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Ensuring the Validity of the Micro Foundation in DSGE Models

Martin Møller Andreasen

School of Economics and Management
University of Aarhus
Building 1322, DK-8000 Aarhus C
Denmark

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Martin Møller Andreasen*
University of Aarhus and CREATES
School of Economics and Management

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Abstract

The presence of i) stochastic trends, ii) deterministic trends, and/or iii) stochastic volatility in DSGE models may imply that the agents' objective functions attain infinite values. We say that such models do not have a valid micro foundation. The paper derives sufficient conditions which ensure that the objective functions of the households and the firms are finite even when various trends and stochastic volatility are included in a standard DSGE model. Based on these conditions we test the validity of the micro foundation in six DSGE models from the literature. The models of Justiniano & Primiceri (*American Economic Review,* forthcoming) and Fernández-Villaverde & Rubio-Ramírez (*Review of Economic Studies,* 2007) do not satisfy these sufficient conditions, or any other known set of conditions ensuring finite values for the objective functions. Thus, the validity of the micro foundation in these models remains to be established.

Keywords: Deterministic trends, DSGE models, Error distributions, Moment generating functions, Stochastic trends, Stochastic volatility, Unit-roots.

JEL: E10, E30

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1 Introduction

Following the pioneering work by Kydland & Prescott (1982), Dynamic Stochastic General Equilibrium (DSGE) models have become a popular framework for developing macroeconomic models. This is so primarily because the framework establishes a clear relationship between optimal microeconomic behavior and changes in macro variables such as GDP, consumption, etc. However, two features in the current literature using DSGE models may undermine the validity of the micro foundation.

First, DSGE models are often specified with stochastic and deterministic trends in order to explain the non-stationary and trending behavior in the time series for GDP, consumption, investments, etc.[1] Recent examples are the DSGE models in Ireland (2004*a*), Ireland (2004*b*), Altig, Christiano, Eichenbaum & Linde (2005), Negro, Schorfheide, Smets & Wouters (2005), An (2005), Justiniano & Primiceri (2005), Schmitt-Grohé & Uribe (2006), Fernández-Villaverde & Rubio-Ramírez (2007*a*), An & Schorfheide (2007), and Gorodnichenko & Ng (2007). However, we show in this paper that including trends in DSGE models may imply infinite values for the households' or the firms' objective functions unless trending variables are properly scaled as in An (2005) and An & Schorfheide (2007). The problem related to objective functions attaining infinite values is simply that agents may mistakenly be considered to be indifferent to clearly different strategies, because all these strategies give an infinite amount of utility or profit. For instance, consider the case where a household always prefers more consumption to less, meaning that the periodic utility function $u(c_t)$ is strictly increasing. If the household's lifetime utility function is infinite for the consumption stream $\{c_t\}_{t=1}^{\infty}$, then it actually indicates that the household is indifferent to $\{c_t\}_{t=1}^{\infty}$ and $\{c_t + k\}_{t=1}^{\infty}$ where $k > 0$, even though $c_t + k$ is prefered to c_t each time period. Thus, such a lifetime utility function fails to represent the household's lifetime preferences for consumption streams. For this reason, we say that models with infinite objective functions do not have a valid micro foundation. Hence, improving the empirical performance of DSGE models by adding stochastic and deterministic trends may undermine the validity of the micro foundation for these models. We henceforth refer to this problem as the "trend problem".

Second, the papers by Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*) introduce stochastic volatility in DSGE models to account for the time-varying volatility of GDP, inflation, interest rates, and other macro variables. This extension may also imply that the objective functions of the households and the firms attain infinite values, as we demonstrate in the present paper. We henceforth refer to this problem as the "stochastic volatility problem". Of course, if stochastic volatility is specified in the processes with stochastic trends, the two problems must be dealt with simultaneously.

None of the papers listed above addresses either of the two problems. Even the two pioneering papers by King, Plosser & Rebelo (1988*a*) and King, Plosser & Rebelo (1988*b*) only discuss the trend problem in case of a deterministic trend, but not with a stochastic trend. The present paper generalizes the result in King et al. (1988*a*) by deriving sufficient conditions which ensure that the objective functions of the households and the firms only attain finite values when deterministic *and* stochastic trends are present in a DSGE model. Following the work of King et al. (1988*a*), many nominal and real frictions are commonly added to the standard neoclassical

[1] Stochastic trends are generated by processes with a unit-root.

growth model (see Smets & Wouters (2003), Christiano, Eichenbaum & Evans (2005), Altig et al. (2005), among others). We therefore derive our sufficient conditions in a DSGE model which contains many of these real and nominal frictions to make our results easily applicable in the present literature.

Having solved the problem of ensuring finite values for objective functions in the presence of stochastic trends, it turns out that we can allow for non-stationary preference shocks in the households' utility function. Hence, we also extend the existing framework for handling long-lasting technology shocks such that the effects of long-lasting preference shocks can be analyzed in DSGE models.

The present paper also addresses the stochastic volatility problem, and here we focus on the case where stochastic volatility is specified in stationary shocks. Extending our results to stochastic volatility in non-stationary shocks requires a specification of the process for the time-varying volatility, and each specification of stochastic volatility will thus give rise to different sufficient conditions. We argue in this paper that the specification of stochastic volatility used in Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*) is ill suited for constructing DSGE models. This is so because the assumed shocks with stochastic volatility in those papers do not have any conditional moments. Hence, we analyze stochastic volatility in stationary shocks and provide guidelines for future research dealing with the specification of stochastic volatility in DSGE models.

The main theoretical results of the paper are as follows. First, in the case of log preferences for the consumption good, the conditions for ensuring finite objective functions for the households and the firms are relatively weak. If long-lasting preference shocks are left out and various boundedness or moment conditions hold, we recover the standard result that a subjective discount factor (β) strictly less than one is sufficient to ensure finite objective functions. Based on this result, none of the papers listed above has an invalid micro foundation due to the trend problem. Second, with power preferences for the consumption good the conditions are more restrictive and the condition $\beta < 1$ may no longer be sufficient. In this case, alternative sufficient conditions are provided. It turns out that the DSGE models of Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*) do not satisfy the sufficient conditions in this paper, or any other known set of conditions for ensuring finite objective functions. This is so due to their specification of stochastic volatility. Therefore, the validity of the micro foundations in the models by Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*) remain to be established.

The rest of this paper is organized as follows. Section 2 presents a small example which illustrates how various trends and stochastic volatility may generate objective functions with infinite values even if $\beta < 1$. We set up our DSGE model in section 3, and section 4 describes the sufficient conditions ensuring that the objective functions of the households and the firms are finite in this DSGE model. To illustrate how to apply the main results from this paper, we then test the validity of the micro foundation in DSGE models from the literature in section 5. In the interest of space, we limite the analysis to six models. Section 6 concludes.

2 A small example

We start with a small example. Consider the standard utility function

$$U_t = \sum_{l=0}^{\infty} \beta^l E_t \left[\xi_{t+l} \frac{(c_{t+l})^{1-\phi} - 1}{1-\phi} \right],$$

where β is the subjective discount factor, $\phi \in \mathbb{R}_+ \setminus \{1\}$ is the relative risk aversion, and ξ_t is a preference shock. Furthermore, assume that consumption (c_t) can be decomposed as $c_t \equiv C_t z_t^*$ where C_t is the stationary part of consumption and z_t^* is the non-stationary part. For simplicity in this example we let $\ln z_t^* - \ln z_{t-1}^* = \ln \mu_{z^*,ss} + \epsilon_t$ where ϵ_t is independent and identically distributed according to the normal distribution, i.e. $\epsilon_t \sim \mathcal{NID}\left(0, \sigma_\epsilon^2\right)$. We start by letting $\xi_t = 1$ for all t and all realizations to solely focus on the trend problem. These assumptions imply that

$$U_t = (z_t^*)^{1-\phi} \sum_{l=0}^{\infty} \left(\beta \mu_{z^*,ss}^{(1-\phi)} \right)^l E_t \left[\frac{\exp\left\{ (1-\phi) \sum_{i=0}^{l} \epsilon_{t+i} \right\} (C_{t+l})^{1-\phi}}{1-\phi} \right] - \frac{1}{1-\phi} \sum_{l=0}^{\infty} \beta^l$$

Assume for simplicity that there exist a function $B\left(z_t^*\right)$ such that $\left| C_{t+l}^{1-\phi} \right| \leq B\left(z_t^*\right) < \infty$ for $l \in \{1, 2, ...\}$ and all realizations. Using standard inequalities and the moment generating function for the normal distribution it follows that

$$|U_t| \leq \left| \frac{B\left(z_t^*\right)}{1-\phi} \right| \sum_{l=0}^{\infty} \left(\beta \mu_{z^*,ss}^{(1-\phi)} \exp\left\{ \frac{1}{2} (1-\phi)^2 \sigma_\epsilon^2 \right\} \right)^l + \left| \frac{1}{1-\phi} \right| \sum_{l=0}^{\infty} \beta^l$$

Hence, $|U_t| < \infty$ if and only if $\beta \mu_{z^*,ss}^{(1-\phi)} \exp\left\{ \frac{(1-\phi)^2}{2} \sigma_\epsilon^2 \right\} < 1$ and $\beta < 1$. Thus, the presence of either a deterministic trend $\left(\mu_{z^*,ss} \neq 1\right)$ or a stochastic trend $\left(\sigma_\epsilon^2 > 0\right)$ implies that the condition $\beta < 1$ may no longer be sufficient to ensure a finite value for the utility function.

To solely consider the stochastic volatility problem, let $z_t^* = 1$ for all t and all realizations and assume that $\xi_t = e^{\sigma_t u_t}$ where $u_t \sim \mathcal{NID}(0,1)$ and σ_t is the time-varying stochastic volatility of preference shocks. This specification of stochastic volatility is the one used by Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*). We show in this paper that this specification of stochastic volatility may easily imply that $E_t\left[\xi_{t+l}\right] = \infty$ for $l \in \{1, 2, ...\}$. If this is the case, $\beta < 1$ is clearly not sufficient to ensure a finite value of the utility function.

The conditions we derive below for ensuring a finite utility function are more general, because we do not impose a particular probability distribution for the structural shocks, and we consider a slightly more general process for the non-stationary shocks.

3 The DSGE model

This section describes our DSGE model where we use the same framework as in Schmitt-Grohé & Uribe (2006). The following two considerations motivate our choice. First, Schmitt-Grohé &

Uribe (2006) show how to derive the exact nonlinear recursive representation of the equilibrium conditions for DSGE models. Thus, when we in section 3 derive sufficient conditions which ensure finiteness of the objective functions in our DSGE model, then these conditions are independent of the approximation method used to solve the model. Second, Schmitt-Grohé & Uribe (2006) use a specification of the labor markets which does not restrict preferences to be separable in consumption and leisure. Hence, their specification offers more flexibility than the specification used in Altig et al. (2005) among others.

The foundation of our DSGE model is the standard neoclassical growth model with four groups of agents: i) households, ii) firms, iii) a government, and iv) a central bank. The economy is driven by mutually independent structural shocks. To this basic structure we add a number of extensions which may be grouped as follows: First, nominal frictions are introduced through: i) sticky wages, ii) sticky prices, iii) a transactional demand for money by households, and iv) a cash-in-advance constraint on a fraction of the firms' wage bill. Second, real frictions are added by assuming: i) adjustment costs related to new investments, ii) a variable capacity utilization rate of the capital stock, iii) habit formation, and iv) imperfect competition in the goods and the labor markets.

3.1 The households

For sake of clarity the presentation of the households' optimization problem is split into three subsections which describe the households' i) preferences, ii) constraints, and iii) first order conditions.[2]

3.1.1 The households' preferences

We start by assuming that the behavior of the households may be described by a representative family with a continuum of members. Each member of this family has the same amount of consumption and hours of work. The family's preferences are specified by a utility function defined over real per capita consumption (c_t) and per capita labor supply (h_t),

$$U_t = E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l} \xi_{t+l} u \left(c_{t+l} - bc_{t-1+l}, h_{t+l} \right). \tag{1}$$

E_t is the conditional expectation given information available at time t and $\beta \in [0, 1[$ is the subjective discount factor. The variable ξ_t denotes stationary preference shocks. We assume that ξ_t is strictly positive for all time periods and all realizations, but we do not impose any particular stochastic process assumption on ξ_t. Thus, the process for ξ_t may include stochastic volatility. The novel feature of our utility function in (1) is the non-stationary exogenous shock, denoted $\varepsilon_{h,t}$, which introduces long lasting preference shocks into the economy. The process for $\varepsilon_{h,t}$ is specified based on the gross growth rate $\mu_{\varepsilon_h,t+1} \equiv \varepsilon_{h,t+1}/\varepsilon_{h,t}$ where we assume

$$\ln \left(\mu_{\varepsilon_h,t+1} \right) = \rho_{\varepsilon_h} \ln \left(\mu_{\varepsilon_h,t} \right) + \epsilon_{\varepsilon_h,t+1}, \tag{2}$$

[2] All the derivations can be found in a technical appendix available on request.

and let $\rho_{\varepsilon_h} \in \left]-1,1\right[$ and $\varepsilon_{h,0} \equiv 1$. The error terms $\{\epsilon_{\varepsilon_h,t}\}_{t=1}^{\infty}$ are assumed to have a mean value of zero and to be independent and identically distributed according to a general probability distribution. We denote this by $\epsilon_{\varepsilon_h,t+1} \sim \mathcal{IID}$. Although the idea of specifying shocks to the households' intertemporal preferences is widely used in literature, the specification in (1) and (2) is new. The process for preference shocks is typically assumed to be stationary in the literature, whereas we assume that the process for $\ln \varepsilon_{h,t}$ is integrated of order one and thus non-stationary. Hence, $\varepsilon_{h,t}$ has a stochastic trend of the form $\exp\left\{\sum_{i=1}^{t} a_i\right\}$ where a_i is a stationary process. This generalization to allow for nonstationary preference shocks is motivated by the findings in Primiceri, Schaumburg & Tambalotti (2006) and Fernández-Villaverde & Rubio-Ramirez (2007*b*), where stationary preference shocks are estimated to be very persistent.[3] In section 2.6 and section 3 of this paper we show that the specification in (1) and (2) is a feasible generalization of the basic framework.

The function $u(\cdot,\cdot)$ in (1) is a period utility index which we assume has the form

$$u\left(c_t - bc_{t-1}, h_t\right) = \frac{\left(\left(c_t - bc_{t-1}\right)^{1-\phi_5}\left(c_t - e_t z_t^*\right)^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)}}{1-\phi_3} \times \left(\phi_6\left(1-h_t\right) + \phi_7 \exp\left\{-\phi_8 \frac{h_t^{1+\phi_9}}{1+\phi_9}\right\}\right)^{\phi_4(1-\phi_3)} - \frac{1}{1-\phi_3}, \tag{3}$$

where $b \in [0,1]$, $\phi_3 \in \left]0,1\right[\cup \left]1,\infty\right[$, $\phi_4 \in \left]0,1\right[$, $\phi_5 = \{0,1\}$, $\phi_6 \geq 0$, $\phi_7 \geq 0$, $\phi_8 \geq 0$, and $\phi_9 \geq 0$. We also require that: i) $\phi_6 \neq 0$ or $\phi_7 \neq 0$, ii) $c_t/c_{t-1} > b$, and iii) $c_t > e_t z_t^*$ for all t and all realizations to ensure that the utility index in (3) is always well-defined. We do emphasize that this utility index should not be considered as an unrestricted function which should be taken to the data. Our purpose is only to set up a utility index which nests the specification typically used in the literature. Table 1 illustrates this point.

Table 1: Restrictions on the Utility Parameters

This table shows the restrictions needed in our utility function in order to get the utility functions in the related papers.

	b	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9
King et al. (1988*b*)	0	$\to 1$	none	0	1	0	none	none
Ireland (2004*a*)	0	$\to 1$	$\frac{1}{2}$	0	0	1	none	0
Altig et al. (2005)	none	$\to 1$	$\frac{1}{2}$	0	0	1	none	1
Schmitt-Grohé & Uribe (2006)	none	none	none	0	1	0	none	none

The parameter b specifies the degree of internal habit effect in the consumption good, and the present habit level is determined by the family's own consumption in the previous period. The variable e_t denotes an external habit effect and differs from the internal habit effect by being exogenous to the representative family. Notice that the external habit is scaled by z_t^* which is an overall measure of technological progress in the economy. Adopting this scaling of e_t ensures

[3] Primiceri et al. (2006) and Fernández-Villaverde & Rubio-Ramirez (2007*b*) specify an AR(1) process for the preference shocks and estimate the persistentcy coefficient in this process to 0.83 and 0.95, respectively.

that the external habit effect does not decline in relation to c_t along the economy's balanced growth path. We leave the form of the external habit effect unspecified, and only require that e_t is a function of stationary variables.[4] Finally, the labor supply in (3) is normalized such that $h_t \in [0, 1[$.

Following the standard assumption in the literature the consumption good is constructed from a continuum of differentiated goods ($c_{i,t}, i \in [0, 1]$) and the aggregation function

$$c_t = \left[\int_0^1 c_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \tag{4}$$

Here, $\eta > 1$ is the intratemporal elasticity of substitution across the differentiated goods. The demand for $c_{i,t}$ with nominal price $P_{i,t}$ is found by solving the following problem

$$\underset{c_{i,t} \geq 0}{Min}\ Cost = \int_0^1 P_{i,t} c_{i,t} di \quad st. \quad \left[\int_0^1 c_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \geq c_t. \tag{5}$$

This implies that the demand for good i is given by

$$c_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} c_t, \tag{6}$$

where $P_t \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di \right]^{1/(1-\eta)}$ is the nominal price index in the ec-onomy. Hence, the inflation rate is given by $\pi_t \equiv P_t / P_{t-1}$.

3.1.2 The constraints on the households

The first constraint on the households originates from basic assumptions about the labor markets. In the framework developed by Schmitt-Grohé & Uribe (2006), labor decisions in the household are assumed to be made by a central authority within the household, which we think of as a union. This union supplies labor monopolistically to a continuum of labor markets, indexed by $j \in [0, 1]$, and faces a labor demand given by $(W_{j,t}/W_t)^{-\tilde{\eta}} h_t^d$ in each market. A derivation of this equation is postponed to the presentation of the firms' optimization problem. At this point it is sufficient to know that: i) $W_{j,t}$ is the nominal wage charged by the union in the j'th labor market, ii) W_t is a nominal wage index, and iii) h_t^d is a measure of the total labor demand in the economy. Both W_t and h_t^d are considered exogenous by the union. Furthermore, we assume that the union determines the wages in each labor market and supplies enough labor to meet demand in all markets. This implies that the total labor supply to market j at time t is given by

$$h_t^j = \left(\frac{w_{j,t}}{w_t} \right)^{-\tilde{\eta}} h_t^d, \tag{7}$$

where $w_{j,t} \equiv W_{j,t}/P_t$ and $w_t \equiv W_t/P_t$. Hence, the total labor supply (h_t) across all markets must satisfy the resource constraint

$$h_t = \int_0^1 h_t^j dj. \tag{8}$$

[4] For instance, we could define $e_t \equiv b \frac{c_{t-1}}{z_t^*}$ since $\frac{c_{t-1}}{z_t^*}$ is stationary. Then $e_t z_t^* = b c_{t-1}$.

The second constraint is also related to the labor markets and describes how the union can change wages. We follow Schmitt-Grohé & Uribe (2006) and assume that in each period the union cannot set the nominal wages optimally in a fraction $\tilde{\alpha} \in [0, 1[$ of randomly chosen labor markets. In these markets the wages are set according to the rule $W_{j,t} = W_{j,t-1} \left(\mu^h_{z^*,t-1} \pi^h_{t-1} \right)^{\tilde{\chi}}$. The parameter $\tilde{\chi} \in [0, 1]$ measures the degree of indexation to $\mu^h_{z^*,t-1} \pi^h_{t-1}$. Here, $\mu^h_{z^*,t-1}$ denotes the households' gross growth rate target in real wages and π^h_{t-1} denotes the households' target for the inflation rate. For instance, if we let $\pi^h_{t-1} = \pi_{t-1}$ and $\mu^h_{z^*,t} = \mu_{z^*,ss}$, then we get the same specification as in Schmitt-Grohé & Uribe (2006). If we further let $\tilde{\chi} = 1$, then we get the specification in Altig et al. (2005).

The third constraint is the law of motion for the physical capital stock (k_t) which is assumed to be owned by the households. We adopt the standard assumption in the literature by letting

$$k_{t+1} = (1 - \delta) k_t + i_t \left(1 - S \left(\frac{i_t}{i_{t-1}} \right) \right). \tag{9}$$

The parameter $\delta \in [0, 1]$ is the depreciation rate for the capital stock and i_t is gross investments. The function $S(\cdot) = \frac{\kappa}{2} (\frac{i_t}{i_{t-1}} - \mu_{i,ss})^2$ with $\kappa \geq 0$ adds investment adjustment costs to the economy. The value of μ_i is determined in such a way that there are no adjustment costs along the economy's balanced growth path.

The fourth constraint is the households' real period by period budget constraint

$$\begin{aligned} & E_t r_{t,t+1} x^h_{t+1} + c_t (1 + l(v_t)) + \Upsilon_t^{-1} (i_t + a(u_t) k_t) + m^h_t + n_t \\ & = \frac{x^h_t + m^h_{t-1}}{\pi_t} + r^k_t u_t k_t + \textstyle\int_0^1 w_{j,t} h_{j,t} dj + \phi_t. \end{aligned} \tag{10}$$

The function $l(\cdot)$ determines the transactional costs imposed on the households based on the velocity $v_t \equiv c_t / m^h_t$. Equation (10) also introduces capital adjustment costs through the function $a(u_t)$, where u_t is the capacity utilization rate of the capital stock. We assume standard functional forms for both functions, i.e.

$$l(v_t) = \phi_1 v_t + \phi_2 / v_t - 2 (\phi_1 \phi_2)^{0,5} \tag{11}$$

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \tag{12}$$

where $\phi_1 \geq 0$ and ϕ_2 are subject to the constraint that $l(\cdot) \geq 0$ and u_t is normalized to 1 in the steady state. Furthermore, we require that $\gamma_1 \geq 0$ and $\gamma_2 \geq 0$. The left hand side of (10) is the households' total expenditures in period t which include: i) state-contingent claims $\left(E_t r_{t,t+1} x^h_{t+1} \right)$, ii) consumption including transaction costs ($c_t [1 + l(v_t)]$), iii) investments and costs of providing capital services to the firms $\left(\Upsilon_t^{-1} (i_t + a(u_t) k_t) \right)$, iv) the real money holdings $\left(m^h_t \right)$, and v) paying transfers (n_t) to the government. Notice that Υ_t^{-1} is the real price in terms of consumption goods for investing and selling capital services to the firms. Changes in Υ_t are often referred to as investment specific shocks or embodied technology shocks, as changes in Υ_t are embodied in the economy's capital stock. The right hand side of (10) is the households' total wealth in period t which consists of: i) pay-off from state-contingent assets purchased in period $t-1$ $\left(x_t^h/\pi_t\right)$, ii) the real money holdings from the previous period $\left(m_{t-1}^h/\pi_t\right)$, iii) income from selling capital services to the firms $\left(r_t^k u_t k_t\right)$, iv) labor income $\left(\int_0^1 w_{j,t}h_{j,t}dj\right)$, and v) dividends received from the firms (ϕ_t). Since all these assumptions and frictions are standard in the literature (see Christiano et al. (2005), Altig et al. (2005), and Schmitt-Grohé & Uribe (2004)), we keep the presentation short and only introduce notation.

The final constraints are a no-Ponzi-game condition and a no-arbitrage restriction on the gross one-period nominal interest rate, $R_{t,1} \geq 1$.

3.1.3 The first order conditions for the households

The households' objective is to maximize the utility function in (1) with respect to the processes for c_t, x_{t+1}^h, h_t, k_{t+1}, i_t, u_t, m_t^h and $w_{j,t}$, given the constraints listed in the previous subsection. The households take the processes for $\varepsilon_{h,t}$, ξ_t, w_t, r_t^k, h_t^d, $r_{t,t+1}$, π_t, π_t^h, ϕ_t, Υ_t, $\mu_{z^*,t}^h$ and n_t as given. This is also the case for the initial conditions for c_0, x_0^h, k_0, i_{-1}, m_{-1}^h, and $w_{j,0}$. We let the Lagrange multipliers for constraints (8), (9), and (10) be $\beta^l \lambda_{t+l} w_{t+l} \tilde{\mu}_{t+l}$, $\beta^l q_{t+l} \lambda_{t+l}$, and $\beta^l \lambda_{t+l}$, respectively, which leads to the following first order conditions:[5]

$$c_t : \lambda_t = \frac{\varepsilon_{h,t}\xi_t u_c\left(c_t - bc_{t-1}, h_t\right) - b\beta E_t \varepsilon_{h,t+1}\xi_{t+1} u_c\left(c_{t+1} - bc_t, h_{t+1}\right)}{1 + l\left(v_t\right) + v_t l'\left(v_t\right)} \tag{13}$$

$$x_{t+1}^h : r_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \text{ for all states} \tag{14}$$

$$h_t : -\varepsilon_{h,t}\xi_t u_h\left(c_t - bc_{t-1}, h_t\right) = \lambda_t w_t \tilde{\mu}_t \tag{15}$$

$$k_{t+1} : \lambda_t = E_t \beta \lambda_{t+1} \left[\frac{r_{t+1}^k u_{t+1} - \Upsilon_{t+l}^{-1} a\left(u_{t+1}\right) + q_{t+1}\left(1-\delta\right)}{q_t}\right] \tag{16}$$

$$\begin{aligned} i_t : \lambda_t = {} & \Upsilon_t q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)\right] \\ & + \beta E_t \Upsilon_t q_{t+1} \lambda_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right) \end{aligned} \tag{17}$$

$$u_t : r_t^k = \Upsilon_t^{-1} a'\left(u_t\right) \tag{18}$$

$$m_t^h : v_t^2 l'\left(v_t\right) = 1 - \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}\right] \tag{19}$$

$$w_{j,t} : w_{j,t} = \begin{cases} \tilde{w}_t & \text{if market } j \text{ is optimizing} \\ w_{j,t-1}\left(\mu_{z^*,t-1}^h \pi_{t-1}^h\right)^{\tilde{\chi}} & \text{else} \end{cases} \tag{20}$$

$$\tilde{w}_t : E_t \sum_{l=0}^{\infty} \left(\beta\tilde{\alpha}\right)^l \lambda_{t+l} h_{t+l}^d \left(\frac{w_{t+l}}{\tilde{w}_t}\right)^{\tilde{\eta}} \left(X_{tl}\right)^{-\tilde{\eta}} \left[\frac{\left(\tilde{\eta}-1\right)}{\tilde{\eta}} \tilde{w}_t X_{tl} + MRS_{t+l}^{h,c}\right] = 0 \tag{21}$$

[5] The variable to the left of each equation denotes the variable for which the equation is a first order condition.

In (21) we use the notation $X_{tl} \equiv \prod_{i=1}^{l} \frac{\left(\mu^h_{z^*,t-1-i}\pi^h_{t+i-1}\right)^{\tilde{\chi}}}{\pi_{t+i}}$ and $MRS_t^{h,c} \equiv \frac{-\varepsilon_{h,t}\xi_t u_h(c_t - bc_{t-1}, h_t)}{\lambda_t}$ to simplify the expression. Equation (13) shows that changes in the households' time preferences through $\varepsilon_{h,t}$ affect the value of λ_t, which may be interpreted as the expected marginal utility of income. The standard expression for the nominal stochastic discount factor appears in (14), and pricing a one-period zero-coupon bond gives the familiar Euler-equation

$$\lambda_t = \beta R_{t,1} E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \right]. \tag{22}$$

We also note that $\tilde{\mu}_t$ is the average markup on wages imposed by the union across labor markets. For an interpretation of the other first order conditions we refer to Schmitt-Grohé & Uribe (2006) and Christiano, Eichenbaum & Evans (2001). Following the procedure described in Schmitt-Grohé & Uribe (2006), the exact recursive representation of (21) is given by $f_t^1 - f_t^2 = 0$, where

$$f_t^1 = \lambda_t h_t^d \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right) \tilde{w}_t + E_t \beta \tilde{\alpha} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} \frac{\pi_{t+1}^{\tilde{\eta}-1} f_{t+1}^1}{\left(\mu^h_{z^*,t}\pi_t^h\right)^{\tilde{\chi}(\tilde{\eta}-1)}} \tag{23}$$

$$f_t^2 = -\varepsilon_{h,t}\xi_t u_h \left(c_t - bc_{t-1}, h_t\right) h_t^d \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} + E_t \beta \tilde{\alpha} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} \frac{\pi_{t+1}^{\tilde{\eta}} f_{t+1}^2}{\left(\mu^h_{z^*,t}\pi_t^h\right)^{\tilde{\chi}\tilde{\eta}}} \tag{24}$$

3.2 The firms

The production in the economy is assumed to be undertaken by a continuum of firms, indexed by $i \in [0,1]$. Here, we adopt the standard assumption that each firm supplies a differentiable good $\left(y_{i,t}^s\right)$ to the goods market which is characterized by monopolistic competition with no exit or entry. Furthermore, all firms have access to the same technology, given as

$$y_{i,t}^s = \begin{cases} F\left(k_{i,t}, z_t h_{i,t}\right) - \psi z_t^* & \text{if } F\left(k_{i,t}, z_t h_{i,t}\right) - \psi z_t^* > 0 \\ 0 & \text{else} \end{cases} \tag{25}$$

where $F(\cdot) \equiv k_{i,t}^{\theta} \left(z_t h_{i,t}\right)^{1-\theta}$ with $\theta \in]0,1[$ and $\psi \geq 0$. Here, $k_{i,t}$ and $h_{i,t}$ denote physical capital and labor services used by the i'th firm, respectively. As in the case of the differentiated consumption goods, firm i's demand in the j'th labor market $\left(h_{i,t}^j\right)$ is given by the solution to the standard problem

$$\underset{h_{i,t}^j \geq 0}{Min}\ Cost = \int_0^1 W_{j,t} h_{i,t}^j dj \quad st. \quad \left[\int_0^1 \left(h_{i,t}^j\right)^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}} \geq h_{i,t}. \tag{26}$$

Here, $W_{j,t}$ is the nominal wage paid to labor services in labor market j. The solution to the problem is

$$h_{i,t}^j = \left(\frac{W_{j,t}}{W_t}\right)^{-\tilde{\eta}} h_{i,t}, \tag{27}$$

where $W_t \equiv \left[\int_0^1 W_t^{1-\tilde{\eta}} dj\right]^{1/(1-\tilde{\eta})}$ is the nominal wage index. Aggregating equation (27) over all producers and defining $\int_0^1 h_{i,t}^j di \equiv h_t^j$ and $\int_0^1 h_{i,t} di \equiv h_t^d$ gives (7), the labor demand faced by the union.

The variable z_t in (25) denotes an aggregate neutral technology shock. We follow Altig et al. (2005) and define z_t^* by the relation $z_t^* \equiv \Upsilon_t^{\theta/(1-\theta)} z_t$. Hence, we may interpret z_t^* as an overall measure of technological progress in the economy, because Υ_t is an embodied technology shock and z_t is a neutral technology shock. Therefore, scaling ψ by z_t^* in (25) ensures that the firms' fixed costs do not decline relative to $F(k_{i,t}, z_t h_{i,t})$ along the economy's balanced growth path. Letting $\mu_{z,t} \equiv z_t/z_{t-1}$ and $\mu_{\Upsilon,t} \equiv \Upsilon_t/\Upsilon_{t-1}$, we assume

$$\ln\left(\frac{\mu_{z,t+1}}{\mu_{z,ss}}\right) = \rho_z \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right) + \epsilon_{z,t+1} \tag{28}$$

$$\ln\left(\frac{\mu_{\Upsilon,t+1}}{\mu_{\Upsilon,ss}}\right) = \rho_\Upsilon \ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right) + \epsilon_{\Upsilon,t+1} \tag{29}$$

and let $z_0 \equiv 1$ and $\Upsilon_0 \equiv 1$. Here, $\epsilon_{z,t+1} \sim \mathcal{IID}$ and $\epsilon_{\Upsilon,t+1} \sim \mathcal{IID}$. We also require $\rho_z \in \,]-1,1[$ and $\rho_\Upsilon \in \,]-1,1[$. Equations (28) and (29) imply that the processes for $\ln z_t$ and $\ln \Upsilon_t$ have stochastic trends and the deterministic trends are $\ln \mu_{z,ss}$ and $\ln \mu_{\Upsilon,ss}$, respectively. These results follow directly from the MA-representations for the processes in (28) and (29). In the case of neutral technology shocks we have $\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right) = a_t$, where a_t is a stationary process. Hence, $\ln z_t = \ln z_{t-1} + \ln \mu_{z,ss} + a_t$.

All firms are assumed to maximize the present value of their nominal dividend payments, denoted $d_{i,t}$. That is, each firm maximizes

$$d_{i,t} \equiv E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} \phi_{i,t+l}, \tag{30}$$

where the expression for the real dividend payments from the i'th firm $(\phi_{i,t})$ is given below in (32). The firms face five constraints when maximizing $d_{i,t}$. The first is related to the good produced by the i'th firm. The total amount of good i is allocated to: i) consumption including transaction costs, ii) public production $(\bar{g}_{i,t})$, iii) investments, and iv) costs of providing capital services to the firms. We make the standard assumption that the aggregation function for the three latter components coincides with the aggregation function for consumption in (4). Hence, with cost minimization in the production of i) aggregate public production, ii) aggregate investments, and iii) aggregate capital services the restriction on the aggregate demand can be written as

$$y_t^d = c_t\left(1 + l\left(v_t\right)\right) + \bar{g}_t + \Upsilon_t^{-1}\left(i_t + a\left(u_t\right) k_t\right). \tag{31}$$

In addition, we assume that the firms satisfy demand, i.e. $y_{i,t}^s \geq y_{i,t}^d \; \forall i \in [0,1]$.

The second restriction is a cash-in-advance constraint on a fraction ν of the firms' payments to workers. Thus, the money demanded by the i'th firm is $m_{i,t}^f = \nu w_t h_{i,t}$. This assumption is also standard in the literature and serves the purpose of motivating demand for money at the firm level.

The third constraint is the budget restriction which gives rise to the expression for real dividends from firm i in period t,

$$\phi_{i,t} = (P_{i,t}/P_t)\, y^d_{i,t} - r^k_t k_{i,t} - w_t h_{i,t} - m^f_{i,t}\left(1 - R^{-1}_{t,1}\right) \tag{32}$$

$$-E_t r_{t,t+1} x^f_{i,t+1} + m^f_{i,t} - \pi^{-1}_t\left(x^f_{i,t} + m^f_{i,t-1}\right).$$

The first term in (32) denotes the real revenue from sales of the i'th good. The firm's expenditures are allocated to: i) purchase of capital services $\left(r^k_t k_{i,t}\right)$, ii) payments to the workers $(w_t h_{i,t})$, and iii) opportunity costs of holding money due to the cash-in-advance constraint $\left(m^f_{i,t}\left(1 - R^{-1}_{t,1}\right)\right)$. The final terms in (32) constitute the change in the firm's real financial wealth.

The fourth constraint introduces staggered price adjustments. We make the standard assumption that in each period a fraction $\alpha \in [0, 1[$ of randomly picked firms are not allowed to set the optimal nominal price of the good they produce. Instead, these firms update their prices according to the rule $P_{i,t} = P_{i,t-1}\left(\pi^f_{t-1}\right)^\chi$, where $\chi \in [0, 1]$ and π^f_{t-1} is the firms' inflation rate target. For instance, if we let $\pi^f_{t-1} = \pi_{t-1}$, then we get the specification used in Schmitt-Grohé & Uribe (2006). If we further let $\chi = 1$, then we get the setup in Altig et al. (2005).

The fifth constraint is a no-Ponze-game condition.

Subject to these constraints, firm i maximizes $d_{i,t}$ with respect to $x^f_{i,t}$, $m^f_{i,t}$, $h_{i,t}$, $k_{i,t}$, and $P_{i,t}$, given the processes for $R_{t,1}$, P_t, w_t, r^k_t, z_t, z^*_t, y^d_t, π^f_t, π_t and the nominal stochastic discount factor between period t and period $t + l$, denoted $r_{t,t+l}$. As in Schmitt-Grohé & Uribe (2006) we assume, without loss of generality that $x^f_{i,t} + m^f_{i,t} = 0$ in all periods and states. Defining $r_{t,t+l} P_{t+l} mc_{i,t+l}$ as the Lagrange multiplier for the constraint $y^s_{i,t} \geq y^d_{i,t}$, the first order conditions are

$$h_{i,t} : mc_{i,t} z_t F_2\left(k_{i,t}, z_t h_{i,t}\right) = w_t\left[1 + \nu\left(1 - \frac{1}{R_{t,1}}\right)\right] \tag{33}$$

$$k_{i,t} : mc_{i,t} F_1\left(k_{i,t}, z_t h_{i,t}\right) = r^k_t \tag{34}$$

$$P_{i,t} : P_{i,t} = \begin{cases} \tilde{P}_t & \text{if firm } i \text{ is optimizing} \\ P_{i,t-1}\left(\pi^f_{t-1}\right)^\chi & \text{else} \end{cases} \tag{35}$$

$$\tilde{P}_t : E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} \alpha^l \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} Y^{-\eta}_{tl} y^d_{t+l}\left[\frac{\eta - 1}{\eta}\frac{\tilde{P}_t}{P_t} Y_{tl} - mc_{i,t+l}\right] = 0 \tag{36}$$

where $Y_{tl} \equiv \prod_{i=1}^{l} \frac{\left(\pi^f_{t+i-1}\right)^\chi}{\pi_{t+i}}$. Following the procedure described in Schmitt-Grohé & Uribe (2006), the exact recursive representation of (36) is given by $\eta x^1_t + (1 - \eta)\, x^2_t = 0$, where

$$x^1_t = y^d_t mc_t \tilde{p}^{-\eta-1}_t + E_t \alpha\beta \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta-1}\left(\frac{\left(\pi^f_t\right)^\chi}{\pi_{t+1}}\right)^{-\eta} x^1_{t+1} \tag{37}$$

$$x_t^2 = y_t^d \tilde{p}_t^{-\eta} + E_t \alpha\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} \left(\frac{\left(\pi_t^f\right)^{\chi}}{\pi_{t+1}}\right)^{1-\eta} x_{t+1}^2 \tag{38}$$

3.3 The government

We follow Schmitt-Grohé & Uribe (2006) and assume that fiscal policy is specified by the following process for aggregate public production

$$\bar{g}_t = z_t^* g_t, \tag{39}$$

where g_t is some unspecified exogenous stationary process. Part of the public production is financed by seigniorage. If we let $m_t \equiv m_t^h + \int_0^1 m_{i,t}^f di$ be the total amount of outstanding real money, then seigniorage is given by $m_t - m_{t-1}/\pi_t$. To keep things simple, we assume that there exist lump-sum transfers (n_t) which are set to ensure that the government's intertemporal budget constraint always holds. Thus, given the process for g_t, this policy regime is Ricardian.

3.4 The central bank

The monetary policy is conducted by the central bank, which adopts a rule for the interest rate or for the money stock. It turns out that the specific nature of these policy rules is unimportant for the validity of the micro foundation, provided that the rules are based on stationary variables. Therefore, we choose not to specify monetary policy explicitly, but simply require that the policy rule should be based on stationary variables.

3.5 Aggregation

Explicit aggregation is necessary in the goods and labor markets. This is due to the differentiated consumption goods and the large number of labor markets. The aggregation in our DSGE model is almost identical to the aggregation described by Schmitt-Grohé & Uribe (2006).

We start by considering the aggregate goods market where the resource constraint reads

$$F\left(u_t k_t, z_t h_t^d\right) - z_t^* \psi = \left[c_t \left(1 + l\left(v_t\right)\right) + \bar{g}_t + \Upsilon_t^{-1}\left(i_t + a\left(u_t\right) k_t\right)\right] s_t \tag{40}$$

$$s_t = (1-\alpha)\, \tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\left(\pi_{t-1}^f\right)^{\tilde{\chi}}}\right)^{\eta} s_{t-1} \tag{41}$$

where $u_t k_t \equiv \int_0^1 k_{i,t} di$ and $h_t^d \equiv \int_0^1 h_{i,t} di$. These equations are derived by summing over all goods while taking into account that i) firms have access to the same technology which is homogenous of degree one, and ii) the ratio $k_{i,t}/h_{i,t}$ is constant across all firms. The state variable s_t is equal to one or greater than one, and in the case of fully flexible prices ($\alpha = 0$) we have $s_t = 1$. Thus, s_t measures the resource costs due to the presence of sticky prices.

The aggregate relations for the firms' first order conditions for labor and capital and total dividend payments, $\phi_t \equiv \int_0^1 \phi_{i,t} di$, are

$$mc_t z_t F_2\left(u_t k_t, z_t h_t^d\right) = w_t \left[1 + \nu\left(1 - \frac{1}{R_{t,1}}\right)\right] \tag{42}$$

$$mc_t F_1\left(u_t k_t, z_t h_t^d\right) = r_t^k \tag{43}$$

$$\phi_t = y_t^d - r_t^k u_t k_t - w_t h_t^d \left[1 + \nu\left(1 - R_{t,1}^{-1}\right)\right] \tag{44}$$

The resource constraint in the aggregate labor market resembles the constraint in the goods market and is given by

$$h_t = h_t^d \tilde{s}_t \tag{45}$$

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}}\left(\frac{\left(\mu_{z^*,t-1}^h \pi_{t-1}^h\right)^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} \tilde{s}_{t-1} \tag{46}$$

Recall that h_t is total labor supply and h_t^d is total labor demand. The state variable $\tilde{s}_t$ is equal to one or greater than one, and in the case of fully flexible wages ($\tilde{\alpha} = 0$) we have $\tilde{s}_t = 1$. Equation (45) therefore implies an unemployment level of $h_t^d(1 - \tilde{s}_t) \geq 0$, a cost of sticky wages.

Aggregating the real money holdings give

$$m_t = m_t^h + \nu w_t h_t^d. \tag{47}$$

Finally, we derive the relationship between the real optimal price $\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}$ and the inflation rate (π_t), and the relationship between the real wage index (w_t) and the optimal real wage ($\tilde{w}_t$)

$$1 = (1-\alpha)\tilde{p}_t^{1-\eta} + \alpha\left(\frac{\left(\pi_{t-1}^f\right)^{\chi}}{\pi_t}\right)^{1-\eta} \tag{48}$$

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}}\left(\frac{\left(\mu_{z^*,t-1}^h \pi_{t-1}^h\right)^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}} \tag{49}$$

3.6 Solving the DSGE model

The three non-stationary exogenous processes in our DSGE model imply that some of the endogenous variables in the model are also non-stationary. One way to deal with this problem is to transform the economy such that we only have equilibrium conditions with stationary variables. The solution to the transformed economy is then easy to approximate by standard methods for DSGE models. We can then transform this approximation back into the original setting and get the desired solution of our DSGE model. Thus, we only need to show how to construct the transformed economy.

We proceed as follows: First, observe that c_t, w_t, $\tilde{w}_t$, $y_{i,t}^d$, y_t^d, $\phi_{i,t}$, ϕ_t, x_t^2, $\bar{g}_t$, n_t, m_t^h, and m_t all are cointegrated with $1/z_t^*$ in such a way that c_t/z_t^*, w_t/z_t^*, etc. are stationary. Likewise,

r_t^k and q_t cointegrate with Υ_t, while i_t, k_{t+1}, and $k_{i,t+1}$ cointegrate with $1/(\Upsilon_t z_t^*)$. Finally, λ_t and f_t^2 cointegrate with $1/(z_t^{*\,(1-\phi_3)(1-\phi_4)-1}\varepsilon_{h,t})$ and $1/(z_t^{*\,(1-\phi_3)(1-\phi_4)}\varepsilon_{h,t})$, respectively. All the remaining variables in the model are stationary – in particular, the labor supply, the interest rate, and the inflation rate. If $\varepsilon_{h,t} = 1$ for all t we obtain the same cointegrating results as in Schmitt-Grohé & Uribe (2006). Next, we transform the variables by multiplying them with their corresponding cointegrating factor. These transformed variables are denoted by the corresponding capital letters, i.e. $C_t \equiv c_t/z_t^*$, $R_t^k \equiv r_t^k \Upsilon_t$, etc. The equilibrium conditions may then be rewritten in terms of the transformed variables in order to get the transformed economy.[6]

It is interesting to note that a long lasting preference shock ($\varepsilon_{h,t}$) does not affect variables such as real consumption and real production in the long run. Only the households' expected marginal value of income (λ_t) and the control variable f_t^2 related to the labor markets are affected in the long run by a shock to $\varepsilon_{h,t}$.

4 Sufficient conditions for a valid micro foundation

This section derives two different set of sufficient conditions which ensure finiteness of the households' and the firms' objective functions. We proceed in the following way. First, we assume that the households' and the firms' objective functions are finite and can be optimized, such that there exists an equilibrium path for the economy. Based on this assumption we derive sufficient conditions which ensure that the objective functions of the households and the firms actually are finite on the equilibrium path. If these sufficient conditions hold, then we are assured that the initial assumption of finite objective functions in the economy hold.

The first set of sufficient conditions we provide in section 4.1 consists of a number of boundedness conditions, integrability conditions, and moment inequalities. Provided these boundedness conditions hold, the moment inequalities for ensuring finite objective functions are the least restrictive among the two different set of sufficient conditions we provide. We then demonstrate in section 4.2 that all but one of these boundedness conditions actually can be verified to hold based on an additional weak assumption. For the remaining case, a second set of sufficient conditions is provided in section 4.3, and here the boundedness conditions are replaced by weak moment requirements. However, this comes at the cost of making the moment inequalities more restrictive than the moment inequalities in the first set of sufficient conditions.

Unless otherwise stated, all proofs are placed in the appendix.

4.1 Sufficient conditions based on boundedness conditions

We introduce the following notation to ease notation below:

$$F_\Upsilon \equiv (1-\phi_4)(1-\phi_3)\frac{\theta}{1-\theta} \tag{50}$$

$$F_z \equiv (1-\phi_4)(1-\phi_3) \tag{51}$$

[6] The list of equilibrium conditions for the untransformed and transformed economies is given in the paper's technical appendix, available on request.

$$f\left(h_t\right) \equiv \left(\phi_6\left(1-h_t\right)+\phi_7 \exp\left\{-\phi_8 \frac{h_t^{1+\phi_9}}{1+\phi_9}\right\}\right)^{\phi_4} \tag{52}$$

We first consider the conditions which ensure that the households' utility function is finite in the case of power preferences for the habit adjusted consumption good.

Proposition 1 (a) *Let $\phi_3 \in \left]0,1\right[\cup \left]1,\infty\right[$. The following conditions are sufficient to ensure that the representative family's utility function is finite:*

1. *For $l \in \{1,2,...\}$ and $\forall$ realizations, there exist a function $B\left(z_t^*\right)$ such that* $\left|\left(\left(C_{t+l}-b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}\left(C_{t+l}-e_{t+l}\right)^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f\left(h_{t+l}\right)^{1-\phi_3}\right| \leq B\left(z_t^*\right) < \infty$
2. $E_t\left[\xi_{t+l}\right] \leq B_1 < \infty$ *for $l \in \{1,2,...\}$*
3. *i)* $E_t\left[\exp\left\{\frac{2\left|\epsilon_{\varepsilon_h,t+1}\right|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$, *ii)* $E_t\left[\exp\left\{\frac{2\left|F_\Upsilon \epsilon_{\Upsilon,t+1}\right|}{1-\rho_\Upsilon}\right\}\right] < \infty$, *and*

   iii) $E_t\left[\exp\left\{\frac{2\left|F_z \epsilon_{z,t+1}\right|}{1-\rho_z}\right\}\right] < \infty$
4. $E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right]\beta < 1$
5. $E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right] E_t\left[\exp\left\{\frac{F_\Upsilon \epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right] E_t\left[\exp\left\{\frac{F_z \epsilon_{z,t+1}}{1-\rho_z}\right\}\right] \beta \mu_{\Upsilon,ss}^{F_\Upsilon} \mu_{z,ss}^{F_z} < 1$

Note first that the bound in condition 1 may dependent on the level of the growth path in the economy (z_t^*). This assumption is imposed in order to show that the lifetime utility function U_t is finite although not necessarily bounded for all parameter values. However, condition 1 is not directly testable for our DSGE model because C_t and h_t are unknown functions of the state variables in the economy. Recall that C_t is the households' consumption expressed in deviation from the stochastic and deterministic growth path in the economy. The variable h_t is the households' labor supply. Nevertheless, in the next section we show that condition 1 is satisfied if the economy is not too far away from its growth path.

All the remaining conditions in proposition 1(a) are easy to check directly given distributional assumptions for the structural shocks.[7] In particular because $M_X(t) \equiv E\left[\exp\{Xt\}\right]$ is known as the moment generating function for X, and the expression for this function is reported in relation to various probability distributions. Note also that the integrability requirments in conditions 3.i) to 3.iii) automatically are satisfied if the moment generating function exists and the probability distribution is symmetric.

In the table below we report some of the most frequently used error distributions and their properties.[8] A survey of more flexible error distributions is given in Hansen, McDonald & Theodossiou (2007). Notice, however, that the Student t-distribution and the Cauchy distribution

[7] In case of normally distributed shocks, $\xi_t = 1$, $\varepsilon_{h,t} = \varepsilon_{\Upsilon,t} = 0$, and $\mu_{\Upsilon,ss} = 1$, our condition 5 in proposition 1(a) reduces to the condition in Burnside (1998) for the price-dividend ratio to be bounded in an asset pricing model. This observation implies that the asset pricing model considered by Burnside (1998) also has a bounded price-dividend ratio in the case of non-normal shocks if condition 3.iii) and condition 5 in proposition 1(a) hold.

[8] The Subbotin distribution is also called the Generalized Error Distribution or the Exponential Power Distribution.

do not have a moment generating function. Hence, conditions 3-5 in proposition 1(a) are never satisfied if these distributions are used for the innovations to long-lasting shocks in our DSGE model.

Table 2: Error distributions

	Logistic	Laplace	Normal	Subbotin
Density, $f(x)$	$\frac{e^{-\frac{x}{\sigma}}}{\sigma\left(1+e^{-\frac{x}{\sigma}}\right)^2}$	$\frac{1}{2\sigma}e^{-\lvert x\rvert/\sigma}$	$\frac{e^{-0.5\left(\frac{x}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$	$\frac{v\exp\{-0.5\lvert z/(\lambda\sigma)\rvert^v\}}{\sigma\lambda 2^{(1+1/v)}\Gamma(1/v)}$
Domain	$x\in\mathbb{R}$	$x\in\mathbb{R}$	$x\in\mathbb{R}$	$x\in\mathbb{R}$
Symmetric	yes	yes	yes	yes
Restrictions	$\sigma>0$	$\sigma>0$	$\sigma>0$	$\sigma>0, v>1, \lambda\equiv\left(\frac{2^{-2/v}\Gamma(1/v)}{\Gamma(3/v)}\right)^{\frac{1}{2}}$
Mean	0	0	0	0
Variance	$\pi^2\sigma^2/3$	$2\sigma^2$	σ^2	σ^2
M.g.f, $M_X(t)$	$\frac{\pi\sigma t}{\sin(\pi\sigma t)}$	$\frac{1}{1-\sigma^2t^2}$	$e^{0.5\sigma^2t^2}$	$\sum_{k=0}^{\infty}\frac{\left(2^{\frac{1}{v}}\lambda\sigma\right)^k\left[1+(-1)^k\right]\Gamma((k+1)/v)}{2\Gamma(1/v)\Gamma(k+1)}$

The fourth condition in proposition 1(a) is a moment inequality, and if there are no long-lasting preference shocks in the model (i.e. $\epsilon_{\varepsilon_h,t}=0$ for t and all realizations), then this inequality reduces to the standard condition $\beta<1$. For the distributions mentioned in table 2 it holds that $E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h}t+1}{1-\rho_{\varepsilon_h}}\right\}\right]>1$. Thus, for these distributions the presence of long-lasting preference shocks imposes a stronger restriction on the value of β. The intuition behind this result is that the households' discount factor (β) now must offset two effects in order to get a finite value for the utility function: i) the infinite utility stream, and ii) the stochastic trend generated by $\varepsilon_{h,t}$.

We interpret the fifth condition in proposition 1(a) by first considering the case with only deterministic trends in the processes for technology. Hence, the condition reduces to the following inequality $\beta\mu_{\Upsilon,ss}^{F_\Upsilon}\mu_{z,ss}^{F_z}<1$. In general, $\mu_{\Upsilon,ss}>1$ and $\mu_{z,ss}>1$, but the sign of F_Υ and F_z depends on the value of ϕ_3. If $\phi_3>1$, which is probably the most realistic case, then $F_\Upsilon,F_z<0$ and the deterministic trends operate as additional discount factors in the households' utility function. If $\phi_3<1$ then $F_\Upsilon,F_z>0$ and the opposite is the case. If our DSGE model does not have deterministic growth in embodied technology ($\mu_{\Upsilon,ss}=1$) then the condition is $\beta\mu_{z,ss}^{F_z}<1$, exactly the same condition as in King et al. (1988*a*) where $\beta^*\equiv\beta\mu_{z,ss}^{F_z}$ is called the effective rate of time preference.[9] On the other hand, if we only have stochastic trends and no deterministic trends in our DSGE model $\left(\text{i.e. } \mu_{\Upsilon,ss}=\ \mu_{z,ss}=1\right)$, then condition 5 reduces to the moment inequality

$$E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h}t+1}{1-\rho_{\varepsilon_h}}\right\}\right]E_t\left[\exp\left\{\frac{F_\Upsilon\epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right]E_t\left[\exp\left\{\frac{F_z\epsilon_{z,t+1}}{1-\rho_z}\right\}\right]\beta<1,$$

which is an even more restrictive condition on β than condition 4. Hence, adding deterministic trends to a model with stochastic trends can be recommended based on proposition 1(a), because the deterministic trends are most likely to operate as additional discount factors.

[9]However, in King et al. (1988*b*) where the neoclassical growth model is extended with stochastic trends, the authors do not state the conditions for finiteness of the household's utility function.

The next proposition considers the limiting case where $\phi_3 \to 1$, which implies log preference for the habit adjusted consumption good and separability between this good and labor supply.

Proposition 2 (a) *Let $\phi_3 \to 1$. The following conditions are sufficient to ensure that the representative family's utility function is finite:*

1. *i)* $\left|\ln\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)\right| \leq B < \infty$, *ii)* $\left|\ln\left(C_{t+l} - e_{t+l}\right)\right| \leq B < \infty$, *and iii)* $\left|\ln f\left(h_{t+l}\right)\right| \leq B < \infty$ *for* $l \in \{1, 2, ...\}$ *and* $\forall$ *realizations*
2. $E_t\left[\xi_{t+l}\right] \leq B_1 < \infty$ *for* $l \in \{1, 2, ...\}$
3. $E_t\left[\exp\left\{\frac{2\left|\epsilon_{\varepsilon_h,t+1}\right|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$
4. $E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h t+1}}{1-\rho_{\varepsilon_h}}\right\}\right]\beta < 1$

Again, conditions 1.i) to 1.iii) in proposition 2(a) are not directly testable, because C_t and h_t are unknown functions of the state variables in the economy. We return to these conditions in the next section. Conditions 3-4 in proposition 2(a) are clearly less restrictive than conditions 3-5 in proposition 1(a). These weaker restrictions arise because the log preferences for the consumption good transform the stochastic trend in consumption from $\exp\{\sum_{i=t}^{\infty} a_i\}$ to $\sum_{i=t}^{\infty} a_i$, where $\{a_i\}_{i=1}^{\infty}$ is a stationary process with zero mean. Thus, with log preferences the requirements on β are less restrictive than with power preferences, because β does not have to offset the effect of a stochastic trend in consumption. Without preference shocks, the requirement is $\beta < 1$, along with conditions 1.i) to 1.iii) and condition 2.

Finally, the conditions for ensuring finiteness of the i'th firm's objective function are stated in proposition 3(a).

Proposition 3 (a) *The following conditions are sufficient to ensure that the present value of the i'te firm's nominal dividend payments is finite:*

1. *i)* $\left|\Lambda_{t+l}\right| \leq B < \infty$, *and ii)* $\left|\Phi_{i,t+l}\right| \leq B < \infty$ $\forall i, l = \{1, 2, ...\}$ *and* $\forall$ *realizations*
2. *i)* $E_t\left[\exp\left\{\frac{2\left|\epsilon_{\varepsilon_h,t+1}\right|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$, *ii)* $E_t\left[\exp\left\{\frac{2\left|F_\Upsilon \epsilon_{\Upsilon,t+1}\right|}{1-\rho_\Upsilon}\right\}\right] < \infty$, *and iii)* $E_t\left[\exp\left\{\frac{2\left|F_z \epsilon_{z,t+1}\right|}{1-\rho_z}\right\}\right] < \infty$
3. $E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right] E_t\left[\exp\left\{\frac{F_\Upsilon \epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right] E_t\left[\exp\left\{\frac{F_z \epsilon_{z,t+1}}{1-\rho_z}\right\}\right] \beta \mu_{\Upsilon,ss}^{F_\Upsilon} \mu_{z,ss}^{F_z} < 1$

In relation to proposition 3(a), recall that Λ_t is the households' expected marginal value of income and $\Phi_{i,t}$ is real dividend payments from firm i. Both variables are expressed in deviation from the stochastic and deterministic growth path in the economy. The important thing to notice is that conditions 2 and 3 in proposition 3(a) are both satisfied if either the conditions in proposition 1(a) or the conditions in proposition 2(a) hold. In the latter case, this follows from the fact that $\phi_3 \to 1$ implies that $F_\Upsilon \to 0$ and $F_z \to 0$.

We summarize this section by noticing that proposition 1(a) to 3(a) only rely on the following five properties from our DSGE model: i) the specification of the households' utility functions, ii) the co-integrating results for consumption (c_t) and firms' real profit $\left(\phi_{i,t}\right)$, iii) a stationary labor supply, iv) the law of motion for z_t^* and $\varepsilon_{h,t}$, and v) a complete market of state contingent claims. Hence, proposition 1(a) to 3(a) may be applied in relation to all DSGE models with these five properties, even though these models differ from our DSGE model along other dimensions. We highlight this result in the following corollary:

Corollary 1 *Proposition 1(a) to 3(a) are valid for all DSGE models with the following characteristics:*

1. *The utility function in (1) and (3)*
2. *It holds that $c_t = C_t z_t^*$ and $\phi_{i,t} = \Phi_{i,t} z_t^*$*
3. *The process for h_t is stationary*
4. $z_t^* \equiv \Upsilon_t^{\theta/(1-\theta)} z_t$, $\ln \mu_{i,t} = \rho_i \ln \mu_{i,t} + \epsilon_{i,t}$ *where $\epsilon_{i,t}$ are iid. and mutually independent and independent of ξ_t, for $i = \{z, \Upsilon, \varepsilon_h\}$*
5. *A complete market of state contingent claims*

4.2 Evaluating the boundedness conditions

This section examines the boundedness conditions in the three propositions from above in greater detail. For this purpose we impose the additional assumption:

Assumption 1 *All variables in the economy, except exogenous state variables, must never be further than the distance $0 < D\left(z_t^*\right) < \infty$ away from the economy's stochastic and deterministic growth path.*

Assumption 1 means that all variables in the transformed economy except exogenous state variables are bounded from above and below by $D\left(z_t^*\right)$. We exclude exogenous state variables in Assumption 1 because assuming that these state variables should be bounded might contradict the specified law of motions for these variables. Note that Assumption 1 does not rule out an infinite consumption level, for instance, since the consumption level (c_t) is given by the relation $c_t = C_t z_t^*$ and the aggregated measure of technology z_t^* may tend to infinity for $t \to \infty$. It is important to realize that Assumption 1 is implicitly used when solving DSGE models by local procedures like the log-linearization approach. This follows from the fact that these solution methods only are reliable if the economy does not get too far away from the approximation point which is the economy's growth path. Hence, Assumption 1 is actually fairly standard.

We proceed with the following three Lemmas.

Lemma 1 *Let $\xi_t \nrightarrow 0$ for all t and all realizations. If Assumption 1 also holds then*

1. $|\Lambda_{t+l}| \leq B < \infty$ *for* $l \in \{1, 2, ...\}$ *and* $\forall$ *realizations*
2. $|\Phi_{i,t+l}| \leq B < \infty \; \forall \; i, l \in \{1, 2, ...\}$ *and* $\forall$ *realizations*
3. $|\ln f(h_{t+l})| \leq B < \infty$ *for* $l \in \{1, 2, ...\}$ *and* $\forall$ *realizations*
4. $\left| \left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right)^{(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{1-\phi_3} \right| \leq B(z_t^*) < \infty$ *for* $l \in \{1, 2, ...\}$ *and* $\forall$ *realizations*

Lemma 2 (Only external habit effect) $|\ln (C_{t+l} - e_{t+l})| \leq B < \infty$ *for* $l \in \{1, 2, ...\}$ *and for all realizations if:*

1. *Assumption 1 holds*
2. $\xi_t \nrightarrow 0$ *for all* t *and all realizations*
3. $\phi_5 = 1$

Lemma 3 (No internal or external habit effect) $\left|\ln \left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)\right| \leq B < \infty$ *for* $l \in \{1, 2, ...\}$ *and for all realizations if:*

1. *Assumption 1 holds*
2. $\xi_t \nrightarrow 0$ *for all* t *and all realizations*
3. $\phi_5 = 0$
4. $b = 0$

In these Lemmas we require in addition to Assumption 1 that the process for the stationary preference shocks (ξ_t) do not tend to 0.

The implication of Lemma 1 to 3 is that all but one of the boundedness conditions from proposition 1(a) to proposition 3(a) are satisfied. The exception is the case with log preference for the consumption good and an internal habit effect ($b > 0$). Here, we cannot show based on Assumption 1 that $\left|\ln \left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)\right| \leq B < \infty$ for $l \in \{1, 2, ...\}$ and for all realizations, because we might have $C_{t+l} \to bC_{t+l-1}/\mu_{z^*,t+l}$, or equivalently, $c_{t+l}/c_{t+l-1} \to b$. Thus, log preferences and an internal habit effect may cause problems. However, this problem can be severely reduced if the researcher restricts the upper limit of b in such a way that the growth rate in consumption (c_t/c_{t-1}) never gets close to b. For instance, the upper limit of b may be restricted to 0.9, implying that we cannot have growth rates in consumption below -10%

during a given time period. Since one time period in DSGE models normally corresponds to one quarter, this is clearly a very weak restriction.

On the other hand, with power preferences for the consumption good, then all the required boundedness conditions are satisfied even if $b > 0$. This comes at the cost that the moment inequalities in proposition 1(a) are more restrictive than those in proposition 2(a). A final alternative is to have log preferences for the consumption good and an external habit effect ($\phi_5 = 1$). This combination has the advantage that proposition 2(a) still applies, and the boundedness conditions can be verified given the stated assumptions.

4.3 Sufficient conditions based on moment conditions

The previous section shows that we are unable to verify one boundedness condition in relation to proposition 2(a) when an internal habit effect is present ($b > 0$). Hence, using proposition 2(a) in this case requires that the bound $\left|\ln\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)\right| \leq B < \infty$ for $l \in \{1, 2, ...\}$ and for all realizations is imposed as an untestable assumption. This can be considered somewhat unsatisfactory because the boundedness condition in itself is quite restrictive. We therefore provide a second set of sufficient conditions in this section, and here we do not require any boundedness conditions to hold. The propositions in this section are denoted proposition 1(b) to 3(b) to emphasize their close connection to the results from section 4.1.

We start by considering the problematic case with log-preferences for the habit adjusted consumption good.

Proposition 2 (b) *Let $\phi_3 \to 1$. The following conditions are sufficient to ensure that the representative family's utility function is finite: For some $p_1, p_2 > 1$ with $\frac{1}{p_1} + \frac{1}{p_2} = 1$, we have*

1. *i)* $E_t\left[\left|\ln\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)\right|^{p_2}\right] \leq B < \infty$, *ii)* $E_t\left[\left|\ln\left(C_{t+l} - e_{t+l}\right)\right|^{p_2}\right] \leq B < \infty$, *and iii)* $E_t\left[\left|\ln f\left(h_{t+l}\right)\right|^{p_2}\right] \leq B < \infty$ *for* $l \in \{1, 2, ...\}$
2. $E_t\left[\xi_{t+l}^{p_1}\right] \leq B_1 < \infty$ *for* $l \in \{1, 2, ...\}$
3. $E_t\left[\exp\left\{\frac{2p_1\left|\epsilon_{\varepsilon_h,t+1}\right|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$
4. $\left(E_t\left[\exp\left\{\frac{p_1\epsilon_{\varepsilon_h}t+1}{1-\rho_{\varepsilon_h}}\right\}\right]\right)^{\frac{1}{p_1}}\beta < 1$

Conditions 1.i) to 1.iii) in proposition 2(b) replaces the boundedness conditions in proposition 2(a) by much weaker moment conditions. The cost of having these weaker conditions are as follows. First, condition 2 in proposition 2(b) requires that the stationary preference shock (ξ_t) has a bounded moment of order p_2 compared to only the first moment being bounded in proposition 2(a). Second, the integrability condition in condition 3 is slightly more restrictive than the corresponding integrability condition in proposition 2(a). Third, the moment inequality in condition 4 is also more restrictive than the corresponding moment inequality in proposition

2(a), because the function $\left(E\left[f\left(X\right)^{p_1}\right]\right)^{\frac{1}{p_1}}$ is increasing in p_1. Thus, the researcher faces a trade-off when setting the values of p_1 and p_2. Small values of p_1 are obviously desirable in relation to condition 2 to 4, but this comes at the cost of having to assume that the fourth ($p_2 = 4$), eighth ($p_2 = 8$) or even higher moments in condition 1 are bounded. Since we cannot verify any of these bounds for the moment requirements in condition 1, we therefore recommend to let $p_1 = p_2 = 2$. In this case, only second moments in condition 1 must be bounded which seems to be a reasonable assumption.

Imposing the boundedness condition in proposition 2(a) avoids this trade-off. Therefore, when it is possible to verify the boundedness conditions in proposition 1(a) to 3(a), we prefer to use these propositions, because they give rise to the weakest moment inequalities. For sake of completeness, the corresponding version of proposition 1(a) and 3(a) are given in the appendix.

5 Testing the validity of the micro foundation in six DSGE models

This section examines the validity of the micro foundations for the DSGE models in the following six papers: i) King et al. (1988*b*), ii) Ireland (2004*a*), iii) Altig et al. (2005), iv) Schmitt-Grohé & Uribe (2006), v) Fernández-Villaverde & Rubio-Ramírez (2007*a*), and vi) Justiniano & Primiceri (2005). To evaluate the boundedness conditions, we use the results in Lemma 1 to 3, assuming that all the appropriate assumptions for these lemmas hold.

First, consider the models by King et al. (1988*b*) and Ireland (2004*a*). These models have log preferences for the consumption good and no internal habit effect ($b = 0$). Thus, the problem with log preferences and internal habit formation is not present in these models. Moreover, the two models do not have preference shocks or embodied technology shocks. Hence, the standard condition $\beta < 1$ is sufficient to ensure finite objective functions in these models. Both papers meet this condition.

The models by Altig et al. (2005) and Schmitt-Grohé & Uribe (2006) also use log preferences for the consumption good, but include an internal habit effect.[10] As mentioned above, this combination may be a problem for one boundedness condition in proposition 2(a) if the internal habit effect is very strong. The highest estimate of b in Altig et al. (2005) is 0.73, and Schmitt-Grohé & Uribe (2006) set the value of b to 0.69. Both papers use quarterly data, and it therefore seems reasonable to assume that the boundedness conditions in proposition 2(a) hold. Alternatively, proposition 2(b) can be used if it assumed that $E_t\left[\left|\ln\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)\right|^2\right] \leq B < \infty$ and $E_t\left[\left|\ln f\left(h_{t+l}\right)\right|^2\right] \leq B < \infty$ for $l = \{1, 2, ...\}$. None of the papers include preference shocks in the utility function. Hence, the models in Altig et al. (2005) and Schmitt-Grohé & Uribe (2006) also satisfy the sufficient conditions in proposition 2(a) or 2(b), because they impose $\beta < 1$.

Finally, we test the validity of the micro foundation in the models by Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*). We cannot use either versions of proposition 1 to 3 for this purpose, because i) preference shocks enter differently in their utility functions, and ii) they include stochastic volatility in the non-stationary technology shocks.

[10] The DSGE model by Schmitt-Grohé & Uribe (2006) is calibrated to log preferences, even though it is set up to encompass also power preferences.

The primary reason we did not nest their DSGE models into our model is their specification of stochastic volatility. As we will show, their specification implies that $E_t\left[\xi_{t+l}^k\right] = \infty$ and $E_t\left[\mu_{z,t+l}^k\right] = \infty$ for $k \in \{1, 2, ...\}$ and $l \in \{1, 2, ...\}$. In particular, $E_t\left[\xi_{t+1}\right] = \infty$ and $E_t\left[\mu_{z,t+1}\right] = \infty$ are unfortunate properties, because they question the existence of a well-defined equilibrium outside the non-stochastic steady state in their models. Hence, we illustrate the critical component in both models by first testing a reduced version of their models where there is a constant labor supply and no stochastic volatility in the non-stationary technology shocks. The general models are tested afterwards. Given these restrictions, our utility function in (1) and (2) nests the utility function used in Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007*a*), because

$$U_t = E_t \sum_{l=0}^{\infty} \beta^l \xi_{t+l} \ln\left(c_{t+l} - bc_{t+l-1}\right), \tag{53}$$

and the stationary preference shocks can be assumed to have the form

$$\ln \xi_{t+1} = \rho_\xi \ln \xi_t + \sigma_{\xi,t+1}\epsilon_{\xi,t+1} \tag{54}$$

$$\ln \sigma_{\xi,t+1} = \rho_{\sigma_\xi} \ln \sigma_{\xi,t} + \epsilon_{\sigma_\xi,t+1} \tag{55}$$

Here, $\epsilon_{\xi,t} \sim \mathcal{NID}\left(0, Var\left(\epsilon_{\xi,t}\right)\right)$ and $\epsilon_{\sigma_\xi,t} \sim \mathcal{NID}\left(0, Var\left(\epsilon_{\sigma_\xi,t}\right)\right)$. Moreover, $\epsilon_{\xi,t+1}$ and $\epsilon_{\sigma_\xi,t+1}$ are mutually independent. In order to apply proposition 2(a), we first need to show that $E_t\left[\xi_{t+l}\right] \leq B_1 < \infty$ for $l \in \{1, 2, ...\}$. For this purpose, consider the following proposition:

Proposition 4 *Let Z_1 and Z_2 be independent. For $k \in \{1, 2, ...\}$, $Z_1 \sim N\left(0, \sigma_1^2\right)$, and $Z_2 \sim N\left(0, \sigma_2^2\right)$ the moment $E\left(\exp\left\{ke^{Z_1}Z_2\right\}\right)$ does not exist.*

Proof. $E\left(\exp\left\{ke^{Z_1}Z_2\right\}\right) = \sum_{\substack{i,j\in\{0,\infty\}\\ i\neq j}} \int_{-j}^{i} \sum_{\substack{n,m\in\{0,\infty\}\\ n\neq m}} \int_{-m}^{n} \frac{1}{2\pi\sigma_1\sigma_2} \exp\left\{g\left(z_1, z_2\right)\right\} dz_1 dz_2$ where $g\left(z_1, z_2\right) \equiv ke^{z_1}z_2 - 0.5\left(z_1/\sigma_1\right)^2 - 0.5\left(z_2/\sigma_2\right)^2$. All four integrals must be finite in order for the moment to be finite. But $g\left(z_1, z_2\right) \to \infty$ for $z_1 \to \infty$ because exponential functions grow faster than power functions. For instance, this imply that the integral $\int_0^\infty \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_1} \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left\{g\left(z_1, z_2\right)\right\} dz_1 dz_2$ is infinite. ∎

Based on (54) and (55) we have

$$\xi_{t+1} = \xi_t^{\rho_\xi} \exp\left\{\sigma_{\xi,t}^{\rho_{\sigma_\xi}} \exp\left\{\epsilon_{\sigma_\xi,t+1}\right\} \epsilon_{\xi,t+1}\right\}. \tag{56}$$

From the result in proposition 4 it follows that $E_t\left[\xi_{t+1}^k\right] = \infty$, and by the law of iterated expectations $E_t\left[\xi_{t+l}^k\right] = \infty$ for $k \in \{1, 2, ...\}$ and $l \in \{1, 2, ...\}$. Thus, the preference shocks in (54) and (55) imply that the sufficient conditions for ensuring a finite utility function in proposition 2(a) or poposition 2(b) do not hold. However, if we exclude stochastic volatility in the preference shock $\left(\epsilon_{\sigma_\xi,t} = 0 \text{ for all } t\right)$ then $E_t\left[\xi_{t+l}^k\right] \leq B_1 < \infty$ for $k \in \{1, 2, ...\}$ and $l \in \{1, 2, ...\}$.

The process for $\mu_{z,t}$ in Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007a) is similar to (54) and (55) with $\xi_t \equiv \mu_{z,t}/\mu_{z,ss}$. Hence, (56) and proposition 4 imply that $E_t\left[\mu_{z,t+l}^k\right] = \infty$ for $k \in \{1, 2, ...\}$ and $l \in \{1, 2, ...\}$, as claimed above.

The unrestricted models in Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007a) are tested in the technical appendix to this paper. However, we reach a similar conclusion, saying that we cannot show that the objective functions in these models are finite.[11] Again, the specification of stochastic volatility in these models generate the negative result, because it implies an infinite expected value for the preference shocks in the next period. Thus, the validity of the micro foundation in the models by Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007a) remains to be established.

Based on these considerations we recommend that new DSGE models with stochastic volatility should be specified such that shocks with stochastic volatility at least have finite conditional mean values.

Summing up, we find that four out of the six DSGE models satisfy the sufficient conditions which ensure finite objective functions for the households and the firms. Hence, we are assured that these four models have a valid micro foundation. On the other hand, the validity of the micro foundation in the two remaining models remains to be established.

6 Conclusion

This paper closes an important gap in the literature by deriving sufficient conditions which ensure the validity of the micro foundation for DSGE models with stochastic trends, deterministic trends, and/or stochastic volatility in stationary shocks. In addition, we show how to introduce long-lasting preference shocks in the households' utility function. This latter feature is new, compared to the existing DSGE models, since these models only specify long-lasting shocks to the economy's production technology.

On an empirical level, future research should be devoted to estimating or calibrating DSGE models with power preferences and trends, because models with trends mostly have been estimated or calibrated based on log preferences for the consumption good. Particularly in finance applications, could this extension be useful, because the degree of relative risk aversion plays an important role for asset decisions. Furthermore, it would also be of great interest to examine whether the introduction of long-lasting preference shocks improves the ability of DSGE models to explain movements in the business cycle. For instance, the results in Primiceri et al. (2006) show that persistent preference shocks are important for explaining movements in the post-war US business cycle.

On a theoretical level, future research should derive sufficient conditions which ensure finite objective functions in DSGE models where the growth rates for the long-lasting shocks evolve according to ARMA(p,q) processes and/or have stochastic volatility.

[11] The stochastic volatility in preference shocks implies that we cannot even verify that the objective functions in Justiniano & Primiceri (2005) and Fernández-Villaverde & Rubio-Ramírez (2007a) are well defined (Stokey, Lucas & Prescott (1989) assumption 9.2)

A Proposition 1(b) and 3(b)

Proposition 1 (b) *Let $\phi_3 \in]0,1[\cup]1,\infty[$. The following conditions are sufficient to ensure that the representative family's utility function is finite: For some $p_1, p_2 > 1$ with $\frac{1}{p_1} + \frac{1}{p_2} = 1$, we have*

1. *For $l \in \{1,2,...\}$ there exist a function $B(z_t^*)$ such that*
$$E_t\left[\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l} - e_{t+l})^{\phi_5}\right)^{p_2(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{p_2(1-\phi_3)}\right] \leq B(z_t^*) < \infty$$
2. $E_t\left[\xi_{t+l}^{p_1}\right] \leq B_1 < \infty$ *for $l \in \{1,2,...\}$*
3. *i)* $E_t\left[\exp\left\{\frac{2p_1|\epsilon_{\varepsilon_h,t+1}|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$, *ii)* $E_t\left[\exp\left\{\frac{2p_1|F_\Upsilon \epsilon_{\Upsilon,t+1}|}{1-\rho_\Upsilon}\right\}\right] < \infty$, *and*
iii) $E_t\left[\exp\left\{\frac{2p_1|F_z \epsilon_{z,t+1}|}{1-\rho_z}\right\}\right] < \infty$
4. $\left(E_t\left[\exp\left\{\frac{\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right]\right)^{\frac{1}{p_1}} \beta < 1$
5. $\left(E_t\left[\exp\left\{\frac{p_1\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right] E_t\left[\exp\left\{\frac{p_1 F_\Upsilon \epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right] E_t\left[\exp\left\{\frac{p_1 F_z \epsilon_{z,t+1}}{1-\rho_z}\right\}\right]\right)^{\frac{1}{p_1}} \beta \mu_{\Upsilon,ss}^{F_\Upsilon} \mu_{z,ss}^{F_z} < 1$

Proposition 3 (b) *The following conditions are sufficient to ensure that the present value of the i'te firm's nominal dividend payments is finite: For some $p_1, p_2 > 1$ with $\frac{1}{p_1} + \frac{1}{p_2} = 1$, we have*

1. $E_t\left[|\Lambda_{t+l}\Phi_{i,t+l}|^{p_2}\right] \leq B < \infty$ *for $l \in \{1,2,...\}$ and $\forall i$*
2. *i)* $E_t\left[\exp\left\{\frac{2p_1|\epsilon_{\varepsilon_h,t+1}|}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$, *ii)* $E_t\left[\exp\left\{\frac{2p_1|F_\Upsilon \epsilon_{\Upsilon,t+1}|}{1-\rho_\Upsilon}\right\}\right] < \infty$, *and*
iii) $E_t\left[\exp\left\{\frac{2p_1|F_z \epsilon_{z,t+1}|}{1-\rho_z}\right\}\right] < \infty$
3. $\left(E_t\left[\exp\left\{\frac{p_1\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right] E_t\left[\exp\left\{\frac{p_1 F_\Upsilon \epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right] E_t\left[\exp\left\{\frac{p_1 F_z \epsilon_{z,t+1}}{1-\rho_z}\right\}\right]\right)^{\frac{1}{p_1}} \beta \mu_{\Upsilon,ss}^{F_\Upsilon} \mu_{z,ss}^{F_z} < 1$

Corollary 2 *Proposition 1(b) to 3(b) are valid for all DSGE models with the following characteristics:*

1. *The utility function in (1) and (3)*
2. *It holds that $c_t = C_t z_t^*$ and $\phi_{i,t} = \Phi_{i,t} z_t^*$*
3. *The process for h_t is stationary*
4. $z_t^* \equiv \Upsilon_t^{\theta/(1-\theta)} z_t$, $\ln \mu_{i,t} = \rho_i \ln \mu_{i,t} + \epsilon_{i,t}$ *where $\epsilon_{i,t}$ are iid. and mutually independent and independent of ξ_t, for $i = \{z, \Upsilon, \varepsilon_h\}$*
5. *A complete market of state contingent claims*

B Proof of proposition 1(a)

It is straightforward to show that

$$U_t = \frac{1}{(1-\phi_3)}\varepsilon_{h,t}\left(z_t^*\right)^{(1-\phi_4)(1-\phi_3)} \times \underbrace{\sum_{l=0}^{\infty}\beta^l\xi_{t+l}\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\left(\left(C_{t+l}-b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}\left(C_{t+l}-e_{t+l}\right)^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} \times f\left(h_{t+l}\right)^{1-\phi_3}}_{D_1}$$

$$-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\underbrace{E_t\sum_{l=0}^{\infty}\beta^l\xi_{t+l}\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}}_{D_2}$$

$$\Updownarrow$$

$$U_t = \frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}D_1 - \varepsilon_{h,t}\frac{1}{(1-\phi_3)}D_2$$

Thus,

$$|U_t| \leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right||D_1| + \left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right||D_2|$$

The remaining part of the proof shows that $|D_1| < \infty$ and $|D_2| \leq B_0 < \infty$. We start with term D_2.

Based on the law of motion for $\varepsilon_{h,t}$ it holds that

$$\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i} = \exp\left\{\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\varepsilon_h}^{i-j}\epsilon_{\varepsilon_h,t+j}\right\}$$

Thus

$$|D_2| \leq \sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right]E_t\left[\xi_{t+l}\right]$$

$$\leq \sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right]B_1$$

By assumption, $E_t\left[\xi_{t+l}\right] \leq B_1 < \infty$ for $l \in \{1, 2, ...\}$

Thus, we only need to show that $\sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] < \infty$. Hence,

$$\sum_{l=0}^{\infty}\beta^l E_t\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}$$

$$= \sum_{l=0}^{\infty}\exp\left\{l\ln\beta+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)\right\}E_t\exp\left\{\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\varepsilon_h}^{i-j}\epsilon_{\varepsilon_h,t+j}\right\}$$

$$= \sum_{l=0}^{\infty}\exp\left\{l\ln\beta+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)\right\}E_t\exp\left\{\sum_{j=1}^{l}\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}$$

because $\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\varepsilon_h}^{i-j}\epsilon_{\varepsilon_h,t+j}=\sum_{j=1}^{l}\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}$

$$=\sum_{l=0}^{\infty}\exp\left\{l\ln\beta+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)\right\}\prod_{j=1}^{l}E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}$$

since $\{\epsilon_{\varepsilon_h,t}\}_{t=1}^{\infty}$ are independent

$$=\sum_{l=0}^{\infty}\exp\left(l\ln\beta+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+\sum_{j=1}^{l}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}\right)$$

We now apply the ratio criterion saying that $\sum_{l=1}^{\infty}a_l$, where $a_l>0$ for all l, is convergent if $q=\lim_{l\to\infty}\frac{a_l}{a_{l-1}}<1$. In our case $a_l\equiv\exp\{X(l)\}$ and the condition is then $\exp\{X(l)-X(l-1)\}<1\iff X(l)-X(l-1)<0$ for $l\to\infty$. Thus we consider:

$X(l)-X(l-1)$

$$=l\ln\beta+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+\sum_{j=1}^{l}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}$$
$$-(l-1)\ln\beta-\sum_{i=1}^{l-1}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)-\sum_{j=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}$$

$$=\ln\beta+\rho_{\varepsilon_h}^{l}\ln\left(\mu_{\varepsilon_h,t}\right)+\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}$$
$$+\sum_{j=2}^{l}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}-\sum_{j=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}$$

$$=\ln\beta+\rho_{\varepsilon_h}^{l}\ln\left(\mu_{\varepsilon_h,t}\right)+\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}$$
$$+\sum_{n=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+n+1}\frac{1-\rho_{\varepsilon_h}^{l+1-(n+1)}}{1-\rho_{\varepsilon_h}}\right\}-\sum_{j=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}$$

change of index: $n=j-1\iff j=n+1$ in the first sum

$$=\ln\beta+\rho_{\varepsilon_h}^{l}\ln\left(\mu_{\varepsilon_h,t}\right)+\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}$$
$$+\sum_{n=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+n+1}\frac{1-\rho_{\varepsilon_h}^{l-n}}{1-\rho_{\varepsilon_h}}\right\}-\sum_{j=1}^{l-1}\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}$$

$$=\ln\beta+\rho_{\varepsilon_h}^{l}\ln\left(\mu_{\varepsilon_h,t}\right)+\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}$$

since $E_t\exp\left\{\epsilon_{\varepsilon_h,t+n+1}\frac{1-\rho_{\varepsilon_h}^{l-n}}{1-\rho_{\varepsilon_h}}\right\}=E_t\exp\left\{\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}=E_t\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l-j}}{1-\rho_{\varepsilon_h}}\right\}$

for $j=n$ because $\{\epsilon_{\varepsilon_h,t}\}_{t=1}^{\infty}$ are identical distributed

and in the limit

$$\lim_{l\to\infty}X(l)-X(l-1)=\lim_{l\to\infty}\left[\ln\beta+\rho_{\varepsilon_h}^{l}\ln\left(\mu_{\varepsilon_h,t}\right)+\ln E_t\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}\right]$$

$$=\ln\beta+\ln\left(\lim_{l\to\infty}E_t\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}\right)$$

where we have used continuity of e^x and Lebesgue's rule of dominated convergence. In detail regarding Lebesgue's rule of dominated convergence, we define

$f_l \equiv \exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^l}{1-\rho_{\varepsilon_h}}\right\}$ and notice that $\lim_{l\to\infty} f_l = \exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1}{1-\rho_{\varepsilon_h}}\right\} \equiv f$ due to continuity of e^x. The requirement for applying Lebesgue's rule of dominated convergence is that there exists a function g where $|f_l| \leq g \;\; a.s.$ where $\int g d\mu < \infty$. Hence consider:

$$|f_l| \leq \exp\left\{\left|\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^l}{1-\rho_{\varepsilon_h}}\right|\right\} \leq \exp\left\{|\epsilon_{\varepsilon_h,t+1}|\,\frac{1+\left|\rho_{\varepsilon_h}\right|^l}{1-\rho_{\varepsilon_h}}\right\} \leq \exp\left\{|\epsilon_{\varepsilon_h,t+1}|\,\frac{2}{1-\rho_{\varepsilon_h}}\right\} \equiv g$$

Thus, $E_t\left[\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1}{1-\rho_{\varepsilon_h}}\right\}\right]\beta < 1$ and $E_t\left[\exp\left\{|\epsilon_{\varepsilon_h,t+1}|\,\frac{2}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$ imply that $|D_2| \leq B_0 < \infty$

Next

$$|D_1| \leq \sum_{l=0}^{\infty}\beta^l E_t \prod_{i=1}^l \mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\xi_{t+l}\times$$
$$\left|\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}\left(C_{t+l}-e_{t+l}\right)^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f\left(h_{t+l}\right)^{1-\phi_3}\right|$$

By assumption, we have for $l \in \{1, 2, ...\}$ and all realizations

$$\left|\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}\left(C_{t+l}-e_{t+l}\right)^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f\left(h_{t+l}\right)^{1-\phi_3}\right| \leq B\left(z_t^*\right) < \infty$$

Hence,$|D_1| \leq B_1 B\left(z_t^*\right)\sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^l \mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\right]$

So, we only need to show that $\sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^l \mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\right] < \infty$. But, the independence of the structural shocks and the previous derivations imply that this infinite sum is finite if

$$E_t\left[\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1}{1-\rho_{\varepsilon_h}}\right\}\right]E_t\left[\exp\left\{\epsilon_{\Upsilon,t+1}\frac{F_\Upsilon}{1-\rho_\Upsilon}\right\}\right]E_t\left[\exp\left\{\epsilon_{z,t+1}\frac{F_z}{1-\rho_z}\right\}\right]\beta\mu_{\Upsilon,ss}^{F_\Upsilon}\mu_{z,ss}^{F_z} < 1$$
$$E_t\left[\exp\left\{|\epsilon_{\varepsilon_h,t+1}|\,\frac{2}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$$
$$E_t\left[\exp\left\{|F_\Upsilon\epsilon_{\Upsilon,t+1}|\,\frac{2}{1-\rho_\Upsilon}\right\}\right] < \infty$$
$$E_t\left[\exp\left\{|F_z\epsilon_{z,t+1}|\,\frac{2}{1-\rho_z}\right\}\right] < \infty$$

Letting $\sum_{l=0}^{\infty}\beta^l E_t\left[\prod_{i=1}^l \mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\right] \equiv M_1 < \infty$ we then have $|D_1| \leq B\left(z_t^*\right)B_1 M_1$.

Summerizing:

We thus have

$$|U_t| \leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right||D_1| + \left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right||D_2|$$

$$\leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right|B\left(z_t^*\right)B_1M_1 + \left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right|B_0 < \infty$$

This completes the proof of $U_t \in \mathbb{R}$. Note that U_t need not be bounded, e.g. let $B\left(z_t^*\right)^{\frac{1}{p_2}} \equiv B \times \left(z_t^*\right)^{1-(1-\phi_4)(1-\phi_3)} > 0$ since $z_t^* > 0$ for all t and all realizations. Then

$|U_t| \leq \left|\frac{\varepsilon_{h,t}}{(1-\phi_3)}\right| B B_1 M_1 z_t^* + \left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right| B_0$

Therefore: increasing z_t^* increases the upper bound for $|U_t|$.

Q.E.D.

C Proof of proposition 2(a)

It is straightforward to show that $\phi_3 \to 1$ implies

$$U_t \to E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\left(1-\phi_5\right)\ln\left(C_{t+l} - bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)$$
$$+E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\left[\ln z_t^* + \textstyle\sum_{i=1}^{l}\ln\mu_{z^*,t+i}\right]$$
$$+E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\phi_5\ln\left(C_{t+l} - e_{t+l}\right)$$
$$+E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\ln f\left(h_{t+l}\right)$$

Using the law of motions for z_t and Υ_t it follows that

$$E_t\left[\textstyle\sum_{i=1}^{l}\ln\mu_{z^*,t+i}\right] = l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss} + \ln\mu_{z,ss}\right) + \frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_z^i$$

So, for ϕ_3 sufficiently close to 1,

$$|U_t| \leq \left|E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\left(1-\phi_5\right)\ln\left(C_{t+l} - bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)\right|$$
$$+\left|\sum_{l=0}^{\infty} \beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right]\left(1-\phi_4\right)\left[\ln z_t^* + l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss} + \ln\mu_{z,ss}\right)\right.\right.$$
$$\left.\left.+\frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_z^i\right]\right|$$
$$+\left|E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\phi_5\ln\left(C_{t+l} - e_{t+l}\right)\right|$$
$$+\left|E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\ln f\left(h_{t+l}\right)\right|$$

$$\leq E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\left(1-\phi_5\right)B$$
$$+\sum_{l=0}^{\infty} \beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right]\left(1-\phi_4\right)\left|\left[\ln z_t^* + l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss} + \ln\mu_{z,ss}\right)\right.\right.$$
$$\left.\left.+\frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_z^i\right]\right|$$

$$+E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l} \xi_{t+l} \left(1-\phi_4\right) \phi_5 B$$
$$+E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l} \xi_{t+l} B$$

since $\left|\ln\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)\right| \leq B < \infty$, $\left|\ln\left(C_{t+l} - e_{t+l}\right)\right| \leq B < \infty$ and $\left|\ln f\left(h_{t+l}\right)\right| \leq B < \infty$ for $l \in \{1, 2, ...\}$ and all realizations. Now recall that $\varepsilon_{h,t+l} = \varepsilon_{h,t}\frac{\varepsilon_{h,t+l}}{\varepsilon_{h,t}} = \varepsilon_{h,t}\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}$. Thus we get

$$= \left(1-\phi_4\right)\left(1-\phi_5\right)\varepsilon_{h,t}E_t \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] E_t\left[\xi_{t+l}\right] B$$
$$+\left(1-\phi_4\right)\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] E_t\left[\xi_{t+l}\right] \left|\left[\ln z_t^* + l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss} + \ln\mu_{z,ss}\right)\right.\right.$$
$$\left.\left.+\ \frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\sum_{i=1}^{l}\rho_{\Upsilon}^{i} + \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i\right]\right|$$
$$+\left(1-\phi_4\right)\phi_5\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] E_t\left[\xi_{t+l}\right] B$$
$$+\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] E_t\left[\xi_{t+l}\right] B$$

$$\leq \left(1-\phi_4\right)\left(1-\phi_5\right)\varepsilon_{h,t}E_t \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] BB_1$$
$$+\left(1-\phi_4\right)\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] B_1 \left|\left[\ln z_t^* + l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss} + \ln\mu_{z,ss}\right)\right.\right.$$
$$\left.\left.+\ \frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\sum_{i=1}^{l}\rho_{\Upsilon}^{i} + \ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i\right]\right|$$
$$+\left(1-\phi_4\right)\phi_5\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] B_1 B$$
$$+\varepsilon_{h,t} \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] B_1 B$$

because $E_t\left[\xi_{t+l}\right] \leq B_1 < \infty$ for $l \in \{1, 2, ...\}$

$$\leq \left(1-\phi_4\right)\left(1-\phi_5\right)\varepsilon_{h,t}BB_1E_t \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right]$$
$$+\left(1-\phi_4\right)\varepsilon_{h,t}B_1 \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right] \left(\left|\ln z_t^*\right| + l\left(\left|\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss}\right| + \left|\ln\mu_{z,ss}\right|\right)\right.$$
$$\left.+\left|\frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\right|\sum_{i=1}^{l}\left|\rho_{\Upsilon}^{i}\right| + \left|\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\right|\sum_{i=1}^{l}\left|\rho_z^i\right|\right)$$
$$+\left(1-\phi_4\right)\phi_5\varepsilon_{h,t}BB_1 \sum_{l=0}^{\infty} \beta^l E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\right]$$

$$+\varepsilon_{h,t} BB_1 \sum_{l=0}^{\infty} \beta^l E_t \left[\prod_{i=1}^{l} \mu_{\varepsilon_h, t+i} \right]$$

All these sum clearly converge if

$$E_t \left[\exp \left\{ \epsilon_{\varepsilon_h, t+1} \frac{1}{1-\rho_{\varepsilon_h}} \right\} \right] \beta < 1$$

and $E_t \left[\exp \left\{ \left| \epsilon_{\varepsilon_h, t+1} \right| \frac{2}{1-\rho_{\varepsilon_h}} \right\} \right] < \infty$. Thus, can write $|U_t| \leq M_0 + |\ln z_t^*| \, M_1 < \infty$ where M_0 and M_1 are constants, and this shows that $|U_t|$ is finite.

Q.E.D.

D Proof of proposition 3(a)

It is straightforward to show that

$$E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} [\left(\frac{P_{i,t+l}}{P_{t+l}} \right) y_{i,t+l}^d - r_{t+l}^k k_{i,t+l} - w_{t+l} h_{i,t+l} - \nu w_{t+l} h_{i,t+l} \left(1 - R_{t+l,1}^{-1} \right)]$$

$$= \frac{P_t z_t^*}{\Lambda_t} E_t \sum_{l=0}^{\infty} \beta^l \Lambda_{t+l} \prod_{i=1}^{l} \mu_{z^*,t+i}^{(1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}} \Phi_{i,t+l}$$

where $\Phi_{i,t+l} \equiv [\left(\frac{P_{i,t+l}}{P_{t+l}} \right) Y_{i,t+l}^d - R_{t+l}^k K_{i,t+l} \mu_{\Upsilon,t+l}^{-1} \mu_{z^*,t+l}^{-1} - W_{t+l} h_{i,t+l} \left(1 + \nu \left(1 - R_{t+l,1}^{-1} \right) \right)]$

Hence

$$\left| E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} [\left(\frac{P_{i,t+l}}{P_{t+l}} \right) y_{i,t+l}^d - r_{t+l}^k k_{i,t+l} - w_{t+l} h_{i,t+l} - \nu w_{t+l} h_{i,t+l} \left(1 - R_{t+l,1}^{-1} \right)] \right|$$

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| \sum_{l=0}^{\infty} \beta^l E_t \left| \Lambda_{t+l} \right| \prod_{i=1}^{l} \mu_{z^*,t+i}^{(1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}} \left| \Phi_{i,t+l} \right|$$

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| \sum_{l=0}^{\infty} \beta^l E_t \left[B^2 \prod_{i=1}^{l} \mu_{z^*,t+i}^{(1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}} \right]$$

since $|\Lambda_{t+l}| \leq B < \infty$ and $|\Phi_{i,t+l}| \leq B < \infty$ for all i, for $l \in \{1, 2, ...\}$ and for all realizations. But, from the proof of proposition 1(a) we know that this sum is convergent if

$$E_t \left[\exp \left\{ \epsilon_{\varepsilon_h,t+1} \frac{1}{1-\rho_{\varepsilon_h}} \right\} \right] E_t \left[\exp \left\{ \epsilon_{\Upsilon,t+1} \frac{F_\Upsilon}{1-\rho_\Upsilon} \right\} \right] E_t \left[\exp \left\{ \epsilon_{z,t+1} \frac{F_z}{1-\rho_z} \right\} \right] \beta \mu_{\Upsilon,ss}^{F_\Upsilon} \mu_{z,ss}^{F_z} < 1$$

and $E_t \left[\exp \left\{ \left| \epsilon_{\varepsilon_h,t+1} \right| \frac{2}{1-\rho_{\varepsilon_h}} \right\} \right] < \infty$, $E_t \left[\exp \left\{ \left| F_\Upsilon \epsilon_{\Upsilon,t+1} \right| \frac{2}{1-\rho_\Upsilon} \right\} \right] < \infty$,
$E_t \left[\exp \left\{ \left| F_z \epsilon_{z,t+1} \right| \frac{2}{1-\rho_z} \right\} \right] < \infty$.

Thus, $\left| E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} \Phi_{i,t} \right| \leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| M < \infty$ which shows that the present value of dividends is finite.

Q.E.D.

E Proof of Lemmas 1 to 3

E.1 Proof of Lemma 1

For 1) and 2)

$|\Lambda_{t+l}| \leq B < \infty$ and $|\Phi_{i,t+l}| \leq B < \infty$ $\forall i$, for $l \in \{1,2,...\}$ and for all realizations follow directly from Assumption 1.

For 3)

Only if $h_t \to 1$, at least through a subsequence, $\phi_6 \neq 0$, and $\phi_7 = 0$ may $\ln f(h_t)$ not be bounded (Recall that $\phi_6 \neq 0$ or $\phi_7 \neq 0$ is maintained throughout). Hence, assume that $h_t \to 1$, $\phi_6 \neq 0$, and $\phi_7 = 0$. The households' first order condition is then given by $\xi_t \phi_4 (1-h_t)^{-1} = \Lambda_t W_t \tilde{\mu}_t$. For a given value of z_t^*, the right hand side of this equation is bounded by Assumption 1. On the other hand, the left hand side tends to infinity for $h_t \to 1$ since $\phi_4 \in]0,1[$ and $\xi_t \nrightarrow 0$. Hence, $h_t \to 1$, $\phi_6 \neq 0$, and $\phi_7 = 0$ violate the first order condition and cannot occur.

For 4)

Note that

$$\left| \left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right)^{(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{1-\phi_3} \right| \leq B(z_t^*) < \infty$$

$$\Updownarrow$$

$$\left| \left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right)^{(1-\phi_4)(1-\phi_3)} \right| \left| f(h_{t+l})^{1-\phi_3} \right| \leq B(z_t^*) < \infty$$

First, consider the case where $\phi_3 < 1$. For a given value of z_t^*, then $\left| \left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right)^{(1-\phi_4)(1-\phi_3)} \right| \leq B(z_t^*) < \infty$ for $l \in \{1,2,...\}$ and all realizations since C_{t+l} is bounded by Assumption 1. Furthermore, $\left| f(h_{t+l})^{1-\phi_3} \right| < \infty$ since $h_{t+l} \in [0,1[$ for $l \in \{1,2,...\}$.

Second, if $\phi_3 > 1$ then $f(h_{t+l})^{1-\phi_3}$ may not be bounded if $h_{t+l} \to 1$, $\phi_6 \neq 0$, and $\phi_7 = 0$, at least through a subsequence. Hence, we assume that $h_{t+l} \to 1$, $\phi_3 > 1$, $\phi_6 \neq 0$ and $\phi_7 = 0$. This implies that the households' first order condition for labor is given by

$$\xi_{t+l} \left[\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right]^{(1-\phi_4)(1-\phi_3)}$$
$$\times (1-h_{t+l})^{\phi_4(1-\phi_3)-1} \phi_4 \phi_6^{\phi_4(1-\phi_3)} = \Lambda_{t+l} W_{t+l} \tilde{\mu}_{t+l}$$

Assume for the moment that $\left| \left[\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right]^{(1-\phi_4)(1-\phi_3)} \right| \leq B(z_t^*) < \infty$, then the left hand side of this equation tends to infinity for $h_{t+l} \to 1$ since $\phi_3 > 1$, $\phi_4 \in]0,1[$ and $\xi_{t+l} \nrightarrow 0$. For a given value of z_t^*, the right hand side of this equation is bounded by Assumption 1. Thus, we cannot have $h_{t+l} \to 1$, $\phi_3 > 1$, $\phi_6 \neq 0$, and $\phi_7 = 0$ if $\left| \left[\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \right]^{(1-\phi_4)(1-\phi_3)} \right| \leq B(z_t^*) < \infty$. Similarly, if we do not have $h_{t+l} \to 1$ but instead $\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}} \right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \to 0$, then the left hand side of this first order condition above tends to infinity, whereas the right hand side is bounded by Assumption 1. Finally, consider the special case where both $h_{t+l} \to 1$ and $\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5} (C_{t+l} - e_{t+l})^{\phi_5} \to 0$. For this purpose notice that

$$\xi_{t+l}\frac{\left[\left(C_{t+l}-b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l}-e_{t+l})^{\phi_5}\right]^{(1-\phi_4)(1-\phi_3)}}{(1-h_{t+l})^{1-\phi_4[1-\phi_3]}}\phi_4\phi_6^{\phi_4(1-\phi_3)} = \Lambda_{t+l}W_{t+l}\tilde{\mu}_{t+l}$$

Thus, this situation could be compatible with the households' first order condition for labor because

$$\frac{\left[\left(C_{t+l}-b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l}-e_{t+l})^{\phi_5}\right]^{(1-\phi_4)(1-\phi_3)}}{(1-h_{t+l})^{1-\phi_4[1-\phi_3]}}\phi_4\phi_6^{\phi_4(1-\phi_3)}$$

would then be bounded provided $(1-\phi_4)(1-\phi_3) = 1-\phi_4(1-\phi_3)$. But, we note that $(1-\phi_4)(1-\phi_3) = 1-\phi_4(1-\phi_3)$ is equivalent to $\phi_3 = 0$. But by assumption $\phi_3 \in]0,1[\cup]1,\infty[$, so this situation cannot happen.

Q.E.D.

E.2 Proof of Lemma 2

If $|\ln(C_{t+l}-e_{t+l})|$ is not bounded, then either $C_{t+l}-e_{t+l} \to \infty$ or $C_{t+l}-e_{t+l} \to 0$, at least through a subsequence. Assumption 1 ensures that C_{t+l} is bounded for a given value of z_t^*, and from the requirement to e_{t+l} it follows that $C_{t+l}-e_{t+l}$ is also bounded. The other case cannot occur either, because $C_{t+l}-e_{t+l} \to 0$ violates the households' first order condition for consumption given by $\frac{\Lambda_{t+l}\left(1+2\left[\phi_1 v_{t+l}-(\phi_1\phi_2)^{0.5}\right]\right)}{(1-\phi_4)} = \frac{\xi_{t+l}}{(C_{t+l}-e_{t+l})}$. Thus, for a given value of z_t^*, the left hand side of this equation is bounded by Assumption 1 and if $C_{t+l}-e_{t+l} \to 0$ then the right hand side tends to infinity.

Q.E.D.

E.3 Proof of Lemma 3

For $b = 0$ we have that $|\ln(C_{t+l})| \leq B < \infty$ is not bounded if $C_{t+l} \to \infty$ or $C_{t+l} \to 0$, at least through a subsequence. Assumption 1 ensures that C_{t+l} is bounded for a given value of z_t^*, and $C_{t+l} \to 0$ violates the households' first order condition for consumption $\frac{\Lambda_{t+l}\left(1+2\left[\phi_1 v_{t+l}-(\phi_1\phi_2)^{0.5}\right]\right)}{(1-\phi_4)} = \xi_{t+l}C_{t+l}^{-1}$. Thus, for a given value of z_t^*, the left hand side of this equation is bounded by Assumption 1. However, the right hand side of the equation tends to infinity for $C_{t+l} \to 0$.

Q.E.D

F Proof of proposition 1(b)

Recall that

$$|U_t| \leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right| |D_1| + \left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right| |D_2|$$

where D_1 and D_2 are defined as in the proof for proposition 1(a). The remaining part of the proof shows that $|D_1| < \infty$ and $|D_2| \leq B_0 < \infty$. We start with term D_2.

Step 1 Proof of $|D_2| \leq B_0 < \infty$

We have directly that from the proof in proposition 1(a) that $|D_2| \leq B_0 < \infty$ if $E_t\left[\exp\left\{\epsilon_{\varepsilon_h,t+1}\frac{1}{1-\rho_{\varepsilon_h}}\right\}\right]\beta < 1$ and $E_t\left[\exp\left\{|\epsilon_{\varepsilon_h,t+1}|\frac{2}{1-\rho_{\varepsilon_h}}\right\}\right] < \infty$

Hence, we have $|D_2| \equiv \left|E_t\sum_{l=0}^{\infty}\beta^l\xi_{t+l}\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}.\right| \leq B_0 < \infty$

Step 2 Proof of $|D_1| < \infty$

$$|D_1| \leq \sum_{l=0}^{\infty}\beta^l E_t\left|\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\xi_{t+l}\right| \times \left|\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l}-e_{t+l})^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{1-\phi_3}\right|$$

$$\leq \sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left|\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)}\xi_{t+l}\right|^{p_1}\right]\right)^{\frac{1}{p_1}} \times \left(E_t\left[\left|\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l}-e_{t+l})^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{1-\phi_3}\right|^{p_2}\right]\right)^{\frac{1}{p_2}}$$

where $\frac{1}{p_1} + \frac{1}{p_2} = 1$ and $p_1, p_2 > 1$ by Hölder's inequality. By assumption, we have for $l \in \{1, 2, ...\}$

$$E_t\left[\left|\left(\left(C_{t+l} - b\frac{C_{t+l-1}}{\mu_{z^*,t+l}}\right)^{1-\phi_5}(C_{t+l}-e_{t+l})^{\phi_5}\right)^{(1-\phi_4)(1-\phi_3)} f(h_{t+l})^{1-\phi_3}\right|^{p_2}\right] \leq B(z_t^*) < \infty$$

Thus

$$\leq \sum_{l=0}^{\infty}\beta^l\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)p_1}\right]E_t\left[\xi_{t+l}^{p_1}\right]\right)^{\frac{1}{p_1}} B(z_t^*)^{\frac{1}{p_2}}$$

since all shocks are mutually independent and $\mu_{\varepsilon_h,t}$, $\mu_{z^*,t}$, and ξ_t are strictly positive

$$\leq B(z_t^*)^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)p_1}\right]B_1\right)^{\frac{1}{p_1}}$$

$E_t\left[\left|\xi_{t+l}\right|^{p_1}\right] = E_t\left[\xi_{t+l}^{p_1}\right] < B_1 < \infty$ for $l \in \{1, 2, ...\}$ and $\forall$ realizations.

So, we only need to show that $\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)p_1}\right]\right)^{\frac{1}{p_1}} < \infty$. Now consider

$$\sum_{l=0}^{\infty}\beta^{l}\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{(1-\phi_4)(1-\phi_3)p_1}\right]\right)^{\frac{1}{p_1}}$$

$$=\sum_{l=0}^{\infty}\exp\{l\ln\beta\}\left(E_t\left[\prod_{i=1}^{l}\exp\left\{p_1\ln\mu_{\varepsilon_h,t+i}+(1-\phi_4)(1-\phi_3)p_1\ln\mu_{z^*,t+i}\right\}\right]\right)^{\frac{1}{p_1}}$$

$$=\sum_{l=0}^{\infty}\exp\{l\ln\beta\}\times$$
$$\left(E_t\left[\prod_{i=1}^{l}\exp\left\{p_1\ln\mu_{\varepsilon_h,t+i}+(1-\phi_4)(1-\phi_3)p_1\ln\left(\mu_{\Upsilon,t+i}^{\frac{\theta}{1-\theta}}\mu_{z,t+i}\right)\right\}\right]\right)^{\frac{1}{p_1}}$$

$$=\sum_{l=0}^{\infty}\exp\{l\ln\beta\}\times$$
$$\left(E_t\left[\prod_{i=1}^{l}\exp\left\{p_1\ln\mu_{\varepsilon_h,t+i}+(1-\phi_4)(1-\phi_3)p_1\left(\tfrac{\theta}{1-\theta}\ln\mu_{\Upsilon,t+i}+\ln\mu_{z,t+i}\right)\right\}\right]\right)^{\frac{1}{p_1}}$$

Let $F_\Upsilon\equiv(1-\phi_4)(1-\phi_3)\frac{\theta}{1-\theta}$ and let $F_z\equiv(1-\phi_4)(1-\phi_3)$ then

$$=\sum_{l=0}^{\infty}\exp\{l\ln\beta\}\left(E_t\left[\prod_{i=1}^{l}\exp\left\{p_1\ln\mu_{\varepsilon_h,t+i}+p_1\left(F_\Upsilon\ln\mu_{\Upsilon,t+i}+F_z\ln\mu_{z,t+i}\right)\right\}\right]\right)^{\frac{1}{p_1}}$$

$$=\sum_{l=0}^{\infty}\exp\{l\left(\ln\beta+F_\Upsilon\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)$$
$$+\textstyle\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+F_\Upsilon\sum_{i=1}^{l}\rho_{\Upsilon}^{i}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)+F_z\sum_{i=1}^{l}\rho_{z}^{i}\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\}$$
$$\times(E_t\left[\exp\left\{\textstyle\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\varepsilon_h}^{i-j}p_1\epsilon_{\varepsilon_h,t+j}\right\}\right]$$
$$\times E_t\left[\exp\left\{\textstyle\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\Upsilon}^{i-j}p_1F_\Upsilon\epsilon_{\Upsilon,t+j}\right\}\right]$$
$$\times E_t\left[\exp\left\{\textstyle\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{z}^{i-j}p_1F_z\epsilon_{z,t+j}\right\}\right])^{\frac{1}{p_1}}$$

$$=\sum_{l=0}^{\infty}\exp\{l\left(\ln\beta+F_\Upsilon\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)$$
$$+\textstyle\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+F_\Upsilon\sum_{i=1}^{l}\rho_{\Upsilon}^{i}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)+F_z\sum_{i=1}^{l}\rho_{z}^{i}\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\}$$
$$\times(E_t\left[\exp\left\{\textstyle\sum_{j=1}^{l}p_1\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}\right]$$
$$\times E_t\left[\exp\left\{\textstyle\sum_{j=1}^{l}p_1F_\Upsilon\epsilon_{\Upsilon,t+j}\frac{1-\rho_{\Upsilon}^{l+1-j}}{1-\rho_{\Upsilon}}\right\}\right]$$
$$\times E_t\left[\exp\left\{\textstyle\sum_{j=1}^{l}p_1F_z\epsilon_{z,t+j}\frac{1-\rho_{z}^{l+1-j}}{1-\rho_{z}}\right\}\right])^{\frac{1}{p_1}}$$

since $\sum_{i=1}^{l}\sum_{j=1}^{i}\rho_{\varepsilon_h}^{i-j}\epsilon_{\varepsilon_h,t+j}=\sum_{j=1}^{l}\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}$ and similarly for the processes for $\mu_{\Upsilon,t}$ and $\mu_{z,t}$

$$=\sum_{l=0}^{\infty}\exp\{l\left(\ln\beta+F_\Upsilon\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)$$

$$
\begin{aligned}
&+\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}\ln\left(\mu_{\varepsilon_h,t}\right)+F_{\Upsilon}\sum_{i=1}^{l}\rho_{\Upsilon}^{i}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)+F_z\sum_{i=1}^{l}\rho_z^{i}\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\\
&+\frac{1}{p_1}\sum_{j=1}^{l}(\ln E_t\exp\left\{p_1\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}+\ln E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\Upsilon}^{l+1-j}}{1-\rho_{\Upsilon}}\right\}\\
&\qquad+\ln E_t\exp\left\{p_1F_z\epsilon_{z,t+j}\frac{1-\rho_z^{l+1-j}}{1-\rho_z}\right\}))\}
\end{aligned}
$$

We now apply the ratio criteria. In our case $a_l\equiv\exp\{X(l)\}$ and the condition is then $\exp\{X(l)-X(l-1)\}<1\iff X(l)-X(l-1)<0$ for $l\to\infty$. Thus we consider:

$$
\begin{aligned}
&X(l)-X(l-1)=\\
&l\left(\ln\beta+F_{\Upsilon}\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)\\
&+\ln\left(\mu_{\varepsilon_h,t}\right)\sum_{i=1}^{l}\rho_{\varepsilon_h}^{i}+F_{\Upsilon}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\sum_{i=1}^{l}\rho_{\Upsilon}^{i}+F_z\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^{i}\\
&+\frac{1}{p_1}\sum_{j=1}^{l}(\ln E_t\exp\left\{p_1\epsilon_{\varepsilon_h,t+j}\frac{1-\rho_{\varepsilon_h}^{l+1-j}}{1-\rho_{\varepsilon_h}}\right\}+\ln E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{\Upsilon,t+j}\frac{1-\rho_{\Upsilon}^{l+1-j}}{1-\rho_{\Upsilon}}\right\}\\
&\qquad+\ln E_t\exp\left\{p_1F_z\epsilon_{z,t+j}\frac{1-\rho_z^{l+1-j}}{1-\rho_z}\right\})\\
&-\\
&(l-1)\left(\ln\beta+F_{\Upsilon}\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)\\
&+\ln\left(\mu_{\varepsilon_h,t}\right)\sum_{i=1}^{l-1}\rho_{\mu_{\varepsilon_h}}^{i}+F_{\Upsilon}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\sum_{i=1}^{l-1}\rho_{\mu_{\Upsilon}}^{i}+F_z\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l-1}\rho_{\mu_z}^{i}\\
&+\frac{1}{p_1}\sum_{j=1}^{l-1}(\ln E_t\exp\left\{p_1\epsilon_{t+j}^{\mu_{\varepsilon_h}}\frac{1-\rho_{\mu_{\varepsilon_h}}^{l-j}}{1-\rho_{\mu_{\varepsilon_h}}}\right\}+\ln E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{t+j}^{\mu_{\Upsilon}}\frac{1-\rho_{\mu_{\Upsilon}}^{l-j}}{1-\rho_{\mu_{\Upsilon}}}\right\}\\
&\qquad+\ln E_t\exp\left\{p_1F_z\epsilon_{t+j}^{\mu_z}\frac{1-\rho_{\mu_z}^{l-j}}{1-\rho_{\mu_z}}\right\})
\end{aligned}
$$

$$
\begin{aligned}
&=\left(\ln\beta+F_{\Upsilon}\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\right)\\
&+\ln\left(\mu_{\varepsilon_h,t}\right)\rho_{\varepsilon_h}^{l}+F_{\Upsilon}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\rho_{\Upsilon}^{l}+F_z\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\rho_z^{l}\\
&+\frac{1}{p_1}\ln E_t\exp\left\{p_1\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}+\frac{1}{p_1}\ln E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{\Upsilon,t+1}\frac{1-\rho_{\Upsilon}^{l}}{1-\rho_{\Upsilon}}\right\}\\
&+\frac{1}{p_1}\ln E_t\exp\left\{p_1F_z\epsilon_{z,t+1}\frac{1-\rho_z^{l}}{1-\rho_z}\right\}
\end{aligned}
$$

using the same arguments as in the proof of proposition 1(a)

Hence

$\lim_{l\to\infty}X(l)-X(l-1)$

$$
\begin{aligned}
&=\ln\beta+F_{\Upsilon}\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}\\
&+\frac{1}{p_1}\ln\left(\lim_{l\to\infty}E_t\exp\left\{p_1\epsilon_{\varepsilon_h,t+1}\frac{1-\rho_{\varepsilon_h}^{l}}{1-\rho_{\varepsilon_h}}\right\}\right)\\
&+\frac{1}{p_1}\ln\left(\lim_{l\to\infty}E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{\Upsilon,t+1}\frac{1-\rho_{\Upsilon}^{l}}{1-\rho_{\Upsilon}}\right\}\right)\\
&+\frac{1}{p_1}\ln\left(\lim_{l\to\infty}E_t\exp\left\{p_1F_z\epsilon_{z,t+1}\frac{1-\rho_z^{l}}{1-\rho_z}\right\}\right)
\end{aligned}
$$

since $\left|\rho_{\varepsilon_h}\right|<1$, $|\rho_{\Upsilon}|<1$, $|\rho_z|<1$ and continuity of the ln-function

$$
\begin{aligned}
&=\ln\beta+F_{\Upsilon}\ln\mu_{\Upsilon,ss}+F_z\ln\mu_{z,ss}+\frac{1}{p_1}\ln\left(E_t\exp\left\{p_1\epsilon_{\varepsilon_h,t+1}\frac{1}{1-\rho_{\varepsilon_h}}\right\}\right)\\
&+\frac{1}{p_1}\ln\left(E_t\exp\left\{p_1F_{\Upsilon}\epsilon_{\Upsilon,t+1}\frac{1}{1-\rho_{\Upsilon}}\right\}\right)+\frac{1}{p_1}\ln\left(E_t\exp\left\{p_1F_z\epsilon_{z,t+1}\frac{1}{1-\rho_z}\right\}\right)
\end{aligned}
$$

by using Lebesgue's rule of dominated convergence where we require that

$E_t\left[\exp\left\{|p_1\epsilon_{\varepsilon_h,t+1}|\,\frac{2}{1-\rho_{\varepsilon_h}}\right\}\right]<\infty,\ E_t\left[\exp\left\{|p_1F_\Upsilon\epsilon_{\Upsilon,t+1}|\,\frac{2}{1-\rho_\Upsilon}\right\}\right]<\infty,$
$E_t\left[\exp\left\{|p_1F_z\epsilon_{z,t+1}|\,\frac{2}{1-\rho_z}\right\}\right]<\infty$

Hence the sum is convergent if
$\left(\beta\mu_{\Upsilon,ss}^{F_\Upsilon}\mu_{z,ss}^{F_z}\right)\left(E_t\left[\exp\left\{\frac{p_1\epsilon_{\varepsilon_h,t+1}}{1-\rho_{\varepsilon_h}}\right\}\right]E_t\left[\exp\left\{\frac{p_1F_\Upsilon\epsilon_{\Upsilon,t+1}}{1-\rho_\Upsilon}\right\}\right]E_t\left[\exp\left\{\frac{p_1F_z\epsilon_{z,t+1}}{1-\rho_z}\right\}\right]\right)^{\frac{1}{p_1}}<1$

For the following, recall that
$|D_1|\leq B\left(z_t^*\right)^{\frac{1}{p_2}}B_1^{\frac{1}{p_1}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{p_1(1-\phi_4)(1-\phi_3)}\right]\right)^{\frac{1}{p_1}}$
and we now know that the the sum convergence, i.e.
$\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\mu_{z^*,t+i}^{p_1(1-\phi_4)(1-\phi_3)}\right]\right)^{\frac{1}{p_1}}\equiv M_1<\infty$

So
$|D_1|\leq B\left(z_t^*\right)B_1M_1$

Summerizing:
We thus have

$$
\begin{aligned}
|U_t| &\leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right||D_1|+\left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right||D_2| \\
&\leq \left|\frac{\varepsilon_{h,t}(z_t^*)^{(1-\phi_4)(1-\phi_3)}}{(1-\phi_3)}\right|B\left(z_t^*\right)^{\frac{1}{p_2}}B_1^{\frac{1}{p_1}}M_1+\left|-\varepsilon_{h,t}\frac{1}{(1-\phi_3)}\right|B_0<\infty
\end{aligned}
$$

Q.E.D.

G Proof of proposition 2(b)

Recall that $\phi_3\to 1$ implies

$$
\begin{aligned}
U_t \to\ & E_t\sum_{l=0}^{\infty}\beta^l\varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\left(1-\phi_5\right)\ln\left(C_{t+l}-bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right) \\
&+\sum_{l=0}^{\infty}\beta^lE_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right]\left(1-\phi_4\right)[\ln z_t^*+l\left(\frac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss}+\ln\mu_{z,ss}\right) \\
&+\frac{\theta}{1-\theta}\ln\left(\frac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\sum_{i=1}^{l}\rho_\Upsilon^i+\ln\left(\frac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i] \\
&+E_t\sum_{l=0}^{\infty}\beta^l\varepsilon_{h,t+l}\xi_{t+l}\left(1-\phi_4\right)\phi_5\ln\left(C_{t+l}-e_{t+l}\right) \\
&+E_t\sum_{l=0}^{\infty}\beta^l\varepsilon_{h,t+l}\xi_{t+l}\ln f\left(h_{t+l}\right)
\end{aligned}
$$

So, for ϕ_3 sufficiently close to 1,

$$|U_t| \leq \left| E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l} \xi_{t+l} (1-\phi_4)(1-\phi_5) \ln\left(C_{t+l} - bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)\right|$$

$$+ \left| \sum_{l=0}^{\infty} \beta^l E_t \left[\varepsilon_{h,t+l}\xi_{t+l}\right](1-\phi_4)\left[\ln z_t^* + l\left(\tfrac{\theta}{1-\theta}\ln \mu_{\Upsilon,ss} + \ln \mu_{z,ss}\right)\right.\right.$$

$$\left.\left. + \tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i\right]\right|$$

$$+ \left| E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l}(1-\phi_4)\phi_5 \ln(C_{t+l} - e_{t+l})\right|$$

$$+ \left| E_t \sum_{l=0}^{\infty} \beta^l \varepsilon_{h,t+l}\xi_{t+l} \ln f(h_{t+l})\right|$$

$$\leq (1-\phi_4)(1-\phi_5)\sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\left|\ln\left(C_{t+l} - bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)\right|\right]$$

$$+ \sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right](1-\phi_4)\left|\left[\ln z_t^* + l\left(\tfrac{\theta}{1-\theta}\ln \mu_{\Upsilon,ss} + \ln \mu_{z,ss}\right)\right.\right.$$

$$\left.\left. + \tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i\right]\right|$$

$$+ (1-\phi_4)\phi_5\sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\left|\ln(C_{t+l} - e_{t+l})\right|\right]$$

$$+ \sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\left|\ln f(h_{t+l})\right|\right]$$

$$\leq (1-\phi_4)(1-\phi_5)\times$$

$$\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}}\left(E_t\left[\left|\ln\left(C_{t+l} - bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)\right|^{p_2}\right]\right)^{\frac{1}{p_2}}$$

$$+ \sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right](1-\phi_4)\left|\left[\ln z_t^* + l\left(\tfrac{\theta}{1-\theta}\ln \mu_{\Upsilon,ss} + \ln \mu_{z,ss}\right)\right.\right.$$

$$\left.\left. + \tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_\Upsilon^i + \ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_z^i\right]\right|$$

$$+ (1-\phi_4)\phi_5\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}}\left(E_t\left[\left|\ln(C_{t+l} - e_{t+l})\right|^{p_2}\right]\right)^{\frac{1}{p_2}}$$

$$+ \sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}}\left(E_t\left[\left|\ln f(h_{t+l})\right|^{p_2}\right]\right)^{\frac{1}{p_2}}$$

by Hölder's inequality where $\frac{1}{p_1} + \frac{1}{p_2} = 1$ and $p_1, p_2 > 1$.

$$\leq (1-\phi_4)(1-\phi_5)\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}} B^{\frac{1}{p_2}}$$

$$+ \sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\xi_{t+l}\right](1-\phi_4)\left|\left[\ln z_t^* + l\left(\tfrac{\theta}{1-\theta}\ln \mu_{\Upsilon,ss} + \ln \mu_{z,ss}\right)\right.\right.$$

$$+\tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_{\Upsilon}^{i}+\ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_{z}^{i}]\Big|$$

$$+(1-\phi_4)\,\phi_5\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}}B^{\frac{1}{p_2}}$$

$$+\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\left(\varepsilon_{h,t+l}\xi_{t+l}\right)^{p_1}\right]\right)^{\frac{1}{p_1}}B^{\frac{1}{p_2}}$$

due to the assumptions that:
$E_t\left[\left|\ln\left(C_{t+l}-bC_{t-1+l}\mu_{z^*,t+l}^{-1}\right)\right|^{p_2}\right]\leq B<\infty$
$E_t\left[\left|\ln\left(C_{t+l}-e_{t+l}\right)\right|^{p_2}\right]\leq B<\infty$
$E_t\left[\left|\ln f\left(h_{t+l}\right)\right|^{p_2}\right]\leq B<\infty$

$$=(1-\phi_4)(1-\phi_5)\,B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]E_t\left[\xi_{t+l}^{p_1}\right]\right)^{\frac{1}{p_1}}$$

$$+\sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\right]E_t\left[\xi_{t+l}\right](1-\phi_4)\left|[\ln z_t^*+l\left(\tfrac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss}+\ln\mu_{z,ss}\right)\right.$$

$$+\tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_{\Upsilon}^{i}+\ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_{z}^{i}]\Big|$$

$$+(1-\phi_4)\,\phi_5 B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]E_t\left[\xi_{t+l}^{p_1}\right]\right)^{\frac{1}{p_1}}$$

$$+B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]E_t\left[\xi_{t+l}^{p_1}\right]\right)^{\frac{1}{p_1}}$$

due to independence of the preference shocks

$$\leq(1-\phi_4)(1-\phi_5)\,B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]B_1\right)^{\frac{1}{p_1}}$$

$$+\sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t+l}\right]B_2(1-\phi_4)\left|[\ln z_t^*+l\left(\tfrac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss}+\ln\mu_{z,ss}\right)\right.$$

$$+\tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_{\Upsilon}^{i}+\ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_{z}^{i}]\Big|$$

$$+(1-\phi_4)\,\phi_5 B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]B_1\right)^{\frac{1}{p_1}}$$

$$+B^{\frac{1}{p_2}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t+l}^{p_1}\right]B_1\right)^{\frac{1}{p_1}}$$

because $E_t\left[\xi_{t+l}^{p_1}\right]\leq B_1<\infty$ for $l\in\{1,2,...\}$ and $E_t\left[\xi_{t+l}\right]\leq E_t\left[\xi_{t+l}^{p_1}\right]+1\leq B_1+1\equiv B_2<\infty$ for $l\in\{1,2,...\}$

$$=(1-\phi_4)(1-\phi_5)\,B^{\frac{1}{p_2}}B_1^{\frac{1}{p_1}}\sum_{l=0}^{\infty}\beta^l\left(E_t\left[\varepsilon_{h,t}^{p_1}\textstyle\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}^{p_1}\cdot\right]\right)^{\frac{1}{p_1}}$$

$$+B_2(1-\phi_4)\sum_{l=0}^{\infty}\beta^l E_t\left[\varepsilon_{h,t}\textstyle\prod_{i=1}^{l}\mu_{\varepsilon_h,t+i}\cdot\right]\left|[\ln z_t^*+l\left(\tfrac{\theta}{1-\theta}\ln\mu_{\Upsilon,ss}+\ln\mu_{z,ss}\right)\right.$$

$$+\tfrac{\theta}{1-\theta}\ln\left(\tfrac{\mu_{\Upsilon,t}}{\mu_{\Upsilon,ss}}\right)\textstyle\sum_{i=1}^{l}\rho_{\Upsilon}^{i}+\ln\left(\tfrac{\mu_{z,t}}{\mu_{z,ss}}\right)\sum_{i=1}^{l}\rho_{z}^{i}]\Big|$$

$$+ (1-\phi_4)\,\phi_5 B^{\frac{1}{p_2}} B_1^{\frac{1}{p_1}} \sum_{l=0}^{\infty} \beta^l \left(E_t \left[\varepsilon_{h,t}^{p_1} \prod_{i=1}^{l} \mu_{\varepsilon_h, t+i}^{p_1} . \right] \right)^{\frac{1}{p_1}}$$

$$+ B^{\frac{1}{p_2}} B_1^{\frac{1}{p_1}} \sum_{l=0}^{\infty} \beta^l \left(E_t \left[\varepsilon_{h,t}^{p_1} \prod_{i=1}^{l} \mu_{\varepsilon_h, t+i}^{p_1} . \right] \right)^{\frac{1}{p_1}}$$

because $\varepsilon_{h,t+l} = \varepsilon_{h,t} \frac{\varepsilon_{h,t+l}}{\varepsilon_{h,t}} = \varepsilon_{h,t} \prod_{i=1}^{l} \mu_{\varepsilon_h, t+i}$. Using the previous results, all these sums clearly converge if

$$\left(E_t \left[\exp \left\{ \frac{p_1 \epsilon_{\varepsilon_h, t+1}}{1-\rho_{\varepsilon_h}} \right\} \right] \right)^{\frac{1}{p_1}} \beta < 1 \text{ and } E_t \left[\exp \left\{ |p_1 \epsilon_{\varepsilon_h, t+1}| \frac{2}{1-\rho_{\varepsilon_h}} \right\} \right] < \infty$$

$$E_t \left[\exp \left\{ \frac{\epsilon_{\varepsilon_h, t+1}}{1-\rho_{\varepsilon_h}} \right\} \right] \beta < 1 \text{ and } E_t \left[\exp \left\{ |\epsilon_{\varepsilon_h, t+1}| \frac{2}{1-\rho_{\varepsilon_h}} \right\} \right] < \infty$$

The first set of conditions also implies that the second set of conditions hold. Hence, we only need to impose the conditions

$$\left(E_t \left[\exp \left\{ \frac{p_1 \epsilon_{\varepsilon_h, t+1}}{1-\rho_{\varepsilon_h}} \right\} \right] \right)^{\frac{1}{p_1}} \beta < 1 \text{ and } E_t \left[\exp \left\{ |p_1 \epsilon_{\varepsilon_h, t+1}| \frac{2}{1-\rho_{\varepsilon_h}} \right\} \right] < \infty$$

Q.E.D.

H Proof of proposition 3(b)

Recall that

$$E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} [\left(\frac{P_{i,t+l}}{P_{t+l}} \right) y_{i,t+l}^d - r_{t+l}^k k_{i,t+l} - w_{t+l} h_{i,t+l} - \nu w_{t+l} h_{i,t+l} \left(1 - R_{t+l,1}^{-1} \right)]$$

$$= \frac{P_t z_t^*}{\Lambda_t} E_t \sum_{l=0}^{\infty} \beta^l \Lambda_{t+l} \prod_{i=1}^{l} \mu_{z^*, t+i}^{(1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}} \Phi_{i,t+l}$$

So

$$\left| E_t \sum_{l=0}^{\infty} r_{t,t+l} P_{t+l} [\left(\frac{P_{i,t+l}}{P_{t+l}} \right) y_{i,t+l}^d - r_{t+l}^k k_{i,t+l} - w_{t+l} h_{i,t+l} - \nu w_{t+l} h_{i,t+l} \left(1 - R_{t+l,1}^{-1} \right)] \right|$$

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| \sum_{l=0}^{\infty} \beta^l E_t \prod_{i=1}^{l} \mu_{z^*, t+i}^{(1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}} \left| \Phi_{i,t+l} \Lambda_{t+l} \right|$$

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| \sum_{l=0}^{\infty} \beta^l \left(E_t \left[\prod_{i=1}^{l} \mu_{z^*, t+i}^{p_1 (1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}}^{p_1} \right] \right)^{\frac{1}{p_1}} \left(E_t \left[\left| \Phi_{i,t+l} \Lambda_{t+l} \right|^{p_2} \right] \right)^{\frac{1}{p_2}}$$

by Hölder's inequality where $\frac{1}{p_1} + \frac{1}{p_2} = 1$ and $p_1, p_2 > 1$.

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| \sum_{l=0}^{\infty} \beta^l \left(E_t \left[\prod_{i=1}^{l} \mu_{z^*, t+i}^{p_1 (1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}}^{p_1} \right] \right)^{\frac{1}{p_1}} B^{\frac{1}{p_2}}$$

by assumption $E_t \left[\left| \Phi_{i,t+l} \Lambda_{t+l} \right|^{p_2} \right] \leq B < \infty$ for $l = \{1, 2, ...\}$

$$\leq \left| \frac{P_t z_t^*}{\Lambda_t} \right| B^{\frac{1}{p_2}} \sum_{l=0}^{\infty} \beta^l \left(E_t \left[\prod_{i=1}^{l} \mu_{z^*, t+i}^{p_1 (1-\phi_3)(1-\phi_4)} \mu_{\varepsilon_{h,t+i}}^{p_1} \right] \right)^{\frac{1}{p_1}}$$

But, from a previous section we know that this sum is convergent if

$$\left(E_t \left[\exp \left\{ p_1 \epsilon_{\varepsilon_h, t+1} \frac{1}{1-\rho_{\varepsilon_h}} \right\} \right] E_t \left[\exp \left\{ p_1 \epsilon_{\Upsilon, t+1} \frac{F_\Upsilon}{1-\rho_\Upsilon} \right\} \right] \right)^{\frac{1}{p_1}} \times$$

$\left(E_t\left[\exp\left\{p_1\epsilon_{z,t+1}\frac{F_z}{1-\rho_z}\right\}\right]\right)^{\frac{1}{p_1}}\beta\mu_{\Upsilon,ss}^{F_\Upsilon}\mu_{z,ss}^{F_z}<1$

and $E_t\left[\exp\left\{|p_1\epsilon_{\varepsilon_h,t+1}|\frac{2}{1-\rho_{\varepsilon_h}}\right\}\right]<\infty$, $E_t\left[\exp\left\{|p_1F_\Upsilon\epsilon_{\Upsilon,t+1}|\frac{2}{1-\rho_\Upsilon}\right\}\right]<\infty$,
$E_t\left[\exp\left\{|p_1F_z\epsilon_{z,t+1}|\frac{2}{1-\rho_z}\right\}\right]<\infty$.

Thus,
$\left|E_t\sum_{l=0}^{\infty}r_{t,t+l}P_{t+l}\Phi_{i,t}\right|\leq\left|\frac{P_tz_t^*}{\Lambda_t}\right|M<\infty$

Q.E.D.

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