003 (0.0010)	0.0088 (0.9601)	0.0098 (1.4615)
$NV(2S^*, 20)$	0.6228 (0.0642)	0.5205 (0.0693)	0.0001 (0.0000)	0.0000 (0.0032)	0.0005 (0.0163)	0.0003 (0.0001)	0.0069 (0.6389)	-0.0061 (0.4596)
$NV(3S^*, 20)$	0.6213 (0.0691)	0.5199 (0.0736)	0.0001 (0.0001)	0.0000 (0.0031)	0.0006 (0.0151)	0.0003 (0.0001)	0.0070 (0.6172)	-0.0079 (0.4618)

Table 14: Monte Carlo simulation results for scenario: **low pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0838 (0.0216)						
$RC(300)$	0.1952 (0.0763)						
$RC(900)$	0.1882 (0.1236)						
$RC(1800)$	0.1844 (0.1682)						
$RC_{1,1}(5)$	0.1593 (0.0298)						
$RC_{1,1}(300)$	0.1971 (0.1250)						
$RC_{1,1}(900)$	0.1928 (0.2032)						
$RC_{1,1}(1800)$	0.1947 (0.2879)						
$RC(\delta^*)$	0.1677 (0.0370)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1967 (0.0476)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1980 (0.0582)			0.0000 (0.0000)			
HY	0.2518 (0.0343)						
$HY(S^*)$	0.2257 (0.0288)						
$NV(S^*, 1, 0, 0)$	0.2050 (0.0354)			0.0001 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2021 (0.0403)			0.0001 (0.0001)			0
$NV(3S^*, 1, 0, 0)$	0.2012 (0.0459)			0.0001 (0.0001)			0
$NV(S^*, 1, 10, 10)$	0.1999 (0.0486)	0.0002 (0.0039)	0.0001 (0.0038)	-0.0000 (0.0018)	0.0001 (0.0032)	-0.0001 (0.0055)	0
$NV(2S^*, 1, 10, 10)$	0.2000 (0.0482)	0.0000 (0.0014)	0.0000 (0.0012)	0.0000 (0.0004)	0.0000 (0.0013)	0.0000 (0.0013)	0
$NV(3S^*, 1, 10, 10)$	0.1999 (0.0538)	0.0000 (0.0009)	0.0000 (0.0009)	0.0000 (0.0003)	0.0001 (0.0009)	0.0000 (0.0011)	0
$NV(S^*, 1, 20, 20)$	0.1999 (0.0486)	0.0002 (0.0039)	0.0001 (0.0038)	-0.0000 (0.0018)	0.0001 (0.0032)	-0.0001 (0.0055)	51
$NV(2S^*, 1, 20, 20)$	0.1995 (0.0549)	0.0001 (0.0035)	0.0001 (0.0029)	0.0000 (0.0005)	-0.0001 (0.0038)	-0.0001 (0.0032)	0
$NV(3S^*, 1, 20, 20)$	0.1997 (0.0594)	-0.0000 (0.0021)	-0.0000 (0.0020)	0.0000 (0.0003)	0.0001 (0.0022)	0.0001 (0.0023)	0

Table 14 ***(cont'd)*:** Monte Carlo simulation results for scenario: **low pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0001	0.0001	0.0000	0.0003	0.0001	0.0001
$RV(5)$	0.9658	0.9078						
	(0.0434)	(0.0565)						
$RV(300)$	0.6354	0.5609						
	(0.1036)	(0.0943)						
$RV(900)$	0.6055	0.5135						
	(0.1684)	(0.1421)						
$RV(1800)$	0.5850	0.4887						
	(0.2396)	(0.1955)						
$RV_1(5)$	0.8707	0.8501						
	(0.0430)	(0.0567)						
$RV_1(300)$	0.6133	0.5161						
	(0.1740)	(0.1462)						
$RV_1(900)$	0.6000	0.4991						
	(0.2934)	(0.2412)						
$RV_1(1800)$	0.5800	0.4981						
	(0.3985)	(0.3405)						
$RV(\delta^*)$	0.6241	0.5470	0.0009			0.0009		
	(0.1091)	(0.0996)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.6006	0.5018	0.0009			0.0009		
	(0.1732)	(0.1570)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6090	0.5075	0.0009			0.0009		
	(0.2148)	(0.1891)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6139	0.5149	0.0009			0.0009		
	(0.1462)	(0.1283)	(0.0001)			(0.0001)		
$TSRV$	0.6293	0.5255	0.0009			0.0009		
	(0.0524)	(0.0645)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6345	0.5318	0.0001			0.0003		
	(0.0500)	(0.0635)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6283	0.5243	0.0001			0.0003		
	(0.0583)	(0.0674)	(0.0000)			(0.0001)		
$NV(3S^*, 0)$	0.6252	0.5229	0.0001			0.0003		
	(0.0659)	(0.0740)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6203	0.5212	0.0001	0.0006	0.0003	0.0003	-0.0004	0.0008
	(0.1490)	(0.1028)	(0.0002)	(0.0151)	(0.0117)	(0.0002)	(0.0307)	(0.0321)
$NV(2S^*, 10)$	0.6230	0.5188	0.0001	0.0004	-0.0008	0.0003	-0.0014	0.0014
	(0.0759)	(0.0828)	(0.0001)	(0.0100)	(0.0359)	(0.0001)	(0.0333)	(0.0243)
$NV(3S^*, 10)$	0.6209	0.5204	0.0001	0.0008	-0.0001	0.0003	0.0002	0.0006
	(0.0821)	(0.0881)	(0.0001)	(0.0090)	(0.0071)	(0.0001)	(0.0456)	(0.0291)
$NV(S^*, 20)$	0.6203	0.5212	0.0001	0.0006	0.0003	0.0003	-0.0004	0.0008
	(0.1490)	(0.1028)	(0.0002)	(0.0151)	(0.0117)	(0.0002)	(0.0307)	(0.0321)
$NV(2S^*, 20)$	0.6230	0.5188	0.0001	0.0004	-0.0008	0.0003	-0.0014	0.0014
	(0.0759)	(0.0828)	(0.0001)	(0.0100)	(0.0359)	(0.0001)	(0.0333)	(0.0243)
$NV(3S^*, 20)$	0.6209	0.5204	0.0001	0.0008	-0.0001	0.0003	0.0002	0.0006
	(0.0821)	(0.0881)	(0.0001)	(0.0090)	(0.0071)	(0.0001)	(0.0456)	(0.0291)

Table 15: Monte Carlo simulation results for scenario: **low pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000	#NonPD
$RC(5)$	0.0249 (0.0141)						
$RC(300)$	0.1808 (0.0744)						
$RC(900)$	0.1831 (0.1234)						
$RC(1800)$	0.1812 (0.1678)						
$RC_{1,1}(5)$	0.0638 (0.0225)						
$RC_{1,1}(300)$	0.1963 (0.1228)						
$RC_{1,1}(900)$	0.1923 (0.2040)						
$RC_{1,1}(1800)$	0.1926 (0.2878)						
$RC(\delta^*)$	0.1333 (0.0450)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1876 (0.0636)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1930 (0.0766)			0.0001 (0.0000)			
HY	0.2106 (0.0433)						
$HY(S^*)$	0.2072 (0.0421)						
$NV(S^*, 1, 0, 0)$	0.2021 (0.0526)			0.0001 (0.0004)			0
$NV(2S^*, 1, 0, 0)$	0.2009 (0.0578)			0.0001 (0.0005)			0
$NV(3S^*, 1, 0, 0)$	0.2005 (0.0635)			0.0001 (0.0006)			0
$NV(S^*, 1, 10, 10)$	0.2019 (0.0524)		0.0004 (0.0083)	0.0000 (0.0017)	-0.0003 (0.0052)		1
$NV(2S^*, 1, 10, 10)$	0.1993 (0.0770)	0.0054 (0.0815)	-0.0048 (0.0977)	-0.0004 (0.0280)	0.0029 (0.1087)	0.0036 (0.0711)	0
$NV(3S^*, 1, 10, 10)$	0.1997 (0.0712)	-0.0002 (0.0052)	0.0001 (0.0043)	-0.0000 (0.0022)	0.0001 (0.0041)	0.0001 (0.0067)	0
$NV(S^*, 1, 20, 20)$	0.2019 (0.0524)		0.0004 (0.0083)	0.0000 (0.0017)	-0.0003 (0.0052)		1
$NV(2S^*, 1, 20, 20)$	0.1993 (0.0770)	0.0054 (0.0815)	-0.0048 (0.0977)	-0.0004 (0.0280)	0.0029 (0.1087)	0.0036 (0.0711)	0
$NV(3S^*, 1, 20, 20)$	0.1957 (0.3362)	-0.0004 (0.0167)	0.0009 (0.0244)	0.0000 (0.0460)	0.0003 (0.1015)	-0.0011 (0.0388)	144

Table 15 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	11.3391 (0.2676)	18.6955 (0.5104)						
$RV(300)$	0.8116 (0.1329)	0.9442 (0.1635)						
$RV(900)$	0.6616 (0.1810)	0.6360 (0.1762)						
$RV(1800)$	0.6111 (0.2479)	0.5487 (0.2173)						
$RV_1(5)$	1.4250 (0.2388)	6.1409 (0.4480)						
$RV_1(300)$	0.6107 (0.1892)	0.5235 (0.1867)						
$RV_1(900)$	0.6069 (0.3001)	0.5054 (0.2663)						
$RV_1(1800)$	0.5819 (0.4058)	0.4997 (0.3538)						
$RV(\delta^*)$	0.7216 (0.1627)	0.6719 (0.1923)	0.0020 (0.0001)			0.0034 (0.0001)		
$RV_1(\delta^*)$	0.6094 (0.2328)	0.5171 (0.2394)	0.0020 (0.0001)			0.0034 (0.0001)		
$RV_2(\delta^*)$	0.6156 (0.2814)	0.5242 (0.2947)	0.0020 (0.0001)			0.0034 (0.0001)		
$K^{TH2}(60)$	0.6115 (0.1496)	0.5137 (0.1511)	0.0020 (0.0001)			0.0034 (0.0001)		
$TSRV$	0.9384 (0.0753)	0.8225 (0.0824)	0.0020 (0.0001)			0.0034 (0.0001)		
$NV(S^*, 0)$	0.9204 (0.0432)	0.7984 (0.0545)	0.0010 (0.0000)			0.0024 (0.0001)		
$NV(2S^*, 0)$	0.7480 (0.0474)	0.6357 (0.0556)	0.0011 (0.0000)			0.0025 (0.0001)		
$NV(3S^*, 0)$	0.6980 (0.0542)	0.5894 (0.0631)	0.0011 (0.0000)			0.0025 (0.0001)		
$NV(S^*, 10)$	0.6259 (0.0570)	0.5207 (0.0672)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0014 (0.0057)	0.0005 (0.0168)
$NV(2S^*, 10)$	0.6247 (0.0582)	0.5208 (0.0647)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0014 (0.0057)	0.0005 (0.0165)
$NV(3S^*, 10)$	0.6232 (0.0647)	0.5203 (0.0725)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0013 (0.0056)	0.0006 (0.0163)
$NV(S^*, 20)$	0.6261 (0.0711)	0.5225 (0.0757)	0.0012 (0.0001)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0085)	0.0008 (0.0255)
$NV(2S^*, 20)$	0.6244 (0.0634)	0.5211 (0.0690)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0082)	0.0008 (0.0244)
$NV(3S^*, 20)$	0.6227 (0.0697)	0.5204 (0.0766)	0.0012 (0.0001)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0081)	0.0010 (0.0237)

Table 16: Monte Carlo simulation results for scenario: **low pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001	#NonPD
$RC(5)$	0.9563 (0.2462)						
$RC(300)$	0.2261 (0.1073)						
$RC(900)$	0.1964 (0.1422)						
$RC(1800)$	0.1897 (0.1808)						
$RC_{1,1}(5)$	0.8454 (0.2263)						
$RC_{1,1}(300)$	0.1960 (0.1448)						
$RC_{1,1}(900)$	0.1958 (0.2180)						
$RC_{1,1}(1800)$	0.1958 (0.2982)						
$RC(\delta^*)$	0.3183 (0.1202)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1985 (0.1141)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1999 (0.1209)			0.0001 (0.0000)			
HY	0.8590 (0.2544)						
$HY(S^*)$	0.5142 (0.0575)						
$NV(S^*, 1, 0, 0)$	0.2882 (0.0407)			0.0002 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2368 (0.0418)			0.0002 (0.0001)			0
$NV(3S^*, 1, 0, 0)$	0.2223 (0.0467)			0.0003 (0.0001)			0
$NV(S^*, 1, 10, 10)$	0.2008 (0.0533)	0.0003 (0.0086)	0.0004 (0.0085)	0.0001 (0.0002)	0.0002 (0.0086)	0.0003 (0.0087)	0
$NV(2S^*, 1, 10, 10)$	0.2001 (0.0493)	0.0003 (0.0064)	0.0005 (0.0063)	0.0001 (0.0002)	0.0002 (0.0063)	0.0002 (0.0063)	0
$NV(3S^*, 1, 10, 10)$	0.2001 (0.0543)	0.0003 (0.0052)	0.0004 (0.0051)	0.0001 (0.0001)	0.0003 (0.0052)	0.0003 (0.0051)	0
$NV(S^*, 1, 20, 20)$	0.2001 (0.0638)	0.0006 (0.0144)	0.0008 (0.0147)	0.0001 (0.0003)	-0.0001 (0.0145)	-0.0001 (0.0148)	0
$NV(2S^*, 1, 20, 20)$	0.2000 (0.0527)	0.0004 (0.0088)	0.0005 (0.0088)	0.0001 (0.0002)	0.0001 (0.0088)	0.0001 (0.0088)	0
$NV(3S^*, 1, 20, 20)$	0.2000 (0.0577)	0.0003 (0.0062)	0.0004 (0.0062)	0.0001 (0.0001)	0.0003 (0.0062)	0.0003 (0.0061)	0

Table 16 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	8.1545 (0.2501)	10.4098 (0.4165)						
$RV(300)$	0.8046 (0.1334)	0.9499 (0.1660)						
$RV(900)$	0.6639 (0.1768)	0.6371 (0.1838)						
$RV(1800)$	0.6098 (0.2413)	0.5510 (0.2206)						
$RV_1(5)$	3.2950 (0.1952)	6.7987 (0.3617)						
$RV_1(300)$	0.6152 (0.1875)	0.5240 (0.1915)						
$RV_1(900)$	0.6048 (0.2972)	0.5067 (0.2639)						
$RV_1(1800)$	0.5837 (0.4059)	0.5000 (0.3542)						
$RV(\delta^*)$	0.7229 (0.1665)	0.6769 (0.1874)	0.0020 (0.0001)			0.0034 (0.0002)		
$RV_1(\delta^*)$	0.5981 (0.2290)	0.5146 (0.2343)	0.0020 (0.0001)			0.0034 (0.0002)		
$RV_2(\delta^*)$	0.6171 (0.2806)	0.5330 (0.2994)	0.0020 (0.0001)			0.0034 (0.0002)		
$K^{TH2}(60)$	0.6113 (0.1553)	0.5139 (0.1545)	0.0020 (0.0001)			0.0034 (0.0002)		
$TSRV$	0.7391 (0.0627)	0.6230 (0.0780)	0.0020 (0.0001)			0.0034 (0.0002)		
$NV(S^*, 0)$	0.7453 (0.0521)	0.6315 (0.0662)	0.0011 (0.0000)			0.0026 (0.0001)		
$NV(2S^*, 0)$	0.6722 (0.0559)	0.5652 (0.0685)	0.0012 (0.0000)			0.0026 (0.0001)		
$NV(3S^*, 0)$	0.6511 (0.0643)	0.5463 (0.0766)	0.0012 (0.0000)			0.0026 (0.0001)		
$NV(S^*, 10)$	0.6253 (0.0719)	0.5213 (0.0890)	0.0012 (0.0001)	0.0001 (0.0062)	0.0026 (0.0339)	0.0028 (0.0002)	-0.0084 (0.5014)	0.0075 (0.9626)
$NV(2S^*, 10)$	0.6219 (0.0686)	0.5201 (0.0813)	0.0012 (0.0001)	0.0000 (0.0060)	0.0026 (0.0316)	0.0028 (0.0001)	-0.0041 (0.4789)	0.0057 (0.9682)
$NV(3S^*, 10)$	0.6207 (0.0767)	0.5193 (0.0888)	0.0012 (0.0001)	0.0000 (0.0059)	0.0021 (0.0302)	0.0028 (0.0002)	-0.0014 (0.4730)	0.0076 (0.9754)
$NV(S^*, 20)$	0.6235 (0.0896)	0.5220 (0.1059)	0.0012 (0.0001)	-0.0002 (0.0154)	0.0046 (0.0740)	0.0028 (0.0002)	0.0793 (2.8558)	0.0002 (2.1718)
$NV(2S^*, 20)$	0.6210 (0.0731)	0.5200 (0.0863)	0.0012 (0.0001)	-0.0001 (0.0144)	0.0040 (0.0636)	0.0028 (0.0002)	0.0683 (2.6919)	-0.0012 (2.0950)
$NV(3S^*, 20)$	0.6201 (0.0809)	0.5191 (0.0937)	0.0012 (0.0001)	0.0001 (0.0141)	0.0030 (0.0596)	0.0028 (0.0002)	0.0672 (2.6625)	-0.0030 (2.1380)

Table 17: Monte Carlo simulation results for scenario: **low pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001	#NonPD
$RC(5)$	0.3405 (0.1459)						
$RC(300)$	0.2121 (0.1091)						
$RC(900)$	0.1964 (0.1432)						
$RC(1800)$	0.1893 (0.1820)						
$RC_{1,1}(5)$	0.4742 (0.1671)						
$RC_{1,1}(300)$	0.1965 (0.1487)						
$RC_{1,1}(900)$	0.1929 (0.2129)						
$RC_{1,1}(1800)$	0.1956 (0.2950)						
$RC(\delta^*)$	0.2434 (0.1148)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.2034 (0.1126)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1966 (0.1241)			0.0001 (0.0000)			
HY	0.7097 (0.1683)						
$HY(S^*)$	0.3830 (0.0594)						
$NV(S^*, 1, 0, 0)$	0.2238 (0.0505)			0.0006 (0.0002)			0
$NV(2S^*, 1, 0, 0)$	0.2106 (0.0515)			0.0006 (0.0002)			0
$NV(3S^*, 1, 0, 0)$	0.2067 (0.0576)			0.0006 (0.0002)			0
$NV(S^*, 1, 10, 10)$	0.2004 (0.0622)	0.0003 (0.0052)	0.0003 (0.0044)	0.0000 (0.0014)	0.0005 (0.0049)	0.0005 (0.0053)	0
$NV(2S^*, 1, 10, 10)$	0.2003 (0.0588)	0.0002 (0.0024)	0.0002 (0.0026)	0.0001 (0.0006)	0.0004 (0.0023)	0.0004 (0.0029)	0
$NV(3S^*, 1, 10, 10)$	0.2002 (0.0646)	0.0002 (0.0020)	0.0003 (0.0020)	0.0001 (0.0005)	0.0004 (0.0019)	0.0003 (0.0022)	0
$NV(S^*, 1, 20, 20)$	0.2012 (0.0842)	0.0016 (0.0244)	0.0014 (0.0227)	0.0002 (0.0023)	-0.0015 (0.0298)	-0.0006 (0.0268)	0
$NV(2S^*, 1, 20, 20)$	0.2002 (0.0650)	0.0003 (0.0050)	0.0004 (0.0049)	0.0001 (0.0006)	0.0003 (0.0050)	0.0002 (0.0053)	0
$NV(3S^*, 1, 20, 20)$	0.2000 (0.0700)	0.0003 (0.0039)	0.0004 (0.0038)	0.0001 (0.0005)	0.0003 (0.0038)	0.0002 (0.0040)	0

Table 17 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	4.0301 (0.1925)	4.3870 (0.2734)						
$RV(300)$	0.8074 (0.1314)	0.9511 (0.1637)						
$RV(900)$	0.6619 (0.1857)	0.6400 (0.1818)						
$RV(1800)$	0.6100 (0.2508)	0.5454 (0.2168)						
$RV_1(5)$	3.0706 (0.1717)	3.8100 (0.2587)						
$RV_1(300)$	0.6139 (0.1908)	0.5181 (0.1910)						
$RV_1(900)$	0.5998 (0.3041)	0.5048 (0.2586)						
$RV_1(1800)$	0.5811 (0.4028)	0.5034 (0.3502)						
$RV(\delta^*)$	0.7141 (0.1613)	0.6912 (0.1896)	0.0020 (0.0001)			0.0031 (0.0002)		
$RV_1(\delta^*)$	0.6074 (0.2261)	0.5078 (0.2309)	0.0020 (0.0001)			0.0031 (0.0002)		
$RV_2(\delta^*)$	0.6201 (0.2751)	0.5151 (0.2824)	0.0020 (0.0001)			0.0031 (0.0002)		
$K^{TH2}(60)$	0.6135 (0.1666)	0.5147 (0.1642)	0.0020 (0.0001)			0.0031 (0.0002)		
$TSRV$	0.6518 (0.0728)	0.5409 (0.0988)	0.0020 (0.0001)			0.0031 (0.0002)		
$NV(S^*, 0)$	0.6642 (0.0685)	0.5532 (0.0924)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(2S^*, 0)$	0.6382 (0.0735)	0.5346 (0.0888)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(3S^*, 0)$	0.6300 (0.0818)	0.5282 (0.0970)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(S^*, 10)$	0.6233 (0.0991)	0.5157 (0.1248)	0.0012 (0.0001)	0.0005 (0.0280)	0.0001 (0.0230)	0.0028 (0.0003)	0.0008 (0.1329)	0.0060 (0.1023)
$NV(2S^*, 10)$	0.6204 (0.0907)	0.5218 (0.1033)	0.0012 (0.0001)	0.0005 (0.0290)	-0.0044 (0.0642)	0.0028 (0.0003)	0.0067 (0.1257)	0.0029 (0.1000)
$NV(3S^*, 10)$	0.6189 (0.0973)	0.5203 (0.1105)	0.0012 (0.0002)	0.0007 (0.0307)	-0.0069 (0.0598)	0.0028 (0.0003)	0.0040 (0.1440)	0.0043 (0.1064)
$NV(S^*, 20)$	0.6233 (0.0991)	0.5157 (0.1248)	0.0012 (0.0001)	0.0005 (0.0280)	0.0001 (0.0230)	0.0028 (0.0003)	0.0008 (0.1329)	0.0060 (0.1023)
$NV(2S^*, 20)$	0.6204 (0.0907)	0.5218 (0.1033)	0.0012 (0.0001)	0.0005 (0.0290)	-0.0044 (0.0642)	0.0028 (0.0003)	0.0067 (0.1257)	0.0029 (0.1000)
$NV(3S^*, 20)$	0.6189 (0.0973)	0.5203 (0.1105)	0.0012 (0.0002)	0.0007 (0.0307)	-0.0069 (0.0598)	0.0028 (0.0003)	0.0040 (0.1440)	0.0043 (0.1064)

Table 18: Monte Carlo simulation results for scenario: **low pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001	#NonPD
$RC(5)$	0.0671 (0.0630)						
$RC(300)$	0.1876 (0.1076)						
$RC(900)$	0.1856 (0.1424)						
$RC(1800)$	0.1812 (0.1802)						
$RC_{1,1}(5)$	0.1331 (0.0946)						
$RC_{1,1}(300)$	0.1950 (0.1438)						
$RC_{1,1}(900)$	0.1913 (0.2156)						
$RC_{1,1}(1800)$	0.1921 (0.2947)						
$RC(\delta^*)$	0.1489 (0.1098)			0.0001 (0.0001)			
$RC_{1,1}(\delta^*)$	0.1810 (0.1128)			0.0001 (0.0001)			
$RC_{2,2}(\delta^*)$	0.1899 (0.1293)			0.0001 (0.0001)			
HY	0.2966 (0.1159)						
$HY(S^*)$	0.2476 (0.0680)						
$NV(S^*, 1, 0, 0)$	0.2063 (0.0719)			0.0008 (0.0011)			0
$NV(2S^*, 1, 0, 0)$	0.2027 (0.0705)			0.0008 (0.0011)			0
$NV(3S^*, 1, 0, 0)$	0.2021 (0.0763)			0.0008 (0.0013)			0
$NV(S^*, 1, 10, 10)$	0.2015 (0.0822)	0.0016 (0.0341)	-0.0007 (0.0195)	0.0001 (0.0119)	0.0017 (0.0201)	0.0004 (0.0144)	1
$NV(2S^*, 1, 10, 10)$	0.1994 (0.0779)	0.0000 (0.0085)	0.0003 (0.0084)	0.0001 (0.0033)	0.0002 (0.0080)	0.0002 (0.0111)	0
$NV(3S^*, 1, 10, 10)$	0.1996 (0.0825)	0.0003 (0.0057)	0.0002 (0.0055)	0.0001 (0.0023)	0.0001 (0.0056)	0.0003 (0.0056)	0
$NV(S^*, 1, 20, 20)$	0.2015 (0.0822)	0.0016 (0.0341)	-0.0007 (0.0195)	0.0001 (0.0119)	0.0017 (0.0201)	0.0004 (0.0144)	1
$NV(2S^*, 1, 20, 20)$	0.1970 (0.1132)	0.0167 (0.2024)	0.0012 (0.0548)	-0.0032 (0.0401)	0.0064 (0.1086)	-0.0024 (0.0463)	0
$NV(3S^*, 1, 20, 20)$	0.1995 (0.0886)	0.0003 (0.0096)	0.0000 (0.0074)	0.0000 (0.0026)	0.0002 (0.0070)	0.0003 (0.0072)	0

Table 18 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV[1] 0.6281	IV[2] 0.5234	$\gamma_1(0)$ 0.0124	$\gamma_1(1)$ 0.0052	$\gamma_1(2)$ 0.0024	$\gamma_2(0)$ 0.0277	$\gamma_2(1)$ 0.0139	$\gamma_2(2)$ 0.0076
$RV(5)$	107.7655 (2.6030)	182.2514 (5.0117)						
$RV(300)$	2.5526 (0.4701)	4.7887 (0.8584)						
$RV(900)$	1.2149 (0.3524)	1.8783 (0.5828)						
$RV(1800)$	0.8762 (0.3630)	1.1518 (0.4897)						
$RV_1(5)$	8.6363 (2.2711)	56.7083 (4.3701)						
$RV_1(300)$	0.6041 (0.4231)	0.5833 (0.7379)						
$RV_1(900)$	0.6382 (0.4203)	0.5598 (0.5714)						
$RV_1(1800)$	0.6019 (0.4802)	0.5225 (0.5249)						
$RV(\delta^*)$	1.0591 (0.3849)	1.2979 (0.5496)	0.0132 (0.0003)			0.0283 (0.0008)		
$RV_1(\delta^*)$	0.6222 (0.4353)	0.5830 (0.5681)	0.0132 (0.0003)			0.0283 (0.0008)		
$RV_2(\delta^*)$	0.6359 (0.5007)	0.5402 (0.5980)	0.0132 (0.0003)			0.0283 (0.0008)		
$K^{TH2}(60)$	0.6173 (0.2134)	0.5069 (0.2471)	0.0132 (0.0003)			0.0283 (0.0008)		
$TSRV$	1.8667 (0.1765)	1.8715 (0.1898)	0.0132 (0.0003)			0.0283 (0.0008)		
$NV(S^*, 0)$	1.6124 (0.0899)	1.6091 (0.1339)	0.0109 (0.0002)			0.0251 (0.0005)		
$NV(2S^*, 0)$	1.0396 (0.0807)	0.9765 (0.1020)	0.0112 (0.0002)			0.0256 (0.0005)		
$NV(3S^*, 0)$	0.8739 (0.0878)	0.7953 (0.1050)	0.0113 (0.0002)			0.0258 (0.0005)		
$NV(S^*, 10)$	0.6250 (0.0938)	0.5245 (0.1368)	0.0123 (0.0002)	0.0051 (0.0006)	0.0024 (0.0028)	0.0277 (0.0006)	0.0129 (0.0549)	0.0079 (0.1584)
$NV(2S^*, 10)$	0.6212 (0.0880)	0.5209 (0.1073)	0.0124 (0.0002)	0.0051 (0.0006)	0.0024 (0.0028)	0.0277 (0.0006)	0.0128 (0.0541)	0.0088 (0.1521)
$NV(3S^*, 10)$	0.6188 (0.0966)	0.5189 (0.1125)	0.0124 (0.0002)	0.0051 (0.0006)	0.0024 (0.0028)	0.0277 (0.0006)	0.0128 (0.0532)	0.0089 (0.1469)
$NV(S^*, 20)$	0.6244 (0.0998)	0.5297 (0.1422)	0.0123 (0.0002)	0.0051 (0.0007)	0.0023 (0.0034)	0.0276 (0.0006)	0.0130 (0.0776)	0.0107 (0.2262)
$NV(2S^*, 20)$	0.6205 (0.0919)	0.5219 (0.1106)	0.0124 (0.0002)	0.0051 (0.0007)	0.0023 (0.0034)	0.0277 (0.0006)	0.0128 (0.0737)	0.0115 (0.2069)
$NV(3S^*, 20)$	0.6182 (0.1007)	0.5192 (0.1162)	0.0124 (0.0002)	0.0051 (0.0007)	0.0023 (0.0033)	0.0277 (0.0006)	0.0128 (0.0710)	0.0112 (0.1936)

Table 19: Monte Carlo simulation results for scenario: **low pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014	#NonPD
$RC(5)$	8.5328 (2.4038)						
$RC(300)$	0.4661 (0.4486)						
$RC(900)$	0.2808 (0.3402)						
$RC(1800)$	0.2397 (0.3111)						
$RC_{1,1}(5)$	6.8554 (2.1534)						
$RC_{1,1}(300)$	0.1955 (0.3965)						
$RC_{1,1}(900)$	0.1979 (0.3638)						
$RC_{1,1}(1800)$	0.2001 (0.3815)						
$RC(\delta^*)$	0.6791 (0.6227)			0.0006 (0.0002)			
$RC_{1,1}(\delta^*)$	0.1559 (0.4956)			0.0006 (0.0002)			
$RC_{2,2}(\delta^*)$	0.2437 (0.4934)			0.0006 (0.0002)			
HY	6.7673 (2.4644)						
$HY(S^*)$	1.9379 (0.2550)						
$NV(S^*, 1, 0, 0)$	0.4965 (0.0849)			0.0025 (0.0004)			0
$NV(2S^*, 1, 0, 0)$	0.3271 (0.0712)			0.0028 (0.0004)			0
$NV(3S^*, 1, 0, 0)$	0.2780 (0.0771)			0.0029 (0.0004)			0
$NV(S^*, 1, 10, 10)$	0.2014 (0.0921)	0.0036 (0.0477)	0.0044 (0.0466)	0.0012 (0.0011)	0.0021 (0.0468)	0.0017 (0.0462)	0
$NV(2S^*, 1, 10, 10)$	0.2007 (0.0782)	0.0036 (0.0278)	0.0043 (0.0266)	0.0012 (0.0008)	0.0022 (0.0268)	0.0017 (0.0263)	0
$NV(3S^*, 1, 10, 10)$	0.1993 (0.0845)	0.0029 (0.0185)	0.0036 (0.0175)	0.0012 (0.0006)	0.0028 (0.0178)	0.0023 (0.0172)	0
$NV(S^*, 1, 20, 20)$	0.2012 (0.0941)	0.0048 (0.0539)	0.0057 (0.0528)	0.0012 (0.0010)	0.0008 (0.0534)	0.0004 (0.0529)	1
$NV(2S^*, 1, 20, 20)$	0.2007 (0.0800)	0.0042 (0.0243)	0.0049 (0.0235)	0.0012 (0.0007)	0.0016 (0.0241)	0.0011 (0.0236)	0
$NV(3S^*, 1, 20, 20)$	0.1991 (0.0871)	0.0030 (0.0151)	0.0037 (0.0146)	0.0012 (0.0006)	0.0027 (0.0151)	0.0022 (0.0147)	0

Table 19 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2		$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0124	0.0052	0.0024	0.0277	0.0139	0.0076	
$RV(5)$	75.9466	99.3658							
	(2.3865)	(3.9946)							
$RV(300)$	2.5028	4.8185							
	(0.4594)	(0.9123)							
$RV(900)$	1.2188	1.8909							
	(0.3453)	(0.6228)							
$RV(1800)$	0.8682	1.1611							
	(0.3463)	(0.4935)							
$RV_1(5)$	27.3156	63.2659							
	(1.7887)	(3.4360)							
$RV_1(300)$	0.6428	0.5822							
	(0.4168)	(0.7503)							
$RV_1(900)$	0.6313	0.5612							
	(0.4064)	(0.5572)							
$RV_1(1800)$	0.6065	0.5328							
	(0.4876)	(0.5251)							
$RV(\delta^*)$	1.0575	1.2969	0.0132			0.0282			
	(0.3842)	(0.5297)	(0.0004)			(0.0013)			
$RV_1(\delta^*)$	0.6164	0.5724	0.0132			0.0282			
	(0.4333)	(0.5464)	(0.0004)			(0.0013)			
$RV_2(\delta^*)$	0.6107	0.6013	0.0132			0.0282			
	(0.4969)	(0.5981)	(0.0004)			(0.0013)			
$K^{TH2}(60)$	0.6099	0.5165	0.0132			0.0282			
	(0.2198)	(0.2590)	(0.0004)			(0.0013)			
$TSRV$	1.0649	0.9872	0.0132			0.0282			
	(0.1294)	(0.1867)	(0.0004)			(0.0013)			
$NV(S^*,0)$	1.0046	0.9436	0.0116			0.0262			
	(0.0971)	(0.1598)	(0.0003)			(0.0009)			
$NV(2S^*,0)$	0.7785	0.6936	0.0117			0.0265			
	(0.0903)	(0.1224)	(0.0003)			(0.0009)			
$NV(3S^*,0)$	0.7126	0.6207	0.0118			0.0267			
	(0.1009)	(0.1235)	(0.0003)			(0.0009)			
$NV(S^*,10)$	0.6221	0.5209	0.0124	0.0021	0.0196	0.0277	-0.0377	0.0661	
	(0.1098)	(0.1735)	(0.0003)	(0.0505)	(0.2563)	(0.0010)	(4.3126)	(8.6675)	
$NV(2S^*,10)$	0.6192	0.5191	0.0124	0.0027	0.0154	0.0277	-0.0103	0.0608	
	(0.1016)	(0.1307)	(0.0003)	(0.0451)	(0.2204)	(0.0010)	(4.2477)	(8.6665)	
$NV(3S^*,10)$	0.6159	0.5153	0.0124	0.0028	0.0132	0.0277	-0.0216	0.0404	
	(0.1127)	(0.1319)	(0.0003)	(0.0423)	(0.2034)	(0.0010)	(4.2457)	(8.6986)	
$NV(S^*,20)$	0.6188	0.5215	0.0124	0.0001	0.0310	0.0276	0.6085	0.0680	
	(0.1156)	(0.1837)	(0.0003)	(0.1224)	(0.5142)	(0.0010)	(23.4384)	(18.2707)	
$NV(2S^*,20)$	0.6181	0.5190	0.0124	0.0015	0.0245	0.0277	0.5898	0.1127	
	(0.1053)	(0.1344)	(0.0003)	(0.1170)	(0.4606)	(0.0010)	(23.1831)	(18.2142)	
$NV(3S^*,20)$	0.6151	0.5148	0.0124	0.0021	0.0216	0.0277	0.5144	-0.0270	
	(0.1167)	(0.1355)	(0.0004)	(0.1149)	(0.4422)	(0.0010)	(23.0993)	(18.2759)	

Table 20: Monte Carlo simulation results for scenario: **low pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014	#NonPD
$RC(5)$	2.9113 (1.3974)						
$RC(300)$	0.3687 (0.4477)						
$RC(900)$	0.2744 (0.3461)						
$RC(1800)$	0.2351 (0.3261)						
$RC_{1,1}(5)$	3.6281 (1.5393)						
$RC_{1,1}(300)$	0.1948 (0.4249)						
$RC_{1,1}(900)$	0.1915 (0.3489)						
$RC_{1,1}(1800)$	0.1971 (0.3819)						
$RC(\delta^*)$	0.4585 (0.6220)			0.0007 (0.0003)			
$RC_{1,1}(\delta^*)$	0.2108 (0.5268)			0.0007 (0.0003)			
$RC_{2,2}(\delta^*)$	0.1991 (0.5234)			0.0007 (0.0003)			
HY	5.2835 (1.5309)						
$HY(S^*)$	1.2017 (0.2341)						
$NV(S^*, 1, 0, 0)$	0.2848 (0.1027)			0.0062 (0.0015)			0
$NV(2S^*, 1, 0, 0)$	0.2392 (0.0850)			0.0064 (0.0015)			0
$NV(3S^*, 1, 0, 0)$	0.2247 (0.0904)			0.0065 (0.0016)			0
$NV(S^*, 1, 10, 10)$	0.2031 (0.1082)	0.0020 (0.0143)	0.0025 (0.0150)	0.0012 (0.0030)	0.0039 (0.0133)	0.0036 (0.0157)	2
$NV(2S^*, 1, 10, 10)$	0.2008 (0.0905)	0.0021 (0.0090)	0.0028 (0.0095)	0.0013 (0.0021)	0.0036 (0.0082)	0.0032 (0.0098)	0
$NV(3S^*, 1, 10, 10)$	0.1988 (0.0966)	0.0022 (0.0070)	0.0028 (0.0072)	0.0013 (0.0017)	0.0035 (0.0064)	0.0032 (0.0076)	0
$NV(S^*, 1, 20, 20)$	0.2036 (0.1200)	0.0033 (0.0275)	0.0038 (0.0268)	0.0013 (0.0031)	0.0026 (0.0268)	0.0024 (0.0279)	5
$NV(2S^*, 1, 20, 20)$	0.2004 (0.0956)	0.0026 (0.0161)	0.0032 (0.0165)	0.0013 (0.0021)	0.0031 (0.0153)	0.0029 (0.0166)	0
$NV(3S^*, 1, 20, 20)$	0.1983 (0.1015)	0.0023 (0.0131)	0.0029 (0.0131)	0.0013 (0.0017)	0.0034 (0.0125)	0.0032 (0.0133)	0

Table 20 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0124	0.0052	0.0024	0.0277	0.0139	0.0076
$RV(5)$	34.7038	39.1797						
	(1.6974)	(2.5328)						
$RV(300)$	2.5239	4.8400						
	(0.4667)	(0.9287)						
$RV(900)$	1.2257	1.9059						
	(0.3792)	(0.6109)						
$RV(1800)$	0.8714	1.1178						
	(0.3670)	(0.4729)						
$RV_1(5)$	25.0801	33.4062						
	(1.4906)	(2.3465)						
$RV_1(300)$	0.6250	0.5417						
	(0.4439)	(0.8086)						
$RV_1(900)$	0.6080	0.5448						
	(0.4284)	(0.5284)						
$RV_1(1800)$	0.5981	0.5567						
	(0.4686)	(0.5189)						
$RV(\delta^*)$	1.0536	1.3873	0.0130			0.0245		
	(0.3671)	(0.5348)	(0.0007)			(0.0018)		
$RV_1(\delta^*)$	0.6345	0.5768	0.0130			0.0245		
	(0.4418)	(0.5647)	(0.0007)			(0.0018)		
$RV_2(\delta^*)$	0.6416	0.5974	0.0130			0.0245		
	(0.4969)	(0.6003)	(0.0007)			(0.0018)		
$K^{TH2}(60)$	0.6138	0.5173	0.0130			0.0245		
	(0.2384)	(0.2769)	(0.0007)			(0.0018)		
$TSRV$	0.7364	0.6363	0.0130			0.0245		
	(0.1396)	(0.2366)	(0.0007)			(0.0018)		
$NV(S^*, 0)$	0.7438	0.6586	0.0120			0.0270		
	(0.1266)	(0.2177)	(0.0005)			(0.0016)		
$NV(2S^*, 0)$	0.6661	0.5757	0.0121			0.0272		
	(0.1142)	(0.1510)	(0.0005)			(0.0015)		
$NV(3S^*, 0)$	0.6432	0.5480	0.0121			0.0273		
	(0.1262)	(0.1494)	(0.0005)			(0.0015)		
$NV(S^*, 10)$	0.6206	0.5298	0.0123	0.0019	-0.0008	0.0276	0.0506	0.0353
	(0.1475)	(0.2475)	(0.0006)	(0.2187)	(0.1801)	(0.0018)	(0.9531)	(0.8190)
$NV(2S^*, 10)$	0.6145	0.5221	0.0124	0.0048	0.0011	0.0277	0.0334	0.0444
	(0.1293)	(0.1642)	(0.0006)	(0.2200)	(0.1807)	(0.0017)	(0.9018)	(0.8016)
$NV(3S^*, 10)$	0.6119	0.5149	0.0124	0.0794	0.6023	0.0277	0.0127	0.0554
	(0.1411)	(0.1623)	(0.0006)	(0.3331)	(1.6924)	(0.0016)	(0.9042)	(0.7986)
$NV(S^*, 20)$	0.6206	0.5298	0.0123	0.0019	-0.0008	0.0276	0.0506	0.0353
	(0.1475)	(0.2475)	(0.0006)	(0.2187)	(0.1801)	(0.0018)	(0.9531)	(0.8190)
$NV(2S^*, 20)$	0.6145	0.5221	0.0124	0.0048	0.0011	0.0277	0.0334	0.0444
	(0.1293)	(0.1642)	(0.0006)	(0.2200)	(0.1807)	(0.0017)	(0.9018)	(0.8016)
$NV(3S^*, 20)$	0.6119	0.5149	0.0124	0.0794	0.6023	0.0277	0.0127	0.0554
	(0.1411)	(0.1623)	(0.0006)	(0.3331)	(1.6924)	(0.0016)	(0.9042)	(0.7986)

Table 21: Monte Carlo simulation results for scenario: **low pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014	#NonPD
$RC(5)$	0.4930 (0.5518)						
$RC(300)$	0.2566 (0.4734)						
$RC(900)$	0.2234 (0.3521)						
$RC(1800)$	0.1943 (0.3110)						
$RC_{1,1}(5)$	0.8211 (0.8158)						
$RC_{1,1}(300)$	0.1919 (0.4102)						
$RC_{1,1}(900)$	0.1870 (0.3517)						
$RC_{1,1}(1800)$	0.1923 (0.3737)						
$RC(\delta^*)$	0.3115 (0.6148)			0.0004 (0.0006)			
$RC_{1,1}(\delta^*)$	0.1869 (0.5257)			0.0004 (0.0006)			
$RC_{2,2}(\delta^*)$	0.2131 (0.5322)			0.0004 (0.0006)			
HY	1.1457 (0.9061)						
$HY(S^*)$	0.4793 (0.2373)						
$NV(S^*, 1, 0, 0)$	0.2263 (0.1446)			0.0076 (0.0072)			3
$NV(2S^*, 1, 0, 0)$	0.2167 (0.1103)			0.0074 (0.0066)			0
$NV(3S^*, 1, 0, 0)$	0.2116 (0.1111)			0.0072 (0.0064)			0
$NV(S^*, 1, 10, 10)$	0.2016 (0.1582)	0.0015 (0.0355)	0.0038 (0.0377)	0.0013 (0.0154)	0.0018 (0.0378)	0.0024 (0.0457)	41
$NV(2S^*, 1, 10, 10)$	0.2006 (0.1153)	0.0025 (0.0172)	0.0029 (0.0193)	0.0012 (0.0075)	0.0023 (0.0187)	0.0026 (0.0186)	0
$NV(3S^*, 1, 10, 10)$	0.1982 (0.1171)	0.0024 (0.0134)	0.0026 (0.0145)	0.0012 (0.0059)	0.0027 (0.0148)	0.0028 (0.0140)	0
$NV(S^*, 1, 20, 20)$	0.1948 (0.1938)	0.0051 (0.1415)	0.0023 (0.1078)	0.0014 (0.0282)	0.0035 (0.0984)	0.0046 (0.1281)	58
$NV(2S^*, 1, 20, 20)$	0.2011 (0.1213)	0.0029 (0.0194)	0.0029 (0.0202)	0.0012 (0.0078)	0.0025 (0.0193)	0.0024 (0.0197)	0
$NV(3S^*, 1, 20, 20)$	0.1981 (0.1221)	0.0024 (0.0144)	0.0028 (0.0145)	0.0012 (0.0059)	0.0029 (0.0149)	0.0026 (0.0144)	0

Table 21 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV[1] 0.6281	IV[2] 0.5234	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	3.2599 (0.0743)	4.0640 (0.1095)						
$RV(300)$	0.6963 (0.1145)	0.6221 (0.1055)						
$RV(900)$	0.6269 (0.1722)	0.5337 (0.1462)						
$RV(1800)$	0.5952 (0.2406)	0.4999 (0.1988)						
$RV_1(5)$	2.2636 (0.0940)	2.6013 (0.1129)						
$RV_1(300)$	0.6132 (0.1801)	0.5159 (0.1544)						
$RV_1(900)$	0.6028 (0.2940)	0.5021 (0.2462)						
$RV_1(1800)$	0.5831 (0.4032)	0.5002 (0.3444)						
$RV(\delta^*)$	0.6677 (0.1325)	0.5772 (0.1235)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_1(\delta^*)$	0.6004 (0.1965)	0.5042 (0.1771)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_2(\delta^*)$	0.6079 (0.2374)	0.5142 (0.2103)	0.0013 (0.0001)			0.0013 (0.0001)		
$K^{TH2}(60)$	0.6129 (0.1373)	0.5121 (0.1230)	0.0013 (0.0001)			0.0013 (0.0001)		
$TSRV$	1.1132 (0.1011)	0.8257 (0.0736)	0.0013 (0.0001)			0.0013 (0.0001)		
$NV(S^*, 0)$	1.2204 (0.0453)	0.8587 (0.0434)	0.0002 (0.0000)			0.0004 (0.0000)		
$NV(2S^*, 0)$	0.8810 (0.0452)	0.6582 (0.0445)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(3S^*, 0)$	0.7799 (0.0499)	0.6022 (0.0498)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(S^*, 10)$	0.5834 (0.0623)	0.4960 (0.0613)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0005 (0.0039)
$NV(2S^*, 10)$	0.6114 (0.0572)	0.5146 (0.0540)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0007 (0.0039)
$NV(3S^*, 10)$	0.6170 (0.0609)	0.5174 (0.0593)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0006 (0.0039)
$NV(S^*, 20)$	0.5958 (0.0829)	0.5188 (0.0847)	0.0005 (0.0001)	0.0005 (0.0001)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0023)	0.0005 (0.0070)
$NV(2S^*, 20)$	0.6222 (0.0641)	0.5206 (0.0591)	0.0005 (0.0000)	0.0004 (0.0000)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0022)	0.0005 (0.0069)
$NV(3S^*, 20)$	0.6225 (0.0661)	0.5203 (0.0638)	0.0005 (0.0000)	0.0004 (0.0001)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0023)	0.0004 (0.0071)

Table 22: Monte Carlo simulation results for scenario: **high pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002	#NonPD
$RC(5)$	0.5631 (0.0576)						
$RC(300)$	0.2241 (0.0837)						
$RC(900)$	0.1950 (0.1285)						
$RC(1800)$	0.1878 (0.1718)						
$RC_{1,1}(5)$	0.8786 (0.0712)						
$RC_{1,1}(300)$	0.1969 (0.1298)						
$RC_{1,1}(900)$	0.1957 (0.2079)						
$RC_{1,1}(1800)$	0.1946 (0.2913)						
$RC(\delta^*)$	0.3306 (0.0625)			0.0000 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1984 (0.0685)			0.0000 (0.0000)			
$RC_{2,2}(\delta^*)$	0.2015 (0.0789)			0.0000 (0.0000)			
HY	0.3401 (0.0614)						
$HY(S^*)$	0.3695 (0.0337)						
$NV(S^*, 1, 0, 0)$	0.3121 (0.0343)			0.0000 (0.0000)			0
$NV(2S^*, 1, 0, 0)$	0.2492 (0.0359)			0.0001 (0.0000)			0
$NV(3S^*, 1, 0, 0)$	0.2299 (0.0400)			0.0001 (0.0000)			0
$NV(S^*, 1, 10, 10)$	0.2012 (0.0479)	0.0002 (0.0035)	0.0003 (0.0034)	0.0001 (0.0001)	-0.0001 (0.0035)	0.0001 (0.0037)	0
$NV(2S^*, 1, 10, 10)$	0.1998 (0.0440)	0.0002 (0.0027)	0.0003 (0.0027)	0.0001 (0.0001)	-0.0001 (0.0027)	0.0000 (0.0028)	0
$NV(3S^*, 1, 10, 10)$	0.1999 (0.0478)	0.0001 (0.0024)	0.0002 (0.0024)	0.0001 (0.0001)	0.0000 (0.0024)	0.0001 (0.0024)	0
$NV(S^*, 1, 20, 20)$	0.1985 (0.0655)	-0.0000 (0.0071)	0.0001 (0.0075)	0.0001 (0.0001)	0.0002 (0.0073)	0.0002 (0.0075)	0
$NV(2S^*, 1, 20, 20)$	0.1995 (0.0481)	0.0001 (0.0050)	0.0002 (0.0052)	0.0001 (0.0001)	0.0000 (0.0051)	0.0001 (0.0051)	0
$NV(3S^*, 1, 20, 20)$	0.1997 (0.0517)	-0.0001 (0.0042)	-0.0000 (0.0042)	0.0001 (0.0001)	0.0003 (0.0042)	0.0003 (0.0042)	0

Table 22 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	2.7999 (0.0859)	2.7188 (0.1054)						
$RV(300)$	0.6935 (0.1143)	0.6252 (0.1058)						
$RV(900)$	0.6281 (0.1708)	0.5355 (0.1488)						
$RV(1800)$	0.5959 (0.2391)	0.5005 (0.2000)						
$RV_1(5)$	2.1592 (0.0913)	2.1995 (0.1023)						
$RV_1(300)$	0.6140 (0.1799)	0.5183 (0.1552)						
$RV_1(900)$	0.6021 (0.2950)	0.5005 (0.2457)						
$RV_1(1800)$	0.5830 (0.4028)	0.4997 (0.3444)						
$RV(\delta^*)$	0.6657 (0.1333)	0.5767 (0.1197)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_1(\delta^*)$	0.6031 (0.1988)	0.5043 (0.1805)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_2(\delta^*)$	0.6103 (0.2380)	0.5129 (0.2132)	0.0013 (0.0001)			0.0013 (0.0001)		
$K^{TH2}(60)$	0.6118 (0.1440)	0.5148 (0.1278)	0.0013 (0.0001)			0.0013 (0.0001)		
$TSRV$	0.8784 (0.0741)	0.6392 (0.0616)	0.0013 (0.0001)			0.0013 (0.0001)		
$NV(S^*, 0)$	0.9141 (0.0495)	0.6481 (0.0508)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(2S^*, 0)$	0.7431 (0.0527)	0.5710 (0.0551)	0.0003 (0.0000)			0.0006 (0.0000)		
$NV(3S^*, 0)$	0.6937 (0.0588)	0.5498 (0.0616)	0.0004 (0.0000)			0.0006 (0.0000)		
$NV(S^*, 10)$	0.6065 (0.0707)	0.5165 (0.0799)	0.0005 (0.0000)	0.0004 (0.0024)	0.0010 (0.0141)	0.0007 (0.0001)	0.0019 (0.1737)	0.0034 (0.2916)
$NV(2S^*, 10)$	0.6191 (0.0660)	0.5190 (0.0689)	0.0005 (0.0000)	0.0006 (0.0025)	0.0012 (0.0134)	0.0007 (0.0001)	0.0014 (0.1548)	0.0015 (0.2976)
$NV(3S^*, 10)$	0.6197 (0.0712)	0.5192 (0.0742)	0.0005 (0.0001)	0.0006 (0.0026)	0.0012 (0.0130)	0.0007 (0.0001)	0.0023 (0.1536)	0.0017 (0.3104)
$NV(S^*, 20)$	0.6150 (0.0936)	0.5241 (0.1704)	0.0005 (0.0001)	-0.0000 (0.0062)	0.0026 (0.0315)	0.0007 (0.0003)	0.0333 (1.1112)	0.0401 (1.0563)
$NV(2S^*, 20)$	0.6222 (0.0708)	0.5199 (0.0748)	0.0005 (0.0001)	-0.0001 (0.0060)	0.0026 (0.0282)	0.0007 (0.0001)	0.0323 (1.0060)	0.0212 (0.7397)
$NV(3S^*, 20)$	0.6208 (0.0754)	0.5196 (0.0795)	0.0005 (0.0001)	-0.0001 (0.0060)	0.0028 (0.0265)	0.0007 (0.0001)	0.0285 (0.9856)	0.0174 (0.7569)

Table 23: Monte Carlo simulation results for scenario: **high pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002	#NonPD
$RC(5)$	0.2460 (0.0425)						
$RC(300)$	0.2124 (0.0832)						
$RC(900)$	0.1955 (0.1279)						
$RC(1800)$	0.1887 (0.1720)						
$RC_{1,1}(5)$	0.5135 (0.0630)						
$RC_{1,1}(300)$	0.1981 (0.1299)						
$RC_{1,1}(900)$	0.1933 (0.2064)						
$RC_{1,1}(1800)$	0.1954 (0.2903)						
$RC(\delta^*)$	0.2463 (0.0584)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1962 (0.0755)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1960 (0.0913)			0.0001 (0.0000)			
HY	0.4196 (0.0628)						
$HY(S^*)$	0.3235 (0.0392)						
$NV(S^*, 1, 0, 0)$	0.2429 (0.0418)			0.0002 (0.0001)			0
$NV(2S^*, 1, 0, 0)$	0.2186 (0.0441)			0.0003 (0.0001)			0
$NV(3S^*, 1, 0, 0)$	0.2112 (0.0495)			0.0003 (0.0001)			0
$NV(S^*, 1, 10, 10)$	0.2018 (0.0637)	0.0001 (0.0047)	0.0001 (0.0031)	0.0000 (0.0031)	0.0002 (0.0056)	0.0003 (0.0070)	0
$NV(2S^*, 1, 10, 10)$	0.2005 (0.0520)	0.0001 (0.0016)	0.0001 (0.0017)	0.0001 (0.0004)	0.0001 (0.0015)	0.0001 (0.0019)	0
$NV(3S^*, 1, 10, 10)$	0.1999 (0.0572)	0.0001 (0.0013)	0.0001 (0.0013)	0.0001 (0.0003)	0.0001 (0.0012)	0.0001 (0.0015)	0
$NV(S^*, 1, 20, 20)$	0.2018 (0.0645)	0.0000 (0.0068)	-0.0000 (0.0058)	0.0000 (0.0049)	0.0003 (0.0101)	0.0005 (0.0145)	6
$NV(2S^*, 1, 20, 20)$	0.2001 (0.0586)	0.0000 (0.0037)	0.0001 (0.0035)	0.0001 (0.0005)	0.0001 (0.0038)	0.0002 (0.0036)	0
$NV(3S^*, 1, 20, 20)$	0.1996 (0.0627)	0.0000 (0.0027)	0.0001 (0.0026)	0.0001 (0.0003)	0.0002 (0.0027)	0.0002 (0.0028)	0

Table 23 ***(cont'd)*:** Monte Carlo simulation results for scenario: **high pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	1.8504 (0.0833)	1.4386 (0.0864)						
$RV(300)$	0.6954 (0.1167)	0.6249 (0.1054)						
$RV(900)$	0.6257 (0.1739)	0.5345 (0.1465)						
$RV(1800)$	0.5953 (0.2443)	0.4984 (0.2001)						
$RV_1(5)$	1.6712 (0.0841)	1.3419 (0.0851)						
$RV_1(300)$	0.6149 (0.1790)	0.5151 (0.1527)						
$RV_1(900)$	0.5997 (0.2996)	0.4999 (0.2461)						
$RV_1(1800)$	0.5815 (0.4001)	0.4979 (0.3416)						
$RV(\delta^*)$	0.6755 (0.1311)	0.5781 (0.1194)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_1(\delta^*)$	0.6062 (0.1997)	0.5033 (0.1745)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_2(\delta^*)$	0.6078 (0.2381)	0.5119 (0.2045)	0.0013 (0.0001)			0.0013 (0.0001)		
$K^{TH2}(60)$	0.6145 (0.1548)	0.5140 (0.1351)	0.0013 (0.0001)			0.0013 (0.0001)		
$TSRV$	0.7116 (0.0700)	0.5532 (0.0769)	0.0013 (0.0001)			0.0013 (0.0001)		
$NV(S^*, 0)$	0.7386 (0.0610)	0.5760 (0.0764)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(2S^*, 0)$	0.6696 (0.0659)	0.5413 (0.0714)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(3S^*, 0)$	0.6492 (0.0730)	0.5328 (0.0762)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(S^*, 10)$	0.6212 (0.0969)	0.5157 (0.1417)	0.0005 (0.0001)	0.0002 (0.0098)	0.0010 (0.0276)	0.0007 (0.0002)	0.0005 (0.0473)	0.0018 (0.0496)
$NV(2S^*, 10)$	0.6218 (0.0840)	0.5188 (0.0882)	0.0005 (0.0001)	0.0002 (0.0136)	0.0007 (0.0118)	0.0007 (0.0002)	0.0021 (0.0484)	0.0009 (0.0360)
$NV(3S^*, 10)$	0.6205 (0.0889)	0.5205 (0.0902)	0.0005 (0.0001)	0.0004 (0.0122)	0.0005 (0.0770)	0.0007 (0.0002)	0.0040 (0.0570)	0.0002 (0.0391)
$NV(S^*, 20)$	0.6212 (0.0969)	0.5157 (0.1417)	0.0005 (0.0001)	0.0002 (0.0098)	0.0010 (0.0276)	0.0007 (0.0002)	0.0005 (0.0473)	0.0018 (0.0496)
$NV(2S^*, 20)$	0.6218 (0.0840)	0.5188 (0.0882)	0.0005 (0.0001)	0.0002 (0.0136)	0.0007 (0.0118)	0.0007 (0.0002)	0.0021 (0.0484)	0.0009 (0.0360)
$NV(3S^*, 20)$	0.6205 (0.0889)	0.5205 (0.0902)	0.0005 (0.0001)	0.0004 (0.0122)	0.0005 (0.0770)	0.0007 (0.0002)	0.0040 (0.0570)	0.0002 (0.0391)

Table 24: Monte Carlo simulation results for scenario: **high pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002	#NonPD
$RC(5)$	0.0460 (0.0251)						
$RC(300)$	0.1854 (0.0810)						
$RC(900)$	0.1853 (0.1266)						
$RC(1800)$	0.1816 (0.1696)						
$RC_{1,1}(5)$	0.1306 (0.0402)						
$RC_{1,1}(300)$	0.1974 (0.1268)						
$RC_{1,1}(900)$	0.1923 (0.2059)						
$RC_{1,1}(1800)$	0.1931 (0.2884)						
$RC(\delta^*)$	0.1648 (0.0602)			0.0001 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1901 (0.0786)			0.0001 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1904 (0.0885)			0.0001 (0.0000)			
HY	0.2696 (0.0588)						
$HY(S^*)$	0.2468 (0.0498)						
$NV(S^*, 1, 0, 0)$	0.2108 (0.0593)			0.0005 (0.0006)			0
$NV(2S^*, 1, 0, 0)$	0.2049 (0.0601)			0.0006 (0.0006)			0
$NV(3S^*, 1, 0, 0)$	0.2033 (0.0646)			0.0006 (0.0007)			0
$NV(S^*, 1, 10, 10)$	0.2096 (0.0593)		0.0014 (0.0122)	0.0001 (0.0024)	-0.0005 (0.0077)		0
$NV(2S^*, 1, 10, 10)$	0.1990 (0.0894)	0.0067 (0.1151)	-0.0051 (0.1418)	-0.0007 (0.0389)	0.0028 (0.1558)	0.0043 (0.1032)	0
$NV(3S^*, 1, 10, 10)$	0.1998 (0.0727)	-0.0003 (0.0065)	0.0002 (0.0054)	-0.0000 (0.0028)	0.0001 (0.0052)	0.0001 (0.0083)	0
$NV(S^*, 1, 20, 20)$	0.2096 (0.0593)		0.0014 (0.0122)	0.0001 (0.0024)	-0.0005 (0.0077)		0
$NV(2S^*, 1, 20, 20)$	0.1990 (0.0894)	0.0067 (0.1151)	-0.0051 (0.1418)	-0.0007 (0.0389)	0.0028 (0.1558)	0.0043 (0.1032)	0
$NV(3S^*, 1, 20, 20)$	0.2038 (0.4233)	-0.0002 (0.0226)	0.0008 (0.0329)	0.0014 (0.0609)	-0.0024 (0.1333)	-0.0002 (0.0522)	256

Table 24 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0051	0.0042	0.0034	0.0068	0.0059	0.0051
$RV(5)$	26.9701	35.9338						
	(0.6282)	(0.9897)						
$RV(300)$	1.4058	1.5682						
	(0.2424)	(0.2771)						
$RV(900)$	0.8539	0.8394						
	(0.2415)	(0.2431)						
$RV(1800)$	0.7022	0.6503						
	(0.2800)	(0.2587)						
$RV_1(5)$	17.0170	21.3042						
	(0.7616)	(1.0097)						
$RV_1(300)$	0.6134	0.5284						
	(0.2649)	(0.2825)						
$RV_1(900)$	0.6099	0.5182						
	(0.3312)	(0.3062)						
$RV_1(1800)$	0.5983	0.5068						
	(0.4372)	(0.3782)						
$RV(\delta^*)$	0.8624	0.7973	0.0059			0.0075		
	(0.2649)	(0.2685)	(0.0002)			(0.0002)		
$RV_1(\delta^*)$	0.6224	0.5358	0.0059			0.0075		
	(0.3131)	(0.3134)	(0.0002)			(0.0002)		
$RV_2(\delta^*)$	0.6220	0.5423	0.0059			0.0075		
	(0.3938)	(0.3889)	(0.0002)			(0.0002)		
$K^{TH2}(60)$	0.6138	0.5170	0.0059			0.0075		
	(0.1808)	(0.1727)	(0.0002)			(0.0002)		
$TSRV$	2.6883	1.7089	0.0059			0.0075		
	(0.3082)	(0.1818)	(0.0002)			(0.0002)		
$NV(S^*, 0)$	2.8376	1.5828	0.0027			0.0050		
	(0.1121)	(0.0884)	(0.0001)			(0.0001)		
$NV(2S^*, 0)$	1.5670	0.9603	0.0031			0.0054		
	(0.0803)	(0.0736)	(0.0001)			(0.0001)		
$NV(3S^*, 0)$	1.1970	0.7846	0.0033			0.0056		
	(0.0801)	(0.0778)	(0.0001)			(0.0001)		
$NV(S^*, 10)$	0.5105	0.4744	0.0052	0.0046	0.0040	0.0069	0.0061	0.0075
	(0.1207)	(0.0959)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0111)	(0.0333)
$NV(2S^*, 10)$	0.5853	0.5062	0.0052	0.0046	0.0039	0.0068	0.0062	0.0076
	(0.0869)	(0.0806)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0110)	(0.0322)
$NV(3S^*, 10)$	0.6012	0.5122	0.0051	0.0046	0.0039	0.0068	0.0063	0.0071
	(0.0880)	(0.0862)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0108)	(0.0312)
$NV(S^*, 20)$	0.6011	0.5192	0.0051	0.0045	0.0039	0.0068	0.0056	0.0050
	(0.1390)	(0.1068)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0184)	(0.0549)
$NV(2S^*, 20)$	0.6169	0.5206	0.0051	0.0045	0.0039	0.0068	0.0056	0.0048
	(0.0911)	(0.0848)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0178)	(0.0514)
$NV(3S^*, 20)$	0.6184	0.5199	0.0051	0.0045	0.0039	0.0068	0.0056	0.0048
	(0.0918)	(0.0901)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0174)	(0.0490)

Table 25: Monte Carlo simulation results for scenario: **high pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020	#NonPD
$RC(5)$	4.6191 (0.5038)						
$RC(300)$	0.4488 (0.1976)						
$RC(900)$	0.2668 (0.1906)						
$RC(1800)$	0.2210 (0.2119)						
$RC_{1,1}(5)$	7.1714 (0.5923)						
$RC_{1,1}(300)$	0.2043 (0.2020)						
$RC_{1,1}(900)$	0.2008 (0.2484)						
$RC_{1,1}(1800)$	0.1951 (0.3157)						
$RC(\delta^*)$	0.6272 (0.2352)			0.0004 (0.0000)			
$RC_{1,1}(\delta^*)$	0.1875 (0.2006)			0.0004 (0.0000)			
$RC_{2,2}(\delta^*)$	0.1921 (0.2218)			0.0004 (0.0000)			
HY	1.5883 (0.5139)						
$HY(S^*)$	1.2577 (0.1277)						
$NV(S^*, 1, 0, 0)$	0.6092 (0.0698)			0.0008 (0.0001)			0
$NV(2S^*, 1, 0, 0)$	0.3756 (0.0571)			0.0011 (0.0001)			0
$NV(3S^*, 1, 0, 0)$	0.3072 (0.0607)			0.0012 (0.0002)			0
$NV(S^*, 1, 10, 10)$	0.1965 (0.0789)	0.0016 (0.0175)	0.0023 (0.0174)	0.0009 (0.0005)	0.0000 (0.0176)	0.0009 (0.0177)	1
$NV(2S^*, 1, 10, 10)$	0.1997 (0.0633)	0.0012 (0.0120)	0.0019 (0.0118)	0.0010 (0.0003)	0.0005 (0.0120)	0.0012 (0.0119)	0
$NV(3S^*, 1, 10, 10)$	0.2000 (0.0680)	0.0009 (0.0087)	0.0015 (0.0084)	0.0010 (0.0003)	0.0008 (0.0086)	0.0014 (0.0085)	0
$NV(S^*, 1, 20, 20)$	0.1955 (0.0868)	-0.0007 (0.0278)	0.0001 (0.0281)	0.0010 (0.0005)	0.0023 (0.0280)	0.0030 (0.0281)	0
$NV(2S^*, 1, 20, 20)$	0.1996 (0.0675)	-0.0003 (0.0147)	0.0004 (0.0147)	0.0010 (0.0003)	0.0019 (0.0149)	0.0026 (0.0147)	0
$NV(3S^*, 1, 20, 20)$	0.1999 (0.0716)	-0.0001 (0.0094)	0.0006 (0.0092)	0.0010 (0.0003)	0.0017 (0.0094)	0.0023 (0.0093)	0

Table 25 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV[1]	IV[2]	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6281	0.5234	0.0051	0.0042	0.0034	0.0068	0.0059	0.0051
$RV(5)$	22.3802	22.4581						
	(0.6980)	(0.9029)						
$RV(300)$	1.3896	1.5797						
	(0.2478)	(0.2877)						
$RV(900)$	0.8573	0.8468						
	(0.2404)	(0.2460)						
$RV(1800)$	0.7029	0.6515						
	(0.2769)	(0.2638)						
$RV_1(5)$	15.9628	17.2698						
	(0.7104)	(0.8567)						
$RV_1(300)$	0.6226	0.5372						
	(0.2667)	(0.2835)						
$RV_1(900)$	0.6068	0.5147						
	(0.3359)	(0.3042)						
$RV_1(1800)$	0.5966	0.5053						
	(0.4379)	(0.3803)						
$RV(\delta^*)$	0.8705	0.7941	0.0059			0.0074		
	(0.2563)	(0.2762)	(0.0002)			(0.0003)		
$RV_1(\delta^*)$	0.6114	0.5207	0.0059			0.0074		
	(0.3184)	(0.2998)	(0.0002)			(0.0003)		
$RV_2(\delta^*)$	0.6149	0.5349	0.0059			0.0074		
	(0.3868)	(0.3842)	(0.0002)			(0.0003)		
$K^{TH2}(60)$	0.6102	0.5121	0.0059			0.0074		
	(0.1882)	(0.1762)	(0.0002)			(0.0003)		
$TSRV$	1.6684	0.9624	0.0059			0.0074		
	(0.1929)	(0.1283)	(0.0002)			(0.0003)		
$NV(S^*, 0)$	1.6352	0.9458	0.0035			0.0058		
	(0.0983)	(0.0951)	(0.0001)			(0.0002)		
$NV(2S^*, 0)$	1.0408	0.6916	0.0039			0.0060		
	(0.0844)	(0.0832)	(0.0001)			(0.0002)		
$NV(3S^*, 0)$	0.8721	0.6210	0.0040			0.0061		
	(0.0892)	(0.0909)	(0.0001)			(0.0002)		
$NV(S^*, 10)$	0.5929	0.5091	0.0051	0.0065	0.0125	0.0068	-0.0030	-0.0285
	(0.1111)	(0.1112)	(0.0002)	(0.0175)	(0.0922)	(0.0003)	(1.1085)	(2.1354)
$NV(2S^*, 10)$	0.6121	0.5159	0.0051	0.0077	0.0086	0.0068	0.0033	-0.0318
	(0.0947)	(0.0934)	(0.0002)	(0.0160)	(0.0790)	(0.0003)	(1.0829)	(2.1622)
$NV(3S^*, 10)$	0.6149	0.5166	0.0051	0.0082	0.0056	0.0068	0.0125	-0.0140
	(0.1001)	(0.1023)	(0.0002)	(0.0153)	(0.0708)	(0.0003)	(1.0737)	(2.1756)
$NV(S^*, 20)$	0.6200	0.5180	0.0051	-0.0010	0.0250	0.0068	0.3361	0.2638
	(0.1237)	(0.1237)	(0.0002)	(0.0438)	(0.1912)	(0.0004)	(6.3876)	(4.9002)
$NV(2S^*, 20)$	0.6203	0.5188	0.0051	-0.0008	0.0237	0.0068	0.2958	0.2368
	(0.0985)	(0.0981)	(0.0002)	(0.0428)	(0.1728)	(0.0003)	(6.1260)	(4.7331)
$NV(3S^*, 20)$	0.6191	0.5181	0.0051	-0.0005	0.0230	0.0068	0.3007	0.2405
	(0.1037)	(0.1072)	(0.0002)	(0.0426)	(0.1659)	(0.0004)	(6.0796)	(4.7725)

Table 26: Monte Carlo simulation results for scenario: **high pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020	#NonPD
$RC(5)$	1.9642 (0.3611)						
$RC(300)$	0.3668 (0.1892)						
$RC(900)$	0.2685 (0.1876)						
$RC(1800)$	0.2257 (0.2128)						
$RC_{1,1}(5)$	4.0174 (0.5001)						
$RC_{1,1}(300)$	0.2033 (0.2041)						
$RC_{1,1}(900)$	0.1940 (0.2446)						
$RC_{1,1}(1800)$	0.1984 (0.3163)						
$RC(\delta^*)$	0.4015 (0.2077)			0.0006 (0.0001)			
$RC_{1,1}(\delta^*)$	0.1969 (0.2017)			0.0006 (0.0001)			
$RC_{2,2}(\delta^*)$	0.1929 (0.2215)			0.0006 (0.0001)			
HY	2.3801 (0.4690)						
$HY(S^*)$	0.9457 (0.1223)						
$NV(S^*, 1, 0, 0)$	0.3547 (0.0739)			0.0030 (0.0006)			0
$NV(2S^*, 1, 0, 0)$	0.2677 (0.0668)			0.0033 (0.0006)			0
$NV(3S^*, 1, 0, 0)$	0.2424 (0.0729)			0.0035 (0.0006)			0
$NV(S^*, 1, 10, 10)$	0.2032 (0.0839)	0.0006 (0.0070)	0.0009 (0.0075)	0.0010 (0.0016)	0.0010 (0.0067)	0.0015 (0.0084)	0
$NV(2S^*, 1, 10, 10)$	0.2014 (0.0734)	0.0006 (0.0044)	0.0009 (0.0046)	0.0010 (0.0010)	0.0011 (0.0040)	0.0016 (0.0048)	0
$NV(3S^*, 1, 10, 10)$	0.2006 (0.0798)	0.0004 (0.0035)	0.0008 (0.0037)	0.0010 (0.0008)	0.0012 (0.0033)	0.0017 (0.0038)	0
$NV(S^*, 1, 20, 20)$	0.2006 (0.0962)	0.0006 (0.0154)	0.0009 (0.0149)	0.0010 (0.0018)	0.0011 (0.0155)	0.0017 (0.0154)	0
$NV(2S^*, 1, 20, 20)$	0.2000 (0.0788)	0.0001 (0.0081)	0.0006 (0.0079)	0.0010 (0.0010)	0.0016 (0.0078)	0.0021 (0.0082)	0
$NV(3S^*, 1, 20, 20)$	0.1996 (0.0848)	-0.0002 (0.0063)	0.0003 (0.0063)	0.0010 (0.0008)	0.0019 (0.0061)	0.0024 (0.0064)	0

Table 26 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0051	$\gamma_1(1)$ 0.0042	$\gamma_1(2)$ 0.0034	$\gamma_2(0)$ 0.0068	$\gamma_2(1)$ 0.0059	$\gamma_2(2)$ 0.0051
$RV(5)$	12.8849 (0.6162)	9.6945 (0.6085)						
$RV(300)$	1.3955 (0.2541)	1.5834 (0.2916)						
$RV(900)$	0.8568 (0.2445)	0.8478 (0.2409)						
$RV(1800)$	0.7032 (0.2906)	0.6424 (0.2607)						
$RV_1(5)$	11.0788 (0.6074)	8.7308 (0.5838)						
$RV_1(300)$	0.6249 (0.2635)	0.5125 (0.2843)						
$RV_1(900)$	0.6008 (0.3481)	0.5097 (0.3074)						
$RV_1(1800)$	0.5917 (0.4236)	0.5080 (0.3735)						
$RV(\delta^*)$	0.8635 (0.2542)	0.8248 (0.2594)	0.0058 (0.0003)			0.0065 (0.0005)		
$RV_1(\delta^*)$	0.6163 (0.3259)	0.5350 (0.3076)	0.0058 (0.0003)			0.0065 (0.0005)		
$RV_2(\delta^*)$	0.6132 (0.3946)	0.5371 (0.3729)	0.0058 (0.0003)			0.0065 (0.0005)		
$K^{TH2}(60)$	0.6166 (0.2023)	0.5142 (0.1878)	0.0058 (0.0003)			0.0065 (0.0005)		
$TSRV$	0.9575 (0.1375)	0.6359 (0.1326)	0.0058 (0.0003)			0.0065 (0.0005)		
$NV(S^*, 0)$	0.9816 (0.1089)	0.6585 (0.1248)	0.0042 (0.0002)			0.0063 (0.0004)		
$NV(2S^*, 0)$	0.7660 (0.0992)	0.5766 (0.1049)	0.0045 (0.0002)			0.0064 (0.0004)		
$NV(3S^*, 0)$	0.7047 (0.1079)	0.5506 (0.1113)	0.0045 (0.0002)			0.0065 (0.0004)		
$NV(S^*, 10)$	0.6168 (0.1379)	0.5190 (0.1556)	0.0051 (0.0003)	0.0007 (0.0839)	0.0199 (0.0769)	0.0068 (0.0006)	0.0362 (0.2637)	0.0007 (0.2127)
$NV(2S^*, 10)$	0.6189 (0.1130)	0.5223 (0.1184)	0.0051 (0.0003)	-0.0017 (0.0857)	0.0038 (0.0740)	0.0068 (0.0005)	0.0510 (0.2537)	-0.0010 (0.2085)
$NV(3S^*, 10)$	0.6170 (0.1220)	0.5175 (0.1244)	0.0051 (0.0003)	-0.0006 (0.0921)	0.0046 (0.0761)	0.0068 (0.0006)	0.0293 (0.2711)	0.0044 (0.2134)
$NV(S^*, 20)$	0.6168 (0.1379)	0.5190 (0.1556)	0.0051 (0.0003)	0.0007 (0.0839)	0.0199 (0.0769)	0.0068 (0.0006)	0.0362 (0.2637)	0.0007 (0.2127)
$NV(2S^*, 20)$	0.6189 (0.1130)	0.5223 (0.1184)	0.0051 (0.0003)	-0.0017 (0.0857)	0.0038 (0.0740)	0.0068 (0.0005)	0.0510 (0.2537)	-0.0010 (0.2085)
$NV(3S^*, 20)$	0.6170 (0.1220)	0.5175 (0.1244)	0.0051 (0.0003)	-0.0006 (0.0921)	0.0046 (0.0761)	0.0068 (0.0006)	0.0293 (0.2711)	0.0044 (0.2134)

Table 27: Monte Carlo simulation results for scenario: **high pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020	#NonPD
$RC(5)$	0.2798 (0.1802)						
$RC(300)$	0.2389 (0.1885)						
$RC(900)$	0.2061 (0.1863)						
$RC(1800)$	0.1808 (0.2037)						
$RC_{1,1}(5)$	0.8025 (0.2700)						
$RC_{1,1}(300)$	0.2015 (0.1996)						
$RC_{1,1}(900)$	0.1892 (0.2420)						
$RC_{1,1}(1800)$	0.1933 (0.3073)						
$RC(\delta^*)$	0.2844 (0.2191)			0.0004 (0.0002)			
$RC_{1,1}(\delta^*)$	0.1931 (0.2065)			0.0004 (0.0002)			
$RC_{2,2}(\delta^*)$	0.1898 (0.2167)			0.0004 (0.0002)			
HY	0.8782 (0.3047)						
$HY(S^*)$	0.4645 (0.1172)						
$NV(S^*, 1, 0, 0)$	0.2356 (0.0934)			0.0056 (0.0026)			0
$NV(2S^*, 1, 0, 0)$	0.2210 (0.0848)			0.0056 (0.0025)			0
$NV(3S^*, 1, 0, 0)$	0.2146 (0.0922)			0.0056 (0.0027)			0
$NV(S^*, 1, 10, 10)$	0.2003 (0.1110)	0.0022 (0.0405)	-0.0007 (0.0351)	0.0010 (0.0144)	-0.0004 (0.0389)	0.0027 (0.0378)	3
$NV(2S^*, 1, 10, 10)$	0.2010 (0.0913)	0.0000 (0.0102)	0.0005 (0.0104)	0.0009 (0.0041)	0.0011 (0.0102)	0.0020 (0.0100)	0
$NV(3S^*, 1, 10, 10)$	0.1996 (0.0983)	0.0000 (0.0074)	0.0007 (0.0076)	0.0009 (0.0031)	0.0013 (0.0076)	0.0019 (0.0073)	0
$NV(S^*, 1, 20, 20)$	0.1965 (0.3727)	0.0067 (0.5665)	0.0039 (0.3407)	-0.0012 (0.3154)	-0.0035 (0.3410)	-0.0011 (0.3587)	212
$NV(2S^*, 1, 20, 20)$	0.2006 (0.0981)	-0.0003 (0.0147)	0.0004 (0.0135)	0.0008 (0.0045)	0.0016 (0.0117)	0.0021 (0.0126)	0
$NV(3S^*, 1, 20, 20)$	0.1989 (0.1040)	-0.0002 (0.0083)	0.0005 (0.0079)	0.0009 (0.0032)	0.0015 (0.0080)	0.0020 (0.0077)	0

Table 27 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	264.1079 (6.1632)	354.6410 (9.8324)						
$RV(300)$	8.4954 (1.6280)	11.0138 (2.0333)						
$RV(900)$	3.1111 (1.0104)	3.9046 (1.2522)						
$RV(1800)$	1.7794 (0.7682)	2.1434 (0.9651)						
$RV_1(5)$	164.5986 (7.4383)	208.3423 (10.0281)						
$RV_1(300)$	0.6369 (1.3168)	0.6572 (1.7584)						
$RV_1(900)$	0.6940 (0.8674)	0.6496 (1.0754)						
$RV_1(1800)$	0.7196 (0.8235)	0.5937 (0.8822)						
$RV(\delta^*)$	1.9176 (0.8335)	2.1943 (0.9999)	0.0515 (0.0015)			0.0685 (0.0021)		
$RV_1(\delta^*)$	0.6989 (0.7897)	0.6379 (0.9012)	0.0515 (0.0015)			0.0685 (0.0021)		
$RV_2(\delta^*)$	0.7000 (0.8332)	0.6396 (0.9186)	0.0515 (0.0015)			0.0685 (0.0021)		
$K^{TH2}(60)$	0.6141 (0.3351)	0.5133 (0.3711)	0.0515 (0.0015)			0.0685 (0.0021)		
$TSRV$	11.1492 (0.6328)	7.3889 (0.4578)	0.0515 (0.0015)			0.0685 (0.0021)		
$NV(S^*, 0)$	10.5990 (0.4172)	6.0135 (0.3495)	0.0310 (0.0007)			0.0532 (0.0013)		
$NV(2S^*, 0)$	4.8993 (0.2205)	2.8206 (0.1928)	0.0343 (0.0008)			0.0559 (0.0013)		
$NV(3S^*, 0)$	3.2367 (0.1746)	1.9130 (0.1655)	0.0358 (0.0008)			0.0571 (0.0014)		
$NV(S^*, 10)$	0.2162 (0.3495)	0.3326 (0.3146)	0.0517 (0.0015)	0.0458 (0.0019)	0.0393 (0.0059)	0.0684 (0.0019)	0.0610 (0.1090)	0.0793 (0.3196)
$NV(2S^*, 10)$	0.4736 (0.1894)	0.4525 (0.1787)	0.0513 (0.0014)	0.0456 (0.0018)	0.0387 (0.0059)	0.0682 (0.0018)	0.0632 (0.1062)	0.0722 (0.3026)
$NV(3S^*, 10)$	0.5320 (0.1645)	0.4804 (0.1642)	0.0512 (0.0014)	0.0455 (0.0018)	0.0385 (0.0059)	0.0681 (0.0018)	0.0651 (0.1039)	0.0648 (0.2889)
$NV(S^*, 20)$	0.5588 (0.3622)	0.5250 (0.3265)	0.0509 (0.0015)	0.0447 (0.0020)	0.0386 (0.0072)	0.0678 (0.0019)	0.0549 (0.1752)	0.0521 (0.5059)
$NV(2S^*, 20)$	0.5983 (0.1860)	0.5187 (0.1815)	0.0508 (0.0014)	0.0447 (0.0019)	0.0386 (0.0072)	0.0678 (0.0018)	0.0546 (0.1662)	0.0523 (0.4580)
$NV(3S^*, 20)$	0.6023 (0.1640)	0.5174 (0.1675)	0.0508 (0.0013)	0.0446 (0.0019)	0.0386 (0.0072)	0.0678 (0.0018)	0.0543 (0.1598)	0.0532 (0.4264)

Table 28: Monte Carlo simulation results for scenario: **high pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200	#NonPD
$RC(5)$	45.1665 (4.9685)						
$RC(300)$	2.6408 (1.3452)						
$RC(900)$	0.9925 (0.8947)						
$RC(1800)$	0.5426 (0.6439)						
$RC_{1,1}(5)$	70.1208 (5.7932)						
$RC_{1,1}(300)$	0.2836 (1.0886)						
$RC_{1,1}(900)$	0.2188 (0.7511)						
$RC_{1,1}(1800)$	0.2021 (0.6232)						
$RC(\delta^*)$	2.3163 (1.2800)			0.0035 (0.0004)			
$RC_{1,1}(\delta^*)$	0.1991 (1.0644)			0.0035 (0.0004)			
$RC_{2,2}(\delta^*)$	0.1749 (1.0137)			0.0035 (0.0004)			
HY	14.0557 (5.0376)						
$HY(S^*)$	7.0170 (0.7391)						
$NV(S^*, 1, 0, 0)$	2.0274 (0.2353)			0.0107 (0.0014)			0
$NV(2S^*, 1, 0, 0)$	0.9889 (0.1376)			0.0125 (0.0015)			0
$NV(3S^*, 1, 0, 0)$	0.6873 (0.1220)			0.0132 (0.0015)			0
$NV(S^*, 1, 10, 10)$	0.1958 (0.2207)	0.0128 (0.1125)	0.0188 (0.1106)	0.0096 (0.0029)	0.0042 (0.1122)	0.0113 (0.1108)	357
$NV(2S^*, 1, 10, 10)$	0.1998 (0.1338)	0.0058 (0.0589)	0.0116 (0.0566)	0.0097 (0.0020)	0.0113 (0.0580)	0.0177 (0.0568)	24
$NV(3S^*, 1, 10, 10)$	0.1998 (0.1247)	0.0029 (0.0380)	0.0084 (0.0357)	0.0098 (0.0017)	0.0142 (0.0371)	0.0204 (0.0360)	1
$NV(S^*, 1, 20, 20)$	0.1958 (0.2383)	-0.0027 (0.1320)	0.0037 (0.1318)	0.0098 (0.0028)	0.0191 (0.1333)	0.0254 (0.1313)	145
$NV(2S^*, 1, 20, 20)$	0.1982 (0.1375)	-0.0024 (0.0552)	0.0036 (0.0541)	0.0099 (0.0019)	0.0191 (0.0557)	0.0250 (0.0545)	2
$NV(3S^*, 1, 20, 20)$	0.1984 (0.1276)	-0.0024 (0.0338)	0.0034 (0.0326)	0.0099 (0.0016)	0.0191 (0.0341)	0.0250 (0.0331)	0

Table 28 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in Equation (8). S^* denotes the optimal number of subgrids as in Equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	218.2343 (6.7813)	219.8221 (8.8830)						
$RV(300)$	8.3591 (1.6902)	11.0751 (2.1377)						
$RV(900)$	3.1117 (1.0416)	3.9440 (1.2736)						
$RV(1800)$	1.7726 (0.7738)	2.1363 (0.9622)						
$RV_1(5)$	154.0234 (6.8669)	167.9505 (8.4132)						
$RV_1(300)$	0.7236 (1.3616)	0.7063 (1.7990)						
$RV_1(900)$	0.6774 (0.9295)	0.6295 (1.1050)						
$RV_1(1800)$	0.7107 (0.8271)	0.5909 (0.8972)						
$RV(\delta^*)$	1.9628 (0.8695)	2.2267 (0.9938)	0.0515 (0.0019)			0.0683 (0.0031)		
$RV_1(\delta^*)$	0.6877 (0.7939)	0.6238 (0.8865)	0.0515 (0.0019)			0.0683 (0.0031)		
$RV_2(\delta^*)$	0.6971 (0.8772)	0.6709 (0.9345)	0.0515 (0.0019)			0.0683 (0.0031)		
$K^{TH2}(60)$	0.6131 (0.3530)	0.5375 (0.3884)	0.0515 (0.0019)			0.0683 (0.0031)		
$TSRV$	5.9128 (0.4520)	3.0458 (0.3993)	0.0515 (0.0019)			0.0683 (0.0031)		
$NV(S^*, 0)$	5.1558 (0.3265)	2.6905 (0.3334)	0.0383 (0.0011)			0.0598 (0.0021)		
$NV(2S^*, 0)$	2.5181 (0.1900)	1.4112 (0.1996)	0.0407 (0.0012)			0.0615 (0.0021)		
$NV(3S^*, 0)$	1.7657 (0.1665)	1.0515 (0.1766)	0.0417 (0.0012)			0.0623 (0.0022)		
$NV(S^*, 10)$	0.5248 (0.3040)	0.4718 (0.3394)	0.0510 (0.0017)	0.0770 (0.1530)	0.0855 (0.7486)	0.0680 (0.0027)	-0.1058 (10.4179)	-0.5215 (20.5528)
$NV(2S^*, 10)$	0.5838 (0.1832)	0.5047 (0.2018)	0.0509 (0.0016)	0.0868 (0.1353)	0.0370 (0.6176)	0.0679 (0.0026)	0.0484 (10.2427)	-0.3311 (20.6180)
$NV(3S^*, 10)$	0.5956 (0.1691)	0.5082 (0.1824)	0.0508 (0.0016)	0.0911 (0.1271)	0.0107 (0.5663)	0.0679 (0.0025)	0.0822 (10.2418)	-0.2348 (20.7695)
$NV(S^*, 20)$	0.6190 (0.3172)	0.5120 (0.3589)	0.0507 (0.0017)	-0.0078 (0.4130)	0.2269 (1.6579)	0.0678 (0.0028)	2.8329 (57.3875)	2.2488 (44.8907)
$NV(2S^*, 20)$	0.6157 (0.1841)	0.5198 (0.2038)	0.0507 (0.0017)	-0.0059 (0.4053)	0.2247 (1.5501)	0.0678 (0.0026)	2.8958 (56.6189)	2.5592 (44.4617)
$NV(3S^*, 20)$	0.6132 (0.1707)	0.5163 (0.1846)	0.0507 (0.0017)	-0.0049 (0.4020)	0.2236 (1.5130)	0.0678 (0.0025)	2.8130 (56.3205)	2.4068 (44.4136)

Table 29: Monte Carlo simulation results for scenario: **high pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200	#NonPD
$RC(5)$	19.1536 (3.5524)						
$RC(300)$	1.8612 (1.3049)						
$RC(900)$	0.9934 (0.8642)						
$RC(1800)$	0.5783 (0.6603)						
$RC_{1,1}(5)$	39.0635 (4.8524)						
$RC_{1,1}(300)$	0.2536 (1.0863)						
$RC_{1,1}(900)$	0.1898 (0.7528)						
$RC_{1,1}(1800)$	0.2117 (0.6448)						
$RC(\delta^*)$	1.2553 (1.1503)			0.0061 (0.0010)			
$RC_{1,1}(\delta^*)$	0.1839 (0.9666)			0.0061 (0.0010)			
$RC_{2,2}(\delta^*)$	0.2349 (0.9483)			0.0061 (0.0010)			
HY	21.9648 (4.5405)						
$HY(S^*)$	4.9298 (0.6753)						
$NV(S^*, 1, 0, 0)$	0.8998 (0.2327)			0.0332 (0.0053)			0
$NV(2S^*, 1, 0, 0)$	0.5225 (0.1484)			0.0352 (0.0053)			0
$NV(3S^*, 1, 0, 0)$	0.4100 (0.1356)			0.0358 (0.0053)			0
$NV(S^*, 1, 10, 10)$	0.2161 (0.2284)	0.0054 (0.0383)	0.0084 (0.0385)	0.0098 (0.0080)	0.0117 (0.0333)	0.0167 (0.0407)	189
$NV(2S^*, 1, 10, 10)$	0.2091 (0.1499)	0.0044 (0.0233)	0.0082 (0.0245)	0.0097 (0.0054)	0.0120 (0.0209)	0.0175 (0.0248)	7
$NV(3S^*, 1, 10, 10)$	0.2044 (0.1396)	0.0042 (0.0178)	0.0080 (0.0189)	0.0097 (0.0045)	0.0123 (0.0162)	0.0178 (0.0192)	1
$NV(S^*, 1, 20, 20)$	0.2031 (0.2419)	0.0006 (0.0660)	0.0047 (0.0642)	0.0100 (0.0082)	0.0164 (0.0630)	0.0225 (0.0660)	154
$NV(2S^*, 1, 20, 20)$	0.2021 (0.1516)	-0.0012 (0.0413)	0.0034 (0.0412)	0.0100 (0.0054)	0.0179 (0.0390)	0.0240 (0.0414)	5
$NV(3S^*, 1, 20, 20)$	0.1995 (0.1419)	-0.0016 (0.0318)	0.0031 (0.0319)	0.0100 (0.0045)	0.0185 (0.0303)	0.0244 (0.0318)	0

Table 29 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

Model \ True	IV^1 0.6281	IV^2 0.5234	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	123.2782 (5.9607)	92.2541 (5.9119)						
$RV(300)$	8.3659 (1.6851)	11.1405 (2.2417)						
$RV(900)$	3.1517 (1.0463)	3.9793 (1.2832)						
$RV(1800)$	1.7719 (0.8028)	2.0867 (0.9352)						
$RV_1(5)$	105.1720 (5.8480)	82.6235 (5.6484)						
$RV_1(300)$	0.7185 (1.3707)	0.4990 (1.8897)						
$RV_1(900)$	0.6300 (0.9256)	0.5852 (1.1162)						
$RV_1(1800)$	0.6928 (0.7630)	0.6217 (0.8696)						
$RV(\delta^*)$	1.9397 (0.8273)	2.3378 (1.0796)	0.0510 (0.0028)			0.0592 (0.0043)		
$RV_1(\delta^*)$	0.7109 (0.7771)	0.6320 (0.9153)	0.0510 (0.0028)			0.0592 (0.0043)		
$RV_2(\delta^*)$	0.6903 (0.8169)	0.6675 (0.9262)	0.0510 (0.0028)			0.0592 (0.0043)		
$K^{TH2}(60)$	0.6144 (0.3715)	0.5148 (0.4203)	0.0510 (0.0028)			0.0592 (0.0043)		
$TSRV$	2.2660 (0.3707)	1.2154 (0.4725)	0.0510 (0.0028)			0.0592 (0.0043)		
$NV(S^*, 0)$	2.1537 (0.3212)	1.2557 (0.4520)	0.0444 (0.0019)			0.0640 (0.0038)		
$NV(2S^*, 0)$	1.2447 (0.2062)	0.8124 (0.2434)	0.0458 (0.0019)			0.0651 (0.0037)		
$NV(3S^*, 0)$	0.9897 (0.1894)	0.6864 (0.2088)	0.0464 (0.0019)			0.0655 (0.0036)		
$NV(S^*, 10)$	0.6104 (0.3203)	0.5408 (0.4869)	0.0507 (0.0024)	-0.0573 (0.8087)	-0.2540 (0.6325)	0.0678 (0.0043)	0.3818 (2.0496)	-0.0098 (1.8975)
$NV(2S^*, 10)$	0.6132 (0.2054)	0.5223 (0.2501)	0.0507 (0.0023)	-0.0241 (0.7943)	0.0274 (0.6869)	0.0679 (0.0040)	0.3461 (2.0940)	0.0101 (1.8482)
$NV(3S^*, 10)$	0.6119 (0.1931)	0.5134 (0.2177)	0.0507 (0.0023)	0.0020 (1.5451)	0.2317 (8.0262)	0.0679 (0.0039)	0.3193 (2.0642)	0.0242 (1.8400)
$NV(S^*, 20)$	0.6104 (0.3203)	0.5408 (0.4869)	0.0507 (0.0024)	-0.0573 (0.8087)	-0.2540 (0.6325)	0.0678 (0.0043)	0.3818 (2.0496)	-0.0098 (1.8975)
$NV(2S^*, 20)$	0.6132 (0.2054)	0.5223 (0.2501)	0.0507 (0.0023)	-0.0241 (0.7943)	0.0274 (0.6869)	0.0679 (0.0040)	0.3461 (2.0940)	0.0101 (1.8482)
$NV(3S^*, 20)$	0.6119 (0.1931)	0.5134 (0.2177)	0.0507 (0.0023)	0.0020 (1.5451)	0.2317 (8.0262)	0.0679 (0.0039)	0.3193 (2.0642)	0.0242 (1.8400)

Table 30: Monte Carlo simulation results for scenario: **high pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.2005	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200	#NonPD
$RC(5)$	2.6245 (1.7307)						
$RC(300)$	0.7847 (1.3532)						
$RC(900)$	0.4244 (0.8777)						
$RC(1800)$	0.1764 (0.6254)						
$RC_{1,1}(5)$	7.5185 (2.5605)						
$RC_{1,1}(300)$	0.2423 (1.1189)						
$RC_{1,1}(900)$	0.1619 (0.7308)						
$RC_{1,1}(1800)$	0.1919 (0.5774)						
$RC(\delta^*)$	0.9079 (1.3896)			0.0035 (0.0019)			
$RC_{1,1}(\delta^*)$	0.1884 (1.1785)			0.0035 (0.0019)			
$RC_{2,2}(\delta^*)$	0.2690 (1.2131)			0.0035 (0.0019)			
HY	6.9342 (2.8044)						
$HY(S^*)$	2.0279 (0.6265)						
$NV(S^*, 1, 0, 0)$	0.4339 (0.2851)			0.0559 (0.0211)			3
$NV(2S^*, 1, 0, 0)$	0.3370 (0.1774)			0.0553 (0.0197)			0
$NV(3S^*, 1, 0, 0)$	0.3082 (0.1591)			0.0535 (0.0188)			0
$NV(S^*, 1, 10, 10)$	0.2132 (0.3050)	-0.0001 (0.0788)	0.0044 (0.0782)	0.0090 (0.0325)	0.0121 (0.0804)	0.0207 (0.0696)	202
$NV(2S^*, 1, 10, 10)$	0.2053 (0.1843)	0.0004 (0.0397)	0.0052 (0.0421)	0.0096 (0.0187)	0.0141 (0.0449)	0.0213 (0.0402)	53
$NV(3S^*, 1, 10, 10)$	0.2015 (0.1673)	0.0013 (0.0308)	0.0057 (0.0337)	0.0099 (0.0143)	0.0150 (0.0344)	0.0221 (0.0312)	11
$NV(S^*, 1, 20, 20)$	0.2030 (0.3445)	0.0002 (0.1485)	0.0051 (0.1085)	0.0101 (0.0384)	0.0160 (0.1003)	0.0215 (0.0891)	240
$NV(2S^*, 1, 20, 20)$	0.1990 (0.1894)	-0.0017 (0.0414)	0.0028 (0.0415)	0.0100 (0.0186)	0.0171 (0.0452)	0.0227 (0.0417)	50
$NV(3S^*, 1, 20, 20)$	0.1968 (0.1704)	-0.0021 (0.0316)	0.0024 (0.0332)	0.0101 (0.0143)	0.0176 (0.0335)	0.0240 (0.0313)	10

Table 30 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9). #NonPD stands for the number of non-positive definite matrices out of the 1000 Monte Carlo replications.

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