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Return predictability and intertemporal asset allocation: Evidence from a bias-adjusted VAR model*

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Abstract

We extend the VAR based intertemporal asset allocation approach from Campbell et al. (2003) to the case where the VAR parameter estimates are adjusted for small-sample bias. We apply the analytical bias formula from Pope (1990) using both Campbell et al.'s dataset, and an extended dataset with quarterly data from 1952 to 2006. The results show that correcting the VAR parameters for small-sample bias has both quantitatively and qualitatively important effects on the strategic intertemporal part of optimal portfolio choice, especially for bonds: for intermediate values of risk-aversion, the intertemporal hedging demand for bonds - and thereby the total demand for bonds - is strongly reduced by the bias-adjustment. We also investigate the robustness of the results by changing the lag-length and one of the state variables of the VAR.

JEL Classification: C32, G11, G12
Keywords: Intertemporal portfolio choice, return predictability, VAR model, small-sample bias

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1 Introduction

One of the most fascinating results of recent research in empirical finance is that asset returns seem to contain predictable components. Until the first half of the 1980s, stock and bond returns were thought to be completely unpredictable, both at short and long horizons, and this unpredictability was taken to imply that asset markets were informationally efficient. However, since the mid 1980s researchers have become increasingly aware of the fact that stock returns are to some extent predictable from lagged valuation ratio's like the dividend yield or price-earnings ratio, and that bond returns are predictable from e.g. lagged yield spreads. The predictability is often found to be insignificant and hard to measure when returns are calculated over short horizons, but when the horizon increases the predictable component shows itself clearly. Thus, the small and insignificant predictability at short horizons build up to large and significant predictability at long horizons. Interestingly, it has also become clear from asset pricing theory that return predictability is not necessarily due to irrationality and market-inefficiency (bubbles, fads, noise traders, etc.), but could be the result of rationally changing risk-aversion and risk premia. Thus, predictable returns are in theory consistent with the efficient markets hypothesis. Cochrane (2005, ch. 20-21) surveys the by now very large literature on stock and bond return predictability, and he relates predictability to the concept of *mean reversion* and to modern consumption-based asset pricing models.[1]

One area where return predictability has profound implications is asset allocation. The old static Markowitz Mean-Variance (MV) model continues to dominate analyses of portfolio choice, especially among practitioners in the financial services industry. However, for long-term investors the static MV model will only be suitable under very strict assumptions, one of them being that investment opportunities are constant over time, meaning that returns are unpredictable. If this is not the case, long-term investors can benefit from the return predictability, both in the form of market-timing and in the form of intertemporal hedging of future return risk. Neither of these effects are captured by the static MV model.

Recent research on dynamic portfolio choice under return predictability has delivered solutions for optimal asset allocations using numerical methods based on discrete-state approximations (see e.g. Balduzzi and Lynch (1999), Barberis (2000), Brennan et al. (1997), and Lynch (2001)), and exact closed-form solutions have been obtained for simple models in a continuous-time setup (see e.g. Kim and Omberg (1996) and Wachter (2002)). However, until Campbell et al. (2003) it was not possible to analyze analytically optimal portfolio choice in a model with more than one risky asset and several predictor variables. Campbell et al. (2003) develop an approximate (based on Taylor

[1] The 'fact' that returns contain predictable components is not uncontroversial. Some have questioned the in-sample statistical significance of predictability (e.g. Boudoukh et al., 2006), and others have questioned whether in-sample predictability also holds out-of-sample (e.g. Goyal and Welch, 2005). Cochrane (2006) analyzes and discusses these objections and he concludes that predictability is present in-sample and that it is both statistically and economically significant, and he reconciles this with poor out-of-sample predictability. Other recent studies in this area are Amihud and Hurvich (2004), Lewellen (2004), Campbell and Yogo (2006), and Ang and Bekaert (2007).

expansions) analytical solution to the long-term investors portfolio choice in a setting where the investor maximizes expected discounted Epstein-Zin utility over an infinite horizon, and where the asset returns and predictor variables are modelled by a linear vector-autoregression.[2] Empirical analyses using Campbell et al.'s (2003) approach have been rather sparse. Campbell et al. themselves apply the methodology on US quarterly and annual stock and bond returns with the dividend-price ratio, interest rate, and yield spread as predictor variables. Rapach and Wohar (2007) use the approach on an international dataset, allowing investors to invest in both domestic and foreign assets. Both studies find evidence of substantial time-variation in optimal asset allocations as well as substantial intertemporal hedging effects coming from the predictability of asset returns.

A potentially important drawback of the empirical approach of Campbell et al. (2003) (and also Rapach and Wohar, 2007) is that the computed optimal allocations are based on standard least squares estimates of the VAR parameters. It is well-known that such estimates are plagued with finite-sample bias that may seriously distort inference based on the VAR model, especially when the model contains variables that are highly persistent, see e.g. Bekaert et al. (1997). This will indeed be the case in the present context where included variables such as interest rates, dividend-price ratio's and yield spreads typically are highly persistent. Campbell et al. (2003) acknowledge the finite-sample bias in their VAR estimates but state that bias corrections are complex in multivariate systems and, hence, they do not attempt to adjust for the bias. In the present paper we extend the Campbell et al. (2003) approach to be based on bias-corrected VAR parameters. We invoke the analytical bias formula from Pope (1990), which holds for general VAR models under quite mild restrictions, and with properties that are comparable to standard Monte Carlo or bootstrap bias-adjustment. Pope's adjustment is straightforward to implement but, surprisingly, it has been left unnoticed in most of the empirical finance literature using VAR models. In an empirical application we compute optimal asset allocations using both adjusted and unadjusted VAR estimates in order to see whether the bias-adjustment is qualitatively important in practice. We use both the original quarterly data from Campbell et al. (2003), which extends from 1952:q1 to 1999:q4, and an updated dataset that ends in 2006:q4. We also analyze the robustness of the results in two other directions: first, by changing the lag length of the VAR, and secondly by changing the definition of one of the state variables of the VAR: the short-term nominal interest rate. This variable is extremely persistent, and in VAR models with two lags the VAR parameter matrix contains unstable roots. We therefore use a stochastically detrended short-term nominal rate, as defined in Campbell (1991), which makes all the roots stable. In Campbell et al.'s (2003) analysis only one-lag models are estimated, and only with the short-term interest rate without detrending.

The main result of our analysis is that bias-correcting the VAR parameters has a quantitatively and qualitatively important effect on optimal asset allocations, in particular for bonds: for intermediate values of risk-aversion, the intertemporal hedging demand

[2] Campbell et al. (2003) is a multivariate extension of the approximate analytical solution by Campbell and Viceira (1999), that allows for only one risky asset whose expected excess return is governed by a single state variable.

for bonds - and thereby the total demand for bonds - is strongly reduced by the bias-adjustment. On the other hand, replacing the nominal interest rate in levels with its detrended version implies a much higher demand for bonds. The most important effect of extending the sample period to 2006 is to decrease the demand for stocks and increase the demand for bonds. Changing the lag-length of the VAR has only minor effects on the results.

The rest of the paper is organized as follows. Section 2 describes the asset allocation model and how the optimal allocations can be computed from a VAR model. Section 3 explains the bias-adjustment procedure. Section 4 reports the empirical results and, finally, section 5 contains some concluding remarks.

2 The asset allocation model

The investor is assumed to set optimal consumption and portfolio plans so as to maximize - over an infinite horizon - an Epstein and Zin (1989, 1991) utility function defined recursively by

$$U_t(C_t, E_t(U_{t+1})) = [(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}},$$

where C_t is consumption at time t, δ is the time discount factor, γ is the coefficient of relative risk aversion, and $\theta \equiv (1-\gamma)/(1-\frac{1}{\psi})$ where ψ is the elasticity of intertemporal substitution. When $\psi = \gamma^{-1}$ the utility function reduces to standard time-separable power utility (CRRA utility), and if in addition $\gamma = 1$ we have log utility. The optimization is done subject to the budget constraint $W_{t+1} = (W_t - C_t)R_{p,t+1}$, where W_{t+1} and $R_{p,t+1}$ are wealth and gross portfolio return, respectively. With n assets, the portfolio return is equal to $R_{p,t+1} = \sum_{i=2}^{n} \alpha_{i,t}(R_{i,t+1} - R_{1,t+1}) + R_{1,t+1}$, where $\alpha_{i,t}$ is the portfolio weight on asset i at time t. $R_{1,t}$ denotes the benchmark return (typically a short-term - but not necessarily riskfree - return).

The above maximization problem leads to an Euler equation for asset i, from which a second-order Taylor expansion gives an approximate log-linear Euler equation in terms of log consumption, $c_t \equiv \log C_t$, and log returns, $r_{i,t} \equiv \log R_{i,t}$, see Campbell et al. (2003). Using also a log-linear approximation of the budget constraint $W_{t+1} = (W_t - C_t)R_{p,t+1}$, and stated in terms of log excess return on asset i, $r_{i,t+1} - r_{1,t+1}$, the log-linear Euler equation becomes

$$E_t(r_{i,t+1}-r_{1,t+1})+\frac{1}{2}Var_t(r_{i,t+1}-r_{1,t+1}) = \frac{\theta}{\psi}(\sigma_{i,c-w,t}-\sigma_{1,c-w,t})+\gamma(\sigma_{i,p,t}-\sigma_{1,p,t})-(\sigma_{i,1,t}-\sigma_{1,1,t}), \quad (1)$$

where $\sigma_{i,c-w,t} = Cov_t(r_{i,t+1}, c_{t+1} - w_{t+1})$, $\sigma_{1,c-w,t} = Cov_t(r_{1,t+1}, c_{t+1} - w_{t+1})$, $\sigma_{i,p,t} = Cov_t(r_{i,t+1}, r_{p,t+1})$, $\sigma_{1,p,t} = Cov_t(r_{1,t+1}, r_{p,t+1})$, $\sigma_{i,1,t} = Cov_t(r_{i,t+1}, r_{1,t+1})$, and $\sigma_{1,1,t} =$

$Var_t(r_{1,t+1})$.

The optimal portfolio and consumption rules must satisfy (1). In order to estimate the conditional moments in (1), we set up a VAR model for the $n-1$ log excess returns, $\mathbf{x}_{t+1} = [(r_{2,t+1}-r_{1,t+1}), ..., (r_{n,t+1}-r_{1,t+1})]'$. We also include the benchmark return, $r_{1,t+1}$, and a number of additional state variables that - in previous studies - have been found to contain significant information about future returns. Denote by $\mathbf{s}_{t+1}$ the vector of these additional state variables. Then the vector $\mathbf{z}_{t+1} = [\mathbf{x}_{t+1},\ r_{1,t+1},\ \mathbf{s}_{t+1}]'$ contains all the variables and a first-order VAR model for $\mathbf{z}_{t+1}$ becomes[3]

$$\mathbf{z}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{z}_t + \mathbf{v}_{t+1}, \tag{2}$$

where $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ are the vector of intercepts and matrix of slope coefficients, respectively. $\mathbf{v}_{t+1}$ is the vector of VAR errors that are assumed to be distributed as $\mathbf{v}_{t+1} \sim niid(\mathbf{0}, \mathbf{\Sigma}_{\text{v}})$, where the error covariance matrix is

$$\mathbf{\Sigma}_{\text{v}} = \begin{bmatrix} \sigma_1^2 & \sigma_{1x}^{'} & \sigma_{1s}^{'} \\ \sigma_{1x} & \mathbf{\Sigma}_{xx} & \mathbf{\Sigma}_{xs}^{'} \\ \sigma_{1s} & \mathbf{\Sigma}_{xs} & \mathbf{\Sigma}_{ss} \end{bmatrix}$$

Based on this formulation of the dynamics of the state variables, the left-hand side of (1) can be written as

$$E_t(\mathbf{x}_{t+1}) + \frac{1}{2} Var_t(\mathbf{x}_{t+1}) = \mathbf{H}_x \mathbf{\Phi}_0 + \mathbf{H}_x \mathbf{\Phi}_1 \mathbf{z}_t + \frac{1}{2}\sigma_x^2, \tag{3}$$

where $\mathbf{H}_x$ is a selection matrix that picks out the excess return vector $\mathbf{x}_t$ from the state vector $\mathbf{z}_t$, and σ_x^2 is the vector of diagonal elements in $\mathbf{\Sigma}_{xx}$. Regarding the conditional covariances on the right-hand side of (1), Campbell et al. (2002, 2003) show, using a log-linear approximation of the portfolio return, $R_{p,t+1}$, that they can all be written as linear functions of the state variables in the following way

$$\begin{aligned} \sigma_{c-w,t} - \sigma_{1,c-w,t}\iota &\equiv [\sigma_{i,c-w,t} - \sigma_{1,c-w,t}]_{i=2,\dots,n} = \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 \mathbf{z}_t, \\ \sigma_{p,t} - \sigma_{1,p,t}\iota &\equiv [\sigma_{i,p,t} - \sigma_{1,p,t}]_{i=2,\dots,n} = \mathbf{\Sigma}_{xx}\alpha_t + \sigma_{1x}, \\ \sigma_{1,t} - \sigma_{1,1,t}\iota &\equiv [\sigma_{i,1,t} - \sigma_{1,1,t}]_{i=2,\dots,n} = \sigma_{1x}, \end{aligned}$$

where ι is a vector of ones, and $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$ are matrices that are defined below.[4] Combining these results, the model can be solved for the log consumption-wealth ratio, $c_t - w_t$, and the optimal asset allocations, α_t. The solution turns out to be

[3] Higher-order VAR models can be stated in the first-order form (2) by using the companion form.

[4] The log-linear approximations of the Euler equation and the budget constraint are exact when $\psi = 1$. The log-linear approximation of the portfolio return is only exact in continuous time, but is highly accurate for short time intervals, c.f. Campbell et al. (2003). In the empirical application we set $\psi = 1$ and hence, the only approximation error in the reported results stems from the log-linear

$$\alpha_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{z}_t, \tag{4}$$

$$c_t - w_t = b_0 + \mathbf{B}_1' \mathbf{z}_t + \mathbf{z}_t' \mathbf{B}_2 \mathbf{z}_t. \tag{5}$$

The parameter b_0 and the parameter matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are complicated non-linear functions of the underlying utility and VAR parameters. As seen, the optimal portfolio rule is linear in the VAR state vector, $\mathbf{z}_t$, while the optimal consumption rule is quadratic in $\mathbf{z}_t$. The precise expressions for $\mathbf{A}_0$ and $\mathbf{A}_1$ in the portfolio rule (4) are

$$\mathbf{A}_0 = \frac{1}{\gamma} \mathbf{\Sigma}_{xx}^{-1} \left(\mathbf{H}_x \mathbf{\Phi}_0 + \frac{1}{2} \sigma_x^2 + (1-\gamma) \sigma_{1x} \right) + \left(1 - \frac{1}{\gamma} \right) \mathbf{\Sigma}_{xx}^{-1} \left(\frac{-\mathbf{\Lambda}_0}{1-\psi} \right), \tag{6}$$

$$\mathbf{A}_1 = \frac{1}{\gamma} \mathbf{\Sigma}_{xx}^{-1} \mathbf{H}_x \mathbf{\Phi}_1 + \left(1 - \frac{1}{\gamma} \right) \mathbf{\Sigma}_{xx}^{-1} \left(\frac{-\mathbf{\Lambda}_1}{1-\psi} \right), \tag{7}$$

where $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$ are matrices that depend on all the utility and VAR parameters as well as b_0, $\mathbf{B}_1$, and $\mathbf{B}_2$, and a parameter $\rho \equiv 1 - \exp(E(c_t - w_t))$ which comes from the log-linear approximation to the budget constraint.[5] The exact expressions for $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$ are given in Campbell et al. (2002).

The economic interpretation of $\mathbf{A}_0$ and $\mathbf{A}_1$ is that the first term in the expressions (6) and (7) measures the myopic component of asset demand, while the second term measures the intertemporal hedging demand. The latter component captures the effect of predictable asset returns which induces a *strategic* motive to hedge future return risk. A simple example can illustrate the intuition: consider the case with only one risky asset whose expected excess return is governed by a single state variable (this is the case considered by Campbell and Viceira, 1999). In this case the second terms in (6) and (7) become negatively related to the covariance between innovations in excess returns and innovations in the state variable. If this covariance is negative, excess returns show mean reversion (i.e. negative autocorrelation) and the demand for the risky asset will be higher than if the covariance is zero or positive. The explanation is that if the risky asset shows mean reversion, then the asset can be used to hedge its own future risk. The overall hedging motive comes from the desire of the long-term risk-averse investor to save (invest in financial assets) with the purpose of consuming at a later date and at the same time smooth consumption over time. Assets that exhibit mean reversion serve this purpose. In general, with multiple risky assets and state variables, the sign and magnitude of the intertemporal hedging component will depend on the VAR parameters

approximation of the portfolio return. In the univariate setup, Campbell et. al (2001) investigate the accuracy of the approximate analytical solution by Campbell and Viceria (1999) using a numerical approach. They find that the two solution methods give very similar results.

[5] Giovannini and Weil (1989) show that with $\psi = 1$, the investor optimally chooses a constant consumption-wealth ratio equal to $1 - \delta$. Hence in this case, $\rho = \delta$.

and the correlations of the VAR innovations, as well as on the relative risk-aversion parameter γ.[6]

The standard static portfolio rule from mean-variance analysis occurs as the special case where there is no predictability in the VAR model, i.e. $\mathbf{\Phi}_1 = \mathbf{0}$ whereby $\mathbf{\Lambda}_0 = \mathbf{\Lambda}_1 = \mathbf{A}_1 = \mathbf{0}$, and $\alpha_t = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}\left(\mathbf{H}_x\mathbf{\Phi}_0 + \frac{1}{2}\sigma_x^2 + (1-\gamma)\sigma_{1x}\right)$. Asset demand depends on the excess returns scaled by the inverse of the covariance matrix of excess returns and the reciprocal of the relative risk aversion parameter. Another special case is $\gamma = 1$ whereby the intertemporal hedging components disappear and asset demand becomes purely myopic. Note that in this special case α_t still depends on $\mathbf{z}_t$ through $\mathbf{A}_1 = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}\mathbf{H}_x\mathbf{\Phi}_1$, which is $\mathbf{0}$ only if $\mathbf{\Phi}_1 = \mathbf{0}$. Thus, with predictable returns the myopic part of asset demand contains a time-varying component.

In general, return predictability induces a *tactical* (timing) motive - in addition to the strategic motive - in portfolio allocation: the investor should change allocation over time depending on the signal that the state vector $\mathbf{z}_t$ sends about future returns. Predictable returns thus leads to both a strategic hedging motive that affects the overall level of asset demand, and a tactical timing motive that changes the optimal allocation over time. These two motives may work in opposite directions such that, for instance, at a particular time a positive intertemporal hedging component - that otherwise would lead to a large demand for a given asset -, is dominated by the state variables signaling poor future returns for the asset, such that the combined effect is a small demand for the asset. The values for the utility parameters and the VAR parameter estimates determine the relative importance of each of these motives. The higher the value of risk-aversion, γ, the more important becomes the strategic motive relative to the tactical motive, all else equal.

Before turning to the econometric and empirical part of the paper, it should be emphasized that the Campbell et al. (2003) approach - like most approaches in this area - are partial equilibrium in nature. (4) and (5) give the optimal consumption and asset allocation for an investor with Epstein-Zin utility and specific utility parameter values *and* who takes the return process, given by the estimated VAR model, as exogenously given. There is nothing in the model that makes this particular return process consistent with *general* equilibrium. As noted by Cochrane (1999), in a general equilibrium model the average investor will always hold the market portfolio and not be engaged in strategic or tactical asset allocation. Thus, the Campbell et al. (2003) model gives the optimal allocation for an investor that somehow deviates from the average investor, for example because of higher or lower risk-aversion than the average investor or higher or lower return covariance with consumption than the average investor.

[6] As seen from (6) and (7), the sign of the intertemporal hedging components shifts between $\gamma > 1$ and $\gamma < 1$. This is due to the well-known fact that a change in expected return has both an income effect and a substitution effect on consumption and asset demands, and these two effects work in opposite directions. When $\gamma > 1$ the substitution effect dominates the income effect, and vice versa when $\gamma < 1$. When $\gamma = 1$ the two effects exactly cancel each other leaving consumption and asset demand unchanged. In the dynamic asset allocation literature it is standard to assume that $\gamma \geq 1$.

3 Bias-adjustment of the VAR parameters

It is well-known that standard least-squares estimates of VAR parameters are biased in finite samples, and that inference based on estimated VAR models may be severely distorted by such biases, especially when (some of) the variables are highly persistent, see e.g. Beakert et al. (1997). However, existing studies using the Campbell et al. (2003) VAR based approach to dynamic asset allocation have not attempted to correct for these small-sample biases. In the present paper we extend the existing literaure by invoking the analytical bias formula derived by Pope (1990) for general VAR models. Surprisingly, Pope's formula has remained relatively unnoticed in the financial econometrics literature despite its easy implementation and quite appealing properties.[7] Usually, in empirical finance, bias-adjustment is done using Monte Carlo or bootstrap procedures. In order to investigate whether bias-adjustment is qualitatively important in practice, in the subsequent empirical section we report results based on both adjusted and unadjusted VAR parameter estimates.

Pope's (1990) analytical bias formula is derived from a higher-order Taylor expansion, and based on the VAR model (2) the bias, $\mathbf{B}_T$, of the OLS estimate of $\mathbf{\Phi}_1$ equals

$$\mathbf{B}_T = -\frac{\mathbf{b}}{T} + O(T^{-3/2})$$

where T is the sample size and

$$\mathbf{b} = \mathbf{G}[(\mathbf{I} - \mathbf{\Phi}_1')^{-1} + \mathbf{\Phi}_1'(\mathbf{I} - (\mathbf{\Phi}_1')^2)^{-1} + \sum \lambda(\mathbf{I} - \lambda\mathbf{\Phi}_1')^{-1}]\mathbf{\Gamma}(0)^{-1},$$

$\mathbf{G}$ is the conditional covariance matrix of $\mathbf{v}_t$, $\mathbf{\Gamma}(j) = E(\mathbf{z}_t\mathbf{z}_{t+j}')$, and the sum is over the eigenvalues λ of $\mathbf{\Phi}_1$. As seen, the approximation error in the bias formula vanishes at the rate $T^{-3/2}$ which is at least as fast as in standard Monte Carlo or bootstrap bias-adjustment. The underlying assumptions are quite mild (see Pope (1990) for details). Among the assumptions are that the VAR system is stationary such that $\mathbf{\Phi}_1$ does not contain unit or explosive roots, and that the VAR innovations $\mathbf{v}_t$ constitute a martingale difference sequence with constant covariance matrix $\mathbf{G}$. Note that we do not have to assume that the innovations are Gaussian.

In the VAR model (2) there is a vector of constant terms, $\mathbf{\Phi}_0$. Pope's bias formula is for a VAR model with 'mean-corrected' variables, i.e. the constant term is zero. However, he notes that this involves no loss of generality since "the estimators ... are invariant under translation of the sample by a constant" (Pope, 1990, p.252). We know that the unconditional sample arithmetic average of a stationary variable is an unbiased estimate

[7] Nicholls and Pope (1988) derive an expression for the least squares bias in Gaussian VAR models. Pope (1990) extends these results to a general VAR model without the restriction of Gaussian innovations. Amihud and Hurvich (2004) apply the Nicholls and Pope (1988) bias adjustment to develop a bias-adjusted predictive return regression with multiple predictors. Engsted and Tanggaard (2004, 2007) use Pope's (1990) bias-adjustment in VAR based variance decompositions for asset returns.

of its true mean, and that standard OLS fits exactly the mean of the variables in the VAR excluding the first observation. Thus, by fitting the VAR under the restriction that the unconditional means of the variables implied by the VAR coefficient estimates are equal to their full-sample arithmetic counterparts, and by bias-correcting $\widehat{\mathbf{\Phi}}_1$, we obtain unbiased estimates of the constant terms in $\mathbf{\Phi}_0$. Campbell et al. (2003) also fit their VAR models under the restriction that the unconditional means of the variables implied by the VAR estimates equal their full-sample arithmetic counterparts. Hence, they obtain unbiased estimates of the VAR implied means, but since their $\mathbf{\Phi}_1$ estimate is biased, so is their estimate of $\mathbf{\Phi}_0$.

In the empirical analysis we restrict the means in this way to obtain unbiased estimates of both $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$. Hence, first we estimate $\mathbf{\Phi}_1$ using standard OLS to obtain the least-squares estimate $\widehat{\mathbf{\Phi}}_1$, and then we bias-adjust this estimate using Pope's formula which yields a bias-corrected coefficient matrix, $\widetilde{\mathbf{\Phi}}_1$. Next, we fit the VAR to give an unbiased estimate of $\mathbf{\Phi}_0$: $\widetilde{\mathbf{\Phi}}_0 = \widehat{\mu}\left(\mathbf{I} - \widetilde{\mathbf{\Phi}}_1\right)$, where $\widehat{\mu}$ is the full-sample arithmetic mean of the state vector. We also use this approach in the part of the empirical application where we do not adjust for bias. This has two implications for the asset allocation. First, when there is no predictability in the VAR model, i.e. $\mathbf{\Phi}_1 = \mathbf{0}$, the optimal asset allocation will be identical whether we use the bias-adjusted or the unadjusted estimates: $\widehat{\alpha}_t = \widetilde{\alpha}_t = \frac{1}{\gamma}\Sigma_{xx}^{-1}\left[\mathbf{H}_x\widetilde{\mathbf{\Phi}}_0 + \frac{1}{2}\sigma_x^2 + (1-\gamma)\,\sigma_{1x}\right] = \frac{1}{\gamma}\Sigma_{xx}^{-1}\left[\mathbf{H}_x\widehat{\mu} + \frac{1}{2}\sigma_x^2 + (1-\gamma)\,\sigma_{1x}\right]$. Second, when $\gamma = 1$, the *average* demand will be identical in the two cases: $\overline{\widehat{\alpha}} = \overline{\widetilde{\alpha}} = \Sigma_{xx}^{-1}\left[\mathbf{H}_x\widetilde{\mathbf{\Phi}}_0 + \mathbf{H}_x\widetilde{\mathbf{\Phi}}_1\widehat{\mu} + \frac{1}{2}\sigma_x^2\right] = \Sigma_{xx}^{-1}\left[\mathbf{H}_x\widehat{\mu} + \frac{1}{2}\sigma_x^2\right]$.[8]

4 Empirical results

We begin the empirical analysis by replicating Campbell et al.'s (2003) results using the same VAR models and a quarterly dataset and sample period similar to theirs, i.e. a sample that ends in 1999:q4. Subsequently we report results for an extended sample period that ends in 2006:q4, and for different VAR models. But first we briefly describe the data.[9]

4.1 Data

In the VAR models we use three asset returns (real short-term bond returns, excess stock returns, and excess long-term bond returns) and three forecasting variables (the dividend-price ratio, the short-term nominal interest rate, and the yield spread). The data are from

[8] This explains why there is no difference between the adjusted and unadjusted estimates in the column "*Constant*" in Table 3 in section 4, and why there is no difference between the two cases when $\gamma = 1$. In general, this approach implies that bias-adjusting the VAR estimates has no effect on the *average* myopic demand.

[9] The program used in the empirical application is based on the MATLAB codes used by Campbell et al. (2003) and made available on John Y. Campbell's website.

the Center for Research in Security Prices (CRSP), begin in 1952:q1 and end in 2006:q4. For the restricted sample period, 1952:q1 - 1999:q4, the data are essentially identical to the data used by Campbell et al. (2003), see the description in their section 4.1.[10] The short-term bond return is measured in real terms as the log gross T-bill return minus log gross inflation. Stock returns are measured by the return on the NYSE, NASDAQ, and AMEX markets, and long-term bond returns are measured by the 5-year Treasury bond return. In the analysis stock and bond returns are measured as excess log returns, i.e. the log gross return minus the log gross T-bill return. The short-term nominal interest rate is given as the 90-day T-bill yield, and the log dividend-price ratio is computed as the log to the sum of dividend payments over the past year minus the log stock price. Finally, the yield spread is the difference between the 5-year bond yield and the 90-day T-bill rate. Table 1 gives summary statistics for the data. We will refer to some of the numbers in this table in subsequent subsections when interpreting the optimal asset allocation results.

4.2 VAR estimation

Table 2 shows VAR parameter estimates for the Campbell et al. (CCV - Campbell, Chen & Viceira) period from a one-lag model, VAR(1), - both the standard least squares estimates (with Newey-West corrected t-statistics in parenthesis) and the bias-adjusted estimates (in bold) using Pope's (1990) correction, as described in section 3. The bottom part of the table reports VAR innovation correlations above the main diagonal, and standard deviations multiplied by 100 on the main diagonal. The $\mathbf{x}_{t+1}$ vector contains three asset returns: excess stock returns (xr_{t+1}), excess long-term bond returns (xb_{t+1}), and the real 90-day T-bill rate (rtb_{t+1}). The vector of additional state variables, $\mathbf{s}_{t+1}$, contains the nominal 90-day T-bill yield (y_{t+1}), the dividend-price ratio ($d_{t+1} - p_{t+1}$), and the long-short yield spread (spr_{t+1}).

Table 2 reveals several interesting points. First, some of the least squares parameter estimates seem to be severely biased, e.g. the spr_t coefficient in the xr_{t+1} equation where the bias-adjustment changes the coefficient from 0.474 to -0.160. Second, the least squares estimates of the first-order autocorrelation coefficients are in general downward biased, as expected, but the multivariate bias-adjustment in many cases leads to quite different parameter values compared to the standard univariate bias-correction from Kendall (1954), which is the most often used approach.[11] For example, Pope's bias for the y_t coefficient

[10] There is a slight difference between our data sample that ends in 1999:q4 and Campbell et al.'s (2003) sample; their data begin in 1952:q2 because they want to have identical sample periods for the analysis with nominal bonds (their section 4) and real bonds (their section 5), and they loose one observation at the beginnning of the sample to compute returns on the real bond. We do not consider such inflation protected bonds in our analysis (our sample period corresponds exactly to the sample period used by Campbell and Viceira (2002, ch.4)). There are some additional minor differences between our data and Campbell et al.'s data concerning the data on stock returns and the dividend-price ratio. These differences have only a very small effect on the results, which can be seen by comparing our unadjusted VAR results in the subsequent tables with the results reported by Campbell et al.

[11] The Kendall bias for the first-order autocorrelation, ϕ_1, is $\frac{-(1+3\phi_1)}{T}$.

in the y_{t+1} equation is -0.036 (=0.955-0.991), while Kendall's bias for a first-order autocorrelation estimate of 0.955 with T=188 (the sample size from 1953:q1 to 1999:q4)[12] is only -0.021. For a process so close to a unit root this is a large difference. Another example that goes in the opposite direction is the first-order autocorrelation of the log dividend-price ratio. Here the least squares estimate of 0.965 is only slightly downward biased according to Pope's formula (bias-adjusted estimate = 0.967). Kendall's formula gives a bias-corrected estimate of 0.986. Third, the *upward* bias of the spr_t coefficient in the xb_{t+1} equation, together with the positive innovation correlation of 0.199, modifies the conclusion reached by Campbell et al. (2003, p.59) regarding bond return predictability. Campbell et al. argue that the least squares estimate of the spr_t coefficient in the xb_{t+1} equation is downward biased due to the positive innovation correlation. The argument builds on Stambaugh's (1999) observation that the small-sample bias of such a coefficient has the opposite sign to the sign of the innovation correlation. However, Stambaugh's observation is for a model where there is only *one* predictor variable and where this variable follows a *univariate* AR(1) process. The results in Table 2 show that in a multivariate context the relation between innovation correlations and the sign of small-sample bias of VAR parameter estimates is more complex than anticipated by Campbell et al.

4.3 Optimal portfolio weights

Next we investigate the effects of bias-adjusting the VAR parameters on the optimal portfolio weights. Table 3 shows the average demands for stocks, bonds and T-bills ("Cash") over the period 1952:q1 - 1999:q4, computed from the formulas (4) to (7). We pick the same preference parameters as in Campbell et al. (2003): $\psi = 1$, $\delta = 0.92^{1/4}$, and $\gamma = 1$, 2, 5, or 20.[13] The final column "spr_t" in Table 3 gives the mean asset demands based on the full VAR(1) system from Table 2. The column "*Constant*" reports mean demands from a system with only a constant term in each regression, i.e. no predictability in any of the variables. Thus, in this case there is by construction no intertemporal hedge effects. The column "AR_t" gives mean demands from a system that contains a constant and the three asset returns, rtb, xr, and xb. Then we add sequentially the additional state variables, y, $d-p$, and spr, to the system until we get the full VAR system in the final column. In what follows we will mostly comment on the full VAR results, but we also briefly comment on the results from the smaller systems when discussing which variables are responsible for the intertemporal hedging demands. Numbers in bold are mean demands based on the bias-adjusted VAR estimates, while the numbers not in bold are based on the unadjusted least squares estimates. In the table we report total demand and the intertemporal hedging demand for each asset; the myopic demand component then follows by subtracting hedging demand from total demand. Figure 1 plots the hedging components of asset allocations, using the full VAR system, for a continuum of values of risk-aversion, γ, from 1 to ∞.

[12] The estimation period begins in 1953:q1 since we use four observations to construct $d_t - p_t$.

[13] Campbell et al. (2003) note that using $\psi = 0.5$ yields very similar results as when $\psi = 1$. However, Rapach and Wohar (2007) show using monthly data, that changing ψ can have a sizable effect on the optimal hedging demand.

For a logarithmic investor, $\gamma = 1$, there is no intertemporal heging demand. Total demand is purely myopic. Table 3 shows that in this case there is a large positive demand for both stocks and long-term bonds - especially stocks - while the investor is short in cash. When risk-aversion increases total demand for stocks decreases, as expected. The hedging demand for stocks first increase with γ, and then decreases for sufficiently high γ values. It reaches its maximum at $\gamma \approx 3$, see Figure 1. As explained in Campbell et al. (2003), the positive hedging demand for stocks is mainly the result of predictable stock returns from the log dividend-price ratio together with the strongly negative correlation between stock return innovations and innovations in the log dividend-price ratio.[14] For bonds the intertemporal hedging component is negative, and most strongly so for $\gamma \approx 4$ based on the unadjusted VAR estimates, see Figure 1. This negative hedging demand for bonds comes mainly from the predictability of bond returns by the yield spread together with a positive correlation between bond return innovations and yield spread innovations, leading to 'mean-aversion' in bond returns.

As seen from Table 3 and Figure 1, adjusting the VAR parameters for small-sample bias does not markedly change the magnitudes and patterns of optimal stock demand. Bias-adjustment induces a smaller intertemporal hedging component, but the effect is not large and is mainly due to the slightly smaller value of the $d_t - p_t$ coefficient in the xr_{t+1} equation in the bias-adjusted system. For bonds, on the other hand, the effect of bias-adjustment is more pronounced. For intermediate values of γ the intertemporal hedging demand for bonds is negative, and bias-adjusting the VAR parameters magnifies this strongly. For example, for $\gamma = 2$ and based on the unadjusted estimates, total demand for bonds is close to 0 (-6.70%) as a result of a negative hedging component (-87.23%) and a positive myopic component (80.53%) that almost cancels each other. However, based on the bias-adjusted estimates, the total allocation to bonds is strongly negative (-117.67%) due to the strongly negative hedging demand of -198.20%. Several forces contribute to the explanation of this effect: bond return innovations are negatively correlated with innovations in the nominal interest rate and the dividend-price ratio, and bias-correction reduces the values of the parameters to y_t and $d_t - p_t$ in the xb_{t+1} equation. Thus, the 'mean-reversion' effect in bond returns, stemming from the interest rate and dividend-price ratio, is reduced. In fact, with respect to y_t we now have a 'mean-aversion' effect on bonds in the bias-adjusted system. Similarly, the 'mean-aversion' effect on bonds, stemming from the real interest rate, is magnified from the bias-adjustment: bond innovations are positively correlated with innovations to the real interest rate, and the rtb_t coefficient in the xb_{t+1} equation increases by the bias-adjustment. The effect from the yield spread pulls in the opposite direction: bond return and yield spread innovations are positively correlated, and the spr_t coefficient in the xb_{t+1} equation is reduced by the bias-adjustment. This induces a smaller 'mean-aversion' effect on bonds. Similarly, the 'mean-reversion' effect on bonds from stocks is slightly magnified by the bias-adjustment. However, apparently these opposite effects are not sufficiently strong to outweight the stronger 'mean-aversion' effects from the real interest rate and the smaller

[14] The intuition is as follows: A negative stock return innovation corresponds to a positive dividend-price innovation, which - through the positive $d-p$ coefficient in the xr equation - leads to higher future stock returns. Thus, the effects from $d-p$ induce 'mean-reversion' in stock returns.

'mean-reversion' effects from the nominal interest rate and the dividend-price ratio. The end result is a markedly stronger negative hedging demand for bonds following the bias-adjustment.

The results in Table 3 and Figure 1 show that adjusting the VAR parameters for small-sample bias does change the optimal asset allocation for intermediate values of the relative risk aversion parameter. In order to asses whether the change has any utility effects, we have calculated the mean value function for these values of γ, using both unadjusted and bias-adjusted VAR estimates, see Table 4. We follow Campbell et al. (2002, 2003) in calculating the mean value function ($E(V_t)$ where $V_t \equiv U_t/W_t$), with one difference. Campbell et al. use the same VAR but change the asset menu in their analysis of the utility effects; we use the same asset menu but two different VAR systems: one that is affected with small-sample bias and one that is adjusted for bias. As noted by Campbell et al., the value function is normalized, meaning in our case that a doubling in $E(V_t)$ from one VAR system to another implies that an investor who bases his asset allocation on the VAR system with the lower $E(V_t)$ requires a doubling of wealth to obtain the same utility as the investor who bases his asset allocation on the system with the higher $E(V_t)$. Table 4 clearly shows that adjusting for small-sample bias has utility effects; for $\gamma = 2$ the mean value function is 0.580 when using the unadjusted VAR estimates and 0.352 when adjusting for bias. This implies that an investor who bases his asset allocation on the bias-adjusted estimates needs an increase in wealth of 64.8% to obtain the same utility as the investor who uses the unadjusted estimates.[15] Note that we should not necessarily expect the bias-adjusted system to produce the highest utility. The point here is that adjusting for small-sample bias leads to quantitatively and qualitatively important *changes* in utility for the investor.

4.4 Robustness analysis

We now do some robustness checks on the above findings. First, we replace the nominal interest rate in levels, y_t, with its 'stochastically detrended' version which is more likely to be stationary. Second, we increase the lag-length of the VAR to two. Finally, we extend the sample period to include the most recent data up to 2006.

As seen in Table 2, the least squares autoregressive coefficient in the y_{t+1} equation is very close to unity (0.955), and after bias-adjustment the coefficient becomes extremely close to unity (0.991). This implies near-nonstationarity of the nominal interest rate. Beginning with Campbell (1991) the standard approach to transforming the interest rate into stationarity is to stochastically detrend it by subtracting its one-year backward moving average.[16] In Table 5 we report results for mean asset allocations based on a VAR

[15]Comparing our results based on unadjusted VAR estimates to the results by Campbell et al. (2003), we obtain fairly similar results. The difference is due to a small difference in the data and the different VAR systems; Campbell et al. include a real consol bond in their VAR system.

[16]Campbell et al. (2003) work with the level of interest rates and do not use the stochastically detrended interest rate in their analyses. Part of the reason is that by using the level of the nominal interest rate together with the real interest rate they can compute inflation to be used in the section of

model with the detrended interest rate ('Detr. y_t') and we compare with the results using the interest rate in levels ('Level y_t').[17] Table A1 in the appendix reports the underlying VAR parameter estimates, and here we only report results based on the full 6-equation VAR models. Again, numbers in bold come from the bias-adjusted VAR estimates. Replacing the nominal interest rate with its detrended version changes somewhat the optimal asset allocations for intermediate values of γ. There is no big change for stocks, but the allocation to bonds increases a lot. For example, for $\gamma = 2$ total demand for bonds changes from -6.70% to 81.10% in the unadjusted VAR(1) model. This is due to a much larger y_t coefficient in the xb_{t+1} equation in the system with the detrended interest rate. Based on the bias-adjusted VAR(1) estimates, the total allocation to bonds changes even more dramatically from -117.67% to 80.70%. Figure 2 shows these differences clearly. Interestingly, however, in the system with the detrended interest rate there is not much difference between optimal asset allocations based on the bias-adjusted and unadjusted VAR(1) estimates, in contrast to the system with the interest rate in levels where the bias-adjustment has a large effect on optimal bond demand.

Next we estimate second-order VAR models, i.e. models with two lags - VAR(2) -, with either 'Level y_t' or 'Detr. y_t' included. Campbell et al. (2003) only estimate first-order models, so it will be interesting to investigate the sensitivity of the results with respect to lag-length. The columns in the right part of Table 5 summarize the asset allocation results based on VAR(2) models. In estimating these models with 'Level y_t' we run into the problem that, due to the nonstationarity of y_t, the VAR coefficient matrix based on least squares contains unstable roots which invalidates the bias-adjustment procedure. Hence, for the VAR(2) model with 'Level y_t' we do not bias-correct the parameters.[18] Tables A2 and A3 in the Appendix contain the VAR(2) parameter estimates. We see that several of the second-lag coefficients are strongly statistically significant. Despite of this, however, there does not seem to be large differences between the optimal allocations from the unadjusted VAR(1) and VAR(2) models, so the results are reasonably robust to changes in the lag-length. However, in the VAR(2) model with the detrended interest rate we see an interesting effect on stocks from bias-correcting the VAR parameters: for γ equal to 2 or 5, the optimal hedging demand for stocks is almost cut in half (from around 100% to a little over 50%), see also Figure 3.

Finally, Table 6 summarizes the results for mean asset allocation when we extend the sample period to 2006:q4. (Tables A4 and A5 in the Appendix report the underlying VAR parameter estimates for the one-lag models). Extending the sample period does not qualitatively change the previous results. The most noticeable change is that in the extended sample the demand for long-term bonds is higher - and the demand for stocks is lower - than in the shorter sample. The main reason for this is the lower correlation

their paper that deals with inflation-indexed bonds. We do not consider such inflation protected bonds in our analysis.

[17] The results with 'Level y_t' for the VAR(1) model are identical to the results in Table 3.

[18] The non-stationarity of y_t also creates a multicollinearity problem in the VAR(2) system because the regressors y_t and y_{t-1} are almost perfectly correlated. This problem manifests itself by the wildly shifting parameters, from very large positive to very large negative (or vice versa) between first- and second-lag regressors, see Table A2 in the Appendix.

between stock and bond innovations in the extended sample, together with a slightly lower Sharpe ratio for stocks and a slightly higher Sharpe ratio for bonds (see Table 1). But besides this, the overall patterns are the same, and the effects of the bias-adjustment are similar in the two datasets.

5 Concluding remarks

In this paper, we have explored the effects of adjusting the VAR parameter estimates for small-sample bias in the VAR based intertemporal asset allocation model from Campbell et al. (2003) on US data over the period 1952-2006.

Using the analytical bias formula from Pope (1990), we find that bias-adjusting the VAR parameters has both quantitatively and qualitatively important effects on the strategic intertemporal part of optimal asset allocation. Thus, neglecting the fact that standard least squares estimates of the VAR parameters are plagued with finite-sample bias can have servere effects on the investor's optimal asset allocation. Futhermore, we find that the choice of state variables has a large effect on optimal asset allocation: replacing the nominal interest rate in levels with its 'stochastically detrended' version increases the optimal demand for bonds dramatically. On the other hand, we find that the results are not especially sensitive to the lag length in the VAR model. With respect to return predictability we find that the bias-adjustment in the multivariate system in general is quite different from the univariate bias-correction from Kendall (1954), and that the observation by Stambaugh (1999) that the small-sample bias has the opposite sign to the sign of the innovation correlation when using one predictor variable that follows a univariate AR(1) process needs to be modified when using a multivariate system.

Of course, our analysis has only addressed one of the limitations of the VAR based intertemporal asset allocation model. Campbell et al. (2003) mention a number of interesting extensions that could be undertaken such as the incorporation of labor income, borrowing and short-sales constraints, and parameter uncertainty and learning effects. Another interesting extension would be to allow for time-varying risk-aversion, for example by modeling utility in the form of habit persistence. We leave that for future research.

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7 Tables and figures

	Sample moment	1952:q1 - 1999:q4	1952:q1 - 2006:q4
(1)	$E[r^{\$}_{1,t} - \pi_t] + \sigma^2(r^{\$}_{1,t} - \pi_t)/2$	1.515	1.360
(2)	$\sigma(r^{\$}_{1,t} - \pi_t)$	1.354	1.433
(3)	$E[r^{\$}_{e,t} - \pi_t] + \sigma^2(r^{\$}_{e,t} - \pi_t)/2$	7.571	6.910
(4)	$\sigma(r^{\$}_{e,t} - \pi_t)$	16.220	16.563
(5)	$SR = (3)/(4)$	0.467	0.417
(6)	$E[r^{\$}_{n,t} - \pi_t] + \sigma^2(r^{\$}_{n,t} - \pi_t)/2$	1.051	1.268
(7)	$\sigma(r^{\$}_{n,t} - \pi_t)$	5.619	5.530
(8)	$SR = (6)/(7)$	0.187	0.229
(9)	$E[y^{\$}_{1,t}]$	5.482	5.160
(10)	$\sigma(y^{\$}_{1,t})$	1.415	1.420
(11)	$E[d_t - p_t]$	-3.419	-3.517
(12)	$\sigma(d_t - p_t)$	0.307	0.389
(13)	$E[y^{\$}_{n,t} - y^{\$}_{1,t}]$	0.948	0.978
(14)	$\sigma(y^{\$}_{n,t} - y^{\$}_{1,t})$	0.506	0.507

Notes: $r^{\$}_{1,t}$ is the log nominal return on T-bills. π_t is the log inflation rate. $r^{\$}_{e,t}$ is the nominal log stock (equity) return. $r^{\$}_{n,t}$ is the log nominal bond return. $y^{\$}_{1,t}$ is the short-term nominal interest rate. $d_t - p_t$ is the log dividend-price ratio. $y^{\$}_{n,t}$ is the nominal bond yield. SR denotes Sharpe ratio. More information about the data sources is given in section 4.1.

Table 1: Summary statistics

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)	R^2 (p)
rtb_{t+1}	0.444	0.004	-0.012	0.248	-0.001	0.450	0.339
	0.478	**0.003**	**-0.015**	**0.200**	**-0.001**	**0.391**	
	(6.559)	(0.764)	(-0.722)	(2.990)	(-0.723)	(2.281)	(0.000)
xr_{t+1}	0.634	0.023	0.441	-1.989	0.044	0.474	0.084
	0.913	**0.018**	**0.414**	**-2.387**	**0.042**	**-0.160**	
	(0.683)	(0.357)	(1.891)	(-2.326)	(1.936)	(0.225)	(0.007)
xb_{t+1}	0.051	-0.055	-0.090	0.326	0.003	3.045	0.096
	0.066	**-0.059**	**-0.090**	**-0.043**	**0.001**	**2.596**	
	(0.209)	(-2.514)	(-0.785)	(0.879)	(0.398)	(3.014)	(0.003)
y_{t+1}	-0.008	0.004	0.005	0.955	0.000	0.116	0.869
	-0.009	**0.004**	**0.005**	**0.991**	**0.000**	**0.128**	
	(-0.247)	(1.517)	(0.435)	(25.79)	(-0.220)	(1.398)	(0.000)
$d_{t+1} - p_{t+1}$	-0.984	-0.020	-0.411	1.340	0.965	-1.045	0.934
	-1.266	**-0.013**	**-0.384**	**1.785**	**0.967**	**-0.393**	
	(-1.044)	-0.302)	(-1.636)	(1.506)	(39.51)	(-0.464)	(0.000)
spr_{t+1}	0.000	-0.001	0.002	0.025	0.000	0.747	0.539
	0.000	**-0.001**	**0.002**	**0.012**	**0.000**	**0.764**	
	(-0.012)	(-0.302)	(0.331)	(1.145)	(-0.204)	(12.22)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	y	$d-p$	spr
rtb	0.549	0.235	0.394	-0.389	-0.235	0.187
xr		7.751	0.225	-0.168	-0.983	0.024
xb			2.670	-0.765	-0.248	0.199
y				0.255	0.200	-0.777
$d-p$					7.900	-0.053
spr						0.172

Notes: The variables are defined at the beginning of section 4.2. The numbers in bold are bias-adjusted estimates. The numbers not in bold are unadjusted estimates. (t) is the Newey-West corrected t-statistic on the unadjusted estimate. (p) denotes p-value in tests of joint significance of the VAR explanatory variables.

Table 2: VAR(1) parameter estimates and innovation correlations. CCV period, 1952:q1 - 1999:q4

State variables:	*Constant*	AR_t	y_t	$d_t - p_t$	spr_t
$\gamma = 1,\ \psi = 1,\ \delta = 0.92^{1/4}$					
Stocks: Total demand	268.47	287.87	291.50	297.77	297.20
	268.47	**287.87**	**291.50**	**297.77**	**297.20**
Hedging demand	0	0	0	0	0
	0	**0**	**0**	**0**	**0**
Bonds: Total demand	167.00	158.52	156.41	158.07	168.45
	167.00	**158.52**	**156.41**	**158.07**	**168.45**
Hedging demand	0	0	0	0	0
	0	**0**	**0**	**0**	**0**
Cash: Total demand	-335.47	-346.39	-347.91	-355.84	-365.65
	-335.47	**-346.39**	**-347.91**	**-355.84**	**-365.65**
Hedging demand	0	0	0	0	0
	0	**0**	**0**	**0**	**0**
$\gamma = 2,\ \psi = 1,\ \delta = 0.92^{1/4}$					
Stocks: Total demand	133.37	143.72	147.13	241.76	242.44
	133.37	**143.06**	**145.86**	**223.93**	**223.58**
Hedging demand	0	0.18	1.87	93.41	94.39
	0	**-0.50**	**0.60**	**75.58**	**75.53**
Bonds: Total demand	79.16	31.82	-7.64	7.21	-6.70
	79.16	**31.49**	**-11.61**	**-36.80**	**-117.67**
Hedging demand	0	-43.46	-81.93	-67.90	-87.23
	0	**-43.80**	**-85.90**	**-111.91**	**-198.20**
Cash: Total demand	-112.53	-75.55	-39.50	-148.98	-135.74
	-112.53	**-74.54**	**-34.25**	**-87.13**	**-5.91**
Hedging demand	0	43.28	80.06	-25.51	-7.16
	0	**44.29**	**85.30**	**36.33**	**122.67**

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State variables:		*Constant*	AR_t	y_t	$d_t - p_t$	spr_t
$\gamma = 5,\ \psi = 1,\ \delta = 0.92^{1/4}$						
Stocks:	Total demand	52.32	55.60	56.40	154.60	158.05
		52.32	**55.17**	**55.89**	**125.72**	**131.67**
	Hedging demand	0	-1.35	-1.12	95.90	99.48
		0	**-1.78**	**-1.93**	**67.02**	**73.10**
Bonds:	Total demand	26.45	-4.54	-15.50	-41.61	-91.28
		26.45	**-4.67**	**-16.71**	**-63.34**	**-165.23**
	Hedging demand	0	-29.88	-40.52	-66.94	-119.06
		0	**-30.01**	**-41.72**	**-88.68**	**-193.01**
Cash:	Total demand	21.23	48.93	59.10	-12.99	33.23
		21.23	**49.49**	**61.12**	**37.62**	**133.56**
	Hedging demand	0	31.23	41.64	-28.95	19.58
		0	**31.79**	**43.65**	**21.66**	**119.90**
$\gamma = 20,\ \psi = 1,\ \delta = 0.92^{1/4}$						
Stocks:	Total demand	11.79	11.78	11.24	58.13	59.31
		11.79	**11.64**	**10.97**	**41.81**	**44.59**
	Hedging demand	0	-1.87	-2.41	44.25	45.49
		0	**-2.01**	**-2.67**	**27.93**	**30.76**
Bonds:	Total demand	0.10	-11.43	3.81	-18.37	-39.86
		0.10	**-11.53**	**4.22**	**-18.58**	**-53.48**
	Hedging demand	0	-11.80	3.43	-18.82	-41.26
		0	**-11.90**	**3.84**	**-19.03**	**-54.88**
Cash:	Total demand	88.11	99.64	84.95	60.24	80.55
		88.11	**99.89**	**84.81**	**76.77**	**108.89**
	Hedging demand	0	13.67	-1.03	-25.44	-4.23
		0	**13.92**	**-1.17**	**-8.90**	**24.12**

Notes: γ, ψ, and δ are the utility parameters described in section 2. The beginning of section 4.3 describes the content of the table in more detail.

Table 3: Mean asset demands from the VAR(1) models. CCV period: 1952:q1 - 1999:q4

γ	$E(V_t)_{\text{unadjusted}}$	$E(V_t)_{\text{adjusted}}$	$\frac{E(V_t)_{\text{unadjusted}} - E(V_t)_{\text{adjusted}}}{E(V_t)_{\text{adjusted}}}$
2	0.580	0.352	0.648
3	0.203	0.126	0.611
4	0.114	0.074	0.541
5	0.078	0.053	0.472
6	0.060	0.042	0.429
7	0.049	0.035	0.400
8	0.042	0.031	0.355
9	0.037	0.028	0.321
10	0.033	0.025	0.320

Notes: $V_t$is the value function defined as U_t/W_t, see Equation (9) in Campbell et al (2003).

Table 4: Mean value function from the VAR(1) models with the short-term interest rate y_t in levels. CCV period 1952:q1 - 1999:q4.

	VAR(1)				VAR(2)			
	Level y_t		Detr. y_t		Level y_t		Detr. y_t	
	Total	Hedge	Total	Hedge	Total	Hedge	Total	Hedge
$\gamma = 1$								
Stocks	297.20	0	294.30	0	314.33	0	304.77	0
	297.20	**0**	**294.30**	**0**	**-**	**-**	**304.77**	**0**
Bonds	168.45	0	174.47	0	191.82	0	200.25	0
	168.45	**0**	**174.47**	**0**	**-**	**-**	**200.25**	**0**
Cash	-365.65	0	-368.78	0	-406.15	0	-405.02	0
	-365.65	**0**	**-368.78**	**0**	**-**	**-**	**-405.02**	**0**
$\gamma = 2$								
Stocks	242.44	94.39	232.57	85.82	260.51	104.03	247.90	96.07
	223.58	**75.53**	**232.49**	**85.74**	**-**	**-**	**210.48**	**58.66**
Bonds	-6.70	-87.23	81.10	-2.17	23.16	-69.77	93.82	-3.28
	-117.67	**-198.20**	**80.70**	**-2.57**	**-**	**-**	**91.02**	**-6.08**
Cash	-135.74	-7.16	-213.68	-83.65	-183.67	-34.27	241.72	-92.79
	-5.91	**122.67**	**-213.19**	**-83.17**	**-**	**-**	**-201.51**	**-52.58**
$\gamma = 5$								
Stocks	158.05	99.48	151.54	93.33	168.08	106.32	163.58	103.51
	131.67	**73.10**	**151.19**	**92.97**	**-**	**-**	**116.25**	**56.18**
Bonds	-91.28	-119.06	30.31	1.75	-64.02	-97.62	37.64	2.43
	-165.23	**-193.01**	**30.32**	**1.76**	**-**	**-**	**40.96**	**5.75**
Cash	33.23	19.58	-81.85	-95.08	-4.06	-8.70	-101.22	-105.95
	133.56	**119.90**	**-81.51**	**-94.73**	**-**	**-**	**-57.21**	**-61.94**

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	VAR(1)				VAR(2)			
	Level y_t		Detr. y_t		Level y_t		Detr. y_t	
	Total	Hedge	Total	Hedge	Total	Hedge	Total	Hedge
$\gamma = 20$								
Stocks	59.31	45.49	60.96	47.01	58.36	43.96	65.46	51.28
	44.59	**30.76**	**60.79**	**46.83**	**-**	**-**	**37.92**	**23.74**
Bonds	-39.86	-41.26	3.54	2.34	-14.81	-18.74	14.04	9.78
	-53.48	**-54.88**	**3.72**	**2.52**	**-**	**-**	**15.65**	**11.39**
Cash	80.55	-4.23	35.50	-49.35	56.45	-25.22	20.50	-61.06
	108.89	**24.12**	**35.49**	**-49.36**	**-**	**-**	**46.43**	**-35.13**

Notes: The numbers in bold are based on the bias-adjusted VAR estimates. The numbers not in bold are based on the unadjusted VAR estimates. The VAR models include all three asset returns and all three predictor variables.

Table 5: Mean asset demands from VAR(1) and VAR(2) models, with the short-term interest rate y_t either in levels or detrended. CCV period 1952:q1 - 1999:q4.

	VAR(1)				VAR(2)			
	Level y_t		Detr. y_t		Level y_t		Detr. y_t	
	Total	Hedge	Total	Hedge	Total	Hedge	Total	Hedge
$\gamma = 1$								
Stocks	257.58	0	254.81	0	265.20	0	259.44	0
	257.58	**0**	**254.81**	**0**	**-**	**-**	**259.44**	**0**
Bonds	367.74	0	375.42	0	393.46	0	402.74	0
	367.74	**0**	**375.42**	**0**	**-**	**-**	**402.74**	**0**
Cash	-525.32	0	-530.23	0	-558.66	0	-562.18	0
	-525.32	**0**	**-530.23**	**0**	**-**	**-**	**-562.18**	**0**
$\gamma = 2$								
Stocks	191.21	63.16	193.87	67.01	202.96	71.21	201.84	72.80
	170.16	**42.12**	**194.13**	**67.28**	**-**	**-**	**179.98**	**50.94**
Bonds	130.85	-49.40	183.26	-0.45	161.56	-32.02	186.92	-11.22
	23.15	**-157.1**	**180.64**	**-3.07**	**-**	**-**	**176.94**	**-21.21**
Cash	-222.06	-13.76	-277.12	-66.56	-264.52	-39.19	-288.77	-61.58
	-93.32	**114.98**	**-274.77**	**-64.20**	**-**	**-**	**-256.91**	**-29.73**
$\gamma = 5$								
Stocks	111.57	61.25	120.87	70.79	118.67	66.99	122.92	72.12
	92.91	**42.58**	**120.84**	**70.75**	**-**	**-**	**98.30**	**47.50**
Bonds	0.83	-66.93	72.18	3.50	19.97	-53.68	68.42	-6.97
	-60.78	**-128.54**	**70.68**	**1.99**	**-**	**-**	**66.56**	**-8.83**
Cash	-12.40	5.68	-93.06	-74.29	-38.64	-13.31	-91.34	-65.15
	67.87	**85.95**	**-91.51**	**-72.75**	**-**	**-**	**-64.86**	**-38.67**

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	VAR(1)				VAR(2)			
	Level y_t		Detr. y_t		Level y_t		Detr. y_t	
	Total	Hedge	Total	Hedge	Total	Hedge	Total	Hedge
$\gamma = 20$								
Stocks	39.29	27.82	47.59	35.90	38.83	27.18	46.20	34.52
	31.80	**20.34**	**47.60**	**35.91**	**-**	**-**	**34.67**	**22.99**
Bonds	3.79	-7.73	13.33	2.16	17.34	3.66	17.45	3.44
	-10.17	**-21.69**	**12.93**	**1.76**	**-**	**-**	**17.78**	**3.77**
Cash	56.93	-20.10	39.08	-38.06	43.83	-30.84	36.36	-37.95
	78.37	**1.34**	**39.47**	**-37.67**	**-**	**-**	**47.55**	**-26.76**

See the notes to Table 5.

Table 6: Mean asset demands from VAR(1) and VAR(2) models, with the short-term interest rate y_t either in levels or detrended. Extended period 1952:q1 - 2006:q4.

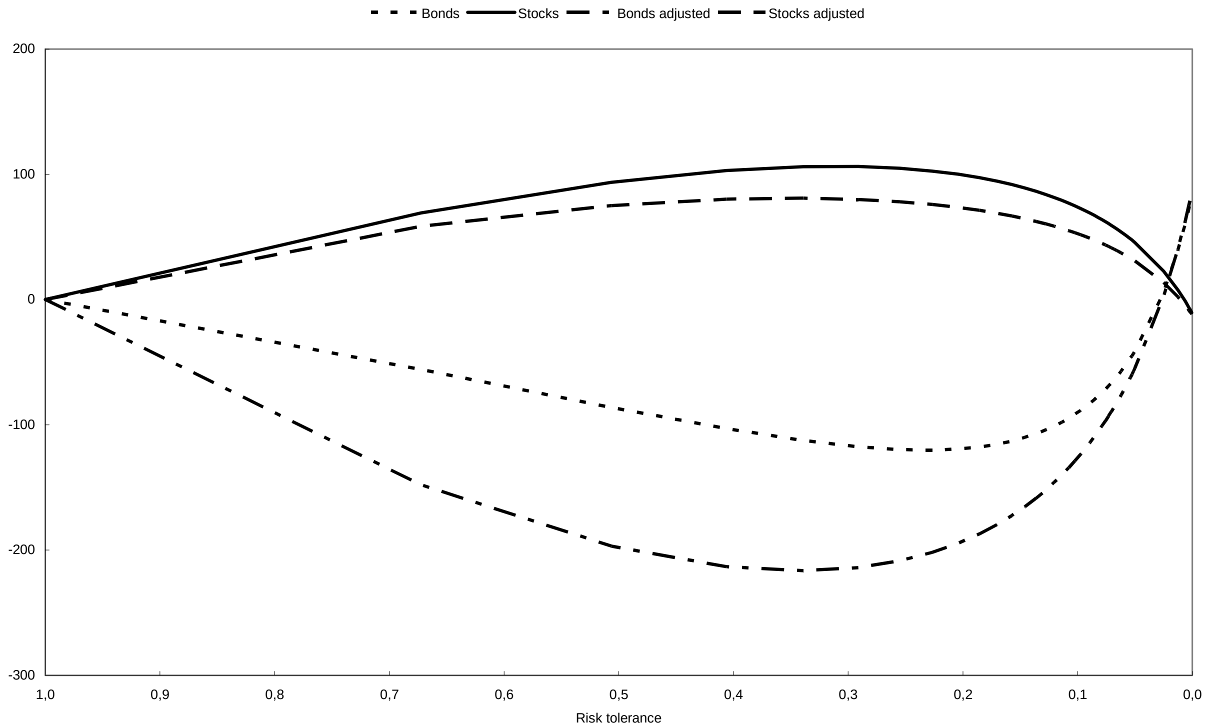

Figure 1: Mean hedging demand from the bias-adjusted and unadjusted VAR(1) models with the short-term interest rate in levels. CCV period: 1952:q1 - 1999:q4

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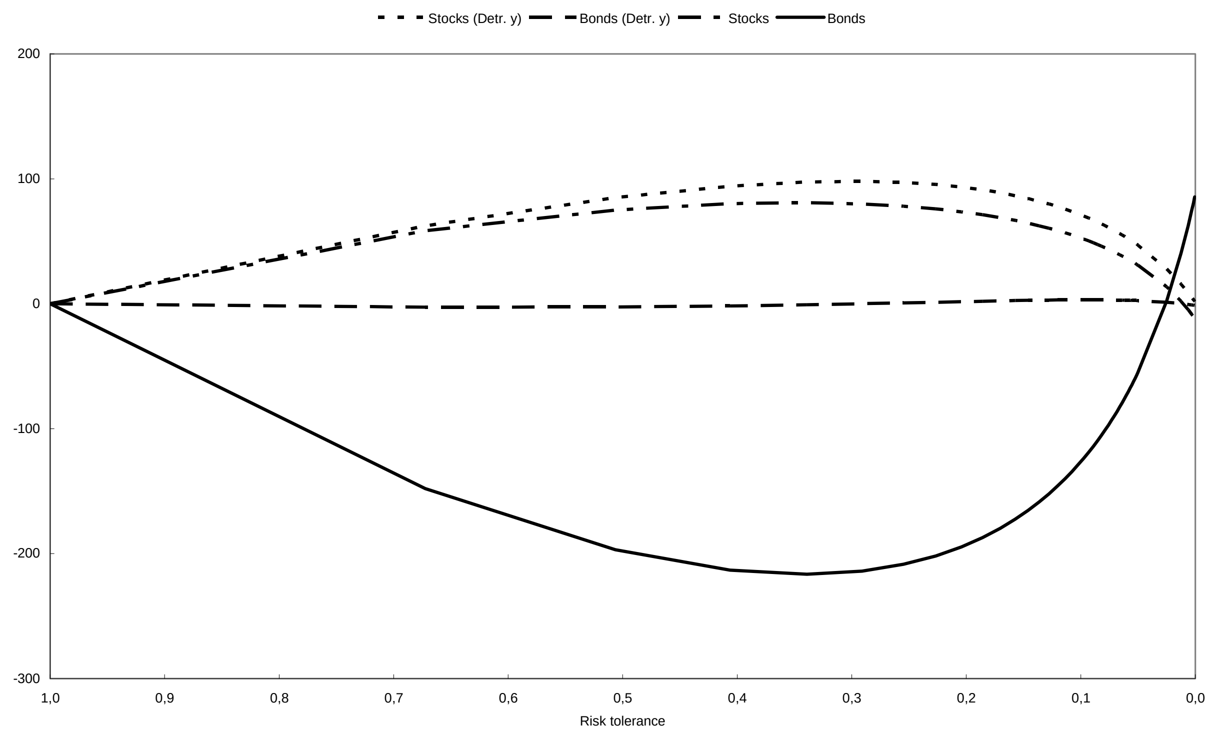

Figure 2: Mean hedging demand from the bias-adjusted VAR(1) models with the short-term interest rate either in levels or detrended. CCV period: 1952:q1 - 1999:q4

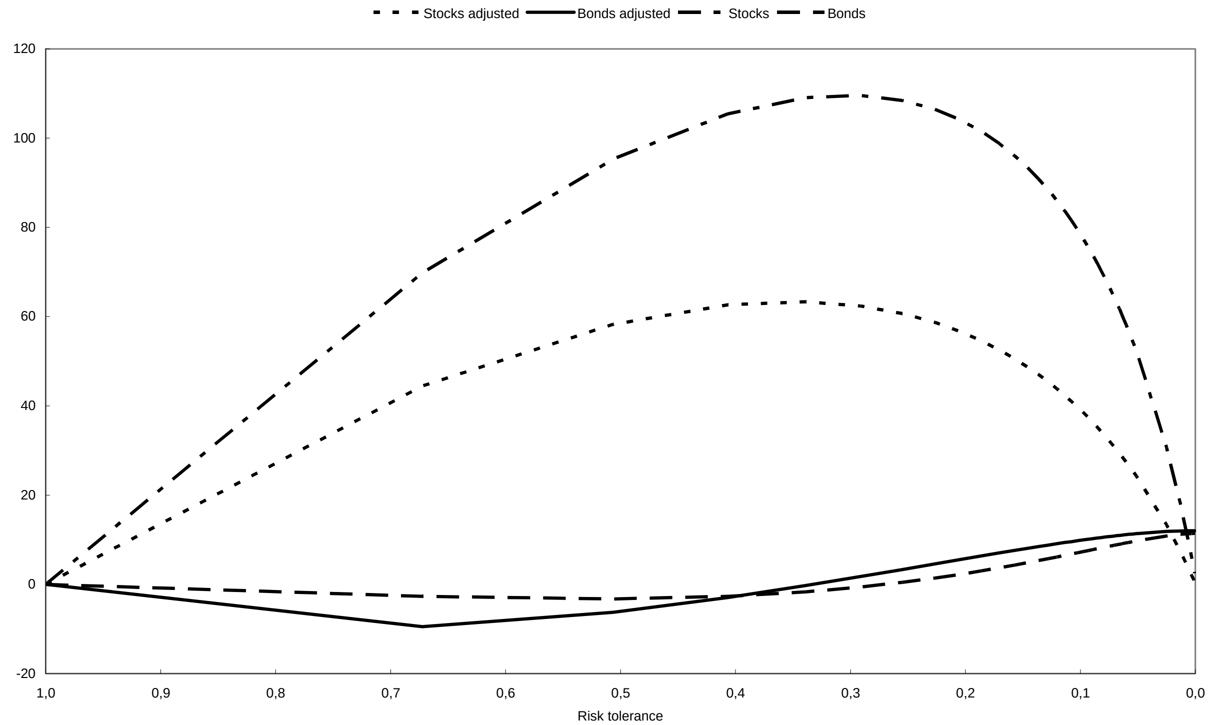

Figure 3: Mean hedging demand from the bias-adjusted and unadjusted VAR(2) models with the detrended short-term interest rate. CCV period: 1952:q1 - 1999:q4

8 Appendix

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	Detr. y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)	R^2 (p)
rtb_{t+1}	0.547	0.002	-0.021	0.116	0.001	0.287	0.293
	0.574	**0.001**	**-0.022**	**0.098**	**0.001**	**0.272**	
	(7.666)	(0.297)	(-0.800)	(0.420)	(0.386)	(1.340)	(0.000)
xr_{t+1}	-0.315	0.029	0.343	-4.454	0.027	-0.014	0.074
	-0.330	**0.037**	**0.346**	**-4.321**	**0.027**	**-0.330**	
	(-0.312)	(0.399)	(1.193)	(-1.419)	(1.344)	(-0.005)	(0.017)
xb_{t+1}	0.207	-0.056	-0.074	0.749	0.005	3.134	0.094
	0.194	**-0.058**	**-0.074**	**0.422**	**0.005**	**2.902**	
	(0.803)	(-2.805)	(-0.785)	(0.624)	(0.733)	(2.788)	(0.003)
Detr. y_{t+1}	-0.031	0.005	-0.003	0.557	-0.001	0.133	0.283
	-0.030	**0.005**	**-0.003**	**0.566**	**-0.001**	**0.114**	
	(-1.293)	(2.306)	(-0.286)	(4.098)	(-0.952)	(1.410)	(0.000)
$d_{t+1} - p_{t+1}$	-0.238	-0.012	-0.206	5.978	0.978	0.795	0.934
	-0.220	**-0.019**	**-0.210**	**5.853**	**0.979**	**1.083**	
	(-0.249)	(-0.172)	(-0.682)	(1.799)	(45.32)	(0.273)	(0.000)
spr_{t+1}	0.012	-0.001	0.004	0.070	0.000	0.760	0.538
	0.012	**-0.001**	**0.005**	**0.085**	**0.000**	**0.799**	
	(0.664)	(-0.370)	(0.488)	(0.620)	(0.340)	(12.34)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	Detr. y	$d - p$	spr
rtb	0.568	0.192	0.397	-0.394	-0.208	0.200
xr		7.795	0.219	-0.143	-0.982	0.019
xb			2.673	-0.755	-0.249	0.201
Detr. y				0.249	0.185	-0.763
$d - p$					7.860	-0.055
spr						0.172

Notes: 'Detr. y' is the stochastically detrended short-term interest rate, defined in the beginning of section 4.4. Otherwise see the notes to Table 2.

Table A1: VAR(1) parameter estimates and innovation correlations, with detrended short rate. CCV period: 1952:q1 - 1999:q4

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)
rtb_{t+1}	0.305	-0.023	-0.093	-1.111	-0.031	-0.863
	-	-	-	-	-	-
	(4.136)	(-0.793)	(-1.064)	(-0.809)	(-1.046)	(-0.715)
xr_{t+1}	0.734	-0.324	2.741	35.94	-0.273	34.25
	-	-	-	-	-	-
	(0.706)	(-0.951)	(1.627)	(1.317)	(-0.821)	(1.351)
xb_{t+1}	-0.354	-0.167	-0.326	-3.447	-0.113	-1.790
	-	-	-	-	-	-
	(-1.020)	(-1.418)	(-0.664)	(-0.439)	(-0.959)	(-0.219)
y_{t+1}	0.036	0.027	-0.017	0.578	0.024	-0.083
	-	-	-	-	-	-
	(0.807)	(2.543)	(-0.324)	(0.684)	(2.290)	(-0.093)
$d_{t+1} - p_{t+1}$	-1.233	0.740	-2.776	-38.37	1.695	-35.92
	-	-	-	-	-	-
	(-1.209)	(1.950)	(-1.582)	(-1.363)	(4.480)	(-1.378)
spr_{t+1}	-0.022	-0.018	0.037	0.607	-0.018	1.195
	-	-	-	-	-	-
	(-0.704)	(-2.489)	(0.959)	(0.935)	(-2.524)	(1.814)

Table continues next page

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Dependent variable	rbt_{t-1} **adj.** (t)	xr_{t-1} **adj.** (t)	xb_{t-1} **adj.** (t)	y_{t-1} **adj.** (t)	$d_{t-1}-p_{t-1}$ **adj.** (t)	spr_{t-1} **adj.** (t)	R^2 (p)
rtb_{t+1}	0.330	-0.006	-0.043	1.259	0.031	1.310	0.423
	-	-	-	-	-	-	
	(4.224)	(-1.239)	(-3.030)	(0.904)	(1.042)	(1.008)	(0.000)
xr_{t+1}	0.553	-0.123	0.523	-38.37	0.322	-37.53	0.149
	-	-	-	-	-	-	
	(0.520)	(-1.958)	(2.731)	(-1.393)	(0.956)	(-1.397)	(0.003)
xb_{t+1}	1.112	-0.059	-0.074	3.390	0.119	5.012	0.176
	-	-	-	-	-	-	
	(2.992)	(-2.801)	(-1.073)	(0.429)	(0.987)	(0.603)	(0.000)
y_{t+1}	-0.137	0.006	0.012	0.431	-0.024	0.230	0.888
	-	-	-	-	-	-	
	(-2.346)	(2.648)	(1.422)	(0.515)	(-2.349)	(0.245)	(0.000)
$d_{t+1}-p_{t+1}$	-0.331	0.156	-0.611	40.42	-0.739	39.18	0.939
	-	-	-	-	-	-	
	(-0.307)	(2.610)	(-3.177)	(1.427)	(-1.921)	(1.414)	(0.000)
spr_{t+1}	0.072	-0.002	-0.008	-0.614	0.018	-0.483	0.589
	-	-	-	-	-	-	
	(1.682)	(-1.312)	(-0.880)	(-0.946)	(2.582)	(-0.685)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	y	$d-p$	spr
rtb	0.514	0.265	0.339	-0.314	-0.263	0.107
xr		7.493	0.216	-0.145	-0.985	-0.011
xb			2.555	-0.743	-0.236	0.126
y				0.237	0.170	-0.752
$d-p$					7.478	-0.008
spr						0.163

Notes: There are no bias-adjusted estimates because the least squares VAR parameter matrix contains unstable roots. Otherwise, see the notes to Table 2.

Table A2: VAR(2) parameter estimates and innovation correlations. CCV period: 1952:q1 - 1999:q4.

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	Detr. y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)
rtb_{t+1}	0.351	-0.041	-0.074	-0.792	-0.048	-0.532
	0.375	**-0.042**	**-0.077**	**-0.923**	**-0.061**	**-0.701**
	(5.769)	(1.531)	(-1.070)	(-0.792)	(-1.760)	(-0.624)
xr_{t+1}	-0.331	-0.036	0.431	-3.325	-0.004	-3.189
	-0.041	**-0.033**	**0.353**	**-5.379**	**-0.535**	**-6.332**
	(-0.305)	(-0.097)	(0.591)	(-0.295)	(-0.011)	(-0.337)
xb_{t+1}	-0.377	-0.175	-0.323	-3.476	-0.126	-1.721
	-0.369	**-0.177**	**-0.324**	**-4.848**	**-0.210**	**-3.525**
	(-1.058)	(-1.607)	(-1.265)	(-0.714)	(-1.146)	(-0.461)
Detr. y_{t+1}	0.039	0.025	-0.009	0.449	0.021	0.118
	0.039	**0.025**	**-0.008**	**0.473**	**0.027**	**0.124**
	(0.898)	(2.343)	(-0.294)	(0.832)	(1.962)	(0.241)
$d_{t+1} - p_{t+1}$	-0.284	0.456	-0.396	2.116	1.419	2.608
	-0.562	**0.454**	**-0.321**	**4.212**	**1.939**	**5.758**
	(-0.266)	(1.202)	(-0.521)	(0.183)	(3.683)	(0.269)
spr_{t+1}	-0.022	-0.016	0.039	0.660	-0.015	1.222
	-0.023	**-0.016**	**0.039**	**0.718**	**-0.017**	**1.321**
	(-0.765)	(-2.140)	(1.789)	(1.790)	(-2.001)	(3.484)

Table continues next page

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Dependent variable	rbt_{t-1} **adj.** (t)	xr_{t-1} **adj.** (t)	xb_{t-1} **adj.** (t)	Detr. y_{t-1} **adj.** (t)	$d_{t-1} - p_{t-1}$ **adj.** (t)	spr_{t-1} **adj.** (t)	R^2 (p)
rtb_{t+1}	0.358	-0.007	-0.048	0.626	0.049	0.906	0.412
	0.375	**0.005**	**-0.039**	**0.744**	**0.062**	**1.021**	
	(4.439)	(-1.402)	(-2.839)	(0.858)	(1.797)	(0.991)	(0.000)
xr_{t+1}	0.024	-0.096	0.545	0.105	0.031	3.515	0.117
	0.004	**0.433**	**0.684**	**1.649**	**0.563**	**5.330**	
	(0.022)	(-1.288)	(2.084)	(0.013)	(0.085)	(0.322)	(0.024)
xb_{t+1}	1.109	-0.053	-0.065	2.831	0.131	5.380	0.180
	1.087	**0.026**	**0.046**	**4.200**	**0.215**	**6.682**	
	(2.744)	(-2.418)	(-0.706)	(0.818)	(1.176)	(1.182)	(0.000)
Detr. y_{t+1}	-0.139	0.007	0.005	0.077	-0.021	0.037	0.395
	-0.136	**0.001**	**0.003**	**0.080**	**-0.027**	**0.026**	
	(-2.496)	(2.950)	(0.672)	(0.210)	(-2.016)	(0.067)	(0.000)
$d_{t+1} - p_{t+1}$	0.166	0.138	-0.564	2.090	-0.442	-1.955	0.937
	0.188	**-0.378**	**-0.706**	**0.515**	**-0.963**	**-3.779**	
	(0.151)	(2.015)	(-2.184)	(0.255)	(-1.149)	(-0.174)	(0.000)
spr_{t+1}	0.070	-0.003	-0.007	-0.488	0.016	-0.556	0.604
	0.069	**-0.001**	**-0.011**	**-0.567**	**0.017**	**-0.617**	
	(1.789)	(-1.730)	(-0.843)	(-1.908)	(2.039)	(-1.434)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	Detr. y	$d-p$	spr
rtb	0.520	0.227	0.330	-0.301	-0.232	0.118
xr		7.626	0.212	-0.130	-0.986	0.003
xb			2.549	-0.733	-0.234	0.143
Detr. y				0.230	0.156	-0.752
$d-p$					7.590	-0.018
spr						0.160

Notes: See the notes to Table 2.

Table A3: VAR(2) parameter estimates and innovation correlations, with detrended short rate. CCV period: 1952:q1 - 1999:q4.

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)	R^2 (p)
rtb_{t+1}	0.286	-0.002	0.008	0.325	-0.001	0.458	0.224
	0.311	**-0.002**	**0.005**	**0.278**	**-0.001**	**0.404**	
	(2.923)	(-0.266)	(0.454)	(3.185)	(-0.796)	(2.081)	(0.000)
xr_{t+1}	0.424	0.014	0.435	-2.201	0.044	0.371	0.086
	0.576	**0.011**	**0.412**	**-2.708**	**0.042**	**-0.351**	
	(0.513)	(0.245)	(2.056)	(-2.667)	(2.553)	(0.164)	(0.002)
xb_{t+1}	-0.031	-0.058	-0.088	0.427	-0.002	3.185	0.099
	-0.034	**-0.059**	**-0.090**	**0.110**	**-0.004**	**2.785**	
	(-0.132)	(-2.744)	(-0.810)	(1.138)	(-0.514)	(3.383)	(0.001)
y_{t+1}	-0.006	0.005	0.003	0.957	0.000	0.104	0.883
	-0.006	**0.005**	**0.004**	**0.988**	**0.000**	**0.115**	
	(-0.240)	(1.961)	(0.313)	(26.95)	(0.460)	(1.368)	(0.000)
$d_{t+1} - p_{t+1}$	-0.780	-0.004	-0.421	1.306	0.965	-0.463	0.957
	-0.940	**0.001**	**-0.396**	**1.880**	**0.968**	**0.325**	
	(-0.919)	-0.071)	(-1.891)	(1.532)	(55.60)	(-0.208)	(0.000)
spr_{t+1}	0.004	-0.001	0.003	0.016	0.000	0.753	0.556
	0.003	**-0.001**	**0.003**	**0.005**	**0.000**	**0.768**	
	(0.191)	(-0.579)	(0.559)	(0.761)	(-0.210)	(13.42)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	y	$d - p$	spr
rtb	0.630	0.221	0.322	-0.304	-0.253	0.109
xr		7.909	0.113	-0.117	-0.971	0.047
xb			2.623	-0.738	-0.144	0.124
y				0.242	0.149	-0.756
$d - p$					8.015	-0.063
spr						0.169

Notes: The variables are defined at the beginning of section 4.2. The numbers in bold are bias-adjusted estimates. The numbers not in bold are unadjusted estimates. (t) is the t-statistic on the unadjusted estimate. (p) denotes p-value in tests of joint significance of the VAR explanatory variables.

Table A4: VAR(1) parameter estimates and innovation correlations. Extended period: 1952:q1 - 2006:q4

Dependent variable	rbt_t **adj.** (t)	xr_t **adj.** (t)	xb_t **adj.** (t)	Detr. y_t **adj.** (t)	$d_t - p_t$ **adj.** (t)	spr_t **adj.** (t)	R^2 (p)
rtb_{t+1}	0.397	-0.006	-0.012	-0.051	0.001	0.136	0.159
	0.418	**-0.006**	**-0.013**	**-0.071**	**0.001**	**0.126**	
	(4.125)	(-0.852)	(-0.474)	(-0.169)	(0.588)	(0.559)	(0.000)
xr_{t+1}	-0.469	0.033	0.416	-3.001	0.031	0.815	0.068
	-0.545	**0.039**	**0.418**	**-3.116**	**0.030**	**0.479**	
	(-0.536)	(0.539)	(1.567)	(-1.093)	(1.818)	(0.292)	(0.012)
xb_{t+1}	0.139	-0.061	-0.089	0.487	0.000	3.050	0.093
	0.123	**-0.063**	**-0.088**	**0.182**	**0.000**	**2.879**	
	(0.597)	(-3.218)	(-0.986)	(0.460)	(0.050)	(2.986)	(0.001)
Detr. y_{t+1}	-0.023	0.005	0.000	0.640	0.000	0.166	0.337
	-0.022	**0.006**	**0.000**	**0.648**	**0.000**	**0.147**	
	(-1.285)	(2.724)	(0.034)	(5.169)	(-0.349)	(1.964)	(0.000)
$d_{t+1} - p_{t+1}$	-0.097	-0.006	-0.249	5.248	0.975	1.076	0.958
	-0.028	**-0.011**	**-0.251**	**5.406**	**0.975**	**1.421**	
	(-0.114)	(-0.103)	(-0.918)	(1.830)	(60.87)	(0.393)	(0.000)
spr_{t+1}	0.009	-0.001	0.002	-0.003	0.000	0.737	0.554
	0.009	**-0.001**	**0.003**	**0.011**	**0.000**	**0.770**	
	(0.598)	(-0.738)	(0.341)	(-0.028)	(0.089)	(12.65)	(0.000)

Cross-corr. of residuals	rtb	xr	xb	Detr. y	$d-p$	spr
rtb	0.656	0.166	0.334	-0.314	-0.216	0.119
xr		7.986	0.099	-0.089	-0.970	0.038
xb			2.632	-0.731	-0.140	0.129
Detr. y				0.237	0.128	-0.740
$d-p$					7.986	-0.058
spr						0.169

Notes: 'Detr. y' is the stochastically detrended short-term interest rate, defined in the beginning of section 4.4. Otherwise see the notes to Table 2.

Table A5: VAR(1) parameter estimates and innovation correlations, with detrended short rate. Extended period: 1952:q1 - 2006:q4

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