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An iterated GMM procedure for estimating the Campbell-Cochrane habit formation model, with an application to Danish stock and bond returns*

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Abstract

We suggest an iterated GMM approach to estimate and test the consumption based habit persistence model of Campbell and Cochrane (1999), and we apply the approach on annual and quarterly Danish stock and bond returns. For comparative purposes we also estimate and test the standard CRRA model. In addition, we compare the pricing errors of the different models using Hansen and Jagannathan's (1997) specification error measure. The main result is that for Denmark the Campbell-Cochrane model does not seem to perform markedly better than the CRRA model. For the long annual sample period covering more than 80 years there is absolutely no evidence of superior performance of the Campbell-Cochrane model. For the shorter and more recent quarterly data over a 20-30 year period, there is some evidence of counter-cyclical time-variation in the degree of risk-aversion, in accordance with the Campbell-Cochrane model, but the model does not produce lower pricing errors or more plausible parameter estimates than the CRRA model.

Keywords: Consumption-based model, habit persistence, GMM, pricing error.

JEL codes: C32, G12

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1 Introduction

Since Mehra and Prescott's (1985) seminal study, explaining the observed high equity premium within the consumption based asset pricing framework has occupied a large number of researchers in finance and macroeconomics. Despite an intense research effort, still no consensus has emerged as to why stocks have given such a high average return compared to bonds. At first sight the natural response to the equity premium puzzle is to dismiss the consumption based framework altogether. However, as emphasized by Cochrane (2005), within the rational equilibrium paradigm of finance, there is really no alternative to the consumption based model, since other models are not alternatives to - but special cases of - the consumption based model. Thus, despite its poor empirical performance, the consumption based framework continues to dominate studies of the equity premium on the aggregate stock market.

In a recent paper Chen and Ludvigson (2006) argue that within the equilibrium consumption based framework, habit formation models are the most promising and successful in describing aggregate stock market behaviour. The most prominent habit model is the one developed by Campbell and Cochrane (1999). In this model people slowly develop habits for a high or low consumption level, such that risk-aversion becomes time-varying and counter-cyclical. The model is able to explain the high US equity premium and a number of other stylized facts for the US stock market. A special feature of the model is that the average risk aversion over time is quite high, but the risk-free rate is low and stable. Thus, the model solves the equity premium puzzle by high risk aversion, but without facing a risk-free rate puzzle.

Campbell and Cochrane (1999) themselves, and most subsequent applications of their model, do not estimate and test the model econometrically. Instead they calibrate the model parameters to match the historical risk-free rate and Sharpe ratio, and then simulate a chosen set of moments which are informally compared to those based on actual historical data. Only a few papers engage in formal econometric estimation and testing of the model. Tallarini and Zhang (2005) use an Efficient Method of Moments technique to estimate and test the model on US data. They statistically reject the model and find that it has strongly counterfactual implications for the risk-free interest rate, although they also find that the model performs well in other dimensions. Fillat and Garduno (2005) and Garcia et al. (2005) use an iterated Generalized Method of Moments approach to estimate and test the model on US data. Fillat and Garduno strongly reject the model by Hansen's (1982) J-test. On the other hand Garcia et al. do not reject the model at conventional significance levels. However, Garcia et al. face the problem that their iterated GMM approach does not lead to convergence with positive values of the risk-aversion parameter. Finally, Møller (2008) estimates the model by GMM in a cross-sectional setting using the Fama-French 25 value and size portfolios. He finds support for the model although it has difficulties in explaining the value premium.

To our knowledge, there have been no formal econometric studies of the Campbell-Cochrane model on data from other countries than the US. Our paper is the first attempt to fill this gap.[1] We examine the Campbell-Cochrane model's ability to explain Danish stock and bond returns. Denmark is interesting because historically over a long period of time the average return on Danish stocks has not been nearly as high as in the US and most other countries, and at the same time the return on Danish bonds has been somewhat higher than in other countries, see e.g. Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002). Thus, the Danish equity premium is not nearly as high as in most other countries, and might not even be regarded a puzzle.

On annual Danish data for the period 1922-2004 and quarterly data for the period 1977-2006 we estimate and test both the standard model based on constant relative risk-aversion (CRRA) and the Campbell-Cochrane model based on habit formation. We basically follow the iterated GMM approach set out in Garcia et al. (2005). However, in contrast to Garcia et al., - who estimate the model parameters in two successive steps - we do a joint GMM estimation of all parameters, thereby properly taking into account sampling error on all parameter estimates. We also compute Hansen and Jagannathan's (1997) specification error measure based on the second moment matrix of returns as weighting matrix. This measure has an intuitively appealing percentage pricing error interpretation, and it allows for direct comparison of the magnitude of pricing errors across models.

Our main findings are as follows. First, neither the CRRA model nor the Campbell-Cochrane model are statistically rejected by Hansen's J-test, and pricing errors are of the same magnitude for both models. Second, both models imply high risk-aversion and a low and plausible value for the real risk-free rate. Third, in most cases the CRRA model produces plausible values for the time discount factor while the Campbell-Cochrane model delivers implausibly low values for this para-

[1] Hyde and Sherif (2005), Hyde et al. (2005), and Li and Zhong (2005) examine the Campbell-Cochrane model using international data, but with the calibrated parameter values from the original US study by Campbell and Cochrane. In Engsted et al. (2008) we apply the iterated GMM approach from the present paper to estimate and test the Campbell-Cochrane model using an international post World War II annual dataset.

meter. These results are quite robust across different data sets and instrument sets. However, when it comes to the variation over time in the degree of relative risk-aversion in the Campbell-Cochrane model, there is some difference between the long annual data set and the shorter quarterly data sets. In the annual data there is no visible counter-cyclical movement in risk-aversion, while in the quarterly data there is some evidence of counter-cyclical variation over time in accordance with the Campbell-Cochrane model.

The rest of the paper is organized as follows. The next section briefly presents the consumption-based models. Section 3 explains the iterated GMM approach used to estimate the models. Section 4 presents the empirical results based on Danish data. Finally, section 5 offers some concluding remarks.

2 The consumption based models

In this section we start by describing the standard CRRA utility version of the consumption based model. Since this version of the model is well-known and familiar to most readers, the description will be very brief. Then we give a more detailed description of the Campbell-Cochrane habit based model.

2.1 The CRRA utility model

Standard asset pricing theory implies that the price of an asset at time t, P_t, is determined by the expected future asset payoff, Y_{t+1}, multiplied by the stochastic discount factor, M_{t+1}: $P_t = E_t(M_{t+1}Y_{t+1})$. The payoff is given as prices plus dividends, $Y_{t+1} = P_{t+1} + D_{t+1}$, and the stochastic discount factor depends on the underlying asset pricing model. In consumption based models M_{t+1} is the intertemporal marginal rate of substitution in consumption. With power utility (constant relative risk aversion), $U(C_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$, where $\gamma \geq 0$ is the degree of relative risk aversion, the stochastic discount factor becomes $M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, where $\delta = (1+t_p)^{-1}$ and t_p is the rate of time-preference. Defining gross return as $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$, the asset pricing relationship can be stated as:

$$0 = E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} - 1\right]. \quad (1)$$

Equation (1) captures the basic idea that risk-adjusted equilibrium returns are unpredictable. In the consumption based model, risk-adjustment takes place by multiplying the raw return with the intertemporal marginal rate of substitution in consumption. Risk-averse consumers want to smooth consumption over time, and for that purpose they use (dis)investments in the asset, thereby making a direct connection between consumption growth and the asset return. The correlation between consumption growth and returns then becomes crucial for the equilibrium expected return. From (1) expected returns are given as:

$$E_t\left[R_{t+1}\right] = \frac{1 - \mathrm{Cov}_t\left[R_{t+1}, \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]} \tag{2}$$

The higher the correlation between consumption growth and returns (the lower the correlation between the stochastic discount factor and returns), the higher will be expected equilibrium return (*ceteris paribus*), because the higher the correlation, the less able the asset will be in helping to smooth consumption over time, which means that the asset will be considered riskier and thereby demand a higher return.

Equation (1) lends itself directly to empirical estimation and testing within the GMM framework, c.f. section 3. Empirically the consumption based power utility model has run into trouble because consumption growth and stock returns are not sufficiently positively correlated to explain the historically observed high return on common stocks, unless the degree of risk aversion γ is extremely high. The basic problem is that unless γ is very high, the variability of the intertemporal marginal rate of substitution cannot match the variability of stock returns. Perhaps people *are* highly risk-averse, but then the power utility model faces another problem, namely that with a high γ, the risk-free rate implied by the model becomes implausibly high. For the risk-free rate the covariance with the stochastic discount factor is zero, thus from (2):

$$R_{f,t+1} = \frac{1}{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]} \tag{3}$$

Thus, within the standard CRRA utility framework, the equity premium puzzle cannot be solved without running into a risk-free rate puzzle. This has led to the development of alternative utility models with a higher volatility of the stochastic discount factor, and with plausible implications for the risk-free rate. The habit persistence model described in the next subsection is one such model.

2.2 The Campbell-Cochrane model

Habit formation models differ from the standard power utility model by letting the utility function be time-nonseparable in the sense that the utility at time t depends not only on consumption at time t, but also on previous periods consumption. The basic idea is that people get used to a certain standard of living and thereby the utility of some consumption level at time t will be higher (lower) if previous periods consumption was low (high) than if previous periods consumption was high (low).

Habit formation can be modelled in a number of different ways. In the Campbell-Cochrane model utility is specified as

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \quad C_t > X_t \tag{4}$$

where X_t is an external habit level that depends on previous periods consumption. Define the *surplus consumption ratio* as $S_t = \frac{C_t - X_t}{C_t}$. Then the stochastic discount factor can be stated as $M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ and the pricing equation becomes

$$0 = E_t \left[\delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} - 1 \right] \tag{5}$$

Compared to the standard power utility model in (1), the Campbell-Cochrane model implies a stochastic discount factor that not only depends on consumption growth but also on growth in the consumption surplus ratio. In this model relative risk-aversion is no longer measured by γ but as $\frac{\gamma}{S_t}$. This shows that relative risk-aversion is time-varying and counter-cyclical: when consumption is high relative to habit, relative risk-aversion is low and expected returns are low. By contrast, when consumption is low and close to habit, relative risk-aversion is high leading to high expected returns. Basically the model explains time-varying and counter-cyclical ex ante returns (which implies pro-cyclical stock prices) as a result of time-varying and counter-cyclical risk-aversion of people. From (5) expected returns are given as:

$$E_t \left[R_{t+1} \right] = \frac{1 - \mathrm{Cov}_t \left[R_{t+1}, \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]}{E_t \left[\delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]} \tag{6}$$

A crucial aspect in operalizing the model is the modelling of the risk-free rate. Campbell and Cochrane specify the model in such a way that the risk-free rate is constant and low by construction. First, assume that consumption is lognormally distributed such that consumption growth is normally distributed and *iid*:

$$\Delta c_{t+1} = g + v_{t+1}, \quad v_{t+1} \sim niid(0, \sigma_v^2) \tag{7}$$

where $c_t \equiv \log(C_t)$. g is the mean consumption growth rate. Next, specify the log surplus consumption ratio $s_t = \log(S_t)$ as a stationary first-order autoregressive process

$$s_{t+1} = (1 - \phi)\overline{s} + \phi s_t + \lambda(s_t) v_{t+1} \tag{8}$$

where $0 < \phi < 1$, $\overline{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function to be specified below. Note that shocks to consumption growth are modelled to have a direct impact on the surplus consumption level, and for ϕ close to one, habit responds slowly to these shocks.

The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda(s_t) = \left\{ \begin{array}{cc} \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s})} - 1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{else} \end{array} \right\} \tag{9}$$

where

$$\overline{S} = \sigma_v \sqrt{\frac{\gamma}{1-\phi}}, \qquad s_{\max} \equiv \overline{s} + \frac{1}{2}(1 - \overline{S}^2), \qquad \overline{s} = \log(\overline{S})$$

Specifying $\lambda(s_t)$ in this way implies the following equation for the log risk-free rate:

$$r_{f,t+1} = -\log(\delta) + \gamma g - \frac{\gamma^2 \sigma_v^2}{2} \left(\frac{1}{\overline{S}} \right)^2 \tag{10}$$

As seen, no time-dependent variables appear in (10), thus the risk-free rate is constant over time. Economically this property of the model is obtained by letting the effects of intertemporal substitution and precautionary saving - which have opposite effects on the risk-free rate - cancel each other out, see Campbell and Cochrane (1999) for details.

Campbell and Cochrane calibrate their model with parameters chosen to match post war US data: mean real consumption growth rate (g), mean real risk-free rate (r_f), volatility (σ_v), etc. Then, based on the calibrated model, simulated time-series for returns, price-dividend ratios, etc., are generated and their properties are compared to the properties of the actually observed post war data. In the present paper we instead estimate the model parameters in a GMM framework. The next section describes how.

3 GMM estimation of the models

The GMM technique developed by Hansen (1982) estimates the model parameters based on the orthogonality conditions implied by the model. Let the asset pricing equation be $0 = E_t\left[M_{t+1}(\theta)R_{t+1} - 1\right]$, where M_{t+1} is the stochastic discount factor, R_{t+1} is a vector of asset returns, and the vector θ contains the model parameters. In the present context this equation corresponds to either (1) or (5) with $\theta = (\delta \quad \gamma)'$. Define a vector of instrumental variables, Z_t, observable at time t. Then the asset pricing equation implies the following orthogonality conditions $E\left[(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t\right] = 0$. GMM estimates θ by making the sample counterpart to these orthogonality conditions as close to zero as possible, by minimizing a quadratic form of the sample orthogonality conditions based on a chosen weighting matrix. Define $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t$ as the sample orthogonality conditions based on T observations. Then the parameter vector θ is estimated by minimizing

$$g_T(\theta)'Wg_T(\theta) \tag{11}$$

where W is the weighting matrix. The statistically optimal (most efficient) weighting matrix is obtained as the inverse of the covariance matrix of the sample orthogonality conditions. Other weighting matrices can be chosen, however, and often a fixed and model-independent weighting matrix (the identity matrix, for example) is used in order to make it possible to compare the magnitude of estimated pricing errors across different models. Such a comparison cannot be done if the statistically optimal weighting matrix is used because this matrix is model-dependent.

GMM estimation of the standard CRRA utility model (1) is straightforward. However, estimation of the Campbell-Cochrane model, equation (5), is complicated by the fact that the surplus consumption ratio, S_t, is not observable in the same way as returns, R_t, and consumption, C_t, are directly observable. Garcia et al. (2005) suggest to generate a process for s_t by initially estimating the parameters ϕ, g and σ_v, and setting γ to some initial value, which then gives $\overline{s}$, from which s_t can be constructed using (8) and a starting value for s_t at $t = 0$. Garcia et al. set $s_0 = \overline{s}$. Having obtained a series for the surplus consumption ratio, GMM can be applied directly. Since the surplus consumption ratio depends on γ, however, the resulting GMM estimate of γ may not correspond to the value initially imposed in generating s_t. Therefore, Garcia et al. iterate over γ by estimating the model in each iteration using GMM with the statistically optimal weighting matrix. Unfortunately, this procedure does not lead to convergence with a positive value of γ in their application. Instead they do a grid search that implies an estimated value of γ close but not identical to the initiallly picked value.

Our procedure differs from Garcia et al.'s in the following way: They estimate ϕ, g and σ_v separately in an inital step. Then, given these parameter estimates, they estimate δ and γ using GMM. Instead we do a joint GMM estimation of all parameters and, hence, take into account sampling error on all parameters. We report results for different instrument sets. However, in order to economize on the number of orthogonality conditions, we fix the instrument set to contain just a constant for the estimation of g and σ_v, and a constant and lagged log price-dividend ratio for the estimation of ϕ. Moreover, following Cochrane's (2005) suggestion, we use the identity matrix as weighting matrix across all GMM estimations. Thereby we attach equal weight to each asset in the estimation.[2] The details of our estimation procedure is as follows:

Following Campbell and Cochrane (1999) and Garcia et al. (2005) we estimate ϕ as the first-order autocorrelation parameter for the log price-dividend ratio:

$$p_t - d_t = \alpha + \phi(p_{t-1} - d_{t-1}) + \varepsilon_t \tag{12}$$

This is feasible since in the Campbell-Cochrane model the surplus consumption ratio is the only state variable, whereby the log price-dividend ratio, $p_t - d_t$, will inherit its dynamic properties from the log surplus consumption ratio, s_t. The mean consumption growth rate, g, and the consumption volatility, σ_v, are estimated from (7).

As starting values in the GMM iterations we use OLS estimates of g, σ_v, and ϕ, and we choose an initial value of $\gamma = 1$ to obtain $\overline{S} = \sigma_v \sqrt{\frac{\gamma}{1-\phi}}$ and set $s_t = \overline{s}$ at $t = 0$. From the chosen parameter values, we obtain the s_t process recursively. Given s_t, S_t is obtained as $\exp(s_t)$. Using this S_t process we minimize (11) jointly with moment conditions of (7) and (12), which gives GMM estimates of all model parameters δ, γ, ϕ, g, and σ_v. The parameter estimates are used to generate a new S_t process and we repeat this procedure until convergence of all estimated parameters.

Since the chosen weighting matrix is not the efficient Hansen (1982) matrix but the identity matrix I, the formula for the covariance matrix of the parameter vector is (c.f. Cochrane (2005), chpt. 11):

$$Var(\widehat{\theta}) = \frac{1}{T}(d'Id)^{-1}d'ISId(d'Id)^{-1} \tag{13}$$

[2]We use a GMM programme written in MatLab. The programme is available upon request.

where $d' = \partial g_T(\theta)/\partial\theta$, and the spectral density matrix $S = \sum_{j=-\infty}^{\infty} E[g_T(\theta)g_{T-j}(\theta)']$ is computed with the usual Newey and West (1987) estimator with a lag truncation. Similarly, the J-test of overidentifying restrictions is computed based on the general formula (c.f. Cochrane (2005) chpt. 11):

$$J_T = Tg_T(\widehat{\theta})' \left[(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')\right]^{-1} g_T(\widehat{\theta}) \quad (14)$$

J_T has an asymptotic χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions. (14) involves the covariance matrix $Var(g_T(\widehat{\theta})) = \frac{1}{T}(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')$, which is singular, so it is inverted using the Moore-Penrose pseudo-inversion.

In addition to formally testing the model using the J-test, we also compute the Hansen and Jagannathan (1997) misspecification measure, HJ, as

$$HJ = \left[E(M_{t+1}(\theta)R_{t+1} - 1)'(E(R_{t+1}R'_{t+1}))^{-1}E(M_{t+1}(\theta)R_{t+1} - 1)\right]^{\frac{1}{2}} \quad (15)$$

HJ measures the minimum distance between the candidate stochastic discount factor M_{t+1} and the set of admissible stochastic discount factors. HJ can be interpreted as the maximum pricing error per unit payoff norm. Thus, it has an intuitively appealing percentage pricing error interpretation. It is a measure of the magnitude of pricing errors that gives a useful *economic* measure of fit, in contrast to the statistical measure of fit given by Hansen's J-test. In addition, since the HJ measure is based on a model-independent weighting matrix, it can be used to compare pricing errors across models. The HJ measure is computed at the GMM estimates of δ and γ. We compute the asymptotic standard error of $\widehat{HJ}$ using the Hansen et al. (1995) procedure.[3]

4 Empirical results

We estimate the models on annual data from 1922 to 2004 and quarterly data from 1977:1 to 2006:3. For the quarterly data we measure consumption as per capita expenditures on non-durables and services from IMF International Financial Statistics and adopt the Campbell (2003) beginning of period timing assumption that consumption during period t takes place at the beginning of period t. We use the dividend-adjusted stock market return from Morgan Stanley Capital International

[3]The asymptotic distribution of $\widehat{HJ}$ is degenerate when $HJ = 0$. Thus, the asymptotic standard error of $\widehat{HJ}$ cannot be used to test whether $HJ = 0$. Instead, the standard error gives a measure of the precision of the estimate of HJ.

and derive the price-dividend ratio from return indices with and without dividend capitalization. We use long-term (10 years) and short-term (3 month) government bond returns from Datastream and Global Financial Data. Nominal returns and nominal consumption are converted to real units using the consumption deflator from IMF International Financial Statistics. Our annual data set is an updated version of the data set in Engsted (2002). As instruments in the GMM estimations, we use lags of stock returns, bond returns, consumption growth, and the price-dividend ratio.

Table 1 reports summary statistics for the real gross stock and bond returns and the instruments. As seen, the average annual arithmetic real stock return, R_S, over the 1922-2004 period is 6.72%, while the long-term, R_{LB}, and short-term, R_{SB}, real bond returns are 4.44% and 2.40%, respectively. The corresponding standard deviations are 20.94%, 12.03%, and 5.23%. Thus, stocks give higher average returns than bonds, but are also more volatile. The average ex post yearly equity premium, i.e. the yearly stock return in excess of the 3-month government bond return, is 4.33%, with a standard deviation of 20.91%. Thus, the Danish equity premium is lower than in most other countries, and in the US in particular, but it is just as volatile as in other countries (in fact, the Danish equity premium is not statistically significant: the standard error of the average premium is 2.31%). This is similar to what Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002) have found using long-term annual data.

Table 1 also reports summary statistics for quarterly data from 1977:1 to 2006:3 and from 1984:4 to 2006:3 (quarterly observations on long-term government bonds start in 1984:4). As seen, over these shorter quarterly sample periods, the average yearly equity premium is 4 x (2.66% - 1.42%) = 4.99% and 4 x (2.86% - 1.22%) = 6.54%, respectively, which is somewhat higher than the average of 4.33% for the annual sample. Table 1 also shows that quarterly real stock returns are slightly positively autocorrelated, whereas real bond returns show strong positive autocorrelation.

In a qualitative sense, the consumption based model implies that the stochastic discount factor should be negatively correlated with stock returns in order to generate a positive equity-premium. Table 2 reports correlations between M_{t+1} and real stock returns $R_{S,t+1}$, where M_{t+1} is either equal to $\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$ (i.e. the standard power utility model, CRRA), or $\delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ (i.e. the Campbell-Cochrane model), and where S_{t+1} has been constructed as described in section 3 from OLS estimates of ϕ, g and σ_v and with values of γ ranging from 1 to 20 in the CRRA utility case, and from 0.5 to 2.0 in the Campbell-Cochrane case corresponding to values of relative risk-aversion γ/S_t ranging from 10 to 40, which is consistent with the GMM estimates reported below. For both models - and across the different values for risk-aversion - stock returns are negatively correlated with the stochastic discount factor in both the annual and quarterly data sets. However, all correlations are close to zero, so although in a qualitative sense this is consistent with the basic consumption-based framework, the evidence does not strongly support it and certainly does not allow us to discriminate between the standard CRRA utility model and the Campbell-Cochrane model.

Now we turn to formal estimation of the parameters and statistical tests of the models. Table 3 reports the iterated GMM estimates and associated test statistics for the long annual data set, while Tables 4 and 5 report the results for the shorter quarterly data. We report results using six different instrument sets for the return moment conditions, see the notes to Table 3. For the annual data, the vector of returns include real returns on stocks, long-term bonds, and short-term bonds. For the standard CRRA utility model, Panel A in Table 3 shows that the annual subjective discount factor δ is precisely estimated at slightly below unity. The estimated risk-aversion parameter γ is around 8-9 and statistically significant. The J-test does not in any case reject the model at conventional significance levels, and the HJ measure indicates pricing errors of around 11%. The annual real risk-free rate, r_f, implied by these estimates is around 6%, which is high but not completely unreasonable.

The estimates in Panel B of Table 3 do not indicate that the Campbell-Cochrane model performs better than the simple CRRA model. The model is not statistically rejected and pricing errors and average risk-aversion are of the same magnitude as for the CRRA model. However, the estimates of δ of around 0.90 (implying an annual rate of time-preference of 10%) is somewhat low. On the other hand, the implied risk-free rate of around 2.7% is more reasonable than the 6% implied by the CRRA model. The estimated average geometric per capita consumption growth rate, g, is 1.6% p.a., with a standard deviation, σ_v, of around 5%, and the estimated persistence parameter of $\phi = 0.88$ implies that the price-dividend ratio and, hence, the surplus consumption ratio are stationary but highly persistent. Figure 1 shows the movement over time in the implied degree of relative risk-aversion, γ/S, computed from column 2 in Table 3, Panel B.[4] There is no systematic strong countercyclical time-variation in relative risk-aversion; the most interesting as-

[4]The time-series movement in γ/S_t is essentially similar to the one in Figure 1 if parameter values from the other columns in Table 3 are used. This also holds for Figures 2 and 3 below.

pect of the figure is the dramatic increase in risk-aversion associated with the decline in real consumption at the outbreak of World War II. Overall, based on these annual results, it is impossible to discriminate between the CRRA and Campbell-Cochrane models.

Turning to the quarterly data, Table 4 reports results for stocks and short-term bonds over the period 1977:1-2006:3. As for the annual data, neither the CRRA model nor the Campbell-Cochrane model are statistically rejected by the J-test, and HJ pricing errors are quite low (below 10%) for both models. The estimated quarterly time discount factor δ is reasonable for the CRRA model, but implausibly low for the Campbell-Cochrane model. The real quarterly risk-free rate is around 1% in both models. In the CRRA model, the degree of risk-aversion is very high - ranging from 13 to 22, depending on the instrument set - but imprecisely estimated. In the Campbell-Cochrane model the estimated values of γ imply an average degree of risk-aversion from 15 to 24, similar to the estimated values for the CRRA model. However, Figure 2 shows that - in contrast to the annual data - the Campbell-Cochrane model now produces visible counter-cyclical time-variation in the degree of risk-aversion: High risk-aversion during the cyclical downturns in the late 1970s, beginning of the 1980s, late 1980's, and start of the new millennium. And low risk-aversion during the booming years of the mid 1980s, mid to late 1990s and the final years of the sample, 2005-2006. (Figure 3 uses the parameter values from column 2 in Table 4, Panel B).

In Table 5 and Figure 3 we include in the return vector long-term bonds in addition to stocks and short-term bonds, and we look at the shorter quarterly sample period, 1984:4-2006:3, since there are no quarterly return data for long-term bonds before 1984:4. The main differences to the quarterly results in Table 4 and Figure 2 are that now δ exceeds one in the CRRA model, r_f is slightly negative in the Campbell-Cochrane model, and HJ pricing errors increase to around 25% for both models even though the J-test still does not reject the models statistically. This is an illustration of the fact emphasized by Hansen and Jagannathan (1997), Cochrane (2005), and others, that a statistical non-rejection by the J-test does not necessarily imply low pricing errors. Figure 3 resembles Figure 2 in showing counter-cyclical time-variation in the degree of risk-aversion, in accordance with the predictions of the Campbell-Cochrane model.

The main conclusion we draw from the empirical analysis is that for Denmark the Campbell-Cochrane habit formation model does not seem to perform markedly better than the standard time-separable power utility model in explaining stock and bond returns. For the long annual sample period covering more than 80 years there is absolutely no evidence of superior performance of the Campbell-Cochrane model. For the shorter and more recent quarterly data over a 20-30 year period, there is some evidence of counter-cyclical time-variation in the degree of risk-aversion, in accordance with the Campbell-Cochrane model, but the model does not produce lower pricing errors than the time-separable model with constant risk-aversion.

5 Concluding remarks

The habit persistence model developed by Campbell and Cochrane (1999) has become one of the most prominent consumption based asset pricing models, in particular with respect to aggregate stock market returns. It explains pro-cyclical stock prices, time-varying and counter-cyclical expected returns, and high and time-varying equity premia as a result of high but time-varying and counter-cyclical risk aversion, and it does this while keeping the risk-free rate low and stable.

When the Campbell-Cochrane model is calibrated to actual historical data from the US, the model is found to match a number of key aspects of the data. However, only a few attempts have been made to formally estimate and test the model, and almost exclusively on US data. These formal estimations and tests generally have led to statistical rejection of the model. Thus, while there is evidence that the Campbell-Cochrane model has empirical content on US data, and that it clearly outperforms the standard CRRA utility model, it is also clear that the model does involve significant pricing errors.[5]

In this paper we have performed a formal econometric estimation and testing of both the standard CRRA model and the Campbell-Cochrane model using Danish stock and bond market returns and aggregate consumption. We have used an iterated GMM procedure that for the Campbell-Cochrane model estimates all parameters in one comprehensive step while generating - within the iterations - a process for the unobservable surplus consumption ratio and, hence, the degree of relative risk-aversion.

The results we obtain using this procedure on Danish asset market returns do not in general support the conclusions from the US studies. Although there is some evidence of time-varying counter-cyclical risk-aversion in recent years, the Campbell-Cochrane model does not produce lower pricing errors or more plausible parameter values than the CRRA model. In Engsted et al. (2008) we present further international evidence on the relative performance of the two models. There seems to be quite

[5] As noted by Campbell and Cochrane (1999) themselves (p.236), the worst performance of the model occurs during the end of their sample period, i.e. the first half of the 1990s.

large cross-country differences in the ability of the Campbell-Cochrane model to explain asset return movements over time. With no doubt, investigations of consumption-based models with habit persistence will continue in the future.

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7 Tables and figures

	Mean (std.dev)	Autocorr. (std.err)
Annual, 1922-2004		
R_S	1.0672 (0.2094)	−0.0963 (0.1104)
R_{LB}	1.0444 (0.1203)	0.0468 (0.1104)
R_{SB}	1.0240 (0.0523)	0.5924 (0.1104)
C/C_{-1}	1.0162 (0.0484)	0.1149 (0.1104)
$p-d$	3.3211 (0.4631)	0.8836 (0.1104)
Quarterly, 1977:1-2006:3		
R_S	1.0266 (0.0979)	0.2462 (0.0921)
R_{SB}	1.0142 (0.0112)	0.7593 (0.0921)
C/C_{-1}	1.0031 (0.0165)	−0.1449 (0.0921)
$p-d$	3.8137 (0.4944)	0.9730 (0.0921)
Quarterly, 1984:4-2006:3		
R_S	1.0286 (0.0941)	0.1806 (0.1072)
R_{LB}	1.0193 (0.0319)	0.4086 (0.1072)
R_{SB}	1.0122 (0.0109)	0.8542 (0.1072)
C/C_{-1}	1.0037 (0.0134)	−0.0682 (0.1072)
$p-d$	4.0434 (0.2407)	0.8769 (0.1072)

Notes: R_S, R_{LB}, and R_{SB} are real gross returns on stocks, long-term bonds, and short-term bonds. C/C_{-1} is the real per capita gross consumption growth rate. $p-d$ is the log price-dividend ratio.

Table 1: Summary statistics for asset returns and instruments.

	Corr(R_S, M^{CRRA})		Corr(R_S, M^{CC})
Annual, 1922-2004			
$\gamma = 1$	-0.1803	$\gamma = 0.5$	-0.1903
$\gamma = 5$	-0.1603	$\gamma = 1$	-0.1649
$\gamma = 10$	-0.1188	$\gamma = 1.5$	-0.1460
$\gamma = 20$	-0.0699	$\gamma = 2$	-0.1309
Quarterly, 1977:1-2006:3			
$\gamma = 1$	-0.1332	$\gamma = 0.5$	-0.1118
$\gamma = 5$	-0.1317	$\gamma = 1$	-0.1323
$\gamma = 10$	-0.1290	$\gamma = 1.5$	-0.1368
$\gamma = 20$	-0.1210	$\gamma = 2$	-0.1371
Quarterly, 1984:4-2006:3			
$\gamma = 1$	-0.1286	$\gamma = 0.5$	-0.0991
$\gamma = 5$	-0.1195	$\gamma = 1$	-0.0810
$\gamma = 10$	-0.1079	$\gamma = 1.5$	-0.0751
$\gamma = 20$	-0.0845	$\gamma = 2$	-0.0763

Notes: M^{CRRA} and M^{CC} are the stochastic discount factors in the CRRA utility model and Campbell-Cochrane model, respectively.

Table 2: Correlations between stock returns and the stochastic discount factor.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	0.9804	0.9876	0.9870	0.9943	0.9772	0.9910
	(0.0559)	(0.0545)	(0.0521)	(0.0479)	(0.0552)	(0.0514)
γ	9.4339	8.9449	8.9982	8.4589	9.5197	8.6616
	(3.7035)	(3.6191)	(3.6723)	(3.3385)	(3.6188)	(3.5970)
J-test	4.1415	7.6336	8.3655	10.3327	5.7790	10.6004
	(0.3872)	(0.3660)	(0.3015)	(0.1705)	(0.5658)	(0.3895)
HJ	0.1182	0.1138	0.1140	0.1135	0.1193	0.1132
	(0.0968)	(0.0803)	(0.0814)	(0.0770)	(0.0998)	(0.0770)
r_f	0.0610	0.0568	0.0571	0.0525	0.0637	0.0548
Panel B: Campbell-Cochrane model						
δ	0.9018	0.9098	0.9051	0.9099	0.8949	0.9157
	(0.0521)	(0.0508)	(0.0500)	(0.0488)	(0.0539)	(0.0488)
γ	1.8599	1.6150	1.7017	1.6166	1.9895	1.5033
	(0.7548)	(0.6723)	(0.7238)	(0.6837)	(0.7965)	(0.6489)
g	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
	(0.0050)	(0.0050)	(0.0050)	(0.0050)	(0.0050)	(0.0050)
σ_v^2	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)
ϕ	0.8854	0.8854	0.8854	0.8854	0.8854	0.8854
	(0.0502)	(0.0502)	(0.0502)	(0.0502)	(0.0502)	(0.0502)
J-test	4.1258	7.1558	7.7175	9.3613	5.5754	9.9166
	(0.3892)	(0.4128)	(0.3582)	(0.2277)	(0.5901)	(0.4478)
HJ	0.1143	0.1127	0.1118	0.1127	0.1188	0.1154
	(0.0857)	(0.0794)	(0.0782)	(0.0794)	(0.1002)	(0.0833)
r_f	0.0258	0.0271	0.0287	0.0270	0.0280	0.0254
γ/S	8.4206	7.9115	8.2066	7.9171	8.7130	7.4951

Notes: The table reports parameter estimates of the CRRA utility and Campbell-Cochrane models using the iterated GMM approach described in section 3, with asymptotic standard errors in parentheses. J-test is Hansen's test of overidentifying restrictions, computed as in (14), with asymptotic p-value in parenthesis. HJ is the Hansen-Jagannathan specification error measure, computed as in (15), with asymptotic standard error in parenthesis. r_f is the log real risk-free rate, computed from (3) and (10). S in γ/S is the average value of S over the sample. The instrument sets for the return moment conditions for (1) and (5) are:

1: Constant, $p - d$.

2: Constant, $p - d$, R_S.

3: Constant, $p - d$, R_{SB}.
4. Constant, $p - d$, C/C_{-1}.
5. Constant, $p - d$, and its lag.
6. Constant, $p - d$, R_S, R_{SB}.

The instrument set for the moment condition for (7) is just a constant, while for (12) the instrument set contains a constant and lagged $p - d$.

Table 3: GMM estimation of the CRRA utility and Campbell-Cochrane models using real annual returns on stocks, long-term bonds, and short-term bonds, 1922-2004.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	0.9930	0.9934	0.9972	0.9972	0.9809	0.9969
	(0.0310)	(0.0296)	(0.0194)	(0.0195)	(0.0547)	(0.0199)
γ	17.2626	16.9044	13.3399	13.4246	21.6905	13.5817
	(13.2284)	(13.2178)	(11.0521)	(11.0774)	(15.7843)	(11.2701)
J-test	1.1357	6.8080	1.3439	5.0895	4.6080	7.2043
	(0.5667)	(0.1464)	(0.8539)	(0.2782)	(0.3299)	(0.3024)
HJ	0.0845	0.0856	0.0967	0.0965	0.0691	0.0960
	(0.1163)	(0.1164)	(0.1162)	(0.1162)	(0.1155)	(0.1163)
r_f	0.0136	0.0138	0.0147	0.0146	0.0150	0.0148
Panel B: Campbell-Cochrane model						
δ	0.9559	0.9572	0.9671	0.9672	0.9376	0.9666
	(0.0471)	(0.0446)	(0.0306)	(0.0305)	(0.0734)	(0.0310)
γ	2.3238	2.2216	1.5362	1.5264	3.4249	1.5626
	(1.6252)	(1.5730)	(1.1300)	(1.1263)	(2.3380)	(1.1538)
g	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)
σ_v^2	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ϕ	0.9628	0.9628	0.9628	0.9628	0.9628	0.9628
	(0.0293)	(0.0293)	(0.0293)	(0.0293)	(0.0293)	(0.0293)
J-test	1.0280	6.9322	1.1747	4.3896	4.6146	7.2783
	(0.5981)	(0.1395)	(0.8823)	(0.3566)	(0.3296)	(0.2959)
HJ	0.0768	0.0785	0.0906	0.0908	0.0605	0.0901
	(0.1176)	(0.1177)	(0.1177)	(0.1176)	(0.1161)	(0.1177)
r_f	0.0080	0.0082	0.0089	0.0089	0.0097	0.0090
γ/S	19.7728	19.3104	15.7690	15.7114	24.0862	15.237

See the notes to Table 3.

Table 4: GMM estimation of the CRRA utility and Campbell-Cochrane models using real quarterly returns on stocks and short-term bonds, 1977:1-2006:3.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	1.0107	1.0179	1.0142	1.0135	1.0165	1.0184
	(0.0839)	(0.0370)	(0.0560)	(0.0614)	(0.0371)	(0.0323)
γ	29.6037	23.7190	27.1305	27.7042	23.2281	22.4344
	(44.5128)	(29.9511)	(36.4666)	(38.1363)	(31.7599)	(27.3983)
J-test	2.8620	5.1253	5.5748	5.6734	4.7891	7.4711
	(0.5812)	(0.6447)	(0.5902)	(0.5784)	(0.6857)	(0.6803)
HJ	0.2677	0.2660	0.2669	0.2671	0.2659	0.2657
	(0.1636)	(0.1590)	(0.1615)	(0.1620)	(0.1586)	(0.1581)
r_f	0.0175	0.0172	0.0177	0.0176	0.0188	0.0174
Panel B: Campbell-Cochrane model						
δ	0.8827	0.9485	0.9056	0.9006	0.9448	0.9489
	(0.1807)	(0.0796)	(0.1330)	(0.1494)	(0.0956)	(0.0765)
γ	2.5159	0.8882	1.9772	2.1094	0.9293	0.8777
	(2.4446)	(1.0266)	(1.8623)	(2.0764)	(1.2098)	(0.9961)
g	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036
	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)
σ_v^2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ϕ	0.8729	0.8729	0.8729	0.8729	0.8729	0.8729
	(0.0584)	(0.0584)	(0.0584)	(0.0584)	(0.0584)	(0.0584)
J-test	2.7031	5.4612	5.6297	6.9086	4.9142	6.9216
	(0.6087)	(0.6039)	(0.5836)	(0.4385)	(0.6704)	(0.7328)
HJ	0.2528	0.2670	0.2547	0.2532	0.2668	0.2671
	(0.1579)	(0.1530)	(0.1546)	(0.1552)	(0.1514)	(0.1512)
r_f	−0.0261	−0.0004	−0.0193	−0.0218	0.0011	−0.0002
γ/S	36.8241	19.7193	32.2148	33.3273	20.3868	19.5562

See the notes to Table 3.

Table 5: GMM estimation of the CRRA utility and Campbell-Cochrane models using real quarterly returns on stocks, long-term bonds, and short-term bonds, 1984:4-2006:3.

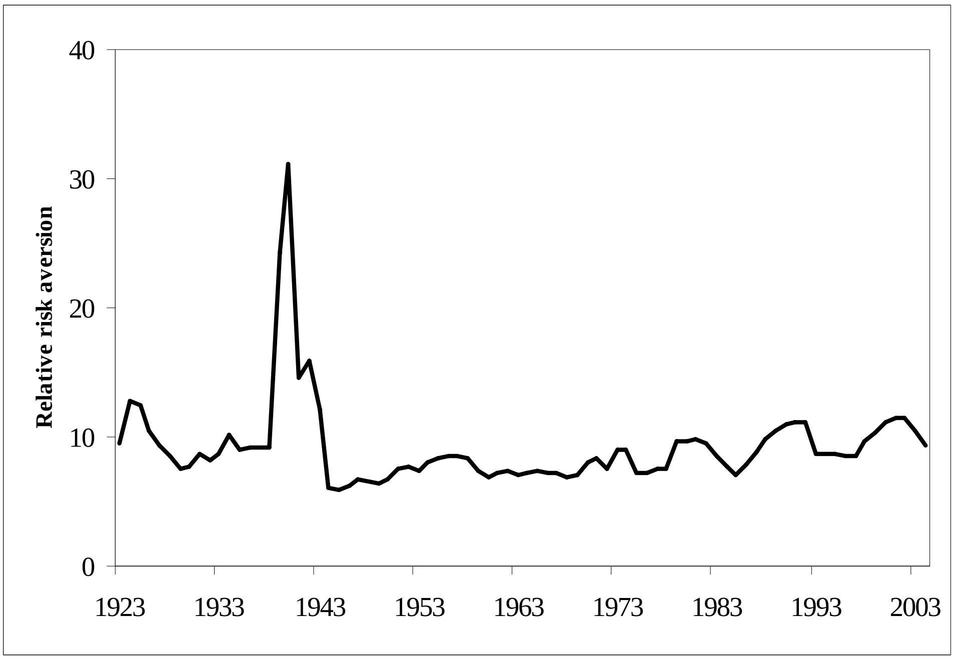

Figure 1: Relative risk-aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1922-2004.

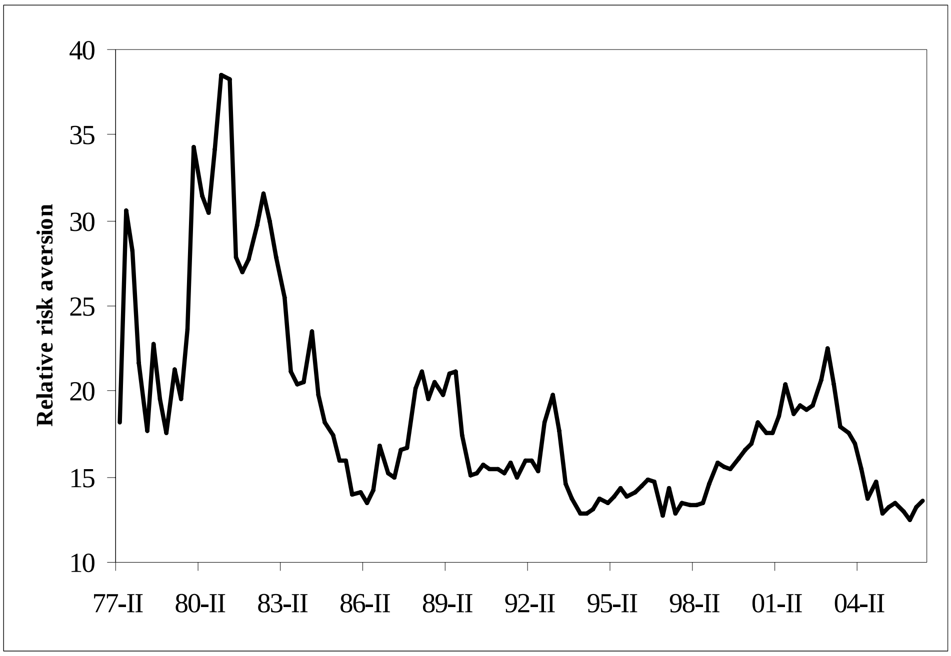

Figure 2: Relative risk-aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1977:1-2006:3.

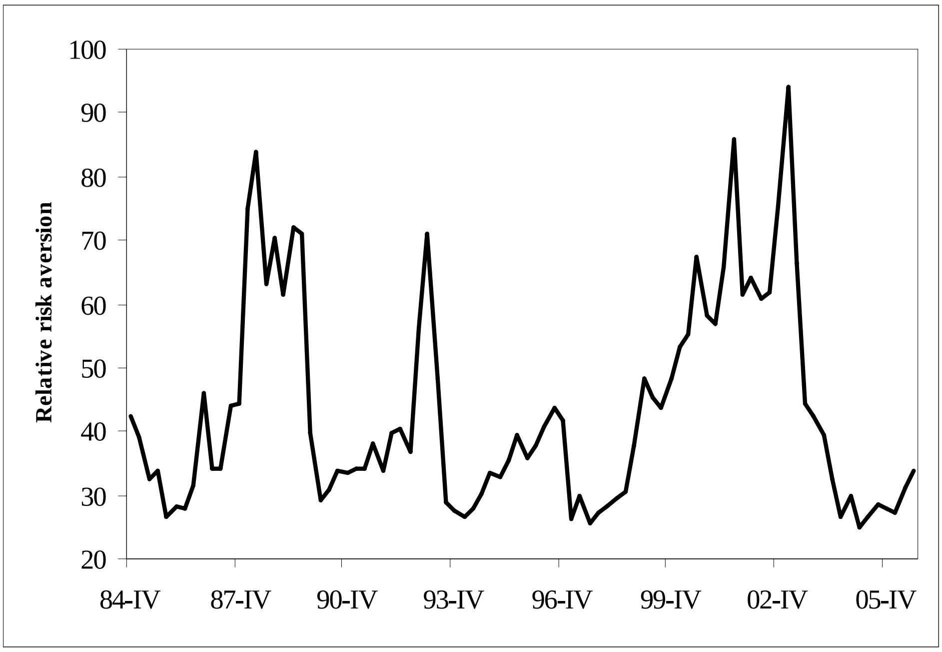

Figure 3: Relative risk-aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1984:4-2006:3.

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