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Volatility Components, Affine Restrictions and Non-Normal Innovations

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Abstract

Recent work by Engle and Lee (1999) shows that allowing for long-run and short-run components greatly enhances a GARCH model's ability fit daily equity return dynamics. Using the risk-neutralization in Duan (1995), we assess the option valuation performance of the Engle-Lee model and compare it to the standard one-component GARCH(1,1) model. We also compare these non-affine GARCH models to one- and two- component models from the class of affine GARCH models developed in Heston and Nandi (2000). Using the option pricing methodology in Duan (1999), we then compare the four conditionally normal GARCH models to four conditionally non-normal versions. As in Hsieh and Ritchken (2005), we find that non-affine models dominate affine models both in terms of fitting return and in terms of option valuation. For the affine models we find strong evidence in favor of the component structure for both returns and options, but for the non-affine models the evidence is much less strong in option valuation. The evidence in favor of the non-normal models is strong when fitting daily returns, but the non-normal models do not provide much improvement when valuing options.

Keywords: Volatility; Component Model; GARCH; Long Memory; Option Valuation; Affine; Normality.

JEL Codes: C22, G13

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1 Introduction

Following the path-breaking work of Engle (1982) and Bollerslev (1986), GARCH models have become an ubiquitous toolkit in empirical finance. In this paper, we assess the ability of eight different GARCH models to fit daily return dynamics and their ability to match market prices of options in a sample of close to 22,000 contracts. The eight models we investigate differ along three dimensions:

First, we consider component models versus GARCH(1,1) models. Engle and Lee (1999) were the first to develop a component GARCH model which they built from the non-affine NGARCH(1,1) model analyzed in Engle and Ng (1993) and Duan (1995). Component GARCH models can be viewed as a convenient way of incorporating long-memory-like features into a short-memory model, at least for the horizons relevant for option valuation. Bollerslev and Mikkelsen (1999) find support for a long memory GARCH option valuation model applied to long maturity options. We consider options with up to one-year maturity where the component models are likely to provide good approximations to long-memory processes. Maheu (2002) presents Monte-Carlo evidence that a component model similar to the ones in this paper can capture long-range volatility dynamics. Adrian and Rosenberg (2005) demonstrate the relevance of the component volatility structure for cross-sectional asset pricing.

Second, we consider non-affine versus affine GARCH models. Most GARCH models are of a non-affine form but Heston and Nandi (2000) developed a class of affine GARCH models. From the affine GARCH(1,1) specification, Christoffersen, Jacobs, Ornthanalai and Wang (2007) develop a non-affine GARCH component model which we also consider in this paper. The affine GARCH(1,1) model has been compared to the non-affine NGARCH(1,1) model in Hsieh and Ritchken (2005) who found strong support for the non-affine specification.

Third, we consider conditionally normal versus conditionally non-normal models. In particular, we modify the four conditionally normal GARCH models above by modeling the return shock using a Generalized Error Distribution (GED). The GED distribution is suggested by Duan (1999) for its tractability in asset return modeling, and it conveniently nests the normal distribution. A skewed version of the GED distribution was developed in Theodossiou (2001) and has been used for option valuation in Lehnert (2003). Lehnert (2003) finds support for a non-affine EGARCH model with skewed GED shocks when comparing its option pricing performance with the affine GARCH(1,1) model in Heston and Nandi (2000), which has normal innovations. However, his analysis does not show if the improvement comes from the non-affine variance dynamic or from the non-normal shocks or from both features.

We estimate these eight models using MLE on S&P500 returns. This empirical comparison allows us to compare the importance of three types of modeling assumptions: first, the importance of the component structure versus the simpler and more parsimonious GARCH(1,1) structure; second, the restrictions of the affine structure; and third, the importance of non-normal return innovations. We find that the likelihood criterion based on return data strongly favors the component models in all cases, as well as the non-normal return innovations. While the affine models are not nested in the non-affine models, comparisons of the likelihoods suggest that the non-affine models fit the return data the best.

Using the MLE estimates, we characterize key properties of each model: multi-day variance forecasting functions, variance impulse response functions, conditional variance of variance paths and conditional correlations between returns and variance. We find important differences between affine and non-affine models, suggesting that the non-affine structure provides more flexibility in a parsimonious fashion. We also find substantial differences between GARCH(1,1) and component models and between models with normal and non-normal innovations.

When we use the estimated model parameters for option valuation, we again find strong support for the non-affine variance specifications, but much less evident support for the non-normal return innovations. The component structure yields great improvements in the affine class of models but not in the non-affine class.

The remainder of the paper is structured as follows. In Section 2, we develop the eight GARCH-based asset models that we will investigate empirically in the paper. In Section 3, we report the model estimates from daily returns and we depict some key dynamic properties of the models. In Section 4, we provide the theoretical mappings from physical to risk-neutral dynamics by applying the general approach in Duan (1999). Section 5 presents the empirical results from using the GARCH models in option valuation, as well as an economic analysis of the option pricing errors. Section 6 concludes and suggests promising avenues for future research.

2 Asset Return Models

In this section we introduce the eight GARCH models to be used for option valuation. The eight models cover all the possibilities in our three-way comparison: GARCH(1,1) versus component GARCH, affine versus non-affine GARCH, and normal versus GED distributed return shocks.

2.1 The Affine GARCH(1,1) Model with Normal Shocks

We first introduce the affine normal GARCH(1,1) model from Heston and Nandi (2000). The return dynamics on the underlying asset are

$$\begin{aligned} R_{t+1} &\equiv \ln \frac{S_{t+1}}{S_t} = r + \lambda h_{t+1} + \sqrt{h_{t+1}} z_{t+1} \\ h_{t+1} &= w + b' h_t + a \left(z_t - c\sqrt{h_t} \right)^2 \end{aligned} \tag{2.1}$$

where S_{t+1} denotes the underlying asset price on the close of day $t+1$, r denotes the risk free rate, λ the price of risk, z_t the *i.i.d.* $N(0,1)$ return shock, and h_{t+1} the daily variance on day $t+1$ which is known at the end of day t. We will refer to this model as AGARCH(1,1)-N below.

Note that c renders the variance response asymmetric to positive versus negative innovations in returns. If c is zero the variance dynamic is symmetric in z_t and the conditional distribution of returns will be largely symmetric at all horizons because the distribution of z_t is symmetric as well. In that case, the only source of asymmetry is the conditional mean return

$$E_t \left[R_{t+1} \right] = r + \lambda h_{t+1} \tag{2.2}$$

and this effect is typically very small in magnitude.

We next derive some key features of the model that are particularly important for its performance in option valuation. The model's unconditional variance can be derived to be

$$E \left[h_{t+1} \right] = \sigma^2 = \frac{w + a}{1 - ac^2 - b'} \tag{2.3}$$

If we use this expression to substitute for w and furthermore define variance persistence as $b = b' + ac^2$, then we can write

$$h_{t+1} = \sigma^2 + b\left(h_t - \sigma^2\right) + a\left(z_t^2 - 1 - 2c\sqrt{h_t}z_t\right) \tag{2.4}$$

The options we analyze below have maturities of between a week and a year. It is therefore important to gauge the model's properties at multi-day horizons. In this regard, consider the conditional variance k days ahead

$$E_t\left[h_{t+k}\right] = \sigma^2 + b^{k-1}\left(h_{t+1} - \sigma^2\right) \tag{2.5}$$

The variance impulse response function can be derived from the ARCH(∞) representation of (2.4). We have

$$h_{t+1} = \sigma^2 + a\sum_{i=0}^{\infty} b^i\left(z_{t-i}^2 - 1 - 2c\sqrt{h_{t-i}}z_{t-i}\right) \tag{2.6}$$

from which we define the variance impulse response k periods ahead relative to the unconditional variance for a z shock today as

$$VIR\left(k\right) = ab^k\left(z^2 - 1 - 2c\sigma z\right)/\sigma^2$$

Below we will consider positive as well as negative shocks which will provide information about the multi-period leverage effect (Black, 1976).

While by design the one-day ahead variance is deterministic in the GARCH models, the multi-day variance is stochastic, and its distribution is important for option pricing as well. For two days ahead the conditional variance of variance is easily derived from (2.4) as

$$Var_t\left[h_{t+2}\right] = 2a^2 + 4a^2c^2h_{t+1} \tag{2.7}$$

Note that the variance of h_{t+2} is linear in h_{t+1}, which is a defining characteristic of the affine GARCH model. Note further that if c is zero then the future variance will have constant conditional variance. This is at odds with the empirical evidence in for example Jones (2003), who finds that the volatility of implied options volatility is higher when the level of implied options volatility is larger. In the affine normal GARCH(1,1) model the c parameter is thus needed both to provide conditional variance of variance dynamics and to provide substantial conditional distribution asymmetry. This doubly duty may cause a tension in the model and we will revisit it again below.

The relationship between future variance and return is also of interest for option valuation. The so-called leverage effect was noted by Black (1976) who observed a negative correlation between volatility and returns. To describe this relationship, we consider the conditional covariance

$$Cov_t\left[R_{t+1}, h_{t+2}\right] = E_t\left[\sqrt{h_{t+1}}z_{t+1}a\left(z_{t+1}^2 - 1 - 2c\sqrt{h_{t+1}}z_{t+1}\right)\right] = -2ach_{t+1} \tag{2.8}$$

Note that since a must be strictly positive to ensure that the GARCH process is identified and positive, the sign of the leverage effect is driven entirely by c.

From the conditional covariance, the conditional correlation is easily derived as

$$Corr_t\left[R_{t+1}, h_{t+2}\right] = \frac{-2c\sqrt{h_t}}{\sqrt{2 + 4c^2h_t}} \tag{2.9}$$

Note that this conditional correlation is time-varying which is a relatively unique property of the affine GARCH model.

2.2 The Affine GARCH Component Model with Normal Shocks

Many papers (see for example Bollerslev and Mikkelsen, 1989) have found that simple exponential decay in (2.5) of the conditional variance to the unconditional value is too fast as the horizon gets large. This motivates the affine normal GARCH component model which is developed in Christoffersen, Jacobs, Ornthanalai and Wang (2007), who build on Engle and Lee (1999). The return and variance dynamics are now

$$\begin{aligned} R_{t+1} &= r + \lambda h_{t+1} + \sqrt{h_{t+1}} z_{t+1} \\ h_{t+1} &= q_{t+1} + \beta(h_t - q_t) + \alpha\left(z_t^2 - 1 - 2\gamma_1\sqrt{h_t} z_t\right) \\ q_{t+1} &= \sigma^2 + \rho\left(q_t - \sigma^2\right) + \varphi\left(z_t^2 - 1 - 2\gamma_2\sqrt{h_t} z_t\right) \end{aligned} \tag{2.10}$$

Instead of mean-reverting to a constant unconditional variance, the conditional variance, h_{t+1}, now moves around a long-run component, q_{t+1}, which itself mean-reverts to the constant unconditional variance, σ^2. Furthermore, the two parameters γ_1 and γ_2 in the component model allow for a different degree of asymmetry in the two components, $h_{t+1} - q_{t+1}$, and q_{t+1}. We will refer to this model as AGARCH(C)-N below.

The added dynamics in this model chiefly serve to generate more flexible dynamics in the multi-day ahead conditional variance. We now have

$$E_t\left[h_{t+k}\right] = E_t\left[q_{t+k} + (h_{t+k} - q_{t+k})\right] = \sigma^2 + \rho^{k-1}\left(q_{t+1} - \sigma^2\right) + \beta^{k-1}\left(h_{t+1} - q_{t+1}\right) \tag{2.11}$$

which clearly allows for slower mean-reversion than does (2.5). We will refer to ρ as long-run persistence and to β as short-run persistence below.

The ARCH representation is

$$h_{t+1} = \sigma^2 + \varphi\sum_{i=0}^{\infty}\rho^i\left(z_{t-i}^2 - 1 - 2\gamma_2\sqrt{h_{t-i}} z_{t-i}\right) + \alpha\sum_{i=0}^{\infty}\beta^i\left(z_{t-i}^2 - 1 - 2\gamma_1\sqrt{h_{t-i}} z_{t-i}\right) \tag{2.12}$$

from which we define the variance impulse response k periods ahead relative to the unconditional variance for a z shock today as

$$VIR(k) = \varphi\rho^k\left(z^2 - 1 - 2\gamma_2\sigma z\right)/\sigma^2 + \alpha\beta^k\left(z^2 - 1 - 2\gamma_1\sigma z\right)/\sigma^2$$

This component model also offers greater flexibility in the conditional variance of variance dynamic, which is now

$$Var_t\left[h_{t+2}\right] = 2\left(\alpha + \varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2 h_{t+1} \tag{2.13}$$

so that the affine structure is preserved. The conditional covariance and correlations are

$$Cov_t\left[R_{t+1}, h_{t+2}\right] = -2\left(\alpha\gamma_1 + \varphi\gamma_2\right) h_{t+1} \tag{2.14}$$

and

$$Corr_t\left[R_{t+1}, h_{t+2}\right] = \frac{-2\left(\alpha\gamma_1 + \varphi\gamma_2\right)\sqrt{h_t}}{\sqrt{2\left(\alpha + \varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2 h_t}} \tag{2.15}$$

respectively. Notice that in these formulas $(\alpha + \varphi)$ has replaced a in the AGARCH(1,1)-N model and similarly $(\gamma_1 \alpha + \gamma_2 \varphi)$ has replaced c. Thus whereas c had to perform double duty (creating asymmetry and variance of variance dynamics) in the AGARCH(1,1)-N model the component model offers much added flexibility. The return asymmetry and variance of variance are now driven by two sources parameterized by γ_1 and γ_2.

2.3 The Non-Affine GARCH(1,1) Model with Normal Shocks

The benchmark so-called NGARCH(1,1)-N model of Engle and Ng (1993) used for option valuation by Duan (1995) is defined as

$$\begin{aligned} R_{t+1} &= r + \lambda \sqrt{h_{t+1}} - \frac{1}{2} h_{t+1} + \sqrt{h_{t+1}} z_{t+1} \\ h_{t+1} &= w + b' h_t + a h_t (z_t - c)^2 \end{aligned} \tag{2.16}$$

The parameter c again renders the variance response asymmetric to positive versus negative return shocks and creates asymmetry in the conditional distribution of multi-day returns beyond that created by the conditional return mean

$$E_t [R_{t+1}] = r + \lambda \sqrt{h_{t+1}} - \frac{1}{2} h_{t+1} \tag{2.17}$$

Although this conditional mean specification is different from the one used in the affine model, we use it because it will generate a risk-neutral conditional variance specification which is similar to the physical one as we will see in Section 4 below. Similarly, the affine conditional-mean specification in (2.2) will generate an affine conditional variance under the risk-neutral measure. This issue will be discussed in a subsequent section

The unconditional variance is now

$$E [h_{t+1}] = \sigma^2 = \frac{w}{1 - b' - a(1 + c^2)} \tag{2.18}$$

and defining variance persistence as $b = b' + a(1 + c^2)$ we can rewrite the conditional variance as

$$h_{t+1} = \sigma^2 + b \left(h_t - \sigma^2 \right) + a h_t \left(z_t^2 - 1 - 2c z_t \right) \tag{2.19}$$

The conditional variance k days ahead is the same as in the affine model

$$E_t [h_{t+k}] = \sigma^2 + b^{k-1} \left(h_{t+1} - \sigma^2 \right) \tag{2.20}$$

The ARCH representation is

$$h_{t+1} = \sigma^2 + a \sum_{i=0}^{\infty} b^i h_{t-i} \left(z_{t-i}^2 - 1 - 2c z_{t-i} \right) \tag{2.21}$$

from which we define the variance impulse response k periods ahead relative to the unconditional variance for a z shock today as

$$VIR(k) = ab^k \left(z^2 - 1 - 2cz \right)$$

The conditional variance of variance can be derived from (2.19) as

$$Var_t\left[h_{t+2}\right] = a^2\left(2+4c^2\right)h_{t+1}^2 \tag{2.22}$$

So that the variance of h_{t+2} is now quadratic in h_{t+1} whereas it was linear in the affine model. Note also that even if c is zero in the non-affine model then the future variance will still have dynamic variance now driven by a.

The conditional covariance in this model is

$$Cov_t\left[R_{t+1}, h_{t+2}\right] = -2ach_{t+1}^{3/2} \tag{2.23}$$

The leverage effect is again driven by c but now the covariance is non-linear in h_{t+1}.

From the conditional covariance, the conditional correlation is

$$Corr_t\left[R_{t+1}, h_{t+2}\right] = \frac{-2c}{\sqrt{2+4c^2}} \tag{2.24}$$

Note that conditional correlation in the non-affine model is constant whereas it was time-varying in the affine model. Thus along this dimension the affine model seemingly offers more flexibility.

2.4 The Non-Affine GARCH Component Model with Normal Shocks

This model is obtained by replacing the constant σ^2 in the NGARCH(1,1)-N model with a time-varying long-run component q_{t+1}. We write

$$\begin{aligned}
R_{t+1} &= r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1} \\
h_{t+1} &= q_{t+1} + \beta(h_t - q_t) + \alpha h_t\left(z_t^2 - 1 - 2\gamma_1 z_t\right) \\
q_{t+1} &= \sigma^2 + \rho\left(q_t - \sigma^2\right) + \varphi h_t\left(z_t^2 - 1 - 2\gamma_2 z_t\right)
\end{aligned} \tag{2.25}$$

and refer to the model as NGARCH(C)-N below.

The added dynamics in this model chiefly serve to generate more flexible dynamics in the multi-day ahead conditional variance. The multi-day conditional variance is

$$E_t\left[h_{t+k}\right] = E_t\left[q_{t+k} + \left(h_{t+k} - q_{t+k}\right)\right] = \sigma^2 + \rho^{k-1}\left(q_{t+1} - \sigma^2\right) + \beta^{k-1}\left(h_{t+1} - q_{t+1}\right) \tag{2.26}$$

The ARCH representation is now

$$h_{t+1} = \sigma^2 + \varphi\sum_{i=0}^{\infty}\rho^i h_{t-i}\left(z_{t-i}^2 - 1 - 2\gamma_2 z_{t-i}\right) + \alpha\sum_{i=0}^{\infty}\beta^i h_{t-i}\left(z_{t-i}^2 - 1 - 2\gamma_1 z_{t-i}\right) \tag{2.27}$$

from which we define the variance impulse response k periods ahead relative to the unconditional variance for a z shock today as

$$VIR\left(k\right) = \varphi\rho^k\left(z^2 - 1 - 2\gamma_2 z\right) + \alpha\beta^k\left(z^2 - 1 - 2\gamma_1 z\right)$$

The conditional variance of variance dynamic is

$$Var_t\left[h_{t+2}\right] = \left[2\left(\alpha+\varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2\right]h_{t+1}^2 \tag{2.28}$$

which has contributions from both components and which is again quadratic in h_{t+1}. The conditional covariance and correlation are

$$Cov_t\left[R_{t+1,}h_{t+2}\right] = -2\left(\alpha\gamma_1 + \varphi\gamma_2\right)h_t^{3/2} \tag{2.29}$$

which is non-linear in h_t and

$$Corr_t\left[R_{t+1,}h_{t+2}\right] = \frac{-2\left(\alpha\gamma_1 + \varphi\gamma_2\right)}{\sqrt{2\left(\alpha+\varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2}} \tag{2.30}$$

which is again constant.

2.5 Generalized Error Distribution Shocks

The assumption that the daily return shock, z_t, is normally distributed is typically rejected empirically for daily asset returns and the empirical analysis below is no exception. Note, however, that while the conditional one-day distribution is normal when z_t is normal, the multi-day distribution is not normal and neither is the unconditional distribution. Thus, the effect of the normal innovation assumption on option valuation in a GARCH model is not straightforward. Our analysis investigates if these dynamics suffice to fit the underlying asset return as well as the option prices on the underlying asset, or whether the conditional normality assumption should be relaxed.

Following Duan (1999), we will assume that the *i.i.d.* return shock which was denoted by z_t in the normal case above now follows the Generalized Error Distribution (GED) and is denoted by ζ_t. Once normalized to get a zero mean and unit variance, we have the probability density function

$$g_v(\zeta) = \frac{v}{2^{1+\frac{1}{v}}\theta\left(v\right)\Gamma\left(\frac{1}{v}\right)}\exp\left(-\frac{1}{2}\left|\frac{\zeta}{\theta\left(v\right)}\right|^v\right) \qquad \text{for } 0 < v \leq \infty$$

where $\Gamma\left(.\right)$ is the gamma function and where $\theta\left(v\right) = \left(\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right)^{\frac{1}{2}}$.

The parameter v determines the thinness of the density tails. For $v < 2$, the density function has tails that are fatter than the normal distribution and vice versa. The expected simple return exists as long as $v > 1$ which therefore provides a natural lower bound in financial return applications.

The GED innovation ζ has a skewness of zero and a kurtosis of

$$\kappa(v) = \frac{\Gamma\left(\frac{5}{v}\right)\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)^2}$$

In the special case where $v = 2$, we get $\kappa(2) = \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2}\right)^2} = 3$, $\theta\left(2\right) = \left(\frac{2^{-1}\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right)^{\frac{1}{2}} = 1$ and because $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ we get

$$g_2(\zeta) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\zeta^2\right)$$

so that standardized GED conveniently nests the standard normal distribution which obtains when $v = 2$. Nelson (1991), Hamilton (1994) and Duan (1999) provide more detail on the properties of the GED distribution.

Replacing the normal distribution by the GED distribution in each of the four models above provides four new models. The resulting eight models will allow us to study the three dimensions of modeling that we are interested in: GARCH(1,1) versus component GARCH, affine versus non-affine GARCH, and normal versus non-normal return shocks.

The GED distribution does not directly impact on the variance persistence and by extension on the multi-day conditional variance in the four models considered above. The functional form for the conditional covariance is also unchanged. However the excess kurtosis of the GED distribution will affect the conditional variance of variance in the four GED models. We have

$$
\begin{aligned}
\text{AGARCH(1,1)-GED} &: \quad Var_t\left(h_{t+2}\right) = \left(\kappa(v) - 1\right) a^2 + 4a^2c^2 h_{t+1} \\
\text{AGARCH(C)-GED} &: \quad Var_t\left(h_{t+2}\right) = \left(\kappa(v) - 1\right) \left(\alpha + \varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2 h_{t+1} \\
\text{NGARCH(1,1)-GED} &: \quad Var_t\left(h_{t+2}\right) = \left(\kappa(v) - 1 + 4c^2\right) a^2 h_{t+1}^2 \\
\text{AGARCH(C)-GED} &: \quad Var_t\left(h_{t+2}\right) = \left[\left(\kappa(v) - 1\right)\left(\alpha + \varphi\right)^2 + 4\left(\alpha\gamma_1 + \varphi\gamma_2\right)^2\right] h_{t+1}^2
\end{aligned}
$$

where $\kappa(v) = \frac{\Gamma\left(\frac{5}{v}\right)\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)^2}$ denotes the kurtosis in the GED distribution.

This in turn will affect the conditional correlation between return and volatility. In each of the conditional correlation function expressions in the four models, the 2 in the denominator will be replaced by a $(\kappa(v) - 1)$ term.

3 Asset Return Empirics

This section presents the empirical results from fitting the GARCH models above to daily returns. We use MLE on a long time series of S&P500 return data to estimate the eight models discussed above: AGARCH(1,1)-N, AGARCH(C)-N, NGARCH(1,1)-N, NGARCH(C)-N, and the four GED based models. We discuss the parameter estimates and their implications for the salient properties of the models. The eight models allow us to make three types of comparisons: component models versus GARCH(1,1) models; affine models versus non-affine models; and non-normal innovations versus normal innovations.

3.1 Parameter Estimates from Daily Return Data

Table 1 presents the Maximum Likelihood estimation results obtained using daily returns data from July 1, 1962 through December 31, 2001. The returns data were obtained from CRSP. Standard errors are calculated from the outer product of the gradient and are given in parentheses. Table 1 reports the physical conditional variance parameters as well as the price of risk, λ. The estimates of λ are required to be positive to guarantee positive excess log returns.

We use variance targeting in order to control the unconditional variance level across models which is important for the subsequent option valuation exercise. We thus force the annualized return standard deviation to be 14.66%. This technique fixes the parameter w in each model, and we therefore do not report on w in Table 1.

In the AGARCH(1,1) cases the unconditional variance is defined by $\sigma^2 = (w + a) / (1 - b)$. Thus if the data warrants a high a (perhaps to match variance of variance) then w will be small in order to match σ^2. If left unconstrained, the w estimate may be negative which yields the possibility of a negative conditional volatility and thus causes problems in the subsequent Monte Carlo computation of option prices. Thus we constrain w to be positive in the estimation.

The fourth row from the bottom reports the total variance persistence in each model. If we substitute out q_{t+1} and q_t from the h_{t+1} equation in the component models, then persistence can be computed as the sum of the coefficients on h_t and h_{t-1}. The total persistence in the component model is thus $\beta + (1 - \beta)\rho$. In the GARCH(1,1) models persistence is simply b. Notice from Table 1 that while the GARCH(1,1) models have high persistence, for each corresponding component model the persistence is even higher–this is particularly true for the affine models. The very large component variance persistence is driven by a large long-run component persistence ρ, plus the contribution from ($(1 - \rho)$ times) the less persistent short-run component β.

In the GARCH(1,1) models the correlation between return and conditional variance is driven by c, which as expected is significantly positive in all cases. In the component models, the correlation is driven by a combination of γ_1 and γ_2, which are both significantly positive in all four component models. Thus, both the long-run and short-run components contribute to the overall correlation with the expected sign. The average conditional correlations between the return and conditional variance are reported in the last row of the table. They are all negative, as expected. The results show that for each set of models, the component model displays a more pronounced leverage effect than the GARCH(1,1) counterpart in that the average correlation is more negative.

The variance of variance is driven mainly by the a parameter in the GARCH(1,1) models and by the α and φ parameters in the component models. In the GED models, the v parameter also contributes. The overall unconditional volatility of variance (annualized square root of variance of variance) is reported in the second row from the bottom. Notice again that in each case the component model displays a larger volatility of variance than its GARCH(1,1) counterpart. Thus three important empirical regularities emerge when comparing component models to their GARCH(1,1) counterparts: the component models allow us to (simultaneously) capture a larger variance persistence, a larger leverage effect, and a larger variance of variance than their GARCH(1,1) counterparts.

Finally, Table 1 presents log likelihood values for each model. In all cases the component model has a significantly larger log-likelihood than the nested GARCH(1,1). When comparing the GED models with their normal counterparts it is also clear that the GED based models have significantly larger log-likelihood values. Thus this return-based analysis strongly favors component models over GARCH(1,1) and GED models over normal models. Although the affine and non-affine models are not nested, a casual comparison of the log-likelihoods suggests that the non-affine GARCH models are also strongly preferred to the affine GARCH models in all four cases.

3.2 Time-Series Properties

In order to explore the asset return models further we now plot various key dynamic properties of the models for the period 1989-2001. This period includes the dates for the option valuation exercise we present in Section 5.

Figure 1 plots the conditional volatility for the period 1989-2001. To be exact, we plot the annualized conditional standard deviation in percent, that is $100 * \sqrt{252 h_{t+1}}$. Notice that the conditional variance patterns across the four GARCH(1,1) models in the left column and the corresponding four component models in the right column display some similarities. The models all capture the low variance during the equity market run-up in 1993-1998, preceded by higher volatility during the first Gulf war and the 1990-1991 recession. The LTCM and Russia debacles in the fall of 1998 are evident, as is the higher volatility during the dot-com bust and 2001 recession in the later part of the sample.

However, Figure 1 also reveals some important differences between models. The non-affine models (in the second and fourth rows) appear to display much more variation in the conditional variance during the more recent period than do the two affine models (in the first and third rows).

We plot the annualized conditional volatility of variance path, $100 * 252 \sqrt{Var_t\left(h_{t+2}\right)}$ for each of the eight models in Figure 2. Figure 2 confirms the findings in Figure 1. The non-affine models in the second and fourth rows of Figure 2 display a much larger volatility of variance than the four affine models in the first and third rows. This is true for both the GARCH(1,1) models in the left column and the component models in the right column. The non-affine models display a much larger volatility of variance during the first Gulf war and the 1990-1991 recession, and particularly during the LTCM and Russia debacles in the fall of 1998, and during the dot-com bust in 2000-2002.

Figure 3 plots the conditional correlation path, $Corr_t\left(R_{t+1}, h_{t+2}\right)$ for each of the eight models we consider. Notice that as derived above, the non-affine models imply a constant conditional correlation and thus show a flat line in the plot. The affine models instead have time-varying correlation and imply a conditional correlation very close to minus one when economic events drive volatility to be high, as for example during the 1990-1991 recession and from 1999 onwards. During the equity market run-up in the mid 1990s the conditional correlation implied by the affine models is much lower in magnitude.

3.3 Variance Term Structure Properties

While Figures 1-3 depict various aspects of the dynamics of the one-day ahead conditional distribution, Figures 4 and 5 capture the properties of the variance dynamics across longer horizons.

In Figure 4 we plot the expected future conditional variance from one to 252 days ahead. The dashed lines in Figure 4 denote multi-day variance forecasts starting from a low current spot variance corresponding to the 25th percentile of the path of variances from 1962 to 2001. The solid lines in Figure 4 denote multi-day variance forecasts starting from a high current spot variance corresponding to the 75th percentile of the path of variances from 1962 to 2001. In the component models the 25th and 75th percentiles are used for the current spot values for both components. The forecasts are normalized by the unconditional variance and shown in percentage terms so that for each model we are plotting $100 * \left(E_t\left[h_{t+k}\right] - \sigma^2\right) / \sigma^2$ against horizon k.

Figure 4 shows that the variance term structure properties vary strongly across models. In the AGARCH(1,1) models the relatively low daily variance persistence implies that the conditional variance forecasts converge to their unconditional levels after around 100 days. This is true for both the normal and GED version of the model. The AGARCH(C) models display some variation in variance forecasts across initial spot variance up until 252 days ahead corresponding roughly to the non-affine GARCH(1,1) models. The non-affine component models display the most variation in variance forecasts at long horizons. This difference across models should have important implications for the option pricing properties below.

Figure 5 plots the variance impulse response functions in percent, that is $100 * VIR(k)$ shown against horizon k. These functions relate how a shock of a certain magnitude today affects the future path of variances when all other shocks are set to zero. The solid lines in Figure 5 depict a positive shock of three standard deviations and the dashed lines depict a negative shock of three standard deviations.

Figure 5 shows that the leverage effect–or variance response asymmetry–is clear in all models. The negative shock increases future variance by as much as 80% but a positive shock never increases future variance by more than 25%. Important differences across models appear again. In the affine GARCH(1,1) models the shocks die out quickly and the positive shocks have virtually no impact at any horizon–even in the affine component models. In the non-affine models the positive shocks have a relatively small but relatively persistent effect across horizons.

4 Option Valuation Methodology

We need to know the mapping between the physical return shocks, z_t, and the risk-neutral return shocks. This mapping will allow us use the physical asset return models developed above to simulate future stock prices from the risk neutral distribution which we in turn can use to compute the hypothetical options payoffs that can be discounted using the risk-free rate to get the current model option price.

Duan (1999) extends the normal case in Duan (1995) and derives a generalized local risk-neutral framework for option valuation in conditionally non-normal GARCH models. As above, let z_{t+1} be i.i.d. Normal under the physical measure and let z^*_{t+1} be i.i.d. normal under the risk neutral measure. Define the mean-shift between the two measures by

$$\eta_{t+1} = z^*_{t+1} - z_{t+1} \tag{4.1}$$

For a GED distributed ζ_{t+1} shock we can write the mapping

$$\eta_{t+1} = z^*_{t+1} - \Phi^{-1}\left(G_v\left(\zeta_{t+1}\right)\right)$$

where $\Phi^{-1}()$ is the standard normal inverse CDF so that $z_{t+1} = \Phi^{-1}\left(G_v\left(\zeta_{t+1}\right)\right)$ is normally distributed. We can then rewrite the linear normal mapping in (4.1) as a nonlinear GED mapping given by

$$\zeta_{t+1} = G_v^{-1}\left(\Phi\left(z^*_{t+1} - \eta_{t+1}\right)\right) \tag{4.2}$$

The derivation of the risk-neutral model requires solving for η_{t+1}. This is done by setting the conditionally expected risk-neutral asset return in each period equal to the risk-free rate. In general we can write

$$\exp(r) = E_t^Q\left[\exp\left\{E_t\left[R_{t+1}\right] + \sqrt{h_{t+1}}G_v^{-1}\left(\Phi\left(z^*_{t+1} - \eta_{t+1}\right)\right)\right\}\right]$$

where the normal case obtains for $v = 2$.

4.1 The Affine Normal Models

Recall from above that in the affine models, the conditional return mean is defined to be

$$E_t[R_{t+1}] = r + \lambda h_{t+1} \tag{4.3}$$

In the normal case we of course have $G_2^{-1} = \Phi^{-1}$ so that the solution for η_{t+1} can be found as

$$\begin{aligned}
\exp(r) = E_t^Q\left[\exp\left\{r + \lambda h_{t+1} + \sqrt{h_{t+1}}\left(z_{t+1}^* - \eta_{t+1}\right)\right\}\right] &\Leftrightarrow \\
1 = \exp(\lambda h_{t+1})\exp\left(-\eta_{t+1}\sqrt{h_{t+1}}\right)E_t^Q\left[\exp\left\{\sqrt{h_{t+1}}z_{t+1}^*\right\}\right] &\Leftrightarrow \\
1 = \exp(\lambda h_{t+1})\exp\left(-\eta_{t+1}\sqrt{h_{t+1}}\right)\exp\left(\frac{1}{2}h_{t+1}\right) &\Leftrightarrow \\
\eta_{t+1} = \left(\lambda + \frac{1}{2}\right)\sqrt{h_{t+1}} &
\end{aligned}$$

so that

$$z_{t+1} = z_{t+1}^* - \left(\lambda + \frac{1}{2}\right)\sqrt{h_{t+1}} \tag{4.4}$$

which corresponds to the mapping in Heston and Nandi (2000).

Future stock returns can now be simulated under the risk-neutral measure by substituting the shock transformation in (4.4) into the asset return models in (2.1) and (2.10). In the affine GARCH(1,1) model we get

$$\begin{aligned}
R_{t+1}^* &= r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^* \qquad (4.5)\\
h_{t+1} &= w + b'h_t + a\left(z_t^* - c^*\sqrt{h_t}\right)^2
\end{aligned}$$

where $z_{t+1}^* \sim N(0,1)$ and where $c^* = c + \lambda + \frac{1}{2}$. Note how the structure of the expected return via its impact on η_{t+1} ultimately provides a risk-neutral volatility dynamic that is similar to the physical one.

4.2 The Non-Affine Normal Models

In these models we have

$$E_t[R_{t+1}] = r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} \tag{4.6}$$

so that the solution for η_{t+1} can be found as $\eta_{t+1} = \lambda$, which gives the mapping

$$z_{t+1} = z_{t+1}^* - \lambda \tag{4.7}$$

which in turn corresponds to the mapping in Duan (1995).

Future stock returns can be simulated under the risk-neutral measure by substituting the shock transformation in (4.7) into the asset return models in (2.16) and (2.25). We get

$$\begin{aligned}
R_{t+1}^* &= r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^* \qquad (4.8)\\
h_{t+1} &= w + b'h_t + ah_t\left(z_t^* - c^*\right)^2
\end{aligned}$$

where $z^*_{t+1} \sim N(0,1)$ and where $c^* = c + \lambda$. Note again how the structure of the expected return, which is different from the affine model, via its impact on η_{t+1} provides a risk-neutral non-affine volatility dynamic that is similar to the physical non-affine volatility dynamic.

4.3 The Affine GED Models

In the non-normal case an exact solution for η_{t+1} involves a prohibitively cumbersome numerical solution for η_{t+1} on every day and on every Monte Carlo path. We therefore develop the following approximation.

In the GED case, the parameter v determines the degree of non-normality in the CDF function G_v. When $v = 2$ we get normality and when $v < 2$ we get fat tails. In the normal special case we have

$$G_2^{-1}(\Phi(z)) = z \text{ for all } z$$

As the Normal and GED are both symmetric we know that

$$G_v^{-1}(\Phi(0)) = 0 \text{ for all } v$$

We use this to suggest the linear approximation

$$G_v^{-1}(\Phi(z)) \approx b_v z$$

where b_v is easily found for a given value of v by fitting $\zeta_i = G_v^{-1}(\Phi(z_i))$ to z_i for a wide grid of z_i values.

Figure 6 shows that the probability integral transform, $\zeta = G_v^{-1}(\Phi(z))$ is very close to linear. The bottom right panel in Figure 6 shows the trivial transformation when $v = 2$ and when the relationship is perfectly linear with a slope of one. The other panels in Figure 6 show the transformation for v values around the empirical estimates in Table 1. The transformation is clearly close to linear in all cases.

With this approximation we can write

$$\begin{aligned}
1 &= \exp(\lambda h_{t+1}) E_t^Q \left[\exp\left\{\sqrt{h_{t+1}} G^{-1}\left(\Phi\left(z^*_{t+1} - \eta_{t+1}\right)\right)\right\}\right] \\
&= \exp(\lambda h_{t+1}) E_t^Q \left[\exp\left\{\sqrt{h_{t+1}} b_v \left(z^*_{t+1} - \eta_{t+1}\right)\right\}\right] \\
&= \exp(\lambda h_{t+1}) \exp\left(-\eta_{t+1}\sqrt{h_{t+1}} b_v\right) E_t^Q \left[\exp\left\{\sqrt{h_{t+1}} b_v z^*_{t+1}\right\}\right] \\
&= \exp(\lambda h_{t+1}) \exp\left(-\eta_{t+1}\sqrt{h_{t+1}} b_v\right) \exp\left(\frac{1}{2} h_{t+1} b_v^2\right)
\end{aligned}$$

Taking logs yields

$$0 = \lambda h_{t+1} - \eta_{t+1}\sqrt{h_{t+1}} b_v + \frac{1}{2} h_{t+1} b_v^2$$

and solving for η_{t+1} yields

$$\eta_{t+1} = \left(\frac{\lambda}{b_v} + \frac{1}{2} b_v\right)\sqrt{h_{t+1}}$$

where of course the normal case obtains when $b_v = 1$.

The mapping between the physical GED and the risk neutral normal shocks are now

$$\zeta_{t+1} = G_v^{-1}\left(\Phi\left(z_{t+1}^* - \left(\frac{\lambda}{b_v} + \frac{1}{2}b_v\right)\sqrt{h_{t+1}}\right)\right)$$

which can be substituted into the return dynamics for the AGARCH(1,1)-GED and AGARCH(C)-GED models to get the risk-neutral processes.

Note that while the linear approximation greatly facilitates the computation of η_{t+1} in the GED models, the GED option prices are still more cumbersome to compute than the normality-based option prices due to the frequent inversion of the GED cumulative distribution function.

4.4 The Non-Affine GED Model

Using the non-affine return drift but the same approximation $G_v^{-1}(\Phi(*))$ as above, we can write

$$\begin{aligned}
1 &= \exp\left(\lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1}\right) E_t^Q\left[\exp\left\{\sqrt{h_{t+1}}G^{-1}\left(\Phi\left(z_{t+1}^* - \eta_{t+1}\right)\right)\right\}\right] \\
&= \exp\left(\lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1}\right) E_t^Q\left[\exp\left\{\sqrt{h_{t+1}}b_v\left(z_{t+1}^* - \eta_{t+1}\right)\right\}\right] \\
&= \exp\left(\lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1}\right) \exp\left(-\eta_{t+1}\sqrt{h_{t+1}}b_v\right) E_t^Q\left[\exp\left\{\sqrt{h_{t+1}}b_v z_{t+1}^*\right\}\right] \\
&= \exp\left(\lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1}\right) \exp\left(-\eta_{t+1}\sqrt{h_{t+1}}b_v\right) \exp\left(\frac{1}{2}h_{t+1}b_v^2\right)
\end{aligned}$$

Taking logs yields

$$0 = \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} - \eta_{t+1}\sqrt{h_{t+1}}b_v + \frac{1}{2}h_{t+1}b_v^2$$

and solving for η_{t+1} yields

$$\eta_{t+1} = \frac{\lambda}{b_v} + \left(\frac{b_v}{2} - \frac{1}{2b_v}\right)\sqrt{h_{t+1}}$$

where of course again the normal case obtains when $b_v = 1$. The mapping between the shocks is then

$$\zeta_{t+1} = G_v^{-1}\left(\Phi\left(z_{t+1}^* - \frac{\lambda}{b_v} - \left(\frac{b_v}{2} - \frac{1}{2b_v}\right)\sqrt{h_{t+1}}\right)\right)$$

which can be substituted into the return dynamics for the NGARCH(1,1)-GED and NGARCH(C)-GED models to get the risk-neutral processes.

4.5 Monte Carlo Simulation

The European call option prices are computed via Monte Carlo, simulating the risk-neutral return process and computing the sample analogue of the discounted risk neutral expectation. For a call option, C, quoted at the close of day t with maturity on day T, and with strike price X we have

$$\begin{aligned}
C_{t,T} &= \exp\left(-r(T-t)\right) E_t^*[Max(S_T - X, 0)] \\
&\approx \exp\left(-r(T-t)\right) \frac{1}{MC}\sum_{i=1}^{MC}\left[Max\left(S_t \exp\left(\sum_{\tau=1}^{T-t} R_{i,t+\tau}^*\right) - X, 0\right)\right]
\end{aligned}$$

where $R^*_{i,t+\tau}$ denotes future daily log-return simulated under the risk-neutral measure. The subscript i refers to the ith out of a total of MC simulated paths.

5 Option Valuation Empirics

We are now ready to use the eight models estimated in Section 2 and the transformation to risk-neutrality in Section 4 to assess the performance of the models for option valuation. In this section we first introduce the options data. We then use each of our eight models to price the option contracts and we compare model and market prices for various maturities, strike prices, and sample years. Subsequently, we conduct an economic analysis of the errors and finally assess the robustness of our results.

5.1 Option Data

We use six years of S&P 500 call option data covering the period 1990-1995. Starting from the raw data from the Berkeley option data base, we apply standard filters following Bakshi, Cao and Chen (1997). We only use options with more than seven days to maturity. We also only use Wednesday options data. Wednesday is the day of the week least likely to be a holiday. It is also less likely than other days such as Monday and Friday to be affected by day-of-the-week effects. If Wednesday is a holiday, we use the next trading day. Using only Wednesday data allows us to study a fairly long time-series, which is useful considering the highly persistent volatility processes.

Table 2 presents descriptive statistics for the options data for 1990-1995 by moneyness and maturity. Panel A reports the number of contracts available after filtering. Our sample consists of 21,752 options with a wide range of moneyness and maturity. Panel B shows the average call price in each of the bins in Panel A. Quite predictably, the average price increases significantly as the moneyness increases (moving down the rows) and as maturity increases (moving from left to right). The average overall price is \$27.91.

In Panel C of Table 2 we report the average Black-Scholes implied volatility for the option contracts in each bin. Panel C clearly documents the volatility smirk evident in quoted equity index option prices. The average implied volatility tends to increase as we move down the rows in each column of Panel C. The effect is most dramatic for the short maturities in the left-hand columns. This empirical regularity illustrates that the Black-Scholes option valuation formula, which assumes a constant per period volatility across time, maturity and strike prices, will result in systematic pricing errors, which motivates the use of GARCH models for option valuation.

5.2 Overall Option Valuation Results

When calculating option prices according to the eight GARCH models, we use the MLE parameters in Table 1 transformed to the risk neutral measure. These risk-neutral parameters as well as the conditional variance paths from Figure 1 are used as inputs into the option pricing formula. In the case of the non-affine models, the formula requires Monte Carlo simulation to calculate the price. In the case of the normal affine models numerical integration solutions exist. But in order to ensure that the results are not driven by the numerical pricing technique we use Monte Carlo simulation using the same set of random numbers for all models.

The overall $RMSE$s for the eight GARCH models are reported in the top row of Table 3. The $RMSE$ is computed as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i,t}\left(C_{i,t}^{Market} - C_{i,t}^{Model}\right)^2}$$

where the summation is over contract i observed on day t and where N is equal to 21,752, the total number of option contracts in the sample. The second row normalizes the $RMSE$ by dividing by the average call price in the sample.

Note first that the best overall model (i.e. the one with the lowest $RMSE$) is the NGARCH(C)-N with an $RMSE$ of 1.38 followed closely by the NGARCH(C)-GED with an $RMSE$ of 1.43. The two non-affine GARCH(1,1) also perform relatively well with $RMSE$s of 1.46 in the GED case and 1.59 in the normal case. The affine models as a group perform worse than the non-affine models. The AGARCH(C)-GED has an $RMSE$ of 1.74, and the $RMSE$ for the AGARCH(C)-N is 1.81. The two affine GARCH(1,1) models perform the worst, showing an $RMSE$ of 2.68 in the GED case and 2.69 in the normal case.

The overall $RMSE$ results in Table 3 allow us to make comparisons in three dimensions: affine versus non-affine variance dynamics, GED versus normal shocks, and GARCH(1,1) versus component variance models.

First, as noted above we see that non-affine models perform much better than affine models. This is true both for GARCH(1,1) and component models and for GED as well as Normal shocks. Thus our results confirm and extend those in Hsieh and Ritchken (2005) who compare an affine and a non-affine model in the GARCH(1,1) case with normal shocks.

Second, we see that GED models perform only marginally better than Normal models: The biggest improvement is in the NGARCH(1,1) case where the $RMSE$ drops from 1.59 to 1.46 going from GED to Normal shocks. In the other pairwise comparisons the difference between the GED and the Normal $RMSE$ is around 5 cents.

Third, we see that the component structure offers large improvements in fit for the affine class of models, but more modest improvements in the non-affine class of models. For the affine models,the $RMSE$ drops from 2.69 to 1.81 (normal shocks) and from 2.68 to 1.74 (GED shocks), where as for the non-affine models the drop is from 1.59 to 1.38 (Normal shocks) and from 1.46 to 1.43 (GED shocks).

Recall now the main findings in terms of daily return log-likelihood values in Table 1. We found that in all cases the component model has a significantly larger log-likelihood than the nested GARCH(1,1). When comparing the GED models with their normal counterparts we also found that the GED based models had significantly larger log-likelihood values. The log-likelihoods also suggested that the non-affine GARCH models were strongly preferred to the affine GARCH models. The option based results support the return-based improvement of non-affine models over affine. It also supports the component model improvements over GARCH(1,1) for affine models but much less so for non-affine models. While the GED offered drastic improvements in the return-based likelihood analysis, the improvements offered in option valuation are much more modest. The normal GARCH models may offer sufficient non-normality in the multi-day distribution, or alternatively the GED specification may not be adequate for the purpose of option valuation.

5.3 Option Valuation Results by Moneyness and Maturity

Table 3 also provides a break-down of the overall $RMSE$ results. In Panel A we report the $RMSE$ for each of six moneyness bins, where the $RMSE$ has been divided by the average market option price for that bin (from Table 2, Panel B). Looking across the rows of Panel A, we see that the non-affine component models generally do well for out-of-the money and at-the-money options, whereas the affine component models do well for the in-the-money options. For deep-in-the-money options the non-affine GARCH(1,1) models do the best. The differences across models are generally larger for out-of-the-money options thus driving the overall $RMSE$ result. The GED models generally do well for out-of-the-money options where the non-normality has more impact. The affine models do particularly poorly for out-of-the money options–perhaps not providing enough non-normality at the relevant horizons.

In Panel B of Table 3, we report the $RMSE$ for each of four maturity bins, where the $RMSE$ again has been divided by the average market option price for that bin. The affine component models do well for the shortest maturities but the non-affine component and GARCH(1,1) models are much better than the affine models at longer maturities. The relative lack of flexibility in the longer-term variance dynamics displayed in Figures 4 and 5 above seems to hurt the affine models in the valuation of long-maturity options.

Finally, Panel C reports the normalized $RMSE$ for each of the years in the option sample. Note that a non-affine model performs best or second best in all of the six years studied. The performance of the models over time will be analyzed in more detail below.

5.4 An Economic Assessment of Option Valuation Performance

We now turn to a more detailed analysis of the option valuation performance of the models over time. To this end we consider weekly observations on some key economic variables, shown in Figure 7.

The top-left panel shows the VIX volatility index from the CBOE and the top right panel shows the weekly log returns on the S&P500 index. Note the sharp increase in VIX in late 1990 and the simultaneous string of large negative returns on the S&P500 index. Towards the end of the sample period the VIX drops to historically low levels and the low return volatility is evident in the top-right panel as well.

The second row of panels shows the weekly crude Brent Oil price (left panel) and the 3-month T-bill rate (right panel). The dramatic spike in the oil price at the start of the First Gulf War and its timing with the VIX spike is clear. The Federal Reserve easing during the 1990-1991 recession and its subsequent tightening in the ensuring expansion is evident in the T-bill panel.

The third row reports in the left panel the credit spread defined as the yield on corporate bonds rated Baa less the yield on Aaa bonds as rated by Moody's. The credit spread clearly widens following the volatility increase and then tighten following the volatility drop and the lowering of interest rates. The right panel shows the term spread defined as the difference between the yield on 10-year T-bond and the 3-month T-bill rate. The term structure is steepening through the first half of the option sample period and then gradually leveling off in the second half.

The bottom row plots in the left panel the weekly average moneyness defined as index value over strike price (S/X) (left panel), and the weekly average maturity in years (right panel). The weekly averages are taken across the option contracts observed during the Wednesday of that week. Note that the average moneyness drops following the string of negative index returns in 1990-1991 but increases towards the end of the sample as the stock market picks up. The average option maturity shows the well-known cycles following the introduction of new contracts on the CBOE.

The economic variables in Figure 7 can be visually compared with the weekly $RMSE$ for each model shown in Figure 8. The spike in the oil price and in VIX in 1990-1991 is evident in the weekly $RMSE$ in all models, but the degree to which the spike is reflected in the option $RMSE$s varies dramatically across models. The GARCH(1,1) models in the left-side panels in Figure 8 show much more of a spike than do the component models in the right-side panels. The affine component models also seem to display more of a spike than do the non-affine component models. The weekly $RMSE$ performance also differs drastically across models in the later part of the sample. The affine GARCH(1,1) models show a strong upward trend in $RMSE$ during 1993-1994 when the VIX is dropping to historically low levels and when the short-term interest rate is increasing and the term spread decreasing. The upward trend in $RMSE$ in 1993-1994 is much less pronounced in the affine component models and the non-affine GARCH(1,1) models and it is virtually absent in the non-affine component models.

The relationship between the economic variables in Figure 7 and the weekly option $RMSE$s in Figure 8 is formalized in Table 4. The top row in Table 4 reports the average weekly $RMSE$ for each model. The time-series of weekly $RMSE$s for each model are then regressed on the economic variables from Figure 7. We also regress the $RMSE$ from each model on the model specific path of volatilities defined as $\sqrt{h_t}$. Table 4 reports the t-statistic for each regressor where the standard deviation for each regressor has been computed using White's robust variance matrix. Any t-statistic larger than two in absolute value is bolded in Table 4. The bottom row of Table 4 shows the regression fit via the R-squared statistic.

Consider first the R-squared statistic. Note that close to 50% of the variation in the weekly RMSE of the affine GARCH(1,1) models is explained by the economic variables. For the affine and non-affine component models the R-squareds are much lower at around 20%, and for the NGARCH(1,1) models the explanatory power is even lower at 13-15%.

Interestingly, the own model volatility is significant in the affine GARCH(1,1) models and only in these models. When the model volatility is high, the $RMSE$ is also high suggesting a misspecification in the volatility dynamic. The coefficient on VIX is negative for the affine GARCH(1,1) models but positive (but insignificant) as expected in all other cases. This suggests that the market volatility effect is captured by the own volatility variable in the affine GARCH(1,1) models.

It is the case for all models that the S&P500 return is insignificant as is the average moneyness. The average maturity is positive and significant for all models. This could simply be due to the fact that longer maturity options are more expensive and thus have higher dollar pricing errors, but the fact that the short-term interest rate and the term spreads are negative and significant in all models suggests that perhaps all models could be enhanced by explicitly modeling interest rate dynamics. In their stochastic volatility models Bakshi, Cao and Chen (1997) found that adding stochastic interest rate dynamics did not improve the performance of the models. However, they were fitting the models to the observed option prices daily which could perhaps explain this finding.

The credit spread is significant (but with differing signs) in all but the two non-affine GARCH(1,1) models which also had the lowest R-squared statistics overall. The oil price is positive and significant in all the affine models. These findings suggest that building option pricing models with variance dynamics driven by key economic variables such as credit spreads and oil prices is a viable avenue for future research.

Figure 9 shows the weekly option price bias for each model defined as the average option price in the market less the average model price each week. Figure 9 shows that all models tend to underprice when market volatility spikes in 1990-1991. The affine GARCH(1,1) models show large overpricing biases in the latter half of the sample when market volatility is very low. Note that the affine component models show much less evidence of this overpricing. The component dynamics seem to be able to generate a much more flexible volatility path which can accommodate the large swings in volatility observed during the sample period. Note that the non-affine GARCH(1,1) models show a sudden overpricing spike in early 1994 which seems to be driven by a spike in the average maturity at that time.

Table 5 reports on regressions of weekly bias on the economic variables used in Table 4. Note again that the explanatory power is very large in the affine GARCH(1,1) models, somewhat large in the affine component models, lower yet in the non-affine GARCH(1,1) models and lowest in the non-affine component models. The biases in the affine models appear to be driven by the VIX which is significant in these models and not in the other models. The increase in the short term rate, the decline in the term and credit spreads seem capable of explaining some of the upward bias in the affine model prices in the second half of the sample. It is also quite interesting to note that the own model volatility path is significant in the component models but not for the other models. This suggest that the path of volatility can be improved upon for option valuation. We will study this topic further in the next section. Finally, note that the spikes in bias in the non-affine GARCH(1,1) models in Figure 9 appear to be captured by the average maturity variable in Table 5.

5.5 Robustness Analysis

So far the GARCH models have been estimated on daily returns only and then used for option valuation without letting the model parameters be driven in any way by the observed option prices. We have argued that as the option prices are not used in estimation, the option valuation exercise is out-of-sample even if the return sample period (1962-2001) overlaps with the option sample period (1990-1995). We now want to check the robustness of our results in two ways. First, we shorten the return sample period to end in 1989 just before the first option price observation. Second, we use options to estimate the weekly spot variance, $h(t)$ minimizing the weekly option $RMSE$, while keeping the model parameters fixed at the MLE values estimated on the 1962-1989 sample of daily index returns. Table 6 and Figure 10 contain the results.

Panel A of Table 6 summarizes the results using the MLE parameters from 1962-2001. The $RMSE$ in the top row of Table 6 is simply repeated from the top row of Table 3. The second row of Table 6 reports the overall bias which is close to zero for all models. The third row reports the average spot volatility across the 313 option sample days. Due to the variance targeting used in MLE these average volatilities are quite similar across models. Finally, the fourth row reports the standard deviation of the 313 spot variances. In keeping with the results in Figure 2 the volatility of variance is highest in the non-affine models.

Panel B of Table 6 reports the same set of results using MLE estimates of the GARCH parameters from daily returns from 1962 through 1989 rather than though 2001. Note that the option $RMSE$ is often lower using when parameters are estimated on returns observed through 1989 than when returns through 2001 are used in estimation. This strongly suggests that using return-based parameters estimated through 2001 to price options during 1990-1995 can indeed be thought of as an out-of-sample exercise. Compared with the 2001 estimates, the 1989 estimates result in higher option $RMSE$s only in the two non-affine component models. Whereas the evidence in favor of non-affine component models thus weakens, the other overall conclusions from the 2001 estimates remain intact. The poorer option valuation performance of the non-affine component models when using the shorter sample could be driven by the fact that the component models need a longer return sample to properly identify the components.

Panel C of Table 6 uses the GARCH parameters from MLE on returns up through 1989 but estimates the GARCH spot variance, h_t, each week by minimizing that week's option $RMSE$ using a nonlinear least squares (NLS) technique. Comparing the pure MLE $RMSE$s in Panel B with the hybrid $RMSE$s in Panel C show that the reduction in $RMSE$ is dramatic in all models. The four non-affine models now all have an $RMSE$ of around \$1 whereas the two affine component models have $RMSE$s of around 1.36 and the affine GARCH(1,1) models around \$2. Thus the overall ranking of models from the pure MLE analysis remains largely intact.

Figure 10 elaborates further on this finding by plotting the weekly spot volatility from NLS shown in dots along with the corresponding MLE based spot volatilities shown in solid lines. The picture is quite striking. In the affine GARCH(1,1) models the NLS optimizer forces the spot volatility to zero in much of the second half of the sample in order to lessen the overpricing apparent in Figure 9. This phenomenon also happens in the affine component models but to a much lesser extend. In the non-affine models the $RMSE$-optimal spot volatility never hits zero and the NLS dots in those panels fall nicely around close to the MLE optimal volatility lines.

The last row in Table 6 formalizes this closeness by reporting the $RMSE$ between the annualized $RMSE$ optimal volatility and its MLE optimal counterpart. These numbers confirm the visual impression in Figure 10 and also corroborate some of our earlier findings: First, the non-affine models have a much closer correspondence between option implied and purely return-generated spot volatility. Second, the component structure reduces the volatility $RMSE$ by well over 50% in the affine models, and now also quite substantially in the non-affine models. Third, the GED shocks do not have much effect when judged by this this metric either.

6 Conclusion and Directions for Future Work

We have assessed the ability of eight different GARCH models to fit daily return dynamics and their ability to match market prices of options. First, we consider component models versus GARCH(1,1) models. As in Engle and Lee (1999) we find strong evidence in favor of component models from the point of view of modeling daily return dynamics. When using option prices to assess the models, we also find strong evidence for the component structure in the affine GARCH models but not in the non-affine models. Second, we consider non-affine versus affine GARCH models. The affine GARCH(1,1) model has been compared to the non-affine NGARCH(1,1) model in Hsieh and Ritchken (2005), who found strong support for the non-affine specification. Our results support their findings and we find that the non-affine models also outperform affine models when allowing for component structures and non-normal shocks. Third, we consider conditionally normal versus conditionally non-normal models. We find that assuming GED shocks for the daily asset returns greatly improves the fit of the all models to daily returns, but the improvement in option valuation is much less evident.

The empirical results suggest some viable directions for future research.

First, it remains to be seen if the differences in performance between models are confirmed when using model parameters estimated from option prices, or when using an integrated analysis that uses option prices as well as underlying returns (see Bates (2000), Chernov and Ghysels (2000), Eraker (2004) and Pan (2002)). The analysis in Table 6 does suggest that the relative performance of the models is comparable when the spot volatility is estimated from options rather than filtered from returns.

Second, it would be interesting to expand the analysis of non-normal shocks to a wider class of distributions. To this end Christoffersen, Heston and Jacobs (2006) develop an Inverse Gaussian GARCH model, Duan, Ritchken and Sun (2006) suggest augmenting GARCH models with jumps, and Lehnert (2003) applies an EGARCH model with skewed GED shocks.

Third, we have restricted attention to European style options on the S&P500 index. It would be interesting to apply the GARCH modeling framework to some of the many American style contracts traded in the derivatives markets. Ritchken and Trevor (1999) and Stentoft (2005) provide fast numerical techniques for GARCH option valuation with early exercise.

Finally, it would be interesting to compare the range of discrete time GARCH models considered here with continuous time stochastic volatility models. Bakshi, Cao and Chen (1997), Bates (1996), and Eraker (2004) study stochastic volatility models with jumps, Alizadeh, Brandt and Diebold (2002), Chernov, Gallant, Ghysels and Tauchen (2003) and Taylor and Xu (1994) study multifactor stochastic volatility models, and Bates (2000) analyses models with Poisson jumps and multiple volatility factors. Recently, Carr and Wu (2004) and Huang and Wu (2004) have considered Levy processes with infinitely many jumps. Comparing GARCH and SV models for the purpose of option valuation may provide more insight into the strengths and weaknesses of the various models.

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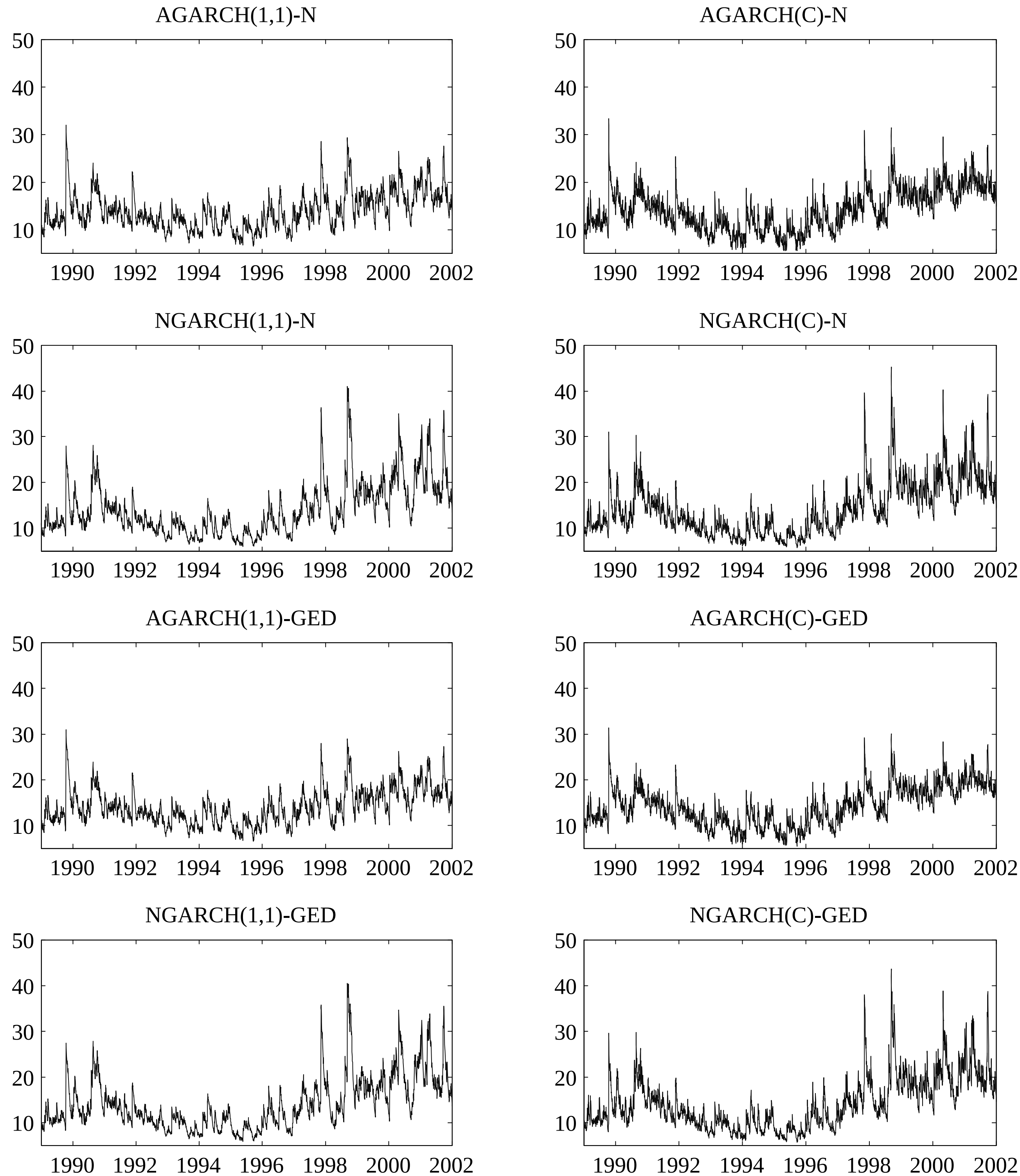
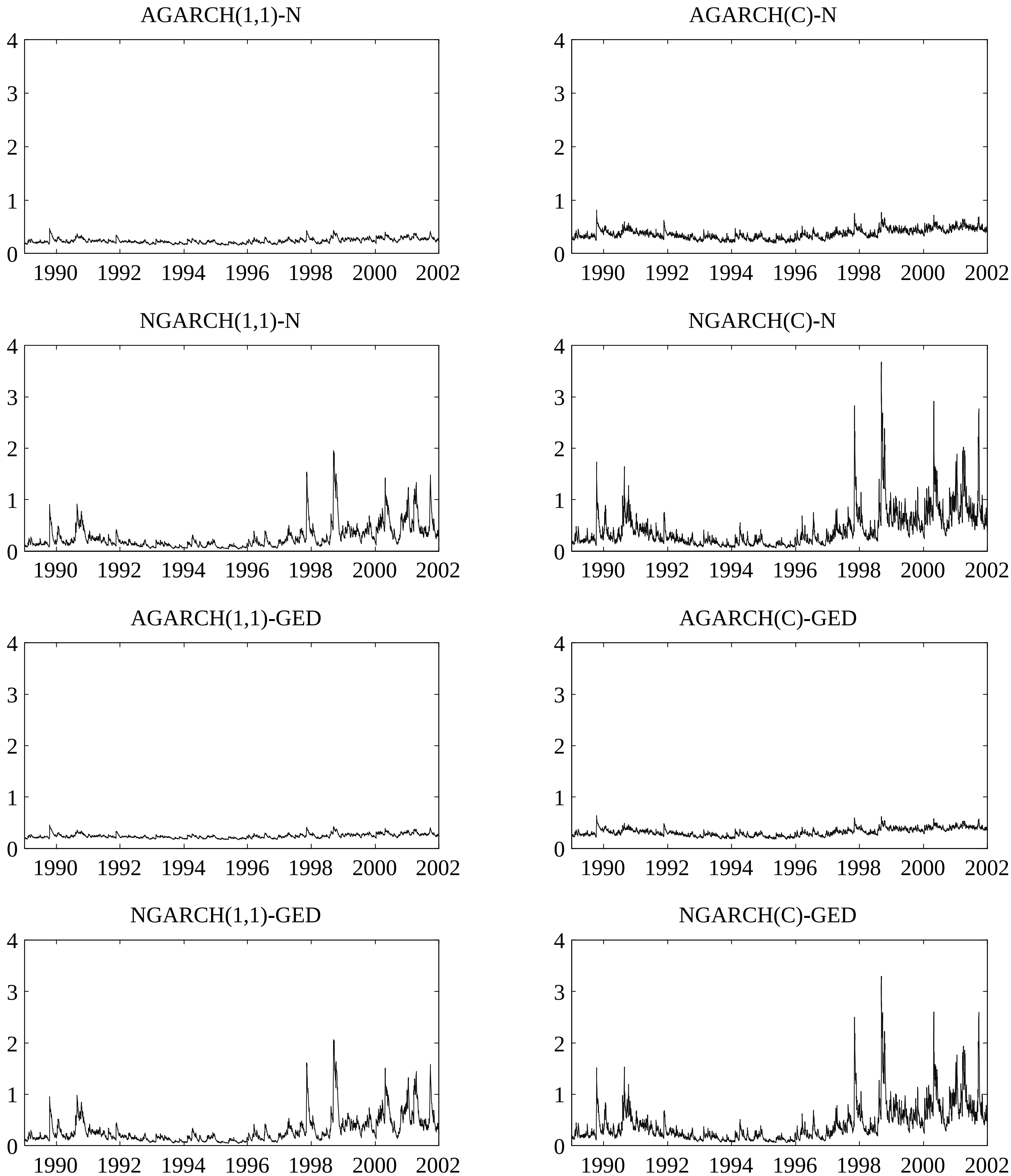
Figure 1. Conditional Volatility Paths



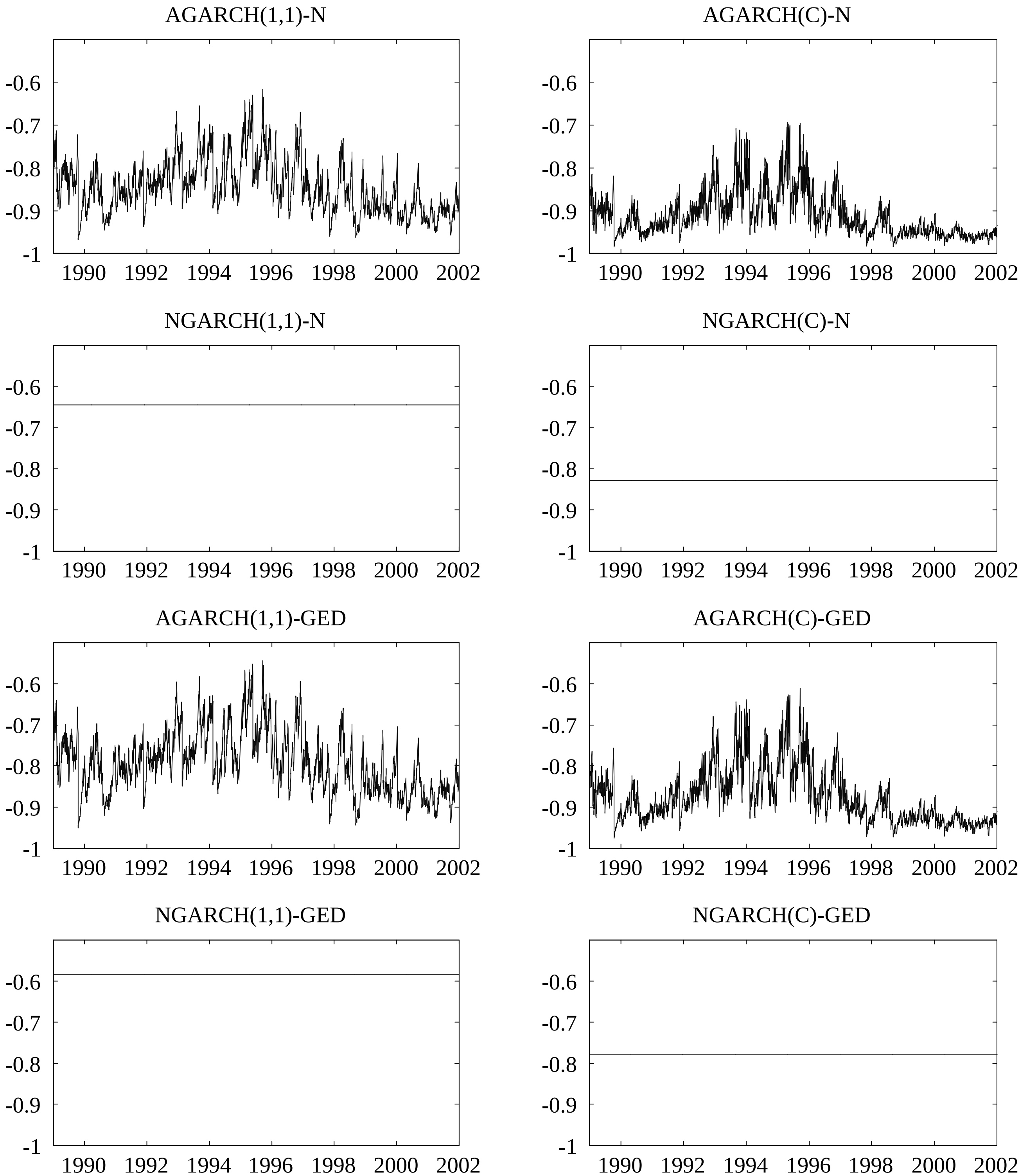
Notes to Figure: We plot the annualized conditional volatility in percent, $100 * \sqrt{252 h_{t+1}}$, for each of the eight models we consider. The parameter values for the underlying GARCH models are obtained from MLE estimation on daily S&P500 returns as reported in Table 1.

Figure 2. Conditional Volatility of Variance Paths



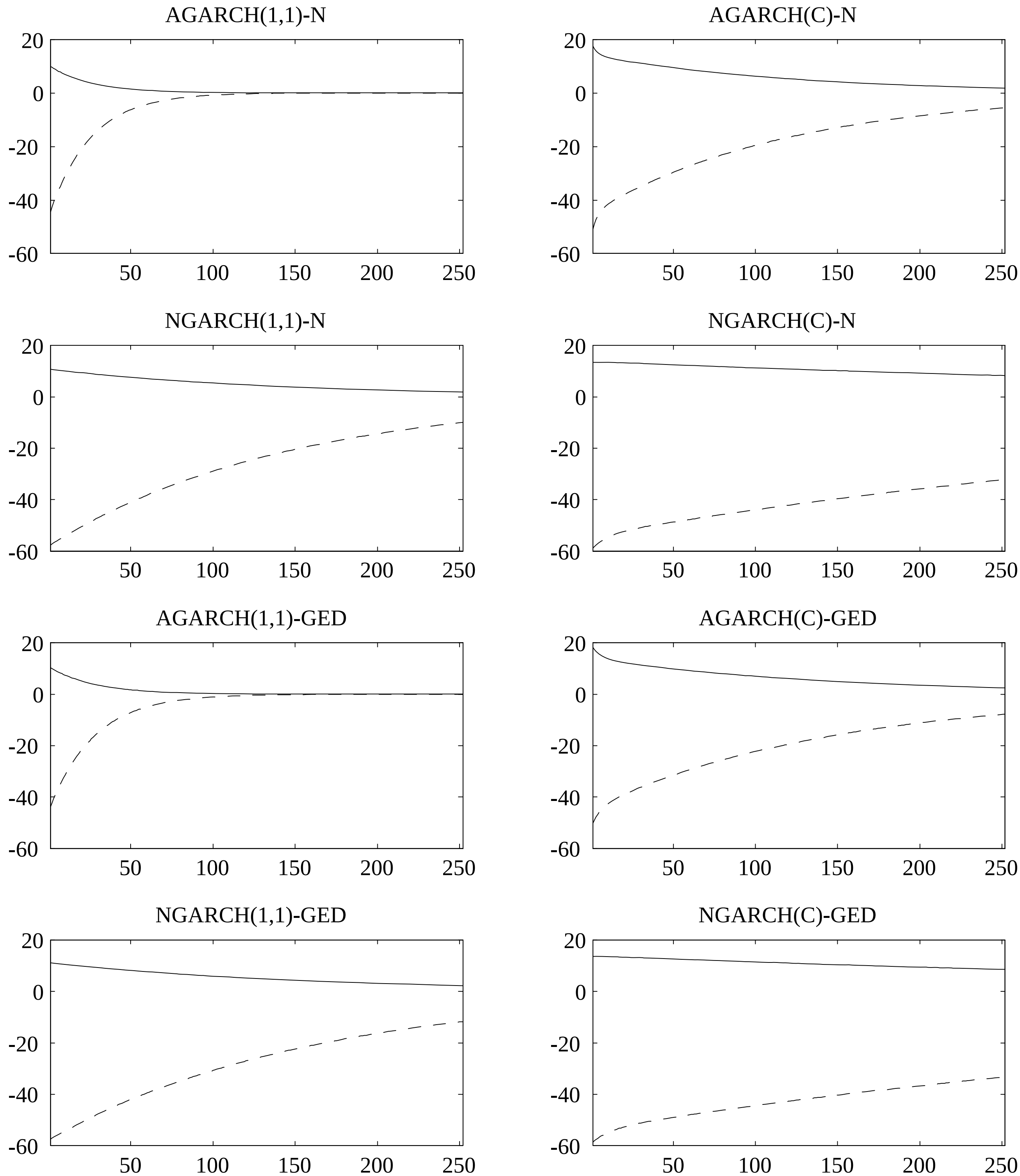
Notes to Figure: We plot the annualized conditional volatility of variance path in percent, $100 * 252 * \sqrt{Var_t\left(h_{t+2}\right)}$, for each of the eight models we consider. The parameter values for the underlying GARCH models are from Table 1.

Figure 3. Conditional Correlation Paths



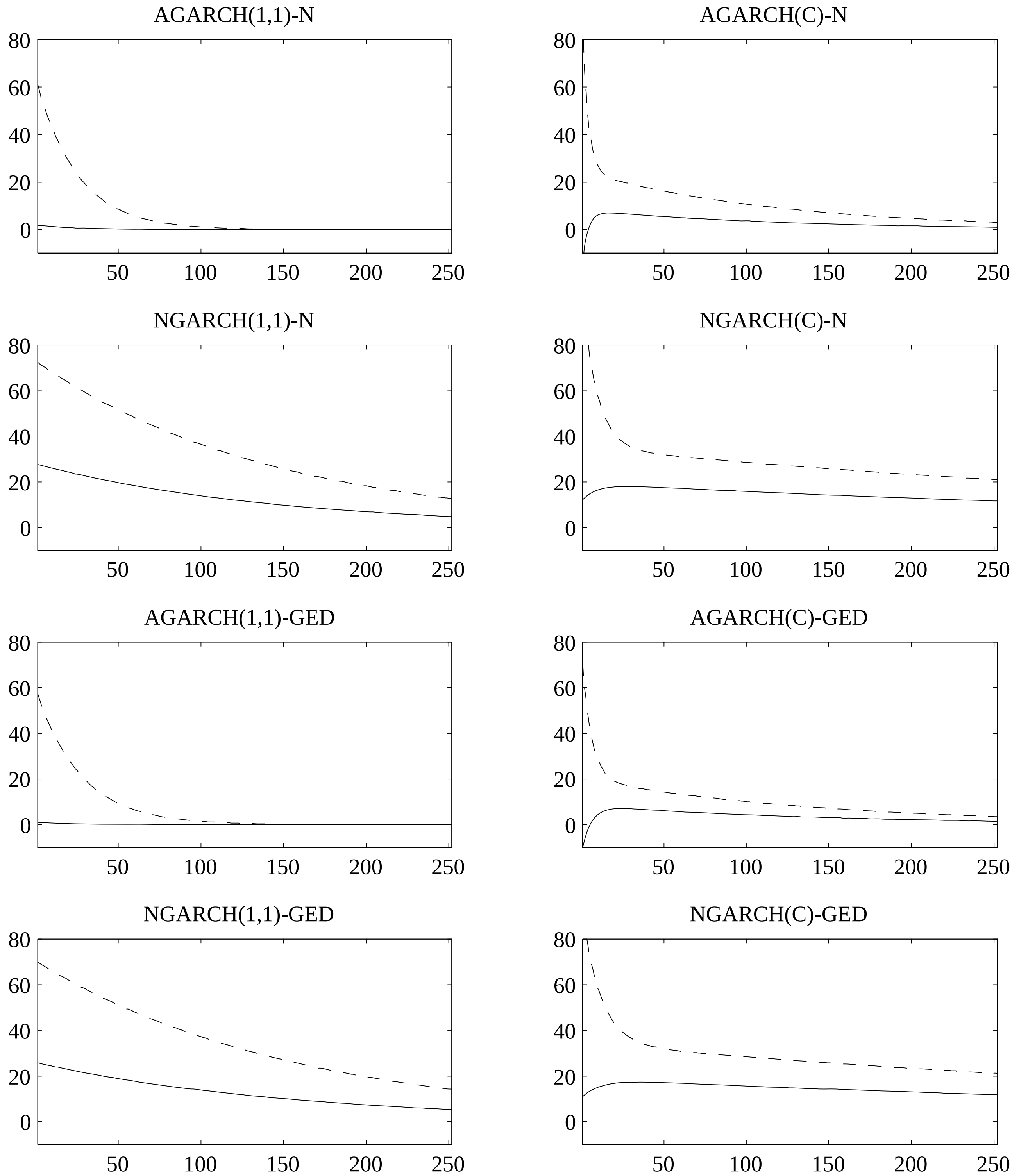
Notes to Figure: We plot the conditional correlation path, $Corr_t\left(R_{t+1}, h_{t+2}\right)$ for each of the eight models we consider. The parameter values for the underlying GARCH models are from Table 1.

Figure 4. Variance Forecasts Across Forecast Horizon



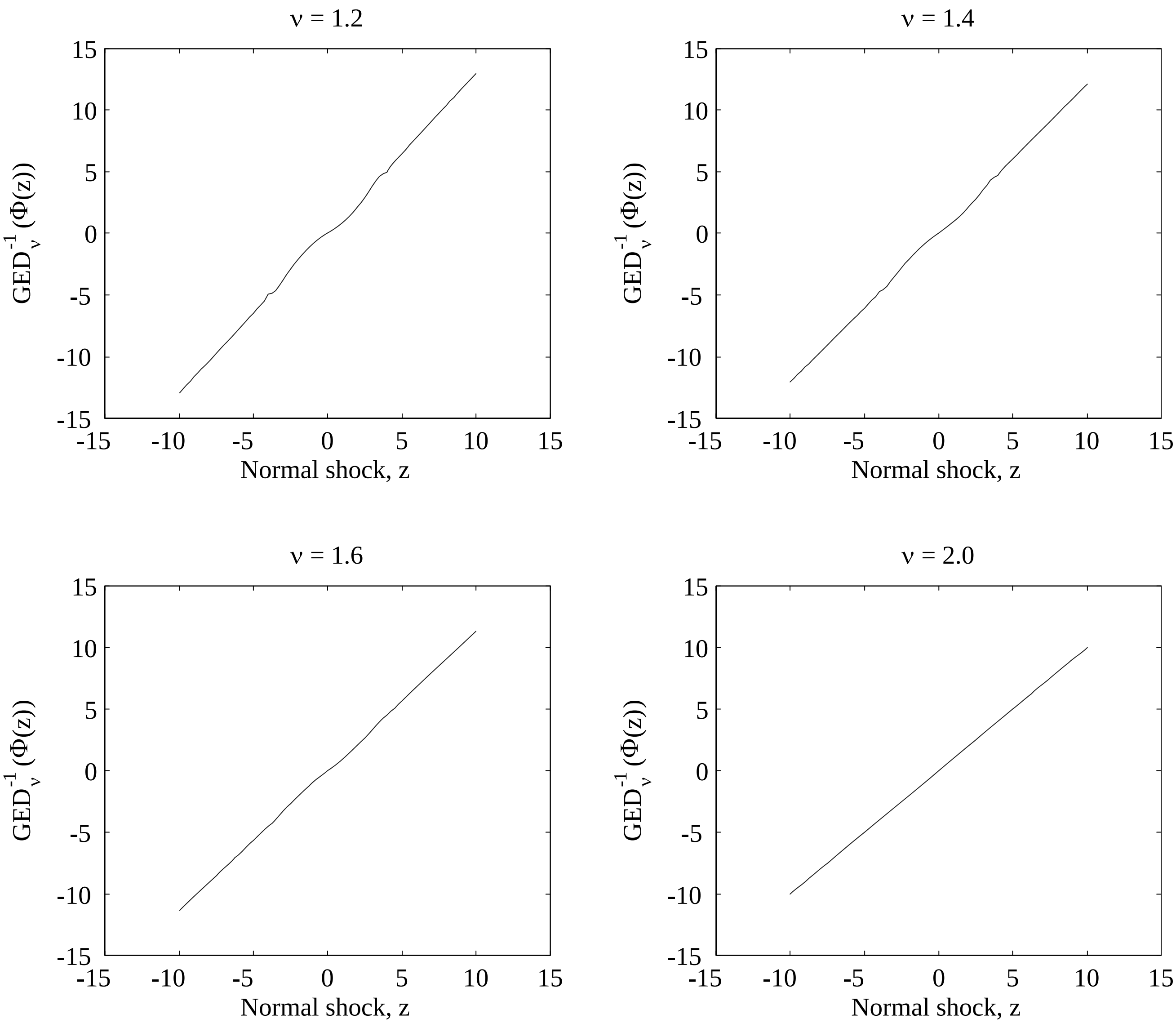
Notes to Figure: We plot the normalized k-day ahead variance forecast $100 * (E_t [h_{t+k}] - \sigma^2) / \sigma^2$ for a low initial spot variance (dashed line) and a high initial spot variance (solid line). Please see the text for details.

Figure 5. Variance Impulse Response Across Horizons



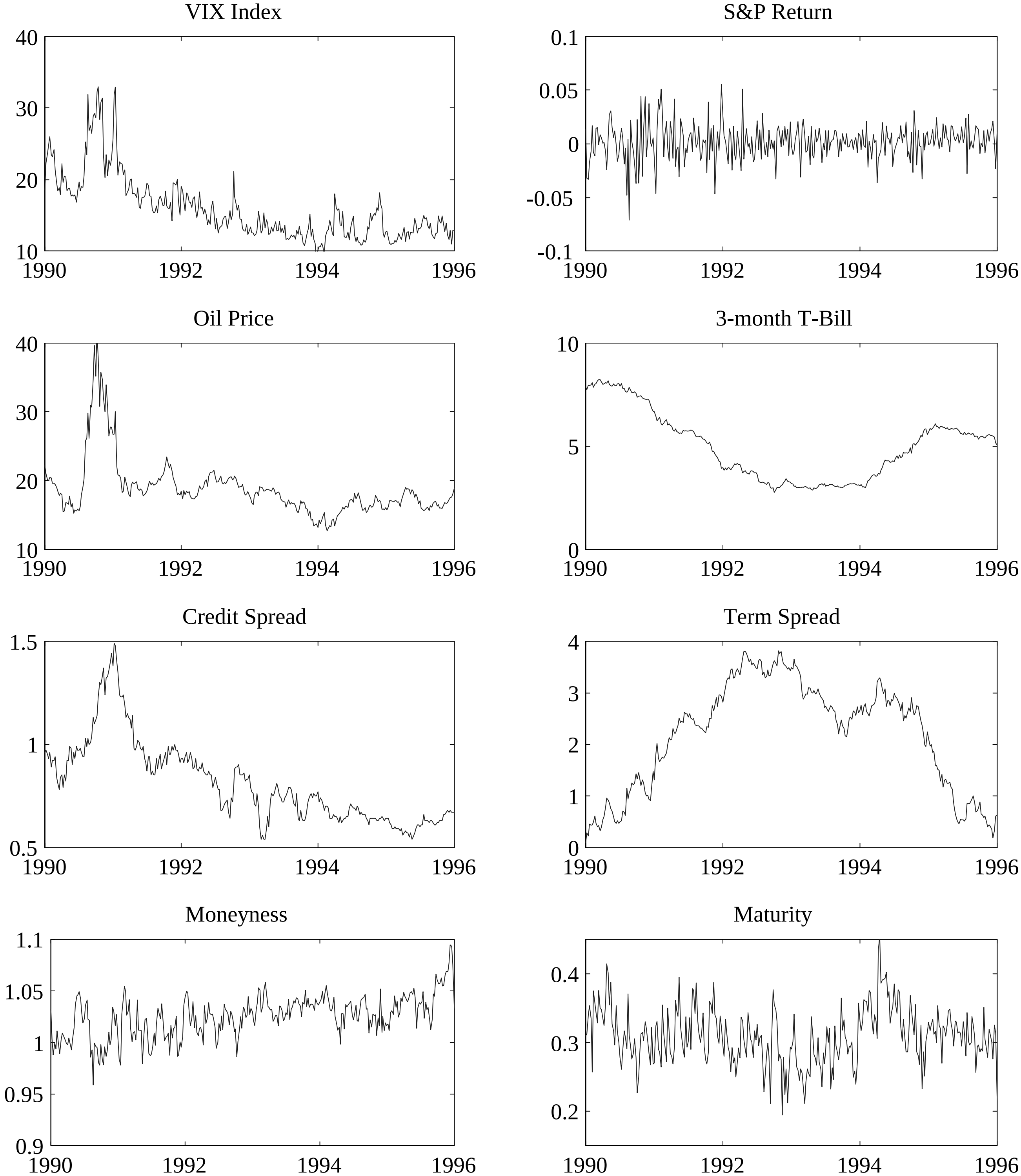
Notes to Figure: We plot the variance impulse response function $VIR(k)$ across k days. The solid line denotes a shock of plus three standard deviations and the dashed line denotes minus three standard deviations. Please see the text for more detail.

Figure 6. The GED Approximation



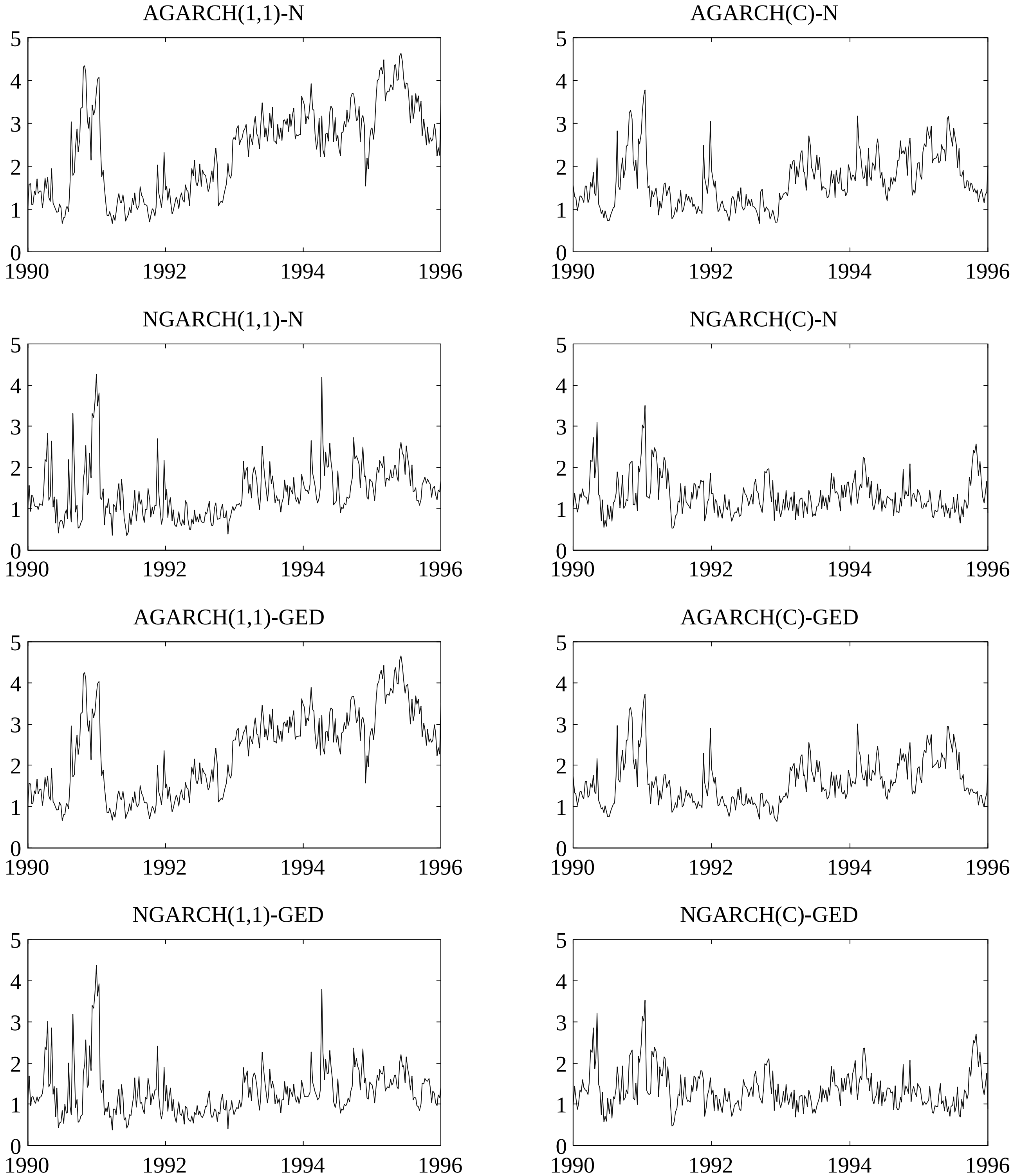
Notes to Figure: For a standard normal shock, z, ranging from -10 to +10 we plot the transformation to a standardized GED variable, ζ, via the function $\zeta = GED_v^{-1}(\Phi(z))$. The four panels correspond to different values of v. The bottom right panel corresponds to the normal special case with $v = 2$ where the transformation is trivial and a straight line with slope 1 obtains.

Figure 7. Weekly Economic Variables



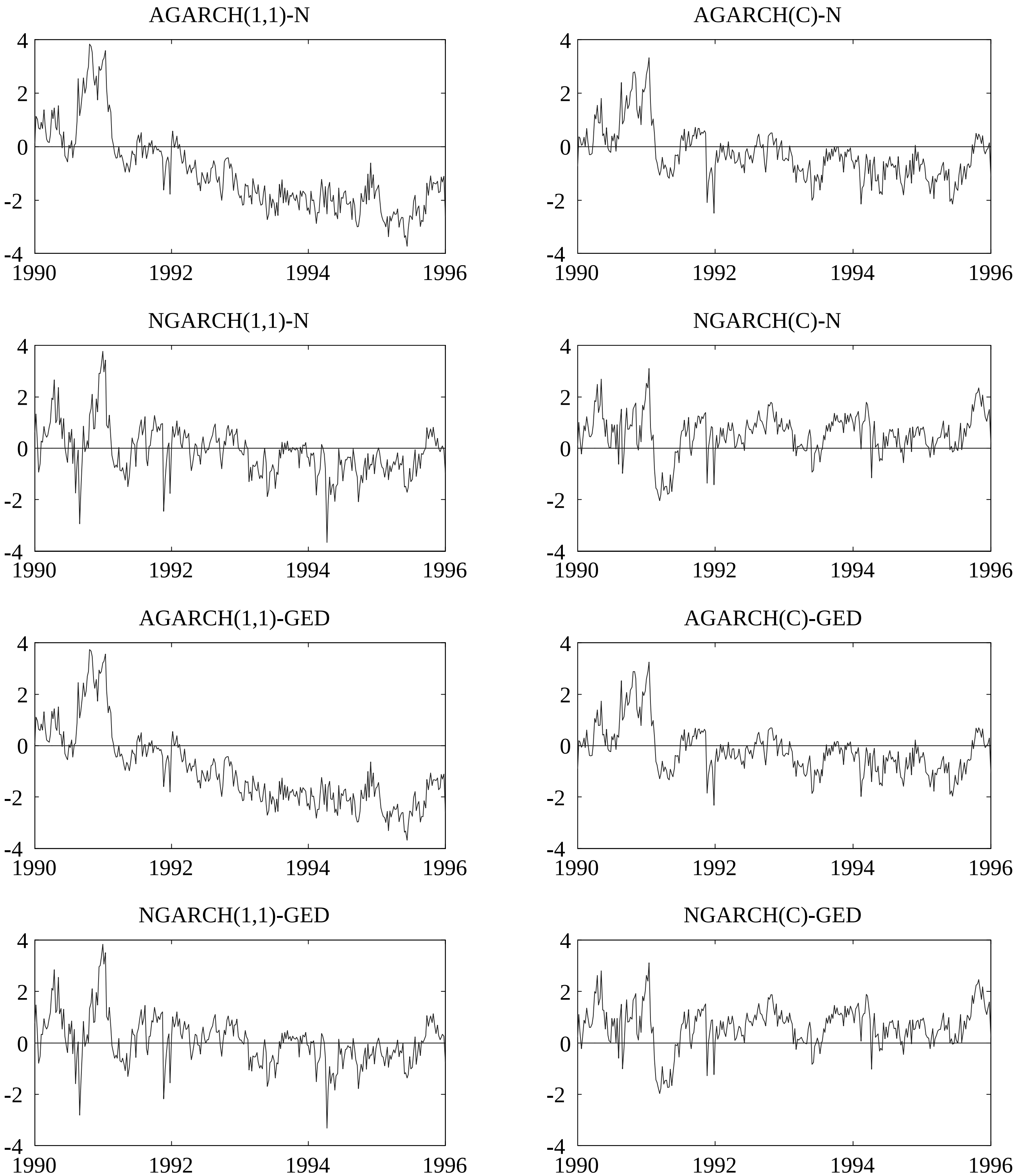
Notes to Figure: We plot weekly observations for various weekly economic variables as well as the weekly average moneyness (S/X) and maturity (in years) from the option data set.

Figure 8. Weekly Option Price $RMSE$



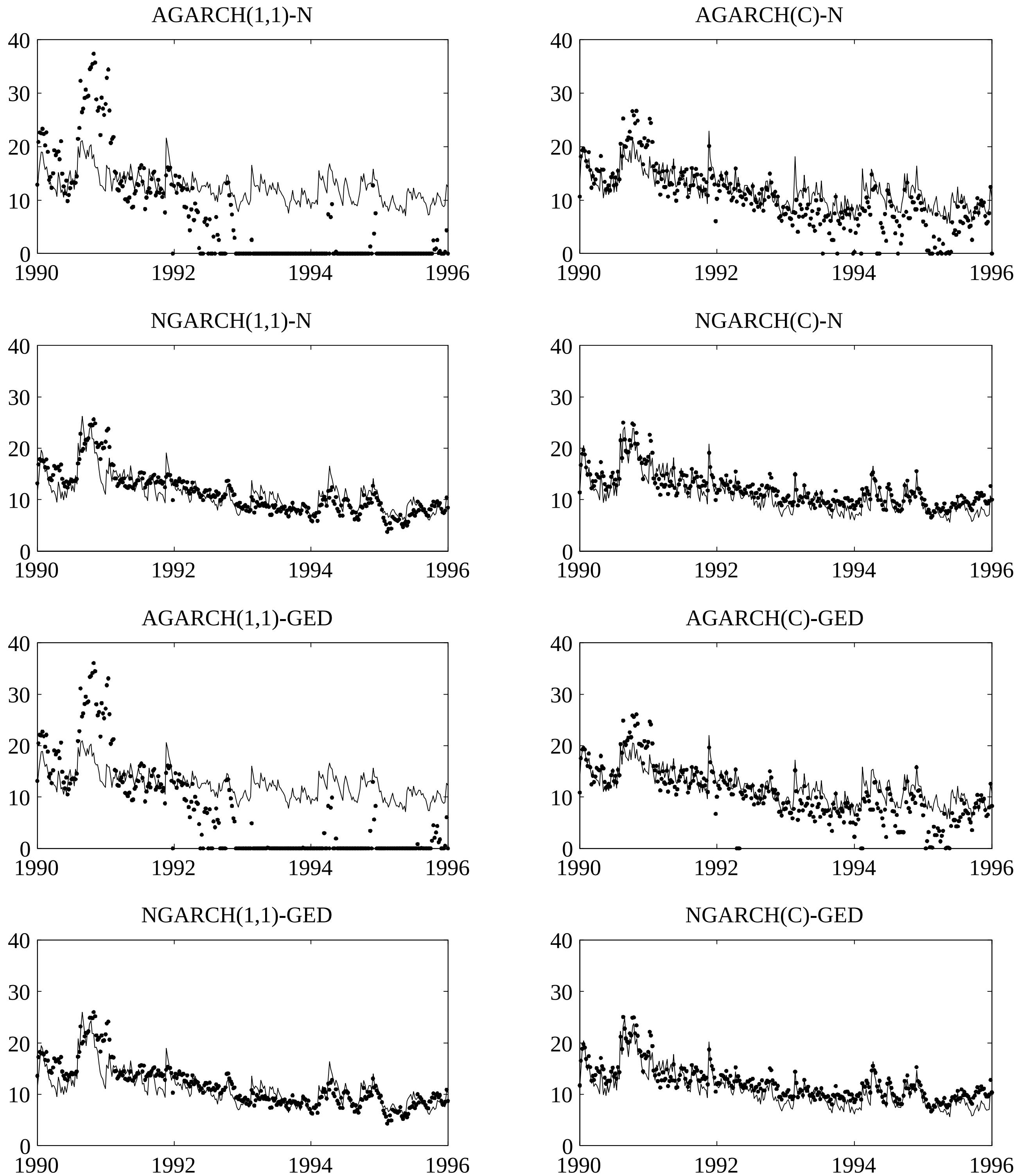
Notes to Figure: We plot the weekly option price root mean squared error ($RMSE$) for each model. The parameter values in the GARCH models are from Table 1.

Figure 9. Weekly Option Price Bias



Notes to Figure: We plot the weekly option price bias (average market price less average model price) for each model. The parameter values in the GARCH models are from Table 1.

Figure 10. $RMSE$ Optimal Spot Volatility Estimated Weekly by NLS



Notes to Figure: The dots show the weekly $RMSE$-optimal GARCH variance $h(t)$ estimated by NLS and reported in annualized standard deviation in percent. The MLE optimal volatility path is show in solid lines.

Table 1: Maximum Likelihood Estimation on Daily S&P500 Returns. 1962-2001

	AGARCH-N		**NGARCH-N**		**AGARCH-GED**		**NGARCH-GED**	
	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components
λ	0.00002	1.00495	0.03768	0.03390	0.56677	1.78607	0.03984	0.03674
	(1.83E-05)	(3.53E-06)	(8.62E-03)	(8.98E-03)	(1.42E-05)	(1.64E-04)	(8.30E-03)	(9.67E-03)
a, α	3.342E-06	2.132E-06	6.253E-02	3.696E-02	3.105E-06	1.705E-06	5.982E-02	3.071E-02
	(1.72E-07)	(1.40E-07)	(3.95E-03)	(3.45E-03)	(2.52E-04)	(1.35E-07)	(4.12E-03)	(4.72E-03)
b', β	0.89921	0.74928	0.90825	0.89262	0.90297	0.83454	0.91133	0.91320
	(2.47E-02)	(1.71E-04)	(5.30E-03)	(2.20E-02)	(6.90E-06)	(2.61E-05)	(5.71E-03)	(1.89E-02)
c, γ_1	135.7520	297.2247	0.5972	1.6588	139.7188	313.8362	0.6136	1.7759
	(2.47E-02)	(5.42E-05)	(4.67E-02)	(4.97E-02)	(1.70E-06)	(8.63E-04)	(5.77E-02)	(1.98E-01)
φ		1.739E-06		3.393E-02		1.524E-06		3.341E-02
		(4.84E-08)		(3.36E-03)		(1.10E-07)		(4.15E-03)
ρ		0.99176		0.99796		0.99309		0.99807
		(2.34E-06)		(5.21E-04)		(4.31E-04)		(6.13E-04)
γ_2		71.40695		0.38247		57.94967		0.38521
		(2.94E-05)		(4.76E-02)		(1.22E-03)		(5.64E-02)
ν					1.34637	1.41600	1.43298	1.45868
					(1.17E-07)	(3.04E-03)	(2.66E-02)	(2.84E-02)
Properties								
Log-Likelihood	33954	34129	34130	34201	34192	34310	34309	34352
Annual Volatility Target	14.66	14.66	14.66	14.66	14.66	14.66	14.66	14.66
Total Variance Persistence	0.9608	0.9979	0.9931	0.9998	0.9636	0.9989	0.9937	0.9998
Empirical z Kurtosis	8.9123	7.4070	6.6153	5.7752	9.1408	7.9727	6.6575	5.9424
Model z Kurtosis	3.0000	3.0000	3.0000	3.0000	4.1797	3.9732	3.9274	3.8612
Annual Vol of Variance	0.2289	0.3507	0.2497	0.3852	0.2341	0.3016	0.2725	0.3726
Average Correlation	-0.8318	-0.8949	-0.6452	-0.8289	-0.7784	-0.8557	-0.5828	-0.7792

We use daily total returns from July 1, 1962 to December 31, 2001 on the S&P500 index to estimate the GARCH models using Maximum Likelihood. Standard errors are calculated from the outer product of the gradient at the optimum parameter values. Annual Vol of Variance refers to the sample mean of the annualized conditional volatility of variance path in each model. Correlation refers to the sample mean of the conditional correlation between variance and return in the affine models and to the constant model implied correlation in the non-affine models.

Table 2: S&P 500 Index Call Option Data. 1990-1995

Panel A. Number of Call Option Contracts

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	101	1,884	1,931	1,769	5,685
0.975<S/X<1.00	283	1,272	706	477	2,738
1.00<S/X<1.025	300	1,212	726	526	2,764
1.025<S/X<1.05	261	1,167	654	409	2,491
1.05<S/X<1.075	245	1,039	582	390	2,256
1.075<S/X	549	2,345	1,679	1,245	5,818
All	1,739	8,919	6,278	4,816	21,752

Panel B. Average Call Price

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.88	2.30	6.25	11.94	6.62
0.975<S/X<1.00	2.29	6.83	15.19	27.50	12.12
1.00<S/X<1.025	8.35	13.60	22.48	34.41	19.32
1.025<S/X<1.05	17.57	22.00	30.11	42.14	26.97
1.05<S/X<1.075	27.11	30.84	38.14	48.83	35.43
1.075<S/X	50.67	52.78	58.98	68.34	57.70
All	24.32	23.66	28.68	36.07	27.91

Panel C. Average Implied Volatility from Call Options

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.1625	0.1269	0.1350	0.1394	0.1342
0.975<S/X<1.00	0.1308	0.1296	0.1448	0.1562	0.1383
1.00<S/X<1.025	0.1527	0.1459	0.1558	0.1605	0.1520
1.025<S/X<1.05	0.1915	0.1647	0.1665	0.1656	0.1681
1.05<S/X<1.075	0.2433	0.1828	0.1775	0.1739	0.1865
1.075<S/X	0.3897	0.2356	0.1961	0.1868	0.2283
All	0.2434	0.1703	0.1622	0.1607	0.1717

Notes to Table: We use European call options on the S&P500 index. The prices are taken from quotes within 30 minutes from closing on each Wednesday during the January 1, 1990 to December 31, 1995 period. We use the moneyness and maturity filters used by Bakshi, Cao and Chen (1997). The implied volatilities are calculated using the Black-Scholes formula. S/X refers to moneyness (index price over strike price), and DTM refers to the number of days to maturity of the option.

Table 3: Root Mean Squared Error (RMSE) over Average Call Price

	AGARCH-N		NGARCH-N		AGARCH-GED		NGARCH-GED	
	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components
Overall RMSE ($)	2.6927	1.8138	1.5875	1.3820	2.6819	1.7404	1.4599	1.4270
RMSE / Avr Call Price	0.0965	0.0650	0.0569	0.0495	0.0961	0.0623	0.0523	0.0511
Moneyness	**Panel A: RMSE over Average Call Price for Options with Various Moneyness**							
S/X<0.975	0.5779	0.3963	0.3200	0.1935	0.5773	0.3787	0.2841	0.1888
0.975<S/X<1.00	0.2673	0.1589	0.1351	0.1022	0.2649	0.1467	0.1199	0.1085
1.00<S/X<1.025	0.1401	0.0799	0.0764	0.0729	0.1390	0.0738	0.0705	0.0785
1.025<S/X<1.05	0.0766	0.0468	0.0504	0.0545	0.0762	0.0448	0.0483	0.0578
1.05<S/X<1.075	0.0455	0.0336	0.0353	0.0425	0.0454	0.0337	0.0349	0.0446
1.075<S/X	0.0220	0.0212	0.0202	0.0250	0.0219	0.0217	0.0200	0.0256
Days to Maturity	**Panel B: RMSE over Average Call Price for Options with Various Maturities**							
DTM<20	0.0304	0.0275	0.0293	0.0282	0.0300	0.0276	0.0293	0.0284
20<DTM<80	0.0731	0.0484	0.0447	0.0430	0.0717	0.0475	0.0446	0.0435
80<DTM<180	0.1002	0.0645	0.0523	0.0527	0.0993	0.0626	0.0503	0.0542
DTM>180	0.1120	0.0776	0.0691	0.0524	0.1125	0.0733	0.0600	0.0549
Sample Year	**Panel C: RMSE over Average Call Price for Various Sample Years**							
1990	0.0967	0.0792	0.0747	0.0682	0.0948	0.0812	0.0775	0.0716
1991	0.0703	0.0727	0.0667	0.0756	0.0700	0.0740	0.0671	0.0757
1992	0.0694	0.0500	0.0379	0.0527	0.0692	0.0502	0.0390	0.0559
1993	0.1073	0.0663	0.0567	0.0462	0.1069	0.0624	0.0504	0.0474
1994	0.1023	0.0691	0.0633	0.0481	0.1025	0.0646	0.0558	0.0498
1995	0.0941	0.0565	0.0469	0.0339	0.0937	0.0526	0.0398	0.0351

Notes to Table: We use the MLE estimates from Table 1 to compute the dollar root mean squared option valuation error (RMSE) divided by the average call price. In Panel A, we show the RMSEs according to moneyness bins. In Panel B, we show the RMSEs according to maturity bins. In Panel C, we show the RMSEs on a year-by-year basis. S/X refers to moneyness (index price over strike price), and DTM refers to the number of days to maturity of the option. The top two rows show the overall RMSE as well as the overall normalized RMSE.

Table 4: Regressing Weekly RMSE on Economic Variables

	AGARCH-N		**NGARCH-N**		**AGARCH-GED**		**NGARCH-GED**	
	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components
Average Weekly RMSE	2.3450	1.6481	1.4028	1.3231	2.3345	1.6008	1.3112	1.3656
Regressor			**t-Statistics using White's Robust Standard Errors**					
Constant	1.130	1.130	-0.180	-0.853	1.120	1.083	-0.188	-0.754
VIX	**-2.170**	1.008	1.588	**3.737**	**-2.201**	1.391	1.695	**3.198**
S&P500 Weekly Return	0.676	1.049	0.169	-0.846	0.673	0.986	0.275	-0.934
Oil Price	**9.715**	**5.043**	0.766	**-2.016**	**9.543**	**5.049**	0.642	-1.246
3-month T-bill Rate	**-5.479**	**-3.670**	**-3.253**	**-4.050**	**-5.454**	**-3.402**	**-2.656**	**-4.263**
Credit Spread	**-4.442**	**-2.838**	-1.017	**2.759**	**-4.393**	**-2.509**	-0.344	**2.912**
Term Spread	**-7.772**	**-4.827**	**-4.300**	**-3.513**	**-7.675**	**-4.524**	**-3.596**	**-3.519**
Average Moneyness (S/K)	-0.197	-0.594	0.698	1.296	-0.184	-0.647	0.529	1.141
Average Maturity (YTM)	**3.485**	**4.411**	**3.813**	**4.609**	**3.531**	**4.308**	**3.480**	**4.525**
Model Volatility	**4.127**	0.302	0.497	0.353	**4.114**	-0.136	0.681	1.191
Regression R-squared	0.4798	0.2156	0.1513	0.2166	0.4796	0.2174	0.1299	0.2087

Notes to Table: For each model we regress the weekly RMSE on a constant and various weekly economic variables as well as the average weekly moneyness and maturity of the options in the sample. We also regress on the model-specific volatility path. We report the t-statistic for each regressor using White's robust standard errors. Numbers in bold are larger than two in absolute value. The top row shows the average weekly RMSE and the bottom row reports the R-squared regression fit.

Table 5: Regressing Weekly BIAS on Economic Variables

	AGARCH-N		**NGARCH-N**		**AGARCH-GED**		**NGARCH-GED**	
	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components
Average Weekly BIAS	-0.9684	-0.3031	-0.1260	0.5500	-0.9745	-0.2186	0.0700	0.6467
Regressor			**t-Statistics using White's Robust Standard Errors**					
Constant	**-5.570**	**-3.350**	-1.235	-0.311	**-5.675**	**-3.429**	-1.377	-0.525
VIX	**10.847**	**4.913**	-0.946	1.952	**10.675**	**4.912**	-0.999	1.744
S&P500 Weekly Return	0.372	-0.881	-0.326	0.019	0.269	-0.736	-0.419	0.061
Oil Price	1.880	**3.989**	0.766	**2.060**	1.863	**4.921**	0.636	**2.363**
3-month T-Bill Rate	**3.706**	-0.658	0.092	**-4.531**	**3.658**	-1.545	-0.143	**-4.449**
Credit Spread	**9.117**	**3.830**	**5.717**	-1.256	**9.311**	**2.405**	**5.600**	-1.211
Term Spread	**2.966**	-0.734	-1.381	**-4.032**	**2.862**	-1.010	-1.664	**-3.920**
Average Moneyness (S/X)	**3.557**	**2.647**	1.052	0.655	**3.691**	**2.793**	1.279	0.852
Average Maturity (YTM)	-1.128	-1.346	**-2.631**	-0.051	-1.329	-0.826	**-2.117**	0.304
Model Volatility	0.300	**2.052**	1.558	**3.147**	0.428	**2.496**	1.535	**3.274**
Regression R-squared	0.8722	0.5880	0.3199	0.1290	0.8699	0.5473	0.2882	0.1294

Notes to Table: For each model we regress the weekly BIAS on a constant and various weekly economic variables as well as the average weekly moneyness and maturity of the options in the sample. We also regress on the model-specific volatility path. We report the t-statistic for each regressor using White's robust standard errors. Numbers in bold are larger than two in absolute value. The top row shows the average weekly BIAS and the bottom row reports the R-squared regression fit.

Table 6: Robustness Analysis

	AGARCH-N		**NGARCH-N**		**AGARCH-GED**		**NGARCH-GED**	
	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components	GARCH(1,1)	Components
				Panel A: Results from MLE on 1962-2001 Sample				
Option RMSE, 1990-1995	2.6927	1.8138	1.5875	1.3820	2.6819	1.7404	1.4599	1.4270
Option BIAS, 1990-1995	-0.0423	-0.0149	-0.0077	0.0193	-0.0425	-0.0114	-0.0004	0.0229
Ann Avr Vol % 1990-1995	12.367	11.884	11.293	11.153	12.398	11.907	11.292	11.157
Ann Vol of Var % 1990-1995	0.7904	0.8763	1.0064	0.9758	0.7819	0.8587	1.0081	0.9753
				Panel B: Results from MLE on 1962-1989 Sample				
Option RMSE, 1990-1995	2.4750	1.6805	1.4347	1.4782	2.4163	1.6438	1.4021	1.5179
Option BIAS, 1990-1995	-0.0314	-0.0060	0.0043	0.0268	-0.0296	-0.0046	0.0095	0.0296
Ann Avr Vol % 1990-1995	12.442	11.871	11.218	11.086	12.428	11.904	11.235	11.107
Ann Vol of Var % 1990-1995	0.7485	0.8481	1.0122	0.9932	0.7283	0.8324	1.0087	0.9876
				Panel C: Results from NLS Estimation of GARCH Spot Variance				
Option RMSE, 1990-1995	2.0362	1.3555	1.0416	1.0076	1.9490	1.3731	1.0273	1.0143
Option BIAS, 1990-1995	-0.0174	0.0107	0.0058	0.0061	-0.0137	0.0118	0.0054	0.0055
Ann Avr Vol % 1990-1995	7.097	10.255	11.306	12.050	7.275	10.500	11.715	12.261
Ann Vol of Var % 1990-1995	2.5754	1.3651	1.1766	0.9814	2.4127	1.3083	1.2098	1.0068
RMSE of Ann Vol %	9.3485	3.7099	2.3055	1.6955	9.0147	3.4056	2.3297	1.7829

Notes to Table: Using the GARCH MLE paramaters from Table 1, Panel A reports the overall option RMSE and BIAS as well as the average model volatility on the option data days and the standard deviation of the model variance path on the option data days. Panel B reports the same statistics but using GARCH parameters estimated on 1962-1989 instead. In Panel C we use NLS to estimate the RMSE optimal spot variance each week. Panel C uses the GARCH MLE parameters from the 1962-2001 sample of returns. The last row of the table reports the RMSE distance between the annualized MLE volatility path and the NLS optimal volatilities.

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