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Pricing Volatility of Stock Returns with Volatile and Persistent Components

Jie Zhu



School of Economics and Management
University of Aarhus
Building 1322, DK-8000 Aarhus C
Denmark



Aarhus School of Business
University of Aarhus

Handelsøjskolen
Aarhus Universitet

UNIVERSITY OF
COPENHAGEN



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Jie Zhu[†]

University of Aarhus and CREATES

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Abstract

In this paper a two-component volatility model based on the component's first moment is introduced to describe the dynamic of speculative return volatility. The two components capture the volatile and persistent part of volatility respectively. Then the model is applied to 10 Asia-Pacific stock markets. Their in-mean effects on return are also tested. The empirical results show that the persistent component accounts much more for volatility dynamic process than the volatile component. However the volatile component is found to be a significant pricing factor of asset returns for most markets, a positive or risk-premium effect exists between return and the volatile component, yet the persistent component is not significantly priced for return dynamic process.

JEL classifications: C14; G12; G15

Keywords: Risk; Return; In-mean effect; Volatile; Persistent; Innovations

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[†]Address correspondence: Jie Zhu, School of Economics and Management, Building 1323, University of Aarhus, DK-8000 Aarhus C, Denmark. E-mail: jzhu@econ.au.dk

1 Introduction

Understanding the relation between speculative return of an asset and its volatility¹ (as a proxy for risk) has long been an important topic in financial literature.² On the one hand, a rational risk-averse investor will require a higher expected return for holding a more risky asset. If volatility is a suitable proxy for risk, there should exist a positive relation between volatility and expected return, as illustrated in the asset pricing literature (see, e.g. Sharpe (1964), Linter (1965) and Mossin (1966)). This positive relation between risk and return should always hold from an *ex ante* point of view. On the other hand, however, it is also well addressed that stock return distribution exhibits skewness. A reasonable explanation is that negative return innovation induces higher volatility than positive innovation of the same magnitude, which in turn generates skewness in return. This feature is consistent to an empirically observed regularity which may be described as a financial leverage effect, see e.g. Black (1976), Engle & Ng (1993) and Yu (2005). The standard argument from Black (1976) is that bad news decreases the stock price, i.e. the equity, and hence increases the debt-to-equity ratio (i.e. financial leverage), making the stock more risky and increasing future expected volatility. Alternatively, a volatility feedback effect may be present, i.e. if volatility is increased, then so is the risk premium in case of a positive risk-return relation, and hence the discount rate, which lowers the current stock price. Both leverage effect and volatility feedback effect indicate a negative relation between return and volatility, although the causality is reversed. The volatility feedback effect should be strongest at the market level, whereas the leverage effect should apply to individual stocks. Thus, from the *ex post* point of view, a negative relation between return and volatility is also possible, although this negative relation will not refute the *ex ante* understanding of the positive relation.

The empirical result is mixed. Most earlier researches are in line with the positive relation. For example, French, Schwert & Stambaugh (1987) find a positive relation between expected market risk premium and predictable volatility of stock return in U.S. market. However more recent studies are in favor of the other side. Baillie & DeGennaro (1990) report a controversial result against French et al. (1987). They argue that in U.S. stock market, a positive relation between stock return and volatility is weak and almost

¹In this paper, the word *Volatility* refers to either conditional standard deviation or conditional variance, depending on context.

²Note that the risk we talk about here is systematic or undiversified risk since unsystematic risk of a stock can be diversified away by putting it into a portfolio.

nonexistent. Bollerslev & Zhou (2005) agree with this conclusion.

In literature it is also suggested that the empirical results depend on model specification of volatility dynamics. For example, Li, Yang, Hsiao & Chang (2005) report that a misspecification in parametric model of volatility dynamics may cause a distorted relation. They implement both parametric and semiparametric approaches to model volatility dynamics and estimate coefficient of volatility in return generating process for twelve stock markets. Although failing to find a significant relation by applying parametric approach, they do find a significantly negative relation in six markets by applying semiparametric approach. They argue the failure to find such a relation in first approach may be caused by model misspecification. However, Shin (2005) applies the same parametric and semiparametric approaches to estimate the relation between stock return and volatility in fourteen emerging markets. She fails to find a significant relation between return and volatility for all markets. And semiparametric approach leads to similar results as in parametric approach.

In addition to one-factor models, recent studies focus on applying multi-factor models to describe volatility dynamics. One advantage of multi-factor models is that they allow volatility process to be driven by shocks with different persistence levels. In empirical work, Ding & Granger (1996), Engle & Lee (1999), Bollerslev & Zhou (2002), Chernov, Gallant, Ghysels & Tauchen (2003), Chacko & Viceira (2003) and Adrian & Rosenberg (2005) find that two-factor volatility specifications significantly outperform one-factor models. For example, Ding & Granger (1996) decompose variance into a IGARCH and GARCH factor, for which the two factors are assigned different weights to volatility. They find that such decomposition performs much better than the one-factor GARCH models. However, although these models successfully capture volatility dynamics in some aspects, they have their own deficiencies. In the case of Ding & Granger's (1996) model, conditional variance is the sum of two weighted factors. However it may find a corner solution for the weights in some cases. Thus it degenerates to one-factor model, which is not interesting for applying this two-factor model. Furthermore, if we include these two components in mean equation and test their impacts on return, the estimates are not significant in most cases. This unsatisfied result is quite similar to the case in one-factor GARCH-M model. Engle & Lee (1999) extend the one-factor GARCH model in another way, yet their setup is similar to Ding & Granger's (1996) and thus suffers from the same problem. Bollerslev & Zhou (2002) apply realized volatility based on high-frequency data, which is a luxury input requirement for some emerging markets. Chacko & Viceira (2003) and Chernov et al.

(2003) include jump process and apply spectral density function to estimate their models, making the estimation procedure complicated. Adrian & Rosenberg (2005) build a two-factor model based on EGARCH-M setup, which is able to capture asymmetry in return. However a significant disadvantage of EGARCH model is that the proper aggregation for long-horizon forecasting is rather inconvenient. It is difficult to compute unbiased forecasts of volatility in multiperiod intervals, as pointed out in Brandt & Jones (2006).

Motivated by Ding & Granger (1996), a new two-component volatility model is introduced in this paper. However, instead of decomposing conditional variance into different components directly, I try to interpret dynamic process of conditional variance by components based on its first moment, and let volatility be a sum of these two components. Then both components are added to the return generating process so as to test whether a significant relation exists between return and volatility. As discussed previously, positive relation implies a risk premium effect and negative relation implies either as a financial leverage effect or as a volatility feedback effect. One component is considered to be a volatile part of volatility and the another one is a persistent part of volatility. This idea is supported by the assumption that investors not only regard volatility level as a price factor, they also care about volatility innovations, see Ang, Hodrick, Xing & Zhang (2006).

Another interesting feature for volatility is its long memory property. As pointed out in Ding & Granger (1996) and Baillie, Bollerslev & Mikkelsen (1996), the autocorrelation for stock absolute return decreases fast at the beginning and then decreases very slowly and remains significantly positive, which is different from an exponentially decreasing function, as implied by GARCH models. This finding motivates the new approaches such as multi-factor model in Ding & Granger (1996) and FIGARCH model in Baillie et al. (1996) respectively. It is shown that the suggested two-component model in this paper is also consistent with the long memory property of return volatility.

The model is then applied to 10 Asia-Pacific stock markets, including both volatility components to return process. It is found that the volatile component is a significant pricing factor for return. There exists a positive relation between the volatile component and return, which implies a risk premium effect. On the contrary, the persistent component doesn't have a significant impact on return. However, the persistent component accounts more for volatility process than the volatile component, indicating a persistent or long memory property. Thus, this paper contributes to the literature by providing some new evidence on empirical analysis of volatility dynamic and its impact on return.

The remainder of the paper is organized as follows. In Section 2, I introduce the model

and testing methodology. Then the model is applied to daily returns of 10 Asia-Pacific stock markets in Section 3, along with empirical results and some Monte Carlo simulations. Section 4 concludes the paper. Technical details are given in Appendices.

2 A Two-factor volatility model

Many studies have been done on relation between asset return and volatility. According to classical asset pricing literature, i.e. Sharpe (1964), Linter (1965) and Mossin (1966), along with others, higher asset risk should be compensated by higher returns. Since volatility is widely regarded as a proxy for risk, so in *ex ante* setup, expected return is positively related to volatility. However, as pointed out by Abel (1988) and Gennotte & Marsh (1993) in one-factor stochastic volatility process, in the equilibrium setting, market return is not necessarily positively related to market volatility. This is due to dynamic optimization of rational investors who hedge changes in the investment opportunities. If volatility provides information about expected return so that changes in volatility change the investment opportunity set, then volatility should be priced. In addition, Black (1976) and others point out that return will also affect volatility in an asymmetric way: negative innovations to stock return tend to increase volatility more than positive innovations of the same magnitude. This asymmetry is partly explained as a result of "leverage effect" due to Black (1976). So from a *ex post* point of view, it seems that a negative correlation between stock return and volatility is possible, which is supported by empirical studies as mentioned previously.

Some empirical evidence also shows the presence of long memory property in realized asset return volatility. This property is well addressed in the literature, see Andersen, Bollerslev, Diebold & Labys (2003) and references therein. Such findings motivate multi-factor stochastic volatility models. A popular approach is to decompose volatility into two components. One reflects short-run effect and the other reflects long-run effect. Relative empirical studies provided by Engle & Lee (1999), Bollerslev & Zhou (2002), Chacko & Viceira (2003) and Chernov et al. (2003) find that two-factor volatility specifications significantly outperform one-factor models. More recently, Adrian & Rosenberg (2005) decompose volatility dynamic as a sum of two components with different rates of mean reversion. Then they add these two components to the return-generating-process and find that the estimated coefficients of both components are negative, but only the short-run component is significant. Their model is based on EGARCH setup. Thus it can capture

the asymmetry in return, yet also suffers from the disadvantage of EGARCH model: it is difficult to compute unbiased forecasts of volatility in long-run period.

The objective of this paper is to introduce a new two-component model which is motivated by Ding & Granger (1996). The two volatility components are then included in return process to test their in-mean effects. The model is applied to 10 Asia-Pacific stock markets. In Ding & Granger (1996), they decompose conditional variance as a weighted sum of two components and the components follow a GARCH and IGARCH process respectively. The difference of the model presented in this paper and in theirs is that I apply generalized autoregressive process to describe the first moment dynamic of these two components, and then construct volatility as the sum of these two components based on their second moments. Thus the model allows more flexibility while in meantime keeps volatility positive, as it is required by definition. More specifically, the model allows the volatile component has zero mean and fluctuates more frequently, yet the persistent component has a non zero mean and moves more smoothly than the volatile component, and thus capturing the persistence in volatility.

The setup of the model is as follows. Some technical details are provided in Appendices.

Assume that dynamic of a stock's log price can be represented by the following diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_{1t} \quad (1)$$

Volatility σ_t^2 is the sum of two components:

$$\sigma_t^2 = s_t^2 + q_t^2 \quad (2)$$

and the dynamics of s_t and q_t are given as follows:

$$ds_t = -\kappa^s s_t dt + \chi^s \sigma_t dW_{2t} \quad (3)$$

$$dq_t = \kappa^q (\bar{q} - q_t) dt + \chi^q \sigma_t dW_{3t} \quad (4)$$

Where W_{2t} and W_{3t} are Wiener processes that may be correlated with each other and also correlated with the return innovation W_{1t} . This specification is similar to the discrete-time model presented in Ding & Granger (1996), except for describing the dynamics of s_t and q_t instead of s_t^2 and q_t^2 as in their paper. This lets the model to be more flexible and

still ensures the positivity of volatility.

Both s_t and q_t follow Ornstein-Uhlenbeck process and thus are conditionally normal. By setting up, s_t is the quick mean-reverting component and its persistence of mean-reverting is captured by the parameter $\kappa^s > 0$. q_t is the slow mean-reverting component and it reverts to a constant \bar{q} at rate κ^q . The two components of volatility s_t^2 and q_t^2 may have potentially different rates of mean reversion. Applying Ito's lemma, we can show that

$$ds_t^2 = 2\kappa^s \left(\frac{\chi^{s^2}}{2\kappa^s} \sigma_t^2 - s_t^2 \right) dt + 2\chi^s \sigma_t s_t dW_{2t} \quad (5)$$

and

$$dq_t^2 = 2\kappa^q \left(\frac{\chi^{q^2}}{2\kappa^q} \sigma_t^2 + \bar{q}q_t - q_t^2 \right) dt + 2\chi^q \sigma_t q_t dW_{3t} \quad (6)$$

From (5) and (6), we can consider that both s_t^2 and q_t^2 follow a general mean-reverting process with time-varying mean. Besides the symmetric factors in (5) and (6), an additional factor $\bar{q}q_t$ appears in time-varying mean for q_t^2 . Since $\sigma_t^2 = s_t^2 + q_t^2$, it is also straight forward to show that

$$d\sigma_t^2 = \chi^* (q_t^* - \sigma_t^2) dt + \chi_t^{s^*} dW_{2t} + \chi_t^{q^*} dW_{3t} \quad (7)$$

where $\chi^* = -(\chi^{s^2} + \chi^{q^2} - 2\kappa^s)$, $q_t^* = \frac{2(\kappa^q - \kappa^s)}{\chi^{s^2} + \chi^{q^2} - 2\kappa^s} q_t^2 - \frac{2\kappa^q \bar{q}}{\chi^{s^2} + \chi^{q^2} - 2\kappa^s} q_t$, $\chi_t^{s^*} = 2s_t \sigma_t \chi^s$ and $\chi_t^{q^*} = 2q_t \sigma_t \chi^q$.

The dynamic of volatility σ_t^2 is complicated, since it is a nonlinear combination of dynamics of s_t and q_t . Nevertheless, intuitively we can interpret (7) as a generalized mean-reverting process with a stochastic mean q_t^* and stochastic diffusions $\chi_t^{s^*}$ and $\chi_t^{q^*}$. Interestingly since the stochastic mean q_t^* doesn't include any s_t term, it implies that conditional variance σ_t^2 fluctuates around a variable which is a quadratic function of q_t . Thus the mean level of σ_t^2 depends only on q_t but not on s_t . In this sense, we can think s_t^2 as the volatile component of volatility and q_t^2 as the persistence component of volatility.

In order to complete the model, we also need to determine the drift term μ_t in the return dynamic (1). Here I adopt a linear setup, that is, let μ_t be a linear function of volatility σ_t^2 . Since σ_t^2 can further be decomposed into s_t^2 and q_t^2 , it is natural to include these two components separately in the drift term in addition to other factors. Thus

$$\mu_t = \mu_0 + \mathbf{x}_t\boldsymbol{\beta} + \delta_1 s_t^2 + \delta_2 q_t^2 \quad (8)$$

\mathbf{x}_t represents other factors that contribute to the return generating process.

To estimate the parameters in (1), as well as in the volatility component dynamics (3) and (4), we can specify the following model:

$$r_t = \mu_0 + \sum_{i=1}^k \mu_{t-i} r_{t-i} + \delta_1 s_t^2 + \delta_2 q_t^2 + \varepsilon_t \quad (9)$$

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0, 1), \quad \sigma_t^2 = s_t^2 + q_t^2 \quad (10)$$

$$s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1} \quad (11)$$

$$q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1} \quad (12)$$

where r_t denotes the daily log price difference, $r_t = \log P_t - \log P_{t-1} = p_t - p_{t-1}$.

The lags of return r_{t-i} are added to the model to reflect possible serial correlation in return. In this paper the lag order i is set to be 1,³ so the empirical model to be estimated is as follows:

$$r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_t^2 + \delta_2 q_t^2 + \varepsilon_t \quad (13)$$

In Ding & Granger (1996), they decompose time-varying volatility in the following way:

$$\sigma_t^2 = w\sigma_{1t}^2 + (1-w)\sigma_{2t}^2 \quad (14)$$

$$\sigma_{1t}^2 = \alpha_1 \varepsilon_{t-1}^2 + (1 - \alpha_1)\sigma_{1t-1}^2 \quad (15)$$

$$\sigma_{2t}^2 = \sigma^2(1 - \alpha_2 - \beta_2) + \alpha_2 \varepsilon_{t-1}^2 + \beta_2 \sigma_{2t-1}^2 \quad (16)$$

We can see that the difference between the model in this paper and in Ding & Granger (1996) is that here s_t and q_t can be considered as the first moment components of volatility,

³It is well documented that there is little serial correlation in return series. In fact the estimated parameters to lag orders higher than 1 are not significant.

yet σ_{1t}^2 and σ_{2t}^2 in (15) and (16) are the components of conditional variance. Thus by model specification, s_t is allowed to have zero-mean and achieves more flexibility.

In Appendix A.1, I apply a result from Nelson (1990) to show that the discrete-time model of (9) – (12) converges to the continuous-time model of (1) – (4). In the continuous-time model, there are three correlated shocks: W_1, W_2 and W_3 . However, using the result of Nelson (1990), we can approximate the discrete-time model by using only a single shock.

One reason to regard s_t and q_t as a volatile and a persistent component of volatility can be seen from their moments:

In Appendix A.2, it is shown that the unconditional first moments of s_t and q_t are as follows:

$$E[s_t] = \mu_s = 0 \quad (17)$$

and

$$E[q_t] = \mu_q = \frac{\omega}{1 - \beta_2} \quad (18)$$

From (17) and (18), it is obvious that s_t is converged to zero, thus it decays out in the long run. That's why it is indicated as a volatile or non-persistent component of volatility. On the other hand, q_t converges to its unconditional mean $\frac{\omega}{1 - \beta_2}$, which is a non-zero constant provided that $\omega \neq 0$. For that reason we can consider it as the persistent component of volatility.

It is also shown in Appendix A.2 that under the stationary condition, the unconditional second moments of s_t and q_t , $E[s_t^2] = \sigma_s^2$ and $E[q_t^2] = \sigma_q^2$ have the following forms:

$$\sigma_s^2 = \frac{\alpha_1^2}{1 - \alpha_1^2 - \beta_1^2} \sigma_q^2 \quad (19)$$

and

$$\sigma_q^2 = \frac{1}{1 - \frac{\alpha_2^2(1 - \beta_1^2)}{(1 - \beta_2^2)(1 - \alpha_1^2 - \beta_1^2)}} \mu_q^2 \quad (20)$$

Since $\sigma_t^2 = s_t^2 + q_t^2$, then the unconditional variance of ε_t , $E[\sigma_t^2] = \sigma^2$ is obtained as follows:

$$\sigma^2 = \frac{\mu_q^2}{1 - \frac{\alpha_1^2}{1-\beta_1^2} - \frac{\alpha_2^2}{1-\beta_2^2}} \quad (21)$$

Note that in order to make (19), (20) and (21) be valid, we need the following conditions be satisfied:

$$(1 - \beta_2^2)(1 - \alpha_1^2 - \beta_1^2) > \alpha_2^2(1 - \beta_1^2) \quad (22)$$

and

$$1 - \alpha_1^2 - \beta_1^2 > 0 \quad (23)$$

(This two conditions also imply that $1 - \beta_1^2 > 0$ and $1 - \beta_2^2 > 0$.)

The covariance between s_t and q_t is as follows, the calculation is shown in Appendix A.2:

$$\text{cov}(s_t, q_t) = \frac{\alpha_1 \alpha_2}{1 - \beta_1 \beta_2} \sigma^2 = \frac{\alpha_1 \alpha_2}{1 - \beta_1 \beta_2} \frac{\mu_q^2}{1 - \frac{\alpha_1^2}{1-\beta_1^2} - \frac{\alpha_2^2}{1-\beta_2^2}} \quad (24)$$

It can be seen from (24) that the sign of $\text{cov}(s_t, q_t)$ depends on the sign of $\alpha_1 \alpha_2$, if both α_1 and α_2 are positive or negative, then the covariance between s_t and q_t is positive, otherwise it will be negative.

It is also possible to obtain the analytical solution for $\text{cov}(s_t^2, q_t^2)$, if the first four moments exist and are finite for both s_t and q_t , yet the calculation is tedious. Instead of presenting the explicit form, we guess that $\text{cov}(s_t^2, q_t^2)$ is likely to be positive since σ_s^2 is proportional to σ_q^2 , as shown in (19).

As a standard approach, the discrete-time model of (9), (11) and (12) can be estimated by Quasi-Maximum Likelihood Method. Some details of this estimation method are given in Appendix A.3.

3 Estimation and Discussion

3.1 Data Description

The data for the empirical study covers 10 stock markets in Asia-Pacific area and is collected from Datastream. It includes 8 Asian stock markets (China, Hong Kong, Japan, Korea, Philippines, Singapore, Taiwan and Thailand) as well as 2 Oceanian stock markets

Table 1: The Selected Stock Market Indices in Asia-Pacific Stock Markets

Market	Market Index
Australia	All Ordinary IDX (AORD)
China	Shenzhen SE Composite Index (SZSEI)
Hong Kong	Heng Seng Index (HSI)
Japan	Osaka Nikkei 225 (N225)
New Zealand	NZX 50 Index Gross (NZ50)
Philippines	PSE Composite Index (PSEI)
Singapore	Strait Times Index (STI)
South Korea	KOSPI (KSII)
Taiwan	TSEC Weighted Index (TWII)
Thailand	SET Index (SETI)

(Australia and New Zealand).⁴ Table 1 presents market indices chosen from these markets.

One reason for choosing these 10 markets is that the capital markets in Asia-Pacific area have developed rapidly in the past three decades and play a more and more important role in the world economy, yet relatively few studies are focusing on these markets compared to US or European markets. Another specific issue is that the stock markets in this region have experienced an increasing integration during the sample period, but on the other hand most of the markets also suffered deep loss from the Asian Financial Crisis in 1997.⁵ It is interesting to study this property too. The sample period is chosen from January 1991 to December 2005. Daily closing price of market index adjusted for dividends is collected, which results in about 3900 daily observations in total for each series.⁶ The log price return (in percentage) is adopted:

$$r_t = 100 \times [\log(P_t) - \log(P_{t-1})] \quad (25)$$

The data set selected for this study has several advantages. Using daily data instead of weekly or monthly data makes more observations available and thus avoids the finite sample bias as discussed in Hwang & Pedro (2006). Also in the estimation process, we apply a discrete-time setup to approximate a continuous-time dynamic model. It is obvious that using a data set with higher frequency will increase the precision of the approximation.

⁴See Comerton & Rydger (2006) for a review on these markets.

⁵South Korea and Thailand are considered to be most heavily hit by the 1997 Asian Financial Crisis; China, Hong Kong, Japan, Philippines, Singapore and Taiwan are also affected by the Crisis. Australia and New Zealand are relatively unaffected, see Radelet & Sachs (1998) for reference.

⁶The data of the first 100 observations is used to generating the starting values for the dynamic process and doesn't include in the estimation procedure. Thus the estimating period is from Jun. 3, 1991 to Dec. 31, 2005 and has 3805 daily observations.

Although it is argued that estimation with lower frequency data, i.e. monthly data, tends to reflect long-term movement in volatility (Baillie & DeGennaro (1990), p. 211), this property only holds if a long period is included in the study. Thus low frequency data requires a much longer period to study in order to get reliable results. This requirement is too luxury for most emerging markets in Asia-Pacific region, since most markets in this region are launched in 1970s and data is only available after then. Second, since the data set includes 10 markets, we can also compare the difference of the parameter estimates among different markets. For example, we can compare results between developed markets and emerging markets. According to International Finance Corporation (IFC) classification, Australia, Japan, Hong Kong, New Zealand as well as Singapore are classified as developed markets and the rest are regarded as emerging markets. Finally since the market index is always available on every trading day, it is ideal to use index data to estimate the parameters in the model (as a approximation for the continuous-time model) by avoiding individual stock's suspending problem.

Table 2 provides some summary statistics for these stock markets. Since data for risk-free rate is not available for some markets during the sample period, in this paper the stock returns are adopted instead of excess stock returns to conduct analysis. This is consistent to the studies in Li et al. (2005) and Shin (2005), and some researchers (i.e. Baillie & DeGennaro (1990), Nelson (1991), Choudhry (1996)) show that using returns instead of excess returns produces little difference in estimation and inference in empirical studies.

Some features from Table 2 are worth noting. According to the preceding classification, Australia, Japan, Hong Kong, New Zealand and Singapore are defined as the developed markets and the rest are defined as the emerging markets. In Table 2, one can see that there is no significant difference for mean return between the developed markets and the emerging markets.⁷ Actually the mean return for the developed markets is higher than the emerging markets. This result is somewhat surprising since during the sample period most emerging markets undergo more rapid economic growth than the developed markets. One possible reason is due to the poor performance of these emerging markets during the 1997 Asian Financial Crisis period. Thailand has even a negative mean return. Japan also suffers a negative mean return due to the collapse of asset bubble in early 1990 and low economic growth thereafter. We should also notice from Table 2 that the emerging markets have much higher volatilities. China and Japan have the highest and lowest volatility for

⁷Two emerging markets, which are not included in this paper,Indonesia and Malaysia, do have higher mean returns but also have much higher volatility.

Table 2: Summary Statistics for Daily Index Return in Asian-Pacific Markets

Market	Mean	Standard Deviation	Skewness	Kurtosis	Normality Test
Australia	0.0299	0.747	-0.507	6.308	1778.8**
China	0.0330	2.383	0.924	18.64	13031.3**
Hong Kong	0.0365	1.599	-0.00848	10.44	4054.6**
Japan	-0.0125	0.0141	0.0748	2.456	538.05**
New Zealand	0.0196	0.843	-0.898	21.55	7853.09**
Philippines	0.0152	1.455	0.668	10.77	3462.2**
Singapore	0.0170	1.262	0.281	11.88	4604.9**
South Korea	0.0214	1.913	-0.0628	4.044	1142.9**
Taiwan	0.0406	1.608	-0.0534	2.646	606.30**
Thailand	-0.0327	1.68	0.377	4.752	1277.1**
Mean for Developed Markets	0.0181	0.893	-0.212	10.52	
Mean for Emerging Markets	0.0155	1.808	0.371	8.17	

Note: This table reports the summary statistics for daily index (in percentage) return in 10 markets from June 3, 1991 to December 31, 2005, i.e. T=3805 return observations. The Developed Markets refer to Australia, Hong Kong, Japan, New Zealand and Singapore and Emerging Markets refer to China, Philippines, South Korea, Taiwan and Thailand. The result for Jarque-Bera Normality Test is also reported, * indicates significance level of 5% and ** indicates significance level of 1%.

the sample period respectively. However the value of skewness and kurtosis doesn't differ much, although the developed markets have negative skewness and the emerging markets have positive skewness. China, Hong Kong, New Zealand and Singapore have the highest kurtosis and three of them are developed markets. The Jarque-Bera Normality Test rejects the normal distribution of return in all markets.

Table 3: Correlation Test for Index Returns

Markets	Correlation with Return on				
	Australia	China	Hong Kong	Japan	New Zealand
Australia	1	0.0504	0.464	0.363	0.476
China	0.0504	1	0.0675	0.0431	0.0400
Hong Kong	0.464	0.0675	1	0.349	0.281
Japan	0.363	0.0431	0.349	1	0.187
New Zealand	0.476	0.0400	0.281	0.187	1

Markets	Correlation with Return on				
	Australia	China	Hong Kong	Japan	New Zealand
Philippines	0.239	0.0204*	0.285	0.123	0.206
Singapore	0.270	0.0222*	0.292	0.250	0.200
South Korea	0.414	0.0342	0.601	0.323	0.286
Taiwan	0.185	0.0466	0.235	0.192	0.150
Thailand	0.235	0.0427	0.342	0.169	0.171

Markets	Correlation with Return on				
	Philippines	Singapore	South Korea	Taiwan	Thailand
Philippines	1	0.164	0.336	0.133	0.259
Singapore	0.164	1	0.312	0.217	0.245
South Korea	0.336	0.312	1	0.245	0.405
Taiwan	0.133	0.217	0.245	1	0.156
Thailand	0.259	0.245	0.405	0.156	1

Note: This table reports the correlation coefficient among the 10 index return series from June 3, 1991 to December 31, 2005, i.e. T=3805 daily observations. The values with * are insignificant at 10%.

Table 3 reports the correlation coefficients among these markets. From Table 3 it is obvious that all of these markets are positively correlated and the coefficients are significant except for the coefficients between China and Philippines, and China and Singapore. Several features need to pay the attention. First it seems that there are higher correlations among the developed markets compared to correlations among the emerging markets or between the developed and the emerging markets. This indicates that the developed markets are more correlated than the emerging markets, maybe partly due to higher market openness and more information sharing. Secondly, although the results are not reported in the table,⁸ it is true that the correlations among different markets are increasing after the 1997 Asian Financial Crisis compared to the period prior to the Crisis. It indicates that the markets are becoming more integrated after the Crisis. Thirdly, it is obvious to see from the table that China has a much lower correlation with all the other markets, which implies that China is still a relatively isolated market.

⁸The result is available upon request.

3.2 Empirical Results

The results of estimating the two-component volatility model using the volatile and persistent component decomposition are presented in Table 4. First note that the parameters for the two components satisfy the moment-existing constraints (22) and (23) for all the markets. The estimate of the parameter for the lagged return μ_1 is positive for most markets and it is significant for Australia, Hong Kong, New Zealand, Philippines and Singapore. But for China, Japan and Thailand, μ_1 is negative although it is not significant for China and Thailand. This result shows that the lagged return has a mixed effect on current return. For most markets it has a positive impact, yet it is not significant and is even negative for some markets. Next turn to the volatility factor loadings. Most of the parameter values of the volatile component δ_1 are positive (9 of 10 markets, except for Thailand), but for most markets the coefficient of the persistent component δ_2 is negative (6 of 10 markets). More interestingly, we can see that all the values for δ_1 are significant, but none of the markets has significant δ_2 .

All of the parameter values for α_1 and β_1 are positive and significant. In all markets the value of β_1 is larger than α_1 , indicating a significant autocorrelation effect in s_t process and lagged s_t contributes more than lagged ε_t to current s_t . For q_t process, like the case for s_t , β_2 is positive and significant for all markets. However most values of α_2 are negative, and the values of α_2 are much less than β_2 . From these numbers, it is obvious that both s_t and q_t have significant serial correlations. Although compared to q_t , s_t is also affected by ε_{t-1} to a larger extent. Notice that for most markets (except for China, New Zealand and Taiwan), the estimation result of the constant intercept ω in q_t process is positive and significant, which reflects that q_t has persistent property. Most values of the Ljung-Box portmanteau statistics for serial correlation in the standardized return innovation $\hat{z}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$, reported as Q_{10} and Q_{100} for 10 and 100 lags respectively, are not significant. It indicates that the model is doing well at capturing heteroskedasticity in return series. Although the Jarque-Bera Normality Test statistics reject the null hypothesis that the standardized errors are normally distributed, the values are much smaller compared to those for the raw data presented in Table 2.

Back to the results for δ_1 and δ_2 . It seems that these two components have opposite effects on return, both for the sign and significance. To assess their empirical relevance of including these two components in the return equation (9), I compute s_t from (11) and q_t from (12) and square them to obtain s_t^2 and q_t^2 , then plot returns against s_t^2 and q_t^2

Table 4: Two-Component Stochastic Volatility Model

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
μ_0	0.0151 (0.041)	-1.61 (2.9)	0.289 (0.24)	-0.101* (0.057)	0.154 (0.13)
μ_1	0.0676** (0.021)	-1.63×10^{-3} (0.030)	0.0612** (0.024)	-0.0280* (0.017)	0.0995** (0.034)
δ_1	0.378** (0.10)	0.0237** (5.2×10^{-3})	0.0454** (0.0084)	0.171** (0.072)	0.220** (0.043)
δ_2	-0.0294 (0.089)	0.729 (1.3)	-0.179 (0.17)	0.0239 (0.033)	-0.360 (0.27)

Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
α_1	0.158** (0.033)	0.324** (0.025)	0.160** (0.019)	0.104** (0.033)	0.442** (0.067)
β_1	0.914** (0.023)	0.925** (0.011)	0.963** (0.0050)	0.961** (0.018)	0.621** (0.11)

Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
ω	0.0512** (0.010)	0.0719 (0.056)	0.214** (0.070)	0.0290** (4.8×10^{-3})	0.174 (0.11)
α_2	-0.0659** (0.014)	0.0124 (0.023)	-0.0469** (0.015)	-0.0385** (7.8×10^{-3})	-0.0709 (0.040)
β_2	0.923** (0.015)	0.952** (0.038)	0.814** (0.062)	0.977** (3.5×10^{-3})	0.745** (0.16)
Q_{10}	6.03	29.53**	8.25	4.93	4.45
Q_{100}	103.10	202.82**	128.07*	88.41	133.01*
Normality Test	307.03**	5705.1**	918.46**	453.41**	1447.9**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from June 3, 1991, to December 31, 2005, i.e. $T = 3,805$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

Table 4: Two-Component Stochastic Volatility Model (Cont)

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
μ_0	0.0618 (0.055)	0.132 (0.072)	-0.183 (2.2)	0.119 (0.14)	-1.46 (1.37)
μ_1	0.182** (0.023)	0.128** (0.021)	0.0484 (0.030)	0.0113 (0.021)	-3.65×10^{-3} (0.095)
δ_1	0.118** (0.028)	0.0846** (0.010)	0.0420** (0.014)	0.0977** (0.028)	-0.0485** (0.021)
δ_2	-0.0621 (0.045)	-0.143 (0.096)	0.125 (1.1)	-0.0829 (0.061)	0.989 (0.87)
Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
α_1	0.350** (0.062)	0.306** (0.033)	0.180** (0.048)	0.161** (0.016)	0.228** (0.039)
β_1	0.787** (0.043)	0.867** (0.021)	0.948** (0.016)	0.948** (0.0082)	0.927** (0.021)
Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
ω	0.0122** (0.0047)	2.61×10^{-3} ** (0.30×10^{-3})	0.150** (0.059)	0.115 (0.072)	0.703* (0.38)
α_2	-9.22×10^{-3} * (4.8×10^{-3})	-1.54×10^{-3} ** (0.28×10^{-3})	8.60×10^{-3} (0.019)	-0.0714** (0.016)	0.0423** (0.018)
β_2	0.988** (0.0040)	0.997** (0.39×10^{-3})	0.893** (0.040)	0.912** (0.057)	0.443 (0.30)
Q_{10}	33.17**	2.22	4.96	23.70**	33.25**
Q_{100}	129.38**	93.35	108.50	114.33	135.75
Normality Test	3451.8**	912.57**	818.77**	375.54**	691.83**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from June 3, 1991, to December 31, 2005, i.e. $T = 3,805$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

respectively. The resulting scatter plots appear in Figure 1 and Figure 2.

Figure 1 ~ Figure 2 about here

From Figure 1, it seems that the relation between return r_t and s_t^2 is positive, especially based on the best-fitting linear regression line minimizing the sum of squared errors (the bold line). Although most points clustering around the origin, the positivity is obvious if compared to Figure 2, which shows the relation between return r_t and q_t^2 . The figures for other markets are also drawn.⁹ For s_t^2 , most of them give the similar results as for Australia: positive relation between returns and s_t^2 , yet for relation between returns and q_t^2 , the results are mixed, either slightly positive or slightly negative. Thus from the figures, s_t^2 has a much stronger evidence of positive relation with return than q_t^2 .

As mentioned earlier, most markets studied in this paper suffered from the 1997 Asian Financial Crisis, which is a special event during the sample period. In order to check consistency of the model estimation prior to and after the crisis, I divide the sample period into two subperiods: one is prior the crisis and the other is after the crisis, and then reestimate the model for both subperiods. The result is presented in Table 5 and Table 6 as follows.

We can compare the estimates obtained from Table 5 and Table 6 for the two subperiods with the estimates from Table 4 for the full sample. Similar to what we have got for the full sample, the estimates for α_1 , β_1 , α_2 and β_2 satisfy the moment-existing conditions (22) and (23) for all markets in both subperiods. All the estimates for α_1 and β_1 are significant for all markets in both subperiods, except β_2 for Australia in subperiod 2. This result is also consistent with the result for the full sample period. The estimates for the lag return μ_1 is positive for most markets in both subperiods, and it is significant for Australia, Hong Kong, Philippines and Singapore in subperiod 1 and is significant for Australia, New Zealand, Philippines, Singapore, South Korea and Taiwan in subperiod 2. This result is a little different compared to what is obtained for the full sample, which reflects the different impacts of lag return in these two subperiods. The interesting thing is that in both subperiods, the estimates for δ_1 is positive and significant for most markets, except for Australia and Philippines in subperiod 1 and South Korea and Thailand in subperiod 2. It indicates that for most markets, the impact of the volatile component

⁹These figures are not presented in the paper in order to save space, they are available upon request.

Table 5: Two-Component Stochastic Volatility Model - subperiod 1

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
μ_0	-0.0869 (0.14)	-2.83 (2.4)	0.143 (0.18)	-0.125 (0.084)	0.384 (0.40)
μ_1	0.107** (0.032)	-0.0937 (0.11)	0.0696* (0.040)	-0.0279 (0.027)	0.0729 (0.053)
δ_1	3.42 (3.84)	0.0362** (0.012)	0.142** (0.071)	0.300** (0.12)	0.441** (0.13)
δ_2	0.119 (0.27)	0.467 (0.45)	-0.107 (0.14)	0.0199 (0.067)	-0.787 (0.76)
Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
α_1	0.154* (0.091)	0.742** (0.11)	0.169** (0.030)	0.116** (0.017)	0.290** (0.056)
β_1	0.550** (0.26)	0.465** (0.055)	0.949** (0.013)	0.939** (0.011)	0.793** (0.073)
Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
ω	0.0784** (0.029)	0.837** (0.37)	0.210** (0.041)	0.0351** (0.010)	0.243** (0.070)
α_2	-0.0346** (0.014)	0.0807 (0.068)	-0.119** (0.021)	-0.0415** (0.012)	-0.0564 (0.038)
β_2	0.891** (0.040)	0.655** (0.16)	0.816** (0.034)	0.971** (8.9×10^{-3})	0.665** (0.087)
Q_{10}	7.16	15.56	11.79	6.05	2.65
Q_{100}	108.14	108.26	98.02	104.01	107.99*
Normality Test	96.62**	1631.5**	460.41**	202.30**	656.16**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from June 3, 1991, to July 18, 1997, i.e. $T = 1596$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

Table 5: Two-Component Stochastic Volatility Model - Subperiod 1 (Cont)

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
μ_0	0.693 (1.5)	0.0857 (0.24)	-1.46 (1.2)	0.0201 (0.12)	1.12 (1.3)
μ_1	0.235** (0.040)	0.129** (0.036)	-0.107 (0.089)	-7.18×10^{-3} (0.034)	2.25×10^{-3} (0.046)
δ_1	0.179 (0.14)	0.191** (0.067)	0.128** (0.039)	0.133** (0.037)	-0.0804** (0.032)
δ_2	-0.660 (1.3)	-0.221 (0.44)	1.21 (0.88)	-0.0427 (0.070)	-0.684 (0.76)

Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
α_1	0.335** (0.050)	0.311** (0.042)	0.220** (0.043)	0.164** (0.018)	0.325** (0.044)
β_1	0.833** (0.031)	0.878** (0.031)	0.914** (0.028)	0.947** (0.0088)	0.836** (0.034)

Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
ω	0.0438 (0.093)	0.0852** (0.049)	0.637** (0.19)	0.297** (0.092)	0.200** (0.053)
α_2	-0.0135 (0.023)	-0.0474 (0.045)	0.0480** (0.021)	-0.117** (0.021)	-0.0367 (0.029)
β_2	0.958** (0.088)	0.880** (0.068)	0.442** (0.16)	0.772** (0.070)	0.843** (0.041)
Q_{10}	15.38	9.42	7.09	15.50	12.97
Q_{100}	110.99	103.62	112.86	132.66*	98.61
Normality Test	187.54**	367.60**	55.94**	129.01**	90.56**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from June 3, 1991, to July 18, 1997, i.e. $T = 1,596$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

Table 6: Two-Component Stochastic Volatility Model - subperiod 2

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
μ_0	-0.0337 (0.032)	0.126 (0.26)	0.0132 (0.046)	0.0506 (0.090)	4.95×10^{-3} (0.028)
μ_1	0.0504* (0.028)	0.0595 (0.052)	0.0840** (0.029)	-6.34×10^{-3} (0.032)	0.122** (0.030)
δ_1	0.465** (0.15)	0.209** (0.037)	0.0885** (0.023)	0.151** (0.052)	0.154** (0.050)
δ_2	0.0700 (0.081)	-0.168 (0.17)	-0.0647** (0.021)	-6.92×10^{-3} (0.053)	0.0497 (0.081)

Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
α_1	0.379** (0.055)	0.447** (0.089)	0.154** (0.022)	0.116** (0.033)	0.384** (0.068)
β_1	0.279 (0.25)	0.632** (0.10)	0.975** (3.9×10^{-3})	0.953** (0.024)	0.656** (0.090)

Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Australia	China	Hong Kong	Japan	New Zealand
ω	0.0276** (7.0×10^{-3})	0.380** (0.17)	0.0405** (7.5×10^{-3})	$1.55 \times 10^{-3**}$ (7.5×10^{-4})	$1.80 \times 10^{-3**}$ (5.2×10^{-4})
α_2	-0.0621** (0.013)	-0.0947* (0.054)	-0.0873** (0.016)	$-4.30 \times 10^{-3**}$ (1.5×10^{-3})	-0.0112** (1.5×10^{-3})
β_2	0.957** (0.010)	0.692** (0.14)	0.970** (5.8×10^{-3})	0.998** (9.2×10^{-3})	0.996** (1.0×10^{-3})
Q_{10}	7.70	24.02**	11.79	9.98	10.25
Q_{100}	86.84	111.33	98.02	92.49	99.87
Normality Test	138.01**	402.04**	460.41**	130.45**	171.20**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from July 20, 1997, to December 31, 2005, i.e. $T = 2,209$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

Table 6: Two-Component Stochastic Volatility Model - Subperiod 2 (Cont)

Mean equation: $r_t = \mu_0 + \mu_1 r_{t-1} + \delta_1 s_{t-1}^2 + \delta_2 q_{t-1}^2 + \varepsilon_t$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
μ_0	-0.109 (0.068)	0.669** (0.30)	0.112 (0.13)	0.416** (0.19)	-1.78 (2.2)
μ_1	0.180** (0.035)	0.113** (0.030)	0.0653** (0.030)	0.0494** (0.024)	-0.0162 (0.14)
δ_1	0.130** (0.038)	0.0712** (0.011)	-0.0122 (0.013)	0.0957** (0.033)	-1.33×10^{-3} (0.026)
δ_2	0.0187 (0.038)	-0.598** (0.29)	-0.0173 (0.049)	-0.226** (0.11)	0.945 (1.1)
Volatile component: $s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
α_1	0.408** (0.10)	0.298** (0.042)	0.209** (0.031)	0.211** (0.023)	0.344** (0.056)
β_1	0.706** (0.084)	0.882** (0.021)	0.948** (0.011)	0.922** (0.016)	0.865** (0.089)
Persistent component: $q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$					
Parameter	Philippines	Singapore	South Korea	Taiwan	Thailand
ω	0.0120 ** (3.6×10^{-3})	0.0173** (3.7×10^{-3})	0.111** (0.026)	0.0276 (0.017)	0.346 (0.96)
α_2	-0.0163** (5.3×10^{-3})	-5.32×10^{-3} * (1.8×10^{-3})	-0.0757** (0.032)	-0.0195 (0.016)	0.0325 (0.027)
β_2	0.990** (2.7×10^{-3})	0.983** (3.7×10^{-3})	0.933** (0.016)	0.978** (0.013)	0.752 (0.69)
Q_{10}	20.32*	10.35	5.64	11.47	15.62
Q_{100}	117.52	117.83	92.32	105.88	142.82**
Normality Test	1777.7**	436.15**	144.34**	126.18**	122.53**

Note: Quasi maximum likelihood estimates (QMLE) estimates are reported for daily percentage returns on the 10 market indices from July 20, 1997, to December 31, 2005, i.e. $T = 2,209$ return observations, with robust standard errors in parentheses. The values of the Ljung-Box portmanteau statistics for up to 10^{th} and 100^{th} order serial dependence in the standardized residuals $\hat{\varepsilon}_t/\hat{\sigma}_t$ are reported, also for the Jarque-Bera Normality Test results. * and ** denote significance level at 10% and 5% respectively.

of volatility s_t^2 is positive in both subperiods. The estimates for δ_2 is not significant for all markets in subperiod 1. However it is significant for Hong Kong, Singapore and Taiwan in the second subperiod and all three are negative. It seems that q_t^2 plays a more important role after the crisis, although these three markets are not the ones that suffered most from the crisis. As a whole, we can see that the estimation results for these two subperiods look quite similar to what have obtained from the full sample, which provides us evidence that the model is consistent for different subperiods. It also confirms that the volatile component of volatility s_t^2 is a significant pricing factor and positively covaries with return.

The empirical results tell us that s_t^2 is a significant pricing factor for all markets and covaries positively with return for most markets (except for Thailand), thus it represents a risk premium effect on return. In contrast, q_t^2 covaries negatively with r_t , but it is not significant for all markets. The intuition behind these results is as follows: since r_t has little serial correlation and on the other hand volatility has long-memory property, as indicated in most empirical literature, it is not convinced to show that volatility is a significant pricing factor to return. However, by decomposing volatility into a volatile component and a persistent component, as what has been done in this paper, we show that the volatile component of volatility is a significant pricing factor to return. The intuition is that the volatile component fluctuates more frequently through time and filters out the long memory effect and the persistent component keeps the persistence or long-memory property. Thus q_t^2 has no significant impact on return since there is no long memory in return. Also consider a rational investor, who knows that volatility includes a volatile and a persistent component. If he observes the volatile component s_t^2 is increased, then he will require a higher return. Thus s_t^2 positively covaries with r_t . However in the long-run, volatility will converge to its mean level. Hence a higher-than-mean q_t^2 implies a decrease in future volatility, which in turn decreases return as well, the result reverses if q_t^2 is lower-than-mean. Thus q_t^2 negatively covaries with return. However the link is weak since r_t has no long memory. Of course in the real world, neither s_t^2 nor q_t^2 is observable. Nevertheless the preceding argument still gives some intuition to explain the sign and significance of δ_1 and δ_2 .

Up to now the model appears to work consistently for different subperiods and be good at capturing the relation between return and components of volatility. One may be tempted to conclude that the volatile component has a significant effect on return. However one may also argue that the significance result is due to data-snooping and deceiving.

Furthermore, as mentioned earlier, there are already a few multi-factor volatility models in the literature. It is for the robust check of the model to compare its performance with similar ones, i.e. the model suggested by Ding & Granger (1996). The following Monte Carlo simulation evidence reveals the performance difference for these two models.

For comparison, I first add the lag return and volatility components to the mean equation in Ding & Granger's (1996) model and perform the estimation.¹⁰ Then I fixed parameter values at the point estimates and constructed a random sample of realizations with size equal to raw data for both models. The procedure is then replicated for $N = 10,000$ times. For each replication, the maximum, minimum and average values are collected from the realized data and their differences with the original sample maximum, minimum and average values are calculated. Finally after these values for differences are calculated for all 10,000 replications, their means and standard errors are collected for both models. This simulation approach is similar to the one implemented by Andersen & Lund (1997). Table 7 and Table 8 report the simulation results for the model introduced in this paper and the Ding & Granger's (1996) model respectively.

It can be seen from these two tables that for the difference of maximum and minimum values from the realized data and sample, the performance of these two models is similar. Both models result in 8 significant differences in these two values, the size differs for different markets. In case of China, New Zealand and Singapore, our model produces a smaller size both for the maximum and minimum differences. But for the case of Australia, Hong Kong, Japan, South Korea, Ding & Granger's (1996) model does a better work with a smaller size. However compare the size for mean differences, our model performs better. Although both models have significant mean differences for 4 markets, our model has smaller size in most markets. Furthermore as mentioned earlier, Ding & Granger's (1996) model fails to reveal the relation between return and risk, yet this model can capture the significant and positive relation between return and volatile component for volatility, in this sense, this model does a better job and add some new evidence to the literature for the relation between return and risk.

Table 9 reports the correlation between s_t and q_t , s_t^2 and q_t^2 . It is no surprise to see that in most markets (except for South Korea and Thailand), s_t and q_t are negatively correlated, as revealed by (24), since for most markets α_1 and α_2 have different signs. Actually, in some markets (Australia, China, New Zealand and Taiwan), the correlation coefficient is

¹⁰The estimates for both volatility components are not significant for all markets in this case, which may indicate that neither of them is good candidate for pricing factor for return.

Table 7: Monte Carlo Simulation Results for Current Model

Market	Max_diff	Min_diff	Average_diff
Australia	-2.583** (0.714)	4.481** (0.422)	7.537×10^{-3} (0.0111)
China	40.76 (398)	-0.9412 (11.0)	0.05070 (0.537)
Hong Kong	-9.084** (3.48)	7.681** (2.15)	0.02865 (0.0267)
Japan	0.5138 (2.86)	0.8249 (1.35)	-2.461×10^{-3} (0.0180)
New Zealand	-2.454 (3.78)	8.376** (1.19)	0.01092 (0.0166)
Philippines	-5.553 (5.98)	3.116* (1.67)	0.05142* (0.0280)
Singapore	-1.025 (8.77)	0.02604 (3.84)	0.06086** (0.0246)
South Korea	-4.333** (0.987)	7.819** (0.685)	0.02540* (0.0135)
Taiwan	5.294 (9.26)	0.2855 (3.09)	9.329×10^{-3} (0.0316)
Thailand	-1.416 (3.34)	-3.481 (8.63)	0.08951** (0.0370)

Note: This table reports the Monte Carlo Simulation results for the two-component model in this paper. The simulation is done for $N = 10,000$ replications. The Max_diff, Min_diff and Average_diff present the difference between the averaged maximum, minimum and mean values from these replications and the sample maximum, minimum and mean values.

Table 8: Monte Carlo Simulation Results for Ding & Granger's (1996) Model

Market	Max_diff	Min_diff	Average_diff
Australia	-2.253** (0.861)	4.034** (0.682)	0.02848** (0.0141)
China	75.14 (796)	-9.836 (23.5)	1.575 (15.5)
Hong Kong	-8.165** (3.14)	6.500** (2.45)	0.06575** (0.0333)
Japan	-0.1356 (1.99)	0.04201 (1.79)	0.04489* (0.0252)
New Zealand	-4.985** (1.15)	8.999** (1.01)	0.01425 (0.0158)
Philippines	-7.498** (2.44)	1.324 (2.37)	0.02464 (0.0324)
Singapore	-5.573** (2.71)	2.650 (2.06)	0.05368* (0.0313)
South Korea	-0.3019 (3.94)	3.370 (3.79)	0.04895 (0.0453)
Taiwan	-0.8202 (3.14)	2.767 (2.78)	0.05980 (0.0582)
Thailand	-1.694 (3.55)	0.8020 (3.25)	0.02382 (0.0466)

Note: This table reports the Monte Carlo Simulation results for Ding & Granger's (1996) model. The simulation is done for $N = 10,000$ replications. The Max_diff, Min_diff and Average_diff present the difference between the averaged maximum, minimum and mean values from these replications and the sample maximum, minimum and mean values.

Table 9: Correlation between Estimated s_t and q_t , s_t^2 and q_t^2

Markets	Corr(s_t, q_t)	Corr(s_t^2, q_t^2)
Australia	-0.998 (1.0×10^{-3})	0.604 (0.013)
China	-0.999 (7.4×10^{-4})	0.196 (0.016)
Hong Kong	-0.725 (0.011)	0.584 (0.013)
Japan	-0.288 (0.016)	0.292 (0.016)
New Zealand	-0.973 (3.7×10^{-3})	0.646 (0.012)
Philippines	-0.254 (0.016)	0.214 (0.016)
Singapore	-0.109 (0.016)	0.184 (0.016)
South Korea	0.941 (5.5×10^{-3})	-0.640 (0.012)
Taiwan	-0.959 (4.6×10^{-3})	0.484 (0.014)
Thailand	0.571 (0.013)	0.125 (0.016)

Note: This table presents the correlation between estimated s_t and q_t , s_t^2 and q_t^2 from (9) to (12), daily returns for 10 markets from June 3, 1991 to December 31, 2005, i.e. $T=3805$ return observations are collected to perform the estimation. Standard errors are reported in the parentheses.

close to -1 , which means s_t and q_t almost always move in opposite directions. It is not straightforward to see the economic intuition behind the negative correlation between s_t and q_t . However since the volatility is a sum of s_t^2 and q_t^2 , it is more relevant see the correlation between s_t^2 and q_t^2 . As we see from Table 9, the correlation between s_t^2 and q_t^2 are almost all positive (except for South Korea) and ranges approximately from 0.15 to 0.65. It indicates that both contribute to the movement of volatility in the same direction. However their contribution may be different in size, which is explored in the following paragraphs.

As is illustrated precedingly, s_t^2 and q_t^2 may contribute differently to volatility, and q_t^2 should contribute more to σ_t^2 as compared to s_t^2 . Table 10 presents the results for averaged volatile and persistent component of volatility $\overline{s^2}$, $\overline{q^2}$ and their sum $\overline{\sigma^2}$. These are obtained by first calculating s_t^2 , q_t^2 and σ_t^2 according to (9) ~ (12) recursively and then taking the average, i.e.:

$$\overline{s^2} = \frac{1}{T} \sum_{t=1}^T s_t^2 \quad (26)$$

Table 10: Estimated Results for Average $\overline{s^2}$, $\overline{q^2}$ and $\overline{\sigma^2}$

Markets	$\overline{s^2}$	$\overline{q^2}$	$\overline{\sigma^2}$	$\frac{1}{T} \sum_{t=1}^T \frac{s_t^2}{\sigma_t^2}$	$\frac{1}{T} \sum_{t=1}^T \frac{q_t^2}{\sigma_t^2}$
Australia	0.0934 (0.23)	0.477 (0.20)	0.570 (0.39)	0.124	0.876
China	3.78 (11.9)	2.65 (1.16)	6.43 (12.2)	0.309	0.691
Hong Kong	1.21 (3.3)	1.38 (0.32)	2.59 (3.5)	0.250	0.750
Japan	0.743 (1.0)	1.27 (0.17)	2.01 (1.1)	0.256	0.744
New Zealand	0.227 (1.1)	0.479 (0.15)	0.705 (1.2)	0.186	0.813
Philippines	0.849 (2.5)	1.39 (0.77)	2.24 (2.8)	0.216	0.784
Singapore	0.852 (3.3)	0.819 (0.18)	1.67 (3.4)	0.250	0.750
South Korea	2.25 (6.9)	1.95 (0.12)	4.20 (6.8)	0.278	0.722
Taiwan	0.796 (1.9)	1.88 (0.86)	2.68 (2.4)	0.197	0.803
Thailand	1.24 (2.6)	1.59 (0.20)	2.83 (2.6)	0.272	0.728
Mean	1.02	1.22	2.24	0.236	0.764

Note: This table reports average of the estimated volatile and persistent component of variance, $\overline{s^2} = \frac{1}{T} \sum_{t=1}^T s_t^2$ and $\overline{q^2} = \frac{1}{T} \sum_{t=1}^T q_t^2$, and average of the estimated variance $\overline{\sigma^2} = \frac{1}{T} \sum_{t=1}^T \sigma_t^2$. The mean contribution of s_t^2 and q_t^2 to σ_t^2 is also reported, the numbers in the parentheses are standard errors.

$$\overline{q^2} = \frac{1}{T} \sum_{t=1}^T q_t^2 \quad (27)$$

$$\overline{\sigma^2} = \frac{1}{T} \sum_{t=1}^T \sigma_t^2 \quad (28)$$

It is clear from Table 10 that s_t^2 and q_t^2 have different contributions to volatility σ_t^2 . The highest value of averaged ratio of s_t^2 to σ_t^2 is 30.9% for China and the lowest value is 12.4% for Australia. The mean of this averaged ratio for all 10 markets is 23.6% and correspondingly the mean of the averaged ratio q_t^2 to σ_t^2 is 76.4%. This means although s_t^2 has a significant impact on return, roughly speaking, it only contributes about 1/4 to the total volatility and q_t^2 contributes about 3/4 to the total volatility. Thus the majority part of volatility is coming from q_t^2 , which is consistent with what the model, i.e. (5) and (6) suggests. Another feature from Table 10 is that volatility in the emerging markets has more impact from s_t^2 compared to the developed markets. The averaged ratio of $\frac{s_t^2}{\sigma_t^2}$ ranges

from 12.4% to 25.6% for the developed markets and ranges from 19.7% to 30.9% for the emerging markets.

Since q_t^2 seems to be more important to volatility, it makes sense to use models including persistent or long memory property to describe dynamic of volatility. However in order to capture the dependence of return on volatility by not introducing the long memory (as implied by the empirical results in this paper), one should choose a model carefully to reflect both effects.

Figure 3 ~ Figure 4 present the estimated process for s_t and q_t for Australia. It is obvious that s_t fluctuates around zero and is more volatile than q_t . On the contrary, although the initial value for q_t is negative, it quickly jumps above zero and keeps positive for the process, which moves more smoothly than s_t . Also it is interesting to see s_t and q_t move in opposite directions at some days.

Figure 3 ~ Figure 4 about here

Figure 5 ~ Figure 14 present the estimated process for s_t^2 and q_t^2 for all markets. To make these figures comparable, different scales for s_t^2 and q_t^2 are preserved in the figures.

Figure 5 ~ Figure 14 about here

From these figures it is obvious that s_t^2 and q_t^2 roughly move in the same directions. For example, between observations 1600 to 2000, which corresponds to the Asian Financial Crisis and recovery period (1997 ~ 1999), s_t^2 in most markets becomes larger and reaches its peak. Correspondingly q_t^2 also moves up and reaches peak in the same period (especially for South Korea and Thailand, which are most heavily hit by the Crisis). However s_t^2 is much more volatile than q_t^2 for all markets. For example, in Australia market, s_t^2 fluctuates in the range from 0 to 14 and q_t^2 moves much narrowly between 0 and 3. Although s_t^2 is truncated, it is also obvious that those markets which are mostly suffered from the 1997 Asian Financial Crisis have much higher maximum value of s_t^2 , although the difference of the maximum values for q_t^2 is not as large as compared to those markets which are not suffered from the Crisis.¹¹ The case for Singapore is a little different from other markets,

¹¹The extreme values of s_t^2 during the Crisis for South Korea and Thailand are 61 and 53 respectively, compared to those of 14, 6 and 41 for Australia, Japan and New Zealand.

it has relatively smaller q_t^2 at the beginning of the period, then it increases and reaches its peak during the crisis, after that it decreases but remains a higher level compared to the period prior to the crisis. Thus the crisis seems to have a persistent impact on the market volatility in the long run.

Also it is true that emerging markets have much higher maximum values of s_t^2 compared to developed markets. Again s_t^2 absorbs the volatile part of volatility and q_t^2 keeps the persistence property. Interestingly, China has much higher s_t^2 and q_t^2 in the first half of the period, which means the Chinese stock market has much higher volatility in its beginning period. As a contrast, Japan has a higher s_t^2 and q_t^2 in early 1990s, 1997 and 2000, which corresponds to the burst of asset bubble, the Asian Financial Crisis and the burst of dotcom bubble respectively.

Figure 15 ~ Figure 16 about here

Figure 15 and Figure 16 plot q_t^2 against s_t^2 for Australia and Hong Kong. It is obvious that s_t^2 and q_t^2 move in the same direction for most time for both markets, i.e. s_t^2 and q_t^2 are positively correlated, although for small values of s_t^2 , q_t^2 will move in the opposite direction in the case of Australia. Since σ_t^2 is the sum of s_t^2 and q_t^2 , the opposite movement between s_t^2 and q_t^2 causes σ_t^2 to be stable when they have small values. However when they become relatively large, they will move in the same direction and both have effects with the same direction on σ_t^2 .

For other markets, the pattern is similar except for South Korea, which is similar to Figure 15, but in a opposite direction.

For a summary, these figures show us the time series movements of s_t^2 and q_t^2 , they are positively correlated, yet s_t^2 is more volatile than q_t^2 . During the 1997 Asian Financial Crisis, both s_t^2 and q_t^2 are increased, but s_t^2 increases to a larger extent than q_t^2 does.

4 Conclusions

This paper explores the relation between stock return and volatility by decomposing volatility into two components: the volatile component and the persistent component. Such decomposition can be expressed by the return dynamic that has both time-varying drift and volatility terms. The volatile component of volatility follows a quicker mean-reverting process with mean zero and the persistent component of volatility follows a slower

mean-reverting process with positive constant mean. This two-component decomposition of volatility and the return process can be approximated by discrete-time model, which is consistent with the continuous-time setup if the time interval goes to infinitely small.

The data analyzed in the paper includes 10 Asia-Pacific stock markets. The empirical results imply that the volatile component has a positive and significant relation to return, i.e. a risk premium effect to return, but the persistent component has negative yet insignificant relation. The persistent component has higher persistence than the volatile component, which reflects the property of long-memory in volatility. The persistent component contributes about 3/4 to volatility and the volatile component contributes only about 1/4. The estimated results also show that the first moment of these two components are negatively correlated, but dynamics of quadratic form of these two components move in the same direction. Both components contribute to the increased volatility during the 1997 Asia Financial Crisis, but the volatile component contributes more than the persistent component. The results are consistent for subperiods prior to and after the crisis. Monte Carlo simulation results show that this model performs as well as some other multi-factor model in describing return generating process and does better in capturing relation between return and risk.

This paper provides a way to analyze volatility by decomposing it into two different components. The results presented are mainly from empirical evidence. As suggested in Ding & Granger (1996), it is also possible to extend the model to more general case including more than two components for volatility dynamic, although it is not clear how to explain the additional components in economic terms. Another possible extension is to explore the relation between these volatility components and other state variables which have impact on return dynamic. Also it is possible to apply the model to individual stocks in addition to market indices. All topics are left for future research.

Appendices

A.1 Derive of Diffusion Approximation

In this Appendix, I show that the continuous-time limit of the GARCH setup (9) ~ (12) is the stochastic volatility model defined by (1) ~ (4) subject to a covariance constraint. The method used in this Appendix is inspired by Nelson (1990). First since $r_t = p_t - p_{t-1}$,

we can rewrite discrete-time model equations (9) \sim (12) as follows¹²:

$$p_t = p_{t-1} + (\mu_0 + \delta_1 s_{t+1}^2 + \delta_2 q_{t+1}^2) + \varepsilon_t \quad (\text{A.1})$$

$$s_{t+1} = \alpha_1 \varepsilon_t + \beta_1 s_t \quad (\text{A.2})$$

$$q_{t+1} = \omega + \alpha_2 \varepsilon_t + \beta_2 q_t \quad (\text{A.3})$$

$$\varepsilon_t \sim N(0, \sigma_t^2), \quad \sigma_t^2 = s_t^2 + q_t^2$$

Now we consider the properties of the discrete stochastic difference equation system (A.1) \sim (A.3) as we partition time t into k increments of length h such that $t = kh$. We allow the parameter vector $(\alpha_1, \beta_1, \omega, \alpha_2, \beta_2)$ to depend on h and make both the drift term in equation (A.1) and the variance of ε_t proportional to h :

$$p_{kh} = p_{(k-1)h} + (\mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2)h + \sqrt{s_{kh}^2 + q_{kh}^2} z_{kh} \quad (\text{A.4})$$

$$s_{(k+1)h} = \alpha_{1h} \varepsilon_{kh} + \beta_{1h} s_{kh} \quad (\text{A.5})$$

$$q_{(k+1)h} = \omega_h + \alpha_{2h} \varepsilon_{kh} + \beta_{2h} q_{kh} \quad (\text{A.6})$$

$$\varepsilon_{kh} = \sqrt{s_{kh}^2 + q_{kh}^2} z_{kh} \quad (\text{A.7})$$

where $z_{kh} \sim N(0, h)$. We can convert the discrete-time process (p_{kh}, s_{kh}, q_{kh}) in (A.4) \sim (A.7) into a continuous-time process by defining:

$$p_{kh} = p_t, \quad s_{kh} = s_t, \quad q_{kh} = q_t \quad \text{for } kh \leq t < (k+1)h \quad (\text{A.8})$$

The purpose of allowing the parameters $(\alpha_1, \beta_1, \omega, \alpha_2, \beta_2)$ to depend on h is to make it possible to find out which sequence of parameters leading the discrete-time process

¹²Equation (A.1) is a little different from (9). The lag returns are not included in the drift to simplify the calculation, however, it is possible to include other explanatory variables into the mean equation (A.1).

converges in distribution to an Ito process as h goes to zero. For more details please refer to Nelson (1990).

The first moment of equations (A.4) \sim (A.7) per unit of time is calculated as follows:

$$E[h^{-1}(p_{kh} - p_{(k-1)h})|M_{kh}] = \mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2 \quad (\text{A.9})$$

$$E[h^{-1}(s_{(k+1)h} - s_{kh})|M_{kh}] = h^{-1}(\beta_{1h} - 1)s_{kh} \quad (\text{A.10})$$

$$E[h^{-1}(q_{(k+1)h} - q_{kh})|M_{kh}] = h^{-1}\omega_h + h^{-1}(\beta_{2h} - 1)q_{kh} \quad (\text{A.11})$$

Where M_{kh} is the σ -algebra generated by the process $(p_{kh}, s_{kh}, q_{kh}, kh)$.

As required by the convergence criteria of Assumption 5 in Nelson (1990), the limits:

$$\lim_{h \rightarrow 0} h^{-1}(1 - \beta_{1h}) = \kappa^s \quad (\text{A.12})$$

$$\lim_{h \rightarrow 0} h^{-1}\omega_h = \omega \quad (\text{A.13})$$

$$\lim_{h \rightarrow 0} h^{-1}(1 - \beta_{2h}) = \theta \quad (\text{A.14})$$

must exist and be finite. We assume that these conditions are satisfied.

The second moment per unit of time is given by

$$E[h^{-1}(p_{kh} - p_{(k-1)h})^2|M_{kh}] = h(\mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2)^2 + (s_{kh}^2 + q_{kh}^2) \quad (\text{A.15})$$

$$E[h^{-1}(s_{(k+1)h} - s_{kh})^2|M_{kh}] = h^{-1}(1 - \beta_{1h})^2 s_{kh}^2 + \alpha_{1h}^2 (s_{kh}^2 + q_{kh}^2) \quad (\text{A.16})$$

$$\begin{aligned} E[h^{-1}(q_{(k+1)h} - q_{kh})^2|M_{kh}] &= h^{-1}\omega_h^2 + h^{-1}(1 - \beta_{2h})^2 q_{kh}^2 \\ &\quad + \alpha_{2h}^2 (s_{kh}^2 + q_{kh}^2) + 2h^{-1}\omega_h(\beta_{2h} - 1) \end{aligned} \quad (\text{A.17})$$

$$\begin{aligned}
& E[h^{-1}(p_{kh} - p_{(k-1)h})(s_{(k+1)h} - s_{kh})|M_{kh}] \\
&= s_k(\beta_{1h} - 1)(\mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2) + \alpha_{1h}(s_{kh}^2 + q_{kh}^2)
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
& E[h^{-1}(p_{kh} - p_{(k-1)h})(q_{(k+1)h} - q_{kh})|M_{kh}] \\
&= \omega_h(\mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2) + (\beta_{2h} - 1)(\mu_0 + \delta_1 s_{kh}^2 + \delta_2 q_{kh}^2)q_{kh} \\
&\quad + \alpha_{2h}(s_{kh}^2 + q_{kh}^2)
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
& E[h^{-1}(s_{(k+1)h} - s_{kh})(q_{(k+1)h} - q_{kh})|M_{kh}] \\
&= h^{-1}\omega_h(\beta_{1h} - 1)s_k + h^{-1}(\beta_{1h} - 1)(\beta_{2h} - 1) \\
&\quad s_{kh}q_{kh} + \alpha_{1h}\alpha_{2h}(s_{kh}^2 + q_{kh}^2)
\end{aligned} \tag{A.20}$$

Now applying the limit conditions (A.12) \sim (A.14) to these moments (A.15) \sim (A.20) we can get

$$E[h^{-1}(p_{kh} - p_{(k-1)h})^2|M_{kh}] = (s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.21}$$

$$E[h^{-1}(s_{(k+1)h} - s_{kh})^2|M_{kh}] = \alpha_{1h}^2(s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.22}$$

$$E[h^{-1}(q_{(k+1)h} - q_{kh})^2|M_{kh}] = \alpha_{2h}^2(s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.23}$$

$$E[h^{-1}(p_{kh} - p_{(k-1)h})(s_{(k+1)h} - s_{kh})|M_{kh}] = \alpha_{1h}(s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.24}$$

$$E[h^{-1}(p_{kh} - p_{(k-1)h})(q_{(k+1)h} - q_{kh})|M_{kh}] = \alpha_{2h}(s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.25}$$

$$E[h^{-1}(s_{(k+1)h} - s_{kh})(q_{(k+1)h} - q_{kh})|M_{kh}] = \alpha_{1h}\alpha_{2h}(s_{kh}^2 + q_{kh}^2) + O(h) \tag{A.26}$$

Where $O(h)$ represents for any terms that are of the same asymptotic order as h . As $h \rightarrow 0$ all the second moments exist and are finite. It is also possible to show that the fourth moments exist and converge to zero, so with $\delta = 2$, Assumption 5 in Nelson (1990) is satisfied. If we also assume that limits

$$\lim_{h \rightarrow 0} \alpha_{1h}^2 = \alpha_1^2 \quad (\text{A.27})$$

$$\lim_{h \rightarrow 0} \alpha_{2h}^2 = \alpha_2^2 \quad (\text{A.28})$$

$$\lim_{h \rightarrow 0} \alpha_{1h}(s_{kh}^2 + q_{kh}^2) = \alpha_1(s^2 + q^2)\rho_{12} \quad (\text{A.29})$$

$$\lim_{h \rightarrow 0} \alpha_{2h}(s_{kh}^2 + q_{kh}^2) = \alpha_2(s^2 + q^2)\rho_{13} \quad (\text{A.30})$$

$$\lim_{h \rightarrow 0} \alpha_{1h}\alpha_{2h} = \alpha_1\alpha_2\rho_{23} \quad (\text{A.31})$$

exist and are finite, then we can define the drift and variance matrix as follows:

$$b(p, s, q) = \begin{bmatrix} \mu_0 + \delta_1 s^2 + \delta_2 q^2 \\ -\kappa^s \\ \omega - \theta q \end{bmatrix} \quad (\text{A.32})$$

$$a(p, s, q) = \begin{bmatrix} s^2 + q^2 & \alpha_1(s^2 + q^2)\rho_{12} & \alpha_2(s^2 + q^2)\rho_{13} \\ \alpha_1(s^2 + q^2)\rho_{12} & \alpha_1^2(s^2 + q^2) & \alpha_1\alpha_2\rho_{23} \\ \alpha_2(s^2 + q^2)\rho_{13} & \alpha_1\alpha_2\rho_{23} & \alpha_2^2(s^2 + q^2) \end{bmatrix} \quad (\text{A.33})$$

Finally, let $\kappa^q = \theta$, $\kappa^q \bar{q} = \omega$, $\chi^s = \alpha_1$, $\chi^q = \alpha_2$ and $\sigma_t^2 = s_t^2 + q_t^2$, we can write the following stochastic differential process as a limit diffusion as $h \rightarrow 0$.

$$dp_t = (\alpha_0 + \alpha_1 s_t^2 + \alpha_2 q_t^2)dt + \sigma_t dW_{1t} \quad (\text{A.34})$$

$$ds_t = -\kappa^s s_t dt + \chi^s \sigma_t dW_{2t} \quad (\text{A.35})$$

$$dq_t = \kappa^q (\bar{q} - q_t) dt + \chi^q \sigma_t dW_{3t} \quad (\text{A.36})$$

This is exactly the diffusion process equation (1) \sim (4) if we let the drift term $\mu_t = \mu_0 + \delta_1 s_t^2 + \delta_2 q_t^2$.

A.2 Derivation of First and Second moments of Volatile and Persistent Components under Stationary Condition

In this appendix, I derive the first and second moments for s_t and q_t under stationary condition.

From (11),

$$s_t = \alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1}$$

and under the stationary condition, $E(s_t) = E(s_{t-1}) = \mu_s$, we obtain

$$\mu_s = \alpha_1 E[\varepsilon_{t-1}] + \beta_1 \mu_s$$

or

$$E[s_t] = \mu_s = \frac{\alpha_1}{1 - \beta_1} E[\varepsilon_{t-1}] = 0 \quad (\text{A.37})$$

since $E[\varepsilon_{t-1}] = 0$ by assumption.

Similarly, under the stationary condition, $E(q_t) = E(q_{t-1}) = \mu_q$, from (12),

$$q_t = \omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1}$$

we obtain

$$\mu_q = \omega + \alpha_2 E[\varepsilon_{t-1}] + \beta_2 \mu_q$$

or

$$E[q_t] = \mu_q = \frac{\omega + \alpha_2 E[\varepsilon_{t-1}]}{1 - \beta_2} = \frac{\omega}{1 - \beta_2} \quad (\text{A.38})$$

We can apply the similar approach to calculate the second moments for s_t and q_t , as long as the stationary condition holds. Take the square of (11) and (12):

$$s_t^2 = \alpha_1^2 \varepsilon_{t-1}^2 + \beta_1^2 s_{t-1}^2 + 2\alpha_1 \beta_1 \varepsilon_{t-1} s_{t-1}$$

and

$$q_t^2 = \omega^2 + \alpha_2^2 \varepsilon_{t-1}^2 + \beta_2^2 q_{t-1}^2 + 2\omega\alpha_2 \varepsilon_{t-1} + 2\omega\beta_2 q_{t-1} + 2\alpha_2\beta_2 \varepsilon_{t-1} q_{t-1}$$

Take the expectation on both sides and under the stationary condition $E[s_t^2] = E[s_{t-1}^2] = \sigma_s^2$ and $E[q_t^2] = E[q_{t-1}^2] = \sigma_q^2$, we obtain that

$$\sigma_s^2 = \alpha_1^2(\sigma_s^2 + \sigma_q^2) + \beta_1^2 \sigma_s^2$$

or

$$\sigma_s^2 = \frac{\alpha_1^2}{1 - \alpha_1^2 - \beta_1^2} \sigma_q^2 \quad (\text{A.39})$$

and

$$\sigma_q^2 = \omega^2 + \alpha_2^2(\sigma_s^2 + \sigma_q^2) + \beta_2^2 \sigma_q^2 + 2\omega\beta_2 \mu_q$$

substitute (A.38) and (A.39), and solve for σ_q^2 , we obtain

$$\sigma_q^2 = \frac{1}{1 - \frac{\alpha_2^2(1-\beta_1^2)}{(1-\beta_2^2)(1-\alpha_1^2-\beta_1^2)}} \mu_q^2 \quad (\text{A.40})$$

since $\sigma_t^2 = s_t^2 + q_t^2$, from (A.39) and (A.40), we obtain (A.41)

$$E[\sigma_t^2] = \sigma^2 = \frac{\mu_q^2}{1 - \frac{\alpha_1^2}{1-\beta_1^2} - \frac{\alpha_2^2}{1-\beta_2^2}} \quad (\text{A.41})$$

To calculate the covariance between s_t and q_t , notice that by definition

$$\text{cov}(s_t, q_t) = E[(s_t - \mu_s)(q_t - \mu_q)]$$

This indicates that

$$\begin{aligned} \text{cov}(s_t, q_t) &= E[s_t q_t] \\ &= E[(\alpha_1 \varepsilon_{t-1} + \beta_1 s_{t-1})(\omega + \alpha_2 \varepsilon_{t-1} + \beta_2 q_{t-1})] \\ &= \alpha_1 \alpha_2 E[\varepsilon_{t-1}^2] + \beta_1 \beta_2 E[s_{t-1} q_{t-1}] \end{aligned}$$

After rearranging the equation, we obtain (24)

$$\text{cov}(s_t, q_t) = \frac{\alpha_1 \alpha_2}{1 - \beta_1 \beta_2} \sigma^2 = \frac{\alpha_1 \alpha_2}{1 - \beta_1 \beta_2} \frac{\mu_q^2}{1 - \frac{\alpha_1^2}{1 - \beta_1^2} - \frac{\alpha_2^2}{1 - \beta_2^2}} \quad (\text{A.42})$$

A.3 Estimation by Quasi-Maximum Likelihood Method

In this Appendix I present the estimation of parameters by Quasi-Maximum Likelihood Method. The application of maximum log likelihood method to GARCH models has been widely discussed in many papers. As a brief introduction, please refer to Chapter 12 in Campbell, Lo & Mackinlay (1997).

Consider the following equations:

$$r_{t+1} = (\mu_0 + \mu_1 r_t + \delta_1 s_{t+1}^2 + \delta_2 q_{t+1}^2) + \varepsilon_{t+1} \quad (\text{A.43})$$

$$s_{t+1} = \alpha_1 \varepsilon_t + \beta_1 s_t \quad (\text{A.44})$$

$$q_{t+1} = \omega + \alpha_2 \varepsilon_t + \beta_2 q_t \quad (\text{A.45})$$

$$\varepsilon_{t+1} = \sigma_{t+1} z_{t+1}, \quad \sigma_{t+1} = \sqrt{s_{t+1}^2 + q_{t+1}^2}, \quad z_{t+1} \sim N(0, 1)$$

First define the parameter vector $\boldsymbol{\theta} = (\mu_0, \mu_1, \delta_1, \delta_2, \alpha_1, \beta_1, \omega, \alpha_2, \beta_2)'$. By definition, when $\boldsymbol{\theta}$ contains true parameters, z_{t+1} is IID with density function $g(z_{t+1}(\boldsymbol{\theta}))$ which we assume to be standard normal:

$$g(z_{t+1}(\boldsymbol{\theta})) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_{t+1}(\boldsymbol{\theta})^2}{2}} \quad (\text{A.46})$$

The conditional log likelihood of ε_{t+1} is therefore:

$$\begin{aligned}
l_t(\varepsilon_{t+1}; \boldsymbol{\theta}) &= \log(g(\varepsilon_{t+1}/\sigma_{t+1}(\boldsymbol{\theta}))) - \log(\sigma_{t+1}^2(\boldsymbol{\theta}))/2 \\
&= -\log(\sqrt{2\pi}) - \varepsilon_{t+1}^2/2\sigma_{t+1}^2(\boldsymbol{\theta}) \\
&\quad - \log(\sigma_{t+1}^2(\boldsymbol{\theta}))/2 \\
&= -\log(\sqrt{2\pi}) - \frac{1}{2}\log(s_{t+1}^2(\varepsilon_{t+1}; \boldsymbol{\theta}) + q_{t+1}^2(\varepsilon_{t+1}; \boldsymbol{\theta})) \\
&\quad - \varepsilon_{t+1}^2/(2(s_{t+1}^2(\varepsilon_{t+1}; \boldsymbol{\theta}) + q_{t+1}^2(\varepsilon_{t+1}; \boldsymbol{\theta})))
\end{aligned} \tag{A.47}$$

and $\varepsilon_{t+1} = r_{t+1} - (\mu_0 + \mu_1 r_t + \delta_1 s_{t+1}^2 + \delta_2 q_{t+1}^2)$

The log likelihood of the whole data set $\varepsilon_1, \dots, \varepsilon_T$ is

$$L(\varepsilon_1, \dots, \varepsilon_T) = \sum_{t=1}^T l_t(\varepsilon_{t+1}; \boldsymbol{\theta}) \tag{A.48}$$

The maximum likelihood estimator is the choice of parameters $\boldsymbol{\theta}$ that maximizes (A.48).

However the specification of the log-likelihood function is depending on our assumption for the error term. If the standardized error term z_t is actually not normally distributed, then the model is misspecified. Fortunately in this case the preceding Maximum Likelihood Method is still available provided that the error terms are not serially correlated and the estimation result is consistent, although it is not the most efficient one (see White (1982) for more details). In this case, the variance of the estimated parameters should be obtained by so-called Sandwich Method or Quasi-Maximum Likelihood Method.

Let $A(\boldsymbol{\theta})$ to be the Hessian matrix of the parameters

$$A(\boldsymbol{\theta}) = E\left[\frac{\partial^2 L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_i \partial \theta_j}\right] \tag{A.49}$$

and $B(\boldsymbol{\theta})$ to be the outer-product of parameters:

$$B(\boldsymbol{\theta}) = E\left[\frac{\partial L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_i} \frac{\partial L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_j}\right] \tag{A.50}$$

where $\boldsymbol{\theta}$ is the parameter vector.

Then $C(\boldsymbol{\theta})$, which is defined as follows:

$$C(\boldsymbol{\theta}) = A(\boldsymbol{\theta})^{-1}B(\boldsymbol{\theta})A(\boldsymbol{\theta})^{-1} \quad (\text{A.51})$$

is the robust estimated variance of parameters and we have

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim N(0, C(\boldsymbol{\theta})) \quad (\text{A.52})$$

and $A(\boldsymbol{\theta})$ and $B(\boldsymbol{\theta})$ can be estimated by their sample counterparts $A_t(\boldsymbol{\theta})$ and $B_t(\boldsymbol{\theta})$:

$$\hat{A}(\boldsymbol{\theta}) = \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \quad (\text{A.53})$$

$$\hat{B}(\boldsymbol{\theta}) = \frac{1}{T} \sum_{t=1}^T \frac{\partial L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_i} \frac{\partial L(\varepsilon_t; \boldsymbol{\theta})}{\partial \theta_j} \quad (\text{A.54})$$

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Figure 1: Scatter plot of r_t vs s_t^2 from equation (9) without in-mean terms for Australia

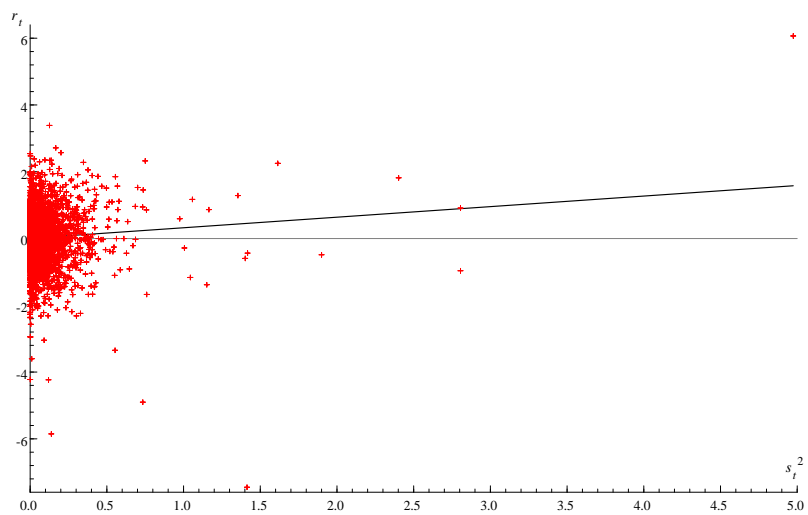


Figure 2: Scatter plot of r_t vs q_t^2 from equation (9) without in-mean terms for Australia

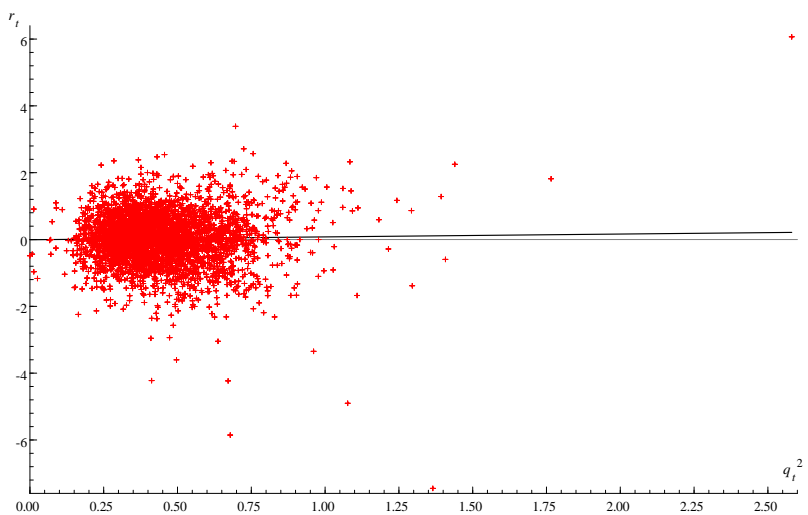


Figure 3: s_t process for Australia, 1991/6/3 – 2005/12/31

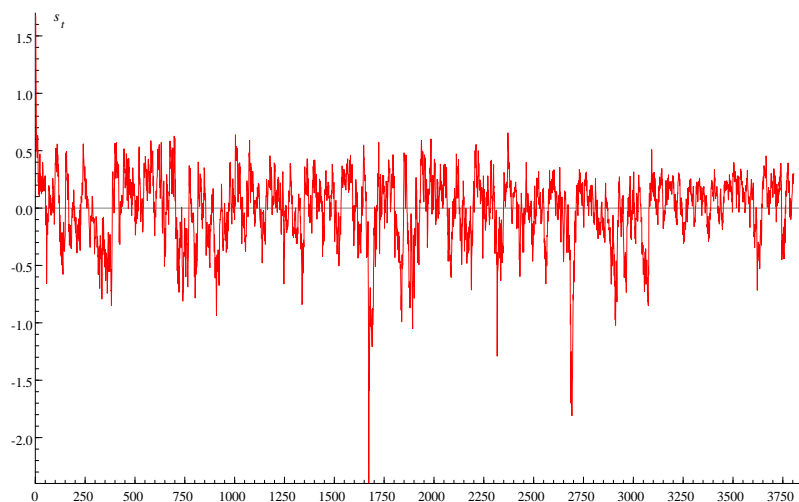


Figure 4: q_t process for Australia, 1991/6/3 – 2005/12/31

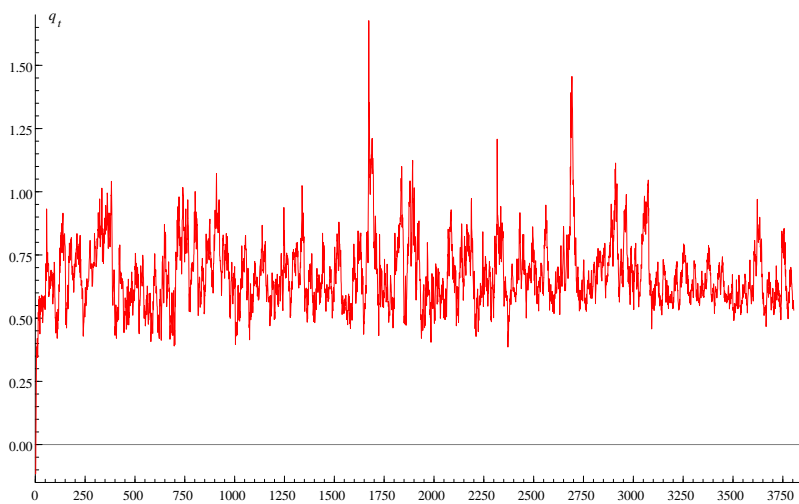


Figure 5: s_t^2 and q_t^2 processes for Australia, 1991/6/3 – 2005/12/31

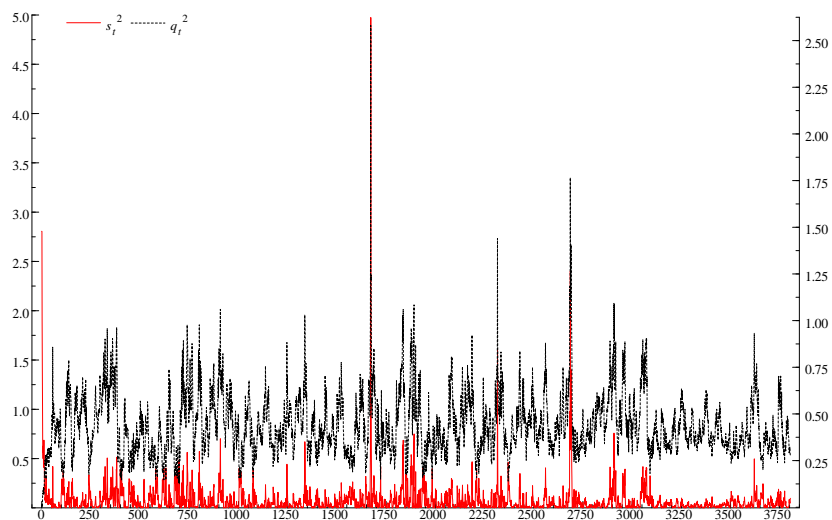


Figure 6: s_t^2 and q_t^2 processes for China, 1991/6/3 – 2005/12/31

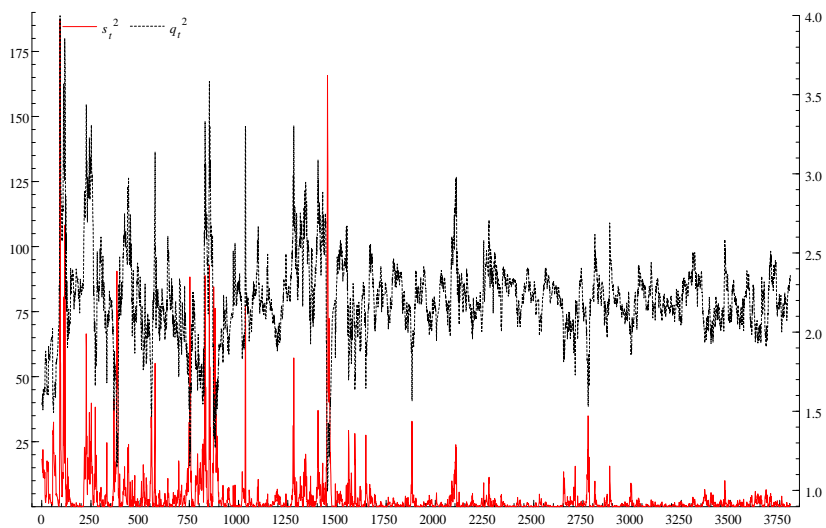


Figure 7: s_t^2 and q_t^2 processes for Hong Kong, 1991/6/3 – 2005/12/31

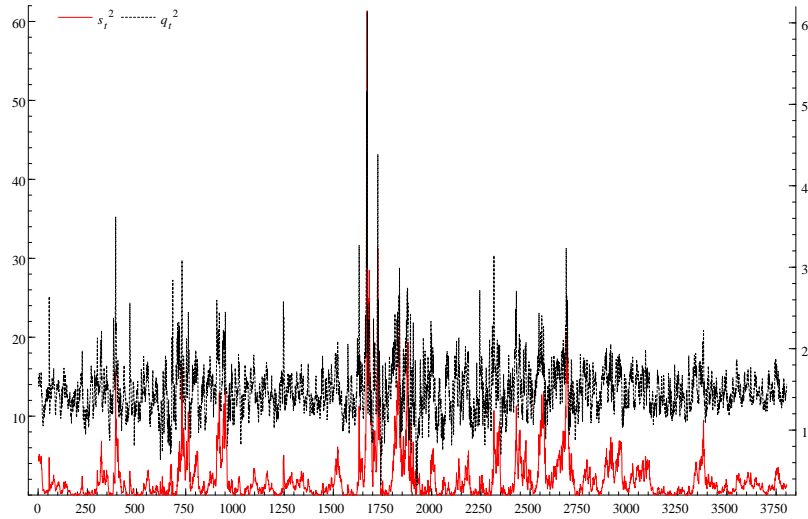


Figure 8: s_t^2 and q_t^2 processes for Japan, 1991/6/3 – 2005/12/31

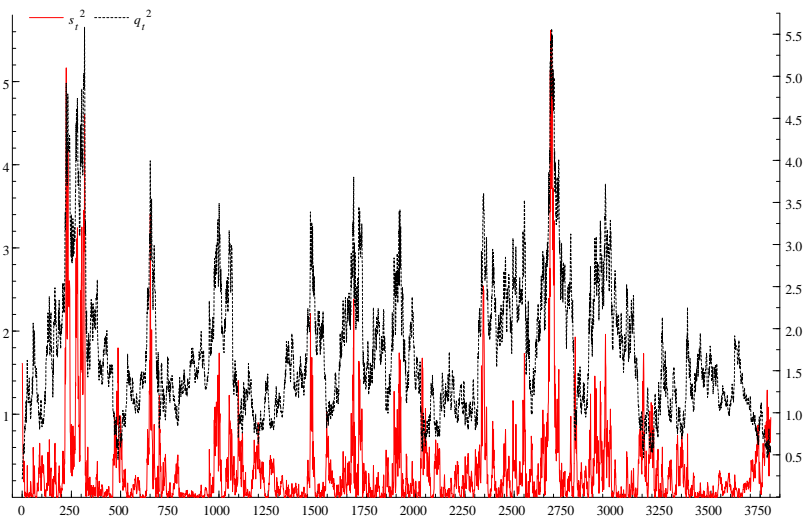


Figure 9: s_t^2 and q_t^2 processes for New Zealand, 1991/6/3 – 2005/12/31

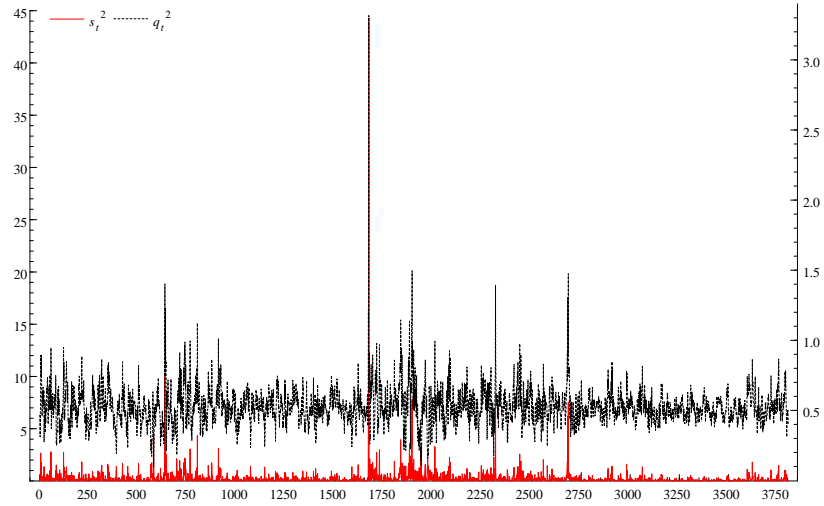


Figure 10: s_t^2 and q_t^2 processes for Philippines, 1991/6/3 – 2005/12/31

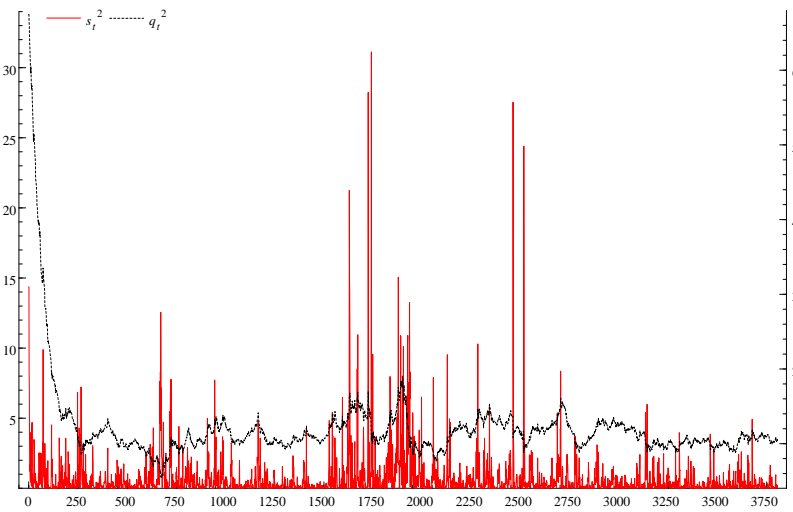


Figure 11: s_t^2 and q_t^2 processes for Singapore, 1991/6/3 – 2005/12/31

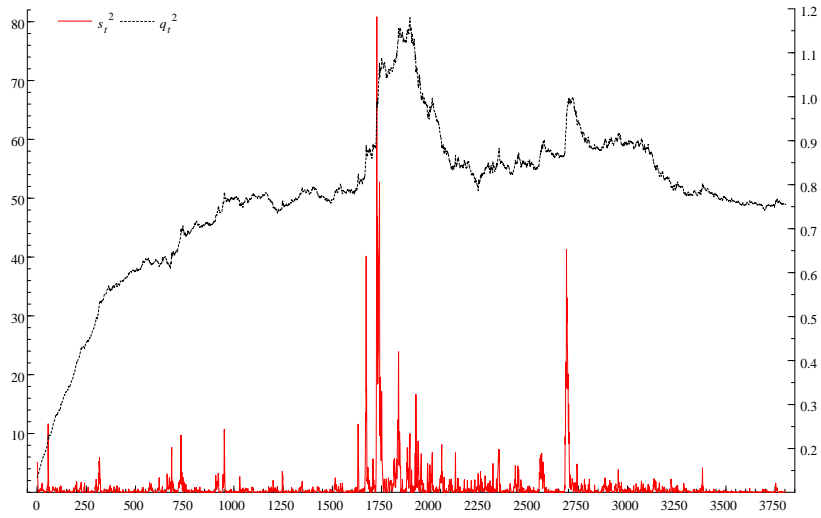


Figure 12: s_t^2 and q_t^2 processes for South Korea, 1991/6/3 – 2005/12/31

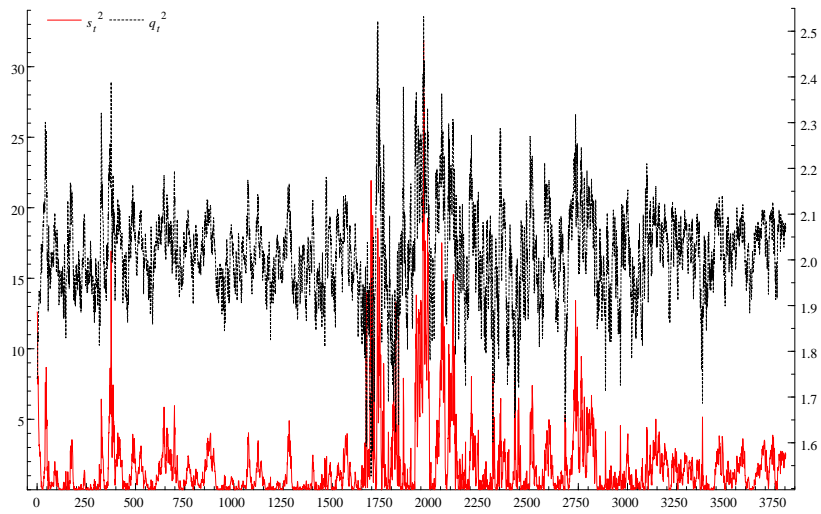


Figure 13: s_t^2 and q_t^2 processes for Taiwan, 1991/6/3 – 2005/12/31

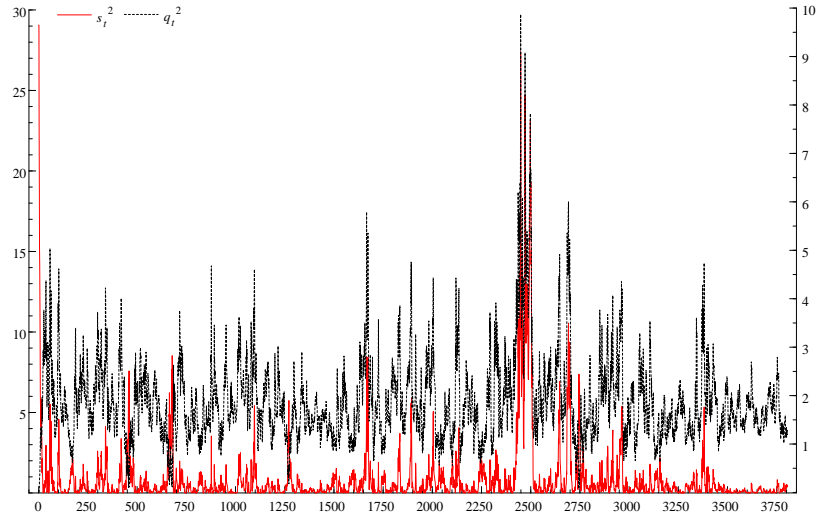


Figure 14: s_t^2 and q_t^2 processes for Thailand, 1991/6/3 – 2005/12/31

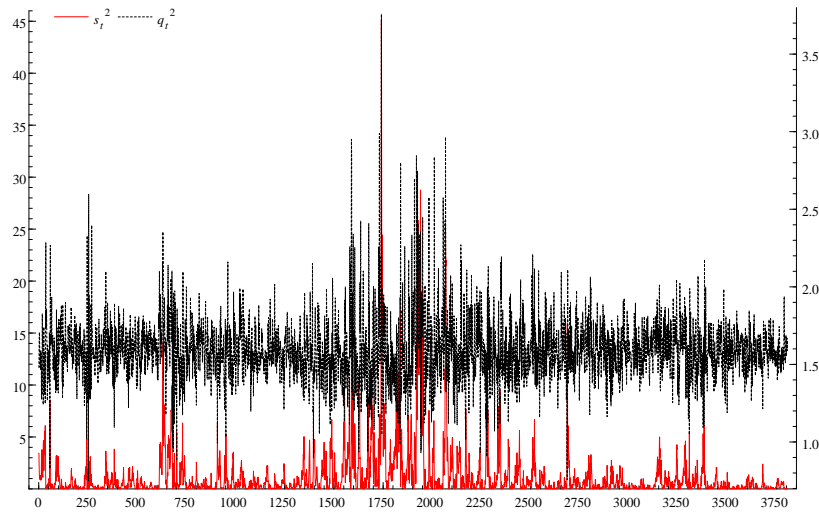


Figure 15: Scatter plot of q_t^2 vs s_t^2 for Australia, 1991/6/3 – 2005/12/31

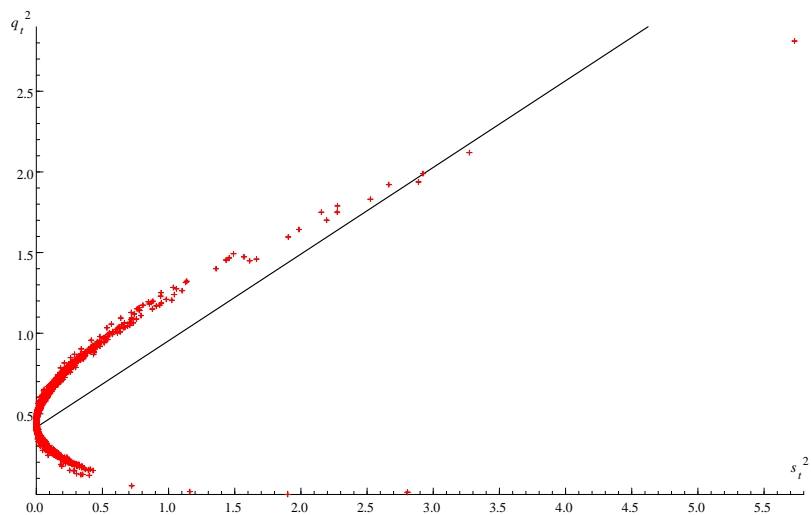
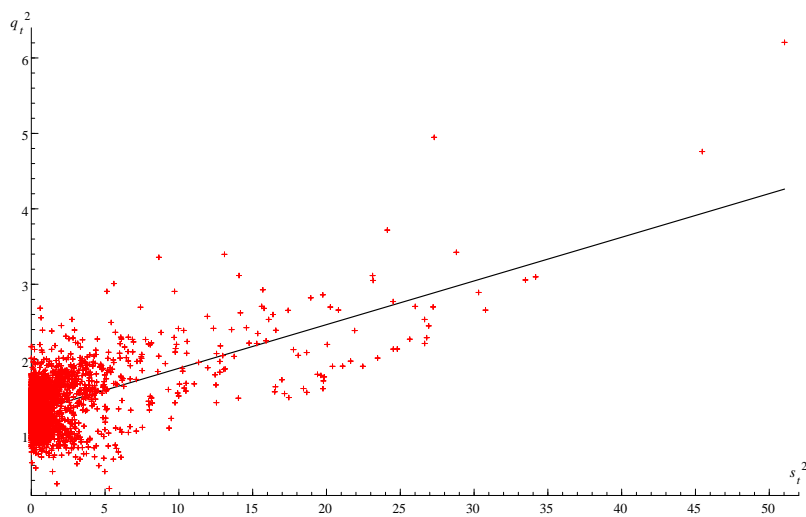


Figure 16: Scatter plot of q_t^2 vs s_t^2 for Hong Kong, 1991/6/3 – 2005/12/31



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